Computer Algebra Independent Integration Tests

Summer 2023 edition

3-Logarithms/64-3.5-Logarithm-functions

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [314]. This is test number [64].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (314)	0.00 (0)
Mathematica	100.00 (314)	0.00 (0)
Fricas	88.85 (279)	11.15 (35)
Maple	85.03 (267)	14.97 (47)
Maxima	70.06 (220)	29.94 (94)
Giac	59.87 (188)	40.13 (126)
Mupad	58.28 (183)	41.72 (131)
Sympy	42.36 (133)	57.64 (181)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

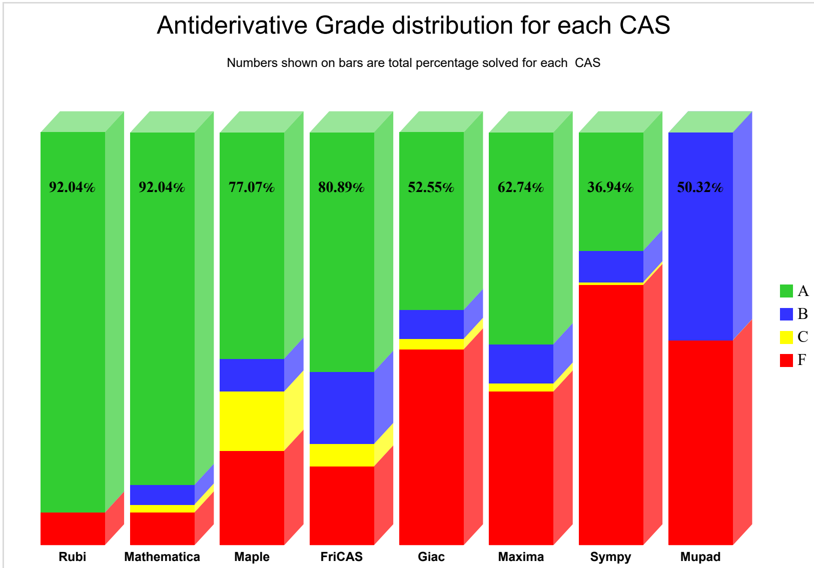
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

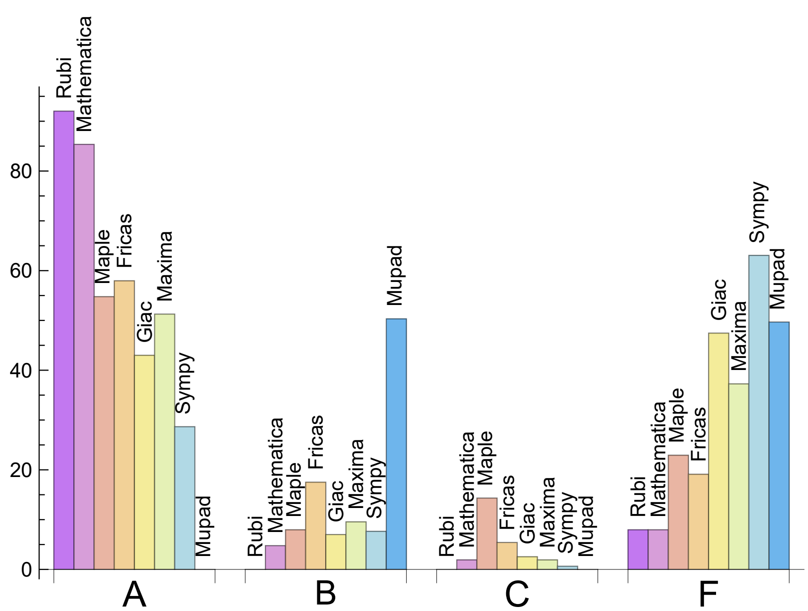
System	% A grade	% B grade	% C grade	% F grade
Rubi	92.038	0.000	0.000	7.962
Mathematica	85.350	4.777	1.911	7.962
Fricas	57.962	17.516	5.414	19.108
Maple	54.777	7.962	14.331	22.930
Maxima	51.274	9.554	1.911	37.261
Giac	42.994	7.006	2.548	47.452
Sympy	28.662	7.643	0.637	63.057
Mupad	0.000	50.318	0.000	49.682

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	35	80.00	0.00	20.00
Maple	47	100.00	0.00	0.00
Maxima	94	55.32	0.00	44.68
Giac	126	92.86	1.59	5.56
Mupad	131	0.00	100.00	0.00
Sympy	181	70.17	24.86	4.97

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.08
Maxima	0.26
Giac	0.35
Fricas	0.54
Mathematica	0.79
Mupad	1.69
Maple	3.00
Sympy	9.00

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	62.42	1.75	26.00	1.00
Rubi	71.36	1.00	41.00	1.00
Maxima	80.40	2.19	43.00	1.05
Mathematica	82.97	1.27	42.00	1.00
Giac	91.66	1.45	34.50	1.06
Mupad	110.57	1.20	26.00	1.00
Fricas	125.56	1.68	48.00	1.15
Maple	130.08	2.15	43.00	1.08

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

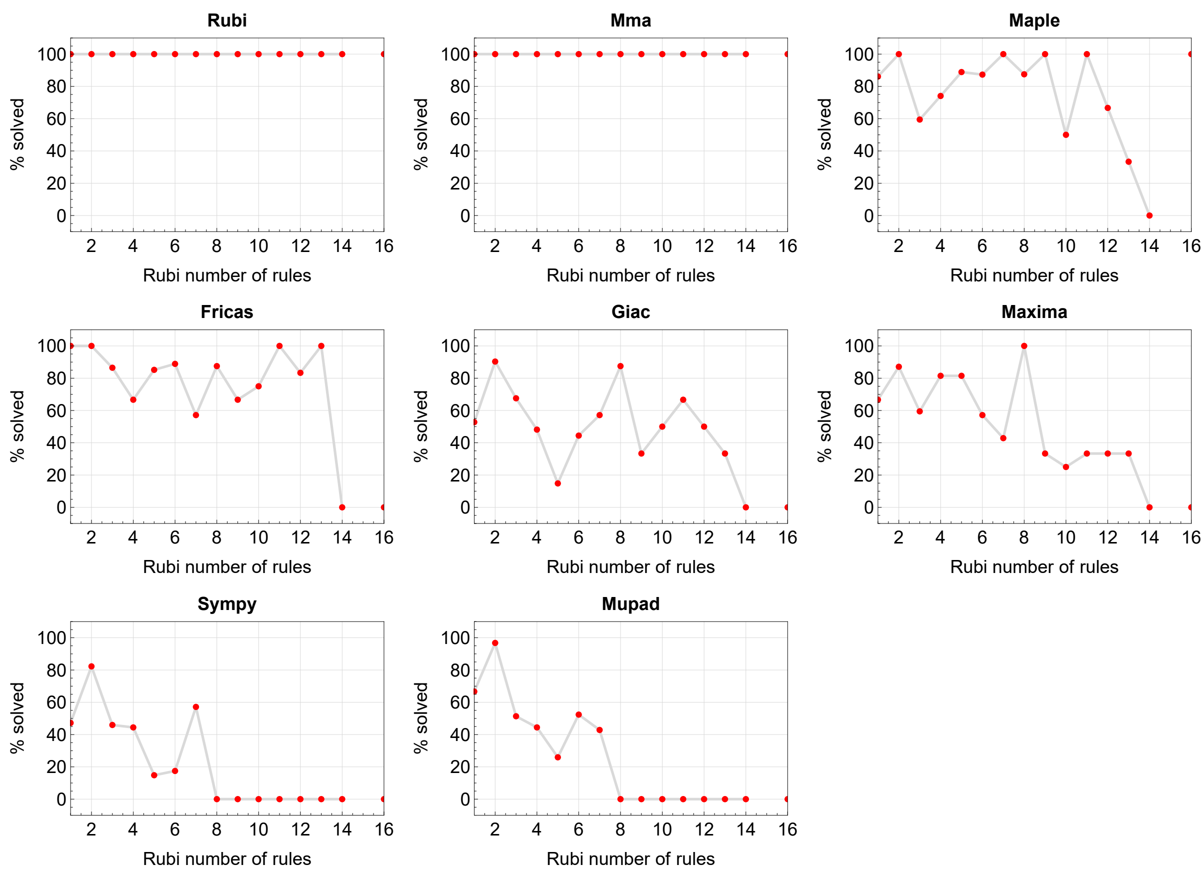


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

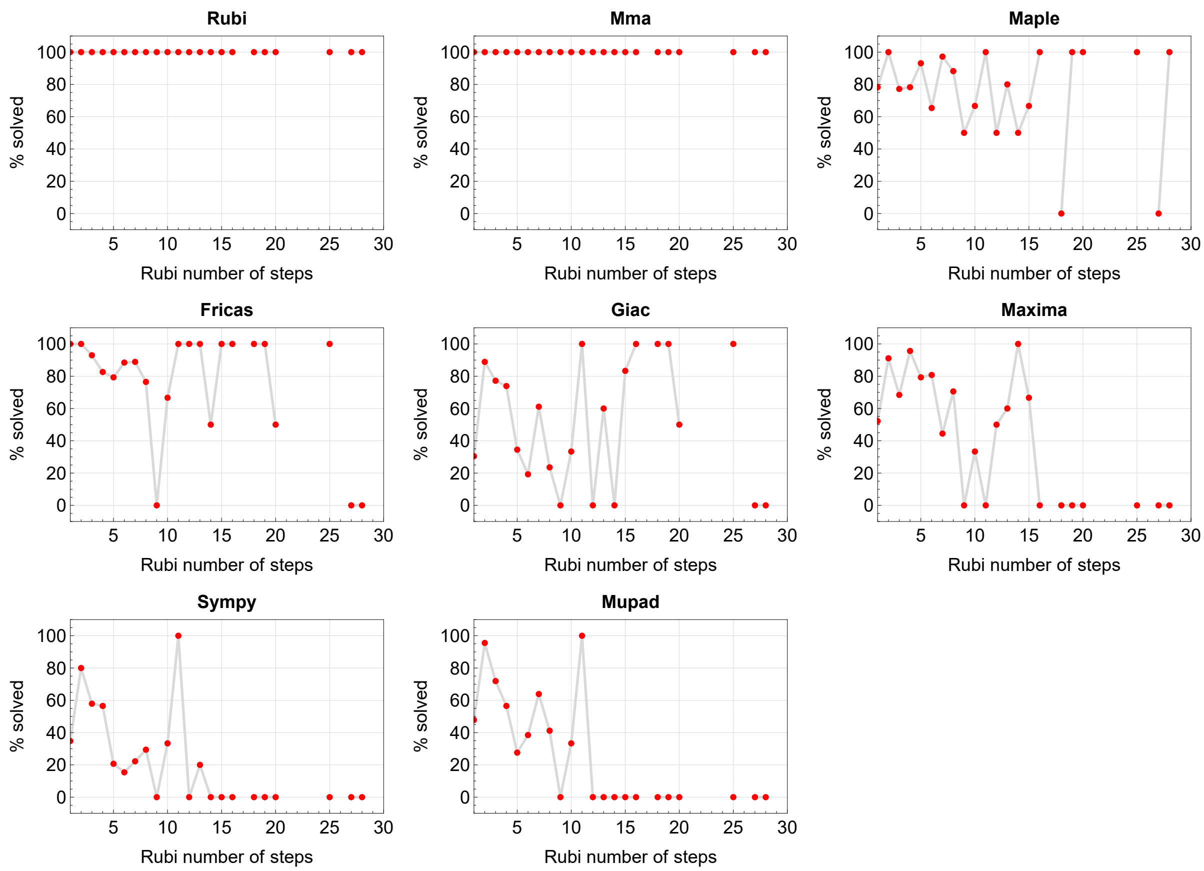


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

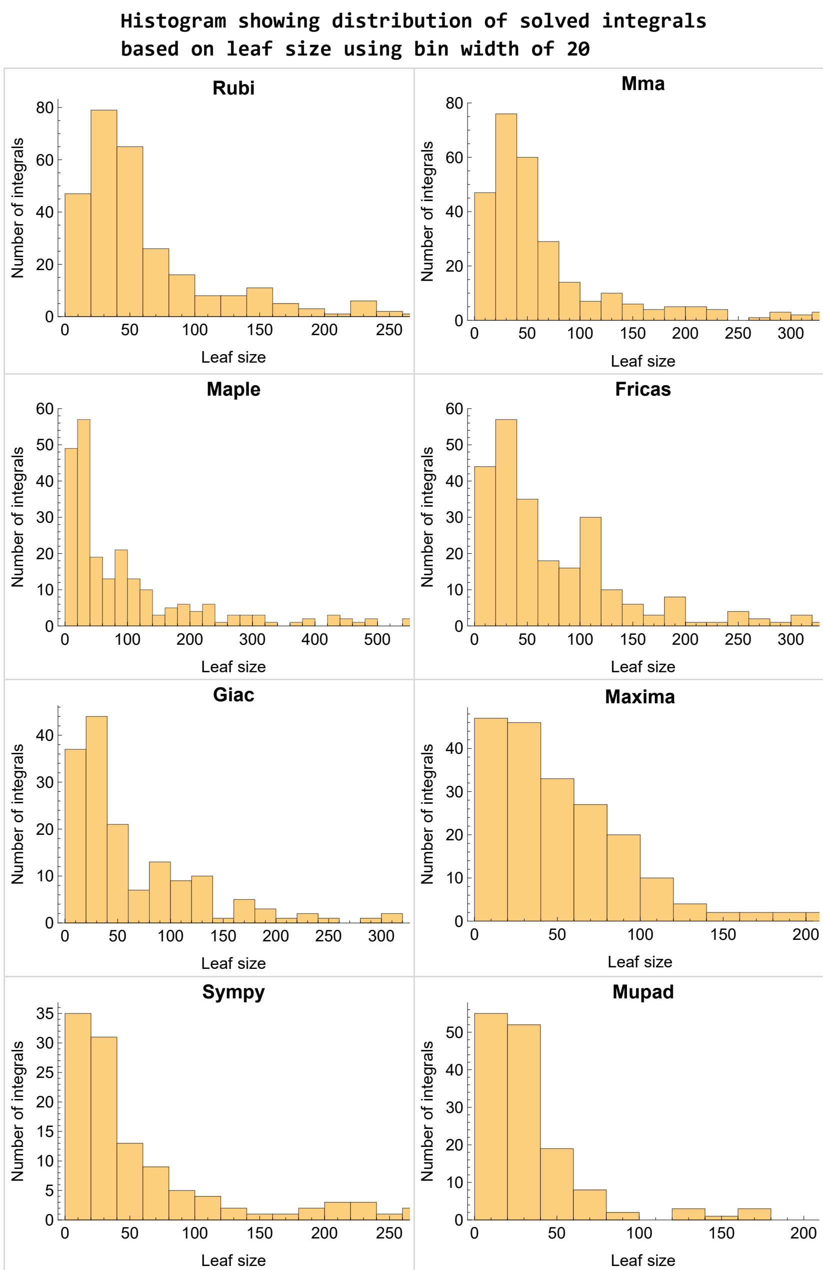


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

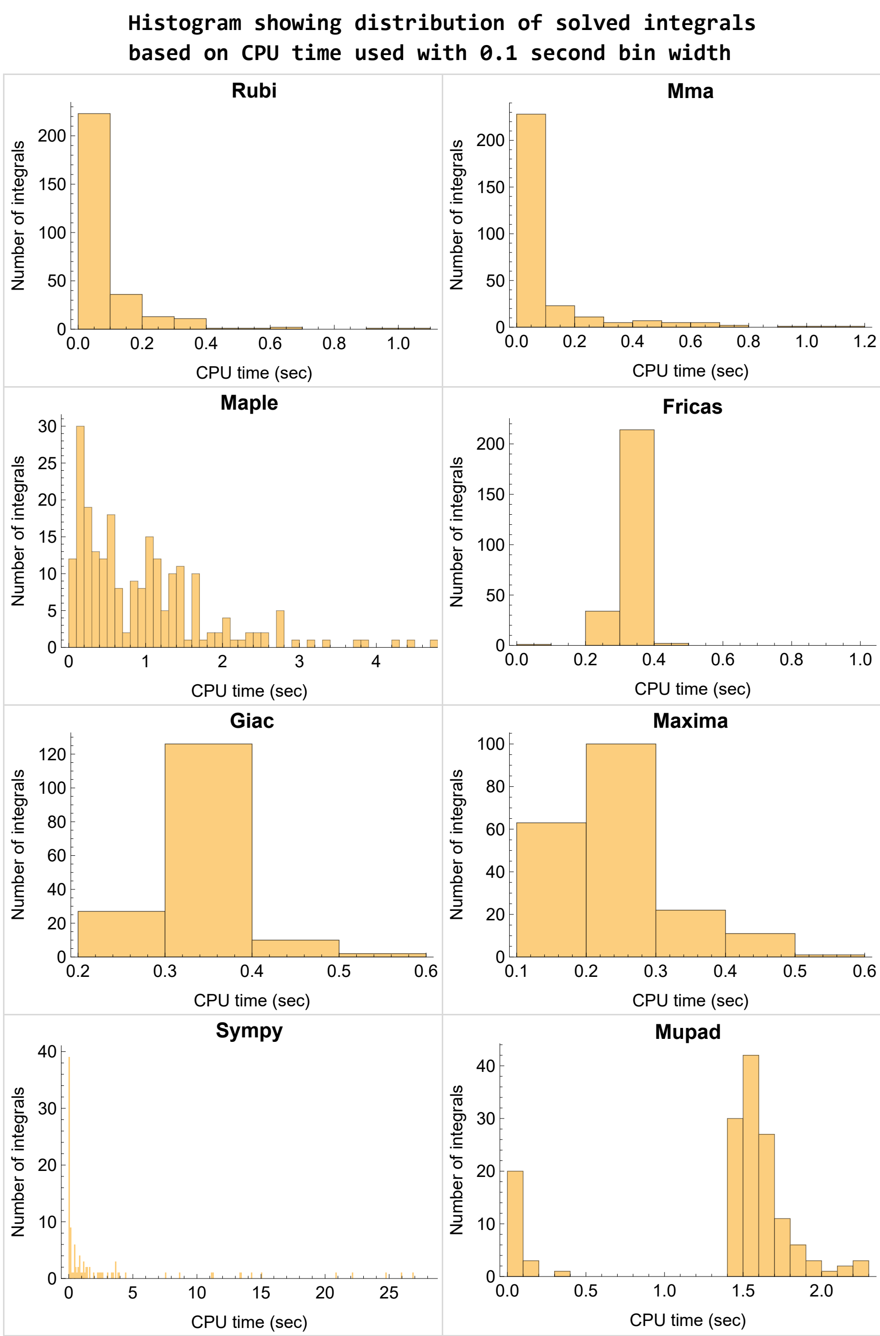


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

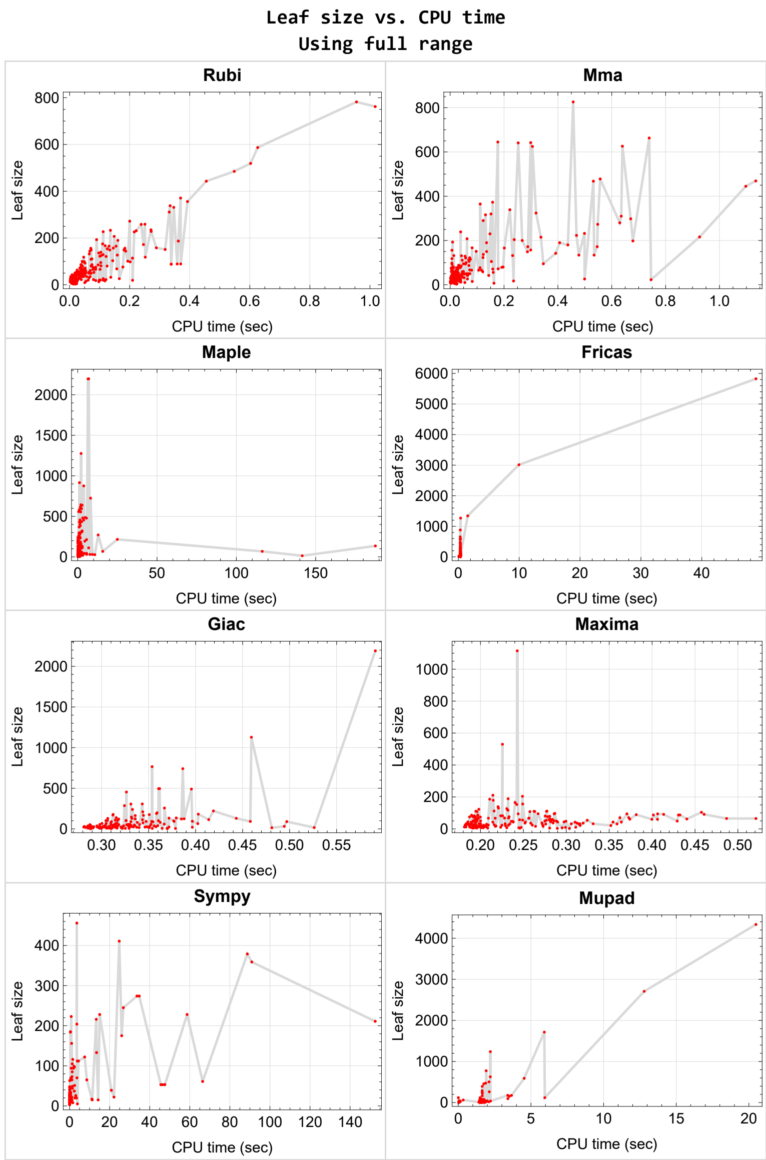


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{1, 6, 7, 8, 13, 14, 15, 30, 35, 36, 37, 39, 105, 117, 122, 127, 286, 287, 288, 289, 290, 308, 309, 313, 314\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{40, 100\}$

Maple $\{17, 18, 20, 21, 22, 29, 92, 93, 94, 95, 151, 154, 155, 156, 157, 158, 159, 169, 172, 201, 202, 204, 205, 215, 216, 218, 219, 252, 253, 256, 257, 305, 306, 307, 310, 311, 312\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

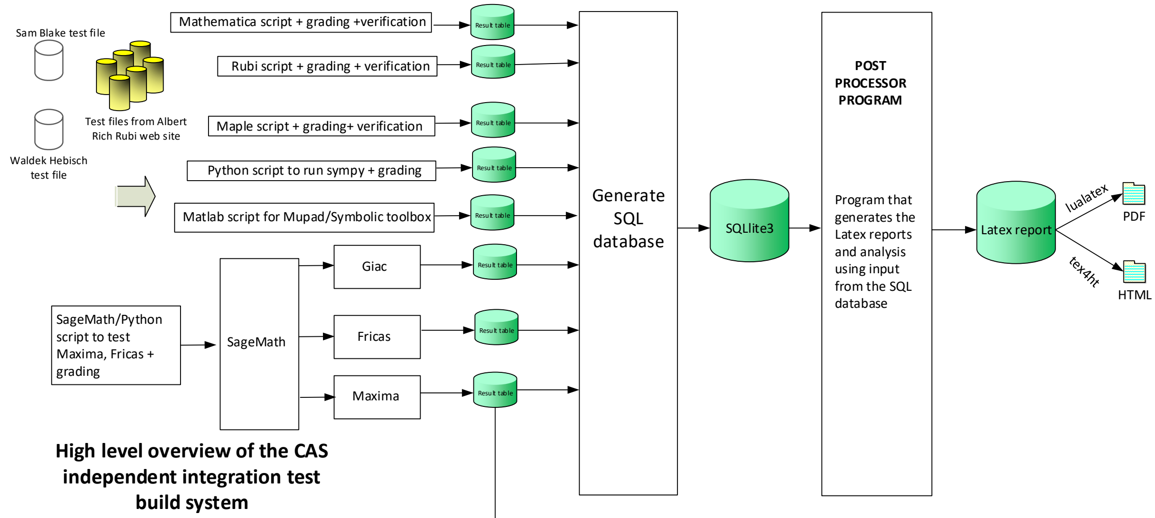
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 27
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2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 23
Maple 23
Fricas 24
Maxima 24
Giac 25
Mupad 25
Sympy 26

Rubi

A grade { 2, 3, 4, 5, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 38, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

B grade { 40, 41, 42, 43, 44, 45, 134, 135, 136, 189, 225, 278, 279, 280, 281 }

C grade { 108, 109, 110, 111, 112, 276 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 5, 9, 10, 11, 12, 19, 24, 25, 26, 27, 28, 34, 38, 51, 55, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 101, 102, 103, 104, 106, 113, 114, 115, 116, 121, 126, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 160, 168, 171, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 192, 194, 195, 196, 197, 198, 199, 200, 207, 209, 210, 211, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 258, 259, 260, 261, 262, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { 23, 107, 118, 119, 120, 123, 124, 125, 128, 161, 162, 164, 165, 167, 170, 173, 174, 176, 177, 189, 191, 208, 212, 277, 278 }

C grade { 17, 18, 20, 21, 22, 29, 40, 41, 44, 45, 92, 93, 94, 95, 100, 137, 151, 154, 155, 156, 157, 158, 159, 169, 172, 179, 201, 202, 204, 205, 215, 216, 218, 219, 252, 253, 256, 257, 279, 305, 306, 307, 310, 311, 312 }

F normal fail { 2, 3, 4, 16, 31, 32, 33, 42, 43, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 70, 96, 97, 99, 108, 109, 110, 111, 112, 163, 166, 175, 178, 193, 203, 206, 217, 220, 263, 264, 265, 280, 281, 300, 301, 302, 303, 304 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 5, 12, 16, 19, 20, 21, 25, 26, 27, 29, 34, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 129, 130, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 194, 201, 203, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 254, 255, 256, 258, 259, 260, 261, 262, 268, 271, 272, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { 9, 10, 11, 17, 18, 22, 23, 24, 28, 89, 90, 91, 128, 131, 135, 136, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 191, 195, 196, 197, 198, 199, 200, 202, 218, 219, 220, 221, 222, 225, 246, 305, 306, 307, 310, 311, 312 }

C grade { 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 253, 257, 300 }

F normal fail { 2, 3, 4, 31, 32, 33, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 193, 266, 267, 269, 270, 277, 278, 279 }

F(-1) timedout fail { }

F(-2) exception fail { 263, 264, 265, 301, 302, 303, 304 }

Maxima

A grade { 5, 12, 19, 20, 21, 25, 27, 29, 34, 38, 51, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 81, 96, 113, 114, 115, 116, 118, 119, 120, 123, 124, 125, 126, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 153, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 254, 259, 262, 267, 268, 270, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 294, 295, 296, 297, 298, 299, 302, 305, 306, 307, 310, 311, 312 }

B grade { 9, 10, 11, 22, 23, 24, 26, 28, 58, 121, 128, 161, 162, 163, 176, 177, 178, 182, 189, 191, 192, 193, 194, 221, 237, 246, 269, 301, 303, 304 }

C grade { 154, 155, 156, 157, 158, 159 }

F normal fail { 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 70, 76, 87, 92, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 231, 232, 233, 234, 251, 252, 253, 255, 256, 257, 258, 260, 261, 263, 264, 265, 266, 277, 278, 279, 280, 281, 293, 300 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 4, 16, 17, 18, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 71, 72, 73, 74, 75, 77, 78, 79, 80, 82, 83, 84, 85, 86, 88, 89, 90, 91, 93, 94, 95, 97, 149, 150, 152 }

Giac

A grade { 5, 11, 12, 19, 25, 26, 27, 34, 48, 49, 50, 51, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 101, 102, 103, 104, 106, 107, 112, 130, 132, 133, 135, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 160, 179, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 194, 195, 196, 197, 198, 199, 200, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 251, 253, 254, 255, 257, 262, 265, 271, 272, 273, 276, 282, 283, 284, 285, 291, 292, 294, 295, 297, 298, 301, 302, 304 }

B grade { 9, 10, 23, 24, 28, 90, 91, 129, 131, 136, 141, 180, 221, 222, 241, 250, 252, 256, 274, 275, 296, 303 }

C grade { 108, 109, 154, 155, 156, 157, 158, 159 }

F normal fail { 2, 3, 4, 17, 18, 20, 21, 22, 29, 31, 32, 33, 38, 40, 41, 42, 43, 46, 47, 52, 53, 54, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 110, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 137, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 191, 193, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 249, 258, 259, 260, 261, 263, 264, 266, 267, 268, 269, 270, 277, 278, 279, 280, 281, 293, 299, 300, 305, 306, 307, 310, 311, 312 }

F(-1) timedout fail { 134, 147 }

F(-2) exception fail { 1, 16, 30, 44, 45, 111, 185 }

Mupad

A grade { }

B grade { 5, 12, 17, 18, 19, 23, 24, 25, 26, 27, 28, 34, 38, 51, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 116, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 194, 207, 211, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

C grade { }

F normal fail { }

F(-1) timedout fail { 2, 3, 4, 9, 10, 11, 16, 20, 21, 22, 29, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 123, 124, 125, 126, 128, 154, 155, 156, 157, 158, 159, 161, 162, 163, 170, 171, 176, 177, 178, 179, 189, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 263, 264, 265, 266, 267, 270, 277, 278, 279, 280, 281, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

F(-2) exception fail { }

Sympy

A grade { 5, 9, 10, 11, 19, 23, 24, 25, 26, 27, 34, 38, 55, 60, 61, 62, 63, 64, 66, 67, 68, 69, 81, 129, 130, 132, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 153, 160, 182, 184, 187, 188, 190, 194, 222, 224, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 239, 241, 242, 243, 244, 245, 247, 248, 252, 253, 256, 257, 258, 259, 260, 261, 262, 272, 273, 274, 275, 276, 282, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { 12, 17, 18, 74, 75, 77, 85, 86, 131, 133, 140, 189, 192, 223, 235, 238, 240, 246, 249, 250, 251, 254, 255, 283 }

C grade { 268, 270 }

F normal fail { 3, 4, 32, 33, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 65, 76, 92, 96, 97, 98, 103, 104, 106, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 134, 135, 136, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 185, 186, 191, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 234, 263, 264, 265, 266, 267, 269, 271, 277, 278, 279, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

F(-1) timedout fail { 1, 2, 6, 7, 8, 16, 20, 21, 22, 29, 30, 31, 35, 36, 37, 70, 71, 72, 73, 78, 79, 80, 82, 83, 84, 87, 88, 89, 90, 91, 93, 94, 95, 101, 102, 107, 108, 109, 110, 111, 112, 183, 280, 281, 284 }

F(-2) exception fail { 28, 40, 41, 42, 43, 44, 45, 99, 100 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	0	34	0	0	34
N.S.	1	1.00	1.06	1.00	0.00	1.06	0.00	0.00	1.06
time (sec)	N/A	0.174	0.900	0.428	0.000	0.313	0.000	0.000	1.666

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	223	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.469	0.000	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	149	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.287	0.000	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.111	0.000	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	25	34	16	15
N.S.	1	1.00	1.00	1.07	1.00	1.67	2.27	1.07	1.00
time (sec)	N/A	0.014	0.003	1.417	0.197	0.308	0.534	0.526	1.438

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	75	33	0	34	34
N.S.	1	1.00	1.06	1.00	2.34	1.03	0.00	1.06	1.06
time (sec)	N/A	0.154	0.172	0.045	0.318	0.315	0.000	0.426	1.428

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	240	57	0	34	34
N.S.	1	1.00	1.06	1.00	7.50	1.78	0.00	1.06	1.06
time (sec)	N/A	0.189	0.450	0.063	0.330	0.393	0.000	0.398	1.472

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	32	1170	81	0	34	34
N.S.	1	1.00	1.06	1.00	36.56	2.53	0.00	1.06	1.06
time (sec)	N/A	0.153	0.966	0.073	0.403	0.739	0.000	0.436	1.547

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	230	271	1115	655	411	766	0
N.S.	1	1.00	0.85	1.00	4.10	2.41	1.51	2.82	0.00
time (sec)	N/A	0.200	0.148	12.893	0.243	0.320	24.793	0.354	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	115	129	530	267	216	286	0
N.S.	1	1.00	0.92	1.03	4.24	2.14	1.73	2.29	0.00
time (sec)	N/A	0.105	0.075	2.968	0.226	0.324	13.323	0.324	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	186	81	65	73	0
N.S.	1	1.00	1.00	1.15	4.54	1.98	1.59	1.78	0.00
time (sec)	N/A	0.050	0.026	0.817	0.211	0.333	8.604	0.316	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	65	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	4.33	0.87	0.87
time (sec)	N/A	0.005	0.002	0.485	0.204	0.315	1.171	0.307	1.425

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	29	24	30	30
N.S.	1	1.00	1.07	1.00	1.07	1.04	0.86	1.07	1.07
time (sec)	N/A	0.126	1.297	0.046	0.258	0.376	5.883	0.337	1.426

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	343	51	26	30	30
N.S.	1	1.00	1.07	1.00	12.25	1.82	0.93	1.07	1.07
time (sec)	N/A	0.125	1.127	0.060	0.275	0.354	10.042	0.375	1.533

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	1467	73	26	30	30
N.S.	1	1.00	1.07	1.00	52.39	2.61	0.93	1.07	1.07
time (sec)	N/A	0.114	2.264	0.050	0.385	0.395	15.993	1.208	1.794

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	42	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.112	0.064	0.000	0.000	0.323	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	B	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	22	22	22	204	0	65	61	0	20
N.S.	1	1.00	1.00	9.27	0.00	2.95	2.77	0.00	0.91
time (sec)	N/A	0.121	0.031	0.081	0.000	0.357	66.404	0.000	1.911

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	B	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	22	22	22	135	0	42	39	0	20
N.S.	1	1.00	1.00	6.14	0.00	1.91	1.77	0.00	0.91
time (sec)	N/A	0.080	0.025	187.626	0.000	0.378	20.866	0.000	2.095

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	28	49	17	16
N.S.	1	1.00	1.00	1.06	1.00	1.75	3.06	1.06	1.00
time (sec)	N/A	0.025	0.015	2.406	0.192	0.347	1.421	0.315	1.470

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	17	17	17	213	22	18	0	0	0
N.S.	1	1.00	1.00	12.53	1.29	1.06	0.00	0.00	0.00
time (sec)	N/A	0.117	0.235	5.581	0.316	0.316	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	20	20	20	68	21	21	0	0	0
N.S.	1	1.00	1.00	3.40	1.05	1.05	0.00	0.00	0.00
time (sec)	N/A	0.112	0.028	15.772	0.352	0.324	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	22	22	22	68	49	45	0	0	0
N.S.	1	1.00	1.00	3.09	2.23	2.05	0.00	0.00	0.00
time (sec)	N/A	0.113	0.030	116.322	0.429	0.337	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	53	211	195	70	198	52
N.S.	1	1.00	1.00	2.65	10.55	9.75	3.50	9.90	2.60
time (sec)	N/A	0.099	0.016	2.341	0.214	0.316	3.668	0.359	1.596

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	38	35	74	89	51	90	18
N.S.	1	1.00	1.90	1.75	3.70	4.45	2.55	4.50	0.90
time (sec)	N/A	0.062	0.007	0.862	0.192	0.328	2.518	0.497	1.524

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	21	73	20	14
N.S.	1	1.00	1.00	1.07	1.00	1.50	5.21	1.43	1.00
time (sec)	N/A	0.006	0.003	0.312	0.183	0.334	1.213	0.337	1.488

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	32	28	24	28	16
N.S.	1	1.00	1.00	1.07	2.13	1.87	1.60	1.87	1.07
time (sec)	N/A	0.055	0.082	0.264	0.229	0.307	0.730	0.332	1.648

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	31	31	15	31	18
N.S.	1	1.00	1.00	1.06	1.72	1.72	0.83	1.72	1.00
time (sec)	N/A	0.087	0.012	0.474	0.234	0.297	11.296	0.331	1.539

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	95	101	0	306	39
N.S.	1	1.00	1.00	0.95	4.75	5.05	0.00	15.30	1.95
time (sec)	N/A	0.102	0.012	1.161	0.267	0.314	0.000	0.331	1.481

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	19	19	23	216	26	23	0	0	0
N.S.	1	1.00	1.21	11.37	1.37	1.21	0.00	0.00	0.00
time (sec)	N/A	0.210	0.746	24.973	0.316	0.299	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	40	0	42	0	0	42
N.S.	1	1.00	1.05	1.00	0.00	1.05	0.00	0.00	1.05
time (sec)	N/A	0.170	1.460	0.530	0.000	0.321	0.000	0.000	1.889

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	331	331	445	0	0	0	0	0	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.347	1.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	235	298	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	0.670	0.000	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	157	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.299	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	42	44	26	25
N.S.	1	1.00	1.00	1.04	1.00	1.68	1.76	1.04	1.00
time (sec)	N/A	0.022	0.018	0.659	0.194	0.338	1.419	0.358	1.541

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	40	95	41	0	42	42
N.S.	1	1.00	1.05	1.00	2.38	1.02	0.00	1.05	1.05
time (sec)	N/A	0.162	6.712	0.095	0.327	0.309	0.000	0.381	1.638

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	40	312	65	0	42	42
N.S.	1	1.00	1.05	1.00	7.80	1.62	0.00	1.05	1.05
time (sec)	N/A	0.171	20.804	0.080	0.380	0.350	0.000	0.429	2.100

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	40	1583	89	0	42	42
N.S.	1	1.00	1.05	1.00	39.58	2.22	0.00	1.05	1.05
time (sec)	N/A	0.168	58.910	0.090	0.496	0.742	0.000	0.472	1.953

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	31	30	22	0	26
N.S.	1	1.00	1.00	1.04	1.19	1.15	0.85	0.00	1.00
time (sec)	N/A	0.166	0.499	10.788	0.359	0.357	22.160	0.000	1.517

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	59	52	26	31	31
N.S.	1	1.00	1.07	1.00	2.03	1.79	0.90	1.07	1.07
time (sec)	N/A	0.102	79.503	0.008	0.355	0.373	15.640	0.364	1.625

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	625	238	0	44	0	0	0
N.S.	1	1.00	12.76	4.86	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.072	0.305	1.491	0.000	0.372	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	642	243	0	45	0	0	0
N.S.	1	1.00	12.84	4.86	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.064	0.299	1.354	0.000	0.341	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	320	0	0	44	0	0	0
N.S.	1	1.00	6.04	0.00	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.064	0.152	0.000	0.000	0.313	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	316	0	0	43	0	0	0
N.S.	1	1.00	6.08	0.00	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.053	0.131	0.000	0.000	0.309	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	A	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	641	233	0	43	0	0	0
N.S.	1	1.00	13.08	4.76	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.070	0.253	1.614	0.000	0.314	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	A	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	645	238	0	44	0	0	0
N.S.	1	1.00	12.90	4.76	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.054	0.177	1.480	0.000	0.355	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	83	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.039	0.160	0.000	0.000	0.352	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	59	0	0	90	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.043	0.121	0.000	0.000	0.333	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	70	0	56	0
N.S.	1	1.00	0.89	0.00	0.00	1.27	0.00	1.02	0.00
time (sec)	N/A	0.032	0.053	0.000	0.000	0.309	0.000	0.353	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	70	0	56	0
N.S.	1	1.00	0.89	0.00	0.00	1.27	0.00	1.02	0.00
time (sec)	N/A	0.023	0.051	0.000	0.000	0.332	0.000	0.330	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	53	0	42	0
N.S.	1	1.00	0.96	0.00	0.00	1.18	0.00	0.93	0.00
time (sec)	N/A	0.019	0.042	0.000	0.000	0.355	0.000	0.335	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	39	64	45	0	54	32
N.S.	1	1.00	1.25	1.22	2.00	1.41	0.00	1.69	1.00
time (sec)	N/A	0.011	0.028	1.661	0.196	0.349	0.000	0.340	1.690

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	45	0	0	46	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.030	0.043	0.000	0.000	0.317	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	53	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.031	0.043	0.000	0.000	0.383	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	53	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.030	0.044	0.000	0.000	0.339	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	23	26	19	26	22
N.S.	1	1.00	1.00	1.18	1.05	1.18	0.86	1.18	1.00
time (sec)	N/A	0.005	0.013	0.404	0.233	0.351	0.420	0.307	1.570

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	22	23	22	24	0	32	20
N.S.	1	1.00	1.10	1.15	1.10	1.20	0.00	1.60	1.00
time (sec)	N/A	0.013	0.006	0.547	0.188	0.295	0.000	0.351	1.493

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	39	0	0	45	0	36	0
N.S.	1	1.00	0.98	0.00	0.00	1.12	0.00	0.90	0.00
time (sec)	N/A	0.014	0.015	0.000	0.000	0.331	0.000	0.331	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	33	55	37	0	43	27
N.S.	1	1.00	1.26	1.22	2.04	1.37	0.00	1.59	1.00
time (sec)	N/A	0.016	0.009	1.289	0.195	0.302	0.000	0.331	1.473

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	48	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	85	86	87	98	112	89	85
N.S.	1	1.00	0.86	0.87	0.88	0.99	1.13	0.90	0.86
time (sec)	N/A	0.049	0.034	0.349	0.197	0.338	4.447	0.331	1.570

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	74	75	75	86	97	75	73
N.S.	1	1.00	0.87	0.88	0.88	1.01	1.14	0.88	0.86
time (sec)	N/A	0.039	0.028	0.225	0.187	0.321	2.437	0.310	1.481

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	64	65	74	85	65	61
N.S.	1	1.00	0.89	0.90	0.92	1.04	1.20	0.92	0.86
time (sec)	N/A	0.035	0.021	0.172	0.197	0.285	1.307	0.315	1.494

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	53	51	59	70	51	49
N.S.	1	1.00	0.86	0.93	0.89	1.04	1.23	0.89	0.86
time (sec)	N/A	0.028	0.016	0.145	0.192	0.283	0.723	0.327	1.522

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	37	36	38	44	37	33
N.S.	1	1.00	0.94	1.12	1.09	1.15	1.33	1.12	1.00
time (sec)	N/A	0.013	0.007	0.105	0.194	0.304	0.368	0.356	1.570

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	62	80	0	0	0	0
N.S.	1	1.00	0.94	1.17	1.51	0.00	0.00	0.00	0.00
time (sec)	N/A	0.081	0.015	0.130	0.190	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	48	46	46	66	47	43
N.S.	1	1.00	0.96	1.02	0.98	0.98	1.40	1.00	0.91
time (sec)	N/A	0.030	0.010	0.122	0.192	0.288	0.981	0.335	1.981

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	65	62	62	70	94	65	54
N.S.	1	1.00	0.90	0.86	0.86	0.97	1.31	0.90	0.75
time (sec)	N/A	0.034	0.022	0.187	0.196	0.330	1.902	0.402	1.661

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	72	75	82	112	80	68
N.S.	1	1.00	0.90	0.84	0.87	0.95	1.30	0.93	0.79
time (sec)	N/A	0.038	0.023	0.237	0.194	0.305	3.846	0.362	1.677

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	87	84	86	94	122	92	79
N.S.	1	1.00	0.87	0.84	0.86	0.94	1.22	0.92	0.79
time (sec)	N/A	0.042	0.033	0.335	0.190	0.333	7.517	0.347	1.759

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	137	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	0.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	190	238	0	444	0	221	395
N.S.	1	1.00	0.92	1.15	0.00	2.14	0.00	1.07	1.91
time (sec)	N/A	0.149	0.135	1.056	0.000	0.324	0.000	0.419	1.652

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	151	190	0	364	0	176	288
N.S.	1	1.00	0.90	1.14	0.00	2.18	0.00	1.05	1.72
time (sec)	N/A	0.118	0.097	0.753	0.000	0.335	0.000	0.351	1.644

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	122	154	0	299	0	146	229
N.S.	1	1.00	0.90	1.13	0.00	2.20	0.00	1.07	1.68
time (sec)	N/A	0.097	0.073	0.623	0.000	0.347	0.000	0.332	1.660

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	122	0	245	359	113	166
N.S.	1	1.00	0.86	1.12	0.00	2.25	3.29	1.04	1.52
time (sec)	N/A	0.071	0.057	0.543	0.000	0.349	90.937	0.343	1.649

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	89	0	190	274	92	120
N.S.	1	1.00	0.99	1.13	0.00	2.41	3.47	1.16	1.52
time (sec)	N/A	0.040	0.038	0.466	0.000	0.333	34.757	0.353	1.504

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	150	166	0	0	0	0	0
N.S.	1	1.00	1.16	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	0.135	0.247	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	87	95	0	199	211	99	262
N.S.	1	1.00	1.01	1.10	0.00	2.31	2.45	1.15	3.05
time (sec)	N/A	0.075	0.070	0.579	0.000	0.368	152.532	0.377	2.133

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	105	145	0	261	0	129	474
N.S.	1	1.00	0.87	1.20	0.00	2.16	0.00	1.07	3.92
time (sec)	N/A	0.121	0.153	0.574	0.000	0.359	0.000	0.332	1.886

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	132	181	0	318	0	164	505
N.S.	1	1.00	0.89	1.21	0.00	2.13	0.00	1.10	3.39
time (sec)	N/A	0.134	0.232	0.850	0.000	0.361	0.000	0.336	2.115

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	172	225	0	404	0	210	627
N.S.	1	1.00	0.91	1.18	0.00	2.13	0.00	1.11	3.30
time (sec)	N/A	0.161	0.288	1.049	0.000	0.357	0.000	0.344	2.202

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	38	37	33	46	37	39
N.S.	1	1.00	0.83	0.90	0.88	0.79	1.10	0.88	0.93
time (sec)	N/A	0.020	0.013	0.248	0.285	0.284	0.072	0.317	0.071

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	468	876	0	1270	0	741	1240
N.S.	1	1.00	0.96	1.81	0.00	2.62	0.00	1.53	2.56
time (sec)	N/A	0.548	0.532	3.810	0.000	0.376	0.000	0.386	2.206

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	324	594	0	880	0	495	775
N.S.	1	1.00	0.96	1.76	0.00	2.60	0.00	1.46	2.29
time (sec)	N/A	0.335	0.319	2.378	0.000	0.329	0.000	0.361	1.920

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	204	384	0	567	0	307	457
N.S.	1	1.00	0.90	1.70	0.00	2.51	0.00	1.36	2.02
time (sec)	N/A	0.215	0.238	1.608	0.000	0.316	0.000	0.343	1.753

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	123	170	0	336	379	167	242
N.S.	1	1.00	0.80	1.10	0.00	2.18	2.46	1.08	1.57
time (sec)	N/A	0.125	0.122	0.674	0.000	0.327	88.720	0.345	1.643

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	89	0	190	274	92	120
N.S.	1	1.00	0.99	1.13	0.00	2.41	3.47	1.16	1.52
time (sec)	N/A	0.041	0.030	0.148	0.000	0.319	33.635	0.458	0.002

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	200	300	0	0	0	0	0
N.S.	1	1.00	0.88	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.268	1.367	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	166	184	0	429	0	189	590
N.S.	1	1.00	1.01	1.12	0.00	2.60	0.00	1.15	3.58
time (sec)	N/A	0.155	0.201	1.747	0.000	0.374	0.000	0.358	4.536

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	215	310	0	1341	0	490	1715
N.S.	1	1.00	0.83	1.20	0.00	5.18	0.00	1.89	6.62
time (sec)	N/A	0.251	0.337	2.734	0.000	1.549	0.000	0.396	5.921

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	310	486	0	3013	0	1128	2707
N.S.	1	1.00	0.87	1.37	0.00	8.46	0.00	3.17	7.60
time (sec)	N/A	0.393	0.635	4.726	0.000	9.965	0.000	0.459	12.796

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	469	725	0	5824	0	2191	4334
N.S.	1	1.00	0.90	1.40	0.00	11.22	0.00	4.22	8.35
time (sec)	N/A	0.603	1.135	8.090	0.000	48.896	0.000	0.591	20.485

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	175	175	131	292	0	0	0	0	0
N.S.	1	1.00	0.75	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.042	1.603	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	258	258	339	433	0	0	0	0	0
N.S.	1	1.00	1.31	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.222	2.460	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	762	762	626	555	0	0	0	0	0
N.S.	1	1.00	0.82	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.017	0.640	1.878	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	782	782	663	637	0	0	0	0	0
N.S.	1	1.00	0.85	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.955	0.739	2.783	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	111	0	123	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.85	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	0.045	0.000	0.198	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	587	587	478	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.626	0.557	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	290	279	0	0	0	0	0
N.S.	1	1.00	0.93	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	0.123	1.385	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	371	365	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	0.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	826	219	0	0	0	0	0
N.S.	1	1.00	1.86	0.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	0.457	0.321	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	273	239	0	134	0	134	0
N.S.	1	1.00	1.59	1.39	0.00	0.78	0.00	0.78	0.00
time (sec)	N/A	0.246	0.548	0.125	0.000	0.318	0.000	0.379	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	232	192	0	124	0	124	0
N.S.	1	1.00	1.56	1.29	0.00	0.83	0.00	0.83	0.00
time (sec)	N/A	0.181	0.499	0.078	0.000	0.329	0.000	0.388	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	190	175	0	114	0	114	0
N.S.	1	1.00	1.50	1.38	0.00	0.90	0.00	0.90	0.00
time (sec)	N/A	0.159	0.406	0.086	0.000	0.346	0.000	0.414	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	80	0	101	0	101	0
N.S.	1	1.00	0.89	0.84	0.00	1.06	0.00	1.06	0.00
time (sec)	N/A	0.104	0.041	0.075	0.000	0.322	0.000	0.360	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	23	19	21	21
N.S.	1	1.00	1.10	0.90	1.00	1.10	0.90	1.00	1.00
time (sec)	N/A	0.051	0.321	0.043	0.272	0.344	44.308	0.365	1.420

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	142	132	0	115	0	92	0
N.S.	1	1.00	1.87	1.74	0.00	1.51	0.00	1.21	0.00
time (sec)	N/A	0.177	0.392	0.076	0.000	0.339	0.000	0.377	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	95	186	0	138	0	130	0
N.S.	1	1.00	0.94	1.84	0.00	1.37	0.00	1.29	0.00
time (sec)	N/A	0.194	0.346	0.079	0.000	0.380	0.000	0.443	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	216	0	0	110	0	132	0
N.S.	1	1.00	1.16	0.00	0.00	0.59	0.00	0.71	0.00
time (sec)	N/A	0.362	0.926	0.000	0.000	0.339	0.000	0.372	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	198	0	0	100	0	122	0
N.S.	1	1.00	1.25	0.00	0.00	0.63	0.00	0.77	0.00
time (sec)	N/A	0.289	0.678	0.000	0.000	0.325	0.000	0.385	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	172	0	0	84	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.252	0.546	0.000	0.000	0.304	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	180	0	0	84	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.209	0.437	0.000	0.000	0.344	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	280	0	0	108	0	181	0
N.S.	1	1.00	1.85	0.00	0.00	0.72	0.00	1.20	0.00
time (sec)	N/A	0.318	0.630	0.000	0.000	0.371	0.000	0.403	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	84	82	88	0	0	0
N.S.	1	1.00	1.00	0.90	0.88	0.95	0.00	0.00	0.00
time (sec)	N/A	0.051	0.007	0.321	0.197	0.325	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	69	67	73	0	0	0
N.S.	1	1.00	1.00	0.90	0.87	0.95	0.00	0.00	0.00
time (sec)	N/A	0.039	0.006	0.268	0.194	0.323	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	52	50	56	0	0	0
N.S.	1	1.00	1.00	0.88	0.85	0.95	0.00	0.00	0.00
time (sec)	N/A	0.026	0.005	0.228	0.200	0.300	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	28	34	40	0	0	35
N.S.	1	1.00	1.00	0.74	0.89	1.05	0.00	0.00	0.92
time (sec)	N/A	0.032	0.003	0.843	0.194	0.315	0.000	0.000	1.469

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	11	13	13	10	13	13
N.S.	1	1.00	1.17	0.92	1.08	1.08	0.83	1.08	1.08
time (sec)	N/A	0.023	0.041	0.060	0.235	0.298	0.348	0.317	1.420

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	601	189	128	0	0	0
N.S.	1	1.00	1.00	4.55	1.43	0.97	0.00	0.00	0.00
time (sec)	N/A	0.066	0.013	2.089	0.232	0.340	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	430	153	93	0	0	0
N.S.	1	1.00	1.00	4.39	1.56	0.95	0.00	0.00	0.00
time (sec)	N/A	0.043	0.006	1.063	0.242	0.336	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	262	117	58	0	0	0
N.S.	1	1.00	1.00	4.16	1.86	0.92	0.00	0.00	0.00
time (sec)	N/A	0.025	0.005	0.630	0.232	0.326	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	76	31	0	0	0
N.S.	1	1.00	1.00	1.03	2.45	1.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.004	1.266	0.233	0.308	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	21	23	19	22	22
N.S.	1	1.00	1.10	1.00	1.05	1.15	0.95	1.10	1.10
time (sec)	N/A	0.046	0.173	0.085	0.356	0.320	1.697	0.462	1.507

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	193	1276	204	245	0	0	0
N.S.	1	1.00	1.00	6.61	1.06	1.27	0.00	0.00	0.00
time (sec)	N/A	0.090	0.009	2.115	0.249	0.350	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	156	916	165	205	0	0	0
N.S.	1	1.00	1.00	5.87	1.06	1.31	0.00	0.00	0.00
time (sec)	N/A	0.071	0.007	1.096	0.240	0.317	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	118	558	126	169	0	0	0
N.S.	1	1.00	1.00	4.73	1.07	1.43	0.00	0.00	0.00
time (sec)	N/A	0.047	0.007	0.579	0.231	0.341	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	69	82	106	0	0	0
N.S.	1	1.00	1.00	0.92	1.09	1.41	0.00	0.00	0.00
time (sec)	N/A	0.087	0.004	1.666	0.226	0.299	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	21	23	19	22	22
N.S.	1	1.00	1.10	1.00	1.05	1.15	0.95	1.10	1.10
time (sec)	N/A	0.045	0.161	0.079	0.326	0.332	1.665	0.524	1.524

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	96	82	106	0	0	0
N.S.	1	1.00	1.00	2.46	2.10	2.72	0.00	0.00	0.00
time (sec)	N/A	0.019	0.009	1.404	0.224	0.307	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	22	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	4.40	1.00
time (sec)	N/A	0.012	0.020	0.153	0.191	0.306	0.044	0.321	1.789

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	8	13
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	0.67	1.08
time (sec)	N/A	0.016	0.004	0.531	0.190	0.314	0.287	0.322	1.840

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	31	39	31	26
N.S.	1	1.00	1.00	0.90	0.80	3.10	3.90	3.10	2.60
time (sec)	N/A	0.011	0.004	0.191	0.201	0.315	0.070	0.494	1.534

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.003	0.306	0.192	0.323	0.115	0.329	0.072

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.013	0.024	1.067	0.290	0.323	0.062	0.370	1.584

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	42	13	16	16	0	0	12
N.S.	1	1.00	3.00	0.93	1.14	1.14	0.00	0.00	0.86
time (sec)	N/A	0.020	0.014	0.211	0.206	0.318	0.000	0.000	1.626

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	8	7	21	0	7	7
N.S.	1	1.00	2.27	0.73	0.64	1.91	0.00	0.64	0.64
time (sec)	N/A	0.021	0.017	0.178	0.295	0.322	0.000	0.370	1.526

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	18	6	5	16	0	16	5
N.S.	1	1.00	2.57	0.86	0.71	2.29	0.00	2.29	0.71
time (sec)	N/A	0.018	0.031	0.173	0.281	0.305	0.000	0.351	1.526

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	23	97	71	17	0	120
N.S.	1	1.00	0.95	0.21	0.87	0.64	0.15	0.00	1.08
time (sec)	N/A	0.074	0.070	0.233	0.287	0.318	0.090	0.000	5.953

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.018	0.005	0.191	0.193	0.313	0.055	0.316	1.591

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.22	0.78	1.00	1.00
time (sec)	N/A	0.010	0.007	0.404	0.200	0.321	0.101	0.306	1.452

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	12	13	156	12	12
N.S.	1	1.00	0.94	0.76	0.71	0.76	9.18	0.71	0.71
time (sec)	N/A	0.014	0.021	0.454	0.201	0.347	1.020	0.300	1.420

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	10	27	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.83	2.25	1.00
time (sec)	N/A	0.023	0.039	0.195	0.196	0.311	0.051	0.285	1.442

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	29	24	23	23	27	23	22
N.S.	1	1.00	1.38	1.14	1.10	1.10	1.29	1.10	1.05
time (sec)	N/A	0.024	0.019	0.172	0.225	0.294	0.060	0.296	1.458

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	31	30	36	27	37	26
N.S.	1	1.00	0.74	0.74	0.71	0.86	0.64	0.88	0.62
time (sec)	N/A	0.042	0.034	0.183	0.276	0.317	0.468	0.295	1.470

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	19	20	26	17	34	18
N.S.	1	1.00	0.83	0.79	0.83	1.08	0.71	1.42	0.75
time (sec)	N/A	0.026	0.028	0.189	0.201	0.346	0.059	0.304	1.511

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	17	12	20	17	13
N.S.	1	1.00	0.70	0.78	0.74	0.52	0.87	0.74	0.57
time (sec)	N/A	0.027	0.034	0.314	0.197	0.329	0.889	0.283	1.487

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	20	22	21	16	22	21	15
N.S.	1	1.00	0.69	0.76	0.72	0.55	0.76	0.72	0.52
time (sec)	N/A	0.032	0.023	0.423	0.196	0.321	1.107	0.298	1.601

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	30	29	29	32	0	18
N.S.	1	1.00	1.00	1.36	1.32	1.32	1.45	0.00	0.82
time (sec)	N/A	0.039	0.037	0.316	0.207	0.317	0.614	0.000	1.489

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	32	32	32	20	18
N.S.	1	1.00	0.81	0.74	1.19	1.19	1.19	0.74	0.67
time (sec)	N/A	0.034	0.032	0.203	0.200	0.312	0.052	0.397	1.560

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	0	38	0	48	27
N.S.	1	1.00	1.00	1.11	0.00	1.41	0.00	1.78	1.00
time (sec)	N/A	0.018	0.011	5.661	0.000	0.315	0.000	0.313	1.479

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	32	0	27	0	24	27
N.S.	1	1.00	1.00	1.19	0.00	1.00	0.00	0.89	1.00
time (sec)	N/A	0.023	0.006	2.560	0.000	0.335	0.000	0.334	1.475

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	20	13	0	27	21
N.S.	1	1.00	1.00	0.84	0.80	0.52	0.00	1.08	0.84
time (sec)	N/A	0.014	0.007	3.313	0.229	0.336	0.000	0.314	1.429

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	0	33	0	35	29
N.S.	1	1.00	1.00	1.17	0.00	1.14	0.00	1.21	1.00
time (sec)	N/A	0.024	0.006	0.560	0.000	0.351	0.000	0.314	1.536

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	29	5	5	3	5	5
N.S.	1	1.00	0.68	0.94	0.16	0.16	0.10	0.16	0.16
time (sec)	N/A	0.017	0.009	0.082	0.208	0.339	0.026	0.310	1.576

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	35	35	30	80	57	30	0	102	0
N.S.	1	1.00	0.86	2.29	1.63	0.86	0.00	2.91	0.00
time (sec)	N/A	0.068	0.037	1.358	0.246	0.369	0.000	0.326	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	66	66	50	132	79	52	0	123	0
N.S.	1	1.00	0.76	2.00	1.20	0.79	0.00	1.86	0.00
time (sec)	N/A	0.078	0.061	1.603	0.281	0.353	0.000	0.336	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	89	89	66	162	110	64	0	454	0
N.S.	1	1.00	0.74	1.82	1.24	0.72	0.00	5.10	0.00
time (sec)	N/A	0.358	0.080	1.900	0.262	0.360	0.000	0.326	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	35	35	30	79	55	30	0	108	0
N.S.	1	1.00	0.86	2.26	1.57	0.86	0.00	3.09	0.00
time (sec)	N/A	0.039	0.035	1.025	0.249	0.310	0.000	0.315	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	66	66	50	132	76	54	0	122	0
N.S.	1	1.00	0.76	2.00	1.15	0.82	0.00	1.85	0.00
time (sec)	N/A	0.087	0.067	1.186	0.278	0.337	0.000	0.318	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	88	88	66	162	109	64	0	495	0
N.S.	1	1.00	0.75	1.84	1.24	0.73	0.00	5.62	0.00
time (sec)	N/A	0.338	0.116	2.058	0.263	0.399	0.000	0.362	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.024	0.025	1.456	0.246	0.305	3.916	0.379	1.498

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	87	87	104	0	0	0
N.S.	1	1.00	0.89	1.85	1.85	2.21	0.00	0.00	0.00
time (sec)	N/A	0.040	0.024	1.436	0.432	0.338	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	86	89	109	0	0	0
N.S.	1	1.00	0.96	1.91	1.98	2.42	0.00	0.00	0.00
time (sec)	N/A	0.041	0.015	1.247	0.382	0.389	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	91	115	0	0	0
N.S.	1	1.00	1.00	0.00	1.75	2.21	0.00	0.00	0.00
time (sec)	N/A	0.042	0.020	0.000	0.413	0.342	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	115	60	104	0	0	37
N.S.	1	1.00	1.00	2.45	1.28	2.21	0.00	0.00	0.79
time (sec)	N/A	0.036	0.011	1.618	0.406	0.409	0.000	0.000	1.499

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	115	62	106	0	0	39
N.S.	1	1.00	0.96	2.56	1.38	2.36	0.00	0.00	0.87
time (sec)	N/A	0.038	0.016	1.444	0.440	0.350	0.000	0.000	0.092

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	65	115	0	0	41
N.S.	1	1.00	1.00	0.00	1.25	2.21	0.00	0.00	0.79
time (sec)	N/A	0.039	0.020	0.000	0.487	0.408	0.000	0.000	1.533

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	75	85	42	184	0	0	39
N.S.	1	1.00	1.47	1.67	0.82	3.61	0.00	0.00	0.76
time (sec)	N/A	0.032	0.012	1.036	0.310	0.358	0.000	0.000	0.095

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	82	44	184	0	0	41
N.S.	1	1.00	1.53	1.67	0.90	3.76	0.00	0.00	0.84
time (sec)	N/A	0.034	0.011	1.135	0.297	0.317	0.000	0.000	0.065

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	56	56	81	2196	48	195	0	0	44
N.S.	1	1.00	1.45	39.21	0.86	3.48	0.00	0.00	0.79
time (sec)	N/A	0.035	0.011	6.515	0.300	0.374	0.000	0.000	0.075

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	75	86	43	147	0	0	0
N.S.	1	1.00	1.47	1.69	0.84	2.88	0.00	0.00	0.00
time (sec)	N/A	0.032	0.010	0.945	0.290	0.336	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	82	44	148	0	0	0
N.S.	1	1.00	1.53	1.67	0.90	3.02	0.00	0.00	0.00
time (sec)	N/A	0.035	0.010	1.086	0.282	0.383	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	56	56	81	2197	49	158	0	0	44
N.S.	1	1.00	1.45	39.23	0.88	2.82	0.00	0.00	0.79
time (sec)	N/A	0.035	0.011	6.917	0.289	0.353	0.000	0.000	0.081

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	108	60	106	0	0	39
N.S.	1	1.00	1.00	2.35	1.30	2.30	0.00	0.00	0.85
time (sec)	N/A	0.033	0.011	1.325	0.400	0.322	0.000	0.000	0.096

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	117	61	102	0	0	39
N.S.	1	1.00	0.96	2.60	1.36	2.27	0.00	0.00	0.87
time (sec)	N/A	0.035	0.017	1.311	0.374	0.361	0.000	0.000	1.492

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	65	117	0	0	43
N.S.	1	1.00	1.00	0.00	1.27	2.29	0.00	0.00	0.84
time (sec)	N/A	0.036	0.018	0.000	0.521	0.337	0.000	0.000	0.093

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	41	83	87	106	0	0	0
N.S.	1	1.00	0.89	1.80	1.89	2.30	0.00	0.00	0.00
time (sec)	N/A	0.038	0.014	1.144	0.431	0.370	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	88	87	107	0	0	0
N.S.	1	1.00	0.93	1.96	1.93	2.38	0.00	0.00	0.00
time (sec)	N/A	0.040	0.015	1.134	0.402	0.340	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	91	117	0	0	0
N.S.	1	1.00	1.00	0.00	1.78	2.29	0.00	0.00	0.00
time (sec)	N/A	0.040	0.019	0.000	0.461	0.361	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	13	111	17	17	0	13	0
N.S.	1	1.00	0.62	5.29	0.81	0.81	0.00	0.62	0.00
time (sec)	N/A	0.016	0.005	6.894	0.199	0.322	0.000	0.288	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	8	7	7	0	24	6
N.S.	1	1.00	1.00	1.33	1.17	1.17	0.00	4.00	1.00
time (sec)	N/A	0.012	0.023	1.084	0.203	0.328	0.000	0.292	1.721

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	35	6	6	0	6	6
N.S.	1	1.00	0.68	0.95	0.16	0.16	0.00	0.16	0.16
time (sec)	N/A	0.019	0.037	2.292	0.195	0.318	0.000	0.287	1.829

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	94	22	15	12	35
N.S.	1	1.00	1.00	1.08	7.83	1.83	1.25	1.00	2.92
time (sec)	N/A	0.019	0.014	1.542	0.286	0.328	14.260	0.300	1.871

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	0	7	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.00	0.78	0.78
time (sec)	N/A	0.012	0.005	1.069	0.245	0.330	0.000	0.298	1.545

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	15	17	16	24	17	16	11
N.S.	1	1.00	0.75	0.85	0.80	1.20	0.85	0.80	0.55
time (sec)	N/A	0.027	0.016	0.575	0.264	0.329	0.573	0.340	1.550

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	33	30	42	65	0	0	29
N.S.	1	1.00	0.66	0.60	0.84	1.30	0.00	0.00	0.58
time (sec)	N/A	0.018	0.010	7.660	0.214	0.357	0.000	0.000	1.697

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	0	7	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.00	1.17	1.00
time (sec)	N/A	0.014	0.015	0.920	0.196	0.370	0.000	0.311	1.593

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.89	0.78	0.78
time (sec)	N/A	0.012	0.004	1.134	0.195	0.353	1.615	0.364	1.676

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	10	10	9
N.S.	1	1.00	1.00	1.10	1.00	1.00	1.00	1.00	0.90
time (sec)	N/A	0.006	0.010	0.605	0.192	0.279	0.190	0.309	1.649

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	43	35	108	27	223	27	0
N.S.	1	1.00	3.07	2.50	7.71	1.93	15.93	1.93	0.00
time (sec)	N/A	0.014	0.010	1.138	0.196	0.324	0.825	0.292	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	11	11	10	11	8
N.S.	1	1.00	1.00	0.82	1.00	1.00	0.91	1.00	0.73
time (sec)	N/A	0.008	0.001	0.598	0.188	0.321	0.180	0.286	1.640

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	59	194	104	120	0	0	0
N.S.	1	1.00	0.80	2.62	1.41	1.62	0.00	0.00	0.00
time (sec)	N/A	0.079	0.045	4.481	0.458	0.343	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	47	53	179	43	456	41	0
N.S.	1	1.00	1.18	1.32	4.48	1.08	11.40	1.02	0.00
time (sec)	N/A	0.046	0.032	1.230	0.215	0.352	3.608	0.289	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	88	0	139	0	0	0	0
N.S.	1	1.00	1.11	0.00	1.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.031	0.000	0.221	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	81	19	17	19	57
N.S.	1	1.00	1.00	1.07	5.40	1.27	1.13	1.27	3.80
time (sec)	N/A	0.016	0.015	1.415	0.275	0.316	11.143	0.350	1.714

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	58	36	134	0	52	0
N.S.	1	1.00	0.86	1.66	1.03	3.83	0.00	1.49	0.00
time (sec)	N/A	0.057	0.035	0.968	0.259	0.320	0.000	0.288	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	99	67	313	0	67	0
N.S.	1	1.00	0.76	1.50	1.02	4.74	0.00	1.02	0.00
time (sec)	N/A	0.102	0.073	1.302	0.256	0.343	0.000	0.305	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	67	120	110	587	0	102	0
N.S.	1	1.00	0.75	1.35	1.24	6.60	0.00	1.15	0.00
time (sec)	N/A	0.370	0.073	1.484	0.277	0.353	0.000	0.305	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	58	37	134	0	54	0
N.S.	1	1.00	0.86	1.66	1.06	3.83	0.00	1.54	0.00
time (sec)	N/A	0.046	0.030	0.953	0.252	0.349	0.000	0.298	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	99	67	305	0	67	0
N.S.	1	1.00	0.76	1.50	1.02	4.62	0.00	1.02	0.00
time (sec)	N/A	0.084	0.052	1.150	0.257	0.377	0.000	0.300	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	66	120	111	587	0	104	0
N.S.	1	1.00	0.75	1.36	1.26	6.67	0.00	1.18	0.00
time (sec)	N/A	0.339	0.106	1.876	0.277	0.338	0.000	0.306	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	39	39	39	295	43	57	0	0	0
N.S.	1	1.00	1.00	7.56	1.10	1.46	0.00	0.00	0.00
time (sec)	N/A	0.040	0.030	1.059	0.252	0.341	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	35	35	33	454	43	69	0	0	0
N.S.	1	1.00	0.94	12.97	1.23	1.97	0.00	0.00	0.00
time (sec)	N/A	0.043	0.018	3.761	0.238	0.322	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	47	65	0	0	0
N.S.	1	1.00	0.98	0.00	1.07	1.48	0.00	0.00	0.00
time (sec)	N/A	0.042	0.023	0.000	0.236	0.307	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	39	39	37	321	32	65	0	0	0
N.S.	1	1.00	0.95	8.23	0.82	1.67	0.00	0.00	0.00
time (sec)	N/A	0.039	0.035	2.759	0.306	0.322	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	35	35	33	478	32	77	0	0	0
N.S.	1	1.00	0.94	13.66	0.91	2.20	0.00	0.00	0.00
time (sec)	N/A	0.037	0.024	5.520	0.331	0.321	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	36	73	0	0	0
N.S.	1	1.00	0.98	0.00	0.82	1.66	0.00	0.00	0.00
time (sec)	N/A	0.038	0.022	0.000	0.313	0.325	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	24	54	101	0	0	20
N.S.	1	1.00	0.90	0.62	1.38	2.59	0.00	0.00	0.51
time (sec)	N/A	0.033	0.007	0.500	0.324	0.359	0.000	0.000	1.655

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	49	76	56	102	0	0	0
N.S.	1	1.00	1.20	1.85	1.37	2.49	0.00	0.00	0.00
time (sec)	N/A	0.035	0.009	1.066	0.280	0.309	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	47	47	57	129	0	0	0
N.S.	1	1.00	1.27	1.27	1.54	3.49	0.00	0.00	0.00
time (sec)	N/A	0.036	0.009	1.154	0.283	0.343	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	55	43	61	116	0	0	0
N.S.	1	1.00	1.20	0.93	1.33	2.52	0.00	0.00	0.00
time (sec)	N/A	0.035	0.010	5.638	0.281	0.311	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	45	24	49	101	0	0	22
N.S.	1	1.00	1.15	0.62	1.26	2.59	0.00	0.00	0.56
time (sec)	N/A	0.032	0.007	0.492	0.275	0.345	0.000	0.000	1.541

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	49	76	51	102	0	0	0
N.S.	1	1.00	1.20	1.85	1.24	2.49	0.00	0.00	0.00
time (sec)	N/A	0.032	0.010	1.076	0.286	0.342	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	47	47	59	127	0	0	0
N.S.	1	1.00	1.27	1.27	1.59	3.43	0.00	0.00	0.00
time (sec)	N/A	0.033	0.010	1.138	0.284	0.330	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	55	43	61	116	0	0	0
N.S.	1	1.00	1.20	0.93	1.33	2.52	0.00	0.00	0.00
time (sec)	N/A	0.034	0.011	4.257	0.280	0.334	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	38	38	38	314	31	84	0	0	0
N.S.	1	1.00	1.00	8.26	0.82	2.21	0.00	0.00	0.00
time (sec)	N/A	0.037	0.012	2.588	0.315	0.346	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	35	35	33	480	32	106	0	0	0
N.S.	1	1.00	0.94	13.71	0.91	3.03	0.00	0.00	0.00
time (sec)	N/A	0.037	0.016	3.154	0.316	0.347	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	36	92	0	0	0
N.S.	1	1.00	1.00	0.00	0.84	2.14	0.00	0.00	0.00
time (sec)	N/A	0.038	0.019	0.000	0.320	0.351	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	38	38	38	293	37	76	0	0	0
N.S.	1	1.00	1.00	7.71	0.97	2.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.012	0.810	0.252	0.348	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	35	35	33	456	45	97	0	0	0
N.S.	1	1.00	0.94	13.03	1.29	2.77	0.00	0.00	0.00
time (sec)	N/A	0.036	0.017	1.439	0.238	0.322	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	47	84	0	0	0
N.S.	1	1.00	1.00	0.00	1.09	1.95	0.00	0.00	0.00
time (sec)	N/A	0.038	0.020	0.000	0.237	0.298	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	33	30	112	258	0	94	31
N.S.	1	1.00	0.66	0.60	2.24	5.16	0.00	1.88	0.62
time (sec)	N/A	0.018	0.009	9.253	0.216	0.301	0.000	0.397	1.688

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	12	62	14	37	9
N.S.	1	1.00	1.00	1.08	0.92	4.77	1.08	2.85	0.69
time (sec)	N/A	0.013	0.014	141.525	0.183	0.337	0.185	0.285	1.591

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	11	14	13	9	94	13	9
N.S.	1	1.00	0.65	0.82	0.76	0.53	5.53	0.76	0.53
time (sec)	N/A	0.004	0.003	0.562	0.198	0.303	0.813	0.297	0.034

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	25	24	23	27	24	25
N.S.	1	1.00	0.96	0.93	0.89	0.85	1.00	0.89	0.93
time (sec)	N/A	0.015	0.004	0.137	0.188	0.312	0.057	0.287	0.144

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	22	4	3	20	0	3	3
N.S.	1	1.00	7.33	1.33	1.00	6.67	0.00	1.00	1.00
time (sec)	N/A	0.020	0.018	0.210	0.304	0.313	0.000	0.289	1.532

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	23	17	22	22	22	17
N.S.	1	1.00	1.25	0.96	0.71	0.92	0.92	0.92	0.71
time (sec)	N/A	0.014	0.001	0.112	0.194	0.335	0.050	0.281	0.042

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	23	22	29	23	25
N.S.	1	1.00	0.92	1.04	0.92	0.88	1.16	0.92	1.00
time (sec)	N/A	0.006	0.013	0.164	0.204	0.346	0.074	0.290	0.072

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	24	20	22	30	20
N.S.	1	1.00	0.88	0.74	0.71	0.59	0.65	0.88	0.59
time (sec)	N/A	0.009	0.006	0.196	0.192	0.307	0.045	0.282	1.543

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	25	28	24	27	42	22
N.S.	1	1.00	1.00	0.62	0.70	0.60	0.68	1.05	0.55
time (sec)	N/A	0.012	0.008	0.254	0.185	0.378	0.061	0.288	0.067

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	26	25	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.84	0.81	0.81
time (sec)	N/A	0.021	0.014	0.115	0.272	0.349	0.059	0.298	1.579

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.003	0.015	0.066	0.000	0.343	3.337	0.346	0.083

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.003	0.014	0.065	0.000	0.355	2.693	0.301	1.895

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.004	0.015	0.088	0.000	0.300	3.474	0.303	1.572

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	52	0	28	0	40	37
N.S.	1	1.00	1.00	1.21	0.00	0.65	0.00	0.93	0.86
time (sec)	N/A	0.009	0.022	0.091	0.000	0.323	0.000	0.360	2.217

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	105	13	9
N.S.	1	1.00	0.71	0.67	0.62	0.67	5.00	0.62	0.43
time (sec)	N/A	0.004	0.005	0.543	0.207	0.346	1.330	0.321	1.482

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	13	24	14	10	14	13
N.S.	1	1.00	0.92	1.00	1.85	1.08	0.77	1.08	1.00
time (sec)	N/A	0.004	0.006	0.119	0.182	0.353	0.101	0.481	1.542

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	14	6	3	6	6
N.S.	1	1.00	1.00	1.17	2.33	1.00	0.50	1.00	1.00
time (sec)	N/A	0.007	0.002	0.083	0.198	0.306	0.042	0.354	1.448

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	38	28	33	38	184	33	26
N.S.	1	1.00	1.19	0.88	1.03	1.19	5.75	1.03	0.81
time (sec)	N/A	0.137	0.013	0.171	0.184	0.385	0.411	0.317	0.116

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	16	15	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.005	0.004	1.171	0.273	0.342	0.066	0.309	1.518

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	22	29	23	34	23	19
N.S.	1	1.00	1.00	1.10	1.45	1.15	1.70	1.15	0.95
time (sec)	N/A	0.003	0.015	0.161	0.187	0.350	0.123	0.321	1.503

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	34	33	33	32	139	29
N.S.	1	1.00	0.89	0.97	0.94	0.94	0.91	3.97	0.83
time (sec)	N/A	0.004	0.006	0.530	0.198	0.339	0.082	0.316	0.104

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	12	12	15	12	12
N.S.	1	1.00	1.00	1.17	1.00	1.00	1.25	1.00	1.00
time (sec)	N/A	0.002	0.003	0.217	0.191	0.370	0.059	0.291	1.499

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	29	28	28	27	29	40
N.S.	1	1.00	0.75	0.81	0.78	0.78	0.75	0.81	1.11
time (sec)	N/A	0.011	0.009	0.333	0.191	0.320	0.058	0.302	1.632

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	43	42	42	48	43	56
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.89	0.80	1.04
time (sec)	N/A	0.028	0.012	0.348	0.189	0.294	0.063	0.311	1.793

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	30	52	42	41	31	46
N.S.	1	1.00	0.94	0.86	1.49	1.20	1.17	0.89	1.31
time (sec)	N/A	0.010	0.019	0.205	0.193	0.303	0.099	0.308	1.609

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	74	64	63	31	73
N.S.	1	1.00	1.26	0.86	2.11	1.83	1.80	0.89	2.09
time (sec)	N/A	0.016	0.015	0.796	0.200	0.321	0.109	0.302	1.651

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.012	0.001	0.806	0.187	0.321	0.048	0.299	1.639

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	31	21	26	32	25
N.S.	1	1.00	0.68	0.84	1.00	0.68	0.84	1.03	0.81
time (sec)	N/A	0.014	0.003	0.859	0.185	0.282	0.091	0.287	1.607

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	30	61	44	47	185	0	52
N.S.	1	1.00	0.68	1.39	1.00	1.07	4.20	0.00	1.18
time (sec)	N/A	0.022	0.013	0.829	0.185	0.364	0.428	0.000	1.595

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	19	31	45	18
N.S.	1	1.00	1.00	1.06	1.33	1.06	1.72	2.50	1.00
time (sec)	N/A	0.008	0.003	0.339	0.209	0.300	0.402	0.304	1.533

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	121	99	26	71
N.S.	1	1.00	1.00	0.75	0.00	3.78	3.09	0.81	2.22
time (sec)	N/A	0.013	0.052	0.664	0.000	0.308	2.205	0.313	1.626

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	144	144	112	112	0	480	175	239	153
N.S.	1	1.00	0.78	0.78	0.00	3.33	1.22	1.66	1.06
time (sec)	N/A	0.067	0.069	0.973	0.000	0.310	25.994	0.333	3.515

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	227	167	112	0	160	133	170	95
N.S.	1	1.00	0.74	0.49	0.00	0.70	0.59	0.75	0.42
time (sec)	N/A	0.112	0.082	1.361	0.000	0.292	13.472	0.312	3.412

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	29	33	28	116	53	27
N.S.	1	1.00	1.26	1.07	1.22	1.04	4.30	1.96	1.00
time (sec)	N/A	0.016	0.047	0.361	0.189	0.311	1.610	0.310	1.563

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	41	0	143	204	38	45
N.S.	1	1.00	1.00	1.02	0.00	3.58	5.10	0.95	1.12
time (sec)	N/A	0.019	0.030	0.638	0.000	0.316	3.634	0.323	1.605

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	149	129	118	0	149	245	257	174
N.S.	1	1.00	0.87	0.79	0.00	1.00	1.64	1.72	1.17
time (sec)	N/A	0.072	0.055	0.980	0.000	0.300	26.839	0.367	3.662

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	233	233	208	118	0	156	228	178	176
N.S.	1	1.00	0.89	0.51	0.00	0.67	0.98	0.76	0.76
time (sec)	N/A	0.136	0.063	1.421	0.000	0.305	15.017	0.312	3.402

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	20	0	21	22	0	19
N.S.	1	1.00	1.00	0.91	0.00	0.95	1.00	0.00	0.86
time (sec)	N/A	0.014	0.039	0.427	0.000	0.291	0.078	0.000	1.556

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	38	37	39	22	0	37
N.S.	1	1.00	1.02	0.93	0.90	0.95	0.54	0.00	0.90
time (sec)	N/A	0.039	0.065	0.186	0.272	0.300	0.086	0.000	1.715

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	38	0	39	22	0	37
N.S.	1	1.00	1.02	0.93	0.00	0.95	0.54	0.00	0.90
time (sec)	N/A	0.025	0.143	0.473	0.000	0.287	0.082	0.000	1.474

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	44	40	0	41	19	0	37
N.S.	1	1.00	1.05	0.95	0.00	0.98	0.45	0.00	0.88
time (sec)	N/A	0.031	0.063	0.453	0.000	0.293	0.086	0.000	1.743

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	17	21	27	22	21
N.S.	1	1.00	1.00	0.66	0.53	0.66	0.84	0.69	0.66
time (sec)	N/A	0.014	0.003	0.193	0.205	0.280	0.056	0.295	1.617

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	59	0	0	0	0	0	0
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	60	0	0	0	0	0	0
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	62	0	0	0	0	32	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.021	0.008	0.000	0.000	0.000	0.000	0.329	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	121	102	0	0	0	0	0
N.S.	1	1.00	0.99	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.058	0.592	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	109	111	0	0	0	0
N.S.	1	1.00	0.90	1.35	1.37	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.030	2.762	0.195	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	5	12	8	10	0	4
N.S.	1	1.00	1.00	0.56	1.33	0.89	1.11	0.00	0.44
time (sec)	N/A	0.007	0.003	0.937	0.181	0.294	0.874	0.000	0.022

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	47	82	0	0	0	59
N.S.	1	1.00	0.81	1.09	1.91	0.00	0.00	0.00	1.37
time (sec)	N/A	0.047	0.015	1.167	0.193	0.000	0.000	0.000	1.770

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	31	31	49	0	228	0	0
N.S.	1	1.00	1.03	1.03	1.63	0.00	7.60	0.00	0.00
time (sec)	N/A	0.065	0.010	0.833	0.196	0.000	58.665	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	0	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.018	0.163	0.527	0.231	0.280	0.000	0.325	1.608

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	57	57	58	46	59	59
N.S.	1	1.00	0.93	0.84	0.84	0.85	0.68	0.87	0.87
time (sec)	N/A	0.157	0.027	1.994	0.283	0.321	0.111	0.367	1.587

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	23	31	37	23	23
N.S.	1	1.00	1.00	0.83	0.79	1.07	1.28	0.79	0.79
time (sec)	N/A	0.012	0.011	0.198	0.193	0.313	2.348	0.372	0.080

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	18	14	80	18
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.78	4.44	1.00
time (sec)	N/A	0.003	0.003	0.504	0.191	0.297	0.048	0.338	1.462

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	21	22	25	21	15	103	21
N.S.	1	1.00	0.78	0.81	0.93	0.78	0.56	3.81	0.78
time (sec)	N/A	0.003	0.004	0.506	0.188	0.305	0.052	0.326	0.086

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	60	56	54	54	41	56	55
N.S.	1	1.00	1.05	0.98	0.95	0.95	0.72	0.98	0.96
time (sec)	N/A	0.038	0.049	2.707	0.277	0.299	0.088	0.328	1.546

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	62	129	0	0	0	0	0
N.S.	1	1.00	1.03	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.009	1.381	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	239	146	0	0	0	0	0
N.S.	1	1.00	3.92	2.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.039	2.062	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	373	121	0	0	0	0	0
N.S.	1	1.00	2.26	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.158	1.680	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	37	0	0	0
N.S.	1	1.00	4.62	0.00	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.022	0.534	0.000	0.000	0.292	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	36	0	0	0
N.S.	1	1.00	4.62	0.00	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.022	0.478	0.000	0.000	0.310	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	10	10	8	10	17
N.S.	1	1.00	1.00	0.88	0.59	0.59	0.47	0.59	1.00
time (sec)	N/A	0.002	0.004	0.623	0.224	0.277	0.029	0.308	0.072

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	16	20	65	16	49
N.S.	1	1.00	0.93	0.96	0.59	0.74	2.41	0.59	1.81
time (sec)	N/A	0.005	0.018	0.221	0.217	0.315	0.424	0.305	1.727

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	31	25	28	26	25	0	26	27
N.S.	1	1.24	1.00	1.12	1.04	1.00	0.00	1.04	1.08
time (sec)	N/A	0.034	0.012	0.397	0.216	0.300	0.000	0.348	1.816

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	22	26	24	23	26	24	20
N.S.	1	1.00	0.85	1.00	0.92	0.88	1.00	0.92	0.77
time (sec)	N/A	0.023	0.012	0.594	0.260	0.302	3.062	0.314	1.477

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.026	14.590	0.470	0.263	0.273	0.256	0.307	1.479

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	8	10	10	7	10	10
N.S.	1	1.00	1.25	1.00	1.25	1.25	0.88	1.25	1.25
time (sec)	N/A	0.016	9.099	0.470	0.242	0.269	0.242	0.309	1.426

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6	6	8	6	8	8	7	8	8
N.S.	1	1.00	1.33	1.00	1.33	1.33	1.17	1.33	1.33
time (sec)	N/A	0.006	0.007	0.473	0.241	0.275	0.221	0.308	1.489

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.022	0.051	0.444	0.231	0.270	0.540	0.330	1.489

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	14	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.40	1.00	1.20	1.20
time (sec)	N/A	0.024	17.235	0.483	0.285	0.277	0.411	0.319	1.529

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	11	10	11	11
N.S.	1	1.00	1.00	0.77	0.69	0.85	0.77	0.85	0.85
time (sec)	N/A	0.017	0.012	0.204	0.299	0.285	0.046	0.305	1.581

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	10	11	10	10	8	14	10
N.S.	1	1.00	1.11	1.22	1.11	1.11	0.89	1.56	1.11
time (sec)	N/A	0.051	0.055	1.043	0.311	0.298	0.048	0.311	1.556

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	0	13	15	0	13
N.S.	1	1.00	1.00	1.54	0.00	1.00	1.15	0.00	1.00
time (sec)	N/A	0.097	0.039	1.234	0.000	0.312	1.185	0.000	1.548

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	53	107	67	48	53	88	63
N.S.	1	1.00	0.79	1.60	1.00	0.72	0.79	1.31	0.94
time (sec)	N/A	0.050	0.026	0.231	0.209	0.313	45.562	0.354	1.775

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	53	80	68	49	53	53	38
N.S.	1	1.00	1.06	1.60	1.36	0.98	1.06	1.06	0.76
time (sec)	N/A	0.039	0.031	0.228	0.228	0.269	47.320	0.318	1.624

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	19	18	18	17	47	18
N.S.	1	1.00	0.81	0.90	0.86	0.86	0.81	2.24	0.86
time (sec)	N/A	0.005	0.004	0.474	0.218	0.276	0.043	0.313	0.063

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	53	80	68	49	53	53	38
N.S.	1	1.00	1.06	1.60	1.36	0.98	1.06	1.06	0.76
time (sec)	N/A	0.033	0.028	0.226	0.233	0.283	47.547	0.315	1.567

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	108	67	48	53	88	63
N.S.	1	1.00	0.93	1.57	0.97	0.70	0.77	1.28	0.91
time (sec)	N/A	0.036	0.022	0.155	0.217	0.299	46.322	0.355	0.340

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	10	0	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	1.11	0.00	1.00
time (sec)	N/A	0.016	0.010	0.107	0.247	0.285	0.067	0.000	1.456

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	17	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.019	0.022	0.000	0.000	0.087	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	72	0	108	0	0	89	0
N.S.	1	1.00	1.20	0.00	1.80	0.00	0.00	1.48	0.00
time (sec)	N/A	0.050	0.178	0.000	0.258	0.000	0.000	0.310	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	71	0	94	0	0	74	0
N.S.	1	1.00	1.11	0.00	1.47	0.00	0.00	1.16	0.00
time (sec)	N/A	0.047	0.158	0.000	0.244	0.000	0.000	0.344	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	156	0	0	129	0
N.S.	1	1.00	1.16	0.00	2.26	0.00	0.00	1.87	0.00
time (sec)	N/A	0.049	0.197	0.000	0.249	0.000	0.000	0.307	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	79	0	130	0	0	106	0
N.S.	1	1.00	1.11	0.00	1.83	0.00	0.00	1.49	0.00
time (sec)	N/A	0.052	0.192	0.000	0.221	0.000	0.000	0.317	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	98	95	426	94	234	0	0	0
N.S.	1	1.00	0.97	4.35	0.96	2.39	0.00	0.00	0.00
time (sec)	N/A	0.202	0.049	1.605	0.406	0.329	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	79	398	70	174	0	0	0
N.S.	1	1.00	0.99	4.98	0.88	2.18	0.00	0.00	0.00
time (sec)	N/A	0.125	0.037	1.381	0.373	0.330	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	52	52	47	368	43	109	0	0	0
N.S.	1	1.00	0.90	7.08	0.83	2.10	0.00	0.00	0.00
time (sec)	N/A	0.044	0.025	0.922	0.354	0.318	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	101	12	10	12	12
N.S.	1	1.00	1.20	1.00	10.10	1.20	1.00	1.20	1.20
time (sec)	N/A	0.013	1.941	0.569	0.575	0.287	3.828	0.345	1.644

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	121	12	12	12	12
N.S.	1	1.00	1.20	1.00	12.10	1.20	1.20	1.20	1.20
time (sec)	N/A	0.260	1.644	0.693	0.399	0.278	20.606	0.374	1.618

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	103	100	643	94	241	0	0	0
N.S.	1	1.00	0.97	6.24	0.91	2.34	0.00	0.00	0.00
time (sec)	N/A	0.144	0.059	2.029	0.371	0.325	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	85	85	82	615	70	181	0	0	0
N.S.	1	1.00	0.96	7.24	0.82	2.13	0.00	0.00	0.00
time (sec)	N/A	0.095	0.032	1.652	0.362	0.319	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	57	57	56	583	43	116	0	0	0
N.S.	1	1.00	0.98	10.23	0.75	2.04	0.00	0.00	0.00
time (sec)	N/A	0.046	0.023	1.025	0.364	0.310	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	13	13	15	12	102	14	14	14	14
N.S.	1	1.00	1.15	0.92	7.85	1.08	1.08	1.08	1.08
time (sec)	N/A	0.016	0.706	0.524	0.574	0.301	13.279	0.366	1.635

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	13	13	15	12	126	14	15	14	14
N.S.	1	1.00	1.15	0.92	9.69	1.08	1.15	1.08	1.08
time (sec)	N/A	0.052	1.604	0.691	0.416	0.304	59.018	0.386	1.609

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [207] had the largest ratio of [1.6670000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	N/A	0	0	1.00	32	0.000
2	A	10	5	1.00	32	0.156
3	A	8	5	1.00	32	0.156
4	A	6	5	1.00	30	0.167
5	A	2	2	1.00	14	0.143
6	N/A	0	0	1.00	32	0.000
7	N/A	0	0	1.00	32	0.000
8	N/A	0	0	1.00	32	0.000
9	A	13	5	1.00	28	0.179
10	A	8	5	1.00	28	0.179
11	A	5	4	1.00	26	0.154
12	A	1	1	1.00	10	0.100
13	N/A	0	0	1.00	28	0.000
14	N/A	0	0	1.00	28	0.000
15	N/A	0	0	1.00	28	0.000
16	A	1	1	1.00	43	0.023
17	A	1	1	1.00	43	0.023
18	A	1	1	1.00	41	0.024
19	A	4	3	1.00	25	0.120
20	A	1	1	1.00	43	0.023
21	A	1	1	1.00	43	0.023
22	A	1	1	1.00	43	0.023
23	A	3	2	1.00	39	0.051

Continued on next page

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	3	2	1.00	37	0.054
25	A	2	1	1.00	15	0.067
26	A	2	2	1.00	34	0.059
27	A	3	2	1.00	37	0.054
28	A	3	2	1.00	39	0.051
29	A	2	2	1.00	45	0.044
30	N/A	0	0	1.00	40	0.000
31	A	9	4	1.00	40	0.100
32	A	7	4	1.00	40	0.100
33	A	5	4	1.00	38	0.105
34	A	4	3	1.00	22	0.136
35	N/A	0	0	1.00	40	0.000
36	N/A	0	0	1.00	40	0.000
37	N/A	0	0	1.00	40	0.000
38	A	1	1	1.00	60	0.017
39	N/A	0	0	1.00	29	0.000
40	A	1	1	1.00	39	0.026
41	A	1	1	1.00	40	0.025
42	A	1	1	1.00	41	0.024
43	A	1	1	1.00	42	0.024
44	A	1	1	1.00	40	0.025
45	A	1	1	1.00	41	0.024
46	A	3	3	1.00	19	0.158
47	A	3	3	1.00	21	0.143
48	A	3	3	1.00	19	0.158
49	A	3	3	1.00	17	0.176
50	A	4	3	1.00	15	0.200
51	A	1	1	1.00	19	0.053
52	A	3	3	1.00	19	0.158
53	A	3	3	1.00	19	0.158
54	A	3	3	1.00	19	0.158
55	A	2	2	1.00	9	0.222
56	A	1	1	1.00	13	0.077
57	A	3	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
58	A	1	1	1.00	15	0.067
59	A	3	3	1.00	18	0.167
60	A	3	2	1.00	18	0.111
61	A	3	2	1.00	18	0.111
62	A	3	2	1.00	18	0.111
63	A	3	2	1.00	16	0.125
64	A	3	2	1.00	14	0.143
65	A	7	6	1.00	18	0.333
66	A	3	2	1.00	18	0.111
67	A	3	2	1.00	18	0.111
68	A	3	2	1.00	18	0.111
69	A	3	2	1.00	18	0.111
70	A	5	3	1.00	19	0.158
71	A	7	6	1.00	19	0.316
72	A	7	6	1.00	19	0.316
73	A	7	6	1.00	19	0.316
74	A	7	6	1.00	17	0.353
75	A	6	6	1.00	15	0.400
76	A	7	4	1.00	19	0.210
77	A	7	6	1.00	19	0.316
78	A	7	6	1.00	19	0.316
79	A	7	6	1.00	19	0.316
80	A	7	6	1.00	19	0.316
81	A	6	6	1.00	7	0.857
82	A	7	6	1.00	23	0.261
83	A	7	6	1.00	23	0.261
84	A	7	6	1.00	23	0.261
85	A	7	6	1.00	21	0.286
86	A	6	6	1.00	15	0.400
87	A	9	5	1.00	23	0.217
88	A	7	6	1.00	23	0.261
89	A	7	6	1.00	23	0.261
90	A	7	6	1.00	23	0.261
91	A	7	6	1.00	23	0.261
92	A	6	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	8	9	1.00	32	0.281
94	A	20	6	1.00	25	0.240
95	A	20	6	1.00	28	0.214
96	A	14	10	1.00	16	0.625
97	A	27	14	1.00	17	0.824
98	A	28	12	1.00	21	0.571
99	A	27	14	1.00	9	1.556
100	A	34	16	1.00	13	1.231
101	A	25	11	1.00	21	0.524
102	A	20	10	1.00	21	0.476
103	A	16	10	1.00	19	0.526
104	A	13	9	1.00	17	0.529
105	N/A	0	0	1.00	21	0.000
106	A	19	11	1.00	21	0.524
107	A	20	12	1.00	21	0.571
108	A	15	12	1.00	23	0.522
109	A	13	12	1.00	23	0.522
110	A	12	10	1.00	23	0.435
111	A	15	13	1.00	23	0.565
112	A	18	13	1.00	23	0.565
113	A	6	5	1.00	12	0.417
114	A	5	5	1.00	12	0.417
115	A	4	4	1.00	10	0.400
116	A	4	4	1.00	8	0.500
117	N/A	0	0	1.00	12	0.000
118	A	5	4	1.00	20	0.200
119	A	4	4	1.00	20	0.200
120	A	3	3	1.00	18	0.167
121	A	2	2	1.00	16	0.125
122	N/A	0	0	1.00	20	0.000
123	A	6	5	1.00	20	0.250
124	A	5	5	1.00	20	0.250
125	A	4	4	1.00	18	0.222
126	A	4	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
127	N/A	0	0	1.00	20	0.000
128	A	3	3	1.00	16	0.188
129	A	2	2	1.00	10	0.200
130	A	2	2	1.00	12	0.167
131	A	2	2	1.00	10	0.200
132	A	2	2	1.00	10	0.200
133	A	2	1	1.00	12	0.083
134	A	3	2	1.00	14	0.143
135	A	2	1	1.00	16	0.062
136	A	2	1	1.00	14	0.071
137	A	7	6	1.00	16	0.375
138	A	2	1	1.00	15	0.067
139	A	2	2	1.00	8	0.250
140	A	3	2	1.00	9	0.222
141	A	3	2	1.00	16	0.125
142	A	3	2	1.00	16	0.125
143	A	4	3	1.00	18	0.167
144	A	3	2	1.00	16	0.125
145	A	3	2	1.00	14	0.143
146	A	3	2	1.00	16	0.125
147	A	4	4	1.00	16	0.250
148	A	3	1	1.00	20	0.050
149	A	3	2	1.00	14	0.143
150	A	3	2	1.00	14	0.143
151	A	3	2	1.00	16	0.125
152	A	3	2	1.00	16	0.125
153	A	4	3	1.00	10	0.300
154	A	5	6	1.00	9	0.667
155	A	5	6	1.00	11	0.546
156	A	15	8	1.00	11	0.727
157	A	5	6	1.00	9	0.667
158	A	7	8	1.00	11	0.727
159	A	15	8	1.00	11	0.727
160	A	4	3	1.00	12	0.250
161	A	5	5	1.00	5	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
162	A	6	6	1.00	7	0.857
163	A	6	6	1.00	7	0.857
164	A	5	5	1.00	5	1.000
165	A	6	6	1.00	7	0.857
166	A	6	6	1.00	7	0.857
167	A	7	5	1.00	5	1.000
168	A	8	6	1.00	7	0.857
169	A	8	6	1.00	7	0.857
170	A	7	5	1.00	5	1.000
171	A	8	6	1.00	7	0.857
172	A	8	6	1.00	7	0.857
173	A	5	5	1.00	5	1.000
174	A	6	6	1.00	7	0.857
175	A	6	6	1.00	7	0.857
176	A	5	5	1.00	5	1.000
177	A	6	6	1.00	7	0.857
178	A	6	6	1.00	7	0.857
179	A	3	3	1.00	16	0.188
180	A	3	2	1.00	10	0.200
181	A	5	4	1.00	10	0.400
182	A	3	4	1.00	8	0.500
183	A	2	3	1.00	6	0.500
184	A	4	4	1.00	10	0.400
185	A	2	2	1.00	35	0.057
186	A	3	3	1.00	8	0.375
187	A	2	3	1.00	6	0.500
188	A	2	2	1.00	6	0.333
189	A	4	5	1.00	6	0.833
190	A	2	2	1.00	6	0.333
191	A	10	9	1.00	8	1.125
192	A	7	7	1.00	8	0.875
193	A	8	8	1.00	7	1.143
194	A	3	4	1.00	8	0.500
195	A	5	6	1.00	9	0.667
196	A	7	8	1.00	11	0.727

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	15	8	1.00	11	0.727
198	A	5	6	1.00	9	0.667
199	A	7	8	1.00	11	0.727
200	A	15	8	1.00	11	0.727
201	A	5	5	1.00	5	1.000
202	A	6	6	1.00	7	0.857
203	A	6	6	1.00	7	0.857
204	A	5	5	1.00	5	1.000
205	A	6	6	1.00	7	0.857
206	A	6	6	1.00	7	0.857
207	A	7	5	1.00	3	1.667
208	A	7	5	1.00	5	1.000
209	A	8	6	1.00	7	0.857
210	A	8	6	1.00	7	0.857
211	A	7	5	1.00	3	1.667
212	A	7	5	1.00	5	1.000
213	A	8	6	1.00	7	0.857
214	A	8	6	1.00	7	0.857
215	A	5	5	1.00	5	1.000
216	A	6	6	1.00	7	0.857
217	A	6	6	1.00	7	0.857
218	A	5	5	1.00	5	1.000
219	A	6	6	1.00	7	0.857
220	A	6	6	1.00	7	0.857
221	A	2	2	1.00	35	0.057
222	A	3	3	1.00	8	0.375
223	A	1	1	1.00	8	0.125
224	A	3	3	1.00	10	0.300
225	A	2	1	1.00	16	0.062
226	A	3	3	1.00	9	0.333
227	A	2	2	1.00	10	0.200
228	A	3	2	1.00	10	0.200
229	A	3	2	1.00	12	0.167
230	A	4	3	1.00	8	0.375
231	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
232	A	2	2	1.00	12	0.167
233	A	2	2	1.00	14	0.143
234	A	6	6	1.00	14	0.429
235	A	1	1	1.00	8	0.125
236	A	2	2	1.00	4	0.500
237	A	1	1	1.00	10	0.100
238	A	3	1	1.00	11	0.091
239	A	3	3	1.00	6	0.500
240	A	2	2	1.00	8	0.250
241	A	2	2	1.00	14	0.143
242	A	2	2	1.00	6	0.333
243	A	4	3	1.00	10	0.300
244	A	4	3	1.00	14	0.214
245	A	2	2	1.00	12	0.167
246	A	2	2	1.00	14	0.143
247	A	2	2	1.00	14	0.143
248	A	2	2	1.00	14	0.143
249	A	2	2	1.00	14	0.143
250	A	2	1	1.00	15	0.067
251	A	2	1	1.00	17	0.059
252	A	7	6	1.00	17	0.353
253	A	10	6	1.00	17	0.353
254	A	3	1	1.00	17	0.059
255	A	3	2	1.00	17	0.118
256	A	8	7	1.00	17	0.412
257	A	11	7	1.00	17	0.412
258	A	3	2	1.00	18	0.111
259	A	5	4	1.00	26	0.154
260	A	5	4	1.00	21	0.190
261	A	7	5	1.00	27	0.185
262	A	2	2	1.00	10	0.200
263	A	3	3	1.00	12	0.250
264	A	3	3	1.00	12	0.250
265	A	3	3	1.00	12	0.250
266	A	8	7	1.00	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
267	A	7	7	1.00	15	0.467
268	A	1	1	1.00	8	0.125
269	A	6	6	1.00	19	0.316
270	A	5	4	1.00	19	0.210
271	A	2	1	1.00	14	0.071
272	A	8	7	1.00	24	0.292
273	A	4	3	1.00	8	0.375
274	A	2	2	1.00	9	0.222
275	A	2	2	1.00	10	0.200
276	A	8	6	1.00	22	0.273
277	A	5	6	1.00	18	0.333
278	A	5	6	1.00	20	0.300
279	A	5	6	1.00	25	0.240
280	A	1	1	1.00	39	0.026
281	A	1	1	1.00	39	0.026
282	A	2	2	1.00	8	0.250
283	A	3	3	1.00	10	0.300
284	A	5	5	1.24	12	0.417
285	A	3	4	1.00	10	0.400
286	N/A	0	0	1.00	10	0.000
287	N/A	0	0	1.00	8	0.000
288	N/A	0	0	1.00	6	0.000
289	N/A	0	0	1.00	10	0.000
290	N/A	0	0	1.00	10	0.000
291	A	2	2	1.00	14	0.143
292	A	2	2	1.00	16	0.125
293	A	8	6	1.00	14	0.429
294	A	5	3	1.00	14	0.214
295	A	6	4	1.00	14	0.286
296	A	4	4	1.00	12	0.333
297	A	5	3	1.00	14	0.214
298	A	7	4	1.00	14	0.286
299	A	2	1	1.00	14	0.071
300	A	3	3	1.00	6	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
301	A	4	4	1.00	13	0.308
302	A	4	4	1.00	14	0.286
303	A	4	4	1.00	17	0.235
304	A	4	4	1.00	18	0.222
305	A	13	13	1.00	10	1.300
306	A	12	12	1.00	8	1.500
307	A	7	6	1.00	6	1.000
308	N/A	0	0	1.00	10	0.000
309	N/A	0	0	1.00	10	0.000
310	A	14	12	1.00	13	0.923
311	A	13	11	1.00	11	1.000
312	A	7	6	1.00	9	0.667
313	N/A	0	0	1.00	13	0.000
314	N/A	0	0	1.00	13	0.000

CHAPTER 3

LISTING OF INTEGRALS

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3.102 $\int x^2\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$ 648
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3.119 $\int x^2\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx$ 748
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3.134 $\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx$ . . . 811
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3.153 $\int \frac{1}{x\log(e^x)}\,dx$ . . . 888
3.154 $\int \log(x)\sin(a+bx)\,dx$ . . . 892
3.155 $\int \log(x)\sin^2(a+bx)\,dx$ . . . 897
3.156 $\int \log(x)\sin^3(a+bx)\,dx$ . . . 902
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3.165 $\int \log\left(a\cos^2(x)\right)\,dx$ . . . 948
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3.168 $\int \log\left(a\tan^2(x)\right)\,dx$ . . . 962
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3.174 $\int \log\left(a\sec^2(x)\right)\,dx$ . . . 994
3.175 $\int \log\left(a\sec^n(x)\right)\,dx$ . . . 999
3.176 $\int \log(a\csc(x))\,dx$ . . . 1003
3.177 $\int \log\left(a\csc^2(x)\right)\,dx$ . . . 1008
3.178 $\int \log\left(a\csc^n(x)\right)\,dx$ . . . 1013
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3.180 $\int \frac{\cot(x)}{\log(e\sin(x))}\,dx$ . . . 1022
3.181 $\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx$ . . . 1026
3.182 $\int \log(\cos(x))\sec^2(x)\,dx$ . . . 1030
3.183 $\int \cot(x)\log(\sin(x))\,dx$ . . . 1034
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3.190 $\int \cos(x)\log(\sin(x))\,dx$ . . . 1063
3.191 $\int \log(\sin(x))\sin^2(x)\,dx$ . . . 1067
3.192 $\int \log(\sin(x))\sin^3(x)\,dx$ . . . 1073
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3.194 $\int \csc^2(x)\log(\sin(x))\,dx$ . . . 1085
3.195 $\int \log(x)\sinh(a+bx)\,dx$ . . . 1089
3.196 $\int \log(x)\sinh^2(a+bx)\,dx$ . . . 1093
3.197 $\int \log(x)\sinh^3(a+bx)\,dx$ . . . 1098
3.198 $\int \cosh(a+bx)\log(x)\,dx$ . . . 1104
3.199 $\int \cosh^2(a+bx)\log(x)\,dx$ . . . 1108
3.200 $\int \cosh^3(a+bx)\log(x)\,dx$ . . . 1113

3.201 $\int \log(a\sinh(x))\,dx$ 1119
3.202 $\int \log\left(a\sinh^2(x)\right)\,dx$ 1123
3.203 $\int \log\left(a\sinh^n(x)\right)\,dx$ 1128
3.204 $\int \log(a\cosh(x))\,dx$ 1132
3.205 $\int \log\left(a\cosh^2(x)\right)\,dx$ 1136
3.206 $\int \log\left(a\cosh^n(x)\right)\,dx$ 1141
3.207 $\int \log(\tanh(x))\,dx$ 1145
3.208 $\int \log(a\tanh(x))\,dx$ 1150
3.209 $\int \log\left(a\tanh^2(x)\right)\,dx$ 1155
3.210 $\int \log\left(a\tanh^n(x)\right)\,dx$ 1160
3.211 $\int \log(\coth(x))\,dx$ 1165
3.212 $\int \log(a\coth(x))\,dx$ 1170
3.213 $\int \log\left(a\coth^2(x)\right)\,dx$ 1175
3.214 $\int \log\left(a\coth^n(x)\right)\,dx$ 1180
3.215 $\int \log(a\text{sech}(x))\,dx$ 1185
3.216 $\int \log\left(a\text{sech}^2(x)\right)\,dx$ 1189
3.217 $\int \log\left(a\text{sech}^n(x)\right)\,dx$ 1194
3.218 $\int \log(a\text{csch}(x))\,dx$ 1198
3.219 $\int \log\left(a\text{csch}^2(x)\right)\,dx$ 1202
3.220 $\int \log\left(a\text{csch}^n(x)\right)\,dx$ 1207
3.221 $\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$ 1211
3.222 $\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx$ 1215
3.223 $\int \frac{\log(x)}{\sqrt{x}}\,dx$ 1219
3.224 $\int x\log\left(2-3x^2\right)\,dx$ 1223
3.225 $\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx$ 1227
3.226 $\int 16x^3\log^2(x)\,dx$ 1230
3.227 $\int \log\left(\sqrt{a+bx}\right)\,dx$ 1234
3.228 $\int x\log\left(\sqrt{2+x}\right)\,dx$ 1238
3.229 $\int x\log\left(\sqrt[3]{1+3x}\right)\,dx$ 1242
3.230 $\int x\log\left(x+x^3\right)\,dx$ 1246
3.231 $\int \log\left(x+\sqrt{1+x^2}\right)\,dx$ 1250
3.232 $\int \log\left(x+\sqrt{-1+x^2}\right)\,dx$ 1253
3.233 $\int \log\left(x-\sqrt{-1+x^2}\right)\,dx$ 1256
3.234 $\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx$ 1259
3.235 $\int \sqrt[3]{x}\log(x)\,dx$ 1263
3.236 $\int 2^{\log(x)}\,dx$ 1267
3.237 $\int \frac{1-\log(x)}{x^2}\,dx$ 1271
3.238 $\int \log\left(1+x+\sqrt{1+x}\right)\,dx$ 1274
3.239 $\int \log\left(x+x^3\right)\,dx$ 1278
3.240 $\int 2^{\log(-8+7x)}\,dx$ 1282
3.241 $\int \log\left(\frac{-11+5x}{5+76x}\right)\,dx$ 1286
3.242 $\int \log\left(\frac{1}{13+x}\right)\,dx$ 1290
3.243 $\int x\log\left(\frac{1+x}{x^2}\right)\,dx$ 1294

3.244 $\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx$ 1298
3.245 $\int (a+bx)\log(a+bx)\,dx$ 1302
3.246 $\int (a+bx)^2\log(a+bx)\,dx$ 1306
3.247 $\int \frac{\log(a+bx)}{a+bx}\,dx$ 1310
3.248 $\int \frac{\log(a+bx)}{(a+bx)^2}\,dx$ 1314
3.249 $\int (a+bx)^n\log(a+bx)\,dx$ 1318
3.250 $\int \frac{1}{ax+bx\log(cx^n)}\,dx$ 1322
3.251 $\int \frac{1}{ax+bx\log^2(cx^n)}\,dx$ 1326
3.252 $\int \frac{1}{ax+bx\log^3(cx^n)}\,dx$ 1330
3.253 $\int \frac{1}{ax+bx\log^4(cx^n)}\,dx$ 1337
3.254 $\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx$ 1344
3.255 $\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$ 1348
3.256 $\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$ 1352
3.257 $\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$ 1359
3.258 $\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx$ 1367
3.259 $\int \frac{-1+\log(3x)}{x(1-\log(3x)+\log^2(3x))}\,dx$ 1371
3.260 $\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx$ 1375
3.261 $\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx$ 1379
3.262 $\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx$ 1383
3.263 $\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$ 1387
3.264 $\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$ 1391
3.265 $\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx$ 1395
3.266 $\int \frac{\log(1+\sqrt{x}-x)}{x}\,dx$ 1399
3.267 $\int \frac{x\log(c+dx)}{a+bx}\,dx$ 1404
3.268 $\int \frac{\log(x)}{-1+x}\,dx$ 1409
3.269 $\int \frac{x\log(1-a-bx)}{a+bx}\,dx$ 1412
3.270 $\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx$ 1417
3.271 $\int (\sin(x\log(x))+\log(x)\sin(x\log(x)))\,dx$ 1422
3.272 $\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx$ 1425
3.273 $\int \log\left(\sqrt{x}+x\right)\,dx$ 1430
3.274 $\int \log\left(-\frac{x}{1+x}\right)\,dx$ 1434
3.275 $\int \log\left(\frac{-1+x}{1+x}\right)\,dx$ 1438
3.276 $\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx$ 1442
3.277 $\int \frac{\log\left(c\left(1+x^2\right)^n\right)}{1+x^2}\,dx$ 1447

3.278 $\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx$ 1452
3.279 $\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$ 1457
3.280 $\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$ 1462
3.281 $\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$ 1466
3.282 $\int \log\left(e^{a+bx}\right)\,dx$ 1470
3.283 $\int \log\left(e^{a+bx^n}\right)\,dx$ 1474
3.284 $\int e^x\log\left(a+be^x\right)\,dx$ 1478
3.285 $\int e^{a+bx}\log(x)\,dx$ 1482
3.286 $\int \frac{x^2}{x+\log(x)}\,dx$ 1486
3.287 $\int \frac{x}{x+\log(x)}\,dx$ 1489
3.288 $\int \frac{1}{x+\log(x)}\,dx$ 1492
3.289 $\int \frac{1}{x(x+\log(x))}\,dx$ 1495
3.290 $\int \frac{1}{x^2(x+\log(x))}\,dx$ 1498
3.291 $\int \frac{\log(x)}{x+4x\log^2(x)}\,dx$ 1501
3.292 $\int \frac{1-\log(x)}{x(x+\log(x))}\,dx$ 1505
3.293 $\int \frac{1+x}{\log(x)(x+\log(x))}\,dx$ 1509
3.294 $\int \log\left(2+\sqrt{\frac{1+x}{x}}\right)\,dx$ 1513
3.295 $\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)\,dx$ 1518
3.296 $\int \log\left(\sqrt{\frac{1+x}{x}}\right)\,dx$ 1523
3.297 $\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)\,dx$ 1527
3.298 $\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx$ 1532
3.299 $\int \left(x^{ax}+x^{ax}\log(x)\right)\,dx$ 1537
3.300 $\int \log^m(x)^p\,dx$ 1540
3.301 $\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx$ 1544
3.302 $\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx$ 1548
3.303 $\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx$ 1552
3.304 $\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx$ 1556
3.305 $\int x^2\log(\log(x)\sin(x))\,dx$ 1560
3.306 $\int x\log(\log(x)\sin(x))\,dx$ 1568
3.307 $\int \log(\log(x)\sin(x))\,dx$ 1575
3.308 $\int \frac{\log(\log(x)\sin(x))}{x}\,dx$ 1580
3.309 $\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$ 1583
3.310 $\int x^2\log\left(e^x\log(x)\sin(x)\right)\,dx$ 1587
3.311 $\int x\log\left(e^x\log(x)\sin(x)\right)\,dx$ 1595
3.312 $\int \log\left(e^x\log(x)\sin(x)\right)\,dx$ 1602

3.313 $\int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx$ 1607
3.314 $\int \frac{\log(e^x \log(x) \sin(x))}{x^2} \, dx$ 1610

3.1 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^p}{x} \, dx$

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Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^p}{x}\,dx = \frac{(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q} - \frac{am\text{Int}(x^{-1+m}(ax^m+b\log^q(cx^n))^p,x)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)*(a*x^m+b*ln(c*x^n)^q)^p,x)/b/n/q+(a*x^m+b*ln(
c*x^n)^q)^(p+1)/b/n/(p+1)/q
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^p}{x}\,dx = \int \frac{\log^{-1+q}(cx^n)\,(ax^m+b\log^q(cx^n))^p}{x}\,dx$$

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(b*n*(1 + p)*q) - (a*m*Defer[Int][x^(-1 +
m)*(a*x^m + b*Log[c*x^n]^q)^p, x])/(b*n*q)
```

Rubi steps

$$\text{integral} = \frac{(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q} - \frac{(am)\int x^{-1+m}(ax^m+b\log^q(cx^n))^p\,dx}{bnq}$$

Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
[Out] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x, x]
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\,x^{n}\right)^{-1+q}\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{p}}{x}dx$$

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\int \frac{(ax^m+b\log(cx^n)^q)^p\log(cx^n)^{q-1}}{x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="fricas
")
[Out] integral((a*x^m + b*log(c*x^n)^q)^p*log(c*x^n)^(q - 1)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m + b\log^q(cx^n))^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m + b\log^q(cx^n))^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m + b\log^q(cx^n))^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,2,5,2,0,5,0,2,1,2,2]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,1,1,2,2]%%%}+%%%{5,[0,0,2
,4,2,0,4,
```

Mupad [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^p}{x}\,dx=\int \frac{\ln(c\,x^n)^{q-1}\,(a\,x^m+b\ln(c\,x^n)^q)^p}{x}\,dx$$

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^p)/x,x)
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^p)/x, x)
```

3.2 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^3}{x}\,dx$

Optimal result . . . 115
Rubi [A] (verified) . . . 115
Mathematica [A] (verified) . . . 118
Maple [F] . . . 118
Fricas [F] . . . 118
Sympy [F(-1)] . . . 119
Maxima [F(-2)] . . . 119
Giac [F] . . . 119
Mupad [F(-1)] . . . 119

Optimal result

Integrand size = 32, antiderivative size = 231

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx = \frac{b^3\log^{4q}(cx^n)}{4nq} - \frac{3ab^2x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}}{n} - \frac{3\,4^{-q}a^2bx^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(2q,-\frac{2m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{n} - \frac{3^{-q}a^3x^{3m}(cx^n)^{-\frac{3m}{n}}\Gamma\left(q,-\frac{3m\log(cx^n)}{n}\right)\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}$$

```
[Out] 1/4*b^3*ln(c*x^n)^(4*q)/n/q-3*a*b^2*x^m*GAMMA(3*q,-m*ln(c*x^n)/n)*ln(c*x^n)
^(3*q)/n/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(3*q))-3*a^2*b*x^(2*m)*GAMMA(2*q
,-2*m*ln(c*x^n)/n)*ln(c*x^n)^(2*q)/(4^q)/n/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)
/n)^(2*q))-a^3*x^(3*m)*GAMMA(q,-3*m*ln(c*x^n)/n)*ln(c*x^n)^q/(3^q)/n/((c*x^
n)^(3*m/n))/((-m*ln(c*x^n)/n)^q)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used

= {2619, 2347, 2212, 2339, 30}

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$$
$$= -\frac{a^3 3^{-q} x^{3m} (cx^n)^{-\frac{3m}{n}} \log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q,-\frac{3m\log(cx^n)}{n}\right)}{n}$$
$$-\frac{3a^2 b 4^{-q} x^{2m} (cx^n)^{-\frac{2m}{n}} \log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\Gamma\left(2q,-\frac{2m\log(cx^n)}{n}\right)}{n}$$
$$-\frac{3ab^2 x^m (cx^n)^{-\frac{m}{n}} \log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)}{n}+\frac{b^3\log^{4q}(cx^n)}{4nq}$$

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (b^3*Log[c*x^n]^(4*q))/(4*n*q) - (3*a*b^2*x^m*Gamma[3*q, -((m*Log[c*x^n])/n
)]*Log[c*x^n]^(3*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q)) - (3*a^2
*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(4^q*n*(c*x^n)
^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^3*x^(3*m)*Gamma[q, (-3*m*Log[c
*x^n])/n]*Log[c*x^n]^q)/(3^q*n*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^3x^{-1+3m}\log^{-1+q}(cx^n) + 3a^2bx^{-1+2m}\log^{-1+2q}(cx^n) + 3ab^2x^{-1+m}\log^{-1+3q}(cx^n) + \frac{b^3\log^{-1+4q}(cx^n)}{x}\right)dx$$

$$= a^3\int x^{-1+3m}\log^{-1+q}(cx^n)\,dx + \left(3a^2b\right)\int x^{-1+2m}\log^{-1+2q}(cx^n)\,dx + \left(3ab^2\right)\int x^{-1+m}\log^{-1+3q}(cx^n)\,dx + b^3\int \frac{\log^{-1+4q}(cx^n)}{x}\,dx$$

$$= \frac{b^3\text{Subst}\left(\int x^{-1+4q}\,dx, x, \log(cx^n)\right)}{n} + \frac{\left(a^3x^{3m}(cx^n)^{-\frac{3m}{n}}\right)\text{Subst}\left(\int e^{\frac{3mx}{n}}x^{-1+q}\,dx, x, \log(cx^n)\right)}{n} + \frac{\left(3a^2bx^{2m}(cx^n)^{-\frac{2m}{n}}\right)\text{Subst}\left(\int e^{\frac{2mx}{n}}x^{-1+2q}\,dx, x, \log(cx^n)\right)}{n} + \frac{\left(3ab^2x^m(cx^n)^{-\frac{m}{n}}\right)\text{Subst}\left(\int e^{\frac{mx}{n}}x^{-1+3q}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{b^3\log^{4q}(cx^n)}{4nq} - \frac{3ab^2x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(3q, -\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}}{n} - \frac{3\ 4^{-q}a^2bx^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(2q, -\frac{2m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{n} - \frac{3^{-q}a^3x^{3m}(cx^n)^{-\frac{3m}{n}}\Gamma\left(q, -\frac{3m\log(cx^n)}{n}\right)\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.97

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m + b\log^q(cx^n))^3}{x}\,dx$$
$$= \frac{\log^q(cx^n)\left(\frac{b^3\log^{3q}(cx^n)}{q} - 12ab^2x^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q} - 3\,4^{1-q}a^2bx^{2m}(c}{}$$

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (Log[c*x^n]^q*((b^3*Log[c*x^n]^(3*q))/q - (12*a*b^2*x^m*Gamma[3*q, -((m*Log
[c*x^n])/n)]*Log[c*x^n]^(2*q))/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q))
- (3*4^(1 - q)*a^2*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/
((c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (4*a^3*x^(3*m)*Gamma[q, (
-3*m*Log[c*x^n])/n])/(3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)))/(4*n
)
```

Maple [F]

$$\int \frac{\ln(c\,x^n)^{-1+q}\,(a\,x^m + b\ln(c\,x^n)^q)^3}{x}dx$$

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

Fricas [F]

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m + b\log^q(cx^n))^3}{x}\,dx = \int \frac{(ax^m + b\log(cx^n)^q)^3\log(cx^n)^{q-1}}{x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="fricas
")
```

```
[Out] integral((3*a*b^2*x^m*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1) + 3*a^2*b*x^(2*m)
*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^3*x^(3*m)*log(c*x^n)^(q - 1) + b^3*log
(c*x^n)^(3*q)*log(c*x^n)^(q - 1))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx=\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)^3\log\left(cx^n\right)^{q-1}}{x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^3*log(c*x^n)^(q - 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx=\int \frac{\ln\left(c\,x^n\right)^{q-1}\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^3}{x}\,dx$$

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^3)/x,x)
```

```
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^3)/x, x)
```

3.3 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^2}{x}\,dx$

Optimal result 120
Rubi [A] (verified) 120
Mathematica [A] (verified) 122
Maple [F] 122
Fricas [F] 123
Sympy [F] 123
Maxima [F(-2)] 123
Giac [F] 123
Mupad [F(-1)] 124

Optimal result

Integrand size = 32, antiderivative size = 156

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx = \frac{b^2\log^{3q}(cx^n)}{3nq} - \frac{2abx^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{n} - \frac{2^{-q}a^2x^{2m}(cx^n)^{-\frac{2m}{n}}\,\Gamma\left(q,-\frac{2m\log(cx^n)}{n}\right)\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}$$

```
[Out] 1/3*b^2*ln(c*x^n)^(3*q)/n/q-2*a*b*x^m*GAMMA(2*q,-m*ln(c*x^n)/n)*ln(c*x^n)^(
2*q)/n/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(2*q))-a^2*x^(2*m)*GAMMA(q,-2*m*ln
(c*x^n)/n)*ln(c*x^n)^q/(2^q)/n/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^q)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2619, 2347, 2212, 2339, 30}

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx = -\frac{a^2 2^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q,-\frac{2m\log(cx^n)}{n}\right)}{n} - \frac{2abx^m(cx^n)^{-\frac{m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)}{n} + \frac{b^2\log^{3q}(cx^n)}{3nq}$$

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (b^2*Log[c*x^n]^(3*q))/(3*n*q) - (2*a*b*x^m*Gamma[2*q, -((m*Log[c*x^n])/n)]
*Log[c*x^n]^(2*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*x^(
2*m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*n*(c*x^n)^((2*m)/n)*(
-((m*Log[c*x^n])/n))^q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2 x^{-1+2m} \log^{-1+q}(cx^n) + 2abx^{-1+m} \log^{-1+2q}(cx^n) + \frac{b^2 \log^{-1+3q}(cx^n)}{x}\right) dx$$

$$= a^2 \int x^{-1+2m} \log^{-1+q}(cx^n)\, dx + (2ab) \int x^{-1+m} \log^{-1+2q}(cx^n)\, dx + b^2 \int \frac{\log^{-1+3q}(cx^n)}{x}\, dx$$

$$
\begin{aligned}
&= \frac{b^2 \text{Subst}\left(\int x^{-1+3q}\,dx, x, \log(cx^n)\right)}{n} \\
&+ \frac{\left(a^2 x^{2m} (cx^n)^{-\frac{2m}{n}}\right) \text{Subst}\left(\int e^{\frac{2mx}{n}} x^{-1+q}\,dx, x, \log(cx^n)\right)}{n} \\
&+ \frac{\left(2abx^m (cx^n)^{-\frac{m}{n}}\right) \text{Subst}\left(\int e^{\frac{mx}{n}} x^{-1+2q}\,dx, x, \log(cx^n)\right)}{n} \\
&= \frac{b^2 \log^{3q}(cx^n)}{3nq} - \frac{2abx^m (cx^n)^{-\frac{m}{n}} \Gamma\left(2q, -\frac{m\log(cx^n)}{n}\right) \log^{2q}(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{n} \\
&- \frac{2^{-q} a^2 x^{2m} (cx^n)^{-\frac{2m}{n}} \Gamma\left(q, -\frac{2m\log(cx^n)}{n}\right) \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$
\int \frac{\log^{-1+q}(cx^n)\left(ax^m + b\log^q(cx^n)\right)^2}{x}\,dx
$$

$$
= \frac{\log^q(cx^n)\left(\frac{b^2\log^{2q}(cx^n)}{q} - 6abx^m(cx^n)^{-\frac{m}{n}}\Gamma\left(2q, -\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q} - 3\ 2^{-q}a^2x^{2m}(cx^n)^{-}\right.}{3n}
$$

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (Log[c*x^n]^q*((b^2*Log[c*x^n]^(2*q))/q - (6*a*b*x^m*Gamma[2*q, -((m*Log[c*
x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (3*a^
2*x^(2*m)*Gamma[q, (-2*m*Log[c*x^n])/n])/(2^q*(c*x^n)^((2*m)/n)*(-((m*Log[c
*x^n])/n))^q)))/(3*n)
```

Maple [F]

$$
\int \frac{\ln(c\,x^n)^{-1+q}\left(a\,x^m + b\ln(c\,x^n)^q\right)^2}{x}dx
$$

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

Fricas [F]

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m + b\log^q(cx^n))^2}{x}\,dx = \int \frac{(ax^m + b\log(cx^n)^q)^2 \log(cx^n)^{q-1}}{x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="fricas
")
```

```
[Out] integral((2*a*b*x^m*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^2*x^(2*m)*log(c*x^n
)^(q - 1) + b^2*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1))/x, x)
```

Sympy [F]

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m + b\log^q(cx^n))^2}{x}\,dx = \int \frac{(ax^m + b\log(cx^n)^q)^2 \log(cx^n)^{q-1}}{x}\,dx$$

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
```

```
[Out] Integral((a*x**m + b*log(c*x**n)**q)**2*log(c*x**n)**(q - 1)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m + b\log^q(cx^n))^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{\log^{-1+q}(cx^n)\,(ax^m + b\log^q(cx^n))^2}{x}\,dx = \int \frac{(ax^m + b\log(cx^n)^q)^2 \log(cx^n)^{q-1}}{x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^2*log(c*x^n)^(q - 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx=\int \frac{\ln\left(c\,x^n\right)^{q-1}\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^2}{x}\,dx$$

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^2)/x,x)
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^2)/x, x)
```

3.4 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))}{x}\,dx$

Optimal result 125
Rubi [A] (verified) 125
Mathematica [A] (verified) 127
Maple [F] 127
Fricas [F] 127
Sympy [F] 127
Maxima [F(-2)] 128
Giac [F] 128
Mupad [F(-1)] 128

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx = \frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}$$

```
[Out] 1/2*b*ln(c*x^n)^(2*q)/n/q-a*x^m*GAMMA(q,-m*ln(c*x^n)/n)*ln(c*x^n)^q/n/((c*x
^n)^(m/n))/((-m*ln(c*x^n)/n)^q)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2619, 2347, 2212, 2339, 30}

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx = \frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)}{n}$$

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (b*Log[c*x^n]^(2*q))/(2*n*q) - (a*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x
^n]^q)/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^{-1+m}\log^{-1+q}(cx^n)+\frac{b\log^{-1+2q}(cx^n)}{x}\right)dx\\
&= a\int x^{-1+m}\log^{-1+q}(cx^n)\,dx+b\int \frac{\log^{-1+2q}(cx^n)}{x}\,dx\\
&= \frac{b\text{Subst}\left(\int x^{-1+2q}\,dx,x,\log(cx^n)\right)}{n}+\frac{\left(ax^m(cx^n)^{-\frac{m}{n}}\right)\text{Subst}\left(\int e^{\frac{mx}{n}}x^{-1+q}\,dx,x,\log(cx^n)\right)}{n}\\
&= \frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))}{x}\,dx = \frac{\log^q(cx^n)\left(\frac{b\log^q(cx^n)}{q}-2ax^m(cx^n)^{-\frac{m}{n}}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\right)}{2n}$$

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
[Out] (Log[c*x^n]^q*((b*Log[c*x^n]^q)/q - (2*a*x^m*Gamma[q, -((m*Log[c*x^n])/n)])
/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)))/(2*n)
```

Maple [F]

$$\int \frac{\ln(c\,x^n)^{-1+q}(a\,x^m+b\ln(c\,x^n)^q)}{x}dx$$

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)/x,x)
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

Fricas [F]

$$\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))}{x}\,dx = \int \frac{(ax^m+b\log(cx^n)^q)\log(cx^n)^{q-1}}{x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="fricas")
[Out] integral((a*x^m*log(c*x^n)^(q - 1) + b*log(c*x^n)^(q - 1)*log(c*x^n)^q)/x,
x)
```

Sympy [F]

$$\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))}{x}\,dx = \int \frac{(ax^m+b\log(cx^n)^q)\log(cx^n)^{q-1}}{x}\,dx$$

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)/x,x)
[Out] Integral((a*x**m + b*log(c*x**n)**q)*log(c*x**n)**(q - 1)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^{-1+q}(cx^n)(ax^m + b\log^q(cx^n))}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{\log^{-1+q}(cx^n)(ax^m + b\log^q(cx^n))}{x}\,dx = \int \frac{(ax^m + b\log(cx^n)^q)\log(cx^n)^{q-1}}{x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)*log(c*x^n)^(q - 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)(ax^m + b\log^q(cx^n))}{x}\,dx = \int \frac{\ln(c\,x^n)^{q-1}\,(a\,x^m + b\ln(c\,x^n)^q)}{x}\,dx$$

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q))/x,x)
```

```
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q))/x, x)
```

3.5 $\int \frac{\log^{-1+q}(cx^n)}{x}\,dx$

Optimal result 129
Rubi [A] (verified) 129
Mathematica [A] (verified) 130
Maple [A] (verified) 130
Fricas [A] (verification not implemented) 130
Sympy [A] (verification not implemented) 131
Maxima [A] (verification not implemented) 131
Giac [A] (verification not implemented) 131
Mupad [B] (verification not implemented) 132

Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{\log^q(cx^n)}{nq}$$

```
[Out] ln(c*x^n)^q/n/q
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2339, 30}

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{\log^q(cx^n)}{nq}$$

```
[In] Int[Log[c*x^n]^(-1 + q)/x,x]
```

```
[Out] Log[c*x^n]^q/(n*q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int x^{-1+q}\,dx, x, \log\left(cx^n\right)\right)}{n} \\ &= \frac{\log^q\left(cx^n\right)}{nq} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x}\,dx = \frac{\log^q\left(cx^n\right)}{nq}$$

```
[In] Integrate[Log[c*x^n]^(-1 + q)/x,x]
[Out] Log[c*x^n]^q/(n*q)
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\ln(c\,x^n)^q}{nq}$
default	$\frac{\ln(c\,x^n)^q}{nq}$
risch	$\frac{\left(\ln(c)+\ln(x^n)-\frac{i\pi\ \mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ix^n))}{2}\right)^{-1+q}\left(\ln(c)+\ln(x^n)-\frac{i\pi\ \mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(\right.}{nq}$

```
[In] int(ln(c*x^n)^(-1+q)/x,x,method=_RETURNVERBOSE)
[Out] ln(c*x^n)^q/n/q
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x}\,dx = \frac{(n\log(x)+\log(c))(n\log(x)+\log(c))^{q-1}}{nq}$$

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="fricas")
[Out] (n*log(x) + log(c))*(n*log(x) + log(c))^(q - 1)/(n*q)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = -\begin{cases} -\log(c)^{q-1}\log(x) & \text{for } n=0 \\ -\frac{\begin{cases} \frac{\log(cx^n)^q}{q} & \text{for } q\neq 0 \\ \log(\log(cx^n)) & \text{otherwise} \end{cases}}{n} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*x**n)**(-1+q)/x,x)
```

```
[Out] -Piecewise((-log(c)**(q - 1)*log(x), Eq(n, 0)), (-Piecewise((log(c*x**n)**q
/q, Ne(q, 0)), (log(log(c*x**n)), True))/n, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{\log(cx^n)^q}{nq}$$

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="maxima")
```

```
[Out] log(c*x^n)^q/(n*q)
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{(n\log(x)+\log(c))^q}{nq}$$

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="giac")
```

```
[Out] (n*log(x) + log(c))^q/(n*q)
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log^{-1+q}(cx^n)}{x}\,dx = \frac{\ln(c\,x^n)^q}{n\,q}$$

```
[In] int(log(c*x^n)^(q - 1)/x,x)
[Out] log(c*x^n)^q/(n*q)
```

3.6 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$

Optimal result 133
Rubi [N/A] 133
Mathematica [N/A] 134
Maple [N/A] 134
Fricas [N/A] 134
Sympy [F(-1)] 134
Maxima [N/A] 135
Giac [N/A] 135
Mupad [N/A] 135

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx = \frac{\log(ax^m+b\log^q(cx^n))}{bnq} - \frac{am\mathrm{Int}\left(\frac{x^{-1+m}}{ax^m+b\log^q(cx^n)},x\right)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q),x)/b/n/q+ln(a*x^m+b*ln(
c*x^n)^q)/b/n/q
```

Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx = \int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$$

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^q), x])/(b*n*q)
```

Rubi steps

$$\text{integral} = \frac{\log(ax^m+b\log^q(cx^n))}{bnq} - \frac{(am)\int \frac{x^{-1+m}}{ax^m+b\log^q(cx^n)}\,dx}{bnq}$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx$$

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\ln(c\,x^n)^{-1+q}}{x\,(a\,x^m + b\ln(c\,x^n)^q)}dx$$

```
[In] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q),x)
[Out] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{\log(cx^n)^{q-1}}{(ax^m + b\log(cx^n)^q)x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="fricas")
[Out] integral(log(c*x^n)^(q - 1)/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{\log(cx^n)^{q-1}}{(ax^m + b\log(cx^n)^q)x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="maxima")
[Out] -a*integrate(x^m/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) + b
^2*x*log(x^n))*(log(c) + log(x^n))^q), x) + log(log(c) + log(x^n))/(b*n)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{\log(cx^n)^{q-1}}{(ax^m + b\log(cx^n)^q)x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="giac")
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{\ln(c\,x^n)^{q-1}}{x\,\left(a\,x^m + b\ln(c\,x^n)^q\right)}\,dx$$

```
[In] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)),x)
[Out] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

3.7 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2} \, dx$

Optimal result 136
Rubi [N/A] 136
Mathematica [N/A] 137
Maple [N/A] 137
Fricas [N/A] 137
Sympy [F(-1)] 138
Maxima [N/A] 138
Giac [N/A] 138
Mupad [N/A] 139

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2} \, dx = -\frac{1}{bnq(ax^m+b\log^q(cx^n))} - \frac{am\text{Int}\left(\frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^2}, x\right)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^2,x)/b/n/q-1/b/n/q/(a*x
^m+b*ln(c*x^n)^q)
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2} \, dx = \int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2} \, dx$$

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(1/(b*n*q*(a*x^m + b*Log[c*x^n]^q))) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m +
 b*Log[c*x^n]^q)^2, x])/(b*n*q)
```

Rubi steps

$$\text{integral} = -\frac{1}{bnq(ax^m+b\log^q(cx^n))} - \frac{(am)\int \frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^2} \, dx}{bnq}$$

Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$$

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2), x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\,x^n\right)^{-1+q}}{x\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^2}dx$$

```
[In] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
[Out] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=\int \frac{\log\left(cx^n\right)^{q-1}}{\left(ax^m+b\log\left(cx^n\right)^q\right)^2x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="fricas
")
[Out] integral(log(c*x^n)^(q - 1)/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*x^(2*m) + b^2
*x*log(c*x^n)^(2*q)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))^2} \, dx = \text{Timed out}$$

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 240, normalized size of antiderivative = 7.50

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))^2} \, dx = \int \frac{\log(cx^n)^{q-1}}{(ax^m + b\log(cx^n)^q)^2 x} \, dx$$

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="maxima
")
```

```
[Out] 1/(a*b*m*x^m*log(x^n) - (n*q - m*log(c))*a*b*x^m + (b^2*m*log(x^n) - (n*q -
 m*log(c))*b^2)*(log(c) + log(x^n))^q) + integrate(-(m*n*(q - 1) - m^2*log(
c) - m^2*log(x^n))/(a*b*m^2*x*x^m*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*a*b*x
*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a*b*x*x^m + (b^2*
m^2*x*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*b^2*x*log(x^n) + (n^2*q^2 - 2*m*n
*q*log(c) + m^2*log(c)^2)*b^2*x)*(log(c) + log(x^n))^q), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))^2} \, dx = \int \frac{\log(cx^n)^{q-1}}{(ax^m + b\log(cx^n)^q)^2 x} \, dx$$

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=\int \frac{\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^2}\,dx$$

```
[In] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
[Out] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

3.8 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$

Optimal result 140
Rubi [N/A] 140
Mathematica [N/A] 141
Maple [N/A] 141
Fricas [N/A] 141
Sympy [F(-1)] 142
Maxima [N/A] 142
Giac [N/A] 143
Mupad [N/A] 143

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=-\frac{1}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2}-\frac{am\mathrm{Int}\left(\frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^3},x\right)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^3,x)/b/n/q-1/2/b/n/q/(a
*x^m+b*ln(c*x^n)^q)^2
```

Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$$

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -1/2*1/(b*n*q*(a*x^m + b*Log[c*x^n]^q)^2) - (a*m*Defer[Int][x^(-1 + m)/(a*x
^m + b*Log[c*x^n]^q)^3, x])/(b*n*q)
```

Rubi steps

$$\text{integral}=-\frac{1}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2}-\frac{(am)\int \frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^3}\,dx}{bnq}$$

Mathematica [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$$

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3), x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\,x^n\right)^{-1+q}}{x\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^3}dx$$

```
[In] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
[Out] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\int \frac{\log\left(cx^n\right)^{q-1}}{\left(ax^m+b\log\left(cx^n\right)^q\right)^3x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="fricas
")
[Out] integral(log(c*x^n)^(q - 1)/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) + 3*a^2*b*x*x^(
2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 1170, normalized size of antiderivative = 36.56

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx=\int \frac{\log(cx^n)^{q-1}}{\left(ax^m+b\log(cx^n)^q\right)^3x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(a*m^2*x^m*log(x^n)^2 + (2*m^2*log(c) + m*n)*a*x^m*log(x^n) - (n^2*q^2
 - m^2*log(c)^2 - m*n*log(c))*a*x^m + (2*b*m^2*log(x^n)^2 - (m*n*(2*q - 1)
- 4*m^2*log(c))*b*log(x^n) - (m*n*(2*q - 1)*log(c) - 2*m^2*log(c)^2)*b)*(lo
g(c) + log(x^n))^q)/(a^3*b*m^3*x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c)
)*a^3*b*x^(3*m)*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2
)*a^3*b*x^(3*m)*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)
^2 - m^3*log(c)^3)*a^3*b*x^(3*m) + (a*b^3*m^3*x^m*log(x^n)^3 - 3*(m^2*n*q -
 m^3*log(c))*a*b^3*x^m*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*l
og(c)^2)*a*b^3*x^m*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log
(c)^2 - m^3*log(c)^3)*a*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a^2*b^2*m^3
*x^(2*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^2*b^2*x^(2*m)*log(x^n)^2 +
 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^2*b^2*x^(2*m)*log(x^n) -
 (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^2*b^2
*x^(2*m))*(log(c) + log(x^n))^q) - integrate(-1/2*(m^3*n*(2*q - 3)*log(c)^2
 - 2*m^4*log(c)^3 - 2*m^4*log(x^n)^3 + 2*(q^2 - 1)*m^2*n^2*log(c) - (2*q^3
- 3*q^2 + q)*m*n^3 + (m^3*n*(2*q - 3) - 6*m^4*log(c))*log(x^n)^2 + 2*(m^3*n
*(2*q - 3)*log(c) - 3*m^4*log(c)^2 + (q^2 - 1)*m^2*n^2)*log(x^n))/(a^2*b*m^
4*x*x^(2*m)*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^2*b*x*x^(2*m)*log(x^n)^
3 + 6*(m^2*n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^2*b*x*x^(2*m)*log(x
^n)^2 - 4*(m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(
c)^3)*a^2*b*x*x^(2*m)*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*
q^2*log(c)^2 - 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^2*b*x*x^(2*m) + (a*b^2*
m^4*x*x^m*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a*b^2*x*x^m*log(x^n)^3 + 6*
```

```
(m^2*n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a*b^2*x*x^m*log(x^n)^2 - 4*
(m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a*b^
2*x*x^m*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^2 -
 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a*b^2*x*x^m)*(log(c) + log(x^n))^q), x)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx=\int \frac{\log(cx^n)^{q-1}}{(ax^m+b\log(cx^n)^q)^3x}\,dx$$

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="giac")
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx=\int \frac{\ln(c\,x^n)^{q-1}}{x\,(a\,x^m+b\ln(c\,x^n)^q)^3}\,dx$$

```
[In] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
[Out] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

3.9 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx$

Optimal result 144
Rubi [A] (verified) 145
Mathematica [A] (verified) 147
Maple [A] (verified) 148
Fricas [B] (verification not implemented) 148
Sympy [A] (verification not implemented) 149
Maxima [B] (verification not implemented) 150
Giac [B] (verification not implemented) 151
Mupad [F(-1)] 153

Optimal result

Integrand size = 28, antiderivative size = 272

$$\begin{aligned}\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx = {} & -\frac{360ab^2n^5x^m}{m^6}-\frac{9a^2bn^3x^{2m}}{8m^4}-\frac{a^3nx^{3m}}{9m^2} \\ & +\frac{360ab^2n^4x^m\log(cx^n)}{m^5}+\frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3} \\ & +\frac{a^3x^{3m}\log(cx^n)}{3m}-\frac{180ab^2n^3x^m\log^2(cx^n)}{m^4} \\ & -\frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2}+\frac{60ab^2n^2x^m\log^3(cx^n)}{m^3} \\ & +\frac{3a^2bx^{2m}\log^3(cx^n)}{2m}-\frac{15ab^2nx^m\log^4(cx^n)}{m^2} \\ & +\frac{3ab^2x^m\log^5(cx^n)}{m}+\frac{b^3\log^8(cx^n)}{8n}\end{aligned}$$

```
[Out] -360*a*b^2*n^5*x^m/m^6-9/8*a^2*b*n^3*x^(2*m)/m^4-1/9*a^3*n*x^(3*m)/m^2+360*
a*b^2*n^4*x^m*ln(c*x^n)/m^5+9/4*a^2*b*n^2*x^(2*m)*ln(c*x^n)/m^3+1/3*a^3*x^(
3*m)*ln(c*x^n)/m-180*a*b^2*n^3*x^m*ln(c*x^n)^2/m^4-9/4*a^2*b*n*x^(2*m)*ln(c
*x^n)^2/m^2+60*a*b^2*n^2*x^m*ln(c*x^n)^3/m^3+3/2*a^2*b*x^(2*m)*ln(c*x^n)^3/
m-15*a*b^2*n*x^m*ln(c*x^n)^4/m^2+3*a*b^2*x^m*ln(c*x^n)^5/m+1/8*b^3*ln(c*x^n
)^8/n
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2619, 2341, 2342, 2339, 30}

$$\int \frac{\log (cx^n) \left(ax^m + b\log^2(cx^n)\right)^3}{x} dx = \frac{a^3x^{3m}\log (cx^n)}{3m} - \frac{a^3nx^{3m}}{9m^2} + \frac{9a^2bn^2x^{2m}\log (cx^n)}{4m^3} - \frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2} + \frac{3a^2bx^{2m}\log^3(cx^n)}{2m} - \frac{9a^2bn^3x^{2m}}{8m^4} + \frac{360ab^2n^4x^m\log (cx^n)}{m^5} - \frac{180ab^2n^3x^m\log^2(cx^n)}{m^4} + \frac{60ab^2n^2x^m\log^3(cx^n)}{m^3} - \frac{15ab^2nx^m\log^4(cx^n)}{m^2} + \frac{3ab^2x^m\log^5(cx^n)}{m} - \frac{360ab^2n^5x^m}{m^6} + \frac{b^3\log^8(cx^n)}{8n}$$

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```

```
[Out] (-360*a*b^2*n^5*x^m)/m^6 - (9*a^2*b*n^3*x^(2*m))/(8*m^4) - (a^3*n*x^(3*m))/
(9*m^2) + (360*a*b^2*n^4*x^m*Log[c*x^n])/m^5 + (9*a^2*b*n^2*x^(2*m)*Log[c*x
^n])/(4*m^3) + (a^3*x^(3*m)*Log[c*x^n])/(3*m) - (180*a*b^2*n^3*x^m*Log[c*x^
n]^2)/m^4 - (9*a^2*b*n*x^(2*m)*Log[c*x^n]^2)/(4*m^2) + (60*a*b^2*n^2*x^m*Lo
g[c*x^n]^3)/m^3 + (3*a^2*b*x^(2*m)*Log[c*x^n]^3)/(2*m) - (15*a*b^2*n*x^m*Lo
g[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(8*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^3x^{-1+3m}\log(cx^n) + 3a^2bx^{-1+2m}\log^3(cx^n) + 3ab^2x^{-1+m}\log^5(cx^n) + \frac{b^3\log^7(cx^n)}{x}\right)dx$$

$$= a^3\int x^{-1+3m}\log(cx^n)\,dx + (3a^2b)\int x^{-1+2m}\log^3(cx^n)\,dx + (3ab^2)\int x^{-1+m}\log^5(cx^n)\,dx + b^3\int \frac{\log^7(cx^n)}{x}\,dx$$

$$= -\frac{a^3nx^{3m}}{9m^2} + \frac{a^3x^{3m}\log(cx^n)}{3m} + \frac{3a^2bx^{2m}\log^3(cx^n)}{2m} + \frac{3ab^2x^m\log^5(cx^n)}{m} + \frac{b^3\text{Subst}\left(\int x^7\,dx, x, \log(cx^n)\right)}{n} - \frac{(9a^2bn)\int x^{-1+2m}\log^2(cx^n)\,dx}{2m} - \frac{(15ab^2n)\int x^{-1+m}\log^4(cx^n)\,dx}{m}$$

$$= -\frac{a^3nx^{3m}}{9m^2} + \frac{a^3x^{3m}\log(cx^n)}{3m} - \frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2} + \frac{3a^2bx^{2m}\log^3(cx^n)}{2m} - \frac{15ab^2nx^m\log^4(cx^n)}{m^2} + \frac{3ab^2x^m\log^5(cx^n)}{m} + \frac{b^3\log^8(cx^n)}{8n} + \frac{(9a^2bn^2)\int x^{-1+2m}\log(cx^n)\,dx}{2m^2} + \frac{(60ab^2n^2)\int x^{-1+m}\log^3(cx^n)\,dx}{m^2}$$

$$= -\frac{9a^2bn^3x^{2m}}{8m^4} - \frac{a^3nx^{3m}}{9m^2} + \frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3} + \frac{a^3x^{3m}\log(cx^n)}{3m} - \frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2} + \frac{60ab^2n^2x^m\log^3(cx^n)}{m^3} + \frac{3a^2bx^{2m}\log^3(cx^n)}{2m} - \frac{15ab^2nx^m\log^4(cx^n)}{m^2} + \frac{3ab^2x^m\log^5(cx^n)}{m} + \frac{b^3\log^8(cx^n)}{8n} - \frac{(180ab^2n^3)\int x^{-1+m}\log^2(cx^n)\,dx}{m^3}$$

$$
\begin{aligned}
&= -\frac{9a^2bn^3x^{2m}}{8m^4} - \frac{a^3nx^{3m}}{9m^2} + \frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3} + \frac{a^3x^{3m}\log(cx^n)}{3m} \\
&\quad - \frac{180ab^2n^3x^m\log^2(cx^n)}{m^4} - \frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2} + \frac{60ab^2n^2x^m\log^3(cx^n)}{m^3} \\
&\quad + \frac{3a^2bx^{2m}\log^3(cx^n)}{2m} - \frac{15ab^2nx^m\log^4(cx^n)}{m^2} + \frac{3ab^2x^m\log^5(cx^n)}{m} \\
&\quad + \frac{b^3\log^8(cx^n)}{8n} + \frac{(360ab^2n^4)\int x^{-1+m}\log(cx^n)\,dx}{m^4} \\
&= -\frac{360ab^2n^5x^m}{m^6} - \frac{9a^2bn^3x^{2m}}{8m^4} - \frac{a^3nx^{3m}}{9m^2} + \frac{360ab^2n^4x^m\log(cx^n)}{m^5} + \frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3} \\
&\quad + \frac{a^3x^{3m}\log(cx^n)}{3m} - \frac{180ab^2n^3x^m\log^2(cx^n)}{m^4} - \frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2} + \frac{60ab^2n^2x^m\log^3(cx^n)}{m^3} \\
&\quad + \frac{3a^2bx^{2m}\log^3(cx^n)}{2m} - \frac{15ab^2nx^m\log^4(cx^n)}{m^2} + \frac{3ab^2x^m\log^5(cx^n)}{m} + \frac{b^3\log^8(cx^n)}{8n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.85

$$
\begin{aligned}
&\int \frac{\log(cx^n)\left(ax^m + b\log^2(cx^n)\right)^3}{x}\,dx \\
&= -\frac{anx^m(25920b^2n^4 + 81abm^2n^2x^m + 8a^2m^4x^{2m})}{72m^6} \\
&\quad + \frac{ax^m(4320b^2n^4 + 27abm^2n^2x^m + 4a^2m^4x^{2m})\log(cx^n)}{12m^5} \\
&\quad - \frac{9abnx^m(80bn^2 + am^2x^m)\log^2(cx^n)}{4m^4} + \frac{3abx^m(40bn^2 + am^2x^m)\log^3(cx^n)}{2m^3} \\
&\quad - \frac{15ab^2nx^m\log^4(cx^n)}{m^2} + \frac{3ab^2x^m\log^5(cx^n)}{m} + \frac{b^3\log^8(cx^n)}{8n}
\end{aligned}
$$

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```

```
[Out] -1/72*(a*n*x^m*(25920*b^2*n^4 + 81*a*b*m^2*n^2*x^m + 8*a^2*m^4*x^(2*m)))/m^
6 + (a*x^m*(4320*b^2*n^4 + 27*a*b*m^2*n^2*x^m + 4*a^2*m^4*x^(2*m))*Log[c*x^
n])/(12*m^5) - (9*a*b*n*x^m*(80*b*n^2 + a*m^2*x^m)*Log[c*x^n]^2)/(4*m^4) +
(3*a*b*x^m*(40*b*n^2 + a*m^2*x^m)*Log[c*x^n]^3)/(2*m^3) - (15*a*b^2*n*x^m*L
og[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(8*n)
```

Maple [A] (verified)

Time = 12.89 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00

method	result
parallelrisch	$-\frac{-9b^3\ln(c\,x^n)^8m^6-216x^m\ln(c\,x^n)^5a\,b^2m^5n-108x^{2m}\ln(c\,x^n)^3a^2b\,m^5n+1080a\,b^2n^2\ln(c\,x^n)^4x^mm^4-24x^{3m}\ln(c\,x^n)a^3m^5n}{}$
risch	Expression too large to display

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/72*(-9*b^3*ln(c*x^n)^8*m^6-216*x^m*ln(c*x^n)^5*a*b^2*m^5*n-108*(x^m)^2*l
n(c*x^n)^3*a^2*b*m^5*n+1080*a*b^2*n^2*ln(c*x^n)^4*x^m*m^4-24*(x^m)^3*ln(c*x
^n)*a^3*m^5*n+162*a^2*b*n^2*ln(c*x^n)^2*(x^m)^2*m^4-4320*a*b^2*n^3*ln(c*x^n
)^3*x^m*m^3+8*a^3*n^2*(x^m)^3*m^4-162*a^2*b*n^3*ln(c*x^n)*(x^m)^2*m^3+12960
*n^4*a*b^2*ln(c*x^n)^2*x^m*m^2+81*a^2*b*n^4*(x^m)^2*m^2-25920*a*b^2*n^5*ln(
c*x^n)*x^m*m+25920*a*b^2*n^6*x^m)/m^6/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(258) = 516.

Time = 0.32 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.41

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx = \frac{9\,b^3m^6n^7\log(x)^8+72\,b^3m^6n^6\log(c)\log(x)^7+252\,b^3m^6n^5\log(c)^2\log(x)^6+504\,b^3m^6n^4\log(c)^3\log(x)^5+}{}$$

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/72*(9*b^3*m^6*n^7*log(x)^8 + 72*b^3*m^6*n^6*log(c)*log(x)^7 + 252*b^3*m^6
*n^5*log(c)^2*log(x)^6 + 504*b^3*m^6*n^4*log(c)^3*log(x)^5 + 630*b^3*m^6*n^
3*log(c)^4*log(x)^4 + 504*b^3*m^6*n^2*log(c)^5*log(x)^3 + 252*b^3*m^6*n*log
(c)^6*log(x)^2 + 72*b^3*m^6*log(c)^7*log(x) + 8*(3*a^3*m^5*n*log(x) + 3*a^3
*m^5*log(c) - a^3*m^4*n)*x^(3*m) + 27*(4*a^2*b*m^5*n^3*log(x)^3 + 4*a^2*b*m
^5*log(c)^3 - 6*a^2*b*m^4*n*log(c)^2 + 6*a^2*b*m^3*n^2*log(c) - 3*a^2*b*m^2
*n^3 + 6*(2*a^2*b*m^5*n^2*log(c) - a^2*b*m^4*n^3)*log(x)^2 + 6*(2*a^2*b*m^5
*n*log(c)^2 - 2*a^2*b*m^4*n^2*log(c) + a^2*b*m^3*n^3)*log(x))*x^(2*m) + 216
*(a*b^2*m^5*n^5*log(x)^5 + a*b^2*m^5*log(c)^5 - 5*a*b^2*m^4*n*log(c)^4 + 20
*a*b^2*m^3*n^2*log(c)^3 - 60*a*b^2*m^2*n^3*log(c)^2 + 120*a*b^2*m*n^4*log(c
) - 120*a*b^2*n^5 + 5*(a*b^2*m^5*n^4*log(c) - a*b^2*m^4*n^5)*log(x)^4 + 10*
(a*b^2*m^5*n^3*log(c)^2 - 2*a*b^2*m^4*n^4*log(c) + 2*a*b^2*m^3*n^5)*log(x)^
3 + 10*(a*b^2*m^5*n^2*log(c)^3 - 3*a*b^2*m^4*n^3*log(c)^2 + 6*a*b^2*m^3*n^4
*log(c) - 6*a*b^2*m^2*n^5)*log(x)^2 + 5*(a*b^2*m^5*n*log(c)^4 - 4*a*b^2*m^4
*n^2*log(c)^3 + 12*a*b^2*m^3*n^3*log(c)^2 - 24*a*b^2*m^2*n^4*log(c) + 24*a*
b^2*m*n^5)*log(x))*x^m)/m^6
```

Sympy [A] (verification not implemented)

Time = 24.79 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.51

$$\int \frac{\log\left(cx^{n}\right)\left(ax^{m}+b\log^{2}\left(cx^{n}\right)\right)^{3}}{x}\,dx$$

$$= -a^{3}n\left(\begin{cases}\frac{\begin{cases}\frac{x^{3m}}{3m} & \text{for}\: m \neq 0\\ \log\left(x\right) & \text{otherwise}\end{cases}}{3m} & \text{for}\: m > -\infty \wedge m < \infty \wedge m \neq 0\\ \frac{\log\left(x\right)^{2}}{2} & \text{otherwise}\end{cases}\right)$$

$$+ a^{3}\left(\begin{cases}\frac{x^{3m}}{3m} & \text{for}\: m \neq 0\\ \log\left(x\right) & \text{otherwise}\end{cases}\right)\log\left(cx^{n}\right)$$

$$+3a^{2}b\left(\begin{cases}\frac{x^{2m}\log\left(cx^{n}\right)^{3}}{2m} - \frac{3nx^{2m}\log\left(cx^{n}\right)^{2}}{4m^{2}} + \frac{3n^{2}x^{2m}\log\left(cx^{n}\right)}{4m^{3}} - \frac{3n^{3}x^{2m}}{8m^{4}}\\ \begin{cases}0 & \text{for}\: \frac{1}{\left|cx^{n}\right|} < 1 \wedge \left|cx^{n}\right| \\ \frac{\log\left(cx^{n}\right)^{4}}{4n} & \text{for}\: \left|cx^{n}\right| < 1\\ \frac{\log\left(\frac{x^{-n}}{c}\right)^{4}}{4n} & \text{for}\: \frac{1}{\left|cx^{n}\right|} < 1\\ \frac{6G_{5,5}^{5,0}\left(\begin{matrix} & 1,1,1,1,1 \\ 0,0,0,0,0 & \end{matrix} \middle| cx^{n}\right)}{n} + \frac{6G_{5,5}^{0,5}\left(\begin{matrix} 1,1,1,1,1 & \\ & 0,0,0,0,0 \end{matrix} \middle| cx^{n}\right)}{n} & \text{otherwise}\end{cases}\end{cases}\right)$$

$$+3ab^{2}\left(\begin{cases}\frac{x^{m}\log\left(cx^{n}\right)^{5}}{m} - \frac{5nx^{m}\log\left(cx^{n}\right)^{4}}{m^{2}} + \frac{20n^{2}x^{m}\log\left(cx^{n}\right)^{3}}{m^{3}} - \frac{60n^{3}x^{m}\log\left(cx^{n}\right)^{2}}{m^{4}} + \frac{120n^{4}x^{m}\log\left(cx^{n}\right)}{m^{5}} - \frac{120n^{5}x^{m}}{m^{6}}\\ \begin{cases}0\\ \frac{\log\left(cx^{n}\right)^{6}}{6n}\\ \frac{\log\left(\frac{x^{-n}}{c}\right)^{6}}{6n}\\ \frac{120G_{7,7}^{7,0}\left(\begin{matrix} & 1,1,1,1,1,1,1 \\ 0,0,0,0,0,0,0 & \end{matrix} \middle| cx^{n}\right)}{n} + \frac{120G_{7,7}^{0,7}\left(\begin{matrix} 1,1,1,1,1,1,1 & \\ & 0,0,0,0,0,0,0 \end{matrix} \middle| cx^{n}\right)}{n}\end{cases}\end{cases}\right)$$

$$- b^{3}\left(\begin{cases}-\log\left(c\right)^{7}\log\left(x\right) & \text{for}\: n = 0\\ -\frac{\log\left(cx^{n}\right)^{8}}{8n} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**3/x,x)
```

```
[Out] -a**3*n*Piecewise((Piecewise((x**(3*m)/(3*m), Ne(m, 0)), (log(x), True))/(3
*m), (m > -oo) & (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a**3*Piecewis
e((x**(3*m)/(3*m), Ne(m, 0)), (log(x), True))*log(c*x**n) + 3*a**2*b*Piecew
```

```
ise((x**(2*m)*log(c*x**n)**3/(2*m) - 3*n*x**(2*m)*log(c*x**n)**2/(4*m**2) +
 3*n**2*x**(2*m)*log(c*x**n)/(4*m**3) - 3*n**3*x**(2*m)/(8*m**4), Ne(m, 0))
, (Piecewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**4/
(4*n), Abs(c*x**n) < 1), (log(1/(c*x**n))**4/(4*n), 1/Abs(c*x**n) < 1), (6*
meijerg(((), (1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0), ()), c*x**n)/n + 6*meijerg
(((1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0)), c*x**n)/n, True)), True)) +
3*a*b**2*Piecewise((x**m*log(c*x**n)**5/m - 5*n*x**m*log(c*x**n)**4/m**2 +
20*n**2*x**m*log(c*x**n)**3/m**3 - 60*n**3*x**m*log(c*x**n)**2/m**4 + 120*n
**4*x**m*log(c*x**n)/m**5 - 120*n**5*x**m/m**6, Ne(m, 0)), (Piecewise((0, (
Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**6/(6*n), Abs(c*x**n)
 < 1), (log(1/(c*x**n))**6/(6*n), 1/Abs(c*x**n) < 1), (120*meijerg(((), (1,
 1, 1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0, 0, 0), ()), c*x**n)/n + 120*meijerg((
(1, 1, 1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0, 0, 0)), c*x**n)/n, True)),
 True)) - b**3*Piecewise((-log(c)**7*log(x), Eq(n, 0)), (-log(c*x**n)**8/(8
*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(258) = 516$.

Time = 0.24 (sec) , antiderivative size = 1115, normalized size of antiderivative = 4.10

$$\int \frac{\log(cx^n)\left(ax^m + b\log^2(cx^n)\right)^3}{x}\,dx = \text{Too large to display}$$

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/84*(12*b^3*log(c*x^n)^7/n + 252*a*b^2*x^m*log(c*x^n)^4/m + 126*a^2*b*x^(2
*m)*log(c*x^n)^2/m - 1008*(n*x^m*log(c*x^n)^3/m^2 - 3*(n*x^m*log(c*x^n)^2/m
^2 - 2*n*(n*x^m*log(c*x^n)/m^2 - n^2*x^m/m^3)/m)*n/m)*a*b^2 - 63*a^2*b*(2*n
*x^(2*m)*log(c*x^n)/m^2 - n^2*x^(2*m)/m^3) + 28*a^3*x^(3*m)/m)*log(c*x^n) +
 1/504*(9*b^3*m^6*n^7*log(x)^8 - 72*b^3*m^6*n^6*log(c)*log(x)^7 + 252*b^3*m
^6*n^5*log(c)^2*log(x)^6 - 504*b^3*m^6*n^4*log(c)^3*log(x)^5 + 630*b^3*m^6*
n^3*log(c)^4*log(x)^4 - 504*b^3*m^6*n^2*log(c)^5*log(x)^3 + 252*b^3*m^6*n*l
og(c)^6*log(x)^2 - 72*b^3*m^6*log(c)^7*log(x) - 72*b^3*m^6*log(x)*log(x^n)^
7 - 56*a^3*m^4*n*x^(3*m) + 252*(b^3*m^6*n*log(x)^2 - 2*b^3*m^6*log(c)*log(x
))*log(x^n)^6 - 504*(b^3*m^6*n^2*log(x)^3 - 3*b^3*m^6*n*log(c)*log(x)^2 + 3
*b^3*m^6*log(c)^2*log(x))*log(x^n)^5 - 189*(2*m^4*n*log(c)^2 - 4*m^3*n^2*lo
g(c) + 3*m^2*n^3)*a^2*b*x^(2*m) - 1512*(m^4*n*log(c)^4 - 8*m^3*n^2*log(c)^3
 + 36*m^2*n^3*log(c)^2 - 96*m*n^4*log(c) + 120*n^5)*a*b^2*x^m + 126*(5*b^3*
m^6*n^3*log(x)^4 - 20*b^3*m^6*n^2*log(c)*log(x)^3 + 30*b^3*m^6*n*log(c)^2*l
og(x)^2 - 20*b^3*m^6*log(c)^3*log(x) - 12*a*b^2*m^4*n*x^m)*log(x^n)^4 - 504
*(b^3*m^6*n^4*log(x)^5 - 5*b^3*m^6*n^3*log(c)*log(x)^4 + 10*b^3*m^6*n^2*log
(c)^2*log(x)^3 - 10*b^3*m^6*n*log(c)^3*log(x)^2 + 5*b^3*m^6*log(c)^4*log(x)
 + 12*(m^4*n*log(c) - 2*m^3*n^2)*a*b^2*x^m)*log(x^n)^3 + 126*(2*b^3*m^6*n^5
```

```
*log(x)^6 - 12*b^3*m^6*n^4*log(c)*log(x)^5 + 30*b^3*m^6*n^3*log(c)^2*log(x)
^4 - 40*b^3*m^6*n^2*log(c)^3*log(x)^3 + 30*b^3*m^6*n*log(c)^4*log(x)^2 - 12
*b^3*m^6*log(c)^5*log(x) - 3*a^2*b*m^4*n*x^(2*m) - 72*(m^4*n*log(c)^2 - 4*m
^3*n^2*log(c) + 6*m^2*n^3)*a*b^2*x^m)*log(x^n)^2 - 36*(2*b^3*m^6*n^6*log(x)
^7 - 14*b^3*m^6*n^5*log(c)*log(x)^6 + 42*b^3*m^6*n^4*log(c)^2*log(x)^5 - 70
*b^3*m^6*n^3*log(c)^3*log(x)^4 + 70*b^3*m^6*n^2*log(c)^4*log(x)^3 - 42*b^3*
m^6*n*log(c)^5*log(x)^2 + 14*b^3*m^6*log(c)^6*log(x) + 21*(m^4*n*log(c) - m
^3*n^2)*a^2*b*x^(2*m) + 168*(m^4*n*log(c)^3 - 6*m^3*n^2*log(c)^2 + 18*m^2*n
^3*log(c) - 24*m*n^4)*a*b^2*x^m)*log(x^n))/m^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(258) = 516$.

Time = 0.35 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.82

$$
\begin{aligned}
\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx &= \frac{1}{8}b^3n^7\log(x)^8+b^3n^6\log(c)\log(x)^7 \\
&+\frac{7}{2}b^3n^5\log(c)^2\log(x)^6+7\,b^3n^4\log(c)^3\log(x)^5 \\
&+\frac{35}{4}b^3n^3\log(c)^4\log(x)^4+7\,b^3n^2\log(c)^5\log(x)^3 \\
&+\frac{3\,ab^2n^5x^m\log(x)^5}{m}+\frac{7}{2}b^3n\log(c)^6\log(x)^2 \\
&+\frac{15\,ab^2n^4x^m\log(c)\log(x)^4}{m}+b^3\log(c)^7\log(x) \\
&+\frac{30\,ab^2n^3x^m\log(c)^2\log(x)^3}{m} \\
&-\frac{15\,ab^2n^5x^m\log(x)^4}{m^2}+\frac{30\,ab^2n^2x^m\log(c)^3\log(x)^2}{m} \\
&-\frac{60\,ab^2n^4x^m\log(c)\log(x)^3}{m^2} \\
&+\frac{15\,ab^2nx^m\log(c)^4\log(x)}{m} \\
&-\frac{90\,ab^2n^3x^m\log(c)^2\log(x)^2}{m^2} \\
&+\frac{3\,a^2bn^3x^{2\,m}\log(x)^3}{2\,m}+\frac{60\,ab^2n^5x^m\log(x)^3}{m^3} \\
&+\frac{3\,ab^2x^m\log(c)^5}{m}-\frac{60\,ab^2n^2x^m\log(c)^3\log(x)}{m^2} \\
&+\frac{9\,a^2bn^2x^{2\,m}\log(c)\log(x)^2}{2\,m} \\
&+\frac{180\,ab^2n^4x^m\log(c)\log(x)^2}{m^3} \\
&-\frac{15\,ab^2nx^m\log(c)^4}{m^2}+\frac{9\,a^2bnx^{2\,m}\log(c)^2\log(x)}{2\,m} \\
&+\frac{180\,ab^2n^3x^m\log(c)^2\log(x)}{m^3}-\frac{9\,a^2bn^3x^{2\,m}\log(x)^2}{4\,m^2} \\
&-\frac{180\,ab^2n^5x^m\log(x)^2}{m^4}+\frac{3\,a^2bx^{2\,m}\log(c)^3}{2\,m} \\
&+\frac{60\,ab^2n^2x^m\log(c)^3}{m^3}-\frac{9\,a^2bn^2x^{2\,m}\log(c)\log(x)}{2\,m^2} \\
&-\frac{360\,ab^2n^4x^m\log(c)\log(x)}{m^4} \\
&-\frac{9\,a^2bnx^{2\,m}\log(c)^2}{4\,m^2}-\frac{180\,ab^2n^3x^m\log(c)^2}{m^4} \\
&+\frac{a^3nx^{3\,m}\log(x)}{3\,m}+\frac{9\,a^2bn^3x^{2\,m}\log(x)}{4\,m^3} \\
&+\frac{360\,ab^2n^5x^m\log(x)}{m^5}+\frac{a^3x^{3\,m}\log(c)}{3\,m} \\
&+\frac{9\,a^2bn^2x^{2\,m}\log(c)}{4\,m^3}+\frac{360\,ab^2n^4x^m\log(c)}{m^5} \\
&\quad a^3nx^{3\,m} \quad 9\,a^2bn^3x^{2\,m} \quad 360\,ab^2n^5x^m
\end{aligned}
$$

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/8*b^3*n^7*log(x)^8 + b^3*n^6*log(c)*log(x)^7 + 7/2*b^3*n^5*log(c)^2*log(x
)^6 + 7*b^3*n^4*log(c)^3*log(x)^5 + 35/4*b^3*n^3*log(c)^4*log(x)^4 + 7*b^3*
n^2*log(c)^5*log(x)^3 + 3*a*b^2*n^5*x^m*log(x)^5/m + 7/2*b^3*n*log(c)^6*log
(x)^2 + 15*a*b^2*n^4*x^m*log(c)*log(x)^4/m + b^3*log(c)^7*log(x) + 30*a*b^2
*n^3*x^m*log(c)^2*log(x)^3/m - 15*a*b^2*n^5*x^m*log(x)^4/m^2 + 30*a*b^2*n^2
*x^m*log(c)^3*log(x)^2/m - 60*a*b^2*n^4*x^m*log(c)*log(x)^3/m^2 + 15*a*b^2*
n*x^m*log(c)^4*log(x)/m - 90*a*b^2*n^3*x^m*log(c)^2*log(x)^2/m^2 + 3/2*a^2*
b*n^3*x^(2*m)*log(x)^3/m + 60*a*b^2*n^5*x^m*log(x)^3/m^3 + 3*a*b^2*x^m*log(
c)^5/m - 60*a*b^2*n^2*x^m*log(c)^3*log(x)/m^2 + 9/2*a^2*b*n^2*x^(2*m)*log(c
)*log(x)^2/m + 180*a*b^2*n^4*x^m*log(c)*log(x)^2/m^3 - 15*a*b^2*n*x^m*log(c
)^4/m^2 + 9/2*a^2*b*n*x^(2*m)*log(c)^2*log(x)/m + 180*a*b^2*n^3*x^m*log(c)^
2*log(x)/m^3 - 9/4*a^2*b*n^3*x^(2*m)*log(x)^2/m^2 - 180*a*b^2*n^5*x^m*log(x
)^2/m^4 + 3/2*a^2*b*x^(2*m)*log(c)^3/m + 60*a*b^2*n^2*x^m*log(c)^3/m^3 - 9/
2*a^2*b*n^2*x^(2*m)*log(c)*log(x)/m^2 - 360*a*b^2*n^4*x^m*log(c)*log(x)/m^4
 - 9/4*a^2*b*n*x^(2*m)*log(c)^2/m^2 - 180*a*b^2*n^3*x^m*log(c)^2/m^4 + 1/3*
a^3*n*x^(3*m)*log(x)/m + 9/4*a^2*b*n^3*x^(2*m)*log(x)/m^3 + 360*a*b^2*n^5*x
^m*log(x)/m^5 + 1/3*a^3*x^(3*m)*log(c)/m + 9/4*a^2*b*n^2*x^(2*m)*log(c)/m^3
 + 360*a*b^2*n^4*x^m*log(c)/m^5 - 1/9*a^3*n*x^(3*m)/m^2 - 9/8*a^2*b*n^3*x^(
2*m)/m^4 - 360*a*b^2*n^5*x^m/m^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx=\int \frac{\ln(c\,x^n)\,\left(a\,x^m+b\ln(c\,x^n)^2\right)^3}{x}\,dx$$

```
[In] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^3)/x,x)
```

```
[Out] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^3)/x, x)
```

3.10 $\int \frac{\log(cx^n)(ax^m+b\log^2(cx^n))^2}{x} dx$

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Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx = -\frac{12abn^3x^m}{m^4}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log(cx^n)}{m^3}+\frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{2abx^m\log^3(cx^n)}{m}+\frac{b^2\log^6(cx^n)}{6n}$$

```
[Out] -12*a*b*n^3*x^m/m^4-1/4*a^2*n*x^(2*m)/m^2+12*a*b*n^2*x^m*ln(c*x^n)/m^3+1/2*
a^2*x^(2*m)*ln(c*x^n)/m-6*a*b*n*x^m*ln(c*x^n)^2/m^2+2*a*b*x^m*ln(c*x^n)^3/m
+1/6*b^2*ln(c*x^n)^6/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2619, 2341, 2342, 2339, 30}

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx = \frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log(cx^n)}{m^3}-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{2abx^m\log^3(cx^n)}{m}-\frac{12abn^3x^m}{m^4}+\frac{b^2\log^6(cx^n)}{6n}$$

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

```
[Out] (-12*a*b*n^3*x^m)/m^4 - (a^2*n*x^(2*m))/(4*m^2) + (12*a*b*n^2*x^m*Log[c*x^n
])/m^3 + (a^2*x^(2*m)*Log[c*x^n])/(2*m) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2 +
(2*a*b*x^m*Log[c*x^n]^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2 x^{-1+2m}\log(cx^n) + 2abx^{-1+m}\log^3(cx^n) + \frac{b^2\log^5(cx^n)}{x}\right)dx \\
&= a^2\int x^{-1+2m}\log(cx^n)\,dx + (2ab)\int x^{-1+m}\log^3(cx^n)\,dx + b^2\int \frac{\log^5(cx^n)}{x}\,dx \\
&= -\frac{a^2nx^{2m}}{4m^2} + \frac{a^2x^{2m}\log(cx^n)}{2m} + \frac{2abx^m\log^3(cx^n)}{m} \\
&\quad + \frac{b^2\text{Subst}\left(\int x^5\,dx, x, \log(cx^n)\right)}{n} - \frac{(6abn)\int x^{-1+m}\log^2(cx^n)\,dx}{m}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a^2nx^{2m}}{4m^2} + \frac{a^2x^{2m}\log\left(cx^n\right)}{2m} - \frac{6abnx^m\log^2\left(cx^n\right)}{m^2} \\
&\quad + \frac{2abx^m\log^3\left(cx^n\right)}{m} + \frac{b^2\log^6\left(cx^n\right)}{6n} + \frac{(12abn^2)\int x^{-1+m}\log\left(cx^n\right)\,dx}{m^2} \\
&= -\frac{12abn^3x^m}{m^4} - \frac{a^2nx^{2m}}{4m^2} + \frac{12abn^2x^m\log\left(cx^n\right)}{m^3} + \frac{a^2x^{2m}\log\left(cx^n\right)}{2m} \\
&\quad - \frac{6abnx^m\log^2\left(cx^n\right)}{m^2} + \frac{2abx^m\log^3\left(cx^n\right)}{m} + \frac{b^2\log^6\left(cx^n\right)}{6n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92

$$
\begin{aligned}
\int \frac{\log\left(cx^n\right)\left(ax^m + b\log^2\left(cx^n\right)\right)^2}{x}\,dx = &-\frac{anx^m(48bn^2 + am^2x^m)}{4m^4} \\
&+ \frac{ax^m(24bn^2 + am^2x^m)\log\left(cx^n\right)}{2m^3} \\
&- \frac{6abnx^m\log^2\left(cx^n\right)}{m^2} \\
&+ \frac{2abx^m\log^3\left(cx^n\right)}{m} + \frac{b^2\log^6\left(cx^n\right)}{6n}
\end{aligned}
$$

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

```
[Out] -1/4*(a*n*x^m*(48*b*n^2 + a*m^2*x^m))/m^4 + (a*x^m*(24*b*n^2 + a*m^2*x^m)*L
og[c*x^n])/(2*m^3) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2 + (2*a*b*x^m*Log[c*x^n]
^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

method	result
parallelrisch	$-\frac{-2b^2\ln(cx^n)^6m^4-24x^m\ln(cx^n)^3ab\,m^3n-6x^{2m}\ln(cx^n)a^2m^3n+72ab\,n^2\ln(cx^n)^2x^mm^2+3a^2n^2x^{2m}m^2-144ab\,n^3\ln(cx^n)}{12m^4n}$
risch	Expression too large to display

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-2*b^2*ln(c*x^n)^6*m^4-24*x^m*ln(c*x^n)^3*a*b*m^3*n-6*(x^m)^2*ln(c*x
^n)*a^2*m^3*n+72*a*b*n^2*ln(c*x^n)^2*x^m*m^2+3*a^2*n^2*(x^m)^2*m^2-144*a*b*
n^3*ln(c*x^n)*x^m*m+144*a*b*n^4*x^m)/m^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(119) = 238$.

Time = 0.32 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.14

$$\int \frac{\log (cx^n) \left(ax^m + b\log^2(cx^n)\right)^2}{x}\,dx$$
$$= \frac{2\,b^2m^4n^5\log (x)^6 + 12\,b^2m^4n^4\log (c)\log (x)^5 + 30\,b^2m^4n^3\log (c)^2\log (x)^4 + 40\,b^2m^4n^2\log (c)^3\log (x)^3 + \cdots}{}$$

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*m^4*n^5*log(x)^6 + 12*b^2*m^4*n^4*log(c)*log(x)^5 + 30*b^2*m^4*
n^3*log(c)^2*log(x)^4 + 40*b^2*m^4*n^2*log(c)^3*log(x)^3 + 30*b^2*m^4*n*log
(c)^4*log(x)^2 + 12*b^2*m^4*log(c)^5*log(x) + 3*(2*a^2*m^3*n*log(x) + 2*a^2
*m^3*log(c) - a^2*m^2*n)*x^(2*m) + 24*(a*b*m^3*n^3*log(x)^3 + a*b*m^3*log(c
)^3 - 3*a*b*m^2*n*log(c)^2 + 6*a*b*m*n^2*log(c) - 6*a*b*n^3 + 3*(a*b*m^3*n^
2*log(c) - a*b*m^2*n^3)*log(x)^2 + 3*(a*b*m^3*n*log(c)^2 - 2*a*b*m^2*n^2*lo
g(c) + 2*a*b*m*n^3)*log(x))*x^m)/m^4
```

Sympy [A] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.73

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx$$

$$= -a^2n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2m}}{2m} & \text{for } m\neq 0\\ \log(x) & \text{otherwise}\end{cases}}{2m} & \text{for } m>-\infty\wedge m<\infty\wedge m\neq 0\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)$$

$$+a^2\left(\begin{cases}\frac{x^{2m}}{2m} & \text{for } m\neq 0\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)$$

$$+2ab\left(\begin{cases}\frac{x^m\log(cx^n)^3}{m}-\frac{3nx^m\log(cx^n)^2}{m^2}+\frac{6n^2x^m\log(cx^n)}{m^3}-\frac{6n^3x^m}{m^4}\\ \begin{cases}0 & \text{for } \frac{1}{|cx^n|}<1\wedge|cx^n|<1\\ \frac{\log(cx^n)^4}{4n} & \text{for } |cx^n|<1\\ \frac{\log\left(\frac{x^{-n}}{c}\right)^4}{4n} & \text{for } \frac{1}{|cx^n|}<1\\ \frac{6G_{5,5}^{5,0}\left(\begin{matrix} & 1,1,1,1,1\\ 0,0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n}+\frac{6G_{5,5}^{0,5}\left(\begin{matrix}1,1,1,1,1 & \\ & 0,0,0,0,0\end{matrix}\middle| cx^n\right)}{n} & \text{otherwise}\end{cases}\end{cases}\right)$$

$$-b^2\left(\begin{cases}-\log(c)^5\log(x) & \text{for } n=0\\ -\frac{\log(cx^n)^6}{6n} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**2/x,x)
```

```
[Out] -a**2*n*Piecewise((Piecewise((x**(2*m)/(2*m), Ne(m, 0)), (log(x), True))/(2
*m), (m > -oo) & (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a**2*Piecewis
e((x**(2*m)/(2*m), Ne(m, 0)), (log(x), True))*log(c*x**n) + 2*a*b*Piecewise
((x**m*log(c*x**n)**3/m - 3*n*x**m*log(c*x**n)**2/m**2 + 6*n**2*x**m*log(c*
x**n)/m**3 - 6*n**3*x**m/m**4, Ne(m, 0)), (Piecewise((0, (Abs(c*x**n) < 1)
& (1/Abs(c*x**n) < 1)), (log(c*x**n)**4/(4*n), Abs(c*x**n) < 1), (log(1/(c*
x**n))**4/(4*n), 1/Abs(c*x**n) < 1), (6*meijerg(((), (1, 1, 1, 1, 1)), ((0,
 0, 0, 0, 0), ()), c*x**n)/n + 6*meijerg(((1, 1, 1, 1, 1), ()), ((), (0, 0,
 0, 0, 0)), c*x**n)/n, True)), True)) - b**2*Piecewise((-log(c)**5*log(x),
Eq(n, 0)), (-log(c*x**n)**6/(6*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(119) = 238$.

Time = 0.23 (sec) , antiderivative size = 530, normalized size of antiderivative = 4.24

$$\int \frac{\log(cx^n)\left(ax^m + b\log^2(cx^n)\right)^2}{x}\,dx$$
$$= \frac{1}{10}\left(\frac{2\,b^2\log(cx^n)^5}{n} + \frac{20\,abx^m\log(cx^n)^2}{m} - 40\,ab\left(\frac{nx^m\log(cx^n)}{m^2} - \frac{n^2x^m}{m^3}\right) + \frac{5\,a^2x^{2\,m}}{m}\right)\log(cx^n)$$
$$+ \frac{2\,b^2m^4n^5\log(x)^6 - 12\,b^2m^4n^4\log(c)\log(x)^5 + 30\,b^2m^4n^3\log(c)^2\log(x)^4 - 40\,b^2m^4n^2\log(c)^3\log(x)^3}{}$$

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*(2*b^2*log(c*x^n)^5/n + 20*a*b*x^m*log(c*x^n)^2/m - 40*a*b*(n*x^m*log(
c*x^n)/m^2 - n^2*x^m/m^3) + 5*a^2*x^(2*m)/m)*log(c*x^n) + 1/60*(2*b^2*m^4*n
^5*log(x)^6 - 12*b^2*m^4*n^4*log(c)*log(x)^5 + 30*b^2*m^4*n^3*log(c)^2*log(
x)^4 - 40*b^2*m^4*n^2*log(c)^3*log(x)^3 + 30*b^2*m^4*n*log(c)^4*log(x)^2 -
12*b^2*m^4*log(c)^5*log(x) - 12*b^2*m^4*log(x)*log(x^n)^5 - 15*a^2*m^2*n*x^
(2*m) + 30*(b^2*m^4*n*log(x)^2 - 2*b^2*m^4*log(c)*log(x))*log(x^n)^4 - 120*
(m^2*n*log(c)^2 - 4*m*n^2*log(c) + 6*n^3)*a*b*x^m - 40*(b^2*m^4*n^2*log(x)^
3 - 3*b^2*m^4*n*log(c)*log(x)^2 + 3*b^2*m^4*log(c)^2*log(x))*log(x^n)^3 + 3
0*(b^2*m^4*n^3*log(x)^4 - 4*b^2*m^4*n^2*log(c)*log(x)^3 + 6*b^2*m^4*n*log(c
)^2*log(x)^2 - 4*b^2*m^4*log(c)^3*log(x) - 4*a*b*m^2*n*x^m)*log(x^n)^2 - 12
*(b^2*m^4*n^4*log(x)^5 - 5*b^2*m^4*n^3*log(c)*log(x)^4 + 10*b^2*m^4*n^2*log
(c)^2*log(x)^3 - 10*b^2*m^4*n*log(c)^3*log(x)^2 + 5*b^2*m^4*log(c)^4*log(x)
 + 20*(m^2*n*log(c) - 2*m*n^2)*a*b*x^m)*log(x^n))/m^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(119) = 238.

Time = 0.32 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.29

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx = \frac{1}{6}\,b^2n^5\log(x)^6+b^2n^4\log(c)\log(x)^5+\frac{5}{2}\,b^2n^3\log(c)^2\log(x)^4+\frac{10}{3}\,b^2n^2\log(c)^3\log(x)^3+\frac{5}{2}\,b^2n\log(c)^4\log(x)^2+b^2\log(c)^5\log(x)+\frac{2\,abn^3x^m\log(x)^3}{m}+\frac{6\,abn^2x^m\log(c)\log(x)^2}{m}+\frac{6\,abnx^m\log(c)^2\log(x)}{m}-\frac{6\,abn^3x^m\log(x)^2}{m^2}+\frac{2\,abx^m\log(c)^3}{m}-\frac{12\,abn^2x^m\log(c)\log(x)}{m^2}-\frac{6\,abnx^m\log(c)^2}{m^2}+\frac{a^2nx^{2\,m}\log(x)}{2\,m}+\frac{12\,abn^3x^m\log(x)}{m^3}+\frac{a^2x^{2\,m}\log(c)}{2\,m}+\frac{12\,abn^2x^m\log(c)}{m^3}-\frac{a^2nx^{2\,m}}{4\,m^2}-\frac{12\,abn^3x^m}{m^4}$$

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*n^5*log(x)^6 + b^2*n^4*log(c)*log(x)^5 + 5/2*b^2*n^3*log(c)^2*log(x
)^4 + 10/3*b^2*n^2*log(c)^3*log(x)^3 + 5/2*b^2*n*log(c)^4*log(x)^2 + b^2*lo
g(c)^5*log(x) + 2*a*b*n^3*x^m*log(x)^3/m + 6*a*b*n^2*x^m*log(c)*log(x)^2/m
+ 6*a*b*n*x^m*log(c)^2*log(x)/m - 6*a*b*n^3*x^m*log(x)^2/m^2 + 2*a*b*x^m*lo
g(c)^3/m - 12*a*b*n^2*x^m*log(c)*log(x)/m^2 - 6*a*b*n*x^m*log(c)^2/m^2 + 1/
2*a^2*n*x^(2*m)*log(x)/m + 12*a*b*n^3*x^m*log(x)/m^3 + 1/2*a^2*x^(2*m)*log(
c)/m + 12*a*b*n^2*x^m*log(c)/m^3 - 1/4*a^2*n*x^(2*m)/m^2 - 12*a*b*n^3*x^m/m
^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx = \int \frac{\ln\left(c\,x^n\right)\,\left(a\,x^m+b\ln\left(c\,x^n\right)^2\right)^2}{x}\,dx$$

```
[In] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^2)/x,x)
```

```
[Out] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^2)/x, x)
```

3.11 $\int \frac{\log(cx^n)(ax^m+b\log^2(cx^n))}{x}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx=-\frac{anx^m}{m^2}+\frac{ax^m\log(cx^n)}{m}+\frac{b\log^4(cx^n)}{4n}$$

```
[Out] -a*n*x^m/m^2+a*x^m*ln(c*x^n)/m+1/4*b*ln(c*x^n)^4/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2619, 2341, 2339, 30}

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx=\frac{ax^m\log(cx^n)}{m}-\frac{anx^m}{m^2}+\frac{b\log^4(cx^n)}{4n}$$

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]
```

```
[Out] -((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^{-1+m}\log\left(cx^n\right)+\frac{b\log^3\left(cx^n\right)}{x}\right)dx\\
&= a\int x^{-1+m}\log\left(cx^n\right)\,dx+b\int \frac{\log^3\left(cx^n\right)}{x}\,dx\\
&= -\frac{anx^m}{m^2}+\frac{ax^m\log\left(cx^n\right)}{m}+\frac{b\text{Subst}\left(\int x^3\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= -\frac{anx^m}{m^2}+\frac{ax^m\log\left(cx^n\right)}{m}+\frac{b\log^4\left(cx^n\right)}{4n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$
\int \frac{\log\left(cx^n\right)\left(ax^m+b\log^2\left(cx^n\right)\right)}{x}\,dx=-\frac{anx^m}{m^2}+\frac{ax^m\log\left(cx^n\right)}{m}+\frac{b\log^4\left(cx^n\right)}{4n}
$$

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]
[Out] -((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$-\frac{-b\ln(c\,x^{n})^{4}m^{2}-4x^{m}\ln(c\,x^{n})amn+4n^{2}a\,x^{m}}{4m^{2}n}$	47
risch	Expression too large to display	2146

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)/x,x,method=_RETURNVERBOSE)
[Out] -1/4*(-b*ln(c*x^n)^4*m^2-4*x^m*ln(c*x^n)*a*m*n+4*n^2*a*x^m)/m^2/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(39) = 78.

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx = \frac{bm^2n^3\log(x)^4+4\,bm^2n^2\log(c)\log(x)^3+6\,bm^2n\log(c)^2\log(x)^2+4\,bm^2\log(c)^3\log(x)+4\,(amn\log(x}{4\,m^2}$$

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="fricas")
[Out] 1/4*(b*m^2*n^3*log(x)^4 + 4*b*m^2*n^2*log(c)*log(x)^3 + 6*b*m^2*n*log(c)^2*
log(x)^2 + 4*b*m^2*log(c)^3*log(x) + 4*(a*m*n*log(x) + a*m*log(c) - a*n)*x^
m)/m^2
```

Sympy [A] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx = -an\left(\begin{cases}\frac{\begin{cases}\frac{x^m}{m} & \text{for } m\neq 0\\ \log(x) & \text{otherwise}\end{cases}}{m} & \text{for } m>-\infty\wedge m<\infty\wedge m\neq 0\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right) + a\left(\begin{cases}\frac{x^m}{m} & \text{for } m\neq 0\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n) - b\left(\begin{cases}-\log(c)^3\log(x) & \text{for } n=0\\ -\frac{\log(cx^n)^4}{4n} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)/x,x)
```

```
[Out] -a*n*Piecewise((Piecewise((x**m/m, Ne(m, 0)), (log(x), True))/m, (m > -oo)
& (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a*Piecewise((x**m/m, Ne(m, 0
)), (log(x), True))*log(c*x**n) - b*Piecewise((-log(c)**3*log(x), Eq(n, 0))
, (-log(c*x**n)**4/(4*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(39) = 78$.

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 4.54

$$\int \frac{\log(cx^n)\left(ax^m + b\log^2(cx^n)\right)}{x}\,dx = \frac{1}{3}\left(\frac{b\log(cx^n)^3}{n} + \frac{3\,ax^m}{m}\right)\log(cx^n) + \frac{bm^2n^3\log(x)^4 - 4\,bm^2n^2\log(c)\log(x)^3 + 6\,bm^2n\log(c)^2\log(x)^2 - 4\,bm^2\log(c)^3\log(x) - 4\,bm^2\log(x)}{}$$

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b*log(c*x^n)^3/n + 3*a*x^m/m)*log(c*x^n) + 1/12*(b*m^2*n^3*log(x)^4 -
4*b*m^2*n^2*log(c)*log(x)^3 + 6*b*m^2*n*log(c)^2*log(x)^2 - 4*b*m^2*log(c)^
3*log(x) - 4*b*m^2*log(x)*log(x^n)^3 - 12*a*n*x^m + 6*(b*m^2*n*log(x)^2 - 2
*b*m^2*log(c)*log(x))*log(x^n)^2 - 4*(b*m^2*n^2*log(x)^3 - 3*b*m^2*n*log(c)
*log(x)^2 + 3*b*m^2*log(c)^2*log(x))*log(x^n))/m^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.78

$$\int \frac{\log(cx^n)\left(ax^m + b\log^2(cx^n)\right)}{x}\,dx = \frac{1}{4}\,bn^3\log(x)^4 + bn^2\log(c)\log(x)^3 + \frac{3}{2}\,bn\log(c)^2\log(x)^2 + b\log(c)^3\log(x) + \frac{anx^m\log(x)}{m} + \frac{ax^m\log(c)}{m} - \frac{anx^m}{m^2}$$

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*b*n^3*log(x)^4 + b*n^2*log(c)*log(x)^3 + 3/2*b*n*log(c)^2*log(x)^2 + b*
log(c)^3*log(x) + a*n*x^m*log(x)/m + a*x^m*log(c)/m - a*n*x^m/m^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (cx^n) \left(ax^m + b\log^2 (cx^n)\right)}{x} \, dx = \int \frac{\ln (c\,x^n) \, \left(a\,x^m + b\ln (c\,x^n)^2\right)}{x} \, dx$$

```
[In] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2))/x,x)
[Out] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2))/x, x)
```

3.12 $\int \frac{\log(cx^n)}{x}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{\log(cx^n)}{x}\,dx = \frac{\log^2(cx^n)}{2n}$$

```
[Out] 1/2*ln(c*x^n)^2/n
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2338}

$$\int \frac{\log(cx^n)}{x}\,dx = \frac{\log^2(cx^n)}{2n}$$

```
[In] Int[Log[c*x^n]/x,x]
```

```
[Out] Log[c*x^n]^2/(2*n)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\log^2(cx^n)}{2n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(cx^n)}{x} dx = \frac{\log^2(cx^n)}{2n}$$

[In] Integrate[Log[c*x^n]/x,x]

[Out] Log[c*x^n]^2/(2*n)

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\ln(c\,x^n)^2}{2n}$
default	$\frac{\ln(c\,x^n)^2}{2n}$
norman	$\frac{\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2}{2n}$
parts	$\ln(c\,x^n)\ln(x) - \frac{n\ln(x)^2}{2}$
risch	$\ln(x)\ln(x^n) - \frac{n\ln(x)^2}{2} - \frac{i\pi\ln(x)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)}{2} + \frac{i\pi\ln(x)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2}{2} + \frac{i\pi\ln(x)}{}$

[In] int(ln(c*x^n)/x,x,method=_RETURNVERBOSE)

[Out] 1/2*ln(c*x^n)^2/n

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(cx^n)}{x} dx = \frac{1}{2} n\log(x)^2 + \log(c)\log(x)$$

[In] integrate(log(c*x^n)/x,x, algorithm="fricas")

[Out] 1/2*n*log(x)^2 + log(c)*log(x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(10) = 20$.

Time = 1.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.33

$$\int \frac{\log(cx^n)}{x}\,dx = \begin{cases} 0 & \text{for } \frac{1}{|cx^n|} < 1 \wedge |cx^n| < 1 \\ \frac{\log(cx^n)^2}{2n} & \text{for } |cx^n| < 1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for } \frac{1}{|cx^n|} < 1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix} \middle| cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix} \middle| cx^n\right)}{n} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*x**n)/x,x)
```

```
[Out] Piecewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**2/(2*
n), Abs(c*x**n) < 1), (log(1/(c*x**n))**2/(2*n), 1/Abs(c*x**n) < 1), (meije
rg(((), (1, 1, 1)), ((0, 0, 0), ()), c*x**n)/n + meijerg(((1, 1, 1), ()), (
(), (0, 0, 0)), c*x**n)/n, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(cx^n)}{x}\,dx = \frac{\log(cx^n)^2}{2\,n}$$

```
[In] integrate(log(c*x^n)/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^n)^2/n
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(cx^n)}{x}\,dx = \frac{1}{2}\,n\log(x)^2 + \log(c)\log(x)$$

```
[In] integrate(log(c*x^n)/x,x, algorithm="giac")
[Out] 1/2*n*log(x)^2 + log(c)*log(x)
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(cx^n)}{x}\,dx = \frac{\ln\left(c\,x^n\right)^2}{2\,n}$$

```
[In] int(log(c*x^n)/x,x)
[Out] log(c*x^n)^2/(2*n)
```

3.13 $\int \frac{\log(cx^n)}{x(ax^m+b\log^2(cx^n))}\,dx$

Optimal result 170
Rubi [N/A] 170
Mathematica [N/A] 171
Maple [N/A] 171
Fricas [N/A] 171
Sympy [N/A] 171
Maxima [N/A] 172
Giac [N/A] 172
Mupad [N/A] 172

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)}\,dx=\frac{\log\left(ax^m+b\log^2\left(cx^n\right)\right)}{2bn}-\frac{am\mathrm{Int}\left(\frac{x^{-1+m}}{ax^m+b\log^2(cx^n)},x\right)}{2bn}$$

```
[Out] -1/2*a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^2),x)/b/n+1/2*ln(a*x^m
+b*ln(c*x^n)^2)/b/n
```

Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)}\,dx=\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)}\,dx$$

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^2]/(2*b*n) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^2), x])/(2*b*n)
```

Rubi steps

$$\text{integral}=\frac{\log\left(ax^m+b\log^2\left(cx^n\right)\right)}{2bn}-\frac{(am)\int \frac{x^{-1+m}}{ax^m+b\log^2(cx^n)}\,dx}{2bn}$$

Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log (cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx=\int \frac{\log (cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx$$

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)), x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln (c\,x^n)}{x\left(a\,x^m+b\ln (c\,x^n)^2\right)}dx$$

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\log (cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx=\int \frac{\log (cx^n)}{\left(b\log (cx^n)^2+ax^m\right)x}\,dx$$

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="fricas")
[Out] integral(log(c*x^n)/(b*x*log(c*x^n)^2 + a*x*x^m), x)
```

Sympy [N/A]

Not integrable

Time = 5.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\log (cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx=\int \frac{\log (cx^n)}{x\left(ax^m+b\log (cx^n)^2\right)}\,dx$$

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2),x)
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))}\,dx = \int \frac{\log(cx^n)}{(b\log(cx^n)^2 + ax^m)x}\,dx$$

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="maxima")
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))}\,dx = \int \frac{\log(cx^n)}{(b\log(cx^n)^2 + ax^m)x}\,dx$$

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="giac")
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))}\,dx = \int \frac{\ln(c\,x^n)}{x\,\left(a\,x^m + b\ln(c\,x^n)^2\right)}\,dx$$

```
[In] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)),x)
[Out] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)), x)
```

3.14 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$

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Rubi [N/A] 173
Mathematica [N/A] 174
Maple [N/A] 174
Fricas [N/A] 174
Sympy [N/A] 175
Maxima [N/A] 175
Giac [N/A] 175
Mupad [N/A] 176

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^2}\,dx=-\frac{1}{2bn\left(ax^m+b\log^2\left(cx^n\right)\right)}-\frac{am\mathrm{Int}\left(\frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^2},x\right)}{2bn}$$

```
[Out] -1/2*a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^2)^2,x)/b/n-1/2/b/n/(a
*x^m+b*ln(c*x^n)^2)
```

Rubi [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^2}\,dx=\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^2}\,dx$$

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
```

```
[Out] -1/2*1/(b*n*(a*x^m + b*Log[c*x^n]^2)) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m +
 b*Log[c*x^n]^2)^2, x])/(2*b*n)
```

Rubi steps

$$\text{integral}=-\frac{1}{2bn\left(ax^m+b\log^2\left(cx^n\right)\right)}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^2}\,dx}{2bn}$$

Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx=\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$$

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2), x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\,x^n\right)}{x\left(a\,x^m+b\ln\left(c\,x^n\right)^2\right)^2}dx$$

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^2,x)
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx=\int \frac{\log(cx^n)}{\left(b\log(cx^n)^2+ax^m\right)^2 x}\,dx$$

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="fricas")
[Out] integral(log(c*x^n)/(b^2*x*log(c*x^n)^4 + 2*a*b*x*x^m*log(c*x^n)^2 + a^2*x*
x^(2*m)), x)
```

Sympy [N/A]

Not integrable

Time = 10.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))^2}\,dx = \int \frac{\log(cx^n)}{x(ax^m + b\log(cx^n)^2)^2}\,dx$$

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**2,x)
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 343, normalized size of antiderivative = 12.25

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))^2}\,dx = \int \frac{\log(cx^n)}{(b\log(cx^n)^2 + ax^m)^2 x}\,dx$$

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="maxima")
[Out] -(m*log(c) + m*log(x^n) + 2*n)/(4*b^2*n^2*log(c)^2 + a^2*m^2*x^(2*m) + (m^2
*log(c)^2 + 4*n^2)*a*b*x^m + (a*b*m^2*x^m + 4*b^2*n^2)*log(x^n)^2 + 2*(a*b*
m^2*x^m*log(c) + 4*b^2*n^2*log(c))*log(x^n)) - integrate((a*m^4*x^m*log(x^n
) + 4*b*m*n^3 + (m^4*log(c) + 3*m^3*n)*a*x^m)/(16*b^3*n^4*x*log(c)^2 + a^3*
m^4*x*x^(3*m) + (m^4*log(c)^2 + 8*m^2*n^2)*a^2*b*x*x^(2*m) + 8*(m^2*n^2*log
(c)^2 + 2*n^4)*a*b^2*x*x^m + (a^2*b*m^4*x*x^(2*m) + 8*a*b^2*m^2*n^2*x*x^m +
 16*b^3*n^4*x)*log(x^n)^2 + 2*(a^2*b*m^4*x*x^(2*m)*log(c) + 8*a*b^2*m^2*n^2
*x*x^m*log(c) + 16*b^3*n^4*x*log(c))*log(x^n)), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))^2}\,dx = \int \frac{\log(cx^n)}{(b\log(cx^n)^2 + ax^m)^2 x}\,dx$$

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="giac")
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx=\int \frac{\ln(c\,x^n)}{x\left(a\,x^m+b\ln(c\,x^n)^2\right)^2}\,dx$$

```
[In] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^2),x)
[Out] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^2), x)
```

3.15 $\int \frac{\log(cx^n)}{x(ax^m+b\log^2(cx^n))^3}\,dx$

Optimal result 177
Rubi [N/A] 177
Mathematica [N/A] 178
Maple [N/A] 178
Fricas [N/A] 178
Sympy [N/A] 179
Maxima [N/A] 179
Giac [N/A] 180
Mupad [N/A] 180

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx = -\frac{1}{4bn\left(ax^m+b\log^2(cx^n)\right)^2} - \frac{am\text{Int}\left(\frac{x^{-1+m}}{(ax^m+b\log^2(cx^n))^3},x\right)}{2bn}$$

```
[Out] -1/2*a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^2)^3,x)/b/n-1/4/b/n/(a
*x^m+b*ln(c*x^n)^2)^2
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx = \int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx$$

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
```

```
[Out] -1/4*1/(b*n*(a*x^m + b*Log[c*x^n]^2)^2) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m
 + b*Log[c*x^n]^2)^3, x])/(2*b*n)
```

Rubi steps

$$\text{integral} = -\frac{1}{4bn\left(ax^m+b\log^2(cx^n)\right)^2} - \frac{(am)\int \frac{x^{-1+m}}{(ax^m+b\log^2(cx^n))^3}\,dx}{2bn}$$

Mathematica [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x\left(ax^m + b\log^2(cx^n)\right)^3}\,dx = \int \frac{\log(cx^n)}{x\left(ax^m + b\log^2(cx^n)\right)^3}\,dx$$

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3), x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln(c\,x^n)}{x\left(a\,x^m + b\ln(c\,x^n)^2\right)^3}dx$$

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^3,x)
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \frac{\log(cx^n)}{x\left(ax^m + b\log^2(cx^n)\right)^3}\,dx = \int \frac{\log(cx^n)}{\left(b\log(cx^n)^2 + ax^m\right)^3 x}\,dx$$

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="fricas")
[Out] integral(log(c*x^n)/(b^3*x*log(c*x^n)^6 + 3*a*b^2*x*x^m*log(c*x^n)^4 + 3*a^
2*b*x*x^(2*m)*log(c*x^n)^2 + a^3*x*x^(3*m)), x)
```

Sympy [N/A]

Not integrable

Time = 15.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))^3} dx = \int \frac{\log(cx^n)}{x(ax^m + b\log(cx^n)^2)^3} dx$$

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**3,x)
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 1467, normalized size of antiderivative = 52.39

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))^3} dx = \int \frac{\log(cx^n)}{(b\log(cx^n)^2 + ax^m)^3 x} dx$$

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="maxima")
[Out] -1/2*(24*b^3*m*n^4*log(c)^3 - 5*a^3*m^4*n*x^(3*m) - (m^5*log(c)^3 + 7*m^4*n
*log(c)^2 - 18*m^3*n^2*log(c) - 4*m^2*n^3)*a^2*b*x^(2*m) + 2*(5*m^3*n^2*log
(c)^3 - 6*m^2*n^3*log(c)^2 + 20*m*n^4*log(c) + 16*n^5)*a*b^2*x^m - (a^2*b*m
^5*x^(2*m) - 10*a*b^2*m^3*n^2*x^m - 24*b^3*m*n^4)*log(x^n)^3 + (72*b^3*m*n^
4*log(c) - (3*m^5*log(c) + 7*m^4*n)*a^2*b*x^(2*m) + 6*(5*m^3*n^2*log(c) - 2
*m^2*n^3)*a*b^2*x^m)*log(x^n)^2 + (72*b^3*m*n^4*log(c)^2 - (3*m^5*log(c)^2
+ 14*m^4*n*log(c) - 18*m^3*n^2)*a^2*b*x^(2*m) + 2*(15*m^3*n^2*log(c)^2 - 12
*m^2*n^3*log(c) + 20*m*n^4)*a*b^2*x^m)*log(x^n))/(64*a*b^5*n^6*x^m*log(c)^4
 + a^6*m^6*x^(6*m) + 2*(m^6*log(c)^2 + 6*m^4*n^2)*a^5*b*x^(5*m) + (m^6*log(
c)^4 + 24*m^4*n^2*log(c)^2 + 48*m^2*n^4)*a^4*b^2*x^(4*m) + 4*(3*m^4*n^2*log
(c)^4 + 24*m^2*n^4*log(c)^2 + 16*n^6)*a^3*b^3*x^(3*m) + 16*(3*m^2*n^4*log(c
)^4 + 8*n^6*log(c)^2)*a^2*b^4*x^(2*m) + (a^4*b^2*m^6*x^(4*m) + 12*a^3*b^3*m
^4*n^2*x^(3*m) + 48*a^2*b^4*m^2*n^4*x^(2*m) + 64*a*b^5*n^6*x^m)*log(x^n)^4
+ 4*(a^4*b^2*m^6*x^(4*m)*log(c) + 12*a^3*b^3*m^4*n^2*x^(3*m)*log(c) + 48*a^
2*b^4*m^2*n^4*x^(2*m)*log(c) + 64*a*b^5*n^6*x^m*log(c))*log(x^n)^3 + 2*(192
*a*b^5*n^6*x^m*log(c)^2 + a^5*b*m^6*x^(5*m) + 3*(m^6*log(c)^2 + 4*m^4*n^2)*
a^4*b^2*x^(4*m) + 12*(3*m^4*n^2*log(c)^2 + 4*m^2*n^4)*a^3*b^3*x^(3*m) + 16*
(9*m^2*n^4*log(c)^2 + 4*n^6)*a^2*b^4*x^(2*m))*log(x^n)^2 + 4*(64*a*b^5*n^6*
x^m*log(c)^3 + a^5*b*m^6*x^(5*m)*log(c) + (m^6*log(c)^3 + 12*m^4*n^2*log(c)
)*a^4*b^2*x^(4*m) + 12*(m^4*n^2*log(c)^3 + 4*m^2*n^4*log(c))*a^3*b^3*x^(3*m
) + 16*(3*m^2*n^4*log(c)^3 + 4*n^6*log(c))*a^2*b^4*x^(2*m))*log(x^n)) + int
egrate(1/2*((2*m^8*log(c) + 15*m^7*n)*a^3*x^(3*m) - 2*(17*m^6*n^2*log(c) -
```

```
m^5*n^3)*a^2*b*x^(2*m) - 32*(3*m^4*n^4*log(c) + 2*m^3*n^5)*a*b^2*x^m - 96*(
m^2*n^6*log(c) + m*n^7)*b^3 + 2*(a^3*m^8*x^(3*m) - 17*a^2*b*m^6*n^2*x^(2*m)
 - 48*a*b^2*m^4*n^4*x^m - 48*b^3*m^2*n^6)*log(x^n))/(256*a*b^5*n^8*x*x^m*lo
g(c)^2 + a^6*m^8*x*x^(6*m) + (m^8*log(c)^2 + 16*m^6*n^2)*a^5*b*x*x^(5*m) +
16*(m^6*n^2*log(c)^2 + 6*m^4*n^4)*a^4*b^2*x*x^(4*m) + 32*(3*m^4*n^4*log(c)^
2 + 8*m^2*n^6)*a^3*b^3*x*x^(3*m) + 256*(m^2*n^6*log(c)^2 + n^8)*a^2*b^4*x*x
^(2*m) + (a^5*b*m^8*x*x^(5*m) + 16*a^4*b^2*m^6*n^2*x*x^(4*m) + 96*a^3*b^3*m
^4*n^4*x*x^(3*m) + 256*a^2*b^4*m^2*n^6*x*x^(2*m) + 256*a*b^5*n^8*x*x^m)*log
(x^n)^2 + 2*(a^5*b*m^8*x*x^(5*m)*log(c) + 16*a^4*b^2*m^6*n^2*x*x^(4*m)*log(
c) + 96*a^3*b^3*m^4*n^4*x*x^(3*m)*log(c) + 256*a^2*b^4*m^2*n^6*x*x^(2*m)*lo
g(c) + 256*a*b^5*n^8*x*x^m*log(c))*log(x^n)), x)
```

Giac [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))^3}\,dx = \int \frac{\log(cx^n)}{\left(b\log(cx^n)^2 + ax^m\right)^3 x}\,dx$$

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="giac")
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(cx^n)}{x(ax^m + b\log^2(cx^n))^3}\,dx = \int \frac{\ln(c\,x^n)}{x\left(a\,x^m + b\ln(c\,x^n)^2\right)^3}\,dx$$

```
[In] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^3),x)
[Out] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^3), x)
```

3.16 $\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x} dx$

Optimal result . . . 181
Rubi [A] (verified) . . . 181
Mathematica [A] (verified) . . . 182
Maple [F] . . . 182
Fricas [A] (verification not implemented) . . . 182
Sympy [F(-1)] . . . 183
Maxima [F(-2)] . . . 183
Giac [F(-2)] . . . 183
Mupad [F(-1)] . . . 184

Optimal result

Integrand size = 43, antiderivative size = 26

$$\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x} dx = \frac{(ax^m+b\log^q(cx^n))^{1+p}}{1+p}$$

```
[Out] (a*x^m+b*ln(c*x^n)^q)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2624}

$$\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x} dx = \frac{(ax^m+b\log^q(cx^n))^{p+1}}{p+1}$$

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\text{integral} = \frac{(ax^m+b\log^q(cx^n))^{1+p}}{1+p}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\left(amx^m + bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \frac{\left(ax^m + b\log^q\left(cx^n\right)\right)^{1+p}}{1+p}$$

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p
)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Maple [F]

$$\int \frac{\left(am\,x^m + bnq\ln\left(c\,x^n\right)^{-1+q}\right)\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^p}{x}dx$$

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{\left(amx^m + bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \frac{\left(\left(n\log\left(x\right) + \log\left(c\right)\right)^q b + ax^m\right)\left(\left(n\log\left(x\right) + \log\left(c\right)\right)^q b + ax^m\right)^p}{p+1}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="fricas")
```

```
[Out] ((n*log(x) + log(c))^q*b + a*x^m)*((n*log(x) + log(c))^q*b + a*x^m)^p/(p +
1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(amx^m + bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/x
,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(amx^m + bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(amx^m + bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,2,5,2,0,5,0,3,1,2,3]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,2,1,2,3]%%%}+%%%{5,[0,0,2
,4,2,0,4,
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^p}{x}\,dx$$
$$= \int \frac{\left(a\,m\,x^m + b\,n\,q\ln\left(c\,x^n\right)^{q-1}\right)\,\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^p}{x}\,dx$$

```
[In] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p)/x,x)
[Out] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p)/x, x)
```

3.17 $\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^2}{x}\,dx$

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Rubi [A] (verified) 185
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Maple [C] (warning: unable to verify) 186
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Giac [F] 187
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Optimal result

Integrand size = 43, antiderivative size = 22

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^2}{x}\,dx=\frac{1}{3}\left(ax^m+b\log^q\left(cx^n\right)\right)^3$$

```
[Out] 1/3*(a*x^m+b*ln(c*x^n)^q)^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2624}

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^2}{x}\,dx=\frac{1}{3}\left(ax^m+b\log^q\left(cx^n\right)\right)^3$$

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^3/3
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\text{integral}=\frac{1}{3}\left(ax^m+b\log^q\left(cx^n\right)\right)^3$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx = \frac{1}{3}(ax^m + b\log^q(cx^n))^3$$

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2
)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^3/3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 9.27

$$\frac{a^3x^{3m}}{3} + \frac{b^3\left(\ln(c) + \ln(x^n) - \frac{i\pi\,\mathrm{csgn}(icx^n)(-\mathrm{csgn}(icx^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(icx^n)+\mathrm{csgn}(ix^n))}{2}\right)^{3q}}{3} + a\,b^2x^m\left(\ln(c) + \ln(x^n) -\right.$$

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] 1/3*a^3*(x^m)^3+1/3*b^3*((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^3+a*b^2*x^m*((ln(c)+ln(x^n)
-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x
^n)))^q)^2+a^2*b*(x^m)^2*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(20) = 40.

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\begin{aligned}&\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx\\ &= (n\log(x) + \log(c))^q a^2bx^{2m} + (n\log(x) + \log(c))^{2q}ab^2x^m\\ &\quad + \frac{1}{3}(n\log(x) + \log(c))^{3q}b^3 + \frac{1}{3}a^3x^{3m}\end{aligned}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="fricas")
```

```
[Out] (n*log(x) + log(c))^q*a^2*b*x^(2*m) + (n*log(x) + log(c))^(2*q)*a*b^2*x^m +
 1/3*(n*log(x) + log(c))^(3*q)*b^3 + 1/3*a^3*x^(3*m)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(17) = 34$.

Time = 66.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx = \frac{a^3x^{3m}}{3} + a^2bx^{2m}\log(cx^n)^q + ab^2x^m\log(cx^n)^{2q} + \frac{b^3\log(cx^n)^{3q}}{3}$$

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/x
,x)
```

```
[Out] a**3*x**(3*m)/3 + a**2*b*x**(2*m)*log(c*x**n)**q + a*b**2*x**m*log(c*x**n)*
*(2*q) + b**3*log(c*x**n)**(3*q)/3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx = \int \frac{(bnq\log(cx^n)^{q-1} + amx^m)(ax^m + b\log(cx^n)^q)^2}{x}\,dx$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)^2/x
, x)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(amx^m + bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^2}{x}\,dx = \frac{\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^3}{3}$$

```
[In] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^2)/x,x)
[Out] (a*x^m + b*log(c*x^n)^q)^3/3
```

3.18 $\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx$

Optimal result . . . 189
Rubi [A] (verified) . . . 189
Mathematica [A] (verified) . . . 190
Maple [C] (warning: unable to verify) . . . 190
Fricas [B] (verification not implemented) . . . 190
Sympy [B] (verification not implemented) . . . 191
Maxima [F(-2)] . . . 191
Giac [F] . . . 191
Mupad [B] (verification not implemented) . . . 192

Optimal result

Integrand size = 41, antiderivative size = 22

$$\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx=\frac{1}{2}(ax^m+b\log^q(cx^n))^2$$

```
[Out] 1/2*(a*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2624}

$$\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx=\frac{1}{2}(ax^m+b\log^q(cx^n))^2$$

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
[Out] (a*x^m + b*Log[c*x^n]^q)^2/2
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\text{integral}=\frac{1}{2}(ax^m+b\log^q(cx^n))^2$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))}{x}\,dx = \frac{1}{2}(ax^m + b\log^q(cx^n))^2$$

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/
x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^2/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 187.63 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.14

method	result
risch	$\frac{a^2x^{2m}}{2}+\frac{b^2\left(\ln(c)+\ln(x^n)-\frac{i\pi\,\mathrm{csgn}(icx^n)(-\mathrm{csgn}(icx^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(icx^n)+\mathrm{csgn}(ix^n))}{2}\right)^{2q}}{2}+ab\,x^m\left(\ln(c)+\ln(x^n)-\right.$

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2*a^2*(x^m)^2+1/2*b^2*((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^2+a*b*x^m*(ln(c)+ln(x^n)-1/
2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)
))^q
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))}{x}\,dx$$
$$= (n\log(x) + \log(c))^q abx^m + \frac{1}{2}(n\log(x) + \log(c))^{2q}b^2 + \frac{1}{2}a^2x^{2m}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="fricas")
```

```
[Out] (n*log(x) + log(c))^q*a*b*x^m + 1/2*(n*log(x) + log(c))^(2*q)*b^2 + 1/2*a^2
*x^(2*m)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 20.87 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{\left(amx^m + bnq\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)}{x}\,dx = \frac{a^2x^{2m}}{2} + abx^m\log(cx^n)^q + \frac{b^2\log(cx^n)^{2q}}{2}$$

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] a**2*x**(2*m)/2 + a*b*x**m*log(c*x**n)**q + b**2*log(c*x**n)**(2*q)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(amx^m + bnq\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{\left(amx^m + bnq\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)}{x}\,dx = \int \frac{\left(bnq\log(cx^n)^{q-1} + amx^m\right)\left(ax^m + b\log(cx^n)^q\right)}{x}\,dx$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)/x,
x)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))\,(ax^m + b\log^q(cx^n))}{x}\,dx = \frac{(a\,x^m + b\ln(c\,x^n)^q)^2}{2}$$

```
[In] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q))/x,x)
[Out] (a*x^m + b*log(c*x^n)^q)^2/2
```

3.19 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x}\,dx$

Optimal result 193
Rubi [A] (verified) 193
Mathematica [A] (verified) 194
Maple [A] (verified) 194
Fricas [A] (verification not implemented) 195
Sympy [A] (verification not implemented) 195
Maxima [A] (verification not implemented) 195
Giac [A] (verification not implemented) 196
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Optimal result

Integrand size = 25, antiderivative size = 16

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x}\,dx = ax^m+b\log^q(cx^n)$$

```
[Out] a*x^m+b*ln(c*x^n)^q
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {14, 2339, 30}

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x}\,dx = ax^m+b\log^q(cx^n)$$

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] a*x^m + b*Log[c*x^n]^q
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(amx^{-1+m} + \frac{bnq\log^{-1+q}(cx^n)}{x}\right) dx\\ &= ax^m + (bnq)\int \frac{\log^{-1+q}(cx^n)}{x}\,dx\\ &= ax^m + (bq)\text{Subst}\left(\int x^{-1+q}\,dx, x, \log(cx^n)\right)\\ &= ax^m + b\log^q(cx^n)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = ax^m + b\log^q(cx^n)$$

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
[Out] a*x^m + b*Log[c*x^n]^q
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
default	$a\,x^m + b\ln(c\,x^n)^q$
parallelrisch	$\ln(c\,x^n)\ln(c\,x^n)^{-1+q}\,b + a\,x^m$
risch	$a\,x^m + b\Big(\ln(c) + \ln(x^n) - \frac{i\pi\ \mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ix^n))}{2}\Big)^{-1+q}\Big(\ln(c) + \mathrm{l}$

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x,x,method=_RETURNVERBOSE)
[Out] a*x^m+b*ln(c*x^n)^q
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = (bn\log(x) + b\log(c))(n\log(x) + \log(c))^{q-1} + ax^m$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
[Out] (b*n*log(x) + b*log(c))*(n*log(x) + log(c))^(q - 1) + a*x^m
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = -am\left(\begin{cases} -\log(x) & \text{for } m = 0 \\ -\frac{x^m}{m} & \text{otherwise} \end{cases}\right) + bnq\left(\begin{cases} \log(c)^{q-1}\log(x) & \text{for } n = 0 \\ \frac{\begin{cases} \frac{\log(cx^n)^q}{q} & \text{for } q \neq 0 \\ \log(\log(cx^n)) & \text{otherwise} \end{cases}}{n} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x,x)
[Out] -a*m*Piecewise((-log(x), Eq(m, 0)), (-x**m/m, True)) + b*n*q*Piecewise((log
(c)**(q - 1)*log(x), Eq(n, 0)), (Piecewise((log(c*x**n)**q/q, Ne(q, 0)), (l
og(log(c*x**n)), True))/n, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = ax^m + b\log(cx^n)^q$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
[Out] a*x^m + b*log(c*x^n)^q
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = (n\log(x) + \log(c))^q b + ax^m$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
[Out] (n*log(x) + log(c))^q*b + a*x^m
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = a\,x^m + b\ln(c\,x^n)^q$$

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/x,x)
[Out] a*x^m + b*log(c*x^n)^q
```

3.20 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$

Optimal result . . . 197
Rubi [A] (verified) . . . 197
Mathematica [A] (verified) . . . 198
Maple [C] (warning: unable to verify) . . . 198
Fricas [A] (verification not implemented) . . . 198
Sympy [F(-1)] . . . 199
Maxima [A] (verification not implemented) . . . 199
Giac [F] . . . 199
Mupad [F(-1)] . . . 199

Optimal result

Integrand size = 43, antiderivative size = 17

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx=\log\left(ax^m+b\log^q(cx^n)\right)$$

```
[Out] ln(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2621}

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx=\log\left(ax^m+b\log^q(cx^n)\right)$$

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]
```

Rule 2621

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rubi steps

$$\text{integral}=\log\left(ax^m+b\log^q(cx^n)\right)$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \log(ax^m + b\log^q(cx^n))$$

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.58 (sec) , antiderivative size = 213, normalized size of antiderivative = 12.53

method	result
risch	$q\ln\left(\ln(x^n) - \frac{i\left(\pi\ \mathrm{csgn}(icx^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n) - \pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2 - \pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2 + \pi\,\mathrm{csgn}(icx^n)^3 + 2i\ln(c)\right)}{2}\right.$

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x,method=_RETU
RNVERBOSE)
```

```
[Out] q*ln(ln(x^n)-1/2*I*(Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)-Pi*csgn(I*c)*csg
n(I*c*x^n)^2-Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+Pi*csgn(I*c*x^n)^3+2*I*ln(c)))-
q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn
(I*c*x^n)+csgn(I*x^n)))+ln((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c
*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q+a*x^m/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \log\left((n\log(x) + \log(c))^q b + ax^m\right)$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="fricas")
```

```
[Out] log((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \text{Timed out}$$

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \log\left(\frac{ax^m + b(\log(c) + \log(x^n))^q}{b}\right)$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="maxima")
[Out] log((a*x^m + b*(log(c) + log(x^n))^q)/b)
```

Giac [F]

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{bnq\log(cx^n)^{q-1} + amx^m}{(ax^m + b\log(cx^n)^q)x}\,dx$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="giac")
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)*x)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{a\,m\,x^m + b\,n\,q\,\ln(c\,x^n)^{q-1}}{x\,\left(a\,x^m + b\,\ln(c\,x^n)^q\right)}\,dx$$

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)),x)
[Out] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

3.21 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal result 200
Rubi [A] (verified) 200
Mathematica [A] (verified) 201
Maple [C] (warning: unable to verify) 201
Fricas [A] (verification not implemented) 201
Sympy [F(-1)] 202
Maxima [A] (verification not implemented) 202
Giac [F] 202
Mupad [F(-1)] 203

Optimal result

Integrand size = 43, antiderivative size = 20

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx=-\frac{1}{ax^m+b\log^q(cx^n)}$$

```
[Out] -1/(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2624}

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx=-\frac{1}{ax^m+b\log^q(cx^n)}$$

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
[Out] -(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\text{integral}=-\frac{1}{ax^m+b\log^q(cx^n)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = -\frac{1}{ax^m + b\log^q(cx^n)}$$

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
^2),x]
```

```
[Out] -(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 15.77 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.40

method	result	size
risch	$-\frac{1}{a\,x^{m}+b\left(\ln(c)+\ln(x^{n})-\frac{i\pi\ \mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ix^{n}))}{2}\right)^{q}}$	68

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c)
)*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = -\frac{1}{(n\log(x) + \log(c))^q b + ax^m}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="fricas")
```

```
[Out] -1/((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**2
,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx = -\frac{1}{ax^m + b(\log(c) + \log(x^n))^q}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="maxima")
```

```
[Out] -1/(a*x^m + b*(log(c) + log(x^n))^q)
```

Giac [F]

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx = \int \frac{bnq\log(cx^n)^{q-1} + amx^m}{(ax^m + b\log(cx^n)^q)^2 x}\,dx$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^2*
x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{a\,m\,x^m + b\,n\,q\ln\left(c\,x^n\right)^{q-1}}{x\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^2}\,dx$$

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
[Out] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

3.22 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$

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Optimal result

Integrand size = 43, antiderivative size = 22

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx=-\frac{1}{2(ax^m+b\log^q(cx^n))^2}$$

```
[Out] -1/2/(a*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2624}

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx=-\frac{1}{2(ax^m+b\log^q(cx^n))^2}$$

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
[Out] -1/2*1/(a*x^m + b*Log[c*x^n]^q)^2
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\text{integral}=-\frac{1}{2(ax^m+b\log^q(cx^n))^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = -\frac{1}{2\left(ax^m + b\log^q(cx^n)\right)^2}$$

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
^3),x]
```

```
[Out] -1/2*1/(a*x^m + b*Log[c*x^n]^q)^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 116.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

method	result	size
risch	$-\frac{1}{2\left(a\,x^{m}+b\left(\ln(c)+\ln(x^{n})-\frac{i\pi\,\operatorname{csgn}(ic\,x^{n})(-\operatorname{csgn}(ic\,x^{n})+\operatorname{csgn}(ic))(-\operatorname{csgn}(ic\,x^{n})+\operatorname{csgn}(ix^{n}))}{2}\right)^{q}\right)^{2}}$	68

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*
c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(20) = 40.

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = -\frac{1}{2\left(2\left(n\log(x) + \log(c)\right)^q abx^m + \left(n\log(x) + \log(c)\right)^{2q}b^2 + a^2x^{2m}\right)}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="fricas")
```

```
[Out] -1/2/(2*(n*log(x) + log(c))^q*a*b*x^m + (n*log(x) + log(c))^(2*q)*b^2 + a^2
*x^(2*m))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**3
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(20) = 40$.

Time = 0.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = -\frac{1}{2\left(a^2x^{2m} + b^2(\log(c) + \log(x^n))^{2q} + 2abe^{(m\log(x) + q\log(\log(c) + \log(x^n)))}\right)}$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="maxima")
```

```
[Out] -1/2/(a^2*x^(2*m) + b^2*(log(c) + log(x^n))^(2*q) + 2*a*b*e^(m*log(x) + q*l
og(log(c) + log(x^n))))
```

Giac [F]

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \int \frac{bnq\log(cx^n)^{q-1} + amx^m}{\left(ax^m + b\log(cx^n)^q\right)^3 x}\,dx$$

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^3*
x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \int \frac{a\,m\,x^m + b\,n\,q\ln\left(c\,x^n\right)^{q-1}}{x\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^3}\,dx$$

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
[Out] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

3.23 $\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx$

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Optimal result

Integrand size = 39, antiderivative size = 20

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx=\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

```
[Out] 1/3*(a*x+b*ln(c*x^n)^2)^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2641, 2624}

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx=\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

```
[In] Int[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^3/3
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{(ax + 2bn\log(cx^n))\left(ax^2 + bx\log^2(cx^n)\right)^2}{x^3}\,dx$$
$$= \int \frac{(ax + 2bn\log(cx^n))\left(ax + b\log^2(cx^n)\right)^2}{x}\,dx$$
$$= \frac{1}{3}\left(ax + b\log^2(cx^n)\right)^3$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x^2} + \frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2 + bx\log^2(cx^n)\right)^2\,dx = \frac{1}{3}\left(ax + b\log^2(cx^n)\right)^3$$

```
[In] Integrate[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
[Out] (a*x + b*Log[c*x^n]^2)^3/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.

Time = 2.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

method	result	size
parallelrisch	$\frac{b^3\ln(c\,x^n)^6}{3} + ax\,b^2\ln(c\,x^n)^4 + a^2x^2b\ln(c\,x^n)^2 + \frac{a^3x^3}{3}$	53
risch	Expression too large to display	20850

```
[In] int((1/x^2*a+2*b*n*ln(c*x^n)/x^3)*(x^2*a+b*x*ln(c*x^n)^2)^2,x,method=_RETUR
NVERBOSE)
[Out] 1/3*b^3*ln(c*x^n)^6+a*x*b^2*ln(c*x^n)^4+a^2*x^2*b*ln(c*x^n)^2+1/3*a^3*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(18) = 36$.

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 9.75

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx$$
$$=\frac{1}{3}b^3n^6\log(x)^6+2\,b^3n^5\log(c)\log(x)^5+ab^2x\log(c)^4+a^2bx^2\log(c)^2+\frac{1}{3}a^3x^3$$
$$+\left(5\,b^3n^4\log(c)^2+ab^2n^4x\right)\log(x)^4+\frac{4}{3}\left(5\,b^3n^3\log(c)^3+3\,ab^2n^3x\log(c)\right)\log(x)^3$$
$$+\left(5\,b^3n^2\log(c)^4+6\,ab^2n^2x\log(c)^2+a^2bn^2x^2\right)\log(x)^2$$
$$+2\left(b^3n\log(c)^5+2\,ab^2nx\log(c)^3+a^2bnx^2\log(c)\right)\log(x)$$

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="fricas")
```

```
[Out] 1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + a*b^2*x*log(c)^4 + a^2*b
*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)^2 + a*b^2*n^4*x)*log(x)^4 +
 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c))*log(x)^3 + (5*b^3*n^2*log(
c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2)*log(x)^2 + 2*(b^3*n*log(c)^5
 + 2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx$$
$$=\frac{a^3x^3}{3}+a^2bx^2\log(cx^n)^2+ab^2x\log(cx^n)^4-2b^3n\left(\begin{cases}-\log(c)^5\log(x) & \text{for } n=0\\ -\frac{\log(cx^n)^6}{6n} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a/x**2+2*b*n*ln(c*x**n)/x**3)*(a*x**2+b*x*ln(c*x**n)**2)**2,x)
```

```
[Out] a**3*x**3/3 + a**2*b*x**2*log(c*x**n)**2 + a*b**2*x*log(c*x**n)**4 - 2*b**3
*n*Piecewise((-log(c)**5*log(x), Eq(n, 0)), (-log(c*x**n)**6/(6*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(18) = 36$.

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 10.55

$$\begin{aligned}&\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx\\&=\frac{1}{3}\,b^3\log(cx^n)^6+4\,ab^2nx\log(cx^n)^3+ab^2x\log(cx^n)^4\\&-\frac{1}{2}\,a^2bn^2x^2+a^2bnx^2\log(cx^n)+a^2bx^2\log(cx^n)^2+\frac{1}{3}\,a^3x^3\\&-12\left(nx\log(cx^n)^2+2\left(n^2x-nx\log(cx^n)\right)n\right)ab^2n+\frac{1}{2}\left(n^2x^2-2\,nx^2\log(cx^n)\right)a^2b\\&-4\left(nx\log(cx^n)^3-3\left(nx\log(cx^n)^2+2\left(n^2x-nx\log(cx^n)\right)n\right)n\right)ab^2\end{aligned}$$

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="maxima")
```

```
[Out] 1/3*b^3*log(c*x^n)^6 + 4*a*b^2*n*x*log(c*x^n)^3 + a*b^2*x*log(c*x^n)^4 - 1/
2*a^2*b*n^2*x^2 + a^2*b*n*x^2*log(c*x^n) + a^2*b*x^2*log(c*x^n)^2 + 1/3*a^3
*x^3 - 12*(n*x*log(c*x^n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*a*b^2*n + 1/2*(
n^2*x^2 - 2*n*x^2*log(c*x^n))*a^2*b - 4*(n*x*log(c*x^n)^3 - 3*(n*x*log(c*x^
n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*n)*a*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(18) = 36$.

Time = 0.36 (sec) , antiderivative size = 198, normalized size of antiderivative = 9.90

$$\begin{aligned}&\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx\\&=\frac{1}{3}\,b^3n^6\log(x)^6+2\,b^3n^5\log(c)\log(x)^5+2\,b^3n\log(c)^5\log(x)+ab^2x\log(c)^4\\&+a^2bx^2\log(c)^2+\frac{1}{3}\,a^3x^3+\left(5\,b^3n^4\log(c)^2+ab^2n^4x\right)\log(x)^4\\&+\frac{4}{3}\left(5\,b^3n^3\log(c)^3+3\,ab^2n^3x\log(c)\right)\log(x)^3\\&+\left(5\,b^3n^2\log(c)^4+6\,ab^2n^2x\log(c)^2+a^2bn^2x^2\right)\log(x)^2\\&+2\left(2\,ab^2nx\log(c)^3+a^2bnx^2\log(c)\right)\log(x)\end{aligned}$$

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="giac")
```

```
[Out] 1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + 2*b^3*n*log(c)^5*log(x)
+ a*b^2*x*log(c)^4 + a^2*b*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)^2
```

```
 + a*b^2*n^4*x)*log(x)^4 + 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c))*
log(x)^3 + (5*b^3*n^2*log(c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2)*lo
g(x)^2 + 2*(2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx$$
$$=\frac{a^3\,x^3}{3}+a^2\,b\,x^2\ln\left(c\,x^n\right)^2+a\,b^2\,x\ln\left(c\,x^n\right)^4+\frac{b^3\ln\left(c\,x^n\right)^6}{3}$$

```
[In] int((a*x^2 + b*x*log(c*x^n)^2)^2*(a/x^2 + (2*b*n*log(c*x^n))/x^3),x)
```

```
[Out] (b^3*log(c*x^n)^6)/3 + (a^3*x^3)/3 + a^2*b*x^2*log(c*x^n)^2 + a*b^2*x*log(c
*x^n)^4
```

3.24 $\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2(cx^n)\right)\,dx$

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Giac [B] (verification not implemented) 216
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Optimal result

Integrand size = 37, antiderivative size = 20

$$\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2(cx^n)\right)\,dx=\frac{1}{2}\left(ax+b\log^2(cx^n)\right)^2$$

```
[Out] 1/2*(a*x+b*ln(c*x^n)^2)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2641, 2624}

$$\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2(cx^n)\right)\,dx=\frac{1}{2}\left(ax+b\log^2(cx^n)\right)^2$$

```
[In] Int[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^2/2
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(ax + 2bn\log(cx^n))\left(ax^2 + bx\log^2(cx^n)\right)}{x^2}\,dx \\ &= \int \frac{(ax + 2bn\log(cx^n))\left(ax + b\log^2(cx^n)\right)}{x}\,dx \\ &= \frac{1}{2}\left(ax + b\log^2(cx^n)\right)^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \left(\frac{a}{x} + \frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2 + bx\log^2(cx^n)\right)\,dx = \frac{a^2x^2}{2} + abx\log^2(cx^n) + \frac{1}{2}b^2\log^4(cx^n)$$

```
[In] Integrate[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
[Out] (a^2*x^2)/2 + a*b*x*Log[c*x^n]^2 + (b^2*Log[c*x^n]^4)/2
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

method	result	size
parallelrisch	$\frac{b^2\ln(cx^n)^4}{2} + axb\ln(cx^n)^2 + \frac{x^2a^2}{2}$	35
default	$\frac{x^2a^2}{2} + abx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2 - 2abnx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right) + \frac{b^2\ln(cx^n)^4}{2} + 2\ln(cx^n)\,abnx$	63
parts	$\frac{x^2a^2}{2} + abx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2 - 2abnx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right) + \frac{b^2\ln(cx^n)^4}{2} + 2\ln(cx^n)\,abnx$	63
risch	Expression too large to display	2698

```
[In] int((a/x+2*b*n*ln(c*x^n)/x^2)*(x^2*a+b*x*ln(c*x^n)^2),x,method=_RETURNVERBO
SE)
[Out] 1/2*b^2*ln(c*x^n)^4+a*x*b*ln(c*x^n)^2+1/2*x^2*a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(18) = 36$.

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.45

$$\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx$$
$$=\frac{1}{2}\,b^2n^4\log\left(x\right)^4+2\,b^2n^3\log\left(c\right)\log\left(x\right)^3+abx\log\left(c\right)^2+\frac{1}{2}\,a^2x^2$$
$$+\left(3\,b^2n^2\log\left(c\right)^2+abn^2x\right)\log\left(x\right)^2+2\,\left(b^2n\log\left(c\right)^3+abnx\log\left(c\right)\right)\log\left(x\right)$$

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"fricas")
```

```
[Out] 1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + a*b*x*log(c)^2 + 1/2*a^2
*x^2 + (3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 2*(b^2*n*log(c)^3 + a*b*
n*x*log(c))*log(x)
```

Sympy [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx$$
$$=\frac{a^2x^2}{2}+abx\log\left(cx^n\right)^2-2b^2n\left(\begin{cases}-\log\left(c\right)^3\log\left(x\right) & \text{for}\ n=0\\ -\frac{\log\left(cx^n\right)^4}{4n} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a/x+2*b*n*ln(c*x**n)/x**2)*(a*x**2+b*x*ln(c*x**n)**2),x)
```

```
[Out] a**2*x**2/2 + a*b*x*log(c*x**n)**2 - 2*b**2*n*Piecewise((-log(c)**3*log(x),
 Eq(n, 0)), (-log(c*x**n)**4/(4*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(18) = 36$.

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.70

$$\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx$$
$$=\frac{1}{2}\,b^2\log\left(cx^n\right)^4-2\,abn^2x+2\,abnx\log\left(cx^n\right)$$
$$+abx\log\left(cx^n\right)^2+\frac{1}{2}\,a^2x^2+2\,\left(n^2x-nx\log\left(cx^n\right)\right)ab$$

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"maxima")
```

```
[Out] 1/2*b^2*log(c*x^n)^4 - 2*a*b*n^2*x + 2*a*b*n*x*log(c*x^n) + a*b*x*log(c*x^n
)^2 + 1/2*a^2*x^2 + 2*(n^2*x - n*x*log(c*x^n))*a*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.50

$$\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2(cx^n)\right)\,dx$$
$$=\frac{1}{2}\,b^2n^4\log(x)^4+2\,b^2n^3\log(c)\log(x)^3+2\,b^2n\log(c)^3\log(x)$$
$$+2\,abnx\log(c)\log(x)+abx\log(c)^2+\frac{1}{2}\,a^2x^2+\left(3\,b^2n^2\log(c)^2+abn^2x\right)\log(x)^2$$

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"giac")
```

```
[Out] 1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + 2*b^2*n*log(c)^3*log(x)
+ 2*a*b*n*x*log(c)*log(x) + a*b*x*log(c)^2 + 1/2*a^2*x^2 + (3*b^2*n^2*log(c
)^2 + a*b*n^2*x)*log(x)^2
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2(cx^n)\right)\,dx=\frac{\left(b\ln\left(c\,x^n\right)^2+a\,x\right)^2}{2}$$

```
[In] int((a*x^2 + b*x*log(c*x^n)^2)*(a/x + (2*b*n*log(c*x^n))/x^2),x)
```

```
[Out] (a*x + b*log(c*x^n)^2)^2/2
```

3.25 $\int \left(a+\frac{2bn\log(cx^n)}{x}\right) dx$

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Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \left(a+\frac{2bn\log(cx^n)}{x}\right) dx = ax+b\log^2(cx^n)$$

```
[Out] a*x+b*ln(c*x^n)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2338}

$$\int \left(a+\frac{2bn\log(cx^n)}{x}\right) dx = ax+b\log^2(cx^n)$$

```
[In] Int[a + (2*b*n*Log[c*x^n])/x,x]
```

```
[Out] a*x + b*Log[c*x^n]^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax+(2bn)\int \frac{\log(cx^n)}{x}\,dx\\ &= ax+b\log^2(cx^n)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)\,dx=ax+b\log^2\left(cx^n\right)$$

[In] Integrate[a + (2*b*n*Log[c*x^n])/x,x]

[Out] a*x + b*Log[c*x^n]^2

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
default	$ax+b\ln\left(c\,x^n\right)^2$
parts	$ax+b\ln\left(c\,x^n\right)^2$
norman	$ax+b\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2$
risch	$ax+2bn\ln\left(x\right)\ln\left(x^n\right)-b\,n^2\ln\left(x\right)^2-ibn\pi\ln\left(x\right)\operatorname{csgn}\left(ic\right)\operatorname{csgn}\left(ix^n\right)\operatorname{csgn}\left(ic\,x^n\right)+ibn\pi\ln\left(x\right)\operatorname{csg}$

[In] int(a+2*b*n*ln(c*x^n)/x,x,method=_RETURNVERBOSE)

[Out] a*x+b*ln(c*x^n)^2

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)\,dx=bn^2\log\left(x\right)^2+2\,bn\log\left(c\right)\log\left(x\right)+ax$$

[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="fricas")

[Out] b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + a*x

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 5.21

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)dx = ax + 2bn\left(\begin{cases} 0 & \text{for } \frac{1}{|cx^n|}<1 \wedge |cx^n|<1 \\ \frac{\log(cx^n)^2}{2n} & \text{for } |cx^n|<1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for } \frac{1}{|cx^n|}<1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(a+2*b*n*ln(c*x**n)/x,x)
[Out] a*x + 2*b*n*Piecewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*
x**n)**2/(2*n), Abs(c*x**n) < 1), (log(1/(c*x**n))**2/(2*n), 1/Abs(c*x**n)
< 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0), ()), c*x**n)/n + meijerg(((1, 1
, 1), ()), ((), (0, 0, 0)), c*x**n)/n, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)dx = b\log\left(cx^n\right)^2 + ax$$

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="maxima")
[Out] b*log(c*x^n)^2 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)dx = \left(n\log\left(x\right)^2 + 2\log\left(c\right)\log\left(x\right)\right)bn + ax$$

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="giac")
[Out] (n*log(x)^2 + 2*log(c)*log(x))*b*n + a*x
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{2bn\log\left(cx^n\right)}{x}\right)\,dx=b\ln\left(c\,x^n\right)^2+a\,x$$

```
[In] int(a + (2*b*n*log(c*x^n))/x,x)
[Out] a*x + b*log(c*x^n)^2
```

3.26 $\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx$

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Maple [A] (verified) 222
Fricas [A] (verification not implemented) 222
Sympy [A] (verification not implemented) 223
Maxima [B] (verification not implemented) 223
Giac [A] (verification not implemented) 223
Mupad [B] (verification not implemented) 224

Optimal result

Integrand size = 34, antiderivative size = 15

$$\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx=\log\left(ax+b\log^2(cx^n)\right)$$

```
[Out] ln(a*x+b*ln(c*x^n)^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2641, 2621}

$$\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx=\log\left(ax+b\log^2(cx^n)\right)$$

```
[In] Int[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] Log[a*x + b*Log[c*x^n]^2]
```

Rule 2621

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
```

```
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{ax + 2bn\log(cx^n)}{x\left(ax + b\log^2(cx^n)\right)}\,dx$$
$$= \log\left(ax + b\log^2(cx^n)\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{ax + 2bn\log(cx^n)}{ax^2 + bx\log^2(cx^n)}\,dx = \log\left(ax + b\log^2(cx^n)\right)$$

```
[In] Integrate[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
[Out] Log[a*x + b*Log[c*x^n]^2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\ln\left(ax + b\ln(c\,x^n)^2\right)$
risch	$\ln\Big(\ln(x^n)^2 + \big(-i\pi\ \mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n) + i\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic) + i\pi\,\mathrm{csgn}(ic\,x$

```
[In] int((a*x+2*b*n*ln(c*x^n))/(x^2*a+b*x*ln(c*x^n)^2),x,method=_RETURNVERBOSE)
[Out] ln(a*x+b*ln(c*x^n)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{ax + 2bn\log(cx^n)}{ax^2 + bx\log^2(cx^n)}\,dx = \log\left(bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + b\log(c)^2 + ax\right)$$

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="fri
cas")
[Out] log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{ax + 2bn \log(cx^n)}{ax^2 + bx \log^2(cx^n)}\, dx = \begin{cases} \log\left(x + \frac{b \log(cx^n)^2}{a}\right) & \text{for } a \neq 0 \\ 2 \log(\log(cx^n)) & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x+2*b*n*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2),x)
```

```
[Out] Piecewise((log(x + b*log(c*x**n)**2/a), Ne(a, 0)), (2*log(log(c*x**n)), Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{ax + 2bn \log(cx^n)}{ax^2 + bx \log^2(cx^n)}\, dx = \log\left(\frac{b \log(c)^2 + 2\, b \log(c) \log(x^n) + b \log(x^n)^2 + ax}{b}\right)$$

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="max
ima")
```

```
[Out] log((b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)/b)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{ax + 2bn \log(cx^n)}{ax^2 + bx \log^2(cx^n)}\, dx = \log\left(bn^2 \log(x)^2 + 2\, bn \log(c) \log(x) + b \log(c)^2 + ax\right)$$

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="gia
c")
```

```
[Out] log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{ax + 2bn\log(cx^n)}{ax^2 + bx\log^2(cx^n)}\,dx = \ln\left(\ln(c\,x^n)^2 + \frac{a\,x}{b}\right)$$

```
[In] int((a*x + 2*b*n*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2),x)
[Out] log(log(c*x^n)^2 + (a*x)/b)
```

3.27 $\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx$

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Optimal result

Integrand size = 37, antiderivative size = 18

$$\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx=-\frac{1}{ax+b\log^2(cx^n)}$$

```
[Out] -1/(a*x+b*ln(c*x^n)^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2641, 2624}

$$\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx=-\frac{1}{ax+b\log^2(cx^n)}$$

```
[In] Int[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] -(a*x + b*Log[c*x^n]^2)^(-1)
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(ax+2bn\log(cx^n))}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx\\ &= \int \frac{ax+2bn\log(cx^n)}{x\left(ax+b\log^2(cx^n)\right)^2}\,dx\\ &= -\frac{1}{ax+b\log^2(cx^n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx = -\frac{1}{ax+b\log^2(cx^n)}$$

```
[In] Integrate[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
[Out] -(a*x + b*Log[c*x^n]^2)^(-1)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{1}{ax+b\ln(c\,x^n)^2}$
risch	$-\frac{}{-b\,\pi^2\,\mathrm{csgn}(ic)^2\,\mathrm{csgn}(ix^n)^2\,\mathrm{csgn}(ic\,x^n)^2+2b\,\pi^2\,\mathrm{csgn}(ic)^2\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^3-b\,\pi^2\,\mathrm{csgn}(ic)^2\,\mathrm{csgn}(ic\,x^n)^4+2b\,\pi^2\,\mathrm{csgn}(ic)\,\mathrm{cs}}$

```
[In] int((x^2*a+2*b*n*x*ln(c*x^n))/(x^2*a+b*x*ln(c*x^n)^2)^2,x,method=_RETURNVER
BOSE)
[Out] -1/(a*x+b*ln(c*x^n)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{ax^2 + 2bnx\log(cx^n)}{\left(ax^2 + bx\log^2(cx^n)\right)^2}\,dx = -\frac{1}{bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + b\log(c)^2 + ax}$$

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="fricas")
[Out] -1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Sympy [A] (verification not implemented)

Time = 11.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{ax^2 + 2bnx\log(cx^n)}{\left(ax^2 + bx\log^2(cx^n)\right)^2}\,dx = -\frac{1}{ax + b\log(cx^n)^2}$$

```
[In] integrate((a*x**2+2*b*n*x*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**2,x)
[Out] -1/(a*x + b*log(c*x**n)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{ax^2 + 2bnx\log(cx^n)}{\left(ax^2 + bx\log^2(cx^n)\right)^2}\,dx = -\frac{1}{b\log(c)^2 + 2\,b\log(c)\log(x^n) + b\log(x^n)^2 + ax}$$

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="maxima")
[Out] -1/(b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{ax^2 + 2bnx\log(cx^n)}{(ax^2 + bx\log^2(cx^n))^2}\,dx = -\frac{1}{bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + b\log(c)^2 + ax}$$

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="giac")
```

```
[Out] -1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ax^2 + 2bnx\log(cx^n)}{(ax^2 + bx\log^2(cx^n))^2}\,dx = -\frac{1}{b\ln(c\,x^n)^2 + a\,x}$$

```
[In] int((a*x^2 + 2*b*n*x*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2)^2,x)
```

```
[Out] -1/(a*x + b*log(c*x^n)^2)
```

3.28 $\int \frac{ax^3+2bnx^2\log(cx^n)}{(ax^2+bx\log^2(cx^n))^3}\,dx$

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Optimal result

Integrand size = 39, antiderivative size = 20

$$\int \frac{ax^3+2bnx^2\log(cx^n)}{(ax^2+bx\log^2(cx^n))^3}\,dx=-\frac{1}{2\left(ax+b\log^2(cx^n)\right)^2}$$

```
[Out] -1/2/(a*x+b*ln(c*x^n)^2)^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2641, 2624}

$$\int \frac{ax^3+2bnx^2\log(cx^n)}{(ax^2+bx\log^2(cx^n))^3}\,dx=-\frac{1}{2\left(ax+b\log^2(cx^n)\right)^2}$$

```
[In] Int[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
[Out] -1/2*1/(a*x + b*Log[c*x^n]^2)^2
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2(ax+2bn\log(cx^n))}{(ax^2+bx\log^2(cx^n))^3}\,dx\\ &= \int \frac{ax+2bn\log(cx^n)}{x\,(ax+b\log^2(cx^n))^3}\,dx\\ &= -\frac{1}{2\,(ax+b\log^2(cx^n))^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{ax^3+2bnx^2\log(cx^n)}{(ax^2+bx\log^2(cx^n))^3}\,dx = -\frac{1}{2\,(ax+b\log^2(cx^n))^2}$$

```
[In] Integrate[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
[Out] -1/2*1/(a*x + b*Log[c*x^n]^2)^2
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result
parallelrisch	$-\frac{1}{2\left(ax+b\ln(c\,x^n)^2\right)^2}$
risch	$-\frac{}{\left(-b\,\pi^2\operatorname{csgn}(ic)^2\operatorname{csgn}(ix^n)^2\operatorname{csgn}(ic\,x^n)^2+2b\,\pi^2\operatorname{csgn}(ic)^2\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^3-b\,\pi^2\operatorname{csgn}(ic)^2\operatorname{csgn}(ic\,x^n)^4+2b\,\pi^2\operatorname{csgn}(ic)\right.}$

```
[In] int((x^3*a+2*b*n*x^2*ln(c*x^n))/(x^2*a+b*x*ln(c*x^n)^2)^3,x,method=_RETURNV
ERBOSE)
[Out] -1/2/(a*x+b*ln(c*x^n)^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(18) = 36.

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.05

$$\int \frac{ax^3 + 2bnx^2 \log(cx^n)}{(ax^2 + bx\log^2(cx^n))^3} \, dx =$$
$$-\frac{1}{2\left(b^2n^4\log(x)^4 + 4\,b^2n^3\log(c)\log(x)^3 + b^2\log(c)^4 + 2\,abx\log(c)^2 + a^2x^2 + 2\left(3\,b^2n^2\log(c)^2 + abn^2x\right)\right.}$$

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="fricas")
```

```
[Out] -1/2/(b^2*n^4*log(x)^4 + 4*b^2*n^3*log(c)*log(x)^3 + b^2*log(c)^4 + 2*a*b*x
*log(c)^2 + a^2*x^2 + 2*(3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 4*(b^2*
n*log(c)^3 + a*b*n*x*log(c))*log(x))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{ax^3 + 2bnx^2 \log(cx^n)}{(ax^2 + bx\log^2(cx^n))^3} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a*x**3+2*b*n*x**2*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(18) = 36.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 4.75

$$\int \frac{ax^3 + 2bnx^2 \log(cx^n)}{(ax^2 + bx\log^2(cx^n))^3} \, dx =$$
$$-\frac{1}{2\left(b^2\log(c)^4 + 4\,b^2\log(c)\log(x^n)^3 + b^2\log(x^n)^4 + 2\,abx\log(c)^2 + a^2x^2 + 2\left(3\,b^2\log(c)^2 + abx\right)\log(x^n\right.}$$

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="maxima")
```

```
[Out] -1/2/(b^2*log(c)^4 + 4*b^2*log(c)*log(x^n)^3 + b^2*log(x^n)^4 + 2*a*b*x*log
(c)^2 + a^2*x^2 + 2*(3*b^2*log(c)^2 + a*b*x)*log(x^n)^2 + 4*(b^2*log(c)^3 +
 a*b*x*log(c))*log(x^n))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(18) = 36$.

Time = 0.33 (sec) , antiderivative size = 306, normalized size of antiderivative = 15.30

$$\int \frac{ax^3 + 2bnx^2\log(cx^n)}{(ax^2 + bx\log^2(cx^n))^3}\,dx =$$

$$-\frac{}{2\left(4\,ab^3n^6x\log(x)^4 + 16\,ab^3n^5x\log(c)\log(x)^3 + a^2b^2n^4x^2\log(x)^4 + 24\,ab^3n^4x\log(c)^2\log(x)^2 + 4\,a^2b^2n\right.}$$

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="giac")
```

```
[Out] -1/2*(4*a*b*n^2*x + a^2*x^2)/(4*a*b^3*n^6*x*log(x)^4 + 16*a*b^3*n^5*x*log(c
)*log(x)^3 + a^2*b^2*n^4*x^2*log(x)^4 + 24*a*b^3*n^4*x*log(c)^2*log(x)^2 +
4*a^2*b^2*n^3*x^2*log(c)*log(x)^3 + 16*a*b^3*n^3*x*log(c)^3*log(x) + 8*a^2*
b^2*n^4*x^2*log(x)^2 + 6*a^2*b^2*n^2*x^2*log(c)^2*log(x)^2 + 4*a*b^3*n^2*x*
log(c)^4 + 16*a^2*b^2*n^3*x^2*log(c)*log(x) + 4*a^2*b^2*n*x^2*log(c)^3*log(
x) + 2*a^3*b*n^2*x^3*log(x)^2 + 8*a^2*b^2*n^2*x^2*log(c)^2 + a^2*b^2*x^2*lo
g(c)^4 + 4*a^3*b*n*x^3*log(c)*log(x) + 4*a^3*b*n^2*x^3 + 2*a^3*b*x^3*log(c)
^2 + a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{ax^3 + 2bnx^2\log(cx^n)}{(ax^2 + bx\log^2(cx^n))^3}\,dx = -\frac{1}{2\,a^2\,x^2 + 4\,a\,b\,x\ln(c\,x^n)^2 + 2\,b^2\ln(c\,x^n)^4}$$

```
[In] int((a*x^3 + 2*b*n*x^2*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2)^3,x)
```

```
[Out] -1/(2*b^2*log(c*x^n)^4 + 2*a^2*x^2 + 4*a*b*x*log(c*x^n)^2)
```

3.29 $\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx$

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Optimal result

Integrand size = 45, antiderivative size = 19

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx=\log\left(ax^{-1+m}+b\log^q(cx^n)\right)$$

```
[Out] ln(a*x^(-1+m)+b*ln(c*x^n)^q)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2641, 2621}

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx=\log\left(ax^{m-1}+b\log^q(cx^n)\right)$$

```
[In] Int[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x*Log[c*
x^n]^q),x]
```

```
[Out] Log[a*x^(-1 + m) + b*Log[c*x^n]^q]
```

Rule 2621

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{a(-1+m)x^{-1+m} + bnq\log^{-1+q}(cx^n)}{x\left(ax^{-1+m} + b\log^q(cx^n)\right)}\,dx$$
$$= \log\left(ax^{-1+m} + b\log^q(cx^n)\right)$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{a(-1+m)x^{-1+m} + bnq\log^{-1+q}(cx^n)}{ax^m + bx\log^q(cx^n)}\,dx = -\log(x) + \log\left(ax^m + bx\log^q(cx^n)\right)$$

```
[In] Integrate[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x*
Log[c*x^n]^q),x]
[Out] -Log[x] + Log[a*x^m + b*x*Log[c*x^n]^q]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 24.97 (sec) , antiderivative size = 216, normalized size of antiderivative = 11.37

method	result
risch	$q\ln\left(\ln(x^n) - \frac{i\left(\pi\,\mathrm{csgn}(icx^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n) - \pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2 - \pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2 + \pi\,\mathrm{csgn}(icx^n)^3 + 2i\ln(c)\right)}{2}\right.$

```
[In] int((a*(m-1)*x^(m-1)+b*n*q*ln(c*x^n)^(-1+q))/(a*x^m+b*x*ln(c*x^n)^q),x,meth
od=_RETURNVERBOSE)
[Out] q*ln(ln(x^n)-1/2*I*(Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)-Pi*csgn(I*c)*csg
n(I*c*x^n)^2-Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+Pi*csgn(I*c*x^n)^3+2*I*ln(c))-
q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn
(I*c*x^n)+csgn(I*x^n)))+ln((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c
*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q+a*x^m/x/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{a(-1+m)x^{-1+m} + bnq\log^{-1+q}(cx^n)}{ax^m + bx\log^q(cx^n)}\,dx = \log\left(\frac{(n\log(x)+\log(c))^q bx + ax^m}{x}\right)$$

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="fricas")
```

```
[Out] log(((n*log(x) + log(c))^q*b*x + a*x^m)/x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a(-1+m)x^{-1+m} + bnq\log^{-1+q}(cx^n)}{ax^m + bx\log^q(cx^n)}\,dx = \text{Timed out}$$

```
[In] integrate((a*(-1+m)*x**(-1+m)+b*n*q*ln(c*x**n)**(-1+q))/(a*x**m+b*x*ln(c*x*
*n)**q),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{a(-1+m)x^{-1+m} + bnq\log^{-1+q}(cx^n)}{ax^m + bx\log^q(cx^n)}\,dx = \log\left(\frac{bx(\log(c)+\log(x^n))^q + ax^m}{bx}\right)$$

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="maxima")
```

```
[Out] log((b*x*(log(c) + log(x^n))^q + a*x^m)/(b*x))
```

Giac [F]

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx=\int \frac{bnq\log(cx^n)^{q-1}+a(m-1)x^{m-1}}{bx\log(cx^n)^q+ax^m}\,dx$$

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*(m - 1)*x^(m - 1))/(b*x*log(c*x^n)^
q + a*x^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx=\int \frac{a\,x^{m-1}\,(m-1)+b\,n\,q\ln(c\,x^n)^{q-1}}{a\,x^m+b\,x\ln(c\,x^n)^q}\,dx$$

```
[In] int((a*x^(m - 1)*(m - 1) + b*n*q*log(c*x^n)^(q - 1))/(a*x^m + b*x*log(c*x^n
)^q),x)
```

```
[Out] int((a*x^(m - 1)*(m - 1) + b*n*q*log(c*x^n)^(q - 1))/(a*x^m + b*x*log(c*x^n
)^q), x)
```

3.30 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx = \frac{e(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q}+\left(d-\frac{aem}{bnq}\right)\text{Int}\left(x^{-1+m}(ax^m+b\log^q(cx^n))^p,x\right)$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)*(a*x^m+b*ln(c*x^n)^q)^p,x)+e*(a*x^
m+b*ln(c*x^n)^q)^(p+1)/b/n/(p+1)/q
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx = \int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (e*(a*x^m + b*Log[c*x^n]^q)^(1 + p))/(b*n*(1 + p)*q) + (d - (a*e*m)/(b*n*q)
)*Defer[Int][x^(-1 + m)*(a*x^m + b*Log[c*x^n]^q)^p, x]
```

Rubi steps

$$\text{integral} = \frac{e(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q}-\left(-d+\frac{aem}{bnq}\right)\int x^{-1+m}(ax^m+b\log^q(cx^n))^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx$$

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
[Out] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x, x
]
```

Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{p}}{x}dx$$

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \int \frac{\left(dx^m+e\log\left(cx^n\right)^{q-1}\right)\left(ax^m+b\log\left(cx^n\right)^q\right)^p}{x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="fricas")
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,2,5,2,0,5,0,2,1,2,2,1]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,1,1,2,2,1]%%%}+%%%{5,[0
,0,2,4,2,
```

Mupad [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(dx^m+e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)^p}{x}\,dx$$
$$=\int \frac{\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^p\,\left(d\,x^m+e\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

```
[In] int(((a*x^m + b*log(c*x^n)^q)^p*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
[Out] int(((a*x^m + b*log(c*x^n)^q)^p*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

3.31 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$

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Optimal result

Integrand size = 40, antiderivative size = 331

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx = -\frac{a^3(aem-bdnq)x^{4m}}{4bmnq} - \frac{b^2(aem-bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1+3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}}{mnq} - \frac{3\,2^{-1-2q}ab(aem-bdnq)x^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(1+2q,-\frac{2m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{mnq} - \frac{3^{-q}a^2(aem-bdnq)x^{3m}(cx^n)^{-\frac{3m}{n}}\Gamma\left(1+q,-\frac{3m\log(cx^n)}{n}\right)\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{mnq} + \frac{e\left(ax^m+b\log^q(cx^n)\right)^4}{4bnq}$$

```
[Out] -1/4*a^3*(-b*d*n*q+a*e*m)*x^(4*m)/b/m/n/q-b^2*(-b*d*n*q+a*e*m)*x^m*GAMMA(1+
3*q,-m*ln(c*x^n)/n)*ln(c*x^n)^(3*q)/m/n/q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)
^(3*q))-3*2^(-1-2*q)*a*b*(-b*d*n*q+a*e*m)*x^(2*m)*GAMMA(1+2*q,-2*m*ln(c*x^n
)/n)*ln(c*x^n)^(2*q)/m/n/q/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^(2*q))-a^2*(
-b*d*n*q+a*e*m)*x^(3*m)*GAMMA(1+q,-3*m*ln(c*x^n)/n)*ln(c*x^n)^q/(3^q)/m/n/q
/((c*x^n)^(3*m/n))/((-m*ln(c*x^n)/n)^q)+1/4*e*(a*x^m+b*ln(c*x^n)^q)^4/b/n/q
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2625, 6874, 2347, 2212}

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx=-\frac{a^3x^{4m}(aem-bdnq)}{4bmnq}$$
$$-\frac{a^2 3^{-q}x^{3m}(cx^n)^{-\frac{3m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}(aem-bdnq)\Gamma\left(q+1,-\frac{3m\log(cx^n)}{n}\right)}{mnq}$$
$$-\frac{b^2x^m(cx^n)^{-\frac{m}{n}}\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}(aem-bdnq)\Gamma\left(3q+1,-\frac{m\log(cx^n)}{n}\right)}{mnq}$$
$$-\frac{3ab2^{-2q-1}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}(aem-bdnq)\Gamma\left(2q+1,-\frac{2m\log(cx^n)}{n}\right)}{mnq}$$
$$+\frac{e\left(ax^m+b\log^q(cx^n)\right)^4}{4bnq}$$

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] -1/4*(a^3*(a*e*m - b*d*n*q)*x^(4*m))/(b*m*n*q) - (b^2*(a*e*m - b*d*n*q)*x^m
*Gamma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q))/(m*n*q*(c*x^n)^(m/n)
*(-((m*Log[c*x^n])/n))^(3*q)) - (3*2^(-1 - 2*q)*a*b*(a*e*m - b*d*n*q)*x^(2*
m)*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^((2
*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*(a*e*m - b*d*n*q)*x^(3*m)*Gamma[
1 + q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((
m*Log[c*x^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^4)/(4*b*n*q)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2625

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
```

```
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e(ax^m + b\log^q(cx^n))^4}{4bnq} - \left(-d + \frac{aem}{bnq}\right)\int x^{-1+m}(ax^m + b\log^q(cx^n))^3\,dx \\
&= \frac{e(ax^m + b\log^q(cx^n))^4}{4bnq} - \left(-d + \frac{aem}{bnq}\right)\int \left(a^3x^{-1+4m} + 3a^2bx^{-1+3m}\log^q(cx^n)\right. \\
&\qquad \left. + 3ab^2x^{-1+2m}\log^{2q}(cx^n) + b^3x^{-1+m}\log^{3q}(cx^n)\right)dx \\
&= \frac{a^3\left(d - \frac{aem}{bnq}\right)x^{4m}}{4m} + \frac{e(ax^m + b\log^q(cx^n))^4}{4bnq} \\
&\quad - \left(3a^2b\left(-d + \frac{aem}{bnq}\right)\right)\int x^{-1+3m}\log^q(cx^n)\,dx \\
&\quad - \left(b^3\left(-d + \frac{aem}{bnq}\right)\right)\int x^{-1+m}\log^{3q}(cx^n)\,dx \\
&\quad - \frac{(3ab(aem - bdnq))\int x^{-1+2m}\log^{2q}(cx^n)\,dx}{nq} \\
&= \frac{a^3\left(d - \frac{aem}{bnq}\right)x^{4m}}{4m} + \frac{e(ax^m + b\log^q(cx^n))^4}{4bnq} \\
&\quad - \frac{\left(3a^2b\left(-d + \frac{aem}{bnq}\right)x^{3m}(cx^n)^{-\frac{3m}{n}}\right)\text{Subst}\left(\int e^{\frac{3mx}{n}}x^q\,dx, x, \log(cx^n)\right)}{n} \\
&\quad - \frac{\left(3ab(aem - bdnq)x^{2m}(cx^n)^{-\frac{2m}{n}}\right)\text{Subst}\left(\int e^{\frac{2mx}{n}}x^{2q}\,dx, x, \log(cx^n)\right)}{n^2q} \\
&\quad - \frac{\left(b^3\left(-d + \frac{aem}{bnq}\right)x^m(cx^n)^{-\frac{m}{n}}\right)\text{Subst}\left(\int e^{\frac{mx}{n}}x^{3q}\,dx, x, \log(cx^n)\right)}{n}
\end{aligned}$$

$$= \frac{a^3\left(d-\frac{aem}{bnq}\right)x^{4m}}{4m} - \frac{b^2(aem-bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1+3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}}{mnq} - \frac{3\,2^{-1-2q}ab(aem-bdnq)x^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(1+2q,-\frac{2m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{mnq} - \frac{3^{-q}a^2(aem-bdnq)x^{3m}(cx^n)^{-\frac{3m}{n}}\Gamma\left(1+q,-\frac{3m\log(cx^n)}{n}\right)\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{mnq} + \frac{e(ax^m+b\log^q(cx^n))^4}{4bnq}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.34

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)(ax^m+b\log^q(cx^n))^3}{x}\,dx$$

$$= \frac{3^{-q}4^{-1-q}(cx^n)^{-\frac{3m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}\left(-12^{1+q}ab^2emqx^m(cx^n)^{\frac{2m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)+3^q4^{1+q}b^3dnc\right.}{}$$

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (4^(-1 - q)*(-(12^(1 + q)*a*b^2*e*m*q*x^m*(c*x^n)^((2*m)/n)*Gamma[3*q, -((m
*Log[c*x^n])/n)]*Log[c*x^n]^(3*q)) + 3^q*4^(1 + q)*b^3*d*n*q*x^m*(c*x^n)^((
2*m)/n)*Gamma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q) + (-((m*Log[c*
x^n])/n))^q*(-4*3^(1 + q)*a^2*b*e*m*q*x^(2*m)*(c*x^n)^(m/n)*Gamma[2*q, (-2*
m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 2*3^(1 + q)*a*b^2*d*n*q*x^(2*m)*(c*x^n)
^(m/n)*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 4^q*(-((m*Log
[c*x^n])/n))^q*(-4*a^3*e*m*q*x^(3*m)*Gamma[q, (-3*m*Log[c*x^n])/n]*Log[c*x^
n]^q + 4*a^2*b*d*n*q*x^(3*m)*Gamma[1 + q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q
 + 3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q*(a^3*d*n*q*x^(4*m) + b^3*e
*m*Log[c*x^n]^(4*q))))))/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))
^(3*q))
```

Maple [F]

$$\int \frac{\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{3}}{x}dx$$

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

Fricas [F]

$$\int \frac{\left(dx^{m}+e\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{3}}{x}\,dx = \int \frac{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{3}\left(dx^{m}+e\log\left(cx^{n}\right)^{q-1}\right)}{x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="fricas")
[Out] integral((a^3*e*x^(3*m)*log(c*x^n)^(q - 1) + a^3*d*x^(4*m) + (b^3*d*x^m + b
^3*e*log(c*x^n)^(q - 1))*log(c*x^n)^(3*q) + 3*(a*b^2*e*x^m*log(c*x^n)^(q -
1) + a*b^2*d*x^(2*m))*log(c*x^n)^(2*q) + 3*(a^2*b*e*x^(2*m)*log(c*x^n)^(q -
 1) + a^2*b*d*x^(3*m))*log(c*x^n)^q)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(dx^{m}+e\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{3}}{x}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(dx^{m}+e\log^{-1+q}\left(cx^{n}\right)\right)\left(ax^{m}+b\log^{q}\left(cx^{n}\right)\right)^{3}}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^3}{x}\,dx = \int \frac{(ax^m + b\log(cx^n)^q)^3(dx^m + e\log(cx^n)^{q-1})}{x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^3}{x}\,dx = \int \frac{(a\,x^m + b\ln(c\,x^n)^q)^3\,(d\,x^m + e\ln(c\,x^n)^{q-1})}{x}\,dx$$

```
[In] int(((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

```
[Out] int(((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

3.32 $\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^2}{x}\,dx$

Optimal result . . . 247
Rubi [A] (verified) . . . 248
Mathematica [A] (verified) . . . 250
Maple [F] . . . 250
Fricas [F] . . . 250
Sympy [F] . . . 251
Maxima [F(-2)] . . . 251
Giac [F] . . . 251
Mupad [F(-1)] . . . 252

Optimal result

Integrand size = 40, antiderivative size = 235

$$\int \frac{(dx^m+e\log^{-1+q}(cx^n))\,(ax^m+b\log^q(cx^n))^2}{x}\,dx = -\frac{a^2(aem-bdnq)x^{3m}}{3bmnq} - \frac{b(aem-bdnq)x^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(1+2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{mnq} - \frac{2^{-q}a(aem-bdnq)x^{2m}(cx^n)^{-\frac{2m}{n}}\,\Gamma\left(1+q,-\frac{2m\log(cx^n)}{n}\right)\log^{q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{mnq} + \frac{e(ax^m+b\log^q(cx^n))^3}{3bnq}$$

```
[Out] -1/3*a^2*(-b*d*n*q+a*e*m)*x^(3*m)/b/m/n/q-b*(-b*d*n*q+a*e*m)*x^m*GAMMA(1+2*
q,-m*ln(c*x^n)/n)*ln(c*x^n)^(2*q)/m/n/q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(
2*q))-a*(-b*d*n*q+a*e*m)*x^(2*m)*GAMMA(1+q,-2*m*ln(c*x^n)/n)*ln(c*x^n)^q/(2
^q)/m/n/q/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^q)+1/3*e*(a*x^m+b*ln(c*x^n)^q
)^3/b/n/q
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2625, 6874, 2347, 2212}

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^2}{x}\,dx$$
$$= -\frac{a^2x^{3m}(aem - bdnq)}{3bmnq}$$
$$- \frac{a2^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^q\left(cx^n\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}(aem - bdnq)\Gamma\left(q+1, -\frac{2m\log(cx^n)}{n}\right)}{mnq}$$
$$- \frac{bx^m(cx^n)^{-\frac{m}{n}}\log^{2q}\left(cx^n\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}(aem - bdnq)\Gamma\left(2q+1, -\frac{m\log(cx^n)}{n}\right)}{mnq}$$
$$+ \frac{e\left(ax^m + b\log^q\left(cx^n\right)\right)^3}{3bnq}$$

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] -1/3*(a^2*(a*e*m - b*d*n*q)*x^(3*m))/(b*m*n*q) - (b*(a*e*m - b*d*n*q)*x^m*G
amma[1 + 2*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^(m/n)*(
-((m*Log[c*x^n])/n))^(2*q)) - (a*(a*e*m - b*d*n*q)*x^(2*m)*Gamma[1 + q, (-2
*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*m*n*q*(c*x^n)^((2*m)/n)*(-((m*Log[c*x^
n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^3)/(3*b*n*q)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2625

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e(ax^m + b\log^q(cx^n))^3}{3bnq} - \left(-d + \frac{aem}{bnq}\right)\int x^{-1+m}(ax^m + b\log^q(cx^n))^2\, dx \\
&= \frac{e(ax^m + b\log^q(cx^n))^3}{3bnq} \\
&\quad - \left(-d + \frac{aem}{bnq}\right)\int \left(a^2x^{-1+3m} + 2abx^{-1+2m}\log^q(cx^n) + b^2x^{-1+m}\log^{2q}(cx^n)\right) dx \\
&= \frac{a^2\left(d - \frac{aem}{bnq}\right)x^{3m}}{3m} + \frac{e(ax^m + b\log^q(cx^n))^3}{3bnq} \\
&\quad - \left(2ab\left(-d + \frac{aem}{bnq}\right)\right)\int x^{-1+2m}\log^q(cx^n)\, dx \\
&\quad - \left(b^2\left(-d + \frac{aem}{bnq}\right)\right)\int x^{-1+m}\log^{2q}(cx^n)\, dx \\
&= \frac{a^2\left(d - \frac{aem}{bnq}\right)x^{3m}}{3m} + \frac{e(ax^m + b\log^q(cx^n))^3}{3bnq} \\
&\quad - \frac{\left(2ab\left(-d + \frac{aem}{bnq}\right)x^{2m}(cx^n)^{-\frac{2m}{n}}\right)\text{Subst}\left(\int e^{\frac{2mx}{n}}x^q\, dx, x, \log(cx^n)\right)}{n} \\
&\quad - \frac{\left(b^2\left(-d + \frac{aem}{bnq}\right)x^m(cx^n)^{-\frac{m}{n}}\right)\text{Subst}\left(\int e^{\frac{mx}{n}}x^{2q}\, dx, x, \log(cx^n)\right)}{n} \\
&= \frac{a^2\left(d - \frac{aem}{bnq}\right)x^{3m}}{3m} \\
&\quad - \frac{b(aem - bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1 + 2q, -\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{mnq} \\
&\quad - \frac{2^{-q}a(aem - bdnq)x^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(1 + q, -\frac{2m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{mnq} \\
&\quad + \frac{e(ax^m + b\log^q(cx^n))^3}{3bnq}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.27

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx$$
$$= \frac{2^{-q}(cx^n)^{-\frac{2m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\left(-32^{1+q}abemqx^m(cx^n)^{m/n}\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)+3\,2^qb^2dnqx^m(cx^n\right.}{}$$

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (-3*2^(1 + q)*a*b*e*m*q*x^m*(c*x^n)^(m/n)*Gamma[2*q, -((m*Log[c*x^n])/n)]*L
og[c*x^n]^(2*q) + 3*2^q*b^2*d*n*q*x^m*(c*x^n)^(m/n)*Gamma[1 + 2*q, -((m*Log
[c*x^n])/n)]*Log[c*x^n]^(2*q) + (-((m*Log[c*x^n])/n))^q*(-3*a^2*e*m*q*x^(2*
m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 3*a*b*d*n*q*x^(2*m)*Gamma[1
 + q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 2^q*(c*x^n)^((2*m)/n)*(-((m*Log[c
*x^n])/n))^q*(a^2*d*n*q*x^(3*m) + b^2*e*m*Log[c*x^n]^(3*q))))/(3*2^q*m*n*q*
(c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q))
```

Maple [F]

$$\int \frac{\left(d\,x^m + e\ln(c\,x^n)^{-1+q}\right)\left(a\,x^m + b\ln(c\,x^n)^q\right)^2}{x}dx$$

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

Fricas [F]

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))^2}{x}\,dx$$
$$= \int \frac{(ax^m + b\log(cx^n)^q)^2(dx^m + e\log(cx^n)^{q-1})}{x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="fricas")
```

```
[Out] integral((a^2*e*x^(2*m)*log(c*x^n)^(q - 1) + a^2*d*x^(3*m) + (b^2*d*x^m + b
^2*e*log(c*x^n)^(q - 1))*log(c*x^n)^(2*q) + 2*(a*b*e*x^m*log(c*x^n)^(q - 1)
 + a*b*d*x^(2*m))*log(c*x^n)^q)/x, x)
```

Sympy [F]

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^2}{x}\,dx$$
$$= \int \frac{\left(ax^m + b\log\left(cx^n\right)^q\right)^2\left(dx^m + e\log\left(cx^n\right)^{q-1}\right)}{x}\,dx$$

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
[Out] Integral((a*x**m + b*log(c*x**n)**q)**2*(d*x**m + e*log(c*x**n)**(q - 1))/x
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)^2}{x}\,dx$$
$$= \int \frac{\left(ax^m + b\log\left(cx^n\right)^q\right)^2\left(dx^m + e\log\left(cx^n\right)^{q-1}\right)}{x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="giac")
[Out] integrate((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(dx^m + e\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)^2}{x}\,dx$$
$$= \int \frac{\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^2\left(d\,x^m + e\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

```
[In] int(((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
[Out] int(((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

3.33 $\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx$

Optimal result 253
Rubi [A] (verified) 253
Mathematica [A] (verified) 255
Maple [F] 255
Fricas [F] 256
Sympy [F] 256
Maxima [F(-2)] 256
Giac [F] 257
Mupad [F(-1)] 257

Optimal result

Integrand size = 38, antiderivative size = 139

$$\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx = -\frac{a(aem-bdnq)x^{2m}}{2bmnq} + \left(\frac{bd}{m}-\frac{ae}{nq}\right)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1+q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q} + \frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}$$

```
[Out] -1/2*a*(-b*d*n*q+a*e*m)*x^(2*m)/b/m/n/q+(b*d/m-a*e/n/q)*x^m*GAMMA(1+q,-m*ln
(c*x^n)/n)*ln(c*x^n)^q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^q)+1/2*e*(a*x^m+b*
ln(c*x^n)^q)^2/b/n/q
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2625, 14, 2347, 2212}

$$\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx = x^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\left(\frac{bd}{m}-\frac{ae}{nq}\right)\Gamma\left(q+1,-\frac{m\log(cx^n)}{n}\right) + \frac{e(ax^m+b\log^q(cx^n))^2}{2bnq} - \frac{ax^{2m}(aem-bdnq)}{2bmnq}$$

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] -1/2*(a*(a*e*m - b*d*n*q)*x^(2*m))/(b*m*n*q) + (((b*d)/m - (a*e)/(n*q))*x^m
*Gamma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c
*x^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^2)/(2*b*n*q)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2625

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}-\left(-d+\frac{aem}{bnq}\right)\int x^{-1+m}(ax^m+b\log^q(cx^n))\,dx\\
&= \frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}-\left(-d+\frac{aem}{bnq}\right)\int \left(ax^{-1+2m}+bx^{-1+m}\log^q(cx^n)\right)\,dx\\
&= \frac{a\left(d-\frac{aem}{bnq}\right)x^{2m}}{2m}+\frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}-\left(b\left(-d+\frac{aem}{bnq}\right)\right)\int x^{-1+m}\log^q(cx^n)\,dx
\end{aligned}
$$

$$= \frac{a\left(d-\frac{aem}{bnq}\right)x^{2m}}{2m}+\frac{e(ax^m+b\log^q(cx^n))^2}{2bnq} - \frac{\left(b\left(-d+\frac{aem}{bnq}\right)x^m(cx^n)^{-\frac{m}{n}}\right)\mathrm{Subst}\left(\int e^{\frac{mx}{n}}x^q\,dx,x,\log(cx^n)\right)}{n}$$

$$= \frac{a\left(d-\frac{aem}{bnq}\right)x^{2m}}{2m} + \left(\frac{bd}{m}-\frac{ae}{nq}\right)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1+q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q} + \frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)(ax^m+b\log^q(cx^n))}{x}\,dx$$
$$= \frac{(cx^n)^{-\frac{m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\left(-2aemqx^m\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)+2bdnqx^m\Gamma\left(1+q,-\frac{m\log(cx^n)}{n}\right)\log^q(c\right.}{2mnq}$$

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (-2*a*e*m*q*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + 2*b*d*n*q*x^m*
Gamma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + (c*x^n)^(m/n)*(-((m*Log[c*
x^n])/n))^q*(a*d*n*q*x^(2*m) + b*e*m*Log[c*x^n]^(2*q)))/(2*m*n*q*(c*x^n)^(m
/n)*(-((m*Log[c*x^n])/n))^q)
```

Maple [F]

$$\int \frac{\left(d\,x^m+e\ln(c\,x^n)^{-1+q}\right)\left(a\,x^m+b\ln(c\,x^n)^q\right)}{x}dx$$

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

Fricas [F]

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)}{x}\,dx = \int \frac{\left(ax^m + b\log\left(cx^n\right)^q\right)\left(dx^m + e\log\left(cx^n\right)^{q-1}\right)}{x}\,dx$$

[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm ="fricas")

[Out] integral((a*e*x^m*log(c*x^n)^(q - 1) + a*d*x^(2*m) + (b*d*x^m + b*e*log(c*x ^n)^(q - 1))*log(c*x^n)^q)/x, x)

Sympy [F]

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)}{x}\,dx = \int \frac{\left(ax^m + b\log\left(cx^n\right)^q\right)\left(dx^m + e\log\left(cx^n\right)^{q-1}\right)}{x}\,dx$$

[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x)

[Out] Integral((a*x**m + b*log(c*x**n)**q)*(d*x**m + e*log(c*x**n)**(q - 1))/x, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(dx^m + e\log^{-1+q}\left(cx^n\right)\right)\left(ax^m + b\log^q\left(cx^n\right)\right)}{x}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm ="maxima")

[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t he first argument is 0which is not of the expected type LIST

Giac [F]

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))\,(ax^m + b\log^q(cx^n))}{x}\,dx = \int \frac{(ax^m + b\log(cx^n)^q)(dx^m + e\log(cx^n)^{q-1})}{x}\,dx$$

[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm ="giac")

[Out] integrate((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx^m + e\log^{-1+q}(cx^n))\,(ax^m + b\log^q(cx^n))}{x}\,dx = \int \frac{(a\,x^m + b\ln(c\,x^n)^q)\,\left(d\,x^m + e\ln(c\,x^n)^{q-1}\right)}{x}\,dx$$

[In] int(((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)

[Out] int(((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)

3.34 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx$

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Mathematica [A] (verified) 259
Maple [A] (verified) 259
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Sympy [A] (verification not implemented) 260
Maxima [A] (verification not implemented) 260
Giac [A] (verification not implemented) 261
Mupad [B] (verification not implemented) 261

Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx=\frac{dx^m}{m}+\frac{e\log^q(cx^n)}{nq}$$

```
[Out] d*x^m/m+e*ln(c*x^n)^q/n/q
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {14, 2339, 30}

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx=\frac{e\log^q(cx^n)}{nq}+\frac{dx^m}{m}$$

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] (d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(dx^{-1+m} + \frac{e\log^{-1+q}(cx^n)}{x}\right) dx \\
&= \frac{dx^m}{m} + e\int \frac{\log^{-1+q}(cx^n)}{x}\,dx \\
&= \frac{dx^m}{m} + \frac{e\text{Subst}\left(\int x^{-1+q}\,dx, x, \log(cx^n)\right)}{n} \\
&= \frac{dx^m}{m} + \frac{e\log^q(cx^n)}{nq}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$
\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = \frac{dx^m}{m} + \frac{e\log^q(cx^n)}{nq}
$$

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
[Out] (d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result
default	$\frac{d\,x^m}{m}+\frac{e\ln(c\,x^n)^q}{nq}$
parallelrisch	$-\frac{-d\,x^m nq-\ln(c\,x^n)\ln(c\,x^n)^{-1+q}em}{mnq}$
risch	$\frac{d\,x^m}{m}+\frac{e\left(\ln(c)+\ln(x^n)-\frac{i\pi\ \text{csgn}(ic\,x^n)(-\text{csgn}(ic\,x^n)+\text{csgn}(ic))(-\text{csgn}(ic\,x^n)+\text{csgn}(ix^n))}{2}\right)^{-1+q}\left(\ln(c)+\ln(x^n)-\frac{i\pi\ \text{csgn}(ic\,x^n)(\cdots}{\cdots}\right.}{nq}$

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x,x,method=_RETURNVERBOSE)
[Out] d*x^m/m+e*ln(c*x^n)^q/n/q
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = \frac{dnqx^m + (emn\log(x) + em\log(c))(n\log(x) + \log(c))^{q-1}}{mnq}$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
```

```
[Out] (d*n*q*x^m + (e*m*n*log(x) + e*m*log(c))*(n*log(x) + log(c))^(q - 1))/(m*n*
q)
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = -d\left(\begin{cases} -\log(x) & \text{for } m = 0 \\ -\frac{x^m}{m} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \log(c)^{q-1}\log(x) & \text{for } n = 0 \\ \frac{\begin{cases} \frac{\log(cx^n)^q}{q} & \text{for } q \neq 0 \\ \log(\log(cx^n)) & \text{otherwise} \end{cases}}{n} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x,x)
```

```
[Out] -d*Piecewise((-log(x), Eq(m, 0)), (-x**m/m, True)) + e*Piecewise((log(c)**(
q - 1)*log(x), Eq(n, 0)), (Piecewise((log(c*x**n)**q/q, Ne(q, 0)), (log(log
(c*x**n)), True))/n, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = \frac{dx^m}{m} + \frac{e\log(cx^n)^q}{nq}$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
```

```
[Out] d*x^m/m + e*log(c*x^n)^q/(n*q)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = \frac{dx^m}{m} + \frac{(n\log(x) + \log(c))^q e}{nq}$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
[Out] d*x^m/m + (n*log(x) + log(c))^q*e/(n*q)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx = \frac{d\,x^m}{m} + \frac{e\ln(c\,x^n)^q}{n\,q}$$

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/x,x)
[Out] (d*x^m)/m + (e*log(c*x^n)^q)/(n*q)
```

3.35 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$

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Maple [N/A] 263
Fricas [N/A] 263
Sympy [F(-1)] 264
Maxima [N/A] 264
Giac [N/A] 264
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Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx = \frac{e\log(ax^m+b\log^q(cx^n))}{bnq} + \left(d-\frac{aem}{bnq}\right)\mathrm{Int}\left(\frac{x^{-1+m}}{ax^m+b\log^q(cx^n)},x\right)$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q),x)+e*ln(a*x^
m+b*ln(c*x^n)^q)/b/n/q
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx = \int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$$

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] (e*Log[a*x^m + b*Log[c*x^n]^q])/(b*n*q) + (d - (a*e*m)/(b*n*q))*Defer[Int][
x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q), x]
```

Rubi steps

$$\text{integral} = \frac{e\log(ax^m+b\log^q(cx^n))}{bnq} - \left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{ax^m+b\log^q(cx^n)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.71 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx=\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$$

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{d\,x^m+e\ln(c\,x^n)^{-1+q}}{x\,(a\,x^m+b\ln(c\,x^n)^q)}dx$$

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x)
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx=\int \frac{dx^m+e\log(cx^n)^{q-1}}{(ax^m+b\log(cx^n)^q)x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="fricas")
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.38

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)}\,dx = \int \frac{dx^m + e\log\left(cx^n\right)^{q-1}}{\left(ax^m + b\log\left(cx^n\right)^q\right)x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="maxima")
[Out] e*log(log(c) + log(x^n))/(b*n) + integrate((b*d*x^m*log(x^n) + (b*d*log(c)
- a*e)*x^m)/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) + b^2*x*
log(x^n))*(log(c) + log(x^n))^q), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)}\,dx = \int \frac{dx^m + e\log\left(cx^n\right)^{q-1}}{\left(ax^m + b\log\left(cx^n\right)^q\right)x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="giac")
[Out] integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x(ax^m + b\log^q(cx^n))}\,dx = \int \frac{d\,x^m + e\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)}\,dx$$

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)),x)
[Out] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

3.36 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

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Maxima [N/A] . . . 268
Giac [N/A] . . . 268
Mupad [N/A] . . . 269

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx = -\frac{e}{bnq(ax^m+b\log^q(cx^n))} + \left(d-\frac{aem}{bnq}\right)\operatorname{Int}\left(\frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^2},x\right)$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^2,x)-e/b/n/q
/(a*x^m+b*ln(c*x^n)^q)
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx = \int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$$

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(e/(b*n*q*(a*x^m + b*Log[c*x^n]^q))) + (d - (a*e*m)/(b*n*q))*Defer[Int][x^
(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^2, x]
```

Rubi steps

$$\text{integral} = -\frac{e}{bnq(ax^m+b\log^q(cx^n))} - \left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 20.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx$$

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2), x
]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{d\,x^m + e\ln(c\,x^n)^{-1+q}}{x\left(a\,x^m + b\ln(c\,x^n)^q\right)^2}dx$$

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{\left(ax^m + b\log(cx^n)^q\right)^2 x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="fricas")
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*x
^(2*m) + b^2*x*log(c*x^n)^(2*q)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 312, normalized size of antiderivative = 7.80

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{\left(ax^m + b\log(cx^n)^q\right)^2 x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="maxima")
[Out] -(b*d*log(c) + b*d*log(x^n) - a*e)/(a^2*b*m*x^m*log(x^n) - (n*q - m*log(c))
*a^2*b*x^m + (a*b^2*m*log(x^n) - (n*q - m*log(c))*a*b^2)*(log(c) + log(x^n)
)^q) + integrate(-((e*m*n*(q - 1) - e*m^2*log(c))*a + (d*m*n*q*log(c) - (q^
2 - q)*d*n^2)*b + (b*d*m*n*q - a*e*m^2)*log(x^n))/(a^2*b*m^2*x*x^m*log(x^n)
^2 - 2*(m*n*q - m^2*log(c))*a^2*b*x*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c
) + m^2*log(c)^2)*a^2*b*x*x^m + (a*b^2*m^2*x*log(x^n)^2 - 2*(m*n*q - m^2*lo
g(c))*a*b^2*x*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a*b^2*x)
*(log(c) + log(x^n))^q), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{\left(ax^m + b\log(cx^n)^q\right)^2 x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="giac")
[Out] integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \int \frac{d\,x^m + e\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^2}\,dx$$

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
[Out] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

3.37 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$

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Mupad [N/A] . . . 273

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx = -\frac{e}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2} + \left(d-\frac{aem}{bnq}\right)\mathrm{Int}\left(\frac{x^{-1+m}}{\left(ax^m+b\log^q(cx^n)\right)^3},x\right)$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^3,x)-1/2*e/b
/n/q/(a*x^m+b*ln(c*x^n)^q)^2
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx = \int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$$

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -1/2*e/(b*n*q*(a*x^m + b*Log[c*x^n]^q)^2) + (d - (a*e*m)/(b*n*q))*Defer[Int
][x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^3, x]
```

Rubi steps

$$\text{integral} = -\frac{e}{2bnq\left(ax^m+b\log^q(cx^n)\right)^2} - \left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 58.91 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx=\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$$

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3), x
]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{d\,x^m+e\ln(c\,x^n)^{-1+q}}{x\,(a\,x^m+b\ln(c\,x^n)^q)^3}dx$$

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.22

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx=\int \frac{dx^m+e\log(cx^n)^{q-1}}{(ax^m+b\log(cx^n)^q)^3x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="fricas")
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) + 3
*a^2*b*x*x^(2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\,(ax^m + b\log^q(cx^n))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 1583, normalized size of antiderivative = 39.58

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\,(ax^m + b\log^q(cx^n))^3}\,dx = \int \frac{dx^m + e\log(cx^n)^{q-1}}{(ax^m + b\log(cx^n)^q)^3 x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="maxima")
[Out] -1/2*(a*b*d*m^2*x^m*log(x^n)^3 + (a^2*e*m^2 - (4*d*m*n*q - 3*d*m^2*log(c))*
a*b)*x^m*log(x^n)^2 + ((2*e*m^2*log(c) + e*m*n)*a^2 - (8*d*m*n*q*log(c) - 3
*d*m^2*log(c)^2 - (3*q^2 - q)*d*n^2)*a*b)*x^m*log(x^n) - ((e*n^2*q^2 - e*m^
2*log(c)^2 - e*m*n*log(c))*a^2 + (4*d*m*n*q*log(c)^2 - d*m^2*log(c)^3 - (3*
q^2 - q)*d*n^2*log(c))*a*b)*x^m - ((e*m*n*(2*q - 1)*log(c) - 2*e*m^2*log(c)
^2)*a*b + (2*d*m*n*q*log(c)^2 - (2*q^2 - q)*d*n^2*log(c))*b^2 + 2*(b^2*d*m*
n*q - a*b*e*m^2)*log(x^n)^2 + ((e*m*n*(2*q - 1) - 4*e*m^2*log(c))*a*b + (4*
d*m*n*q*log(c) - (2*q^2 - q)*d*n^2)*b^2)*log(x^n))*(log(c) + log(x^n))^q)/(
a^4*b*m^3*x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^4*b*x^(3*m)*log(x
^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^4*b*x^(3*m)*log(x
^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^
4*b*x^(3*m) + (a^2*b^3*m^3*x^m*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^2*b^
3*x^m*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^2*b^3*
x^m*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log
(c)^3)*a^2*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a^3*b^2*m^3*x^(2*m)*log(
x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^3*b^2*x^(2*m)*log(x^n)^2 + 3*(m*n^2*q^2
 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^3*b^2*x^(2*m)*log(x^n) - (n^3*q^3 - 3
*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^3*b^2*x^(2*m))*(lo
g(c) + log(x^n))^q) - integrate(-1/2*(2*(b*d*m^3*n*q - a*e*m^4)*log(x^n)^3
+ ((e*m^3*n*(2*q - 3) - 6*e*m^4*log(c))*a + (6*d*m^3*n*q*log(c) - (2*q^2 -
3*q)*d*m^2*n^2)*b)*log(x^n)^2 + (e*m^3*n*(2*q - 3)*log(c)^2 - 2*e*m^4*log(c
)^3 + 2*(q^2 - 1)*e*m^2*n^2*log(c) - (2*q^3 - 3*q^2 + q)*e*m*n^3)*a + (2*d*
m^3*n*q*log(c)^3 - (2*q^2 - 3*q)*d*m^2*n^2*log(c)^2 - 2*(q^3 - q)*d*m*n^3*l
```

```
og(c) + (2*q^4 - 3*q^3 + q^2)*d*n^4)*b + 2*((e*m^3*n*(2*q - 3)*log(c) - 3*e
*m^4*log(c)^2 + (q^2 - 1)*e*m^2*n^2)*a + (3*d*m^3*n*q*log(c)^2 - (2*q^2 - 3
*q)*d*m^2*n^2*log(c) - (q^3 - q)*d*m*n^3)*b)*log(x^n))/(a^3*b*m^4*x*x^(2*m)
*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^3*b*x*x^(2*m)*log(x^n)^3 + 6*(m^2*
n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^3*b*x*x^(2*m)*log(x^n)^2 - 4*(
m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a^3*b
*x*x^(2*m)*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^
2 - 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^3*b*x*x^(2*m) + (a^2*b^2*m^4*x*x^m
*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^2*b^2*x*x^m*log(x^n)^3 + 6*(m^2*n^
2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^2*b^2*x*x^m*log(x^n)^2 - 4*(m*n^
3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a^2*b^2*x
*x^m*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^2 - 4*
m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^2*b^2*x*x^m)*(log(c) + log(x^n))^q), x)
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \int \frac{dx^m + e\log\left(cx^n\right)^{q-1}}{\left(ax^m + b\log\left(cx^n\right)^q\right)^3 x}\,dx$$

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="giac")
```

```
[Out] integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^3}\,dx = \int \frac{d\,x^m + e\ln\left(c\,x^n\right)^{q-1}}{x\left(a\,x^m + b\ln\left(c\,x^n\right)^q\right)^3}\,dx$$

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
```

```
[Out] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

3.38 $\int \frac{adnx^m-admx^m\log(cx^n)-bdn(-1+q)\log^q(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

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Optimal result

Integrand size = 60, antiderivative size = 26

$$\int \frac{adnx^m-admx^m\log(cx^n)-bdn(-1+q)\log^q(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx=\frac{d\log(cx^n)}{ax^m+b\log^q(cx^n)}$$

```
[Out] d*ln(c*x^n)/(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.017, Rules used = {2626}

$$\int \frac{adnx^m-admx^m\log(cx^n)-bdn(-1+q)\log^q(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx=\frac{d\log(cx^n)}{ax^m+b\log^q(cx^n)}$$

```
[In] Int[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)/(x*(a*
x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] (d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Rule 2626

```
Int[(Log[(c_.)*(x_)^(n_.)]^(q_.)*(f_.) + (d_.)*(x_)^(m_.) + Log[(c_.)*(x_)^
(n_.)]*(e_.)*(x_)^(m_.))/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x
_)^(m_.))^2), x_Symbol] :> Simp[d*(Log[c*x^n]/(a*n*(a*x^m + b*Log[c*x^n]^q)
)), x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[e*n + d*m, 0] && EqQ
[a*f + b*d*(q - 1), 0]
```

Rubi steps

$$\text{integral}=\frac{d\log(cx^n)}{ax^m+b\log^q(cx^n)}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \frac{d\log(cx^n)}{ax^m + b\log^q(cx^n)}$$

```
[In] Integrate[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)/
(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] (d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Maple [A] (verified)

Time = 10.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{d\ln(c\,x^n)}{a\,x^m+b\ln(c\,x^n)^q}$
risch	$\frac{\left(-i\pi\,\mathrm{csgn}(ic\,x^n)^3+i\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic)+i\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ix^n)-i\pi\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)+2\ln(c)+2\ln(x^n)\right)}{2a\,x^m+2b\left(\ln(c)+\ln(x^n)-\frac{i\pi\,\mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ix^n))}{2}\right)^q}$

```
[In] int((a*d*n*x^m-a*d*m*x^m*ln(c*x^n)-b*d*n*(-1+q)*ln(c*x^n)^q)/x/(a*x^m+b*ln(
c*x^n)^q)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*ln(c*x^n)/(a*x^m+b*ln(c*x^n)^q)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x\left(ax^m + b\log^q(cx^n)\right)^2}\,dx = \frac{dn\log(x) + d\log(c)}{(n\log(x) + \log(c))^q b + ax^m}$$

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="fricas")
```

```
[Out] (d*n*log(x) + d*log(c))/((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [A] (verification not implemented)

Time = 22.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx = \frac{d\log(cx^n)}{ax^m + b\log(cx^n)^q}$$

```
[In] integrate((a*d*n*x**m-a*d*m*x**m*ln(c*x**n)-b*d*n*(-1+q)*ln(c*x**n)**q)/x/(
a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] d*log(c*x**n)/(a*x**m + b*log(c*x**n)**q)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx = \frac{d\log(c) + d\log(x^n)}{ax^m + b(\log(c) + \log(x^n))^q}$$

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="maxima")
```

```
[Out] (d*log(c) + d*log(x^n))/(a*x^m + b*(log(c) + log(x^n))^q)
```

Giac [F]

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx = \int -\frac{bdn(q-1)\log(cx^n)^q + admx^m\log(cx^n) - adnx^m}{(ax^m + b\log(cx^n)^q)^2 x}\,dx$$

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

```
[Out] integrate(-(b*d*n*(q - 1)*log(c*x^n)^q + a*d*m*x^m*log(c*x^n) - a*d*n*x^m)/
((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{adnx^m - admx^m \log(cx^n) - bdn(-1+q)\log^q(cx^n)}{x(ax^m + b\log^q(cx^n))^2}\,dx = \frac{d\,\ln(c\,x^n)}{a\,x^m + b\ln(c\,x^n)^q}$$

```
[In] int(-(a*d*m*x^m*log(c*x^n) - a*d*n*x^m + b*d*n*log(c*x^n)^q*(q - 1))/(x*(a*
x^m + b*log(c*x^n)^q)^2),x)
[Out] (d*log(c*x^n))/(a*x^m + b*log(c*x^n)^q)
```

3.39 $\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx$

Optimal result 278
Rubi [N/A] 278
Mathematica [N/A] 279
Maple [N/A] 279
Fricas [N/A] 279
Sympy [N/A] 280
Maxima [N/A] 280
Giac [N/A] 280
Mupad [N/A] 281

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx = \frac{\log(cx^n)}{a(ax+b\log^q(cx^n))} - \frac{n(1-q)\text{Int}\left(\frac{1}{x(ax+b\log^q(cx^n))},x\right)}{a}$$

```
[Out] -n*(1-q)*CannotIntegrate(1/x/(a*x+b*ln(c*x^n)^q),x)/a+ln(c*x^n)/a/(a*x+b*ln
(c*x^n)^q)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx = \int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx$$

```
[In] Int[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
```

```
[Out] Log[c*x^n]/(a*(a*x + b*Log[c*x^n]^q)) - (n*(1 - q)*Defer[Int][1/(x*(a*x + b
*Log[c*x^n]^q)), x])/a
```

Rubi steps

$$\text{integral} = \frac{\log(cx^n)}{a(ax+b\log^q(cx^n))} - \frac{(n(1-q))\int \frac{1}{x(ax+b\log^q(cx^n))}\,dx}{a}$$

Mathematica [N/A]

Not integrable

Time = 79.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{nq - \log(cx^n)}{(ax + b\log^q(cx^n))^2}\,dx = \int \frac{nq - \log(cx^n)}{(ax + b\log^q(cx^n))^2}\,dx$$

```
[In] Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
[Out] Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{nq - \ln(c\,x^n)}{(ax + b\ln(c\,x^n)^q)^2}dx$$

```
[In] int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
[Out] int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int \frac{nq - \log(cx^n)}{(ax + b\log^q(cx^n))^2}\,dx = \int \frac{nq - \log(cx^n)}{(ax + b\log(cx^n)^q)^2}\,dx$$

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="fricas")
[Out] integral((n*q - log(c*x^n))/(a^2*x^2 + 2*a*b*x*log(c*x^n)^q + b^2*log(c*x^n
)^(2*q)), x)
```

Sympy [N/A]

Not integrable

Time = 15.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{nq - \log(cx^n)}{(ax + b\log^q(cx^n))^2}\,dx = \int \frac{nq - \log(cx^n)}{(ax + b\log(cx^n)^q)^2}\,dx$$

```
[In] integrate((n*q-ln(c*x**n))/(a*x+b*ln(c*x**n)**q)**2,x)
[Out] Integral((n*q - log(c*x**n))/(a*x + b*log(c*x**n)**q)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \frac{nq - \log(cx^n)}{(ax + b\log^q(cx^n))^2}\,dx = \int \frac{nq - \log(cx^n)}{(ax + b\log(cx^n)^q)^2}\,dx$$

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="maxima")
[Out] n*(q - 1)*integrate(1/(a^2*x^2 + a*b*x*(log(c) + log(x^n))^q), x) + (log(c)
 + log(x^n))/(a^2*x + a*b*(log(c) + log(x^n))^q)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{nq - \log(cx^n)}{(ax + b\log^q(cx^n))^2}\,dx = \int \frac{nq - \log(cx^n)}{(ax + b\log(cx^n)^q)^2}\,dx$$

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="giac")
[Out] integrate((n*q - log(c*x^n))/(a*x + b*log(c*x^n)^q)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx=\int -\frac{\ln(c\,x^n)-n\,q}{\left(b\ln(c\,x^n)^q+a\,x\right)^2}\,dx$$

```
[In] int(-(log(c*x^n) - n*q)/(b*log(c*x^n)^q + a*x)^2,x)
[Out] int(-(log(c*x^n) - n*q)/(b*log(c*x^n)^q + a*x)^2, x)
```

3.40 $\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result 282
Rubi [A] (verified) 282
Mathematica [B] (warning: unable to verify) 283
Maple [C] (verified) 284
Fricas [A] (verification not implemented) 284
Sympy [F(-2)] 285
Maxima [F(-2)] 285
Giac [F] 285
Mupad [F(-1)] 286

Optimal result

Integrand size = 39, antiderivative size = 49

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

```
[Out] -1/2*polylog(2,1-2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))*(-e/d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2497}

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,1-\frac{2x\left(\sqrt{-\frac{e}{d}}d+ex\right)}{ex^2+d}\right)}{2e}$$

```
[In] Int[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -1/2*(Sqrt[-(e/d)]*PolyLog[2, 1 - (2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]
)/e
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{-\frac{e}{d}}\,\text{Li}_2\left(1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 625 vs. 2(49) = 98.

Time = 0.31 (sec) , antiderivative size = 625, normalized size of antiderivative = 12.76

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$

$$= \frac{-2\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}x\right)+2\log\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)-\log^2\left(\sqrt{-d}-\right.}{}$$

```
[In] Integrate[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d*Sqrt[-(e/d)] + e*x)/(Sqrt[-d]*
Sqrt[e] + d*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(e + d*(-(e/d)
)^(3/2)*x)/(e + Sqrt[-d]*Sqrt[e]*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] - Sqrt[e]*
x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d] + Sqrt[e]
*x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] + 2*PolyLog[2, (Sqrt[-d]
+ Sqrt[e]*x)/(Sqrt[-d] + Sqrt[e]/Sqrt[-(e/d)])] + 2*PolyLog[2, 1 + (Sqrt[e]
*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*PolyLog[2,
 (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2
)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[-d]*Sqrt[e] + d*Sqrt[-(e/d
)])])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.49 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.86

method	result
risch	$\frac{\ln(2)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^2+d\right)}\frac{2\ln(x-\_\alpha)\ln\left(\frac{x\left(ex+d\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)+e\left(\frac{\ln(x-\_\alpha)^2}{\_\alpha e}+\frac{2\_\alpha\ln(x-\_\alpha)\ln\left(\frac{x+\_\alpha}{2\_\alpha}\right)}{d}+\frac{2\_\alpha\,\mathrm{dilog}\left(\frac{x+}{2}\right.}{d}\right.}{}}{}$

[In] int(ln(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] ln(2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/e*sum(1/_alpha*(2*ln(x-_alpha)*ln(x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))+e*(1/_alpha/e*ln(x-_alpha)^2+2*_alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/d*dilog(1/2*(x+_alpha)/_alpha))-2*dilog(x/_alpha)-2*ln(x-_alpha)*ln(x/_alpha)-2*dilog((_alpha*e+d*(-e/d)^(1/2)+(x-_alpha)*e)/(_alpha*e+d*(-e/d)^(1/2)))-2*ln(x-_alpha)*ln((_alpha*e+d*(-e/d)^(1/2)+(x-_alpha)*e)/(_alpha*e+d*(-e/d)^(1/2)))),_alpha=RootOf(_Z^2*e+d))

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2+dx\sqrt{-\frac{e}{d}}\right)}{ex^2+d}+1\right)}{2\,e}$$

[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="fricas")

[Out] -1/2*sqrt(-e/d)*dilog(-2*(e*x^2 + d*x*sqrt(-e/d))/(e*x^2 + d) + 1)/e

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(ln(2*x*(e*x+d*(-e/d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\log\left(\frac{2\left(ex+d\sqrt{-\frac{e}{d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="g
iac")
[Out] integrate(log(2*(e*x + d*sqrt(-e/d))*x/(e*x^2 + d))/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x+d\,\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

```
[In] int(log((2*x*(e*x + d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x + d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.41 $\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result 287
Rubi [A] (verified) 287
Mathematica [B] (verified) 288
Maple [C] (verified) 289
Fricas [A] (verification not implemented) 289
Sympy [F(-2)] 290
Maxima [F(-2)] 290
Giac [F] 290
Mupad [F(-1)] 291

Optimal result

Integrand size = 40, antiderivative size = 50

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,1+\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{2e}$$

```
[Out] 1/2*polylog(2,1+2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))*(-e/d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2497}

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{ex^2+d}+1\right)}{2e}$$

```
[In] Int[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (Sqrt[-(e/d)]*PolyLog[2, 1 + (2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)])/(2*
e)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-\frac{e}{d}}\text{Li}_2\left(1+\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{2e}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 642 vs. 2(50) = 100.

Time = 0.30 (sec) , antiderivative size = 642, normalized size of antiderivative = 12.84

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$

$$= \frac{-2\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}x\right)+2\log\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)-\log^2\left(\sqrt{-d}+\right.}{}$$

```
[In] Integrate[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[e]*(1 + Sqrt[-(e/d)]*x))/(S
qrt[e] - Sqrt[-d]*Sqrt[-(e/d)])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[e]
*(1 + Sqrt[-(e/d)]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d]
- Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*Log[Sqrt[-d]
 + Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*PolyLog[2,
(Sqrt[-(e/d)]*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(e/d)])] +
2*PolyLog[2, (Sqrt[-(e/d)]*(Sqrt[-d] + Sqrt[e]*x))/(-Sqrt[e] + Sqrt[-d]*Sqr
t[-(e/d)])] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sq
rt[-d]*Sqrt[e]*x)/(2*d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2
*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.35 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.86

method	result
risch	$\frac{\ln\left(2\right)\arctan\left(\frac{x e}{\sqrt{d e}}\right)}{\sqrt{d e}}+\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(e\_Z^{2}+d\right)}\frac{2\ln\left(x-\_\alpha\right)\ln\left(\frac{x\left(e x-d\sqrt{-\frac{e}{d}}\right)}{e\,x^{2}+d}\right)+e\left(\frac{\ln\left(x-\_\alpha\right)^{2}}{\_\alpha e}+\frac{2\_\alpha\ln\left(x-\_\alpha\right)\ln\left(\frac{x+\_\alpha}{2\_\alpha}\right)}{d}+\frac{2\_\alpha\operatorname{dilog}\left(\cdots\right.}{d}\cdots\right.}{\cdots}}{\cdots}$

[In] int(ln(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] ln(2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/e*sum(1/_alpha*(2*ln(x-_alpha)*ln(x*(e*x-d*(-e/d)^(1/2))/(e*x^2+d))+e*(1/_alpha/e*ln(x-_alpha)^2+2*_alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/d*dilog(1/2*(x+_alpha)/_alpha))-2*dilog((-d*(-e/d)^(1/2)+_alpha*e+(x-_alpha)*e)/(_alpha*e-d*(-e/d)^(1/2)))-2*ln(x-_alpha)*ln((-d*(-e/d)^(1/2)+_alpha*e+(x-_alpha)*e)/(_alpha*e-d*(-e/d)^(1/2)))-2*dilog(x/_alpha)-2*ln(x-_alpha)*ln(x/_alpha)),_alpha=RootOf(_Z^2*e+d))

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\frac{\sqrt{-\frac{e}{d}}\operatorname{Li}_2\left(-\frac{2\left(ex^2-dx\sqrt{-\frac{e}{d}}\right)}{ex^2+d}+1\right)}{2\,e}$$

[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="fricas")

[Out] 1/2*sqrt(-e/d)*dilog(-2*(e*x^2 - d*x*sqrt(-e/d))/(e*x^2 + d) + 1)/e

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(ln(-2*x*(-e*x+d*(-e/d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\log\left(\frac{2\left(ex-d\sqrt{-\frac{e}{d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"giac")
```

```
[Out] integrate(log(2*(e*x - d*sqrt(-e/d))*x/(e*x^2 + d))/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\int \frac{\ln\left(\frac{2\,x\,\left(e\,x-d\,\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

```
[In] int(log((2*x*(e*x - d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x - d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.42 $\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result 292
Rubi [A] (verified) 292
Mathematica [B] (verified) 293
Maple [F] 293
Fricas [A] (verification not implemented) 294
Sympy [F(-2)] 294
Maxima [F(-2)] 294
Giac [F] 295
Mupad [F(-1)] 295

Optimal result

Integrand size = 41, antiderivative size = 53

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\operatorname{PolyLog}\left(2,1+\frac{2\sqrt{e}x(\sqrt{-d}-\sqrt{e}x)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] -1/2*polylog(2,1+2*x*e^(1/2)*((-d)^(1/2)-x*e^(1/2))/(e*x^2+d))/(-d)^(1/2)/e
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2497}

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\operatorname{PolyLog}\left(2,\frac{2\sqrt{e}x(\sqrt{-d}-\sqrt{e}x)}{ex^2+d}+1\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[In] Int[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -1/2*PolyLog[2, 1 + (2*Sqrt[e]*x*(Sqrt[-d] - Sqrt[e]*x))/(d + e*x^2)]/(Sqrt
[-d]*Sqrt[e])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = -\frac{\text{Li}_2\left(1+\frac{2\sqrt{e}x\left(\sqrt{-d}-\sqrt{e}x\right)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 320 vs. 2(53) = 106.

Time = 0.15 (sec) , antiderivative size = 320, normalized size of antiderivative = 6.04

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{-2\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)+2\log\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)-\log^2\left(\sqrt{-d}+\sqrt{e}x\right)+2\log\left(\sqrt{-d}\right.}{}$$

```
[In] Integrate[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),x
]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + 2*Log[(d*Sqrt[e]*
x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Log[Sqrt[-d] + Sqrt[e]*x]^2 + 2*
Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sqrt[
-d] - Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d + e*x^2)] - 2*L
og[Sqrt[-d] + Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d + e*x^2
)] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] + 2*PolyLog[2, (d + Sqrt[-d]*Sq
rt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*S
qrt[e])
```

Maple [F]

$$\int \frac{\ln\left(\frac{2x\left(ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}dx$$

```
[In] int(ln(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{-d}\,\mathrm{Li}_2\left(-\frac{2\,(ex^2-\sqrt{-d}\sqrt{e}x)}{ex^2+d}+1\right)}{2\,d\sqrt{e}}$$

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="fricas")
```

```
[Out] 1/2*sqrt(-d)*dilog(-2*(e*x^2 - sqrt(-d)*sqrt(e)*x)/(e*x^2 + d) + 1)/(d*sqrt
(e))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(ln(2*x*(e*x+d*e**(1/2)/(-d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\log\left(\frac{2\left(ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="giac")
```

```
[Out] sage2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x-\sqrt{-d}\,\sqrt{e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

```
[In] int(log((2*x*(e*x - (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
```

```
[Out] int(log((2*x*(e*x - (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.43 $\int \frac{\log \left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result 296
Rubi [A] (verified) 296
Mathematica [B] (verified) 297
Maple [F] 297
Fricas [A] (verification not implemented) 298
Sympy [F(-2)] 298
Maxima [F(-2)] 298
Giac [F] 299
Mupad [F(-1)] 299

Optimal result

Integrand size = 42, antiderivative size = 52

$$\int \frac{\log \left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\frac{\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{e}x\left(\sqrt{-d}+\sqrt{e}x\right)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] 1/2*polylog(2,1-2*x*e^(1/2)*((-d)^(1/2)+x*e^(1/2))/(e*x^2+d))/(-d)^(1/2)/e^
(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2497}

$$\int \frac{\log \left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\frac{\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{e}x\left(\sqrt{e}x+\sqrt{-d}\right)}{ex^2+d}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[In] Int[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] PolyLog[2, 1 - (2*Sqrt[e]*x*(Sqrt[-d] + Sqrt[e]*x))/(d + e*x^2)]/(2*Sqrt[-d
]*Sqrt[e])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Li}_2\left(1-\frac{2\sqrt{e}x(\sqrt{-d}+\sqrt{e}x)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 316 vs. 2(52) = 104.

Time = 0.13 (sec) , antiderivative size = 316, normalized size of antiderivative = 6.08

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{-2\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}x\right)+2\log\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)-2\log\left(\sqrt{-d}\right.}{}$$

```
[In] Integrate[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2),
x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - 2*
Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sqrt[
-d] - Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)] - 2*Log[
Sqrt[-d] + Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)] + 2
*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*
x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*Sqrt[e]
)
```

Maple [F]

$$\int \frac{\ln\left(-\frac{2x\left(-ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}dx$$

```
[In] int(ln(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{\log \left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\sqrt{-d}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2+\sqrt{-d}\sqrt{e}x\right)}{ex^2+d}+1\right)}{2\,d\sqrt{e}}$$

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="fricas")
```

```
[Out] -1/2*sqrt(-d)*dilog(-2*(e*x^2 + sqrt(-d)*sqrt(e)*x)/(e*x^2 + d) + 1)/(d*sqr
t(e))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log \left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(ln(-2*x*(-e*x+d*e**(1/2)/(-d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\log\left(\frac{2\left(ex-\frac{d\sqrt{e}}{\sqrt{-d}}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="giac")

[Out] sage2

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x+\sqrt{-d}\,\sqrt{e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

[In] int(log((2*x*(e*x + (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2),x)

[Out] int(log((2*x*(e*x + (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2), x)

3.44 $\int \frac{\log \left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2} dx$

Optimal result 300
Rubi [A] (verified) 300
Mathematica [B] (verified) 301
Maple [C] (verified) 302
Fricas [A] (verification not implemented) 302
Sympy [F(-2)] 303
Maxima [F(-2)] 303
Giac [F(-2)] 303
Mupad [F(-1)] 304

Optimal result

Integrand size = 40, antiderivative size = 49

$$\int \frac{\log \left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2} dx = \frac{\text{PolyLog}\left(2, 1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[Out] 1/2*polylog(2,1-2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/d^(1/2)/(-e)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2497}

$$\int \frac{\log \left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2} dx = \frac{\text{PolyLog}\left(2, 1-\frac{2x\left(ex+\sqrt{d}\sqrt{-e}\right)}{ex^2+d}\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[In] Int[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] PolyLog[2, 1 - (2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(2*Sqrt[d]*Sqrt[
-e])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x]][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Li}_2\left(1 - \frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 641 vs. 2(49) = 98.

Time = 0.25 (sec) , antiderivative size = 641, normalized size of antiderivative = 13.08

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{-2\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}x\right)+2\log\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)-\log^2\left(\sqrt{-d}-\right.}{}$$

```
[In] Integrate[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] + e*x)/(Sqrt[d]
*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[d]*S
qrt[-e] + e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] - Sq
rt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d] +
 Sqrt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] + 2*PolyLog[2,
1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] +
2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*
x)/(-d)^(3/2)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[d]*Sqrt[-e] +
Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] + e*x)/(-(Sqrt[d]*Sqrt[
-e]) + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.61 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.76

method	result
risch	$\frac{\ln(2)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^2+d\right)}\frac{2\ln(x-\_\alpha)\ln\left(\frac{x\left(ex+\sqrt{d}\sqrt{-e}\right)}{e x^2+d}\right)+e\left(\frac{\ln(x-\_\alpha)^2}{\_\alpha e}+\frac{2\_\alpha\ln(x-\_\alpha)\ln\left(\frac{x+\_\alpha}{2\_\alpha}\right)}{d}+\frac{2\_\alpha\,\mathrm{dilog}\left(\frac{x+}{2}\right.}{d}\right.}{}}{}$

[In] int(ln(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] ln(2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/e*sum(1/_alpha*(2*ln(x-_alpha)*ln(x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))+e*(1/_alpha/e*ln(x-_alpha)^2+2*_alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/d*dilog(1/2*(x+_alpha)/_alpha))-2*dilog((_alpha*e+d^(1/2)*(-e)^(1/2)+(x-_alpha)*e)/(_alpha*e+d^(1/2)*(-e)^(1/2)))-2*ln(x-_alpha)*ln((_alpha*e+d^(1/2)*(-e)^(1/2)+(x-_alpha)*e)/(_alpha*e+d^(1/2)*(-e)^(1/2)))-2*dilog(x/_alpha)-2*ln(x-_alpha)*ln(x/_alpha)),_alpha=RootOf(_Z^2*e+d))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\sqrt{-e}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2+\sqrt{d}\sqrt{-e}x\right)}{ex^2+d}+1\right)}{2\sqrt{d}e}$$

[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="fricas")

[Out] -1/2*sqrt(-e)*dilog(-2*(e*x^2 + sqrt(d)*sqrt(-e)*x)/(e*x^2 + d) + 1)/(sqrt(d)*e)

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(ln(2*x*(e*x+d**(1/2)*(-e)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x+\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

```
[In] int(log((2*x*(e*x + d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x + d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.45 $\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal result 305
Rubi [A] (verified) 305
Mathematica [B] (verified) 306
Maple [C] (verified) 307
Fricas [A] (verification not implemented) 307
Sympy [F(-2)] 308
Maxima [F(-2)] 308
Giac [F(-2)] 308
Mupad [F(-1)] 309

Optimal result

Integrand size = 41, antiderivative size = 50

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\operatorname{PolyLog}\left(2,1+\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[Out] -1/2*polylog(2,1+2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/d^(1/2)/(-e)^(1/2
)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2497}

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\operatorname{PolyLog}\left(2,\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{ex^2+d}+1\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[In] Int[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -1/2*PolyLog[2, 1 + (2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(Sqrt[d]*Sq
rt[-e])
```

<u>Rule 2497</u>

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = -\frac{\text{Li}_2\left(1+\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 645 vs. 2(50) = 100.

Time = 0.18 (sec) , antiderivative size = 645, normalized size of antiderivative = 12.90

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$$
$$= \frac{-2\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}x\right)+2\log\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)-\log^2\left(\sqrt{-d}+\right.}{}$$

```
[In] Integrate[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] - e*x)/(Sqrt[d]
*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[d]*S
qrt[-e] - e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] - Sq
rt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d
] + Sqrt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)] + 2*PolyL
og[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*
d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sq
rt[e]*x)/(-d)^(3/2)] - 2*PolyLog[2, (-(Sqrt[-d]*Sqrt[e]) + e*x)/(Sqrt[d]*Sq
rt[-e] - Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] + e*x)/(Sqrt[d
]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.48 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.76

method	result
risch	$\frac{\ln(2)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^2+d\right)} \frac{2\ln(x-\_\alpha)\ln\left(\frac{x\left(ex-\sqrt{d}\sqrt{-e}\right)}{e\,x^2+d}\right)+e\left(\frac{\ln(x-\_\alpha)^2}{\_\alpha e}+\frac{2\_\alpha\ln(x-\_\alpha)\ln\left(\frac{x+\_\alpha}{2\_\alpha}\right)}{d}+\frac{2\_\alpha\,\mathrm{dilog}\left(\right.}{d}\right.}{}}{}$

```
[In] int(ln(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x,method=_RETURN
VERBOSE)
```

```
[Out] ln(2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/e*sum(1/_alpha*(2*ln(x-_alpha
)*ln(x*(e*x-d^(1/2)*(-e)^(1/2))/(e*x^2+d))+e*(1/_alpha/e*ln(x-_alpha)^2+2*_
alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/d*dilog(1/2*(x+_alp
ha)/_alpha))-2*dilog((_alpha*e-d^(1/2)*(-e)^(1/2)+(x-_alpha)*e)/(_alpha*e-d
^(1/2)*(-e)^(1/2)))-2*ln(x-_alpha)*ln((_alpha*e-d^(1/2)*(-e)^(1/2)+(x-_alph
a)*e)/(_alpha*e-d^(1/2)*(-e)^(1/2)))-2*dilog(x/_alpha)-2*ln(x-_alpha)*ln(x/
_alpha)),_alpha=RootOf(_Z^2*e+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{-e}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2-\sqrt{d}\sqrt{-e}x\right)}{ex^2+d}+1\right)}{2\sqrt{d}e}$$

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="fricas")
```

```
[Out] 1/2*sqrt(-e)*dilog(-2*(e*x^2 - sqrt(d)*sqrt(-e)*x)/(e*x^2 + d) + 1)/(sqrt(d
)*e)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(ln(-2*x*(-e*x+d**(1/2)*(-e)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \int \frac{\ln\left(\frac{2\,x\,\left(e\,x-\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

```
[In] int(log((2*x*(e*x - d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x - d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.46 $\int (ex)^m (a+b\log(c\log^p(dx)))\,dx$

Optimal result 310
Rubi [A] (verified) 310
Mathematica [A] (verified) 311
Maple [F] 312
Fricas [A] (verification not implemented) 312
Sympy [F] 312
Maxima [F] 312
Giac [F] 313
Mupad [F(-1)] 313

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int (ex)^m (a+b\log(c\log^p(dx)))\,dx = -\frac{bp(dx)^{-1-m}(ex)^{1+m}\,\text{ExpIntegralEi}((1+m)\log(dx))}{e(1+m)} + \frac{(ex)^{1+m}(a+b\log(c\log^p(dx)))}{e(1+m)}$$

```
[Out] -b*p*(d*x)^(-1-m)*(e*x)^(1+m)*Ei((1+m)*ln(d*x))/e/(1+m)+(e*x)^(1+m)*(a+b*ln
(c*ln(d*x)^p))/e/(1+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2602, 2347, 2209}

$$\int (ex)^m (a+b\log(c\log^p(dx)))\,dx = \frac{(ex)^{m+1}(a+b\log(c\log^p(dx)))}{e(m+1)} - \frac{bp(dx)^{-m-1}(ex)^{m+1}\,\text{ExpIntegralEi}((m+1)\log(dx))}{e(m+1)}$$

```
[In] Int[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

```
[Out] -((b*p*(d*x)^(-1 - m)*(e*x)^(1 + m)*ExpIntegralEi[(1 + m)*Log[d*x]])/(e*(1
+ m))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x]^p]))/(e*(1 + m))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}-\frac{(bp)\int\frac{(ex)^m}{\log(dx)}\,dx}{1+m}\\
&= \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}-\frac{\left(bp(dx)^{-1-m}(ex)^{1+m}\right)\operatorname{Subst}\left(\int\frac{e^{(1+m)x}}{x}\,dx,x,\log(dx)\right)}{e(1+m)}\\
&= -\frac{bp(dx)^{-1-m}(ex)^{1+m}\operatorname{Ei}((1+m)\log(dx))}{e(1+m)}+\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$
\int (ex)^m\left(a+b\log\left(c\log^p(dx)\right)\right)\,dx = \frac{(dx)^{-m}(ex)^m\left(-bp\,\text{ExpIntegralEi}((1+m)\log(dx))+dx(dx)^m\left(a+b\log\left(c\log^p(dx)\right)\right)\right)}{d(1+m)}
$$

```
[In] Integrate[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

```
[Out] ((e*x)^m*(-(b*p*ExpIntegralEi[(1 + m)*Log[d*x]]) + d*x*(d*x)^m*(a + b*Log[c
*Log[d*x]^p])))/(d*(1 + m)*(d*x)^m)
```

Maple [F]

$$\int (ex)^m (a + b\ln(c\ln(dx)^p))\,dx$$

```
[In] int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

```
[Out] int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int (ex)^m (a + b\log(c\log^p(dx)))\,dx = \frac{bdpxe^{\left(m\log(dx)+m\log\left(\frac{e}{d}\right)\right)}\log(\log(dx)) - bp\left(\frac{e}{d}\right)^m \mathrm{Ei}((m+1)\log(dx)) + (bdx\log(c) + adx)e^{\left(m\log(dx)+m\log\left(\frac{e}{d}\right)\right)}}{dm + d}$$

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="fricas")
```

```
[Out] (b*d*p*x*e^(m*log(d*x) + m*log(e/d))*log(log(d*x)) - b*p*(e/d)^m*Ei((m + 1)
*log(d*x)) + (b*d*x*log(c) + a*d*x)*e^(m*log(d*x) + m*log(e/d)))/(d*m + d)
```

Sympy [F]

$$\int (ex)^m (a + b\log(c\log^p(dx)))\,dx = \int (ex)^m (a + b\log(c\log(dx)^p))\,dx$$

```
[In] integrate((e*x)**m*(a+b*ln(c*ln(d*x)**p)),x)
```

```
[Out] Integral((e*x)**m*(a + b*log(c*log(d*x)**p)), x)
```

Maxima [F]

$$\int (ex)^m (a + b\log(c\log^p(dx)))\,dx = \int (b\log(c\log(dx)^p) + a)(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="maxima")
```

```
[Out] -(e^m*p*integrate(x^m/((m^2 + 2*m + 1)*log(d)^2 + 2*(m^2 + 2*m + 1)*log(d)*
log(x) + (m^2 + 2*m + 1)*log(x)^2), x) - ((e^m*(m + 1)*x*log(d) + e^m*(m +
1)*x*log(x))*x^m*log((log(d) + log(x))^p) + (e^m*(m + 1)*x*log(c)*log(x) +
(e^m*(m + 1)*log(c)*log(d) - e^m*p)*x)*x^m)/((m^2 + 2*m + 1)*log(d) + (m^2
+ 2*m + 1)*log(x)))*b + (e*x)^(m + 1)*a/(e*(m + 1))
```

Giac [F]

$$\int (ex)^m (a + b\log(c\log^p(dx)))\,dx = \int (b\log(c\log(dx)^p) + a)(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="giac")
[Out] integrate((b*log(c*log(d*x)^p) + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a + b\log(c\log^p(dx)))\,dx = \int (a + b\,\ln(c\ln(d\,x)^p))\,(e\,x)^m\,dx$$

```
[In] int((a + b*log(c*log(d*x)^p))*(e*x)^m,x)
[Out] int((a + b*log(c*log(d*x)^p))*(e*x)^m, x)
```

3.47 $\int (ex)^m (a+b\log(c\log^p(dx^n)))\,dx$

Optimal result 314
Rubi [A] (verified) 314
Mathematica [A] (verified) 315
Maple [F] 316
Fricas [A] (verification not implemented) 316
Sympy [F] 316
Maxima [F] 316
Giac [F] 317
Mupad [F(-1)] 317

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int (ex)^m (a+b\log(c\log^p(dx^n)))\,dx = -\frac{bp(ex)^{1+m}(dx^n)^{-\frac{1+m}{n}}\operatorname{ExpIntegralEi}\left(\frac{(1+m)\log(dx^n)}{n}\right)}{e(1+m)} + \frac{(ex)^{1+m}(a+b\log(c\log^p(dx^n)))}{e(1+m)}$$

```
[Out] -b*p*(e*x)^(1+m)*Ei((1+m)*ln(d*x^n)/n)/e/(1+m)/((d*x^n)^((1+m)/n))+(e*x)^(1
+m)*(a+b*ln(c*ln(d*x^n)^p))/e/(1+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2602, 2347, 2209}

$$\int (ex)^m (a+b\log(c\log^p(dx^n)))\,dx = \frac{(ex)^{m+1}(a+b\log(c\log^p(dx^n)))}{e(m+1)} - \frac{bp(ex)^{m+1}(dx^n)^{-\frac{m+1}{n}}\operatorname{ExpIntegralEi}\left(\frac{(m+1)\log(dx^n)}{n}\right)}{e(m+1)}$$

```
[In] Int[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -((b*p*(e*x)^(1 + m)*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(e*(1 + m)*(d*x
^n)^((1 + m)/n))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(1 + m))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)}-\frac{(bnp)\int\frac{(ex)^m}{\log(dx^n)}\,dx}{1+m}\\
&= \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)}-\frac{\left(bp(ex)^{1+m}\left(dx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(1+m)x}{n}}}{x}\,dx,x,\log\left(dx^n\right)\right)}{e(1+m)}\\
&= -\frac{bp(ex)^{1+m}\left(dx^n\right)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m)\log(dx^n)}{n}\right)}{e(1+m)}+\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$
\int(ex)^m\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx
=\frac{x(ex)^m\left(a-bp(dx^n)^{-\frac{1+m}{n}}\text{ExpIntegralEi}\left(\frac{(1+m)\log(dx^n)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{1+m}
$$

```
[In] Integrate[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] (x*(e*x)^m*(a - (b*p*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(d*x^n)^((1 + m
)/n) + b*Log[c*Log[d*x^n]^p]))/(1 + m)
```

Maple [F]

$$\int (ex)^m \left(a + b\ln\left(c\ln\left(d\,x^n\right)^p\right)\right) dx$$

[In] int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)

[Out] int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int (ex)^m \left(a + b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = \frac{bpxe^{(m\log(e)+m\log(x))}\log\left(n\log\left(x\right)+\log\left(d\right)\right) - bp\mathrm{Ei}\left(\frac{(m+1)n\log(x)+(m+1)\log(d)}{n}\right)e^{\left(\frac{mn\log(e)-(m+1)\log(d)}{n}\right)} + (bx\log(c$$

[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")

[Out] (b*p*x*e^(m*log(e) + m*log(x))*log(n*log(x) + log(d)) - b*p*Ei(((m + 1)*n*l
og(x) + (m + 1)*log(d))/n)*e^((m*n*log(e) - (m + 1)*log(d))/n) + (b*x*log(c
) + a*x)*e^(m*log(e) + m*log(x)))/(m + 1)

Sympy [F]

$$\int (ex)^m \left(a + b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = \int (ex)^m \left(a + b\log\left(c\log\left(dx^n\right)^p\right)\right)\,dx$$

[In] integrate((e*x)**m*(a+b*ln(c*ln(d*x**n)**p)),x)

[Out] Integral((e*x)**m*(a + b*log(c*log(d*x**n)**p)), x)

Maxima [F]

$$\int (ex)^m \left(a + b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = \int (b\log\left(c\log\left(dx^n\right)^p\right) + a)(ex)^m\,dx$$

[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")

[Out] -(e^m*n*p*integrate(x^m/((m + 1)*log(d) + (m + 1)*log(x^n)), x) - (e^m*x*x^
m*log(c) + e^m*x*x^m*log((log(d) + log(x^n))^p))/(m + 1))*b + (e*x)^(m + 1)
a/(e(m + 1))

Giac [F]

$$\int (ex)^m \left(a + b\log \left(c\log^p \left(dx^n\right)\right)\right) \, dx = \int (b\log \left(c\log \left(dx^n\right)^p\right) + a)(ex)^m \, dx$$

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
[Out] integrate((b*log(c*log(d*x^n)^p) + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a + b\log \left(c\log^p \left(dx^n\right)\right)\right) \, dx = \int (e\,x)^m \left(a + b\,\ln \left(c\ln \left(d\,x^n\right)^p\right)\right) \, dx$$

```
[In] int((e*x)^m*(a + b*log(c*log(d*x^n)^p)),x)
[Out] int((e*x)^m*(a + b*log(c*log(d*x^n)^p)), x)
```

3.48 $\int x^2(a+b\log(c\log^p(dx^n)))\,dx$

Optimal result 318
Rubi [A] (verified) 318
Mathematica [A] (verified) 319
Maple [F] 320
Fricas [A] (verification not implemented) 320
Sympy [F] 320
Maxima [F] 320
Giac [A] (verification not implemented) 321
Mupad [F(-1)] 321

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx = -\frac{1}{3}bpx^3(dx^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{3\log(dx^n)}{n}\right) + \frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))$$

```
[Out] -1/3*b*p*x^3*Ei(3*ln(d*x^n)/n)/((d*x^n)^(3/n))+1/3*x^3*(a+b*ln(c*ln(d*x^n)^
p))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2602, 2347, 2209}

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx = \frac{1}{3}x^3(a+b\log(c\log^p(dx^n))) - \frac{1}{3}bpx^3(dx^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{3\log(dx^n)}{n}\right)$$

```
[In] Int[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -1/3*(b*p*x^3*ExpIntegralEi[(3*Log[d*x^n])/n])/(d*x^n)^(3/n) + (x^3*(a + b*
Log[c*Log[d*x^n]^p]))/3
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))-\frac{1}{3}(bnp)\int\frac{x^2}{\log(dx^n)}\,dx\\
&= \frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))-\frac{1}{3}\left(bpx^3(dx^n)^{-3/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{x}\,dx,x,\log(dx^n)\right)\\
&= -\frac{1}{3}bpx^3(dx^n)^{-3/n}\,\text{Ei}\left(\frac{3\log(dx^n)}{n}\right)+\frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$
\int x^2(a+b\log(c\log^p(dx^n)))\,dx=\frac{1}{3}x^3\left(a-bp(dx^n)^{-3/n}\,\text{ExpIntegralEi}\left(\frac{3\log(dx^n)}{n}\right)+b\log(c\log^p(dx^n))\right)
$$

```
[In] Integrate[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
[Out] (x^3*(a - (b*p*ExpIntegralEi[(3*Log[d*x^n])/n])/(d*x^n)^(3/n) + b*Log[c*Log
[d*x^n]^p]))/3
```

Maple [F]

$$\int x^2(a+b\ln(c\ln(d\,x^n)^p))\,dx$$

```
[In] int(x^2*(a+b*ln(c*ln(d*x^n)^p)),x)
```

```
[Out] int(x^2*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int x^2(a+b\log(c\log^p(dx^n)))\;dx = \frac{bd^{\frac{3}{n}}px^3\log(n\log(x)+\log(d)) - bp\,\mathrm{log\_integral}\left(d^{\frac{3}{n}}x^3\right) + (bx^3\log(c)+ax^3)d^{\frac{3}{n}}}{3\,d^{\frac{3}{n}}}$$

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^(3/n)*p*x^3*log(n*log(x) + log(d)) - b*p*log_integral(d^(3/n)*x^3)
 + (b*x^3*log(c) + a*x^3)*d^(3/n))/d^(3/n)
```

Sympy [F]

$$\int x^2(a+b\log(c\log^p(dx^n)))\;dx = \int x^2(a+b\log(c\log(dx^n)^p))\;dx$$

```
[In] integrate(x**2*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral(x**2*(a + b*log(c*log(d*x**n)**p)), x)
```

Maxima [F]

$$\int x^2(a+b\log(c\log^p(dx^n)))\;dx = \int (b\log(c\log(dx^n)^p)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/3*(x^3*log(c) + x^3*log((log(d) + log(x^n))^p) - 3*n*p*integr
ate(1/3*x^2/(log(d) + log(x^n)), x))*b
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx=\frac{1}{3}\,bpx^3\log\left(n\log\left(x\right)+\log\left(d\right)\right)+\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3-\frac{bp\mathrm{Ei}\left(\frac{3\,\log(d)}{n}+3\,\log\left(x\right)\right)}{3\,d^{\frac{3}{n}}}$$

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
[Out] 1/3*b*p*x^3*log(n*log(x) + log(d)) + 1/3*b*x^3*log(c) + 1/3*a*x^3 - 1/3*b*p
*Ei(3*log(d)/n + 3*log(x))/d^(3/n)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx=\int x^2\,\left(a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\right)\,dx$$

```
[In] int(x^2*(a + b*log(c*log(d*x^n)^p)),x)
[Out] int(x^2*(a + b*log(c*log(d*x^n)^p)), x)
```

3.49 $\int x(a+b\log(c\log^p(dx^n)))\,dx$

Optimal result 322
Rubi [A] (verified) 322
Mathematica [A] (verified) 323
Maple [F] 324
Fricas [A] (verification not implemented) 324
Sympy [F] 324
Maxima [F] 324
Giac [A] (verification not implemented) 325
Mupad [F(-1)] 325

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int x(a+b\log(c\log^p(dx^n)))\,dx = -\frac{1}{2}bpx^2(dx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2\log(dx^n)}{n}\right)+\frac{1}{2}x^2(a+b\log(c\log^p(dx^n)))$$

```
[Out] -1/2*b*p*x^2*Ei(2*ln(d*x^n)/n)/((d*x^n)^(2/n))+1/2*x^2*(a+b*ln(c*ln(d*x^n)^
p))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2602, 2347, 2209}

$$\int x(a+b\log(c\log^p(dx^n)))\,dx = \frac{1}{2}x^2(a+b\log(c\log^p(dx^n)))-\frac{1}{2}bpx^2(dx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2\log(dx^n)}{n}\right)$$

```
[In] Int[x*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -1/2*(b*p*x^2*ExpIntegralEi[(2*Log[d*x^n])/n])/(d*x^n)^(2/n) + (x^2*(a + b*
Log[c*Log[d*x^n]^p]))/2
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2(a+b\log\left(c\log^p\left(dx^n\right)\right)) - \frac{1}{2}(bnp)\int\frac{x}{\log\left(dx^n\right)}\,dx\\ &= \frac{1}{2}x^2(a+b\log\left(c\log^p\left(dx^n\right)\right)) - \frac{1}{2}\left(bpx^2\left(dx^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{x}\,dx, x, \log\left(dx^n\right)\right)\\ &= -\frac{1}{2}bpx^2\left(dx^n\right)^{-2/n}\text{Ei}\left(\frac{2\log\left(dx^n\right)}{n}\right) + \frac{1}{2}x^2(a+b\log\left(c\log^p\left(dx^n\right)\right))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int x(a+b\log\left(c\log^p\left(dx^n\right)\right))\,dx = \frac{1}{2}x^2\left(a - bp\left(dx^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\log\left(dx^n\right)}{n}\right) + b\log\left(c\log^p\left(dx^n\right)\right)\right)$$

```
[In] Integrate[x*(a + b*Log[c*Log[d*x^n]^p]),x]
[Out] (x^2*(a - (b*p*ExpIntegralEi[(2*Log[d*x^n])/n])/(d*x^n)^(2/n) + b*Log[c*Log
[d*x^n]^p]))/2
```

Maple [F]

$$\int x(a+b\ln(c\ln(d\,x^{n})^{p}))\,dx$$

```
[In] int(x*(a+b*ln(c*ln(d*x^n)^p)),x)
[Out] int(x*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int x(a+b\log(c\log^{p}(dx^{n})))\,dx = \frac{bd^{\frac{2}{n}}px^{2}\log(n\log(x)+\log(d)) - bp\operatorname{log\_integral}\left(d^{\frac{2}{n}}x^{2}\right) + (bx^{2}\log(c)+ax^{2})d^{\frac{2}{n}}}{2\,d^{\frac{2}{n}}}$$

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
[Out] 1/2*(b*d^(2/n)*p*x^2*log(n*log(x) + log(d)) - b*p*log_integral(d^(2/n)*x^2)
 + (b*x^2*log(c) + a*x^2)*d^(2/n))/d^(2/n)
```

Sympy [F]

$$\int x(a+b\log(c\log^{p}(dx^{n})))\,dx = \int x(a+b\log(c\log(dx^{n})^{p}))\,dx$$

```
[In] integrate(x*(a+b*ln(c*ln(d*x**n)**p)),x)
[Out] Integral(x*(a + b*log(c*log(d*x**n)**p)), x)
```

Maxima [F]

$$\int x(a+b\log(c\log^{p}(dx^{n})))\,dx = \int (b\log(c\log(dx^{n})^{p})+a)x\,dx$$

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
[Out] 1/2*a*x^2 - 1/2*(2*n*p*integrate(1/2*x/(log(d) + log(x^n)), x) - x^2*log(c)
 - x^2*log((log(d) + log(x^n))^p))*b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x(a+b\log(c\log^p(dx^n)))\,dx = \frac{1}{2}bpx^2\log(n\log(x)+\log(d)) + \frac{1}{2}bx^2\log(c) + \frac{1}{2}ax^2 - \frac{bp\text{Ei}\left(\frac{2\log(d)}{n}+2\log(x)\right)}{2d^{\frac{2}{n}}}$$

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
[Out] 1/2*b*p*x^2*log(n*log(x) + log(d)) + 1/2*b*x^2*log(c) + 1/2*a*x^2 - 1/2*b*p
*Ei(2*log(d)/n + 2*log(x))/d^(2/n)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(c\log^p(dx^n)))\,dx = \int x\left(a+b\ln\left(c\ln\left(d\,x^n\right)^p\right)\right)\,dx$$

```
[In] int(x*(a + b*log(c*log(d*x^n)^p)),x)
[Out] int(x*(a + b*log(c*log(d*x^n)^p)), x)
```

3.50 $\int (a+b\log (c\log ^p(dx^n))) \, dx$

Optimal result 326
Rubi [A] (verified) 326
Mathematica [A] (verified) 327
Maple [F] 328
Fricas [A] (verification not implemented) 328
Sympy [F] 328
Maxima [F] 328
Giac [A] (verification not implemented) 329
Mupad [F(-1)] 329

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int (a+b\log (c\log ^p(dx^n))) \, dx = ax-bpx(dx^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{\log (dx^n)}{n}\right)+bx\log (c\log ^p(dx^n))$$

```
[Out] a*x-b*p*x*Ei(ln(d*x^n)/n)/((d*x^n)^(1/n))+b*x*ln(c*ln(d*x^n)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2600, 2337, 2209}

$$\int (a+b\log (c\log ^p(dx^n))) \, dx = ax+bx\log (c\log ^p(dx^n))-bpx(dx^n)^{-1/n}\text{ExpIntegralEi}\left(\frac{\log (dx^n)}{n}\right)$$

```
[In] Int[a + b*Log[c*Log[d*x^n]^p],x]
```

```
[Out] a*x - (b*p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*x*Log[c*Log[d*
x^n]^p]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2600

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \log\left(c\log^p\left(dx^n\right)\right)\,dx\\
&= ax + bx\log\left(c\log^p\left(dx^n\right)\right) - (bnp)\int \frac{1}{\log\left(dx^n\right)}\,dx\\
&= ax + bx\log\left(c\log^p\left(dx^n\right)\right) - \left(bpx(dx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{x}\,dx, x, \log\left(dx^n\right)\right)\\
&= ax - bpx(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log\left(dx^n\right)}{n}\right) + bx\log\left(c\log^p\left(dx^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \left(a + b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = x\left(a - bp(dx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{\log\left(dx^n\right)}{n}\right) + b\log\left(c\log^p\left(dx^n\right)\right)\right)$$

```
[In] Integrate[a + b*Log[c*Log[d*x^n]^p],x]
[Out] x*(a - (b*p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*Log[c*Log[d*x^n
]^p])
```

Maple [F]

$$\int (a+b\ln(c\ln(d\,x^n)^p))\,dx$$

```
[In] int(a+b*ln(c*ln(d*x^n)^p),x)
[Out] int(a+b*ln(c*ln(d*x^n)^p),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int (a+b\log(c\log^p(dx^n)))\,dx = \frac{bd^{\left(\frac{1}{n}\right)}px\log(n\log(x)+\log(d)) - bp\log\_integral\left(d^{\left(\frac{1}{n}\right)}x\right) + (bx\log(c)+ax)d^{\left(\frac{1}{n}\right)}}{d^{\left(\frac{1}{n}\right)}}$$

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="fricas")
[Out] (b*d^(1/n)*p*x*log(n*log(x) + log(d)) - b*p*log_integral(d^(1/n)*x) + (b*x*
log(c) + a*x)*d^(1/n))/d^(1/n)
```

Sympy [F]

$$\int (a+b\log(c\log^p(dx^n)))\,dx = \int (a+b\log(c\log(dx^n)^p))\,dx$$

```
[In] integrate(a+b*ln(c*ln(d*x**n)**p),x)
[Out] Integral(a + b*log(c*log(d*x**n)**p), x)
```

Maxima [F]

$$\int (a+b\log(c\log^p(dx^n)))\,dx = \int b\log(c\log(dx^n)^p)+a\,dx$$

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="maxima")
[Out] -(n*p*integrate(1/(log(d) + log(x^n)), x) - x*log(c) - x*log((log(d) + log(
x^n))^p))*b + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int (a + b\log (c\log^p (dx^n)))\,dx = \left(px\log (n\log (x) + \log (d)) + x\log (c) - \frac{p\text{Ei}\left(\frac{\log(d)}{n} + \log (x)\right)}{d^{\left(\frac{1}{n}\right)}}\right)b + ax$$

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="giac")
[Out] (p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n))*b
 + a*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\log (c\log^p (dx^n)))\,dx = \int a + b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\,dx$$

```
[In] int(a + b*log(c*log(d*x^n)^p),x)
[Out] int(a + b*log(c*log(d*x^n)^p), x)
```

3.51 $\int \frac{a+b\log(c\log^p(dx^n))}{x}\,dx$

Optimal result . . . 330
Rubi [A] (verified) . . . 330
Mathematica [A] (verified) . . . 331
Maple [A] (verified) . . . 331
Fricas [A] (verification not implemented) . . . 331
Sympy [F] . . . 332
Maxima [A] (verification not implemented) . . . 332
Giac [A] (verification not implemented) . . . 332
Mupad [B] (verification not implemented) . . . 333

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx = -bp\log(x)+\frac{\log\left(dx^n\right)\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{n}$$

```
[Out] -b*p*ln(x)+ln(d*x^n)*(a+b*ln(c*ln(d*x^n)^p))/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2601}

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx = \frac{\log\left(dx^n\right)\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{n}-bp\log(x)$$

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x,x]
```

```
[Out] -(b*p*Log[x]) + (Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = -bp\log(x)+\frac{\log\left(dx^n\right)\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{n}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=a\log(x)-\frac{bp\log\left(dx^n\right)}{n}+\frac{b\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}$$

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x,x]
[Out] a*Log[x] - (b*p*Log[d*x^n])/n + (b*Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

method	result	size
parts	$\ln\left(x\right)a+\frac{b\left(\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\ln\left(d\,x^{n}\right)-p\ln\left(d\,x^{n}\right)\right)}{n}$	39
derivativedivides	$\frac{\ln\left(d\,x^{n}\right)a+\ln\left(d\,x^{n}\right)\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)b-bp\ln\left(d\,x^{n}\right)}{n}$	43
default	$\frac{\ln\left(d\,x^{n}\right)a+\ln\left(d\,x^{n}\right)\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)b-bp\ln\left(d\,x^{n}\right)}{n}$	43
parallelrisch	$\frac{\ln\left(d\,x^{n}\right)a+\ln\left(d\,x^{n}\right)\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)b-bp\ln\left(d\,x^{n}\right)}{n}$	43

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*a+b/n*(ln(c*ln(d*x^n)^p)*ln(d*x^n)-p*ln(d*x^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=\frac{\left(bnp\log\left(x\right)+bp\log\left(d\right)\right)\log\left(n\log\left(x\right)+\log\left(d\right)\right)-\left(bnp-bn\log\left(c\right)-an\right)\log\left(x\right)}{n}$$

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="fricas")
[Out] ((b*n*p*log(x) + b*p*log(d))*log(n*log(x) + log(d)) - (b*n*p - b*n*log(c) -
 a*n)*log(x))/n
```

Sympy [F]

$$\int \frac{a + b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x}\,dx = \int \frac{a + b\log\left(c\log\left(dx^{n}\right)^{p}\right)}{x}\,dx$$

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x,x)
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{a + b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x}\,dx = b\log\left(c\log\left(dx^{n}\right)^{p}\right)\log\left(x\right) - \left(p\log\left(x\right)\log\left(\log\left(dx^{n}\right)\right) - \frac{\left(\log\left(dx^{n}\right)\log\left(\log\left(dx^{n}\right)\right) - \log\left(dx^{n}\right)\right)p}{n}\right)b + a\log\left(x\right)$$

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="maxima")
[Out] b*log(c*log(d*x^n)^p)*log(x) - (p*log(x)*log(log(d*x^n)) - (log(d*x^n)*log(
log(d*x^n)) - log(d*x^n))*p/n)*b + a*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{a + b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x}\,dx = \frac{\left(\left(n\log\left(x\right) + \log\left(d\right)\right)\log\left(n\log\left(x\right) + \log\left(d\right)\right) - n\log\left(x\right) - \log\left(d\right)\right)bp + \left(n\log\left(x\right) + \log\left(d\right)\right)b\log\left(c\right) + (n\log}{n}$$

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="giac")
[Out] (((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*b*p + (n*
log(x) + log(d))*b*log(c) + (n*log(x) + log(d))*a)/n
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=\ln\left(x\right)\,\left(a-b\,p\right)+\frac{b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\,\ln\left(d\,x^n\right)}{n}$$

```
[In] int((a + b*log(c*log(d*x^n)^p))/x,x)
[Out] log(x)*(a - b*p) + (b*log(c*log(d*x^n)^p)*log(d*x^n))/n
```

3.52 $\int \frac{a+b\log(c\log^p(dx^n))}{x^2}\,dx$

Optimal result . . . 334
Rubi [A] (verified) . . . 334
Mathematica [A] (verified) . . . 335
Maple [F] . . . 336
Fricas [A] (verification not implemented) . . . 336
Sympy [F] . . . 336
Maxima [F] . . . 336
Giac [F] . . . 337
Mupad [F(-1)] . . . 337

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx=\frac{bp(dx^n)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

```
[Out] b*p*(d*x^n)^(1/n)*Ei(-ln(d*x^n)/n)/x+(-a-b*ln(c*ln(d*x^n)^p))/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2602, 2347, 2209}

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx=\frac{bp(dx^n)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]
```

```
[Out] (b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)])/x - (a + b*Log[c*Log[d*
x^n]^p])/x
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}+(bnp)\int\frac{1}{x^2\log\left(dx^n\right)}\,dx\\
&= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}+\frac{\left(bp(dx^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int\frac{e^{-\frac{x}{n}}}{x}\,dx,x,\log\left(dx^n\right)\right)}{x}\\
&= \frac{bp(dx^n)^{\frac{1}{n}}\,\text{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx = -\frac{a-bp(dx^n)^{\frac{1}{n}}\,\text{ExpIntegralEi}\left(-\frac{\log(dx^n)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]
[Out] -((a - b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)] + b*Log[c*Log[d*x^
n]^p])/x)
```

Maple [F]

$$\int \frac{a + b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)}{x^{2}}dx$$

[In] int((a+b*ln(c*ln(d*x^n)^p))/x^2,x)

[Out] int((a+b*ln(c*ln(d*x^n)^p))/x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a + b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{2}}\,dx = \frac{bd^{\left(\frac{1}{n}\right)}px\,\mathrm{log\_integral}\left(\frac{1}{d^{\left(\frac{1}{n}\right)}x}\right) - bp\log\left(n\log\left(x\right) + \log\left(d\right)\right) - b\log\left(c\right) - a}{x}$$

[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="fricas")

[Out] (b*d^(1/n)*p*x*log_integral(1/(d^(1/n)*x)) - b*p*log(n*log(x) + log(d)) - b
*log(c) - a)/x

Sympy [F]

$$\int \frac{a + b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{2}}\,dx = \int \frac{a + b\log\left(c\log\left(dx^{n}\right)^{p}\right)}{x^{2}}\,dx$$

[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**2,x)

[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**2, x)

Maxima [F]

$$\int \frac{a + b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{2}}\,dx = \int \frac{b\log\left(c\log\left(dx^{n}\right)^{p}\right) + a}{x^{2}}\,dx$$

[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="maxima")

[Out] (n*p*integrate(1/(x^2*log(d) + x^2*log(x^n)), x) - (log(c) + log((log(d) +
log(x^n))^p))/x)*b - a/x

Giac [F]

$$\int \frac{a + b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx = \int \frac{b\log\left(c\log\left(dx^n\right)^p\right) + a}{x^2}\,dx$$

[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="giac")

[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx = \int \frac{a + b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)}{x^2}\,dx$$

[In] int((a + b*log(c*log(d*x^n)^p))/x^2,x)

[Out] int((a + b*log(c*log(d*x^n)^p))/x^2, x)

3.53 $\int \frac{a+b\log(c\log^p(dx^n))}{x^3}\,dx$

Optimal result 338
Rubi [A] (verified) 338
Mathematica [A] (verified) 339
Maple [F] 340
Fricas [A] (verification not implemented) 340
Sympy [F] 340
Maxima [F] 340
Giac [F] 341
Mupad [F(-1)] 341

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx = \frac{bp(dx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2} - \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

```
[Out] 1/2*b*p*(d*x^n)^(2/n)*Ei(-2*ln(d*x^n)/n)/x^2+1/2*(-a-b*ln(c*ln(d*x^n)^p))/x
^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2602, 2347, 2209}

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx = \frac{bp(dx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2} - \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
```

```
[Out] (b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n])/(2*x^2) - (a + b*Log[c
*Log[d*x^n]^p])/(2*x^2)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}+\frac{1}{2}(bnp)\int\frac{1}{x^3\log\left(dx^n\right)}\,dx\\ &= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}+\frac{\left(bp\left(dx^n\right)^{2/n}\right)\text{Subst}\left(\int\frac{e^{-\frac{2x}{n}}}{x}\,dx,x,\log\left(dx^n\right)\right)}{2x^2}\\ &= \frac{bp\left(dx^n\right)^{2/n}\text{Ei}\left(-\frac{2\log\left(dx^n\right)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx = -\frac{a-bp\left(dx^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2\log\left(dx^n\right)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
[Out] -1/2*(a - b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n] + b*Log[c*Log[
d*x^n]^p])/x^2
```

Maple [F]

$$\int \frac{a+b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)}{x^{3}}dx$$

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x^3,x)
```

```
[Out] int((a+b*ln(c*ln(d*x^n)^p))/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{3}}\,dx = \frac{bd^{\frac{2}{n}}px^{2}\,\text{log\_integral}\left(\frac{1}{d^{\frac{2}{n}}x^{2}}\right)-bp\log\left(n\log\left(x\right)+\log\left(d\right)\right)-b\log\left(c\right)-a}{2\,x^{2}}$$

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^(2/n)*p*x^2*log_integral(1/(d^(2/n)*x^2)) - b*p*log(n*log(x) + log
(d)) - b*log(c) - a)/x^2
```

Sympy [F]

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{3}}\,dx = \int \frac{a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)}{x^{3}}\,dx$$

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**3,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**3, x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{3}}\,dx = \int \frac{b\log\left(c\log\left(dx^{n}\right)^{p}\right)+a}{x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*n*p*integrate(1/2/(x^3*log(d) + x^3*log(x^n)), x) - (log(c) + log((l
og(d) + log(x^n))^p))/x^2)*b - 1/2*a/x^2
```

Giac [F]

$$\int \frac{a + b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx = \int \frac{b\log\left(c\log\left(dx^n\right)^p\right) + a}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx = \int \frac{a + b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)}{x^3}\,dx$$

```
[In] int((a + b*log(c*log(d*x^n)^p))/x^3,x)
[Out] int((a + b*log(c*log(d*x^n)^p))/x^3, x)
```

3.54 $\int \frac{a+b\log(c\log^p(dx^n))}{x^4}\,dx$

Optimal result . . . 342
Rubi [A] (verified) . . . 342
Mathematica [A] (verified) . . . 343
Maple [F] . . . 344
Fricas [A] (verification not implemented) . . . 344
Sympy [F] . . . 344
Maxima [F] . . . 344
Giac [F] . . . 345
Mupad [F(-1)] . . . 345

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^4}\,dx = \frac{bp(dx^n)^{3/n}\,\text{ExpIntegralEi}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{3x^3}$$

```
[Out] 1/3*b*p*(d*x^n)^(3/n)*Ei(-3*ln(d*x^n)/n)/x^3+1/3*(-a-b*ln(c*ln(d*x^n)^p))/x
^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2602, 2347, 2209}

$$\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^4}\,dx = \frac{bp(dx^n)^{3/n}\,\text{ExpIntegralEi}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{3x^3}$$

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
```

```
[Out] (b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n])/(3*x^3) - (a + b*Log[c
*Log[d*x^n]^p])/(3*x^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}+\frac{1}{3}(bnp)\int\frac{1}{x^4\log\left(dx^n\right)}\,dx\\ &= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}+\frac{\left(bp\left(dx^n\right)^{3/n}\right)\text{Subst}\left(\int\frac{e^{-\frac{3x}{n}}}{x}\,dx,x,\log\left(dx^n\right)\right)}{3x^3}\\ &= \frac{bp\left(dx^n\right)^{3/n}\text{Ei}\left(-\frac{3\log\left(dx^n\right)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^4}\,dx = -\frac{a-bp\left(dx^n\right)^{3/n}\text{ExpIntegralEi}\left(-\frac{3\log\left(dx^n\right)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}$$

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
[Out] -1/3*(a - b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n] + b*Log[c*Log[
d*x^n]^p])/x^3
```

Maple [F]

$$\int \frac{a+b\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)}{x^{4}}dx$$

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x^4,x)
[Out] int((a+b*ln(c*ln(d*x^n)^p))/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{4}}\,dx = \frac{bd^{\frac{3}{n}}px^{3}\operatorname{log\_integral}\left(\frac{1}{d^{\frac{3}{n}}x^{3}}\right)-bp\log\left(n\log\left(x\right)+\log\left(d\right)\right)-b\log\left(c\right)-a}{3\,x^{3}}$$

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="fricas")
[Out] 1/3*(b*d^(3/n)*p*x^3*log_integral(1/(d^(3/n)*x^3)) - b*p*log(n*log(x) + log
(d)) - b*log(c) - a)/x^3
```

Sympy [F]

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{4}}\,dx = \int \frac{a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)}{x^{4}}\,dx$$

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**4,x)
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**4, x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c\log^{p}\left(dx^{n}\right)\right)}{x^{4}}\,dx = \int \frac{b\log\left(c\log\left(dx^{n}\right)^{p}\right)+a}{x^{4}}\,dx$$

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="maxima")
[Out] 1/3*(3*n*p*integrate(1/3/(x^4*log(d) + x^4*log(x^n)), x) - (log(c) + log((l
og(d) + log(x^n))^p))/x^3)*b - 1/3*a/x^3
```

Giac [F]

$$\int \frac{a + b \log \left(c \log^p \left(dx^n\right)\right)}{x^4} \, dx = \int \frac{b \log \left(c \log \left(dx^n\right)^p\right) + a}{x^4} \, dx$$

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \log \left(c \log^p \left(dx^n\right)\right)}{x^4} \, dx = \int \frac{a + b \ln \left(c \ln \left(d\, x^n\right)^p\right)}{x^4} \, dx$$

```
[In] int((a + b*log(c*log(d*x^n)^p))/x^4,x)
[Out] int((a + b*log(c*log(d*x^n)^p))/x^4, x)
```

3.55 $\int \log \left(c \log ^p(d x)\right) d x$

Optimal result 346
Rubi [A] (verified) 346
Mathematica [A] (verified) 347
Maple [A] (verified) 347
Fricas [A] (verification not implemented) 347
Sympy [A] (verification not implemented) 348
Maxima [A] (verification not implemented) 348
Giac [A] (verification not implemented) 348
Mupad [B] (verification not implemented) 348

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int \log \left(c \log ^p(d x)\right) d x=x \log \left(c \log ^p(d x)\right)-\frac{p \operatorname{LogIntegral}(d x)}{d}$$

```
[Out] -p*Li(d*x)/d+x*ln(c*ln(d*x)^p)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2600, 2335}

$$\int \log \left(c \log ^p(d x)\right) d x=x \log \left(c \log ^p(d x)\right)-\frac{p \operatorname{LogIntegral}(d x)}{d}$$

```
[In] Int[Log[c*Log[d*x]^p],x]
```

```
[Out] x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2600

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = x\log\left(c\log^p(dx)\right) - p\int \frac{1}{\log(dx)}\,dx$$
$$= x\log\left(c\log^p(dx)\right) - \frac{p\text{li}(dx)}{d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log\left(c\log^p(dx)\right)\,dx = x\log\left(c\log^p(dx)\right) - \frac{p\,\text{LogIntegral}(dx)}{d}$$

```
[In] Integrate[Log[c*Log[d*x]^p],x]
[Out] x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

method	result	size
default	$x\ln\left(c\ln\left(dx\right)^{p}\right)+\frac{p\,\text{Ei}_{1}\left(-\ln\left(dx\right)\right)}{d}$	26

```
[In] int(ln(c*ln(d*x)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*ln(d*x)^p)+p/d*Ei(1,-ln(d*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \log\left(c\log^p(dx)\right)\,dx = \frac{dpx\log\left(\log\left(dx\right)\right) + dx\log\left(c\right) - p\,\text{log\_integral}\left(dx\right)}{d}$$

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="fricas")
[Out] (d*p*x*log(log(d*x)) + d*x*log(c) - p*log_integral(d*x))/d
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \log \left(c\log^p(dx)\right)\,dx = x\log \left(c\log \left(dx\right)^p\right) - \frac{p\operatorname{li}\left(dx\right)}{d}$$

```
[In] integrate(ln(c*ln(d*x)**p),x)
[Out] x*log(c*log(d*x)**p) - p*li(d*x)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \log \left(c\log^p(dx)\right)\,dx = x\log \left(c\log \left(dx\right)^p\right) - \frac{p\mathrm{Ei}(\log \left(dx\right))}{d}$$

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="maxima")
[Out] x*log(c*log(d*x)^p) - p*Ei(log(d*x))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \log \left(c\log^p(dx)\right)\,dx = px\log \left(\log \left(d\right) + \log \left(x\right)\right) + x\log \left(c\right) - \frac{p\mathrm{Ei}(\log \left(d\right) + \log \left(x\right))}{d}$$

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="giac")
[Out] p*x*log(log(d) + log(x)) + x*log(c) - p*Ei(log(d) + log(x))/d
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \log \left(c\log^p(dx)\right)\,dx = x\,\ln\left(c\ln\left(d\,x\right)^p\right) - \frac{p\,\mathrm{logint}(d\,x)}{d}$$

```
[In] int(log(c*log(d*x)^p),x)
[Out] x*log(c*log(d*x)^p) - (p*logint(d*x))/d
```

3.56 $\int \frac{\log(c\log^p(dx))}{x} \, dx$

Optimal result 349
Rubi [A] (verified) 349
Mathematica [A] (verified) 350
Maple [A] (verified) 350
Fricas [A] (verification not implemented) 350
Sympy [F] 351
Maxima [A] (verification not implemented) 351
Giac [A] (verification not implemented) 351
Mupad [B] (verification not implemented) 352

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\log(c\log^p(dx))}{x} \, dx = -p\log(x) + \log(dx)\log(c\log^p(dx))$$

```
[Out] -p*ln(x)+ln(d*x)*ln(c*ln(d*x)^p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2601}

$$\int \frac{\log(c\log^p(dx))}{x} \, dx = \log(dx)\log(c\log^p(dx)) - p\log(x)$$

```
[In] Int[Log[c*Log[d*x]^p]/x,x]
```

```
[Out] -(p*Log[x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = -p\log(x) + \log(dx)\log(c\log^p(dx))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(c \log^p(dx)\right)}{x} \, dx = -p \log(dx) + \log(dx) \log \left(c \log^p(dx)\right)$$

```
[In] Integrate[Log[c*Log[d*x]^p]/x,x]
[Out] -(p*Log[d*x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\ln(dx)\ln(c\ln(dx)^p) - \ln(dx)\,p$	23
default	$\ln(dx)\ln(c\ln(dx)^p) - \ln(dx)\,p$	23

```
[In] int(ln(c*ln(d*x)^p)/x,x,method=_RETURNVERBOSE)
[Out] ln(d*x)*ln(c*ln(d*x)^p)-ln(d*x)*p
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\log \left(c \log^p(dx)\right)}{x} \, dx = p \log(dx) \log(\log(dx)) - (p - \log(c)) \log(dx)$$

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="fricas")
[Out] p*log(d*x)*log(log(d*x)) - (p - log(c))*log(d*x)
```

Sympy [F]

$$\int \frac{\log(c\log^p(dx))}{x}\,dx = \int \frac{\log\left(c\log(dx)^p\right)}{x}\,dx$$

```
[In] integrate(ln(c*ln(d*x)**p)/x,x)
[Out] Integral(log(c*log(d*x)**p)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c\log^p(dx))}{x}\,dx = -p\log(dx) + \log(dx)\log\left(c\log(dx)^p\right)$$

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="maxima")
[Out] -p*log(d*x) + log(d*x)*log(c*log(d*x)^p)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\log(c\log^p(dx))}{x}\,dx = ((\log(d) + \log(x))\log(\log(d) + \log(x)) - \log(d) - \log(x))p + (\log(d) + \log(x))\log(c)$$

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="giac")
[Out] ((log(d) + log(x))*log(log(d) + log(x)) - log(d) - log(x))*p + (log(d) + lo
g(x))*log(c)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log (c \log^p(dx))}{x} \, dx = \ln (c \ln (d\, x)^p) \, \ln (d\, x) - p \, \ln (x)$$

[In] int(log(c*log(d*x)^p)/x,x)

[Out] log(c*log(d*x)^p)*log(d*x) - p*log(x)

3.57 $\int \log(c\log^p(dx^n))\,dx$

Optimal result 353
Rubi [A] (verified) 353
Mathematica [A] (verified) 354
Maple [F] 354
Fricas [A] (verification not implemented) 355
Sympy [F] 355
Maxima [F] 355
Giac [A] (verification not implemented) 356
Mupad [F(-1)] 356

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \log(c\log^p(dx^n))\,dx = -px(dx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{\log(dx^n)}{n}\right)+x\log(c\log^p(dx^n))$$

[Out] -p*x*Ei(ln(d*x^n)/n)/((d*x^n)^(1/n))+x*ln(c*ln(d*x^n)^p)

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2600, 2337, 2209}

$$\int \log(c\log^p(dx^n))\,dx = x\log(c\log^p(dx^n))-px(dx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{\log(dx^n)}{n}\right)$$

[In] Int[Log[c*Log[d*x^n]^p],x]

[Out] -((p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + x*Log[c*Log[d*x^n]^p]

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2600

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(c\log^p\left(dx^n\right)\right)-(np)\int\frac{1}{\log\left(dx^n\right)}\,dx\\
&= x\log\left(c\log^p\left(dx^n\right)\right)-\left(px\left(dx^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{x}\,dx,x,\log\left(dx^n\right)\right)\\
&= -px\left(dx^n\right)^{-1/n}\text{Ei}\left(\frac{\log\left(dx^n\right)}{n}\right)+x\log\left(c\log^p\left(dx^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int\log\left(c\log^p\left(dx^n\right)\right)\,dx=x\left(-p\left(dx^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{\log\left(dx^n\right)}{n}\right)+\log\left(c\log^p\left(dx^n\right)\right)\right)$$

```
[In] Integrate[Log[c*Log[d*x^n]^p],x]
[Out] x*(-((p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + Log[c*Log[d*x^n]^p])
```

Maple [F]

$$\int\ln\left(c\ln\left(d\,x^n\right)^p\right)dx$$

```
[In] int(ln(c*ln(d*x^n)^p),x)
[Out] int(ln(c*ln(d*x^n)^p),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \log \left(c\log^p\left(dx^n\right)\right)\,dx = \frac{d^{\left(\frac{1}{n}\right)}px\log\left(n\log\left(x\right)+\log\left(d\right)\right)+d^{\left(\frac{1}{n}\right)}x\log\left(c\right)-p\,\mathrm{log\_integral}\left(d^{\left(\frac{1}{n}\right)}x\right)}{d^{\left(\frac{1}{n}\right)}}$$

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="fricas")
[Out] (d^(1/n)*p*x*log(n*log(x) + log(d)) + d^(1/n)*x*log(c) - p*log_integral(d^(
1/n)*x))/d^(1/n)
```

Sympy [F]

$$\int \log \left(c\log^p\left(dx^n\right)\right)\,dx = \int \log\left(c\log\left(dx^n\right)^p\right)dx$$

```
[In] integrate(ln(c*ln(d*x**n)**p),x)
[Out] Integral(log(c*log(d*x**n)**p), x)
```

Maxima [F]

$$\int \log \left(c\log^p\left(dx^n\right)\right)\,dx = \int \log\left(c\log\left(dx^n\right)^p\right)\,dx$$

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="maxima")
[Out] -n*p*integrate(1/(log(d) + log(x^n)), x) + x*log(c) + x*log((log(d) + log(x
^n))^p)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \log\left(c\log^p\left(dx^n\right)\right)\,dx = px\log\left(n\log\left(x\right)+\log\left(d\right)\right)+x\log\left(c\right)-\frac{p\mathrm{Ei}\left(\frac{\log(d)}{n}+\log\left(x\right)\right)}{d^{\left(\frac{1}{n}\right)}}$$

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="giac")
[Out] p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(c\log^p\left(dx^n\right)\right)\,dx = \int \ln\left(c\ln\left(d\,x^n\right)^p\right)\,dx$$

```
[In] int(log(c*log(d*x^n)^p),x)
[Out] int(log(c*log(d*x^n)^p), x)
```

3.58 $\int \frac{\log(c\log^p(dx^n))}{x} \, dx$

Optimal result 357
Rubi [A] (verified) 357
Mathematica [A] (verified) 358
Maple [A] (verified) 358
Fricas [A] (verification not implemented) 358
Sympy [F] 359
Maxima [B] (verification not implemented) 359
Giac [A] (verification not implemented) 359
Mupad [B] (verification not implemented) 360

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{\log(c\log^p(dx^n))}{x} \, dx = -p\log(x) + \frac{\log(dx^n)\log(c\log^p(dx^n))}{n}$$

```
[Out] -p*ln(x)+ln(d*x^n)*ln(c*ln(d*x^n)^p)/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2601}

$$\int \frac{\log(c\log^p(dx^n))}{x} \, dx = \frac{\log(dx^n)\log(c\log^p(dx^n))}{n} - p\log(x)$$

```
[In] Int[Log[c*Log[d*x^n]^p]/x,x]
```

```
[Out] -(p*Log[x]) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = -p\log(x) + \frac{\log(dx^n)\log(c\log^p(dx^n))}{n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\log (c\log^p(dx^n))}{x}\,dx = -\frac{p\log (dx^n)}{n} + \frac{\log (dx^n)\log (c\log^p(dx^n))}{n}$$

```
[In] Integrate[Log[c*Log[d*x^n]^p]/x,x]
[Out] -((p*Log[d*x^n])/n) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{\ln(c\ln(dx^n)^p)\ln(dx^n)-p\ln(dx^n)}{n}$	33
default	$\frac{\ln(c\ln(dx^n)^p)\ln(dx^n)-p\ln(dx^n)}{n}$	33

```
[In] int(ln(c*ln(d*x^n)^p)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(ln(c*ln(d*x^n)^p)*ln(d*x^n)-p*ln(d*x^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\log (c\log^p(dx^n))}{x}\,dx = \frac{(np\log (x) + p\log (d))\log (n\log (x) + \log (d)) - (np - n\log (c))\log (x)}{n}$$

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="fricas")
[Out] ((n*p*log(x) + p*log(d))*log(n*log(x) + log(d)) - (n*p - n*log(c))*log(x))/
n
```

Sympy [F]

$$\int \frac{\log \left(c \log ^p\left(d x^n\right)\right)}{x} d x=\int \frac{\log \left(c \log \left(d x^n\right)^p\right)}{x} d x$$

```
[In] integrate(ln(c*ln(d*x**n)**p)/x,x)
[Out] Integral(log(c*log(d*x**n)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(27) = 54.

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\log \left(c \log ^p\left(d x^n\right)\right)}{x} d x=-p \log (x) \log \left(\log \left(d x^n\right)\right)+\log \left(c \log \left(d x^n\right)^p\right) \log (x)+\frac{\left(\log \left(d x^n\right) \log \left(\log \left(d x^n\right)\right)-\log \left(d x^n\right)\right) p}{n}$$

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="maxima")
[Out] -p*log(x)*log(log(d*x^n)) + log(c*log(d*x^n)^p)*log(x) + (log(d*x^n)*log(lo
g(d*x^n)) - log(d*x^n))*p/n
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{\log \left(c \log ^p\left(d x^n\right)\right)}{x} d x=\frac{((n \log (x)+\log (d)) \log (n \log (x)+\log (d))-n \log (x)-\log (d)) p+(n \log (x)+\log (d)) \log (c)}{n}$$

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="giac")
[Out] (((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*p + (n*lo
g(x) + log(d))*log(c))/n
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log(c\log^p(dx^n))}{x}\,dx = \frac{\ln\left(c\ln\left(d\,x^n\right)^p\right)\,\ln\left(d\,x^n\right)}{n} - p\,\ln(x)$$

```
[In] int(log(c*log(d*x^n)^p)/x,x)
[Out] (log(c*log(d*x^n)^p)*log(d*x^n))/n - p*log(x)
```

3.59 $\int x^m \log \left(d\left(bx+cx^2\right)^n\right) \, dx$

Optimal result 361
Rubi [A] (verified) 361
Mathematica [A] (verified) 362
Maple [F] 363
Fricas [F] 363
Sympy [F] 363
Maxima [F] 363
Giac [F] 364
Mupad [F(-1)] 364

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int x^m \log \left(d\left(bx+cx^2\right)^n\right) \, dx = -\frac{2nx^{1+m}}{(1+m)^2} + \frac{nx^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{cx}{b}\right)}{(1+m)^2} + \frac{x^{1+m} \log \left(d\left(bx+cx^2\right)^n\right)}{1+m}$$

```
[Out] -2*n*x^(1+m)/(1+m)^2+n*x^(1+m)*hypergeom([1, 1+m],[2+m],-c*x/b)/(1+m)^2+x^(
1+m)*ln(d*(c*x^2+b*x)^n)/(1+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2605, 81, 66}

$$\int x^m \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{x^{m+1} \log \left(d\left(bx+cx^2\right)^n\right)}{m+1} + \frac{nx^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{cx}{b}\right)}{(m+1)^2} - \frac{2nx^{m+1}}{(m+1)^2}$$

```
[In] Int[x^m*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (-2*n*x^(1 + m))/(1 + m)^2 + (n*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, -((c*x)/b)])/(1 + m)^2 + (x^(1 + m)*Log[d*(b*x + c*x^2)^n])/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m} - \frac{n\int \frac{x^m(b+2cx)}{b+cx}\,dx}{1+m} \\
&= -\frac{2nx^{1+m}}{(1+m)^2} + \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m} + \frac{(bn)\int \frac{x^m}{b+cx}\,dx}{1+m} \\
&= -\frac{2nx^{1+m}}{(1+m)^2} + \frac{nx^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{cx}{b}\right)}{(1+m)^2} + \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{x^{1+m}\left(-2n+n\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{cx}{b}\right)+(1+m)\log\left(d(x(b+cx))^n\right)\right)}{(1+m)^2}$$

```
[In] Integrate[x^m*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (x^(1 + m)*(-2*n + n*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*x)/b)] + (1 +
m)*Log[d*(x*(b + c*x))^n]))/(1 + m)^2
```

Maple [F]

$$\int x^m \ln\left(d\left(c\,x^2+bx\right)^n\right)dx$$

```
[In] int(x^m*ln(d*(c*x^2+b*x)^n),x)
[Out] int(x^m*ln(d*(c*x^2+b*x)^n),x)
```

Fricas [F]

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)\,dx=\int x^m \log\left(\left(cx^2+bx\right)^n d\right)\,dx$$

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
[Out] integral(x^m*log((c*x^2 + b*x)^n*d), x)
```

Sympy [F]

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)\,dx=\int x^m \log\left(d\left(bx+cx^2\right)^n\right)dx$$

```
[In] integrate(x**m*ln(d*(c*x**2+b*x)**n),x)
[Out] Integral(x**m*log(d*(b*x + c*x**2)**n), x)
```

Maxima [F]

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)\,dx=\int x^m \log\left(\left(cx^2+bx\right)^n d\right)\,dx$$

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
[Out] (x*x^m*log((c*x + b)^n) + x*x^m*log(x^n))/(m + 1) + integrate((((m + 1)*log
(d) - 2*n)*c*x + ((m + 1)*log(d) - n)*b)*x^m/(c*(m + 1)*x + b*(m + 1)), x)
```

Giac [F]

$$\int x^m \log \left(d(bx + cx^2)^n\right) \, dx = \int x^m \log \left((cx^2 + bx)^n d\right) \, dx$$

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
[Out] integrate(x^m*log((c*x^2 + b*x)^n*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \log \left(d(bx + cx^2)^n\right) \, dx = \int x^m \ln \left(d \left(c\, x^2 + b\, x\right)^n\right) \, dx$$

```
[In] int(x^m*log(d*(b*x + c*x^2)^n),x)
[Out] int(x^m*log(d*(b*x + c*x^2)^n), x)
```

3.60 $\int x^4 \log \left(d(bx+cx^2)^n\right) dx$

Optimal result 365
Rubi [A] (verified) 365
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Optimal result

Integrand size = 18, antiderivative size = 99

$$\int x^4 \log \left(d(bx+cx^2)^n\right) dx = -\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}+\frac{b^5n\log(b+cx)}{5c^5}+\frac{1}{5}x^5\log\left(d(bx+cx^2)^n\right)$$

```
[Out] -1/5*b^4*n*x/c^4+1/10*b^3*n*x^2/c^3-1/15*b^2*n*x^3/c^2+1/20*b*n*x^4/c-2/25*
n*x^5+1/5*b^5*n*ln(c*x+b)/c^5+1/5*x^5*ln(d*(c*x^2+b*x)^n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2605, 78}

$$\int x^4 \log \left(d(bx+cx^2)^n\right) dx = \frac{b^5n\log(b+cx)}{5c^5}-\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{1}{5}x^5\log\left(d(bx+cx^2)^n\right)+\frac{bnx^4}{20c}-\frac{2nx^5}{25}$$

```
[In] Int[x^4*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -1/5*(b^4*n*x)/c^4 + (b^3*n*x^2)/(10*c^3) - (b^2*n*x^3)/(15*c^2) + (b*n*x^4
)/(20*c) - (2*n*x^5)/25 + (b^5*n*Log[b + c*x])/(5*c^5) + (x^5*Log[d*(b*x +
c*x^2)^n])/5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{5}n\int\frac{x^4(b+2cx)}{b+cx}\,dx\\ &= \frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{5}n\int\left(\frac{b^4}{c^4}-\frac{b^3x}{c^3}+\frac{b^2x^2}{c^2}-\frac{bx^3}{c}+2x^4-\frac{b^5}{c^4(b+cx)}\right)dx\\ &= -\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}+\frac{b^5n\log(b+cx)}{5c^5}+\frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int x^4\log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{cnx(-60b^4+30b^3cx-20b^2c^2x^2+15bc^3x^3-24c^4x^4)+60b^5n\log(b+cx)+60c^5x^5\log\left(d(x(b+cx))^n\right)}{300c^5}$$

```
[In] Integrate[x^4*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c^2*x^2 + 15*b*c^3*x^3 - 24*c^4*x^4)
+ 60*b^5*n*Log[b + c*x] + 60*c^5*x^5*Log[d*(x*(b + c*x))^n])/(300*c^5)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

method	result
parts	$\frac{x^5 \ln\left(d\left(c\,x^2+bx\right)^n\right)}{5}-\frac{n\left(\frac{\frac{2}{5}c^4x^5-\frac{1}{4}b\,x^4c^3+\frac{1}{3}b^2c^2x^3-\frac{1}{2}b^3c\,x^2+b^4x}{c^4}-\frac{b^5\ln(xc+b)}{c^5}\right)}{5}$
parallelrisch	$-\frac{-60x^5\ln(d(x(xc+b))^n)c^5n+24x^5c^5n^2-15x^4b\,c^4n^2+20x^3b^2c^3n^2-30x^2b^3c^2n^2+60\ln(x)b^5n^2+60x\,b^4c\,n^2-60\ln(d(x(xc+b) \cdots}{300c^5n}$

```
[In] int(x^4*ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln(d*(c*x^2+b*x)^n)-1/5*n*(1/c^4*(2/5*c^4*x^5-1/4*b*x^4*c^3+1/3*b^2
*c^2*x^3-1/2*b^3*c*x^2+b^4*x)-b^5/c^5*ln(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int x^4 \log\left(d\left(bx+cx^2\right)^n\right)\,dx$$
$$=\frac{60\,c^5nx^5\log\left(cx^2+bx\right)-24\,c^5nx^5+60\,c^5x^5\log\left(d\right)+15\,bc^4nx^4-20\,b^2c^3nx^3+30\,b^3c^2nx^2-60\,b^4cnx+6 \cdots}{300\,c^5}$$

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/300*(60*c^5*n*x^5*log(c*x^2 + b*x) - 24*c^5*n*x^5 + 60*c^5*x^5*log(d) + 1
5*b*c^4*n*x^4 - 20*b^2*c^3*n*x^3 + 30*b^3*c^2*n*x^2 - 60*b^4*c*n*x + 60*b^5
*n*log(c*x + b))/c^5
```

Sympy [A] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int x^4 \log\left(d\left(bx+cx^2\right)^n\right)\,dx$$
$$=\begin{cases} \frac{b^5n\log(b+cx)}{5c^5}-\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}+\frac{x^5\log\left(d\left(bx+cx^2\right)^n\right)}{5} & \text{for } c\neq 0 \\ -\frac{nx^5}{25}+\frac{x^5\log\left(d(bx)^n\right)}{5} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b**5*n*log(b + c*x)/(5*c**5) - b**4*n*x/(5*c**4) + b**3*n*x**2/(
10*c**3) - b**2*n*x**3/(15*c**2) + b*n*x**4/(20*c) - 2*n*x**5/25 + x**5*log
(d*(b*x + c*x**2)**n)/5, Ne(c, 0)), (-n*x**5/25 + x**5*log(d*(b*x)**n)/5, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int x^4 \log \left(d(bx+cx^2)^n\right) \, dx = \frac{1}{5} x^5 \log \left((cx^2+bx)^n d\right) + \frac{1}{300} n\left(\frac{60 b^5 \log (cx+b)}{c^5} - \frac{24 c^4 x^5 - 15 bc^3 x^4 + 20 b^2 c^2 x^3 - 30 b^3 cx^2 + 60 b^4 x}{c^4}\right)$$

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
[Out] 1/5*x^5*log((c*x^2 + b*x)^n*d) + 1/300*n*(60*b^5*log(c*x + b)/c^5 - (24*c^4
*x^5 - 15*b*c^3*x^4 + 20*b^2*c^2*x^3 - 30*b^3*c*x^2 + 60*b^4*x)/c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int x^4 \log \left(d(bx+cx^2)^n\right) \, dx = \frac{1}{5} nx^5 \log \left(cx^2+bx\right) - \frac{1}{25} (2n - 5 \log (d))x^5 + \frac{bnx^4}{20c} - \frac{b^2nx^3}{15c^2} + \frac{b^3nx^2}{10c^3} - \frac{b^4nx}{5c^4} + \frac{b^5 n \log (cx+b)}{5c^5}$$

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
[Out] 1/5*n*x^5*log(c*x^2 + b*x) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/c - 1
/15*b^2*n*x^3/c^2 + 1/10*b^3*n*x^2/c^3 - 1/5*b^4*n*x/c^4 + 1/5*b^5*n*log(c*
x + b)/c^5
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int x^4 \log \left(d(bx+cx^2)^n\right) \, dx = \frac{x^5 \ln \left(d\,(c\,x^2+b\,x)^n\right)}{5} - \frac{2\,n\,x^5}{25} - \frac{b^2\,n\,x^3}{15\,c^2} + \frac{b^3\,n\,x^2}{10\,c^3} + \frac{b^5\,n\,\ln (b+c\,x)}{5\,c^5} + \frac{b\,n\,x^4}{20\,c} - \frac{b^4\,n\,x}{5\,c^4}$$

```
[In] int(x^4*log(d*(b*x + c*x^2)^n),x)
[Out] (x^5*log(d*(b*x + c*x^2)^n))/5 - (2*n*x^5)/25 - (b^2*n*x^3)/(15*c^2) + (b^3
*n*x^2)/(10*c^3) + (b^5*n*log(b + c*x))/(5*c^5) + (b*n*x^4)/(20*c) - (b^4*n
*x)/(5*c^4)
```

3.61 $\int x^3 \log \left(d(bx+cx^2)^n\right) \, dx$

Optimal result 369
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Mathematica [A] (verified) 370
Maple [A] (verified) 370
Fricas [A] (verification not implemented) 371
Sympy [A] (verification not implemented) 371
Maxima [A] (verification not implemented) 372
Giac [A] (verification not implemented) 372
Mupad [B] (verification not implemented) 372

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int x^3 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{b^3nx}{4c^3}-\frac{b^2nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8} -\frac{b^4n\log(b+cx)}{4c^4}+\frac{1}{4}x^4\log \left(d\left(bx+cx^2\right)^n\right)$$

```
[Out] 1/4*b^3*n*x/c^3-1/8*b^2*n*x^2/c^2+1/12*b*n*x^3/c-1/8*n*x^4-1/4*b^4*n*ln(c*x
+b)/c^4+1/4*x^4*ln(d*(c*x^2+b*x)^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2605, 78}

$$\int x^3 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = -\frac{b^4n\log(b+cx)}{4c^4}+\frac{b^3nx}{4c^3}-\frac{b^2nx^2}{8c^2} +\frac{1}{4}x^4\log \left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^3}{12c}-\frac{nx^4}{8}$$

```
[In] Int[x^3*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (b^3*n*x)/(4*c^3) - (b^2*n*x^2)/(8*c^2) + (b*n*x^3)/(12*c) - (n*x^4)/8 - (b
^4*n*Log[b + c*x])/(4*c^4) + (x^4*Log[d*(b*x + c*x^2)^n])/4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{4}n\int\frac{x^3(b+2cx)}{b+cx}\,dx\\
&= \frac{1}{4}x^4\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{4}n\int\left(-\frac{b^3}{c^3}+\frac{b^2x}{c^2}-\frac{bx^2}{c}+2x^3+\frac{b^4}{c^3(b+cx)}\right)dx\\
&= \frac{b^3nx}{4c^3}-\frac{b^2nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}-\frac{b^4n\log(b+cx)}{4c^4}+\frac{1}{4}x^4\log\left(d\left(bx+cx^2\right)^n\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int x^3\log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{cnx(6b^3-3b^2cx+2bc^2x^2-3c^3x^3)-6b^4n\log(b+cx)+6c^4x^4\log\left(d(x(b+cx))^n\right)}{24c^4}$$

```
[In] Integrate[x^3*Log[d*(b*x + c*x^2)^n],x]
[Out] (c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c^2*x^2 - 3*c^3*x^3) - 6*b^4*n*Log[b + c*x]
 + 6*c^4*x^4*Log[d*(x*(b + c*x))^n])/(24*c^4)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{x^{4}\ln\left(d\left(c\,x^{2}+bx\right)^{n}\right)}{4}-\frac{n\left(-\frac{-\frac{1}{2}c^{3}x^{4}+\frac{1}{3}b\,x^{3}c^{2}-\frac{1}{2}b^{2}c\,x^{2}+x\,b^{3}}{c^{3}}+\frac{b^{4}\ln\left(xc+b\right)}{c^{4}}\right)}{4}$	75
parallelrisch	$\frac{6x^{4}\ln\left(d\left(x\left(xc+b\right)\right)^{n}\right)c^{4}n-3x^{4}c^{4}n^{2}+2x^{3}b\,c^{3}n^{2}-3x^{2}b^{2}c^{2}n^{2}+6\ln\left(x\right)b^{4}n^{2}+6x\,b^{3}c\,n^{2}-6\ln\left(d\left(x\left(xc+b\right)\right)^{n}\right)b^{4}n-6b^{4}n^{2}}{24c^{4}n}$	114

```
[In] int(x^3*ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(d*(c*x^2+b*x)^n)-1/4*n*(-1/c^3*(-1/2*c^3*x^4+1/3*b*x^3*c^2-1/2*b
^2*c*x^2+x*b^3)+b^4/c^4*ln(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int x^{3}\log\left(d\left(bx+cx^{2}\right)^{n}\right)\,dx$$
$$=\frac{6\,c^{4}nx^{4}\log\left(cx^{2}+bx\right)-3\,c^{4}nx^{4}+6\,c^{4}x^{4}\log\left(d\right)+2\,bc^{3}nx^{3}-3\,b^{2}c^{2}nx^{2}+6\,b^{3}cnx-6\,b^{4}n\log\left(cx+b\right)}{24\,c^{4}}$$

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/24*(6*c^4*n*x^4*log(c*x^2 + b*x) - 3*c^4*n*x^4 + 6*c^4*x^4*log(d) + 2*b*c
^3*n*x^3 - 3*b^2*c^2*n*x^2 + 6*b^3*c*n*x - 6*b^4*n*log(c*x + b))/c^4
```

Sympy [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int x^{3}\log\left(d\left(bx+cx^{2}\right)^{n}\right)\,dx$$
$$=\begin{cases}-\frac{b^{4}n\log\left(b+cx\right)}{4c^{4}}+\frac{b^{3}nx}{4c^{3}}-\frac{b^{2}nx^{2}}{8c^{2}}+\frac{bnx^{3}}{12c}-\frac{nx^{4}}{8}+\frac{x^{4}\log\left(d\left(bx+cx^{2}\right)^{n}\right)}{4} & \text{for } c\neq 0\\ -\frac{nx^{4}}{16}+\frac{x^{4}\log\left(d\left(bx\right)^{n}\right)}{4} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((-b**4*n*log(b + c*x)/(4*c**4) + b**3*n*x/(4*c**3) - b**2*n*x**2/
(8*c**2) + b*n*x**3/(12*c) - n*x**4/8 + x**4*log(d*(b*x + c*x**2)**n)/4, Ne
(c, 0)), (-n*x**4/16 + x**4*log(d*(b*x)**n)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int x^3 \log \left(d(bx+cx^2)^n\right)\, dx = \frac{1}{4}\, x^4 \log \left((cx^2+bx)^n d\right) - \frac{1}{24}\, n\left(\frac{6\, b^4 \log (cx+b)}{c^4} + \frac{3\, c^3 x^4 - 2\, bc^2 x^3 + 3\, b^2 c x^2 - 6\, b^3 x}{c^3}\right)$$

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((c*x^2 + b*x)^n*d) - 1/24*n*(6*b^4*log(c*x + b)/c^4 + (3*c^3*x^
4 - 2*b*c^2*x^3 + 3*b^2*c*x^2 - 6*b^3*x)/c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int x^3 \log \left(d(bx+cx^2)^n\right)\, dx = \frac{1}{4}\, nx^4 \log \left(cx^2+bx\right) - \frac{1}{8}\, (n - 2\, \log (d))x^4 + \frac{bnx^3}{12\, c} - \frac{b^2 nx^2}{8\, c^2} + \frac{b^3 nx}{4\, c^3} - \frac{b^4 n \log (cx+b)}{4\, c^4}$$

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/4*n*x^4*log(c*x^2 + b*x) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c - 1/8*
b^2*n*x^2/c^2 + 1/4*b^3*n*x/c^3 - 1/4*b^4*n*log(c*x + b)/c^4
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^3 \log \left(d(bx+cx^2)^n\right)\, dx = \frac{x^4\, \ln \left(d\, (c\, x^2 + b\, x)^n\right)}{4} - \frac{n\, x^4}{8} - \frac{b^2\, n\, x^2}{8\, c^2} - \frac{b^4\, n\, \ln (b + c\, x)}{4\, c^4} + \frac{b\, n\, x^3}{12\, c} + \frac{b^3\, n\, x}{4\, c^3}$$

```
[In] int(x^3*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^4*log(d*(b*x + c*x^2)^n))/4 - (n*x^4)/8 - (b^2*n*x^2)/(8*c^2) - (b^4*n*l
og(b + c*x))/(4*c^4) + (b*n*x^3)/(12*c) + (b^3*n*x)/(4*c^3)
```

3.62 $\int x^2 \log \left(d(bx+cx^2)^n\right) \, dx$

Optimal result 373
Rubi [A] (verified) 373
Mathematica [A] (verified) 374
Maple [A] (verified) 374
Fricas [A] (verification not implemented) 375
Sympy [A] (verification not implemented) 375
Maxima [A] (verification not implemented) 375
Giac [A] (verification not implemented) 376
Mupad [B] (verification not implemented) 376

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = -\frac{b^2 n x}{3c^2}+\frac{b n x^2}{6c}-\frac{2 n x^3}{9}+\frac{b^3 n \log(b+cx)}{3c^3}+\frac{1}{3}x^3 \log \left(d\left(bx+cx^2\right)^n\right)$$

```
[Out] -1/3*b^2*n*x/c^2+1/6*b*n*x^2/c-2/9*n*x^3+1/3*b^3*n*ln(c*x+b)/c^3+1/3*x^3*ln
(d*(c*x^2+b*x)^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2605, 78}

$$\int x^2 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{b^3 n \log(b+cx)}{3c^3}-\frac{b^2 n x}{3c^2}+\frac{1}{3}x^3 \log \left(d\left(bx+cx^2\right)^n\right)+\frac{b n x^2}{6c}-\frac{2 n x^3}{9}$$

```
[In] Int[x^2*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -1/3*(b^2*n*x)/c^2 + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (b^3*n*Log[b + c*x])/(
3*c^3) + (x^3*Log[d*(b*x + c*x^2)^n])/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{3}n\int\frac{x^2(b+2cx)}{b+cx}\,dx\\
&= \frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{3}n\int\left(\frac{b^2}{c^2}-\frac{bx}{c}+2x^2-\frac{b^3}{c^2(b+cx)}\right)dx\\
&= -\frac{b^2nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{b^3n\log(b+cx)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int x^2\log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{cnx(-6b^2+3bcx-4c^2x^2)+6b^3n\log(b+cx)+6c^3x^3\log\left(d(x(b+cx))^n\right)}{18c^3}$$

```
[In] Integrate[x^2*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-6*b^2 + 3*b*c*x - 4*c^2*x^2) + 6*b^3*n*Log[b + c*x] + 6*c^3*x^3*Lo
g[d*(x*(b + c*x))^n])/(18*c^3)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result	size
parts	$\frac{x^3\ln\left(d\left(cx^2+bx\right)^n\right)}{3}-\frac{n\left(\frac{\frac{2}{3}x^3c^2-\frac{1}{2}cbx^2+b^2x}{c^2}-\frac{b^3\ln(xc+b)}{c^3}\right)}{3}$	64
parallelrisch	$-\frac{-6x^3\ln(d(x(xc+b))^n)c^3n+4x^3c^3n^2-3x^2bc^2n^2+6\ln(x)b^3n^2+6xb^2cn^2-6\ln(d(x(xc+b))^n)b^3n-6b^3n^2}{18c^3n}$	100

```
[In] int(x^2*ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(d*(c*x^2+b*x)^n)-1/3*n*(1/c^2*(2/3*x^3*c^2-1/2*c*b*x^2+b^2*x)-b^
3/c^3*ln(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int x^2\log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{6\,c^3nx^3\log\left(cx^2+bx\right)-4\,c^3nx^3+6\,c^3x^3\log\left(d\right)+3\,bc^2nx^2-6\,b^2cnx+6\,b^3n\log\left(cx+b\right)}{18\,c^3}$$

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/18*(6*c^3*n*x^3*log(c*x^2 + b*x) - 4*c^3*n*x^3 + 6*c^3*x^3*log(d) + 3*b*c
^2*n*x^2 - 6*b^2*c*n*x + 6*b^3*n*log(c*x + b))/c^3
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int x^2\log\left(d\left(bx+cx^2\right)^n\right)\,dx = \begin{cases} \frac{b^3n\log\left(b+cx\right)}{3c^3}-\frac{b^2nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{x^3\log\left(d\left(bx+cx^2\right)^n\right)}{3} & \text{for } c\neq 0 \\ -\frac{nx^3}{9}+\frac{x^3\log\left(d\left(bx\right)^n\right)}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b**3*n*log(b + c*x)/(3*c**3) - b**2*n*x/(3*c**2) + b*n*x**2/(6*c
) - 2*n*x**3/9 + x**3*log(d*(b*x + c*x**2)**n)/3, Ne(c, 0)), (-n*x**3/9 + x
**3*log(d*(b*x)**n)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int x^2\log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{1}{3}\,x^3\log\left(\left(cx^2+bx\right)^nd\right)+\frac{1}{18}\,n\left(\frac{6\,b^3\log\left(cx+b\right)}{c^3}-\frac{4\,c^2x^3-3\,bcx^2+6\,b^2x}{c^2}\right)$$

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((c*x^2 + b*x)^n*d) + 1/18*n*(6*b^3*log(c*x + b)/c^3 - (4*c^2*x^
3 - 3*b*c*x^2 + 6*b^2*x)/c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int x^2 \log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{1}{3}nx^3\log\left(cx^2+bx\right) - \frac{1}{9}\left(2n-3\log\left(d\right)\right)x^3 + \frac{bnx^2}{6c} - \frac{b^2nx}{3c^2} + \frac{b^3n\log\left(cx+b\right)}{3c^3}$$

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/3*n*x^3*log(c*x^2 + b*x) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c - 1/3
*b^2*n*x/c^2 + 1/3*b^3*n*log(c*x + b)/c^3
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int x^2 \log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{x^3\,\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{3} - \frac{2\,n\,x^3}{9} + \frac{b^3\,n\,\ln\left(b+c\,x\right)}{3\,c^3} + \frac{b\,n\,x^2}{6\,c} - \frac{b^2\,n\,x}{3\,c^2}$$

```
[In] int(x^2*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^3*log(d*(b*x + c*x^2)^n))/3 - (2*n*x^3)/9 + (b^3*n*log(b + c*x))/(3*c^3)
 + (b*n*x^2)/(6*c) - (b^2*n*x)/(3*c^2)
```

3.63 $\int x \log \left(d(bx+cx^2)^n\right) \, dx$

Optimal result 377
Rubi [A] (verified) 377
Mathematica [A] (verified) 378
Maple [A] (verified) 378
Fricas [A] (verification not implemented) 379
Sympy [A] (verification not implemented) 379
Maxima [A] (verification not implemented) 379
Giac [A] (verification not implemented) 380
Mupad [B] (verification not implemented) 380

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int x \log \left(d(bx+cx^2)^n\right) \, dx = \frac{bnx}{2c} - \frac{nx^2}{2} - \frac{b^2 n \log(b+cx)}{2c^2} + \frac{1}{2} x^2 \log \left(d(bx+cx^2)^n\right)$$

```
[Out] 1/2*b*n*x/c-1/2*n*x^2-1/2*b^2*n*ln(c*x+b)/c^2+1/2*x^2*ln(d*(c*x^2+b*x)^n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2605, 78}

$$\int x \log \left(d(bx+cx^2)^n\right) \, dx = -\frac{b^2 n \log(b+cx)}{2c^2} + \frac{1}{2} x^2 \log \left(d(bx+cx^2)^n\right) + \frac{bnx}{2c} - \frac{nx^2}{2}$$

```
[In] Int[x*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b^2*n*Log[b + c*x])/(2*c^2) + (x^2*Log[d*(b*x
+ c*x^2)^n])/2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{2}n\int\frac{x(b+2cx)}{b+cx}\,dx\\
&= \frac{1}{2}x^2\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{2}n\int\left(-\frac{b}{c}+2x+\frac{b^2}{c(b+cx)}\right)dx\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b^2n\log(b+cx)}{2c^2}+\frac{1}{2}x^2\log\left(d\left(bx+cx^2\right)^n\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x\log\left(d\left(bx+cx^2\right)^n\right)\,dx=-\frac{1}{2}n\left(-\frac{bx}{c}+x^2+\frac{b^2\log(b+cx)}{c^2}\right)+\frac{1}{2}x^2\log\left(d(x(b+cx))^n\right)$$

```
[In] Integrate[x*Log[d*(b*x + c*x^2)^n],x]
[Out] -1/2*(n*(-((b*x)/c) + x^2 + (b^2*Log[b + c*x])/c^2)) + (x^2*Log[d*(x*(b + c
*x))^n])/2
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result	size
parts	$\frac{x^2\ln\left(d\left(c\,x^2+bx\right)^n\right)}{2}-\frac{n\left(-\frac{-c\,x^2+bx}{c}+\frac{b^2\ln(xc+b)}{c^2}\right)}{2}$	53
parallelrisch	$\frac{x^2\ln(d(x(xc+b))^n)c^2n-x^2c^2n^2+\ln(x)b^2n^2+xbc\,n^2-\ln(d(x(xc+b))^n)b^2n-b^2n^2}{2c^2n}$	83

```
[In] int(x*ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*ln(d*(c*x^2+b*x)^n)-1/2*n*(-1/c*(-c*x^2+b*x)+1/c^2*b^2*ln(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int x \log\left(d(bx+cx^2)^n\right) dx = \frac{c^2nx^2\log(cx^2+bx) - c^2nx^2 + c^2x^2\log(d) + bcnx - b^2n\log(cx+b)}{2\,c^2}$$

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
[Out] 1/2*(c^2*n*x^2*log(c*x^2 + b*x) - c^2*n*x^2 + c^2*x^2*log(d) + b*c*n*x - b^
2*n*log(c*x + b))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int x \log\left(d(bx+cx^2)^n\right) dx = \begin{cases} -\frac{b^2n\log(b+cx)}{2c^2} + \frac{bnx}{2c} - \frac{nx^2}{2} + \frac{x^2\log\left(d(bx+cx^2)^n\right)}{2} & \text{for } c \neq 0 \\ -\frac{nx^2}{4} + \frac{x^2\log(d(bx)^n)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*ln(d*(c*x**2+b*x)**n),x)
[Out] Piecewise((-b**2*n*log(b + c*x)/(2*c**2) + b*n*x/(2*c) - n*x**2/2 + x**2*lo
g(d*(b*x + c*x**2)**n)/2, Ne(c, 0)), (-n*x**2/4 + x**2*log(d*(b*x)**n)/2, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int x \log\left(d(bx+cx^2)^n\right) dx = \frac{1}{2}x^2\log\left((cx^2+bx)^n d\right) - \frac{1}{2}n\left(\frac{b^2\log(cx+b)}{c^2} + \frac{cx^2-bx}{c}\right)$$

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
[Out] 1/2*x^2*log((c*x^2 + b*x)^n*d) - 1/2*n*(b^2*log(c*x + b)/c^2 + (c*x^2 - b*x
)/c)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int x \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{1}{2} nx^2 \log \left(cx^2+bx\right) - \frac{1}{2} (n - \log (d))x^2 + \frac{bnx}{2 c} - \frac{b^2 n \log (cx + b)}{2 c^2}$$

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/2*n*x^2*log(c*x^2 + b*x) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/2*b^2*n
*log(c*x + b)/c^2
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{x^2 \ln \left(d \left(c x^2 + b x\right)^n\right)}{2} - \frac{n x^2}{2} + \frac{b n x}{2 c} - \frac{b^2 n \ln (b + c x)}{2 c^2}$$

```
[In] int(x*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^2*log(d*(b*x + c*x^2)^n))/2 - (n*x^2)/2 + (b*n*x)/(2*c) - (b^2*n*log(b +
 c*x))/(2*c^2)
```

3.64 $\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx$

Optimal result 381
Rubi [A] (verified) 381
Mathematica [A] (verified) 382
Maple [A] (verified) 382
Fricas [A] (verification not implemented) 383
Sympy [A] (verification not implemented) 383
Maxima [A] (verification not implemented) 383
Giac [A] (verification not implemented) 384
Mupad [B] (verification not implemented) 384

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx = -2nx+\frac{bn\log(b+cx)}{c}+x\log \left(d\left(bx+cx^2\right)^n\right)$$

```
[Out] -2*n*x+b*n*ln(c*x+b)/c+x*ln(d*(c*x^2+b*x)^n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2603, 45}

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx = x\log \left(d\left(bx+cx^2\right)^n\right)+\frac{bn\log(b+cx)}{c}-2nx$$

```
[In] Int[Log[d*(b*x + c*x^2)^n],x]
[Out] -2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(b*x + c*x^2)^n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
```

```
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(d\left(bx+cx^2\right)^n\right)-n\int\frac{b+2cx}{b+cx}\,dx\\ &= x\log\left(d\left(bx+cx^2\right)^n\right)-n\int\left(2-\frac{b}{b+cx}\right)\,dx\\ &= -2nx+\frac{bn\log(b+cx)}{c}+x\log\left(d\left(bx+cx^2\right)^n\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int\log\left(d\left(bx+cx^2\right)^n\right)\,dx=-2nx+\frac{bn\log(b+cx)}{c}+x\log\left(d(x(b+cx))^n\right)$$

```
[In] Integrate[Log[d*(b*x + c*x^2)^n],x]
[Out] -2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(x*(b + c*x))^n]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

method	result	size
default	$x\ln\left(d\left(c\,x^2+bx\right)^n\right)-n\left(2x-\frac{b\ln(xc+b)}{c}\right)$	37
parts	$x\ln\left(d\left(c\,x^2+bx\right)^n\right)-n\left(2x-\frac{b\ln(xc+b)}{c}\right)$	37
parallelrisch	$-\frac{\ln(x)b\,n^2-x\ln(d(x(xc+b))^n)cn+2xc\,n^2-\ln(d(x(xc+b))^n)bn-2b\,n^2}{cn}$	63

```
[In] int(ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
[Out] x*ln(d*(c*x^2+b*x)^n)-n*(2*x-b/c*ln(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx=\frac{cnx\log \left(cx^2+bx\right)-2\,cnx+bn\log \left(cx+b\right)+cx\log \left(d\right)}{c}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
[Out] (c*n*x*log(c*x^2 + b*x) - 2*c*n*x + b*n*log(c*x + b) + c*x*log(d))/c
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx=\begin{cases} \frac{bn\log \left(b+cx\right)}{c}-2nx+x\log \left(d\left(bx+cx^2\right)^n\right) & \text{for}\: c\neq 0 \\ -nx+x\log \left(d(bx)^n\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(d*(c*x**2+b*x)**n),x)
[Out] Piecewise((b*n*log(b + c*x)/c - 2*n*x + x*log(d*(b*x + c*x**2)**n), Ne(c, 0
)), (-n*x + x*log(d*(b*x)**n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \log \left(d\left(bx+cx^2\right)^n\right) \, dx=-n\left(2\,x-\frac{b\log \left(cx+b\right)}{c}\right)+x\log \left(\left(cx^2+bx\right)^n d\right)$$

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
[Out] -n*(2*x - b*log(c*x + b)/c) + x*log((c*x^2 + b*x)^n*d)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \log \left(d\left(bx+cx^2\right)^n\right)\,dx = nx\log\left(cx^2+bx\right) - (2\,n - \log\left(d\right))x + \frac{bn\log\left(cx+b\right)}{c}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="giac")
[Out] n*x*log(c*x^2 + b*x) - (2*n - log(d))*x + b*n*log(c*x + b)/c
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \log \left(d\left(bx+cx^2\right)^n\right)\,dx = x\,\ln\left(d\left(c\,x^2+b\,x\right)^n\right) - 2\,n\,x + \frac{b\,n\,\ln\left(b+c\,x\right)}{c}$$

```
[In] int(log(d*(b*x + c*x^2)^n),x)
[Out] x*log(d*(b*x + c*x^2)^n) - 2*n*x + (b*n*log(b + c*x))/c
```

3.65 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x} \, dx$

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Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x} \, dx = -\frac{1}{2}n\log^2(x) - n\log(x)\log\left(1+\frac{cx}{b}\right) + \log(x)\log\left(d\left(bx+cx^2\right)^n\right) - n\,\text{PolyLog}\left(2,-\frac{cx}{b}\right)$$

```
[Out] -1/2*n*ln(x)^2-n*ln(x)*ln(1+c*x/b)+ln(x)*ln(d*(c*x^2+b*x)^n)-n*polylog(2,-c
*x/b)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2604, 1607, 2404, 2338, 2354, 2438}

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x} \, dx = \log(x)\log\left(d\left(bx+cx^2\right)^n\right) - n\,\text{PolyLog}\left(2,-\frac{cx}{b}\right) - n\log(x)\log\left(\frac{cx}{b}+1\right) - \frac{1}{2}n\log^2(x)$$

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x,x]
```

```
[Out] -1/2*(n*Log[x]^2) - n*Log[x]*Log[1 + (c*x)/b] + Log[x]*Log[d*(b*x + c*x^2)^
n] - n*PolyLog[2, -((c*x)/b)]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int\frac{(b+2cx)\log(x)}{bx+cx^2}\,dx\\
&= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int\frac{(b+2cx)\log(x)}{x(b+cx)}\,dx\\
&= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int\left(\frac{\log(x)}{x}+\frac{c\log(x)}{b+cx}\right)\,dx\\
&= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int\frac{\log(x)}{x}\,dx-(cn)\int\frac{\log(x)}{b+cx}\,dx\\
&= -\frac{1}{2}n\log^2(x)-n\log(x)\log\left(1+\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)+n\int\frac{\log\left(1+\frac{cx}{b}\right)}{x}\,dx
\end{aligned}$$

$$= -\frac{1}{2}n\log^2(x) - n\log(x)\log\left(1+\frac{cx}{b}\right) + \log(x)\log\left(d\left(bx+cx^2\right)^n\right) - n\mathrm{Li}_2\left(-\frac{cx}{b}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx = \log(x)\log\left(d(x(b+cx))^n\right) - n\left(\frac{\log^2(x)}{2} + \log(x)\log\left(\frac{b+cx}{b}\right) + \mathrm{PolyLog}\left(2,-\frac{cx}{b}\right)\right)$$

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x,x]
[Out] Log[x]*Log[d*(x*(b + c*x))^n] - n*(Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] + P
olyLog[2, -((c*x)/b)])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result	size
parts	$\ln(x)\ln\left(d(c\,x^2+bx)^n\right) - n\left(\frac{\ln(x)^2}{2} + c\left(\frac{\mathrm{dilog}\left(\frac{xc+b}{b}\right)}{c} + \frac{\ln(x)\ln\left(\frac{xc+b}{b}\right)}{c}\right)\right)$	62

```
[In] int(ln(d*(c*x^2+b*x)^n)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*ln(d*(c*x^2+b*x)^n)-n*(1/2*ln(x)^2+c*(dilog((c*x+b)/b)/c+ln(x)*ln((c*
x+b)/b)/c))
```

Fricas [F]

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx = \int \frac{\log\left((cx^2+bx)^n d\right)}{x}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="fricas")
[Out] integral(log((c*x^2 + b*x)^n*d)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx = \int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx$$

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x,x)
[Out] Integral(log(d*(b*x + c*x**2)**n)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx &= -n\log\left(cx^2+bx\right)\log(x)\\ &\quad+\frac{1}{2}\left(2\,\log\left(cx^2+bx\right)\log(x)-2\,\log\left(\frac{cx}{b}+1\right)\log(x)-\log(x)^2-2\,\mathrm{Li}_2\left(-\frac{cx}{b}\right)\right)n\\ &\quad+\log\left(\left(cx^2+bx\right)^n d\right)\log(x)\end{aligned}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="maxima")
[Out] -n*log(c*x^2 + b*x)*log(x) + 1/2*(2*log(c*x^2 + b*x)*log(x) - 2*log(c*x/b +
 1)*log(x) - log(x)^2 - 2*dilog(-c*x/b))*n + log((c*x^2 + b*x)^n*d)*log(x)
```

Giac [F]

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx = \int \frac{\log\left(\left(cx^2+bx\right)^n d\right)}{x}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x)^n*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x}\,dx=\int \frac{\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{x}\,dx$$

```
[In] int(log(d*(b*x + c*x^2)^n)/x,x)
```

```
[Out] int(log(d*(b*x + c*x^2)^n)/x, x)
```

3.66 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^2}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^2}\,dx = -\frac{n}{x} + \frac{cn\log(x)}{b} - \frac{cn\log(b+cx)}{b} - \frac{\log\left(d(bx+cx^2)^n\right)}{x}$$

```
[Out] -n/x+c*n*ln(x)/b-c*n*ln(c*x+b)/b-ln(d*(c*x^2+b*x)^n)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2605, 78}

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^2}\,dx = -\frac{\log\left(d(bx+cx^2)^n\right)}{x} + \frac{cn\log(x)}{b} - \frac{cn\log(b+cx)}{b} - \frac{n}{x}$$

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^2,x]
```

```
[Out] -(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(b*x + c*x^2)^n]/x
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(d(bx+cx^2)^n\right)}{x} + n\int \frac{b+2cx}{x^2(b+cx)}\,dx \\
&= -\frac{\log\left(d(bx+cx^2)^n\right)}{x} + n\int \left(\frac{1}{x^2}+\frac{c}{bx}-\frac{c^2}{b(b+cx)}\right)dx \\
&= -\frac{n}{x} + \frac{cn\log(x)}{b} - \frac{cn\log(b+cx)}{b} - \frac{\log\left(d(bx+cx^2)^n\right)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^2}\,dx = -\frac{n}{x} + \frac{cn\log(x)}{b} - \frac{cn\log(b+cx)}{b} - \frac{\log\left(d(x(b+cx))^n\right)}{x}$$

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^2,x]
[Out] -(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(x*(b + c*x))^n]/x
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
parts	$-\frac{\ln\left(d\left(c\,x^2+bx\right)^n\right)}{x}+n\left(-\frac{1}{x}+\frac{c\ln(x)}{b}-\frac{c\ln(xc+b)}{b}\right)$	48
parallelrisch	$\frac{2\ln(x)x\,c^2n^2-x\ln(d(x(xc+b))^n)c^2n-\ln(d(x(xc+b))^n)bcn-bc\,n^2}{xbcn}$	69

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^2,x,method=_RETURNVERBOSE)
[Out] -ln(d*(c*x^2+b*x)^n)/x+n*(-1/x+c/b*ln(x)-c/b*ln(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^2}\,dx$$
$$= -\frac{cnx\log(cx+b) - cnx\log(x) + bn\log(cx^2+bx) + bn + b\log(d)}{bx}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="fricas")
```

```
[Out] -(c*n*x*log(c*x + b) - c*n*x*log(x) + b*n*log(c*x^2 + b*x) + b*n + b*log(d)
)/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^2}\,dx = \begin{cases} -\frac{n}{x} - \frac{\log\left(d(bx+cx^2)^n\right)}{x} - \frac{2cn\log(b+cx)}{b} + \frac{c\log\left(d(bx+cx^2)^n\right)}{b} & \text{for } b \neq 0 \\ -\frac{2n}{x} - \frac{\log\left(d(cx^2)^n\right)}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**2,x)
```

```
[Out] Piecewise((-n/x - log(d*(b*x + c*x**2)**n)/x - 2*c*n*log(b + c*x)/b + c*log
(d*(b*x + c*x**2)**n)/b, Ne(b, 0)), (-2*n/x - log(d*(c*x**2)**n)/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^2}\,dx = -n\left(\frac{c\log(cx+b)}{b} - \frac{c\log(x)}{b} + \frac{1}{x}\right) - \frac{\log\left((cx^2+bx)^n d\right)}{x}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="maxima")
```

```
[Out] -n*(c*log(c*x + b)/b - c*log(x)/b + 1/x) - log((c*x^2 + b*x)^n*d)/x
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^2}\,dx = -\frac{cn\log\left(cx+b\right)}{b} + \frac{cn\log\left(x\right)}{b} - \frac{n\log\left(cx^2+bx\right)}{x} - \frac{n+\log\left(d\right)}{x}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="giac")
[Out] -c*n*log(c*x + b)/b + c*n*log(x)/b - n*log(c*x^2 + b*x)/x - (n + log(d))/x
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^2}\,dx = -\frac{\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{x} - \frac{n}{x} - \frac{2\,c\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b}$$

```
[In] int(log(d*(b*x + c*x^2)^n)/x^2,x)
[Out] - log(d*(b*x + c*x^2)^n)/x - n/x - (2*c*n*atanh((2*c*x)/b + 1))/b
```

3.67 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3}\,dx$

Optimal result 394
Rubi [A] (verified) 394
Mathematica [A] (verified) 395
Maple [A] (verified) 395
Fricas [A] (verification not implemented) 396
Sympy [A] (verification not implemented) 396
Maxima [A] (verification not implemented) 397
Giac [A] (verification not implemented) 397
Mupad [B] (verification not implemented) 397

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^3}\,dx = -\frac{n}{4x^2} - \frac{cn}{2bx} - \frac{c^2 n\log(x)}{2b^2} + \frac{c^2 n\log(b+cx)}{2b^2} - \frac{\log\left(d(bx+cx^2)^n\right)}{2x^2}$$

```
[Out] -1/4*n/x^2-1/2*c*n/b/x-1/2*c^2*n*ln(x)/b^2+1/2*c^2*n*ln(c*x+b)/b^2-1/2*ln(d
*(c*x^2+b*x)^n)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2605, 78}

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^3}\,dx = -\frac{c^2 n\log(x)}{2b^2} + \frac{c^2 n\log(b+cx)}{2b^2} - \frac{\log\left(d(bx+cx^2)^n\right)}{2x^2} - \frac{cn}{2bx} - \frac{n}{4x^2}$$

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -1/4*n/x^2 - (c*n)/(2*b*x) - (c^2*n*Log[x])/(2*b^2) + (c^2*n*Log[b + c*x])/
(2*b^2) - Log[d*(b*x + c*x^2)^n]/(2*x^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(d(bx+cx^2)^n\right)}{2x^2}+\frac{1}{2}n\int\frac{b+2cx}{x^3(b+cx)}\,dx\\
&= -\frac{\log\left(d(bx+cx^2)^n\right)}{2x^2}+\frac{1}{2}n\int\left(\frac{1}{x^3}+\frac{c}{bx^2}-\frac{c^2}{b^2x}+\frac{c^3}{b^2(b+cx)}\right)dx\\
&= -\frac{n}{4x^2}-\frac{cn}{2bx}-\frac{c^2n\log(x)}{2b^2}+\frac{c^2n\log(b+cx)}{2b^2}-\frac{\log\left(d(bx+cx^2)^n\right)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int\frac{\log\left(d(bx+cx^2)^n\right)}{x^3}\,dx=\frac{1}{2}n\left(-\frac{1}{2x^2}-\frac{c}{bx}-\frac{c^2\log(x)}{b^2}+\frac{c^2\log(b+cx)}{b^2}\right)-\frac{\log\left(d(x(b+cx))^n\right)}{2x^2}$$

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^3,x]
[Out] (n*(-1/2*1/x^2 - c/(b*x) - (c^2*Log[x])/b^2 + (c^2*Log[b + c*x])/b^2))/2 -
Log[d*(x*(b + c*x))^n]/(2*x^2)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

method	result	size
parts	$-\frac{\ln\left(d\left(c\,x^2+bx\right)^n\right)}{2x^2}+\frac{n\left(-\frac{1}{2x^2}-\frac{c}{bx}-\frac{c^2\ln(x)}{b^2}+\frac{c^2\ln(xc+b)}{b^2}\right)}{2}$	62
parallelrisch	$-\frac{2\ln(x)x^2c^2n-2\ln(xc+b)x^2c^2n-2x^2c^2n+2xbcn+2\ln\left(d\left(x(xc+b)\right)^n\right)b^2+b^2n}{4x^2b^2}$	73

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(d*(c*x^2+b*x)^n)/x^2+1/2*n*(-1/2/x^2-c/b/x-c^2/b^2*ln(x)+c^2/b^2*ln
(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx = \frac{2\,c^2nx^2\log\left(cx+b\right)-2\,c^2nx^2\log\left(x\right)-2\,bcnx-2\,b^2n\log\left(cx^2+bx\right)-b^2n-2\,b^2\log\left(d\right)}{4\,b^2x^2}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^2*n*x^2*log(c*x + b) - 2*c^2*n*x^2*log(x) - 2*b*c*n*x - 2*b^2*n*lo
g(c*x^2 + b*x) - b^2*n - 2*b^2*log(d))/(b^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx = \begin{cases} -\frac{n}{4x^2}-\frac{\log\left(d(bx+cx^2)^n\right)}{2x^2}-\frac{cn}{2bx}+\frac{c^2n\log\left(b+cx\right)}{b^2}-\frac{c^2\log\left(d(bx+cx^2)^n\right)}{2b^2} & \text{for}\ b\neq 0 \\ -\frac{n}{2x^2}-\frac{\log\left(d(cx^2)^n\right)}{2x^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**3,x)
```

```
[Out] Piecewise((-n/(4*x**2) - log(d*(b*x + c*x**2)**n)/(2*x**2) - c*n/(2*b*x) +
c**2*n*log(b + c*x)/b**2 - c**2*log(d*(b*x + c*x**2)**n)/(2*b**2), Ne(b, 0)
), (-n/(2*x**2) - log(d*(c*x**2)**n)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx = \frac{1}{4}\,n\left(\frac{2\,c^2\log\left(cx+b\right)}{b^2} - \frac{2\,c^2\log\left(x\right)}{b^2} - \frac{2\,cx+b}{bx^2}\right) - \frac{\log\left(\left(cx^2+bx\right)^n d\right)}{2\,x^2}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*n*(2*c^2*log(c*x + b)/b^2 - 2*c^2*log(x)/b^2 - (2*c*x + b)/(b*x^2)) - 1
/2*log((c*x^2 + b*x)^n*d)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx = \frac{c^2n\log\left(cx+b\right)}{2\,b^2} - \frac{c^2n\log\left(x\right)}{2\,b^2} - \frac{n\log\left(cx^2+bx\right)}{2\,x^2} - \frac{2\,cnx+bn+2\,b\log\left(d\right)}{4\,bx^2}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c^2*n*log(c*x + b)/b^2 - 1/2*c^2*n*log(x)/b^2 - 1/2*n*log(c*x^2 + b*x)/
x^2 - 1/4*(2*c*n*x + b*n + 2*b*log(d))/(b*x^2)
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int \frac{\log \left(d(bx+cx^2)^n\right)}{x^3}\,dx = \frac{c^2\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^2} - \frac{\frac{n}{2}+\frac{c\,n\,x}{b}}{2\,x^2} - \frac{\ln\left(d\,\left(c\,x^2+b\,x\right)^n\right)}{2\,x^2}$$

```
[In] int(log(d*(b*x + c*x^2)^n)/x^3,x)
```

```
[Out] (c^2*n*atanh((2*c*x)/b + 1))/b^2 - (n/2 + (c*n*x)/b)/(2*x^2) - log(d*(b*x +
 c*x^2)^n)/(2*x^2)
```

3.68 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^4}\,dx$

Optimal result 398
Rubi [A] (verified) 398
Mathematica [A] (verified) 399
Maple [A] (verified) 400
Fricas [A] (verification not implemented) 400
Sympy [A] (verification not implemented) 400
Maxima [A] (verification not implemented) 401
Giac [A] (verification not implemented) 401
Mupad [B] (verification not implemented) 401

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = -\frac{n}{9x^3}-\frac{cn}{6bx^2}+\frac{c^2n}{3b^2x}+\frac{c^3n\log(x)}{3b^3}-\frac{c^3n\log(b+cx)}{3b^3}-\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}$$

```
[Out] -1/9*n/x^3-1/6*c*n/b/x^2+1/3*c^2*n/b^2/x+1/3*c^3*n*ln(x)/b^3-1/3*c^3*n*ln(c
*x+b)/b^3-1/3*ln(d*(c*x^2+b*x)^n)/x^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2605, 78}

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = \frac{c^3n\log(x)}{3b^3}-\frac{c^3n\log(b+cx)}{3b^3}+\frac{c^2n}{3b^2x}-\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}-\frac{cn}{6bx^2}-\frac{n}{9x^3}$$

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -1/9*n/x^3 - (c*n)/(6*b*x^2) + (c^2*n)/(3*b^2*x) + (c^3*n*Log[x])/(3*b^3) -
 (c^3*n*Log[b + c*x])/(3*b^3) - Log[d*(b*x + c*x^2)^n]/(3*x^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}+\frac{1}{3}n\int\frac{b+2cx}{x^4(b+cx)}\,dx\\
&= -\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}+\frac{1}{3}n\int\left(\frac{1}{x^4}+\frac{c}{bx^3}-\frac{c^2}{b^2x^2}+\frac{c^3}{b^3x}-\frac{c^4}{b^3(b+cx)}\right)dx\\
&= -\frac{n}{9x^3}-\frac{cn}{6bx^2}+\frac{c^2n}{3b^2x}+\frac{c^3n\log(x)}{3b^3}-\frac{c^3n\log(b+cx)}{3b^3}-\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int\frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx=\frac{1}{3}n\left(-\frac{1}{3x^3}-\frac{c}{2bx^2}+\frac{c^2}{b^2x}+\frac{c^3\log(x)}{b^3}-\frac{c^3\log(b+cx)}{b^3}\right)-\frac{\log\left(d(x(b+cx))^n\right)}{3x^3}$$

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^4,x]
```

```
[Out] (n*(-1/3*1/x^3 - c/(2*b*x^2) + c^2/(b^2*x) + (c^3*Log[x])/b^3 - (c^3*Log[b
+ c*x])/b^3))/3 - Log[d*(x*(b + c*x))^n]/(3*x^3)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

method	result	size
parts	$-\frac{\ln\left(d\left(c\,x^{2}+b x\right)^{n}\right)}{3 x^{3}}+\frac{n\left(-\frac{1}{3 x^{3}}-\frac{c}{2 b\,x^{2}}+\frac{c^{3}\ln\left(x\right)}{b^{3}}+\frac{c^{2}}{b^{2} x}-\frac{c^{3}\ln\left(x c+b\right)}{b^{3}}\right)}{3}$	72
parallelrisch	$-\frac{-6\ln\left(x\right)x^{3}c^{3}n+6\ln\left(x c+b\right)x^{3}c^{3}n+6x^{3}c^{3}n-6x^{2}b\,c^{2}n+3x\,b^{2}c n+6\ln\left(d\left(x\left(x c+b\right)\right)^{n}\right)b^{3}+2b^{3}n}{18x^{3}b^{3}}$	86

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(d*(c*x^2+b*x)^n)/x^3+1/3*n*(-1/3/x^3-1/2*c/b/x^2+c^3/b^3*ln(x)+c^2/
b^2/x-c^3/b^3*ln(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = -\frac{6\,c^3nx^3\log\left(cx+b\right)-6\,c^3nx^3\log\left(x\right)-6\,bc^2nx^2+3\,b^2cnx+6\,b^3n\log\left(cx^2+bx\right)+2\,b^3n+6\,b^3\log\left(d\right)}{18\,b^3x^3}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*c^3*n*x^3*log(c*x + b) - 6*c^3*n*x^3*log(x) - 6*b*c^2*n*x^2 + 3*b^
2*c*n*x + 6*b^3*n*log(c*x^2 + b*x) + 2*b^3*n + 6*b^3*log(d))/(b^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = \begin{cases} -\frac{n}{9x^3}-\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}-\frac{cn}{6bx^2}+\frac{c^2n}{3b^2x}-\frac{2c^3n\log\left(b+cx\right)}{3b^3}+\frac{c^3\log\left(d(bx+cx^2)^n\right)}{3b^3} & \text{for}\ b\neq 0\\ -\frac{2n}{9x^3}-\frac{\log\left(d(cx^2)^n\right)}{3x^3} & \text{otherwise}\end{cases}$$

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**4,x)
```

```
[Out] Piecewise((-n/(9*x**3) - log(d*(b*x + c*x**2)**n)/(3*x**3) - c*n/(6*b*x**2)
 + c**2*n/(3*b**2*x) - 2*c**3*n*log(b + c*x)/(3*b**3) + c**3*log(d*(b*x + c
*x**2)**n)/(3*b**3), Ne(b, 0)), (-2*n/(9*x**3) - log(d*(c*x**2)**n)/(3*x**3
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = -\frac{1}{18}n\left(\frac{6\,c^3\log\left(cx+b\right)}{b^3}-\frac{6\,c^3\log\left(x\right)}{b^3}-\frac{6\,c^2x^2-3\,bcx-2\,b^2}{b^2x^3}\right)-\frac{\log\left(\left(cx^2+bx\right)^n d\right)}{3\,x^3}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="maxima")
```

```
[Out] -1/18*n*(6*c^3*log(c*x + b)/b^3 - 6*c^3*log(x)/b^3 - (6*c^2*x^2 - 3*b*c*x -
 2*b^2)/(b^2*x^3)) - 1/3*log((c*x^2 + b*x)^n*d)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = -\frac{c^3n\log\left(cx+b\right)}{3\,b^3}+\frac{c^3n\log\left(x\right)}{3\,b^3}-\frac{n\log\left(cx^2+bx\right)}{3\,x^3}+\frac{6\,c^2nx^2-3\,bcnx-2\,b^2n-6\,b^2\log\left(d\right)}{18\,b^2x^3}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*c^3*n*log(c*x + b)/b^3 + 1/3*c^3*n*log(x)/b^3 - 1/3*n*log(c*x^2 + b*x)
/x^3 + 1/18*(6*c^2*n*x^2 - 3*b*c*n*x - 2*b^2*n - 6*b^2*log(d))/(b^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = -\frac{\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{3\,x^3}-\frac{\frac{n}{3}-\frac{c^2\,n\,x^2}{b^2}+\frac{c\,n\,x}{2\,b}}{3\,x^3}-\frac{2\,c^3\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{3\,b^3}$$

```
[In] int(log(d*(b*x + c*x^2)^n)/x^4,x)
```

```
[Out] - log(d*(b*x + c*x^2)^n)/(3*x^3) - (n/3 - (c^2*n*x^2)/b^2 + (c*n*x)/(2*b))/
(3*x^3) - (2*c^3*n*atanh((2*c*x)/b + 1))/(3*b^3)
```

3.69 $\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx$

Optimal result 402
Rubi [A] (verified) 402
Mathematica [A] (verified) 403
Maple [A] (verified) 404
Fricas [A] (verification not implemented) 404
Sympy [A] (verification not implemented) 404
Maxima [A] (verification not implemented) 405
Giac [A] (verification not implemented) 405
Mupad [B] (verification not implemented) 405

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx = -\frac{n}{16x^4} - \frac{cn}{12bx^3} + \frac{c^2n}{8b^2x^2} - \frac{c^3n}{4b^3x} - \frac{c^4n\log(x)}{4b^4} + \frac{c^4n\log(b+cx)}{4b^4} - \frac{\log\left(d(bx+cx^2)^n\right)}{4x^4}$$

```
[Out] -1/16*n/x^4-1/12*c*n/b/x^3+1/8*c^2*n/b^2/x^2-1/4*c^3*n/b^3/x-1/4*c^4*n*ln(x
)/b^4+1/4*c^4*n*ln(c*x+b)/b^4-1/4*ln(d*(c*x^2+b*x)^n)/x^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2605, 78}

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx = -\frac{c^4n\log(x)}{4b^4} + \frac{c^4n\log(b+cx)}{4b^4} - \frac{c^3n}{4b^3x} + \frac{c^2n}{8b^2x^2} - \frac{\log\left(d(bx+cx^2)^n\right)}{4x^4} - \frac{cn}{12bx^3} - \frac{n}{16x^4}$$

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -1/16*n/x^4 - (c*n)/(12*b*x^3) + (c^2*n)/(8*b^2*x^2) - (c^3*n)/(4*b^3*x) -
(c^4*n*Log[x])/(4*b^4) + (c^4*n*Log[b + c*x])/(4*b^4) - Log[d*(b*x + c*x^2)
^n]/(4*x^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(d(bx+cx^2)^n\right)}{4x^4}+\frac{1}{4}n\int\frac{b+2cx}{x^5(b+cx)}\,dx\\
&= -\frac{\log\left(d(bx+cx^2)^n\right)}{4x^4}+\frac{1}{4}n\int\left(\frac{1}{x^5}+\frac{c}{bx^4}-\frac{c^2}{b^2x^3}+\frac{c^3}{b^3x^2}-\frac{c^4}{b^4x}+\frac{c^5}{b^4(b+cx)}\right)dx\\
&= -\frac{n}{16x^4}-\frac{cn}{12bx^3}+\frac{c^2n}{8b^2x^2}-\frac{c^3n}{4b^3x}-\frac{c^4n\log(x)}{4b^4}+\frac{c^4n\log(b+cx)}{4b^4}-\frac{\log\left(d(bx+cx^2)^n\right)}{4x^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int\frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx = -\frac{bn(3b^3+4b^2cx-6bc^2x^2+12c^3x^3)+12c^4nx^4\log(x)-12c^4nx^4\log(b+cx)+12b^4\log\left(d(x(b+cx))^n\right)}{48b^4x^4}$$

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -1/48*(b*n*(3*b^3 + 4*b^2*c*x - 6*b*c^2*x^2 + 12*c^3*x^3) + 12*c^4*n*x^4*Lo
g[x] - 12*c^4*n*x^4*Log[b + c*x] + 12*b^4*Log[d*(x*(b + c*x))^n])/(b^4*x^4)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

method	result	size
parts	$-\frac{\ln\left(d\left(c\,x^{2}+b x\right)^{n}\right)}{4 x^{4}}+\frac{n\left(-\frac{1}{4 x^{4}}-\frac{c}{3 b\,x^{3}}-\frac{c^{3}}{b^{3} x}+\frac{c^{2}}{2 b^{2} x^{2}}-\frac{c^{4} \ln\left(x\right)}{b^{4}}+\frac{c^{4} \ln\left(x c+b\right)}{b^{4}}\right)}{4}$	84
parallelrisch	$-\frac{12 \ln\left(x\right) x^{4} c^{4} n-12 \ln\left(x c+b\right) x^{4} c^{4} n-12 x^{4} c^{4} n+12 x^{3} b\,c^{3} n-6 x^{2} b^{2} c^{2} n+4 x\,b^{3} c n+12 \ln\left(d \left(x \left(x c+b\right)\right)^{n}\right) b^{4}+3 b^{4} n}{48 x^{4} b^{4}}$	98

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(d*(c*x^2+b*x)^n)/x^4+1/4*n*(-1/4/x^4-1/3*c/b/x^3-c^3/b^3/x+1/2*c^2/
b^2/x^2-c^4/b^4*ln(x)+c^4/b^4*ln(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx$$
$$=\frac{12\,c^4nx^4\log\left(cx+b\right)-12\,c^4nx^4\log\left(x\right)-12\,bc^3nx^3+6\,b^2c^2nx^2-4\,b^3cnx-12\,b^4n\log\left(cx^2+bx\right)-3\,b^4n-}{48\,b^4x^4}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(12*c^4*n*x^4*log(c*x + b) - 12*c^4*n*x^4*log(x) - 12*b*c^3*n*x^3 + 6*
b^2*c^2*n*x^2 - 4*b^3*c*n*x - 12*b^4*n*log(c*x^2 + b*x) - 3*b^4*n - 12*b^4*
log(d))/(b^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 7.52 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx$$
$$=\begin{cases}-\frac{n}{16x^4}-\frac{\log\left(d(bx+cx^2)^n\right)}{4x^4}-\frac{cn}{12bx^3}+\frac{c^2n}{8b^2x^2}-\frac{c^3n}{4b^3x}+\frac{c^4n\log\left(b+cx\right)}{2b^4}-\frac{c^4\log\left(d(bx+cx^2)^n\right)}{4b^4} & \text{for}\ b\neq 0\\ -\frac{n}{8x^4}-\frac{\log\left(d(cx^2)^n\right)}{4x^4} & \text{otherwise}\end{cases}$$

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**5,x)
```

```
[Out] Piecewise((-n/(16*x**4) - log(d*(b*x + c*x**2)**n)/(4*x**4) - c*n/(12*b*x**
3) + c**2*n/(8*b**2*x**2) - c**3*n/(4*b**3*x) + c**4*n*log(b + c*x)/(2*b**4
) - c**4*log(d*(b*x + c*x**2)**n)/(4*b**4), Ne(b, 0)), (-n/(8*x**4) - log(d
*(c*x**2)**n)/(4*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx = \frac{1}{48}\,n\left(\frac{12\,c^4\log\left(cx+b\right)}{b^4} - \frac{12\,c^4\log\left(x\right)}{b^4} - \frac{12\,c^3x^3 - 6\,bc^2x^2 + 4\,b^2cx + 3\,b^3}{b^3x^4}\right) - \frac{\log\left((cx^2+bx)^n d\right)}{4\,x^4}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*n*(12*c^4*log(c*x + b)/b^4 - 12*c^4*log(x)/b^4 - (12*c^3*x^3 - 6*b*c^2
*x^2 + 4*b^2*c*x + 3*b^3)/(b^3*x^4)) - 1/4*log((c*x^2 + b*x)^n*d)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx = \frac{c^4n\log\left(cx+b\right)}{4\,b^4} - \frac{c^4n\log\left(x\right)}{4\,b^4} - \frac{n\log\left(cx^2+bx\right)}{4\,x^4} - \frac{12\,c^3nx^3 - 6\,bc^2nx^2 + 4\,b^2cnx + 3\,b^3n + 12\,b^3\log\left(d\right)}{48\,b^3x^4}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^4*n*log(c*x + b)/b^4 - 1/4*c^4*n*log(x)/b^4 - 1/4*n*log(c*x^2 + b*x)/
x^4 - 1/48*(12*c^3*n*x^3 - 6*b*c^2*n*x^2 + 4*b^2*c*n*x + 3*b^3*n + 12*b^3*l
og(d))/(b^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx = \frac{c^4\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{2\,b^4} - \frac{\ln\left(d\,(c\,x^2+b\,x)^n\right)}{4\,x^4} - \frac{\frac{n}{4} - \frac{c^2\,n\,x^2}{2\,b^2} + \frac{c^3\,n\,x^3}{b^3} + \frac{c\,n\,x}{3\,b}}{4\,x^4}$$

```
[In] int(log(d*(b*x + c*x^2)^n)/x^5,x)
```

```
[Out] (c^4*n*atanh((2*c*x)/b + 1))/(2*b^4) - log(d*(b*x + c*x^2)^n)/(4*x^4) - (n/
4 - (c^2*n*x^2)/(2*b^2) + (c^3*n*x^3)/b^3 + (c*n*x)/(3*b))/(4*x^4)
```

3.70 $\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal result 406
Rubi [A] (verified) 406
Mathematica [A] (verified) 408
Maple [F] 408
Fricas [F] 409
Sympy [F(-1)] 409
Maxima [F] 409
Giac [F] 409
Mupad [F(-1)] 410

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = -\frac{2cnx^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(1+m)(2+m)} - \frac{2cnx^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)(1+m)(2+m)} + \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m}$$

```
[Out] x^(1+m)*ln(d*(c*x^2+b*x+a)^n)/(1+m)-2*c*n*x^(2+m)*hypergeom([1, 2+m],[3+m],
-2*c*x/(b-(-4*a*c+b^2)^(1/2)))/(1+m)/(2+m)/(b-(-4*a*c+b^2)^(1/2))-2*c*n*x^(
2+m)*hypergeom([1, 2+m],[3+m],-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/(1+m)/(2+m)/(b
+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used

= {2605, 844, 66}

$$\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$= -\frac{2cnx^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(b-\sqrt{b^2-4ac}\right)} - \frac{2cnx^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(\sqrt{b^2-4ac}+b\right)} + \frac{x^{m+1}\log\left(d\left(a+bx+cx^2\right)^n\right)}{m+1}$$

```
[In] Int[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-2*c*n*x^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b - Sqrt[b^2
 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) - (2*c*n*x^(2 + m)*H
ypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/((b +
Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) + (x^(1 + m)*Log[d*(a + b*x + c*x^2)^n]
)/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{x^{1+m}\log\left(d(a+bx+cx^2)^n\right)}{1+m} - \frac{n\int\frac{x^{1+m}(b+2cx)}{a+bx+cx^2}\,dx}{1+m}$$

$$= \frac{x^{1+m}\log\left(d(a+bx+cx^2)^n\right)}{1+m} - \frac{n\int\left(\frac{2cx^{1+m}}{b-\sqrt{b^2-4ac}+2cx}+\frac{2cx^{1+m}}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{1+m}$$

$$= \frac{x^{1+m}\log\left(d(a+bx+cx^2)^n\right)}{1+m} - \frac{(2cn)\int\frac{x^{1+m}}{b-\sqrt{b^2-4ac}+2cx}\,dx}{1+m} - \frac{(2cn)\int\frac{x^{1+m}}{b+\sqrt{b^2-4ac}+2cx}\,dx}{1+m}$$

$$= -\frac{2cnx^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(1+m)(2+m)} - \frac{2cnx^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)(1+m)(2+m)} + \frac{x^{1+m}\log\left(d(a+bx+cx^2)^n\right)}{1+m}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

$$\int x^m\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = -\frac{x^{1+m}\left(\left(b+\sqrt{b^2-4ac}\right)nx\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)+\left(b-\sqrt{b^2-4ac}\right)nx\,\text{Hyp}\right.}{2a\left(2+3m+m^2\right)}$$

```
[In] Integrate[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/2*(x^(1 + m)*((b + Sqrt[b^2 - 4*a*c])*n*x*Hypergeometric2F1[1, 2 + m, 3
+ m, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*n*x*Hyperg
eometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])] - 2*a*(2 + m
)*Log[d*(a + x*(b + c*x))^n]))/(a*(2 + 3*m + m^2))
```

Maple [F]

$$\int x^m\ln\left(d\left(c\,x^2+bx+a\right)^n\right)dx$$

```
[In] int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

Fricas [F]

$$\int x^m \log \left(d(a+bx+cx^2)^n\right) dx = \int x^m \log \left((cx^2+bx+a)^n d\right) dx$$

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
[Out] integral(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \log \left(d(a+bx+cx^2)^n\right) dx = \text{Timed out}$$

```
[In] integrate(x**m*ln(d*(c*x**2+b*x+a)**n),x)
[Out] Timed out
```

Maxima [F]

$$\int x^m \log \left(d(a+bx+cx^2)^n\right) dx = \int x^m \log \left((cx^2+bx+a)^n d\right) dx$$

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
[Out] x*x^m*log((c*x^2 + b*x + a)^n)/(m + 1) + integrate((((m + 1)*log(d) - 2*n)*
c*x^2 + ((m + 1)*log(d) - n)*b*x + a*(m + 1)*log(d))*x^m/(c*(m + 1)*x^2 + b
*(m + 1)*x + a*(m + 1)), x)
```

Giac [F]

$$\int x^m \log \left(d(a+bx+cx^2)^n\right) dx = \int x^m \log \left((cx^2+bx+a)^n d\right) dx$$

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
[Out] integrate(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \log \left(d\left(a+bx+cx^2\right)^n\right) \,dx = \int x^m\,\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right) \,dx$$

```
[In] int(x^m*log(d*(a + b*x + c*x^2)^n),x)
[Out] int(x^m*log(d*(a + b*x + c*x^2)^n), x)
```

3.71 $\int x^4 \log\left(d(a+bx+cx^2)^n\right)\,dx$

Optimal result 411
Rubi [A] (verified) 411
Mathematica [A] (verified) 414
Maple [A] (verified) 414
Fricas [A] (verification not implemented) 415
Sympy [F(-1)] 415
Maxima [F(-2)] 416
Giac [A] (verification not implemented) 416
Mupad [B] (verification not implemented) 417

Optimal result

Integrand size = 19, antiderivative size = 207

$$\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = -\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4}+\frac{b\left(b^2-3ac\right)nx^2}{10c^3} -\frac{\left(b^2-2ac\right)nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25} +\frac{\sqrt{b^2-4ac}\left(b^4-3ab^2c+a^2c^2\right)n\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{5c^5} +\frac{b\left(b^4-5ab^2c+5a^2c^2\right)n\log\left(a+bx+cx^2\right)}{10c^5} +\frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)$$

```
[Out] -1/5*(2*a^2*c^2-4*a*b^2*c+b^4)*n*x/c^4+1/10*b*(-3*a*c+b^2)*n*x^2/c^3-1/15*(
-2*a*c+b^2)*n*x^3/c^2+1/20*b*n*x^4/c-2/25*n*x^5+1/10*b*(5*a^2*c^2-5*a*b^2*c
+b^4)*n*ln(c*x^2+b*x+a)/c^5+1/5*x^5*ln(d*(c*x^2+b*x+a)^n)+1/5*(a^2*c^2-3*a*
b^2*c+b^4)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

$= \{2605, 814, 648, 632, 212, 642\}$

$$\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{n\sqrt{b^2-4ac}(a^2c^2-3ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{5c^5} + \frac{bn(5a^2c^2-5ab^2c+b^4)\log\left(a+bx+cx^2\right)}{10c^5} - \frac{nx(2a^2c^2-4ab^2c+b^4)}{5c^4} + \frac{bnx^2(b^2-3ac)}{10c^3} - \frac{nx^3(b^2-2ac)}{15c^2} + \frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{bnx^4}{20c} - \frac{2nx^5}{25}$$

```
[In] Int[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/5*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x)/c^4 + (b*(b^2 - 3*a*c)*n*x^2)/(10*
c^3) - ((b^2 - 2*a*c)*n*x^3)/(15*c^2) + (b*n*x^4)/(20*c) - (2*n*x^5)/25 + (
Sqrt[b^2 - 4*a*c]*(b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(5*c^5) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + b*x + c*x
^2])/(10*c^5) + (x^5*Log[d*(a + b*x + c*x^2)^n])/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{5}n\int\frac{x^5(b+2cx)}{a+bx+cx^2}\,dx\\
&= \frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{5}n\int\left(\frac{b^4-4ab^2c+2a^2c^2}{c^4}-\frac{b(b^2-3ac)\,x}{c^3}\right.\\
&\quad\left.+\frac{(b^2-2ac)\,x^2}{c^2}-\frac{bx^3}{c}+2x^4-\frac{a(b^4-4ab^2c+2a^2c^2)+b(b^4-5ab^2c+5a^2c^2)\,x}{c^4\,(a+bx+cx^2)}\right)dx\\
&= -\frac{(b^4-4ab^2c+2a^2c^2)\,nx}{5c^4}+\frac{b(b^2-3ac)\,nx^2}{10c^3}-\frac{(b^2-2ac)\,nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}\\
&\quad+\frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{n\int\frac{a(b^4-4ab^2c+2a^2c^2)+b(b^4-5ab^2c+5a^2c^2)x}{a+bx+cx^2}\,dx}{5c^4}\\
&= -\frac{(b^4-4ab^2c+2a^2c^2)\,nx}{5c^4}+\frac{b(b^2-3ac)\,nx^2}{10c^3}-\frac{(b^2-2ac)\,nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}\\
&\quad+\frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{((b^2-4ac)\,(b^4-3ab^2c+a^2c^2)\,n)\int\frac{1}{a+bx+cx^2}\,dx}{10c^5}\\
&\quad+\frac{(b(b^4-5ab^2c+5a^2c^2)\,n)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{10c^5}\\
&= -\frac{(b^4-4ab^2c+2a^2c^2)\,nx}{5c^4}+\frac{b(b^2-3ac)\,nx^2}{10c^3}-\frac{(b^2-2ac)\,nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}\\
&\quad+\frac{b(b^4-5ab^2c+5a^2c^2)\,n\log\left(a+bx+cx^2\right)}{10c^5}+\frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)\\
&\quad+\frac{((b^2-4ac)\,(b^4-3ab^2c+a^2c^2)\,n)\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{5c^5}
\end{aligned}
$$

$$= -\frac{(b^4-4ab^2c+2a^2c^2)\,nx}{5c^4}+\frac{b(b^2-3ac)\,nx^2}{10c^3}-\frac{(b^2-2ac)\,nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}+\frac{\sqrt{b^2-4ac}(b^4-3ab^2c+a^2c^2)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{5c^5}+\frac{b(b^4-5ab^2c+5a^2c^2)\,n\log\left(a+bx+cx^2\right)}{10c^5}+\frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

$$\int x^4\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \frac{cnx(-60b^4+30b^3cx-20b^2c(-12a+cx^2)+15bc^2x(-6a+cx^2)-8c^2(15a^2-5acx^2+3c^2x^4))+60\sqrt{b^2-}}{}$$

```
[In] Integrate[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c*(-12*a + c*x^2) + 15*b*c^2*x*(-6*a
+ c*x^2) - 8*c^2*(15*a^2 - 5*a*c*x^2 + 3*c^2*x^4)) + 60*Sqrt[b^2 - 4*a*c]*(
b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 30*b*
(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + x*(b + c*x)] + 60*c^5*x^5*Log[d*(a
+ x*(b + c*x))^n])/(300*c^5)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.15

method	result
parts	$\frac{x^5\ln\left(d\left(cx^2+bx+a\right)^n\right)}{5}-\frac{n\left(\frac{\frac{2}{5}c^4x^5-\frac{1}{4}bx^4c^3-\frac{2}{3}ac^3x^3+\frac{1}{3}b^2c^2x^3+\frac{3}{2}abc^2x^2-\frac{1}{2}b^3cx^2+2a^2xc^2-4ab^2cx+b^4x}{c^4}+\frac{\left(-5a^2bc^2+5ab^3c-b^5\right)\ln\left(c\right.}{2c}\right.}{5}$
risch	Expression too large to display

```
[In] int(x^4*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln(d*(c*x^2+b*x+a)^n)-1/5*n*(1/c^4*(2/5*c^4*x^5-1/4*b*x^4*c^3-2/3*a
*c^3*x^3+1/3*b^2*c^2*x^3+3/2*a*b*c^2*x^2-1/2*b^3*c*x^2+2*a^2*x*c^2-4*a*b^2*
c*x+b^4*x)+1/c^4*(1/2*(-5*a^2*b*c^2+5*a*b^3*c-b^5)/c*ln(c*x^2+b*x+a)+2*(-2*
c^2*a^3+4*a^2*b^2*c-b^4*a-1/2*(-5*a^2*b*c^2+5*a*b^3*c-b^5)*b/c)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.14

$$\int x^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$= \left[-\frac{24\,c^5nx^5 - 60\,c^5x^5\log\left(d\right) - 15\,bc^4nx^4 + 20\,(b^2c^3 - 2\,ac^4)nx^3 - 30\,(b^3c^2 - 3\,abc^3)nx^2 - 30\,(b^4 - 3\,ab^2c}{}\right.$$

$$-\frac{24\,c^5nx^5 - 60\,c^5x^5\log\left(d\right) - 15\,bc^4nx^4 + 20\,(b^2c^3 - 2\,ac^4)nx^3 - 30\,(b^3c^2 - 3\,abc^3)nx^2 - 60\,(b^4 - 3\,ab^2c}{}$$

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*n*x^4 + 20*(b^2*c^3 -
2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 30*(b^4 - 3*a*b^2*c + a^2
*c^2)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2
*c^3)*n*x - 30*(2*c^5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c*x^2
+ b*x + a))/c^5, -1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*n*x^4
+ 20*(b^2*c^3 - 2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 60*(b^4 -
 3*a*b^2*c + a^2*c^2)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*n*x - 30*(2*c^
5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c*x^2 + b*x + a))/c^5]
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Timed out}$$

```
[In] integrate(x**4*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^4 \log\left(d(a + bx + cx^2)^n\right) dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07

$$\begin{aligned}&\int x^4 \log\left(d(a + bx + cx^2)^n\right) dx \\ &= \frac{1}{5} nx^5 \log\left(cx^2 + bx + a\right) - \frac{1}{25} (2n - 5\log(d))x^5 + \frac{bnx^4}{20c} \\ &\quad - \frac{(b^2n - 2acn)x^3}{15c^2} + \frac{(b^3n - 3abcn)x^2}{10c^3} - \frac{(b^4n - 4ab^2cn + 2a^2c^2n)x}{5c^4} \\ &\quad + \frac{(b^5n - 5ab^3cn + 5a^2bc^2n)\log\left(cx^2 + bx + a\right)}{10c^5} \\ &\quad - \frac{(b^6n - 7ab^4cn + 13a^2b^2c^2n - 4a^3c^3n)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{5\sqrt{-b^2 + 4ac}c^5}\end{aligned}$$

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/5*n*x^5*log(c*x^2 + b*x + a) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/c
 - 1/15*(b^2*n - 2*a*c*n)*x^3/c^2 + 1/10*(b^3*n - 3*a*b*c*n)*x^2/c^3 - 1/5*
(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*x/c^4 + 1/10*(b^5*n - 5*a*b^3*c*n + 5*a
^2*b*c^2*n)*log(c*x^2 + b*x + a)/c^5 - 1/5*(b^6*n - 7*a*b^4*c*n + 13*a^2*b^
2*c^2*n - 4*a^3*c^3*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.91

$$\int x^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$= x^2 \left(\frac{b\left(\frac{b^2 n}{5 c^2} - \frac{2 a n}{5 c}\right)}{2 c} - \frac{a b n}{10 c^2}\right) - \frac{2 n x^5}{25} + x \left(\frac{a\left(\frac{b^2 n}{5 c^2} - \frac{2 a n}{5 c}\right)}{c} - \frac{b\left(\frac{b\left(\frac{b^2 n}{5 c^2} - \frac{2 a n}{5 c}\right)}{c} - \frac{a b n}{5 c^2}\right)}{c}\right)$$

$$+ \frac{x^5 \ln\left(d\left(c x^2 + b x + a\right)^n\right)}{5} - x^3 \left(\frac{b^2 n}{15 c^2} - \frac{2 a n}{15 c}\right)$$

$$+ \frac{\ln\left(b\sqrt{b^2-4 a c} - 4 a c + b^2 + 2 c x \sqrt{b^2-4 a c}\right) \left(\frac{b^5 n}{10} + c^2 \left(\frac{a^2 n \sqrt{b^2-4 a c}}{10} + \frac{a^2 b n}{2}\right) - c \left(\frac{a b^3 n}{2} + \frac{3 a b^2 n \sqrt{b^2\cdots}}{10}\right.\right.}{c^5}$$

$$- \frac{\ln\left(4 a c + b\sqrt{b^2-4 a c} - b^2 + 2 c x \sqrt{b^2-4 a c}\right) \left(c^2 \left(\frac{a^2 n \sqrt{b^2-4 a c}}{10} - \frac{a^2 b n}{2}\right) - \frac{b^5 n}{10} + c \left(\frac{a b^3 n}{2} - \frac{3 a b^2 n \sqrt{b^2\cdots}}{10}\right.\right.}{c^5}$$

$$+ \frac{b n x^4}{20 c}$$

```
[In] int(x^4*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x^2*((b*((b^2*n)/(5*c^2) - (2*a*n)/(5*c)))/(2*c) - (a*b*n)/(10*c^2)) - (2*n
*x^5)/25 + x*((a*((b^2*n)/(5*c^2) - (2*a*n)/(5*c)))/c - (b*((b*((b^2*n)/(5*
c^2) - (2*a*n)/(5*c)))/c - (a*b*n)/(5*c^2)))/c) + (x^5*log(d*(a + b*x + c*x
^2)^n))/5 - x^3*((b^2*n)/(15*c^2) - (2*a*n)/(15*c)) + (log(b*(b^2 - 4*a*c)^
(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^5*n)/10 + c^2*((a^2*n*
(b^2 - 4*a*c)^(1/2))/10 + (a^2*b*n)/2) - c*((a*b^3*n)/2 + (3*a*b^2*n*(b^2 -
 4*a*c)^(1/2))/10) + (b^4*n*(b^2 - 4*a*c)^(1/2))/10))/c^5 - (log(4*a*c + b*
(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(c^2*((a^2*n*(b^2 -
4*a*c)^(1/2))/10 - (a^2*b*n)/2) - (b^5*n)/10 + c*((a*b^3*n)/2 - (3*a*b^2*n*
(b^2 - 4*a*c)^(1/2))/10) + (b^4*n*(b^2 - 4*a*c)^(1/2))/10))/c^5 + (b*n*x^4)
/(20*c)
```

3.72 $\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 167

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \frac{b(b^2-3ac)\,nx}{4c^3} - \frac{(b^2-2ac)\,nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} - \frac{b\sqrt{b^2-4ac}(b^2-2ac)\,n\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4} - \frac{(b^4-4ab^2c+2a^2c^2)\,n\log\left(a+bx+cx^2\right)}{8c^4} + \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)$$

```
[Out] 1/4*b*(-3*a*c+b^2)*n*x/c^3-1/8*(-2*a*c+b^2)*n*x^2/c^2+1/12*b*n*x^3/c-1/8*n*
x^4-1/8*(2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(c*x^2+b*x+a)/c^4+1/4*x^4*ln(d*(c*x^2
+b*x+a)^n)-1/4*b*(-2*a*c+b^2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a
*c+b^2)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = -\frac{n(2a^2c^2-4ab^2c+b^4)\log\left(a+bx+cx^2\right)}{8c^4} - \frac{bn\sqrt{b^2-4ac}(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4} + \frac{bnx(b^2-3ac)}{4c^3} - \frac{nx^2(b^2-2ac)}{8c^2} + \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{bnx^3}{12c} - \frac{nx^4}{8}$$

```
[In] Int[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (b*(b^2 - 3*a*c)*n*x)/(4*c^3) - ((b^2 - 2*a*c)*n*x^2)/(8*c^2) + (b*n*x^3)/(
12*c) - (n*x^4)/8 - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*
x + c*x^2])/(8*c^4) + (x^4*Log[d*(a + b*x + c*x^2)^n])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{4}n\int\frac{x^4(b+2cx)}{a+bx+cx^2}\,dx\\
&=\frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{4}n\int\left(-\frac{b(b^2-3ac)}{c^3}+\frac{(b^2-2ac)\,x}{c^2}-\frac{bx^2}{c}+2x^3\right.\\
&\qquad\left.+\frac{ab(b^2-3ac)+(b^4-4ab^2c+2a^2c^2)\,x}{c^3\,(a+bx+cx^2)}\right)dx\\
&=\frac{b(b^2-3ac)\,nx}{4c^3}-\frac{(b^2-2ac)\,nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}\\
&\quad+\frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{ab(b^2-3ac)+(b^4-4ab^2c+2a^2c^2)x}{a+bx+cx^2}\,dx}{4c^3}\\
&=\frac{b(b^2-3ac)\,nx}{4c^3}-\frac{(b^2-2ac)\,nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}+\frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)\\
&\quad+\frac{(b(b^2-4ac)\,(b^2-2ac)\,n)\int\frac{1}{a+bx+cx^2}\,dx}{8c^4}-\frac{((b^4-4ab^2c+2a^2c^2)\,n)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{8c^4}\\
&=\frac{b(b^2-3ac)\,nx}{4c^3}-\frac{(b^2-2ac)\,nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}-\frac{(b^4-4ab^2c+2a^2c^2)\,n\log(a+bx+cx^2)}{8c^4}\\
&\quad+\frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{(b(b^2-4ac)\,(b^2-2ac)\,n)\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{4c^4}\\
&=\frac{b(b^2-3ac)\,nx}{4c^3}-\frac{(b^2-2ac)\,nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}\\
&\quad-\frac{b\sqrt{b^2-4ac}(b^2-2ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4}-\frac{(b^4-4ab^2c+2a^2c^2)\,n\log(a+bx+cx^2)}{8c^4}\\
&\quad+\frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$
\int x^3\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx
$$

$$
=\frac{cnx(6b^3-3b^2cx+2bc(-9a+cx^2)-3c^2x(-2a+cx^2))-6b\sqrt{b^2-4ac}(b^2-2ac)\,n\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-3(b
}{24c^4}
$$

```
[In] Integrate[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c*(-9*a + c*x^2) - 3*c^2*x*(-2*a + c*x^2))
- 6*b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]] - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + x*(b + c*x)] + 6*c^4*x^4*Lo
g[d*(a + x*(b + c*x))^n])/(24*c^4)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.14

method	result
parts	$\frac{x^4 \ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{4} - \frac{n\left(\frac{\frac{1}{2}c^3x^4-\frac{1}{3}b\,x^3c^2-a\,c^2x^2+\frac{1}{2}b^2c\,x^2+3abcx-x\,b^3}{c^3}+\frac{\frac{\left(2c^2a^2-4a\,b^2c+b^4\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-3a^2bc+a\,b^3-\frac{(2c}{}\right.}{}}{c^3}\right.}{4}$
risch	Expression too large to display

```
[In] int(x^3*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(d*(c*x^2+b*x+a)^n)-1/4*n*(1/c^3*(1/2*c^3*x^4-1/3*b*x^3*c^2-a*c^2
*x^2+1/2*b^2*c*x^2+3*a*b*c*x-x*b^3)+1/c^3*(1/2*(2*a^2*c^2-4*a*b^2*c+b^4)/c*
ln(c*x^2+b*x+a)+2*(-3*a^2*b*c+a*b^3-1/2*(2*a^2*c^2-4*a*b^2*c+b^4)*b/c)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.18

$$\int x^3 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$= \left[-\frac{3\,c^4nx^4 - 6\,c^4x^4\log(d) - 2\,bc^3nx^3 + 3\,(b^2c^2 - 2\,ac^3)nx^2 + 3\,(b^3 - 2\,abc)\sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2 \cdots}{\cdots}\right.}{24\,c \cdots}\right.$$
$$-\frac{3\,c^4nx^4 - 6\,c^4x^4\log(d) - 2\,bc^3nx^3 + 3\,(b^2c^2 - 2\,ac^3)nx^2 + 6\,(b^3 - 2\,abc)\sqrt{-b^2+4\,ac}n\arctan\left(-\frac{\sqrt{-b^2+\cdots}}{b^{\cdots}}\right.}{24\,c^4}$$

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3*n*x^3 + 3*(b^2*c^2 - 2*a*c
^3)*n*x^2 + 3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 6*(b^3*
c - 3*a*b*c^2)*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*log(
c*x^2 + b*x + a))/c^4, -1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3*n*x^
3 + 3*(b^2*c^2 - 2*a*c^3)*n*x^2 + 6*(b^3 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*ar
ctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 6*(b^3*c - 3*a*b*c^2)
*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*log(c*x^2 + b*x +
a))/c^4]
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \log \left(d(a+bx+cx^2)^n\right) \, dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(d*(c*x**2+b*x+a)**n),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \log \left(d(a+bx+cx^2)^n\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\begin{aligned}\int x^3 \log \left(d(a+bx+cx^2)^n\right) \, dx =& \frac{1}{4} n x^4 \log \left(cx^2+bx+a\right) - \frac{1}{8} (n-2 \log (d)) x^4 \\ &+ \frac{bnx^3}{12\,c} - \frac{(b^2n-2\,acn)x^2}{8\,c^2} + \frac{(b^3n-3\,abcn)x}{4\,c^3} \\ &- \frac{(b^4n-4\,ab^2cn+2\,a^2c^2n) \log \left(cx^2+bx+a\right)}{8\,c^4} \\ &+ \frac{(b^5n-6\,ab^3cn+8\,a^2bc^2n) \arctan \left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c^4}\end{aligned}$$

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
[Out] 1/4*n*x^4*log(c*x^2 + b*x + a) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c -
1/8*(b^2*n - 2*a*c*n)*x^2/c^2 + 1/4*(b^3*n - 3*a*b*c*n)*x/c^3 - 1/8*(b^4*n
- 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/c^4 + 1/4*(b^5*n - 6*a*b^
3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.72

$$\int x^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$
$$= x\left(\frac{b\left(\frac{b^2 n}{4 c^2}-\frac{a n}{2 c}\right)}{c}-\frac{a b n}{4 c^2}\right)-\frac{n x^4}{8}+\frac{x^4 \ln\left(d\left(c x^2+b x+a\right)^n\right)}{4}-x^2\left(\frac{b^2 n}{8 c^2}-\frac{a n}{4 c}\right)$$
$$+\frac{\ln\left(4 a c+b\sqrt{b^2-4 a c}-b^2+2 c x\sqrt{b^2-4 a c}\right)\left(c\left(\frac{a b^2 n}{2}-\frac{a b n\sqrt{b^2-4 a c}}{4}\right)-\frac{b^4 n}{8}+\frac{b^3 n\sqrt{b^2-4 a c}}{8}-\frac{a^2 c^2 n}{4}\right)}{c^4}$$
$$-\frac{\ln\left(b\sqrt{b^2-4 a c}-4 a c+b^2+2 c x\sqrt{b^2-4 a c}\right)\left(\frac{b^4 n}{8}-c\left(\frac{a b^2 n}{2}+\frac{a b n\sqrt{b^2-4 a c}}{4}\right)+\frac{b^3 n\sqrt{b^2-4 a c}}{8}+\frac{a^2 c^2 n}{4}\right)}{c^4}$$
$$+\frac{b n x^3}{12 c}$$

```
[In] int(x^3*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x*((b*((b^2*n)/(4*c^2) - (a*n)/(2*c)))/c - (a*b*n)/(4*c^2)) - (n*x^4)/8 + (
x^4*log(d*(a + b*x + c*x^2)^n))/4 - x^2*((b^2*n)/(8*c^2) - (a*n)/(4*c)) + (
log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(c*((a
*b^2*n)/2 - (a*b*n*(b^2 - 4*a*c)^(1/2))/4) - (b^4*n)/8 + (b^3*n*(b^2 - 4*a*
c)^(1/2))/8 - (a^2*c^2*n)/4))/c^4 - (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^
2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^4*n)/8 - c*((a*b^2*n)/2 + (a*b*n*(b^2 -
4*a*c)^(1/2))/4) + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 + (a^2*c^2*n)/4))/c^4 + (b
*n*x^3)/(12*c)
```

3.73 $\int x^2 \log\left(d(a+bx+cx^2)^n\right) dx$

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Optimal result

Integrand size = 19, antiderivative size = 136

$$\int x^2 \log\left(d(a+bx+cx^2)^n\right) dx = -\frac{(b^2-2ac)\,nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{\sqrt{b^2-4ac}(b^2-ac)\,n\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3} + \frac{b(b^2-3ac)\,n\log\left(a+bx+cx^2\right)}{6c^3} + \frac{1}{3}x^3\log\left(d(a+bx+cx^2)^n\right)$$

```
[Out] -1/3*(-2*a*c+b^2)*n*x/c^2+1/6*b*n*x^2/c-2/9*n*x^3+1/6*b*(-3*a*c+b^2)*n*ln(c
*x^2+b*x+a)/c^3+1/3*x^3*ln(d*(c*x^2+b*x+a)^n)+1/3*(-a*c+b^2)*n*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int x^2 \log\left(d(a+bx+cx^2)^n\right) dx = \frac{n\sqrt{b^2-4ac}(b^2-ac)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3} + \frac{bn(b^2-3ac)\log\left(a+bx+cx^2\right)}{6c^3} - \frac{nx(b^2-2ac)}{3c^2} + \frac{1}{3}x^3\log\left(d(a+bx+cx^2)^n\right) + \frac{bnx^2}{6c} - \frac{2nx^3}{9}$$

```
[In] Int[x^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/3*((b^2 - 2*a*c)*n*x)/c^2 + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (Sqrt[b^2 -
4*a*c]*(b^2 - a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) + (b*(
b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*c^3) + (x^3*Log[d*(a + b*x + c*x^2)
^n])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{3}n\int\frac{x^3(b+2cx)}{a+bx+cx^2}\,dx\\
&= \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{3}n\int\left(\frac{b^2-2ac}{c^2}-\frac{bx}{c}+2x^2-\frac{a(b^2-2ac)+b(b^2-3ac)\,x}{c^2\,(a+bx+cx^2)}\right)dx\\
&= -\frac{(b^2-2ac)\,nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{n\int\frac{a(b^2-2ac)+b(b^2-3ac)x}{a+bx+cx^2}\,dx}{3c^2}\\
&= -\frac{(b^2-2ac)\,nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)\\
&\quad+\frac{(b(b^2-3ac)\,n)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{6c^3}-\frac{((b^4-5ab^2c+4a^2c^2)\,n)\int\frac{1}{a+bx+cx^2}\,dx}{6c^3}\\
&= -\frac{(b^2-2ac)\,nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}\\
&\quad+\frac{b(b^2-3ac)\,n\log\left(a+bx+cx^2\right)}{6c^3}+\frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)\\
&\quad+\frac{((b^4-5ab^2c+4a^2c^2)\,n)\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{3c^3}\\
&= -\frac{(b^2-2ac)\,nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{\sqrt{b^2-4ac}(b^2-ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3}\\
&\quad+\frac{b(b^2-3ac)\,n\log\left(a+bx+cx^2\right)}{6c^3}+\frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$
\int x^2\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx
= \frac{cnx(-6b^2+3bcx-4c(-3a+cx^2))+6\sqrt{b^2-4ac}(b^2-ac)\,n\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+3b(b^2-3ac)\,n\log(a+x(}{18c^3}
$$

```
[In] Integrate[x^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-6*b^2 + 3*b*c*x - 4*c*(-3*a + c*x^2)) + 6*Sqrt[b^2 - 4*a*c]*(b^2 -
 a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*b*(b^2 - 3*a*c)*n*Log[a
+ x*(b + c*x)] + 6*c^3*x^3*Log[d*(a + x*(b + c*x))^n])/(18*c^3)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13

method	result
parts	$\frac{x^3 \ln\left(d\left(c x^2+bx+a\right)^n\right)}{3} - \frac{n\left(-\frac{-\frac{2}{3}x^3c^2+\frac{1}{2}cb\,x^2+2xca-b^2x}{c^2}+\frac{\frac{(3abc-b^3)\ln\left(c x^2+bx+a\right)}{2c}+\frac{2\left(2a^2c-a\,b^2-\frac{(3abc-b^3)b}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}}}{c^2}\right)}{3}$
risch	$\frac{x^3 \ln\left(\left(c x^2+bx+a\right)^n\right)}{3}+\frac{i\pi\,x^3\operatorname{csgn}\left(i\left(c x^2+bx+a\right)^n\right)\operatorname{csgn}\left(id\left(c x^2+bx+a\right)^n\right)^2}{6}+\frac{i\pi\,x^3\operatorname{csgn}\left(id\left(c x^2+bx+a\right)^n\right)^2\operatorname{csgn}(id)}{6}-\frac{i\pi\,x^3}{}$

```
[In] int(x^2*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(d*(c*x^2+b*x+a)^n)-1/3*n*(-1/c^2*(-2/3*x^3*c^2+1/2*c*b*x^2+2*x*c
*a-b^2*x)+1/c^2*(1/2*(3*a*b*c-b^3)/c*ln(c*x^2+b*x+a)+2*(2*a^2*c-a*b^2-1/2*(
3*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.20

$$\int x^2 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$= \left[-\frac{4\,c^3nx^3-6\,c^3x^3\log\left(d\right)-3\,bc^2nx^2+3\,(b^2-ac)\sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+6}{18\,c^3}\right.$$
$$\left.-\frac{4\,c^3nx^3-6\,c^3x^3\log\left(d\right)-3\,bc^2nx^2-6\,(b^2-ac)\sqrt{-b^2+4\,ac}n\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+6\,(b^2c-2}{18\,c^3}\right.$$

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/18*(4*c^3*n*x^3 - 6*c^3*x^3*log(d) - 3*b*c^2*n*x^2 + 3*(b^2 - a*c)*sqrt
(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*
(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^2*c - 2*a*c^2)*n*x - 3*(2*c^3*n*x^3
+ (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3, -1/18*(4*c^3*n*x^3 - 6*c^3*
x^3*log(d) - 3*b*c^2*n*x^2 - 6*(b^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqr
t(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^2*c - 2*a*c^2)*n*x - 3*(2
*c^3*n*x^3 + (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3]
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \log \left(d(a+bx+cx^2)^n\right) \, dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \log \left(d(a+bx+cx^2)^n\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\begin{aligned}\int x^2 \log \left(d(a+bx+cx^2)^n\right) \, dx = {} & \frac{1}{3} nx^3 \log \left(cx^2+bx+a\right) - \frac{1}{9} \left(2 n - 3 \log \left(d\right)\right) x^3 + \frac{bnx^2}{6 c} \\ & - \frac{(b^2n - 2 acn)x}{3 c^2} + \frac{(b^3n - 3 abcn) \log \left(cx^2+bx+a\right)}{6 c^3} \\ & - \frac{(b^4n - 5 ab^2cn + 4 a^2c^2n) \arctan \left(\frac{2 cx+b}{\sqrt{-b^2+4 ac}}\right)}{3 \sqrt{-b^2+4 ac}c^3}\end{aligned}$$

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/3*n*x^3*log(c*x^2 + b*x + a) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c -
 1/3*(b^2*n - 2*a*c*n)*x/c^2 + 1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x + a)
/c^3 - 1/3*(b^4*n - 5*a*b^2*c*n + 4*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2
 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.68

$$\int x^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \frac{x^3 \ln \left(d\,(c\,x^2+b\,x+a)^n\right)}{3} - \frac{2\,n\,x^3}{9} - x\left(\frac{b^2\,n}{3\,c^2}-\frac{2\,a\,n}{3\,c}\right) - \frac{\ln \left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\left(\frac{a\,b\,n}{2}-\frac{a\,n\,\sqrt{b^2-4\,a\,c}}{6}\right)-\frac{b^3\,n}{6}+\frac{b^2\,n\,\sqrt{b^2-4\,a\,c}}{6}\right)}{c^3} + \frac{\ln \left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^3\,n}{6}-c\left(\frac{a\,b\,n}{2}+\frac{a\,n\,\sqrt{b^2-4\,a\,c}}{6}\right)+\frac{b^2\,n\,\sqrt{b^2-4\,a\,c}}{6}\right)}{c^3} + \frac{b\,n\,x^2}{6\,c}$$

```
[In] int(x^2*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] (x^3*log(d*(a + b*x + c*x^2)^n))/3 - (2*n*x^3)/9 - x*((b^2*n)/(3*c^2) - (2*
a*n)/(3*c)) - (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c
)^(1/2))*(c*((a*b*n)/2 - (a*n*(b^2 - 4*a*c)^(1/2))/6) - (b^3*n)/6 + (b^2*n*
(b^2 - 4*a*c)^(1/2))/6))/c^3 + (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2
*c*x*(b^2 - 4*a*c)^(1/2))*((b^3*n)/6 - c*((a*b*n)/2 + (a*n*(b^2 - 4*a*c)^(1
/2))/6) + (b^2*n*(b^2 - 4*a*c)^(1/2))/6))/c^3 + (b*n*x^2)/(6*c)
```

3.74 $\int x \log \left(d(a+bx+cx^2)^n\right) \, dx$

Optimal result 430
Rubi [A] (verified) 430
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Optimal result

Integrand size = 17, antiderivative size = 109

$$\int x \log \left(d(a+bx+cx^2)^n\right) \, dx = \frac{bnx}{2c} - \frac{nx^2}{2} - \frac{b\sqrt{b^2-4ac}n\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{(b^2-2ac)\,n\log(a+bx+cx^2)}{4c^2} + \frac{1}{2}x^2\log\left(d(a+bx+cx^2)^n\right)$$

```
[Out] 1/2*b*n*x/c-1/2*n*x^2-1/4*(-2*a*c+b^2)*n*ln(c*x^2+b*x+a)/c^2+1/2*x^2*ln(d*(
c*x^2+b*x+a)^n)-1/2*b*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^
(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int x \log \left(d(a+bx+cx^2)^n\right) \, dx = -\frac{bn\sqrt{b^2-4ac}\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{n(b^2-2ac)\log(a+bx+cx^2)}{4c^2} + \frac{1}{2}x^2\log\left(d(a+bx+cx^2)^n\right) + \frac{bnx}{2c} - \frac{nx^2}{2}$$

```
[In] Int[x*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(2*c^2) - ((b^2 - 2*a*c)*n*Log[a + b*x + c*x^2])/(4*c^2) +
(x^2*Log[d*(a + b*x + c*x^2)^n])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{2}n\int\frac{x^2(b+2cx)}{a+bx+cx^2}\,dx\\
&= \frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{2}n\int\left(-\frac{b}{c}+2x+\frac{ab+\left(b^2-2ac\right)x}{c\left(a+bx+cx^2\right)}\right)dx\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{ab+\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{2c}\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)\\
&\quad+\frac{\left(b\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{4c^2}-\frac{\left(\left(b^2-2ac\right)n\right)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{4c^2}\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4c^2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)\\
&\quad-\frac{\left(b\left(b^2-4ac\right)n\right)\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{2c^2}\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}\\
&\quad-\frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4c^2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int x\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx =$$

$$-\frac{2b\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+\left(b^2-2ac\right)n\log(a+x(b+cx))-2cx(n(b-cx)+cx\log\left(d(a+x(b+\right.}{4c^2}$$

```
[In] Integrate[x*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/4*(2*b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + (b^2
 - 2*a*c)*n*Log[a + x*(b + c*x)] - 2*c*x*(n*(b - c*x) + c*x*Log[d*(a + x*(b
 + c*x))^n]))/c^2
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

method	result
parts	$\frac{x^2\ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{2}-\frac{n\left(-\frac{-c\,x^2+bx}{c}+\frac{\frac{\left(-2ca+b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-2ca+b^2\right)b}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}}}{c}\right)}{2}$
risch	$\frac{x^2\ln\left(\left(c\,x^2+bx+a\right)^n\right)}{2}+\frac{i\,\mathrm{csgn}\left(id\left(c\,x^2+bx+a\right)^n\right)^2\mathrm{csgn}\left(i\left(c\,x^2+bx+a\right)^n\right)x^2\pi}{4}-\frac{i\pi\,x^2\,\mathrm{csgn}\left(i\left(c\,x^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(c\,x^2+bx+a\right)\right.}{4}$

```
[In] int(x*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(d*(c*x^2+b*x+a)^n)-1/2*n*(-1/c*(-c*x^2+b*x)+1/c*(1/2*(-2*a*c+b^2
)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2*(-2*a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.25

$$\int x\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$=\left[-\frac{2\,c^2nx^2-2\,c^2x^2\log\left(d\right)-2\,bcnx-\sqrt{b^2-4\,ac}bn\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-\left(2\,c^2nx^2-\left(\right.\right.}{4\,c^2}\right.$$

$$-\frac{2\,c^2nx^2-2\,c^2x^2\log\left(d\right)-2\,bcnx+2\,\sqrt{-b^2+4\,ac}bn\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-\left(2\,c^2nx^2-\left(b^2-2\,ac\right.\right.}{4\,c^2}$$

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x - sqrt(b^2 - 4*a*c)*b*n*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x
^2 + b*x + a)) - (2*c^2*n*x^2 - (b^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c^2,
 -1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x + 2*sqrt(-b^2 + 4*a*c)*b*
n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c^2*n*x^2 - (b
^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(102) = 204.

Time = 90.94 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.29

$$\int x \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$
$$= \begin{cases} -\frac{a^2 \log\left(d(a+bx)^n\right)}{2b^2} + \frac{anx}{2b} - \frac{nx^2}{4} + \frac{x^2 \log\left(d(a+bx)^n\right)}{2} \\ -\frac{b^2 \log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{8c^2} + \frac{bnx}{2c} - \frac{nx^2}{2} + \frac{x^2 \log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{2} \\ \frac{2abn \log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}} - \frac{ab \log\left(d(a+bx+cx^2)^n\right)}{c\sqrt{-4ac+b^2}} + \frac{a \log\left(d(a+bx+cx^2)^n\right)}{2c} - \frac{b^3 n \log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2c^2\sqrt{-4ac+b^2}} + \frac{b^3 \log\left(d(a+bx+cx\right.}{4c^2\sqrt{-4ac+b^2}} \end{cases}$$

```
[In] integrate(x*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((-a**2*log(d*(a + b*x)**n)/(2*b**2) + a*n*x/(2*b) - n*x**2/4 + x*
*2*log(d*(a + b*x)**n)/2, Eq(c, 0)), (-b**2*log(d*(b**2/(4*c) + b*x + c*x**
2)**n)/(8*c**2) + b*n*x/(2*c) - n*x**2/2 + x**2*log(d*(b**2/(4*c) + b*x + c
*x**2)**n)/2, Eq(a, b**2/(4*c))), (2*a*b*n*log(b/(2*c) + x + sqrt(-4*a*c +
b**2)/(2*c))/(c*sqrt(-4*a*c + b**2)) - a*b*log(d*(a + b*x + c*x**2)**n)/(c*
sqrt(-4*a*c + b**2)) + a*log(d*(a + b*x + c*x**2)**n)/(2*c) - b**3*n*log(b/
(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/(2*c**2*sqrt(-4*a*c + b**2)) + b**3*
log(d*(a + b*x + c*x**2)**n)/(4*c**2*sqrt(-4*a*c + b**2)) - b**2*log(d*(a +
 b*x + c*x**2)**n)/(4*c**2) + b*n*x/(2*c) - n*x**2/2 + x**2*log(d*(a + b*x
+ c*x**2)**n)/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int x \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04

$$\int x\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx=\frac{1}{2}\,nx^2\log\left(cx^2+bx+a\right)-\frac{1}{2}\,(n-\log\left(d\right))x^2+\frac{bnx}{2\,c}-\frac{\left(b^2n-2\,acn\right)\log\left(cx^2+bx+a\right)}{4\,c^2}+\frac{\left(b^3n-4\,abcn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*n*x^2*log(c*x^2 + b*x + a) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/4*(
b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/c^2 + 1/2*(b^3*n - 4*a*b*c*n)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.52

$$\int x\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx=\frac{x^2\,\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{2}-\frac{n\,x^2}{2}-\frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,n-2\,a\,c\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2}+\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(2\,a\,c\,n-b^2\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2}+\frac{b\,n\,x}{2\,c}$$

```
[In] int(x*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] (x^2*log(d*(a + b*x + c*x^2)^n))/2 - (n*x^2)/2 - (log(b*(b^2 - 4*a*c)^(1/2)
 - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^2*n - 2*a*c*n + b*n*(b^2 - 4
*a*c)^(1/2)))/(4*c^2) + (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b
^2 - 4*a*c)^(1/2))*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*c^2) + (
b*n*x)/(2*c)
```

3.75 $\int \log \left(d(a+bx+cx^2)^n\right) dx$

Optimal result 436
Rubi [A] (verified) 436
Mathematica [A] (verified) 438
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Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) dx = -2nx+\frac{\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log \left(a+bx+cx^2\right)}{2c}+x\log \left(d\left(a+bx+cx^2\right)^n\right)$$

```
[Out] -2*n*x+1/2*b*n*ln(c*x^2+b*x+a)/c+x*ln(d*(c*x^2+b*x+a)^n)+n*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2603, 787, 648, 632, 212, 642}

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) dx = \frac{n\sqrt{b^2-4ac}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log \left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log \left(a+bx+cx^2\right)}{2c}-2nx$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int\frac{x(b+2cx)}{a+bx+cx^2}\,dx\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{-2ac-bcx}{a+bx+cx^2}\,dx}{c}\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{(bn)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2c}-\frac{\left(\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{2c}
\end{aligned}
$$

$$= -2nx + \frac{bn\log\left(a+bx+cx^2\right)}{2c} + x\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(\left(b^2-4ac\right)n\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}$$

$$= -2nx + \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} + \frac{bn\log\left(a+bx+cx^2\right)}{2c} + x\log\left(d\left(a+bx+cx^2\right)^n\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{2\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) + bn\log(a+x(b+cx)) + 2cx(-2n+\log\left(d(a+x(b+cx))^n\right))}{2c}$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a +
 x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.13

method	result
default	$x\ln\left(d\left(c\,x^2+bx+a\right)^n\right)-n\left(2x-\frac{b\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-2a+\frac{b^2}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}}\right)$
parts	$x\ln\left(d\left(c\,x^2+bx+a\right)^n\right)-n\left(2x-\frac{b\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-2a+\frac{b^2}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}}\right)$
risch	$x\ln\left(\left(c\,x^2+bx+a\right)^n\right)+\frac{i\text{csgn}\left(id\left(c\,x^2+bx+a\right)^n\right)^2\text{csgn}\left(i\left(c\,x^2+bx+a\right)^n\right)x\pi}{2}-\frac{i\pi x\ \text{csgn}\left(i\left(c\,x^2+bx+a\right)^n\right)\text{csgn}\left(id\left(c\,x^2+bx\right.\right.}{2}$

```
[In] int(ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(d*(c*x^2+b*x+a)^n)-n*(2*x-1/2*b/c*ln(c*x^2+b*x+a)+2*(-2*a+1/2*b^2/c)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.41

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\left[-\frac{4\,cnx-2\,cx\log\left(d\right)-\sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(2\,cnx+bn)\log\left(cx^2+bx\right.}{2\,c}\right.$$
$$\left.-\frac{4\,cnx-2\,cx\log\left(d\right)-2\,\sqrt{-b^2+4\,ac}n\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-(2\,cnx+bn)\log\left(cx^2+bx+a\right)}{2\,c}\right]$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*c*
n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqrt(-
b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c
*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(75) = 150$.

Time = 34.76 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.47

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\begin{cases}\frac{a\log\left(d(a+bx)^n\right)}{b}-nx+x\log\left(d(a+bx)^n\right)\\ \frac{b\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{2c}-2nx+x\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)\\ -\frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}}+\frac{2a\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{-4ac+b^2}}+\frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}}-\frac{b^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2c\sqrt{-4ac+b^2}}+\frac{b\log\left(d(a+bx\right.}{2c}\end{cases}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((a*log(d*(a + b*x)**n)/b - n*x + x*log(d*(a + b*x)**n), Eq(c, 0))
, (b*log(d*(b**2/(4*c) + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(b**2/(4
*c) + b*x + c*x**2)**n), Eq(a, b**2/(4*c))), (-4*a*n*log(b/(2*c) + x + sqrt
(-4*a*c + b**2)/(2*c))/sqrt(-4*a*c + b**2) + 2*a*log(d*(a + b*x + c*x**2)**
n)/sqrt(-4*a*c + b**2) + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)
)/(c*sqrt(-4*a*c + b**2)) - b**2*log(d*(a + b*x + c*x**2)**n)/(2*c*sqrt(-4*
a*c + b**2)) + b*log(d*(a + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(a +
b*x + c*x**2)**n), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Exception raised: ValueError}$$

[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = nx\log \left(cx^2+bx+a\right) - (2\,n - \log \left(d\right))x + \frac{bn\log \left(cx^2+bx+a\right)}{2\,c} - \frac{\left(b^2n - 4\,acn\right)\arctan \left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}$$

[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")

[Out] n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)/
c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c)

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.52

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = x\,\ln \left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right) - 2\,n\,x - \frac{n\,\mathrm{atan}\left(\frac{b\,n\,\sqrt{4\,a\,c-b^2}}{2\,\left(\frac{b^2\,n}{2}-2\,a\,c\,n\right)} - \frac{n\,x\,\sqrt{4\,a\,c-b^2}}{2\,a\,n-\frac{b^2\,n}{2\,c}}\right)\,\sqrt{4\,a\,c-b^2}}{c} + \frac{b\,n\,\ln \left(c\,x^2+b\,x+a\right)}{2\,c}$$

[In] int(log(d*(a + b*x + c*x^2)^n),x)

[Out] x*log(d*(a + b*x + c*x^2)^n) - 2*n*x - (n*atan((b*n*(4*a*c - b^2)^(1/2))/(2
*((b^2*n)/2 - 2*a*c*n)) - (n*x*(4*a*c - b^2)^(1/2))/(2*a*n - (b^2*n)/(2*c))
)*(4*a*c - b^2)^(1/2))/c + (b*n*log(a + b*x + c*x^2))/(2*c)

3.76 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx$

Optimal result . . . 441
Rubi [A] (verified) . . . 441
Mathematica [A] (verified) . . . 443
Maple [A] (verified) . . . 444
Fricas [F] . . . 444
Sympy [F] . . . 444
Maxima [F] . . . 445
Giac [F] . . . 445
Mupad [F(-1)] . . . 445

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx = -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right) + \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right) - n\,\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\,\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] ln(x)*ln(d*(c*x^2+b*x+a)^n)-n*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))-n*ln
(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))-n*polylog(2,-2*c*x/(b-(-4*a*c+b^2)^(
1/2)))-n*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used

= {2604, 2404, 2354, 2438}

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}\,dx = -n\,\text{PolyLog}\left(2, -\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\,\text{PolyLog}\left(2, -\frac{2cx}{b+\sqrt{b^2-4ac}}\right) - n\log(x)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right) - n\log(x)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right) + \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

```
[Out] -(n*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]) - n*Log[x]*Log[1 + (2*
c*x)/(b + Sqrt[b^2 - 4*a*c])] + Log[x]*Log[d*(a + b*x + c*x^2)^n] - n*PolyL
og[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])] - n*PolyLog[2, (-2*c*x)/(b + Sqrt[b
^2 - 4*a*c])]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int\frac{(b+2cx)\log(x)}{a+bx+cx^2}\,dx\\
&= \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int\left(\frac{2c\log(x)}{b-\sqrt{b^2-4ac}+2cx}+\frac{2c\log(x)}{b+\sqrt{b^2-4ac}+2cx}\right)dx\\
&= \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)-(2cn)\int\frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx\\
&\quad-(2cn)\int\frac{\log(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx\\
&= -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)+\log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)\\
&\quad+n\int\frac{\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{x}\,dx+n\int\frac{\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{x}\,dx\\
&= -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)+\log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)\\
&\quad-n\,\mathrm{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\,\mathrm{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.16

$$\int\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx=\log(x)\log\left(d(a+x(b+cx))^n\right)-n\left(\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)+\log(x)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)+\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)+\text{PolyLog}\left(2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)\right)$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

```
[Out] Log[x]*Log[d*(a + x*(b + c*x))^n] - n*(Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] +
2*c*x)/(b - Sqrt[b^2 - 4*a*c])] + Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
)/(b + Sqrt[b^2 - 4*a*c])] + PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])] +
 PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.29

method	result
parts	$\ln(x)\ln\left(d(c\,x^2+bx+a)^n\right)-n\Big(\ln(x)\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)+\ln(x)\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)+\text{dilog}$
risch	$\ln\left((c\,x^2+bx+a)^n\right)\ln(x)-\ln\left(\frac{-2xc+\sqrt{-4ca+b^2}-b}{-b+\sqrt{-4ca+b^2}}\right)\ln(x)\,n-\ln\left(\frac{2xc+\sqrt{-4ca+b^2}+b}{b+\sqrt{-4ca+b^2}}\right)\ln(x)\,n-\text{dilog}$

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*ln(d*(c*x^2+b*x+a)^n)-n*(ln(x)*ln((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(
-4*a*c+b^2)^(1/2)))+ln(x)*ln((2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(
1/2)))+dilog((-2*x*c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))+dilog((
2*x*c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))))
```

Fricas [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}\,dx=\int \frac{\log\left((cx^2+bx+a)^n d\right)}{x}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}\,dx=\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}\,dx$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x,x)
```

```
[Out] Integral(log(d*(a + b*x + c*x**2)**n)/x, x)
```

Maxima [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}\,dx = \int \frac{\log\left((cx^2+bx+a)^n d\right)}{x}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="maxima")
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Giac [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}\,dx = \int \frac{\log\left((cx^2+bx+a)^n d\right)}{x}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}\,dx = \int \frac{\ln\left(d\,(c\,x^2+b\,x+a)^n\right)}{x}\,dx$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x,x)
[Out] int(log(d*(a + b*x + c*x^2)^n)/x, x)
```

3.77 $\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx$

Optimal result 446
Rubi [A] (verified) 446
Mathematica [A] (verified) 448
Maple [A] (verified) 448
Fricas [A] (verification not implemented) 449
Sympy [B] (verification not implemented) 449
Maxima [F(-2)] 450
Giac [A] (verification not implemented) 450
Mupad [B] (verification not implemented) 450

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \frac{\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}+\frac{bn\log(x)}{a}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{x}$$

```
[Out] b*n*ln(x)/a-1/2*b*n*ln(c*x^2+b*x+a)/a-ln(d*(c*x^2+b*x+a)^n)/x+n*arctanh((2*
c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \frac{n\sqrt{b^2-4ac}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{x}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}+\frac{bn\log(x)}{a}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/a + (b*n*Log[x
])/a - (b*n*Log[a + b*x + c*x^2])/(2*a) - Log[d*(a + b*x + c*x^2)^n]/x
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(d(a+bx+cx^2)^n\right)}{x} + n\int \frac{b+2cx}{x\,(a+bx+cx^2)}\,dx$$

$$= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{x} + n\int \left(\frac{b}{ax} + \frac{-b^2+2ac-bcx}{a\,(a+bx+cx^2)}\right)\,dx$$

$$
\begin{aligned}
&= \frac{bn\log(x)}{a} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{x} + \frac{n\int \frac{-b^2+2ac-bcx}{a+bx+cx^2}\,dx}{a} \\
&= \frac{bn\log(x)}{a} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{x} - \frac{(bn)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a} - \frac{((b^2-4ac)\,n)\int \frac{1}{a+bx+cx^2}\,dx}{2a} \\
&= \frac{bn\log(x)}{a} - \frac{bn\log\left(a+bx+cx^2\right)}{2a} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{x} \\
&\quad + \frac{((b^2-4ac)\,n)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a} \\
&= \frac{\sqrt{b^2-4ac}n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a} + \frac{bn\log(x)}{a} - \frac{bn\log\left(a+bx+cx^2\right)}{2a} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$
\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx
= \frac{2\sqrt{-b^2+4ac}n\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right) + 2bn\log(x) - bn\log(a+x(b+cx)) - \frac{2a\log(d(a+x(b+cx))^n)}{x}}{2a}
$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

```
[Out] (2*Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*b*n*Log[
x] - b*n*Log[a + x*(b + c*x)] - (2*a*Log[d*(a + x*(b + c*x))^n])/x)/(2*a)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

method	result
parts	$-\frac{\ln\left(d(c\,x^2+bx+a)^n\right)}{x}+n\left(\frac{b\ln(x)}{a}+\frac{-\frac{b\ln\left(c\,x^2+bx+a\right)}{2}+\frac{2\left(2ca-\frac{b^2}{2}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}}}{a}\right)$
risch	$-\frac{\ln\left((c\,x^2+bx+a)^n\right)}{x}-\frac{i\pi a\,\text{csgn}\left(i(c\,x^2+bx+a)^n\right)\text{csgn}\left(id(c\,x^2+bx+a)^n\right)^2-i\pi a\,\text{csgn}\left(i(c\,x^2+bx+a)^n\right)\text{csgn}\left(id(c\,x^2+bx+a)^n\right)\text{csgn}}{}$

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(d*(c*x^2+b*x+a)^n)/x+n*(1/a*b*ln(x)+1/a*(-1/2*b*ln(c*x^2+b*x+a)+2*(2*c*
a-1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.31

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^2} \, dx$$
$$= \left[\frac{2\,bnx\log\left(x\right)+\sqrt{b^2-4\,ac}nx\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(bnx+2\,an)\log\left(cx^2+bx+a\right)-\right.$$

[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="fricas")

[Out] [1/2*(2*b*n*x*log(x) + sqrt(b^2 - 4*a*c)*n*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b*n*x + 2*a*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x), 1/2*(2*b*n*x*log(x) + 2*sqrt(-b^2 + 4*a*c)*n*x*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b*n*x + 2*a*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(78) = 156$.

Time = 152.53 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.45

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^2} \, dx$$
$$= \begin{cases} -\frac{n}{x}-\frac{\log\left(d(bx)^n\right)}{x} \\ -\frac{n}{x}-\frac{\log\left(d(bx+cx^2)^n\right)}{x}-\frac{2cn\log\left(b+cx\right)}{b}+\frac{c\log\left(d(bx+cx^2)^n\right)}{b} \\ -\frac{\log\left(d(a+bx)^n\right)}{x}+\frac{bn\log\left(x\right)}{a}-\frac{b\log\left(d(a+bx)^n\right)}{a} \\ -\frac{\log\left(d(a+bx+cx^2)^n\right)}{x}+\frac{bn\log\left(x\right)}{a}-\frac{b\log\left(d(a+bx+cx^2)^n\right)}{2a}+\frac{n\sqrt{-4ac+b^2}\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{a}-\frac{\sqrt{-4ac+b^2}\log\left(d(a+bx+cx^2\right.}{2a} \end{cases}$$

[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**2,x)

[Out] Piecewise((-n/x - log(d*(b*x)**n)/x, Eq(a, 0) & Eq(c, 0)), (-n/x - log(d*(b*x + c*x**2)**n)/x - 2*c*n*log(b + c*x)/b + c*log(d*(b*x + c*x**2)**n)/b, Eq(a, 0)), (-log(d*(a + b*x)**n)/x + b*n*log(x)/a - b*log(d*(a + b*x)**n)/a, Eq(c, 0)), (-log(d*(a + b*x + c*x**2)**n)/x + b*n*log(x)/a - b*log(d*(a + b*x + c*x**2)**n)/(2*a) + n*sqrt(-4*a*c + b**2)*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/a - sqrt(-4*a*c + b**2)*log(d*(a + b*x + c*x**2)**n)/(2*a), True))

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = -\frac{bn\log \left(cx^2+bx+a\right)}{2\,a} + \frac{bn\log \left(x\right)}{a} - \frac{n\log \left(cx^2+bx+a\right)}{x} - \frac{\left(b^2n-4\,acn\right)\arctan \left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\log \left(d\right)}{x}$$

[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="giac")

[Out] -1/2*b*n*log(c*x^2 + b*x + a)/a + b*n*log(x)/a - n*log(c*x^2 + b*x + a)/x -
 (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a) - log(d)/x

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.05

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^2}\,dx = \frac{b\,n\,\ln \left(x\right)}{a} - \frac{\ln \left(2\,b\,c^2\,n^2+4\,c^3\,n^2\,x-\frac{n\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,c\,n-2\,a\,c^2\,n+b\,c^2\,n\,x+\frac{c\,n\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,\left(2\,x\,b^2+a\,b-6\,a\,c\,x\right)}{2\,a}\right)}{2\,a}\right)\,\left(b\,n-n\,\sqrt{b^2-\right.}{2\,a} - \frac{\ln \left(2\,b\,c^2\,n^2+4\,c^3\,n^2\,x-\frac{n\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,c\,n-2\,a\,c^2\,n+b\,c^2\,n\,x+\frac{c\,n\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\left(2\,x\,b^2+a\,b-6\,a\,c\,x\right)}{2\,a}\right)}{2\,a}\right)\,\left(b\,n+n\,\sqrt{b^2-\right.}{2\,a} - \frac{\ln \left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{x}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^2,x)
```

```
[Out] (b*n*log(x))/a - (log(2*b*c^2*n^2 + 4*c^3*n^2*x - (n*(b - (b^2 - 4*a*c)^(1/
2))*(b^2*c*n - 2*a*c^2*n + b*c^2*n*x + (c*n*(b - (b^2 - 4*a*c)^(1/2))*(a*b
+ 2*b^2*x - 6*a*c*x))/(2*a)))/(2*a))*(b*n - n*(b^2 - 4*a*c)^(1/2)))/(2*a) -
 (log(2*b*c^2*n^2 + 4*c^3*n^2*x - (n*(b + (b^2 - 4*a*c)^(1/2))*(b^2*c*n - 2
*a*c^2*n + b*c^2*n*x + (c*n*(b + (b^2 - 4*a*c)^(1/2))*(a*b + 2*b^2*x - 6*a*
c*x))/(2*a)))/(2*a))*(b*n + n*(b^2 - 4*a*c)^(1/2)))/(2*a) - log(d*(a + b*x
+ c*x^2)^n)/x
```

3.78 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx = -\frac{bn}{2ax}-\frac{b\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2}-\frac{(b^2-2ac)\,n\log(x)}{2a^2}+\frac{(b^2-2ac)\,n\log\left(a+bx+cx^2\right)}{4a^2}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}$$

```
[Out] -1/2*b*n/a/x-1/2*(-2*a*c+b^2)*n*ln(x)/a^2+1/4*(-2*a*c+b^2)*n*ln(c*x^2+b*x+a
)/a^2-1/2*ln(d*(c*x^2+b*x+a)^n)/x^2-1/2*b*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))*(-4*a*c+b^2)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx = -\frac{bn\sqrt{b^2-4ac}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2}+\frac{n(b^2-2ac)\log\left(a+bx+cx^2\right)}{4a^2}-\frac{n\log(x)\left(b^2-2ac\right)}{2a^2}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}-\frac{bn}{2ax}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -1/2*(b*n)/(a*x) - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(2*a^2) - ((b^2 - 2*a*c)*n*Log[x])/(2*a^2) + ((b^2 - 2*a*c)*n*Log[a
+ b*x + c*x^2])/(4*a^2) - Log[d*(a + b*x + c*x^2)^n]/(2*x^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2} + \frac{1}{2}n\int \frac{b+2cx}{x^2\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2} + \frac{1}{2}n\int \left(\frac{b}{ax^2} + \frac{-b^2+2ac}{a^2x} + \frac{b(b^2-3ac)+c(b^2-2ac)\,x}{a^2\left(a+bx+cx^2\right)}\right)\,dx$$

$$= -\frac{bn}{2ax} - \frac{(b^2-2ac)\,n\log(x)}{2a^2} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2} + \frac{n\int \frac{b(b^2-3ac)+c(b^2-2ac)x}{a+bx+cx^2}\,dx}{2a^2}$$

$$= -\frac{bn}{2ax} - \frac{(b^2-2ac)\,n\log(x)}{2a^2} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2} + \frac{(b(b^2-4ac)\,n)\int \frac{1}{a+bx+cx^2}\,dx}{4a^2} + \frac{((b^2-2ac)\,n)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{4a^2}$$

$$= -\frac{bn}{2ax} - \frac{(b^2-2ac)\,n\log(x)}{2a^2} + \frac{(b^2-2ac)\,n\log\left(a+bx+cx^2\right)}{4a^2} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2} - \frac{(b(b^2-4ac)\,n)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{2a^2}$$

$$= -\frac{bn}{2ax} - \frac{b\sqrt{b^2-4ac}n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2} - \frac{(b^2-2ac)\,n\log(x)}{2a^2} + \frac{(b^2-2ac)\,n\log\left(a+bx+cx^2\right)}{4a^2} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^3}\,dx = -\frac{\frac{nx\left(2ab+2b\sqrt{b^2-4ac}x\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2(b^2-2ac)x\log(x)-(b^2-2ac)x\log(a+x(b+cx))\right)}{a^2}+2\log\left(d(a+x(b+cx))^n\right)}{4x^2}$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -1/4*((n*x*(2*a*b + 2*b*Sqrt[b^2 - 4*a*c]*x*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]] + 2*(b^2 - 2*a*c)*x*Log[x] - (b^2 - 2*a*c)*x*Log[a + x*(b + c*x)]))
/a^2 + 2*Log[d*(a + x*(b + c*x))^n])/x^2
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.20

method	result
parts	$-\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)}{2x^{2}}+\frac{n\left(-\frac{b}{ax}+\frac{\left(2ca-b^{2}\right)\ln\left(x\right)}{a^{2}}+\frac{\frac{\left(-2a\,c^{2}+b^{2}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-3abc+b^{3}-\frac{\left(-2a\,c^{2}+b^{2}c\right)b}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^{2}}}\right)}{\sqrt{4ca-b^{2}}}}{a^{2}}\right)}{2}$
risch	Expression too large to display

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(d*(c*x^2+b*x+a)^n)/x^2+1/2*n*(-b/a/x+(2*a*c-b^2)/a^2*ln(x)+1/a^2*(1
/2*(-2*a*c^2+b^2*c)/c*ln(c*x^2+b*x+a)+2*(-3*a*b*c+b^3-1/2*(-2*a*c^2+b^2*c)*
b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.16

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^3}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}bnx^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^2-2\,ac)nx^2\log\left(x\right)-2\,abnx-2\,a^2\log\left(d\right)}{4\,a^2x^2}\right.$$
$$-\frac{2\,\sqrt{-b^2+4\,ac}bnx^2\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2-2\,ac)nx^2\log\left(x\right)+2\,abnx+2\,a^2\log\left(d\right)-((b^2}{4\,a^2x^2}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*n*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sq
rt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2 - 2*a*c)*n*x^2*log
(x) - 2*a*b*n*x - 2*a^2*log(d) + ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x^2
+ b*x + a))/(a^2*x^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*n*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2 - 2*a*c)*n*x^2*log(x) + 2*a*b
*n*x + 2*a^2*log(d) - ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x^2 + b*x + a))
/(a^2*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^3}\,dx = \frac{(b^2n-2\,acn)\log\left(cx^2+bx+a\right)}{4\,a^2} - \frac{n\log\left(cx^2+bx+a\right)}{2\,x^2} - \frac{(b^2n-2\,acn)\log(x)}{2\,a^2} + \frac{(b^3n-4\,abcn)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2} - \frac{bnx+a\log(d)}{2\,ax^2}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="giac")
[Out] 1/4*(b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/a^2 - 1/2*n*log(c*x^2 + b*x + a)
/x^2 - 1/2*(b^2*n - 2*a*c*n)*log(x)/a^2 + 1/2*(b^3*n - 4*a*b*c*n)*arctan((2
*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(b*n*x + a*log
(d))/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.92

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^3}\,dx$$

$$= \frac{\ln\left(\frac{b^3\,c^2\,n^2-2\,a\,b\,c^3\,n^2}{4\,a^2}+\frac{\left(b^2\,n-2\,a\,c\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{\left(\frac{x\,\left(24\,a^3\,c^2-8\,a^2\,b^2\,c\right)}{4\,a^2}-a\,b\,c\right)\left(b^2\,n-2\,a\,c\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)}{4\,a^2}-\frac{2\,a\,b^3\,c\,n-6\,a^2\,b\,c^2\,n}{4\,a^2}+\frac{x\,\cdots}{\cdots}\right.}{4\,a^2}\cdots\right.}{4\,a^2}\cdots$$

$$-\frac{\ln\left(x\right)\,\left(b^2\,n-2\,a\,c\,n\right)}{2\,a^2}-\frac{\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{2\,x^2}$$

$$-\frac{\ln\left(\frac{b^3\,c^2\,n^2-2\,a\,b\,c^3\,n^2}{4\,a^2}+\frac{\left(2\,a\,c\,n-b^2\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{2\,a\,b^3\,c\,n-6\,a^2\,b\,c^2\,n}{4\,a^2}+\frac{\left(\frac{x\,\left(24\,a^3\,c^2-8\,a^2\,b^2\,c\right)}{4\,a^2}-a\,b\,c\right)\left(2\,a\,c\,n-b^2\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)}{4\,a^2}-\cdots\right.}{4\,a^2}\cdots\right.}{4\,a^2}\cdots$$

$$-\frac{b\,n}{2\,a\,x}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^3,x)
```

```
[Out] (log((b^3*c^2*n^2 - 2*a*b*c^3*n^2)/(4*a^2) + ((b^2*n - 2*a*c*n + b*n*(b^2 -
 4*a*c)^(1/2))*((((x*(24*a^3*c^2 - 8*a^2*b^2*c))/(4*a^2) - a*b*c)*(b^2*n -
2*a*c*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (2*a*b^3*c*n - 6*a^2*b*c^2*n)
/(4*a^2) + (x*(12*a^2*c^3*n - 4*a*b^2*c^2*n))/(4*a^2)))/(4*a^2) + (b^2*c^3*
n^2*x)/(4*a^2))*(b^2*n - 2*a*c*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (log
(x)*(b^2*n - 2*a*c*n))/(2*a^2) - log(d*(a + b*x + c*x^2)^n)/(2*x^2) - (log(
(b^3*c^2*n^2 - 2*a*b*c^3*n^2)/(4*a^2) + ((2*a*c*n - b^2*n + b*n*(b^2 - 4*a*
c)^(1/2))*((2*a*b^3*c*n - 6*a^2*b*c^2*n)/(4*a^2) + (((x*(24*a^3*c^2 - 8*a^2
*b^2*c))/(4*a^2) - a*b*c)*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a
^2) - (x*(12*a^2*c^3*n - 4*a*b^2*c^2*n))/(4*a^2)))/(4*a^2) + (b^2*c^3*n^2*x
)/(4*a^2))*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (b*n)/(2*
a*x)
```

3.79 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^4}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 149

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^4}\,dx = -\frac{bn}{6ax^2}+\frac{\left(b^2-2ac\right)n}{3a^2x} + \frac{\sqrt{b^2-4ac}\left(b^2-ac\right)n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3} + \frac{b\left(b^2-3ac\right)n\log(x)}{3a^3}-\frac{b\left(b^2-3ac\right)n\log\left(a+bx+cx^2\right)}{6a^3} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}$$

```
[Out] -1/6*b*n/a/x^2+1/3*(-2*a*c+b^2)*n/a^2/x+1/3*b*(-3*a*c+b^2)*n*ln(x)/a^3-1/6*
b*(-3*a*c+b^2)*n*ln(c*x^2+b*x+a)/a^3-1/3*ln(d*(c*x^2+b*x+a)^n)/x^3+1/3*(-a*
c+b^2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^4}\,dx = \frac{n\sqrt{b^2-4ac}\left(b^2-ac\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3} - \frac{bn\left(b^2-3ac\right)\log\left(a+bx+cx^2\right)}{6a^3}+\frac{bn\log(x)\left(b^2-3ac\right)}{3a^3} + \frac{n\left(b^2-2ac\right)}{3a^2x}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}-\frac{bn}{6ax^2}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -1/6*(b*n)/(a*x^2) + ((b^2 - 2*a*c)*n)/(3*a^2*x) + (Sqrt[b^2 - 4*a*c]*(b^2
- a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*a^3) + (b*(b^2 - 3*a*c)
*n*Log[x])/(3*a^3) - (b*(b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*a^3) - Log
[d*(a + b*x + c*x^2)^n]/(3*x^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}+\frac{1}{3}n\int\frac{b+2cx}{x^3\left(a+bx+cx^2\right)}\,dx\\
&= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}+\frac{1}{3}n\int\left(\frac{b}{ax^3}+\frac{-b^2+2ac}{a^2x^2}+\frac{b^3-3abc}{a^3x}+\frac{-b^4+4ab^2c-2a^2c^2-bc(b^2-3ac)\,x}{a^3\left(a+bx+cx^2\right)}\right)dx\\
&= -\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x}+\frac{b(b^2-3ac)\,n\log(x)}{3a^3}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}+\frac{n\int\frac{-b^4+4ab^2c-2a^2c^2-bc(b^2-3ac)x}{a+bx+cx^2}\,dx}{3a^3}\\
&= -\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x}+\frac{b(b^2-3ac)\,n\log(x)}{3a^3}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}-\frac{(b(b^2-3ac)\,n)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{6a^3}-\frac{((b^2-4ac)\,(b^2-ac)\,n)\int\frac{1}{a+bx+cx^2}\,dx}{6a^3}\\
&= -\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x}+\frac{b(b^2-3ac)\,n\log(x)}{3a^3}-\frac{b(b^2-3ac)\,n\log\left(a+bx+cx^2\right)}{6a^3}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}+\frac{((b^2-4ac)\,(b^2-ac)\,n)\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{3a^3}\\
&= -\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x}+\frac{\sqrt{b^2-4ac}(b^2-ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3}+\frac{b(b^2-3ac)\,n\log(x)}{3a^3}-\frac{b(b^2-3ac)\,n\log\left(a+bx+cx^2\right)}{6a^3}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$
\int\frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx = -\frac{\frac{nx\left(a^2b-2a(b^2-2ac)x-2\sqrt{b^2-4ac}(b^2-ac)x^2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2b(b^2-3ac)x^2\log(x)+b(b^2-3ac)x^2\log(a+x(b+cx))\right)}{a^3}+2\log\left(d(a
$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -1/6*((n*x*(a^2*b - 2*a*(b^2 - 2*a*c)*x - 2*Sqrt[b^2 - 4*a*c]*(b^2 - a*c)*x
^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*b*(b^2 - 3*a*c)*x^2*Log[x] +
b*(b^2 - 3*a*c)*x^2*Log[a + x*(b + c*x)]))/a^3 + 2*Log[d*(a + x*(b + c*x))^
n])/x^3
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.21

method	result
parts	$-\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)}{3x^{3}}+\frac{n\left(-\frac{b}{2a\,x^{2}}-\frac{2ca-b^{2}}{a^{2}x}-\frac{b\left(3ca-b^{2}\right)\ln\left(x\right)}{a^{3}}+\frac{\frac{\left(3ab\,c^{2}-c\,b^{3}\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-2c^{2}a^{2}+4a\,b^{2}c-b^{4}-\frac{\left(3ab\,c^{2}-c\,b^{3}\right)b}{2c}\right.}{\sqrt{4ca-b^{2}}}}{a^{3}}\right.}{3}$
risch	$-\frac{\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)}{3x^{3}}-\frac{i\pi\,a^{3}\,\mathrm{csgn}\left(i\left(c\,x^{2}+bx+a\right)^{n}\right)\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)^{2}-i\pi\,a^{3}\,\mathrm{csgn}\left(i\left(c\,x^{2}+bx+a\right)^{n}\right)\,\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)}{}$

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(d*(c*x^2+b*x+a)^n)/x^3+1/3*n*(-1/2*b/a/x^2-(2*a*c-b^2)/a^2/x-b*(3*a
*c-b^2)/a^3*ln(x)+1/a^3*(1/2*(3*a*b*c^2-b^3*c)/c*ln(c*x^2+b*x+a)+2*(-2*c^2*
a^2+4*a*b^2*c-b^4-1/2*(3*a*b*c^2-b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.13

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx$$
$$=\left[-\frac{(b^2-ac)\sqrt{b^2-4\,ac}nx^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3-3\,abc)nx^3\log\left(x\right)+a^2bnx-2}{6\,a^3x^3}\right.$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - a*c)*sqrt(b^2 - 4*a*c)*n*x^3*log((2*c^2*x^2 + 2*b*c*x + b^2 -
 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^3 - 3*a*b
*c)*n*x^3*log(x) + a^2*b*n*x - 2*(a*b^2 - 2*a^2*c)*n*x^2 + 2*a^3*log(d) + (
(b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c*x^2 + b*x + a))/(a^3*x^3), 1/6*(2*(b
^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*x^3*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) + 2*(b^3 - 3*a*b*c)*n*x^3*log(x) - a^2*b*n*x + 2*(a*b^2 - 2*a
^2*c)*n*x^2 - 2*a^3*log(d) - ((b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c*x^2 +
b*x + a))/(a^3*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**4,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.10

$$\begin{aligned}\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx = & -\frac{(b^3n-3\,abcn)\log\left(cx^2+bx+a\right)}{6\,a^3} \\ & -\frac{n\log\left(cx^2+bx+a\right)}{3\,x^3}+\frac{(b^3n-3\,abcn)\log\left(x\right)}{3\,a^3} \\ & -\frac{(b^4n-5\,ab^2cn+4\,a^2c^2n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a^3} \\ & +\frac{2\,b^2nx^2-4\,acnx^2-abnx-2\,a^2\log\left(d\right)}{6\,a^2x^3}\end{aligned}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="giac")
[Out] -1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x + a)/a^3 - 1/3*n*log(c*x^2 + b*x +
 a)/x^3 + 1/3*(b^3*n - 3*a*b*c*n)*log(x)/a^3 - 1/3*(b^4*n - 5*a*b^2*c*n + 4
*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3)
 + 1/6*(2*b^2*n*x^2 - 4*a*c*n*x^2 - a*b*n*x - 2*a^2*log(d))/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.39

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx$$

$$= \frac{\ln\left(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+13\,a^2\,b^3\,c-20\,a^3\,b\,c^2+4\,a^3\,c^3\,x+2\,a^3\,c^2\,\sqrt{b^2-\cdots}\right.}{\cdots}$$

$$-\frac{\ln\left(d\,(c\,x^2+b\,x+a)^n\right)}{3\,x^3}-\frac{\frac{b\,n}{2\,a}+\frac{n\,x\,(2\,a\,c-b^2)}{a^2}}{3\,x^2}$$

$$-\frac{\ln\left(2\,a\,b^5+2\,b^6\,x+2\,a\,b^4\,\sqrt{b^2-4\,a\,c}+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}-13\,a^2\,b^3\,c+20\,a^3\,b\,c^2-4\,a^3\,c^3\,x+2\,a^3\,c^2\,\sqrt{b\cdots}\right.}{\cdots}$$

$$+\frac{\ln\left(x\right)\,\left(b^3\,n-3\,a\,b\,c\,n\right)}{3\,a^3}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^4,x)
```

```
[Out] (log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c
)^(1/2) + 13*a^2*b^3*c - 20*a^3*b*c^2 + 4*a^3*c^3*x + 2*a^3*c^2*(b^2 - 4*a*
c)^(1/2) - 25*a^2*b^2*c^2*x + 14*a*b^4*c*x - 7*a^2*b^2*c*(b^2 - 4*a*c)^(1/2
) - 10*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 11*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*
(a*((b*c*n)/2 - (c*n*(b^2 - 4*a*c)^(1/2))/6) - (b^3*n)/6 + (b^2*n*(b^2 - 4*
a*c)^(1/2))/6))/a^3 - log(d*(a + b*x + c*x^2)^n)/(3*x^3) - ((b*n)/(2*a) + (
n*x*(2*a*c - b^2))/a^2)/(3*x^2) - (log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 - 4
*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c + 20*a^3*b*c^2 - 4
*a^3*c^3*x + 2*a^3*c^2*(b^2 - 4*a*c)^(1/2) + 25*a^2*b^2*c^2*x - 14*a*b^4*c*
x - 7*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 11
*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*((b^3*n)/6 - a*((b*c*n)/2 + (c*n*(b^2 - 4
*a*c)^(1/2))/6) + (b^2*n*(b^2 - 4*a*c)^(1/2))/6))/a^3 + (log(x)*(b^3*n - 3*
a*b*c*n))/(3*a^3)
```

3.80 $\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx$

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Mathematica [A] (verified) 467
Maple [A] (verified) 467
Fricas [A] (verification not implemented) 468
Sympy [F(-1)] 468
Maxima [F(-2)] 469
Giac [A] (verification not implemented) 469
Mupad [B] (verification not implemented) 470

Optimal result

Integrand size = 19, antiderivative size = 190

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx = -\frac{bn}{12ax^3}+\frac{(b^2-2ac)\,n}{8a^2x^2}-\frac{b(b^2-3ac)\,n}{4a^3x} - \frac{b\sqrt{b^2-4ac}(b^2-2ac)\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4a^4} - \frac{(b^4-4ab^2c+2a^2c^2)\,n\log(x)}{4a^4} + \frac{(b^4-4ab^2c+2a^2c^2)\,n\log\left(a+bx+cx^2\right)}{8a^4} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}$$

```
[Out] -1/12*b*n/a/x^3+1/8*(-2*a*c+b^2)*n/a^2/x^2-1/4*b*(-3*a*c+b^2)*n/a^3/x-1/4*(
2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(x)/a^4+1/8*(2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(c*x
^2+b*x+a)/a^4-1/4*ln(d*(c*x^2+b*x+a)^n)/x^4-1/4*b*(-2*a*c+b^2)*n*arctanh((2
*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

= {2605, 814, 648, 632, 212, 642}

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx = -\frac{bn\sqrt{b^2-4ac}(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4a^4} - \frac{bn(b^2-3ac)}{4a^3x} + \frac{n(b^2-2ac)}{8a^2x^2} + \frac{n(2a^2c^2-4ab^2c+b^4)\log\left(a+bx+cx^2\right)}{8a^4} - \frac{n\log(x)\left(2a^2c^2-4ab^2c+b^4\right)}{4a^4} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4} - \frac{bn}{12ax^3}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -1/12*(b*n)/(a*x^3) + ((b^2 - 2*a*c)*n)/(8*a^2*x^2) - (b*(b^2 - 3*a*c)*n)/(
4*a^3*x) - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(4*a^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[x])/(4*a^4) + (
(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*x + c*x^2])/(8*a^4) - Log[d*(a +
b*x + c*x^2)^n]/(4*x^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}+\frac{1}{4}n\int\frac{b+2cx}{x^4\left(a+bx+cx^2\right)}\,dx\\
&= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}+\frac{1}{4}n\int\left(\frac{b}{ax^4}+\frac{-b^2+2ac}{a^2x^3}+\frac{b^3-3abc}{a^3x^2}\right.\\
&\quad\left.+\frac{-b^4+4ab^2c-2a^2c^2}{a^4x}+\frac{b(b^4-5ab^2c+5a^2c^2)+c(b^4-4ab^2c+2a^2c^2)\,x}{a^4\left(a+bx+cx^2\right)}\right)dx\\
&= -\frac{bn}{12ax^3}+\frac{(b^2-2ac)\,n}{8a^2x^2}-\frac{b(b^2-3ac)\,n}{4a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\,n\log(x)}{4a^4}\\
&\quad-\frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}+\frac{n\int\frac{b(b^4-5ab^2c+5a^2c^2)+c(b^4-4ab^2c+2a^2c^2)x}{a+bx+cx^2}\,dx}{4a^4}\\
&= -\frac{bn}{12ax^3}+\frac{(b^2-2ac)\,n}{8a^2x^2}-\frac{b(b^2-3ac)\,n}{4a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\,n\log(x)}{4a^4}\\
&\quad-\frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}+\frac{(b(b^2-4ac)\,(b^2-2ac)\,n)\int\frac{1}{a+bx+cx^2}\,dx}{8a^4}\\
&\quad+\frac{((b^4-4ab^2c+2a^2c^2)\,n)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{8a^4}\\
&= -\frac{bn}{12ax^3}+\frac{(b^2-2ac)\,n}{8a^2x^2}-\frac{b(b^2-3ac)\,n}{4a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\,n\log(x)}{4a^4}\\
&\quad+\frac{(b^4-4ab^2c+2a^2c^2)\,n\log\left(a+bx+cx^2\right)}{8a^4}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}\\
&\quad-\frac{(b(b^2-4ac)\,(b^2-2ac)\,n)\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{4a^4}
\end{aligned}$$

$$= -\frac{bn}{12ax^3} + \frac{(b^2-2ac)\,n}{8a^2x^2} - \frac{b(b^2-3ac)\,n}{4a^3x} - \frac{b\sqrt{b^2-4ac}(b^2-2ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4a^4} - \frac{(b^4-4ab^2c+2a^2c^2)\,n\log(x)}{4a^4} + \frac{(b^4-4ab^2c+2a^2c^2)\,n\log\left(a+bx+cx^2\right)}{8a^4} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx = -\frac{\frac{nx\left(2a^3b-3a^2(b^2-2ac)x+6ab(b^2-3ac)x^2+6b\sqrt{b^2-4ac}(b^2-2ac)x^3\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+6(b^4-4ab^2c+2a^2c^2)x^3\log(x)-3(b^4-4ab^2c+2a}{a^4}}{24x^4}$$

[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^5,x]

[Out] -1/24*((n*x*(2*a^3*b - 3*a^2*(b^2 - 2*a*c)*x + 6*a*b*(b^2 - 3*a*c)*x^2 + 6*b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*x^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*Log[x] - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*Log[a + x*(b + c*x)]))/a^4 + 6*Log[d*(a + x*(b + c*x))^n])/x^4

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18

method	result
parts	$-\frac{\ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{4x^4}+\frac{n\left(-\frac{b}{3a\,x^3}-\frac{2ca-b^2}{2a^2x^2}+\frac{\left(-2c^2a^2+4a\,b^2c-b^4\right)\ln(x)}{a^4}+\frac{b\left(3ca-b^2\right)}{a^3x}+\frac{\frac{\left(2c^3a^2-4a\,b^2c^2+b^4c\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(5a^2\right.}{}}{}\right.}{4}$
risch	Expression too large to display

[In] int(ln(d*(c*x^2+b*x+a)^n)/x^5,x,method=_RETURNVERBOSE)

[Out] -1/4*ln(d*(c*x^2+b*x+a)^n)/x^4+1/4*n*(-1/3*b/a/x^3-1/2*(2*a*c-b^2)/a^2/x^2+1/a^4*(-2*a^2*c^2+4*a*b^2*c-b^4)*ln(x)+b*(3*a*c-b^2)/a^3/x+1/a^4*(1/2*(2*a^2*c^3-4*a*b^2*c^2+b^4*c)/c*ln(c*x^2+b*x+a)+2*(5*a^2*b*c^2-5*a*b^3*c+b^5-1/2*(2*a^2*c^3-4*a*b^2*c^2+b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.13

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^5}\,dx$$
$$= \left[-\frac{3\,(b^3-2\,abc)\sqrt{b^2-4\,ac}nx^4\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+6\,(b^4-4\,ab^2c+2\,a^2c^2)nx^4\log(x)}{}\right.$$
$$-\frac{6\,(b^3-2\,abc)\sqrt{-b^2+4\,ac}nx^4\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+6\,(b^4-4\,ab^2c+2\,a^2c^2)nx^4\log(x)+2\,a^3bnx}{}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*x^4*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4
- 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a^3*b*n*x + 6*(a*b^3 - 3*a^2*b*c)
*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3*c)*n*x^2 - 3*((b^4 - 4*a*b^2*c +
 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 + b*x + a))/(a^4*x^4), -1/24*(6*(b^3
 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*x^4*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a^3*b*n*
x + 6*(a*b^3 - 3*a^2*b*c)*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3*c)*n*x^
2 - 3*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 + b*x + a))
/(a^4*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.11

$$\begin{aligned}
&\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx \\
&= \frac{(b^4n - 4\,ab^2cn + 2\,a^2c^2n)\log\left(cx^2+bx+a\right)}{8\,a^4} - \frac{n\log\left(cx^2+bx+a\right)}{4\,x^4} \\
&\quad - \frac{(b^4n - 4\,ab^2cn + 2\,a^2c^2n)\log\left(x\right)}{4\,a^4} + \frac{(b^5n - 6\,ab^3cn + 8\,a^2bc^2n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{4\sqrt{-b^2+4\,ac}\,a^4} \\
&\quad - \frac{6\,b^3nx^3 - 18\,abcnx^3 - 3\,ab^2nx^2 + 6\,a^2cnx^2 + 2\,a^2bnx + 6\,a^3\log\left(d\right)}{24\,a^3x^4}
\end{aligned}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/a^4 - 1/4*n*lo
g(c*x^2 + b*x + a)/x^4 - 1/4*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(x)/a^4
 + 1/4*(b^5*n - 6*a*b^3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^4) - 1/24*(6*b^3*n*x^3 - 18*a*b*c*n*x^3 - 3*
a*b^2*n*x^2 + 6*a^2*c*n*x^2 + 2*a^2*b*n*x + 6*a^3*log(d))/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 627, normalized size of antiderivative = 3.30

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x^5}\,dx$$

$$= \frac{\ln \left(2\,a\,b^6+2\,b^7\,x-12\,a^4\,c^3+2\,a\,b^5\,\sqrt{b^2-4\,a\,c}+2\,b^6\,x\,\sqrt{b^2-4\,a\,c}-15\,a^2\,b^4\,c+31\,a^3\,b^2\,c^2+37\,a^2\,b^3\,c^2\,x-\cdots\right.}{\cdots}$$

$$-\frac{\ln \left(d\,(c\,x^2+b\,x+a)^n\right)}{4\,x^4}-\frac{\ln (x)\,\left(2\,n\,a^2\,c^2-4\,n\,a\,b^2\,c+n\,b^4\right)}{4\,a^4}$$

$$-\frac{\ln \left(12\,a^4\,c^3-2\,b^7\,x-2\,a\,b^6+2\,a\,b^5\,\sqrt{b^2-4\,a\,c}+2\,b^6\,x\,\sqrt{b^2-4\,a\,c}+15\,a^2\,b^4\,c-31\,a^3\,b^2\,c^2-37\,a^2\,b^3\,c^2\cdots\right.}{\cdots}$$

$$-\frac{\frac{b\,n}{3\,a}+\frac{n\,x\,(2\,a\,c-b^2)}{2\,a^2}-\frac{b\,n\,x^2\,(3\,a\,c-b^2)}{a^3}}{4\,x^3}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^5,x)
```

```
[Out] (log(2*a*b^6 + 2*b^7*x - 12*a^4*c^3 + 2*a*b^5*(b^2 - 4*a*c)^(1/2) + 2*b^6*x
*(b^2 - 4*a*c)^(1/2) - 15*a^2*b^4*c + 31*a^3*b^2*c^2 + 37*a^2*b^3*c^2*x - 1
6*a*b^5*c*x - 20*a^3*b*c^3*x - 9*a^2*b^3*c*(b^2 - 4*a*c)^(1/2) + 7*a^3*b*c^
2*(b^2 - 4*a*c)^(1/2) - 6*a^3*c^3*x*(b^2 - 4*a*c)^(1/2) - 12*a*b^4*c*x*(b^2
 - 4*a*c)^(1/2) + 19*a^2*b^2*c^2*x*(b^2 - 4*a*c)^(1/2))*((b^4*n)/8 - a*((b^
2*c*n)/2 + (b*c*n*(b^2 - 4*a*c)^(1/2))/4) + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 +
 (a^2*c^2*n)/4))/a^4 - log(d*(a + b*x + c*x^2)^n)/(4*x^4) - (log(x)*(b^4*n
+ 2*a^2*c^2*n - 4*a*b^2*c*n))/(4*a^4) - (log(12*a^4*c^3 - 2*b^7*x - 2*a*b^6
 + 2*a*b^5*(b^2 - 4*a*c)^(1/2) + 2*b^6*x*(b^2 - 4*a*c)^(1/2) + 15*a^2*b^4*c
 - 31*a^3*b^2*c^2 - 37*a^2*b^3*c^2*x + 16*a*b^5*c*x + 20*a^3*b*c^3*x - 9*a^
2*b^3*c*(b^2 - 4*a*c)^(1/2) + 7*a^3*b*c^2*(b^2 - 4*a*c)^(1/2) - 6*a^3*c^3*x
*(b^2 - 4*a*c)^(1/2) - 12*a*b^4*c*x*(b^2 - 4*a*c)^(1/2) + 19*a^2*b^2*c^2*x*
(b^2 - 4*a*c)^(1/2))*(a*((b^2*c*n)/2 - (b*c*n*(b^2 - 4*a*c)^(1/2))/4) - (b^
4*n)/8 + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 - (a^2*c^2*n)/4))/a^4 - ((b*n)/(3*a)
 + (n*x*(2*a*c - b^2))/(2*a^2) - (b*n*x^2*(3*a*c - b^2))/a^3)/(4*x^3)
```

3.81 $\int \log(1+x+x^2)\,dx$

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Optimal result

Integrand size = 7, antiderivative size = 42

$$\int \log(1+x+x^2)\,dx = -2x+\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+\frac{1}{2}\log(1+x+x^2)+x\log(1+x+x^2)$$

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2603, 787, 648, 632, 210, 642}

$$\int \log(1+x+x^2)\,dx = \sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+x\log(x^2+x+1)+\frac{1}{2}\log(x^2+x+1)-2x$$

```
[In] Int[Log[1 + x + x^2],x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2 + x*Log[1 + x
 + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(1+x+x^2\right)-\int\frac{x(1+2x)}{1+x+x^2}\,dx\\
&= -2x+x\log\left(1+x+x^2\right)-\int\frac{-2-x}{1+x+x^2}\,dx\\
&= -2x+x\log\left(1+x+x^2\right)+\frac{1}{2}\int\frac{1+2x}{1+x+x^2}\,dx+\frac{3}{2}\int\frac{1}{1+x+x^2}\,dx\\
&= -2x+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)-3\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \log \left(1+x+x^2\right) \, dx = -2x+\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+\left(\frac{1}{2}+x\right)\log \left(1+x+x^2\right)$$

```
[In] Integrate[Log[1 + x + x^2],x]
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + (1/2 + x)*Log[1 + x + x^2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$-2x+\frac{\ln\left(x^{2}+x+1\right)}{2}+x\ln\left(x^{2}+x+1\right)+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}$	38
parts	$-2x+\frac{\ln\left(x^{2}+x+1\right)}{2}+x\ln\left(x^{2}+x+1\right)+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}$	38
risch	$x\ln\left(x^{2}+x+1\right)-2x+\frac{\ln\left(4x^{2}+4x+4\right)}{2}+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}$	42

```
[In] int(ln(x^2+x+1),x,method=_RETURNVERBOSE)
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \log \left(1+x+x^2\right) \, dx = \frac{1}{2} \, (2 \, x + 1) \log \left(x^{2} + x + 1\right) + \sqrt{3} \arctan \left(\frac{1}{3} \, \sqrt{3} (2 \, x + 1)\right) - 2 \, x$$

```
[In] integrate(log(x^2+x+1),x, algorithm="fricas")
[Out] 1/2*(2*x + 1)*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*
x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \log\left(1+x+x^2\right)\,dx = x\log\left(x^2+x+1\right)-2x+\frac{\log\left(x^2+x+1\right)}{2}+\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)$$

```
[In] integrate(ln(x**2+x+1),x)
```

```
[Out] x*log(x**2 + x + 1) - 2*x + log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/
3 + sqrt(3)/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \log\left(1+x+x^2\right)\,dx = x\log\left(x^2+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x+\frac{1}{2}\log\left(x^2+x+1\right)$$

```
[In] integrate(log(x^2+x+1),x, algorithm="maxima")
```

```
[Out] x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log(
x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \log\left(1+x+x^2\right)\,dx = x\log\left(x^2+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x+\frac{1}{2}\log\left(x^2+x+1\right)$$

```
[In] integrate(log(x^2+x+1),x, algorithm="giac")
```

```
[Out] x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log(
x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \log \left(1+x+x^2\right) \, dx = \frac{\ln \left(x^2+x+1\right)}{2} - 2\,x + \sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3} + \frac{\sqrt{3}}{3}\right) + x\,\ln \left(x^2+x+1\right)$$

```
[In] int(log(x + x^2 + 1),x)
[Out] log(x + x^2 + 1)/2 - 2*x + 3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3) + x*lo
g(x + x^2 + 1)
```

3.82 $\int (d+ex)^4 \log \left(d(a+bx+cx^2)^n\right) \, dx$

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Optimal result

Integrand size = 23, antiderivative size = 485

$$\begin{aligned}
\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \\
&-\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2d^2+15abde+2a^2e^2\right)\right)nx}{5c^4} \\
&-\frac{e\left(20c^3d^3-b^3e^3-10c^2de(bd+ae)+bce^2(5bd+3ae)\right)nx^2}{10c^3} \\
&-\frac{e^2\left(20c^2d^2+b^2e^2-ce(5bd+2ae)\right)nx^3}{15c^2}-\frac{e^3(10cd-be)nx^4}{20c}-\frac{2}{25}e^4nx^5 \\
&+\frac{\sqrt{b^2-4ac}\left(5c^4d^4+b^4e^4-10c^3d^2e(bd+ae)-b^2ce^3(5bd+3ae)+c^2e^2\left(10b^2d^2+10abde+a^2e^2\right)\right)n\operatorname{arctan}\ldots}{5c^5} \\
&-\frac{(2cd-be)\left(c^4d^4+b^4e^4-2c^3d^2e(bd+5ae)-b^2ce^3(3bd+5ae)+c^2e^2\left(4b^2d^2+10abde+5a^2e^2\right)\right)n\log(a\ldots}{10c^5e} \\
&+\frac{(d+ex)^5\log\left(d\left(a+bx+cx^2\right)^n\right)}{5e}
\end{aligned}$$

```
[Out] -1/5*(10*c^4*d^4+b^4*e^4-10*c^3*d^2*e*(2*a*e+b*d)-b^2*c*e^3*(4*a*e+5*b*d)+c
^2*e^2*(2*a^2*e^2+15*a*b*d*e+10*b^2*d^2))*n*x/c^4-1/10*e*(20*c^3*d^3-b^3*e^
3-10*c^2*d*e*(a*e+b*d)+b*c*e^2*(3*a*e+5*b*d))*n*x^2/c^3-1/15*e^2*(20*c^2*d^
2+b^2*e^2-c*e*(2*a*e+5*b*d))*n*x^3/c^2-1/20*e^3*(-b*e+10*c*d)*n*x^4/c-2/25*
e^4*n*x^5-1/10*(-b*e+2*c*d)*(c^4*d^4+b^4*e^4-2*c^3*d^2*e*(5*a*e+b*d)-b^2*c*
e^3*(5*a*e+3*b*d)+c^2*e^2*(5*a^2*e^2+10*a*b*d*e+4*b^2*d^2))*n*ln(c*x^2+b*x+
a)/c^5/e+1/5*(e*x+d)^5*ln(d*(c*x^2+b*x+a)^n)/e+1/5*(5*c^4*d^4+b^4*e^4-10*c^
3*d^2*e*(a*e+b*d)-b^2*c*e^3*(3*a*e+5*b*d)+c^2*e^2*(a^2*e^2+10*a*b*d*e+10*b^
2*d^2))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$= \frac{n\sqrt{b^2-4ac}\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2e^2(a^2e^2+10abde+10b^2d^2)-b^2ce^3(3ae+5bd)-10c^3d^2e(ae+bd)+b^4\right.}{5c^5}$$

$$-\frac{nx(c^2e^2(2a^2e^2+15abde+10b^2d^2)-b^2ce^3(4ae+5bd)-10c^3d^2e(2ae+bd)+b^4e^4+10c^4d^4)}{5c^4}$$

$$-\frac{n(2cd-be)\left(c^2e^2(5a^2e^2+10abde+4b^2d^2)-b^2ce^3(5ae+3bd)-2c^3d^2e(5ae+bd)+b^4e^4+c^4d^4\right)\log(a}{10c^5e}$$

$$-\frac{enx^2(-10c^2de(ae+bd)+bce^2(3ae+5bd)-b^3e^3+20c^3d^3)}{10c^3}$$

$$-\frac{e^2nx^3(-ce(2ae+5bd)+b^2e^2+20c^2d^2)}{15c^2}$$

$$+\frac{(d+ex)^5\log\left(d\left(a+bx+cx^2\right)^n\right)}{5e}-\frac{e^3nx^4(10cd-be)}{20c}-\frac{2}{25}e^4nx^5$$

```
[In] Int[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/5*((10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*c*e^3*(5*b*d
 + 4*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*n*x)/c^4 - (e*(2
0*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d + 3*a*e))*n*x
^2)/(10*c^3) - (e^2*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*n*x^3)/(15
*c^2) - (e^3*(10*c*d - b*e)*n*x^4)/(20*c) - (2*e^4*n*x^5)/25 + (Sqrt[b^2 -
4*a*c]*(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d +
 3*a*e) + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(5*c^5) - ((2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d
^2*e*(b*d + 5*a*e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*
b*d*e + 5*a^2*e^2))*n*Log[a + b*x + c*x^2])/(10*c^5*e) + ((d + e*x)^5*Log[d
*(a + b*x + c*x^2)^n])/(5*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^5\log\left(d(a+bx+cx^2)^n\right)}{5e} - \frac{n\int\frac{(b+2cx)(d+ex)^5}{a+bx+cx^2}\,dx}{5e}$$

$$= \frac{(d+ex)^5\log\left(d(a+bx+cx^2)^n\right)}{5e} - \frac{n\int\left(\frac{e\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2d^2+15abde+2a^2e^2\right)\right)}{c^4}+\frac{e^2\left(20c^3d^3-b^3e^3-10c^2de(bd+ae)+\cdots\right.}{c^3}\cdots\right.}{\cdots}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10b^2d^2+15abde+2a^2e^2)\right)nx}{5c^4}$$
$$-\frac{e\left(20c^3d^3-b^3e^3-10c^2de(bd+ae)+bce^2(5bd+3ae)\right)nx^2}{10c^3}$$
$$-\frac{e^2\left(20c^2d^2+b^2e^2-ce(5bd+2ae)\right)nx^3}{15c^2}-\frac{e^3(10cd-be)nx^4}{20c}$$
$$-\frac{2}{25}e^4nx^5+\frac{(d+ex)^5\log\left(d(a+bx+cx^2)^n\right)}{5e}$$
$$-\frac{n\int\frac{5ab^3cde^4-ab^4e^5-2ab^2ce^3(5cd^2-2ae^2)+bc^2d(c^2d^4+10acd^2e^2-15a^2e^4)-2ac^2e(5c^2d^4-10acd^2e^2+a^2e^4)+(2cd-be)(c^4d^4+b^4}{a+bx+cx^2}}{5c^4e}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10b^2d^2+15abde+2a^2e^2)\right)nx}{5c^4}$$
$$-\frac{e\left(20c^3d^3-b^3e^3-10c^2de(bd+ae)+bce^2(5bd+3ae)\right)nx^2}{10c^3}$$
$$-\frac{e^2\left(20c^2d^2+b^2e^2-ce(5bd+2ae)\right)nx^3}{15c^2}-\frac{e^3(10cd-be)nx^4}{20c}$$
$$-\frac{2}{25}e^4nx^5+\frac{(d+ex)^5\log\left(d(a+bx+cx^2)^n\right)}{5e}$$
$$-\frac{((b^2-4ac)\left(5c^4d^4+b^4e^4-10c^3d^2e(bd+ae)-b^2ce^3(5bd+3ae)+c^2e^2(10b^2d^2+10abde+a^2e^2}{10c^5}$$
$$-\frac{((2cd-be)\left(c^4d^4+b^4e^4-2c^3d^2e(bd+5ae)-b^2ce^3(3bd+5ae)+c^2e^2(4b^2d^2+10abde+5a^2e^2)\right.}{10c^5e}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10b^2d^2+15abde+2a^2e^2)\right)nx}{5c^4}$$
$$-\frac{e\left(20c^3d^3-b^3e^3-10c^2de(bd+ae)+bce^2(5bd+3ae)\right)nx^2}{10c^3}$$
$$-\frac{e^2\left(20c^2d^2+b^2e^2-ce(5bd+2ae)\right)nx^3}{15c^2}-\frac{e^3(10cd-be)nx^4}{20c}-\frac{2}{25}e^4nx^5$$
$$-\frac{(2cd-be)\left(c^4d^4+b^4e^4-2c^3d^2e(bd+5ae)-b^2ce^3(3bd+5ae)+c^2e^2(4b^2d^2+10abde+5a^2e^2)\right)}{10c^5e}$$
$$+\frac{(d+ex)^5\log\left(d(a+bx+cx^2)^n\right)}{5e}$$
$$+\frac{((b^2-4ac)\left(5c^4d^4+b^4e^4-10c^3d^2e(bd+ae)-b^2ce^3(5bd+3ae)+c^2e^2(10b^2d^2+10abde+a^2e^2}{5c^5}$$

$$= -\frac{(10c^4d^4 + b^4e^4 - 10c^3d^2e(bd + 2ae) - b^2ce^3(5bd + 4ae) + c^2e^2(10b^2d^2 + 15abde + 2a^2e^2))\,nx}{5c^4}$$
$$- \frac{e(20c^3d^3 - b^3e^3 - 10c^2de(bd + ae) + bce^2(5bd + 3ae))\,nx^2}{10c^3}$$
$$- \frac{e^2(20c^2d^2 + b^2e^2 - ce(5bd + 2ae))\,nx^3}{15c^2} - \frac{e^3(10cd - be)nx^4}{20c} - \frac{2}{25}e^4nx^5$$
$$+ \frac{\sqrt{b^2 - 4ac}(5c^4d^4 + b^4e^4 - 10c^3d^2e(bd + ae) - b^2ce^3(5bd + 3ae) + c^2e^2(10b^2d^2 + 10abde + a^2e^2))}{5c^5}$$
$$- \frac{(2cd - be)\,(c^4d^4 + b^4e^4 - 2c^3d^2e(bd + 5ae) - b^2ce^3(3bd + 5ae) + c^2e^2(4b^2d^2 + 10abde + 5a^2e^2))\,n}{10c^5e}$$
$$+ \frac{(d + ex)^5 \log\left(d(a + bx + cx^2)^n\right)}{5e}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.96

$$\int (d + ex)^4 \log\left(d(a + bx + cx^2)^n\right)\,dx$$
$$= \frac{-\frac{n\left(60ce(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10b^2d^2+15abde+2a^2e^2))x+30c^2e^2(20c^3d^3-b^3e^3-10c^2de(bd+ae)+bce^2(5bd-\right.}{}}{}$$

```
[In] Integrate[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-1/60*(n*(60*c*e*(10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*
c*e^3*(5*b*d + 4*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*x +
30*c^2*e^2*(20*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d
+ 3*a*e))*x^2 + 20*c^3*e^3*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*x^3
 + 15*c^4*e^4*(10*c*d - b*e)*x^4 + 24*c^5*e^5*x^5 - 60*Sqrt[b^2 - 4*a*c]*e*
(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d + 3*a*e)
 + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]] + 30*(2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d^2*e*(b*d + 5*a*
e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*b*d*e + 5*a^2*e^
2))*Log[a + x*(b + c*x)]))/c^5 + (d + e*x)^5*Log[d*(a + x*(b + c*x))^n])/(5
*e)
```

Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.81

method	result
parts	$\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)e^{4}x^{5}}{5}+\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)e^{3}d\,x^{4}+2\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)e^{2}d^{2}x^{3}+2\ln\left(d(c\,x\right.$
risch	Expression too large to display

```
[In] int((e*x+d)^4*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*ln(d*(c*x^2+b*x+a)^n)*e^4*x^5+ln(d*(c*x^2+b*x+a)^n)*e^3*d*x^4+2*ln(d*(c
*x^2+b*x+a)^n)*e^2*d^2*x^3+2*ln(d*(c*x^2+b*x+a)^n)*e*d^3*x^2+ln(d*(c*x^2+b*
x+a)^n)*d^4*x+1/5*ln(d*(c*x^2+b*x+a)^n)/e*d^5-1/5/e*n*(e/c^4*(2/5*c^4*e^4*x
^5-1/4*b*c^3*e^4*x^4+5/2*c^4*d*e^3*x^4-2/3*a*c^3*e^4*x^3+1/3*b^2*c^2*e^4*x^
3-5/3*b*c^3*d*e^3*x^3+20/3*c^4*d^2*e^2*x^3+3/2*a*b*c^2*e^4*x^2-5*a*c^3*d*e^
3*x^2-1/2*b^3*c*e^4*x^2+5/2*b^2*c^2*d*e^3*x^2-5*b*c^3*d^2*e^2*x^2+10*c^4*d^
3*e*x^2+2*a^2*x*c^2*e^4-4*a*b^2*c*x*e^4+15*a*b*c^2*d*x*e^3-20*a*x*c^3*d^2*e
^2+b^4*x*e^4-5*x*b^3*c*d*e^3+10*b^2*c^2*x*d^2*e^2-10*x*b*c^3*d^3*e+10*x*c^4
*d^4)+1/c^4*(1/2*(-5*a^2*b*c^2*e^5+10*a^2*c^3*d*e^4+5*a*b^3*c*e^5-20*a*b^2*
c^2*d*e^4+30*a*b*c^3*d^2*e^3-20*a*c^4*d^3*e^2-b^5*e^5+5*b^4*c*d*e^4-10*b^3*
c^2*d^2*e^3+10*b^2*c^3*d^3*e^2-5*b*c^4*d^4*e+2*c^5*d^5)/c*ln(c*x^2+b*x+a)+2
*(-2*a^3*c^2*e^5+4*a^2*b^2*c*e^5-15*a^2*b*c^2*d*e^4+20*a^2*c^3*d^2*e^3-a*b^
4*e^5+5*a*b^3*c*d*e^4-10*a*b^2*c^2*d^2*e^3+10*a*b*c^3*d^3*e^2-10*a*c^4*d^4*
e+b*c^4*d^5-1/2*(-5*a^2*b*c^2*e^5+10*a^2*c^3*d*e^4+5*a*b^3*c*e^5-20*a*b^2*c
^2*d*e^4+30*a*b*c^3*d^2*e^3-20*a*c^4*d^3*e^2-b^5*e^5+5*b^4*c*d*e^4-10*b^3*c
^2*d^2*e^3+10*b^2*c^3*d^3*e^2-5*b*c^4*d^4*e+2*c^5*d^5)*b/c)/(4*a*c-b^2)^(1/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.62

$$\int (d+ex)^4\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/300*(24*c^5*e^4*n*x^5 + 15*(10*c^5*d*e^3 - b*c^4*e^4)*n*x^4 + 20*(20*c^
5*d^2*e^2 - 5*b*c^4*d*e^3 + (b^2*c^3 - 2*a*c^4)*e^4)*n*x^3 + 30*(20*c^5*d^3
*e - 10*b*c^4*d^2*e^2 + 5*(b^2*c^3 - 2*a*c^4)*d*e^3 - (b^3*c^2 - 3*a*b*c^3)
```

```
*e^4)*n*x^2 - 30*(5*c^4*d^4 - 10*b*c^3*d^3*e + 10*(b^2*c^2 - a*c^3)*d^2*e^2
 - 5*(b^3*c - 2*a*b*c^2)*d*e^3 + (b^4 - 3*a*b^2*c + a^2*c^2)*e^4)*sqrt(b^2
- 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) + 60*(10*c^5*d^4 - 10*b*c^4*d^3*e + 10*(b^2*c^3
- 2*a*c^4)*d^2*e^2 - 5*(b^3*c^2 - 3*a*b*c^3)*d*e^3 + (b^4*c - 4*a*b^2*c^2 +
 2*a^2*c^3)*e^4)*n*x - 30*(2*c^5*e^4*n*x^5 + 10*c^5*d*e^3*n*x^4 + 20*c^5*d^
2*e^2*n*x^3 + 20*c^5*d^3*e*n*x^2 + 10*c^5*d^4*n*x + (5*b*c^4*d^4 - 10*(b^2*
c^3 - 2*a*c^4)*d^3*e + 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 5*(b^4*c - 4*a*b^
2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4)*n)*log(c*x^
2 + b*x + a) - 60*(c^5*e^4*x^5 + 5*c^5*d*e^3*x^4 + 10*c^5*d^2*e^2*x^3 + 10*
c^5*d^3*e*x^2 + 5*c^5*d^4*x)*log(d))/c^5, -1/300*(24*c^5*e^4*n*x^5 + 15*(10
*c^5*d*e^3 - b*c^4*e^4)*n*x^4 + 20*(20*c^5*d^2*e^2 - 5*b*c^4*d*e^3 + (b^2*c
^3 - 2*a*c^4)*e^4)*n*x^3 + 30*(20*c^5*d^3*e - 10*b*c^4*d^2*e^2 + 5*(b^2*c^3
 - 2*a*c^4)*d*e^3 - (b^3*c^2 - 3*a*b*c^3)*e^4)*n*x^2 - 60*(5*c^4*d^4 - 10*b
*c^3*d^3*e + 10*(b^2*c^2 - a*c^3)*d^2*e^2 - 5*(b^3*c - 2*a*b*c^2)*d*e^3 + (
b^4 - 3*a*b^2*c + a^2*c^2)*e^4)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*
a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 60*(10*c^5*d^4 - 10*b*c^4*d^3*e + 10*(b^2
*c^3 - 2*a*c^4)*d^2*e^2 - 5*(b^3*c^2 - 3*a*b*c^3)*d*e^3 + (b^4*c - 4*a*b^2*
c^2 + 2*a^2*c^3)*e^4)*n*x - 30*(2*c^5*e^4*n*x^5 + 10*c^5*d*e^3*n*x^4 + 20*c
^5*d^2*e^2*n*x^3 + 20*c^5*d^3*e*n*x^2 + 10*c^5*d^4*n*x + (5*b*c^4*d^4 - 10*
(b^2*c^3 - 2*a*c^4)*d^3*e + 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 5*(b^4*c - 4
*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4)*n)*log
(c*x^2 + b*x + a) - 60*(c^5*e^4*x^5 + 5*c^5*d*e^3*x^4 + 10*c^5*d^2*e^2*x^3
+ 10*c^5*d^3*e*x^2 + 5*c^5*d^4*x)*log(d))/c^5]
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^4 \log \left(d(a+bx+cx^2)^n\right)\, dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**4*ln(d*(c*x**2+b*x+a)**n),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^4 \log \left(d(a+bx+cx^2)^n\right)\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.53

$$\int (d+ex)^4 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$
$$= -\frac{1}{25}\left(2e^4n - 5e^4\log(d)\right)x^5 - \frac{(10cde^3n - be^4n - 20cde^3\log(d))x^4}{20c} - \frac{(20c^2d^2e^2n - 5bcde^3n + b^2e^4n - 2ace^4n - 30c^2d^2e^2\log(d))x^3}{15c^2} + \frac{1}{5}\left(e^4nx^5 + 5de^3nx^4 + 10d^2e^2nx^3 + 10d^3enx^2 + 5d^4nx\right)\log\left(cx^2+bx+a\right) - \frac{(20c^3d^3en - 10bc^2d^2e^2n + 5b^2cde^3n - 10ac^2de^3n - b^3e^4n + 3abce^4n - 20c^3d^3e\log(d))x^2}{10c^3} - \frac{(10c^4d^4n - 10bc^3d^3en + 10b^2c^2d^2e^2n - 20ac^3d^2e^2n - 5b^3cde^3n + 15abc^2de^3n + b^4e^4n - 4ab^2ce^4n + \cdots}{5c^4} + \frac{(5bc^4d^4n - 10b^2c^3d^3en + 20ac^4d^3en + 10b^3c^2d^2e^2n - 30abc^3d^2e^2n - 5b^4cde^3n + 20ab^2c^2de^3n - 10a\cdots}{10c^5} - \frac{(5b^2c^4d^4n - 20ac^5d^4n - 10b^3c^3d^3en + 40abc^4d^3en + 10b^4c^2d^2e^2n - 50ab^2c^3d^2e^2n + 40a^2c^4d^2e^2n - 5\cdots}{5\sqrt{-\cdots}}$$

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] -1/25*(2*e^4*n - 5*e^4*log(d))*x^5 - 1/20*(10*c*d*e^3*n - b*e^4*n - 20*c*d*
e^3*log(d))*x^4/c - 1/15*(20*c^2*d^2*e^2*n - 5*b*c*d*e^3*n + b^2*e^4*n - 2*
a*c*e^4*n - 30*c^2*d^2*e^2*log(d))*x^3/c^2 + 1/5*(e^4*n*x^5 + 5*d*e^3*n*x^4
 + 10*d^2*e^2*n*x^3 + 10*d^3*e*n*x^2 + 5*d^4*n*x)*log(c*x^2 + b*x + a) - 1/
10*(20*c^3*d^3*e*n - 10*b*c^2*d^2*e^2*n + 5*b^2*c*d*e^3*n - 10*a*c^2*d*e^3*
n - b^3*e^4*n + 3*a*b*c*e^4*n - 20*c^3*d^3*e*log(d))*x^2/c^3 - 1/5*(10*c^4*
d^4*n - 10*b*c^3*d^3*e*n + 10*b^2*c^2*d^2*e^2*n - 20*a*c^3*d^2*e^2*n - 5*b^
3*c*d*e^3*n + 15*a*b*c^2*d*e^3*n + b^4*e^4*n - 4*a*b^2*c*e^4*n + 2*a^2*c^2*
e^4*n - 5*c^4*d^4*log(d))*x/c^4 + 1/10*(5*b*c^4*d^4*n - 10*b^2*c^3*d^3*e*n
+ 20*a*c^4*d^3*e*n + 10*b^3*c^2*d^2*e^2*n - 30*a*b*c^3*d^2*e^2*n - 5*b^4*c*
d*e^3*n + 20*a*b^2*c^2*d*e^3*n - 10*a^2*c^3*d*e^3*n + b^5*e^4*n - 5*a*b^3*c
*e^4*n + 5*a^2*b*c^2*e^4*n)*log(c*x^2 + b*x + a)/c^5 - 1/5*(5*b^2*c^4*d^4*n
 - 20*a*c^5*d^4*n - 10*b^3*c^3*d^3*e*n + 40*a*b*c^4*d^3*e*n + 10*b^4*c^2*d^
2*e^2*n - 50*a*b^2*c^3*d^2*e^2*n + 40*a^2*c^4*d^2*e^2*n - 5*b^5*c*d*e^3*n +
 30*a*b^3*c^2*d*e^3*n - 40*a^2*b*c^3*d*e^3*n + b^6*e^4*n - 7*a*b^4*c*e^4*n
+ 13*a^2*b^2*c^2*e^4*n - 4*a^3*c^3*e^4*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 1240, normalized size of antiderivative = 2.56

$$\int (d+ex)^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= x^3\left(\frac{b\left(\frac{e^3\,n\,(b\,e+10\,c\,d)}{5\,c}-\frac{2\,b\,e^4\,n}{5\,c}\right)}{3\,c}+\frac{2\,a\,e^4\,n}{15\,c}-\frac{d\,e^2\,n\,(b\,e+4\,c\,d)}{3\,c}\right)$$

$$-x\left(\frac{a\left(\frac{b\left(\frac{e^3\,n\,(b\,e+10\,c\,d)}{5\,c}-\frac{2\,b\,e^4\,n}{5\,c}\right)}{c}+\frac{2\,a\,e^4\,n}{5\,c}-\frac{d\,e^2\,n\,(b\,e+4\,c\,d)}{c}\right)}{c}-\frac{b\left(\frac{b\left(\frac{b\left(\frac{e^3\,n\,(b\,e+10\,c\,d)}{5\,c}-\frac{2\,b\,e^4\,n}{5\,c}\right)}{c}+\frac{2\,a\,e^4\,n}{5\,c}-\frac{d\,e^2\,n\,(b\,e+4\,c\,d)}{c}\right)}{c}-\frac{a\left(\frac{e^3\,n\,(b\,e+10\,c\,d)}{5\,c}-\frac{2\,b\,e^4\,n}{5\,c}\right)}{c}+\frac{2\,d^2\,e\,n\,(b\,e+2\,c\,d)}{c}\right)}{c}+\frac{2\,d^3\,n\,(b\,e+c\,d)}{c}\right)$$

$$-x^2\left(\frac{b\left(\frac{b\left(\frac{e^3\,n\,(b\,e+10\,c\,d)}{5\,c}-\frac{2\,b\,e^4\,n}{5\,c}\right)}{c}+\frac{2\,a\,e^4\,n}{5\,c}-\frac{d\,e^2\,n\,(b\,e+4\,c\,d)}{c}\right)}{2\,c}-\frac{a\left(\frac{e^3\,n\,(b\,e+10\,c\,d)}{5\,c}-\frac{2\,b\,e^4\,n}{5\,c}\right)}{2\,c}+\frac{d^2\,e\,n\,(b\,e+2\,c\,d)}{c}\right)-x^4\left(\frac{e^3\,n\,(b\,e+10\,c\,d)}{20\,c}-\frac{b\,e^4\,n}{10\,c}\right)$$

$$+\ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)\left(d^4\,x+2\,d^3\,e\,x^2+2\,d^2\,e^2\,x^3+d\,e^3\,x^4+\frac{e^4\,x^5}{5}\right)$$

$$+\frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^5\,e^4\,n+5\,b\,c^4\,d^4\,n+b^4\,e^4\,n\,\sqrt{b^2-4\,a\,c}+5\,c^4\,d^4\,n\,\sqrt{b^2\right.}{}$$

$$-\frac{2\,e^4\,n\,x^5}{25}$$

$$+\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^5\,e^4\,n+5\,b\,c^4\,d^4\,n-b^4\,e^4\,n\,\sqrt{b^2-4\,a\,c}-5\,c^4\,d^4\,n\,\sqrt{b^2\right.}{}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^4,x)
```

```
[Out] x^3*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/(3*c) + (2*a*e^
4*n)/(15*c) - (d*e^2*n*(b*e + 4*c*d))/(3*c)) - x*((a*((b*((e^3*n*(b*e + 10*
c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4
*c*d))/c))/c - (b*((b*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)
))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4*c*d))/c))/c - (a*((e^3*n*(b*e
+ 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/c + (2*d^2*e*n*(b*e + 2*c*d))/c))/c
+ (2*d^3*n*(b*e + c*d))/c) - x^2*((b*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2
*b*e^4*n)/(5*c)))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4*c*d))/c))/(2*c)
 - (a*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/(2*c) + (d^2*e*n*
(b*e + 2*c*d))/c) - x^4*((e^3*n*(b*e + 10*c*d))/(20*c) - (b*e^4*n)/(10*c))
+ log(d*(a + b*x + c*x^2)^n)*(d^4*x + (e^4*x^5)/5 + 2*d^3*e*x^2 + d*e^3*x^4
 + 2*d^2*e^2*x^3) + (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 -
 4*a*c)^(1/2))*(b^5*e^4*n + 5*b*c^4*d^4*n + b^4*e^4*n*(b^2 - 4*a*c)^(1/2) +
 5*c^4*d^4*n*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^4*n + 20*a*c^4*d^3*e*n - 5*b
^4*c*d*e^3*n + 5*a^2*b*c^2*e^4*n - 10*a^2*c^3*d*e^3*n - 10*b^2*c^3*d^3*e*n
+ a^2*c^2*e^4*n*(b^2 - 4*a*c)^(1/2) + 10*b^3*c^2*d^2*e^2*n - 10*a*c^3*d^2*e
^2*n*(b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*n*(b^2 - 4*a*c)^(1/2) - 3*a*b
^2*c*e^4*n*(b^2 - 4*a*c)^(1/2) - 10*b*c^3*d^3*e*n*(b^2 - 4*a*c)^(1/2) - 5*b
^3*c*d*e^3*n*(b^2 - 4*a*c)^(1/2) - 30*a*b*c^3*d^2*e^2*n + 20*a*b^2*c^2*d*e^
3*n + 10*a*b*c^2*d*e^3*n*(b^2 - 4*a*c)^(1/2)))/(10*c^5) - (2*e^4*n*x^5)/25
+ (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^
5*e^4*n + 5*b*c^4*d^4*n - b^4*e^4*n*(b^2 - 4*a*c)^(1/2) - 5*c^4*d^4*n*(b^2
- 4*a*c)^(1/2) - 5*a*b^3*c*e^4*n + 20*a*c^4*d^3*e*n - 5*b^4*c*d*e^3*n + 5*a
^2*b*c^2*e^4*n - 10*a^2*c^3*d*e^3*n - 10*b^2*c^3*d^3*e*n - a^2*c^2*e^4*n*(b
^2 - 4*a*c)^(1/2) + 10*b^3*c^2*d^2*e^2*n + 10*a*c^3*d^2*e^2*n*(b^2 - 4*a*c)
^(1/2) - 10*b^2*c^2*d^2*e^2*n*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c*e^4*n*(b^2 -
4*a*c)^(1/2) + 10*b*c^3*d^3*e*n*(b^2 - 4*a*c)^(1/2) + 5*b^3*c*d*e^3*n*(b^2
- 4*a*c)^(1/2) - 30*a*b*c^3*d^2*e^2*n + 20*a*b^2*c^2*d*e^3*n - 10*a*b*c^2*d
*e^3*n*(b^2 - 4*a*c)^(1/2)))/(10*c^5)
```

3.83 $\int (d+ex)^3 \log \left(d(a+bx+cx^2)^n\right) \, dx$

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Optimal result

Integrand size = 23, antiderivative size = 338

$$\begin{aligned}
&\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx \\
&= -\frac{(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae))\,nx}{4c^3} \\
&\quad -\frac{e(12c^2d^2+b^2e^2-2ce(2bd+ae))\,nx^2}{8c^2}-\frac{e^2(8cd-be)nx^3}{12c}-\frac{1}{8}e^3nx^4 \\
&\quad +\frac{\sqrt{b^2-4ac}(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4} \\
&\quad -\frac{(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+a^2e^2))\,n\log\left(a+bx+cx^2\right)}{8c^4e} \\
&\quad +\frac{(d+ex)^4\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e}
\end{aligned}$$

```
[Out] -1/4*(8*c^3*d^3-b^3*e^3+b*c*e^2*(3*a*e+4*b*d)-2*c^2*d*e*(4*a*e+3*b*d))*n*x/
c^3-1/8*e*(12*c^2*d^2+b^2*e^2-2*c*e*(a*e+2*b*d))*n*x^2/c^2-1/12*e^2*(-b*e+8
*c*d)*n*x^3/c-1/8*e^3*n*x^4-1/8*(2*c^4*d^4+b^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*
c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))*n*ln(c*x^2+b
*x+a)/c^4/e+1/4*(e*x+d)^4*ln(d*(c*x^2+b*x+a)^n)/e+1/4*(-b*e+2*c*d)*(2*c^2*d
^2+b^2*e^2-2*c*e*(a*e+b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c
+b^2)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2605, 814, 648, 632, 212, 642}

$$\begin{aligned}\int (d+ex)^3\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx =\\ &-\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8c^4e}\\ &+\frac{n\sqrt{b^2-4ac}(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{4c^4}\\ &-\frac{nx\left(-2c^2de(4ae+3bd)+bce^2(3ae+4bd)-b^3e^3+8c^3d^3\right)}{4c^3}\\ &-\frac{enx^2\left(-2ce(ae+2bd)+b^2e^2+12c^2d^2\right)}{8c^2}\\ &+\frac{(d+ex)^4\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e}-\frac{e^2nx^3(8cd-be)}{12c}-\frac{1}{8}e^3nx^4\end{aligned}$$

```
[In] Int[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/4*((8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*(3*b*d + 4
*a*e))*n*x)/c^3 - (e*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))*n*x^2)/(8
*c^2) - (e^2*(8*c*d - b*e)*n*x^3)/(12*c) - (e^3*n*x^4)/8 + (Sqrt[b^2 - 4*a*
c]*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*
d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a
^2*e^2))*n*Log[a + b*x + c*x^2])/(8*c^4*e) + ((d + e*x)^4*Log[d*(a + b*x +
c*x^2)^n])/(4*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^4\log\left(d(a+bx+cx^2)^n\right)}{4e} - \frac{n\int\frac{(b+2cx)(d+ex)^4}{a+bx+cx^2}\,dx}{4e}$$

$$= \frac{(d+ex)^4\log\left(d(a+bx+cx^2)^n\right)}{4e} - \frac{n\int\left(\frac{e\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)}{c^3}+\frac{e^2\left(12c^2d^2+b^2e^2-2ce(2bd+ae)\right)x}{c^2}+\frac{e^3(8cd-be)x^2}{c}+2e^4x^3+\frac{-4}{}\right.}{}$$

$$= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} - \frac{e\left(12c^2d^2+b^2e^2-2ce(2bd+ae)\right)nx^2}{8c^2} - \frac{e^2(8cd-be)nx^3}{12c} - \frac{1}{8}e^3nx^4 + \frac{(d+ex)^4\log\left(d(a+bx+cx^2)^n\right)}{4e} - \frac{n\int\frac{-4ab^2cde^3+ab^3e^4-8ac^2de\left(cd^2-ae^2\right)+bc\left(c^2d^4+6acd^2e^2-3a^2e^4\right)+\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2\right.\right.}{a+bx+cx^2}}{4c^3e}$$

$$= -\frac{(8c^3d^3 - b^3e^3 + bce^2(4bd + 3ae) - 2c^2de(3bd + 4ae))\,nx}{4c^3}$$
$$- \frac{e(12c^2d^2 + b^2e^2 - 2ce(2bd + ae))\,nx^2}{8c^2} - \frac{e^2(8cd - be)nx^3}{12c}$$
$$- \frac{1}{8}e^3nx^4 + \frac{(d + ex)^4 \log\left(d(a + bx + cx^2)^n\right)}{4e}$$
$$- \frac{((b^2 - 4ac)\,(2cd - be)\,(2c^2d^2 + b^2e^2 - 2ce(bd + ae))\,n) \int \frac{1}{a+bx+cx^2}\,dx}{8c^4}$$
$$- \frac{((2c^4d^4 + b^4e^4 - 4b^2ce^3(bd + ae) - 4c^3d^2e(bd + 3ae) + 2c^2e^2(3b^2d^2 + 6abde + a^2e^2))\,n) \int \frac{b+2c}{a+bx+}}{8c^4e}$$

$$= -\frac{(8c^3d^3 - b^3e^3 + bce^2(4bd + 3ae) - 2c^2de(3bd + 4ae))\,nx}{4c^3}$$
$$- \frac{e(12c^2d^2 + b^2e^2 - 2ce(2bd + ae))\,nx^2}{8c^2} - \frac{e^2(8cd - be)nx^3}{12c} - \frac{1}{8}e^3nx^4$$
$$- \frac{(2c^4d^4 + b^4e^4 - 4b^2ce^3(bd + ae) - 4c^3d^2e(bd + 3ae) + 2c^2e^2(3b^2d^2 + 6abde + a^2e^2))\,n\log(a + b}{8c^4e}$$
$$+ \frac{(d + ex)^4 \log\left(d(a + bx + cx^2)^n\right)}{4e}$$
$$+ \frac{((b^2 - 4ac)\,(2cd - be)\,(2c^2d^2 + b^2e^2 - 2ce(bd + ae))\,n)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2cx\right)}{4c^4}$$

$$= -\frac{(8c^3d^3 - b^3e^3 + bce^2(4bd + 3ae) - 2c^2de(3bd + 4ae))\,nx}{4c^3}$$
$$- \frac{e(12c^2d^2 + b^2e^2 - 2ce(2bd + ae))\,nx^2}{8c^2} - \frac{e^2(8cd - be)nx^3}{12c}$$
$$- \frac{1}{8}e^3nx^4 + \frac{\sqrt{b^2 - 4ac}(2cd - be)\,(2c^2d^2 + b^2e^2 - 2ce(bd + ae))\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4}$$
$$- \frac{(2c^4d^4 + b^4e^4 - 4b^2ce^3(bd + ae) - 4c^3d^2e(bd + 3ae) + 2c^2e^2(3b^2d^2 + 6abde + a^2e^2))\,n\log(a + b}{8c^4e}$$
$$+ \frac{(d + ex)^4 \log\left(d(a + bx + cx^2)^n\right)}{4e}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.96

$$\int (d + ex)^3 \log\left(d\left(a + bx + cx^2\right)^n\right)\,dx$$
$$= \frac{-\frac{n\left(6ce\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)x+3c^2e^2\left(12c^2d^2+b^2e^2-2ce(2bd+ae)\right)x^2+2c^3e^3(8cd-be)x^3+3c^4e^4x^4-6\sqrt{b^2-4ac}e(2c}{}}{}$$

```
[In] Integrate[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

[Out] (-1/6*(n*(6*c*e*(8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*
(3*b*d + 4*a*e))*x + 3*c^2*e^2*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))
*x^2 + 2*c^3*e^3*(8*c*d - b*e)*x^3 + 3*c^4*e^4*x^4 - 6*Sqrt[b^2 - 4*a*c]*e*
(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]] + 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*
c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*Log[
a + x*(b + c*x)]))/c^4 + (d + e*x)^4*Log[d*(a + x*(b + c*x))^n])/(4*e)

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.76

method	result
parts	$\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right) e^{3} x^{4}}{4}+\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right) e^{2} d\,x^{3}+\frac{3\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right) e\,d^{2} x^{2}}{2}+\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)$
risch	Expression too large to display

[In] int((e*x+d)^3*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)

[Out] 1/4*ln(d*(c*x^2+b*x+a)^n)*e^3*x^4+ln(d*(c*x^2+b*x+a)^n)*e^2*d*x^3+3/2*ln(d*
(c*x^2+b*x+a)^n)*e*d^2*x^2+ln(d*(c*x^2+b*x+a)^n)*d^3*x+1/4*ln(d*(c*x^2+b*x+
a)^n)/e*d^4-1/4/e*n*(e/c^3*(1/2*c^3*e^3*x^4-1/3*b*c^2*e^3*x^3+8/3*c^3*d*e^2
*x^3-a*c^2*e^3*x^2+1/2*b^2*c*e^3*x^2-2*b*c^2*d*e^2*x^2+6*c^3*d^2*e*x^2+3*a*
b*c*x*e^3-8*a*c^2*d*x*e^2-x*b^3*e^3+4*b^2*c*d*x*e^2-6*x*b*c^2*d^2*e+8*x*c^3
d^3)+1/c^3(1/2*(2*a^2*c^2*e^4-4*a*b^2*c*e^4+12*a*b*c^2*d*e^3-12*a*c^3*d^2
*e^2+b^4*e^4-4*b^3*c*d*e^3+6*b^2*c^2*d^2*e^2-4*b*c^3*d^3*e+2*c^4*d^4)/c*ln(
c*x^2+b*x+a)+2*(-3*a^2*b*c*e^4+8*a^2*c^2*d*e^3+a*b^3*e^4-4*a*b^2*c*d*e^3+6*
a*b*c^2*d^2*e^2-8*a*c^3*d^3*e+b*c^3*d^4-1/2*(2*a^2*c^2*e^4-4*a*b^2*c*e^4+12
*a*b*c^2*d*e^3-12*a*c^3*d^2*e^2+b^4*e^4-4*b^3*c*d*e^3+6*b^2*c^2*d^2*e^2-4*b
*c^3*d^3*e+2*c^4*d^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 880, normalized size of antiderivative = 2.60

$$\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$
$$= \left[-\frac{3\,c^4e^3nx^4+2\,(8\,c^4de^2-bc^3e^3)nx^3+3\,(12\,c^4d^2e-4\,bc^3de^2+(b^2c^2-2\,ac^3)e^3)nx^2-3\,(4\,c^3d^3-6\,bc^2d\right.$$
$$-\frac{3\,c^4e^3nx^4+2\,(8\,c^4de^2-bc^3e^3)nx^3+3\,(12\,c^4d^2e-4\,bc^3de^2+(b^2c^2-2\,ac^3)e^3)nx^2-6\,(4\,c^3d^3-6\,bc^2d^2$$

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*c^4*e^3*n*x^4 + 2*(8*c^4*d*e^2 - b*c^3*e^3)*n*x^3 + 3*(12*c^4*d^2
*e - 4*b*c^3*d*e^2 + (b^2*c^2 - 2*a*c^3)*e^3)*n*x^2 - 3*(4*c^3*d^3 - 6*b*c^
2*d^2*e + 4*(b^2*c - a*c^2)*d*e^2 - (b^3 - 2*a*b*c)*e^3)*sqrt(b^2 - 4*a*c)*
n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a)) + 6*(8*c^4*d^3 - 6*b*c^3*d^2*e + 4*(b^2*c^2 - 2*a*c^3)*d*
e^2 - (b^3*c - 3*a*b*c^2)*e^3)*n*x - 3*(2*c^4*e^3*n*x^4 + 8*c^4*d*e^2*n*x^3
 + 12*c^4*d^2*e*n*x^2 + 8*c^4*d^3*n*x + (4*b*c^3*d^3 - 6*(b^2*c^2 - 2*a*c^3
)*d^2*e + 4*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3)*
n)*log(c*x^2 + b*x + a) - 6*(c^4*e^3*x^4 + 4*c^4*d*e^2*x^3 + 6*c^4*d^2*e*x^
2 + 4*c^4*d^3*x)*log(d))/c^4, -1/24*(3*c^4*e^3*n*x^4 + 2*(8*c^4*d*e^2 - b*c
^3*e^3)*n*x^3 + 3*(12*c^4*d^2*e - 4*b*c^3*d*e^2 + (b^2*c^2 - 2*a*c^3)*e^3)*
n*x^2 - 6*(4*c^3*d^3 - 6*b*c^2*d^2*e + 4*(b^2*c - a*c^2)*d*e^2 - (b^3 - 2*a
*b*c)*e^3)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
 - 4*a*c)) + 6*(8*c^4*d^3 - 6*b*c^3*d^2*e + 4*(b^2*c^2 - 2*a*c^3)*d*e^2 - (
b^3*c - 3*a*b*c^2)*e^3)*n*x - 3*(2*c^4*e^3*n*x^4 + 8*c^4*d*e^2*n*x^3 + 12*c
^4*d^2*e*n*x^2 + 8*c^4*d^3*n*x + (4*b*c^3*d^3 - 6*(b^2*c^2 - 2*a*c^3)*d^2*e
 + 4*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3)*n)*log(
c*x^2 + b*x + a) - 6*(c^4*e^3*x^4 + 4*c^4*d*e^2*x^3 + 6*c^4*d^2*e*x^2 + 4*c
^4*d^3*x)*log(d))/c^4]
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^3 \log \left(d(a+bx+cx^2)^n\right) \, dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**3*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^3 \log \left(d(a+bx+cx^2)^n\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.46

$$\begin{aligned}
&\int (d+ex)^3 \log \left(d(a+bx+cx^2)^n\right) \, dx \\
&= -\frac{1}{8}\left(e^3n - 2e^3\log(d)\right)x^4 - \frac{(8cde^2n - be^3n - 12cde^2\log(d))x^3}{12c} \\
&\quad + \frac{1}{4}\left(e^3nx^4 + 4de^2nx^3 + 6d^2enx^2 + 4d^3nx\right)\log\left(cx^2+bx+a\right) \\
&\quad - \frac{(12c^2d^2en - 4bcde^2n + b^2e^3n - 2ace^3n - 12c^2d^2e\log(d))x^2}{8c^2} \\
&\quad - \frac{(8c^3d^3n - 6bc^2d^2en + 4b^2cde^2n - 8ac^2de^2n - b^3e^3n + 3abce^3n - 4c^3d^3\log(d))x}{4c^3} \\
&\quad + \frac{(4bc^3d^3n - 6b^2c^2d^2en + 12ac^3d^2en + 4b^3cde^2n - 12abc^2de^2n - b^4e^3n + 4ab^2ce^3n - 2a^2c^2e^3n)\log(cx^2}{8c^4} \\
&\quad - \frac{(4b^2c^3d^3n - 16ac^4d^3n - 6b^3c^2d^2en + 24abc^3d^2en + 4b^4cde^2n - 20ab^2c^2de^2n + 16a^2c^3de^2n - b^5e^3n +}{4\sqrt{-b^2+4ac}c^4}
\end{aligned}$$

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] -1/8*(e^3*n - 2*e^3*log(d))*x^4 - 1/12*(8*c*d*e^2*n - b*e^3*n - 12*c*d*e^2*
log(d))*x^3/c + 1/4*(e^3*n*x^4 + 4*d*e^2*n*x^3 + 6*d^2*e*n*x^2 + 4*d^3*n*x)
*log(c*x^2 + b*x + a) - 1/8*(12*c^2*d^2*e*n - 4*b*c*d*e^2*n + b^2*e^3*n - 2
*a*c*e^3*n - 12*c^2*d^2*e*log(d))*x^2/c^2 - 1/4*(8*c^3*d^3*n - 6*b*c^2*d^2*
e*n + 4*b^2*c*d*e^2*n - 8*a*c^2*d*e^2*n - b^3*e^3*n + 3*a*b*c*e^3*n - 4*c^3
*d^3*log(d))*x/c^3 + 1/8*(4*b*c^3*d^3*n - 6*b^2*c^2*d^2*e*n + 12*a*c^3*d^2*
e*n + 4*b^3*c*d*e^2*n - 12*a*b*c^2*d*e^2*n - b^4*e^3*n + 4*a*b^2*c*e^3*n -
2*a^2*c^2*e^3*n)*log(c*x^2 + b*x + a)/c^4 - 1/4*(4*b^2*c^3*d^3*n - 16*a*c^4
*d^3*n - 6*b^3*c^2*d^2*e*n + 24*a*b*c^3*d^2*e*n + 4*b^4*c*d*e^2*n - 20*a*b^
2*c^2*d*e^2*n + 16*a^2*c^3*d*e^2*n - b^5*e^3*n + 6*a*b^3*c*e^3*n - 8*a^2*b*
c^2*e^3*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 775, normalized size of antiderivative = 2.29

$$\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx$$

$$= \ln \left(d\left(c\,x^2+b\,x+a\right)^n\right) \left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)-x^3\left(\frac{e^2\,n\,(b\,e+8\,c\,d)}{12\,c}-\frac{b\,e^3\,n}{6\,c}\right)$$

$$-x\left(\frac{b\left(\frac{b\left(\frac{e^2\,n\,(b\,e+8\,c\,d)}{4\,c}-\frac{b\,e^3\,n}{2\,c}\right)}{c}+\frac{a\,e^3\,n}{2\,c}-\frac{d\,e\,n\,(b\,e+3\,c\,d)}{c}\right)}{c}-\frac{a\left(\frac{e^2\,n\,(b\,e+8\,c\,d)}{4\,c}-\frac{b\,e^3\,n}{2\,c}\right)}{c}\right.$$

$$\left.+\frac{d^2\,n\,(3\,b\,e+4\,c\,d)}{2\,c}\right)+x^2\left(\frac{b\left(\frac{e^2\,n\,(b\,e+8\,c\,d)}{4\,c}-\frac{b\,e^3\,n}{2\,c}\right)}{2\,c}+\frac{a\,e^3\,n}{4\,c}-\frac{d\,e\,n\,(b\,e+3\,c\,d)}{2\,c}\right)$$

$$-\frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^4\,e^3\,n+2\,a^2\,c^2\,e^3\,n-4\,b\,c^3\,d^3\,n+b^3\,e^3\,n\,\sqrt{b^2-4\,a\,c}\right.}{}$$

$$-\frac{e^3\,n\,x^4}{8}$$

$$-\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^4\,e^3\,n+2\,a^2\,c^2\,e^3\,n-4\,b\,c^3\,d^3\,n-b^3\,e^3\,n\,\sqrt{b^2-4\,a\,c}\right.}{}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^3,x)
```

```
[Out] log(d*(a + b*x + c*x^2)^n)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x
^3) - x^3*((e^2*n*(b*e + 8*c*d))/(12*c) - (b*e^3*n)/(6*c)) - x*((b*((b*((e^
2*n*(b*e + 8*c*d))/(4*c) - (b*e^3*n)/(2*c)))/c + (a*e^3*n)/(2*c) - (d*e*n*(
b*e + 3*c*d))/c))/c - (a*((e^2*n*(b*e + 8*c*d))/(4*c) - (b*e^3*n)/(2*c)))/c
```

```
 + (d^2*n*(3*b*e + 4*c*d))/(2*c)) + x^2*((b*((e^2*n*(b*e + 8*c*d))/(4*c) -
(b*e^3*n)/(2*c)))/(2*c) + (a*e^3*n)/(4*c) - (d*e*n*(b*e + 3*c*d))/(2*c)) -
(log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^4*
e^3*n + 2*a^2*c^2*e^3*n - 4*b*c^3*d^3*n + b^3*e^3*n*(b^2 - 4*a*c)^(1/2) - 4
*c^3*d^3*n*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e^3*n - 12*a*c^3*d^2*e*n - 4*b^3
*c*d*e^2*n + 6*b^2*c^2*d^2*e*n - 2*a*b*c*e^3*n*(b^2 - 4*a*c)^(1/2) + 12*a*b
*c^2*d*e^2*n + 4*a*c^2*d*e^2*n*(b^2 - 4*a*c)^(1/2) + 6*b*c^2*d^2*e*n*(b^2 -
 4*a*c)^(1/2) - 4*b^2*c*d*e^2*n*(b^2 - 4*a*c)^(1/2)))/(8*c^4) - (e^3*n*x^4)
/8 - (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*
(b^4*e^3*n + 2*a^2*c^2*e^3*n - 4*b*c^3*d^3*n - b^3*e^3*n*(b^2 - 4*a*c)^(1/2
) + 4*c^3*d^3*n*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e^3*n - 12*a*c^3*d^2*e*n -
4*b^3*c*d*e^2*n + 6*b^2*c^2*d^2*e*n + 2*a*b*c*e^3*n*(b^2 - 4*a*c)^(1/2) + 1
2*a*b*c^2*d*e^2*n - 4*a*c^2*d*e^2*n*(b^2 - 4*a*c)^(1/2) - 6*b*c^2*d^2*e*n*(
b^2 - 4*a*c)^(1/2) + 4*b^2*c*d*e^2*n*(b^2 - 4*a*c)^(1/2)))/(8*c^4)
```

3.84 $\int (d+ex)^2 \log \left(d(a+bx+cx^2)^n\right) \, dx$

Optimal result 495
Rubi [A] (verified) 495
Mathematica [A] (verified) 498
Maple [A] (verified) 499
Fricas [A] (verification not implemented) 499
Sympy [F(-1)] 500
Maxima [F(-2)] 500
Giac [A] (verification not implemented) 500
Mupad [B] (verification not implemented) 501

Optimal result

Integrand size = 23, antiderivative size = 226

$$\begin{aligned}
&\int (d+ex)^2 \log \left(d(a+bx+cx^2)^n\right) \, dx \\
&= -\frac{(6c^2d^2+b^2e^2-ce(3bd+2ae))\,nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 \\
&\quad + \frac{\sqrt{b^2-4ac}(3c^2d^2+b^2e^2-ce(3bd+ae))\,n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3} \\
&\quad - \frac{(2cd-be)\,(c^2d^2+b^2e^2-ce(bd+3ae))\,n\log(a+bx+cx^2)}{6c^3e} \\
&\quad + \frac{(d+ex)^3\log\left(d(a+bx+cx^2)^n\right)}{3e}
\end{aligned}$$

```
[Out] -1/3*(6*c^2*d^2+b^2*e^2-c*e*(2*a*e+3*b*d))*n*x/c^2-1/6*e*(-b*e+6*c*d)*n*x^2
/c-2/9*e^2*n*x^3-1/6*(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))*n*ln(c*
x^2+b*x+a)/c^3/e+1/3*(e*x+d)^3*ln(d*(c*x^2+b*x+a)^n)/e+1/3*(3*c^2*d^2+b^2*e
^2-c*e*(a*e+3*b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1
/2)/c^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{2605, 814, 648, 632, 212, 642\}$

$$\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx$$
$$= \frac{n\sqrt{b^2-4ac}\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)}{3c^3} - \frac{nx(-ce(2ae+3bd)+b^2e^2+6c^2d^2)}{3c^2} - \frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6c^3e} + \frac{(d+ex)^3\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e} - \frac{enx^2(6cd-be)}{6c} - \frac{2}{9}e^2nx^3$$

```
[In] Int[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/3*((6*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + 2*a*e))*n*x)/c^2 - (e*(6*c*d - b*
e)*n*x^2)/(6*c) - (2*e^2*n*x^3)/9 + (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2
 - c*e*(3*b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) - (
(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[a + b*x + c*x^2
])/(6*c^3*e) + ((d + e*x)^3*Log[d*(a + b*x + c*x^2)^n])/(3*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^3\log\left(d(a+bx+cx^2)^n\right)}{3e} - \frac{n\int \frac{(b+2cx)(d+ex)^3}{a+bx+cx^2}\,dx}{3e}$$

$$= \frac{(d+ex)^3\log\left(d(a+bx+cx^2)^n\right)}{3e} - \frac{n\int \left(\frac{e(6c^2d^2+b^2e^2-ce(3bd+2ae))}{c^2} + \frac{e^2(6cd-be)x}{c} + 2e^3x^2 + \frac{-ab^2e^3-2ace(3cd^2-ae^2)+bcd(cd^2+3ae^2)+(2cd-be)(c^2d^2}{c^2(a+bx+cx^2)}}{3e}$$

$$= -\frac{(6c^2d^2+b^2e^2-ce(3bd+2ae))\,nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 + \frac{(d+ex)^3\log\left(d(a+bx+cx^2)^n\right)}{3e} - \frac{n\int \frac{-ab^2e^3-2ace(3cd^2-ae^2)+bcd(cd^2+3ae^2)+(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))x}{a+bx+cx^2}\,dx}{3c^2e}$$

$$= -\frac{(6c^2d^2+b^2e^2-ce(3bd+2ae))\,nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 + \frac{(d+ex)^3\log\left(d(a+bx+cx^2)^n\right)}{3e} - \frac{\left((b^2-4ac)\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)n\right)\int \frac{1}{a+bx+cx^2}\,dx}{6c^3} - \frac{\left((2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{6c^3e}$$

$$= -\frac{(6c^2d^2+b^2e^2-ce(3bd+2ae))\,nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 - \frac{(2cd-be)\,(c^2d^2+b^2e^2-ce(bd+3ae))\,n\log\left(a+bx+cx^2\right)}{6c^3e} + \frac{(d+ex)^3\log\left(d(a+bx+cx^2)^n\right)}{3e} + \frac{((b^2-4ac)\,(3c^2d^2+b^2e^2-ce(3bd+ae))\,n)\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{3c^3}$$

$$= -\frac{(6c^2d^2+b^2e^2-ce(3bd+2ae))\,nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 + \frac{\sqrt{b^2-4ac}(3c^2d^2+b^2e^2-ce(3bd+ae))\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3} - \frac{(2cd-be)\,(c^2d^2+b^2e^2-ce(bd+3ae))\,n\log\left(a+bx+cx^2\right)}{6c^3e} + \frac{(d+ex)^3\log\left(d(a+bx+cx^2)^n\right)}{3e}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.90

$$\int (d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= \frac{-\frac{n\left(cex\left(6b^2e^2-3ce(6bd+4ae+bex)+2c^2\left(18d^2+9dex+2e^2x^2\right)\right)-6\sqrt{b^2-4ac}e(3c^2d^2+b^2e^2-ce(3bd+ae))\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+3(2cd-be)(c^2d^2}{6c^3}}{3e}$$

```
[In] Integrate[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-1/6*(n*(c*e*x*(6*b^2*e^2 - 3*c*e*(6*b*d + 4*a*e + b*e*x) + 2*c^2*(18*d^2
+ 9*d*e*x + 2*e^2*x^2)) - 6*Sqrt[b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*
(3*b*d + a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*(2*c*d - b*e)*(c^
2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Log[a + x*(b + c*x)]))/c^3 + (d + e*x)
^3*Log[d*(a + x*(b + c*x))^n])/(3*e)
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.70

method	result
parts	$\frac{\ln\left(d\left(c\,x^2+bx+a\right)^n\right)e^2x^3}{3}+\ln\left(d\left(c\,x^2+bx+a\right)^n\right)ed\,x^2+\ln\left(d\left(c\,x^2+bx+a\right)^n\right)d^2x+\frac{\ln\left(d\left(c\,x^2+bx+a\right)^n\right)d}{3e}$
risch	Expression too large to display

```
[In] int((e*x+d)^2*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(d*(c*x^2+b*x+a)^n)*e^2*x^3+ln(d*(c*x^2+b*x+a)^n)*e*d*x^2+ln(d*(c*x^2
+b*x+a)^n)*d^2*x+1/3*ln(d*(c*x^2+b*x+a)^n)/e*d^3-1/3/e*n*(-e/c^2*(-2/3*c^2*
e^2*x^3+1/2*b*c*e^2*x^2-3*c^2*d*e*x^2+2*x*c*a*e^2-x*e^2*b^2+3*x*b*c*d*e-6*c
^2*d^2*x)+1/c^2*(1/2*(3*a*b*c*e^3-6*a*c^2*d*e^2-b^3*e^3+3*b^2*c*d*e^2-3*b*c
^2*d^2*e+2*c^3*d^3)/c*ln(c*x^2+b*x+a)+2*(2*a^2*c*e^3-a*b^2*e^3+3*a*b*c*d*e^
2-6*a*c^2*d^2*e+b*c^2*d^3-1/2*(3*a*b*c*e^3-6*a*c^2*d*e^2-b^3*e^3+3*b^2*c*d*
e^2-3*b*c^2*d^2*e+2*c^3*d^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.51

$$\int (d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$=\Bigg[-\frac{4\,c^3e^2nx^3+3\,(6\,c^3de-bc^2e^2)nx^2+3\,(3\,c^2d^2-3\,bcde+(b^2-ac)e^2)\sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,a}{cx^2+}\right.}{}$$

$$-\frac{4\,c^3e^2nx^3+3\,(6\,c^3de-bc^2e^2)nx^2-6\,(3\,c^2d^2-3\,bcde+(b^2-ac)e^2)\sqrt{-b^2+4\,ac}n\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(}{b^2-4\,a}\right.}{}$$

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/18*(4*c^3*e^2*n*x^3 + 3*(6*c^3*d*e - b*c^2*e^2)*n*x^2 + 3*(3*c^2*d^2 -
3*b*c*d*e + (b^2 - a*c)*e^2)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x +
 b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(6*c^3
*d^2 - 3*b*c^2*d*e + (b^2*c - 2*a*c^2)*e^2)*n*x - 3*(2*c^3*e^2*n*x^3 + 6*c^
3*d*e*n*x^2 + 6*c^3*d^2*n*x + (3*b*c^2*d^2 - 3*(b^2*c - 2*a*c^2)*d*e + (b^3
 - 3*a*b*c)*e^2)*n)*log(c*x^2 + b*x + a) - 6*(c^3*e^2*x^3 + 3*c^3*d*e*x^2 +
```

```
3*c^3*d^2*x)*log(d))/c^3, -1/18*(4*c^3*e^2*n*x^3 + 3*(6*c^3*d*e - b*c^2*e^
2)*n*x^2 - 6*(3*c^2*d^2 - 3*b*c*d*e + (b^2 - a*c)*e^2)*sqrt(-b^2 + 4*a*c)*n
*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(6*c^3*d^2 - 3*b
*c^2*d*e + (b^2*c - 2*a*c^2)*e^2)*n*x - 3*(2*c^3*e^2*n*x^3 + 6*c^3*d*e*n*x^
2 + 6*c^3*d^2*n*x + (3*b*c^2*d^2 - 3*(b^2*c - 2*a*c^2)*d*e + (b^3 - 3*a*b*c
)*e^2)*n)*log(c*x^2 + b*x + a) - 6*(c^3*e^2*x^3 + 3*c^3*d*e*x^2 + 3*c^3*d^2
*x)*log(d))/c^3]
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**2*ln(d*(c*x**2+b*x+a)**n),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.36

$$\begin{aligned}\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx &= -\frac{1}{9}\left(2\,e^2n-3\,e^2\log\left(d\right)\right)x^3 \\ &\quad -\frac{\left(6\,cden-be^2n-6\,cde\log\left(d\right)\right)x^2}{6\,c}+\frac{1}{3}\left(e^2nx^3+3\,denx^2+3\,d^2nx\right)\log\left(cx^2+bx+a\right) \\ &\quad -\frac{\left(6\,c^2d^2n-3\,bcden+b^2e^2n-2\,ace^2n-3\,c^2d^2\log\left(d\right)\right)x}{3\,c^2} \\ &\quad +\frac{\left(3\,bc^2d^2n-3\,b^2cden+6\,ac^2den+b^3e^2n-3\,abce^2n\right)\log\left(cx^2+bx+a\right)}{6\,c^3} \\ &\quad -\frac{\left(3\,b^2c^2d^2n-12\,ac^3d^2n-3\,b^3cden+12\,abc^2den+b^4e^2n-5\,ab^2ce^2n+4\,a^2c^2e^2n\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^3}\end{aligned}$$

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] -1/9*(2*e^2*n - 3*e^2*log(d))*x^3 - 1/6*(6*c*d*e*n - b*e^2*n - 6*c*d*e*log(
d))*x^2/c + 1/3*(e^2*n*x^3 + 3*d*e*n*x^2 + 3*d^2*n*x)*log(c*x^2 + b*x + a)
- 1/3*(6*c^2*d^2*n - 3*b*c*d*e*n + b^2*e^2*n - 2*a*c*e^2*n - 3*c^2*d^2*log(
d))*x/c^2 + 1/6*(3*b*c^2*d^2*n - 3*b^2*c*d*e*n + 6*a*c^2*d*e*n + b^3*e^2*n
- 3*a*b*c*e^2*n)*log(c*x^2 + b*x + a)/c^3 - 1/3*(3*b^2*c^2*d^2*n - 12*a*c^3
*d^2*n - 3*b^3*c*d*e*n + 12*a*b*c^2*d*e*n + b^4*e^2*n - 5*a*b^2*c*e^2*n + 4
*a^2*c^2*e^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^3)
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.02

$$\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = \ln\left(b\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{\frac{d^2\,n\,\sqrt{b^2-4\,a\,c}}{2}+\frac{b\,d^2\,n}{2}+a\,d\,e\,n}{c}-\frac{\frac{a\,b\,e^2\,n}{2}+\frac{b^2\,d\,e\,n}{2}+\frac{a\,e^2\,n\,\sqrt{b^2-4\,a\,c}}{6}+\frac{b\,d\,e\,n\,\sqrt{b^2-4\,a\,c}}{2}}{c^2}+\frac{b^3\,e^2\,n}{6\,c^3}+\frac{b^2\,e^2\,n\,\sqrt{b^2-4\,a\,c}}{6\,c^3}\right)+x\left(\frac{b\left(\frac{e\,n\,(b\,e+6\,c\,d)}{3\,c}-\frac{2\,b\,e^2\,n}{3\,c}\right)}{c}-\frac{d\,n\,(b\,e+2\,c\,d)}{c}+\frac{2\,a\,e^2\,n}{3\,c}\right)-\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{\frac{a\,b\,e^2\,n}{2}+\frac{b^2\,d\,e\,n}{2}-\frac{a\,e^2\,n\,\sqrt{b^2-4\,a\,c}}{6}-\frac{b\,d\,e\,n\,\sqrt{b^2-4\,a\,c}}{2}}{c^2}-\frac{\frac{b\,d^2\,n}{2}-\frac{d^2\,n\,\sqrt{b^2-4\,a\,c}}{2}+a\,d\,e\,n}{c}-\frac{b^3\,e^2\,n}{6\,c^3}+\frac{b^2\,e^2\,n\,\sqrt{b^2-4\,a\,c}}{6\,c^3}\right)+\ln\left(d\,\left(c\,x^2+b\,x+a\right)^n\right)\,\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right)-x^2\,\left(\frac{e\,n\,(b\,e+6\,c\,d)}{6\,c}-\frac{b\,e^2\,n}{3\,c}\right)-\frac{2\,e^2\,n\,x^3}{9}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^2,x)
```

```
[Out] log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(((d^2
*n*(b^2 - 4*a*c)^(1/2))/2 + (b*d^2*n)/2 + a*d*e*n)/c - ((a*b*e^2*n)/2 + (b^
2*d*e*n)/2 + (a*e^2*n*(b^2 - 4*a*c)^(1/2))/6 + (b*d*e*n*(b^2 - 4*a*c)^(1/2)
)/2)/c^2 + (b^3*e^2*n)/(6*c^3) + (b^2*e^2*n*(b^2 - 4*a*c)^(1/2))/(6*c^3)) +
 x*((b*((e*n*(b*e + 6*c*d))/(3*c) - (2*b*e^2*n)/(3*c)))/c - (d*n*(b*e + 2*c
*d))/c + (2*a*e^2*n)/(3*c)) - log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c
*x*(b^2 - 4*a*c)^(1/2))*(((a*b*e^2*n)/2 + (b^2*d*e*n)/2 - (a*e^2*n*(b^2 - 4
```

```
*a*c)^(1/2))/6 - (b*d*e*n*(b^2 - 4*a*c)^(1/2))/2)/c^2 - ((b*d^2*n)/2 - (d^2
*n*(b^2 - 4*a*c)^(1/2))/2 + a*d*e*n)/c - (b^3*e^2*n)/(6*c^3) + (b^2*e^2*n*(
b^2 - 4*a*c)^(1/2))/(6*c^3)) + log(d*(a + b*x + c*x^2)^n)*(d^2*x + (e^2*x^3
)/3 + d*e*x^2) - x^2*((e*n*(b*e + 6*c*d))/(6*c) - (b*e^2*n)/(3*c)) - (2*e^2
*n*x^3)/9
```

3.85 $\int (d+ex)\log\left(d(a+bx+cx^2)^n\right)\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 154

$$\int (d+ex)\log\left(d(a+bx+cx^2)^n\right)\,dx = -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx-\frac{1}{2}enx^2 + \frac{\sqrt{b^2-4ac}(2cd-be)n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{(2c^2d^2+b^2e^2-2ce(bd+ae))\,n\log(a+bx+cx^2)}{4c^2e} + \frac{(d+ex)^2\log\left(d(a+bx+cx^2)^n\right)}{2e}$$

```
[Out] -1/2*(4*d-b*e/c)*n*x-1/2*e*n*x^2-1/4*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))*n*
ln(c*x^2+b*x+a)/c^2/e+1/2*(e*x+d)^2*ln(d*(c*x^2+b*x+a)^n)/e+1/2*(-b*e+2*c*d
)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int (d+ex)\log\left(d(a+bx+cx^2)^n\right)\,dx = \frac{n\sqrt{b^2-4ac}(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{n(-2ce(ae+bd)+b^2e^2+2c^2d^2)\log(a+bx+cx^2)}{4c^2e} + \frac{(d+ex)^2\log\left(d(a+bx+cx^2)^n\right)}{2e} - \frac{1}{2}nx\left(4d-\frac{be}{c}\right)-\frac{1}{2}enx^2$$

```
[In] Int[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/2*((4*d - (b*e)/c)*n*x) - (e*n*x^2)/2 + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)
*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*c^2) - ((2*c^2*d^2 + b^2*e^2
- 2*c*e*(b*d + a*e))*n*Log[a + b*x + c*x^2])/(4*c^2*e) + ((d + e*x)^2*Log[d
*(a + b*x + c*x^2)^n])/(2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}-\frac{n\int\frac{(b+2cx)(d+ex)^2}{a+bx+cx^2}\,dx}{2e}\\
&= \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}\\
&\quad-\frac{n\int\left(e\left(4d-\frac{be}{c}\right)+2e^2x+\frac{bcd^2-4acde+abe^2+\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)x}{c\left(a+bx+cx^2\right)}\right)dx}{2e}\\
&= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx-\frac{1}{2}enx^2+\frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}\\
&\quad-\frac{n\int\frac{bcd^2-4acde+abe^2+\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)x}{a+bx+cx^2}\,dx}{2ce}\\
&= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx-\frac{1}{2}enx^2+\frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}\\
&\quad-\frac{\left(\left(b^2-4ac\right)(2cd-be)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{4c^2}\\
&\quad-\frac{\left(\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\right)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{4c^2e}\\
&= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx-\frac{1}{2}enx^2-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log\left(a+bx+cx^2\right)}{4c^2e}\\
&\quad+\frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}\\
&\quad+\frac{\left(\left(b^2-4ac\right)(2cd-be)n\right)\operatorname{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{2c^2}\\
&= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx-\frac{1}{2}enx^2+\frac{\sqrt{b^2-4ac}(2cd-be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}\\
&\quad-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log\left(a+bx+cx^2\right)}{4c^2e}\\
&\quad+\frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int (d+ex)\log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\frac{-2\sqrt{b^2-4ac}(-2cd+be)n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+(2bcd-b^2e+2ace)\,n\log(a+x(b+cx))+2cx(ben-cn(4}{4c^2}$$

```
[In] Integrate[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-2*Sqrt[b^2 - 4*a*c]*(-2*c*d + b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]] + (2*b*c*d - b^2*e + 2*a*c*e)*n*Log[a + x*(b + c*x)] + 2*c*x*(b*e*n - c*
n*(4*d + e*x) + c*(2*d + e*x)*Log[d*(a + x*(b + c*x))^n]))/(4*c^2)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

method	result
parts	$\frac{\ln\left(d\left(c\,x^2+bx+a\right)^n\right)e\,x^2}{2}+\ln\left(d\left(c\,x^2+bx+a\right)^n\right)dx-\frac{n\left(-\frac{-ce\,x^2+bex-4cdx}{c}+\frac{\frac{\left(-2ace+e\,b^2-2bcd\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(abe-\right.}{}}{}\right.}{2}$
risch	Expression too large to display

```
[In] int((e*x+d)*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(d*(c*x^2+b*x+a)^n)*e*x^2+ln(d*(c*x^2+b*x+a)^n)*d*x-1/2*n*(-1/c*(-c*e
*x^2+b*e*x-4*c*d*x)+1/c*(1/2*(-2*a*c*e+b^2*e-2*b*c*d)/c*ln(c*x^2+b*x+a)+2*(
a*b*e-4*a*c*d-1/2*(-2*a*c*e+b^2*e-2*b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.18

$$\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\left[-\frac{2\,c^2enx^2+\sqrt{b^2-4\,ac}(2\,cd-be)n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(4\,c^2d-bce)nx-(2\,c^2en}{4\,c^2}\right.$$
$$-\frac{2\,c^2enx^2-2\,\sqrt{-b^2+4\,ac}(2\,cd-be)n\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(4\,c^2d-bce)nx-(2\,c^2enx^2+4\,c}{4\,c^2}$$

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*c^2*e*n*x^2 + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*n*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
2*(4*c^2*d - b*c*e)*n*x - (2*c^2*e*n*x^2 + 4*c^2*d*n*x + (2*b*c*d - (b^2 -
2*a*c)*e)*n)*log(c*x^2 + b*x + a) - 2*(c^2*e*x^2 + 2*c^2*d*x)*log(d))/c^2,
-1/4*(2*c^2*e*n*x^2 - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*n*arctan(-sqrt(-b^
2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(4*c^2*d - b*c*e)*n*x - (2*c^2*e*
n*x^2 + 4*c^2*d*n*x + (2*b*c*d - (b^2 - 2*a*c)*e)*n)*log(c*x^2 + b*x + a) -
 2*(c^2*e*x^2 + 2*c^2*d*x)*log(d))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(139) = 278.

Time = 88.72 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.46

$$\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\begin{cases}\frac{ae\log\left(d\left(a+bx+cx^2\right)^n\right)}{2c}-\frac{b^2e\log\left(d\left(a+bx+cx^2\right)^n\right)}{4c^2}+\frac{bd\log\left(d\left(a+bx+cx^2\right)^n\right)}{2c}+\frac{benx}{2c}-\frac{ben\sqrt{-4ac+b^2}\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2c^2}+\frac{b}{} \\ -\frac{a^2e\log\left(d(a+bx)^n\right)}{2b^2}+\frac{ad\log\left(d(a+bx)^n\right)}{b}+\frac{aenx}{2b}-dnx+dx\log\left(d(a+bx)^n\right)-\frac{enx^2}{4}+\frac{ex^2\log\left(d(a+bx)^n\right)}{2}\end{cases}$$

```
[In] integrate((e*x+d)*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((a*e*log(d*(a + b*x + c*x**2)**n)/(2*c) - b**2*e*log(d*(a + b*x +
 c*x**2)**n)/(4*c**2) + b*d*log(d*(a + b*x + c*x**2)**n)/(2*c) + b*e*n*x/(2
*c) - b*e*n*sqrt(-4*a*c + b**2)*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)
)/(2*c**2) + b*e*sqrt(-4*a*c + b**2)*log(d*(a + b*x + c*x**2)**n)/(4*c**2)
- 2*d*n*x + d*x*log(d*(a + b*x + c*x**2)**n) - e*n*x**2/2 + e*x**2*log(d*(a
 + b*x + c*x**2)**n)/2 + d*n*sqrt(-4*a*c + b**2)*log(b/(2*c) + x + sqrt(-4*
a*c + b**2)/(2*c))/c - d*sqrt(-4*a*c + b**2)*log(d*(a + b*x + c*x**2)**n)/(
2*c), Ne(c, 0)), (-a**2*e*log(d*(a + b*x)**n)/(2*b**2) + a*d*log(d*(a + b*x
)**n)/b + a*e*n*x/(2*b) - d*n*x + d*x*log(d*(a + b*x)**n) - e*n*x**2/4 + e*
x**2*log(d*(a + b*x)**n)/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)\log \left(d\left(a+bx+cx^2\right)^n\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$\begin{aligned}\int (d+ex)\log \left(d\left(a+bx+cx^2\right)^n\right)\,dx &= -\frac{1}{2}\,(en-e\log(d))x^2+\frac{1}{2}\,(enx^2+2\,dnx)\log\left(cx^2+bx+a\right) \\ &\quad -\frac{(4\,cdn-ben-2\,cd\log(d))x}{2\,c}+\frac{(2\,bcdn-b^2en+2\,acen)\log\left(cx^2+bx+a\right)}{4\,c^2} \\ &\quad -\frac{(2\,b^2cdn-8\,ac^2dn-b^3en+4\,abcen)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}\end{aligned}$$

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] -1/2*(e*n - e*log(d))*x^2 + 1/2*(e*n*x^2 + 2*d*n*x)*log(c*x^2 + b*x + a) -
1/2*(4*c*d*n - b*e*n - 2*c*d*log(d))*x/c + 1/4*(2*b*c*d*n - b^2*e*n + 2*a*c
*e*n)*log(c*x^2 + b*x + a)/c^2 - 1/2*(2*b^2*c*d*n - 8*a*c^2*d*n - b^3*e*n +
 4*a*b*c*e*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^
2)
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.57

$$\int (d+ex)\log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\ln \left(d\left(c\,x^2+b\,x+a\right)^n\right)\,\left(\frac{e\,x^2}{2}+d\,x\right)-x\,\left(\frac{n\,(b\,e+4\,c\,d)}{2\,c}-\frac{b\,e\,n}{c}\right)-\frac{e\,n\,x^2}{2}$$
$$+\frac{\ln \left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\,\left(\frac{a\,e\,n}{2}+\frac{b\,d\,n}{2}-\frac{d\,n\,\sqrt{b^2-4\,a\,c}}{2}\right)-\frac{b^2\,e\,n}{4}+\frac{b\,e\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)}{c^2}$$
$$-\frac{\ln \left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^2\,e\,n}{4}-c\,\left(\frac{a\,e\,n}{2}+\frac{b\,d\,n}{2}+\frac{d\,n\,\sqrt{b^2-4\,a\,c}}{2}\right)+\frac{b\,e\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)}{c^2}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x),x)
```

```
[Out] log(d*(a + b*x + c*x^2)^n)*(d*x + (e*x^2)/2) - x*((n*(b*e + 4*c*d))/(2*c) -
 (b*e*n)/c) - (e*n*x^2)/2 + (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*
x*(b^2 - 4*a*c)^(1/2))*(c*((a*e*n)/2 + (b*d*n)/2 - (d*n*(b^2 - 4*a*c)^(1/2)
)/2) - (b^2*e*n)/4 + (b*e*n*(b^2 - 4*a*c)^(1/2))/4))/c^2 - (log(b*(b^2 - 4*
a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^2*e*n)/4 - c*((a*
e*n)/2 + (b*d*n)/2 + (d*n*(b^2 - 4*a*c)^(1/2))/2) + (b*e*n*(b^2 - 4*a*c)^(1
/2))/4))/c^2
```

3.86 $\int \log \left(d(a+bx+cx^2)^n\right) \, dx$

Optimal result 510
Rubi [A] (verified) 510
Mathematica [A] (verified) 512
Maple [A] (verified) 512
Fricas [A] (verification not implemented) 513
Sympy [B] (verification not implemented) 513
Maxima [F(-2)] 514
Giac [A] (verification not implemented) 514
Mupad [B] (verification not implemented) 514

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = -2nx+\frac{\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log \left(a+bx+cx^2\right)}{2c}+x\log \left(d\left(a+bx+cx^2\right)^n\right)$$

```
[Out] -2*n*x+1/2*b*n*ln(c*x^2+b*x+a)/c+x*ln(d*(c*x^2+b*x+a)^n)+n*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2603, 787, 648, 632, 212, 642}

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \frac{n\sqrt{b^2-4ac}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log \left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log \left(a+bx+cx^2\right)}{2c}-2nx$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x \log \left(d\left(a+bx+cx^2\right)^n\right)-n \int \frac{x(b+2cx)}{a+bx+cx^2}\,dx\\
&= -2nx+x \log \left(d\left(a+bx+cx^2\right)^n\right)-\frac{n \int \frac{-2ac-bcx}{a+bx+cx^2}\,dx}{c}\\
&= -2nx+x \log \left(d\left(a+bx+cx^2\right)^n\right)+\frac{(bn) \int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c}-\frac{\left(\left(b^2-4ac\right)n\right) \int \frac{1}{a+bx+cx^2}\,dx}{2c}
\end{aligned}$$

$$= -2nx + \frac{bn\log(a+bx+cx^2)}{2c} + x\log\left(d(a+bx+cx^2)^n\right) + \frac{((b^2-4ac)\,n)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}$$

$$= -2nx + \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} + \frac{bn\log(a+bx+cx^2)}{2c} + x\log\left(d(a+bx+cx^2)^n\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \log\left(d(a+bx+cx^2)^n\right)\,dx = \frac{2\sqrt{b^2-4ac}\,n\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) + bn\log(a+x(b+cx)) + 2cx(-2n+\log(d(a+x(b+cx))^n))}{2c}$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a +
 x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.13

method	result
default	$x\ln\left(d(c\,x^2+bx+a)^n\right) - n\left(2x - \frac{b\ln(c\,x^2+bx+a)}{2c} + \frac{2\left(-2a+\frac{b^2}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}}\right)$
parts	$x\ln\left(d(c\,x^2+bx+a)^n\right) - n\left(2x - \frac{b\ln(c\,x^2+bx+a)}{2c} + \frac{2\left(-2a+\frac{b^2}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}}\right)$
risch	$x\ln\left((c\,x^2+bx+a)^n\right) + \frac{i\,\mathrm{csgn}\left(id(c\,x^2+bx+a)^n\right)^2\mathrm{csgn}\left(i(c\,x^2+bx+a)^n\right)x\pi}{2} - \frac{i\pi x\,\mathrm{csgn}\left(i(c\,x^2+bx+a)^n\right)\mathrm{csgn}\left(id(c\,x^2+bx\right.}{2}$

```
[In] int(ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(d*(c*x^2+b*x+a)^n)-n*(2*x-1/2*b/c*ln(c*x^2+b*x+a)+2*(-2*a+1/2*b^2/c)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.41

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\left[-\frac{4\,cnx-2\,cx\log\left(d\right)-\sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(2\,cnx+bn)\log\left(cx^2+bx\right.}{2\,c}\right.$$
$$\left.-\frac{4\,cnx-2\,cx\log\left(d\right)-2\sqrt{-b^2+4\,ac}n\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-(2\,cnx+bn)\log\left(cx^2+bx+a\right)}{2\,c}\right]$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*c*
n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqrt(-
b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c
*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(75) = 150$.

Time = 33.64 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.47

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)\,dx$$
$$=\begin{cases}\frac{a\log\left(d(a+bx)^n\right)}{b}-nx+x\log\left(d(a+bx)^n\right)\\ \frac{b\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{2c}-2nx+x\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)\\ -\frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}}+\frac{2a\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{-4ac+b^2}}+\frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}}-\frac{b^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2c\sqrt{-4ac+b^2}}+\frac{b\log\left(d(a+bx\right.}{2c}\end{cases}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((a*log(d*(a + b*x)**n)/b - n*x + x*log(d*(a + b*x)**n), Eq(c, 0))
, (b*log(d*(b**2/(4*c) + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(b**2/(4
*c) + b*x + c*x**2)**n), Eq(a, b**2/(4*c))), (-4*a*n*log(b/(2*c) + x + sqrt
(-4*a*c + b**2)/(2*c))/sqrt(-4*a*c + b**2) + 2*a*log(d*(a + b*x + c*x**2)**
n)/sqrt(-4*a*c + b**2) + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)
)/(c*sqrt(-4*a*c + b**2)) - b**2*log(d*(a + b*x + c*x**2)**n)/(2*c*sqrt(-4*
a*c + b**2)) + b*log(d*(a + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(a +
b*x + c*x**2)**n), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = nx\log \left(cx^2+bx+a\right) - (2\,n - \log \left(d\right))x + \frac{bn\log \left(cx^2+bx+a\right)}{2\,c} - \frac{\left(b^2n - 4\,acn\right)\arctan \left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)/
c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.52

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = x\,\ln \left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right) - 2\,n\,x - \frac{n\,\mathrm{atan}\left(\frac{b\,n\,\sqrt{4\,a\,c-b^2}}{2\,\left(\frac{b^2\,n}{2}-2\,a\,c\,n\right)} - \frac{n\,x\,\sqrt{4\,a\,c-b^2}}{2\,a\,n-\frac{b^2\,n}{2\,c}}\right)\,\sqrt{4\,a\,c-b^2}}{c} + \frac{b\,n\,\ln \left(c\,x^2+b\,x+a\right)}{2\,c}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x*log(d*(a + b*x + c*x^2)^n) - 2*n*x - (n*atan((b*n*(4*a*c - b^2)^(1/2))/(2
*((b^2*n)/2 - 2*a*c*n)) - (n*x*(4*a*c - b^2)^(1/2))/(2*a*n - (b^2*n)/(2*c))
)*(4*a*c - b^2)^(1/2))/c + (b*n*log(a + b*x + c*x^2))/(2*c)
```

3.87 $\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx$

Optimal result . . . 515
Rubi [A] (verified) . . . 516
Mathematica [A] (verified) . . . 518
Maple [A] (verified) . . . 519
Fricas [F] . . . 519
Sympy [F(-1)] . . . 519
Maxima [F] . . . 520
Giac [F] . . . 520
Mupad [F(-1)] . . . 520

Optimal result

Integrand size = 23, antiderivative size = 228

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx = -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} - \frac{n\,\mathrm{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{e} - \frac{n\,\mathrm{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e}$$

```
[Out] ln(e*x+d)*ln(d*(c*x^2+b*x+a)^n)/e-n*ln(e*x+d)*ln(-e*(b+2*c*x-(-4*a*c+b^2)^(
1/2))/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))/e-n*ln(e*x+d)*ln(-e*(b+2*c*x+(-4*a*
c+b^2)^(1/2))/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e-n*polylog(2,2*c*(e*x+d)/(
2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))/e-n*polylog(2,2*c*(e*x+d)/(2*c*d-e*(b+(-4*
a*c+b^2)^(1/2))))/e
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2604, 2465, 2441, 2440, 2438}

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx = -\frac{n\,\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{e} - \frac{n\,\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e} - \frac{n\log(d+ex)\log\left(-\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e} - \frac{n\log(d+ex)\log\left(-\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{e} + \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

```
[Out] -((n*Log[-((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))]*Log[d + e*x])/e) - (n*Log[-((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))]*Log[d + e*x])/e + (Log[d + e*x]*Log[d*
(a + b*x + c*x^2)^n])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/e
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} - \frac{n\int\frac{(b+2cx)\log(d+ex)}{a+bx+cx^2}\,dx}{e} \\
&= \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} - \frac{n\int\left(\frac{2c\log(d+ex)}{b-\sqrt{b^2-4ac}+2cx}+\frac{2c\log(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{e} \\
&= \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} - \frac{(2cn)\int\frac{\log(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{e} - \frac{(2cn)\int\frac{\log(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{e} \\
&= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} \\
&\quad - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} \\
&\quad + n\int\frac{\log\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\right)}{d+ex}\,dx + n\int\frac{\log\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\right)}{d+ex}\,dx
\end{aligned}
$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} + \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\right)}{x}\,dx, x, d+ex\right)}{e} + \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\right)}{x}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} - \frac{n\text{Li}_2\left(\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{e} - \frac{n\text{Li}_2\left(\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.88

$$\int\frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx = \frac{\log(d+ex)\left(-n\log\left(\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd-be+\sqrt{b^2-4ac}e}\right) - n\log\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\right) + \log\left(d(a+x(b+cx))^n\right)\right) - n\,\text{Pol}}{e}$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

```
[Out] (Log[d + e*x]*(-(n*Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d - b*e +
Sqrt[b^2 - 4*a*c]*e)]) - n*Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d
+ (b + Sqrt[b^2 - 4*a*c])*e)] + Log[d*(a + x*(b + c*x))^n]) - n*PolyLog[2,
(2*c*(d + e*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)] - n*PolyLog[2, (2*c*(
d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/e
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.32

method	result
parts	$\frac{\ln(ex+d)\ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{e}-\frac{n\left(\ln(ex+d)\ln\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ac\,e^2+e^2b^2}}{-be+2cd+\sqrt{-4ac\,e^2+e^2b^2}}\right)+\ln(ex+d)\ln\left(\frac{be-2cd+2c(ex+d)+\sqrt{-4ac\,e^2+e^2b^2}}{be-2cd+\sqrt{-4ac\,e^2+e^2b^2}}\right)\right.}{e}$
risch	$\frac{\ln\left(\left(c\,x^2+bx+a\right)^n\right)\ln(ex+d)}{e}-\frac{n\ln(ex+d)\ln\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ac\,e^2+e^2b^2}}{-be+2cd+\sqrt{-4ac\,e^2+e^2b^2}}\right)}{e}-\frac{n\ln(ex+d)\ln\left(\frac{be-2cd+2c(ex+d)+\sqrt{-4ac\,e^2+e^2b^2}}{be-2cd+\sqrt{-4ac\,e^2+e^2b^2}}\right)}{e}$

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)*ln(d*(c*x^2+b*x+a)^n)/e-1/e*n*(ln(e*x+d)*ln((-b*e+2*c*d-2*c*(e*x+
d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))+ln(
e*x+d)*ln((b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/(b*e-2*c*d+(-4
*a*c*e^2+b^2*e^2)^(1/2)))+dilog((-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2
)^(1/2))/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))+dilog((b*e-2*c*d+2*c*(e*x
+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))))
```

Fricas [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx=\int \frac{\log\left((cx^2+bx+a)^n d\right)}{ex+d}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx=\text{Timed out}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx = \int \frac{\log\left((cx^2+bx+a)^n d\right)}{ex+d}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx = \int \frac{\log\left((cx^2+bx+a)^n d\right)}{ex+d}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx = \int \frac{\ln\left(d\,(c\,x^2+b\,x+a)^n\right)}{d+e\,x}\,dx$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x),x)
[Out] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x), x)
```

3.88 $\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx$

Optimal result 521
Rubi [A] (verified) 521
Mathematica [A] (verified) 523
Maple [A] (verified) 524
Fricas [A] (verification not implemented) 524
Sympy [F(-1)] 525
Maxima [F(-2)] 525
Giac [A] (verification not implemented) 525
Mupad [B] (verification not implemented) 526

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \frac{\sqrt{b^2-4ac}\,n\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd^2-bde+ae^2} - \frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)n\log\left(a+bx+cx^2\right)}{2e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)}$$

```
[Out] -(-b*e+2*c*d)*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)+1/2*(-b*e+2*c*d)*n*ln(c*x^2
+b*x+a)/e/(a*e^2-b*d*e+c*d^2)-ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)+n*arctanh((2*
c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \frac{n\sqrt{b^2-4ac}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{ae^2-bde+cd^2} + \frac{n(2cd-be)\log\left(a+bx+cx^2\right)}{2e\left(ae^2-bde+cd^2\right)} - \frac{n(2cd-be)\log(d+ex)}{e\left(ae^2-bde+cd^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*d^2 - b*d*e
 + a*e^2) - ((2*c*d - b*e)*n*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^2)) + ((
2*c*d - b*e)*n*Log[a + b*x + c*x^2])/(2*e*(c*d^2 - b*d*e + a*e^2)) - Log[d*
(a + b*x + c*x^2)^n]/(e*(d + e*x))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)} + \frac{n\int\frac{b+2cx}{(d+ex)(a+bx+cx^2)}\,dx}{e}$$

$$= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)} + \frac{n\int\left(\frac{e(-2cd+be)}{(cd^2-bde+ae^2)(d+ex)}+\frac{bcd-b^2e+2ace+c(2cd-be)x}{(cd^2-bde+ae^2)(a+bx+cx^2)}\right)dx}{e}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)} + \frac{n\int\frac{bcd-b^2e+2ace+c(2cd-be)x}{a+bx+cx^2}\,dx}{e\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)} - \frac{\left(\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)} + \frac{((2cd-be)n)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2e\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)n\log\left(a+bx+cx^2\right)}{2e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)} + \frac{\left(\left(b^2-4ac\right)n\right)\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{cd^2-bde+ae^2}$$

$$= \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd^2-bde+ae^2} - \frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)n\log\left(a+bx+cx^2\right)}{2e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01

$$\int\frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = -\frac{\sqrt{-b^2+4ac}\,n\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{-cd^2+e(bd-ae)} + \frac{(-2cd+be)n\log(d+ex)}{e\left(cd^2+e(-bd+ae)\right)} - \frac{(-2cd+be)n\log(a+x(b+cx))}{2e\left(cd^2+e(-bd+ae)\right)} - \frac{\log\left(d(a+x(b+cx))^n\right)}{e(d+ex)}$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-(c*d^2) +
 e*(b*d - a*e))) + ((-2*c*d + b*e)*n*Log[d + e*x])/(e*(c*d^2 + e*(-(b*d) +
a*e))) - ((-2*c*d + b*e)*n*Log[a + x*(b + c*x)])/(2*e*(c*d^2 + e*(-(b*d) +
a*e))) - Log[d*(a + x*(b + c*x))^n]/(e*(d + e*x))
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12

method	result
parts	$-\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)}{e(ex+d)}+\frac{n\left(\frac{(be-2cd)\ln(ex+d)}{a\,e^{2}-bde+c\,d^{2}}+\frac{\frac{\left(-bce+2c^{2}d\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(2ace-e\,b^{2}+bcd-\frac{\left(-bce+2c^{2}d\right)b}{2c}\right)\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^{2}}}\right)}{\sqrt{4ca-b^{2}}}}{a\,e^{2}-bde+c\,d^{2}}\right)}{e}$
risch	Expression too large to display

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)+1/e*n*((b*e-2*c*d)/(a*e^2-b*d*e+c*d^2)*ln(
e*x+d)+1/(a*e^2-b*d*e+c*d^2)*(1/2*(-b*c*e+2*c^2*d)/c*ln(c*x^2+b*x+a)+2*(2*a
*c*e-e*b^2+b*c*d-1/2*(-b*c*e+2*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.60

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx$$
$$=\left[\frac{(e^2nx+den)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+((2\,cde-be^2)nx+(bde-2\,ae^2)n)\log\left(\right.}{2\left(cd^3e-bd^2e^2+ade^3+(cd^2\right.}\right.$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((e^2*n*x + d*e*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((2*c*d*e - b*e
^2)*n*x + (b*d*e - 2*a*e^2)*n)*log(c*x^2 + b*x + a) - 2*((2*c*d*e - b*e^2)*
n*x + (2*c*d^2 - b*d*e)*n)*log(e*x + d) - 2*(c*d^2 - b*d*e + a*e^2)*log(d))
/(c*d^3*e - b*d^2*e^2 + a*d*e^3 + (c*d^2*e^2 - b*d*e^3 + a*e^4)*x), 1/2*(2*
(e^2*n*x + d*e*n)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + ((2*c*d*e - b*e^2)*n*x + (b*d*e - 2*a*e^2)*n)*log(c*x^2 +
 b*x + a) - 2*((2*c*d*e - b*e^2)*n*x + (2*c*d^2 - b*d*e)*n)*log(e*x + d) -
2*(c*d^2 - b*d*e + a*e^2)*log(d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3 + (c*d^2*e
^2 - b*d*e^3 + a*e^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \frac{(2\,cdn-ben)\log\left(cx^2+bx+a\right)}{2\,(cd^2e-bde^2+ae^3)} - \frac{n\log\left(cx^2+bx+a\right)}{e^2x+de} - \frac{(2\,cdn-ben)\log\left(ex+d\right)}{cd^2e-bde^2+ae^3} - \frac{(b^2n-4\,acn)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(cd^2-bde+ae^2)\sqrt{-b^2+4\,ac}} - \frac{\log\left(d\right)}{e^2x+de}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*c*d*n - b*e*n)*log(c*x^2 + b*x + a)/(c*d^2*e - b*d*e^2 + a*e^3) - n*
log(c*x^2 + b*x + a)/(e^2*x + d*e) - (2*c*d*n - b*e*n)*log(e*x + d)/(c*d^2*
e - b*d*e^2 + a*e^3) - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a
*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c)) - log(d)/(e^2*x + d*e)
```

Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 590, normalized size of antiderivative = 3.58

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = \frac{\ln (d+e\,x)\,\left(b\,e\,n-2\,c\,d\,n\right)}{c\,d^2\,e-b\,d\,e^2+a\,e^3}-\frac{\ln \left(d\,\left(c\,x^2+b\,x+a\right)^n\right)}{e\,\left(d+e\,x\right)}$$
$$-\frac{\ln \left(\frac{2\,b\,c^2\,n^2}{e}+\frac{4\,c^3\,n^2\,x}{e}-\frac{n\,\left(b\,e-2\,c\,d+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(c^2\,n\,x\,(b\,e-2\,c\,d)-c\,n\,\left(-e\,b^2+c\,d\,b+2\,a\,c\,e\right)+\frac{c\,e\,n\,\left(b\,e-2\,c\,d+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,d\,e+2\,x\,b\right.}{2}\right.}{2\,\left(c\,d^2\,e-b\,d\,e^2+a\,e^3\right)}\right.}{c\,d^2\,e-b\,d\,e^2+a\,e^3}$$
$$-\frac{\ln \left(\frac{2\,b\,c^2\,n^2}{e}+\frac{4\,c^3\,n^2\,x}{e}-\frac{n\,\left(2\,c\,d-b\,e+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\,n\,\left(-e\,b^2+c\,d\,b+2\,a\,c\,e\right)-c^2\,n\,x\,(b\,e-2\,c\,d)+\frac{c\,e\,n\,\left(2\,c\,d-b\,e+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,d\,e+2\,x\,b\right.}{2}\right.}{2\,\left(c\,d^2\,e-b\,d\,e^2+a\,e^3\right)}\right.}{c\,d^2\,e-b\,d\,e^2+a\,e^3}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^2,x)
```

```
[Out] (log(d + e*x)*(b*e*n - 2*c*d*n))/(a*e^3 - b*d*e^2 + c*d^2*e) - log(d*(a + b
*x + c*x^2)^n)/(e*(d + e*x)) - (log((2*b*c^2*n^2)/e + (4*c^3*n^2*x)/e - (n*
(b*e - 2*c*d + e*(b^2 - 4*a*c)^(1/2))*(c^2*n*x*(b*e - 2*c*d) - c*n*(2*a*c*e
 - b^2*e + b*c*d) + (c*e*n*(b*e - 2*c*d + e*(b^2 - 4*a*c)^(1/2))*(2*b^2*e^2
*x + 2*c^2*d^2*x + a*b*e^2 + b*c*d^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e -
2*b*c*d*e*x))/(2*(a*e^3 - b*d*e^2 + c*d^2*e))))/(2*(a*e^3 - b*d*e^2 + c*d^2
*e)))*(e*((b*n)/2 + (n*(b^2 - 4*a*c)^(1/2))/2) - c*d*n))/(a*e^3 - b*d*e^2 +
 c*d^2*e) - (log((2*b*c^2*n^2)/e + (4*c^3*n^2*x)/e - (n*(2*c*d - b*e + e*(b
^2 - 4*a*c)^(1/2))*(c*n*(2*a*c*e - b^2*e + b*c*d) - c^2*n*x*(b*e - 2*c*d) +
 (c*e*n*(2*c*d - b*e + e*(b^2 - 4*a*c)^(1/2))*(2*b^2*e^2*x + 2*c^2*d^2*x +
a*b*e^2 + b*c*d^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e - 2*b*c*d*e*x))/(2*(a
*e^3 - b*d*e^2 + c*d^2*e))))/(2*(a*e^3 - b*d*e^2 + c*d^2*e)))*(e*((b*n)/2 -
 (n*(b^2 - 4*a*c)^(1/2))/2) - c*d*n))/(a*e^3 - b*d*e^2 + c*d^2*e)
```

3.89 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3} dx$

Optimal result 527
Rubi [A] (verified) 528
Mathematica [A] (verified) 530
Maple [A] (verified) 530
Fricas [B] (verification not implemented) 531
Sympy [F(-1)] 532
Maxima [F(-2)] 532
Giac [A] (verification not implemented) 533
Mupad [B] (verification not implemented) 533

Optimal result

Integrand size = 23, antiderivative size = 259

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^3} dx = \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{b^2-4ac}(2cd-be)n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2\left(cd^2-bde+ae^2\right)^2} - \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log\left(a+bx+cx^2\right)}{4e\left(cd^2-bde+ae^2\right)^2} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{2e(d+ex)^2}$$

```
[Out] 1/2*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)-1/2*(2*c^2*d^2+b^2*e^2-2*c
*e*(a*e+b*d))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^2+1/4*(2*c^2*d^2+b^2*e^2-2*
c*e*(a*e+b*d))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^2)^2-1/2*ln(d*(c*x^2+b*
x+a)^n)/e/(e*x+d)^2+1/2*(-b*e+2*c*d)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)*(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \frac{n\sqrt{b^2-4ac}(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2\left(ae^2-bde+cd^2\right)^2} + \frac{n(-2ce(ae+bd)+b^2e^2+2c^2d^2)\log\left(a+bx+cx^2\right)}{4e\left(ae^2-bde+cd^2\right)^2} - \frac{n\log(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{2e\left(ae^2-bde+cd^2\right)^2} + \frac{n(2cd-be)}{2e(d+ex)\left(ae^2-bde+cd^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{2e(d+ex)^2}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```

```
[Out] ((2*c*d - b*e)*n)/(2*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)) + (Sqrt[b^2 - 4*a
*c]*(2*c*d - b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*(c*d^2 - b*d
*e + a*e^2)^2) - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*Log[d + e*x])
/(2*e*(c*d^2 - b*d*e + a*e^2)^2) + ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e
))*n*Log[a + b*x + c*x^2])/(4*e*(c*d^2 - b*d*e + a*e^2)^2) - Log[d*(a + b*x
 + c*x^2)^n]/(2*e*(d + e*x)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}+\frac{n\int\frac{b+2cx}{(d+ex)^2(a+bx+cx^2)}\,dx}{2e}\\
&= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}\\
&\quad+\frac{n\int\left(\frac{e(-2cd+be)}{(cd^2-bde+ae^2)(d+ex)^2}+\frac{e\left(-2c^2d^2-b^2e^2+2ce(bd+ae)\right)}{(cd^2-bde+ae^2)^2(d+ex)}+\frac{-2b^2cde+4ac^2de+b^3e^2+bc\left(cd^2-3ae^2\right)+c\left(2c^2d^2+b^2e^2-2c\cdots\right)}{(cd^2-bde+ae^2)^2(a+bx+cx^2)}\cdots\right)}{2e}\\
&= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2}\\
&\quad-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}+\frac{n\int\frac{-2b^2cde+4ac^2de+b^3e^2+bc\left(cd^2-3ae^2\right)+c\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)x}{a+bx+cx^2}\,dx}{2e\left(cd^2-bde+ae^2\right)^2}\\
&= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2}\\
&\quad-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}-\frac{\left(\left(b^2-4ac\right)(2cd-be)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{4\left(cd^2-bde+ae^2\right)^2}\\
&\quad+\frac{\left(\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\right)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{4e\left(cd^2-bde+ae^2\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2}\\
&\quad+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log\left(a+bx+cx^2\right)}{4e\left(cd^2-bde+ae^2\right)^2}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}\\
&\quad+\frac{\left(\left(b^2-4ac\right)(2cd-be)n\right)\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{2\left(cd^2-bde+ae^2\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\sqrt{b^2-4ac}(2cd-be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2\left(cd^2-bde+ae^2\right)^2}\\
&\quad-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2}\\
&\quad+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log\left(a+bx+cx^2\right)}{4e\left(cd^2-bde+ae^2\right)^2}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.83

$$
\int\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3}\,dx
$$

$$
=\frac{\frac{n(d+ex)\left(2(2cd-be)\left(cd^2+e(-bd+ae)\right)-2\sqrt{b^2-4ac}e(-2cd+be)(d+ex)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)(d+ex)\log(d+ex)+(\cdots}{\left(cd^2+e(-bd+ae)\right)^2}}{4e(d+ex)^2}
$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```

```
[Out] ((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e)) - 2*Sqrt[b^2 - 4*
a*c]*e*(-2*c*d + b*e)*(d + e*x)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*
(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[d + e*x] + (2*c^2*d
^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[a + x*(b + c*x)]))/(c*d^2 +
 e*(-(b*d) + a*e))^2 - 2*Log[d*(a + x*(b + c*x))^n])/(4*e*(d + e*x)^2)
```

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.20

method	result
parts	$-\frac{\ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{2e(ex+d)^2}+\frac{n\left(\frac{\left(2ac\,e^2-e^2b^2+2bcde-2c^2d^2\right)\ln(ex+d)}{\left(a\,e^2-bde+c\,d^2\right)^2}-\frac{be-2cd}{\left(a\,e^2-bde+c\,d^2\right)(ex+d)}+\frac{\frac{\left(-2c^2a\,e^2+b^2c\,e^2-2b\,c^2de+2c^3d^2\right)\ln\left(c\,x\right.}{2c}}{}\right.}{}$
risch	Expression too large to display

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^2+1/2/e*n*((2*a*c*e^2-b^2*e^2+2*b*c*d*
e-2*c^2*d^2)/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)-(b*e-2*c*d)/(a*e^2-b*d*e+c*d^2
)/(e*x+d)+1/(a*e^2-b*d*e+c*d^2)^2*(1/2*(-2*a*c^2*e^2+b^2*c*e^2-2*b*c^2*d*e+
2*c^3*d^2)/c*ln(c*x^2+b*x+a)+2*(-3*a*b*c*e^2+4*a*c^2*d*e+b^3*e^2-2*b^2*c*d*
e+b*c^2*d^2-1/2*(-2*a*c^2*e^2+b^2*c*e^2-2*b*c^2*d*e+2*c^3*d^2)*b/c)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(245) = 490$.

Time = 1.55 (sec) , antiderivative size = 1341, normalized size of antiderivative = 5.18

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(2*c^2*d^3*e - 3*b*c*d^2*e^2 - a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*n*x -
 ((2*c*d*e^3 - b*e^4)*n*x^2 + 2*(2*c*d^2*e^2 - b*d*e^3)*n*x + (2*c*d^3*e -
b*d^2*e^2)*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sq
rt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(2*c^2*d^4 - 3*b*c*d^3*
e - a*b*d*e^3 + (b^2 + 2*a*c)*d^2*e^2)*n + ((2*c^2*d^2*e^2 - 2*b*c*d*e^3 +
(b^2 - 2*a*c)*e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d
*e^3)*n*x + (2*b*c*d^3*e + 4*a*b*d*e^3 - 2*a^2*e^4 - (b^2 + 6*a*c)*d^2*e^2)
*n)*log(c*x^2 + b*x + a) - 2*((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 - 2*a*c)*
e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)*n*x + (2
*c^2*d^4 - 2*b*c*d^3*e + (b^2 - 2*a*c)*d^2*e^2)*n)*log(e*x + d) - 2*(c^2*d^
4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log(d))/(c
^2*d^6*e - 2*b*c*d^5*e^2 - 2*a*b*d^3*e^4 + a^2*d^2*e^5 + (b^2 + 2*a*c)*d^4*
e^3 + (c^2*d^4*e^3 - 2*b*c*d^3*e^4 - 2*a*b*d*e^6 + a^2*e^7 + (b^2 + 2*a*c)*
d^2*e^5)*x^2 + 2*(c^2*d^5*e^2 - 2*b*c*d^4*e^3 - 2*a*b*d^2*e^5 + a^2*d*e^6 +
 (b^2 + 2*a*c)*d^3*e^4)*x), 1/4*(2*(2*c^2*d^3*e - 3*b*c*d^2*e^2 - a*b*e^4 +
 (b^2 + 2*a*c)*d*e^3)*n*x + 2*((2*c*d*e^3 - b*e^4)*n*x^2 + 2*(2*c*d^2*e^2 -
 b*d*e^3)*n*x + (2*c*d^3*e - b*d^2*e^2)*n)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(
```

```
-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(2*c^2*d^4 - 3*b*c*d^3*e - a*b
*d*e^3 + (b^2 + 2*a*c)*d^2*e^2)*n + ((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 -
2*a*c)*e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)*n
*x + (2*b*c*d^3*e + 4*a*b*d*e^3 - 2*a^2*e^4 - (b^2 + 6*a*c)*d^2*e^2)*n)*log
(c*x^2 + b*x + a) - 2*((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 - 2*a*c)*e^4)*n*
x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)*n*x + (2*c^2*d^
4 - 2*b*c*d^3*e + (b^2 - 2*a*c)*d^2*e^2)*n)*log(e*x + d) - 2*(c^2*d^4 - 2*b
*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log(d))/(c^2*d^6*
e - 2*b*c*d^5*e^2 - 2*a*b*d^3*e^4 + a^2*d^2*e^5 + (b^2 + 2*a*c)*d^4*e^3 + (
c^2*d^4*e^3 - 2*b*c*d^3*e^4 - 2*a*b*d*e^6 + a^2*e^7 + (b^2 + 2*a*c)*d^2*e^5
)*x^2 + 2*(c^2*d^5*e^2 - 2*b*c*d^4*e^3 - 2*a*b*d^2*e^5 + a^2*d*e^6 + (b^2 +
 2*a*c)*d^3*e^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.89

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \frac{(2\,c^2d^2n-2\,bcden+b^2e^2n-2\,ace^2n)\log\left(cx^2+bx+a\right)}{4\,(c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5)} - \frac{n\log\left(cx^2+bx+a\right)}{2\,(e^3x^2+2\,de^2x+d^2e)} - \frac{(2\,c^2d^2n-2\,bcden+b^2e^2n-2\,ace^2n)\log\left(ex+d\right)}{2\,(c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5)} - \frac{(2\,b^2cdn-8\,ac^2dn-b^3en+4\,abcen)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)\sqrt{-b^2+4\,ac}} + \frac{2\,cdenx-be^2nx+2\,cd^2n-bden-cd^2\log\left(d\right)+bde\log\left(d\right)-ae^2\log\left(d\right)}{2\,(cd^2e^3x^2-bde^4x^2+ae^5x^2+2\,cd^3e^2x-2\,bd^2e^3x+2\,ade^4x+cd^4e-bd^3e^2+ad^2e^3)}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*c^2*d^2*n - 2*b*c*d*e*n + b^2*e^2*n - 2*a*c*e^2*n)*log(c*x^2 + b*x +
 a)/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4
+ a^2*e^5) - 1/2*n*log(c*x^2 + b*x + a)/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/2
*(2*c^2*d^2*n - 2*b*c*d*e*n + b^2*e^2*n - 2*a*c*e^2*n)*log(e*x + d)/(c^2*d^
4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5)
- 1/2*(2*b^2*c*d*n - 8*a*c^2*d*n - b^3*e*n + 4*a*b*c*e*n)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*c*d*e*n*x - b*e^2*n*
x + 2*c*d^2*n - b*d*e*n - c*d^2*log(d) + b*d*e*log(d) - a*e^2*log(d))/(c*d^
2*e^3*x^2 - b*d*e^4*x^2 + a*e^5*x^2 + 2*c*d^3*e^2*x - 2*b*d^2*e^3*x + 2*a*d
*e^4*x + c*d^4*e - b*d^3*e^2 + a*d^2*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 1715, normalized size of antiderivative = 6.62

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx = \text{Too large to display}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^3,x)
```

```
[Out] (log(3*b^2*c^3*d^4 - 12*a*c^4*d^4 - 2*b^5*e^4*x - 12*a^3*c^2*e^4 - 2*a*b^4*
e^4 + 2*b^4*e^4*x*(b^2 - 4*a*c)^(1/2) + 6*c^4*d^4*x*(b^2 - 4*a*c)^(1/2) + 1
1*a^2*b^2*c*e^4 - 2*b^3*c^2*d^3*e + b^4*c*d^2*e^2 + 40*a^2*c^3*d^2*e^2 + 2*
a*b^3*e^4*(b^2 - 4*a*c)^(1/2) + 3*b*c^3*d^4*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^3
```

*d^3*e + 6*a*b^3*c*d*e^3 + 12*a*b^3*c*e^4*x - 32*a*c^4*d^3*e*x + 8*b^4*c*d*
e^3*x - 5*a^2*b*c*e^4*(b^2 - 4*a*c)^(1/2) - 16*a*c^3*d^3*e*(b^2 - 4*a*c)^(1
/2) - 24*a^2*b*c^2*d*e^3 - 16*a^2*b*c^2*e^4*x + 32*a^2*c^3*d*e^3*x + 8*b^2*
c^3*d^3*e*x + 16*a^2*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^2*d^3*e*(b^2 -
 4*a*c)^(1/2) + b^3*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*e^4*x*(b^2 -
4*a*c)^(1/2) - 14*a*b^2*c^2*d^2*e^2 - 12*b^3*c^2*d^2*e^2*x + 14*a*b*c^2*d^2
e^2(b^2 - 4*a*c)^(1/2) - 20*a*c^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 14*b^2*
c^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 10*a*b^2*c*d*e^3*(b^2 - 4*a*c)^(1/2) -
8*a*b^2*c*e^4*x*(b^2 - 4*a*c)^(1/2) - 12*b*c^3*d^3*e*x*(b^2 - 4*a*c)^(1/2)
- 8*b^3*c*d*e^3*x*(b^2 - 4*a*c)^(1/2) + 48*a*b*c^3*d^2*e^2*x - 40*a*b^2*c^2
*d*e^3*x + 20*a*b*c^2*d*e^3*x*(b^2 - 4*a*c)^(1/2))*(e*((c*d*n*(b^2 - 4*a*c)
^(1/2))/2 - (b*c*d*n)/2) - e^2*((a*c*n)/2 - (b^2*n)/4 + (b*n*(b^2 - 4*a*c)^
(1/2))/4) + (c^2*d^2*n)/2))/(a^2*e^5 + c^2*d^4*e + b^2*d^2*e^3 - 2*a*b*d*e^
4 + 2*a*c*d^2*e^3 - 2*b*c*d^3*e^2) - (log(d + e*x)*(e^2*(b^2*n - 2*a*c*n) +
 2*c^2*d^2*n - 2*b*c*d*e*n))/(2*a^2*e^5 + 2*c^2*d^4*e + 2*b^2*d^2*e^3 - 4*a
*b*d*e^4 + 4*a*c*d^2*e^3 - 4*b*c*d^3*e^2) + (log(2*a*b^4*e^4 + 12*a*c^4*d^4
 + 2*b^5*e^4*x + 12*a^3*c^2*e^4 - 3*b^2*c^3*d^4 + 2*b^4*e^4*x*(b^2 - 4*a*c)
^(1/2) + 6*c^4*d^4*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^2*c*e^4 + 2*b^3*c^2*d^3
*e - b^4*c*d^2*e^2 - 40*a^2*c^3*d^2*e^2 + 2*a*b^3*e^4*(b^2 - 4*a*c)^(1/2) +
 3*b*c^3*d^4*(b^2 - 4*a*c)^(1/2) - 8*a*b*c^3*d^3*e - 6*a*b^3*c*d*e^3 - 12*a
*b^3*c*e^4*x + 32*a*c^4*d^3*e*x - 8*b^4*c*d*e^3*x - 5*a^2*b*c*e^4*(b^2 - 4*
a*c)^(1/2) - 16*a*c^3*d^3*e*(b^2 - 4*a*c)^(1/2) + 24*a^2*b*c^2*d*e^3 + 16*a
^2*b*c^2*e^4*x - 32*a^2*c^3*d*e^3*x - 8*b^2*c^3*d^3*e*x + 16*a^2*c^2*d*e^3*
(b^2 - 4*a*c)^(1/2) - 2*b^2*c^2*d^3*e*(b^2 - 4*a*c)^(1/2) + b^3*c*d^2*e^2*(
b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*e^4*x*(b^2 - 4*a*c)^(1/2) + 14*a*b^2*c^2*d^2
*e^2 + 12*b^3*c^2*d^2*e^2*x + 14*a*b*c^2*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 20*a
*c^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 14*b^2*c^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/
2) - 10*a*b^2*c*d*e^3*(b^2 - 4*a*c)^(1/2) - 8*a*b^2*c*e^4*x*(b^2 - 4*a*c)^(
1/2) - 12*b*c^3*d^3*e*x*(b^2 - 4*a*c)^(1/2) - 8*b^3*c*d*e^3*x*(b^2 - 4*a*c)
^(1/2) - 48*a*b*c^3*d^2*e^2*x + 40*a*b^2*c^2*d*e^3*x + 20*a*b*c^2*d*e^3*x*(
b^2 - 4*a*c)^(1/2))*(e^2*((b^2*n)/4 - (a*c*n)/2 + (b*n*(b^2 - 4*a*c)^(1/2))
/4) - e*((c*d*n*(b^2 - 4*a*c)^(1/2))/2 + (b*c*d*n)/2) + (c^2*d^2*n)/2))/(a^
2*e^5 + c^2*d^4*e + b^2*d^2*e^3 - 2*a*b*d*e^4 + 2*a*c*d^2*e^3 - 2*b*c*d^3*e
^2) - log(d*(a + b*x + c*x^2)^n)/(2*e*(d^2 + e^2*x^2 + 2*d*e*x)) - (n*(b*e
- 2*c*d))/((2*d*e + 2*e^2*x)*(a*e^2 + c*d^2 - b*d*e))

3.90 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^4}\,dx$

Optimal result 535
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Optimal result

Integrand size = 23, antiderivative size = 356

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{\sqrt{b^2-4ac}\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)n\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3\left(cd^2-bde+ae^2\right)^3} - \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log(d+ex)}{3e\left(cd^2-bde+ae^2\right)^3} + \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log\left(a+bx+cx^2\right)}{6e\left(cd^2-bde+ae^2\right)^3}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{3e(d+ex)^3}$$

```
[Out] 1/6*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+1/3*(2*c^2*d^2+b^2*e^2-2
*c*e*(a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)-1/3*(-b*e+2*c*d)*(c^2*d^2
+b^2*e^2-c*e*(3*a*e+b*d))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^3+1/6*(-b*e+2*c
*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^
2)^3-1/3*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^3+1/3*(3*c^2*d^2+b^2*e^2-c*e*(a*e+
3*b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/(a*e^2-b
*d*e+c*d^2)^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx$$
$$= \frac{n\sqrt{b^2-4ac}\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)}{3\left(ae^2-bde+cd^2\right)^3}$$
$$+\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6e\left(ae^2-bde+cd^2\right)^3}$$
$$+\frac{n(-2ce(ae+bd)+b^2e^2+2c^2d^2)}{3e(d+ex)\left(ae^2-bde+cd^2\right)^2}$$
$$-\frac{n(2cd-be)\log(d+ex)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{3e\left(ae^2-bde+cd^2\right)^3}$$
$$+\frac{n(2cd-be)}{6e(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{3e(d+ex)^3}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```

```
[Out] ((2*c*d - b*e)*n)/(6*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) + ((2*c^2*d^2 +
 b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(3*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x))
+ (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*n*ArcTanh[(b
 + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*(c*d^2 - b*d*e + a*e^2)^3) - ((2*c*d - b*e
)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[d + e*x])/(3*e*(c*d^2 - b*d
*e + a*e^2)^3) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*L
og[a + b*x + c*x^2])/(6*e*(c*d^2 - b*d*e + a*e^2)^3) - Log[d*(a + b*x + c*x
^2)^n]/(3*e*(d + e*x)^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(d(a+bx+cx^2)^n\right)}{3e(d+ex)^3} + \frac{n\int \frac{b+2cx}{(d+ex)^3(a+bx+cx^2)}\,dx}{3e}$$

$$= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{3e(d+ex)^3} + \frac{n\int \left(\frac{e(-2cd+be)}{(cd^2-bde+ae^2)(d+ex)^3} + \frac{e\left(-2c^2d^2-b^2e^2+2ce(bd+ae)\right)}{(cd^2-bde+ae^2)^2(d+ex)^2} + \frac{e(2cd-be)\left(-c^2d^2-b^2e^2+ce(bd+3ae)\right)}{(cd^2-bde+ae^2)^3(d+ex)} + \frac{3b^3cde^2-b^4e^3+b}{}\right.}{3e}$$

$$= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log(d+ex)}{3e\left(cd^2-bde+ae^2\right)^3} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{3e(d+ex)^3} + \frac{n\int \frac{3b^3cde^2-b^4e^3+bc^2d(cd^2-9ae^2)-b^2ce(3cd^2-4ae^2)+2ac^2e(3cd^2-ae^2)+c(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)x}{a+bx+cx^2}\,dx}{3e\left(cd^2-bde+ae^2\right)^3}$$

$$
\begin{aligned}
&= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} \\
&\quad - \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log(d+ex)}{3e\left(cd^2-bde+ae^2\right)^3} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3} \\
&\quad - \frac{\left(\left(b^2-4ac\right)\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)n\right)\int \frac{1}{a+bx+cx^2}\,dx}{6\left(cd^2-bde+ae^2\right)^3} \\
&\quad + \frac{\left((2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{6e\left(cd^2-bde+ae^2\right)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} \\
&\quad - \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log(d+ex)}{3e\left(cd^2-bde+ae^2\right)^3} \\
&\quad + \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log\left(a+bx+cx^2\right)}{6e\left(cd^2-bde+ae^2\right)^3} \\
&\quad - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3} \\
&\quad + \frac{\left(\left(b^2-4ac\right)\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)n\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{3\left(cd^2-bde+ae^2\right)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} \\
&\quad + \frac{\sqrt{b^2-4ac}\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3\left(cd^2-bde+ae^2\right)^3} \\
&\quad - \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log(d+ex)}{3e\left(cd^2-bde+ae^2\right)^3} \\
&\quad + \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n\log\left(a+bx+cx^2\right)}{6e\left(cd^2-bde+ae^2\right)^3} \\
&\quad - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.87

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx$$
$$= \frac{\frac{n(d+ex)\left((2cd-be)\left(cd^2+e(-bd+ae)\right)^2+2\left(cd^2+e(-bd+ae)\right)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)(d+ex)+2\sqrt{b^2-4ac}e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)(d+ex)^2 \cdots}{(cd^2+e(\cdots}}{\cdots}$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```

```
[Out] ((n*(d + e*x)*((2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^2 + 2*(c*d^2 + e*(-
(b*d) + a*e))*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 2*Sqrt[
b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*(d + e*x)^2*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e
*(b*d + 3*a*e))*(d + e*x)^2*Log[d + e*x] + (2*c*d - b*e)*(c^2*d^2 + b^2*e^2
 - c*e*(b*d + 3*a*e))*(d + e*x)^2*Log[a + x*(b + c*x)]))/(c*d^2 + e*(-(b*d)
 + a*e))^3 - 2*Log[d*(a + x*(b + c*x))^n])/(6*e*(d + e*x)^3)
```

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.37

method	result
parts	$-\frac{\ln\left(d\left(c\,x^2+bx+a\right)^n\right)}{3e(ex+d)^3}+\frac{n\left(-\frac{\left(3abc\,e^3-6a\,c^2d\,e^2-b^3e^3+3b^2cd\,e^2-3b\,c^2d^2e+2c^3d^3\right)\ln(ex+d)}{\left(a\,e^2-bde+c\,d^2\right)^3}-\frac{be-2cd}{2\left(a\,e^2-bde+c\,d^2\right)(ex+d)^2}-\frac{2ac\,e^2-e^2b^2 \cdots}{\left(a\,e^2-bde+ \cdots\right.} \cdots\right.}{\cdots}$
risch	Expression too large to display

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^3+1/3/e*n*(-(3*a*b*c*e^3-6*a*c^2*d*e^2
-b^3*e^3+3*b^2*c*d*e^2-3*b*c^2*d^2*e+2*c^3*d^3)/(a*e^2-b*d*e+c*d^2)^3*ln(e*
x+d)-1/2*(b*e-2*c*d)/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2-(2*a*c*e^2-b^2*e^2+2*b*c
*d*e-2*c^2*d^2)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+1/(a*e^2-b*d*e+c*d^2)^3*(1/2*
(3*a*b*c^2*e^3-6*a*c^3*d*e^2-b^3*c*e^3+3*b^2*c^2*d*e^2-3*b*c^3*d^2*e+2*c^4*
d^3)/c*ln(c*x^2+b*x+a)+2*(-2*a^2*c^2*e^3+4*a*b^2*e^3*c-9*a*b*c^2*d*e^2+6*a*
c^3*d^2*e-b^4*e^3+3*b^3*c*d*e^2-3*b^2*c^2*d^2*e+b*c^3*d^3-1/2*(3*a*b*c^2*e^
3-6*a*c^3*d*e^2-b^3*c*e^3+3*b^2*c^2*d*e^2-3*b*c^3*d^2*e+2*c^4*d^3)*b/c)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1496 vs. 2(340) = 680.

Time = 9.97 (sec) , antiderivative size = 3013, normalized size of antiderivative = 8.46

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4} \, dx = \text{Too large to display}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [1/6*(2*(2*c^3*d^4*e^2 - 4*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e^5 + (a
*b^2 - 2*a^2*c)*e^6)*n*x^2 + (10*c^3*d^5*e - 21*b*c^2*d^4*e^2 - a^2*b*e^6 +
 4*(4*b^2*c + a*c^2)*d^3*e^3 - (5*b^3 + 6*a*b*c)*d^2*e^4 + 6*(a*b^2 - a^2*c
)*d*e^5)*n*x - ((3*c^2*d^2*e^4 - 3*b*c*d*e^5 + (b^2 - a*c)*e^6)*n*x^3 + 3*(
3*c^2*d^3*e^3 - 3*b*c*d^2*e^4 + (b^2 - a*c)*d*e^5)*n*x^2 + 3*(3*c^2*d^4*e^2
 - 3*b*c*d^3*e^3 + (b^2 - a*c)*d^2*e^4)*n*x + (3*c^2*d^5*e - 3*b*c*d^4*e^2
+ (b^2 - a*c)*d^3*e^3)*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (6*c^3*d^6 -
13*b*c^2*d^5*e - a^2*b*d*e^5 + 2*(5*b^2*c + 2*a*c^2)*d^4*e^2 - 3*(b^3 + 2*a
*b*c)*d^3*e^3 + 2*(2*a*b^2 - a^2*c)*d^2*e^4)*n + ((2*c^3*d^3*e^3 - 3*b*c^2*
d^2*e^4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^3 + 3*(2*c^3
*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3 - 3*a*b*c)*
d*e^5)*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*a*c^2)*d^3*e
^3 - (b^3 - 3*a*b*c)*d^2*e^4)*n*x + (3*b*c^2*d^5*e + 6*a^2*b*d*e^5 - 2*a^3*
e^6 - 3*(b^2*c + 4*a*c^2)*d^4*e^2 + (b^3 + 15*a*b*c)*d^3*e^3 - 6*(a*b^2 + a
^2*c)*d^2*e^4)*n)*log(c*x^2 + b*x + a) - 2*((2*c^3*d^3*e^3 - 3*b*c^2*d^2*e^
4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^3 + 3*(2*c^3*d^4*e
^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3 - 3*a*b*c)*d*e^5)
*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*a*c^2)*d^3*e^3 - (
b^3 - 3*a*b*c)*d^2*e^4)*n*x + (2*c^3*d^6 - 3*b*c^2*d^5*e + 3*(b^2*c - 2*a*c
^2)*d^4*e^2 - (b^3 - 3*a*b*c)*d^3*e^3)*n)*log(e*x + d) - 2*(c^3*d^6 - 3*b*c
^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*e^2 - (b^3 + 6*a
*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(d))/(c^3*d^9*e - 3*b*c^2*d^8
*e^2 - 3*a^2*b*d^4*e^6 + a^3*d^3*e^7 + 3*(b^2*c + a*c^2)*d^7*e^3 - (b^3 + 6
*a*b*c)*d^6*e^4 + 3*(a*b^2 + a^2*c)*d^5*e^5 + (c^3*d^6*e^4 - 3*b*c^2*d^5*e^
5 - 3*a^2*b*d*e^9 + a^3*e^10 + 3*(b^2*c + a*c^2)*d^4*e^6 - (b^3 + 6*a*b*c)*
d^3*e^7 + 3*(a*b^2 + a^2*c)*d^2*e^8)*x^3 + 3*(c^3*d^7*e^3 - 3*b*c^2*d^6*e^4
 - 3*a^2*b*d^2*e^8 + a^3*d*e^9 + 3*(b^2*c + a*c^2)*d^5*e^5 - (b^3 + 6*a*b*c
)*d^4*e^6 + 3*(a*b^2 + a^2*c)*d^3*e^7)*x^2 + 3*(c^3*d^8*e^2 - 3*b*c^2*d^7*e
^3 - 3*a^2*b*d^3*e^7 + a^3*d^2*e^8 + 3*(b^2*c + a*c^2)*d^6*e^4 - (b^3 + 6*a
*b*c)*d^5*e^5 + 3*(a*b^2 + a^2*c)*d^4*e^6)*x), 1/6*(2*(2*c^3*d^4*e^2 - 4*b*
c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e^5 + (a*b^2 - 2*a^2*c)*e^6)*n*x^2 +
(10*c^3*d^5*e - 21*b*c^2*d^4*e^2 - a^2*b*e^6 + 4*(4*b^2*c + a*c^2)*d^3*e^3
- (5*b^3 + 6*a*b*c)*d^2*e^4 + 6*(a*b^2 - a^2*c)*d*e^5)*n*x + 2*((3*c^2*d^2*
e^4 - 3*b*c*d*e^5 + (b^2 - a*c)*e^6)*n*x^3 + 3*(3*c^2*d^3*e^3 - 3*b*c*d^2*e
```

```
^4 + (b^2 - a*c)*d*e^5)*n*x^2 + 3*(3*c^2*d^4*e^2 - 3*b*c*d^3*e^3 + (b^2 - a
*c)*d^2*e^4)*n*x + (3*c^2*d^5*e - 3*b*c*d^4*e^2 + (b^2 - a*c)*d^3*e^3)*n)*s
qrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (
6*c^3*d^6 - 13*b*c^2*d^5*e - a^2*b*d*e^5 + 2*(5*b^2*c + 2*a*c^2)*d^4*e^2 -
3*(b^3 + 2*a*b*c)*d^3*e^3 + 2*(2*a*b^2 - a^2*c)*d^2*e^4)*n + ((2*c^3*d^3*e^
3 - 3*b*c^2*d^2*e^4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^
3 + 3*(2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3
 - 3*a*b*c)*d*e^5)*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*
a*c^2)*d^3*e^3 - (b^3 - 3*a*b*c)*d^2*e^4)*n*x + (3*b*c^2*d^5*e + 6*a^2*b*d*
e^5 - 2*a^3*e^6 - 3*(b^2*c + 4*a*c^2)*d^4*e^2 + (b^3 + 15*a*b*c)*d^3*e^3 -
6*(a*b^2 + a^2*c)*d^2*e^4)*n)*log(c*x^2 + b*x + a) - 2*((2*c^3*d^3*e^3 - 3*
b*c^2*d^2*e^4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^3 + 3*
(2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3 - 3*a
*b*c)*d*e^5)*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*a*c^2)
*d^3*e^3 - (b^3 - 3*a*b*c)*d^2*e^4)*n*x + (2*c^3*d^6 - 3*b*c^2*d^5*e + 3*(b
^2*c - 2*a*c^2)*d^4*e^2 - (b^3 - 3*a*b*c)*d^3*e^3)*n)*log(e*x + d) - 2*(c^3
*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*e^2
- (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(d))/(c^3*d^9*e -
 3*b*c^2*d^8*e^2 - 3*a^2*b*d^4*e^6 + a^3*d^3*e^7 + 3*(b^2*c + a*c^2)*d^7*e^
3 - (b^3 + 6*a*b*c)*d^6*e^4 + 3*(a*b^2 + a^2*c)*d^5*e^5 + (c^3*d^6*e^4 - 3*
b*c^2*d^5*e^5 - 3*a^2*b*d*e^9 + a^3*e^10 + 3*(b^2*c + a*c^2)*d^4*e^6 - (b^3
 + 6*a*b*c)*d^3*e^7 + 3*(a*b^2 + a^2*c)*d^2*e^8)*x^3 + 3*(c^3*d^7*e^3 - 3*b
*c^2*d^6*e^4 - 3*a^2*b*d^2*e^8 + a^3*d*e^9 + 3*(b^2*c + a*c^2)*d^5*e^5 - (b
^3 + 6*a*b*c)*d^4*e^6 + 3*(a*b^2 + a^2*c)*d^3*e^7)*x^2 + 3*(c^3*d^8*e^2 - 3
*b*c^2*d^7*e^3 - 3*a^2*b*d^3*e^7 + a^3*d^2*e^8 + 3*(b^2*c + a*c^2)*d^6*e^4
- (b^3 + 6*a*b*c)*d^5*e^5 + 3*(a*b^2 + a^2*c)*d^4*e^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**4,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(340) = 680$.

Time = 0.46 (sec) , antiderivative size = 1128, normalized size of antiderivative = 3.17

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx$$
$$= \frac{(2c^3d^3n - 3bc^2d^2en + 3b^2cde^2n - 6ac^2de^2n - b^3e^3n + 3abce^3n)\log(cx^2+bx+a)}{6(c^3d^6e - 3bc^2d^5e^2 + 3b^2cd^4e^3 + 3ac^2d^4e^3 - b^3d^3e^4 - 6abcd^3e^4 + 3ab^2d^2e^5 + 3a^2cd^2e^5 - 3a^2bde^6 + a^3e^7)}$$
$$- \frac{n\log(cx^2+bx+a)}{3(e^4x^3 + 3de^3x^2 + 3d^2e^2x + d^3e)}$$
$$- \frac{(2c^3d^3n - 3bc^2d^2en + 3b^2cde^2n - 6ac^2de^2n - b^3e^3n + 3abce^3n)\log(ex+d)}{3(c^3d^6e - 3bc^2d^5e^2 + 3b^2cd^4e^3 + 3ac^2d^4e^3 - b^3d^3e^4 - 6abcd^3e^4 + 3ab^2d^2e^5 + 3a^2cd^2e^5 - 3a^2bde^6 + a^3}$$
$$- \frac{(3b^2c^2d^2n - 12ac^3d^2n - 3b^3cden + 12abc^2den + b^4e^2n - 5ab^2ce^2n + 4a^2c^2e^2n)\arctan\left(\frac{2cx+}{\sqrt{-b^2+}}\right)}{3(c^3d^6 - 3bc^2d^5e + 3b^2cd^4e^2 + 3ac^2d^4e^2 - b^3d^3e^3 - 6abcd^3e^3 + 3ab^2d^2e^4 + 3a^2cd^2e^4 - 3a^2bde^5 + a^3e^6}$$
$$+ \frac{4c^2d^2e^2nx^2 - 4bcde^3nx^2 + 2b^2e^4nx^2 - 4ace^4nx^2 + 10c^2d^3enx - 11bcd^2e^2nx + 5b^2de^3nx - 6acd}{6(c^2d^4e^4x^3 - 2bcd^3e^5x^3 + b^2d^2e^6x^3 + 2acd^2e^6x^3 - 2abde^7x^3 + a^2e^8x^3 + 3c^2d^5e^3x^2 - 6bcd^4e^4x^2 + 3b^2d}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*c^3*d^3*n - 3*b*c^2*d^2*e*n + 3*b^2*c*d*e^2*n - 6*a*c^2*d*e^2*n - b^
3*e^3*n + 3*a*b*c*e^3*n)*log(c*x^2 + b*x + a)/(c^3*d^6*e - 3*b*c^2*d^5*e^2
+ 3*b^2*c*d^4*e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b
^2*d^2*e^5 + 3*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) - 1/3*n*log(c*x^2 +
 b*x + a)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) - 1/3*(2*c^3*d^3*n
- 3*b*c^2*d^2*e*n + 3*b^2*c*d*e^2*n - 6*a*c^2*d*e^2*n - b^3*e^3*n + 3*a*b*c
*e^3*n)*log(e*x + d)/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 + 3*a*c
^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a^2*c*d^2*
e^5 - 3*a^2*b*d*e^6 + a^3*e^7) - 1/3*(3*b^2*c^2*d^2*n - 12*a*c^3*d^2*n - 3*
```

```
b^3*c*d*e*n + 12*a*b*c^2*d*e*n + b^4*e^2*n - 5*a*b^2*c*e^2*n + 4*a^2*c^2*e^
2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b
^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^
2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^2 + 4*a*c)) + 1/
6*(4*c^2*d^2*e^2*n*x^2 - 4*b*c*d*e^3*n*x^2 + 2*b^2*e^4*n*x^2 - 4*a*c*e^4*n*
x^2 + 10*c^2*d^3*e*n*x - 11*b*c*d^2*e^2*n*x + 5*b^2*d*e^3*n*x - 6*a*c*d*e^3
*n*x - a*b*e^4*n*x + 6*c^2*d^4*n - 7*b*c*d^3*e*n + 3*b^2*d^2*e^2*n - 2*a*c*
d^2*e^2*n - a*b*d*e^3*n - 2*c^2*d^4*log(d) + 4*b*c*d^3*e*log(d) - 2*b^2*d^2
*e^2*log(d) - 4*a*c*d^2*e^2*log(d) + 4*a*b*d*e^3*log(d) - 2*a^2*e^4*log(d))
/(c^2*d^4*e^4*x^3 - 2*b*c*d^3*e^5*x^3 + b^2*d^2*e^6*x^3 + 2*a*c*d^2*e^6*x^3
 - 2*a*b*d*e^7*x^3 + a^2*e^8*x^3 + 3*c^2*d^5*e^3*x^2 - 6*b*c*d^4*e^4*x^2 +
3*b^2*d^3*e^5*x^2 + 6*a*c*d^3*e^5*x^2 - 6*a*b*d^2*e^6*x^2 + 3*a^2*d*e^7*x^2
 + 3*c^2*d^6*e^2*x - 6*b*c*d^5*e^3*x + 3*b^2*d^4*e^4*x + 6*a*c*d^4*e^4*x -
6*a*b*d^3*e^5*x + 3*a^2*d^2*e^6*x + c^2*d^7*e - 2*b*c*d^6*e^2 + b^2*d^5*e^3
 + 2*a*c*d^5*e^3 - 2*a*b*d^4*e^4 + a^2*d^3*e^5)
```

Mupad [B] (verification not implemented)

Time = 12.80 (sec) , antiderivative size = 2707, normalized size of antiderivative = 7.60

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = \text{Too large to display}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^4,x)
```

```
[Out] (log(d + e*x)*(e^3*(b^3*n - 3*a*b*c*n) + e^2*(6*a*c^2*d*n - 3*b^2*c*d*n) -
2*c^3*d^3*n + 3*b*c^2*d^2*e*n))/(3*a^3*e^7 + 3*c^3*d^6*e - 3*b^3*d^3*e^4 +
9*a*b^2*d^2*e^5 + 9*a*c^2*d^4*e^3 + 9*a^2*c*d^2*e^5 - 9*b*c^2*d^5*e^2 + 9*b
^2*c*d^4*e^3 - 9*a^2*b*d*e^6 - 18*a*b*c*d^3*e^4) - (log(32*a*b^5*e^5 - 2*a*
e^5*(b^2 - 4*a*c)^(5/2) - 192*a*c^5*d^5 + 32*b^6*e^5*x + 48*b^2*c^4*d^5 - 1
8*b^3*e^5*x*(b^2 - 4*a*c)^(3/2) - 3*b^5*e^5*x*(b^2 - 4*a*c)^(1/2) + 96*c^5*
d^5*x*(b^2 - 4*a*c)^(1/2) - 208*a^2*b^3*c*e^5 + 320*a^3*b*c^2*e^5 - 704*a^3
*c^3*d*e^4 - 48*b^3*c^3*d^4*e - 16*b^5*c*d^2*e^3 - 64*a^3*c^3*e^5*x + 1152*
a^2*c^4*d^3*e^2 + 48*b^4*c^2*d^3*e^2 - 33*b*d*e^4*(b^2 - 4*a*c)^(5/2) - 11*
b*e^5*x*(b^2 - 4*a*c)^(5/2) - 24*a*b^2*e^5*(b^2 - 4*a*c)^(3/2) - 6*a*b^4*e^
5*(b^2 - 4*a*c)^(1/2) + 48*b*c^4*d^5*(b^2 - 4*a*c)^(1/2) + 18*b^3*d*e^4*(b^
2 - 4*a*c)^(3/2) + 15*b^5*d*e^4*(b^2 - 4*a*c)^(1/2) + 44*c*d^2*e^3*(b^2 - 4
*a*c)^(5/2) + 72*c^3*d^4*e*(b^2 - 4*a*c)^(3/2) + 22*c*d*e^4*x*(b^2 - 4*a*c)
^(5/2) + 192*a*b*c^4*d^4*e - 128*a*b^4*c*d*e^4 + 120*b^3*c^2*d^3*e^2*(b^2 -
 4*a*c)^(1/2) - 224*a*b^4*c*e^5*x - 576*a*c^5*d^4*e*x - 160*b^5*c*d*e^4*x +
 144*b^2*c^4*d^4*e*x - 72*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(3/2) - 120*b^2*c^3*d
^4*e*(b^2 - 4*a*c)^(1/2) - 60*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 144*c^3*d
^3*e^2*x*(b^2 - 4*a*c)^(3/2) - 480*a*b^2*c^3*d^3*e^2 + 320*a*b^3*c^2*d^2*e^
3 - 1024*a^2*b*c^3*d^2*e^3 + 688*a^2*b^2*c^2*d*e^4 + 400*a^2*b^2*c^2*e^5*x
+ 1408*a^2*c^4*d^2*e^3*x - 288*b^3*c^3*d^3*e^2*x + 304*b^4*c^2*d^2*e^3*x -
```

216*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(3/2) - 1568*a*b^2*c^3*d^2*e^3*x + 240*b^2*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 120*b^3*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 240*b*c^4*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 108*b^2*c*d*e^4*x*(b^2 - 4*a*c)^(3/2) + 30*b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 1152*a*b*c^4*d^3*e^2*x + 992*a*b^3*c^2*d*e^4*x - 1408*a^2*b*c^3*d*e^4*x)*(e^3*((b^3*n)/6 - (b^2*n*(b^2 - 4*a*c)^(1/2))/6 + (a*c*n*(b^2 - 4*a*c)^(1/2))/6 - (a*b*c*n)/2) + e^2*(a*c^2*d*n - (b^2*c*d*n)/2 + (b*c*d*n*(b^2 - 4*a*c)^(1/2))/2) + e*((b*c^2*d^2*n)/2 - (c^2*d^2*n*(b^2 - 4*a*c)^(1/2))/2) - (c^3*d^3*n)/3))/(a^3*e^7 + c^3*d^6*e - b^3*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - 3*a^2*b*d*e^6 - 6*a*b*c*d^3*e^4) - (log(2*a*e^5*(b^2 - 4*a*c)^(5/2) + 32*a*b^5*e^5 - 192*a*c^5*d^5 + 32*b^6*e^5*x + 48*b^2*c^4*d^5 + 18*b^3*e^5*x*(b^2 - 4*a*c)^(3/2) + 3*b^5*e^5*x*(b^2 - 4*a*c)^(1/2) - 96*c^5*d^5*x*(b^2 - 4*a*c)^(1/2) - 208*a^2*b^3*c*e^5 + 320*a^3*b*c^2*e^5 - 704*a^3*c^3*d*e^4 - 48*b^3*c^3*d^4*e - 16*b^5*c*d^2*e^3 - 64*a^3*c^3*e^5*x + 1152*a^2*c^4*d^3*e^2 + 48*b^4*c^2*d^3*e^2 + 33*b*d*e^4*(b^2 - 4*a*c)^(5/2) + 11*b*e^5*x*(b^2 - 4*a*c)^(5/2) + 24*a*b^2*e^5*(b^2 - 4*a*c)^(3/2) + 6*a*b^4*e^5*(b^2 - 4*a*c)^(1/2) - 48*b*c^4*d^5*(b^2 - 4*a*c)^(1/2) - 18*b^3*d*e^4*(b^2 - 4*a*c)^(3/2) - 15*b^5*d*e^4*(b^2 - 4*a*c)^(1/2) - 44*c*d^2*e^3*(b^2 - 4*a*c)^(5/2) - 72*c^3*d^4*e*(b^2 - 4*a*c)^(3/2) - 22*c*d*e^4*x*(b^2 - 4*a*c)^(5/2) + 192*a*b*c^4*d^4*e - 128*a*b^4*c*d*e^4 - 120*b^3*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 224*a*b^4*c*e^5*x - 576*a*c^5*d^4*e*x - 160*b^5*c*d*e^4*x + 144*b^2*c^4*d^4*e*x + 72*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(3/2) + 120*b^2*c^3*d^4*e*(b^2 - 4*a*c)^(1/2) + 60*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 144*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(3/2) - 480*a*b^2*c^3*d^3*e^2 + 320*a*b^3*c^2*d^2*e^3 - 1024*a^2*b*c^3*d^2*e^3 + 688*a^2*b^2*c^2*d*e^4 + 400*a^2*b^2*c^2*e^5*x + 1408*a^2*c^4*d^2*e^3*x - 288*b^3*c^3*d^3*e^2*x + 304*b^4*c^2*d^2*e^3*x + 216*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(3/2) - 1568*a*b^2*c^3*d^2*e^3*x - 240*b^2*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 120*b^3*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) + 240*b*c^4*d^4*e*x*(b^2 - 4*a*c)^(1/2) - 108*b^2*c*d*e^4*x*(b^2 - 4*a*c)^(3/2) - 30*b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 1152*a*b*c^4*d^3*e^2*x + 992*a*b^3*c^2*d*e^4*x - 1408*a^2*b*c^3*d*e^4*x)*(e^3*((b^3*n)/6 + (b^2*n*(b^2 - 4*a*c)^(1/2))/6 - (a*c*n*(b^2 - 4*a*c)^(1/2))/6 - (a*b*c*n)/2) - e^2*((b^2*c*d*n)/2 - a*c^2*d*n + (b*c*d*n*(b^2 - 4*a*c)^(1/2))/2) + e*((b*c^2*d^2*n)/2 + (c^2*d^2*n*(b^2 - 4*a*c)^(1/2))/2) - (c^3*d^3*n)/3))/(a^3*e^7 + c^3*d^6*e - b^3*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - 3*a^2*b*d*e^6 - 6*a*b*c*d^3*e^4) - ((a*b*e^3*n - 6*c^2*d^3*n - 3*b^2*d*e^2*n + 2*a*c*d*e^2*n + 7*b*c*d^2*e*n)/(2*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)) - (n*x*(b^2*e^3 + 2*c^2*d^2*e - 2*a*c*e^3 - 2*b*c*d*e^2))/(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2))/(3*d^2*e + 3*e^3*x^2 + 6*d*e^2*x) - log(d*(a + b*x + c*x^2)^n)/(3*e*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x))

3.91 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}\,dx$

Optimal result 545
Rubi [A] (verified) 546
Mathematica [A] (verified) 549
Maple [A] (verified) 550
Fricas [B] (verification not implemented) 550
Sympy [F(-1)] 551
Maxima [F(-2)] 551
Giac [B] (verification not implemented) 551
Mupad [B] (verification not implemented) 553

Optimal result

Integrand size = 23, antiderivative size = 519

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx = \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3}$$
$$+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n}{4e\left(cd^2-bde+ae^2\right)^3(d+ex)}$$
$$+\frac{\sqrt{b^2-4ac}(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4\left(cd^2-bde+ae^2\right)^4}$$
$$-\frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+a^2e^2)\right)n\log(d+ex)}{4e\left(cd^2-bde+ae^2\right)^4}$$
$$+\frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+a^2e^2)\right)n\log(a+bx+cx^2}{8e\left(cd^2-bde+ae^2\right)^4}$$
$$-\frac{\log\left(d(a+bx+cx^2)^n\right)}{4e(d+ex)^4}$$

```
[Out] 1/12*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^3+1/8*(2*c^2*d^2+b^2*e^2-
2*c*e*(a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^2+1/4*(-b*e+2*c*d)*(c^2*
d^2+b^2*e^2-c*e*(3*a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)-1/4*(2*c^4*d
^4+b^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2
+6*a*b*d*e+3*b^2*d^2))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^4+1/8*(2*c^4*d^4+b
^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a
*b*d*e+3*b^2*d^2))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^2)^4-1/4*ln(d*(c*x^
2+b*x+a)^n)/e/(e*x+d)^4+1/4*(-b*e+2*c*d)*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d)
)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c
*d^2)^4
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2605, 814, 648, 632, 212, 642}

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx$$
$$= \frac{n(2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4)\log(a+bx+cx^2)}{8e\,(ae^2-bde+cd^2)^4}$$
$$-\frac{n\log(d+ex)\,(2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4)}{4e\,(ae^2-bde+cd^2)^4}$$
$$+\frac{n\sqrt{b^2-4ac}(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-2ce(ae+bd)+b^2e^2+2c^2d^2)}{4\,(ae^2-bde+cd^2)^4}$$
$$+\frac{n(2cd-be)\,(-ce(3ae+bd)+b^2e^2+c^2d^2)}{4e(d+ex)\,(ae^2-bde+cd^2)^3}+\frac{n(-2ce(ae+bd)+b^2e^2+2c^2d^2)}{8e(d+ex)^2\,(ae^2-bde+cd^2)^2}$$
$$+\frac{n(2cd-be)}{12e(d+ex)^3\,(ae^2-bde+cd^2)}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{4e(d+ex)^4}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]
```

```
[Out] ((2*c*d - b*e)*n)/(12*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^3) + ((2*c^2*d^2
+ b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(8*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^
2) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n)/(4*e*(c*d^2
- b*d*e + a*e^2)^3*(d + e*x)) + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(2*c^2*d^2
 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(
4*(c*d^2 - b*d*e + a*e^2)^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a
*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^
2))*n*Log[d + e*x])/(4*e*(c*d^2 - b*d*e + a*e^2)^4) + ((2*c^4*d^4 + b^4*e^4
 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d
^2 + 6*a*b*d*e + a^2*e^2))*n*Log[a + b*x + c*x^2])/(8*e*(c*d^2 - b*d*e + a*
e^2)^4) - Log[d*(a + b*x + c*x^2)^n]/(4*e*(d + e*x)^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(d(a+bx+cx^2)^n\right)}{4e(d+ex)^4} + \frac{n\int \frac{b+2cx}{(d+ex)^4(a+bx+cx^2)}\,dx}{4e}$$

$$= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{4e(d+ex)^4} + \frac{n\int\left(\frac{e(-2cd+be)}{(cd^2-bde+ae^2)(d+ex)^4} + \frac{e\left(-2c^2d^2-b^2e^2+2ce(bd+ae)\right)}{(cd^2-bde+ae^2)^2(d+ex)^3} + \frac{e(2cd-be)\left(-c^2d^2-b^2e^2+ce(bd+3ae)\right)}{(cd^2-bde+ae^2)^3(d+ex)^2} + \frac{e(-2c^4d^4-b^4e^4 \ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}$$
$$+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n}{4e\left(cd^2-bde+ae^2\right)^3(d+ex)}$$
$$-\frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)\right)n\log(d+ex}{4e\left(cd^2-bde+ae^2\right)^4}$$
$$-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4}$$
$$+\frac{n\int\frac{-4b^4cde^3+b^5e^4+b^3ce^2\left(6cd^2-5ae^2\right)-4b^2c^2de\left(cd^2-4ae^2\right)+8ac^3de\left(cd^2-ae^2\right)+bc^2\left(c^2d^4-18acd^2e^2+5a^2e^4\right)+c\left(2c^4d^4+b^4e^4-4\right.}{a+bx+cx^2}}{4e\left(cd^2-bde+ae^2\right)^4}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}$$
$$+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n}{4e\left(cd^2-bde+ae^2\right)^3(d+ex)}$$
$$-\frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)\right)n\log(d+ex}{4e\left(cd^2-bde+ae^2\right)^4}$$
$$-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4}$$
$$-\frac{\left(\left(b^2-4ac\right)(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{8\left(cd^2-bde+ae^2\right)^4}$$
$$+\frac{\left(\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)\right)n\right)\int\frac{b+2cx}{a+bx+cx}}{8e\left(cd^2-bde+ae^2\right)^4}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}$$
$$+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n}{4e\left(cd^2-bde+ae^2\right)^3(d+ex)}$$
$$-\frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)\right)n\log(d+ex}{4e\left(cd^2-bde+ae^2\right)^4}$$
$$+\frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)\right)n\log\left(a+bx\right.}{8e\left(cd^2-bde+ae^2\right)^4}$$
$$-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4}$$
$$+\frac{\left(\left(b^2-4ac\right)(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\right)\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{4\left(cd^2-bde+ae^2\right)^4}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}$$
$$+ \frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n}{4e\left(cd^2-bde+ae^2\right)^3(d+ex)}$$
$$+ \frac{\sqrt{b^2-4ac}(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4\left(cd^2-bde+ae^2\right)^4}$$
$$- \frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+a^2e^2)\right)n\log(d+e}{4e\left(cd^2-bde+ae^2\right)^4}$$
$$+ \frac{\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+a^2e^2)\right)n\log\left(a+b\right.}{8e\left(cd^2-bde+ae^2\right)^4}$$
$$- \frac{\log\left(d(a+bx+cx^2)^n\right)}{4e(d+ex)^4}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx$$
$$= \frac{\frac{n(d+ex)\left(2(2cd-be)\left(cd^2+e(-bd+ae)\right)^3+3\left(cd^2+e(-bd+ae)\right)^2\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)(d+ex)+6(2cd-be)\left(cd^2+e(-bd+ae)\right)\left(c^2d^2+b^2e^2-ce\right.}{}}{}$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]
```

```
[Out] ((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^3 + 3*(c*d^2 + e*
(-(b*d) + a*e))^2*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 6*(
2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a
*e))*(d + e*x)^2 + 6*Sqrt[b^2 - 4*a*c]*e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2
 - 2*c*e*(b*d + a*e))*(d + e*x)^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] -
6*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e
) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[d + e*x] +
 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*
e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[a + x*(b
+ c*x)]))/(c*d^2 + e*(-(b*d) + a*e))^4 - 6*Log[d*(a + x*(b + c*x))^n])/(24*
e*(d + e*x)^4)
```

Maple [A] (verified)

Time = 8.09 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.40

method	result
parts	$-\frac{\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)}{4e\left(ex+d\right)^{4}}+\frac{n\left(-\frac{\left(2a^{2}c^{2}e^{4}-4a\,b^{2}c\,e^{4}+12ab\,c^{2}d\,e^{3}-12a\,c^{3}d^{2}e^{2}+b^{4}e^{4}-4b^{3}cd\,e^{3}+6b^{2}c^{2}d^{2}e^{2}-4b\,c^{3}d^{3}e+2c^{4}d^{4}\right)\ln\left(ex+d\right)}{\left(a\,e^{2}-bde+c\,d^{2}\right)^{4}}-\frac{2ac\,e^{2}}{2\left(a\,e^{2}\right.}\right.}{}$
risch	Expression too large to display

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^4+1/4/e*n*(-(2*a^2*c^2*e^4-4*a*b^2*c*e
^4+12*a*b*c^2*d*e^3-12*a*c^3*d^2*e^2+b^4*e^4-4*b^3*c*d*e^3+6*b^2*c^2*d^2*e^
2-4*b*c^3*d^3*e+2*c^4*d^4)/(a*e^2-b*d*e+c*d^2)^4*ln(e*x+d)-1/2*(2*a*c*e^2-b
^2*e^2+2*b*c*d*e-2*c^2*d^2)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^2-1/3*(b*e-2*c*d)
/(a*e^2-b*d*e+c*d^2)/(e*x+d)^3+(3*a*b*c*e^3-6*a*c^2*d*e^2-b^3*e^3+3*b^2*c*d
*e^2-3*b*c^2*d^2*e+2*c^3*d^3)/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)+1/(a*e^2-b*d*e+
c*d^2)^4*(1/2*(2*a^2*c^3*e^4-4*a*b^2*c^2*e^4+12*a*b*c^3*d*e^3-12*a*c^4*d^2*
e^2+b^4*c*e^4-4*b^3*c^2*d*e^3+6*b^2*c^3*d^2*e^2-4*b*c^4*d^3*e+2*c^5*d^4)/c*
ln(c*x^2+b*x+a)+2*(5*a^2*b*c^2*e^4-8*a^2*c^3*d*e^3-5*a*b^3*e^4*c+16*a*b^2*c
^2*d*e^3-18*a*b*c^3*d^2*e^2+8*a*c^4*d^3*e+b^5*e^4-4*b^4*c*d*e^3+6*b^3*c^2*d
^2*e^2-4*b^2*c^3*d^3*e+b*c^4*d^4-1/2*(2*a^2*c^3*e^4-4*a*b^2*c^2*e^4+12*a*b*
c^3*d*e^3-12*a*c^4*d^2*e^2+b^4*c*e^4-4*b^3*c^2*d*e^3+6*b^2*c^3*d^2*e^2-4*b*
c^4*d^3*e+2*c^5*d^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2902 vs. 2(501) = 1002.

Time = 48.90 (sec) , antiderivative size = 5824, normalized size of antiderivative = 11.22

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx = \text{Too large to display}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^5} \, dx = \text{Timed out}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^5} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2191 vs. $2(501) = 1002$.

Time = 0.59 (sec) , antiderivative size = 2191, normalized size of antiderivative = 4.22

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^5} \, dx = \text{Too large to display}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/8*(2*c^4*d^4*n - 4*b*c^3*d^3*e*n + 6*b^2*c^2*d^2*e^2*n - 12*a*c^3*d^2*e^2
*n - 4*b^3*c*d*e^3*n + 12*a*b*c^2*d*e^3*n + b^4*e^4*n - 4*a*b^2*c*e^4*n + 2
*a^2*c^2*e^4*n)*log(c*x^2 + b*x + a)/(c^4*d^8*e - 4*b*c^3*d^7*e^2 + 6*b^2*c
^2*d^6*e^3 + 4*a*c^3*d^6*e^3 - 4*b^3*c*d^5*e^4 - 12*a*b*c^2*d^5*e^4 + b^4*d
^4*e^5 + 12*a*b^2*c*d^4*e^5 + 6*a^2*c^2*d^4*e^5 - 4*a*b^3*d^3*e^6 - 12*a^2*
b*c*d^3*e^6 + 6*a^2*b^2*d^2*e^7 + 4*a^3*c*d^2*e^7 - 4*a^3*b*d*e^8 + a^4*e^9
) - 1/4*n*log(c*x^2 + b*x + a)/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d
^3*e^2*x + d^4*e) - 1/4*(2*c^4*d^4*n - 4*b*c^3*d^3*e*n + 6*b^2*c^2*d^2*e^2*
n - 12*a*c^3*d^2*e^2*n - 4*b^3*c*d*e^3*n + 12*a*b*c^2*d*e^3*n + b^4*e^4*n -
 4*a*b^2*c*e^4*n + 2*a^2*c^2*e^4*n)*log(e*x + d)/(c^4*d^8*e - 4*b*c^3*d^7*e
```

```
^2 + 6*b^2*c^2*d^6*e^3 + 4*a*c^3*d^6*e^3 - 4*b^3*c*d^5*e^4 - 12*a*b*c^2*d^5
*e^4 + b^4*d^4*e^5 + 12*a*b^2*c*d^4*e^5 + 6*a^2*c^2*d^4*e^5 - 4*a*b^3*d^3*e
^6 - 12*a^2*b*c*d^3*e^6 + 6*a^2*b^2*d^2*e^7 + 4*a^3*c*d^2*e^7 - 4*a^3*b*d*e
^8 + a^4*e^9) - 1/4*(4*b^2*c^3*d^3*n - 16*a*c^4*d^3*n - 6*b^3*c^2*d^2*e*n +
 24*a*b*c^3*d^2*e*n + 4*b^4*c*d*e^2*n - 20*a*b^2*c^2*d*e^2*n + 16*a^2*c^3*d
*e^2*n - b^5*e^3*n + 6*a*b^3*c*e^3*n - 8*a^2*b*c^2*e^3*n)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/((c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c
^3*d^6*e^2 - 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*
c*d^4*e^4 + 6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^
2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*sqrt(-b^2 + 4*a*
c)) + 1/24*(12*c^3*d^3*e^3*n*x^3 - 18*b*c^2*d^2*e^4*n*x^3 + 18*b^2*c*d*e^5*
n*x^3 - 36*a*c^2*d*e^5*n*x^3 - 6*b^3*e^6*n*x^3 + 18*a*b*c*e^6*n*x^3 + 42*c^
3*d^4*e^2*n*x^2 - 66*b*c^2*d^3*e^3*n*x^2 + 63*b^2*c*d^2*e^4*n*x^2 - 108*a*c
^2*d^2*e^4*n*x^2 - 21*b^3*d*e^5*n*x^2 + 54*a*b*c*d*e^5*n*x^2 + 3*a*b^2*e^6*
n*x^2 - 6*a^2*c*e^6*n*x^2 + 52*c^3*d^5*e*n*x - 88*b*c^2*d^4*e^2*n*x + 80*b^
2*c*d^3*e^3*n*x - 100*a*c^2*d^3*e^3*n*x - 26*b^3*d^2*e^4*n*x + 42*a*b*c*d^2
*e^4*n*x + 10*a*b^2*d*e^5*n*x - 8*a^2*c*d*e^5*n*x - 2*a^2*b*e^6*n*x + 22*c^
3*d^6*n - 40*b*c^2*d^5*e*n + 35*b^2*c*d^4*e^2*n - 28*a*c^2*d^4*e^2*n - 11*b
^3*d^3*e^3*n + 6*a*b*c*d^3*e^3*n + 7*a*b^2*d^2*e^4*n - 2*a^2*c*d^2*e^4*n -
2*a^2*b*d*e^5*n - 6*c^3*d^6*log(d) + 18*b*c^2*d^5*e*log(d) - 18*b^2*c*d^4*e
^2*log(d) - 18*a*c^2*d^4*e^2*log(d) + 6*b^3*d^3*e^3*log(d) + 36*a*b*c*d^3*e
^3*log(d) - 18*a*b^2*d^2*e^4*log(d) - 18*a^2*c*d^2*e^4*log(d) + 18*a^2*b*d*
e^5*log(d) - 6*a^3*e^6*log(d))/(c^3*d^6*e^5*x^4 - 3*b*c^2*d^5*e^6*x^4 + 3*b
^2*c*d^4*e^7*x^4 + 3*a*c^2*d^4*e^7*x^4 - b^3*d^3*e^8*x^4 - 6*a*b*c*d^3*e^8*
x^4 + 3*a*b^2*d^2*e^9*x^4 + 3*a^2*c*d^2*e^9*x^4 - 3*a^2*b*d*e^10*x^4 + a^3*
e^11*x^4 + 4*c^3*d^7*e^4*x^3 - 12*b*c^2*d^6*e^5*x^3 + 12*b^2*c*d^5*e^6*x^3
+ 12*a*c^2*d^5*e^6*x^3 - 4*b^3*d^4*e^7*x^3 - 24*a*b*c*d^4*e^7*x^3 + 12*a*b^
2*d^3*e^8*x^3 + 12*a^2*c*d^3*e^8*x^3 - 12*a^2*b*d^2*e^9*x^3 + 4*a^3*d*e^10*
x^3 + 6*c^3*d^8*e^3*x^2 - 18*b*c^2*d^7*e^4*x^2 + 18*b^2*c*d^6*e^5*x^2 + 18*
a*c^2*d^6*e^5*x^2 - 6*b^3*d^5*e^6*x^2 - 36*a*b*c*d^5*e^6*x^2 + 18*a*b^2*d^4
*e^7*x^2 + 18*a^2*c*d^4*e^7*x^2 - 18*a^2*b*d^3*e^8*x^2 + 6*a^3*d^2*e^9*x^2
+ 4*c^3*d^9*e^2*x - 12*b*c^2*d^8*e^3*x + 12*b^2*c*d^7*e^4*x + 12*a*c^2*d^7*
e^4*x - 4*b^3*d^6*e^5*x - 24*a*b*c*d^6*e^5*x + 12*a*b^2*d^5*e^6*x + 12*a^2*
c*d^5*e^6*x - 12*a^2*b*d^4*e^7*x + 4*a^3*d^3*e^8*x + c^3*d^10*e - 3*b*c^2*d
^9*e^2 + 3*b^2*c*d^8*e^3 + 3*a*c^2*d^8*e^3 - b^3*d^7*e^4 - 6*a*b*c*d^7*e^4
+ 3*a*b^2*d^6*e^5 + 3*a^2*c*d^6*e^5 - 3*a^2*b*d^5*e^6 + a^3*d^4*e^7)
```

Mupad [B] (verification not implemented)

Time = 20.48 (sec) , antiderivative size = 4334, normalized size of antiderivative = 8.35

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx = \text{Too large to display}$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^5,x)
```

```
[Out] (log(10*d*e^5*(b^2 - 4*a*c)^(7/2) + 3*e^6*x*(b^2 - 4*a*c)^(7/2) - 6*a*e^6*(
4*a*c - b^2)^3 + 96*c^5*d^6*(4*a*c - b^2) - 10*b*e^6*x*(4*a*c - b^2)^3 - 10
*b^5*e^6*x*(4*a*c - b^2) + 29*b^2*e^6*x*(b^2 - 4*a*c)^(5/2) + 29*b^4*e^6*x*
(b^2 - 4*a*c)^(3/2) + 3*b^6*e^6*x*(b^2 - 4*a*c)^(1/2) + 192*c^6*d^6*x*(b^2
- 4*a*c)^(1/2) + 44*a*b^2*e^6*(4*a*c - b^2)^2 - 16*b^3*d*e^5*(4*a*c - b^2)^
2 + 58*c*d^2*e^4*(4*a*c - b^2)^3 + 176*c^2*d^3*e^3*(b^2 - 4*a*c)^(5/2) + 44
*b^3*e^6*x*(4*a*c - b^2)^2 + 14*a*b*e^6*(b^2 - 4*a*c)^(5/2) - 232*c^3*d^4*e
^2*(4*a*c - b^2)^2 - 14*a*b^4*e^6*(4*a*c - b^2) + 44*a*b^3*e^6*(b^2 - 4*a*c
)^(3/2) + 6*a*b^5*e^6*(b^2 - 4*a*c)^(1/2) + 96*b*c^5*d^6*(b^2 - 4*a*c)^(1/2
) - 48*b*d*e^5*(4*a*c - b^2)^3 + 32*b^5*d*e^5*(4*a*c - b^2) + 74*b^2*d*e^5*
(b^2 - 4*a*c)^(5/2) - 66*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 18*b^6*d*e^5*(b^2
- 4*a*c)^(1/2) + 160*c^4*d^5*e*(b^2 - 4*a*c)^(3/2) + 288*b*c^2*d^3*e^3*(4*a
*c - b^2)^2 - 84*b^2*c*d^2*e^4*(4*a*c - b^2)^2 - 40*b^2*c^3*d^4*e^2*(4*a*c
- b^2) + 160*b^3*c^2*d^3*e^3*(4*a*c - b^2) - 64*b^2*c^2*d^3*e^3*(b^2 - 4*a*
c)^(3/2) + 360*b^3*c^3*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 240*b^4*c^2*d^3*e^3*(b
^2 - 4*a*c)^(1/2) - 352*c^3*d^3*e^3*x*(4*a*c - b^2)^2 - 128*b*c^4*d^5*e*(4*
a*c - b^2) - 206*b*c*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*c*d*e^5*x*(4*a*c - b^
2)^3 + 320*c^5*d^5*e*x*(4*a*c - b^2) - 110*b^4*c*d^2*e^4*(4*a*c - b^2) - 16
8*b*c^3*d^4*e^2*(b^2 - 4*a*c)^(3/2) - 288*b^2*c^4*d^5*e*(b^2 - 4*a*c)^(1/2)
 + 148*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 90*b^5*c*d^2*e^4*(b^2 - 4*a*c)^(
1/2) + 116*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(5/2) + 464*c^4*d^4*e^2*x*(b^2 - 4*a
*c)^(3/2) - 264*b^2*c*d*e^5*x*(4*a*c - b^2)^2 - 800*b*c^4*d^4*e^2*x*(4*a*c
- b^2) - 928*b*c^3*d^3*e^3*x*(b^2 - 4*a*c)^(3/2) - 116*b*c*d*e^5*x*(b^2 - 4
*a*c)^(5/2) + 528*b*c^2*d^2*e^4*x*(4*a*c - b^2)^2 + 800*b^2*c^3*d^3*e^3*x*(
4*a*c - b^2) - 400*b^3*c^2*d^2*e^4*x*(4*a*c - b^2) + 696*b^2*c^2*d^2*e^4*x*
(b^2 - 4*a*c)^(3/2) + 720*b^2*c^4*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 480*b^3*c
^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) + 180*b^4*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2
) + 100*b^4*c*d*e^5*x*(4*a*c - b^2) - 576*b*c^5*d^5*e*x*(b^2 - 4*a*c)^(1/2)
 - 232*b^3*c*d*e^5*x*(b^2 - 4*a*c)^(3/2) - 36*b^5*c*d*e^5*x*(b^2 - 4*a*c)^(
1/2))*(e^4*((b^4*n)/8 + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 + (a^2*c^2*n)/4 - (a*
b^2*c*n)/2 - (a*b*c*n*(b^2 - 4*a*c)^(1/2))/4) - e^3*((b^3*c*d*n)/2 - (3*a*b
*c^2*d*n)/2 - (a*c^2*d*n*(b^2 - 4*a*c)^(1/2))/2 + (b^2*c*d*n*(b^2 - 4*a*c)^
(1/2))/2) + e^2*((3*b^2*c^2*d^2*n)/4 - (3*a*c^3*d^2*n)/2 + (3*b*c^2*d^2*n*(
b^2 - 4*a*c)^(1/2))/4) - e*((b*c^3*d^3*n)/2 + (c^3*d^3*n*(b^2 - 4*a*c)^(1/2
))/2) + (c^4*d^4*n)/4))/(a^4*e^9 + c^4*d^8*e + b^4*d^4*e^5 - 4*a*b^3*d^3*e^
6 + 4*a*c^3*d^6*e^3 + 4*a^3*c*d^2*e^7 - 4*b*c^3*d^7*e^2 - 4*b^3*c*d^5*e^4 +
```

6*a^2*b^2*d^2*e^7 + 6*a^2*c^2*d^4*e^5 + 6*b^2*c^2*d^6*e^3 - 4*a^3*b*d*e^8 - 12*a*b*c^2*d^5*e^4 + 12*a*b^2*c*d^4*e^5 - 12*a^2*b*c*d^3*e^6) - (log(d + e*x)*(e^2*(6*b^2*c^2*d^2*n - 12*a*c^3*d^2*n) - e^3*(4*b^3*c*d*n - 12*a*b*c^2*d*n) + e^4*(b^4*n + 2*a^2*c^2*n - 4*a*b^2*c*n) + 2*c^4*d^4*n - 4*b*c^3*d^3*e*n))/(4*a^4*e^9 + 4*c^4*d^8*e + 4*b^4*d^4*e^5 - 16*a*b^3*d^3*e^6 + 16*a*c^3*d^6*e^3 + 16*a^3*c*d^2*e^7 - 16*b*c^3*d^7*e^2 - 16*b^3*c*d^5*e^4 + 24*a^2*b^2*d^2*e^7 + 24*a^2*c^2*d^4*e^5 + 24*b^2*c^2*d^6*e^3 - 16*a^3*b*d*e^8 - 48*a*b*c^2*d^5*e^4 + 48*a*b^2*c*d^4*e^5 - 48*a^2*b*c*d^3*e^6) - log(d*(a + b*x + c*x^2)^n)/(4*e*(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)) - (log(10*d*e^5*(b^2 - 4*a*c)^(7/2) + 3*e^6*x*(b^2 - 4*a*c)^(7/2) + 6*a*e^6*(4*a*c - b^2)^3 - 96*c^5*d^6*(4*a*c - b^2) + 10*b*e^6*x*(4*a*c - b^2)^3 + 10*b^5*e^6*x*(4*a*c - b^2) + 29*b^2*e^6*x*(b^2 - 4*a*c)^(5/2) + 29*b^4*e^6*x*(b^2 - 4*a*c)^(3/2) + 3*b^6*e^6*x*(b^2 - 4*a*c)^(1/2) + 192*c^6*d^6*x*(b^2 - 4*a*c)^(1/2) - 44*a*b^2*e^6*(4*a*c - b^2)^2 + 16*b^3*d*e^5*(4*a*c - b^2)^2 - 58*c*d^2*e^4*(4*a*c - b^2)^3 + 176*c^2*d^3*e^3*(b^2 - 4*a*c)^(5/2) - 44*b^3*e^6*x*(4*a*c - b^2)^2 + 14*a*b*e^6*(b^2 - 4*a*c)^(5/2) + 232*c^3*d^4*e^2*(4*a*c - b^2)^2 + 14*a*b^4*e^6*(4*a*c - b^2) + 44*a*b^3*e^6*(b^2 - 4*a*c)^(3/2) + 6*a*b^5*e^6*(b^2 - 4*a*c)^(1/2) + 96*b*c^5*d^6*(b^2 - 4*a*c)^(1/2) + 48*b*d*e^5*(4*a*c - b^2)^3 - 32*b^5*d*e^5*(4*a*c - b^2) + 74*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 66*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 18*b^6*d*e^5*(b^2 - 4*a*c)^(1/2) + 160*c^4*d^5*e*(b^2 - 4*a*c)^(3/2) - 288*b*c^2*d^3*e^3*(4*a*c - b^2)^2 + 84*b^2*c*d^2*e^4*(4*a*c - b^2)^2 + 40*b^2*c^3*d^4*e^2*(4*a*c - b^2) - 160*b^3*c^2*d^3*e^3*(4*a*c - b^2) - 64*b^2*c^2*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 360*b^3*c^3*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 240*b^4*c^2*d^3*e^3*(b^2 - 4*a*c)^(1/2) + 352*c^3*d^3*e^3*x*(4*a*c - b^2)^2 + 128*b*c^4*d^5*e*(4*a*c - b^2) - 206*b*c*d^2*e^4*(b^2 - 4*a*c)^(5/2) - 20*c*d*e^5*x*(4*a*c - b^2)^3 - 320*c^5*d^5*e*x*(4*a*c - b^2) + 110*b^4*c*d^2*e^4*(4*a*c - b^2) - 168*b*c^3*d^4*e^2*(b^2 - 4*a*c)^(3/2) - 288*b^2*c^4*d^5*e*(b^2 - 4*a*c)^(1/2) + 148*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 90*b^5*c*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 116*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(5/2) + 464*c^4*d^4*e^2*x*(b^2 - 4*a*c)^(3/2) + 264*b^2*c*d*e^5*x*(4*a*c - b^2)^2 + 800*b*c^4*d^4*e^2*x*(4*a*c - b^2) - 928*b*c^3*d^3*e^3*x*(b^2 - 4*a*c)^(3/2) - 116*b*c*d*e^5*x*(b^2 - 4*a*c)^(5/2) - 528*b*c^2*d^2*e^4*x*(4*a*c - b^2)^2 - 800*b^2*c^3*d^3*e^3*x*(4*a*c - b^2) + 400*b^3*c^2*d^2*e^4*x*(4*a*c - b^2) + 696*b^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(3/2) + 720*b^2*c^4*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 480*b^3*c^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) + 180*b^4*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 100*b^4*c*d*e^5*x*(4*a*c - b^2) - 576*b*c^5*d^5*e*x*(b^2 - 4*a*c)^(1/2) - 232*b^3*c*d*e^5*x*(b^2 - 4*a*c)^(3/2) - 36*b^5*c*d*e^5*x*(b^2 - 4*a*c)^(1/2))*(e^3*((b^3*c*d*n)/2 - (3*a*b*c^2*d*n)/2 + (a*c^2*d*n*(b^2 - 4*a*c)^(1/2))/2 - (b^2*c*d*n*(b^2 - 4*a*c)^(1/2))/2) - e^4*((b^4*n)/8 - (b^3*n*(b^2 - 4*a*c)^(1/2))/8 + (a^2*c^2*n)/4 - (a*b^2*c*n)/2 + (a*b*c*n*(b^2 - 4*a*c)^(1/2))/4) + e^2*((3*a*c^3*d^2*n)/2 - (3*b^2*c^2*d^2*n)/4 + (3*b*c^2*d^2*n*(b^2 - 4*a*c)^(1/2))/4) + e*((b*c^3*d^3*n)/2 - (c^3*d^3*n*(b^2 - 4*a*c)^(1/2))/2 - (c^4*d^4*n)/4))/(a^4*e^9 + c^4*d^8*e + b^4*d^4*e^5 - 4*a*b^3*d^3*e^6 + 4*a*c^3*d^6*e^3 + 4*a^3*c*d^2*e^7 - 4*b*c^3*d^7*e^2 - 4*b^3*c*d

```
^5*e^4 + 6*a^2*b^2*d^2*e^7 + 6*a^2*c^2*d^4*e^5 + 6*b^2*c^2*d^6*e^3 - 4*a^3*
b*d*e^8 - 12*a*b*c^2*d^5*e^4 + 12*a*b^2*c*d^4*e^5 - 12*a^2*b*c*d^3*e^6) - (
(11*b^3*d^2*e^3*n - 22*c^3*d^5*n + 2*a^2*b*e^5*n - 7*a*b^2*d*e^4*n + 2*a^2*
c*d*e^4*n + 40*b*c^2*d^4*e*n + 28*a*c^2*d^3*e^2*n - 35*b^2*c*d^3*e^2*n - 6*
a*b*c*d^2*e^3*n)/(6*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*
a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2
*d^5*e - 6*a*b*c*d^3*e^3)) - (x*(a*b^2*e^5*n - 2*a^2*c*e^5*n - 5*b^3*d*e^4*
n + 10*c^3*d^4*e*n - 24*a*c^2*d^2*e^3*n - 16*b*c^2*d^3*e^2*n + 15*b^2*c*d^2
*e^3*n + 12*a*b*c*d*e^4*n))/(2*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d
^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^
5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)) + (n*x^2*(b^3*e^5 - 2*c^3*d^3*e^2 + 3
*b*c^2*d^2*e^3 - 3*a*b*c*e^5 + 6*a*c^2*d*e^4 - 3*b^2*c*d*e^4))/(a^3*e^6 + c
^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4
+ 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))/(4*d^
3*e + 4*e^4*x^3 + 12*d^2*e^2*x + 12*d*e^3*x^2)
```

3.92 $\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2} dx$

Optimal result 556
Rubi [A] (verified) 556
Mathematica [A] (verified) 558
Maple [C] (warning: unable to verify) 559
Fricas [F] 559
Sympy [F] 560
Maxima [F] 560
Giac [F] 560
Mupad [F(-1)] 560

Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2} dx = \frac{in\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{ce}} + \frac{2n\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{ce}} + \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\sqrt{ce}} + \frac{in\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{ce}}$$

```
[Out] I*n*arctan(x*c^(1/2)/a^(1/2))^2/e/a^(1/2)/c^(1/2)+arctan(x*c^(1/2)/a^(1/2))
*ln(d*(c*x^2+a)^n)/e/a^(1/2)/c^(1/2)+2*n*arctan(x*c^(1/2)/a^(1/2))*ln(2*a^(
1/2)/(a^(1/2)+I*x*c^(1/2)))/e/a^(1/2)/c^(1/2)+I*n*polylog(2,1-2*a^(1/2)/(a^
(1/2)+I*x*c^(1/2)))/e/a^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {211, 2520, 12, 5040, 4964, 2449, 2352}

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2} dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\sqrt{ce}} + \frac{in\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{ce}} + \frac{2n\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{ce}} + \frac{in\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{c}x+\sqrt{a}}\right)}{\sqrt{a}\sqrt{ce}}$$

```
[In] Int[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

```
[Out] (I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[c]*e) + (2*n*ArcTan[(Sqrt
[c]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*
e) + (ArcTan[(Sqrt[c]*x)/Sqrt[a]]*Log[d*(a + c*x^2)^n])/(Sqrt[a]*Sqrt[c]*e)
 + (I*n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[
c]*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\sqrt{c}e} - (2cn)\int \frac{x\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}e\,(a+cx^2)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\sqrt{c}e} - \frac{\left(2\sqrt{c}n\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a+cx^2}\,dx}{\sqrt{a}e}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{c}e} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\sqrt{c}e} + \frac{(2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{c}x}{\sqrt{a}}}\,dx}{ae}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{c}e} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}e}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\sqrt{c}e} - \frac{(2n)\int \frac{\log\left(\frac{2}{1+\frac{i\sqrt{c}x}{\sqrt{a}}}\right)}{1+\frac{cx^2}{a}}\,dx}{ae}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{c}e} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}e}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\sqrt{c}e} + \frac{(2in)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{c}x}{\sqrt{a}}}\right)}{\sqrt{a}\sqrt{c}e}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{c}e} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}e}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\sqrt{c}e} + \frac{in\text{Li}_2\left(1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}e}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.75

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx$$
$$= \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(in\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) + 2n\log\left(\frac{2i}{i-\frac{\sqrt{c}x}{\sqrt{a}}}\right) + \log\left(d(a+cx^2)^n\right)\right) + in\,\text{PolyLog}\left(2, \frac{i\sqrt{a}+\sqrt{c}x}{-i\sqrt{a}+\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}e}$$

```
[In] Integrate[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

[Out] (ArcTan[(Sqrt[c]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + 2*n*Log[(2*I)/(I - (Sqrt[c]*x)/Sqrt[a])] + Log[d*(a + c*x^2)^n]) + I*n*PolyLog[2, (I*Sqrt[a] + Sqrt[c]*x)/((-I)*Sqrt[a] + Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*e)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.60 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{\arctan\left(\frac{xc}{\sqrt{ca}}\right) n \ln\left(c\,x^{2}+a\right)}{e\sqrt{ca}}+\frac{\arctan\left(\frac{xc}{\sqrt{ca}}\right) \ln\left(\left(c\,x^{2}+a\right)^{n}\right)}{e\sqrt{ca}}+\frac{n\left(\sum_{\_\alpha=\operatorname{RootOf}\left(c\_Z^{2}+a\right)}\frac{2\ln(x-\_\alpha)\ln\left(c\,x^{2}+a\right)-c\left(\frac{\ln(x-\_\alpha)^{2}}{\_\alpha c}+\right.}{}\right.}{4ec}$

[In] int(ln(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x,method=_RETURNVERBOSE)

[Out] -1/e/(c*a)^(1/2)*arctan(x*c/(c*a)^(1/2))*n*ln(c*x^2+a)+1/e/(c*a)^(1/2)*arctan(x*c/(c*a)^(1/2))*ln((c*x^2+a)^n)+1/4/e*n/c*sum(1/_alpha*(2*ln(x-_alpha)*ln(c*x^2+a)-c*(1/_alpha/c*ln(x-_alpha)^2+2*_alpha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/a*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootOf(_Z^2*c+a))+1/2*(I*Pi*csgn(I*(c*x^2+a)^n)*csgn(I*d*(c*x^2+a)^n)^2-I*Pi*csgn(I*(c*x^2+a)^n)*csgn(I*d*(c*x^2+a)^n)*csgn(I*d)-I*Pi*csgn(I*d*(c*x^2+a)^n)^3+I*Pi*csgn(I*d*(c*x^2+a)^n)^2*csgn(I*d)+2*ln(d))/e/(c*a)^(1/2)*arctan(x*c/(c*a)^(1/2))

Fricas [F]

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \int \frac{\log\left(\left(cx^2+a\right)^n d\right)}{cex^2+ae}\,dx$$

[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="fricas")

[Out] integral(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)

Sympy [F]

$$\int \frac{\log \left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \frac{\int \frac{\log \left(d(a+cx^2)^n\right)}{a+cx^2}\,dx}{e}$$

```
[In] integrate(ln(d*(c*x**2+a)**n)/(c*e*x**2+a*e),x)
[Out] Integral(log(d*(a + c*x**2)**n)/(a + c*x**2), x)/e
```

Maxima [F]

$$\int \frac{\log \left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \int \frac{\log \left((cx^2+a)^n d\right)}{cex^2+ae}\,dx$$

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="maxima")
[Out] integrate(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

Giac [F]

$$\int \frac{\log \left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \int \frac{\log \left((cx^2+a)^n d\right)}{cex^2+ae}\,dx$$

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="giac")
[Out] integrate(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \int \frac{\ln \left(d\,(c\,x^2+a)^n\right)}{c\,e\,x^2+a\,e}\,dx$$

```
[In] int(log(d*(a + c*x^2)^n)/(a*e + c*e*x^2),x)
[Out] int(log(d*(a + c*x^2)^n)/(a*e + c*e*x^2), x)
```

3.93 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx$

Optimal result 561
Rubi [A] (verified) 562
Mathematica [A] (verified) 565
Maple [C] (warning: unable to verify) 565
Fricas [F] 566
Sympy [F(-1)] 566
Maxima [F(-2)] 566
Giac [F] 567
Mupad [F(-1)] 567

Optimal result

Integrand size = 32, antiderivative size = 258

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \frac{2n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e} - \frac{4n\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,e} - \frac{2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}\,e} - \frac{2n\operatorname{PolyLog}\left(2,-\frac{1+\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,e}$$

```
[Out] 2*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))^2/e/(-4*a*c+b^2)^(1/2)-2*arctanh(
(2*c*x+b)/(-4*a*c+b^2)^(1/2))*ln(d*(c*x^2+b*x+a)^n)/e/(-4*a*c+b^2)^(1/2)-4*
n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*ln(2/(1-b/(-4*a*c+b^2)^(1/2)-2*c*x/
(-4*a*c+b^2)^(1/2)))/e/(-4*a*c+b^2)^(1/2)-2*n*polylog(2,(-1-b/(-4*a*c+b^2)^
(1/2)-2*c*x/(-4*a*c+b^2)^(1/2))/(1-b/(-4*a*c+b^2)^(1/2)-2*c*x/(-4*a*c+b^2)^
(1/2)))/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {632, 212, 2607, 12, 6256, 6131, 6055, 2449, 2352}

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = -\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log \left(d(a+bx+cx^2)^n\right)}{e\sqrt{b^2-4ac}} + \frac{2n\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{e\sqrt{b^2-4ac}} - \frac{4n\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log \left(\frac{2}{-\frac{2cx}{\sqrt{b^2-4ac}}-\frac{b}{\sqrt{b^2-4ac}}+1}\right)}{e\sqrt{b^2-4ac}} - \frac{2n\,\text{PolyLog}\left(2,-\frac{\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}+1}{-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}+1}\right)}{e\sqrt{b^2-4ac}}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

```
[Out] (2*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]^2)/(Sqrt[b^2 - 4*a*c]*e) - (4*n
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]*Log[2/(1 - b/Sqrt[b^2 - 4*a*c] - (2
*c*x)/Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*e) - (2*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]]*Log[d*(a + b*x + c*x^2)^n])/(Sqrt[b^2 - 4*a*c]*e) - (2*n*
PolyLog[2, -((1 + b/Sqrt[b^2 - 4*a*c] + (2*c*x)/Sqrt[b^2 - 4*a*c])/(1 - b/S
qrt[b^2 - 4*a*c] - (2*c*x)/Sqrt[b^2 - 4*a*c]))])/(Sqrt[b^2 - 4*a*c]*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2607

```
Int[Log[(c_.)*(Px_)^(n_.)]/(Qx_), x_Symbol] :> With[{u = IntHide[1/Qx, x]},
 Simp[u*Log[c*Px^n], x] - Dist[n, Int[SimplifyIntegrand[u*(D[Px, x]/Px), x]
, x], x]] /; FreeQ[{c, n}, x] && QuadraticQ[{Qx, Px}, x] && EqQ[D[Px/Qx, x]
, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6256

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}e} - n\int\frac{2(-b-2cx)\tanh^{-1}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}e\,(a+bx+cx^2)}\,dx$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}e} - \frac{(2n)\int\frac{(-b-2cx)\tanh^{-1}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{a+bx+cx^2}dx}{\sqrt{b^2-4ac}e}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}e} + \frac{n\text{Subst}\left(\int\frac{\sqrt{b^2-4ac}x\tanh^{-1}(x)}{-\frac{b^2-4ac}{4c}+\frac{(b^2-4ac)x^2}{4c}}dx, x, \frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{ce}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}e} + \frac{\left(\sqrt{b^2-4ac}n\right)\text{Subst}\left(\int\frac{x\tanh^{-1}(x)}{-\frac{b^2-4ac}{4c}+\frac{(b^2-4ac)x^2}{4c}}dx, x, \frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{ce}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}e} - \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}e} - \frac{(4n)\text{Subst}\left(\int\frac{\tanh^{-1}(x)}{1-x}dx, x, \frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}e}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e} - \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}e} + \frac{(4n)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-x}\right)}{1-x^2}dx, x, \frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}e}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e} - \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}e} - \frac{(4n)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e}$$
$$- \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}e} - \frac{2n\mathrm{Li}_2\left(1-\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.31

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx$$
$$= \frac{-n\log^2\left(b-\sqrt{b^2-4ac}+2cx\right)+2n\log\left(\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right)\log\left(b+\sqrt{b^2-4ac}+2cx\right)+n\log^2\left(b+\sqrt{b^2-\right.}{}$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

```
[Out] (-(n*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]^2) + 2*n*Log[(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x] + n*Lo
g[b + Sqrt[b^2 - 4*a*c] + 2*c*x]^2 - 2*n*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]
*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + 2*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] - 2*Log[b + Sqrt[b^2 - 4
*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] - 2*n*PolyLog[2, (-b + Sqrt[b^2 -
 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + 2*n*PolyLog[2, (b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(2*Sqrt[b^2 - 4*a*c]*e)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.46 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.68

method	result
risch	$-\frac{2\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^2}}\right)n\ln(cx^2+bx+a)}{e\sqrt{4ca-b^2}} + \frac{2\arctan\left(\frac{2xc+b}{\sqrt{4ca-b^2}}\right)\ln\left((cx^2+bx+a)^n\right)}{e\sqrt{4ca-b^2}} + \frac{n\left(\sum_{\_\alpha=\mathrm{RootOf}\left(c\_Z^2+\_Zb+a\right)} \frac{2\ln(x-\_\alpha)}{}\right.}{}$

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x,method=_RETURNVERBOSE)
```

```
[Out] -2/e/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*n*ln(c*x^2+b*x+a
)+2/e/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*ln((c*x^2+b*x+a
)^n)+1/2/e*n*sum(1/(2*_alpha*c+b)*(2*ln(x-_alpha)*ln(c*x^2+b*x+a)-1/(2*_alp
```

```
ha*c+b)*ln(x-_alpha)^2-2*(2*_alpha*c+b)/(4*a*c-b^2)*ln(x-_alpha)*ln((2*_alp
ha*c+(x-_alpha)*c+b)/(2*_alpha*c+b))-2*(2*_alpha*c+b)/(4*a*c-b^2)*dilog((2*
_alpha*c+(x-_alpha)*c+b)/(2*_alpha*c+b))),_alpha=RootOf(_Z^2*c+_Z*b+a))+(I*
Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*csgn(I*(c*x^2+b
*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)*csgn(I*d)-I*Pi*csgn(I*d*(c*x^2+b*x+a)^n)
^3+I*Pi*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*d)+2*ln(d))/e/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \int \frac{\log\left((cx^2+bx+a)^n d\right)}{cex^2+bex+ae}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="fricas")
[Out] integral(log((c*x^2 + b*x + a)^n*d)/(c*e*x^2 + b*e*x + a*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(c*e*x**2+b*e*x+a*e),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \int \frac{\log\left((cx^2+bx+a)^n d\right)}{cex^2+bex+ae}\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(c*e*x^2 + b*e*x + a*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = \int \frac{\ln\left(d\,(c\,x^2+b\,x+a)^n\right)}{c\,e\,x^2+b\,e\,x+a\,e}\,dx$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(a*e + b*e*x + c*e*x^2),x)
[Out] int(log(d*(a + b*x + c*x^2)^n)/(a*e + b*e*x + c*e*x^2), x)
```

3.94 $\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex^2}\,dx$

Optimal result 569
Rubi [A] (verified) 570
Mathematica [A] (verified) 576
Maple [C] (warning: unable to verify) 577
Fricas [F] 577
Sympy [F(-1)] 578
Maxima [F(-2)] 578
Giac [F] 578
Mupad [F(-1)] 578

Optimal result

Integrand size = 25, antiderivative size = 762

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\log\left(-\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\log\left(-\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\,\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\,\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\,\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\,\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] 1/2*ln(g*(c*x^2+b*x+a)^n)*ln((-d)^(1/2)-x*e^(1/2))/(-d)^(1/2)/e^(1/2)-1/2*l
n(g*(c*x^2+b*x+a)^n)*ln((-d)^(1/2)+x*e^(1/2))/(-d)^(1/2)/e^(1/2)+1/2*n*ln((
-d)^(1/2)+x*e^(1/2))*ln(-(b+2*c*x-(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/
2)-(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*ln((-d)^(1/2)-
x*e^(1/2))*ln((b+2*c*x-(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/2)+(b-(-4*a
*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*n*ln((-d)^(1/2)+x*e^(1/2))*
ln(-(b+2*c*x+(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/2)-(b+(-4*a*c+b^2)^(1
/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*ln((-d)^(1/2)-x*e^(1/2))*ln((b+2*c*
```

```
x+(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/2)+(b+(-4*a*c+b^2)^(1/2))*e^(1/2
)))/(-d)^(1/2)/e^(1/2)+1/2*n*polylog(2,2*c*((-d)^(1/2)+x*e^(1/2))/(2*c*(-d)
^(1/2)-(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*polylog(2,
2*c*((-d)^(1/2)-x*e^(1/2))/(2*c*(-d)^(1/2)+(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))
/(-d)^(1/2)/e^(1/2)+1/2*n*polylog(2,2*c*((-d)^(1/2)+x*e^(1/2))/(2*c*(-d)^(1
/2)-(b+(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*polylog(2,2*c
*((-d)^(1/2)-x*e^(1/2))/(2*c*(-d)^(1/2)+(b+(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-
d)^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

= {2608, 2604, 2465, 2441, 2440, 2438}

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = -\frac{n\,\text{PolyLog}\left(2,\frac{2c(\sqrt{-d}-\sqrt{e}x)}{2\sqrt{-d}c+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\,\text{PolyLog}\left(2,\frac{2c(\sqrt{-d}-\sqrt{e}x)}{2\sqrt{-d}c+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\,\text{PolyLog}\left(2,\frac{2c(\sqrt{e}x+\sqrt{-d})}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\,\text{PolyLog}\left(2,\frac{2c(\sqrt{e}x+\sqrt{-d})}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(\frac{\sqrt{e}\left(-\sqrt{b^2-4ac}+b+2cx\right)}{\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{b^2-4ac}+b+2cx\right)}{\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(-\frac{\sqrt{e}\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2c\sqrt{-d}-\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(-\frac{\sqrt{e}\left(\sqrt{b^2-4ac}+b+2cx\right)}{2c\sqrt{-d}-\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[In] Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

```
[Out] -1/2*(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b -
Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(Sqrt[-d]*Sqrt[e])
- (n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqr
t[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) +
 (n*Log[-((Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b - Sq
rt[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e])
 + (n*Log[-((Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b +
Sqrt[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e
```

```
]) + (Log[Sqrt[-d] - Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*Sqr
t[e]) - (Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*
Sqrt[e]) - (n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b -
Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (n*PolyLog[2, (2*c*(Sq
rt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*
Sqrt[-d]*Sqrt[e]) + (n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d
] - (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) + (n*PolyLog[2,
 (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2 - 4*a*c])*Sqrt[
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
```

```
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-d}\log\left(g(a+bx+cx^2)^n\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log\left(g(a+bx+cx^2)^n\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{-d}-\sqrt{e}x}dx}{2\sqrt{-d}}-\frac{\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{-d}+\sqrt{e}x}dx}{2\sqrt{-d}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\int \frac{(b+2cx)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{a+bx+cx^2}dx}{2\sqrt{-d}\sqrt{e}} + \frac{n\int \frac{(b+2cx)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{a+bx+cx^2}dx}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\int\left(\frac{2c\log\left(\sqrt{-d}-\sqrt{e}x\right)}{b-\sqrt{b^2-4ac}+2cx}+\frac{2c\log\left(\sqrt{-d}-\sqrt{e}x\right)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{2\sqrt{-d}\sqrt{e}} + \frac{n\int\left(\frac{2c\log\left(\sqrt{-d}+\sqrt{e}x\right)}{b-\sqrt{b^2-4ac}+2cx}+\frac{2c\log\left(\sqrt{-d}+\sqrt{e}x\right)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(cn)\int \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)}{b-\sqrt{b^2-4ac}+2cx}dx}{\sqrt{-d}\sqrt{e}} - \frac{(cn)\int \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)}{b+\sqrt{b^2-4ac}+2cx}dx}{\sqrt{-d}\sqrt{e}} + \frac{(cn)\int \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)}{b-\sqrt{b^2-4ac}+2cx}dx}{\sqrt{-d}\sqrt{e}} + \frac{(cn)\int \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)}{b+\sqrt{b^2-4ac}+2cx}dx}{\sqrt{-d}\sqrt{e}}$$

$$
\begin{aligned}
&= -\frac{n \log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{n \log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{n \log\left(-\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{n \log\left(-\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{n\int \frac{\log\left(-\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{-2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}} - \frac{n\int \frac{\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{-2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}} \\
&\quad - \frac{n\int \frac{\log\left(-\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}} - \frac{n\int \frac{\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}
\end{aligned}
$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\log\left(-\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\log\left(-\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{-2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{x}\,dx, x, \sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{-2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{x}\,dx, x, \sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{-2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{x}\,dx, x, \sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{-2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{x}\,dx, x, \sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\log\left(-\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\log\left(-\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\operatorname{Li}_2\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\operatorname{Li}_2\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\operatorname{Li}_2\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{2c\sqrt{-d}-\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{n\operatorname{Li}_2\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 626, normalized size of antiderivative = 0.82

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \frac{-n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right) - n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right) + n\log}{}$$

```
[In] Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

```
[Out] (-(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b - Sqr
t[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x]) - n*Log[(Sqrt[e]*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e]
)]*Log[Sqrt[-d] - Sqrt[e]*x] + n*Log[(Sqrt[e]*(-b + Sqrt[b^2 - 4*a*c] - 2*c
*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] + Sqrt
[e]*x] + n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*Sqrt[-d] + (
```

```
b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] + Sqrt[e]*x] + Log[Sqrt[-d] -
 Sqrt[e]*x]*Log[g*(a + x*(b + c*x))^n] - Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a
 + x*(b + c*x))^n] - n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d
] + (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])] - n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[
e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])] + n*PolyLog[2, (2*
c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[e])
] + n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2
 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.88 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.73

method	result	size
risch	Expression too large to display	555

```
[In] int(ln(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((c*x^2+b*x+a)^n)-n*ln(c*x^2+b*x+a))/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
+1/2*n/e*sum(1/_alpha*(ln(x-_alpha)*ln(c*x^2+b*x+a)-ln(x-_alpha)*ln((RootOf
(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=1)-x+_alpha)/RootO
f(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=1))-ln(x-_alpha)*
ln((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=2)-x+_al
pha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=2))-dil
og((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=1)-x+_al
pha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=1))-dil
og((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=2)-x+_al
pha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+a*e-c*d,index=2))),_a
lpha=RootOf(_Z^2*e+d))+(1/2*I*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*
x+a)^n)^2-1/2*I*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)*csgn(I
*g)-1/2*I*Pi*csgn(I*g*(c*x^2+b*x+a)^n)^3+1/2*I*Pi*csgn(I*g*(c*x^2+b*x+a)^n)
^2*csgn(I*g)+ln(g))/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Fricas [F]

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \int \frac{\log \left((cx^2+bx+a)^n g\right)}{ex^2+d}\,dx$$

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*g)/(e*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(g*(c*x**2+b*x+a)**n)/(e*x**2+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \int \frac{\log \left((cx^2+bx+a)^n g\right)}{ex^2+d}\,dx$$

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x + a)^n*g)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \int \frac{\ln \left(g\,(c\,x^2+b\,x+a)^n\right)}{e\,x^2+d}\,dx$$

```
[In] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x^2),x)
[Out] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x^2), x)
```

3.95 $\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx$

Optimal result 580
Rubi [A] (verified) 581
Mathematica [A] (verified) 587
Maple [C] (warning: unable to verify) 588
Fricas [F] 589
Sympy [F(-1)] 589
Maxima [F(-2)] 589
Giac [F] 589
Mupad [F(-1)] 590

Optimal result

Integrand size = 28, antiderivative size = 782

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx = -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} - \frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} + \frac{n\log\left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)\log\left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} + \frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)\log\left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} + \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}} - \frac{n\,\text{PolyLog}\left(2,-\frac{c\left(e-\sqrt{e^2-4df}+2fx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{n\,\text{PolyLog}\left(2,-\frac{c\left(e-\sqrt{e^2-4df}+2fx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\,\text{PolyLog}\left(2,-\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\,\text{PolyLog}\left(2,-\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}$$

```
[Out] ln(g*(c*x^2+b*x+a)^n)*ln(e+2*f*x-(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)-n*l
n(f*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e-(-4*d*f+e^2
)^(1/2))))*ln(e+2*f*x-(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)-ln(g*(c*x^2+b*
x+a)^n)*ln(e+2*f*x+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)-n*ln(e+2*f*x-(-4*
d*f+e^2)^(1/2))*ln(-f*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(c*e-b*f+f*(-4*a*c+b^2)^
(1/2)-c*(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)+n*ln(e+2*f*x+(-4*d*f+e^2)^(
```

```
1/2))*ln(f*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(f*(b-(-4*a*c+b^2)^(1/2))-c*(e+(-4*
d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n*ln(e+2*f*x+(-4*d*f+e^2)^(1/2))*ln(f*
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e+(-4*d*f+e^2)^(1
/2))))/(-4*d*f+e^2)^(1/2)-n*polylog(2,-c*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(f*(b
-(-4*a*c+b^2)^(1/2))-c*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)-n*polylo
g(2,-c*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e-(-4*d*f+
e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n*polylog(2,-c*(e+2*f*x+(-4*d*f+e^2)^(1/2)
)/(f*(b-(-4*a*c+b^2)^(1/2))-c*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n
*polylog(2,-c*(e+2*f*x+(-4*d*f+e^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e+(
-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used

$= \{2608, 2604, 2465, 2441, 2440, 2438\}$

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx = -\frac{n\,\mathrm{PolyLog}\left(2,-\frac{c\left(e+2fx-\sqrt{e^2-4df}\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{n\,\mathrm{PolyLog}\left(2,-\frac{c\left(e+2fx-\sqrt{e^2-4df}\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\,\mathrm{PolyLog}\left(2,-\frac{c\left(e+2fx+\sqrt{e^2-4df}\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\,\mathrm{PolyLog}\left(2,-\frac{c\left(e+2fx+\sqrt{e^2-4df}\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} - \frac{n\log\left(-\sqrt{e^2-4df}+e+2fx\right)\log\left(-\frac{f\left(-\sqrt{b^2-4ac}+b+2cx\right)}{f\sqrt{b^2-4ac}-bf-c\sqrt{e^2-4df}+ce}\right)}{\sqrt{e^2-4df}} - \frac{n\log\left(-\sqrt{e^2-4df}+e+2fx\right)\log\left(\frac{f\left(\sqrt{b^2-4ac}+b+2cx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\log\left(\sqrt{e^2-4df}+e+2fx\right)\log\left(\frac{f\left(-\sqrt{b^2-4ac}+b+2cx\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}} + \frac{n\log\left(\sqrt{e^2-4df}+e+2fx\right)\log\left(\frac{f\left(\sqrt{b^2-4ac}+b+2cx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}} + \frac{\log\left(-\sqrt{e^2-4df}+e+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(\sqrt{e^2-4df}+e+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}$$

```
[In] Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]
```

```
[Out] -((n*Log[-((f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*e - b*f + Sqrt[b^2 - 4*a*
c]*f - c*Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sqrt[e^2
- 4*d*f]) - (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a
*c])*f - c*(e - Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sq
rt[e^2 - 4*d*f] + (n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b - Sqrt[b^2
 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*
```

```
x])/Sqrt[e^2 - 4*d*f] + (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sq
rt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f]
+ 2*f*x])/Sqrt[e^2 - 4*d*f] + (Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a
+ b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]*
Log[g*(a + b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqr
t[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d
*f])))])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqrt[e^2 - 4*d*f] + 2*
f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 -
 4*d*f] + (n*PolyLog[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^
2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f] + (n*PolyLo
g[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(
e + Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2f\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}+2fx\right)}-\frac{2f\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}+2fx\right)}\right)dx\\
&= \frac{(2f)\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{e-\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}}-\frac{(2f)\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{e+\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}}\\
&= \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}\\
&\quad-\frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}\\
&\quad-\frac{n\int \frac{(b+2cx)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{a+bx+cx^2}\,dx}{\sqrt{e^2-4df}}+\frac{n\int \frac{(b+2cx)\log\left(e+\sqrt{e^2-4df}+2fx\right)}{a+bx+cx^2}\,dx}{\sqrt{e^2-4df}}\\
&= \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}\\
&\quad-\frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}\\
&\quad-\frac{n\int \left(\frac{2c\log\left(e-\sqrt{e^2-4df}+2fx\right)}{b-\sqrt{b^2-4ac}+2cx}+\frac{2c\log\left(e-\sqrt{e^2-4df}+2fx\right)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{\sqrt{e^2-4df}}\\
&\quad+\frac{n\int \left(\frac{2c\log\left(e+\sqrt{e^2-4df}+2fx\right)}{b-\sqrt{b^2-4ac}+2cx}+\frac{2c\log\left(e+\sqrt{e^2-4df}+2fx\right)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{\sqrt{e^2-4df}}\\
&= \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}\\
&\quad-\frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}\\
&\quad-\frac{(2cn)\int \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{e^2-4df}}-\frac{(2cn)\int \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{e^2-4df}}\\
&\quad+\frac{(2cn)\int \frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{e^2-4df}}+\frac{(2cn)\int \frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{e^2-4df}}
\end{aligned}$$

$$= -\frac{n \log \left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}f-c\sqrt{e^2-4df}}\right) \log \left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$
$$- \frac{n \log \left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right) \log \left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$
$$+ \frac{n \log \left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right) \log \left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$
$$+ \frac{n \log \left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right) \log \left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$
$$+ \frac{\log \left(e-\sqrt{e^2-4df}+2fx\right) \log \left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}$$
$$- \frac{\log \left(e+\sqrt{e^2-4df}+2fx\right) \log \left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}$$
$$+ \frac{(2fn) \int \frac{\log \left(\frac{2f\left(b-\sqrt{b^2-4ac}+2cx\right)}{2\left(b-\sqrt{b^2-4ac}\right)f-2c\left(e-\sqrt{e^2-4df}\right)}\right)}{e-\sqrt{e^2-4df}+2fx} \, dx}{\sqrt{e^2-4df}}$$
$$- \frac{(2fn) \int \frac{\log \left(\frac{2f\left(b-\sqrt{b^2-4ac}+2cx\right)}{2\left(b-\sqrt{b^2-4ac}\right)f-2c\left(e+\sqrt{e^2-4df}\right)}\right)}{e+\sqrt{e^2-4df}+2fx} \, dx}{\sqrt{e^2-4df}}$$
$$+ \frac{(2fn) \int \frac{\log \left(\frac{2f\left(b+\sqrt{b^2-4ac}+2cx\right)}{2\left(b+\sqrt{b^2-4ac}\right)f-2c\left(e-\sqrt{e^2-4df}\right)}\right)}{e-\sqrt{e^2-4df}+2fx} \, dx}{\sqrt{e^2-4df}}$$
$$- \frac{(2fn) \int \frac{\log \left(\frac{2f\left(b+\sqrt{b^2-4ac}+2cx\right)}{2\left(b+\sqrt{b^2-4ac}\right)f-2c\left(e+\sqrt{e^2-4df}\right)}\right)}{e+\sqrt{e^2-4df}+2fx} \, dx}{\sqrt{e^2-4df}}$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$

$$-\frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$

$$+\frac{n\log\left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)\log\left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$

$$+\frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)\log\left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$

$$+\frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}$$

$$-\frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}$$

$$+\frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{2\left(b-\sqrt{b^2-4ac}\right)f-2c\left(e-\sqrt{e^2-4df}\right)}\right)}{x}dx,x,e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$

$$+\frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{2\left(b+\sqrt{b^2-4ac}\right)f-2c\left(e-\sqrt{e^2-4df}\right)}\right)}{x}dx,x,e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$

$$-\frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{2\left(b-\sqrt{b^2-4ac}\right)f-2c\left(e+\sqrt{e^2-4df}\right)}\right)}{x}dx,x,e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$

$$-\frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{2\left(b+\sqrt{b^2-4ac}\right)f-2c\left(e+\sqrt{e^2-4df}\right)}\right)}{x}dx,x,e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}$$

$$
= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}
- \frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}
+ \frac{n\log\left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)\log\left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}
+ \frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)\log\left(e+\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}
+ \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}
- \frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}}
- \frac{n\mathrm{Li}_2\left(-\frac{c\left(e-\sqrt{e^2-4df}+2fx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}
- \frac{n\mathrm{Li}_2\left(-\frac{c\left(e-\sqrt{e^2-4df}+2fx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}
+ \frac{n\mathrm{Li}_2\left(-\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(b-\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}
+ \frac{n\mathrm{Li}_2\left(-\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 663, normalized size of antiderivative = 0.85

$$
\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx
= \frac{-n\log\left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{-ce+bf-\sqrt{b^2-4ac}f+c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)-n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f+c\left(-e+\sqrt{e^2-4df}\right)}\right)\log}{}
$$

```
[In] Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]
```

```
[Out] (-(n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(-(c*e) + b*f - Sqrt[b^2 - 4*a
*c]*f + c*Sqrt[e^2 - 4*d*f])]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]) - n*Log[(
f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqr
t[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x] + n*Log[(f*(-b + Sqrt[
b^2 - 4*a*c] - 2*c*x))/((-b + Sqrt[b^2 - 4*a*c])*f + c*(e + Sqrt[e^2 - 4*d*
f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x] + n*Log[(f*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e +
```

```
Sqrt[e^2 - 4*d*f] + 2*f*x] + Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a +
x*(b + c*x))^n] - Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a + x*(b + c*x)
)^n] - n*PolyLog[2, (c*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/((b - Sqrt[b^2 - 4
*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))] - n*PolyLog[2, (c*(-e + Sqrt[e^2 -
4*d*f] - 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))]
+ n*PolyLog[2, (c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((-b + Sqrt[b^2 - 4*a*c]
)*f + c*(e + Sqrt[e^2 - 4*d*f]))] + n*PolyLog[2, (c*(e + Sqrt[e^2 - 4*d*f]
+ 2*f*x))/(-((b + Sqrt[b^2 - 4*a*c])*f) + c*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt
[e^2 - 4*d*f]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.78 (sec) , antiderivative size = 637, normalized size of antiderivative = 0.81

method	result	size
risch	Expression too large to display	637

```
[In] int(ln(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(ln((c*x^2+b*x+a)^n)-n*ln(c*x^2+b*x+a))/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e
)/(4*d*f-e^2)^(1/2))+n*sum((ln(x-_alpha)*ln(c*x^2+b*x+a)-ln(x-_alpha)*ln((R
ootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1)
-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-
c*d,index=1))-ln(x-_alpha)*ln((RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alp
ha*f-_alpha*c*e+a*f-c*d,index=2)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*
f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=2))-dilog((RootOf(_Z^2*c*f+(2*_al
pha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1)-x+_alpha)/RootOf(_Z^
2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1))-dilog((
RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=2
)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f
-c*d,index=2)))/(2*_alpha*f+e),_alpha=RootOf(_Z^2*f+_Z*e+d))+2*(1/2*I*Pi*cs
gn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)^2-1/2*I*Pi*csgn(I*(c*x^2+b*
x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)*csgn(I*g)-1/2*I*Pi*csgn(I*g*(c*x^2+b*x+a)
^n)^3+1/2*I*Pi*csgn(I*g*(c*x^2+b*x+a)^n)^2*csgn(I*g)+ln(g))/(4*d*f-e^2)^(1/
2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))
```

Fricas [F]

$$\int \frac{\log (g(a+bx+cx^2)^n)}{d+ex+fx^2}\, dx = \int \frac{\log ((cx^2+bx+a)^n g)}{fx^2+ex+d}\, dx$$

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log (g(a+bx+cx^2)^n)}{d+ex+fx^2}\, dx = \text{Timed out}$$

```
[In] integrate(ln(g*(c*x**2+b*x+a)**n)/(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log (g(a+bx+cx^2)^n)}{d+ex+fx^2}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\log (g(a+bx+cx^2)^n)}{d+ex+fx^2}\, dx = \int \frac{\log ((cx^2+bx+a)^n g)}{fx^2+ex+d}\, dx$$

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx = \int \frac{\ln \left(g\,(c\,x^2+b\,x+a)^n\right)}{f\,x^2+e\,x+d}\,dx$$

```
[In] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x + f*x^2),x)
[Out] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x + f*x^2), x)
```

3.96 $\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal result 591
Rubi [A] (verified) 591
Mathematica [A] (verified) 594
Maple [F] 595
Fricas [F] 595
Sympy [F] 595
Maxima [A] (verification not implemented) 595
Giac [F] 596
Mupad [F(-1)] 596

Optimal result

Integrand size = 16, antiderivative size = 144

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx = 8n^2x-\frac{4bn^2\log(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-\frac{bn^2\log^2(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+x\log^2\left(d\left(bx+cx^2\right)^n\right)-\frac{2bn^2\operatorname{PolyLog}\left(2,1+\frac{cx}{b}\right)}{c}$$

```
[Out] 8*n^2*x-4*b*n^2*ln(c*x+b)/c-2*b*n^2*ln(-c*x/b)*ln(c*x+b)/c-b*n^2*ln(c*x+b)^
2/c-4*n*x*ln(d*(c*x^2+b*x)^n)+2*b*n*ln(c*x+b)*ln(d*(c*x^2+b*x)^n)/c+x*ln(d*
(c*x^2+b*x)^n)^2-2*b*n^2*polylog(2,1+c*x/b)/c
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2603, 2608, 45, 2604, 1607, 2465, 2441, 2352, 2437, 2338}

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx = x\log^2\left(d\left(bx+cx^2\right)^n\right)-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}-\frac{2bn^2\operatorname{PolyLog}\left(2,\frac{cx}{b}+1\right)}{c}-\frac{bn^2\log^2(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-\frac{4bn^2\log(b+cx)}{c}+8n^2x$$

```
[In] Int[Log[d*(b*x + c*x^2)^n]^2,x]
```

```
[Out] 8*n^2*x - (4*b*n^2*Log[b + c*x])/c - (2*b*n^2*Log[-((c*x)/b)]*Log[b + c*x])
/c - (b*n^2*Log[b + c*x]^2)/c - 4*n*x*Log[d*(b*x + c*x^2)^n] + (2*b*n*Log[b
 + c*x]*Log[d*(b*x + c*x^2)^n])/c + x*Log[d*(b*x + c*x^2)^n]^2 - (2*b*n^2*P
olyLog[2, 1 + (c*x)/b])/c
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log^2\left(d\left(bx+cx^2\right)^n\right)-(2n)\int\frac{(b+2cx)\log\left(d(bx+cx^2)^n\right)}{b+cx}\,dx\\
&= x\log^2\left(d\left(bx+cx^2\right)^n\right)-(2n)\int\left(2\log\left(d\left(bx+cx^2\right)^n\right)-\frac{b\log\left(d(bx+cx^2)^n\right)}{b+cx}\right)dx\\
&= x\log^2\left(d\left(bx+cx^2\right)^n\right)-(4n)\int\log\left(d\left(bx+cx^2\right)^n\right)\,dx+(2bn)\int\frac{\log\left(d(bx+cx^2)^n\right)}{b+cx}\,dx\\
&= -4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d(bx+cx^2)^n\right)}{c}\\
&\quad+x\log^2\left(d\left(bx+cx^2\right)^n\right)+\left(4n^2\right)\int\frac{b+2cx}{b+cx}\,dx-\frac{\left(2bn^2\right)\int\frac{(b+2cx)\log(b+cx)}{bx+cx^2}\,dx}{c}\\
&= -4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d(bx+cx^2)^n\right)}{c}\\
&\quad+x\log^2\left(d\left(bx+cx^2\right)^n\right)+\left(4n^2\right)\int\left(2-\frac{b}{b+cx}\right)dx-\frac{\left(2bn^2\right)\int\frac{(b+2cx)\log(b+cx)}{x(b+cx)}\,dx}{c}
\end{aligned}$$

$$
\begin{aligned}
&= 8n^2x - \frac{4bn^2\log(b+cx)}{c} - 4nx\log\left(d\left(bx+cx^2\right)^n\right) \\
&\quad + \frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c} + x\log^2\left(d\left(bx+cx^2\right)^n\right) \\
&\quad - \frac{(2bn^2)\int\left(\frac{\log(b+cx)}{x}+\frac{c\log(b+cx)}{b+cx}\right)dx}{c} \\
&= 8n^2x - \frac{4bn^2\log(b+cx)}{c} - 4nx\log\left(d\left(bx+cx^2\right)^n\right) \\
&\quad + \frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c} + x\log^2\left(d\left(bx+cx^2\right)^n\right) \\
&\quad - (2bn^2)\int\frac{\log(b+cx)}{b+cx}\,dx - \frac{(2bn^2)\int\frac{\log(b+cx)}{x}\,dx}{c} \\
&= 8n^2x - \frac{4bn^2\log(b+cx)}{c} - \frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c} - 4nx\log\left(d\left(bx+cx^2\right)^n\right) \\
&\quad + \frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c} + x\log^2\left(d\left(bx+cx^2\right)^n\right) \\
&\quad + (2bn^2)\int\frac{\log\left(-\frac{cx}{b}\right)}{b+cx}\,dx - \frac{(2bn^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,b+cx\right)}{c} \\
&= 8n^2x - \frac{4bn^2\log(b+cx)}{c} - \frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c} - \frac{bn^2\log^2(b+cx)}{c} \\
&\quad - 4nx\log\left(d\left(bx+cx^2\right)^n\right) + \frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c} \\
&\quad + x\log^2\left(d\left(bx+cx^2\right)^n\right) - \frac{2bn^2\text{Li}_2\left(1+\frac{cx}{b}\right)}{c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$
\int\log^2\left(d\left(bx+cx^2\right)^n\right)dx
$$

$$
= \frac{-bn^2\log^2(b+cx) - 2bn\log(b+cx)\left(2n+n\log\left(-\frac{cx}{b}\right)-\log\left(d(x(b+cx))^n\right)\right) + cx\left(8n^2-4n\log\left(d(x(b+c\right.\right.}{c}
$$

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]^2,x]
```

```
[Out] (-(b*n^2*Log[b + c*x]^2) - 2*b*n*Log[b + c*x]*(2*n + n*Log[-((c*x)/b)] - Lo
g[d*(x*(b + c*x))^n]) + c*x*(8*n^2 - 4*n*Log[d*(x*(b + c*x))^n] + Log[d*(x*
(b + c*x))^n]^2) - 2*b*n^2*PolyLog[2, 1 + (c*x)/b])/c
```

Maple [F]

$$\int \ln \left(d\left(c\,x^{2}+b x\right)^{n}\right)^{2}dx$$

```
[In] int(ln(d*(c*x^2+b*x)^n)^2,x)
[Out] int(ln(d*(c*x^2+b*x)^n)^2,x)
```

Fricas [F]

$$\int \log^2 \left(d\left(bx+cx^2\right)^n\right)\,dx = \int \log \left(\left(cx^2+bx\right)^n d\right)^2\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="fricas")
[Out] integral(log((c*x^2 + b*x)^n*d)^2, x)
```

Sympy [F]

$$\int \log^2 \left(d\left(bx+cx^2\right)^n\right)\,dx = \int \log \left(d\left(bx+cx^2\right)^n\right)^2 dx$$

```
[In] integrate(ln(d*(c*x**2+b*x)**n)**2,x)
[Out] Integral(log(d*(b*x + c*x**2)**n)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \log^2 \left(d\left(bx+cx^2\right)^n\right)\,dx =& \\ &-\left(\frac{2\left(\log\left(cx+b\right)\log\left(-\frac{cx+b}{b}+1\right)+\mathrm{Li}_2\left(\frac{cx+b}{b}\right)\right)b}{c}+\frac{b\log\left(cx+b\right)^2-8\,cx+4\,b\log\left(cx+b\right)}{c}\right)n^2 \\ &-2\,n\left(2\,x-\frac{b\log\left(cx+b\right)}{c}\right)\log\left(\left(cx^2+bx\right)^n d\right)+x\log\left(\left(cx^2+bx\right)^n d\right)^2\end{aligned}$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="maxima")
[Out] -(2*(log(c*x + b)*log(-(c*x + b)/b + 1) + dilog((c*x + b)/b))*b/c + (b*log(
c*x + b)^2 - 8*c*x + 4*b*log(c*x + b))/c)*n^2 - 2*n*(2*x - b*log(c*x + b)/c
)*log((c*x^2 + b*x)^n*d) + x*log((c*x^2 + b*x)^n*d)^2
```

Giac [F]

$$\int \log^2\left(d(bx+cx^2)^n\right)\,dx = \int \log\left((cx^2+bx)^n d\right)^2\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x)^n*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(d(bx+cx^2)^n\right)\,dx = \int \ln\left(d\left(c\,x^2+b\,x\right)^n\right)^2\,dx$$

```
[In] int(log(d*(b*x + c*x^2)^n)^2,x)
[Out] int(log(d*(b*x + c*x^2)^n)^2, x)
```

3.97 $\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal result . . . 597
Rubi [A] (verified) . . . 598
Mathematica [A] (verified) . . . 606
Maple [F] . . . 607
Fricas [F] . . . 607
Sympy [F] . . . 607
Maxima [F(-2)] . . . 607
Giac [F] . . . 608
Mupad [F(-1)] . . . 608

Optimal result

Integrand size = 17, antiderivative size = 587

$$\begin{aligned}
&\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx \\
&= 8n^2x-\frac{4\sqrt{b^2-4ac}\,n^2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} \\
&\quad-\frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c} \\
&\quad-\frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{c} \\
&\quad-\frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log^2\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c} \\
&\quad-\frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c} \\
&\quad-\frac{2bn^2\log\left(a+bx+cx^2\right)}{c}-4nx\log\left(d\left(a+bx+cx^2\right)^n\right) \\
&\quad+\frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} \\
&\quad+\frac{\left(b+\sqrt{b^2-4ac}\right)n\log\left(b+\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} \\
&\quad+x\log^2\left(d\left(a+bx+cx^2\right)^n\right)-\frac{\left(b-\sqrt{b^2-4ac}\right)n^2\operatorname{PolyLog}\left(2,-\frac{b-\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c} \\
&\quad-\frac{\left(b+\sqrt{b^2-4ac}\right)n^2\operatorname{PolyLog}\left(2,\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c}
\end{aligned}$$

```
[Out] 8*n^2*x-2*b*n^2*ln(c*x^2+b*x+a)/c-4*n*x*ln(d*(c*x^2+b*x+a)^n)+x*ln(d*(c*x^2
+b*x+a)^n)^2+n*ln(d*(c*x^2+b*x+a)^n)*ln(b+2*c*x-(-4*a*c+b^2)^(1/2))*(b-(-4*
```

```
a*c+b^2)^(1/2))/c-1/2*n^2*ln(b+2*c*x-(-4*a*c+b^2)^(1/2))^2*(b-(-4*a*c+b^2)^
(1/2))/c-n^2*ln(b+2*c*x-(-4*a*c+b^2)^(1/2))*ln(1/2*(b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))/c-n^2*polylog(2,1/2*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))/c-4*n^2*a
rctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c+n*ln(d*(c*x^2+b*x
+a)^n)*ln(b+2*c*x+(-4*a*c+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2))/c-n^2*ln(1/2*(
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*ln(b+2*c*x+(-4*a*c+b^2)^(1
/2))*(b+(-4*a*c+b^2)^(1/2))/c-1/2*n^2*ln(b+2*c*x+(-4*a*c+b^2)^(1/2))^2*(b+(
-4*a*c+b^2)^(1/2))/c-n^2*polylog(2,1/2*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {2603, 2608, 787, 648, 632, 212, 642, 2604, 2465, 2437, 2338, 2441, 2440, 2438}

$$\begin{aligned}
&\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx \\
&= -\frac{4n^2\sqrt{b^2-4ac}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} \\
&+\frac{n\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} \\
&+\frac{n\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}+b+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} \\
&-\frac{n^2\left(b-\sqrt{b^2-4ac}\right)\text{PolyLog}\left(2,-\frac{b+2cx-\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c} \\
&-\frac{n^2\left(\sqrt{b^2-4ac}+b\right)\text{PolyLog}\left(2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c} \\
&-\frac{n^2\left(b-\sqrt{b^2-4ac}\right)\log^2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2c} \\
&-\frac{n^2\left(\sqrt{b^2-4ac}+b\right)\log^2\left(\sqrt{b^2-4ac}+b+2cx\right)}{2c} \\
&-\frac{n^2\left(\sqrt{b^2-4ac}+b\right)\log\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)\log\left(\sqrt{b^2-4ac}+b+2cx\right)}{c} \\
&-\frac{n^2\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\log\left(\frac{\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c} \\
&+x\log^2\left(d\left(a+bx+cx^2\right)^n\right)-4nx\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{2bn^2\log\left(a+bx+cx^2\right)}{c}+8n^2x
\end{aligned}$$

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

```
[Out] 8*n^2*x - (4*Sqrt[b^2 - 4*a*c]*n^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
c - ((b - Sqrt[b^2 - 4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c
) - ((b + Sqrt[b^2 - 4*a*c])*n^2*Log[-1/2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/c - ((b + Sqrt[b^2 -
4*a*c])*n^2*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c) - ((b - Sqrt[b^2 -
4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(b + Sqrt[b^2 - 4*a*c] +
 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c - (2*b*n^2*Log[a + b*x + c*x^2])/c - 4*n*
x*Log[d*(a + b*x + c*x^2)^n] + ((b - Sqrt[b^2 - 4*a*c])*n*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + ((b + Sqrt[b^2 - 4*a*c])*
n*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + x*Log[
d*(a + b*x + c*x^2)^n]^2 - ((b - Sqrt[b^2 - 4*a*c])*n^2*PolyLog[2, -1/2*(b
- Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c - ((b + Sqrt[b^2 - 4*a*c
])*n^2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
```

```
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = x\log^2\left(d\left(a+bx+cx^2\right)^n\right) - (2n)\int \frac{x(b+2cx)\log\left(d\left(a+bx+cx^2\right)^n\right)}{a+bx+cx^2}\,dx$$

$$= x\log^2\left(d\left(a+bx+cx^2\right)^n\right) - (2n)\int \left(2\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{(2a+bx)\log\left(d\left(a+bx+cx^2\right)^n\right)}{a+bx+cx^2}\right)dx$$

$$= x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + (2n)\int \frac{(2a+bx)\log\left(d\left(a+bx+cx^2\right)^n\right)}{a+bx+cx^2}\,dx - (4n)\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$$

$$= -4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + (2n)\int \left(\frac{\left(b-\sqrt{b^2-4ac}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(b+\sqrt{b^2-4ac}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{b+\sqrt{b^2-4ac}+2cx}\right)dx + \left(4n^2\right)\int \frac{x(b+2cx)}{a+bx+cx^2}\,dx$$

$$= 8n^2x - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + \left(2\left(b-\sqrt{b^2-4ac}\right)n\right)\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{b-\sqrt{b^2-4ac}+2cx}\,dx + \left(2\left(b+\sqrt{b^2-4ac}\right)n\right)\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{b+\sqrt{b^2-4ac}+2cx}\,dx + \frac{\left(4n^2\right)\int \frac{-2ac-bcx}{a+bx+cx^2}\,dx}{c}$$

$$= 8n^2x - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(d(a+bx+cx^2)^n\right)}{c} + \frac{\left(b+\sqrt{b^2-4ac}\right)n\log\left(b+\sqrt{b^2-4ac}+2cx\right)\log\left(d(a+bx+cx^2)^n\right)}{c} + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) - \frac{(2bn^2)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{c} + \frac{(2(b^2-4ac)\,n^2)\int\frac{1}{a+bx+cx^2}\,dx}{c} - \frac{\left(\left(b-\sqrt{b^2-4ac}\right)n^2\right)\int\frac{(b+2cx)\log\left(b-\sqrt{b^2-4ac}+2cx\right)}{a+bx+cx^2}\,dx}{c} - \frac{\left(\left(b+\sqrt{b^2-4ac}\right)n^2\right)\int\frac{(b+2cx)\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{a+bx+cx^2}\,dx}{c}$$

$$= 8n^2x - \frac{2bn^2\log\left(a+bx+cx^2\right)}{c} - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(d(a+bx+cx^2)^n\right)}{c} + \frac{\left(b+\sqrt{b^2-4ac}\right)n\log\left(b+\sqrt{b^2-4ac}+2cx\right)\log\left(d(a+bx+cx^2)^n\right)}{c} + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) - \frac{(4(b^2-4ac)\,n^2)\,\mathrm{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c} - \frac{\left(\left(b-\sqrt{b^2-4ac}\right)n^2\right)\int\left(\frac{2c\log\left(b-\sqrt{b^2-4ac}+2cx\right)}{b-\sqrt{b^2-4ac}+2cx} + \frac{2c\log\left(b-\sqrt{b^2-4ac}+2cx\right)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{c} - \frac{\left(\left(b+\sqrt{b^2-4ac}\right)n^2\right)\int\left(\frac{2c\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{b-\sqrt{b^2-4ac}+2cx} + \frac{2c\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{c}$$

$$
\begin{aligned}
&= 8n^2x - \frac{4\sqrt{b^2-4ac}\,n^2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} \\
&\quad - \frac{2bn^2 \log\left(a+bx+cx^2\right)}{c} - 4nx \log\left(d\left(a+bx+cx^2\right)^n\right) \\
&\quad + \frac{\left(b-\sqrt{b^2-4ac}\right) n \log\left(b-\sqrt{b^2-4ac}+2cx\right) \log\left(d\left(a+bx+cx^2\right)^n\right)}{c} \\
&\quad + \frac{\left(b+\sqrt{b^2-4ac}\right) n \log\left(b+\sqrt{b^2-4ac}+2cx\right) \log\left(d\left(a+bx+cx^2\right)^n\right)}{c} \\
&\quad + x \log^2\left(d\left(a+bx+cx^2\right)^n\right) \\
&\quad - \left(2\left(b-\sqrt{b^2-4ac}\right) n^2\right) \int \frac{\log\left(b-\sqrt{b^2-4ac}+2cx\right)}{b-\sqrt{b^2-4ac}+2cx}\, dx \\
&\quad - \left(2\left(b-\sqrt{b^2-4ac}\right) n^2\right) \int \frac{\log\left(b-\sqrt{b^2-4ac}+2cx\right)}{b+\sqrt{b^2-4ac}+2cx}\, dx \\
&\quad - \left(2\left(b+\sqrt{b^2-4ac}\right) n^2\right) \int \frac{\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{b-\sqrt{b^2-4ac}+2cx}\, dx \\
&\quad - \left(2\left(b+\sqrt{b^2-4ac}\right) n^2\right) \int \frac{\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{b+\sqrt{b^2-4ac}+2cx}\, dx
\end{aligned}
$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}$$
$$- \frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{c}$$
$$- \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c}$$
$$- \frac{2bn^2\log\left(a+bx+cx^2\right)}{c} - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right)$$
$$+ \frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c}$$
$$+ \frac{\left(b+\sqrt{b^2-4ac}\right)n\log\left(b+\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c}$$
$$+ x\log^2\left(d\left(a+bx+cx^2\right)^n\right)$$
$$+ \left(2\left(b-\sqrt{b^2-4ac}\right)n^2\right)\int\frac{\log\left(\frac{2c\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2c\left(b-\sqrt{b^2-4ac}\right)+2c\left(b+\sqrt{b^2-4ac}\right)}\right)}{b-\sqrt{b^2-4ac}+2cx}\,dx$$
$$- \frac{\left(\left(b-\sqrt{b^2-4ac}\right)n^2\right)\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, b-\sqrt{b^2-4ac}+2cx\right)}{c}$$
$$+ \left(2\left(b+\sqrt{b^2-4ac}\right)n^2\right)\int\frac{\log\left(\frac{2c\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\left(b-\sqrt{b^2-4ac}\right)-2c\left(b+\sqrt{b^2-4ac}\right)}\right)}{b+\sqrt{b^2-4ac}+2cx}\,dx$$
$$- \frac{\left(\left(b+\sqrt{b^2-4ac}\right)n^2\right)\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, b+\sqrt{b^2-4ac}+2cx\right)}{c}$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c} - \frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{c} - \frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log^2\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c} - \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c} - \frac{2bn^2\log\left(a+bx+cx^2\right)}{c} - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} + \frac{\left(b+\sqrt{b^2-4ac}\right)n\log\left(b+\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(\left(b-\sqrt{b^2-4ac}\right)n^2\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{-2c\left(b-\sqrt{b^2-4ac}\right)+2c\left(b+\sqrt{b^2-4ac}\right)}\right)}{x}dx, x, b-\sqrt{b^2-4ac}+2cx\right)}{c} + \frac{\left(\left(b+\sqrt{b^2-4ac}\right)n^2\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2cx}{2c\left(b-\sqrt{b^2-4ac}\right)-2c\left(b+\sqrt{b^2-4ac}\right)}\right)}{x}dx, x, b+\sqrt{b^2-4ac}+2cx\right)}{c}$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c} - \frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{c} - \frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log^2\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c} - \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c} - \frac{2bn^2\log\left(a+bx+cx^2\right)}{c} - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} + \frac{\left(b+\sqrt{b^2-4ac}\right)n\log\left(b+\sqrt{b^2-4ac}+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c} + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) - \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\mathrm{Li}_2\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c} - \frac{\left(b+\sqrt{b^2-4ac}\right)n^2\mathrm{Li}_2\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.81

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx = x\log^2\left(d(a+x(b+cx))^n\right) + \frac{n\left(4n\left(4cx-2\sqrt{b^2-4ac}\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-b\log(a+x(b+cx))\right)-8cx\log\left(d(a+x(b+cx))^n\right)+2(b\cdots}{}$$

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

```
[Out] x*Log[d*(a + x*(b + c*x))^n]^2 + (n*(4*n*(4*c*x - 2*Sqrt[b^2 - 4*a*c]*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - b*Log[a + x*(b + c*x)]) - 8*c*x*Log[d*(
a + x*(b + c*x))^n] + 2*(b - Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] +
 2*c*x]*Log[d*(a + x*(b + c*x))^n] + 2*(b + Sqrt[b^2 - 4*a*c])*Log[b + Sqrt
[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] + (-b + Sqrt[b^2 - 4*a*c]
)*n*(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]
 + 2*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) + 2*PolyLo
g[2, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) - (b + Sqrt[b
^2 - 4*a*c])*n*(Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*(2*Log[(-b + Sqrt[b^2 -
```

```
4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]
) + 2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])))/
(2*c)
```

Maple [F]

$$\int \ln \left(d \left(c \,x^{2}+b x +a \right)^{n}\right)^{2}d x$$

```
[In] int(ln(d*(c*x^2+b*x+a)^n)^2,x)
[Out] int(ln(d*(c*x^2+b*x+a)^n)^2,x)
```

Fricas [F]

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx=\int \log\left(\left(cx^2+bx+a\right)^n d\right)^2\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="fricas")
[Out] integral(log((c*x^2 + b*x + a)^n*d)^2, x)
```

Sympy [F]

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx=\int \log\left(d\left(a+bx+cx^2\right)^n\right)^2\,dx$$

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)**2,x)
[Out] Integral(log(d*(a + b*x + c*x**2)**n)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx=\int \log\left(\left(cx^2+bx+a\right)^n d\right)^2\,dx$$

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x + a)^n*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx=\int \ln\left(d\left(c\,x^2+b\,x+a\right)^n\right)^2\,dx$$

```
[In] int(log(d*(a + b*x + c*x^2)^n)^2,x)
[Out] int(log(d*(a + b*x + c*x^2)^n)^2, x)
```

3.98 $\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} \, dx$

Optimal result 609
Rubi [A] (verified) 610
Mathematica [A] (verified) 615
Maple [A] (verified) 615
Fricas [F] 616
Sympy [F] 616
Maxima [F] 616
Giac [F] 617
Mupad [F(-1)] 617

Optimal result

Integrand size = 21, antiderivative size = 311

$$\begin{aligned}\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} \, dx = {} & -2x+\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-\log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) \\ & +4\log(4+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{3+i\sqrt{3}}\right) \\ & -\log(2+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{1-i\sqrt{3}}\right) \\ & +4\log(4+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{3-i\sqrt{3}}\right) \\ & +\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right) \\ & +\log(2+2x)\log\left(1+x+x^2\right)-4\log(4+2x)\log\left(1+x+x^2\right) \\ & -\operatorname{PolyLog}\left(2,\frac{2(1+x)}{1-i\sqrt{3}}\right)-\operatorname{PolyLog}\left(2,\frac{2(1+x)}{1+i\sqrt{3}}\right) \\ & +4\operatorname{PolyLog}\left(2,\frac{2(2+x)}{3-i\sqrt{3}}\right)+4\operatorname{PolyLog}\left(2,\frac{2(2+x)}{3+i\sqrt{3}}\right)\end{aligned}$$

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+ln(2+2*x)*ln(x^2+x+1)-4*ln(4+2*x)*ln(x^2
+x+1)-ln(2+2*x)*ln((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))+4*ln(4+2*x)*ln((-1-2*x
+I*3^(1/2))/(3+I*3^(1/2)))-ln(2+2*x)*ln((-1-2*x-I*3^(1/2))/(1-I*3^(1/2)))+4
*ln(4+2*x)*ln((-1-2*x-I*3^(1/2))/(3-I*3^(1/2)))-polylog(2,2*(1+x)/(1-I*3^(1
/2)))+4*polylog(2,2*(2+x)/(3-I*3^(1/2)))-polylog(2,2*(1+x)/(1+I*3^(1/2)))+4
*polylog(2,2*(2+x)/(3+I*3^(1/2)))+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2608, 2603, 787, 648, 632, 210, 642, 2604, 2465, 2441, 2440, 2438}

$$\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2}\,dx = \sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right) - \text{PolyLog}\left(2,\frac{2(x+1)}{1-i\sqrt{3}}\right) - \text{PolyLog}\left(2,\frac{2(x+1)}{1+i\sqrt{3}}\right) + 4\,\text{PolyLog}\left(2,\frac{2(x+2)}{3-i\sqrt{3}}\right) + 4\,\text{PolyLog}\left(2,\frac{2(x+2)}{3+i\sqrt{3}}\right) + x\log\left(x^2+x+1\right) + \log(2x+2)\log\left(x^2+x+1\right) - 4\log(2x+4)\log\left(x^2+x+1\right) + \frac{1}{2}\log\left(x^2+x+1\right) - 2x - \log(2x+2)\log\left(-\frac{2x-i\sqrt{3}+1}{1+i\sqrt{3}}\right) + 4\log(2x+4)\log\left(-\frac{2x-i\sqrt{3}+1}{3+i\sqrt{3}}\right) - \log(2x+2)\log\left(-\frac{2x+i\sqrt{3}+1}{1-i\sqrt{3}}\right) + 4\log(2x+4)\log\left(-\frac{2x+i\sqrt{3}+1}{3-i\sqrt{3}}\right)$$

```
[In] Int[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[2 + 2*x]*Log[-((1 - I*Sqrt[3
] + 2*x)/(1 + I*Sqrt[3]))] + 4*Log[4 + 2*x]*Log[-((1 - I*Sqrt[3] + 2*x)/(3
+ I*Sqrt[3]))] - Log[2 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(1 - I*Sqrt[3]))]
 + 4*Log[4 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(3 - I*Sqrt[3]))] + Log[1 + x
 + x^2]/2 + x*Log[1 + x + x^2] + Log[2 + 2*x]*Log[1 + x + x^2] - 4*Log[4 +
2*x]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/(1 - I*Sqrt[3])] - PolyLog[2
, (2*(1 + x))/(1 + I*Sqrt[3])] + 4*PolyLog[2, (2*(2 + x))/(3 - I*Sqrt[3])]
+ 4*PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
```

```
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\log\left(1+x+x^2\right)-\frac{(2+3x)\log\left(1+x+x^2\right)}{2+3x+x^2}\right) dx\\
&= \int \log\left(1+x+x^2\right) dx-\int \frac{(2+3x)\log\left(1+x+x^2\right)}{2+3x+x^2}\,dx\\
&= x\log\left(1+x+x^2\right)-\int \frac{x(1+2x)}{1+x+x^2}\,dx-\int \left(-\frac{2\log\left(1+x+x^2\right)}{2+2x}+\frac{8\log\left(1+x+x^2\right)}{4+2x}\right) dx\\
&= -2x+x\log\left(1+x+x^2\right)+2\int \frac{\log\left(1+x+x^2\right)}{2+2x}\,dx\\
&\quad -8\int \frac{\log\left(1+x+x^2\right)}{4+2x}\,dx-\int \frac{-2-x}{1+x+x^2}\,dx\\
&= -2x+x\log\left(1+x+x^2\right)+\log(2+2x)\log\left(1+x+x^2\right)\\
&\quad -4\log(4+2x)\log\left(1+x+x^2\right)+\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{3}{2}\int \frac{1}{1+x+x^2}\,dx\\
&\quad +4\int \frac{(1+2x)\log(4+2x)}{1+x+x^2}\,dx-\int \frac{(1+2x)\log(2+2x)}{1+x+x^2}\,dx
\end{aligned}
$$

$$= -2x + \frac{1}{2}\log\left(1+x+x^2\right) + x\log\left(1+x+x^2\right)$$
$$+ \log(2+2x)\log\left(1+x+x^2\right) - 4\log(4+2x)\log\left(1+x+x^2\right)$$
$$- 3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) + 4\int \left(\frac{2\log(4+2x)}{1-i\sqrt{3}+2x} + \frac{2\log(4+2x)}{1+i\sqrt{3}+2x}\right)dx$$
$$- \int \left(\frac{2\log(2+2x)}{1-i\sqrt{3}+2x} + \frac{2\log(2+2x)}{1+i\sqrt{3}+2x}\right)dx$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) + \frac{1}{2}\log\left(1+x+x^2\right) + x\log\left(1+x+x^2\right)$$
$$+ \log(2+2x)\log\left(1+x+x^2\right) - 4\log(4+2x)\log\left(1+x+x^2\right) - 2\int \frac{\log(2+2x)}{1-i\sqrt{3}+2x}\,dx$$
$$- 2\int \frac{\log(2+2x)}{1+i\sqrt{3}+2x}\,dx + 8\int \frac{\log(4+2x)}{1-i\sqrt{3}+2x}\,dx + 8\int \frac{\log(4+2x)}{1+i\sqrt{3}+2x}\,dx$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right)$$
$$+ 4\log(4+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{3+i\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{1-i\sqrt{3}}\right)$$
$$+ 4\log(4+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{3-i\sqrt{3}}\right) + \frac{1}{2}\log\left(1+x+x^2\right) + x\log\left(1+x+x^2\right)$$
$$+ \log(2+2x)\log\left(1+x+x^2\right) - 4\log(4+2x)\log\left(1+x+x^2\right)$$
$$+ 2\int \frac{\log\left(\frac{2\left(1-i\sqrt{3}+2x\right)}{-4+2\left(1-i\sqrt{3}\right)}\right)}{2+2x}\,dx + 2\int \frac{\log\left(\frac{2\left(1+i\sqrt{3}+2x\right)}{-4+2\left(1+i\sqrt{3}\right)}\right)}{2+2x}\,dx$$
$$- 8\int \frac{\log\left(\frac{2\left(1-i\sqrt{3}+2x\right)}{-8+2\left(1-i\sqrt{3}\right)}\right)}{4+2x}\,dx - 8\int \frac{\log\left(\frac{2\left(1+i\sqrt{3}+2x\right)}{-8+2\left(1+i\sqrt{3}\right)}\right)}{4+2x}\,dx$$

$$= -2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right)$$
$$+4\log(4+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{3+i\sqrt{3}}\right)-\log(2+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{1-i\sqrt{3}}\right)$$
$$+4\log(4+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{3-i\sqrt{3}}\right)+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)$$
$$+\log(2+2x)\log\left(1+x+x^2\right)-4\log(4+2x)\log\left(1+x+x^2\right)$$
$$-4\text{Subst}\left(\int\frac{\log\left(1+\frac{2x}{-8+2\left(1-i\sqrt{3}\right)}\right)}{x}dx,x,4+2x\right)$$
$$-4\text{Subst}\left(\int\frac{\log\left(1+\frac{2x}{-8+2\left(1+i\sqrt{3}\right)}\right)}{x}dx,x,4+2x\right)$$
$$+\text{Subst}\left(\int\frac{\log\left(1+\frac{2x}{-4+2\left(1-i\sqrt{3}\right)}\right)}{x}dx,x,2+2x\right)$$
$$+\text{Subst}\left(\int\frac{\log\left(1+\frac{2x}{-4+2\left(1+i\sqrt{3}\right)}\right)}{x}dx,x,2+2x\right)$$

$$= -2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right)$$
$$+4\log(4+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{3+i\sqrt{3}}\right)-\log(2+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{1-i\sqrt{3}}\right)$$
$$+4\log(4+2x)\log\left(-\frac{1+i\sqrt{3}+2x}{3-i\sqrt{3}}\right)+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)$$
$$+\log(2+2x)\log\left(1+x+x^2\right)-4\log(4+2x)\log\left(1+x+x^2\right)$$
$$-\text{Li}_2\left(\frac{2(1+x)}{1+i\sqrt{3}}\right)-\text{Li}_2\left(\frac{2i(1+x)}{i+\sqrt{3}}\right)+4\text{Li}_2\left(\frac{2(2+x)}{3-i\sqrt{3}}\right)+4\text{Li}_2\left(\frac{2(2+x)}{3+i\sqrt{3}}\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.93

$$\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} dx = -2x + \sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - \log\left(\frac{-i+\sqrt{3}-2ix}{i+\sqrt{3}}\right)\log(2(1+x)) - \log\left(\frac{i+\sqrt{3}+2ix}{-i+\sqrt{3}}\right)\log(2(1+x)) + \frac{1}{2}\log(1+x+x^2) + x\log(1+x+x^2) + \log(2(1+x))\log(1+x+x^2) - 4\log(2(2+x))\log(1+x+x^2) - \text{PolyLog}\left(2,\frac{2(1+x)}{1+i\sqrt{3}}\right) - \text{PolyLog}\left(2,\frac{2i(1+x)}{i+\sqrt{3}}\right) + 4\left(\left(\log\left(\frac{-i+\sqrt{3}-2ix}{3i+\sqrt{3}}\right) + \log\left(\frac{i+\sqrt{3}+2ix}{-3i+\sqrt{3}}\right)\right)\log(2(2+x)) + \text{PolyLog}\left(2,\frac{2(2+x)}{3+i\sqrt{3}}\right) + \text{PolyLog}\left(2,\frac{2i(2+x)}{3i+\sqrt{3}}\right)\right)$$

```
[In] Integrate[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[(-I + Sqrt[3] - (2*I)*x)/(I
+ Sqrt[3])]*Log[2*(1 + x)] - Log[(I + Sqrt[3] + (2*I)*x)/(-I + Sqrt[3])]*Lo
g[2*(1 + x)] + Log[1 + x + x^2]/2 + x*Log[1 + x + x^2] + Log[2*(1 + x)]*Log
[1 + x + x^2] - 4*Log[2*(2 + x)]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/
(1 + I*Sqrt[3])] - PolyLog[2, ((2*I)*(1 + x))/(I + Sqrt[3])] + 4*((Log[(-I
+ Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])] + Log[(I + Sqrt[3] + (2*I)*x)/(-3*I +
 Sqrt[3])])*Log[2*(2 + x)] + PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])] + Poly
Log[2, ((2*I)*(2 + x))/(3*I + Sqrt[3])])
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.90

method	result
default	$-2x+\frac{\ln(x^2+x+1)}{2}+x\ln(x^2+x+1)+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}-4\ln(x+2)\ln(x^2+x+1)+4\ln$
risch	$-2x+\frac{\ln(x^2+x+1)}{2}+x\ln(x^2+x+1)+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}-4\ln(x+2)\ln(x^2+x+1)+4\ln$
parts	$-2x+\frac{\ln(x^2+x+1)}{2}+x\ln(x^2+x+1)+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}-4\ln(x+2)\ln(x^2+x+1)+4\ln$

```
[In] int(x^2*ln(x^2+x+1)/(x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-4*ln
(x+2)*ln(x^2+x+1)+4*ln(x+2)*ln((-1-2*x+I*3^(1/2))/(3+I*3^(1/2)))+4*ln(x+2)*
ln((1+2*x+I*3^(1/2))/(-3+I*3^(1/2)))+4*dilog((-1-2*x+I*3^(1/2))/(3+I*3^(1/2
)))+4*dilog((1+2*x+I*3^(1/2))/(-3+I*3^(1/2)))+ln(x+1)*ln(x^2+x+1)-ln(x+1)*l
n((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))-ln(x+1)*ln((1+2*x+I*3^(1/2))/(I*3^(1/2)
-1))-dilog((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))-dilog((1+2*x+I*3^(1/2))/(I*3^(
1/2)-1))
```

Fricas [F]

$$\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} \, dx = \int \frac{x^2 \log\left(x^2+x+1\right)}{x^2+3\,x+2} \, dx$$

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Sympy [F]

$$\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} \, dx = \int \frac{x^2 \log\left(x^2+x+1\right)}{(x+1)\,(x+2)} \, dx$$

```
[In] integrate(x**2*ln(x**2+x+1)/(x**2+3*x+2),x)
```

```
[Out] Integral(x**2*log(x**2 + x + 1)/((x + 1)*(x + 2)), x)
```

Maxima [F]

$$\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} \, dx = \int \frac{x^2 \log\left(x^2+x+1\right)}{x^2+3\,x+2} \, dx$$

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Giac [F]

$$\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2}\,dx = \int \frac{x^2 \log(x^2+x+1)}{x^2+3\,x+2}\,dx$$

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="giac")
[Out] integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2}\,dx = \int \frac{x^2\,\ln(x^2+x+1)}{x^2+3\,x+2}\,dx$$

```
[In] int((x^2*log(x + x^2 + 1))/(3*x + x^2 + 2),x)
[Out] int((x^2*log(x + x^2 + 1))/(3*x + x^2 + 2), x)
```

3.99 $\int \log^2(1+x+x^2)\,dx$

Optimal result 618
Rubi [A] (verified) 619
Mathematica [A] (verified) 625
Maple [F] 626
Fricas [F] 626
Sympy [F(-2)] 626
Maxima [F] 626
Giac [F] 627
Mupad [F(-1)] 627

Optimal result

Integrand size = 9, antiderivative size = 371

$$
\begin{aligned}
\int \log^2\left(1+x+x^2\right)\,dx &= 8x-4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right)\\
&\quad-\left(1+i\sqrt{3}\right)\log\left(\frac{i\left(1-i\sqrt{3}+2x\right)}{2\sqrt{3}}\right)\log\left(1+i\sqrt{3}+2x\right)\\
&\quad-\frac{1}{2}\left(1+i\sqrt{3}\right)\log^2\left(1+i\sqrt{3}+2x\right)\\
&\quad-\left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(-\frac{i\left(1+i\sqrt{3}+2x\right)}{2\sqrt{3}}\right)\\
&\quad-2\log\left(1+x+x^2\right)-4x\log\left(1+x+x^2\right)\\
&\quad+\left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)\\
&\quad+\left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)\\
&\quad+x\log^2\left(1+x+x^2\right)-\left(1+i\sqrt{3}\right)\operatorname{PolyLog}\left(2,-\frac{i-\sqrt{3}+2ix}{2\sqrt{3}}\right)\\
&\quad-\left(1-i\sqrt{3}\right)\operatorname{PolyLog}\left(2,\frac{i+\sqrt{3}+2ix}{2\sqrt{3}}\right)
\end{aligned}
$$

```
[Out] 8*x-2*ln(x^2+x+1)-4*x*ln(x^2+x+1)+x*ln(x^2+x+1)^2+ln(x^2+x+1)*ln(1+2*x-I*3^
(1/2))*(1-I*3^(1/2))-1/2*ln(1+2*x-I*3^(1/2))^2*(1-I*3^(1/2))-ln(1+2*x-I*3^(
1/2))*ln(-1/6*I*(1+2*x+I*3^(1/2))*3^(1/2))*(1-I*3^(1/2))-polylog(2,1/6*(I+2
*I*x+3^(1/2))*3^(1/2))*(1-I*3^(1/2))+ln(x^2+x+1)*ln(1+2*x+I*3^(1/2))*(1+I*3
^(1/2))-1/2*ln(1+2*x+I*3^(1/2))^2*(1+I*3^(1/2))-ln(1+2*x+I*3^(1/2))*ln(1/6*
I*(1+2*x-I*3^(1/2))*3^(1/2))*(1+I*3^(1/2))-polylog(2,1/6*(-I-2*I*x+3^(1/2))
*3^(1/2))*(1+I*3^(1/2))-4*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.556, Rules used = {2603, 2608, 787, 648, 632, 210, 642, 2604, 2465, 2437, 2338, 2441, 2440, 2438}

$$\int \log^2\left(1+x+x^2\right)\,dx=-4\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\left(1+i\sqrt{3}\right)\text{PolyLog}\left(2,-\frac{2ix-\sqrt{3}+i}{2\sqrt{3}}\right)-\left(1-i\sqrt{3}\right)\text{PolyLog}\left(2,\frac{2ix+\sqrt{3}+i}{2\sqrt{3}}\right)+x\log^2\left(x^2+x+1\right)+\left(1-i\sqrt{3}\right)\log\left(x^2+x+1\right)\log\left(2x-i\sqrt{3}+1\right)-4x\log\left(x^2+x+1\right)+\left(1+i\sqrt{3}\right)\log\left(2x+i\sqrt{3}+1\right)\log\left(x^2+x+1\right)-2\log\left(x^2+x+1\right)+8x-\frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(2x-i\sqrt{3}+1\right)-\frac{1}{2}\left(1+i\sqrt{3}\right)\log^2\left(2x+i\sqrt{3}+1\right)-\left(1-i\sqrt{3}\right)\log\left(-\frac{i\left(2x+i\sqrt{3}+1\right)}{2\sqrt{3}}\right)\log\left(2x-i\sqrt{3}+1\right)-\left(1+i\sqrt{3}\right)\log\left(\frac{i\left(2x-i\sqrt{3}+1\right)}{2\sqrt{3}}\right)\log\left(2x+i\sqrt{3}+1\right)$$

```
[In] Int[Log[1 + x + x^2]^2,x]
```

```
[Out] 8*x - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - ((1 - I*Sqrt[3])*Log[1 - I*Sqrt
[3] + 2*x]^2)/2 - (1 + I*Sqrt[3])*Log[((I/2)*(1 - I*Sqrt[3] + 2*x))/Sqrt[3]
]*Log[1 + I*Sqrt[3] + 2*x] - ((1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x]^2)/2
 - (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*x]*Log[((-1/2*I)*(1 + I*Sqrt[3] +
2*x))/Sqrt[3]] - 2*Log[1 + x + x^2] - 4*x*Log[1 + x + x^2] + (1 - I*Sqrt[3]
)*Log[1 - I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqr
t[3] + 2*x]*Log[1 + x + x^2] + x*Log[1 + x + x^2]^2 - (1 + I*Sqrt[3])*PolyL
og[2, -1/2*(I - Sqrt[3] + (2*I)*x)/Sqrt[3]] - (1 - I*Sqrt[3])*PolyLog[2, (I
 + Sqrt[3] + (2*I)*x)/(2*Sqrt[3])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log^2\left(1+x+x^2\right)-2\int\frac{x(1+2x)\log\left(1+x+x^2\right)}{1+x+x^2}\,dx\\
&= x\log^2\left(1+x+x^2\right)-2\int\left(2\log\left(1+x+x^2\right)-\frac{(2+x)\log\left(1+x+x^2\right)}{1+x+x^2}\right)dx\\
&= x\log^2\left(1+x+x^2\right)+2\int\frac{(2+x)\log\left(1+x+x^2\right)}{1+x+x^2}\,dx-4\int\log\left(1+x+x^2\right)\,dx
\end{aligned}
$$

$$= -4x\log\left(1+x+x^2\right)+x\log^2\left(1+x+x^2\right)$$
$$+2\int\left(\frac{(1-i\sqrt{3})\log(1+x+x^2)}{1-i\sqrt{3}+2x}+\frac{(1+i\sqrt{3})\log(1+x+x^2)}{1+i\sqrt{3}+2x}\right)dx$$
$$+4\int\frac{x(1+2x)}{1+x+x^2}\,dx$$
$$= 8x-4x\log\left(1+x+x^2\right)+x\log^2\left(1+x+x^2\right)+4\int\frac{-2-x}{1+x+x^2}\,dx$$
$$+\left(2\left(1-i\sqrt{3}\right)\right)\int\frac{\log(1+x+x^2)}{1-i\sqrt{3}+2x}\,dx+\left(2\left(1+i\sqrt{3}\right)\right)\int\frac{\log(1+x+x^2)}{1+i\sqrt{3}+2x}\,dx$$
$$= 8x-4x\log\left(1+x+x^2\right)+\left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)$$
$$+\left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)$$
$$+x\log^2\left(1+x+x^2\right)-2\int\frac{1+2x}{1+x+x^2}\,dx-6\int\frac{1}{1+x+x^2}\,dx$$
$$+\left(-1-i\sqrt{3}\right)\int\frac{(1+2x)\log\left(1+i\sqrt{3}+2x\right)}{1+x+x^2}\,dx$$
$$+\left(-1+i\sqrt{3}\right)\int\frac{(1+2x)\log\left(1-i\sqrt{3}+2x\right)}{1+x+x^2}\,dx$$
$$= 8x-2\log\left(1+x+x^2\right)-4x\log\left(1+x+x^2\right)$$
$$+\left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)$$
$$+\left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)$$
$$+x\log^2\left(1+x+x^2\right)+12\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2x\right)$$
$$+\left(-1-i\sqrt{3}\right)\int\left(\frac{2\log\left(1+i\sqrt{3}+2x\right)}{1-i\sqrt{3}+2x}+\frac{2\log\left(1+i\sqrt{3}+2x\right)}{1+i\sqrt{3}+2x}\right)dx$$
$$+\left(-1+i\sqrt{3}\right)\int\left(\frac{2\log\left(1-i\sqrt{3}+2x\right)}{1-i\sqrt{3}+2x}+\frac{2\log\left(1-i\sqrt{3}+2x\right)}{1+i\sqrt{3}+2x}\right)dx$$

$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - 2\log\left(1+x+x^2\right) - 4x\log\left(1+x+x^2\right)$$
$$+ \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)$$
$$+ \left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)$$
$$+ x\log^2\left(1+x+x^2\right) - \left(2\left(1-i\sqrt{3}\right)\right)\int \frac{\log\left(1-i\sqrt{3}+2x\right)}{1-i\sqrt{3}+2x}\,dx$$
$$- \left(2\left(1-i\sqrt{3}\right)\right)\int \frac{\log\left(1-i\sqrt{3}+2x\right)}{1+i\sqrt{3}+2x}\,dx$$
$$- \left(2\left(1+i\sqrt{3}\right)\right)\int \frac{\log\left(1+i\sqrt{3}+2x\right)}{1-i\sqrt{3}+2x}\,dx$$
$$- \left(2\left(1+i\sqrt{3}\right)\right)\int \frac{\log\left(1+i\sqrt{3}+2x\right)}{1+i\sqrt{3}+2x}\,dx$$

$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \left(1+i\sqrt{3}\right)\log\left(\frac{i(1-i\sqrt{3}+2x)}{2\sqrt{3}}\right)\log\left(1+i\sqrt{3}+2x\right)$$
$$- \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(-\frac{i(1+i\sqrt{3}+2x)}{2\sqrt{3}}\right) - 2\log\left(1+x+x^2\right)$$
$$- 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)$$
$$+ \left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) + x\log^2\left(1+x+x^2\right)$$
$$- \left(1-i\sqrt{3}\right)\mathrm{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, 1-i\sqrt{3}+2x\right)$$
$$+ \left(2\left(1-i\sqrt{3}\right)\right)\int \frac{\log\left(\frac{2\left(1+i\sqrt{3}+2x\right)}{-2\left(1-i\sqrt{3}\right)+2\left(1+i\sqrt{3}\right)}\right)}{1-i\sqrt{3}+2x}\,dx$$
$$- \left(1+i\sqrt{3}\right)\mathrm{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, 1+i\sqrt{3}+2x\right)$$
$$+ \left(2\left(1+i\sqrt{3}\right)\right)\int \frac{\log\left(\frac{2\left(1-i\sqrt{3}+2x\right)}{2\left(1-i\sqrt{3}\right)-2\left(1+i\sqrt{3}\right)}\right)}{1+i\sqrt{3}+2x}\,dx$$

$$
\begin{aligned}
&= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right) \\
&\quad - \left(1+i\sqrt{3}\right)\log\left(\frac{i(1-i\sqrt{3}+2x)}{2\sqrt{3}}\right)\log\left(1+i\sqrt{3}+2x\right) \\
&\quad - \frac{1}{2}\left(1+i\sqrt{3}\right)\log^2\left(1+i\sqrt{3}+2x\right) \\
&\quad - \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(-\frac{i(1+i\sqrt{3}+2x)}{2\sqrt{3}}\right) - 2\log\left(1+x+x^2\right) \\
&\quad - 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) \\
&\quad + \left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) + x\log^2\left(1+x+x^2\right) \\
&\quad + \left(1-i\sqrt{3}\right)\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{2x}{-2\left(1-i\sqrt{3}\right)+2\left(1+i\sqrt{3}\right)}\right)}{x}\,dx, x, 1-i\sqrt{3}+2x\right) \\
&\quad + \left(1+i\sqrt{3}\right)\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{2x}{2\left(1-i\sqrt{3}\right)-2\left(1+i\sqrt{3}\right)}\right)}{x}\,dx, x, 1+i\sqrt{3}+2x\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right) \\
&\quad - \left(1+i\sqrt{3}\right)\log\left(\frac{i(1-i\sqrt{3}+2x)}{2\sqrt{3}}\right)\log\left(1+i\sqrt{3}+2x\right) \\
&\quad - \frac{1}{2}\left(1+i\sqrt{3}\right)\log^2\left(1+i\sqrt{3}+2x\right) \\
&\quad - \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(-\frac{i(1+i\sqrt{3}+2x)}{2\sqrt{3}}\right) - 2\log\left(1+x+x^2\right) \\
&\quad - 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) \\
&\quad + \left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) + x\log^2\left(1+x+x^2\right) \\
&\quad - \left(1+i\sqrt{3}\right)\mathrm{Li}_2\left(-\frac{i-\sqrt{3}+2ix}{2\sqrt{3}}\right) - \left(1-i\sqrt{3}\right)\mathrm{Li}_2\left(\frac{i+\sqrt{3}+2ix}{2\sqrt{3}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.98

$$\int \log^2\left(1+x+x^2\right)\,dx = 8x-4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+i\left(i+\sqrt{3}\right)\log\left(\frac{-i+\sqrt{3}-2ix}{2\sqrt{3}}\right)\log\left(1-i\sqrt{3}+2x\right)+\frac{1}{2}i\left(i+\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right)-\left(1+i\sqrt{3}\right)\log\left(\frac{i+\sqrt{3}+2ix}{2\sqrt{3}}\right)\log\left(1+i\sqrt{3}+2x\right)-\frac{1}{2}\left(1+i\sqrt{3}\right)\log^2\left(1+i\sqrt{3}+2x\right)-2\log\left(1+x+x^2\right)-4x\log\left(1+x+x^2\right)+\left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)+\left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)+x\log^2\left(1+x+x^2\right)-\left(1+i\sqrt{3}\right)\text{PolyLog}\left(2,\frac{-i+\sqrt{3}-2ix}{2\sqrt{3}}\right)+i\left(i+\sqrt{3}\right)\text{PolyLog}\left(2,\frac{i+\sqrt{3}+2ix}{2\sqrt{3}}\right)$$

```
[In] Integrate[Log[1 + x + x^2]^2,x]
```

```
[Out] 8*x - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + I*(I + Sqrt[3])*Log[(-I + Sqrt[
3] - (2*I)*x)/(2*Sqrt[3])]*Log[1 - I*Sqrt[3] + 2*x] + (I/2)*(I + Sqrt[3])*L
og[1 - I*Sqrt[3] + 2*x]^2 - (1 + I*Sqrt[3])*Log[(I + Sqrt[3] + (2*I)*x)/(2*
Sqrt[3])]*Log[1 + I*Sqrt[3] + 2*x] - ((1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2
*x]^2)/2 - 2*Log[1 + x + x^2] - 4*x*Log[1 + x + x^2] + (1 - I*Sqrt[3])*Log[
1 - I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] +
 2*x]*Log[1 + x + x^2] + x*Log[1 + x + x^2]^2 - (1 + I*Sqrt[3])*PolyLog[2,
(-I + Sqrt[3] - (2*I)*x)/(2*Sqrt[3])] + I*(I + Sqrt[3])*PolyLog[2, (I + Sqr
t[3] + (2*I)*x)/(2*Sqrt[3])]
```

Maple [F]

$$\int \ln\left(x^2+x+1\right)^2 dx$$

```
[In] int(ln(x^2+x+1)^2,x)
[Out] int(ln(x^2+x+1)^2,x)
```

Fricas [F]

$$\int \log^2\left(1+x+x^2\right)\,dx = \int \log\left(x^2+x+1\right)^2\,dx$$

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="fricas")
[Out] integral(log(x^2 + x + 1)^2, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \log^2\left(1+x+x^2\right)\,dx = \text{Exception raised: RecursionError}$$

```
[In] integrate(ln(x**2+x+1)**2,x)
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded in com
parison
```

Maxima [F]

$$\int \log^2\left(1+x+x^2\right)\,dx = \int \log\left(x^2+x+1\right)^2\,dx$$

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="maxima")
[Out] x*log(x^2 + x + 1)^2 - integrate(2*(2*x^2 + x)*log(x^2 + x + 1)/(x^2 + x +
1), x)
```

Giac [F]

$$\int \log^2(1+x+x^2)\,dx = \int \log\left(x^2+x+1\right)^2\,dx$$

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="giac")
[Out] integrate(log(x^2 + x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2(1+x+x^2)\,dx = \int \ln\left(x^2+x+1\right)^2\,dx$$

```
[In] int(log(x + x^2 + 1)^2,x)
[Out] int(log(x + x^2 + 1)^2, x)
```

3.100 $\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx$

Optimal result 629
Rubi [A] (verified) 630
Mathematica [A] (warning: unable to verify) 637
Maple [C] (verified) 638
Fricas [F] 638
Sympy [F(-2)] 639
Maxima [F] 639
Giac [F] 639
Mupad [F(-1)] 639

Optimal result

Integrand size = 13, antiderivative size = 443

$$
\begin{aligned}
\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx = {} & \log(x)-\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) \\
& +3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) \\
& -\frac{1}{4}\left(3+\sqrt{5}\right)\log^2\left(1-\sqrt{5}+2x\right)-\frac{1}{2}\left(1-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right) \\
& -\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(-\frac{1-\sqrt{5}+2x}{2\sqrt{5}}\right)\log\left(1+\sqrt{5}+2x\right) \\
& -\frac{1}{4}\left(3-\sqrt{5}\right)\log^2\left(1+\sqrt{5}+2x\right) \\
& -\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(\frac{1+\sqrt{5}+2x}{2\sqrt{5}}\right) \\
& +3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right) \\
& +\frac{\log\left(-1+x+x^2\right)}{x}-3\log(x)\log\left(-1+x+x^2\right) \\
& +\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) \\
& +\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) \\
& -\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+3\,\mathrm{PolyLog}\left(2,-\frac{2x}{1+\sqrt{5}}\right) \\
& -\frac{1}{2}\left(3+\sqrt{5}\right)\mathrm{PolyLog}\left(2,-\frac{1-\sqrt{5}+2x}{2\sqrt{5}}\right) \\
& -\frac{1}{2}\left(3-\sqrt{5}\right)\mathrm{PolyLog}\left(2,\frac{1+\sqrt{5}+2x}{2\sqrt{5}}\right) \\
& -3\,\mathrm{PolyLog}\left(2,1+\frac{2x}{1-\sqrt{5}}\right)
\end{aligned}
$$

```
[Out] ln(x)+ln(x^2+x-1)/x-3*ln(x)*ln(x^2+x-1)-1/2*ln(x^2+x-1)^2/x^2+3*ln(1+2*x-5^
(1/2))*ln(1/2*5^(1/2)-1/2)+3*ln(x)*ln(1+2*x/(5^(1/2)+1))-3*polylog(2,1+2*x/
(-5^(1/2)+1))+3*polylog(2,-2*x/(5^(1/2)+1))-1/2*ln(1+2*x+5^(1/2))*(-5^(1/2)
+1)+1/2*ln(x^2+x-1)*ln(1+2*x+5^(1/2))*(3-5^(1/2))-1/2*ln(1/10*(-1-2*x+5^(1/
2))*5^(1/2))*ln(1+2*x+5^(1/2))*(3-5^(1/2))-1/4*ln(1+2*x+5^(1/2))^2*(3-5^(1/
2))-1/2*polylog(2,1/10*(1+2*x+5^(1/2))*5^(1/2))*(3-5^(1/2))-1/2*ln(1+2*x-5^
(1/2))*(5^(1/2)+1)+1/2*ln(x^2+x-1)*ln(1+2*x-5^(1/2))*(3+5^(1/2))-1/4*ln(1+2
*x-5^(1/2))^2*(3+5^(1/2))-1/2*ln(1+2*x-5^(1/2))*ln(1/10*(1+2*x+5^(1/2))*5^(
1/2))*(3+5^(1/2))-1/2*polylog(2,1/10*(-1-2*x+5^(1/2))*5^(1/2))*(3+5^(1/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.231, Rules used = {2605, 2608, 814, 646, 31, 2604, 2404, 2353, 2352, 2354, 2438, 2465, 2437, 2338, 2441, 2440}

$$\begin{aligned}\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx = {} & 3\,\text{PolyLog}\left(2,-\frac{2x}{1+\sqrt{5}}\right) \\ & -\frac{1}{2}\left(3+\sqrt{5}\right)\text{PolyLog}\left(2,-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right) \\ & -\frac{1}{2}\left(3-\sqrt{5}\right)\text{PolyLog}\left(2,\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right) \\ & -3\,\text{PolyLog}\left(2,\frac{2x}{1-\sqrt{5}}+1\right)-\frac{\log^2\left(x^2+x-1\right)}{2x^2} \\ & +\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(x^2+x-1\right)\log\left(2x-\sqrt{5}+1\right) \\ & -3\log(x)\log\left(x^2+x-1\right) \\ & +\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)\log\left(x^2+x-1\right)+\frac{\log\left(x^2+x-1\right)}{x} \\ & -\frac{1}{4}\left(3+\sqrt{5}\right)\log^2\left(2x-\sqrt{5}+1\right)-\frac{1}{4}\left(3-\sqrt{5}\right)\log^2\left(2x+\sqrt{5}+1\right) \\ & -\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right)\log\left(2x-\sqrt{5}+1\right) \\ & +3\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)\log\left(2x-\sqrt{5}+1\right) \\ & -\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\log(x) \\ & -\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right)\log\left(2x+\sqrt{5}+1\right) \\ & -\frac{1}{2}\left(1-\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)+3\log(x)\log\left(\frac{2x}{1+\sqrt{5}}+1\right)\end{aligned}$$

```
[In] Int[Log[-1 + x + x^2]^2/x^3,x]
```

```
[Out] Log[x] - ((1 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/2 + 3*Log[(-1 + Sqrt[5])/2]
*Log[1 - Sqrt[5] + 2*x] - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]^2)/4 - ((1
- Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[-1/2*(1 - Sqrt[5]
 + 2*x)/Sqrt[5]]*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[1 + Sqrt[5]
 + 2*x]^2)/4 - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*Log[(1 + Sqrt[5] + 2*x
)/(2*Sqrt[5])])/2 + 3*Log[x]*Log[1 + (2*x)/(1 + Sqrt[5])] + Log[-1 + x + x^
2]/x - 3*Log[x]*Log[-1 + x + x^2] + ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*L
```

```
og[-1 + x + x^2])/2 + ((3 - Sqrt[5])*Log[1 + Sqrt[5] + 2*x]*Log[-1 + x + x^
2])/2 - Log[-1 + x + x^2]^2/(2*x^2) + 3*PolyLog[2, (-2*x)/(1 + Sqrt[5])] -
((3 + Sqrt[5])*PolyLog[2, -1/2*(1 - Sqrt[5] + 2*x)/Sqrt[5]])/2 - ((3 - Sqrt
[5])*PolyLog[2, (1 + Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 - 3*PolyLog[2, 1 + (2*x
)/(1 - Sqrt[5])]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
(a + b*Log[(-c)*(d/e)])*(Log[d + e*x]/e), x] + Dist[b, Int[Log[(-e)*(x/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[(-c)*(d/e), 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
```

```
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+\int \frac{(1+2x)\log\left(-1+x+x^2\right)}{x^2\left(-1+x+x^2\right)}\,dx\\
&= -\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+\int \left(-\frac{\log\left(-1+x+x^2\right)}{x^2}-\frac{3\log\left(-1+x+x^2\right)}{x}+\frac{(4+3x)\log\left(-1+x+x^2\right)}{-1+x+x^2}\right)dx\\
&= -\frac{\log^2\left(-1+x+x^2\right)}{2x^2}-3\int \frac{\log\left(-1+x+x^2\right)}{x}\,dx-\int \frac{\log\left(-1+x+x^2\right)}{x^2}\,dx+\int \frac{(4+3x)\log\left(-1+x+x^2\right)}{-1+x+x^2}\,dx\\
&= \frac{\log\left(-1+x+x^2\right)}{x}-3\log(x)\log\left(-1+x+x^2\right)-\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+3\int \frac{(1+2x)\log(x)}{-1+x+x^2}\,dx-\int \frac{1+2x}{x\left(-1+x+x^2\right)}\,dx\\
&\quad+\int \left(\frac{\left(3+\sqrt{5}\right)\log\left(-1+x+x^2\right)}{1-\sqrt{5}+2x}+\frac{\left(3-\sqrt{5}\right)\log\left(-1+x+x^2\right)}{1+\sqrt{5}+2x}\right)dx\\
&= \frac{\log\left(-1+x+x^2\right)}{x}-3\log(x)\log\left(-1+x+x^2\right)-\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+3\int \left(\frac{2\log(x)}{1-\sqrt{5}+2x}+\frac{2\log(x)}{1+\sqrt{5}+2x}\right)dx+\left(3-\sqrt{5}\right)\int \frac{\log\left(-1+x+x^2\right)}{1+\sqrt{5}+2x}\,dx\\
&\quad+\left(3+\sqrt{5}\right)\int \frac{\log\left(-1+x+x^2\right)}{1-\sqrt{5}+2x}\,dx-\int \left(-\frac{1}{x}+\frac{3+x}{-1+x+x^2}\right)dx
\end{aligned}$$

$$
\begin{aligned}
&= \log(x) + \frac{\log\left(-1+x+x^2\right)}{x} - 3\log(x)\log\left(-1+x+x^2\right) \\
&\quad + \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) \\
&\quad + \frac{1}{2}\left(3-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) \\
&\quad - \frac{\log^2\left(-1+x+x^2\right)}{2x^2} + 6\int \frac{\log(x)}{1-\sqrt{5}+2x}\,dx + 6\int \frac{\log(x)}{1+\sqrt{5}+2x}\,dx \\
&\quad + \frac{1}{2}\left(-3-\sqrt{5}\right)\int \frac{(1+2x)\log\left(1-\sqrt{5}+2x\right)}{-1+x+x^2}\,dx \\
&\quad + \frac{1}{2}\left(-3+\sqrt{5}\right)\int \frac{(1+2x)\log\left(1+\sqrt{5}+2x\right)}{-1+x+x^2}\,dx - \int \frac{3+x}{-1+x+x^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \log(x) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) \\
&\quad + 3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right) + \frac{\log\left(-1+x+x^2\right)}{x} - 3\log(x)\log\left(-1+x+x^2\right) \\
&\quad + \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) \\
&\quad + \frac{1}{2}\left(3-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) \\
&\quad - \frac{\log^2\left(-1+x+x^2\right)}{2x^2} - 3\int \frac{\log\left(1+\frac{2x}{1+\sqrt{5}}\right)}{x}\,dx + 6\int \frac{\log\left(-\frac{2x}{1-\sqrt{5}}\right)}{1-\sqrt{5}+2x}\,dx \\
&\quad + \frac{1}{2}\left(-3-\sqrt{5}\right)\int \left(\frac{2\log\left(1-\sqrt{5}+2x\right)}{1-\sqrt{5}+2x} + \frac{2\log\left(1-\sqrt{5}+2x\right)}{1+\sqrt{5}+2x}\right)dx \\
&\quad + \frac{1}{2}\left(-3+\sqrt{5}\right)\int \left(\frac{2\log\left(1+\sqrt{5}+2x\right)}{1-\sqrt{5}+2x} + \frac{2\log\left(1+\sqrt{5}+2x\right)}{1+\sqrt{5}+2x}\right)dx \\
&\quad + \frac{1}{2}\left(-1+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx - \frac{1}{2}\left(1+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx
\end{aligned}
$$

$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right)$$
$$- \frac{1}{2}\left(1-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right) + 3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right) + \frac{\log\left(-1+x+x^2\right)}{x}$$
$$- 3\log(x)\log\left(-1+x+x^2\right) + \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(-1+x+x^2\right)$$
$$+ \frac{1}{2}\left(3-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\log\left(-1+x+x^2\right)$$
$$- \frac{\log^2\left(-1+x+x^2\right)}{2x^2} + 3\text{Li}_2\left(-\frac{2x}{1+\sqrt{5}}\right) - 3\text{Li}_2\left(1+\frac{2x}{1-\sqrt{5}}\right)$$
$$+ \left(-3-\sqrt{5}\right)\int \frac{\log\left(1-\sqrt{5}+2x\right)}{1-\sqrt{5}+2x}\,dx + \left(-3-\sqrt{5}\right)\int \frac{\log\left(1-\sqrt{5}+2x\right)}{1+\sqrt{5}+2x}\,dx$$
$$+ \left(-3+\sqrt{5}\right)\int \frac{\log\left(1+\sqrt{5}+2x\right)}{1-\sqrt{5}+2x}\,dx + \left(-3+\sqrt{5}\right)\int \frac{\log\left(1+\sqrt{5}+2x\right)}{1+\sqrt{5}+2x}\,dx$$

$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)$$
$$+ 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) - \frac{1}{2}\left(1-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)$$
$$- \frac{1}{2}\left(3-\sqrt{5}\right)\log\left(-\frac{1-\sqrt{5}+2x}{2\sqrt{5}}\right)\log\left(1+\sqrt{5}+2x\right)$$
$$- \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(\frac{1+\sqrt{5}+2x}{2\sqrt{5}}\right)$$
$$+ 3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right) + \frac{\log\left(-1+x+x^2\right)}{x} - 3\log(x)\log\left(-1+x+x^2\right)$$
$$+ \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(-1+x+x^2\right)$$
$$+ \frac{1}{2}\left(3-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\log\left(-1+x+x^2\right)$$
$$- \frac{\log^2\left(-1+x+x^2\right)}{2x^2} + 3\text{Li}_2\left(-\frac{2x}{1+\sqrt{5}}\right) - 3\text{Li}_2\left(1+\frac{2x}{1-\sqrt{5}}\right)$$
$$+ \frac{1}{2}\left(-3-\sqrt{5}\right)\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, 1-\sqrt{5}+2x\right)$$
$$+ \left(3-\sqrt{5}\right)\int \frac{\log\left(\frac{2\left(1-\sqrt{5}+2x\right)}{2\left(1-\sqrt{5}\right)-2\left(1+\sqrt{5}\right)}\right)}{1+\sqrt{5}+2x}\,dx$$
$$+ \frac{1}{2}\left(-3+\sqrt{5}\right)\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, 1+\sqrt{5}+2x\right)$$
$$+ \left(3+\sqrt{5}\right)\int \frac{\log\left(\frac{2\left(1+\sqrt{5}+2x\right)}{-2\left(1-\sqrt{5}\right)+2\left(1+\sqrt{5}\right)}\right)}{1-\sqrt{5}+2x}\,dx$$

$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right)$$
$$- \frac{1}{4}\left(3+\sqrt{5}\right)\log^2\left(1-\sqrt{5}+2x\right) - \frac{1}{2}\left(1-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)$$
$$- \frac{1}{2}\left(3-\sqrt{5}\right)\log\left(-\frac{1-\sqrt{5}+2x}{2\sqrt{5}}\right)\log\left(1+\sqrt{5}+2x\right)$$
$$- \frac{1}{4}\left(3-\sqrt{5}\right)\log^2\left(1+\sqrt{5}+2x\right)$$
$$- \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(\frac{1+\sqrt{5}+2x}{2\sqrt{5}}\right)$$
$$+ 3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right) + \frac{\log\left(-1+x+x^2\right)}{x} - 3\log(x)\log\left(-1+x+x^2\right)$$
$$+ \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(-1+x+x^2\right)$$
$$+ \frac{1}{2}\left(3-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\log\left(-1+x+x^2\right)$$
$$- \frac{\log^2\left(-1+x+x^2\right)}{2x^2} + 3\text{Li}_2\left(-\frac{2x}{1+\sqrt{5}}\right) - 3\text{Li}_2\left(1+\frac{2x}{1-\sqrt{5}}\right)$$
$$+ \frac{1}{2}\left(3-\sqrt{5}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2x}{2\left(1-\sqrt{5}\right)-2\left(1+\sqrt{5}\right)}\right)}{x}\,dx, x, 1+\sqrt{5}+2x\right)$$
$$+ \frac{1}{2}\left(3+\sqrt{5}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2x}{-2\left(1-\sqrt{5}\right)+2\left(1+\sqrt{5}\right)}\right)}{x}\,dx, x, 1-\sqrt{5}+2x\right)$$

$$
\begin{aligned}
&= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) \\
&\quad - \frac{1}{4}\left(3+\sqrt{5}\right)\log^2\left(1-\sqrt{5}+2x\right) - \frac{1}{2}\left(1-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right) \\
&\quad - \frac{1}{2}\left(3-\sqrt{5}\right)\log\left(-\frac{1-\sqrt{5}+2x}{2\sqrt{5}}\right)\log\left(1+\sqrt{5}+2x\right) \\
&\quad - \frac{1}{4}\left(3-\sqrt{5}\right)\log^2\left(1+\sqrt{5}+2x\right) \\
&\quad - \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(\frac{1+\sqrt{5}+2x}{2\sqrt{5}}\right) \\
&\quad + 3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right) + \frac{\log\left(-1+x+x^2\right)}{x} - 3\log(x)\log\left(-1+x+x^2\right) \\
&\quad + \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) \\
&\quad + \frac{1}{2}\left(3-\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\log\left(-1+x+x^2\right) - \frac{\log^2\left(-1+x+x^2\right)}{2x^2} \\
&\quad + 3\mathrm{Li}_2\left(-\frac{2x}{1+\sqrt{5}}\right) - \frac{1}{2}\left(3+\sqrt{5}\right)\mathrm{Li}_2\left(-\frac{1-\sqrt{5}+2x}{2\sqrt{5}}\right) \\
&\quad - \frac{1}{2}\left(3-\sqrt{5}\right)\mathrm{Li}_2\left(\frac{1+\sqrt{5}+2x}{2\sqrt{5}}\right) - 3\mathrm{Li}_2\left(1+\frac{2x}{1-\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.86

$$
\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx
$$

$$
= \frac{-2\log^2\left(-1+x+x^2\right) + x\Big(4x\log(x) - 12x\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log(x) - 6x\log\left(-1+\sqrt{5}-2x\right)\log\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right.}{}
$$

```
[In] Integrate[Log[-1 + x + x^2]^2/x^3,x]
```

```
[Out] (-2*Log[-1 + x + x^2]^2 + x*(4*x*Log[x] - 12*x*Log[(1 + Sqrt[5])/2]*Log[x]
- 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] - 2*Sqrt[5]*x*Log[-1
 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] + 12*x*Log[x]*Log[1/2 - Sqrt[5]/
2 + x] - 12*x*Log[(2*x)/(-1 + Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + x] + 3*x*Log[
1/2 - Sqrt[5]/2 + x]^2 + Sqrt[5]*x*Log[1/2 - Sqrt[5]/2 + x]^2 - 6*x*Log[-1
+ Sqrt[5] - 2*x]*Log[(1 + Sqrt[5])/2 + x] - 2*Sqrt[5]*x*Log[-1 + Sqrt[5] -
2*x]*Log[(1 + Sqrt[5])/2 + x] + 12*x*Log[x]*Log[(1 + Sqrt[5])/2 + x] + 3*x*
Log[(1 + Sqrt[5])/2 + x]^2 - Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]^2 - 2*x*Log
[1 - Sqrt[5] + 2*x] - 2*Sqrt[5]*x*Log[1 - Sqrt[5] + 2*x] + 3*x*Log[5]*Log[1
 - Sqrt[5] + 2*x] + Sqrt[5]*x*Log[5]*Log[1 - Sqrt[5] + 2*x] - 2*x*Log[1 + S
```

```
qrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1 + Sqrt[5] + 2*x] - 6*x*Log[1/2 - Sqrt[5]/
2 + x]*Log[1 + Sqrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1/2 - Sqrt[5]/2 + x]*Log[1
+ Sqrt[5] + 2*x] - 6*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5] + 2*x] + 2*
Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5] + 2*x] + 6*x*Log[1/2 - S
qrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] - 2*Sqrt[5]*x*Log[1/2 -
Sqrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] + 4*Log[-1 + x + x^2] +
 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[-1 + x + x^2] + 2*Sqrt[5]*x*Log[-1 + Sqrt[
5] - 2*x]*Log[-1 + x + x^2] - 12*x*Log[x]*Log[-1 + x + x^2] + 6*x*Log[1 + S
qrt[5] + 2*x]*Log[-1 + x + x^2] - 2*Sqrt[5]*x*Log[1 + Sqrt[5] + 2*x]*Log[-1
 + x + x^2] - 4*Sqrt[5]*x*PolyLog[2, (-1 + Sqrt[5] - 2*x)/(2*Sqrt[5])] - 12
*x*PolyLog[2, (-1 + Sqrt[5] - 2*x)/(-1 + Sqrt[5])] + 12*x*PolyLog[2, (-2*x)
/(1 + Sqrt[5])]))/(4*x^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.49

method	result
parts	$-\frac{\ln\left(x^2+x-1\right)^2}{2x^2}-3\ln\left(x\right)\ln\left(x^2+x-1\right)+3\ln\left(x\right)\ln\left(\frac{-1-2x+\sqrt{5}}{\sqrt{5}-1}\right)+3\ln\left(x\right)\ln\left(\frac{1+2x+\sqrt{5}}{\sqrt{5}+1}\right)+3\,\mathrm{dilog}\,($

```
[In] int(ln(x^2+x-1)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x^2+x-1)^2/x^2-3*ln(x)*ln(x^2+x-1)+3*ln(x)*ln((-1-2*x+5^(1/2))/(5^(
1/2)-1))+3*ln(x)*ln((1+2*x+5^(1/2))/(5^(1/2)+1))+3*dilog((-1-2*x+5^(1/2))/(
5^(1/2)-1))+3*dilog((1+2*x+5^(1/2))/(5^(1/2)+1))+Sum((ln(x-_alpha)*ln(x^2+x
-1)-dilog((_alpha+x+1)/(2*_alpha+1))-ln(x-_alpha)*ln((_alpha+x+1)/(2*_alpha
+1))-1/2*ln(x-_alpha)^2)*(_alpha+2),_alpha=RootOf(_Z^2+_Z-1))+ln(x^2+x-1)/x
+ln(x)-1/2*ln(x^2+x-1)+5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))
```

Fricas [F]

$$\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx=\int \frac{\log\left(x^2+x-1\right)^2}{x^3}\,dx$$

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(log(x^2 + x - 1)^2/x^3, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx = \text{Exception raised: RecursionError}$$

```
[In] integrate(ln(x**2+x-1)**2/x**3,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded while
calling a Python object
```

Maxima [F]

$$\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx = \int \frac{\log(x^2+x-1)^2}{x^3}\,dx$$

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 + x - 1)^2/x^2 + integrate((2*x + 1)*log(x^2 + x - 1)/(x^4 + x
^3 - x^2), x)
```

Giac [F]

$$\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx = \int \frac{\log(x^2+x-1)^2}{x^3}\,dx$$

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(log(x^2 + x - 1)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(-1+x+x^2)}{x^3}\,dx = \int \frac{\ln(x^2+x-1)^2}{x^3}\,dx$$

```
[In] int(log(x + x^2 - 1)^2/x^3,x)
```

```
[Out] int(log(x + x^2 - 1)^2/x^3, x)
```

3.101 $\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx$

Optimal result 640
Rubi [A] (verified) 640
Mathematica [A] (verified) 644
Maple [A] (verified) 645
Fricas [A] (verification not implemented) 645
Sympy [F(-1)] 646
Maxima [F] 646
Giac [A] (verification not implemented) 646
Mupad [F(-1)] 647

Optimal result

Integrand size = 21, antiderivative size = 172

$$\begin{aligned}\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx &= \frac{x}{4096}-\frac{x^2}{1024}+\frac{x^3}{192}-\frac{x^4}{32}-\frac{683\sqrt{-x+x^2}}{4096} \\ &+\frac{149(1-2x)\sqrt{-x+x^2}}{2048}-\frac{1}{12}\left(-x+x^2\right)^{3/2} \\ &-\frac{1}{32}x\left(-x+x^2\right)^{3/2}+\frac{\operatorname{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)}{32768} \\ &-\frac{1537\operatorname{arctanh}\left(\frac{x}{\sqrt{-x+x^2}}\right)}{16384}-\frac{\log(1+8x)}{32768} \\ &+\frac{1}{4}x^4\log\left(-1+4x+4\sqrt{-x+x^2}\right)\end{aligned}$$

```
[Out] 1/4096*x-1/1024*x^2+1/192*x^3-1/32*x^4-1/12*(x^2-x)^(3/2)-1/32*x*(x^2-x)^(3
/2)+1/32768*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-1537/16384*arctanh(x/(x^2-x
)^(1/2))-1/32768*ln(1+8*x)+1/4*x^4*ln(-1+4*x+4*(x^2-x)^(1/2))-683/4096*(x^2
-x)^(1/2)+149/2048*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules

used = {2617, 2615, 6874, 654, 634, 212, 626, 748, 857, 738, 684}

$$\int x^3 \log\left(-1+4x+4\sqrt{(-1+x)x}\right)dx = \frac{\operatorname{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{32768} - \frac{1537\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right)}{16384} - \frac{x^4}{32} + \frac{x^3}{192} - \frac{x^2}{1024} - \frac{1}{32}\left(x^2-x\right)^{3/2}x - \frac{1}{12}\left(x^2-x\right)^{3/2} + \frac{149(1-2x)\sqrt{x^2-x}}{2048} - \frac{683\sqrt{x^2-x}}{4096} + \frac{1}{4}x^4\log\left(4\sqrt{x^2-x}+4x-1\right) + \frac{x}{4096} - \frac{\log(8x+1)}{32768}$$

```
[In] Int[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] x/4096 - x^2/1024 + x^3/192 - x^4/32 - (683*Sqrt[-x + x^2])/4096 + (149*(1
- 2*x)*Sqrt[-x + x^2])/2048 - (-x + x^2)^(3/2)/12 - (x*(-x + x^2)^(3/2))/32
 + ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/32768 - (1537*ArcTanh[x/Sqrt[-x +
 x^2]])/16384 - Log[1 + 8*x]/32768 + (x^4*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])
/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
```

```
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) dx \\
&= \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + 2\int \frac{x^4}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)} dx \\
&= \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + 2\int \left(\frac{1}{8192} - \frac{x}{1024} + \frac{x^2}{128} - \frac{x^3}{16} - \frac{1}{8192(1+8x)} \right. \\
&\qquad \left. - \frac{x}{12\sqrt{-x+x^2}} - \frac{85\sqrt{-x+x^2}}{1024} + \frac{\sqrt{-x+x^2}}{3072(-1-8x)} - \frac{11}{128}x\sqrt{-x+x^2} - \frac{1}{16}x^2\sqrt{-x+x^2}\right) dx \\
&= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{\log(1+8x)}{32768} + \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{\int \frac{\sqrt{-x+x^2}}{-1-8x} dx}{1536} \\
&\qquad - \frac{1}{8}\int x^2\sqrt{-x+x^2} dx - \frac{85}{512}\int \sqrt{-x+x^2} dx - \frac{1}{6}\int \frac{x}{\sqrt{-x+x^2}} dx - \frac{11}{64}\int x\sqrt{-x+x^2} dx \\
&= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{85(1-2x)\sqrt{-x+x^2}}{2048} - \frac{11}{192}\left(-x+x^2\right)^{3/2} \\
&\qquad - \frac{1}{32}x\left(-x+x^2\right)^{3/2} - \frac{\log(1+8x)}{32768} + \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{\int \frac{1-10x}{(-1-8x)\sqrt{-x+x^2}} dx}{24576} \\
&\qquad + \frac{85\int \frac{1}{\sqrt{-x+x^2}} dx}{4096} - \frac{5}{64}\int x\sqrt{-x+x^2} dx - \frac{1}{12}\int \frac{1}{\sqrt{-x+x^2}} dx - \frac{11}{128}\int \sqrt{-x+x^2} dx \\
&= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{129(1-2x)\sqrt{-x+x^2}}{2048} - \frac{1}{12}\left(-x+x^2\right)^{3/2} \\
&\qquad - \frac{1}{32}x\left(-x+x^2\right)^{3/2} - \frac{\log(1+8x)}{32768} + \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-x+x^2}\right) \\
&\qquad + \frac{5\int \frac{1}{\sqrt{-x+x^2}} dx}{98304} + \frac{3\int \frac{1}{(-1-8x)\sqrt{-x+x^2}} dx}{32768} + \frac{11\int \frac{1}{\sqrt{-x+x^2}} dx}{1024} - \frac{5}{128}\int \sqrt{-x+x^2} dx \\
&\qquad + \frac{85\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-x+x^2}}\right)}{2048} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-x+x^2}}\right)
\end{aligned}$$

$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{149(1-2x)\sqrt{-x+x^2}}{2048} - \frac{1}{12}(-x+x^2)^{3/2} - \frac{1}{32}x(-x+x^2)^{3/2} - \frac{769\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)}{6144} - \frac{\log(1+8x)}{32768} + \frac{1}{4}x^4\log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right)}{49152} - \frac{3\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{-1+10x}{\sqrt{-x+x^2}}\right)}{16384} + \frac{5\int \frac{1}{\sqrt{-x+x^2}}\,dx}{1024} + \frac{11}{512}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right)$$

$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{149(1-2x)\sqrt{-x+x^2}}{2048} - \frac{1}{12}(-x+x^2)^{3/2} - \frac{1}{32}x(-x+x^2)^{3/2} + \frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)}{32768} - \frac{1697\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)}{16384} - \frac{\log(1+8x)}{32768} + \frac{1}{4}x^4\log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{5}{512}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right)$$

$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{149(1-2x)\sqrt{-x+x^2}}{2048} - \frac{1}{12}(-x+x^2)^{3/2} - \frac{1}{32}x(-x+x^2)^{3/2} + \frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)}{32768} - \frac{1537\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)}{16384} - \frac{\log(1+8x)}{32768} + \frac{1}{4}x^4\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.59

$$\int x^3\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \frac{24\sqrt{1-x}x^{3/2} - 96\sqrt{1-x}x^{5/2} + 512\sqrt{1-x}x^{7/2} - 3072\sqrt{1-x}x^{9/2} - 6112\sqrt{1-x}x^{3/2}\sqrt{(-1+x)x} - 512}{}$$

```
[In] Integrate[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (24*Sqrt[1 - x]*x^(3/2) - 96*Sqrt[1 - x]*x^(5/2) + 512*Sqrt[1 - x]*x^(7/2)
- 3072*Sqrt[1 - x]*x^(9/2) - 6112*Sqrt[1 - x]*x^(3/2)*Sqrt[(-1 + x)*x] - 51
20*Sqrt[1 - x]*x^(5/2)*Sqrt[(-1 + x)*x] - 3072*Sqrt[1 - x]*x^(7/2)*Sqrt[(-1
 + x)*x] - 9240*Sqrt[-((-1 + x)^2*x^2)] - 9222*Sqrt[(-1 + x)*x]*ArcSin[Sqrt
[1 - x]] + 3*Sqrt[-((-1 + x)*x)]*ArcTanh[(1 - 10*x)/(6*Sqrt[(-1 + x)*x])] -
 3*Sqrt[-((-1 + x)*x)]*Log[1 + 8*x] + 24576*Sqrt[1 - x]*x^(9/2)*Log[-1 + 4*
x + 4*Sqrt[(-1 + x)*x]])/(98304*Sqrt[-((-1 + x)*x)])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.39

method	result
parts	$\frac{\ln\left(-1+4x+4\sqrt{(-1+x)x}\right)x^4}{4}+\frac{x}{4096}+\frac{x^3}{192}-\frac{x^4}{32}+\frac{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}{65536}-\frac{5\ln\left(-\frac{1}{2}+x+\sqrt{\left(x+\frac{1}{8}\right)^2-\frac{5x}{4}-\frac{1}{64}}\right)}{65536}-\frac{41x^2\sqrt{}}{96}$

```
[In] int(x^3*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(-1+4*x+4*((-1+x)*x)^(1/2))*x^4+1/4096*x+1/192*x^3-1/32*x^4+1/65536*(
64*(x+1/8)^2-80*x-1)^(1/2)-5/65536*ln(-1/2+x+((x+1/8)^2-5/4*x-1/64)^(1/2))-
41/960*x^2*(x^2-x)^(1/2)-283/6144*x*(x^2-x)^(1/2)-1/1024*x^2-3069/65536*ln(
-1/2+x+(x^2-x)^(1/2))+1/32768*arctanh(32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1
)^(1/2))+1/16*(x^2-x)^(3/2)-581/8192*(x^2-x)^(1/2)-1/32768*ln(1+8*x)+95/409
6*(2*x-1)*(x^2-x)^(1/2)+1/10*x^2*(x^2-x)^(3/2)-1/10*x^4*(x^2-x)^(1/2)-1/320
*x^3*(x^2-x)^(1/2)+23/320*x*(x^2-x)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78

$$\begin{aligned}\int x^3\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= -\frac{1}{32}\,x^4+\frac{1}{192}\,x^3-\frac{1}{1024}\,x^2+\frac{1}{4}\left(x^4-1\right)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)\\ &\quad-\frac{1}{12288}\left(384\,x^3+640\,x^2+764\,x+1155\right)\sqrt{x^2-x}+\frac{1}{4096}\,x\\ &\quad+\frac{4095}{32768}\,\log\left(8\,x+1\right)-\frac{2559}{32768}\,\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)\\ &\quad+\frac{4095}{32768}\,\log\left(-2\,x+2\,\sqrt{x^2-x}-1\right)-\frac{4095}{32768}\,\log\left(-4\,x+4\,\sqrt{x^2-x}+1\right)\end{aligned}$$

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/32*x^4 + 1/192*x^3 - 1/1024*x^2 + 1/4*(x^4 - 1)*log(4*x + 4*sqrt(x^2 - x
) - 1) - 1/12288*(384*x^3 + 640*x^2 + 764*x + 1155)*sqrt(x^2 - x) + 1/4096*
x + 4095/32768*log(8*x + 1) - 2559/32768*log(-2*x + 2*sqrt(x^2 - x) + 1) +
4095/32768*log(-2*x + 2*sqrt(x^2 - x) - 1) - 4095/32768*log(-4*x + 4*sqrt(x
^2 - x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
[Out] Timed out
```

Maxima [F]

$$\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int x^3 \log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^3*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78

$$\begin{aligned}\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= \frac{1}{4}\,x^4 \log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{1}{32}\,x^4+\frac{1}{192}\,x^3-\frac{1}{1024}\,x^2 \\ &\quad -\frac{1}{12288}\,(4\,(32\,(3\,x+5)x+191)x+1155)\sqrt{x^2-x}+\frac{1}{4096}\,x \\ &\quad -\frac{1}{32768}\,\log\left(|8\,x+1|\right)+\frac{1537}{32768}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right) \\ &\quad -\frac{1}{32768}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)+\frac{1}{32768}\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)\end{aligned}$$

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
[Out] 1/4*x^4*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/32*x^4 + 1/192*x^3 - 1/1024*x^
2 - 1/12288*(4*(32*(3*x + 5)*x + 191)*x + 1155)*sqrt(x^2 - x) + 1/4096*x -
1/32768*log(abs(8*x + 1)) + 1537/32768*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))
 - 1/32768*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/32768*log(abs(-4*x + 4*
sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx = \int x^3 \ln \left(4\,x+4\,\sqrt{x\,(x-1)}-1\right) \,dx$$

```
[In] int(x^3*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
[Out] int(x^3*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.102 $\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx$

Optimal result 648
Rubi [A] (verified) 648
Mathematica [A] (verified) 652
Maple [A] (verified) 652
Fricas [A] (verification not implemented) 653
Sympy [F(-1)] 653
Maxima [F] 654
Giac [A] (verification not implemented) 654
Mupad [F(-1)] 654

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx = -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x+x^2\right)^{3/2}-\frac{\operatorname{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)}{3072}-\frac{223\operatorname{arctanh}\left(\frac{x}{\sqrt{-x+x^2}}\right)}{1536}+\frac{\log(1+8x)}{3072}+\frac{1}{3}x^3\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

```
[Out] -1/384*x+1/96*x^2-1/18*x^3-1/18*(x^2-x)^(3/2)-1/3072*arctanh(1/6*(1-10*x)/(
x^2-x)^(1/2))-223/1536*arctanh(x/(x^2-x)^(1/2))+1/3072*ln(1+8*x)+1/3*x^3*ln
(-1+4*x+4*(x^2-x)^(1/2))-85/384*(x^2-x)^(1/2)+5/64*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2617, 2615, 6874, 654, 634, 212, 626, 748, 857, 738}

$$\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx = -\frac{\operatorname{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{3072}-\frac{223\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right)}{1536}-\frac{x^3}{18}+\frac{x^2}{96}-\frac{1}{18}\left(x^2-x\right)^{3/2}+\frac{5}{64}(1-2x)\sqrt{x^2-x}-\frac{85\sqrt{x^2-x}}{384}+\frac{1}{3}x^3\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{x}{384}+\frac{\log(8x+1)}{3072}$$

```
[In] Int[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -1/384*x + x^2/96 - x^3/18 - (85*Sqrt[-x + x^2])/384 + (5*(1 - 2*x)*Sqrt[-x
 + x^2])/64 - (-x + x^2)^(3/2)/18 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/
3072 - (223*ArcTanh[x/Sqrt[-x + x^2]])/1536 + Log[1 + 8*x]/3072 + (x^3*Log[
-1 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
```

```
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2 \log\left(-1+4x+4\sqrt{-x+x^2}\right) dx \\
&= \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{8}{3}\int \frac{x^3}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)} dx \\
&= \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{8}{3}\int \left(-\frac{1}{1024}+\frac{x}{128}-\frac{x^2}{16}+\frac{1}{1024(1+8x)}\right. \\
&\qquad \left. -\frac{x}{12\sqrt{-x+x^2}}-\frac{11}{128}\sqrt{-x+x^2}-\frac{1}{16}x\sqrt{-x+x^2}+\frac{\sqrt{-x+x^2}}{384(1+8x)}\right) dx
\end{aligned}$$

$$= -\frac{x}{384} + \frac{x^2}{96} - \frac{x^3}{18} + \frac{\log(1+8x)}{3072} + \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{1}{144}\int \frac{\sqrt{-x+x^2}}{1+8x}\,dx - \frac{1}{6}\int x\sqrt{-x+x^2}\,dx - \frac{2}{9}\int \frac{x}{\sqrt{-x+x^2}}\,dx - \frac{11}{48}\int \sqrt{-x+x^2}\,dx$$

$$= -\frac{x}{384} + \frac{x^2}{96} - \frac{x^3}{18} - \frac{85}{384}\sqrt{-x+x^2} + \frac{11}{192}(1-2x)\sqrt{-x+x^2} - \frac{1}{18}\left(-x+x^2\right)^{3/2} + \frac{\log(1+8x)}{3072} + \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) - \frac{\int \frac{-1+10x}{(1+8x)\sqrt{-x+x^2}}\,dx}{2304} + \frac{11}{384}\int \frac{1}{\sqrt{-x+x^2}}\,dx - \frac{1}{12}\int \sqrt{-x+x^2}\,dx - \frac{1}{9}\int \frac{1}{\sqrt{-x+x^2}}\,dx$$

$$= -\frac{x}{384} + \frac{x^2}{96} - \frac{x^3}{18} - \frac{85}{384}\sqrt{-x+x^2} + \frac{5}{64}(1-2x)\sqrt{-x+x^2} - \frac{1}{18}\left(-x+x^2\right)^{3/2} + \frac{\log(1+8x)}{3072} + \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) - \frac{5\int \frac{1}{\sqrt{-x+x^2}}\,dx}{9216} + \frac{\int \frac{1}{(1+8x)\sqrt{-x+x^2}}\,dx}{1024} + \frac{1}{96}\int \frac{1}{\sqrt{-x+x^2}}\,dx + \frac{11}{192}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right) - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right)$$

$$= -\frac{x}{384} + \frac{x^2}{96} - \frac{x^3}{18} - \frac{85}{384}\sqrt{-x+x^2} + \frac{5}{64}(1-2x)\sqrt{-x+x^2} - \frac{1}{18}\left(-x+x^2\right)^{3/2} - \frac{95}{576}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right) + \frac{\log(1+8x)}{3072} + \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) - \frac{5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right)}{4608} - \frac{1}{512}\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{1-10x}{\sqrt{-x+x^2}}\right) + \frac{1}{48}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x+x^2}}\right)$$

$$= -\frac{x}{384} + \frac{x^2}{96} - \frac{x^3}{18} - \frac{85}{384}\sqrt{-x+x^2} + \frac{5}{64}(1-2x)\sqrt{-x+x^2} - \frac{1}{18}\left(-x+x^2\right)^{3/2} - \frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)}{3072} - \frac{223\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)}{1536} + \frac{\log(1+8x)}{3072} + \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.56

$$\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = -\frac{24\sqrt{1-x}x^{3/2}-96\sqrt{1-x}x^{5/2}+512\sqrt{1-x}x^{7/2}+928\sqrt{1-x}x^{3/2}\sqrt{(-1+x)x}+512\sqrt{1-x}x^{5/2}\sqrt{(-1}{}$$

[In] Integrate[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]

[Out] -1/9216*(24*Sqrt[1 - x]*x^(3/2) - 96*Sqrt[1 - x]*x^(5/2) + 512*Sqrt[1 - x]*x^(7/2) + 928*Sqrt[1 - x]*x^(3/2)*Sqrt[(-1 + x)*x] + 512*Sqrt[1 - x]*x^(5/2)*Sqrt[(-1 + x)*x] + 1320*Sqrt[-((-1 + x)^2*x^2)] + 1338*Sqrt[(-1 + x)*x]*ArcSin[Sqrt[1 - x]] + 3*Sqrt[-((-1 + x)*x)]*ArcTanh[(1 - 10*x)/(6*Sqrt[(-1 + x)*x])] - 3*Sqrt[-((-1 + x)*x)]*Log[1 + 8*x] - 3072*Sqrt[1 - x]*x^(7/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])/Sqrt[-((-1 + x)*x)]

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.29

method	result
parts	$\frac{x^3 \ln\left(-1+4x+4\sqrt{(-1+x)x}\right)}{3}-\frac{451\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)}{6144}-\frac{25\sqrt{x^2-x}}{256}-\frac{5x\sqrt{x^2-x}}{64}-\frac{\operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}\right)}{3072}-\frac{x^3\sqrt{}}{}$

[In] int(x^2*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x,method=_RETURNVERBOSE)

[Out] 1/3*x^3*ln(-1+4*x+4*((-1+x)*x)^(1/2))-451/6144*ln(-1/2+x+(x^2-x)^(1/2))-25/256*(x^2-x)^(1/2)-5/64*x*(x^2-x)^(1/2)-1/3072*arctanh(32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))-1/6*x^3*(x^2-x)^(1/2)-1/18*x^3+1/96*x^2-1/384*x+1/3072*ln(1+8*x)+17/384*(2*x-1)*(x^2-x)^(1/2)+1/9*(x^2-x)^(3/2)+1/6*x*(x^2-x)^(3/2)-1/6144*(64*(x+1/8)^2-80*x-1)^(1/2)+5/6144*ln(-1/2+x+((x+1/8)^2-5/4*x-1/64)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = -\frac{1}{18}x^3 + \frac{1}{96}x^2 + \frac{1}{3}(x^3+1)\log\left(4x+4\sqrt{x^2-x}-1\right) - \frac{1}{1152}(64x^2+116x+165)\sqrt{x^2-x} - \frac{1}{384}x - \frac{511}{3072}\log(8x+1) + \frac{245}{1024}\log\left(-2x+2\sqrt{x^2-x}+1\right) - \frac{511}{3072}\log\left(-2x+2\sqrt{x^2-x}-1\right) + \frac{511}{3072}\log\left(-4x+4\sqrt{x^2-x}+1\right)$$

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/18*x^3 + 1/96*x^2 + 1/3*(x^3 + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/115
2*(64*x^2 + 116*x + 165)*sqrt(x^2 - x) - 1/384*x - 511/3072*log(8*x + 1) +
245/1024*log(-2*x + 2*sqrt(x^2 - x) + 1) - 511/3072*log(-2*x + 2*sqrt(x^2 -
 x) - 1) + 511/3072*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2 \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int x^2 \log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^2*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$\begin{aligned}\int x^2 \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= \frac{1}{3}\,x^3\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{1}{18}\,x^3\\&\quad+\frac{1}{96}\,x^2-\frac{1}{1152}\,(4\,(16\,x+29)x+165)\sqrt{x^2-x}\\&\quad-\frac{1}{384}\,x+\frac{1}{3072}\,\log\left(|8\,x+1|\right)\\&\quad+\frac{223}{3072}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\\&\quad+\frac{1}{3072}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)\\&\quad-\frac{1}{3072}\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)\end{aligned}$$

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
[Out] 1/3*x^3*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/18*x^3 + 1/96*x^2 - 1/1152*(4*
(16*x + 29)*x + 165)*sqrt(x^2 - x) - 1/384*x + 1/3072*log(abs(8*x + 1)) + 2
23/3072*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/3072*log(abs(-2*x + 2*sqrt
(x^2 - x) - 1)) - 1/3072*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int x^2\,\ln\left(4\,x+4\,\sqrt{x\,(x-1)}-1\right)\,dx$$

```
[In] int(x^2*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
[Out] int(x^2*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.103 $\int x \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal result 655
Rubi [A] (verified) 656
Mathematica [A] (verified) 659
Maple [A] (verified) 659
Fricas [A] (verification not implemented) 660
Sympy [F] 660
Maxima [F] 660
Giac [A] (verification not implemented) 661
Mupad [F(-1)] 661

Optimal result

Integrand size = 19, antiderivative size = 127

$$\begin{aligned}\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}\\ &\quad+\frac{1}{256}\text{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)\\ &\quad-\frac{33}{128}\text{arctanh}\left(\frac{x}{\sqrt{-x+x^2}}\right)-\frac{1}{256}\log(1+8x)\\ &\quad+\frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)\end{aligned}$$

```
[Out] 1/32*x-1/8*x^2+1/256*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-33/128*arctanh(x/(
x^2-x)^(1/2))-1/256*ln(1+8*x)+1/2*x^2*ln(-1+4*x+4*(x^2-x)^(1/2))-11/32*(x^2
-x)^(1/2)+1/16*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {2617, 2615, 6874, 654, 634, 212, 626, 748, 857, 738}

$$\int x \log\left(-1+4x+4\sqrt{(-1+x)x}\right) dx = \frac{1}{256}\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right) - \frac{33}{128}\text{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right) - \frac{x^2}{8} + \frac{1}{16}(1-2x)\sqrt{x^2-x} - \frac{11\sqrt{x^2-x}}{32} + \frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right) + \frac{x}{32} - \frac{1}{256}\log(8x+1)$$

```
[In] Int[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] x/32 - x^2/8 - (11*Sqrt[-x + x^2])/32 + ((1 - 2*x)*Sqrt[-x + x^2])/16 + Arc
Tanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/256 - (33*ArcTanh[x/Sqrt[-x + x^2]])/12
8 - Log[1 + 8*x]/256 + (x^2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\log\left(-1+4x+4\sqrt{-x+x^2}\right)dx\\
&= \frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)+4\int\frac{x^2}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,dx\\
&= \frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)+4\int\left(\frac{1}{128}-\frac{x}{16}-\frac{1}{128(1+8x)}-\frac{x}{12\sqrt{-x+x^2}}\right.\\
&\qquad\left.-\frac{1}{16}\sqrt{-x+x^2}+\frac{\sqrt{-x+x^2}}{48(-1-8x)}\right)dx\\
&= \frac{x}{32}-\frac{x^2}{8}-\frac{1}{256}\log(1+8x)+\frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)\\
&\quad+\frac{1}{12}\int\frac{\sqrt{-x+x^2}}{-1-8x}\,dx-\frac{1}{4}\int\sqrt{-x+x^2}\,dx-\frac{1}{3}\int\frac{x}{\sqrt{-x+x^2}}\,dx\\
&= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}\\
&\quad-\frac{1}{256}\log(1+8x)+\frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)\\
&\quad+\frac{1}{192}\int\frac{1-10x}{(-1-8x)\sqrt{-x+x^2}}\,dx+\frac{1}{32}\int\frac{1}{\sqrt{-x+x^2}}\,dx-\frac{1}{6}\int\frac{1}{\sqrt{-x+x^2}}\,dx\\
&= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}\\
&\quad-\frac{1}{256}\log(1+8x)+\frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)\\
&\quad+\frac{5}{768}\int\frac{1}{\sqrt{-x+x^2}}\,dx+\frac{3}{256}\int\frac{1}{(-1-8x)\sqrt{-x+x^2}}\,dx\\
&\quad+\frac{1}{16}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right)-\frac{1}{3}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right)\\
&= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{13}{48}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)\\
&\quad-\frac{1}{256}\log(1+8x)+\frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)\\
&\quad+\frac{5}{384}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right)\\
&\quad-\frac{3}{128}\text{Subst}\left(\int\frac{1}{36-x^2}\,dx,x,\frac{-1+10x}{\sqrt{-x+x^2}}\right)
\end{aligned}$$

$$= \frac{x}{32} - \frac{x^2}{8} - \frac{11}{32}\sqrt{-x+x^2} + \frac{1}{16}(1-2x)\sqrt{-x+x^2} + \frac{1}{256}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) - \frac{33}{128}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right) - \frac{1}{256}\log(1+8x) + \frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.50

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$$
$$= \frac{8\sqrt{1-x}x^{3/2} - 32\sqrt{1-x}x^{5/2} - 32\sqrt{1-x}x^{3/2}\sqrt{(-1+x)x} - 72\sqrt{-(-1+x)^2x^2} - 66\sqrt{(-1+x)x}\,\text{arcsin}}{}$$

```
[In] Integrate[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (8*Sqrt[1 - x]*x^(3/2) - 32*Sqrt[1 - x]*x^(5/2) - 32*Sqrt[1 - x]*x^(3/2)*Sq
rt[(-1 + x)*x] - 72*Sqrt[-((-1 + x)^2*x^2)] - 66*Sqrt[(-1 + x)*x]*ArcSin[Sq
rt[1 - x]] + Sqrt[-((-1 + x)*x)]*ArcTanh[(1 - 10*x)/(6*Sqrt[(-1 + x)*x])] -
 Sqrt[-((-1 + x)*x)]*Log[1 + 8*x] + 128*Sqrt[1 - x]*x^(5/2)*Log[-1 + 4*x +
4*Sqrt[(-1 + x)*x]])/(256*Sqrt[-((-1 + x)*x)])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.38

method	result
parts	$\frac{x^2\ln\left(-1+4x+4\sqrt{(-1+x)x}\right)}{2} - \frac{61\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)}{512} - \frac{13\sqrt{x^2-x}}{64} + \frac{\text{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}\right)}{256} + \frac{x\sqrt{x^2-x}}{48} - \frac{x^2\sqrt{}}{}$

```
[In] int(x*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(-1+4*x+4*((-1+x)*x)^(1/2))-61/512*ln(-1/2+x+(x^2-x)^(1/2))-13/64
*(x^2-x)^(1/2)+1/256*arctanh(32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))+
1/48*x*(x^2-x)^(1/2)-1/3*x^2*(x^2-x)^(1/2)-1/8*x^2+1/32*x-1/256*ln(1+8*x)+3
/32*(2*x-1)*(x^2-x)^(1/2)+1/3*(x^2-x)^(3/2)+1/512*(64*(x+1/8)^2-80*x-1)^(1/
2)-5/512*ln(-1/2+x+((x+1/8)^2-5/4*x-1/64)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = -\frac{1}{8}\,x^2+\frac{1}{2}\,(x^2-1)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right) - \frac{1}{32}\,\sqrt{x^2-x}(4\,x+9)+\frac{1}{32}\,x+\frac{63}{256}\,\log\left(8\,x+1\right) - \frac{31}{256}\,\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right) + \frac{63}{256}\,\log\left(-2\,x+2\,\sqrt{x^2-x}-1\right) - \frac{63}{256}\,\log\left(-4\,x+4\,\sqrt{x^2-x}+1\right)$$

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
[Out] -1/8*x^2 + 1/2*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/32*sqrt(x^2 - x
)*(4*x + 9) + 1/32*x + 63/256*log(8*x + 1) - 31/256*log(-2*x + 2*sqrt(x^2 -
 x) + 1) + 63/256*log(-2*x + 2*sqrt(x^2 - x) - 1) - 63/256*log(-4*x + 4*sqr
t(x^2 - x) + 1)
```

Sympy [F]

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int x\log\left(4x+4\sqrt{x^2-x}-1\right)\,dx$$

```
[In] integrate(x*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
[Out] Integral(x*log(4*x + 4*sqrt(x**2 - x) - 1), x)
```

Maxima [F]

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int x\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
[Out] integrate(x*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \frac{1}{2}\,x^2\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right) - \frac{1}{8}\,x^2 - \frac{1}{32}\,\sqrt{x^2-x}(4\,x+9) + \frac{1}{32}\,x - \frac{1}{256}\,\log\left(|8\,x+1|\right) + \frac{33}{256}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right) - \frac{1}{256}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right) + \frac{1}{256}\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)$$

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/8*x^2 - 1/32*sqrt(x^2 - x)*(4*
x + 9) + 1/32*x - 1/256*log(abs(8*x + 1)) + 33/256*log(abs(-2*x + 2*sqrt(x^
2 - x) + 1)) - 1/256*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/256*log(abs(-
4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int x\,\ln\left(4\,x+4\,\sqrt{x\,(x-1)}-1\right)\,dx$$

```
[In] int(x*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.104 $\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx$

Optimal result . . . 662
Rubi [A] (verified) . . . 662
Mathematica [A] (verified) . . . 665
Maple [A] (verified) . . . 665
Fricas [A] (verification not implemented) . . . 666
Sympy [F] . . . 666
Maxima [F] . . . 666
Giac [A] (verification not implemented) . . . 667
Mupad [F(-1)] . . . 667

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{1}{16}\text{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) -\frac{7}{8}\text{arctanh}\left(\frac{x}{\sqrt{-x+x^2}}\right)+\frac{1}{16}\log(1+8x) +x\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

```
[Out] -1/2*x-1/16*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-7/8*arctanh(x/(x^2-x)^(1/2)
)+1/16*ln(1+8*x)+x*ln(-1+4*x+4*(x^2-x)^(1/2))-1/2*(x^2-x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2617, 2613, 6874, 654, 634, 212, 748, 857, 738}

$$\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = -\frac{1}{16}\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right) -\frac{7}{8}\text{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right)-\frac{\sqrt{x^2-x}}{2} +x\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{x}{2}+\frac{1}{16}\log(8x+1)$$

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -1/2*x - Sqrt[-x + x^2]/2 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/16 - (7*
ArcTanh[x/Sqrt[-x + x^2]])/8 + Log[1 + 8*x]/16 + x*Log[-1 + 4*x + 4*Sqrt[-x
 + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2613

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
, x_Symbol] :> Simp[x*Log[d + e*x + f*Sqrt[a + b*x + c*x^2]], x] + Dist[f^2
```

```
*((b^2 - 4*a*c)/2), Int[x/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a
*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx\\
&= x\log\left(-1+4x+4\sqrt{-x+x^2}\right)+8\int \frac{x}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,dx\\
&= x\log\left(-1+4x+4\sqrt{-x+x^2}\right)+8\int \left(-\frac{1}{16}+\frac{1}{16(1+8x)}-\frac{x}{12\sqrt{-x+x^2}}+\frac{\sqrt{-x+x^2}}{6(1+8x)}\right)dx\\
&= -\frac{x}{2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)\\
&\quad -\frac{2}{3}\int \frac{x}{\sqrt{-x+x^2}}\,dx+\frac{4}{3}\int \frac{\sqrt{-x+x^2}}{1+8x}\,dx\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)\\
&\quad -\frac{1}{12}\int \frac{-1+10x}{(1+8x)\sqrt{-x+x^2}}\,dx-\frac{1}{3}\int \frac{1}{\sqrt{-x+x^2}}\,dx\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x\right.\\
&\qquad\qquad \left.+4\sqrt{-x+x^2}\right)-\frac{5}{48}\int \frac{1}{\sqrt{-x+x^2}}\,dx\\
&\quad +\frac{3}{16}\int \frac{1}{(1+8x)\sqrt{-x+x^2}}\,dx-\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right)\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)\\
&\quad -\frac{5}{24}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right)-\frac{3}{8}\text{Subst}\left(\int \frac{1}{36-x^2}\,dx,x,\frac{1-10x}{\sqrt{-x+x^2}}\right)
\end{aligned}
$$

$$= -\frac{x}{2} - \frac{1}{2}\sqrt{-x+x^2} - \frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) - \frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right) + \frac{1}{16}\log(1+8x) + x\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \frac{1}{16}\Big(-8x-8\sqrt{(-1+x)x}+2\log(1+8x) - 7\log\left(1-2x-2\sqrt{(-1+x)x}\right) + 16x\log\left(-1+4x+4\sqrt{(-1+x)x}\right) - \log\left(1-10x+6\sqrt{(-1+x)x}\right)\Big)$$

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-8*x - 8*Sqrt[(-1 + x)*x] + 2*Log[1 + 8*x] - 7*Log[1 - 2*x - 2*Sqrt[(-1 +
x)*x]] + 16*x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] - Log[1 - 10*x + 6*Sqrt[(-
1 + x)*x]])/16
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

method	result
default	$x\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)-\frac{7\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)}{16}-\frac{\operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}\right)}{16}-\frac{\sqrt{x^2-x}}{2}-\frac{x}{2}+$
parts	$x\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)-\frac{19\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)}{32}-\frac{\operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}\right)}{16}+\frac{\sqrt{x^2-x}}{4}-x\sqrt{}$

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(-1+4*x+4*((-1+x)*x)^(1/2))-7/16*ln(-1/2+x+(x^2-x)^(1/2))-1/16*arctanh(
32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))-1/2*(x^2-x)^(1/2)-1/2*x+1/16*
ln(1+8*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = (x+1)\log \left(4x+4\sqrt{x^2-x}-1\right)-\frac{1}{2}x-\frac{1}{2}\sqrt{x^2-x} -\frac{7}{16}\log(8x+1)+\frac{15}{16}\log\left(-2x+2\sqrt{x^2-x}+1\right) -\frac{7}{16}\log\left(-2x+2\sqrt{x^2-x}-1\right) +\frac{7}{16}\log\left(-4x+4\sqrt{x^2-x}+1\right)$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) - 7/16*l
og(8*x + 1) + 15/16*log(-2*x + 2*sqrt(x^2 - x) + 1) - 7/16*log(-2*x + 2*sqr
t(x^2 - x) - 1) + 7/16*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

Sympy [F]

$$\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = \int \log \left(4x+4\sqrt{x(x-1)}-1\right) dx$$

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Integral(log(4*x + 4*sqrt(x*(x - 1)) - 1), x)
```

Maxima [F]

$$\int \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = \int \log \left(4x+4\sqrt{(x-1)x}-1\right) dx$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 1/2*x + integrate(1/2*(2*x^2 + x)/
(4*x^3 - 5*x^2 + 4*(x^(5/2) - x^(3/2))*sqrt(x - 1) + x), x) - 1/2*log(sqrt(
x) + 1) - 1/2*log(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = x\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-x}+\frac{1}{16}\,\log\left(|8\,x+1|\right)+\frac{7}{16}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)+\frac{1}{16}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)-\frac{1}{16}\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) + 1/16*log(a
bs(8*x + 1)) + 7/16*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/16*log(abs(-2*
x + 2*sqrt(x^2 - x) - 1)) - 1/16*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int \ln\left(4\,x+4\,\sqrt{x\,(x-1)}-1\right)\,dx$$

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.105 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$

Optimal result 668
Rubi [N/A] 668
Mathematica [N/A] 669
Maple [N/A] 669
Fricas [N/A] 669
Sympy [N/A] 670
Maxima [N/A] 670
Giac [N/A] 670
Mupad [N/A] 671

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx = \text{Int}\left(\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(-1+4*x+4*(x^2-x)^(1/2))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$$

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
```

```
[Out] Defer[Int][Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$$

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
[Out] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\ln \left(-1+4x+4\sqrt{\left(-1+x\right)x}\right)}{x}dx$$

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x,x)
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}\,dx$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="fricas")
[Out] integral(log(4*x + 4*sqrt(x^2 - x) - 1)/x, x)
```

Sympy [N/A]

Not integrable

Time = 44.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log \left(4x+4\sqrt{x^2-x}-1\right)}{x}\,dx$$

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x,x)
[Out] Integral(log(4*x + 4*sqrt(x**2 - x) - 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}\,dx$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="maxima")
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}\,dx$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="giac")
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\ln \left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x}\,dx$$

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x,x)
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x, x)
```

3.106 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx$

Optimal result 672
Rubi [A] (verified) 672
Mathematica [A] (verified) 676
Maple [A] (verified) 676
Fricas [A] (verification not implemented) 676
Sympy [F] 677
Maxima [F] 677
Giac [A] (verification not implemented) 677
Mupad [F(-1)] 678

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = \frac{4\sqrt{-x+x^2}}{x}+4\text{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)+4\log(x)-4\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}$$

```
[Out] 4*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))+4*ln(x)-4*ln(1+8*x)-ln(-1+4*x+4*(x^2-
x)^(1/2))/x+4*(x^2-x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2617, 2615, 6874, 654, 634, 212, 676, 678, 748, 857, 738}

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = 4\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)+\frac{4\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x}+4\log(x)-4\log(8x+1)$$

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

```
[Out] (4*Sqrt[-x + x^2])/x + 4*ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])] + 4*Log[x]
- 4*Log[1 + 8*x] - Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^2}\,dx$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} - 8\int \frac{1}{x\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)\right)}\,dx$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} - 8\int\left(-\frac{1}{2x}+\frac{4}{1+8x}-\frac{x}{12\sqrt{-x+x^2}} + \frac{\sqrt{-x+x^2}}{4x^2}-\frac{5\sqrt{-x+x^2}}{4x}+\frac{32\sqrt{-x+x^2}}{3(1+8x)}\right)dx$$

$$= 4\log(x)-4\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}+\frac{2}{3}\int\frac{x}{\sqrt{-x+x^2}}\,dx - 2\int\frac{\sqrt{-x+x^2}}{x^2}\,dx+10\int\frac{\sqrt{-x+x^2}}{x}\,dx-\frac{256}{3}\int\frac{\sqrt{-x+x^2}}{1+8x}\,dx$$

$$= \frac{4\sqrt{-x+x^2}}{x}+4\log(x)-4\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} + \frac{1}{3}\int\frac{1}{\sqrt{-x+x^2}}\,dx-2\int\frac{1}{\sqrt{-x+x^2}}\,dx-5\int\frac{1}{\sqrt{-x+x^2}}\,dx + \frac{16}{3}\int\frac{-1+10x}{(1+8x)\sqrt{-x+x^2}}\,dx$$

$$= \frac{4\sqrt{-x+x^2}}{x}+4\log(x)-4\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} + \frac{2}{3}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right) - 4\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right)+\frac{20}{3}\int\frac{1}{\sqrt{-x+x^2}}\,dx - 10\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right)-12\int\frac{1}{(1+8x)\sqrt{-x+x^2}}\,dx$$

$$= \frac{4\sqrt{-x+x^2}}{x}-\frac{40}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)+4\log(x)-4\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}+\frac{40}{3}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right) + 24\text{Subst}\left(\int\frac{1}{36-x^2}\,dx,x,\frac{1-10x}{\sqrt{-x+x^2}}\right)$$

$$= \frac{4\sqrt{-x+x^2}}{x}+4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)+4\log(x) - 4\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.87

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = 2\text{arctanh}\left(\frac{1-10x}{6\sqrt{(-1+x)x}}\right) + \frac{4\sqrt{(-1+x)x}+\frac{4\sqrt{-(-1+x)^2x^2}\arcsin(\sqrt{1-x})}{-1+x}+4x\log(x)-2x\log(1+8x)-4x\log\left(1-4x-4\sqrt{(-1+x)x}\right)}{x}$$

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

```
[Out] 2*ArcTanh[(1 - 10*x)/(6*Sqrt[(-1 + x)*x])] + (4*Sqrt[(-1 + x)*x] + (4*Sqrt[
-((-1 + x)^2*x^2)]*ArcSin[Sqrt[1 - x]])/(-1 + x) + 4*x*Log[x] - 2*x*Log[1 +
 8*x] - 4*x*Log[1 - 4*x - 4*Sqrt[(-1 + x)*x]] + 4*x*Log[1 - 2*x - 2*Sqrt[(-
1 + x)*x]] - Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])/x
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

method	result
parts	$-\frac{\ln\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}+\frac{4\sqrt{x^2-x}}{x}+4\,\text{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}\right)+10\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)-4$

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(-1+4*x+4*((-1+x)*x)^(1/2))/x+4*(x^2-x)^(1/2)/x+4*arctanh(32/3*(1/8-5/4*
x)/(64*(x+1/8)^2-80*x-1)^(1/2))+10*ln(-1/2+x+(x^2-x)^(1/2))-4*ln(1+8*x)+4*l
n(x)-16*(x^2-x)^(1/2)+2*(64*(x+1/8)^2-80*x-1)^(1/2)-10*ln(-1/2+x+((x+1/8)^2
-5/4*x-1/64)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.51

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = -\frac{7\,x\log\left(8\,x+1\right)+2\,(x+1)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-8\,x\log\left(x\right)+x\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)+7\,x}{2\,x}$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(7*x*log(8*x + 1) + 2*(x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 8*x*log
(x) + x*log(-2*x + 2*sqrt(x^2 - x) + 1) + 7*x*log(-2*x + 2*sqrt(x^2 - x) -
1) - 7*x*log(-4*x + 4*sqrt(x^2 - x) + 1) - 8*x - 8*sqrt(x^2 - x))/x
```

Sympy [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = \int \frac{\log\left(4x+4\sqrt{x^2-x}-1\right)}{x^2}\,dx$$

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**2,x)
[Out] Integral(log(4*x + 4*sqrt(x**2 - x) - 1)/x**2, x)
```

Maxima [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = \int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x^2}\,dx$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="maxima")
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx &= -\frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x} \\ &\quad+\frac{4}{x-\sqrt{x^2-x}}-4\,\log\left(|8\,x+1|\right) \\ &\quad+4\,\log\left(|x|\right)-4\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right) \\ &\quad+4\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)\end{aligned}$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="giac")
[Out] -log(4*x + 4*sqrt((x - 1)*x) - 1)/x + 4/(x - sqrt(x^2 - x)) - 4*log(abs(8*x
 + 1)) + 4*log(abs(x)) - 4*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 4*log(abs
(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx=\int \frac{\ln \left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^2}\,dx$$

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^2,x)
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^2, x)
```

3.107 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3} dx$

Optimal result 679
Rubi [A] (verified) 679
Mathematica [A] (verified) 683
Maple [B] (verified) 683
Fricas [A] (verification not implemented) 684
Sympy [F(-1)] 684
Maxima [F] 684
Giac [A] (verification not implemented) 685
Mupad [F(-1)] 685

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3} dx = -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2(-x+x^2)^{3/2}}{3x^3} - 16\text{arctanh}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) - 16\log(x) + 16\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2}$$

```
[Out] -2/x-2/3*(x^2-x)^(3/2)/x^3-16*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-16*ln(x)+
16*ln(1+8*x)-1/2*ln(-1+4*x+4*(x^2-x)^(1/2))/x^2-10*(x^2-x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2617, 2615, 6874, 654, 634, 212, 748, 857, 738, 664, 676, 678}

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3} dx = -16\text{arctanh}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right) - \frac{10\sqrt{x^2-x}}{x} - \frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{2x^2} - \frac{2(x^2-x)^{3/2}}{3x^3} - \frac{2}{x} - 16\log(x) + 16\log(8x+1)$$

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

```
[Out] -2/x - (10*Sqrt[-x + x^2])/x - (2*(-x + x^2)^(3/2))/(3*x^3) - 16*ArcTanh[(1
 - 10*x)/(6*Sqrt[-x + x^2])] - 16*Log[x] + 16*Log[1 + 8*x] - Log[-1 + 4*x +
 4*Sqrt[-x + x^2]]/(2*x^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^3}\,dx$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} - 4\int \frac{1}{x^2\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)\right)}\,dx$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} - 4\int \left(-\frac{1}{2x^2}+\frac{4}{x}-\frac{32}{1+8x}-\frac{x}{12\sqrt{-x+x^2}} + \frac{256\sqrt{-x+x^2}}{3(-1-8x)}+\frac{\sqrt{-x+x^2}}{4x^3}-\frac{5\sqrt{-x+x^2}}{4x^2}+\frac{43\sqrt{-x+x^2}}{4x}\right)dx$$

$$= -\frac{2}{x}-16\log(x)+16\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2}+\frac{1}{3}\int \frac{x}{\sqrt{-x+x^2}}\,dx + 5\int \frac{\sqrt{-x+x^2}}{x^2}\,dx-43\int \frac{\sqrt{-x+x^2}}{x}\,dx-\frac{1024}{3}\int \frac{\sqrt{-x+x^2}}{-1-8x}\,dx-\int \frac{\sqrt{-x+x^2}}{x^3}\,dx$$

$$= -\frac{2}{x}-\frac{10\sqrt{-x+x^2}}{x}-\frac{2\left(-x+x^2\right)^{3/2}}{3x^3} - 16\log(x)+16\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2}+\frac{1}{6}\int \frac{1}{\sqrt{-x+x^2}}\,dx + 5\int \frac{1}{\sqrt{-x+x^2}}\,dx-\frac{64}{3}\int \frac{1-10x}{(-1-8x)\sqrt{-x+x^2}}\,dx+\frac{43}{2}\int \frac{1}{\sqrt{-x+x^2}}\,dx$$

$$= -\frac{2}{x}-\frac{10\sqrt{-x+x^2}}{x}-\frac{2\left(-x+x^2\right)^{3/2}}{3x^3}-16\log(x)+16\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right) + 10\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right)-\frac{80}{3}\int \frac{1}{\sqrt{-x+x^2}}\,dx + 43\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right)-48\int \frac{1}{(-1-8x)\sqrt{-x+x^2}}\,dx$$

$$= -\frac{2}{x}-\frac{10\sqrt{-x+x^2}}{x}-\frac{2\left(-x+x^2\right)^{3/2}}{3x^3} + \frac{160}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)-16\log(x)+16\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2}-\frac{160}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-x+x^2}}\right) + 96\text{Subst}\left(\int \frac{1}{36-x^2}\,dx,x,\frac{-1+10x}{\sqrt{-x+x^2}}\right)$$

$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2(-x+x^2)^{3/2}}{3x^3} - 16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) - 16\log(x) + 16\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = -\frac{12x - 4\sqrt{(-1+x)x} + 64x\sqrt{(-1+x)x} + 96x^2\operatorname{arctanh}\left(\frac{1-10x}{6\sqrt{(-1+x)x}}\right) + 96x^2\log(x) - 96x^2\log(1+8x)}{6x^2}$$

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

```
[Out] -1/6*(12*x - 4*Sqrt[(-1 + x)*x] + 64*x*Sqrt[(-1 + x)*x] + 96*x^2*ArcTanh[(1
 - 10*x)/(6*Sqrt[(-1 + x)*x])] + 96*x^2*Log[x] - 96*x^2*Log[1 + 8*x] + 3*Lo
g[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])/x^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(87) = 174.

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.84

method	result
parts	$-\frac{\ln\left(-1+4x+4\sqrt{(-1+x)x}\right)}{2x^2} + \frac{2\sqrt{x^2-x}}{3x^2} - \frac{80\sqrt{x^2-x}}{3x} - 16\ \operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}\right) + \frac{32\ln(1+8x)x-32\ln(x)}{x}$

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^2+2/3*(x^2-x)^(1/2)/x^2-80/3*(x^2-x)^(
1/2)/x-16*arctanh(32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))+4/x*(8*ln(1
+8*x)*x-8*ln(x)*x-1)-16*ln(1+8*x)+16*ln(x)+2/x-16/x^2*(x^2-x)^(3/2)+80*(x^2
-x)^(1/2)-40*ln(-1/2+x+(x^2-x)^(1/2))-8*(64*(x+1/8)^2-80*x-1)^(1/2)+40*ln(-
1/2+x+((x+1/8)^2-5/4*x-1/64)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.37

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \frac{189\,x^2\log\left(8\,x+1\right)-192\,x^2\log\left(x\right)+3\,x^2\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)+189\,x^2\log\left(-2\,x+2\,\sqrt{x^2-x}-1\right)$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(189*x^2*log(8*x + 1) - 192*x^2*log(x) + 3*x^2*log(-2*x + 2*sqrt(x^2 -
 x) + 1) + 189*x^2*log(-2*x + 2*sqrt(x^2 - x) - 1) - 189*x^2*log(-4*x + 4*s
qrt(x^2 - x) + 1) - 128*x^2 + 6*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1) -
8*sqrt(x^2 - x)*(16*x - 1) - 24*x)/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x^3}\,dx$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^3, x)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.29

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = -\frac{2}{x} - \frac{\log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{2\,x^2} - \frac{2\left(18\,\left(x-\sqrt{x^2-x}\right)^2-3\,x+3\,\sqrt{x^2-x}+1\right)}{3\,\left(x-\sqrt{x^2-x}\right)^3} + 16\,\log\left(|8\,x+1|\right) - 16\,\log\left(|x|\right) + 16\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right) - 16\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -2/x - 1/2*log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2 - 2/3*(18*(x - sqrt(x^2 - x
))^2 - 3*x + 3*sqrt(x^2 - x) + 1)/(x - sqrt(x^2 - x))^3 + 16*log(abs(8*x +
1)) - 16*log(abs(x)) + 16*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) - 16*log(abs
(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \int \frac{\ln \left(4\,x+4\,\sqrt{x\,(x-1)}-1\right)}{x^3}\,dx$$

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^3,x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^3, x)
```

3.108 $\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal result 686
Rubi [A] (verified) 686
Mathematica [C] (verified) 690
Maple [F] 690
Fricas [A] (verification not implemented) 691
Sympy [F(-1)] 691
Maxima [F] 691
Giac [C] (verification not implemented) 692
Mupad [F(-1)] 692

Optimal result

Integrand size = 23, antiderivative size = 187

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=-\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}}-\frac{\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{320\sqrt{2}\sqrt{-1+x}\sqrt{x}}+\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{320\sqrt{2}}+\frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

```
[Out] 1/60*x^(3/2)-2/25*x^(5/2)-71/300*(x^2-x)^(3/2)/x^(3/2)+2/5*x^(5/2)*ln(-1+4*
x+4*(x^2-x)^(1/2))+1/640*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-2/25*(x^2-x)^(3/
2)/x^(1/2)-1/160*x^(1/2)-17/32/x^(1/2)*(x^2-x)^(1/2)-1/640*arctan(2/3*2^(1/
2)*(-1+x)^(1/2))*(x^2-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2617, 2615, 6865, 6874, 209, 1602, 2025, 2041, 1160, 455, 52, 65}

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=-\frac{\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{320\sqrt{2}\sqrt{x-1}\sqrt{x}}+\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{320\sqrt{2}}-\frac{2x^{5/2}}{25}+\frac{x^{3/2}}{60}-\frac{2\left(x^2-x\right)^{3/2}}{25\sqrt{x}}-\frac{17\sqrt{x^2-x}}{32\sqrt{x}}-\frac{71\left(x^2-x\right)^{3/2}}{300x^{3/2}}+\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\sqrt{x}}{160}$$

```
[In] Int[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -1/160*Sqrt[x] + x^(3/2)/60 - (2*x^(5/2))/25 - (17*Sqrt[-x + x^2])/(32*Sqrt
[x]) - (71*(-x + x^2)^(3/2))/(300*x^(3/2)) - (2*(-x + x^2)^(3/2))/(25*Sqrt[
x]) - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(320*Sqrt[2]*Sqrt
[-1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x]]/(320*Sqrt[2]) + (2*x^(5/2)*Lo
g[-1 + 4*x + 4*Sqrt[-x + x^2]])/5
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx$$

$$= \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{16}{5}\int\frac{x^{5/2}}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,dx$$

$$= \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{32}{5}\text{Subst}\left(\int\frac{x^6}{-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\left(-x^2+x^4\right)}\,dx,x,\sqrt{x}\right)$$

$$= \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{32}{5}\text{Subst}\left(\int\left(-\frac{1}{1024}+\frac{x^2}{128}-\frac{x^4}{16}+\frac{1}{1024\left(1+8x^2\right)}-\frac{x^2}{12\sqrt{-x^2+x^4}}-\frac{11}{128}\sqrt{-x^2+x^4}-\frac{1}{16}x^2\sqrt{-\right.\right.$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25} + \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{1}{160}\text{Subst}\left(\int\frac{1}{1+8x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{60}\text{Subst}\left(\int\frac{\sqrt{-x^2+x}}{1+8x^2}\right.$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{8\sqrt{-x+x^2}}{15\sqrt{x}}-\frac{11\left(-x+x^2\right)^{3/2}}{60x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{320\sqrt{2}} + \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{4}{25}\text{Subst}\left(\int\sqrt{-x^2+x^4}\,dx,x,\sqrt{x}\right)+\frac{\sqrt{-x+x^2}\,\text{Subst}\left(\int x\right.}{60\sqrt{-1-}}$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{8\sqrt{-x+x^2}}{15\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{320\sqrt{2}}+\frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{\sqrt{-x+x^2}\,\text{Subst}\left(\int\frac{\sqrt{-1+x}}{1+8x}\,dx,x,x\right)}{120\sqrt{-1+x}\sqrt{x}}$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{320\sqrt{2}} + \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{\left(3\sqrt{-x+x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+x}(1+8x)}\,dx,x,x\right)}{320\sqrt{-1+x}\sqrt{x}}$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{320\sqrt{2}} + \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{\left(3\sqrt{-x+x^2}\right)\text{Subst}\left(\int\frac{1}{9+8x^2}\,dx,x,\sqrt{-1+x}\right)}{160\sqrt{-1+x}\sqrt{x}}$$

$$= -\frac{\sqrt{x}}{160} + \frac{x^{3/2}}{60} - \frac{2x^{5/2}}{25} - \frac{17\sqrt{-x+x^2}}{32\sqrt{x}} - \frac{71(-x+x^2)^{3/2}}{300x^{3/2}} - \frac{2(-x+x^2)^{3/2}}{25\sqrt{x}} - \frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{320\sqrt{2}\sqrt{-1+x}\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{320\sqrt{2}} + \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.16

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)dx = \frac{15\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}-i\sqrt{x}}{3\sqrt{-1+x}}\right) + 15\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}+i\sqrt{x}}{3\sqrt{-1+x}}\right) - 2\sqrt{-1}$$

```
[In] Integrate[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (15*Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x]
)] + 15*Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3*Sqrt[-1
+ x])] - 2*Sqrt[-1 + x]*(-15*Sqrt[2]*Sqrt[x]*ArcTan[2*Sqrt[2]*Sqrt[x]] + 4*
(192*x^3 + 707*Sqrt[(-1 + x)*x] + 8*x^2*(-5 + 24*Sqrt[(-1 + x)*x]) + x*(15
+ 376*Sqrt[(-1 + x)*x]) - 960*x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])))/(19
200*Sqrt[-1 + x]*Sqrt[x])
```

Maple [F]

$$\int x^{\frac{3}{2}}\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)dx$$

```
[In] int(x^(3/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^(3/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.59

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\frac{3840\,x^{\frac{7}{2}}\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)+15\,\sqrt{2}x\arctan\left(2\,\sqrt{2}\sqrt{x}\right)+15\,\sqrt{2}x\arctan\left(\frac{3\,\sqrt{}}{4\,\sqrt{}}\right.}{9600\,x}$$

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/9600*(3840*x^(7/2)*log(4*x + 4*sqrt(x^2 - x) - 1) + 15*sqrt(2)*x*arctan(2
*sqrt(2)*sqrt(x)) + 15*sqrt(2)*x*arctan(3/4*sqrt(2)*sqrt(x)/sqrt(x^2 - x))
- 4*(192*x^2 + 376*x + 707)*sqrt(x^2 - x)*sqrt(x) - 4*(192*x^3 - 40*x^2 + 1
5*x)*sqrt(x))/x
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**(3/2)*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x^{\frac{3}{2}}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 2/25*(2*x^2 + 5)*sqrt(x)
 - 2/15*x^(3/2) + integrate(1/5*(2*x^(5/2) + x^(3/2))/(4*x^2 + 4*(x^(3/2) -
 sqrt(x))*sqrt(x - 1) - 5*x + 1), x) + 1/5*log(sqrt(x) + 1) - 1/5*log(sqrt(
x) - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.71

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{2}{25}\,x^{\frac{5}{2}}+\frac{1}{1280}\,\sqrt{2}\left(\pi-2\,\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1\right)}{3\,\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)+\frac{1}{19200}\,\sqrt{2}\left(15i\,\pi+2828i\,\sqrt{2}+30\,\arctan\left(\frac{2}{3}i\,\sqrt{2}\right)\right)-\frac{1}{2400}\,(8\,(24\,x+47)x+707)\sqrt{x-1}+\frac{1}{60}\,x^{\frac{3}{2}}+\frac{1}{640}\,\sqrt{2}\arctan\left(2\,\sqrt{2}\sqrt{x}\right)-\frac{1}{160}\,\sqrt{x}$$

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)*log(4*x + 4*sqrt((x - 1)*x) - 1) - 2/25*x^(5/2) + 1/1280*sqrt(2
)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^2 - 1)/(sqrt(x - 1) -
 sqrt(x)))) + 1/19200*sqrt(2)*(15*I*pi + 2828*I*sqrt(2) + 30*arctan(2/3*I*s
qrt(2))) - 1/2400*(8*(24*x + 47)*x + 707)*sqrt(x - 1) + 1/60*x^(3/2) + 1/64
0*sqrt(2)*arctan(2*sqrt(2)*sqrt(x)) - 1/160*sqrt(x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int x^{3/2}\,\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

```
[In] int(x^(3/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x^(3/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.109 $\int \sqrt{x} \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx$

Optimal result 693
Rubi [A] (verified) 693
Mathematica [C] (verified) 697
Maple [F] 697
Fricas [A] (verification not implemented) 698
Sympy [F(-1)] 698
Maxima [F] 698
Giac [C] (verification not implemented) 699
Mupad [F(-1)] 699

Optimal result

Integrand size = 23, antiderivative size = 158

$$\int \sqrt{x} \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx = \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{-x+x^2}}{12\sqrt{x}}-\frac{2(-x+x^2)^{3/2}}{9x^{3/2}} +\frac{\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{24\sqrt{2}\sqrt{-1+x}\sqrt{x}} -\frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}} +\frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

```
[Out] -2/9*x^(3/2)-2/9*(x^2-x)^(3/2)/x^(3/2)+2/3*x^(3/2)*ln(-1+4*x+4*(x^2-x)^(1/2
))-1/48*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)+1/12*x^(1/2)-11/12/x^(1/2)*(x^2-x
)^(1/2)+1/48*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2-x)^(1/2)*2^(1/2)/(-1+x)^
(1/2)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules

used = {2617, 2615, 6865, 6874, 209, 1602, 2025, 1160, 455, 52, 65, 210}

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \frac{\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{24\sqrt{2}\sqrt{x-1}\sqrt{x}} - \frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}} - \frac{2x^{3/2}}{9} - \frac{11\sqrt{x^2-x}}{12\sqrt{x}} - \frac{2(x^2-x)^{3/2}}{9x^{3/2}} + \frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right) + \frac{\sqrt{x}}{12}$$

```
[In] Int[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] Sqrt[x]/12 - (2*x^(3/2))/9 - (11*Sqrt[-x + x^2])/(12*Sqrt[x]) - (2*(-x + x^
2)^(3/2))/(9*x^(3/2)) + (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])
/(24*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) - ArcTan[2*Sqrt[2]*Sqrt[x]]/(24*Sqrt[2])
 + (2*x^(3/2)*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx$$

$$= \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{16}{3}\int\frac{x^{3/2}}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,dx$$

$$= \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{3}\text{Subst}\left(\int\frac{x^4}{-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\left(-x^2+x^4\right)}\,dx,x,\sqrt{x}\right)$$

$$= \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{3}\text{Subst}\left(\int\left(\frac{1}{128}-\frac{x^2}{16}-\frac{1}{128\left(1+8x^2\right)}-\frac{x^2}{12\sqrt{-x^2+x^4}}-\frac{1}{16}\sqrt{-x^2+x^4}+\frac{\sqrt{-x^2+x^4}}{48\left(-1-8x^2\right)}\right)\,dx,x$$

$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}+\frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{1}{12}\text{Subst}\left(\int\frac{1}{1+8x^2}\,dx,x,\sqrt{x}\right)+\frac{2}{9}\text{Subst}\left(\int\frac{\sqrt{-x^2+x^4}}{-1-8x^2}\,dx,x,\sqrt{x}\right)-\frac{2}{3}\text{Subst}\left(\int\sqrt{-x^2+x^4}\,dx,x,\sqrt{x}\right)-\frac{8}{9}\text{Subst}\left(\int\frac{x^2}{\sqrt{-x^2+x^4}}\,dx,x,\sqrt{x}\right)$$

$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{8\sqrt{-x+x^2}}{9\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{\left(2\sqrt{-x+x^2}\right)\text{Subst}\left(\int\frac{x\sqrt{-1+x^2}}{-1-8x^2}\,dx,x,\sqrt{x}\right)}{9\sqrt{-1+x}\sqrt{x}}$$

$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{8\sqrt{-x+x^2}}{9\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{\sqrt{-x+x^2}\text{Subst}\left(\int\frac{\sqrt{-1+x}}{-1-8x}\,dx,x,x\right)}{9\sqrt{-1+x}\sqrt{x}}$$

$$= \frac{\sqrt{x}}{12} - \frac{2x^{3/2}}{9} - \frac{11\sqrt{-x+x^2}}{12\sqrt{x}} - \frac{2(-x+x^2)^{3/2}}{9x^{3/2}} - \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}} + \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right) - \frac{\sqrt{-x+x^2}\,\text{Subst}\left(\int \frac{1}{(-1-8x)\sqrt{-1+x}}\,dx, x, x\right)}{8\sqrt{-1+x}\sqrt{x}}$$

$$= \frac{\sqrt{x}}{12} - \frac{2x^{3/2}}{9} - \frac{11\sqrt{-x+x^2}}{12\sqrt{x}} - \frac{2(-x+x^2)^{3/2}}{9x^{3/2}} - \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}} + \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right) - \frac{\sqrt{-x+x^2}\,\text{Subst}\left(\int \frac{1}{-9-8x^2}\,dx, x, \sqrt{-1+x}\right)}{4\sqrt{-1+x}\sqrt{x}}$$

$$= \frac{\sqrt{x}}{12} - \frac{2x^{3/2}}{9} - \frac{11\sqrt{-x+x^2}}{12\sqrt{x}} - \frac{2(-x+x^2)^{3/2}}{9x^{3/2}} + \frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{24\sqrt{2}\sqrt{-1+x}\sqrt{x}} - \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}} + \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.25

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \frac{-3\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}-i\sqrt{x}}{3\sqrt{-1+x}}\right) - 3\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}+i\sqrt{x}}{3\sqrt{-1+x}}\right) + 2\sqrt{-1+x}\left(-3\sqrt{2}\sqrt{x}\,\text{arcta}\right.}{288\sqrt{-}}$$

```
[In] Integrate[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-3*Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x]
)] - 3*Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3*Sqrt[-1 +
 x])] + 2*Sqrt[-1 + x]*(-3*Sqrt[2]*Sqrt[x]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 4*(-
3*x + 8*x^2 + 25*Sqrt[(-1 + x)*x] + 8*x*Sqrt[(-1 + x)*x] - 24*x^2*Log[-1 +
4*x + 4*Sqrt[(-1 + x)*x]])))/(288*Sqrt[-1 + x]*Sqrt[x])
```

Maple [F]

$$\int \sqrt{x}\,\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)dx$$

```
[In] int(x^(1/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^(1/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$$
$$=\frac{96\,x^{\frac{5}{2}}\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-3\,\sqrt{2}x\arctan\left(2\,\sqrt{2}\sqrt{x}\right)-3\,\sqrt{2}x\arctan\left(\frac{3\,\sqrt{2}\sqrt{x}}{4\,\sqrt{x^2-x}}\right)-4\,\sqrt{x^2-x}(8\,x+25}{144\,x}$$

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/144*(96*x^(5/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 3*sqrt(2)*x*arctan(2*sqr
t(2)*sqrt(x)) - 3*sqrt(2)*x*arctan(3/4*sqrt(2)*sqrt(x)/sqrt(x^2 - x)) - 4*s
qrt(x^2 - x)*(8*x + 25)*sqrt(x) - 4*(8*x^2 - 3*x)*sqrt(x))/x
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**(1/2)*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx=\int \sqrt{x}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4/9*x^(3/2) - 2/3*sqrt(x
) + integrate(1/3*(2*x^2 + x)/(4*x^(5/2) + 4*(x^2 - x)*sqrt(x - 1) - 5*x^(3
/2) + sqrt(x)), x) + 1/3*log(sqrt(x) + 1) - 1/3*log(sqrt(x) - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$$
$$= \frac{2}{3}x^{\frac{3}{2}}\log\left(4x+4\sqrt{(x-1)x}-1\right) - \frac{1}{96}\sqrt{2}\left(\pi - 2\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2-1\right)}{3\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right) + \frac{1}{288}\sqrt{2}\left(-3i\pi + 100i\sqrt{2} - 6\arctan\left(\frac{2}{3}i\sqrt{2}\right)\right) - \frac{1}{36}(8x+25)\sqrt{x-1} - \frac{2}{9}x^{\frac{3}{2}} - \frac{1}{48}\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right) + \frac{1}{12}\sqrt{x}$$

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/96*sqrt(2)*(pi - 2*arctan(
1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^2 - 1)/(sqrt(x - 1) - sqrt(x)))) + 1/2
88*sqrt(2)*(-3*I*pi + 100*I*sqrt(2) - 6*arctan(2/3*I*sqrt(2))) - 1/36*(8*x
+ 25)*sqrt(x - 1) - 2/9*x^(3/2) - 1/48*sqrt(2)*arctan(2*sqrt(2)*sqrt(x)) +
1/12*sqrt(x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int \sqrt{x}\,\ln\left(4x+4\sqrt{x\,(x-1)}-1\right)\,dx$$

```
[In] int(x^(1/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x^(1/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.110 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx$

Optimal result 700
Rubi [A] (verified) 700
Mathematica [C] (verified) 703
Maple [F] 704
Fricas [A] (verification not implemented) 704
Sympy [F(-1)] 704
Maxima [F] 705
Giac [F] 705
Mupad [F(-1)] 705

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x}-\frac{2\sqrt{-x+x^2}}{\sqrt{x}} - \frac{\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{2}\sqrt{-1+x}\sqrt{x}} + \frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

```
[Out] 1/2*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-2*x^(1/2)+2*ln(-1+4*x+4*(x^2-x)^(1/2)
)*x^(1/2)-2/x^(1/2)*(x^2-x)^(1/2)-1/2*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2
-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2617, 2615, 6865, 6874, 209, 1602, 1160, 455, 52, 65}

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = -\frac{\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{2}\sqrt{x-1}\sqrt{x}} + \frac{\arctan\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{2\sqrt{x^2-x}}{\sqrt{x}} + 2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right) - 2\sqrt{x}$$

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] - (2*Sqrt[-x + x^2])/Sqrt[x] - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]
*Sqrt[-1 + x])/3])/(Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x
]]/Sqrt[2] + 2*Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[-x + x^2]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
```

```
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}\,dx \\
&= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+16\int \frac{\sqrt{x}}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,dx \\
&= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) \\
&\quad +32\text{Subst}\left(\int \frac{x^2}{-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\left(-x^2+x^4\right)}\,dx,x,\sqrt{x}\right) \\
&= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) \\
&\quad +32\text{Subst}\left(\int \left(-\frac{1}{16}+\frac{1}{16\left(1+8x^2\right)}-\frac{x^2}{12\sqrt{-x^2+x^4}}+\frac{\sqrt{-x^2+x^4}}{6\left(1+8x^2\right)}\right)\,dx,x,\sqrt{x}\right)
\end{aligned}$$

$$= -2\sqrt{x}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+2\text{Subst}\left(\int\frac{1}{1+8x^2}\,dx,x,\sqrt{x}\right) -\frac{8}{3}\text{Subst}\left(\int\frac{x^2}{\sqrt{-x^2+x^4}}\,dx,x,\sqrt{x}\right)+\frac{16}{3}\text{Subst}\left(\int\frac{\sqrt{-x^2+x^4}}{1+8x^2}\,dx,x,\sqrt{x}\right)$$

$$= -2\sqrt{x}-\frac{8\sqrt{-x+x^2}}{3\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) +\frac{\left(16\sqrt{-x+x^2}\right)\text{Subst}\left(\int\frac{x\sqrt{-1+x^2}}{1+8x^2}\,dx,x,\sqrt{x}\right)}{3\sqrt{-1+x}\sqrt{x}}$$

$$= -2\sqrt{x}-\frac{8\sqrt{-x+x^2}}{3\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) +\frac{\left(8\sqrt{-x+x^2}\right)\text{Subst}\left(\int\frac{\sqrt{-1+x}}{1+8x}\,dx,x,x\right)}{3\sqrt{-1+x}\sqrt{x}}$$

$$= -2\sqrt{x}-\frac{2\sqrt{-x+x^2}}{\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) -\frac{\left(3\sqrt{-x+x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+x}(1+8x)}\,dx,x,x\right)}{\sqrt{-1+x}\sqrt{x}}$$

$$= -2\sqrt{x}-\frac{2\sqrt{-x+x^2}}{\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) -\frac{\left(6\sqrt{-x+x^2}\right)\text{Subst}\left(\int\frac{1}{9+8x^2}\,dx,x,\sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}}$$

$$= -2\sqrt{x}-\frac{2\sqrt{-x+x^2}}{\sqrt{x}}-\frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{2}\sqrt{-1+x}\sqrt{x}} +\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.46

$$\int\frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx$$

$$= \frac{\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}-i\sqrt{x}}{3\sqrt{-1+x}}\right)+\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}+i\sqrt{x}}{3\sqrt{-1+x}}\right)+2\sqrt{-1+x}\left(\sqrt{2}\sqrt{x}\arctan\left(2\sqrt{2}\right.\right.}{4\sqrt{-1+x}\sqrt{x}}$$

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

```
[Out] (Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x])]
+ Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3*Sqrt[-1 + x])]
 + 2*Sqrt[-1 + x]*(Sqrt[2]*Sqrt[x]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 4*(x + Sqrt[
(-1 + x)*x] - x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])))/(4*Sqrt[-1 + x]*Sqrt[
x])
```

Maple [F]

$$\int \frac{\ln \left(-1+4x+4\sqrt{(-1+x)\,x}\right)}{\sqrt{x}}dx$$

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = \frac{\sqrt{2}x\arctan\left(2\sqrt{2}\sqrt{x}\right)+\sqrt{2}x\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)+4\,x^{\frac{3}{2}}\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-4\,x^{\frac{3}{2}}-4\,\sqrt{x^2-x}\sqrt{x}}{2\,x}$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) + sqrt(2)*x*arctan(3/4*sqrt(2)*sqr
t(x)/sqrt(x^2 - x)) + 4*x^(3/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 4*x^(3/2)
- 4*sqrt(x^2 - x)*sqrt(x))/x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = \text{Timed out}$$

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx=\int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{\sqrt{x}}\,dx$$

[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(x)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4*sqrt(x) + integrate((2*x^2 + x)/(4*x^(7/2) - 5*x^(5/2) + 4*(x^3 - x^2)*sqrt(x - 1) + x^(3/2)), x) + log(sqrt(x) + 1) - log(sqrt(x) - 1)

Giac [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx=\int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{\sqrt{x}}\,dx$$

[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="giac")

[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/sqrt(x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx=\int \frac{\ln\left(4\,x+4\,\sqrt{x\,(x-1)}-1\right)}{\sqrt{x}}\,dx$$

[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(1/2),x)

[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(1/2), x)

3.111 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx$

Optimal result 706
Rubi [A] (verified) 706
Mathematica [C] (verified) 710
Maple [F] 710
Fricas [A] (verification not implemented) 711
Sympy [F(-1)] 711
Maxima [F] 711
Giac [F(-2)] 712
Mupad [F(-1)] 712

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = -\frac{4\sqrt{2}\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}} + 4\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right) - 8\arctan\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}$$

```
[Out] -8*arctan(x^(1/2)/(x^2-x)^(1/2))+4*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-2*ln(-
1+4*x+4*(x^2-x)^(1/2))/x^(1/2)-4*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2-x)^(
1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {2617, 2615, 6865, 6874, 209, 1602, 2046, 2033, 1160, 455, 52, 65, 210}

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = -\frac{4\sqrt{2}\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{x-1}\sqrt{x}} - 8\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right) + 4\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}$$

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(Sqrt[-1 + x
]*Sqrt[x]) + 4*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 8*ArcTan[Sqrt[x]/Sqrt[-x
 + x^2]] - (2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/Sqrt[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
```

```
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^{3/2}}\,dx\\
&= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 16\int \frac{1}{\sqrt{x}\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)\right)}\,dx\\
&= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}\\
&\quad - 32\text{Subst}\left(\int \frac{1}{-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\left(-x^2+x^4\right)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 32\text{Subst}\left(\int \left(-\frac{1}{2\left(1+8x^2\right)} - \frac{x^2}{12\sqrt{-x^2+x^4}}\right.\right.\\
&\qquad \left.\left. + \frac{\sqrt{-x^2+x^4}}{4x^2} + \frac{4\sqrt{-x^2+x^4}}{3\left(-1-8x^2\right)}\right)\,dx, x, \sqrt{x}\right)\\
&= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} + \frac{8}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{-x^2+x^4}}\,dx, x, \sqrt{x}\right)\\
&\quad - 8\text{Subst}\left(\int \frac{\sqrt{-x^2+x^4}}{x^2}\,dx, x, \sqrt{x}\right) + 16\text{Subst}\left(\int \frac{1}{1+8x^2}\,dx, x, \sqrt{x}\right)\\
&\quad - \frac{128}{3}\text{Subst}\left(\int \frac{\sqrt{-x^2+x^4}}{-1-8x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{16\sqrt{-x+x^2}}{3\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}\\
&\quad + 8\text{Subst}\left(\int \frac{1}{\sqrt{-x^2+x^4}}\,dx, x, \sqrt{x}\right) - \frac{\left(128\sqrt{-x+x^2}\right)\text{Subst}\left(\int \frac{x\sqrt{-1+x^2}}{-1-8x^2}\,dx, x, \sqrt{x}\right)}{3\sqrt{-1+x}\sqrt{x}}\\
&= -\frac{16\sqrt{-x+x^2}}{3\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}\\
&\quad - 8\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{\left(64\sqrt{-x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x}}{-1-8x}\,dx, x, x\right)}{3\sqrt{-1+x}\sqrt{x}}\\
&= 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}\\
&\quad + \frac{\left(24\sqrt{-x+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1-8x)\sqrt{-1+x}}\,dx, x, x\right)}{\sqrt{-1+x}\sqrt{x}}
\end{aligned}$$

$$= 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} + \frac{\left(48\sqrt{-x+x^2}\right)\text{Subst}\left(\int\frac{1}{-9-8x^2}\,dx, x, \sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}}$$

$$= -\frac{4\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.58

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \frac{2\left(\sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}-i\sqrt{x}}{3\sqrt{-1+x}}\right) + \sqrt{2}\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}}{3\sqrt{-}}\right.\right.}{}$$

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

```
[Out] (2*(Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x]
)] + Sqrt[2]*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3*Sqrt[-1 + x
])] + 4*Sqrt[(-1 + x)*x]*ArcTan[Sqrt[-1 + x]] + 2*Sqrt[2]*Sqrt[-1 + x]*Sqrt
[x]*ArcTan[2*Sqrt[2]*Sqrt[x]] - Sqrt[-1 + x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)
*x]]))/(Sqrt[-1 + x]*Sqrt[x])
```

Maple [F]

$$\int \frac{\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)}{x^{\frac{3}{2}}}dx$$

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \frac{2\left(2\sqrt{2}x\arctan\left(2\sqrt{2}\sqrt{x}\right)+2\sqrt{2}x\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)-4\,x\arctan\right.}{x}$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(2*sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) + 2*sqrt(2)*x*arctan(3/4*sqrt(2)*s
qrt(x)/sqrt(x^2 - x)) - 4*x*arctan(sqrt(x)/sqrt(x^2 - x)) - sqrt(x)*log(4*x
 + 4*sqrt(x^2 - x) - 1))/x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log \left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \int \frac{\log \left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/sqrt(x) - 2/sqrt(x) - integrate((2*
x^2 + x)/(4*x^(9/2) - 5*x^(7/2) + x^(5/2) + 4*(x^4 - x^3)*sqrt(x - 1)), x)
- log(sqrt(x) + 1) + log(sqrt(x) - 1)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: 2*(2*
sqrt(2)*atan(4*sqrt(sageVARx)/sqrt(2))-2*(-2*(1/2*pi*sign(-sqrt(sageVARx)+s
qrt(sageVARx-1))+atan(1/2*((-sqrt(sageVARx)+sqrt(sageVARx-1))^2-1)/(-sqrt(s
ageVARx)+sqrt(sa
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^{3/2}}\,dx$$

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(3/2),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(3/2), x)
```

3.112 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx$

Optimal result 713
Rubi [A] (verified) 713
Mathematica [C] (verified) 717
Maple [F] 718
Fricas [A] (verification not implemented) 718
Sympy [F(-1)] 718
Maxima [F] 719
Giac [A] (verification not implemented) 719
Mupad [F(-1)] 720

Optimal result

Integrand size = 23, antiderivative size = 151

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{32\sqrt{2}\sqrt{-x+x^2}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{3\sqrt{-1+x}\sqrt{x}} - \frac{32}{3}\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\arctan\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}}$$

```
[Out] 44/3*arctan(x^(1/2)/(x^2-x)^(1/2))-2/3*ln(-1+4*x+4*(x^2-x)^(1/2))/x^(3/2)-3
2/3*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-16/3/x^(1/2)+4/3*(x^2-x)^(1/2)/x^(3/2
)+32/3*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/
x^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2617, 2615, 6865, 6874, 209, 1602, 2045, 2033, 2046, 1160, 455, 52, 65}

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \frac{32\sqrt{2}\sqrt{x^2-x}\arctan\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{3\sqrt{x-1}\sqrt{x}} + \frac{44}{3}\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right) - \frac{32}{3}\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right) + \frac{4\sqrt{x^2-x}}{3x^{3/2}} - \frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}} - \frac{16}{3\sqrt{x}}$$

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```

```
[Out] -16/(3*Sqrt[x]) + (4*Sqrt[-x + x^2])/(3*x^(3/2)) + (32*Sqrt[2]*Sqrt[-x + x^
2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(3*Sqrt[-1 + x]*Sqrt[x]) - (32*Sqrt[
2]*ArcTan[2*Sqrt[2]*Sqrt[x]])/3 + (44*ArcTan[Sqrt[x]/Sqrt[-x + x^2]])/3 - (
2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/(3*x^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
```

```
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^{5/2}}\,dx$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{16}{3}\int \frac{1}{x^{3/2}\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)\right)}\,dx$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{32}{3}\text{Subst}\left(\int \frac{1}{x^2\left(-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\left(-x^2+x^4\right)\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{32}{3}\text{Subst}\left(\int \left(-\frac{1}{2x^2}+\frac{4}{1+8x^2}-\frac{x^2}{12\sqrt{-x^2+x^4}}+\frac{\sqrt{-x^2+x^4}}{4x^4}-\frac{5\sqrt{-x^2+x^4}}{4x^2}+\frac{32\sqrt{-x^2+x^4}}{3\left(1+8x^2\right)}\right) d$$

$$= -\frac{16}{3\sqrt{x}} - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} + \frac{8}{9}\text{Subst}\left(\int \frac{x^2}{\sqrt{-x^2+x^4}}\,dx, x, \sqrt{x}\right) - \frac{8}{3}\text{Subst}\left(\int \frac{\sqrt{-x^2+x^4}}{x^4}\,dx, x, \sqrt{x}\right) + \frac{40}{3}\text{Subst}\left(\int \frac{\sqrt{-x^2+x^4}}{x^2}\,dx, x, \sqrt{x}\right) - \frac{128}{3}\text{Subst}\left(\int \frac{1}{1+8x^2}\,dx, x, \sqrt{x}\right) - \frac{1024}{9}\text{Subst}\left(\int \frac{\sqrt{-x^2+x^4}}{1+8x^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{128\sqrt{-x+x^2}}{9\sqrt{x}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{4}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-x^2+x^4}}\,dx, x, \sqrt{x}\right) - \frac{40}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-x^2+x^4}}\,dx, x, \sqrt{x}\right) - \frac{\left(1024\sqrt{-x+x^2}\right)\text{Subst}\left(\int \frac{x\sqrt{-1+x^2}}{1+8x^2}\,dx, x, \sqrt{x}\right)}{9\sqrt{-1+x}\sqrt{x}}$$

$$
\begin{aligned}
&= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{128\sqrt{-x+x^2}}{9\sqrt{x}} \\
&\quad - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} \\
&\quad + \frac{4}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) + \frac{40}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) \\
&\quad - \frac{\left(512\sqrt{-x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x}}{1+8x}\,dx, x, x\right)}{9\sqrt{-1+x}\sqrt{x}} \\
&= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) \\
&\quad - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} + \frac{\left(64\sqrt{-x+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}(1+8x)}\,dx, x, x\right)}{\sqrt{-1+x}\sqrt{x}} \\
&= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) \\
&\quad - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} + \frac{\left(128\sqrt{-x+x^2}\right)\text{Subst}\left(\int \frac{1}{9+8x^2}\,dx, x, \sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}} \\
&= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{32\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{3\sqrt{-1+x}\sqrt{x}} \\
&\quad - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) \\
&\quad - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.85

$$
\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx =
$$

$$
-\frac{2\left(8\sqrt{-(-1+x)^2}x - 2\sqrt{-(-1+x)^2}\sqrt{(-1+x)x} + 8\sqrt{2-2x}x\sqrt{(-1+x)x}\arctan\left(\frac{2\sqrt{2}-i\sqrt{x}}{3\sqrt{-1+x}}\right) + 8\sqrt{2-\right.}{}
$$

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```

```
[Out] (-2*(8*Sqrt[-(-1 + x)^2]*x - 2*Sqrt[-(-1 + x)^2]*Sqrt[(-1 + x)*x] + 8*Sqrt[
2 - 2*x]*x*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] - I*Sqrt[x])/(3*Sqrt[-1 + x])
] + 8*Sqrt[2 - 2*x]*x*Sqrt[(-1 + x)*x]*ArcTan[(2*Sqrt[2] + I*Sqrt[x])/(3*Sq
```

```
rt[-1 + x])] + 24*Sqrt[1 - x]*x*Sqrt[(-1 + x)*x]*ArcTan[Sqrt[-1 + x]] + 16*
Sqrt[2]*Sqrt[-(-1 + x)^2]*x^(3/2)*ArcTan[2*Sqrt[2]*Sqrt[x]] - 2*Sqrt[-1 + x
]*x*Sqrt[(-1 + x)*x]*ArcTanh[Sqrt[1 - x]] + Sqrt[-(-1 + x)^2]*Log[-1 + 4*x
+ 4*Sqrt[(-1 + x)*x]]))/(3*Sqrt[-(-1 + x)^2]*x^(3/2))
```

Maple [F]

$$\int \frac{\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)}{x^{\frac{5}{2}}}dx$$

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x)
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = -\frac{2\left(16\sqrt{2}x^2\arctan\left(2\sqrt{2}\sqrt{x}\right)+16\sqrt{2}x^2\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)-22\,x^2\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+8\,x^{\frac{3}{2}}+\sqrt{x}\log\left(4\,x+4\right.\right.}{3\,x^2}$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="fricas")
[Out] -2/3*(16*sqrt(2)*x^2*arctan(2*sqrt(2)*sqrt(x)) + 16*sqrt(2)*x^2*arctan(3/4*
sqrt(2)*sqrt(x)/sqrt(x^2 - x)) - 22*x^2*arctan(sqrt(x)/sqrt(x^2 - x)) + 8*x
^(3/2) + sqrt(x)*log(4*x + 4*sqrt(x^2 - x) - 1) - 2*sqrt(x^2 - x)*sqrt(x))/
x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3/sqrt(x) - 2/3*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/x^(3/2) - 2/9/x^(3/2
) - integrate(1/3*(2*x^2 + x)/(4*x^(11/2) - 5*x^(9/2) + x^(7/2) + 4*(x^5 -
x^4)*sqrt(x - 1)), x) - 1/3*log(sqrt(x) + 1) + 1/3*log(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.20

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \frac{22}{3}\,\pi - \frac{16}{3}\,\sqrt{2}\left(\pi - 2\,\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1\right)}{3\,\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right) - \frac{32}{3}\,\sqrt{2}\arctan\left(2\,\sqrt{2}\sqrt{x}\right) + \frac{8\left(\sqrt{x-1}-\sqrt{x}-\frac{1}{\sqrt{x-1}-\sqrt{x}}\right)}{3\left(\left(\sqrt{x-1}-\sqrt{x}-\frac{1}{\sqrt{x-1}-\sqrt{x}}\right)^{2}+4\right)} - \frac{16}{3\,\sqrt{x}} - \frac{2\,\log\left(4\,x+4\,\sqrt{x^{2}-x}-1\right)}{3\,x^{\frac{3}{2}}} - \frac{44}{3}\,\arctan\left(\frac{\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1}{2\,\left(\sqrt{x-1}-\sqrt{x}\right)}\right)$$

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] 22/3*pi - 16/3*sqrt(2)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^
2 - 1)/(sqrt(x - 1) - sqrt(x)))) - 32/3*sqrt(2)*arctan(2*sqrt(2)*sqrt(x)) +
 8/3*(sqrt(x - 1) - sqrt(x) - 1/(sqrt(x - 1) - sqrt(x)))/((sqrt(x - 1) - sq
rt(x) - 1/(sqrt(x - 1) - sqrt(x)))^2 + 4) - 16/3/sqrt(x) - 2/3*log(4*x + 4*
sqrt(x^2 - x) - 1)/x^(3/2) - 44/3*arctan(1/2*((sqrt(x - 1) - sqrt(x))^2 - 1
)/(sqrt(x - 1) - sqrt(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^{5/2}}\,dx$$

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(5/2),x)
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(5/2), x)
```

3.113 $\int x^3 \log(a + be^x) \, dx$

Optimal result . . . 721
Rubi [A] (verified) . . . 721
Mathematica [A] (verified) . . . 723
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Fricas [A] (verification not implemented) . . . 724
Sympy [F] . . . 725
Maxima [A] (verification not implemented) . . . 725
Giac [F] . . . 725
Mupad [F(-1)] . . . 725

Optimal result

Integrand size = 12, antiderivative size = 93

$$\int x^3 \log(a + be^x) \, dx = \frac{1}{4}x^4 \log(a + be^x) - \frac{1}{4}x^4 \log\left(1 + \frac{be^x}{a}\right) - x^3 \operatorname{PolyLog}\left(2, -\frac{be^x}{a}\right) + 3x^2 \operatorname{PolyLog}\left(3, -\frac{be^x}{a}\right) - 6x \operatorname{PolyLog}\left(4, -\frac{be^x}{a}\right) + 6 \operatorname{PolyLog}\left(5, -\frac{be^x}{a}\right)$$

```
[Out] 1/4*x^4*ln(a+b*exp(x))-1/4*x^4*ln(1+b*exp(x)/a)-x^3*polylog(2,-b*exp(x)/a)+
3*x^2*polylog(3,-b*exp(x)/a)-6*x*polylog(4,-b*exp(x)/a)+6*polylog(5,-b*exp(
x)/a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2612, 2611, 6744, 2320, 6724}

$$\int x^3 \log(a + be^x) \, dx = -x^3 \operatorname{PolyLog}\left(2, -\frac{be^x}{a}\right) + 3x^2 \operatorname{PolyLog}\left(3, -\frac{be^x}{a}\right) - 6x \operatorname{PolyLog}\left(4, -\frac{be^x}{a}\right) + 6 \operatorname{PolyLog}\left(5, -\frac{be^x}{a}\right) + \frac{1}{4}x^4 \log(a + be^x) - \frac{1}{4}x^4 \log\left(\frac{be^x}{a} + 1\right)$$

```
[In] Int[x^3*Log[a + b*E^x],x]
```

```
[Out] (x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*E
^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] + 6
*PolyLog[5, -((b*E^x)/a)]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x^4\log\left(a+be^x\right)-\frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right)+\int x^3\log\left(1+\frac{be^x}{a}\right)\,dx$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3\int x^2\text{Li}_2\left(-\frac{be^x}{a}\right)dx$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6\int x\text{Li}_3\left(-\frac{be^x}{a}\right)dx$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6x\text{Li}_4\left(-\frac{be^x}{a}\right) + 6\int \text{Li}_4\left(-\frac{be^x}{a}\right)dx$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6x\text{Li}_4\left(-\frac{be^x}{a}\right) + 6\text{Subst}\left(\int \frac{\text{Li}_4\left(-\frac{bx}{a}\right)}{x}dx, x, e^x\right)$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6x\text{Li}_4\left(-\frac{be^x}{a}\right) + 6\text{Li}_5\left(-\frac{be^x}{a}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int x^3\log(a+be^x)\,dx = \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\,\text{PolyLog}\left(2,-\frac{be^x}{a}\right) + 3x^2\,\text{PolyLog}\left(3,-\frac{be^x}{a}\right) - 6x\,\text{PolyLog}\left(4,-\frac{be^x}{a}\right) + 6\,\text{PolyLog}\left(5,-\frac{be^x}{a}\right)$$

```
[In] Integrate[x^3*Log[a + b*E^x],x]
```

```
[Out] (x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*E
^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] + 6
*PolyLog[5, -((b*E^x)/a)]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{x^4 \ln(a+b\,\mathrm{e}^x)}{4}-\frac{x^4 \ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{4}-x^3 \operatorname{Li}_2\left(-\frac{b\,\mathrm{e}^x}{a}\right)+3x^2 \operatorname{Li}_3\left(-\frac{b\,\mathrm{e}^x}{a}\right)-6x \operatorname{Li}_4\left(-\frac{b\,\mathrm{e}^x}{a}\right)+6 \operatorname{Li}_5\left(-\frac{b\,\mathrm{e}^x}{a}\right)$	84
risch	$\frac{x^4 \ln(a+b\,\mathrm{e}^x)}{4}-\frac{x^4 \ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{4}-x^3 \operatorname{Li}_2\left(-\frac{b\,\mathrm{e}^x}{a}\right)+3x^2 \operatorname{Li}_3\left(-\frac{b\,\mathrm{e}^x}{a}\right)-6x \operatorname{Li}_4\left(-\frac{b\,\mathrm{e}^x}{a}\right)+6 \operatorname{Li}_5\left(-\frac{b\,\mathrm{e}^x}{a}\right)$	84
parts	$\frac{x^4 \ln(a+b\,\mathrm{e}^x)}{4}-\frac{x^4 \ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{4}-x^3 \operatorname{Li}_2\left(-\frac{b\,\mathrm{e}^x}{a}\right)+3x^2 \operatorname{Li}_3\left(-\frac{b\,\mathrm{e}^x}{a}\right)-6x \operatorname{Li}_4\left(-\frac{b\,\mathrm{e}^x}{a}\right)+6 \operatorname{Li}_5\left(-\frac{b\,\mathrm{e}^x}{a}\right)$	84

```
[In] int(x^3*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(a+b*exp(x))-1/4*x^4*ln(1+b*exp(x)/a)-x^3*polylog(2,-b*exp(x)/a)+
3*x^2*polylog(3,-b*exp(x)/a)-6*x*polylog(4,-b*exp(x)/a)+6*polylog(5,-b*exp(
x)/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int x^3 \log(a+be^x)\,dx = \frac{1}{4}\,x^4 \log(be^x+a) - \frac{1}{4}\,x^4 \log\left(\frac{be^x+a}{a}\right) - x^3 \mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right) + 3\,x^2 \mathrm{polylog}\left(3,-\frac{be^x}{a}\right) - 6\,x \mathrm{polylog}\left(4,-\frac{be^x}{a}\right) + 6\,\mathrm{polylog}\left(5,-\frac{be^x}{a}\right)$$

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log(b*e^x + a) - 1/4*x^4*log((b*e^x + a)/a) - x^3*dilog(-(b*e^x + a
)/a + 1) + 3*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*polylo
g(5, -b*e^x/a)
```

Sympy [F]

$$\int x^3 \log(a + be^x)\, dx = -\frac{b \int \frac{x^4 e^x}{a+be^x}\, dx}{4} + \frac{x^4 \log(a + be^x)}{4}$$

```
[In] integrate(x**3*ln(a+b*exp(x)),x)
[Out] -b*Integral(x**4*exp(x)/(a + b*exp(x)), x)/4 + x**4*log(a + b*exp(x))/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^3 \log(a + be^x)\, dx = \frac{1}{4}x^4 \log(be^x + a) - \frac{1}{4}x^4 \log\left(\frac{be^x}{a} + 1\right) - x^3 \mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2 \mathrm{Li}_3\left(-\frac{be^x}{a}\right) - 6x \mathrm{Li}_4\left(-\frac{be^x}{a}\right) + 6\,\mathrm{Li}_5\left(-\frac{be^x}{a}\right)$$

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="maxima")
[Out] 1/4*x^4*log(b*e^x + a) - 1/4*x^4*log(b*e^x/a + 1) - x^3*dilog(-b*e^x/a) + 3
*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*polylog(5, -b*e^x/
a)
```

Giac [F]

$$\int x^3 \log(a + be^x)\, dx = \int x^3 \log(be^x + a)\, dx$$

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(x^3*log(b*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \log(a + be^x)\, dx = \int x^3 \ln\left(a + b\,\mathrm{e}^x\right) dx$$

```
[In] int(x^3*log(a + b*exp(x)),x)
[Out] int(x^3*log(a + b*exp(x)), x)
```

3.114 $\int x^2 \log(a+be^x)\,dx$

Optimal result . . . 726
Rubi [A] (verified) . . . 726
Mathematica [A] (verified) . . . 728
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Fricas [A] (verification not implemented) . . . 729
Sympy [F] . . . 729
Maxima [A] (verification not implemented) . . . 729
Giac [F] . . . 730
Mupad [F(-1)] . . . 730

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int x^2 \log(a+be^x)\,dx = \frac{1}{3}x^3\log(a+be^x) - \frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right) - x^2\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right) + 2x\operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right) - 2\operatorname{PolyLog}\left(4,-\frac{be^x}{a}\right)$$

```
[Out] 1/3*x^3*ln(a+b*exp(x))-1/3*x^3*ln(1+b*exp(x)/a)-x^2*polylog(2,-b*exp(x)/a)+
2*x*polylog(3,-b*exp(x)/a)-2*polylog(4,-b*exp(x)/a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2612, 2611, 6744, 2320, 6724}

$$\int x^2 \log(a+be^x)\,dx = -x^2\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right) + 2x\operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right) - 2\operatorname{PolyLog}\left(4,-\frac{be^x}{a}\right) + \frac{1}{3}x^3\log(a+be^x) - \frac{1}{3}x^3\log\left(\frac{be^x}{a}+1\right)$$

```
[In] Int[x^2*Log[a + b*E^x],x]
```

```
[Out] (x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*E
^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log\left(a+be^x\right)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)+\int x^2\log\left(1+\frac{be^x}{a}\right)\,dx\\
&= \frac{1}{3}x^3\log\left(a+be^x\right)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)-x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+2\int x\mathrm{Li}_2\left(-\frac{be^x}{a}\right)\,dx\\
&= \frac{1}{3}x^3\log\left(a+be^x\right)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)-x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)\\
&\quad +2x\mathrm{Li}_3\left(-\frac{be^x}{a}\right)-2\int \mathrm{Li}_3\left(-\frac{be^x}{a}\right)\,dx
\end{aligned}
$$

$$= \frac{1}{3}x^3 \log(a + be^x) - \frac{1}{3}x^3 \log\left(1 + \frac{be^x}{a}\right) - x^2 \text{Li}_2\left(-\frac{be^x}{a}\right) + 2x\text{Li}_3\left(-\frac{be^x}{a}\right) - 2\text{Subst}\left(\int \frac{\text{Li}_3\left(-\frac{bx}{a}\right)}{x} dx, x, e^x\right)$$

$$= \frac{1}{3}x^3 \log(a+be^x) - \frac{1}{3}x^3 \log\left(1+\frac{be^x}{a}\right) - x^2\text{Li}_2\left(-\frac{be^x}{a}\right) + 2x\text{Li}_3\left(-\frac{be^x}{a}\right) - 2\text{Li}_4\left(-\frac{be^x}{a}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int x^2 \log(a + be^x)\, dx = \frac{1}{3}x^3 \log(a + be^x) - \frac{1}{3}x^3 \log\left(1 + \frac{be^x}{a}\right) - x^2 \,\text{PolyLog}\left(2, -\frac{be^x}{a}\right) + 2x \,\text{PolyLog}\left(3, -\frac{be^x}{a}\right) - 2 \,\text{PolyLog}\left(4, -\frac{be^x}{a}\right)$$

```
[In] Integrate[x^2*Log[a + b*E^x],x]
[Out] (x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*E
^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{x^3 \ln(a+b\,e^x)}{3} - \frac{x^3 \ln\left(1+\frac{b\,e^x}{a}\right)}{3} - x^2 \,\text{Li}_2\left(-\frac{b\,e^x}{a}\right) + 2x \,\text{Li}_3\left(-\frac{b\,e^x}{a}\right) - 2 \,\text{Li}_4\left(-\frac{b\,e^x}{a}\right)$	69
risch	$\frac{x^3 \ln(a+b\,e^x)}{3} - \frac{x^3 \ln\left(1+\frac{b\,e^x}{a}\right)}{3} - x^2 \,\text{Li}_2\left(-\frac{b\,e^x}{a}\right) + 2x \,\text{Li}_3\left(-\frac{b\,e^x}{a}\right) - 2 \,\text{Li}_4\left(-\frac{b\,e^x}{a}\right)$	69
parts	$\frac{x^3 \ln(a+b\,e^x)}{3} - \frac{x^3 \ln\left(1+\frac{b\,e^x}{a}\right)}{3} - x^2 \,\text{Li}_2\left(-\frac{b\,e^x}{a}\right) + 2x \,\text{Li}_3\left(-\frac{b\,e^x}{a}\right) - 2 \,\text{Li}_4\left(-\frac{b\,e^x}{a}\right)$	69

```
[In] int(x^2*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*ln(a+b*exp(x))-1/3*x^3*ln(1+b*exp(x)/a)-x^2*polylog(2,-b*exp(x)/a)+
2*x*polylog(3,-b*exp(x)/a)-2*polylog(4,-b*exp(x)/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int x^2 \log \left(a+b e^x\right) \, dx=\frac{1}{3} \, x^{3} \log \left(b e^{x} + a\right) - \frac{1}{3} \, x^{3} \log \left(\frac{b e^{x} + a}{a}\right) - x^{2} {\rm Li}_2\left(-\frac{b e^{x} + a}{a} + 1\right) + 2 \, x {\rm polylog}\left(3, -\frac{b e^{x}}{a}\right) - 2 \, {\rm polylog}\left(4, -\frac{b e^{x}}{a}\right)$$

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(b*e^x + a) - 1/3*x^3*log((b*e^x + a)/a) - x^2*dilog(-(b*e^x + a
)/a + 1) + 2*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

Sympy [F]

$$\int x^2 \log \left(a+b e^x\right) \, dx=- \frac{b \int \frac{x^{3} e^{x}}{a + b e^{x}}\, dx}{3} + \frac{x^{3} \log {\left(a + b e^{x} \right)}}{3}$$

```
[In] integrate(x**2*ln(a+b*exp(x)),x)
```

```
[Out] -b*Integral(x**3*exp(x)/(a + b*exp(x)), x)/3 + x**3*log(a + b*exp(x))/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int x^2 \log \left(a+b e^x\right) \, dx=\frac{1}{3} \, x^{3} \log \left(b e^{x} + a\right) - \frac{1}{3} \, x^{3} \log \left(\frac{b e^{x}}{a} + 1\right) - x^{2} {\rm Li}_2\left(-\frac{b e^{x}}{a}\right) + 2 \, x {\rm Li}_{3}(-\frac{b e^{x}}{a}) - 2 \, {\rm Li}_{4}(-\frac{b e^{x}}{a})$$

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*e^x + a) - 1/3*x^3*log(b*e^x/a + 1) - x^2*dilog(-b*e^x/a) + 2
*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

Giac [F]

$$\int x^2 \log(a + be^x) \, dx = \int x^2 \log(be^x + a) \, dx$$

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(x^2*log(b*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log(a + be^x) \, dx = \int x^2 \ln(a + b\,e^x) \, dx$$

```
[In] int(x^2*log(a + b*exp(x)),x)
[Out] int(x^2*log(a + b*exp(x)), x)
```

3.115 $\int x\log\left(a+be^x\right)\,dx$

Optimal result 731
Rubi [A] (verified) 731
Mathematica [A] (verified) 733
Maple [A] (verified) 733
Fricas [A] (verification not implemented) 733
Sympy [F] 734
Maxima [A] (verification not implemented) 734
Giac [F] 734
Mupad [F(-1)] 734

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int x\log\left(a+be^x\right)\,dx=\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)+\operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right)$$

```
[Out] 1/2*x^2*ln(a+b*exp(x))-1/2*x^2*ln(1+b*exp(x)/a)-x*polylog(2,-b*exp(x)/a)+po
lylog(3,-b*exp(x)/a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2612, 2611, 2320, 6724}

$$\int x\log\left(a+be^x\right)\,dx=-x\operatorname{PolyLog}\left(2,-\frac{be^x}{a}\right)+\operatorname{PolyLog}\left(3,-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

```
[In] Int[x*Log[a + b*E^x],x]
```

```
[Out] (x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^x
)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)+\int x\log\left(1+\frac{be^x}{a}\right)\,dx\\
&= \frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\int \text{Li}_2\left(-\frac{be^x}{a}\right)\,dx\\
&= \frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx,x,e^x\right)\\
&= \frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Li}_3\left(-\frac{be^x}{a}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x\log\left(a+be^x\right)\,dx=\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\,\mathrm{PolyLog}\left(2,-\frac{be^x}{a}\right)+\mathrm{PolyLog}\left(3,-\frac{be^x}{a}\right)$$

```
[In] Integrate[x*Log[a + b*E^x],x]
[Out] (x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^x
)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^2\ln(a+b\,\mathrm{e}^x)}{2}-\frac{x^2\ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{2}-x\,\mathrm{Li}_2\left(-\frac{b\,\mathrm{e}^x}{a}\right)+\mathrm{Li}_3\left(-\frac{b\,\mathrm{e}^x}{a}\right)$	52
risch	$\frac{x^2\ln(a+b\,\mathrm{e}^x)}{2}-\frac{x^2\ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{2}-x\,\mathrm{Li}_2\left(-\frac{b\,\mathrm{e}^x}{a}\right)+\mathrm{Li}_3\left(-\frac{b\,\mathrm{e}^x}{a}\right)$	52
parts	$\frac{x^2\ln(a+b\,\mathrm{e}^x)}{2}-\frac{x^2\ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{2}-x\,\mathrm{Li}_2\left(-\frac{b\,\mathrm{e}^x}{a}\right)+\mathrm{Li}_3\left(-\frac{b\,\mathrm{e}^x}{a}\right)$	52

```
[In] int(x*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*ln(a+b*exp(x))-1/2*x^2*ln(1+b*exp(x)/a)-x*polylog(2,-b*exp(x)/a)+po
lylog(3,-b*exp(x)/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int x\log\left(a+be^x\right)\,dx=\frac{1}{2}\,x^2\log\left(be^x+a\right)-\frac{1}{2}\,x^2\log\left(\frac{be^x+a}{a}\right)-x\mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right)+\mathrm{polylog}\left(3,-\frac{be^x}{a}\right)$$

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="fricas")
[Out] 1/2*x^2*log(b*e^x + a) - 1/2*x^2*log((b*e^x + a)/a) - x*dilog(-(b*e^x + a)/
a + 1) + polylog(3, -b*e^x/a)
```

Sympy [F]

$$\int x \log(a+be^x)\,dx = -\frac{b\int\frac{x^2e^x}{a+be^x}\,dx}{2} + \frac{x^2\log(a+be^x)}{2}$$

```
[In] integrate(x*ln(a+b*exp(x)),x)
[Out] -b*Integral(x**2*exp(x)/(a + b*exp(x)), x)/2 + x**2*log(a + b*exp(x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int x \log(a+be^x)\,dx = \frac{1}{2}x^2\log(be^x+a) - \frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right) - x\mathrm{Li}_2\left(-\frac{be^x}{a}\right) + \mathrm{Li}_3(-\frac{be^x}{a})$$

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="maxima")
[Out] 1/2*x^2*log(b*e^x + a) - 1/2*x^2*log(b*e^x/a + 1) - x*dilog(-b*e^x/a) + pol
ylog(3, -b*e^x/a)
```

Giac [F]

$$\int x \log(a+be^x)\,dx = \int x\log(be^x+a)\,dx$$

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(x*log(b*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \log(a+be^x)\,dx = \int x\,\ln\left(a+b\,\mathrm{e}^x\right)\,dx$$

```
[In] int(x*log(a + b*exp(x)),x)
[Out] int(x*log(a + b*exp(x)), x)
```

3.116 $\int \log(a+be^x)\, dx$

Optimal result . . . 735
Rubi [A] (verified) . . . 735
Mathematica [A] (verified) . . . 736
Maple [A] (verified) . . . 737
Fricas [A] (verification not implemented) . . . 737
Sympy [F] . . . 737
Maxima [A] (verification not implemented) . . . 738
Giac [F] . . . 738
Mupad [B] (verification not implemented) . . . 738

Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \log(a+be^x)\, dx = x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) - \text{PolyLog}\left(2,-\frac{be^x}{a}\right)$$

```
[Out] x*ln(a+b*exp(x))-x*ln(1+b*exp(x)/a)-polylog(2,-b*exp(x)/a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2318, 2221, 2317, 2438}

$$\int \log(a+be^x)\, dx = -\text{PolyLog}\left(2,-\frac{be^x}{a}\right) + x\log(a+be^x) - x\log\left(\frac{be^x}{a}+1\right)$$

```
[In] Int[Log[a + b*E^x],x]
[Out] x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2318

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Dist[b*d*e*n*Log[F], Int[x*(
(F^(e*(c + d*x)))^n/(a + b*(F^(e*(c + d*x)))^n)), x], x] /; FreeQ[{F, a, b,
 c, d, e, n}, x] &&  !GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a+be^x\right)-b\int\frac{e^x x}{a+be^x}\,dx\\
&= x\log\left(a+be^x\right)-x\log\left(1+\frac{be^x}{a}\right)+\int\log\left(1+\frac{be^x}{a}\right)\,dx\\
&= x\log\left(a+be^x\right)-x\log\left(1+\frac{be^x}{a}\right)+\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx,x,e^x\right)\\
&= x\log\left(a+be^x\right)-x\log\left(1+\frac{be^x}{a}\right)-\text{Li}_2\left(-\frac{be^x}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int\log\left(a+be^x\right)\,dx=x\log\left(a+be^x\right)-x\log\left(1+\frac{be^x}{a}\right)-\text{PolyLog}\left(2,-\frac{be^x}{a}\right)$$

```
[In] Integrate[Log[a + b*E^x],x]
[Out] x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\operatorname{dilog}\left(-\frac{b\,\mathrm{e}^{x}}{a}\right)+\ln\left(a+b\,\mathrm{e}^{x}\right)\ln\left(-\frac{b\,\mathrm{e}^{x}}{a}\right)$	28
default	$\operatorname{dilog}\left(-\frac{b\,\mathrm{e}^{x}}{a}\right)+\ln\left(a+b\,\mathrm{e}^{x}\right)\ln\left(-\frac{b\,\mathrm{e}^{x}}{a}\right)$	28
risch	$-\ln\left(\frac{a+b\,\mathrm{e}^{x}}{a}\right)x+x\ln\left(a+b\,\mathrm{e}^{x}\right)-\operatorname{dilog}\left(\frac{a+b\,\mathrm{e}^{x}}{a}\right)$	38
parts	$x\ln\left(a+b\,\mathrm{e}^{x}\right)-b\left(\frac{\operatorname{dilog}\left(\frac{a+b\,\mathrm{e}^{x}}{a}\right)}{b}+\frac{x\ln\left(\frac{a+b\,\mathrm{e}^{x}}{a}\right)}{b}\right)$	46

```
[In] int(ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
[Out] dilog(-b*exp(x)/a)+ln(a+b*exp(x))*ln(-b*exp(x)/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \log\left(a+be^{x}\right)\,dx = x\log\left(be^{x}+a\right)-x\log\left(\frac{be^{x}+a}{a}\right)-\mathrm{Li}_2\left(-\frac{be^{x}+a}{a}+1\right)$$

```
[In] integrate(log(a+b*exp(x)),x, algorithm="fricas")
[Out] x*log(b*e^x + a) - x*log((b*e^x + a)/a) - dilog(-(b*e^x + a)/a + 1)
```

Sympy [F]

$$\int \log\left(a+be^{x}\right)\,dx = -b\int \frac{xe^{x}}{a+be^{x}}\,dx+x\log\left(a+be^{x}\right)$$

```
[In] integrate(ln(a+b*exp(x)),x)
[Out] -b*Integral(x*exp(x)/(a + b*exp(x)), x) + x*log(a + b*exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \log \left(a+be^x\right)\,dx = \log \left(be^x+a\right)\log \left(-\frac{be^x+a}{a}+1\right)+\text{Li}_2\left(\frac{be^x+a}{a}\right)$$

```
[In] integrate(log(a+b*exp(x)),x, algorithm="maxima")
[Out] log(b*e^x + a)*log(-(b*e^x + a)/a + 1) + dilog((b*e^x + a)/a)
```

Giac [F]

$$\int \log \left(a+be^x\right)\,dx = \int \log \left(be^x+a\right)\,dx$$

```
[In] integrate(log(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(log(b*e^x + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \log \left(a+be^x\right)\,dx = x\,\ln \left(a+b\,\mathrm{e}^x\right)-x\,\ln \left(\frac{b\,\mathrm{e}^x}{a}+1\right)-\mathrm{polylog}\left(2,-\frac{b\,\mathrm{e}^x}{a}\right)$$

```
[In] int(log(a + b*exp(x)),x)
[Out] x*log(a + b*exp(x)) - x*log((b*exp(x))/a + 1) - polylog(2, -(b*exp(x))/a)
```

3.117 $\int \frac{\log(a+be^x)}{x} \, dx$

Optimal result 739
Rubi [N/A] 739
Mathematica [N/A] 740
Maple [N/A] 740
Fricas [N/A] 740
Sympy [N/A] 740
Maxima [N/A] 741
Giac [N/A] 741
Mupad [N/A] 741

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\log(a+be^x)}{x} \, dx = \mathrm{Int}\left(\frac{\log(a+be^x)}{x}, x\right)$$

```
[Out] CannotIntegrate(ln(a+b*exp(x))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(a+be^x)}{x} \, dx = \int \frac{\log(a+be^x)}{x} \, dx$$

```
[In] Int[Log[a + b*E^x]/x,x]
```

```
[Out] Defer[Int][Log[a + b*E^x]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log(a+be^x)}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\log (a+be^x)}{x} \, dx = \int \frac{\log (a+be^x)}{x} \, dx$$

```
[In] Integrate[Log[a + b*E^x]/x,x]
[Out] Integrate[Log[a + b*E^x]/x, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{\ln \left(a +b \,{\mathrm e}^{x}\right)}{x}d x$$

```
[In] int(ln(a+b*exp(x))/x,x)
[Out] int(ln(a+b*exp(x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\log (a+be^x)}{x} \, dx = \int \frac{\log (be^x + a)}{x} \, dx$$

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="fricas")
[Out] integral(log(b*e^x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log (a+be^x)}{x} \, dx = \int \frac{\log (a+be^x)}{x} \, dx$$

```
[In] integrate(ln(a+b*exp(x))/x,x)
[Out] Integral(log(a + b*exp(x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\log (a+be^x)}{x} \, dx = \int \frac{\log (be^x+a)}{x} \, dx$$

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="maxima")
[Out] integrate(log(b*e^x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\log (a+be^x)}{x} \, dx = \int \frac{\log (be^x+a)}{x} \, dx$$

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="giac")
[Out] integrate(log(b*e^x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\log (a+be^x)}{x} \, dx = \int \frac{\ln (a+b\,\mathrm{e}^x)}{x} \, dx$$

```
[In] int(log(a + b*exp(x))/x,x)
[Out] int(log(a + b*exp(x))/x, x)
```

3.118 $\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal result . . . 742
Rubi [A] (verified) . . . 743
Mathematica [A] (verified) . . . 744
Maple [B] (verified) . . . 745
Fricas [A] (verification not implemented) . . . 745
Sympy [F] . . . 746
Maxima [A] (verification not implemented) . . . 746
Giac [F] . . . 746
Mupad [F(-1)] . . . 747

Optimal result

Integrand size = 20, antiderivative size = 132

$$\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = -\frac{x^3 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)} + \frac{3x^2 \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{6x \operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)} + \frac{6 \operatorname{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}$$

```
[Out] -x^3*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+3*x^2*polylog(3,-e*(f^(c*(
b*x+a)))^n)/b^2/c^2/n^2/ln(f)^2-6*x*polylog(4,-e*(f^(c*(b*x+a)))^n)/b^3/c^3
/n^3/ln(f)^3+6*polylog(5,-e*(f^(c*(b*x+a)))^n)/b^4/c^4/n^4/ln(f)^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2611, 6744, 2320, 6724}

$$\int x^3 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{6 \operatorname{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)} - \frac{6x \operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)} + \frac{3x^2 \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)} - \frac{x^3 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[In] Int[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*PolyL
og[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -
(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^(c*
(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3\int x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)\,dx}{bcn\log(f)}\\
&= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6\int x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)\,dx}{b^2c^2n^2\log^2(f)}\\
&= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\\
&\quad-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\int \text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)\,dx}{b^3c^3n^3\log^3(f)}\\
&= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\\
&\quad-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{Subst}\left(\int\frac{\text{Li}_4(-ex^n)}{x}\,dx,x,f^{c(a+bx)}\right)}{b^4c^4n^3\log^4(f)}\\
&= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\\
&\quad-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{Li}_5\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int x^3\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx &= -\frac{x^3\,\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}\\
&\quad+\frac{3x^2\,\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\\
&\quad-\frac{6x\,\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}\\
&\quad+\frac{6\,\text{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}
\end{aligned}$$

```
[In] Integrate[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*PolyL
og[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -
(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^(c*
(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(132) = 264$.

Time = 2.09 (sec) , antiderivative size = 601, normalized size of antiderivative = 4.55

method	result
risch	$\frac{x^4 \ln\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{4} - \frac{\mathrm{Li}_2\left(-f^{xbcn} f^{-xbcn} \left(f^{c(bx+a)}\right)^n e\right) \ln\left(f^{c(bx+a)}\right)^3}{c^4 b^4 \ln(f)^4 n} + \frac{\mathrm{dilog}\left(1+f^{xbcn} f^{-xbcn} \left(f^{c(bx+a)}\right)^n e\right) \ln\left(f^{c(bx+a)}\right.}{c^4 b^4 \ln(f)^4 n}$

```
[In] int(x^3*ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(1+e*(f^(c*(b*x+a)))^n)-1/c^4/b^4/ln(f)^4/n*polylog(2,-f^(x*b*c*n
)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))^3+1/c^4/b^4/ln(f)^4/n
*dilog(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))^3+
3/c^2/b^2/ln(f)^2/n^2*polylog(3,-f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n
*e)*x^2-1/4*ln(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x^4-6/c^3/b^
3/ln(f)^3/n^3*polylog(4,-f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x-1/
c/b/ln(f)/n*dilog(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x^3+6/c^4
/b^4/ln(f)^4/n^4*polylog(5,-f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)+3
/c^2/b^2/ln(f)^2/n*dilog(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln
(f^(c*(b*x+a)))*x^2-3/c^3/b^3/ln(f)^3/n*dilog(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f
^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))^2*x+3/c^3/b^3/ln(f)^3/n*polylog(2,-f^(
x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))^2*x-3/c^2/b^2/
ln(f)^2/n*polylog(2,-f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*
(b*x+a)))*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right) dx = -\frac{b^3c^3n^3x^3\mathrm{Li}_2\left(-ef^{bcnx+acn}\right)\log(f)^3 - 3\,b^2c^2n^2x^2\log(f)^2\,\mathrm{polylog}\left(3,-ef^{bcnx+acn}\right) + 6\,bcnx\log(f)\,\mathrm{polylo}}{b^4c^4n^4\log(f)^4}$$

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x + a*c*n))*log(f)^3 - 3*b^2*c^2*n^2*x^
2*log(f)^2*polylog(3, -e*f^(b*c*n*x + a*c*n)) + 6*b*c*n*x*log(f)*polylog(4,
 -e*f^(b*c*n*x + a*c*n)) - 6*polylog(5, -e*f^(b*c*n*x + a*c*n)))/(b^4*c^4*n
^4*log(f)^4)
```

Sympy [F]

$$\int x^3 \log\left(1 + e\left(f^{c(a+bx)}\right)^n\right) dx = \int x^3 \log\left(e\left(f^{ac+bcx}\right)^n + 1\right) dx$$

```
[In] integrate(x**3*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] Integral(x**3*log(e*(f**(a*c + b*c*x))**n + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.43

$$\int x^3 \log\left(1 + e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{4} x^4 \log\left(ef^{(bx+a)cn} + 1\right) - \frac{b^4c^4n^4x^4 \log\left(ef^{bcnx}f^{acn} + 1\right)\log(f)^4 + 4\,b^3c^3n^3x^3 \mathrm{Li}_2\left(-ef^{bcnx}f^{acn}\right)\log(f)^3 - 12\,b^2c^2n^2x^2\log(f)^2\,\mathrm{Li}_3(-}{4\,b^4c^4n^4\log(f)^4}$$

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(e*f^((b*x + a)*c*n) + 1) - 1/4*(b^4*c^4*n^4*x^4*log(e*f^(b*c*n*
x)*f^(a*c*n) + 1)*log(f)^4 + 4*b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x)*f^(a*c*
n))*log(f)^3 - 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x)*f^(a*c
*n)) + 24*b*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)) - 24*polylog(
5, -e*f^(b*c*n*x)*f^(a*c*n)))/(b^4*c^4*n^4*log(f)^4)
```

Giac [F]

$$\int x^3 \log\left(1 + e\left(f^{c(a+bx)}\right)^n\right) dx = \int x^3 \log\left(e\left(f^{(bx+a)c}\right)^n + 1\right) dx$$

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(e*(f^((b*x + a)*c))^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int x^3 \ln \left(e\left(f^{c\,(a+b\,x)}\right)^n+1\right) \, dx$$

```
[In] int(x^3*log(e*(f^(c*(a + b*x)))^n + 1),x)
[Out] int(x^3*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

3.119 $\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx$

Optimal result . . . 748
Rubi [A] (verified) . . . 748
Mathematica [A] (verified) . . . 750
Maple [B] (verified) . . . 750
Fricas [A] (verification not implemented) . . . 751
Sympy [F] . . . 751
Maxima [A] (verification not implemented) . . . 751
Giac [F] . . . 752
Mupad [F(-1)] . . . 752

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx = -\frac{x^2 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn \log(f)} + \frac{2x \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2 \log^2(f)} - \frac{2 \operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3 \log^3(f)}$$

```
[Out] -x^2*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+2*x*polylog(3,-e*(f^(c*(b*
x+a)))^n)/b^2/c^2/n^2/ln(f)^2-2*polylog(4,-e*(f^(c*(b*x+a)))^n)/b^3/c^3/n^3
/ln(f)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2611, 6744, 2320, 6724}

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx = -\frac{2 \operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3 \log^3(f)} + \frac{2x \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2 \log^2(f)} - \frac{x^2 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn \log(f)}$$

```
[In] Int[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLog
[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e*(
f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2\int x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)\,dx}{bcn\log(f)}\\
&= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\int \text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)\,dx}{b^2c^2n^2\log^2(f)}\\
&= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Subst}\left(\int \frac{\text{Li}_3(-ex^n)}{x}\,dx,x,f^{c(a+bx)}\right)}{b^3c^3n^2\log^3(f)}\\
&= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx = -\frac{x^2 \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn \log(f)} + \frac{2x \operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2 \log^2(f)} - \frac{2 \operatorname{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3 \log^3(f)}$$

```
[In] Integrate[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLog
[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e*(
f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(98) = 196.

Time = 1.06 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.39

method	result
risch	$\frac{x^3 \ln\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{3}+\frac{2\,\mathrm{Li}_3\left(-f^{xbcn} f^{-xbcn}\left(f^{c(bx+a)}\right)^n e\right) x}{c^2 b^2 \ln(f)^2 n^2}-\frac{2\,\mathrm{Li}_2\left(-f^{xbcn} f^{-xbcn}\left(f^{c(bx+a)}\right)^n e\right) \ln\left(f^{c(bx+a)}\right) x}{c^2 b^2 \ln(f)^2 n}+\frac{\mathrm{Li}_2(-$

```
[In] int(x^2*ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(1+e*(f^(c*(b*x+a)))^n)+2/c^2/b^2/ln(f)^2/n^2*polylog(3,-f^(x*b*c
*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x-2/c^2/b^2/ln(f)^2/n*polylog(2,-f^(x
*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))*x+1/c^3/b^3/ln(
f)^3/n*polylog(2,-f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*
x+a)))^2-1/3*ln(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x^3-1/c/b/l
n(f)/n*dilog(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x^2+2/c^2/b^2/
ln(f)^2/n*dilog(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*
x+a)))*x-1/c^3/b^3/ln(f)^3/n*dilog(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)
))^n*e)*ln(f^(c*(b*x+a)))^2-2/c^3/b^3/ln(f)^3/n^3*polylog(4,-f^(x*b*c*n)*f^
(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx = -\frac{b^2c^2n^2x^2\mathrm{Li}_2\left(-ef^{bcnx+acn}\right)\log\left(f\right)^2 - 2\,bcnx\log\left(f\right)\mathrm{polylog}\left(3,-ef^{bcnx+acn}\right) + 2\,\mathrm{polylog}\left(4,-ef^{bcnx+acn}\right)}{b^3c^3n^3\log\left(f\right)^3}$$

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x + a*c*n))*log(f)^2 - 2*b*c*n*x*log(f)
*polylog(3, -e*f^(b*c*n*x + a*c*n)) + 2*polylog(4, -e*f^(b*c*n*x + a*c*n)))
/(b^3*c^3*n^3*log(f)^3)
```

Sympy [F]

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx = \int x^2 \log \left(e\left(f^{ac+bcx}\right)^n + 1\right) dx$$

```
[In] integrate(x**2*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] Integral(x**2*log(e*(f**(a*c + b*c*x))**n + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.56

$$\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{3}x^3\log\left(ef^{(bx+a)cn}+1\right) - \frac{b^3c^3n^3x^3\log\left(ef^{bcnx}f^{acn}+1\right)\log\left(f\right)^3 + 3\,b^2c^2n^2x^2\mathrm{Li}_2\left(-ef^{bcnx}f^{acn}\right)\log\left(f\right)^2 - 6\,bcnx\log\left(f\right)\mathrm{Li}_3\left(-ef^{bc\cdots}\right.}{3\,b^3c^3n^3\log\left(f\right)^3}$$

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(e*f^((b*x + a)*c*n) + 1) - 1/3*(b^3*c^3*n^3*x^3*log(e*f^(b*c*n*
x)*f^(a*c*n) + 1)*log(f)^3 + 3*b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x)*f^(a*c*
n))*log(f)^2 - 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)) + 6*po
lylog(4, -e*f^(b*c*n*x)*f^(a*c*n)))/(b^3*c^3*n^3*log(f)^3)
```

Giac [F]

$$\int x^2 \log\left(1 + e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int x^2 \log\left(e\left(f^{(bx+a)c}\right)^n + 1\right) \, dx$$

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
[Out] integrate(x^2*log(e*(f^((b*x + a)*c))^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(1 + e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int x^2 \ln\left(e\left(f^{c\,(a+b\,x)}\right)^n + 1\right) \, dx$$

```
[In] int(x^2*log(e*(f^(c*(a + b*x)))^n + 1),x)
[Out] int(x^2*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

3.120 $\int x \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx$

Optimal result 753
Rubi [A] (verified) 753
Mathematica [A] (verified) 754
Maple [B] (verified) 755
Fricas [A] (verification not implemented) 755
Sympy [F] 755
Maxima [A] (verification not implemented) 756
Giac [F] 756
Mupad [F(-1)] 756

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int x \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx=-\frac{x \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}$$

```
[Out] -x*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+polylog(3,-e*(f^(c*(b*x+a)))
^n)/b^2/c^2/n^2/ln(f)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2611, 2320, 6724}

$$\int x \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx=\frac{\operatorname{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x \operatorname{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[In] Int[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(e
*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\int \text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)\,dx}{bcn\log(f)}\\ &= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Subst}\left(\int\frac{\text{Li}_2(-ex^n)}{x}\,dx,x,f^{c(a+bx)}\right)}{b^2c^2n\log^2(f)}\\ &= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx=-\frac{x\,\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}$$

```
[In] Integrate[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
[Out] -((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(e
*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(63) = 126.

Time = 0.63 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.16

method	result
risch	$\frac{x^2 \ln\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{2} - \frac{\ln\left(1+f^{xbcn} f^{-xbcn} \left(f^{c(bx+a)}\right)^n e\right) x^2}{2} - \frac{\mathrm{Li}_2\left(-f^{xbcn} f^{-xbcn} \left(f^{c(bx+a)}\right)^n e\right) \ln\left(f^{c(bx+a)}\right)}{c^2 b^2 \ln(f)^2 n} + \frac{\mathrm{Li}_3\left(-f^{x}\right.}{}$

```
[In] int(x*ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(1+e*(f^(c*(b*x+a)))^n)-1/2*ln(1+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(
b*x+a)))^n*e)*x^2-1/c^2/b^2/ln(f)^2/n*polylog(2,-f^(x*b*c*n)*f^(-x*b*c*n)*(
f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))+1/c^2/b^2/ln(f)^2/n^2*polylog(3,-f^(x
*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)-1/c/b/ln(f)/n*dilog(1+f^(x*b*c*n)
*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x+1/c^2/b^2/ln(f)^2/n*dilog(1+f^(x*b*c*n
)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int x \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = -\frac{bcnx\mathrm{Li}_2\left(-ef^{bcnx+acn}\right)\log(f) - \mathrm{polylog}\left(3,-ef^{bcnx+acn}\right)}{b^2c^2n^2\log(f)^2}$$

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b*c*n*x*dilog(-e*f^(b*c*n*x + a*c*n))*log(f) - polylog(3, -e*f^(b*c*n*x +
 a*c*n)))/(b^2*c^2*n^2*log(f)^2)
```

Sympy [F]

$$\int x \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x \log\left(e\left(f^{ac+bcx}\right)^n+1\right)dx$$

```
[In] integrate(x*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] Integral(x*log(e*(f**(a*c + b*c*x))**n + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \frac{1}{2}\,x^2\log\left(ef^{(bx+a)cn}+1\right) - \frac{b^2c^2n^2x^2\log\left(ef^{bcnx}f^{acn}+1\right)\log\left(f\right)^2+2\,bcnx\mathrm{Li}_2\left(-ef^{bcnx}f^{acn}\right)\log\left(f\right)-2\,\mathrm{Li}_3\left(-ef^{bcnx}f^{acn}\right)}{2\,b^2c^2n^2\log\left(f\right)^2}$$

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
[Out] 1/2*x^2*log(e*f^((b*x + a)*c*n) + 1) - 1/2*(b^2*c^2*n^2*x^2*log(e*f^(b*c*n*
x)*f^(a*c*n) + 1)*log(f)^2 + 2*b*c*n*x*dilog(-e*f^(b*c*n*x)*f^(a*c*n))*log(
f) - 2*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)))/(b^2*c^2*n^2*log(f)^2)
```

Giac [F]

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)\,dx$$

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
[Out] integrate(x*log(e*(f^((b*x + a)*c))^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int x\,\ln\left(e\left(f^{c\,(a+b\,x)}\right)^n+1\right)\,dx$$

```
[In] int(x*log(e*(f^(c*(a + b*x)))^n + 1),x)
[Out] int(x*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

3.121 $\int \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx$

Optimal result . . . 757
Rubi [A] (verified) . . . 757
Mathematica [A] (verified) . . . 758
Maple [A] (verified) . . . 758
Fricas [A] (verification not implemented) . . . 759
Sympy [F] . . . 759
Maxima [B] (verification not implemented) . . . 759
Giac [F] . . . 760
Mupad [F(-1)] . . . 760

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx=-\frac{\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[Out] -polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2317, 2438}

$$\int \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx=-\frac{\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[In] Int[Log[1 + e*(f^(c*(a + b*x)))^n],x]
[Out] -(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log(1+ex)}{x}\,dx, x, \left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$
$$= -\frac{\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = -\frac{\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[In] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n],x]
[Out] -(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{\text{dilog}\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{bc\ln(f)n}$
default	$-\frac{\text{dilog}\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{bc\ln(f)n}$
risch	$x\ln\left(1+e\left(f^{c(bx+a)}\right)^n\right)-\frac{\text{dilog}\left(1+f^{xbcn}f^{-xbcn}\left(f^{c(bx+a)}\right)^n e\right)}{cb\ln(f)n}-\ln\left(1+f^{xbcn}f^{-xbcn}\left(f^{c(bx+a)}\right)^n e\right)$

```
[In] int(ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
[Out] -1/b/c/ln(f)/n*dilog(1+e*(f^(c*(b*x+a)))^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = -\frac{\text{Li}_2\left(-ef^{bcnx+acn}\right)}{bcn\log\left(f\right)}$$

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
[Out] -dilog(-e*f^(b*c*n*x + a*c*n))/(b*c*n*log(f))
```

Sympy [F]

$$\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \int \log\left(e\left(f^{c(a+bx)}\right)^n+1\right)\,dx$$

```
[In] integrate(ln(1+e*(f**(c*(b*x+a)))**n),x)
[Out] Integral(log(e*(f**(c*(a + b*x)))**n + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = x\log\left(ef^{(bx+a)cn}+1\right) - \frac{bcnx\log\left(ef^{bcnx}f^{acn}+1\right)\log\left(f\right)+\text{Li}_2\left(-ef^{bcnx}f^{acn}\right)}{bcn\log\left(f\right)}$$

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
[Out] x*log(e*f^((b*x + a)*c*n) + 1) - (b*c*n*x*log(e*f^(b*c*n*x)*f^(a*c*n) + 1)*
log(f) + dilog(-e*f^(b*c*n*x)*f^(a*c*n)))/(b*c*n*log(f))
```

Giac [F]

$$\int \log \left(1 + e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int \log \left(e\left(f^{(bx+a)c}\right)^n + 1\right) \, dx$$

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
[Out] integrate(log(e*(f^((b*x + a)*c))^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(1 + e\left(f^{c(a+bx)}\right)^n\right) \, dx = \int \ln \left(e\left(f^{c\,(a+b\,x)}\right)^n + 1\right) \, dx$$

```
[In] int(log(e*(f^(c*(a + b*x)))^n + 1),x)
[Out] int(log(e*(f^(c*(a + b*x)))^n + 1), x)
```

3.122 $\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$

Optimal result 761
Rubi [N/A] 761
Mathematica [N/A] 762
Maple [N/A] 762
Fricas [N/A] 762
Sympy [N/A] 762
Maxima [N/A] 763
Giac [N/A] 763
Mupad [N/A] 763

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \mathrm{Int}\left(\frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

```
[In] Int[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]
```

```
[Out] Defer[Int][Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

```
[In] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{x}dx$$

```
[In] int(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
[Out] int(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)}{x}\,dx$$

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="fricas")
[Out] integral(log(e*(f^(b*c*x + a*c))^n + 1)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(e\left(f^{ac+bcx}\right)^n+1\right)}{x}\,dx$$

```
[In] integrate(ln(1+e*(f**(c*(b*x+a)))**n)/x,x)
[Out] Integral(log(e*(f**(a*c + b*c*x))**n + 1)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log \left(e\left(f^{(bx+a)c}\right)^n+1\right)}{x}\,dx$$

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="maxima")
[Out] integrate(log(e*f^((b*x + a)*c*n) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log \left(e\left(f^{(bx+a)c}\right)^n+1\right)}{x}\,dx$$

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="giac")
[Out] integrate(log(e*(f^((b*x + a)*c))^n + 1)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\ln \left(e\,\left(f^{c\,(a+b\,x)}\right)^n+1\right)}{x}\,dx$$

```
[In] int(log(e*(f^(c*(a + b*x)))^n + 1)/x,x)
[Out] int(log(e*(f^(c*(a + b*x)))^n + 1)/x, x)
```

3.123 $\int x^3 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal result 764
Rubi [A] (verified) 765
Mathematica [A] (verified) 767
Maple [B] (verified) 768
Fricas [A] (verification not implemented) 769
Sympy [F] 769
Maxima [A] (verification not implemented) 769
Giac [F] 770
Mupad [F(-1)] 770

Optimal result

Integrand size = 20, antiderivative size = 193

$$\int x^3 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx = \frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^3\operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{3x^2\operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{6\operatorname{PolyLog}\left(5,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4\log^4(f)}$$

```
[Out] 1/4*x^4*ln(d+e*(f^(c*(b*x+a)))^n)-1/4*x^4*ln(1+e*(f^(c*(b*x+a)))^n/d)-x^3*p
olylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+3*x^2*polylog(3,-e*(f^(c*(b*x+
a)))^n/d)/b^2/c^2/n^2/ln(f)^2-6*x*polylog(4,-e*(f^(c*(b*x+a)))^n/d)/b^3/c^3
/n^3/ln(f)^3+6*polylog(5,-e*(f^(c*(b*x+a)))^n/d)/b^4/c^4/n^4/ln(f)^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2612, 2611, 6744, 2320, 6724}

$$\int x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\frac{6\,\text{PolyLog}\left(5,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4\log^4(f)}-\frac{6x\,\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{3x^2\,\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x^3\,\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{4}x^4\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{4}x^4\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

```
[In] Int[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (
6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) + (6*P
olyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\
&\quad+\int x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\,dx\\
&=\frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\
&\quad-\frac{x^3\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{3\int x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\,dx}{bcn\log(f)}\\
&=\frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\
&\quad-\frac{x^3\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{6\int x\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\,dx}{b^2c^2n^2\log^2(f)}\\
&=\frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^3\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}\\
&\quad+\frac{3x^2\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{6\int\text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\,dx}{b^3c^3n^3\log^3(f)}
\end{aligned}$$

$$= \frac{1}{4}x^4 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1 + \frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn \log(f)} + \frac{3x^2 \text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2 \log^2(f)} - \frac{6x \text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3 \log^3(f)} + \frac{6 \text{Subst}\left(\int \frac{\text{Li}_4\left(-\frac{ex^n}{d}\right)}{x} \, dx, x, f^{c(a+bx)}\right)}{b^4c^4n^3 \log^4(f)}$$

$$= \frac{1}{4}x^4 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1 + \frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn \log(f)} + \frac{3x^2 \text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2 \log^2(f)} - \frac{6x \text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3 \log^3(f)} + \frac{6 \text{Li}_5\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4 \log^4(f)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00

$$\int x^3 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) \, dx = \frac{1}{4}x^4 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1 + \frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \operatorname{PolyLog}\left(2, -\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn \log(f)} + \frac{3x^2 \operatorname{PolyLog}\left(3, -\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2 \log^2(f)} - \frac{6x \operatorname{PolyLog}\left(4, -\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3 \log^3(f)} + \frac{6 \operatorname{PolyLog}\left(5, -\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4 \log^4(f)}$$

```
[In] Integrate[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (
6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) + (6*P
olyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. 2(189) = 378.

Time = 2.12 (sec) , antiderivative size = 1276, normalized size of antiderivative = 6.61

method	result	size
risch	Expression too large to display	1276

```
[In] int(x^3*ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(d+e*(f^(c*(b*x+a)))^n)-3/4/c^4/b^4/ln(f)^4*ln(1+e*f^(x*b*c*n)*f^
(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^4+1/c^4/b^4/ln(f)^4*ln((d
+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)/d)*ln(f^(c*(b*x+a)))^4-1/4*l
n(d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x^4-1/4/c^4/b^4/ln(f)^4*l
n(d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))^4+6/c^4
/b^4/ln(f)^4/n^4*polylog(5,-e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)
-1/c^4/b^4/ln(f)^4/n*polylog(2,-e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^
n/d)*ln(f^(c*(b*x+a)))^3-3/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(x*b*c*n)*f^(-x
*b*c*n)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))*x^2+3/c^3/b^3/ln(f)^3/n*poly
log(2,-e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^2*
x+3/c^2/b^2/ln(f)^2/n*dilog((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e
)/d)*ln(f^(c*(b*x+a)))*x^2-3/c^3/b^3/ln(f)^3/n*dilog((d+f^(x*b*c*n)*f^(-x*b
*c*n)*(f^(c*(b*x+a)))^n*e)/d)*ln(f^(c*(b*x+a)))^2*x-6/c^3/b^3/ln(f)^3/n^3*p
olylog(4,-e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)*x+1/c/b/ln(f)*ln(
d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))*x^3-3/2/c
^2/b^2/ln(f)^2*ln(d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(
b*x+a)))^2*x^2+1/c^3/b^3/ln(f)^3*ln(d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a
)))^n*e)*ln(f^(c*(b*x+a)))^3*x-3/2/c^2/b^2/ln(f)^2*ln(1+e*f^(x*b*c*n)*f^(-x
*b*c*n)*(f^(c*(b*x+a)))^n/d)*x^2*ln(f^(c*(b*x+a)))^2+2/c^3/b^3/ln(f)^3*ln(1
+e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)*x*ln(f^(c*(b*x+a)))^3+3/c^
2/b^2/ln(f)^2*ln((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)/d)*x^2*ln
(f^(c*(b*x+a)))^2-3/c^3/b^3/ln(f)^3*ln((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b
*x+a)))^n*e)/d)*x*ln(f^(c*(b*x+a)))^3-1/c/b/ln(f)*ln((d+f^(x*b*c*n)*f^(-x*b
*c*n)*(f^(c*(b*x+a)))^n*e)/d)*x^3*ln(f^(c*(b*x+a)))+3/c^2/b^2/ln(f)^2/n^2*p
olylog(3,-e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)*x^2-1/c/b/ln(f)/n
*dilog((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)/d)*x^3+1/c^4/b^4/ln
(f)^4/n*dilog((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)/d)*ln(f^(c*(
b*x+a)))^3
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.27

$$\int x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) dx =$$
$$-\frac{4 b^3c^3n^3x^3\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log(f)^3-12 b^2c^2n^2x^2\log(f)^2\,\mathrm{polylog}\left(3,-\frac{ef^{bcnx+acn}}{d}\right)-(b^4c^4n^4x^4-$$

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*c^3*n^3*x^3*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f)^3
- 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x + a*c*n)/d) - (b^4*c
^4*n^4*x^4 - a^4*c^4*n^4)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^4 + (b^4*c^
4*n^4*x^4 - a^4*c^4*n^4)*log(f)^4*log((e*f^(b*c*n*x + a*c*n) + d)/d) + 24*b
*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x + a*c*n)/d) - 24*polylog(5, -e*f^(b*
c*n*x + a*c*n)/d))/(b^4*c^4*n^4*log(f)^4)
```

Sympy [F]

$$\int x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = \int x^3 \log \left(d+e\left(f^{ac+bcx}\right)^n\right) dx$$

```
[In] integrate(x**3*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] Integral(x**3*log(d + e*(f**(a*c + b*c*x))**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.06

$$\int x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{4}x^4\log\left(ef^{(bx+a)cn}+d\right)$$
$$-\frac{b^4c^4n^4x^4\log\left(\frac{ef^{bcnx}f^{acn}}{d}+1\right)\log(f)^4+4 b^3c^3n^3x^3\mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right)\log(f)^3-12 b^2c^2n^2x^2\log(f)^2\,\mathrm{Li}_3(-}{4 b^4c^4n^4\log(f)^4}$$

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(e*f^((b*x + a)*c*n) + d) - 1/4*(b^4*c^4*n^4*x^4*log(e*f^(b*c*n*
x)*f^(a*c*n)/d + 1)*log(f)^4 + 4*b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x)*f^(a*
c*n)/d)*log(f)^3 - 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x)*f^
(a*c*n)/d) + 24*b*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)/d) - 24*
polylog(5, -e*f^(b*c*n*x)*f^(a*c*n)/d))/(b^4*c^4*n^4*log(f)^4)
```

Giac [F]

$$\int x^3 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) dx = \int x^3 \log\left(e\left(f^{(bx+a)c}\right)^n + d\right) dx$$

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
[Out] integrate(x^3*log(e*(f^((b*x + a)*c))^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) dx = \int x^3 \ln\left(d + e\left(f^{c\,(a+b\,x)}\right)^n\right) dx$$

```
[In] int(x^3*log(d + e*(f^(c*(a + b*x)))^n),x)
[Out] int(x^3*log(d + e*(f^(c*(a + b*x)))^n), x)
```

3.124 $\int x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx$

Optimal result 771
Rubi [A] (verified) 771
Mathematica [A] (verified) 774
Maple [B] (verified) 774
Fricas [A] (verification not implemented) 775
Sympy [F] 775
Maxima [A] (verification not implemented) 776
Giac [F] 776
Mupad [F(-1)] 776

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \frac{1}{3}x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{3}x^3 \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^2 \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{2x \operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)} - \frac{2\operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}$$

```
[Out] 1/3*x^3*ln(d+e*(f^(c*(b*x+a)))^n)-1/3*x^3*ln(1+e*(f^(c*(b*x+a)))^n/d)-x^2*p
olylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+2*x*polylog(3,-e*(f^(c*(b*x+a)
))^n/d)/b^2/c^2/n^2/ln(f)^2-2*polylog(4,-e*(f^(c*(b*x+a)))^n/d)/b^3/c^3/n^3
/ln(f)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{2612, 2611, 6744, 2320, 6724\}$

$$\int x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = -\frac{2 \operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)} + \frac{2x \operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)} - \frac{x^2 \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right) - \frac{1}{3}x^3\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

```
[In] Int[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (2*
PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\
&\quad+\int x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx\\
&=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\
&\quad-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{2\int x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(f)}\\
&=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\
&\quad-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{2\int\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx}{b^2c^2n^2\log^2(f)}\\
&=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}\\
&\quad+\frac{2x\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Subst}\left(\int\frac{\text{Li}_3\left(-\frac{ex^n}{d}\right)}{x}dx,x,f^{c(a+bx)}\right)}{b^3c^3n^2\log^3(f)}\\
&=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\
&\quad-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00

$$\int x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{2x\operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{2\operatorname{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}$$

```
[In] Integrate[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (2*
PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(152) = 304.

Time = 1.10 (sec) , antiderivative size = 916, normalized size of antiderivative = 5.87

method	result	size
risch	Expression too large to display	916

```
[In] int(x^2*ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(d+e*(f^(c*(b*x+a)))^n)+1/3/ln(f)^3/b^3/c^3*ln(d+f^(x*b*c*n)*f^(-
x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))^3-1/n/ln(f)^3/b^3/c^3*dilog
((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)/d)*ln(f^(c*(b*x+a)))^2-1/
c^3/b^3/ln(f)^3*ln((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)/d)*ln(f
^(c*(b*x+a)))^3-1/3*ln(d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x^3-
1/n/ln(f)/b/c*dilog((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)/d)*x^2
+2/n/ln(f)^2/b^2/c^2*dilog((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)
/d)*ln(f^(c*(b*x+a)))*x-1/c/b/ln(f)*ln((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b
*x+a)))^n*e)/d)*x^2*ln(f^(c*(b*x+a)))+2/c^2/b^2/ln(f)^2*ln((d+f^(x*b*c*n)*f
^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)/d)*x*ln(f^(c*(b*x+a)))^2-1/ln(f)^2/b^2/c^2
*ln(f^(c*(b*x+a)))^2*ln(1+e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)*x
-2/n/ln(f)^2/b^2/c^2*ln(f^(c*(b*x+a)))*polylog(2,-e*f^(x*b*c*n)*f^(-x*b*c*n
```

```
)*(f^(c*(b*x+a)))^n/d)*x+1/ln(f)/b/c*ln(d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b
*x+a)))^n*e)*ln(f^(c*(b*x+a)))*x^2-1/ln(f)^2/b^2/c^2*ln(d+f^(x*b*c*n)*f^(-x
*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))^2*x+2/n^2/ln(f)^2/b^2/c^2*po
lylog(3,-e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)*x+2/3/ln(f)^3/b^3/
c^3*ln(f^(c*(b*x+a)))^3*ln(1+e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d
)+1/n/ln(f)^3/b^3/c^3*ln(f^(c*(b*x+a)))^2*polylog(2,-e*f^(x*b*c*n)*f^(-x*b*
c*n)*(f^(c*(b*x+a)))^n/d)-2/n^3/ln(f)^3/b^3/c^3*polylog(4,-e*f^(x*b*c*n)*f^
(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.31

$$\int x^2 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) dx =$$
$$-\frac{3\,b^2c^2n^2x^2\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log(f)^2 - 6\,bcnx\log(f)\,\mathrm{polylog}\left(3,-\frac{ef^{bcnx+acn}}{d}\right) - (b^3c^3n^3x^3 + a^3c^3n}{3\,b^3c^3}$$

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*c^2*n^2*x^2*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f)^2
- 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x + a*c*n)/d) - (b^3*c^3*n^3*x^3
+ a^3*c^3*n^3)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^3 + (b^3*c^3*n^3*x^3 +
 a^3*c^3*n^3)*log(f)^3*log((e*f^(b*c*n*x + a*c*n) + d)/d) + 6*polylog(4, -e
*f^(b*c*n*x + a*c*n)/d))/(b^3*c^3*n^3*log(f)^3)
```

Sympy [F]

$$\int x^2 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) dx = \int x^2 \log\left(d + e\left(f^{ac+bcx}\right)^n\right) dx$$

```
[In] integrate(x**2*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] Integral(x**2*log(d + e*(f**(a*c + b*c*x))**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06

$$\int x^2 \log \left(d + e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{3} x^3 \log \left(ef^{(bx+a)cn} + d\right) - \frac{b^3c^3n^3x^3 \log \left(\frac{ef^{bcnx}f^{acn}}{d} + 1\right) \log (f)^3 + 3 b^2c^2n^2x^2\mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right) \log (f)^2 - 6 bcnx \log (f) \mathrm{Li}_3(-\frac{ef^{bcnx}fa}{d}}{3 b^3c^3n^3 \log (f)^3}$$

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(e*f^((b*x + a)*c*n) + d) - 1/3*(b^3*c^3*n^3*x^3*log(e*f^(b*c*n*
x)*f^(a*c*n)/d + 1)*log(f)^3 + 3*b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x)*f^(a*
c*n)/d)*log(f)^2 - 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)/d)
+ 6*polylog(4, -e*f^(b*c*n*x)*f^(a*c*n)/d))/(b^3*c^3*n^3*log(f)^3)
```

Giac [F]

$$\int x^2 \log \left(d + e\left(f^{c(a+bx)}\right)^n\right) dx = \int x^2 \log \left(e\left(f^{(bx+a)c}\right)^n + d\right) dx$$

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(e*(f^((b*x + a)*c))^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log \left(d + e\left(f^{c(a+bx)}\right)^n\right) dx = \int x^2 \ln \left(d + e \left(f^{c\,(a+b\,x)}\right)^n\right) dx$$

```
[In] int(x^2*log(d + e*(f^(c*(a + b*x)))^n),x)
```

```
[Out] int(x^2*log(d + e*(f^(c*(a + b*x)))^n), x)
```

3.125 $\int x \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) dx$

Optimal result . . . 777
Rubi [A] (verified) . . . 777
Mathematica [A] (verified) . . . 779
Maple [B] (verified) . . . 779
Fricas [A] (verification not implemented) . . . 780
Sympy [F] . . . 780
Maxima [A] (verification not implemented) . . . 781
Giac [F] . . . 781
Mupad [F(-1)] . . . 781

Optimal result

Integrand size = 18, antiderivative size = 118

$$\int x \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{2}x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{2}x^2 \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{\operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}$$

```
[Out] 1/2*x^2*ln(d+e*(f^(c*(b*x+a)))^n)-1/2*x^2*ln(1+e*(f^(c*(b*x+a)))^n/d)-x*pol
ylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+polylog(3,-e*(f^(c*(b*x+a)))^n/d
)/b^2/c^2/n^2/ln(f)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2612, 2611, 2320, 6724}

$$\int x \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{\operatorname{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)} - \frac{x \operatorname{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{1}{2}x^2 \log \left(e\left(f^{c(a+bx)}\right)^n+d\right) - \frac{1}{2}x^2 \log \left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

```
[In] Int[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + Po
lyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\
&\quad+\int x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\,dx\\
&=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\\
&\quad-\frac{x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{\int \text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\,dx}{bcn\log(f)}
\end{aligned}
$$

$$= \frac{1}{2}x^2 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{2}x^2 \log\left(1 + \frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{ex^n}{d}\right)}{x}\,dx, x, f^{c(a+bx)}\right)}{b^2c^2n\log^2(f)}$$

$$= \frac{1}{2}x^2 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{2}x^2 \log\left(1 + \frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int x \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{2}x^2 \log\left(d + e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{2}x^2 \log\left(1 + \frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x\,\text{PolyLog}\left(2, -\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{\text{PolyLog}\left(3, -\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}$$

```
[In] Integrate[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
[Out] (x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + Po
lyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(114) = 228.

Time = 0.58 (sec) , antiderivative size = 558, normalized size of antiderivative = 4.73

method	result
risch	$\frac{x^2 \ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)}{2} - \frac{\ln\left(f^{c(bx+a)}\right)^2 \ln\left(1+\frac{e\,f^{xbcn} f^{-xbcn}\left(f^{c(bx+a)}\right)^n}{d}\right)}{2\ln(f)^2 b^2 c^2} - \frac{\ln\left(f^{c(bx+a)}\right) \text{Li}_2\left(-\frac{e\,f^{xbcn} f^{-xbcn}\left(f^{c(bx+a)}\right)^n}{d}\right)}{n\ln(f)^2 b^2 c^2}$

```
[In] int(x*ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*ln(d+e*(f^(c*(b*x+a)))^n)-1/2/ln(f)^2/b^2/c^2*ln(f^(c*(b*x+a)))^2*l
n(1+e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)-1/n/ln(f)^2/b^2/c^2*ln(
f^(c*(b*x+a)))*polylog(2,-e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n/d)+1
/n^2/ln(f)^2/b^2/c^2*polylog(3,-e*f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^
```

```
n/d)-1/2*ln(d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*x^2+1/ln(f)/b/c
*ln(d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c*(b*x+a)))*x-1/2
/ln(f)^2/b^2/c^2*ln(d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)*ln(f^(c
*(b*x+a)))^2-1/n/ln(f)/b/c*dilog((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a))
)^n*e)/d)*x+1/n/ln(f)^2/b^2/c^2*dilog((d+f^(x*b*c*n)*f^(-x*b*c*n)*(f^(c*(b*
x+a)))^n*e)/d)*ln(f^(c*(b*x+a)))-1/c/b/ln(f)*ln((d+f^(x*b*c*n)*f^(-x*b*c*n)
*(f^(c*(b*x+a)))^n*e)/d)*x*ln(f^(c*(b*x+a)))+1/c^2/b^2/ln(f)^2*ln((d+f^(x*b
*c*n)*f^(-x*b*c*n)*(f^(c*(b*x+a)))^n*e)/d)*ln(f^(c*(b*x+a)))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.43

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^{n}\right)\,dx = -\frac{2\,bcnx\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log\left(f\right)-\left(b^2c^2n^2x^2-a^2c^2n^2\right)\log\left(ef^{bcnx+acn}+d\right)\log\left(f\right)^2+\left(b^2c^2n^2x^2-a\right.}{2\,b^2c^2n^2\log\left(f\right)^2}$$

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*n*x*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f) - (b^2*c^2
*n^2*x^2 - a^2*c^2*n^2)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^2 + (b^2*c^2*
n^2*x^2 - a^2*c^2*n^2)*log(f)^2*log((e*f^(b*c*n*x + a*c*n) + d)/d) - 2*poly
log(3, -e*f^(b*c*n*x + a*c*n)/d))/(b^2*c^2*n^2*log(f)^2)
```

Sympy [F]

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^{n}\right)\,dx=\int x\log\left(d+e\left(f^{ac+bcx}\right)^{n}\right)dx$$

```
[In] integrate(x*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] Integral(x*log(d + e*(f**(a*c + b*c*x))**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\frac{1}{2}\,x^2\log\left(ef^{(bx+a)cn}+d\right)-\frac{b^2c^2n^2x^2\log\left(\frac{ef^{bcnx}f^{acn}}{d}+1\right)\log\left(f\right)^2+2\,bcnx\mathrm{Li}_2\left(-\frac{ef^{bcnx}f^{acn}}{d}\right)\log\left(f\right)-2\,\mathrm{Li}_3\left(-\frac{ef^{bcnx}f^{acn}}{d}\right)}{2\,b^2c^2n^2\log\left(f\right)^2}$$

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(e*f^((b*x + a)*c*n) + d) - 1/2*(b^2*c^2*n^2*x^2*log(e*f^(b*c*n*
x)*f^(a*c*n)/d + 1)*log(f)^2 + 2*b*c*n*x*dilog(-e*f^(b*c*n*x)*f^(a*c*n)/d)*
log(f) - 2*polylog(3, -e*f^(b*c*n*x)*f^(a*c*n)/d))/(b^2*c^2*n^2*log(f)^2)
```

Giac [F]

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)\,dx$$

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x*log(e*(f^((b*x + a)*c))^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int x\,\ln\left(d+e\,{\left(f^{c\,(a+b\,x)}\right)}^n\right)\,dx$$

```
[In] int(x*log(d + e*(f^(c*(a + b*x)))^n),x)
```

```
[Out] int(x*log(d + e*(f^(c*(a + b*x)))^n), x)
```

3.126 $\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal result 782
Rubi [A] (verified) 782
Mathematica [A] (verified) 784
Maple [A] (verified) 784
Fricas [A] (verification not implemented) 784
Sympy [F] 785
Maxima [A] (verification not implemented) 785
Giac [F] 785
Mupad [F(-1)] 786

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=x\log \left(d+e\left(f^{c(a+bx)}\right)^n\right)-x\log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}$$

```
[Out] x*ln(d+e*(f^(c*(b*x+a)))^n)-x*ln(1+e*(f^(c*(b*x+a)))^n/d)-polylog(2,-e*(f^(
c*(b*x+a)))^n/d)/b/c/n/ln(f)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2318, 2221, 2317, 2438}

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=-\frac{\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+x\log \left(e\left(f^{c(a+bx)}\right)^n+d\right)-x\log \left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

```
[In] Int[Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] - P
olyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2318

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Dist[b*d*e*n*Log[F], Int[x*(
(F^(e*(c + d*x)))^n/(a + b*(F^(e*(c + d*x)))^n)), x], x] /; FreeQ[{F, a, b,
 c, d, e, n}, x] &&  !GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-(bcen\log(f))\int\frac{\left(f^{c(a+bx)}\right)^n x}{d+e\left(f^{c(a+bx)}\right)^n}\,dx\\
&= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)+\int\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)\,dx\\
&= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{d}\right)}{x}\,dx,x,\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}\\
&= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx = x \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) - x \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn \log(f)}$$

```
[In] Integrate[Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] - P
olyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(-\frac{e\left(f^{c(bx+a)}\right)^n}{d}\right)+\ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)\ln\left(-\frac{e\left(f^{c(bx+a)}\right)^n}{d}\right)}{bc\ln(f)n}$
default	$\frac{\operatorname{dilog}\left(-\frac{e\left(f^{c(bx+a)}\right)^n}{d}\right)+\ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)\ln\left(-\frac{e\left(f^{c(bx+a)}\right)^n}{d}\right)}{bc\ln(f)n}$
risch	$x\ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)-\frac{\operatorname{dilog}\left(\frac{d+f^{xbcn}f^{-xbcn}\left(f^{c(bx+a)}\right)^n e}{d}\right)}{cb\ln(f)n}-\frac{\ln\left(\frac{d+f^{xbcn}f^{-xbcn}\left(f^{c(bx+a)}\right)^n e}{d}\right)\ln\left(f^{c(b}\right.}{cb\ln(f)}$

```
[In] int(ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c/ln(f)/n*(dilog(-e*(f^(c*(b*x+a)))^n/d)+ln(d+e*(f^(c*(b*x+a)))^n)*ln(-
e*(f^(c*(b*x+a)))^n/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.41

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx = \frac{(bcnx+acn)\log\left(ef^{bcnx+acn}+d\right)\log(f)-(bcnx+acn)\log(f)\log\left(\frac{ef^{bcnx+acn}+d}{d}\right)-\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right.}{bcn\log(f)}$$

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] ((b*c*n*x + a*c*n)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f) - (b*c*n*x + a*c*n
)*log(f)*log((e*f^(b*c*n*x + a*c*n) + d)/d) - dilog(-(e*f^(b*c*n*x + a*c*n)
 + d)/d + 1))/(b*c*n*log(f))
```

Sympy [F]

$$\int \log \left(d + e\left(f^{c(a+bx)}\right)^n\right)\, dx = \int \log \left(d + e\left(f^{c(a+bx)}\right)^n\right) dx$$

```
[In] integrate(ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] Integral(log(d + e*(f**(c*(a + b*x)))**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \log \left(d + e\left(f^{c(a+bx)}\right)^n\right)\, dx = x \log \left(e f^{(bx+a)cn} + d\right) - \frac{bcnx \log \left(\frac{e f^{bcnx} f^{acn}}{d} + 1\right) \log (f) + \mathrm{Li}_2\left(-\frac{e f^{bcnx} f^{acn}}{d}\right)}{bcn \log (f)}$$

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] x*log(e*f^((b*x + a)*c*n) + d) - (b*c*n*x*log(e*f^(b*c*n*x)*f^(a*c*n)/d + 1
)*log(f) + dilog(-e*f^(b*c*n*x)*f^(a*c*n)/d))/(b*c*n*log(f))
```

Giac [F]

$$\int \log \left(d + e\left(f^{c(a+bx)}\right)^n\right)\, dx = \int \log \left(e\left(f^{(bx+a)c}\right)^n + d\right)\, dx$$

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(log(e*(f^((b*x + a)*c))^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\int \ln \left(d+e\,\left(f^{c\,(a+b\,x)}\right)^n\right)\,dx$$

```
[In] int(log(d + e*(f^(c*(a + b*x)))^n),x)
[Out] int(log(d + e*(f^(c*(a + b*x)))^n), x)
```

3.127 $\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$

Optimal result 787
Rubi [N/A] 787
Mathematica [N/A] 788
Maple [N/A] 788
Fricas [N/A] 788
Sympy [N/A] 788
Maxima [N/A] 789
Giac [N/A] 789
Mupad [N/A] 789

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\text{Int}\left(\frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

```
[In] Int[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Defer[Int][Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

```
[In] Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)}{x}dx$$

```
[In] int(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
[Out] int(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)}{x}\,dx$$

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="fricas")
[Out] integral(log(e*(f^(b*c*x + a*c))^n + d)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(d+e\left(f^{ac+bcx}\right)^n\right)}{x}\,dx$$

```
[In] integrate(ln(d+e*(f**(c*(b*x+a)))**n)/x,x)
[Out] Integral(log(d + e*(f**(a*c + b*c*x))**n)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)}{x}\,dx$$

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="maxima")
[Out] integrate(log(e*f^((b*x + a)*c*n) + d)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)}{x}\,dx$$

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="giac")
[Out] integrate(log(e*(f^((b*x + a)*c))^n + d)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx=\int \frac{\ln\left(d+e\left(f^{c\,(a+b\,x)}\right)^n\right)}{x}\,dx$$

```
[In] int(log(d + e*(f^(c*(a + b*x)))^n)/x,x)
[Out] int(log(d + e*(f^(c*(a + b*x)))^n)/x, x)
```

3.128 $\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi \right) \, dx$

Optimal result 790
Rubi [A] (verified) 790
Mathematica [A] (verified) 791
Maple [B] (verified) 791
Fricas [B] (verification not implemented) 792
Sympy [F] 792
Maxima [B] (verification not implemented) 792
Giac [F] 793
Mupad [F(-1)] 793

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi \right) \, dx=x \log (\pi)-\frac{\text{PolyLog}\left(2,-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi }\right)}{den \log (F)}$$

```
[Out] x*ln(Pi)-polylog(2,-b*(F^(e*(d*x+c)))^n/Pi)/d/e/n/ln(F)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2317, 2439, 2438}

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi \right) \, dx=x \log (\pi)-\frac{\text{PolyLog}\left(2,-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi }\right)}{den \log (F)}$$

```
[In] Int[Log[b*(F^(e*(c + d*x)))^n + Pi],x]
[Out] x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(\pi+bx)}{x}\,dx, x, \left(F^{e(c+dx)}\right)^n\right)}{den\log(F)}\\ &= x\log(\pi) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{\pi}\right)}{x}\,dx, x, \left(F^{e(c+dx)}\right)^n\right)}{den\log(F)}\\ &= x\log(\pi) - \frac{\text{Li}_2\left(-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den\log(F)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = x\log(\pi) - \frac{\text{PolyLog}\left(2, -\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den\log(F)}$$

```
[In] Integrate[Log[b*(F^(e*(c + d*x)))^n + Pi],x]
[Out] x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(39) = 78.

Time = 1.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.46

method	result
derivativedivides	$\frac{\left(\ln\left(b\left(F^{e(dx+c)}\right)^n+\pi\right)-\ln\left(\frac{b\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)\right)\ln\left(-\frac{b\left(F^{e(dx+c)}\right)^n}{\pi}\right)-\text{dilog}\left(\frac{b\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)}{de\ln(F)n}$
default	$\frac{\left(\ln\left(b\left(F^{e(dx+c)}\right)^n+\pi\right)-\ln\left(\frac{b\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)\right)\ln\left(-\frac{b\left(F^{e(dx+c)}\right)^n}{\pi}\right)-\text{dilog}\left(\frac{b\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)}{de\ln(F)n}$
risch	$x\ln\left(b\left(F^{e(dx+c)}\right)^n+\pi\right)-\frac{\text{dilog}\left(\frac{b\,F^{xned}F^{-xned}\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)}{\ln(F)den}-\frac{\ln\left(F^{e(dx+c)}\right)\ln\left(\frac{b\,F^{xned}F^{-xned}\left(F^{e(c}\right.\right.}{\pi}}{\ln(F)de}$

```
[In] int(ln(b*(F^(e*(d*x+c)))^n+Pi),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/e/ln(F)/n*((ln(b*(F^(e*(d*x+c)))^n+Pi)-ln((b*(F^(e*(d*x+c)))^n+Pi)/Pi))
*ln(-b*(F^(e*(d*x+c)))^n/Pi)-dilog((b*(F^(e*(d*x+c)))^n+Pi)/Pi))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(38) = 76$.

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.72

$$\int \log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = \frac{(denx+cen)\log\left(\pi+F^{denx+cen}b\right)\log(F)-(denx+cen)\log(F)\log\left(\frac{\pi+F^{denx+cen}b}{\pi}\right)-\mathrm{Li}_2\left(-\frac{\pi+F^{denx+cen}b}{\pi}\right.}{den\log(F)}$$

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="fricas")
```

```
[Out] ((d*e*n*x + c*e*n)*log(pi + F^(d*e*n*x + c*e*n)*b)*log(F) - (d*e*n*x + c*e*
n)*log(F)*log((pi + F^(d*e*n*x + c*e*n)*b)/pi) - dilog(-(pi + F^(d*e*n*x +
c*e*n)*b)/pi + 1))/(d*e*n*log(F))
```

Sympy [F]

$$\int \log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = \int \log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx$$

```
[In] integrate(ln(b*(F**(e*(d*x+c)))**n+pi),x)
```

```
[Out] Integral(log(b*(F**(e*(c + d*x)))**n + pi), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = x\log\left(\pi+F^{(dx+c)en}b\right) - \frac{denx\log\left(\frac{F^{denx}F^{cen}b}{\pi}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{denx}F^{cen}b}{\pi}\right)}{den\log(F)}$$

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="maxima")
```

```
[Out] x*log(pi + F^((d*x + c)*e*n)*b) - (d*e*n*x*log(F^(d*e*n*x)*F^(c*e*n)*b/pi +
 1)*log(F) + dilog(-F^(d*e*n*x)*F^(c*e*n)*b/pi))/(d*e*n*log(F))
```

Giac [F]

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right) dx = \int \log \left(\pi + \left(F^{(dx+c)e}\right)^n b\right) dx$$

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="giac")
[Out] integrate(log(pi + (F^((d*x + c)*e))^n*b), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi\right) dx = \int \ln \left(\Pi + b \left(F^{e\,(c+d\,x)}\right)^n\right) dx$$

```
[In] int(log(Pi + b*(F^(e*(c + d*x)))^n),x)
[Out] int(log(Pi + b*(F^(e*(c + d*x)))^n), x)
```

3.129 $\int \frac{1}{x(3+\log(x))} dx$

Optimal result . . . 794
Rubi [A] (verified) . . . 794
Mathematica [A] (verified) . . . 795
Maple [A] (verified) . . . 795
Fricas [A] (verification not implemented) . . . 795
Sympy [A] (verification not implemented) . . . 796
Maxima [A] (verification not implemented) . . . 796
Giac [B] (verification not implemented) . . . 796
Mupad [B] (verification not implemented) . . . 796

Optimal result

Integrand size = 10, antiderivative size = 5

$$\int \frac{1}{x(3+\log(x))} dx = \log(3+\log(x))$$

```
[Out] ln(3+ln(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2339, 29}

$$\int \frac{1}{x(3+\log(x))} dx = \log(\log(x)+3)$$

```
[In] Int[1/(x*(3 + Log[x])),x]
```

```
[Out] Log[3 + Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, 3+\log(x)\right)\\ &= \log(3+\log(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx = \log(3+\log(x))$$

```
[In] Integrate[1/(x*(3 + Log[x])),x]
[Out] Log[3 + Log[x]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln(3+\ln(x))$	6
default	$\ln(3+\ln(x))$	6
norman	$\ln(3+\ln(x))$	6
risch	$\ln(3+\ln(x))$	6
parallelrisch	$\ln(3+\ln(x))$	6

```
[In] int(1/x/(3+ln(x)),x,method=_RETURNVERBOSE)
[Out] ln(3+ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx = \log(\log(x)+3)$$

```
[In] integrate(1/x/(3+log(x)),x, algorithm="fricas")
[Out] log(log(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx=\log\left(\log\left(x\right)+3\right)$$

```
[In] integrate(1/x/(3+ln(x)),x)
[Out] log(log(x) + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx=\log\left(\log\left(x\right)+3\right)$$

```
[In] integrate(1/x/(3+log(x)),x, algorithm="maxima")
[Out] log(log(x) + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(5) = 10.

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 4.40

$$\int \frac{1}{x(3+\log(x))}\,dx=\frac{1}{2}\,\log\left(\frac{1}{4}\,\pi^2(\mathrm{sgn}(x)-1)^2+(\log\left(|x|\right)+3)^2\right)$$

```
[In] integrate(1/x/(3+log(x)),x, algorithm="giac")
[Out] 1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2)
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+\log(x))}\,dx=\ln\left(\ln\left(x\right)+3\right)$$

```
[In] int(1/(x*(log(x) + 3)),x)
[Out] log(log(x) + 3)
```

3.130 $\int \frac{\sqrt{1+\log(x)}}{x}\,dx$

Optimal result 797
Rubi [A] (verified) 797
Mathematica [A] (verified) 798
Maple [A] (verified) 798
Fricas [A] (verification not implemented) 798
Sympy [A] (verification not implemented) 799
Maxima [A] (verification not implemented) 799
Giac [A] (verification not implemented) 799
Mupad [B] (verification not implemented) 799

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2}{3}(1+\log(x))^{3/2}$$

```
[Out] 2/3*(1+ln(x))^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2339, 30}

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2}{3}(\log(x)+1)^{3/2}$$

```
[In] Int[Sqrt[1 + Log[x]]/x,x]
```

```
[Out] (2*(1 + Log[x])^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \sqrt{x}\,dx, x, 1+\log(x)\right)\\ &= \frac{2}{3}(1+\log(x))^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2}{3}(1+\log(x))^{3/2}$$

```
[In] Integrate[Sqrt[1 + Log[x]]/x,x]
[Out] (2*(1 + Log[x])^(3/2))/3
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}$	9
default	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}$	9

```
[In] int((1+ln(x))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/3*(1+ln(x))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}$$

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="fricas")
[Out] 2/3*(log(x) + 1)^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2(\log{(x)}+1)^{\frac{3}{2}}}{3}$$

```
[In] integrate((1+ln(x))**(1/2)/x,x)
[Out] 2*(log(x) + 1)**(3/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}}$$

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="maxima")
[Out] 2/3*(log(x) + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}}$$

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="giac")
[Out] 2/3*(log(x) + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{1+\log(x)}}{x}\,dx = \sqrt{\ln{(x)}+1}\,\left(\frac{2\,\ln{(x)}}{3}+\frac{2}{3}\right)$$

```
[In] int((log(x) + 1)^(1/2)/x,x)
[Out] (log(x) + 1)^(1/2)*((2*log(x))/3 + 2/3)
```

3.131 $\int \frac{(1+\log(x))^5}{x} dx$

Optimal result 800
Rubi [A] (verified) 800
Mathematica [A] (verified) 801
Maple [A] (verified) 801
Fricas [B] (verification not implemented) 802
Sympy [B] (verification not implemented) 802
Maxima [A] (verification not implemented) 802
Giac [B] (verification not implemented) 803
Mupad [B] (verification not implemented) 803

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{(1+\log(x))^5}{x} dx = \frac{1}{6}(1+\log(x))^6$$

```
[Out] 1/6*(1+ln(x))^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2339, 30}

$$\int \frac{(1+\log(x))^5}{x} dx = \frac{1}{6}(\log(x)+1)^6$$

```
[In] Int[(1 + Log[x])^5/x,x]
```

```
[Out] (1 + Log[x])^6/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x^5\,dx, x, 1+\log(x)\right)$$
$$= \frac{1}{6}(1+\log(x))^6$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{1}{6}(1+\log(x))^6$$

```
[In] Integrate[(1 + Log[x])^5/x,x]
[Out] (1 + Log[x])^6/6
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{(1+\ln(x))^6}{6}$	9
default	$\frac{(1+\ln(x))^6}{6}$	9
norman	$\ln(x)^5+\ln(x)+\frac{5\ln(x)^2}{2}+\frac{10\ln(x)^3}{3}+\frac{5\ln(x)^4}{2}+\frac{\ln(x)^6}{6}$	32
risch	$\ln(x)^5+\ln(x)+\frac{5\ln(x)^2}{2}+\frac{10\ln(x)^3}{3}+\frac{5\ln(x)^4}{2}+\frac{\ln(x)^6}{6}$	32
parts	$\ln(x)^5+\ln(x)+\frac{5\ln(x)^2}{2}+\frac{10\ln(x)^3}{3}+\frac{5\ln(x)^4}{2}+\frac{\ln(x)^6}{6}$	32

```
[In] int((1+ln(x))^5/x,x,method=_RETURNVERBOSE)
[Out] 1/6*(1+ln(x))^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{1}{6}\log(x)^6 + \log(x)^5 + \frac{5}{2}\log(x)^4 + \frac{10}{3}\log(x)^3 + \frac{5}{2}\log(x)^2 + \log(x)$$

```
[In] integrate((1+log(x))^5/x,x, algorithm="fricas")
```

```
[Out] 1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + log
(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(7) = 14$.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.90

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{\log(x)^6}{6} + \log(x)^5 + \frac{5\log(x)^4}{2} + \frac{10\log(x)^3}{3} + \frac{5\log(x)^2}{2} + \log(x)$$

```
[In] integrate((1+ln(x))**5/x,x)
```

```
[Out] log(x)**6/6 + log(x)**5 + 5*log(x)**4/2 + 10*log(x)**3/3 + 5*log(x)**2/2 +
log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{1}{6}(\log(x)+1)^6$$

```
[In] integrate((1+log(x))^5/x,x, algorithm="maxima")
```

```
[Out] 1/6*(log(x) + 1)^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{1}{6}\log(x)^6 + \log(x)^5 + \frac{5}{2}\log(x)^4 + \frac{10}{3}\log(x)^3 + \frac{5}{2}\log(x)^2 + \log(x)$$

```
[In] integrate((1+log(x))^5/x,x, algorithm="giac")
[Out] 1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + log
(x)
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \frac{(1+\log(x))^5}{x}\,dx = \frac{\ln(x)\,(\ln(x)+2)\,\left(\ln(x)^2+\ln(x)+1\right)\,\left(\ln(x)^2+3\ln(x)+3\right)}{6}$$

```
[In] int((log(x) + 1)^5/x,x)
[Out] (log(x)*(log(x) + 2)*(log(x) + log(x)^2 + 1)*(3*log(x) + log(x)^2 + 3))/6
```

3.132 $\int \frac{1}{x\sqrt{\log(x)}} \, dx$

Optimal result 804
Rubi [A] (verified) 804
Mathematica [A] (verified) 805
Maple [A] (verified) 805
Fricas [A] (verification not implemented) 805
Sympy [A] (verification not implemented) 806
Maxima [A] (verification not implemented) 806
Giac [A] (verification not implemented) 806
Mupad [B] (verification not implemented) 806

Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{1}{x\sqrt{\log(x)}} \, dx = 2\sqrt{\log(x)}$$

```
[Out] 2*ln(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2339, 30}

$$\int \frac{1}{x\sqrt{\log(x)}} \, dx = 2\sqrt{\log(x)}$$

```
[In] Int[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \log(x)\right)\\ &= 2\sqrt{\log(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log(x)}$$

```
[In] Integrate[1/(x*Sqrt[Log[x]]),x]
[Out] 2*Sqrt[Log[x]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\sqrt{\ln(x)}$	7
default	$2\sqrt{\ln(x)}$	7

```
[In] int(1/x/ln(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*ln(x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log(x)}$$

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\sqrt{\log{(x)}}$$

```
[In] integrate(1/x/ln(x)**(1/2),x)
[Out] 2*sqrt(log(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\,\sqrt{\log\left(x\right)}$$

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\,\sqrt{\log\left(x\right)}$$

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{\log(x)}}\,dx = 2\,\sqrt{\ln\left(x\right)}$$

```
[In] int(1/(x*log(x)^(1/2)),x)
[Out] 2*log(x)^(1/2)
```

3.133 $\int \frac{1}{x(1+\log^2(x))}\,dx$

Optimal result 807
Rubi [A] (verified) 807
Mathematica [A] (verified) 808
Maple [A] (verified) 808
Fricas [A] (verification not implemented) 808
Sympy [B] (verification not implemented) 809
Maxima [A] (verification not implemented) 809
Giac [A] (verification not implemented) 809
Mupad [B] (verification not implemented) 810

Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {209}

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan(\log(x))$$

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \log(x)\right) \\ &= \tan^{-1}(\log(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan(\log(x))$$

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
[Out] ArcTan[Log[x]]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan\left(\ln\left(x\right)\right)$	4
default	$\arctan\left(\ln\left(x\right)\right)$	4
risch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20
parallelrisch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20

```
[In] int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
[Out] arctan(log(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

```
[In] integrate(1/x/(1+ln(x)**2),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
[Out] arctan(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
[Out] arctan(log(x))
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{atan}(\ln{(x)})$$

```
[In] int(1/(x*(log(x)^2 + 1)),x)
[Out] atan(log(x))
```

3.134 $\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx$

Optimal result . . . 811
Rubi [A] (verified) . . . 811
Mathematica [B] (verified) . . . 812
Maple [A] (verified) . . . 812
Fricas [A] (verification not implemented) . . . 813
Sympy [F] . . . 813
Maxima [A] (verification not implemented) . . . 813
Giac [F(-1)] . . . 814
Mupad [B] (verification not implemented) . . . 814

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$

```
[Out] arctanh(ln(x)/(-3+ln(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {223, 212}

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{\log^2(x)-3}}\right)$$

```
[In] Int[1/(x*Sqrt[-3 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[-3 + Log[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{-3+x^2}}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)\\ &= \tanh^{-1}\left(\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = -\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)+\frac{1}{2}\log\left(1+\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$

```
[In] Integrate[1/(x*Sqrt[-3 + Log[x]^2]),x]
[Out] -1/2*Log[1 - Log[x]/Sqrt[-3 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[-3 + Log[x]^
2]]/2
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\ln\left(\ln(x)+\sqrt{-3+\ln(x)^2}\right)$	13
default	$\ln\left(\ln(x)+\sqrt{-3+\ln(x)^2}\right)$	13

```
[In] int(1/x/(-3+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(ln(x)+(-3+ln(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = -\log\left(\sqrt{\log(x)^2-3}-\log(x)\right)$$

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(log(x)^2 - 3) - log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{\log(x)^2-3}}\,dx$$

```
[In] integrate(1/x/(-3+ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(log(x)**2 - 3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \log\left(2\sqrt{\log(x)^2-3}+2\log(x)\right)$$

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] log(2*sqrt(log(x)^2 - 3) + 2*log(x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \ln\left(\ln(x)+\sqrt{\ln(x)^2-3}\right)$$

```
[In] int(1/(x*(log(x)^2 - 3)^(1/2)),x)
[Out] log(log(x) + (log(x)^2 - 3)^(1/2))
```

3.135 $\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx$

Optimal result . . . 815
Rubi [A] (verified) . . . 815
Mathematica [B] (verified) . . . 816
Maple [A] (verified) . . . 816
Fricas [B] (verification not implemented) . . . 816
Sympy [F] . . . 817
Maxima [A] (verification not implemented) . . . 817
Giac [A] (verification not implemented) . . . 817
Mupad [B] (verification not implemented) . . . 818

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3\log(x)}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {222}

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3\log(x)}{2}\right)$$

```
[In] Int[1/(x*Sqrt[4 - 9*Log[x]^2]),x]
[Out] ArcSin[(3*Log[x])/2]/3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{4-9x^2}}\,dx, x, \log(x)\right) \\ &= \frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \frac{2}{3}\arctan\left(\frac{3\log(x)}{-2+\sqrt{4-9\log^2(x)}}\right)$$

```
[In] Integrate[1/(x*Sqrt[4 - 9*Log[x]^2]),x]
[Out] (2*ArcTan[(3*Log[x])/(-2 + Sqrt[4 - 9*Log[x]^2])])/3
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\arcsin\left(\frac{3\ln(x)}{2}\right)}{3}$	8
default	$\frac{\arcsin\left(\frac{3\ln(x)}{2}\right)}{3}$	8

```
[In] int(1/x/(4-9*ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsin(3/2*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = -\frac{2}{3}\arctan\left(\frac{\sqrt{-9\log(x)^2+4}-2}{3\log(x)}\right)$$

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -2/3*arctan(1/3*(sqrt(-9*log(x)^2 + 4) - 2)/log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{-\left(3\log\left(x\right)-2\right)\left(3\log\left(x\right)+2\right)}}\,dx$$

```
[In] integrate(1/x/(4-9*ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(3*log(x) - 2)*(3*log(x) + 2))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3}{2}\log\left(x\right)\right)$$

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*arcsin(3/2*log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3}{2}\log\left(x\right)\right)$$

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*arcsin(3/2*log(x))
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx = \frac{\operatorname{asin}\left(\frac{3\ln(x)}{2}\right)}{3}$$

```
[In] int(1/(x*(4 - 9*log(x)^2)^(1/2)),x)
[Out] asin((3*log(x))/2)/3
```

3.136 $\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx$

Optimal result 819
Rubi [A] (verified) 819
Mathematica [B] (verified) 820
Maple [A] (verified) 820
Fricas [B] (verification not implemented) 820
Sympy [F] 821
Maxima [A] (verification not implemented) 821
Giac [B] (verification not implemented) 821
Mupad [B] (verification not implemented) 822

Optimal result

Integrand size = 14, antiderivative size = 7

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \operatorname{arcsinh}\left(\frac{\log(x)}{2}\right)$$

```
[Out] arcsinh(1/2*ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {221}

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \operatorname{arcsinh}\left(\frac{\log(x)}{2}\right)$$

```
[In] Int[1/(x*Sqrt[4 + Log[x]^2]),x]
[Out] ArcSinh[Log[x]/2]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \log(x)\right) \\ &= \sinh^{-1}\left(\frac{\log(x)}{2}\right) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = -\log\left(-\log(x)+\sqrt{4+\log^2(x)}\right)$$

```
[In] Integrate[1/(x*Sqrt[4 + Log[x]^2]),x]
[Out] -Log[-Log[x] + Sqrt[4 + Log[x]^2]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\operatorname{arcsinh}\left(\frac{\ln(x)}{2}\right)$	6
default	$\operatorname{arcsinh}\left(\frac{\ln(x)}{2}\right)$	6

```
[In] int(1/x/(4+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(1/2*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = -\log\left(\sqrt{\log(x)^2+4}-\log(x)\right)$$

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -log(sqrt(log(x)^2 + 4) - log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{\log{(x)}^2+4}}\,dx$$

```
[In] integrate(1/x/(4+ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(log(x)**2 + 4)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \operatorname{arsinh}\left(\frac{1}{2}\log{(x)}\right)$$

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsinh(1/2*log(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = -\log\left(\sqrt{\log{(x)}^2+4}-\log{(x)}\right)$$

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(sqrt(log(x)^2 + 4) - log(x))
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx = \operatorname{asinh}\left(\frac{\ln(x)}{2}\right)$$

```
[In] int(1/(x*(log(x)^2 + 4)^(1/2)),x)
[Out] asinh(log(x)/2)
```

3.137 $\int \frac{1}{x(2+3\log^3(6x))}\,dx$

Optimal result . . . 823
Rubi [A] (verified) . . . 823
Mathematica [A] (verified) . . . 825
Maple [C] (verified) . . . 826
Fricas [A] (verification not implemented) . . . 826
Sympy [A] (verification not implemented) . . . 827
Maxima [A] (verification not implemented) . . . 827
Giac [F] . . . 827
Mupad [B] (verification not implemented) . . . 828

Optimal result

Integrand size = 16, antiderivative size = 111

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\log(6x)}{\sqrt[6]{3}}\right)}{2^{2/3}3^{5/6}}+\frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} - \frac{\log\left(2^{2/3}-\sqrt[3]{6}\log(6x)+3^{2/3}\log^2(6x)\right)}{6\ 2^{2/3}\sqrt[3]{3}}$$

```
[Out] 1/6*arctan(1/3*2^(2/3)*ln(6*x)*3^(5/6)-1/3*3^(1/2))*2^(1/3)*3^(1/6)+1/18*ln
(2^(1/3)+3^(1/3)*ln(6*x))*2^(1/3)*3^(2/3)-1/36*ln(2^(2/3)-6^(1/3)*ln(6*x)+3
^(2/3)*ln(6*x)^2)*2^(1/3)*3^(2/3)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\log(6x)}{\sqrt[6]{3}}\right)}{2^{2/3}3^{5/6}} - \frac{\log\left(3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}\right)}{6\ 2^{2/3}\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}$$

```
[In] Int[1/(x*(2 + 3*Log[6*x]^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] - (2^(2/3)*Log[6*x])/3^(1/6)]/(2^(2/3)*3^(5/6))) + Log[2
^(1/3) + 3^(1/3)*Log[6*x]]/(3*2^(2/3)*3^(1/3)) - Log[2^(2/3) - 6^(1/3)*Log[
6*x] + 3^(2/3)*Log[6*x]^2]/(6*2^(2/3)*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{2+3x^3}\,dx, x, \log(6x)\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}+\sqrt[3]{3}x}\,dx, x, \log(6x)\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{2}-\sqrt[3]{3}x}{2^{2/3}-\sqrt[3]{6}x+3^{2/3}x^2}\,dx, x, \log(6x)\right)}{3\ 2^{2/3}} \\
&= \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{6}x+3^{2/3}x^2}\,dx, x, \log(6x)\right)}{2\sqrt[3]{2}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{6}+2\ 3^{2/3}x}{2^{2/3}-\sqrt[3]{6}x+3^{2/3}x^2}\,dx, x, \log(6x)\right)}{6\ 2^{2/3}\sqrt[3]{3}} \\
&= \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} - \frac{\log\left(2^{2/3}-\sqrt[3]{6}\log(6x)+3^{2/3}\log^2(6x)\right)}{6\ 2^{2/3}\sqrt[3]{3}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-2^{2/3}\sqrt[3]{3}\log(6x)\right)}{2^{2/3}\sqrt[3]{3}} \\
&= -\frac{\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{3}\log(6x)}{\sqrt{3}}\right)}{2^{2/3}3^{5/6}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} \\
&\quad - \frac{\log\left(2^{2/3}-\sqrt[3]{6}\log(6x)+3^{2/3}\log^2(6x)\right)}{6\ 2^{2/3}\sqrt[3]{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$
\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx
= \frac{6\arctan\left(\frac{-1+2^{2/3}\sqrt[3]{3}\log(6x)}{\sqrt{3}}\right)+\sqrt{3}\left(2\log\left(2+2^{2/3}\sqrt[3]{3}\log(6x)\right)-\log\left(2-2^{2/3}\sqrt[3]{3}\log(6x)+\sqrt[3]{2}3^{2/3}\log^2(\cdots\right.\right.}{6\ 2^{2/3}3^{5/6}}
$$

```
[In] Integrate[1/(x*(2 + 3*Log[6*x]^3)),x]
```

```
[Out] (6*ArcTan[(-1 + 2^(2/3)*3^(1/3)*Log[6*x])/Sqrt[3]] + Sqrt[3]*(2*Log[2 + 2^(
2/3)*3^(1/3)*Log[6*x]] - Log[2 - 2^(2/3)*3^(1/3)*Log[6*x] + 2^(1/3)*3^(2/3)
*Log[6*x]^2]))/(6*2^(2/3)*3^(5/6))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.21

method	result	si
risch	$\sum_{\_R=\mathrm{RootOf}(324\_Z^3-1)} \_R\ln(\ln(6x)+6\_R)$	2
derivativedivides	$\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{18}-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)^2-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln(6x)}{3}+\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{36}+\frac{2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}3^{\frac{1}{3}}\ln(6x)-1\right)}{3}\right)}{6}$	8
default	$\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{18}-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)^2-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln(6x)}{3}+\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{36}+\frac{2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}3^{\frac{1}{3}}\ln(6x)-1\right)}{3}\right)}{6}$	8

```
[In] int(1/x/(2+3*ln(6*x)^3),x,method=_RETURNVERBOSE)
[Out] sum(_R*ln(ln(6*x)+6*_R),_R=RootOf(324*_Z^3-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = -\frac{1}{72}\cdot 12^{\frac{2}{3}}\log\left(6\log(6x)^2-12^{\frac{2}{3}}\log(6x)+2\cdot 12^{\frac{1}{3}}\right)+\frac{1}{36}\cdot 12^{\frac{2}{3}}\log\left(12^{\frac{2}{3}}+6\log(6x)\right)+\frac{1}{6}\cdot 12^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 12^{\frac{1}{6}}\left(12^{\frac{2}{3}}\log(6x)-12^{\frac{1}{3}}\right)\right)$$

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="fricas")
[Out] -1/72*12^(2/3)*log(6*log(6*x)^2 - 12^(2/3)*log(6*x) + 2*12^(1/3)) + 1/36*12
^(2/3)*log(12^(2/3) + 6*log(6*x)) + 1/6*12^(1/6)*arctan(1/6*12^(1/6)*(12^(2
/3)*log(6*x) - 12^(1/3)))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.15

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = \text{RootSum}\left(324z^3-1,\left(i\mapsto i\log\left(6i+\log\left(6x\right)\right)\right)\right)$$

```
[In] integrate(1/x/(2+3*ln(6*x)**3),x)
[Out] RootSum(324*_z**3 - 1, Lambda(_i, _i*log(6*_i + log(6*x))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}\log\left(6\,x\right)^2-3^{\frac{1}{3}}2^{\frac{1}{3}}\log\left(6\,x\right)+2^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(\frac{1}{3}\cdot 3^{\frac{2}{3}}\left(3^{\frac{1}{3}}\log\left(6\,x\right)+2^{\frac{1}{3}}\right)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{1}{6}}2^{\frac{2}{3}}\left(2\cdot 3^{\frac{2}{3}}\log\left(6\,x\right)-3^{\frac{1}{3}}2^{\frac{1}{3}}\right)\right)$$

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="maxima")
[Out] -1/36*3^(2/3)*2^(1/3)*log(3^(2/3)*log(6*x)^2 - 3^(1/3)*2^(1/3)*log(6*x) + 2
^(2/3)) + 1/18*3^(2/3)*2^(1/3)*log(1/3*3^(2/3)*(3^(1/3)*log(6*x) + 2^(1/3))
) + 1/6*3^(1/6)*2^(1/3)*arctan(1/6*3^(1/6)*2^(2/3)*(2*3^(2/3)*log(6*x) - 3^
(1/3)*2^(1/3)))
```

Giac [F]

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = \int \frac{1}{\left(3\log\left(6\,x\right)^3+2\right)x}\,dx$$

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="giac")
[Out] integrate(1/((3*log(6*x)^3 + 2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx=\frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}+\frac{2^{1/3}\,3^{2/3}}{3\,x^2}\right)}{18}+\frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}+\frac{2^{1/3}\,3^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,x^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18}-\frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}-\frac{2^{1/3}\,3^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,x^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18}$$

```
[In] int(1/(x*(3*log(6*x)^3 + 2)),x)
```

```
[Out] (2^(1/3)*3^(2/3)*log(log(6*x)/x^2 + (2^(1/3)*3^(2/3))/(3*x^2)))/18 + (2^(1/
3)*3^(2/3)*log(log(6*x)/x^2 + (2^(1/3)*3^(2/3)*((3^(1/2)*1i)/2 - 1/2))/(3*x
^2))*((3^(1/2)*1i)/2 - 1/2))/18 - (2^(1/3)*3^(2/3)*log(log(6*x)/x^2 - (2^(1
/3)*3^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(3*x^2))*((3^(1/2)*1i)/2 + 1/2))/18
```

3.138 $\int \frac{\log(\log(6x))}{x\log(6x)}\,dx$

Optimal result 829
Rubi [A] (verified) 829
Mathematica [A] (verified) 830
Maple [A] (verified) 830
Fricas [A] (verification not implemented) 830
Sympy [A] (verification not implemented) 831
Maxima [A] (verification not implemented) 831
Giac [A] (verification not implemented) 831
Mupad [B] (verification not implemented) 831

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\log^2(\log(6x))$$

```
[Out] 1/2*ln(ln(6*x))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2338}

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\log^2(\log(6x))$$

```
[In] Int[Log[Log[6*x]]/(x*Log[6*x]),x]
```

```
[Out] Log[Log[6*x]]^2/2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \log(6x)\right) \\ &= \frac{1}{2}\log^2(\log(6x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\log^2(\log(6x))$$

```
[In] Integrate[Log[Log[6*x]]/(x*Log[6*x]),x]
[Out] Log[Log[6*x]]^2/2
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\ln(\ln(6x))^2}{2}$	10
default	$\frac{\ln(\ln(6x))^2}{2}$	10
norman	$\frac{\ln(\ln(6x))^2}{2}$	10
risch	$\frac{\ln(\ln(6x))^2}{2}$	10

```
[In] int(ln(ln(6*x))/x/ln(6*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(ln(6*x))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\log\left(\log\left(6\,x\right)\right)^2$$

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="fricas")
[Out] 1/2*log(log(6*x))^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{\log\left(\log\left(6x\right)\right)^2}{2}$$

```
[In] integrate(ln(ln(6*x))/x/ln(6*x),x)
[Out] log(log(6*x))**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\,\log\left(\log\left(6\,x\right)\right)^2$$

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="maxima")
[Out] 1/2*log(log(6*x))^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{1}{2}\,\log\left(\log\left(6\,x\right)\right)^2$$

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="giac")
[Out] 1/2*log(log(6*x))^2
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(\log(6x))}{x\log(6x)}\,dx = \frac{\ln\left(\ln\left(6\,x\right)\right)^2}{2}$$

```
[In] int(log(log(6*x))/(x*log(6*x)),x)
[Out] log(log(6*x))^2/2
```

3.139 $\int \frac{2^{\log(x)}}{x}\,dx$

Optimal result . . . 832
Rubi [A] (verified) . . . 832
Mathematica [A] (verified) . . . 833
Maple [A] (verified) . . . 833
Fricas [A] (verification not implemented) . . . 834
Sympy [A] (verification not implemented) . . . 834
Maxima [A] (verification not implemented) . . . 834
Giac [A] (verification not implemented) . . . 834
Mupad [B] (verification not implemented) . . . 835

Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{2^{\log(x)}}{\log(2)}$$

```
[Out] 2^ln(x)/ln(2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2306, 30}

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{x^{\log(2)}}{\log(2)}$$

```
[In] Int[2^Log[x]/x,x]
```

```
[Out] x^Log[2]/Log[2]
```

<u>Rule 30</u>

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

<u>Rule 2306</u>

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{-1+\log(2)}\,dx\\ &= \frac{x^{\log(2)}}{\log(2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{2^{\log(x)}}{\log(2)}$$

```
[In] Integrate[2^Log[x]/x,x]
[Out] 2^Log[x]/Log[2]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
gosper	$\frac{2^{\ln(x)}}{\ln(2)}$	10
derivativedivides	$\frac{2^{\ln(x)}}{\ln(2)}$	10
default	$\frac{2^{\ln(x)}}{\ln(2)}$	10
risch	$\frac{x^{\ln(2)}}{\ln(2)}$	10
parallelrisch	$\frac{2^{\ln(x)}}{\ln(2)}$	10
norman	$\frac{e^{\ln(2)\ln(x)}}{\ln(2)}$	12

```
[In] int(2^ln(x)/x,x,method=_RETURNVERBOSE)
[Out] 2^ln(x)/ln(2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{e^{(\log(2)\log(x))}}{\log(2)}$$

```
[In] integrate(2^log(x)/x,x, algorithm="fricas")
[Out] e^(log(2)*log(x))/log(2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{2^{\log(x)}}{\log(2)}$$

```
[In] integrate(2**ln(x)/x,x)
[Out] 2**log(x)/log(2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{2^{\log(x)}}{\log(2)}$$

```
[In] integrate(2^log(x)/x,x, algorithm="maxima")
[Out] 2^log(x)/log(2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{2^{\log(x)}}{x}\,dx = \frac{2^{\log(x)}}{\log(2)}$$

```
[In] integrate(2^log(x)/x,x, algorithm="giac")
[Out] 2^log(x)/log(2)
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{2^{\log(x)}}{x} \, dx = \frac{x^{\ln(2)}}{\ln(2)}$$

```
[In] int(2^log(x)/x,x)
```

```
[Out] x^log(2)/log(2)
```

3.140 $\int \frac{\sin^2(\log(x))}{x} \, dx$

Optimal result . . . 836
Rubi [A] (verified) . . . 836
Mathematica [A] (verified) . . . 837
Maple [A] (verified) . . . 837
Fricas [A] (verification not implemented) . . . 838
Sympy [B] (verification not implemented) . . . 838
Maxima [A] (verification not implemented) . . . 839
Giac [A] (verification not implemented) . . . 839
Mupad [B] (verification not implemented) . . . 839

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{\sin^2(\log(x))}{x} \, dx = \frac{\log(x)}{2} - \frac{1}{2}\cos(\log(x))\sin(\log(x))$$

```
[Out] 1/2*ln(x)-1/2*cos(ln(x))*sin(ln(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2715, 8}

$$\int \frac{\sin^2(\log(x))}{x} \, dx = \frac{\log(x)}{2} - \frac{1}{2}\sin(\log(x))\cos(\log(x))$$

```
[In] Int[Sin[Log[x]]^2/x,x]
```

```
[Out] Log[x]/2 - (Cos[Log[x]]*Sin[Log[x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \sin^2(x)\,dx, x, \log(x)\right) \\ &= -\frac{1}{2}\cos(\log(x))\sin(\log(x)) + \frac{1}{2}\text{Subst}(\int 1\,dx, x, \log(x)) \\ &= \frac{\log(x)}{2} - \frac{1}{2}\cos(\log(x))\sin(\log(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\sin^2(\log(x))}{x}\,dx = \frac{\log(x)}{2} - \frac{1}{4}\sin(2\log(x))$$

```
[In] Integrate[Sin[Log[x]]^2/x,x]
[Out] Log[x]/2 - Sin[2*Log[x]]/4
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\ln\left(\sqrt{x}\right) - \frac{\sin(2\ln(x))}{4}$	13
derivativedivides	$\frac{\ln(x)}{2} - \frac{\cos(\ln(x))\sin(\ln(x))}{2}$	14
default	$\frac{\ln(x)}{2} - \frac{\cos(\ln(x))\sin(\ln(x))}{2}$	14
risch	$\frac{\ln(x)}{2} + \frac{i x^{2i}}{8} - \frac{i x^{-2i}}{8}$	24
norman	$\frac{\tan^3\left(\frac{\ln(x)}{2}\right)+\frac{\ln(x)}{2}+\ln(x)\left(\tan^2\left(\frac{\ln(x)}{2}\right)\right)+\frac{\ln(x)\left(\tan^4\left(\frac{\ln(x)}{2}\right)\right)}{2}-\tan\left(\frac{\ln(x)}{2}\right)}{\left(1+\tan^2\left(\frac{\ln(x)}{2}\right)\right)^2}$	53

```
[In] int(sin(ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] ln(x^(1/2))-1/4*sin(2*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\sin^2(\log(x))}{x}\,dx = -\frac{1}{2}\cos(\log(x))\sin(\log(x)) + \frac{1}{2}\log(x)$$

```
[In] integrate(sin(log(x))^2/x,x, algorithm="fricas")
[Out] -1/2*cos(log(x))*sin(log(x)) + 1/2*log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(15) = 30.

Time = 1.02 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.18

$$\int \frac{\sin^2(\log(x))}{x}\,dx = \frac{\log(x)\tan^4\left(\frac{\log(x)}{2}\right)}{2\tan^4\left(\frac{\log(x)}{2}\right) + 4\tan^2\left(\frac{\log(x)}{2}\right) + 2} + \frac{2\log(x)\tan^2\left(\frac{\log(x)}{2}\right)}{2\tan^4\left(\frac{\log(x)}{2}\right) + 4\tan^2\left(\frac{\log(x)}{2}\right) + 2} + \frac{\log(x)}{2\tan^4\left(\frac{\log(x)}{2}\right) + 4\tan^2\left(\frac{\log(x)}{2}\right) + 2} + \frac{2\tan^3\left(\frac{\log(x)}{2}\right)}{2\tan^4\left(\frac{\log(x)}{2}\right) + 4\tan^2\left(\frac{\log(x)}{2}\right) + 2} - \frac{2\tan\left(\frac{\log(x)}{2}\right)}{2\tan^4\left(\frac{\log(x)}{2}\right) + 4\tan^2\left(\frac{\log(x)}{2}\right) + 2}$$

```
[In] integrate(sin(ln(x))**2/x,x)
[Out] log(x)*tan(log(x)/2)**4/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*l
og(x)*tan(log(x)/2)**2/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + log(
x)/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*tan(log(x)/2)**3/(2*ta
n(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) - 2*tan(log(x)/2)/(2*tan(log(x)/2)
**4 + 4*tan(log(x)/2)**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(\log(x))}{x} \, dx = \frac{1}{2} \log(x) - \frac{1}{4} \sin(2 \log(x))$$

```
[In] integrate(sin(log(x))^2/x,x, algorithm="maxima")
[Out] 1/2*log(x) - 1/4*sin(2*log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(\log(x))}{x} \, dx = \frac{1}{2} \log(x) - \frac{1}{4} \sin(2 \log(x))$$

```
[In] integrate(sin(log(x))^2/x,x, algorithm="giac")
[Out] 1/2*log(x) - 1/4*sin(2*log(x))
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(\log(x))}{x} \, dx = \frac{\ln(x)}{2} - \frac{\sin(2 \ln(x))}{4}$$

```
[In] int(sin(log(x))^2/x,x)
[Out] log(x)/2 - sin(2*log(x))/4
```

3.141 $\int \frac{7-\log(x)}{x(3+\log(x))} \, dx$

Optimal result 840
Rubi [A] (verified) 840
Mathematica [A] (verified) 841
Maple [A] (verified) 841
Fricas [A] (verification not implemented) 842
Sympy [A] (verification not implemented) 842
Maxima [A] (verification not implemented) 842
Giac [B] (verification not implemented) 842
Mupad [B] (verification not implemented) 843

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{7-\log(x)}{x(3+\log(x))} \, dx = -\log(x)+10\log(3+\log(x))$$

```
[Out] -ln(x)+10*ln(3+ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2412, 45}

$$\int \frac{7-\log(x)}{x(3+\log(x))} \, dx = 10\log(\log(x)+3)-\log(x)$$

```
[In] Int[(7 - Log[x])/(x*(3 + Log[x])),x]
```

```
[Out] -Log[x] + 10*Log[3 + Log[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
```

```
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{7-x}{3+x}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(-1+\frac{10}{3+x}\right)\,dx, x, \log(x)\right)\\ &= -\log(x)+10\log(3+\log(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = -\log(x)+10\log(3+\log(x))$$

```
[In] Integrate[(7 - Log[x])/(x*(3 + Log[x])),x]
[Out] -Log[x] + 10*Log[3 + Log[x]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\ln(x)+10\ln(3+\ln(x))$	13
default	$-\ln(x)+10\ln(3+\ln(x))$	13
norman	$-\ln(x)+10\ln(3+\ln(x))$	13
risch	$-\ln(x)+10\ln(3+\ln(x))$	13
parallelrisch	$-\ln(x)+10\ln(3+\ln(x))$	13

```
[In] int((7-ln(x))/x/(3+ln(x)),x,method=_RETURNVERBOSE)
[Out] -ln(x)+10*ln(3+ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = -\log(x) + 10\log(\log(x) + 3)$$

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="fricas")
[Out] -log(x) + 10*log(log(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = -\log(x) + 10\log(\log(x) + 3)$$

```
[In] integrate((7-ln(x))/x/(3+ln(x)),x)
[Out] -log(x) + 10*log(log(x) + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = -\log(x) + 10\log(\log(x) + 3)$$

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="maxima")
[Out] -log(x) + 10*log(log(x) + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = 5\log\left(\frac{1}{4}\pi^2(\mathrm{sgn}(x) - 1)^2 + (\log(|x|) + 3)^2\right) - \log(x)$$

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="giac")
[Out] 5*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2) - log(x)
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = 10\,\ln\left(\ln\left(x\right)+3\right)-\ln\left(x\right)$$

```
[In] int(-(log(x) - 7)/(x*(log(x) + 3)),x)
[Out] 10*log(log(x) + 3) - log(x)
```

3.142 $\int \frac{(2-\log(x))(3+\log(x))^2}{x} \, dx$

Optimal result 844
Rubi [A] (verified) 844
Mathematica [A] (verified) 845
Maple [A] (verified) 845
Fricas [A] (verification not implemented) 846
Sympy [A] (verification not implemented) 846
Maxima [A] (verification not implemented) 846
Giac [A] (verification not implemented) 846
Mupad [B] (verification not implemented) 847

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x} \, dx = \frac{5}{3}(3+\log(x))^3 - \frac{1}{4}(3+\log(x))^4$$

```
[Out] 5/3*(3+ln(x))^3-1/4*(3+ln(x))^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2412, 45}

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x} \, dx = \frac{5}{3}(\log(x)+3)^3 - \frac{1}{4}(\log(x)+3)^4$$

```
[In] Int[((2 - Log[x])*(3 + Log[x])^2)/x,x]
```

```
[Out] (5*(3 + Log[x])^3)/3 - (3 + Log[x])^4/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
```

```
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int (2-x)(3+x)^2\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(5(3+x)^2-(3+x)^3\right)\,dx, x, \log(x)\right)\\ &= \frac{5}{3}(3+\log(x))^3-\frac{1}{4}(3+\log(x))^4\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = 18\log(x)+\frac{3\log^2(x)}{2}-\frac{4\log^3(x)}{3}-\frac{\log^4(x)}{4}$$

```
[In] Integrate[((2 - Log[x])*(3 + Log[x])^2)/x,x]
[Out] 18*Log[x] + (3*Log[x]^2)/2 - (4*Log[x]^3)/3 - Log[x]^4/4
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
default	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
norman	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
risch	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
parts	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24

```
[In] int((2-ln(x))*(3+ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] -1/4*ln(x)^4-4/3*ln(x)^3+3/2*ln(x)^2+18*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = -\frac{1}{4}\,\log{(x)}^4 - \frac{4}{3}\,\log{(x)}^3 + \frac{3}{2}\,\log{(x)}^2 + 18\,\log{(x)}$$

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="fricas")
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = -\frac{\log{(x)}^4}{4} - \frac{4\log{(x)}^3}{3} + \frac{3\log{(x)}^2}{2} + 18\log{(x)}$$

```
[In] integrate((2-ln(x))*(3+ln(x))**2/x,x)
[Out] -log(x)**4/4 - 4*log(x)**3/3 + 3*log(x)**2/2 + 18*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = -\frac{1}{4}\,\log{(x)}^4 - \frac{4}{3}\,\log{(x)}^3 + \frac{3}{2}\,\log{(x)}^2 + 18\,\log{(x)}$$

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="maxima")
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = -\frac{1}{4}\,\log{(x)}^4 - \frac{4}{3}\,\log{(x)}^3 + \frac{3}{2}\,\log{(x)}^2 + 18\,\log{(x)}$$

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="giac")
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = \frac{\ln(x)\,\left(-3\ln(x)^3 - 16\ln(x)^2 + 18\,\ln(x) + 216\right)}{12}$$

```
[In] int(-((log(x) - 2)*(log(x) + 3)^2)/x,x)
[Out] (log(x)*(18*log(x) - 16*log(x)^2 - 3*log(x)^3 + 216))/12
```

3.143 $\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx$

Optimal result 848
Rubi [A] (verified) 848
Mathematica [A] (verified) 849
Maple [A] (verified) 850
Fricas [A] (verification not implemented) 850
Sympy [A] (verification not implemented) 850
Maxima [A] (verification not implemented) 851
Giac [A] (verification not implemented) 851
Mupad [B] (verification not implemented) 851

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = -\frac{1}{8}\operatorname{arcsinh}(\log(x)) + \frac{1}{8}\log(x)\sqrt{1+\log^2(x)} + \frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}$$

```
[Out] -1/8*arcsinh(ln(x))+1/8*ln(x)*(1+ln(x)^2)^(1/2)+1/4*ln(x)^3*(1+ln(x)^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {285, 327, 221}

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = -\frac{1}{8}\operatorname{arcsinh}(\log(x)) + \frac{1}{8}\sqrt{\log^2(x)+1}\log(x) + \frac{1}{4}\sqrt{\log^2(x)+1}\log^3(x)$$

```
[In] Int[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
```

```
[Out] -1/8*ArcSinh[Log[x]] + (Log[x]*Sqrt[1 + Log[x]^2])/8 + (Log[x]^3*Sqrt[1 + L
og[x]^2])/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x^2\sqrt{1+x^2}\,dx, x, \log(x)\right)\\
&= \frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, \log(x)\right)\\
&= \frac{1}{8}\log(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \log(x)\right)\\
&= -\frac{1}{8}\sinh^{-1}(\log(x))+\frac{1}{8}\log(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$
\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \frac{1}{8}\left(-\text{arcsinh}(\log(x))+\log(x)\sqrt{1+\log^2(x)}\left(1+2\log^2(x)\right)\right)
$$

```
[In] Integrate[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
[Out] (-ArcSinh[Log[x]] + Log[x]*Sqrt[1 + Log[x]^2]*(1 + 2*Log[x]^2))/8
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{\ln(x)\left(1+\ln(x)^{2}\right)^{\frac{3}{2}}}{4}-\frac{\ln(x)\sqrt{1+\ln(x)^{2}}}{8}-\frac{\operatorname{arcsinh}(\ln(x))}{8}$	31
default	$\frac{\ln(x)\left(1+\ln(x)^{2}\right)^{\frac{3}{2}}}{4}-\frac{\ln(x)\sqrt{1+\ln(x)^{2}}}{8}-\frac{\operatorname{arcsinh}(\ln(x))}{8}$	31

```
[In] int(ln(x)^2*(1+ln(x)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x)*(1+ln(x)^2)^(3/2)-1/8*ln(x)*(1+ln(x)^2)^(1/2)-1/8*arcsinh(ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \frac{1}{8}\left(2\log(x)^3+\log(x)\right)\sqrt{\log(x)^2+1} + \frac{1}{8}\log\left(\sqrt{\log(x)^2+1}-\log(x)\right)$$

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/8*(2*log(x)^3 + log(x))*sqrt(log(x)^2 + 1) + 1/8*log(sqrt(log(x)^2 + 1) -
 log(x))
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \sqrt{\log(x)^2+1}\left(\frac{\log(x)^3}{4}+\frac{\log(x)}{8}\right)-\frac{\operatorname{asinh}(\log(x))}{8}$$

```
[In] integrate(ln(x)**2*(1+ln(x)**2)**(1/2)/x,x)
[Out] sqrt(log(x)**2 + 1)*(log(x)**3/4 + log(x)/8) - asinh(log(x))/8
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \frac{1}{4}\left(\log\left(x\right)^2+1\right)^{\frac{3}{2}}\log\left(x\right) - \frac{1}{8}\sqrt{\log\left(x\right)^2+1}\log\left(x\right) - \frac{1}{8}\,\operatorname{arsinh}\left(\log\left(x\right)\right)$$

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="maxima")
[Out] 1/4*(log(x)^2 + 1)^(3/2)*log(x) - 1/8*sqrt(log(x)^2 + 1)*log(x) - 1/8*arcsi
nh(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \frac{1}{8}\left(2\log\left(x\right)^2+1\right)\sqrt{\log\left(x\right)^2+1}\log\left(x\right) + \frac{1}{8}\log\left(\sqrt{\log\left(x\right)^2+1}-\log\left(x\right)\right)$$

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/8*(2*log(x)^2 + 1)*sqrt(log(x)^2 + 1)*log(x) + 1/8*log(sqrt(log(x)^2 + 1)
 - log(x))
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \left(\frac{\ln\left(x\right)^3}{4}+\frac{\ln\left(x\right)}{8}\right)\sqrt{\ln\left(x\right)^2+1} - \frac{\operatorname{asinh}(\ln\left(x\right))}{8}$$

```
[In] int((log(x)^2*(log(x)^2 + 1)^(1/2))/x,x)
[Out] (log(x)/8 + log(x)^3/4)*(log(x)^2 + 1)^(1/2) - asinh(log(x))/8
```

3.144 $\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx$

Optimal result 852
Rubi [A] (verified) 852
Mathematica [A] (verified) 853
Maple [A] (verified) 853
Fricas [A] (verification not implemented) 854
Sympy [A] (verification not implemented) 854
Maxima [A] (verification not implemented) 854
Giac [A] (verification not implemented) 854
Mupad [B] (verification not implemented) 855

Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx=\frac{1}{4(3+2\log(x))}+\frac{1}{4}\log(3+2\log(x))$$

```
[Out] 1/4/(3+2*ln(x))+1/4*ln(3+2*ln(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2412, 45}

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx=\frac{1}{4}\log(2\log(x)+3)+\frac{1}{4(2\log(x)+3)}$$

```
[In] Int[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
```

```
[Out] 1/(4*(3 + 2*Log[x])) + Log[3 + 2*Log[x]]/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
```

```
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1+x}{(3+2x)^2}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{2(3+2x)^2}+\frac{1}{2(3+2x)}\right)\,dx, x, \log(x)\right)\\ &= \frac{1}{4(3+2\log(x))}+\frac{1}{4}\log(3+2\log(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{1}{4}\left(\frac{1}{3+2\log(x)}+\log(3+2\log(x))\right)$$

```
[In] Integrate[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
[Out] ((3 + 2*Log[x])^(-1) + Log[3 + 2*Log[x]])/4
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{1}{12+8\ln(x)}+\frac{\ln\left(\frac{3}{2}+\ln(x)\right)}{4}$	19
derivativedivides	$\frac{1}{12+8\ln(x)}+\frac{\ln(3+2\ln(x))}{4}$	21
default	$\frac{1}{12+8\ln(x)}+\frac{\ln(3+2\ln(x))}{4}$	21
norman	$\frac{1}{12+8\ln(x)}+\frac{\ln(3+2\ln(x))}{4}$	21
parallelrisch	$\frac{1+2\ln\left(\frac{3}{2}+\ln(x)\right)\ln(x)+3\ln\left(\frac{3}{2}+\ln(x)\right)}{12+8\ln(x)}$	29

```
[In] int((1+ln(x))/x/(3+2*ln(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/4/(3+2*ln(x))+1/4*ln(3/2+ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{(2\,\log\left(x\right)+3)\log\left(2\,\log\left(x\right)+3\right)+1}{4\left(2\,\log\left(x\right)+3\right)}$$

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="fricas")
[Out] 1/4*((2*log(x) + 3)*log(2*log(x) + 3) + 1)/(2*log(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{\log\left(\log\left(x\right)+\frac{3}{2}\right)}{4}+\frac{1}{8\log\left(x\right)+12}$$

```
[In] integrate((1+ln(x))/x/(3+2*ln(x))**2,x)
[Out] log(log(x) + 3/2)/4 + 1/(8*log(x) + 12)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{1}{4\left(2\,\log\left(x\right)+3\right)}+\frac{1}{4}\,\log\left(2\,\log\left(x\right)+3\right)$$

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="maxima")
[Out] 1/4/(2*log(x) + 3) + 1/4*log(2*log(x) + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{1}{4\left(2\,\log\left(x\right)+3\right)}+\frac{1}{8}\,\log\left(\pi^2(\mathrm{sgn}(x)-1)^2+\left(2\,\log\left(|x|\right)+3\right)^2\right)$$

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="giac")
[Out] 1/4/(2*log(x) + 3) + 1/8*log(pi^2*(sgn(x) - 1)^2 + (2*log(abs(x)) + 3)^2)
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \frac{\ln\left(2\,\ln\left(x\right)+3\right)}{4}+\frac{1}{4\,\left(2\,\ln\left(x\right)+3\right)}$$

```
[In] int((log(x) + 1)/(x*(2*log(x) + 3)^2),x)
[Out] log(2*log(x) + 3)/4 + 1/(4*(2*log(x) + 3))
```

3.145 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

Optimal result . . . 856
Rubi [A] (verified) . . . 856
Mathematica [A] (verified) . . . 857
Maple [A] (verified) . . . 857
Fricas [A] (verification not implemented) . . . 858
Sympy [A] (verification not implemented) . . . 858
Maxima [A] (verification not implemented) . . . 858
Giac [A] (verification not implemented) . . . 858
Mupad [B] (verification not implemented) . . . 859

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = -2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}$$

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2412, 45}

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

```
[In] Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
```

```
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx, x, \log(x)\right)\\ &= -2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}(-2+\log(x))\sqrt{1+\log(x)}$$

```
[In] Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
[Out] (2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\ln(x)}$	18
default	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\ln(x)}$	18

```
[In] int(ln(x)/x/(1+ln(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,\sqrt{\log{(x)}+1}(\log{(x)}-2)$$

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2(\log{(x)}+1)^{\frac{3}{2}}}{3} - 2\sqrt{\log{(x)}+1}$$

```
[In] integrate(ln(x)/x/(1+ln(x))**(1/2),x)
[Out] 2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}} - 2\,\sqrt{\log{(x)}+1}$$

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \frac{2}{3}\,(\log{(x)}+1)^{\frac{3}{2}} - 2\,\sqrt{\log{(x)}+1}$$

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \sqrt{\ln(x)+1}\left(\frac{2\ln(x)}{3}-\frac{4}{3}\right)$$

```
[In] int(log(x)/(x*(log(x) + 1)^(1/2)),x)
[Out] (log(x) + 1)^(1/2)*((2*log(x))/3 - 4/3)
```

3.146 $\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx$

Optimal result 860
Rubi [A] (verified) 860
Mathematica [A] (verified) 861
Maple [A] (verified) 861
Fricas [A] (verification not implemented) 862
Sympy [A] (verification not implemented) 862
Maxima [A] (verification not implemented) 862
Giac [A] (verification not implemented) 862
Mupad [B] (verification not implemented) 863

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{8}\sqrt{-1+4\log(x)}+\frac{1}{24}(-1+4\log(x))^{3/2}$$

```
[Out] 1/24*(-1+4*ln(x))^(3/2)+1/8*(-1+4*ln(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2412, 45}

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{24}(4\log(x)-1)^{3/2}+\frac{1}{8}\sqrt{4\log(x)-1}$$

```
[In] Int[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]
```

```
[Out] Sqrt[-1 + 4*Log[x]]/8 + (-1 + 4*Log[x])^(3/2)/24
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
```

```
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x}{\sqrt{-1+4x}}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{4\sqrt{-1+4x}}+\frac{1}{4}\sqrt{-1+4x}\right)\,dx, x, \log(x)\right)\\ &= \frac{1}{8}\sqrt{-1+4\log(x)}+\frac{1}{24}(-1+4\log(x))^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{12}(1+2\log(x))\sqrt{-1+4\log(x)}$$

```
[In] Integrate[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]
[Out] ((1 + 2*Log[x])*Sqrt[-1 + 4*Log[x]])/12
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{(-1+4\ln(x))^{\frac{3}{2}}}{24}+\frac{\sqrt{-1+4\ln(x)}}{8}$	22
default	$\frac{(-1+4\ln(x))^{\frac{3}{2}}}{24}+\frac{\sqrt{-1+4\ln(x)}}{8}$	22

```
[In] int(ln(x)/x/(-1+4*ln(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/24*(-1+4*ln(x))^(3/2)+1/8*(-1+4*ln(x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.55

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{12}\,\sqrt{4\,\log\left(x\right)-1}(2\,\log\left(x\right)+1)$$

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="fricas")
[Out] 1/12*sqrt(4*log(x) - 1)*(2*log(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{\left(4\log\left(x\right)-1\right)^{\frac{3}{2}}}{24}+\frac{\sqrt{4\log\left(x\right)-1}}{8}$$

```
[In] integrate(ln(x)/x/(-1+4*ln(x))**(1/2),x)
[Out] (4*log(x) - 1)**(3/2)/24 + sqrt(4*log(x) - 1)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{24}\left(4\,\log\left(x\right)-1\right)^{\frac{3}{2}}+\frac{1}{8}\,\sqrt{4\,\log\left(x\right)-1}$$

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="maxima")
[Out] 1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \frac{1}{24}\left(4\,\log\left(x\right)-1\right)^{\frac{3}{2}}+\frac{1}{8}\,\sqrt{4\,\log\left(x\right)-1}$$

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="giac")
[Out] 1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \sqrt{4\,\ln(x)-1}\,\left(\frac{\ln(x)}{6}+\frac{1}{12}\right)$$

```
[In] int(log(x)/(x*(4*log(x) - 1)^(1/2)),x)
```

```
[Out] (4*log(x) - 1)^(1/2)*(log(x)/6 + 1/12)
```

3.147 $\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx$

Optimal result 864
Rubi [A] (verified) 864
Mathematica [A] (verified) 865
Maple [A] (verified) 866
Fricas [A] (verification not implemented) 866
Sympy [A] (verification not implemented) 866
Maxima [A] (verification not implemented) 867
Giac [F(-1)] 867
Mupad [B] (verification not implemented) 867

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = -2\text{arctanh}\left(\sqrt{1+\log(x)}\right)+2\sqrt{1+\log(x)}$$

```
[Out] -2*arctanh((1+ln(x))^(1/2))+2*(1+ln(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2412, 52, 65, 213}

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log(x)+1}-2\text{arctanh}\left(\sqrt{\log(x)+1}\right)$$

```
[In] Int[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \log(x)\right)\\
&= 2\sqrt{1+\log(x)} + \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \log(x)\right)\\
&= 2\sqrt{1+\log(x)} + 2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\log(x)}\right)\\
&= -2\tanh^{-1}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = -2\text{arctanh}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}$$

```
[In] Integrate[Sqrt[1 + Log[x]]/(x*Log[x]),x]
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$2\sqrt{1+\ln \left(x \right)}+\ln \left(\sqrt{1+\ln \left(x \right)}-1\right)-\ln \left(\sqrt{1+\ln \left(x \right)}+1\right)$	30
default	$2\sqrt{1+\ln \left(x \right)}+\ln \left(\sqrt{1+\ln \left(x \right)}-1\right)-\ln \left(\sqrt{1+\ln \left(x \right)}+1\right)$	30

```
[In] int((1+ln(x))^(1/2)/x/ln(x),x,method=_RETURNVERBOSE)
[Out] 2*(1+ln(x))^(1/2)+ln((1+ln(x))^(1/2)-1)-ln((1+ln(x))^(1/2)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx=2\,\sqrt{\log\left(x\right)+1}-\log\left(\sqrt{\log\left(x\right)+1}+1\right)+\log\left(\sqrt{\log\left(x\right)+1}-1\right)$$

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="fricas")
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx=2\sqrt{\log\left(x\right)+1}+\log\left(\sqrt{\log\left(x\right)+1}-1\right)-\log\left(\sqrt{\log\left(x\right)+1}+1\right)$$

```
[In] integrate((1+ln(x))**(1/2)/x/ln(x),x)
[Out] 2*sqrt(log(x) + 1) + log(sqrt(log(x) + 1) - 1) - log(sqrt(log(x) + 1) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\log\left(x\right)+1} - \log\left(\sqrt{\log\left(x\right)+1}+1\right) + \log\left(\sqrt{\log\left(x\right)+1}-1\right)$$

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="maxima")
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = \text{Timed out}$$

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx = 2\sqrt{\ln\left(x\right)+1} - 2\,\mathrm{atanh}\left(\sqrt{\ln\left(x\right)+1}\right)$$

```
[In] int((log(x) + 1)^(1/2)/(x*log(x)),x)
[Out] 2*(log(x) + 1)^(1/2) - 2*atanh((log(x) + 1)^(1/2))
```

3.148 $\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx$

Optimal result 868
Rubi [A] (verified) 868
Mathematica [A] (verified) 869
Maple [A] (verified) 869
Fricas [A] (verification not implemented) 870
Sympy [A] (verification not implemented) 870
Maxima [A] (verification not implemented) 870
Giac [A] (verification not implemented) 870
Mupad [B] (verification not implemented) 871

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = -\frac{2}{3(1-\log(x))^3}+\frac{1}{1-\log(x)}+\frac{1}{(-1+\log(x))^2}$$

```
[Out] -2/3/(1-ln(x))^3+1/(1-ln(x))+1/(-1+ln(x))^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {712}

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = \frac{1}{(\log(x)-1)^2}+\frac{1}{1-\log(x)}-\frac{2}{3(1-\log(x))^3}$$

```
[In] Int[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
```

```
[Out] -2/(3*(1 - Log[x])^3) + (1 - Log[x])^(-1) + (-1 + Log[x])^(-2)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1-4x+x^2}{(-1+x)^4}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{2}{(-1+x)^4}-\frac{2}{(-1+x)^3}+\frac{1}{(-1+x)^2}\right)dx, x, \log(x)\right)\\
&= -\frac{2}{3(1-\log(x))^3}+\frac{1}{1-\log(x)}+\frac{1}{(-1+\log(x))^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$
\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = \frac{-4+9\log(x)-3\log^2(x)}{3(-1+\log(x))^3}
$$

```
[In] Integrate[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
[Out] (-4 + 9*Log[x] - 3*Log[x]^2)/(3*(-1 + Log[x])^3)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{-\ln(x)^2+3\ln(x)-\frac{4}{3}}{(-1+\ln(x))^3}$	20
risch	$-\frac{3\ln(x)^2-9\ln(x)+4}{3(-1+\ln(x))^3}$	21
parallelrisch	$\frac{-4-3\ln(x)^2+9\ln(x)}{3(-1+\ln(x))^3}$	21
derivativedivides	$\frac{2}{3(-1+\ln(x))^3}-\frac{1}{-1+\ln(x)}+\frac{1}{(-1+\ln(x))^2}$	24
default	$\frac{2}{3(-1+\ln(x))^3}-\frac{1}{-1+\ln(x)}+\frac{1}{(-1+\ln(x))^2}$	24

```
[In] int((1-4*ln(x)+ln(x)^2)/x/(-1+ln(x))^4,x,method=_RETURNVERBOSE)
[Out] (-ln(x)^2+3*ln(x)-4/3)/(-1+ln(x))^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = -\frac{3\,\log\left(x\right)^{2}-9\,\log\left(x\right)+4}{3\,\left(\log\left(x\right)^{3}-3\,\log\left(x\right)^{2}+3\,\log\left(x\right)-1\right)}$$

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="fricas")
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = \frac{-3\log\left(x\right)^{2}+9\log\left(x\right)-4}{3\log\left(x\right)^{3}-9\log\left(x\right)^{2}+9\log\left(x\right)-3}$$

```
[In] integrate((1-4*ln(x)+ln(x)**2)/x/(-1+ln(x))**4,x)
[Out] (-3*log(x)**2 + 9*log(x) - 4)/(3*log(x)**3 - 9*log(x)**2 + 9*log(x) - 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = -\frac{3\,\log\left(x\right)^{2}-9\,\log\left(x\right)+4}{3\,\left(\log\left(x\right)^{3}-3\,\log\left(x\right)^{2}+3\,\log\left(x\right)-1\right)}$$

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="maxima")
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = -\frac{3\,\log\left(x\right)^{2}-9\,\log\left(x\right)+4}{3\,\left(\log\left(x\right)-1\right)^{3}}$$

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="giac")
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x) - 1)^3
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx = -\frac{\ln{(x)}^2-3\,\ln{(x)}+\frac{4}{3}}{{(\ln{(x)}-1)}^3}$$

```
[In] int((log(x)^2 - 4*log(x) + 1)/(x*(log(x) - 1)^4),x)
[Out] -(log(x)^2 - 3*log(x) + 4/3)/(log(x) - 1)^3
```

3.149 $\int \frac{\log^2(ax^n)^p}{x} \, dx$

Optimal result . . . 872
Rubi [A] (verified) . . . 872
Mathematica [A] (verified) . . . 873
Maple [A] (verified) . . . 873
Fricas [A] (verification not implemented) . . . 874
Sympy [F] . . . 874
Maxima [F(-2)] . . . 874
Giac [A] (verification not implemented) . . . 874
Mupad [B] (verification not implemented) . . . 875

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{\log^2(ax^n)^p}{x} \, dx = \frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}$$

```
[Out] ln(a*x^n)*(ln(a*x^n)^2)^p/n/(1+2*p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {15, 30}

$$\int \frac{\log^2(ax^n)^p}{x} \, dx = \frac{\log(ax^n)\log^2(ax^n)^p}{n(2p+1)}$$

```
[In] Int[(Log[a*x^n]^2)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (x^2)^p \, dx, x, \log(ax^n)\right)}{n}$$
$$= \frac{\left(\log^{-2p}(ax^n)\log^2(ax^n)^p\right)\text{Subst}\left(\int x^{2p}\, dx, x, \log(ax^n)\right)}{n}$$
$$= \frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(ax^n)^p}{x}\, dx = \frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}$$

```
[In] Integrate[(Log[a*x^n]^2)^p/x,x]
[Out] (Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

Maple [A] (verified)

Time = 5.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(a\,x^n)\mathrm{e}^{p\ln\left(\ln(a\,x^n)^2\right)}}{n(1+2p)}$	30
default	$\frac{\ln(a\,x^n)\mathrm{e}^{p\ln\left(\ln(a\,x^n)^2\right)}}{n(1+2p)}$	30

```
[In] int((ln(a*x^n)^2)^p/x,x,method=_RETURNVERBOSE)
[Out] 1/n/(1+2*p)*ln(a*x^n)*exp(p*ln(ln(a*x^n)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \frac{(n\log(x)+\log(a))\left(n^2\log(x)^2+2\,n\log(a)\log(x)+\log(a)^2\right)^p}{2\,np+n}$$

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="fricas")
[Out] (n*log(x) + log(a))*(n^2*log(x)^2 + 2*n*log(a)*log(x) + log(a)^2)^p/(2*n*p
+ n)
```

Sympy [F]

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \int \frac{\left(\log(ax^n)^2\right)^p}{x}\,dx$$

```
[In] integrate((ln(a*x**n)**2)**p/x,x)
[Out] Integral((log(a*x**n)**2)**p/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \frac{(n\log(x)\,\mathrm{sgn}(\log(ax^n))+\log(a)\,\mathrm{sgn}(\log(ax^n)))^{2\,p+1}}{n(2\,p+1)\mathrm{sgn}(\log(ax^n))}$$

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="giac")
[Out] (n*log(x)*sgn(log(a*x^n)) + log(a)*sgn(log(a*x^n)))^(2*p + 1)/(n*(2*p + 1)*
sgn(log(a*x^n)))
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(ax^n)^p}{x} \, dx = \frac{\ln(a\,x^n)\,\left(\ln(a\,x^n)^2\right)^p}{n\,(2\,p+1)}$$

```
[In] int((log(a*x^n)^2)^p/x,x)
[Out] (log(a*x^n)*(log(a*x^n)^2)^p)/(n*(2*p + 1))
```

3.150 $\int \frac{\log^m(ax^n)^p}{x}\,dx$

Optimal result 876
Rubi [A] (verified) 876
Mathematica [A] (verified) 877
Maple [A] (verified) 877
Fricas [A] (verification not implemented) 878
Sympy [F] 878
Maxima [F(-2)] 878
Giac [A] (verification not implemented) 878
Mupad [B] (verification not implemented) 879

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{\log^m(ax^n)^p}{x}\,dx = \frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}$$

```
[Out] ln(a*x^n)*(ln(a*x^n)^m)^p/n/(m*p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {15, 30}

$$\int \frac{\log^m(ax^n)^p}{x}\,dx = \frac{\log(ax^n)\log^m(ax^n)^p}{n(mp+1)}$$

```
[In] Int[(Log[a*x^n]^m)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (x^m)^p \, dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\left(\log^{-mp}(ax^n)\log^m(ax^n)^p\right)\text{Subst}\left(\int x^{mp}\, dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^m(ax^n)^p}{x}\, dx = \frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}$$

```
[In] Integrate[(Log[a*x^n]^m)^p/x,x]
[Out] (Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\ln(a\,x^n)\mathrm{e}^{p\ln\left(\mathrm{e}^{m\ln(\ln(a\,x^n))}\right)}}{n(mp+1)}$
default	$\frac{\ln(a\,x^n)\mathrm{e}^{p\ln\left(\mathrm{e}^{m\ln(\ln(a\,x^n))}\right)}}{n(mp+1)}$
risch	$\frac{\left(\ln(a)+\ln(x^n)-\frac{i\pi\ \operatorname{csgn}(ia\,x^n)(-\operatorname{csgn}(ia\,x^n)+\operatorname{csgn}(ia))(-\operatorname{csgn}(ia\,x^n)+\operatorname{csgn}(ix^n))}{2}\right)^{mp}\left(\ln(a)+\ln(x^n)-\frac{i\pi\ \operatorname{csgn}(ia\,x^n)(-\operatorname{csg}}{}\right.}{n(mp+1)}$

```
[In] int((ln(a*x^n)^m)^p/x,x,method=_RETURNVERBOSE)
[Out] 1/n/(m*p+1)*ln(a*x^n)*exp(p*ln(exp(m*ln(ln(a*x^n)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \frac{(n\log{(x)} + \log{(a)})(n\log{(x)} + \log{(a)})^{mp}}{mnp + n}$$

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="fricas")
[Out] (n*log(x) + log(a))*(n*log(x) + log(a))^(m*p)/(m*n*p + n)
```

Sympy [F]

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \int \frac{(\log{(ax^n)}^m)^p}{x} \, dx$$

```
[In] integrate((ln(a*x**n)**m)**p/x,x)
[Out] Integral((log(a*x**n)**m)**p/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \frac{(n\log{(x)} + \log{(a)})^{mp+1}}{(mp + 1)n}$$

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="giac")
[Out] (n*log(x) + log(a))^(m*p + 1)/((m*p + 1)*n)
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\log^m (ax^n)^p}{x} \, dx = \frac{\ln (a\,x^n)\ (\ln (a\,x^n)^m)^p}{n\ (m\,p+1)}$$

```
[In] int((log(a*x^n)^m)^p/x,x)
[Out] (log(a*x^n)*(log(a*x^n)^m)^p)/(n*(m*p + 1))
```

3.151 $\int \frac{\sqrt{\log^2(ax^n)}}{x} dx$

Optimal result . . . 880
Rubi [A] (verified) . . . 880
Mathematica [A] (verified) . . . 881
Maple [C] (warning: unable to verify) . . . 881
Fricas [A] (verification not implemented) . . . 882
Sympy [F] . . . 882
Maxima [A] (verification not implemented) . . . 882
Giac [A] (verification not implemented) . . . 882
Mupad [B] (verification not implemented) . . . 883

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{\sqrt{\log^2(ax^n)}}{x} dx = \frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

```
[Out] 1/2*ln(a*x^n)*(ln(a*x^n)^2)^(1/2)/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 30}

$$\int \frac{\sqrt{\log^2(ax^n)}}{x} dx = \frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

```
[In] Int[Sqrt[Log[a*x^n]^2]/x,x]
[Out] (Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \sqrt{x^2}\,dx, x, \log(ax^n)\right)}{n}$$

$$= \frac{\sqrt{\log^2(ax^n)}\,\text{Subst}\left(\int x\,dx, x, \log(ax^n)\right)}{n\log(ax^n)}$$

$$= \frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx = \frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

```
[In] Integrate[Sqrt[Log[a*x^n]^2]/x,x]
[Out] (Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\operatorname{csgn}(\ln(a\,x^n))\ln(a\,x^n)^2}{2n}$	21
default	$\frac{\operatorname{csgn}(\ln(a\,x^n))\ln(a\,x^n)^2}{2n}$	21

```
[In] int((ln(a*x^n)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/2/n*csgn(ln(a*x^n))*ln(a*x^n)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx = \frac{1}{2} n \log(x)^2 + \log(a) \log(x)$$

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*n*log(x)^2 + log(a)*log(x)
```

Sympy [F]

$$\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx = \int \frac{\sqrt{\log(ax^n)^2}}{x} \, dx$$

```
[In] integrate((ln(a*x**n)**2)**(1/2)/x,x)
[Out] Integral(sqrt(log(a*x**n)**2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx = -\frac{1}{2} n \log(x)^2 + \log(a) \log(x) + \log(x) \log(x^n)$$

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="maxima")
[Out] -1/2*n*log(x)^2 + log(a)*log(x) + log(x)*log(x^n)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx = \frac{1}{2} n \log(x)^2 \operatorname{sgn}(\log(ax^n)) + \log(a) \log(x) \operatorname{sgn}(\log(ax^n))$$

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/2*n*log(x)^2*sgn(log(a*x^n)) + log(a)*log(x)*sgn(log(a*x^n))
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx = \frac{\ln(a\,x^n)\,\sqrt{\ln(a\,x^n)^2}}{2\,n}$$

```
[In] int((log(a*x^n)^2)^(1/2)/x,x)
[Out] (log(a*x^n)*(log(a*x^n)^2)^(1/2))/(2*n)
```

3.152 $\int \frac{(b\log^m(ax^n))^p}{x}\,dx$

Optimal result 884
Rubi [A] (verified) 884
Mathematica [A] (verified) 885
Maple [A] (verified) 885
Fricas [A] (verification not implemented) 886
Sympy [F] 886
Maxima [F(-2)] 886
Giac [A] (verification not implemented) 886
Mupad [B] (verification not implemented) 887

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx=\frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(1+mp)}$$

```
[Out] ln(a*x^n)*(b*ln(a*x^n)^m)^p/n/(m*p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 30}

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx=\frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(mp+1)}$$

```
[In] Int[(b*Log[a*x^n]^m)^p/x,x]
```

```
[Out] (Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (bx^m)^p \, dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\left(\log^{-mp}(ax^n)\,(b\log^m(ax^n))^p\right)\text{Subst}\left(\int x^{mp}\,dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(1+mp)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$
\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(1+mp)}
$$

```
[In] Integrate[(b*Log[a*x^n]^m)^p/x,x]
[Out] (Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\ln(a\,x^n)\mathrm{e}^{p\ln\left(b\,\mathrm{e}^{m\ln(\ln(a\,x^n))}\right)}}{n(mp+1)}$	34
default	$\frac{\ln(a\,x^n)\mathrm{e}^{p\ln\left(b\,\mathrm{e}^{m\ln(\ln(a\,x^n))}\right)}}{n(mp+1)}$	34

```
[In] int((b*ln(a*x^n)^m)^p/x,x,method=_RETURNVERBOSE)
[Out] 1/n/(m*p+1)*ln(a*x^n)*exp(p*ln(b*exp(m*ln(ln(a*x^n)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \frac{(n\log(x)+\log(a))e^{(mp\log(n\log(x)+\log(a))+p\log(b))}}{mnp+n}$$

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="fricas")
[Out] (n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/(m*n*p + n)
```

Sympy [F]

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \int \frac{(b\log(ax^n)^m)^p}{x}\,dx$$

```
[In] integrate((b*ln(a*x**n)**m)**p/x,x)
[Out] Integral((b*log(a*x**n)**m)**p/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{(b\log^m(ax^n))^p}{x}\,dx = \frac{(n\log(x)+\log(a))e^{(mp\log(n\log(x)+\log(a))+p\log(b))}}{(mp+1)n}$$

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="giac")
[Out] (n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/((m*p + 1)*n)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(b\log^m{(ax^n)})^p}{x}\,dx = \frac{\ln{(a\,x^n)}\,\left(b\ln{(a\,x^n)}^m\right)^p}{n\,\left(m\,p+1\right)}$$

```
[In] int((b*log(a*x^n)^m)^p/x,x)
[Out] (log(a*x^n)*(b*log(a*x^n)^m)^p)/(n*(m*p + 1))
```

3.153 $\int \frac{1}{x \log(e^x)} \, dx$

Optimal result . . . 888
Rubi [A] (verified) . . . 888
Mathematica [A] (verified) . . . 889
Maple [A] (verified) . . . 889
Fricas [A] (verification not implemented) . . . 890
Sympy [A] (verification not implemented) . . . 890
Maxima [A] (verification not implemented) . . . 890
Giac [A] (verification not implemented) . . . 890
Mupad [B] (verification not implemented) . . . 891

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{1}{x \log(e^x)} \, dx = -\frac{\log(x)}{x - \log(e^x)} + \frac{\log(\log(e^x))}{x - \log(e^x)}$$

```
[Out] -ln(x)/(x-ln(exp(x)))+ln(ln(exp(x)))/(x-ln(exp(x)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2191, 2188, 29}

$$\int \frac{1}{x \log(e^x)} \, dx = \frac{\log(\log(e^x))}{x - \log(e^x)} - \frac{\log(x)}{x - \log(e^x)}$$

```
[In] Int[1/(x*Log[E^x]),x]
```

```
[Out] -(Log[x]/(x - Log[E^x])) + Log[Log[E^x]]/(x - Log[E^x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{1}{x}\,dx}{x-\log(e^x)}+\frac{\int \frac{1}{\log(e^x)}\,dx}{x-\log(e^x)}\\ &= -\frac{\log(x)}{x-\log(e^x)}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,\log(e^x)\right)}{x-\log(e^x)}\\ &= -\frac{\log(x)}{x-\log(e^x)}+\frac{\log(\log(e^x))}{x-\log(e^x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\log(e^x)}\,dx=\frac{-\log(x)+\log(\log(e^x))}{x-\log(e^x)}$$

```
[In] Integrate[1/(x*Log[E^x]),x]
[Out] (-Log[x] + Log[Log[E^x]])/(x - Log[E^x])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\ln(\ln(e^x))}{\ln(e^x)-x}+\frac{\ln(x)}{\ln(e^x)-x}$	29
risch	$-\frac{\ln(\ln(e^x))}{\ln(e^x)-x}+\frac{\ln(x)}{\ln(e^x)-x}$	29

```
[In] int(1/x/ln(exp(x)),x,method=_RETURNVERBOSE)
[Out] -1/(ln(exp(x))-x)*ln(ln(exp(x)))+1/(ln(exp(x))-x)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{1}{x \log(e^x)} dx = -\frac{1}{x}$$

```
[In] integrate(1/x/log(exp(x)),x, algorithm="fricas")
[Out] -1/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.10

$$\int \frac{1}{x \log(e^x)} dx = -\frac{1}{x}$$

```
[In] integrate(1/x/ln(exp(x)),x)
[Out] -1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{1}{x \log(e^x)} dx = -\frac{1}{x}$$

```
[In] integrate(1/x/log(exp(x)),x, algorithm="maxima")
[Out] -1/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{1}{x \log(e^x)} dx = -\frac{1}{x}$$

```
[In] integrate(1/x/log(exp(x)),x, algorithm="giac")
[Out] -1/x
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{1}{x \log (e^x)} \, dx = -\frac{1}{x}$$

```
[In] int(1/(x*log(exp(x))),x)
[Out] -1/x
```

3.154 $\int \log(x)\sin(a+bx)\,dx$

Optimal result 892
Rubi [A] (verified) 892
Mathematica [A] (verified) 894
Maple [C] (warning: unable to verify) 894
Fricas [A] (verification not implemented) 894
Sympy [F] 895
Maxima [C] (verification not implemented) 895
Giac [C] (verification not implemented) 895
Mupad [F(-1)] 896

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \log(x)\sin(a+bx)\,dx=\frac{\cos(a)\,\mathrm{CosIntegral}(bx)}{b}-\frac{\cos(a+bx)\log(x)}{b}-\frac{\sin(a)\mathrm{Si}(bx)}{b}$$

```
[Out] Ci(b*x)*cos(a)/b-cos(b*x+a)*ln(x)/b-Si(b*x)*sin(a)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2718, 2634, 12, 3384, 3380, 3383}

$$\int \log(x)\sin(a+bx)\,dx=\frac{\cos(a)\,\mathrm{CosIntegral}(bx)}{b}-\frac{\sin(a)\mathrm{Si}(bx)}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

```
[In] Int[Log[x]*Sin[a + b*x],x]
```

```
[Out] (Cos[a]*CosIntegral[b*x])/b - (Cos[a + b*x]*Log[x])/b - (Sin[a]*SinIntegral
[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
```

```
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cos(a+bx)\log(x)}{b} + \int \frac{\cos(a+bx)}{bx}\,dx \\
&= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\int \frac{\cos(a+bx)}{x}\,dx}{b} \\
&= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos(a)\int \frac{\cos(bx)}{x}\,dx}{b} - \frac{\sin(a)\int \frac{\sin(bx)}{x}\,dx}{b} \\
&= \frac{\cos(a)\text{Ci}(bx)}{b} - \frac{\cos(a+bx)\log(x)}{b} - \frac{\sin(a)\text{Si}(bx)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \log(x)\sin(a+bx)\,dx = \frac{\cos(a)\,\text{CosIntegral}(bx) - \cos(a+bx)\log(x) - \sin(a)\text{Si}(bx)}{b}$$

```
[In] Integrate[Log[x]*Sin[a + b*x],x]
```

```
[Out] (Cos[a]*CosIntegral[b*x] - Cos[a + b*x]*Log[x] - Sin[a]*SinIntegral[b*x])/b
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.29

method	result	size
risch	$-\frac{\cos(bx+a)\ln(x)}{b}+\frac{i\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(bx)}{2b}-\frac{i\mathrm{e}^{-ia}\ \mathrm{Si}(bx)}{b}-\frac{\mathrm{e}^{-ia}\ \mathrm{Ei}_1(-ibx)}{2b}-\frac{\mathrm{e}^{ia}\ \mathrm{Ei}_1(-ibx)}{2b}$	80

```
[In] int(ln(x)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(b*x+a)*ln(x)/b+1/2*I/b*exp(-I*a)*Pi*csgn(b*x)-I/b*exp(-I*a)*Si(b*x)-1/
2/b*exp(-I*a)*Ei(1,-I*b*x)-1/2/b*exp(I*a)*Ei(1,-I*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \log(x)\sin(a+bx)\,dx = \frac{\cos(a)\,\text{Ci}(bx) - \cos(bx+a)\log(x) - \sin(a)\,\text{Si}(bx)}{b}$$

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] (cos(a)*cos_integral(b*x) - cos(b*x + a)*log(x) - sin(a)*sin_integral(b*x))
/b
```

Sympy [F]

$$\int \log(x)\sin(a+bx)\,dx = \int \log{(x)}\sin{(a+bx)}\,dx$$

```
[In] integrate(ln(x)*sin(b*x+a),x)
[Out] Integral(log(x)*sin(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \log(x)\sin(a+bx)\,dx = -\frac{\cos{(bx+a)}\log{(x)}}{b} - \frac{(E_1(i\,bx)+E_1(-i\,bx))\cos{(a)} - (i\,E_1(i\,bx) - i\,E_1(-i\,bx))\sin{(a)}}{2\,b}$$

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="maxima")
[Out] -cos(b*x + a)*log(x)/b - 1/2*((exp_integral_e(1, I*b*x) + exp_integral_e(1,
 -I*b*x))*cos(a) - (I*exp_integral_e(1, I*b*x) - I*exp_integral_e(1, -I*b*x
))*sin(a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.91

$$\int \log(x)\sin(a+bx)\,dx = -\frac{\cos{(bx+a)}\log{(x)}}{b} - \frac{\Re(\mathrm{Ci}\,(bx))\tan{\left(\frac{1}{2}\,a\right)}^2 + \Re(\mathrm{Ci}\,(-bx))\tan{\left(\frac{1}{2}\,a\right)}^2 + 2\,\Im(\mathrm{Ci}\,(bx))\tan{\left(\frac{1}{2}\,a\right)} - 2\,\Im(\mathrm{Ci}\,(-bx))\tan{\left(\frac{1}{2}\,a\right)} + 4\,\mathrm{S}}{2\left(b\tan{\left(\frac{1}{2}\,a\right)}^2 + b\right)}$$

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="giac")
[Out] -cos(b*x + a)*log(x)/b - 1/2*(real_part(cos_integral(b*x))*tan(1/2*a)^2 + r
eal_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(b*x))*
tan(1/2*a) - 2*imag_part(cos_integral(-b*x))*tan(1/2*a) + 4*sin_integral(b*
x)*tan(1/2*a) - real_part(cos_integral(b*x)) - real_part(cos_integral(-b*x)
))/(b*tan(1/2*a)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sin(a+bx)\,dx = \int \sin\left(a+b\,x\right)\,\ln\left(x\right)\,dx$$

```
[In] int(sin(a + b*x)*log(x),x)
[Out] int(sin(a + b*x)*log(x), x)
```

3.155 $\int \log(x)\sin^2(a+bx)\,dx$

Optimal result 897
Rubi [A] (verified) 897
Mathematica [A] (verified) 899
Maple [C] (warning: unable to verify) 899
Fricas [A] (verification not implemented) 899
Sympy [F] 900
Maxima [C] (verification not implemented) 900
Giac [C] (verification not implemented) 900
Mupad [F(-1)] 901

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \log(x)\sin^2(a+bx)\,dx = -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\text{CosIntegral}(2bx)\sin(2a)}{4b} - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}+\frac{\cos(2a)\text{Si}(2bx)}{4b}$$

```
[Out] -1/2*x+1/2*x*ln(x)+1/4*cos(2*a)*Si(2*b*x)/b+1/4*Ci(2*b*x)*sin(2*a)/b-1/2*co
s(b*x+a)*ln(x)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {2715, 8, 2634, 3384, 3380, 3383}

$$\int \log(x)\sin^2(a+bx)\,dx = \frac{\sin(2a)\,\text{CosIntegral}(2bx)}{4b}+\frac{\cos(2a)\text{Si}(2bx)}{4b} - \frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

```
[In] Int[Log[x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/2*x + (x*Log[x])/2 + (CosIntegral[2*b*x]*Sin[2*a])/(4*b) - (Cos[a + b*x]
*Log[x]*Sin[a + b*x])/(2*b) + (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \int \left(\frac{1}{2} - \frac{\sin(2a+2bx)}{4bx}\right)\,dx \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\int \frac{\sin(2a+2bx)}{x}\,dx}{4b} \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\cos(2a)\int \frac{\sin(2bx)}{x}\,dx}{4b} + \frac{\sin(2a)\int \frac{\cos(2bx)}{x}\,dx}{4b} \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\text{Ci}(2bx)\sin(2a)}{4b} - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\cos(2a)\text{Si}(2bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \log(x)\sin^2(a+bx)\,dx = \frac{-2bx+2bx\log(x)+\text{CosIntegral}(2bx)\sin(2a)-\log(x)\sin(2(a+bx))+\cos(2a)\text{Si}(2bx)}{4b}$$

```
[In] Integrate[Log[x]*Sin[a + b*x]^2,x]
[Out] (-2*b*x + 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a + b*
x)] + Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.60 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

method	result
risch	$\frac{\ln(x)x}{2}-\frac{\sin(2bx+2a)\ln(x)}{4b}-\frac{\mathrm{e}^{-2ia}\pi\,\mathrm{csgn}(bx)}{8b}+\frac{\mathrm{e}^{-2ia}\,\mathrm{Si}(2bx)}{4b}-\frac{i\mathrm{e}^{-2ia}\,\mathrm{Ei}_1(-2ibx)}{8b}+\frac{a\ln(ibx)}{2b}-\frac{\ln(a+i(ibx+ia))a}{2b}-$

```
[In] int(ln(x)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)*x-1/4/b*sin(2*b*x+2*a)*ln(x)-1/8/b*exp(-2*I*a)*Pi*csgn(b*x)+1/4/b
*exp(-2*I*a)*Si(2*b*x)-1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2/b*a*ln(I*b*x)
-1/2/b*ln(a+I*(I*b*x+I*a))*a-1/2*x-1/2*a/b+1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x
)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \log(x)\sin^2(a+bx)\,dx = \frac{2\,bx\log(x)-2\,\cos(bx+a)\log(x)\sin(bx+a)-2\,bx+\text{Ci}(2\,bx)\sin(2\,a)+\cos(2\,a)\,\text{Si}(2\,bx)}{4\,b}$$

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/4*(2*b*x*log(x) - 2*cos(b*x + a)*log(x)*sin(b*x + a) - 2*b*x + cos_integr
al(2*b*x)*sin(2*a) + cos(2*a)*sin_integral(2*b*x))/b
```

Sympy [F]

$$\int \log(x)\sin^2(a+bx)\,dx = \int \log{(x)}\sin^2{(a+bx)}\,dx$$

```
[In] integrate(ln(x)*sin(b*x+a)**2,x)
```

```
[Out] Integral(log(x)*sin(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \log(x)\sin^2(a+bx)\,dx = \frac{(2\,bx+2\,a-\sin{(2\,bx+2\,a)})\log{(x)}}{4\,b} - \frac{4\,bx+(i\,\mathrm{Ei}(2i\,bx)-i\,\mathrm{Ei}(-2i\,bx))\cos{(2\,a)}+4\,a\log{(x)}-(\mathrm{Ei}(2i\,bx)+\mathrm{Ei}(-2i\,bx))\sin{(2\,a)}}{8\,b}$$

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (I*Ei(2*I*b*x)
 - I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) - (Ei(2*I*b*x) + Ei(-2*I*b*x))*sin
(2*a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.86

$$\int \log(x)\sin^2(a+bx)\,dx = \frac{1}{4}\left(2\,x-\frac{\sin{(2\,bx+2\,a)}}{b}\right)\log{(x)} - \frac{4\,bx\tan{(a)}^2+\Im(\mathrm{Ci}\,(2\,bx))\tan{(a)}^2-\Im(\mathrm{Ci}\,(-2\,bx))\tan{(a)}^2+2\,\mathrm{Si}\,(2\,bx)\tan{(a)}^2+4\,bx-2\,\Re(\mathrm{Ci}\,(2\,bx)}{8\,\left(b\tan{(a)}^2+b\right)}$$

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*x - sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 + imag_part(cos
_integral(2*b*x))*tan(a)^2 - imag_part(cos_integral(-2*b*x))*tan(a)^2 + 2*s
in_integral(2*b*x)*tan(a)^2 + 4*b*x - 2*real_part(cos_integral(2*b*x))*tan(
a) - 2*real_part(cos_integral(-2*b*x))*tan(a) - imag_part(cos_integral(2*b*
x)) + imag_part(cos_integral(-2*b*x)) - 2*sin_integral(2*b*x))/(b*tan(a)^2
+ b)
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sin^2(a+bx)\,dx = \int \sin\left(a+b\,x\right)^2\,\ln\left(x\right)\,dx$$

```
[In] int(sin(a + b*x)^2*log(x),x)
[Out] int(sin(a + b*x)^2*log(x), x)
```

3.156 $\int \log(x)\sin^3(a+bx)\,dx$

Optimal result 902
Rubi [A] (verified) 902
Mathematica [A] (verified) 904
Maple [C] (warning: unable to verify) 905
Fricas [A] (verification not implemented) 905
Sympy [F] 905
Maxima [C] (verification not implemented) 906
Giac [C] (verification not implemented) 906
Mupad [F(-1)] 907

Optimal result

Integrand size = 11, antiderivative size = 89

$$\int \log(x)\sin^3(a+bx)\,dx = \frac{3\cos(a)\,\text{CosIntegral}(bx)}{4b}-\frac{\cos(3a)\,\text{CosIntegral}(3bx)}{12b}-\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{3\sin(a)\text{Si}(bx)}{4b}+\frac{\sin(3a)\text{Si}(3bx)}{12b}$$

```
[Out] 3/4*Ci(b*x)*cos(a)/b-1/12*Ci(3*b*x)*cos(3*a)/b-cos(b*x+a)*ln(x)/b+1/3*cos(b
*x+a)^3*ln(x)/b-3/4*Si(b*x)*sin(a)/b+1/12*Si(3*b*x)*sin(3*a)/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2713, 2634, 12, 6874, 3384, 3380, 3383, 3393}

$$\int \log(x)\sin^3(a+bx)\,dx = \frac{3\cos(a)\,\text{CosIntegral}(bx)}{4b}-\frac{\cos(3a)\,\text{CosIntegral}(3bx)}{12b}-\frac{3\sin(a)\text{Si}(bx)}{4b}+\frac{\sin(3a)\text{Si}(3bx)}{12b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\frac{\log(x)\cos(a+bx)}{b}$$

```
[In] Int[Log[x]*Sin[a + b*x]^3,x]
```

```
[Out] (3*Cos[a]*CosIntegral[b*x])/(4*b) - (Cos[3*a]*CosIntegral[3*b*x])/(12*b) -
(Cos[a + b*x]*Log[x])/b + (Cos[a + b*x]^3*Log[x])/(3*b) - (3*Sin[a]*SinInte
gral[b*x])/(4*b) + (Sin[3*a]*SinIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \int \frac{\cos(a+bx)\left(-3+\cos^2(a+bx)\right)}{3bx}\,dx$$

$$= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\int \frac{\cos(a+bx)(-3+\cos^2(a+bx))}{x}\,dx}{3b}$$

$$= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\int \left(-\frac{3\cos(a+bx)}{x} + \frac{\cos^3(a+bx)}{x}\right)dx}{3b}$$

$$= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\int \frac{\cos^3(a+bx)}{x}\,dx}{3b} + \frac{\int \frac{\cos(a+bx)}{x}\,dx}{b}$$

$$= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\int \left(\frac{3\cos(a+bx)}{4x} + \frac{\cos(3a+3bx)}{4x}\right)dx}{3b} + \frac{\cos(a)\int \frac{\cos(bx)}{x}\,dx}{b} - \frac{\sin(a)\int \frac{\sin(bx)}{x}\,dx}{b}$$

$$= \frac{\cos(a)\text{Ci}(bx)}{b} - \frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\sin(a)\text{Si}(bx)}{b} - \frac{\int \frac{\cos(3a+3bx)}{x}\,dx}{12b} - \frac{\int \frac{\cos(a+bx)}{x}\,dx}{4b}$$

$$= \frac{\cos(a)\text{Ci}(bx)}{b} - \frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\sin(a)\text{Si}(bx)}{b} - \frac{\cos(a)\int \frac{\cos(bx)}{x}\,dx}{4b} - \frac{\cos(3a)\int \frac{\cos(3bx)}{x}\,dx}{12b} + \frac{\sin(a)\int \frac{\sin(bx)}{x}\,dx}{4b} + \frac{\sin(3a)\int \frac{\sin(3bx)}{x}\,dx}{12b}$$

$$= \frac{3\cos(a)\text{Ci}(bx)}{4b} - \frac{\cos(3a)\text{Ci}(3bx)}{12b} - \frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{3\sin(a)\text{Si}(bx)}{4b} + \frac{\sin(3a)\text{Si}(3bx)}{12b}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \log(x)\sin^3(a+bx)\,dx$$

$$= \frac{9\cos(a)\,\text{CosIntegral}(bx) - \cos(3a)\,\text{CosIntegral}(3bx) - 9\cos(a+bx)\log(x) + \cos(3(a+bx))\log(x) - 9\sin}{12b}$$

```
[In] Integrate[Log[x]*Sin[a + b*x]^3,x]
```

```
[Out] (9*Cos[a]*CosIntegral[b*x] - Cos[3*a]*CosIntegral[3*b*x] - 9*Cos[a + b*x]*L
og[x] + Cos[3*(a + b*x)]*Log[x] - 9*Sin[a]*SinIntegral[b*x] + Sin[3*a]*SinI
ntegral[3*b*x])/(12*b)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.90 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{3\cos(bx+a)\ln(x)}{4b}+\frac{\ln(x)\cos(3bx+3a)}{12b}-\frac{i\mathrm{e}^{-3ia}\pi\ \mathrm{csgn}(bx)}{24b}+\frac{i\mathrm{e}^{-3ia}\ \mathrm{Si}(3bx)}{12b}+\frac{\mathrm{e}^{-3ia}\ \mathrm{Ei}_1(-3ibx)}{24b}+\frac{3i\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(bx)}{8b}-$

```
[In] int(ln(x)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*cos(b*x+a)*ln(x)/b+1/12/b*ln(x)*cos(3*b*x+3*a)-1/24*I/b*exp(-3*I*a)*Pi
*csgn(b*x)+1/12*I/b*exp(-3*I*a)*Si(3*b*x)+1/24/b*exp(-3*I*a)*Ei(1,-3*I*b*x)
+3/8*I/b*exp(-I*a)*Pi*csgn(b*x)-3/4*I/b*exp(-I*a)*Si(b*x)-3/8/b*exp(-I*a)*E
i(1,-I*b*x)-3/8/b*exp(I*a)*Ei(1,-I*b*x)+1/24/b*exp(3*I*a)*Ei(1,-3*I*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int \log(x)\sin^3(a+bx)\,dx = -\frac{\cos(3a)\,\mathrm{Ci}(3bx) - 9\cos(a)\,\mathrm{Ci}(bx) - 4\left(\cos(bx+a)^3 - 3\cos(bx+a)\right)\log(x) - \sin(3a)\,\mathrm{Si}(3bx) + 9}{12b}$$

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(cos(3*a)*cos_integral(3*b*x) - 9*cos(a)*cos_integral(b*x) - 4*(cos(b
*x + a)^3 - 3*cos(b*x + a))*log(x) - sin(3*a)*sin_integral(3*b*x) + 9*sin(a
)*sin_integral(b*x))/b
```

Sympy [F]

$$\int \log(x)\sin^3(a+bx)\,dx = \int \log(x)\sin^3(a+bx)\,dx$$

```
[In] integrate(ln(x)*sin(b*x+a)**3,x)
```

```
[Out] Integral(log(x)*sin(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \log(x)\sin^3(a+bx)\,dx = \frac{\left(\cos\left(bx+a\right)^3-3\,\cos\left(bx+a\right)\right)\log\left(x\right)}{3\,b} + \frac{\left(E_1(3i\,bx)+E_1(-3i\,bx)\right)\cos\left(3\,a\right)-9\left(E_1(i\,bx)+E_1(-i\,bx)\right)\cos\left(a\right)-\left(i\,E_1(3i\,bx)-i\,E_1(-3i\,bx)\right)\sin\left(3\right.}{24\,b}$$

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))*log(x)/b + 1/24*((exp_integral_e(1, 3
*I*b*x) + exp_integral_e(1, -3*I*b*x))*cos(3*a) - 9*(exp_integral_e(1, I*b*
x) + exp_integral_e(1, -I*b*x))*cos(a) - (I*exp_integral_e(1, 3*I*b*x) - I*
exp_integral_e(1, -3*I*b*x))*sin(3*a) + 9*(I*exp_integral_e(1, I*b*x) - I*e
xp_integral_e(1, -I*b*x))*sin(a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.10

$$\int \log(x)\sin^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(cos(b*x + a)^3/b - 3*cos(b*x + a)/b)*log(x) + 1/24*(real_part(cos_inte
gral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))*tan
(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1
/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 18*im
ag_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/2*a) + 18*imag_part(cos_integ
ral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 36*sin_integral(b*x)*tan(3/2*a)^2*tan(
1/2*a) + 2*imag_part(cos_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 - 2*imag_
part(cos_integral(-3*b*x))*tan(3/2*a)*tan(1/2*a)^2 + 4*sin_integral(3*b*x)*
tan(3/2*a)*tan(1/2*a)^2 + real_part(cos_integral(3*b*x))*tan(3/2*a)^2 + 9*r
eal_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*real_part(cos_integral(-b*x))*
tan(3/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2 - real_part(cos
_integral(3*b*x))*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))*tan(1/2*a)^
2 - 9*real_part(cos_integral(-b*x))*tan(1/2*a)^2 - real_part(cos_integral(-
3*b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(3*b*x))*tan(3/2*a) - 2*imag
_part(cos_integral(-3*b*x))*tan(3/2*a) + 4*sin_integral(3*b*x)*tan(3/2*a) -
 18*imag_part(cos_integral(b*x))*tan(1/2*a) + 18*imag_part(cos_integral(-b*
```

```
x))*tan(1/2*a) - 36*sin_integral(b*x)*tan(1/2*a) - real_part(cos_integral(3
*b*x)) + 9*real_part(cos_integral(b*x)) + 9*real_part(cos_integral(-b*x)) -
 real_part(cos_integral(-3*b*x)))/(b*tan(3/2*a)^2*tan(1/2*a)^2 + b*tan(3/2*
a)^2 + b*tan(1/2*a)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sin^3(a+bx)\,dx=\int \sin\left(a+b\,x\right)^3\,\ln\left(x\right)\,dx$$

```
[In] int(sin(a + b*x)^3*log(x),x)
```

```
[Out] int(sin(a + b*x)^3*log(x), x)
```

3.157 $\int \cos(a+bx)\log(x)\,dx$

Optimal result 908
Rubi [A] (verified) 908
Mathematica [A] (verified) 910
Maple [C] (warning: unable to verify) 910
Fricas [A] (verification not implemented) 910
Sympy [F] 911
Maxima [C] (verification not implemented) 911
Giac [C] (verification not implemented) 911
Mupad [F(-1)] 912

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \cos(a+bx)\log(x)\,dx = -\frac{\text{CosIntegral}(bx)\sin(a)}{b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\cos(a)\text{Si}(bx)}{b}$$

```
[Out] -cos(a)*Si(b*x)/b-Ci(b*x)*sin(a)/b+ln(x)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2717, 2634, 12, 3384, 3380, 3383}

$$\int \cos(a+bx)\log(x)\,dx = -\frac{\sin(a)\,\text{CosIntegral}(bx)}{b} - \frac{\cos(a)\text{Si}(bx)}{b} + \frac{\log(x)\sin(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]*Log[x],x]
```

```
[Out] -((CosIntegral[b*x]*Sin[a])/b) + (Log[x]*Sin[a + b*x])/b - (Cos[a]*SinInteg
ral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
```

```
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log(x)\sin(a+bx)}{b} - \int \frac{\sin(a+bx)}{bx}\,dx\\
&= \frac{\log(x)\sin(a+bx)}{b} - \frac{\int \frac{\sin(a+bx)}{x}\,dx}{b}\\
&= \frac{\log(x)\sin(a+bx)}{b} - \frac{\cos(a)\int \frac{\sin(bx)}{x}\,dx}{b} - \frac{\sin(a)\int \frac{\cos(bx)}{x}\,dx}{b}\\
&= -\frac{\text{Ci}(bx)\sin(a)}{b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\cos(a)\text{Si}(bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \cos(a+bx)\log(x)\,dx = -\frac{\text{CosIntegral}(bx)\sin(a)-\log(x)\sin(a+bx)+\cos(a)\text{Si}(bx)}{b}$$

```
[In] Integrate[Cos[a + b*x]*Log[x],x]
```

```
[Out] -((CosIntegral[b*x]*Sin[a] - Log[x]*Sin[a + b*x] + Cos[a]*SinIntegral[b*x])
/b)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.26

method	result	size
risch	$\frac{\ln(x)\sin(bx+a)}{b}+\frac{\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(bx)}{2b}-\frac{\mathrm{e}^{-ia}\,\mathrm{Si}(bx)}{b}+\frac{i\mathrm{e}^{-ia}\,\mathrm{Ei}_1(-ibx)}{2b}-\frac{i\mathrm{e}^{ia}\,\mathrm{Ei}_1(-ibx)}{2b}$	79

```
[In] int(cos(b*x+a)*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*sin(b*x+a)/b+1/2/b*exp(-I*a)*Pi*csgn(b*x)-1/b*exp(-I*a)*Si(b*x)+1/2*I
/b*exp(-I*a)*Ei(1,-I*b*x)-1/2*I/b*exp(I*a)*Ei(1,-I*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \cos(a+bx)\log(x)\,dx = \frac{\log(x)\sin(bx+a)-\text{Ci}(bx)\sin(a)-\cos(a)\,\text{Si}(bx)}{b}$$

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] (log(x)*sin(b*x + a) - cos_integral(b*x)*sin(a) - cos(a)*sin_integral(b*x))
/b
```

Sympy [F]

$$\int \cos(a+bx)\log(x)\,dx = \int \log\left(x\right)\cos\left(a+bx\right)\,dx$$

```
[In] integrate(cos(b*x+a)*ln(x),x)
[Out] Integral(log(x)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \cos(a+bx)\log(x)\,dx = \frac{\log\left(x\right)\sin\left(bx+a\right)}{b} + \frac{\left(i\,E_1(i\,bx) - i\,E_1(-i\,bx)\right)\cos\left(a\right) + \left(E_1(i\,bx) + E_1(-i\,bx)\right)\sin\left(a\right)}{2\,b}$$

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="maxima")
[Out] log(x)*sin(b*x + a)/b + 1/2*((I*exp_integral_e(1, I*b*x) - I*exp_integral_e
(1, -I*b*x))*cos(a) + (exp_integral_e(1, I*b*x) + exp_integral_e(1, -I*b*x)
)*sin(a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.09

$$\int \cos(a+bx)\log(x)\,dx = \frac{\log\left(x\right)\sin\left(bx+a\right)}{b} + \frac{\Im(\mathrm{Ci}\left(bx\right))\tan\left(\frac{1}{2}\,a\right)^2 - \Im(\mathrm{Ci}\left(-bx\right))\tan\left(\frac{1}{2}\,a\right)^2 + 2\,\mathrm{Si}\left(bx\right)\tan\left(\frac{1}{2}\,a\right)^2 - 2\,\Re(\mathrm{Ci}\left(bx\right))\tan\left(\frac{1}{2}\,a\right) - 2\,\Re(\mathrm{Ci}}{2\left(b\tan\left(\frac{1}{2}\,a\right)^2 + b\right)}$$

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="giac")
[Out] log(x)*sin(b*x + a)/b + 1/2*(imag_part(cos_integral(b*x))*tan(1/2*a)^2 - im
ag_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*sin_integral(b*x)*tan(1/2*a)^2
 - 2*real_part(cos_integral(b*x))*tan(1/2*a) - 2*real_part(cos_integral(-b*
x))*tan(1/2*a) - imag_part(cos_integral(b*x)) + imag_part(cos_integral(-b*x
)) - 2*sin_integral(b*x))/(b*tan(1/2*a)^2 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)\log(x)\,dx = \int \cos\left(a+b\,x\right)\,\ln\left(x\right)\,dx$$

```
[In] int(cos(a + b*x)*log(x),x)
[Out] int(cos(a + b*x)*log(x), x)
```

3.158 $\int \cos^2(a+bx)\log(x)\,dx$

Optimal result 913
Rubi [A] (verified) 913
Mathematica [A] (verified) 915
Maple [C] (warning: unable to verify) 915
Fricas [A] (verification not implemented) 916
Sympy [F] 916
Maxima [C] (verification not implemented) 916
Giac [C] (verification not implemented) 917
Mupad [F(-1)] 917

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \cos^2(a+bx)\log(x)\,dx = -\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\text{CosIntegral}(2bx)\sin(2a)}{4b}+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\cos(2a)\text{Si}(2bx)}{4b}$$

```
[Out] -1/2*x+1/2*x*ln(x)-1/4*cos(2*a)*Si(2*b*x)/b-1/4*Ci(2*b*x)*sin(2*a)/b+1/2*co
s(b*x+a)*ln(x)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2715, 8, 2634, 12, 3408, 3384, 3380, 3383}

$$\int \cos^2(a+bx)\log(x)\,dx = -\frac{\sin(2a)\,\text{CosIntegral}(2bx)}{4b}-\frac{\cos(2a)\text{Si}(2bx)}{4b}+\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

```
[In] Int[Cos[a + b*x]^2*Log[x],x]
```

```
[Out] -1/2*x + (x*Log[x])/2 - (CosIntegral[2*b*x]*Sin[2*a])/(4*b) + (Cos[a + b*x]
*Log[x]*Sin[a + b*x])/(2*b) - (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3408

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[ExpandToSum[
u, x]^m*(a + b*Sin[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] && L
inearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \int \frac{1}{4}\left(2+\frac{\sin(2(a+bx))}{bx}\right)\,dx \\ &= \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{1}{4}\int \left(2+\frac{\sin(2(a+bx))}{bx}\right)\,dx \end{aligned}$$

$$= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{\int \frac{\sin(2(a+bx))}{x}\,dx}{4b}$$

$$= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{\int \frac{\sin(2a+2bx)}{x}\,dx}{4b}$$

$$= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{\cos(2a)\int \frac{\sin(2bx)}{x}\,dx}{4b} - \frac{\sin(2a)\int \frac{\cos(2bx)}{x}\,dx}{4b}$$

$$= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\text{Ci}(2bx)\sin(2a)}{4b} + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{\cos(2a)\text{Si}(2bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \cos^2(a+bx)\log(x)\,dx = -\frac{2bx - 2bx\log(x) + \text{CosIntegral}(2bx)\sin(2a) - \log(x)\sin(2(a+bx)) + \cos(2a)\text{Si}(2bx)}{4b}$$

```
[In] Integrate[Cos[a + b*x]^2*Log[x],x]
[Out] -1/4*(2*b*x - 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a
+ b*x)] + Cos[2*a]*SinIntegral[2*b*x])/b
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

method	result
risch	$\frac{\ln(x)x}{2} + \frac{\sin(2bx+2a)\ln(x)}{4b} + \frac{e^{-2ia}\pi\ \text{csgn}(bx)}{8b} - \frac{e^{-2ia}\ \text{Si}(2bx)}{4b} + \frac{ie^{-2ia}\ \text{Ei}_1(-2ibx)}{8b} + \frac{a\ln(ibx)}{2b} - \frac{\ln(a+i(ibx+ia))a}{2b} -$

```
[In] int(cos(b*x+a)^2*ln(x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)*x+1/4/b*sin(2*b*x+2*a)*ln(x)+1/8/b*exp(-2*I*a)*Pi*csgn(b*x)-1/4/b
*exp(-2*I*a)*Si(2*b*x)+1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2/b*a*ln(I*b*x)
-1/2/b*ln(a+I*(I*b*x+I*a))*a-1/2*x-1/2*a/b-1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x
)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \cos^2(a+bx)\log(x)\,dx$$
$$=\frac{2\,bx\log\left(x\right)+2\,\cos\left(bx+a\right)\log\left(x\right)\sin\left(bx+a\right)-2\,bx-\operatorname{Ci}\left(2\,bx\right)\sin\left(2\,a\right)-\cos\left(2\,a\right)\operatorname{Si}\left(2\,bx\right)}{4\,b}$$

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="fricas")
[Out] 1/4*(2*b*x*log(x) + 2*cos(b*x + a)*log(x)*sin(b*x + a) - 2*b*x - cos_integr
al(2*b*x)*sin(2*a) - cos(2*a)*sin_integral(2*b*x))/b
```

Sympy [F]

$$\int \cos^2(a+bx)\log(x)\,dx=\int \log\left(x\right)\cos^2\left(a+bx\right)\,dx$$

```
[In] integrate(cos(b*x+a)**2*ln(x),x)
[Out] Integral(log(x)*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \cos^2(a+bx)\log(x)\,dx=\frac{\left(2\,bx+2\,a+\sin\left(2\,bx+2\,a\right)\right)\log\left(x\right)}{4\,b}$$
$$-\frac{4\,bx+\left(-i\,\operatorname{Ei}(2i\,bx)+i\,\operatorname{Ei}(-2i\,bx)\right)\cos\left(2\,a\right)+4\,a\log\left(x\right)+\left(\operatorname{Ei}(2i\,bx)+\operatorname{Ei}(-2i\,bx)\right)\sin\left(2\,a\right)}{8\,b}$$

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="maxima")
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (-I*Ei(2*I*b*x
) + I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) + (Ei(2*I*b*x) + Ei(-2*I*b*x))*si
n(2*a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.85

$$\int \cos^2(a+bx)\log(x)\,dx = \frac{1}{4}\left(2\,x + \frac{\sin\left(2\,bx + 2\,a\right)}{b}\right)\log\left(x\right) - \frac{4\,bx\tan\left(a\right)^2 - \Im(\mathrm{Ci}\left(2\,bx\right))\tan\left(a\right)^2 + \Im(\mathrm{Ci}\left(-2\,bx\right))\tan\left(a\right)^2 - 2\,\mathrm{Si}\left(2\,bx\right)\tan\left(a\right)^2 + 4\,bx + 2\,\Re(\mathrm{Ci}\left(2\,b\right.}{8\left(b\tan\left(a\right)^2 + b\right)}$$

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 - imag_part(cos
_integral(2*b*x))*tan(a)^2 + imag_part(cos_integral(-2*b*x))*tan(a)^2 - 2*s
in_integral(2*b*x)*tan(a)^2 + 4*b*x + 2*real_part(cos_integral(2*b*x))*tan(
a) + 2*real_part(cos_integral(-2*b*x))*tan(a) + imag_part(cos_integral(2*b*
x)) - imag_part(cos_integral(-2*b*x)) + 2*sin_integral(2*b*x))/(b*tan(a)^2
+ b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\log(x)\,dx = \int \cos\left(a+b\,x\right)^2\,\ln\left(x\right)\,dx$$

```
[In] int(cos(a + b*x)^2*log(x),x)
```

```
[Out] int(cos(a + b*x)^2*log(x), x)
```

3.159 $\int \cos^3(a+bx)\log(x)\,dx$

Optimal result 918
Rubi [A] (verified) 918
Mathematica [A] (verified) 920
Maple [C] (warning: unable to verify) 921
Fricas [A] (verification not implemented) 921
Sympy [F] 921
Maxima [C] (verification not implemented) 922
Giac [C] (verification not implemented) 922
Mupad [F(-1)] 923

Optimal result

Integrand size = 11, antiderivative size = 88

$$\int \cos^3(a+bx)\log(x)\,dx = -\frac{3\,\text{CosIntegral}(bx)\sin(a)}{4b} - \frac{\text{CosIntegral}(3bx)\sin(3a)}{12b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{3\cos(a)\text{Si}(bx)}{4b} - \frac{\cos(3a)\text{Si}(3bx)}{12b}$$

```
[Out] -3/4*cos(a)*Si(b*x)/b-1/12*cos(3*a)*Si(3*b*x)/b-3/4*Ci(b*x)*sin(a)/b-1/12*C
i(3*b*x)*sin(3*a)/b+ln(x)*sin(b*x+a)/b-1/3*ln(x)*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2713, 2634, 12, 6874, 3384, 3380, 3383, 4515}

$$\int \cos^3(a+bx)\log(x)\,dx = -\frac{3\sin(a)\,\text{CosIntegral}(bx)}{4b} - \frac{\sin(3a)\,\text{CosIntegral}(3bx)}{12b} - \frac{3\cos(a)\text{Si}(bx)}{4b} - \frac{\cos(3a)\text{Si}(3bx)}{12b} - \frac{\log(x)\sin^3(a+bx)}{3b} + \frac{\log(x)\sin(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]^3*Log[x],x]
```

```
[Out] (-3*CosIntegral[b*x]*Sin[a])/(4*b) - (CosIntegral[3*b*x]*Sin[3*a])/(12*b) +
 (Log[x]*Sin[a + b*x])/b - (Log[x]*Sin[a + b*x]^3)/(3*b) - (3*Cos[a]*SinInt
egral[b*x])/(4*b) - (Cos[3*a]*SinIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4515

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Cos[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IGtQ[q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \int \frac{(5+\cos(2(a+bx)))\sin(a+bx)}{6bx}\,dx$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \frac{(5+\cos(2(a+bx)))\sin(a+bx)}{x}\,dx}{6b}$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \left(\frac{5\sin(a+bx)}{x} + \frac{\cos(2a+2bx)\sin(a+bx)}{x}\right) dx}{6b}$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \frac{\cos(2a+2bx)\sin(a+bx)}{x}\,dx}{6b} - \frac{5\int \frac{\sin(a+bx)}{x}\,dx}{6b}$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \left(-\frac{\sin(a+bx)}{2x} + \frac{\sin(3a+3bx)}{2x}\right) dx}{6b} - \frac{(5\cos(a))\int \frac{\sin(bx)}{x}\,dx}{6b} - \frac{(5\sin(a))\int \frac{\cos(bx)}{x}\,dx}{6b}$$
$$= -\frac{5\text{Ci}(bx)\sin(a)}{6b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{5\cos(a)\text{Si}(bx)}{6b} + \frac{\int \frac{\sin(a+bx)}{x}\,dx}{12b} - \frac{\int \frac{\sin(3a+3bx)}{x}\,dx}{12b}$$
$$= -\frac{5\text{Ci}(bx)\sin(a)}{6b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{5\cos(a)\text{Si}(bx)}{6b} + \frac{\cos(a)\int \frac{\sin(bx)}{x}\,dx}{12b} - \frac{\cos(3a)\int \frac{\sin(3bx)}{x}\,dx}{12b} + \frac{\sin(a)\int \frac{\cos(bx)}{x}\,dx}{12b} - \frac{\sin(3a)\int \frac{\cos(3bx)}{x}\,dx}{12b}$$
$$= -\frac{3\text{Ci}(bx)\sin(a)}{4b} - \frac{\text{Ci}(3bx)\sin(3a)}{12b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{3\cos(a)\text{Si}(bx)}{4b} - \frac{\cos(3a)\text{Si}(3bx)}{12b}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \cos^3(a+bx)\log(x)\,dx =$$
$$-\frac{9\,\text{CosIntegral}(bx)\sin(a) + \text{CosIntegral}(3bx)\sin(3a) - 9\log(x)\sin(a+bx) - \log(x)\sin(3(a+bx)) + 9\,\text{co}}{12b}$$

```
[In] Integrate[Cos[a + b*x]^3*Log[x],x]
[Out] -1/12*(9*CosIntegral[b*x]*Sin[a] + CosIntegral[3*b*x]*Sin[3*a] - 9*Log[x]*S
in[a + b*x] - Log[x]*Sin[3*(a + b*x)] + 9*Cos[a]*SinIntegral[b*x] + Cos[3*a
]*SinIntegral[3*b*x])/b
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

method	result
risch	$\frac{3\ln(x)\sin(bx+a)}{4b}+\frac{\ln(x)\sin(3bx+3a)}{12b}+\frac{\mathrm{e}^{-3ia}\pi\ \mathrm{csgn}(bx)}{24b}-\frac{\mathrm{e}^{-3ia}\ \mathrm{Si}(3bx)}{12b}+\frac{i\mathrm{e}^{-3ia}\ \mathrm{Ei}_1(-3ibx)}{24b}+\frac{3\,\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(bx)}{8b}-\frac{3\,\mathrm{e}^{-}}{}$

```
[In] int(cos(b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
[Out] 3/4*ln(x)*sin(b*x+a)/b+1/12/b*ln(x)*sin(3*b*x+3*a)+1/24/b*exp(-3*I*a)*Pi*cs
gn(b*x)-1/12/b*exp(-3*I*a)*Si(3*b*x)+1/24*I/b*exp(-3*I*a)*Ei(1,-3*I*b*x)+3/
8/b*exp(-I*a)*Pi*csgn(b*x)-3/4/b*exp(-I*a)*Si(b*x)+3/8*I/b*exp(-I*a)*Ei(1,-
I*b*x)-3/8*I/b*exp(I*a)*Ei(1,-I*b*x)-1/24*I/b*exp(3*I*a)*Ei(1,-3*I*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \cos^3(a+bx)\log(x)\,dx = \frac{4\left(\cos\left(bx+a\right)^2+2\right)\log\left(x\right)\sin\left(bx+a\right)-\mathrm{Ci}\left(3\,bx\right)\sin\left(3\,a\right)-9\ \mathrm{Ci}\left(bx\right)\sin\left(a\right)-\cos\left(3\,a\right)\mathrm{Si}\left(3\,bx\right)-9\ \mathrm{co}}{12\,b}$$

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="fricas")
[Out] 1/12*(4*(cos(b*x + a)^2 + 2)*log(x)*sin(b*x + a) - cos_integral(3*b*x)*sin(
3*a) - 9*cos_integral(b*x)*sin(a) - cos(3*a)*sin_integral(3*b*x) - 9*cos(a)
*sin_integral(b*x))/b
```

Sympy [F]

$$\int \cos^3(a+bx)\log(x)\,dx = \int \log\left(x\right)\cos^3\left(a+bx\right)\,dx$$

```
[In] integrate(cos(b*x+a)**3*ln(x),x)
[Out] Integral(log(x)*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \cos^3(a+bx)\log(x)\,dx = -\frac{\left(\sin\left(bx+a\right)^3-3\,\sin\left(bx+a\right)\right)\log\left(x\right)}{3\,b} + \frac{\left(i\,E_1(3i\,bx)-i\,E_1(-3i\,bx)\right)\cos\left(3\,a\right)-9\left(-i\,E_1(i\,bx)+i\,E_1(-i\,bx)\right)\cos\left(a\right)+\left(E_1(3i\,bx)+E_1(-3i\,bx)\right)\mathrm{s}}{24\,b}$$

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*((I*exp_integral_e(1
, 3*I*b*x) - I*exp_integral_e(1, -3*I*b*x))*cos(3*a) - 9*(-I*exp_integral_e
(1, I*b*x) + I*exp_integral_e(1, -I*b*x))*cos(a) + (exp_integral_e(1, 3*I*b
*x) + exp_integral_e(1, -3*I*b*x))*sin(3*a) + 9*(exp_integral_e(1, I*b*x) +
 exp_integral_e(1, -I*b*x))*sin(a))/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 495, normalized size of antiderivative = 5.62

$$\int \cos^3(a+bx)\log(x)\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*(imag_part(cos_integ
ral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x))*tan(
3/2*a)^2*tan(1/2*a)^2 - 9*imag_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1/
2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 2*sin_
integral(3*b*x)*tan(3/2*a)^2*tan(1/2*a)^2 + 18*sin_integral(b*x)*tan(3/2*a)
^2*tan(1/2*a)^2 - 18*real_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/2*a) -
 18*real_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 2*real_part(cos
_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 - 2*real_part(cos_integral(-3*b*x
))*tan(3/2*a)*tan(1/2*a)^2 + imag_part(cos_integral(3*b*x))*tan(3/2*a)^2 -
9*imag_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*imag_part(cos_integral(-b*x
))*tan(3/2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2 + 2*sin_inte
gral(3*b*x)*tan(3/2*a)^2 - 18*sin_integral(b*x)*tan(3/2*a)^2 - imag_part(co
s_integral(3*b*x))*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x))*tan(1/2*a)
^2 - 9*imag_part(cos_integral(-b*x))*tan(1/2*a)^2 + imag_part(cos_integral(
-3*b*x))*tan(1/2*a)^2 - 2*sin_integral(3*b*x)*tan(1/2*a)^2 + 18*sin_integra
l(b*x)*tan(1/2*a)^2 - 2*real_part(cos_integral(3*b*x))*tan(3/2*a) - 2*real_
```

```
part(cos_integral(-3*b*x))*tan(3/2*a) - 18*real_part(cos_integral(b*x))*tan
(1/2*a) - 18*real_part(cos_integral(-b*x))*tan(1/2*a) - imag_part(cos_integ
ral(3*b*x)) - 9*imag_part(cos_integral(b*x)) + 9*imag_part(cos_integral(-b*
x)) + imag_part(cos_integral(-3*b*x)) - 2*sin_integral(3*b*x) - 18*sin_inte
gral(b*x))/(b*tan(3/2*a)^2*tan(1/2*a)^2 + b*tan(3/2*a)^2 + b*tan(1/2*a)^2 +
 b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\log(x)\,dx=\int \cos\left(a+b\,x\right)^3\,\ln\left(x\right)\,dx$$

```
[In] int(cos(a + b*x)^3*log(x),x)
[Out] int(cos(a + b*x)^3*log(x), x)
```

3.160 $\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx$

Optimal result . . . 924
Rubi [A] (verified) . . . 924
Mathematica [A] (verified) . . . 925
Maple [A] (verified) . . . 925
Fricas [A] (verification not implemented) . . . 926
Sympy [A] (verification not implemented) . . . 926
Maxima [A] (verification not implemented) . . . 926
Giac [A] (verification not implemented) . . . 926
Mupad [B] (verification not implemented) . . . 927

Optimal result

Integrand size = 12, antiderivative size = 5

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log(x)\sin(x)$$

```
[Out] ln(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2717, 2634, 3380}

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log(x)\sin(x)$$

```
[In] Int[Cos[x]*Log[x] + Sin[x]/x,x]
```

```
[Out] Log[x]*Sin[x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \cos(x)\log(x)\,dx + \int \frac{\sin(x)}{x}\,dx\\ &= \log(x)\sin(x) + \text{Si}(x) - \int \frac{\sin(x)}{x}\,dx\\ &= \log(x)\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right)\,dx = \log(x)\sin(x)$$

```
[In] Integrate[Cos[x]*Log[x] + Sin[x]/x,x]
[Out] Log[x]*Sin[x]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
risch	$\ln(x)\sin(x)$	6
parallelrisch	$\ln(x)\sin(x)$	6
norman	$\frac{2\ln(x)\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	19

```
[In] int(cos(x)*ln(x)+sin(x)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log{(x)}\sin{(x)}$$

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="fricas")
[Out] log(x)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log{(x)}\sin{(x)}$$

```
[In] integrate(cos(x)*ln(x)+sin(x)/x,x)
[Out] log(x)*sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log{(x)}\sin{(x)}$$

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="maxima")
[Out] log(x)*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log{(x)}\sin{(x)}$$

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="giac")
[Out] log(x)*sin(x)
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\ln{(x)}\,\sin{(x)}$$

```
[In] int(cos(x)*log(x) + sin(x)/x,x)
[Out] log(x)*sin(x)
```

3.161 $\int \log(a\sin(x))\,dx$

Optimal result 928
Rubi [A] (verified) 928
Mathematica [A] (verified) 929
Maple [B] (verified) 930
Fricas [B] (verification not implemented) 930
Sympy [F] 931
Maxima [B] (verification not implemented) 931
Giac [F] 931
Mupad [F(-1)] 932

Optimal result

Integrand size = 5, antiderivative size = 47

$$\int \log(a\sin(x))\,dx = \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] 1/2*I*x^2-x*ln(1-exp(2*I*x))+x*ln(a*sin(x))+1/2*I*polylog(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3798, 2221, 2317, 2438}

$$\int \log(a\sin(x))\,dx = x\log(a\sin(x)) + \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right) + \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right)$$

```
[In] Int[Log[a*Sin[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]] + (I/2)*PolyLog[2, E^(
(2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log(a\sin(x)) - \int x\cot(x)\,dx \\
&= \frac{ix^2}{2} + x\log(a\sin(x)) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \int \log\left(1-e^{2ix}\right)\,dx \\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \log(a\sin(x))\,dx = -x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\left(x^2 + \text{PolyLog}\left(2, e^{2ix}\right)\right)$$

```
[In] Integrate[Log[a*Sin[x]],x]
[Out] -(x*Log[1 - E^((2*I)*x)]) + x*Log[a*Sin[x]] + (I/2)*(x^2 + PolyLog[2, E^((2
*I)*x)])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(37) = 74.

Time = 1.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

method	result
default	$-i\left(\ln\left(e^{ix}\right)\ln\left(ia\left(1-e^{2ix}\right)e^{-ix}\right)+\frac{\ln\left(e^{ix}\right)^{2}}{2}+\operatorname{dilog}\left(e^{ix}\right)-\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right)-\operatorname{dilog}\left(e^{ix}+1\right)-\ln\right.$
risch	$-x\ln\left(e^{ix}\right)+\frac{i\pi\,\operatorname{csgn}(a\sin(x))^{3}x}{2}-\frac{i\pi\,\operatorname{csgn}(a\sin(x))\operatorname{csgn}(ia\sin(x))^{2}x}{2}+\frac{ix^{2}}{2}-\frac{i\pi\,\operatorname{csgn}(\sin(x))\operatorname{csgn}(a\sin(x))^{2}x}{2}+\frac{i\pi\operatorname{csgn}}{}$

```
[In] int(ln(a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(I*a*(-exp(I*x)^2+1)/exp(I*x))+1/2*ln(exp(I*x))^2+dilog(
exp(I*x))-ln(exp(I*x))*ln(exp(I*x)+1)-dilog(exp(I*x)+1)-ln(2)*ln(exp(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(32) = 64.

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.21

$$\begin{aligned}\int\log(a\sin(x))\,dx=&\ x\log\left(a\sin\left(x\right)\right)-\frac{1}{2}\,x\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\&-\frac{1}{2}\,x\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)-\frac{1}{2}\,x\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\&-\frac{1}{2}\,x\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\&+\frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))\\&-\frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))+\frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))\end{aligned}$$

```
[In] integrate(log(a*sin(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x) - I*s
in(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) - I*sin(
x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*sin(x)) -
 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log(a\sin(x))\,dx = \int \log\left(a\sin\left(x\right)\right)dx$$

```
[In] integrate(ln(a*sin(x)),x)
```

```
[Out] Integral(log(a*sin(x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(32) = 64$.

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\begin{aligned}\int \log(a\sin(x))\,dx = &\frac{1}{2}i\,x^2 - i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right) + i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)\\ &-\frac{1}{2}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)\\ &-\frac{1}{2}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right)\\ &+x\log\left(a\sin\left(x\right)\right)+i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right)+i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\end{aligned}$$

```
[In] integrate(log(a*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(x), cos(x) + 1) + I*x*arctan2(sin(x), -cos(x) +
 1) - 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/2*x*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) + x*log(a*sin(x)) + I*dilog(-e^(I*x)) + I*dilog(e^
(I*x))
```

Giac [F]

$$\int \log(a\sin(x))\,dx = \int \log\left(a\sin\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\sin(x))\,dx = \int \ln\left(a\ \sin\left(x\right)\right)\,dx$$

```
[In] int(log(a*sin(x)),x)
[Out] int(log(a*sin(x)), x)
```

3.162 $\int \log \left(a \sin^2(x)\right) dx$

Optimal result 933
Rubi [A] (verified) 933
Mathematica [A] (verified) 935
Maple [B] (verified) 935
Fricas [B] (verification not implemented) 935
Sympy [F] 936
Maxima [B] (verification not implemented) 936
Giac [F] 937
Mupad [F(-1)] 937

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \log \left(a \sin^2(x)\right) dx = ix^2 - 2x \log \left(1 - e^{2ix}\right) + x \log \left(a \sin^2(x)\right) + i \operatorname{PolyLog}\left(2, e^{2ix}\right)$$

```
[Out] I*x^2-2*x*ln(1-exp(2*I*x))+x*ln(a*sin(x)^2)+I*polylog(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3798, 2221, 2317, 2438}

$$\int \log \left(a \sin^2(x)\right) dx = x \log \left(a \sin^2(x)\right) + i \operatorname{PolyLog}\left(2, e^{2ix}\right) + ix^2 - 2x \log \left(1 - e^{2ix}\right)$$

```
[In] Int[Log[a*Sin[x]^2],x]
```

```
[Out] I*x^2 - 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^2] + I*PolyLog[2, E^((2*I
)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\sin^2(x)\right) - \int 2x\cot(x)\,dx \\
&= x\log\left(a\sin^2(x)\right) - 2\int x\cot(x)\,dx \\
&= ix^2 + x\log\left(a\sin^2(x)\right) + 4i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= ix^2 - 2x\log\left(1-e^{2ix}\right) + x\log\left(a\sin^2(x)\right) + 2\int \log\left(1-e^{2ix}\right)\,dx \\
&= ix^2 - 2x\log\left(1-e^{2ix}\right) + x\log\left(a\sin^2(x)\right) - i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&= ix^2 - 2x\log\left(1-e^{2ix}\right) + x\log\left(a\sin^2(x)\right) + i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \log \left(a \sin^2(x)\right) \, dx = x\left(ix - 2\log\left(1 - e^{2ix}\right) + \log\left(a \sin^2(x)\right)\right) + i\,\text{PolyLog}\left(2, e^{2ix}\right)$$

```
[In] Integrate[Log[a*Sin[x]^2],x]
```

```
[Out] x*(I*x - 2*Log[1 - E^((2*I)*x)] + Log[a*Sin[x]^2]) + I*PolyLog[2, E^((2*I)*
x)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(39) = 78.

Time = 1.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.91

method	result
default	$-i\left(\ln\left(\mathrm{e}^{ix}\right)\ln\left(-a\left(\mathrm{e}^{2ix}-1\right)^{2}\mathrm{e}^{-2ix}\right)+\ln\left(\mathrm{e}^{ix}\right)^{2}-2\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{ix}+1\right)-2\,\mathrm{dilog}\left(\mathrm{e}^{ix}+1\right)+2\,\mathrm{dilo}$
risch	$-2x\ln\left(\mathrm{e}^{ix}\right)+ix^{2}-2i\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{2ix}-1\right)+\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-2ix}\right)\mathrm{csgn}\left(i\mathrm{e}^{-2ix}\left(\mathrm{e}^{2ix}-1\right)^{2}\right)^{2}x}{2}+\frac{i\pi\mathrm{csgn}\left(ia\left(\mathrm{e}^{2ix}-1\right)^{2}\mathrm{e}\right.}{2}$

```
[In] int(ln(a*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(-a*(exp(I*x)^2-1)^2/exp(I*x)^2)+ln(exp(I*x))^2-2*ln(exp
(I*x))*ln(exp(I*x)+1)-2*dilog(exp(I*x)+1)+2*dilog(exp(I*x))-2*ln(2)*ln(exp(
I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(34) = 68.

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.42

$$\begin{aligned}\int \log \left(a \sin^2(x)\right) \, dx = {} & x\log\left(-a\cos(x)^2 + a\right) - x\log\left(\cos(x) + i\sin(x) + 1\right) \\ & - x\log\left(\cos(x) - i\sin(x) + 1\right) - x\log\left(-\cos(x) + i\sin(x) + 1\right) \\ & - x\log\left(-\cos(x) - i\sin(x) + 1\right) \\ & + i\,\mathrm{Li}_2(\cos(x) + i\sin(x)) - i\,\mathrm{Li}_2(\cos(x) - i\sin(x)) \\ & - i\,\mathrm{Li}_2(-\cos(x) + i\sin(x)) + i\,\mathrm{Li}_2(-\cos(x) - i\sin(x))\end{aligned}$$

```
[In] integrate(log(a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-a*cos(x)^2 + a) - x*log(cos(x) + I*sin(x) + 1) - x*log(cos(x) - I*si
n(x) + 1) - x*log(-cos(x) + I*sin(x) + 1) - x*log(-cos(x) - I*sin(x) + 1) +
 I*dilog(cos(x) + I*sin(x)) - I*dilog(cos(x) - I*sin(x)) - I*dilog(-cos(x)
+ I*sin(x)) + I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log \left(a\sin^2(x)\right)\,dx = \int \log \left(a\sin^2\left(x\right)\right)dx$$

```
[In] integrate(ln(a*sin(x)**2),x)
```

```
[Out] Integral(log(a*sin(x)**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.98

$$\begin{aligned}\int \log \left(a\sin^2(x)\right)\,dx = {} & i\,x^2 - 2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right) \\ & + 2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right) + x\log\left(a\sin\left(x\right)^2\right) \\ & - x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right) \\ & - x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right) \\ & + 2i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + 2i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\end{aligned}$$

```
[In] integrate(log(a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] I*x^2 - 2*I*x*arctan2(sin(x), cos(x) + 1) + 2*I*x*arctan2(sin(x), -cos(x) +
 1) + x*log(a*sin(x)^2) - x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x*log
(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 2*I*dilog(-e^(I*x)) + 2*I*dilog(e^(I
*x))
```

Giac [F]

$$\int \log \left(a \sin^2(x)\right) \, dx = \int \log \left(a \sin{(x)}^2\right) \, dx$$

```
[In] integrate(log(a*sin(x)^2),x, algorithm="giac")
[Out] integrate(log(a*sin(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \sin^2(x)\right) \, dx = \int \ln \left(a \sin{(x)}^2\right) \, dx$$

```
[In] int(log(a*sin(x)^2),x)
[Out] int(log(a*sin(x)^2), x)
```

3.163 $\int \log(a\sin^n(x))\,dx$

Optimal result 938
Rubi [A] (verified) 938
Mathematica [A] (verified) 940
Maple [F] 940
Fricas [B] (verification not implemented) 940
Sympy [F] 941
Maxima [B] (verification not implemented) 941
Giac [F] 941
Mupad [F(-1)] 942

Optimal result

Integrand size = 7, antiderivative size = 52

$$\int \log(a\sin^n(x))\,dx = \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right) + x\log(a\sin^n(x)) + \frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] 1/2*I*n*x^2-n*x*ln(1-exp(2*I*x))+x*ln(a*sin(x)^n)+1/2*I*n*polylog(2,exp(2*I
*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3798, 2221, 2317, 2438}

$$\int \log(a\sin^n(x))\,dx = x\log(a\sin^n(x)) + \frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right) + \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right)$$

```
[In] Int[Log[a*Sin[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyLo
g[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\sin^n(x)\right)-\int nx\cot(x)\,dx\\
&= x\log\left(a\sin^n(x)\right)-n\int x\cot(x)\,dx\\
&= \frac{1}{2}inx^2+x\log\left(a\sin^n(x)\right)+(2in)\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)+x\log\left(a\sin^n(x)\right)+n\int\log\left(1-e^{2ix}\right)\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)+x\log\left(a\sin^n(x)\right)-\frac{1}{2}(in)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&= \frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)+x\log\left(a\sin^n(x)\right)+\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \log(a\sin^n(x))\,dx = \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right) + x\log(a\sin^n(x)) + \frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[In] Integrate[Log[a*Sin[x]^n],x]
[Out] (I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyLo
g[2, E^((2*I)*x)]
```

Maple [F]

$$\int \ln\left(a(\sin^n(x))\right)dx$$

```
[In] int(ln(a*sin(x)^n),x)
[Out] int(ln(a*sin(x)^n),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(37) = 74.

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \log(a\sin^n(x))\,dx = &-\frac{1}{2}nx\log(\cos(x)+i\sin(x)+1) - \frac{1}{2}nx\log(\cos(x)-i\sin(x)+1) \\ &- \frac{1}{2}nx\log(-\cos(x)+i\sin(x)+1) \\ &- \frac{1}{2}nx\log(-\cos(x)-i\sin(x)+1) \\ &+ nx\log(\sin(x)) + \frac{1}{2}in\text{Li}_2(\cos(x)+i\sin(x)) \\ &- \frac{1}{2}in\text{Li}_2(\cos(x)-i\sin(x)) - \frac{1}{2}in\text{Li}_2(-\cos(x)+i\sin(x)) \\ &+ \frac{1}{2}in\text{Li}_2(-\cos(x)-i\sin(x)) + x\log(a)\end{aligned}$$

```
[In] integrate(log(a*sin(x)^n),x, algorithm="fricas")
[Out] -1/2*n*x*log(cos(x) + I*sin(x) + 1) - 1/2*n*x*log(cos(x) - I*sin(x) + 1) -
1/2*n*x*log(-cos(x) + I*sin(x) + 1) - 1/2*n*x*log(-cos(x) - I*sin(x) + 1) +
 n*x*log(sin(x)) + 1/2*I*n*dilog(cos(x) + I*sin(x)) - 1/2*I*n*dilog(cos(x)
- I*sin(x)) - 1/2*I*n*dilog(-cos(x) + I*sin(x)) + 1/2*I*n*dilog(-cos(x) - I
*sin(x)) + x*log(a)
```

Sympy [F]

$$\int \log \left(a \sin^n(x)\right) \, dx = \int \log \left(a \sin^{n}\left(x\right)\right)\, dx$$

```
[In] integrate(ln(a*sin(x)**n),x)
```

```
[Out] Integral(log(a*sin(x)**n), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(37) = 74.

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.75

$$\int \log \left(a \sin^n(x)\right) \, dx =$$
$$-\frac{1}{2}\left(-i\,x^2 + 2i\,x \arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) - 2i\,x \arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) + x \log\left(\cos\left(x\right)^2 + \sin\right.$$
$$+ x \log\left(a \sin\left(x\right)^n\right)$$

```
[In] integrate(log(a*sin(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -c
os(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n + x*l
og(a*sin(x)^n)
```

Giac [F]

$$\int \log \left(a \sin^n(x)\right) \, dx = \int \log \left(a \sin\left(x\right)^{n}\right) \, dx$$

```
[In] integrate(log(a*sin(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log (a \sin^n(x)) \, dx = \int \ln (a \sin (x)^n) \, dx$$

```
[In] int(log(a*sin(x)^n),x)
[Out] int(log(a*sin(x)^n), x)
```

3.164 $\int \log(a\cos(x))\,dx$

Optimal result 943
Rubi [A] (verified) 943
Mathematica [A] (verified) 944
Maple [B] (verified) 945
Fricas [B] (verification not implemented) 945
Sympy [F] 946
Maxima [A] (verification not implemented) 946
Giac [F] 946
Mupad [B] (verification not implemented) 947

Optimal result

Integrand size = 5, antiderivative size = 47

$$\int \log(a\cos(x))\,dx = \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) + x\log(a\cos(x)) + \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

```
[Out] 1/2*I*x^2-x*ln(1+exp(2*I*x))+x*ln(a*cos(x))+1/2*I*polylog(2,-exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3800, 2221, 2317, 2438}

$$\int \log(a\cos(x))\,dx = x\log(a\cos(x)) + \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right) + \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right)$$

```
[In] Int[Log[a*Cos[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E^
((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log(a\cos(x))+\int x\tan(x)\,dx\\
&=\frac{ix^2}{2}+x\log(a\cos(x))-2i\int\frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&=\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))+\int\log\left(1+e^{2ix}\right)\,dx\\
&=\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))-\frac{1}{2}i\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\
&=\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))+\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int\log(a\cos(x))\,dx=\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))+\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

```
[In] Integrate[Log[a*Cos[x]],x]
[Out] (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E^
((2*I)*x)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(37) = 74.

Time = 1.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

method	result
default	$-i\left(\ln\left(\mathrm{e}^{ix}\right)\ln\left(a\left(1+\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)+\frac{\ln\left(\mathrm{e}^{ix}\right)^{2}}{2}-\ln\left(\mathrm{e}^{ix}\right)\ln\left(1+i\mathrm{e}^{ix}\right)-\ln\left(\mathrm{e}^{ix}\right)\ln\left(1-i\mathrm{e}^{ix}\right)-\operatorname{dilog}\left(1\right.\right.$
risch	$-x\ln\left(\mathrm{e}^{ix}\right)-i\ln\left(\mathrm{e}^{ix}\right)\ln\left(1+\mathrm{e}^{2ix}\right)+\frac{ix^{2}}{2}+\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}\left(i\cos\left(x\right)\right)^{2}x}{2}-\frac{i\pi\,\mathrm{csgn}\left(ia\cos\left(x\right)\right)^{3}x}{2}+i\ln\left(\mathrm{e}^{ix}\right)$

```
[In] int(ln(a*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(a*(exp(I*x)^2+1)/exp(I*x))+1/2*ln(exp(I*x))^2-ln(exp(I*
x))*ln(1+I*exp(I*x))-ln(exp(I*x))*ln(1-I*exp(I*x))-dilog(1+I*exp(I*x))-dilo
g(1-I*exp(I*x))-ln(2)*ln(exp(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(32) = 64.

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \log(a\cos(x))\,dx = {} & x\log\left(a\cos\left(x\right)\right)-\frac{1}{2}\,x\log\left(i\ \cos\left(x\right)+\sin\left(x\right)+1\right) \\ & -\frac{1}{2}\,x\log\left(i\ \cos\left(x\right)-\sin\left(x\right)+1\right)-\frac{1}{2}\,x\log\left(-i\ \cos\left(x\right)+\sin\left(x\right)+1\right) \\ & -\frac{1}{2}\,x\log\left(-i\ \cos\left(x\right)-\sin\left(x\right)+1\right) \\ & -\frac{1}{2}i\,\mathrm{Li}_2(i\ \cos\left(x\right)+\sin\left(x\right))+\frac{1}{2}i\,\mathrm{Li}_2(i\ \cos\left(x\right)-\sin\left(x\right)) \\ & +\frac{1}{2}i\,\mathrm{Li}_2(-i\ \cos\left(x\right)+\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_2(-i\ \cos\left(x\right)-\sin\left(x\right))\end{aligned}$$

```
[In] integrate(log(a*cos(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*cos(x)) - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) - s
in(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - sin(
x) + 1) - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) - sin(x)) +
 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]

$$\int \log(a\cos(x))\,dx = \int \log\left(a\cos\left(x\right)\right)dx$$

```
[In] integrate(ln(a*cos(x)),x)
[Out] Integral(log(a*cos(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \log(a\cos(x))\,dx = \frac{1}{2}i\,x^2 - i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right) - \frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right) + x\log\left(a\cos\left(x\right)\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)$$

```
[In] integrate(log(a*cos(x)),x, algorithm="maxima")
[Out] 1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) - 1/2*x*log(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*cos(x)) + 1/2*I*dilog(-e^(2*I*x))
```

Giac [F]

$$\int \log(a\cos(x))\,dx = \int \log\left(a\cos\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*cos(x)),x, algorithm="giac")
[Out] integrate(log(a*cos(x)), x)
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \log(a\cos(x))\,dx = x\,\ln\left(a\,\cos\left(x\right)\right) + \frac{\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\,\,1\mathrm{i}}{2} + \frac{x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

```
[In] int(log(a*cos(x)),x)
[Out] (polylog(2, -exp(x*2i))*1i)/2 + (x*(x + log(exp(x*2i) + 1)*2i)*1i)/2 + x*lo
g(a*cos(x))
```

3.165 $\int \log\left(a\cos^2(x)\right)\,dx$

Optimal result 948
Rubi [A] (verified) 948
Mathematica [A] (verified) 950
Maple [B] (verified) 950
Fricas [B] (verification not implemented) 950
Sympy [F] 951
Maxima [A] (verification not implemented) 951
Giac [F] 951
Mupad [B] (verification not implemented) 952

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \log\left(a\cos^2(x)\right)\,dx = ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) + i\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

```
[Out] I*x^2-2*x*ln(1+exp(2*I*x))+x*ln(a*cos(x)^2)+I*polylog(2,-exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3800, 2221, 2317, 2438}

$$\int \log\left(a\cos^2(x)\right)\,dx = x\log\left(a\cos^2(x)\right) + i\,\text{PolyLog}\left(2,-e^{2ix}\right) + ix^2 - 2x\log\left(1+e^{2ix}\right)$$

```
[In] Int[Log[a*Cos[x]^2],x]
```

```
[Out] I*x^2 - 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^2] + I*PolyLog[2, -E^((2*
I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\cos^2(x)\right) - \int -2x\tan(x)\,dx \\
&= x\log\left(a\cos^2(x)\right) + 2\int x\tan(x)\,dx \\
&= ix^2 + x\log\left(a\cos^2(x)\right) - 4i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx \\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) + 2\int \log\left(1+e^{2ix}\right)\,dx \\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) - i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) + i\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \log \left(a \cos^2(x)\right) \, dx = x\left(ix - 2\log\left(1 + e^{2ix}\right) + \log\left(a\cos^2(x)\right)\right) + i\,\text{PolyLog}\left(2, -e^{2ix}\right)$$

```
[In] Integrate[Log[a*Cos[x]^2],x]
[Out] x*(I*x - 2*Log[1 + E^((2*I)*x)] + Log[a*Cos[x]^2]) + I*PolyLog[2, -E^((2*I)
*x)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(39) = 78.

Time = 1.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.56

method	result
default	$-i\left(\ln\left(\mathrm{e}^{ix}\right)\ln\left(a\left(1+\mathrm{e}^{2ix}\right)^{2}\mathrm{e}^{-2ix}\right)-2\ln\left(\mathrm{e}^{ix}\right)\ln\left(1+i\mathrm{e}^{ix}\right)-2\ln\left(\mathrm{e}^{ix}\right)\ln\left(1-i\mathrm{e}^{ix}\right)+\ln\left(\mathrm{e}^{ix}\right)^{2}-2\,\mathrm{d}$
risch	$-2x\ln\left(\mathrm{e}^{ix}\right)+ix^{2}-i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{ix}\right)\mathrm{csgn}\left(i\mathrm{e}^{2ix}\right)^{2}x-\frac{i\pi \mathrm{csgn}\left(ia\left(1+\mathrm{e}^{2ix}\right)^{2}\mathrm{e}^{-2ix}\right)^{3}x}{2}+2i\ln\left(\mathrm{e}^{ix}\right)\ln\left(1-i\mathrm{e}^{ix}\right.$

```
[In] int(ln(a*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -I*(ln(exp(I*x))*ln(a*(exp(I*x)^2+1)^2/exp(I*x)^2)-2*ln(exp(I*x))*ln(1+I*ex
p(I*x))-2*ln(exp(I*x))*ln(1-I*exp(I*x))+ln(exp(I*x))^2-2*dilog(1+I*exp(I*x)
)-2*dilog(1-I*exp(I*x))-2*ln(2)*ln(exp(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(34) = 68.

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.36

$$\begin{aligned}\int \log \left(a \cos^2(x)\right) \, dx = {} & x\log\left(a\cos\left(x\right)^{2}\right) - x\log\left(i\cos\left(x\right) + \sin\left(x\right) + 1\right) \\ & - x\log\left(i\cos\left(x\right) - \sin\left(x\right) + 1\right) - x\log\left(-i\cos\left(x\right) + \sin\left(x\right) + 1\right) \\ & - x\log\left(-i\cos\left(x\right) - \sin\left(x\right) + 1\right) \\ & - i\,\mathrm{Li}_2(i\cos\left(x\right) + \sin\left(x\right)) + i\,\mathrm{Li}_2(i\cos\left(x\right) - \sin\left(x\right)) \\ & + i\,\mathrm{Li}_2(-i\cos\left(x\right) + \sin\left(x\right)) - i\,\mathrm{Li}_2(-i\cos\left(x\right) - \sin\left(x\right))\end{aligned}$$

```
[In] integrate(log(a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a*cos(x)^2) - x*log(I*cos(x) + sin(x) + 1) - x*log(I*cos(x) - sin(x)
+ 1) - x*log(-I*cos(x) + sin(x) + 1) - x*log(-I*cos(x) - sin(x) + 1) - I*di
log(I*cos(x) + sin(x)) + I*dilog(I*cos(x) - sin(x)) + I*dilog(-I*cos(x) + s
in(x)) - I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]

$$\int \log \left(a\cos^2(x)\right) \, dx = \int \log \left(a\cos^2{(x)}\right) dx$$

```
[In] integrate(ln(a*cos(x)**2),x)
```

```
[Out] Integral(log(a*cos(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \log \left(a\cos^2(x)\right) \, dx = i\,x^2 - 2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right) + 1\right) + x\log\left(a\cos\left(x\right)^2\right) - x\log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right) + i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)$$

```
[In] integrate(log(a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] I*x^2 - 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*cos(x)^2) - x*log(c
os(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + I*dilog(-e^(2*I*x))
```

Giac [F]

$$\int \log \left(a\cos^2(x)\right) \, dx = \int \log \left(a\cos\left(x\right)^2\right) \, dx$$

```
[In] integrate(log(a*cos(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cos(x)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \log \left(a\cos^2(x)\right)\,dx = x\,\ln\left(a\cos\left(x\right)^2\right) + \mathrm{polylog}\left(2, -\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i} + x\,\left(x + \ln\left(\mathrm{e}^{x\,2\mathrm{i}} + 1\right)\,2\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int(log(a*cos(x)^2),x)
```

```
[Out] polylog(2, -exp(x*2i))*1i + x*(x + log(exp(x*2i) + 1)*2i)*1i + x*log(a*cos(
x)^2)
```

3.166 $\int \log(a\cos^n(x))\,dx$

Optimal result 953
Rubi [A] (verified) 953
Mathematica [A] (verified) 955
Maple [F] 955
Fricas [B] (verification not implemented) 955
Sympy [F] 956
Maxima [A] (verification not implemented) 956
Giac [F] 956
Mupad [B] (verification not implemented) 956

Optimal result

Integrand size = 7, antiderivative size = 52

$$\int \log(a\cos^n(x))\,dx = \frac{1}{2}inx^2 - nx\log\left(1+e^{2ix}\right) + x\log(a\cos^n(x)) + \frac{1}{2}in\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

```
[Out] 1/2*I*n*x^2-n*x*ln(1+exp(2*I*x))+x*ln(a*cos(x)^n)+1/2*I*n*polylog(2,-exp(2*
I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3800, 2221, 2317, 2438}

$$\int \log(a\cos^n(x))\,dx = x\log(a\cos^n(x)) + \frac{1}{2}in\,\text{PolyLog}\left(2,-e^{2ix}\right) + \frac{1}{2}inx^2 - nx\log\left(1+e^{2ix}\right)$$

```
[In] Int[Log[a*Cos[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyLo
g[2, -E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\cos^n(x)\right)+\int nx\tan(x)\,dx\\
&= x\log\left(a\cos^n(x)\right)+n\int x\tan(x)\,dx\\
&= \frac{1}{2}inx^2+x\log\left(a\cos^n(x)\right)-(2in)\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+n\int \log\left(1+e^{2ix}\right)\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)-\frac{1}{2}(in)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\
&= \frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \log \left(a \cos ^n(x)\right) \, dx=\frac{1}{2} i n x^2-n x \log \left(1+e^{2 i x}\right)+x \log \left(a \cos ^n(x)\right)+\frac{1}{2} i n \operatorname{PolyLog}\left(2,-e^{2 i x}\right)$$

```
[In] Integrate[Log[a*Cos[x]^n],x]
[Out] (I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyLo
g[2, -E^((2*I)*x)]
```

Maple [F]

$$\int \ln \left(a \left(\cos ^n(x)\right)\right) dx$$

```
[In] int(ln(a*cos(x)^n),x)
[Out] int(ln(a*cos(x)^n),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(37) = 74.

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \log \left(a \cos ^n(x)\right) \, dx= & -\frac{1}{2} n x \log (i \cos (x)+\sin (x)+1)-\frac{1}{2} n x \log (i \cos (x)-\sin (x)+1) \\ & -\frac{1}{2} n x \log (-i \cos (x)+\sin (x)+1) \\ & -\frac{1}{2} n x \log (-i \cos (x)-\sin (x)+1) \\ & +n x \log (\cos (x))-\frac{1}{2} i n \operatorname{Li}_2(i \cos (x)+\sin (x)) \\ & +\frac{1}{2} i n \operatorname{Li}_2(i \cos (x)-\sin (x))+\frac{1}{2} i n \operatorname{Li}_2(-i \cos (x)+\sin (x)) \\ & -\frac{1}{2} i n \operatorname{Li}_2(-i \cos (x)-\sin (x))+x \log (a)\end{aligned}$$

```
[In] integrate(log(a*cos(x)^n),x, algorithm="fricas")
[Out] -1/2*n*x*log(I*cos(x) + sin(x) + 1) - 1/2*n*x*log(I*cos(x) - sin(x) + 1) -
1/2*n*x*log(-I*cos(x) + sin(x) + 1) - 1/2*n*x*log(-I*cos(x) - sin(x) + 1) +
 n*x*log(cos(x)) - 1/2*I*n*dilog(I*cos(x) + sin(x)) + 1/2*I*n*dilog(I*cos(x
) - sin(x)) + 1/2*I*n*dilog(-I*cos(x) + sin(x)) - 1/2*I*n*dilog(-I*cos(x) -
 sin(x)) + x*log(a)
```

Sympy [F]

$$\int \log\left(a\cos^n(x)\right)\,dx = \int \log\left(a\cos^n\left(x\right)\right)dx$$

```
[In] integrate(ln(a*cos(x)**n),x)
[Out] Integral(log(a*cos(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \log\left(a\cos^n(x)\right)\,dx = -\frac{1}{2}\left(-i\,x^2 + 2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right) + x\log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right)+1\right) - i\,\mathrm{Li}_2\left(-\right.\right.$$
$$+\,x\log\left(a\cos\left(x\right)^n\right)$$

```
[In] integrate(log(a*cos(x)^n),x, algorithm="maxima")
[Out] -1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*cos(x)^n)
```

Giac [F]

$$\int \log\left(a\cos^n(x)\right)\,dx = \int \log\left(a\cos\left(x\right)^n\right)\,dx$$

```
[In] integrate(log(a*cos(x)^n),x, algorithm="giac")
[Out] integrate(log(a*cos(x)^n), x)
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \log\left(a\cos^n(x)\right)\,dx = x\,\ln\left(a\cos\left(x\right)^n\right) + \frac{n\,\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\ 1\mathrm{i}}{2} + \frac{n\,x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\ 1\mathrm{i}}{2}$$

```
[In] int(log(a*cos(x)^n),x)
[Out] x*log(a*cos(x)^n) + (n*polylog(2, -exp(x*2i))*1i)/2 + (n*x*(x + log(exp(x*2
i) + 1)*2i)*1i)/2
```

3.167 $\int \log(a\tan(x))\,dx$

Optimal result 957
Rubi [A] (verified) 957
Mathematica [A] (verified) 959
Maple [B] (verified) 959
Fricas [B] (verification not implemented) 960
Sympy [F] 960
Maxima [A] (verification not implemented) 961
Giac [F] 961
Mupad [B] (verification not implemented) 961

Optimal result

Integrand size = 5, antiderivative size = 51

$$\int \log(a\tan(x))\,dx = 2x\text{arctanh}\left(e^{2ix}\right) + x\log(a\tan(x)) - \frac{1}{2}i\,\text{PolyLog}\left(2, -e^{2ix}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2, e^{2ix}\right)$$

```
[Out] 2*x*arctanh(exp(2*I*x))+x*ln(a*tan(x))-1/2*I*polylog(2,-exp(2*I*x))+1/2*I*p
olylog(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 4504, 4268, 2317, 2438}

$$\int \log(a\tan(x))\,dx = x\log(a\tan(x)) + 2x\text{arctanh}\left(e^{2ix}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2, -e^{2ix}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2, e^{2ix}\right)$$

```
[In] Int[Log[a*Tan[x]],x]
```

```
[Out] 2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]] - (I/2)*PolyLog[2, -E^((2*I)*x)]
 + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log(a\tan(x)) - \int x\csc(x)\sec(x)\,dx \\
&= x\log(a\tan(x)) - 2\int x\csc(2x)\,dx \\
&= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) + \int \log\left(1-e^{2ix}\right)\,dx - \int \log\left(1+e^{2ix}\right)\,dx \\
&= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&\quad + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) - \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \log(a\tan(x))\,dx = -\frac{1}{2}i\log(-i(i-\tan(x)))\log(a\tan(x)) + \frac{1}{2}i\log(a\tan(x))\log(-i(i+\tan(x))) - \frac{1}{2}i\,\text{PolyLog}(2,-i\tan(x)) + \frac{1}{2}i\,\text{PolyLog}(2,i\tan(x))$$

```
[In] Integrate[Log[a*Tan[x]],x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]] + (I/2)*Log[a*Tan[x]]*Log[(-I
)*(I + Tan[x])] - (I/2)*PolyLog[2, (-I)*Tan[x]] + (I/2)*PolyLog[2, I*Tan[x]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(39) = 78.

Time = 1.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

method	result
derivativedivides	$a\left(-\frac{i\ln(a\tan(x))\left(\ln\left(\frac{i\tan(x)a+a}{a}\right)-\ln\left(-\frac{i\tan(x)a-a}{a}\right)\right)}{2a}-\frac{i\left(\text{dilog}\left(\frac{i\tan(x)a+a}{a}\right)-\text{dilog}\left(-\frac{i\tan(x)a-a}{a}\right)\right)}{2a}\right)$
default	$a\left(-\frac{i\ln(a\tan(x))\left(\ln\left(\frac{i\tan(x)a+a}{a}\right)-\ln\left(-\frac{i\tan(x)a-a}{a}\right)\right)}{2a}-\frac{i\left(\text{dilog}\left(\frac{i\tan(x)a+a}{a}\right)-\text{dilog}\left(-\frac{i\tan(x)a-a}{a}\right)\right)}{2a}\right)$
risch	$-x\ln\left(1+e^{2ix}\right)-\frac{i\pi\,\text{csgn}\left(i\left(e^{2ix}-1\right)\right)\text{csgn}\left(\frac{i}{1+e^{2ix}}\right)\text{csgn}\left(\frac{i\left(e^{2ix}-1\right)}{1+e^{2ix}}\right)x}{2}-\frac{i\pi\text{csgn}\left(\frac{ia\left(e^{2ix}-1\right)}{1+e^{2ix}}\right)^{3}x}{2}-i\ln($

```
[In] int(ln(a*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2*I*ln(a*tan(x))*(ln((I*tan(x)*a+a)/a)-ln(-(I*tan(x)*a-a)/a))/a-1/2*I
*(dilog((I*tan(x)*a+a)/a)-dilog(-(I*tan(x)*a-a)/a))/a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(32) = 64$.

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.61

$$
\begin{aligned}
\int \log(a\tan(x))\,dx = {} & x\log\left(a\tan\left(x\right)\right) - \frac{1}{2}\,x\log\left(\frac{2\,\left(\tan\left(x\right)^{2} + i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2} + 1}\right) \\
& - \frac{1}{2}\,x\log\left(\frac{2\,\left(\tan\left(x\right)^{2} - i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2} + 1}\right) \\
& + \frac{1}{2}\,x\log\left(-\frac{2\,\left(i\,\tan\left(x\right) - 1\right)}{\tan\left(x\right)^{2} + 1}\right) + \frac{1}{2}\,x\log\left(-\frac{2\,\left(-i\,\tan\left(x\right) - 1\right)}{\tan\left(x\right)^{2} + 1}\right) \\
& - \frac{1}{4}i\,\mathrm{Li}_{2}\left(-\frac{2\,\left(\tan\left(x\right)^{2} + i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2} + 1} + 1\right) \\
& + \frac{1}{4}i\,\mathrm{Li}_{2}\left(-\frac{2\,\left(\tan\left(x\right)^{2} - i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2} + 1} + 1\right) \\
& + \frac{1}{4}i\,\mathrm{Li}_{2}\left(\frac{2\,\left(i\,\tan\left(x\right) - 1\right)}{\tan\left(x\right)^{2} + 1} + 1\right) - \frac{1}{4}i\,\mathrm{Li}_{2}\left(\frac{2\,\left(-i\,\tan\left(x\right) - 1\right)}{\tan\left(x\right)^{2} + 1} + 1\right)
\end{aligned}
$$

```
[In] integrate(log(a*tan(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*tan(x)) - 1/2*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*x
*log(2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*x*log(-2*(I*tan(x) - 1)/
(tan(x)^2 + 1)) + 1/2*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/4*I*dilo
g(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(-2*(tan(x)^2 -
 I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1
) + 1) - 1/4*I*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F]

$$\int \log(a\tan(x))\,dx = \int \log\left(a\tan\left(x\right)\right)dx$$

```
[In] integrate(ln(a*tan(x)),x)
```

```
[Out] Integral(log(a*tan(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \log(a\tan(x))\,dx = x\log\left(a\tan\left(x\right)\right) + \frac{1}{4}\,\pi\log\left(\tan\left(x\right)^{2} + 1\right) - x\log\left(\tan\left(x\right)\right) + \frac{1}{2}i\,\mathrm{Li}_2(i\,\tan\left(x\right) + 1) - \frac{1}{2}i\,\mathrm{Li}_2(-i\,\tan\left(x\right) + 1)$$

```
[In] integrate(log(a*tan(x)),x, algorithm="maxima")
[Out] x*log(a*tan(x)) + 1/4*pi*log(tan(x)^2 + 1) - x*log(tan(x)) + 1/2*I*dilog(I*
tan(x) + 1) - 1/2*I*dilog(-I*tan(x) + 1)
```

Giac [F]

$$\int \log(a\tan(x))\,dx = \int \log\left(a\tan\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*tan(x)),x, algorithm="giac")
[Out] integrate(log(a*tan(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \log(a\tan(x))\,dx = 2\,x\,\mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right) + x\,\ln\left(a\tan(x)\right) - \frac{\mathrm{polylog}(2, -\mathrm{e}^{x\,2\mathrm{i}})\ 1\mathrm{i}}{2} + \frac{\mathrm{polylog}(2, \mathrm{e}^{x\,2\mathrm{i}})\ 1\mathrm{i}}{2}$$

```
[In] int(log(a*tan(x)),x)
[Out] 2*x*atanh(exp(x*2i)) - (polylog(2, -exp(x*2i))*1i)/2 + (polylog(2, exp(x*2i
))*1i)/2 + x*log(a*tan(x))
```

3.168 $\int \log(a\tan^2(x))\,dx$

Optimal result 962
Rubi [A] (verified) 962
Mathematica [A] (verified) 964
Maple [A] (verified) 964
Fricas [B] (verification not implemented) 964
Sympy [F] 965
Maxima [A] (verification not implemented) 966
Giac [F] 966
Mupad [B] (verification not implemented) 966

Optimal result

Integrand size = 7, antiderivative size = 49

$$\int \log\left(a\tan^2(x)\right)\,dx = 4x\text{arctanh}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) - i\,\text{PolyLog}\left(2,-e^{2ix}\right) + i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] 4*x*arctanh(exp(2*I*x))+x*ln(a*tan(x)^2)-I*polylog(2,-exp(2*I*x))+I*polylog
(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 4504, 4268, 2317, 2438}

$$\int \log\left(a\tan^2(x)\right)\,dx = x\log\left(a\tan^2(x)\right) + 4x\text{arctanh}\left(e^{2ix}\right) - i\,\text{PolyLog}\left(2,-e^{2ix}\right) + i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[In] Int[Log[a*Tan[x]^2],x]
```

```
[Out] 4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^2] - I*PolyLog[2, -E^((2*I)*x)] +
 I*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\tan^2(x)\right) - \int 2x\csc(x)\sec(x)\,dx \\
&= x\log\left(a\tan^2(x)\right) - 2\int x\csc(x)\sec(x)\,dx \\
&= x\log\left(a\tan^2(x)\right) - 4\int x\csc(2x)\,dx \\
&= 4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) + 2\int \log\left(1-e^{2ix}\right)\,dx - 2\int \log\left(1+e^{2ix}\right)\,dx \\
&= 4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) - i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&\quad + i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= 4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) - i\text{Li}_2\left(-e^{2ix}\right) + i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\begin{aligned}\int \log \left(a \tan^2(x)\right) \, dx = & -\frac{1}{2} i \log(-i(i-\tan(x))) \log \left(a \tan^2(x)\right) \\ & + \frac{1}{2} i \log \left(a \tan^2(x)\right) \log(-i(i+\tan(x))) \\ & - i \operatorname{PolyLog}(2,-i \tan(x)) + i \operatorname{PolyLog}(2,i \tan(x))\end{aligned}$$

```
[In] Integrate[Log[a*Tan[x]^2],x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^2] + (I/2)*Log[a*Tan[x]^2]*Log
[(-I)*(I + Tan[x])] - I*PolyLog[2, (-I)*Tan[x]] + I*PolyLog[2, I*Tan[x]]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

method	result
derivativedivides	$-\frac{i\left(\ln(\tan(x)-i)\ln\left(a\left(\tan^2(x)\right)\right)-2\,\mathrm{dilog}(-i\tan(x))-2\ln(\tan(x)-i)\ln(-i\tan(x))\right)}{2}+\frac{i\left(\ln(\tan(x)+i)\ln\left(a\left(\tan^2(x)\right)\right)-\right.}{}$
default	$-\frac{i\left(\ln(\tan(x)-i)\ln\left(a\left(\tan^2(x)\right)\right)-2\,\mathrm{dilog}(-i\tan(x))-2\ln(\tan(x)-i)\ln(-i\tan(x))\right)}{2}+\frac{i\left(\ln(\tan(x)+i)\ln\left(a\left(\tan^2(x)\right)\right)-\right.}{}$
risch	Expression too large to display

```
[In] int(ln(a*tan(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(ln(tan(x)-I)*ln(a*tan(x)^2)-2*dilog(-I*tan(x))-2*ln(tan(x)-I)*ln(-I
*tan(x)))+1/2*I*(ln(tan(x)+I)*ln(a*tan(x)^2)-2*dilog(I*tan(x))-2*ln(tan(x)+
I)*ln(I*tan(x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(34) = 68.

Time = 0.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.76

$$\int \log\left(a\tan^2(x)\right)\,dx = x\log\left(a\tan(x)^2\right) - x\log\left(\frac{2\left(\tan(x)^2 + i\tan(x)\right)}{\tan(x)^2+1}\right) - x\log\left(\frac{2\left(\tan(x)^2 - i\tan(x)\right)}{\tan(x)^2+1}\right) + x\log\left(-\frac{2\,(i\tan(x)-1)}{\tan(x)^2+1}\right) + x\log\left(-\frac{2\,(-i\tan(x)-1)}{\tan(x)^2+1}\right) - \frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{2\left(\tan(x)^2+i\tan(x)\right)}{\tan(x)^2+1}+1\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{2\left(\tan(x)^2-i\tan(x)\right)}{\tan(x)^2+1}+1\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{2\,(i\tan(x)-1)}{\tan(x)^2+1}+1\right) - \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{2\,(-i\tan(x)-1)}{\tan(x)^2+1}+1\right)$$

```
[In] integrate(log(a*tan(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a*tan(x)^2) - x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - x*log(2
*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + x*log(-2*(I*tan(x) - 1)/(tan(x)^2
+ 1)) + x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/2*I*dilog(-2*(tan(x)^2
 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/2*I*dilog(-2*(tan(x)^2 - I*tan(x))/(ta
n(x)^2 + 1) + 1) + 1/2*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1) + 1) - 1/2*I
*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F]

$$\int \log\left(a\tan^2(x)\right)\,dx = \int \log\left(a\tan^2(x)\right)dx$$

```
[In] integrate(ln(a*tan(x)**2),x)
```

```
[Out] Integral(log(a*tan(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \log\left(a\tan^2(x)\right)\,dx = x\log\left(a\tan\left(x\right)^2\right) + \frac{1}{2}\pi\log\left(\tan\left(x\right)^2+1\right) - 2\,x\log\left(\tan\left(x\right)\right) + i\,\mathrm{Li}_2(i\,\tan\left(x\right)+1) - i\,\mathrm{Li}_2(-i\,\tan\left(x\right)+1)$$

```
[In] integrate(log(a*tan(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*tan(x)^2) + 1/2*pi*log(tan(x)^2 + 1) - 2*x*log(tan(x)) + I*dilog(I*
tan(x) + 1) - I*dilog(-I*tan(x) + 1)
```

Giac [F]

$$\int \log\left(a\tan^2(x)\right)\,dx = \int \log\left(a\tan\left(x\right)^2\right)\,dx$$

```
[In] integrate(log(a*tan(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*tan(x)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \log\left(a\tan^2(x)\right)\,dx = x\,\ln\left(a\tan(x)^2\right) - \mathrm{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i} + 4\,x\,\mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right) + \mathrm{polylog}\left(2,\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}$$

```
[In] int(log(a*tan(x)^2),x)
```

```
[Out] x*log(a*tan(x)^2) - polylog(2, -exp(x*2i))*1i + 4*x*atanh(exp(x*2i)) + poly
log(2, exp(x*2i))*1i
```

3.169 $\int \log(a\tan^n(x))\,dx$

Optimal result 967
Rubi [A] (verified) 967
Mathematica [A] (verified) 969
Maple [C] (warning: unable to verify) 969
Fricas [B] (verification not implemented) 971
Sympy [F] 971
Maxima [A] (verification not implemented) 972
Giac [F] 972
Mupad [B] (verification not implemented) 972

Optimal result

Integrand size = 7, antiderivative size = 56

$$\int \log(a\tan^n(x))\,dx = 2nx\operatorname{arctanh}\left(e^{2ix}\right)+x\log(a\tan^n(x)) -\frac{1}{2}in\operatorname{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}in\operatorname{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] 2*n*x*arctanh(exp(2*I*x))+x*ln(a*tan(x)^n)-1/2*I*n*polylog(2,-exp(2*I*x))+1
/2*I*n*polylog(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 4504, 4268, 2317, 2438}

$$\int \log(a\tan^n(x))\,dx = x\log(a\tan^n(x))+2nx\operatorname{arctanh}\left(e^{2ix}\right) -\frac{1}{2}in\operatorname{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}in\operatorname{PolyLog}\left(2,e^{2ix}\right)$$

```
[In] Int[Log[a*Tan[x]^n],x]
```

```
[Out] 2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^n] - (I/2)*n*PolyLog[2, -E^((2*
I)*x)] + (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\tan^n(x)\right) - \int nx\csc(x)\sec(x)\,dx \\
&= x\log\left(a\tan^n(x)\right) - n\int x\csc(x)\sec(x)\,dx \\
&= x\log\left(a\tan^n(x)\right) - (2n)\int x\csc(2x)\,dx \\
&= 2nx\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^n(x)\right) + n\int\log\left(1-e^{2ix}\right)\,dx - n\int\log\left(1+e^{2ix}\right)\,dx \\
&= 2nx\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^n(x)\right) - \frac{1}{2}(in)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&\quad + \frac{1}{2}(in)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= 2nx\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^n(x)\right) - \frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right) + \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \log \left(a \tan ^n(x)\right) \, dx = -\frac{1}{2} i \log (-i (i-\tan (x))) \log \left(a \tan ^n(x)\right) + \frac{1}{2} i \log \left(a \tan ^n(x)\right) \log (-i (i+\tan (x))) - \frac{1}{2} i n \text{PolyLog}(2,-i \tan (x))+\frac{1}{2} i n \text{PolyLog}(2,i \tan (x))$$

```
[In] Integrate[Log[a*Tan[x]^n],x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^n] + (I/2)*Log[a*Tan[x]^n]*Log
[(-I)*(I + Tan[x])] - (I/2)*n*PolyLog[2, (-I)*Tan[x]] + (I/2)*n*PolyLog[2,
I*Tan[x]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.52 (sec) , antiderivative size = 2196, normalized size of antiderivative = 39.21

method	result	size
risch	Expression too large to display	2196

```
[In] int(ln(a*tan(x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln((exp(2*I*x)-1)^n*(1+exp(2*I*x))^(-n)*exp(-1/2*I*Pi*n*(csgn(I*(exp(2*I*
x)-1)/(1+exp(2*I*x)))^3-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(exp
(2*I*x)-1))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x)))+
csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+exp(
2*I*x)))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2
*I*x)))^2+csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^3+csgn((exp(2*I*x)-1)/(1+exp(
2*I*x)))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))-csgn((exp(2*I*x)-1)/(1+exp(2
*I*x)))^2+1)))+1/2*I*Pi*csgn(I*(exp(2*I*x)-1)^n*(1+exp(2*I*x))^(-n)*exp(-1/
2*I*Pi*n*(csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3-csgn(I*(exp(2*I*x)-1)/(1+
exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))
^2*csgn(I/(1+exp(2*I*x)))+csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp
(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*cs
gn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^3+c
sgn((exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))-cs
gn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+1)))*csgn(I*a*(exp(2*I*x)-1)^n*(1+exp(2
*I*x))^(-n)*exp(-1/2*I*Pi*n*(csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3-csgn(I
*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))-csgn(I*(exp(2*I*x)
-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x)))+csgn(I*(exp(2*I*x)-1)/(1+exp(2
```

```
*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))-csgn(I*(exp(2*I*x)-1)
/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+csgn((exp(2*I*x)-1)/
(1+exp(2*I*x)))^3+csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1)
/(1+exp(2*I*x)))-csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+1)))^2*x+1/2*I*Pi*cs
gn(I*a*(exp(2*I*x)-1)^n*(1+exp(2*I*x))^(-n)*exp(-1/2*I*Pi*n*(csgn(I*(exp(2*
I*x)-1)/(1+exp(2*I*x)))^3-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(e
xp(2*I*x)-1))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x))
)+csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+ex
p(2*I*x)))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp
(2*I*x)))^2+csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^3+csgn((exp(2*I*x)-1)/(1+ex
p(2*I*x)))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))-csgn((exp(2*I*x)-1)/(1+exp
(2*I*x)))^2+1)))^2*csgn(I*a)*x+ln(a)*x+I*n*dilog(exp(I*x)+1)-n*x*ln(exp(I*x
)+1)-I*n*dilog(exp(I*x))+n*x*ln(1+I*exp(I*x))+n*x*ln(1-I*exp(I*x))-I*n*dilo
g(1+I*exp(I*x))-I*n*dilog(1-I*exp(I*x))-1/2*I*Pi*csgn(I*a*(exp(2*I*x)-1)^n*
(1+exp(2*I*x))^(-n)*exp(-1/2*I*Pi*n*(csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^
3-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))-csgn(I*(ex
p(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x)))+csgn(I*(exp(2*I*x)-1)/
(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))-csgn(I*(exp(2
*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+csgn((exp(2*
I*x)-1)/(1+exp(2*I*x)))^3+csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2
*I*x)-1)/(1+exp(2*I*x)))-csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+1)))^3*x-1/2
*I*Pi*csgn(I*(exp(2*I*x)-1)^n*(1+exp(2*I*x))^(-n)*exp(-1/2*I*Pi*n*(csgn(I*(
exp(2*I*x)-1)/(1+exp(2*I*x)))^3-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csg
n(I*(exp(2*I*x)-1))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2
*I*x)))+csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I
/(1+exp(2*I*x)))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/
(1+exp(2*I*x)))^2+csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^3+csgn((exp(2*I*x)-1)
/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))-csgn((exp(2*I*x)-1)/
(1+exp(2*I*x)))^2+1)))*csgn(I*a*(exp(2*I*x)-1)^n*(1+exp(2*I*x))^(-n)*exp(-1
/2*I*Pi*n*(csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3-csgn(I*(exp(2*I*x)-1)/(1
+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x))
)^2*csgn(I/(1+exp(2*I*x)))+csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(ex
p(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))-csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*c
sgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^3+
csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))-c
sgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2+1)))*csgn(I*a)*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(37) = 74.

Time = 0.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.48

$$\int \log(a\tan^n(x))\,dx = -\frac{1}{2}nx\log\left(\frac{2\left(\tan(x)^2+i\tan(x)\right)}{\tan(x)^2+1}\right) - \frac{1}{2}nx\log\left(\frac{2\left(\tan(x)^2-i\tan(x)\right)}{\tan(x)^2+1}\right) + \frac{1}{2}nx\log\left(-\frac{2\left(i\tan(x)-1\right)}{\tan(x)^2+1}\right) + \frac{1}{2}nx\log\left(-\frac{2\left(-i\tan(x)-1\right)}{\tan(x)^2+1}\right) + nx\log(\tan(x)) - \frac{1}{4}i\,n\mathrm{Li}_2\left(-\frac{2\left(\tan(x)^2+i\tan(x)\right)}{\tan(x)^2+1}+1\right) + \frac{1}{4}i\,n\mathrm{Li}_2\left(-\frac{2\left(\tan(x)^2-i\tan(x)\right)}{\tan(x)^2+1}+1\right) + \frac{1}{4}i\,n\mathrm{Li}_2\left(\frac{2\left(i\tan(x)-1\right)}{\tan(x)^2+1}+1\right) - \frac{1}{4}i\,n\mathrm{Li}_2\left(\frac{2\left(-i\tan(x)-1\right)}{\tan(x)^2+1}+1\right) + x\log(a)$$

```
[In] integrate(log(a*tan(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*n*x*log(2*(tan(x
)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*n*x*log(-2*(I*tan(x) - 1)/(tan(x)^2 +
 1)) + 1/2*n*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + n*x*log(tan(x)) - 1
/4*I*n*dilog(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dilog(-
2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dilog(2*(I*tan(x) - 1
)/(tan(x)^2 + 1) + 1) - 1/4*I*n*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
 + x*log(a)
```

Sympy [F]

$$\int \log(a\tan^n(x))\,dx = \int \log(a\tan^n(x))\,dx$$

```
[In] integrate(ln(a*tan(x)**n),x)
```

```
[Out] Integral(log(a*tan(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \log\left(a\tan^{n}(x)\right)\,dx = -nx\log\left(\tan\left(x\right)\right) + \frac{1}{4}\left(\pi\log\left(\tan\left(x\right)^{2}+1\right) + 2i\operatorname{Li}_2(i\tan\left(x\right)+1) - 2i\operatorname{Li}_2(-i\tan\left(x\right)+1)\right)n + x\log\left(a\tan\left(x\right)^{n}\right)$$

```
[In] integrate(log(a*tan(x)^n),x, algorithm="maxima")
[Out] -n*x*log(tan(x)) + 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2*
I*dilog(-I*tan(x) + 1))*n + x*log(a*tan(x)^n)
```

Giac [F]

$$\int \log\left(a\tan^{n}(x)\right)\,dx = \int \log\left(a\tan\left(x\right)^{n}\right)\,dx$$

```
[In] integrate(log(a*tan(x)^n),x, algorithm="giac")
[Out] integrate(log(a*tan(x)^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \log\left(a\tan^{n}(x)\right)\,dx = \frac{n\,\mathrm{polylog}\left(2,\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}}{2} + x\,\ln\left(a\tan(x)^{n}\right) - \frac{n\,\mathrm{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}}{2} + 2\,n\,x\,\mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right)$$

```
[In] int(log(a*tan(x)^n),x)
[Out] (n*polylog(2, exp(x*2i))*1i)/2 + x*log(a*tan(x)^n) - (n*polylog(2, -exp(x*2
i))*1i)/2 + 2*n*x*atanh(exp(x*2i))
```

3.170 $\int \log(a\cot(x))\,dx$

Optimal result 973
Rubi [A] (verified) 973
Mathematica [A] (verified) 975
Maple [B] (verified) 975
Fricas [B] (verification not implemented) 976
Sympy [F] 976
Maxima [A] (verification not implemented) 977
Giac [F] 977
Mupad [F(-1)] 977

Optimal result

Integrand size = 5, antiderivative size = 51

$$\int \log(a\cot(x))\,dx = -2x\text{arctanh}\left(e^{2ix}\right) + x\log(a\cot(x)) + \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] -2*x*arctanh(exp(2*I*x))+x*ln(a*cot(x))+1/2*I*polylog(2,-exp(2*I*x))-1/2*I*
polylog(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 4504, 4268, 2317, 2438}

$$\int \log(a\cot(x))\,dx = x\log(a\cot(x)) - 2x\text{arctanh}\left(e^{2ix}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[In] Int[Log[a*Cot[x]],x]
```

```
[Out] -2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]] + (I/2)*PolyLog[2, -E^((2*I)*x)
] - (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log(a\cot(x)) + \int x\csc(x)\sec(x)\,dx \\
&= x\log(a\cot(x)) + 2\int x\csc(2x)\,dx \\
&= -2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot(x)) - \int \log\left(1-e^{2ix}\right)\,dx + \int \log\left(1+e^{2ix}\right)\,dx \\
&= -2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot(x)) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&\quad - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= -2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot(x)) + \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) - \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \log(a\cot(x))\,dx = & -\frac{1}{2}i\log(a\cot(x))\log(-i(i-\tan(x))) \\ & +\frac{1}{2}i\log(a\cot(x))\log(-i(i+\tan(x))) \\ & +\frac{1}{2}i\,\text{PolyLog}(2,-i\tan(x))-\frac{1}{2}i\,\text{PolyLog}(2,i\tan(x))\end{aligned}$$

```
[In] Integrate[Log[a*Cot[x]],x]
```

```
[Out] (-1/2*I)*Log[a*Cot[x]]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]]*Log[(-I
)*(I + Tan[x])] + (I/2)*PolyLog[2, (-I)*Tan[x]] - (I/2)*PolyLog[2, I*Tan[x]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(39) = 78.

Time = 0.94 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.69

method	result
derivativedivides	$-a\left(-\frac{i\ln(a\cot(x))\left(\ln\left(\frac{i\cot(x)a+a}{a}\right)-\ln\left(-\frac{i\cot(x)a-a}{a}\right)\right)}{2a}-\frac{i\left(\text{dilog}\left(\frac{i\cot(x)a+a}{a}\right)-\text{dilog}\left(-\frac{i\cot(x)a-a}{a}\right)\right)}{2a}\right)$
default	$-a\left(-\frac{i\ln(a\cot(x))\left(\ln\left(\frac{i\cot(x)a+a}{a}\right)-\ln\left(-\frac{i\cot(x)a-a}{a}\right)\right)}{2a}-\frac{i\left(\text{dilog}\left(\frac{i\cot(x)a+a}{a}\right)-\text{dilog}\left(-\frac{i\cot(x)a-a}{a}\right)\right)}{2a}\right)$
risch	$x\ln\left(1+\mathrm{e}^{2ix}\right)-\frac{i\pi\ \text{csgn}\left(i\left(1+\mathrm{e}^{2ix}\right)\right)\text{csgn}\left(\frac{i}{\mathrm{e}^{2ix}-1}\right)\text{csgn}\left(\frac{i\left(1+\mathrm{e}^{2ix}\right)}{\mathrm{e}^{2ix}-1}\right)x}{2}-\frac{i\pi\ \text{csgn}\left(\frac{i\left(1+\mathrm{e}^{2ix}\right)}{\mathrm{e}^{2ix}-1}\right)\text{csgn}\left(\frac{ia\left(1+\mathrm{e}^{2i}\right.}{\mathrm{e}^{2ix}-1}\right.}{2}$

```
[In] int(ln(a*cot(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(-1/2*I*ln(a*cot(x))*(ln((I*cot(x)*a+a)/a)-ln(-(I*cot(x)*a-a)/a))/a-1/2*
I*(dilog((I*cot(x)*a+a)/a)-dilog(-(I*cot(x)*a-a)/a))/a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(32) = 64$.

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.88

$$\begin{aligned}\int \log(a\cot(x))\,dx = {} & x\log\left(\frac{a\cos(2x)+a}{\sin(2x)}\right) - \frac{1}{2}x\log(\cos(2x)+i\sin(2x)+1) \\ & - \frac{1}{2}x\log(\cos(2x)-i\sin(2x)+1) \\ & + \frac{1}{2}x\log(-\cos(2x)+i\sin(2x)+1) \\ & + \frac{1}{2}x\log(-\cos(2x)-i\sin(2x)+1) \\ & - \frac{1}{4}i\,\mathrm{Li}_2(\cos(2x)+i\sin(2x)) + \frac{1}{4}i\,\mathrm{Li}_2(\cos(2x)-i\sin(2x)) \\ & - \frac{1}{4}i\,\mathrm{Li}_2(-\cos(2x)+i\sin(2x)) + \frac{1}{4}i\,\mathrm{Li}_2(-\cos(2x)-i\sin(2x))\end{aligned}$$

```
[In] integrate(log(a*cot(x)),x, algorithm="fricas")
```

```
[Out] x*log((a*cos(2*x) + a)/sin(2*x)) - 1/2*x*log(cos(2*x) + I*sin(2*x) + 1) - 1
/2*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*x*log(-cos(2*x) + I*sin(2*x) + 1)
 + 1/2*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*dilog(cos(2*x) + I*sin(2*x
)) + 1/4*I*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*dilog(-cos(2*x) + I*sin(2*x
)) + 1/4*I*dilog(-cos(2*x) - I*sin(2*x))
```

Sympy [F]

$$\int \log(a\cot(x))\,dx = \int \log(a\cot(x))\,dx$$

```
[In] integrate(ln(a*cot(x)),x)
```

```
[Out] Integral(log(a*cot(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \log(a\cot(x))\,dx = -\frac{1}{4}\pi\log\left(\tan\left(x\right)^{2}+1\right) + x\log\left(\frac{a}{\tan\left(x\right)}\right) + x\log\left(\tan\left(x\right)\right) - \frac{1}{2}i\,\mathrm{Li}_2(i\,\tan\left(x\right)+1) + \frac{1}{2}i\,\mathrm{Li}_2(-i\,\tan\left(x\right)+1)$$

```
[In] integrate(log(a*cot(x)),x, algorithm="maxima")
[Out] -1/4*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)) + x*log(tan(x)) - 1/2*I*dilog(I
*tan(x) + 1) + 1/2*I*dilog(-I*tan(x) + 1)
```

Giac [F]

$$\int \log(a\cot(x))\,dx = \int \log\left(a\cot\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*cot(x)),x, algorithm="giac")
[Out] integrate(log(a*cot(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\cot(x))\,dx = \int \ln\left(a\cot(x)\right)\,dx$$

```
[In] int(log(a*cot(x)),x)
[Out] int(log(a*cot(x)), x)
```

3.171 $\int \log \left(a\cot^2(x)\right) \, dx$

Optimal result 978
Rubi [A] (verified) 978
Mathematica [A] (verified) 980
Maple [A] (verified) 980
Fricas [B] (verification not implemented) 980
Sympy [F] 981
Maxima [A] (verification not implemented) 981
Giac [F] 981
Mupad [F(-1)] 982

Optimal result

Integrand size = 7, antiderivative size = 49

$$\int \log \left(a\cot^2(x)\right) \, dx = -4x\text{arctanh}\left(e^{2ix}\right) + x\log\left(a\cot^2(x)\right) + i\,\text{PolyLog}\left(2,-e^{2ix}\right) - i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] -4*x*arctanh(exp(2*I*x))+x*ln(a*cot(x)^2)+I*polylog(2,-exp(2*I*x))-I*polylo
g(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 4504, 4268, 2317, 2438}

$$\int \log \left(a\cot^2(x)\right) \, dx = x\log\left(a\cot^2(x)\right) - 4x\text{arctanh}\left(e^{2ix}\right) + i\,\text{PolyLog}\left(2,-e^{2ix}\right) - i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[In] Int[Log[a*Cot[x]^2],x]
```

```
[Out] -4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^2] + I*PolyLog[2, -E^((2*I)*x)]
- I*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\cot^2(x)\right) - \int -2x\csc(x)\sec(x)\,dx \\
&= x\log\left(a\cot^2(x)\right) + 2\int x\csc(x)\sec(x)\,dx \\
&= x\log\left(a\cot^2(x)\right) + 4\int x\csc(2x)\,dx \\
&= -4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\cot^2(x)\right) - 2\int \log\left(1-e^{2ix}\right)\,dx + 2\int \log\left(1+e^{2ix}\right)\,dx \\
&= -4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\cot^2(x)\right) \\
&\quad + i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) - i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= -4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\cot^2(x)\right) + i\text{Li}_2\left(-e^{2ix}\right) - i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \log\left(a\cot^2(x)\right)\,dx = -\frac{1}{2}i\log\left(a\cot^2(x)\right)\log(-i(i-\tan(x))) + \frac{1}{2}i\log\left(a\cot^2(x)\right)\log(-i(i+\tan(x))) + i\,\text{PolyLog}(2,-i\tan(x)) - i\,\text{PolyLog}(2,i\tan(x))$$

```
[In] Integrate[Log[a*Cot[x]^2],x]
[Out] (-1/2*I)*Log[a*Cot[x]^2]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^2]*Log
[(-I)*(I + Tan[x])] + I*PolyLog[2, (-I)*Tan[x]] - I*PolyLog[2, I*Tan[x]]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{i(\ln(\cot(x)-i)\ln(a(\cot^2(x)))-2\,\text{dilog}(-i\cot(x))-2\ln(\cot(x)-i)\ln(-i\cot(x)))}{2} - \frac{i(\ln(\cot(x)+i)\ln(a(\cot^2(x)))-2\,\text{dil}$
default	$\frac{i(\ln(\cot(x)-i)\ln(a(\cot^2(x)))-2\,\text{dilog}(-i\cot(x))-2\ln(\cot(x)-i)\ln(-i\cot(x)))}{2} - \frac{i(\ln(\cot(x)+i)\ln(a(\cot^2(x)))-2\,\text{dil}$
risch	Expression too large to display

```
[In] int(ln(a*cot(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*(ln(cot(x)-I)*ln(a*cot(x)^2)-2*dilog(-I*cot(x))-2*ln(cot(x)-I)*ln(-I*
cot(x)))-1/2*I*(ln(cot(x)+I)*ln(a*cot(x)^2)-2*dilog(I*cot(x))-2*ln(cot(x)+I
)*ln(I*cot(x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(34) = 68.

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.02

$$\int \log\left(a\cot^2(x)\right)\,dx = x\log\left(-\frac{a\cos(2x)+a}{\cos(2x)-1}\right) - x\log(\cos(2x)+i\sin(2x)+1) - x\log(\cos(2x)-i\sin(2x)+1) + x\log(-\cos(2x)+i\sin(2x)+1) + x\log(-\cos(2x)-i\sin(2x)+1) - \frac{1}{2}i\,\text{Li}_2(\cos(2x)+i\sin(2x)) + \frac{1}{2}i\,\text{Li}_2(\cos(2x)-i\sin(2x)) - \frac{1}{2}i\,\text{Li}_2(-\cos(2x)+i\sin(2x)) + \frac{1}{2}i\,\text{Li}_2(-\cos(2x)-i\sin(2x))$$

```
[In] integrate(log(a*cot(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-(a*cos(2*x) + a)/(cos(2*x) - 1)) - x*log(cos(2*x) + I*sin(2*x) + 1)
- x*log(cos(2*x) - I*sin(2*x) + 1) + x*log(-cos(2*x) + I*sin(2*x) + 1) + x*
log(-cos(2*x) - I*sin(2*x) + 1) - 1/2*I*dilog(cos(2*x) + I*sin(2*x)) + 1/2*
I*dilog(cos(2*x) - I*sin(2*x)) - 1/2*I*dilog(-cos(2*x) + I*sin(2*x)) + 1/2*
I*dilog(-cos(2*x) - I*sin(2*x))
```

Sympy [F]

$$\int \log \left(a \cot^2(x)\right) \, dx = \int \log \left(a \cot^{2}{\left(x \right)}\right)\, dx$$

```
[In] integrate(ln(a*cot(x)**2),x)
```

```
[Out] Integral(log(a*cot(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \log \left(a \cot^2(x)\right) \, dx = -\frac{1}{2} \, \pi \log \left(\tan \left(x\right)^{2} + 1\right) + x \log \left(\frac{a}{\tan \left(x\right)^{2}}\right) + 2 \, x \log \left(\tan \left(x\right)\right) - i \, {\rm Li}_2\left(i \, \tan \left(x\right) + 1\right) + i \, {\rm Li}_2\left(-i \, \tan \left(x\right) + 1\right)$$

```
[In] integrate(log(a*cot(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)^2) + 2*x*log(tan(x)) - I*dilog(I
*tan(x) + 1) + I*dilog(-I*tan(x) + 1)
```

Giac [F]

$$\int \log \left(a \cot^2(x)\right) \, dx = \int { \log \left(a \cot \left(x\right)^{2}\right) \,d x }$$

```
[In] integrate(log(a*cot(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \cot^2(x)\right) \, dx = \int \ln \left(a \cot(x)^2\right) \, dx$$

```
[In] int(log(a*cot(x)^2),x)
[Out] int(log(a*cot(x)^2), x)
```

3.172 $\int \log(a\cot^n(x))\,dx$

Optimal result . . . 983
Rubi [A] (verified) . . . 983
Mathematica [A] (verified) . . . 985
Maple [C] (warning: unable to verify) . . . 985
Fricas [B] (verification not implemented) . . . 987
Sympy [F] . . . 987
Maxima [A] (verification not implemented) . . . 988
Giac [F] . . . 988
Mupad [B] (verification not implemented) . . . 988

Optimal result

Integrand size = 7, antiderivative size = 56

$$\int \log(a\cot^n(x))\,dx = -2nx\operatorname{arctanh}(e^{2ix}) + x\log(a\cot^n(x)) + \frac{1}{2}in\operatorname{PolyLog}(2,-e^{2ix}) - \frac{1}{2}in\operatorname{PolyLog}(2,e^{2ix})$$

```
[Out] -2*n*x*arctanh(exp(2*I*x))+x*ln(a*cot(x)^n)+1/2*I*n*polylog(2,-exp(2*I*x))-
1/2*I*n*polylog(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 4504, 4268, 2317, 2438}

$$\int \log(a\cot^n(x))\,dx = x\log(a\cot^n(x)) - 2nx\operatorname{arctanh}(e^{2ix}) + \frac{1}{2}in\operatorname{PolyLog}(2,-e^{2ix}) - \frac{1}{2}in\operatorname{PolyLog}(2,e^{2ix})$$

```
[In] Int[Log[a*Cot[x]^n],x]
```

```
[Out] -2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^n] + (I/2)*n*PolyLog[2, -E^((2
*I)*x)] - (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\cot^n(x)\right)+\int nx\csc(x)\sec(x)\,dx\\
&= x\log\left(a\cot^n(x)\right)+n\int x\csc(x)\sec(x)\,dx\\
&= x\log\left(a\cot^n(x)\right)+(2n)\int x\csc(2x)\,dx\\
&= -2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)-n\int\log\left(1-e^{2ix}\right)\,dx+n\int\log\left(1+e^{2ix}\right)\,dx\\
&= -2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)+\frac{1}{2}(in)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&\quad-\frac{1}{2}(in)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\
&= -2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)+\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \log\left(a\cot^n(x)\right)\,dx = -\frac{1}{2}i\log\left(a\cot^n(x)\right)\log(-i(i-\tan(x))) + \frac{1}{2}i\log\left(a\cot^n(x)\right)\log(-i(i+\tan(x))) + \frac{1}{2}in\,\mathrm{PolyLog}(2,-i\tan(x)) - \frac{1}{2}in\,\mathrm{PolyLog}(2,i\tan(x))$$

```
[In] Integrate[Log[a*Cot[x]^n],x]
```

```
[Out] (-1/2*I)*Log[a*Cot[x]^n]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^n]*Log
[(-I)*(I + Tan[x])] + (I/2)*n*PolyLog[2, (-I)*Tan[x]] - (I/2)*n*PolyLog[2,
I*Tan[x]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.92 (sec) , antiderivative size = 2197, normalized size of antiderivative = 39.23

method	result	size
risch	Expression too large to display	2197

```
[In] int(ln(a*cot(x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln((exp(2*I*x)-1)^(-n)*(1+exp(2*I*x))^n*exp(-1/2*I*Pi*n*(-csgn(I*(1+exp(2
*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2+csgn(I*(1+exp(2*I*x)))*csgn
(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))+csgn(I/(exp(2*I*x)
-1)*(1+exp(2*I*x)))^3-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2
*I*x)-1))-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+ex
p(2*I*x)))^2-csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3+csgn(1/(exp(2*I*x)-1)*
(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))+csgn(1/(exp(2*I*x)-1)
*(1+exp(2*I*x)))^2-1))+1/2*I*Pi*csgn(I*(exp(2*I*x)-1)^(-n)*(1+exp(2*I*x))^
n*exp(-1/2*I*Pi*n*(-csgn(I*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I
*x)))^2+csgn(I*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I
/(exp(2*I*x)-1))+csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3-csgn(I/(exp(2*I*x)
-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))-csgn(I/(exp(2*I*x)-1)*(1+exp(2
*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2-csgn(1/(exp(2*I*x)-1)*(1+ex
p(2*I*x)))^3+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1
+exp(2*I*x)))+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2-1)))*csgn(I*a*(exp(2*
I*x)-1)^(-n)*(1+exp(2*I*x))^n*exp(-1/2*I*Pi*n*(-csgn(I*(1+exp(2*I*x)))*csgn
(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2+csgn(I*(1+exp(2*I*x)))*csgn(I/(exp(2*I*
x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))+csgn(I/(exp(2*I*x)-1)*(1+exp(2
```

```
*I*x)))^3-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))-cs
gn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2
-csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x
)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*
x)))^2-1)))^2*x+1/2*I*Pi*csgn(I*a*(exp(2*I*x)-1)^(-n)*(1+exp(2*I*x))^n*exp(
-1/2*I*Pi*n*(-csgn(I*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^
2+csgn(I*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(
2*I*x)-1))+csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3-csgn(I/(exp(2*I*x)-1)*(1
+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x))
)*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2-csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*
x)))^3+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2
*I*x)))+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2-1)))^2*csgn(I*a)*x+ln(a)*x-
1/2*I*Pi*csgn(I*a*(exp(2*I*x)-1)^(-n)*(1+exp(2*I*x))^n*exp(-1/2*I*Pi*n*(-cs
gn(I*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2+csgn(I*(1+exp(
2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))+csgn(
I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*
csgn(I/(exp(2*I*x)-1))-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*
I*x)-1)*(1+exp(2*I*x)))^2-csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3+csgn(1/(e
xp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))+csgn(1/(
exp(2*I*x)-1)*(1+exp(2*I*x)))^2-1)))^3*x-n*x*ln(1+I*exp(I*x))-n*x*ln(1-I*ex
p(I*x))+I*n*dilog(1+I*exp(I*x))+I*n*dilog(1-I*exp(I*x))-I*n*dilog(exp(I*x)+
1)+n*x*ln(exp(I*x)+1)+I*n*dilog(exp(I*x))-1/2*I*Pi*csgn(I*(exp(2*I*x)-1)^(-
n)*(1+exp(2*I*x))^n*exp(-1/2*I*Pi*n*(-csgn(I*(1+exp(2*I*x)))*csgn(I/(exp(2*
I*x)-1)*(1+exp(2*I*x)))^2+csgn(I*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+e
xp(2*I*x)))*csgn(I/(exp(2*I*x)-1))+csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3-
csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))-csgn(I/(exp(
2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2-csgn(1/(e
xp(2*I*x)-1)*(1+exp(2*I*x)))^3+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I
/(exp(2*I*x)-1)*(1+exp(2*I*x)))+csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2-1))
)*csgn(I*a*(exp(2*I*x)-1)^(-n)*(1+exp(2*I*x))^n*exp(-1/2*I*Pi*n*(-csgn(I*(1
+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2+csgn(I*(1+exp(2*I*x))
)*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))+csgn(I/(exp(
2*I*x)-1)*(1+exp(2*I*x)))^3-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/
(exp(2*I*x)-1))-csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)
*(1+exp(2*I*x)))^2-csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3+csgn(1/(exp(2*I*
x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))+csgn(1/(exp(2*I
*x)-1)*(1+exp(2*I*x)))^2-1)))*csgn(I*a)*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(37) = 74$.

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.82

$$\begin{aligned}\int \log (a\cot ^n(x))\, dx &= nx\log \left(\frac{\cos (2\,x)+1}{\sin (2\,x)}\right)-\frac{1}{2}\,nx\log (\cos (2\,x)+i\,\sin (2\,x)+1)\\&\quad -\frac{1}{2}\,nx\log (\cos (2\,x)-i\,\sin (2\,x)+1)\\&\quad +\frac{1}{2}\,nx\log (-\cos (2\,x)+i\,\sin (2\,x)+1)\\&\quad +\frac{1}{2}\,nx\log (-\cos (2\,x)-i\,\sin (2\,x)+1)\\&\quad -\frac{1}{4}i\,n\mathrm{Li}_2(\cos (2\,x)+i\,\sin (2\,x))+\frac{1}{4}i\,n\mathrm{Li}_2(\cos (2\,x)-i\,\sin (2\,x))\\&\quad -\frac{1}{4}i\,n\mathrm{Li}_2(-\cos (2\,x)+i\,\sin (2\,x))\\&\quad +\frac{1}{4}i\,n\mathrm{Li}_2(-\cos (2\,x)-i\,\sin (2\,x))+x\log (a)\end{aligned}$$

```
[In] integrate(log(a*cot(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log((cos(2*x) + 1)/sin(2*x)) - 1/2*n*x*log(cos(2*x) + I*sin(2*x) + 1) -
 1/2*n*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*n*x*log(-cos(2*x) + I*sin(2*x
) + 1) + 1/2*n*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*n*dilog(cos(2*x) +
 I*sin(2*x)) + 1/4*I*n*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*n*dilog(-cos(2*
x) + I*sin(2*x)) + 1/4*I*n*dilog(-cos(2*x) - I*sin(2*x)) + x*log(a)
```

Sympy [F]

$$\int \log (a\cot ^n(x))\, dx = \int \log \left(a\cot ^{n}\left(x\right)\right)dx$$

```
[In] integrate(ln(a*cot(x)**n),x)
```

```
[Out] Integral(log(a*cot(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \log\left(a\cot^n(x)\right)\,dx &= nx\log\left(\tan\left(x\right)\right)\\ &\quad - \frac{1}{4}\left(\pi\log\left(\tan\left(x\right)^2+1\right)+2i\,\mathrm{Li}_2(i\,\tan\left(x\right)+1)-2i\,\mathrm{Li}_2(-i\,\tan\left(x\right)+1)\right)n\\ &\quad + x\log\left(a\frac{1}{\tan\left(x\right)}^n\right)\end{aligned}$$

```
[In] integrate(log(a*cot(x)^n),x, algorithm="maxima")
```

```
[Out] n*x*log(tan(x)) - 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2*I
*dilog(-I*tan(x) + 1))*n + x*log(a*(1/tan(x))^n)
```

Giac [F]

$$\int \log\left(a\cot^n(x)\right)\,dx = \int \log\left(a\cot\left(x\right)^n\right)\,dx$$

```
[In] integrate(log(a*cot(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \log\left(a\cot^n(x)\right)\,dx &= x\,\ln\left(a\cot(x)^n\right)-\frac{n\,\mathrm{polylog}(2,\mathrm{e}^{x\,2\mathrm{i}})\,1\mathrm{i}}{2}\\ &\quad +\frac{n\,\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\,1\mathrm{i}}{2}-2\,n\,x\,\mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right)\end{aligned}$$

```
[In] int(log(a*cot(x)^n),x)
```

```
[Out] x*log(a*cot(x)^n) - (n*polylog(2, exp(x*2i))*1i)/2 + (n*polylog(2, -exp(x*2
i))*1i)/2 - 2*n*x*atanh(exp(x*2i))
```

3.173 $\int \log(a\sec(x))\,dx$

Optimal result 989
Rubi [A] (verified) 989
Mathematica [A] (verified) 990
Maple [B] (verified) 991
Fricas [B] (verification not implemented) 991
Sympy [F] 992
Maxima [A] (verification not implemented) 992
Giac [F] 992
Mupad [B] (verification not implemented) 993

Optimal result

Integrand size = 5, antiderivative size = 46

$$\int \log(a\sec(x))\,dx = -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

```
[Out] -1/2*I*x^2+x*ln(1+exp(2*I*x))+x*ln(a*sec(x))-1/2*I*polylog(2,-exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3800, 2221, 2317, 2438}

$$\int \log(a\sec(x))\,dx = x\log(a\sec(x)) - \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right)$$

```
[In] Int[Log[a*Sec[x]],x]
```

```
[Out] (-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2,
-E^((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log(a\sec(x)) - \int x\tan(x)\,dx \\
&= -\frac{ix^2}{2} + x\log(a\sec(x)) + 2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \int \log\left(1+e^{2ix}\right)\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \log(a\sec(x))\,dx = -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \frac{1}{2}i\,\text{PolyLog}\left(2, -e^{2ix}\right)$$

```
[In] Integrate[Log[a*Sec[x]],x]
[Out] (-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2,
-E^((2*I)*x)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(36) = 72.

Time = 1.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.35

method	result
default	$-i\left(\ln(2)\ln(\mathrm{e}^{ix})+\ln(\mathrm{e}^{ix})\ln\left(\frac{a\,\mathrm{e}^{ix}}{1+\mathrm{e}^{2ix}}\right)+\ln(\mathrm{e}^{ix})\ln(1+i\mathrm{e}^{ix})+\ln(\mathrm{e}^{ix})\ln(1-i\mathrm{e}^{ix})+\mathrm{dilog}(1+i\right.$
risch	$x\ln(\mathrm{e}^{ix})-\frac{i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{ix}}{1+\mathrm{e}^{2ix}}\right)^3 x}{2}-\frac{ix^2}{2}-\frac{i\pi\,\mathrm{csgn}(i\mathrm{e}^{ix})\,\mathrm{csgn}\left(\frac{i}{1+\mathrm{e}^{2ix}}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{ix}}{1+\mathrm{e}^{2ix}}\right)x}{2}-i\ln(\mathrm{e}^{ix})\ln(1-i\mathrm{e}^{ix})+\frac{i\pi}{}$

```
[In] int(ln(a*sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(2)*ln(exp(I*x))+ln(exp(I*x))*ln(a*exp(I*x)/(exp(I*x)^2+1))+ln(exp(I*
x))*ln(1+I*exp(I*x))+ln(exp(I*x))*ln(1-I*exp(I*x))+dilog(1+I*exp(I*x))+dilo
g(1-I*exp(I*x))-1/2*ln(exp(I*x))^2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(31) = 62.

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\begin{aligned}\int \log(a\sec(x))\,dx = {} & x\log\left(\frac{a}{\cos(x)}\right)+\frac{1}{2}\,x\log\left(i\,\cos(x)+\sin(x)+1\right)\\ & +\frac{1}{2}\,x\log\left(i\,\cos(x)-\sin(x)+1\right)+\frac{1}{2}\,x\log\left(-i\,\cos(x)+\sin(x)+1\right)\\ & +\frac{1}{2}\,x\log\left(-i\,\cos(x)-\sin(x)+1\right)\\ & +\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos(x)+\sin(x))-\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos(x)-\sin(x))\\ & -\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos(x)+\sin(x))+\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos(x)-\sin(x))\end{aligned}$$

```
[In] integrate(log(a*sec(x)),x, algorithm="fricas")
```

```
[Out] x*log(a/cos(x)) + 1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - s
in(x) + 1) + 1/2*x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(
x) + 1) + 1/2*I*dilog(I*cos(x) + sin(x)) - 1/2*I*dilog(I*cos(x) - sin(x)) -
 1/2*I*dilog(-I*cos(x) + sin(x)) + 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]

$$\int \log(a\sec(x))\,dx = \int \log\left(a\sec\left(x\right)\right)dx$$

```
[In] integrate(ln(a*sec(x)),x)
```

```
[Out] Integral(log(a*sec(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \log(a\sec(x))\,dx = -\frac{1}{2}i\,x^2 + i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right) + \frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right) + x\log\left(a\sec\left(x\right)\right) - \frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)$$

```
[In] integrate(log(a*sec(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/2*x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*sec(x)) - 1/2*I*dilog(-e^(2*I*x))
```

Giac [F]

$$\int \log(a\sec(x))\,dx = \int \log\left(a\sec\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*sec(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \log(a\sec(x))\,dx = x\,\ln\left(\frac{a}{\cos\left(x\right)}\right)-\frac{\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\,\,1\mathrm{i}}{2}-\frac{x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

```
[In] int(log(a/cos(x)),x)
[Out] x*log(a/cos(x)) - (x*(x + log(exp(x*2i) + 1)*2i)*1i)/2 - (polylog(2, -exp(x
*2i))*1i)/2
```

3.174 $\int \log(a\sec^2(x))\,dx$

Optimal result 994
Rubi [A] (verified) 994
Mathematica [A] (verified) 996
Maple [B] (verified) 996
Fricas [B] (verification not implemented) 996
Sympy [F] 997
Maxima [A] (verification not implemented) 997
Giac [F] 997
Mupad [B] (verification not implemented) 998

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \log\left(a\sec^2(x)\right)\,dx = -ix^2 + 2x\log\left(1+e^{2ix}\right) + x\log\left(a\sec^2(x)\right) - i\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

```
[Out] -I*x^2+2*x*ln(1+exp(2*I*x))+x*ln(a*sec(x)^2)-I*polylog(2,-exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3800, 2221, 2317, 2438}

$$\int \log\left(a\sec^2(x)\right)\,dx = x\log\left(a\sec^2(x)\right) - i\,\text{PolyLog}\left(2,-e^{2ix}\right) - ix^2 + 2x\log\left(1+e^{2ix}\right)$$

```
[In] Int[Log[a*Sec[x]^2],x]
```

```
[Out] (-I)*x^2 + 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^2] - I*PolyLog[2, -E^(
(2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\sec^2(x)\right)-\int 2x\tan(x)\,dx\\
&= x\log\left(a\sec^2(x)\right)-2\int x\tan(x)\,dx\\
&= -ix^2+x\log\left(a\sec^2(x)\right)+4i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= -ix^2+2x\log\left(1+e^{2ix}\right)+x\log\left(a\sec^2(x)\right)-2\int \log\left(1+e^{2ix}\right)\,dx\\
&= -ix^2+2x\log\left(1+e^{2ix}\right)+x\log\left(a\sec^2(x)\right)+i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\
&= -ix^2+2x\log\left(1+e^{2ix}\right)+x\log\left(a\sec^2(x)\right)-i\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \log \left(a \sec^2(x)\right) \, dx = x\left(-ix + 2\log \left(1 + e^{2ix}\right) + \log \left(a \sec^2(x)\right)\right) - i \operatorname{PolyLog}\left(2, -e^{2ix}\right)$$

```
[In] Integrate[Log[a*Sec[x]^2],x]
```

```
[Out] x*((-I)*x + 2*Log[1 + E^((2*I)*x)] + Log[a*Sec[x]^2]) - I*PolyLog[2, -E^((2
*I)*x)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(39) = 78.

Time = 1.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.60

method	result
default	$-i\left(\ln \left(\mathrm{e}^{ix}\right) \ln \left(\frac{a\,\mathrm{e}^{2ix}}{\left(1+\mathrm{e}^{2ix}\right)^2}\right) - \ln \left(\mathrm{e}^{ix}\right)^2 + 2\ln \left(\mathrm{e}^{ix}\right) \ln \left(1 + i\mathrm{e}^{ix}\right) + 2\ln \left(\mathrm{e}^{ix}\right) \ln \left(1 - i\mathrm{e}^{ix}\right) + 2\,\mathrm{dilog}\,(1 +$
risch	$2x \ln \left(\mathrm{e}^{ix}\right) - ix^2 - \frac{i\pi \operatorname{csgn}\left(i\mathrm{e}^{2ix}\right) \operatorname{csgn}\left(\frac{i}{\left(1+\mathrm{e}^{2ix}\right)^2}\right) \operatorname{csgn}\left(\frac{i\mathrm{e}^{2ix}}{\left(1+\mathrm{e}^{2ix}\right)^2}\right) x}{2} - i\pi \operatorname{csgn}\left(i(1+\mathrm{e}^{2ix})\right) \operatorname{csgn}\left(i(1+\mathrm{e}^{2ix}\right.$

```
[In] int(ln(a*sec(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(a*exp(I*x)^2/(exp(I*x)^2+1)^2)-ln(exp(I*x))^2+2*ln(exp(
I*x))*ln(1+I*exp(I*x))+2*ln(exp(I*x))*ln(1-I*exp(I*x))+2*dilog(1+I*exp(I*x)
)+2*dilog(1-I*exp(I*x))+2*ln(2)*ln(exp(I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(34) = 68.

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\begin{aligned} \int \log \left(a \sec^2(x)\right) \, dx = {} & x \log \left(\frac{a}{\cos(x)^2}\right) + x \log \left(i \cos(x) + \sin(x) + 1\right) \\ & + x \log \left(i \cos(x) - \sin(x) + 1\right) + x \log \left(-i \cos(x) + \sin(x) + 1\right) \\ & + x \log \left(-i \cos(x) - \sin(x) + 1\right) \\ & + i \operatorname{Li}_2(i \cos(x) + \sin(x)) - i \operatorname{Li}_2(i \cos(x) - \sin(x)) \\ & - i \operatorname{Li}_2(-i \cos(x) + \sin(x)) + i \operatorname{Li}_2(-i \cos(x) - \sin(x)) \end{aligned}$$

```
[In] integrate(log(a*sec(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a/cos(x)^2) + x*log(I*cos(x) + sin(x) + 1) + x*log(I*cos(x) - sin(x)
+ 1) + x*log(-I*cos(x) + sin(x) + 1) + x*log(-I*cos(x) - sin(x) + 1) + I*di
log(I*cos(x) + sin(x)) - I*dilog(I*cos(x) - sin(x)) - I*dilog(-I*cos(x) + s
in(x)) + I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]

$$\int \log \left(a \sec^2(x)\right) \, dx = \int \log \left(a \sec^2 (x)\right) dx$$

```
[In] integrate(ln(a*sec(x)**2),x)
```

```
[Out] Integral(log(a*sec(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \log \left(a \sec^2(x)\right) \, dx = -i\,x^2 + 2i\,x \arctan\left(\sin\left(2\,x\right), \cos\left(2\,x\right) + 1\right) + x \log\left(a \sec\left(x\right)^2\right) + x \log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right) - i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)$$

```
[In] integrate(log(a*sec(x)^2),x, algorithm="maxima")
```

```
[Out] -I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*sec(x)^2) + x*log(
cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x))
```

Giac [F]

$$\int \log \left(a \sec^2(x)\right) \, dx = \int \log \left(a \sec \left(x\right)^2\right) \, dx$$

```
[In] integrate(log(a*sec(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)^2), x)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \log \left(a\sec^2(x)\right)\,dx = x\,\ln\left(\frac{a}{\cos\left(x\right)^2}\right)-\mathrm{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}-x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int(log(a/cos(x)^2),x)
```

```
[Out] x*log(a/cos(x)^2) - x*(x + log(exp(x*2i) + 1)*2i)*1i - polylog(2, -exp(x*2i
))*1i
```

3.175 $\int \log(a\sec^n(x))\,dx$

Optimal result 999
Rubi [A] (verified) 999
Mathematica [A] (verified) 1001
Maple [F] 1001
Fricas [B] (verification not implemented) 1001
Sympy [F] 1002
Maxima [A] (verification not implemented) 1002
Giac [F] 1002
Mupad [B] (verification not implemented) 1002

Optimal result

Integrand size = 7, antiderivative size = 51

$$\int \log(a\sec^n(x))\,dx = -\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log(a\sec^n(x)) - \frac{1}{2}in\,\text{PolyLog}\left(2,-e^{2ix}\right)$$

```
[Out] -1/2*I*n*x^2+n*x*ln(1+exp(2*I*x))+x*ln(a*sec(x)^n)-1/2*I*n*polylog(2,-exp(2
*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3800, 2221, 2317, 2438}

$$\int \log(a\sec^n(x))\,dx = x\log(a\sec^n(x)) - \frac{1}{2}in\,\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right)$$

```
[In] Int[Log[a*Sec[x]^n],x]
```

```
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*Pol
yLog[2, -E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\sec^n(x)\right) - \int nx\tan(x)\,dx \\
&= x\log\left(a\sec^n(x)\right) - n\int x\tan(x)\,dx \\
&= -\frac{1}{2}inx^2 + x\log\left(a\sec^n(x)\right) + (2in)\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx \\
&= -\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log\left(a\sec^n(x)\right) - n\int \log\left(1+e^{2ix}\right)\,dx \\
&= -\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log\left(a\sec^n(x)\right) + \frac{1}{2}(in)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= -\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log\left(a\sec^n(x)\right) - \frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \log \left(a \sec ^n(x)\right) \, dx=-\frac{1}{2} i n x^2+n x \log \left(1+e^{2 i x}\right)+x \log \left(a \sec ^n(x)\right)-\frac{1}{2} i n \operatorname{PolyLog}\left(2,-e^{2 i x}\right)$$

```
[In] Integrate[Log[a*Sec[x]^n],x]
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*Pol
yLog[2, -E^((2*I)*x)]
```

Maple [F]

$$\int \ln \left(a \left(\sec ^n(x)\right)\right) dx$$

```
[In] int(ln(a*sec(x)^n),x)
[Out] int(ln(a*sec(x)^n),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(36) = 72.

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.29

$$\begin{aligned}\int \log \left(a \sec ^n(x)\right) \, dx = {} & n x \log \left(\frac{1}{\cos (x)}\right)+\frac{1}{2} n x \log (i \cos (x)+\sin (x)+1) \\ & +\frac{1}{2} n x \log (i \cos (x)-\sin (x)+1)+\frac{1}{2} n x \log (-i \cos (x)+\sin (x)+1) \\ & +\frac{1}{2} n x \log (-i \cos (x)-\sin (x)+1)+\frac{1}{2} i n \mathrm{Li}_2(i \cos (x)+\sin (x)) \\ & -\frac{1}{2} i n \mathrm{Li}_2(i \cos (x)-\sin (x))-\frac{1}{2} i n \mathrm{Li}_2(-i \cos (x)+\sin (x)) \\ & +\frac{1}{2} i n \mathrm{Li}_2(-i \cos (x)-\sin (x))+x \log (a)\end{aligned}$$

```
[In] integrate(log(a*sec(x)^n),x, algorithm="fricas")
[Out] n*x*log(1/cos(x)) + 1/2*n*x*log(I*cos(x) + sin(x) + 1) + 1/2*n*x*log(I*cos(
x) - sin(x) + 1) + 1/2*n*x*log(-I*cos(x) + sin(x) + 1) + 1/2*n*x*log(-I*cos
(x) - sin(x) + 1) + 1/2*I*n*dilog(I*cos(x) + sin(x)) - 1/2*I*n*dilog(I*cos(
x) - sin(x)) - 1/2*I*n*dilog(-I*cos(x) + sin(x)) + 1/2*I*n*dilog(-I*cos(x)
- sin(x)) + x*log(a)
```

Sympy [F]

$$\int \log \left(a \sec^{n}(x)\right) \, dx = \int \log \left(a \sec^{n}\left(x\right)\right)\, dx$$

```
[In] integrate(ln(a*sec(x)**n),x)
[Out] Integral(log(a*sec(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \log \left(a \sec^{n}(x)\right) \, dx$$
$$= \frac{1}{2} \left(-i\, x^{2} + 2 i\, x \arctan\left(\sin\left(2\, x\right), \cos\left(2\, x\right) + 1\right) + x \log\left(\cos\left(2\, x\right)^{2} + \sin\left(2\, x\right)^{2} + 2\, \cos\left(2\, x\right) + 1\right) - i\, \mathrm{Li}_{2}(-e$$
$$+ x \log\left(a \sec\left(x\right)^{n}\right)$$

```
[In] integrate(log(a*sec(x)^n),x, algorithm="maxima")
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*sec(x)^n)
```

Giac [F]

$$\int \log \left(a \sec^{n}(x)\right) \, dx = \int \log \left(a \sec\left(x\right)^{n}\right) \, dx$$

```
[In] integrate(log(a*sec(x)^n),x, algorithm="giac")
[Out] integrate(log(a*sec(x)^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \log \left(a \sec^{n}(x)\right) \, dx = x\, \ln\left(a \left(\frac{1}{\cos\left(x\right)}\right)^{n}\right) - \frac{n\, \mathrm{polylog}(2, -\mathrm{e}^{x\, 2\mathrm{i}})\, 1\mathrm{i}}{2} - \frac{n\, x\, \left(x + \ln\left(\mathrm{e}^{x\, 2\mathrm{i}} + 1\right)\, 2\mathrm{i}\right)\, 1\mathrm{i}}{2}$$

```
[In] int(log(a*(1/cos(x))^n),x)
[Out] x*log(a*(1/cos(x))^n) - (n*polylog(2, -exp(x*2i))*1i)/2 - (n*x*(x + log(exp
(x*2i) + 1)*2i)*1i)/2
```

3.176 $\int \log(a\csc(x))\,dx$

Optimal result 1003
Rubi [A] (verified) 1003
Mathematica [A] (verified) 1004
Maple [B] (verified) 1005
Fricas [B] (verification not implemented) 1005
Sympy [F] 1006
Maxima [B] (verification not implemented) 1006
Giac [F] 1006
Mupad [F(-1)] 1007

Optimal result

Integrand size = 5, antiderivative size = 46

$$\int \log(a\csc(x))\,dx=-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)+x\log(a\csc(x))-\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] -1/2*I*x^2+x*ln(1-exp(2*I*x))+x*ln(a*csc(x))-1/2*I*polylog(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3798, 2221, 2317, 2438}

$$\int \log(a\csc(x))\,dx=x\log(a\csc(x))-\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)$$

```
[In] Int[Log[a*Csc[x]],x]
```

```
[Out] (-1/2*I)*x^2 + x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*PolyLog[2,
E^((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log(a\csc(x)) + \int x\cot(x)\,dx \\
&= -\frac{ix^2}{2} + x\log(a\csc(x)) - 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \int \log\left(1-e^{2ix}\right)\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \log(a\csc(x))\,dx = x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \frac{1}{2}i\left(x^2 + \text{PolyLog}\left(2, e^{2ix}\right)\right)$$

```
[In] Integrate[Log[a*Csc[x]],x]
[Out] x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*(x^2 + PolyLog[2, E^((2*I)
*x)])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.

Time = 1.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

method	result
default	$-i\left(\ln\left(2\right)\ln\left(\mathrm{e}^{ix}\right)+\ln\left(\mathrm{e}^{ix}\right)\ln\left(\frac{ia\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)-\frac{\ln\left(\mathrm{e}^{ix}\right)^{2}}{2}-\operatorname{dilog}\left(\mathrm{e}^{ix}\right)+\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{ix}+1\right)+\operatorname{dilog}\left(\mathrm{e}^{ix}+\right.\right.$
risch	$x\ln\left(\mathrm{e}^{ix}\right)-\frac{ix^{2}}{2}+\frac{i\pi x}{2}+\frac{i\pi\ \operatorname{csgn}\left(\frac{i\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)\operatorname{csgn}\left(\frac{ia\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)^{2}x}{2}-\frac{i\pi\operatorname{csgn}\left(\frac{i\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)^{3}x}{2}+\frac{i\pi\ \operatorname{csgn}\left(\frac{ia\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)\operatorname{csgn}\left(\frac{a\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1}\right)^{2}x}{2}$

```
[In] int(ln(a*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(2)*ln(exp(I*x))+ln(exp(I*x))*ln(I*a*exp(I*x)/(exp(I*x)^2-1))-1/2*ln(
exp(I*x))^2-dilog(exp(I*x))+ln(exp(I*x))*ln(exp(I*x)+1)+dilog(exp(I*x)+1))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(31) = 62$.

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\begin{aligned}\int \log(a\csc(x))\,dx = {} & x\log\left(\frac{a}{\sin\left(x\right)}\right)+\frac{1}{2}\,x\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\ & +\frac{1}{2}\,x\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)+\frac{1}{2}\,x\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\ & +\frac{1}{2}\,x\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\ & -\frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))+\frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))\\ & +\frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))\end{aligned}$$

```
[In] integrate(log(a*csc(x)),x, algorithm="fricas")
```

```
[Out] x*log(a/sin(x)) + 1/2*x*log(cos(x) + I*sin(x) + 1) + 1/2*x*log(cos(x) - I*s
in(x) + 1) + 1/2*x*log(-cos(x) + I*sin(x) + 1) + 1/2*x*log(-cos(x) - I*sin(
x) + 1) - 1/2*I*dilog(cos(x) + I*sin(x)) + 1/2*I*dilog(cos(x) - I*sin(x)) +
 1/2*I*dilog(-cos(x) + I*sin(x)) - 1/2*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log(a\csc(x))\,dx = \int \log\left(a\csc\left(x\right)\right)dx$$

```
[In] integrate(ln(a*csc(x)),x)
```

```
[Out] Integral(log(a*csc(x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(31) = 62.

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.89

$$\begin{aligned}\int \log(a\csc(x))\,dx = &-\frac{1}{2}i\,x^2 + i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)\\ &- i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)\\ &+ \frac{1}{2}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)\\ &+ \frac{1}{2}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right)\\ &+ x\log\left(a\csc\left(x\right)\right) - i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) - i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\end{aligned}$$

```
[In] integrate(log(a*csc(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + I*x*arctan2(sin(x), cos(x) + 1) - I*x*arctan2(sin(x), -cos(x)
+ 1) + 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*x*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1) + x*log(a*csc(x)) - I*dilog(-e^(I*x)) - I*dilog(e
^(I*x))
```

Giac [F]

$$\int \log(a\csc(x))\,dx = \int \log\left(a\csc\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*csc(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\csc(x))\,dx = \int \ln\left(\frac{a}{\sin(x)}\right)\,dx$$

```
[In] int(log(a/sin(x)),x)
[Out] int(log(a/sin(x)), x)
```

3.177 $\int \log(a\csc^2(x)) \, dx$

Optimal result 1008
Rubi [A] (verified) 1008
Mathematica [A] (verified) 1010
Maple [B] (verified) 1010
Fricas [B] (verification not implemented) 1010
Sympy [F] 1011
Maxima [B] (verification not implemented) 1011
Giac [F] 1012
Mupad [F(-1)] 1012

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \log\left(a\csc^2(x)\right) \, dx = -ix^2 + 2x\log\left(1 - e^{2ix}\right) + x\log\left(a\csc^2(x)\right) - i\,\text{PolyLog}\left(2, e^{2ix}\right)$$

```
[Out] -I*x^2+2*x*ln(1-exp(2*I*x))+x*ln(a*csc(x)^2)-I*polylog(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3798, 2221, 2317, 2438}

$$\int \log\left(a\csc^2(x)\right) \, dx = x\log\left(a\csc^2(x)\right) - i\,\text{PolyLog}\left(2, e^{2ix}\right) - ix^2 + 2x\log\left(1 - e^{2ix}\right)$$

```
[In] Int[Log[a*Csc[x]^2],x]
```

```
[Out] (-I)*x^2 + 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*PolyLog[2, E^((
2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\csc^2(x)\right)-\int -2x\cot(x)\,dx\\
&= x\log\left(a\csc^2(x)\right)+2\int x\cot(x)\,dx\\
&= -ix^2+x\log\left(a\csc^2(x)\right)-4i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-2\int \log\left(1-e^{2ix}\right)\,dx\\
&= -ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)+i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&= -ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \log \left(a\csc^2(x)\right) \, dx = 2x\log \left(1-e^{2ix}\right)+x\log \left(a\csc^2(x)\right)-i\left(x^2+\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

```
[In] Integrate[Log[a*Csc[x]^2],x]
[Out] 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*(x^2 + PolyLog[2, E^((2*I)
*x)])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 1.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

method	result
default	$-i\left(\ln\left(e^{ix}\right)\ln\left(-\frac{a\,e^{2ix}}{\left(e^{2ix}-1\right)^2}\right)-\ln\left(e^{ix}\right)^2+2\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right)+2\,\text{dilog}\left(e^{ix}+1\right)-2\,\text{dilog}\left(e^{ix}\right)+2$
risch	$2x\ln\left(e^{ix}\right)+2i\ln\left(e^{ix}\right)\ln\left(e^{2ix}-1\right)-ix^2+\frac{i\pi\text{csgn}\left(i\left(e^{2ix}-1\right)^2\right)^3 x}{2}-\frac{i\pi\,\text{csgn}\left(ie^{2ix}\right)^3 x}{2}-\frac{i\pi\,\text{csgn}\left(ie^{2ix}\right)\text{csgn}\left(\right.}{}$

```
[In] int(ln(a*csc(x)^2),x,method=_RETURNVERBOSE)
[Out] -I*(ln(exp(I*x))*ln(-a*exp(I*x)^2/(exp(I*x)^2-1)^2)-ln(exp(I*x))^2+2*ln(exp
(I*x))*ln(exp(I*x)+1)+2*dilog(exp(I*x)+1)-2*dilog(exp(I*x))+2*ln(2)*ln(exp(
I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(34) = 68.

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\begin{aligned}\int \log \left(a\csc^2(x)\right) \, dx = {} & x\log \left(-\frac{a}{\cos(x)^2-1}\right)+x\log\left(\cos(x)+i\sin(x)+1\right) \\ & +x\log\left(\cos(x)-i\sin(x)+1\right)+x\log\left(-\cos(x)+i\sin(x)+1\right) \\ & +x\log\left(-\cos(x)-i\sin(x)+1\right) \\ & -i\,\text{Li}_2(\cos(x)+i\sin(x))+i\,\text{Li}_2(\cos(x)-i\sin(x)) \\ & +i\,\text{Li}_2(-\cos(x)+i\sin(x))-i\,\text{Li}_2(-\cos(x)-i\sin(x))\end{aligned}$$

```
[In] integrate(log(a*csc(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-a/(cos(x)^2 - 1)) + x*log(cos(x) + I*sin(x) + 1) + x*log(cos(x) - I*
sin(x) + 1) + x*log(-cos(x) + I*sin(x) + 1) + x*log(-cos(x) - I*sin(x) + 1)
 - I*dilog(cos(x) + I*sin(x)) + I*dilog(cos(x) - I*sin(x)) + I*dilog(-cos(x
) + I*sin(x)) - I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log \left(a \csc^2(x)\right) \, dx = \int \log \left(a \csc^{2}\left(x\right)\right)\, dx$$

```
[In] integrate(ln(a*csc(x)**2),x)
```

```
[Out] Integral(log(a*csc(x)**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(34) = 68$.

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\begin{aligned}\int \log \left(a \csc^2(x)\right) \, dx = & -i\,x^{2} + 2i\,x \arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) \\ & - 2i\,x \arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) + x \log\left(a \csc\left(x\right)^{2}\right) \\ & + x \log\left(\cos\left(x\right)^{2} + \sin\left(x\right)^{2} + 2\,\cos\left(x\right) + 1\right) \\ & + x \log\left(\cos\left(x\right)^{2} + \sin\left(x\right)^{2} - 2\,\cos\left(x\right) + 1\right) \\ & - 2i\,\mathrm{Li}_2\left(-e^{\left(i\,x\right)}\right) - 2i\,\mathrm{Li}_2\left(e^{\left(i\,x\right)}\right)\end{aligned}$$

```
[In] integrate(log(a*csc(x)^2),x, algorithm="maxima")
```

```
[Out] -I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -cos(x)
+ 1) + x*log(a*csc(x)^2) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*lo
g(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(
I*x))
```

Giac [F]

$$\int \log \left(a \csc^2(x)\right) \, dx = \int \log \left(a \csc (x)^2\right) \, dx$$

[In] `integrate(log(a*csc(x)^2),x, algorithm="giac")`

[Out] `integrate(log(a*csc(x)^2), x)`

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \csc^2(x)\right) \, dx = \int \ln \left(\frac{a}{\sin (x)^2}\right) \, dx$$

[In] `int(log(a/sin(x)^2),x)`

[Out] `int(log(a/sin(x)^2), x)`

3.178 $\int \log(a\csc^n(x))\,dx$

Optimal result 1013
Rubi [A] (verified) 1013
Mathematica [A] (verified) 1015
Maple [F] 1015
Fricas [B] (verification not implemented) 1015
Sympy [F] 1016
Maxima [B] (verification not implemented) 1016
Giac [F] 1016
Mupad [F(-1)] 1017

Optimal result

Integrand size = 7, antiderivative size = 51

$$\int \log(a\csc^n(x))\,dx = -\frac{1}{2}inx^2 + nx\log\left(1-e^{2ix}\right) + x\log(a\csc^n(x)) - \frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] -1/2*I*n*x^2+n*x*ln(1-exp(2*I*x))+x*ln(a*csc(x)^n)-1/2*I*n*polylog(2,exp(2*
I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3798, 2221, 2317, 2438}

$$\int \log(a\csc^n(x))\,dx = x\log(a\csc^n(x)) - \frac{1}{2}in\,\text{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{2}inx^2 + nx\log\left(1-e^{2ix}\right)$$

```
[In] Int[Log[a*Csc[x]^n],x]
```

```
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*Pol
yLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\csc^n(x)\right)+\int nx\cot(x)\,dx\\
&= x\log\left(a\csc^n(x)\right)+n\int x\cot(x)\,dx\\
&= -\frac{1}{2}inx^2+x\log\left(a\csc^n(x)\right)-(2in)\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-n\int\log\left(1-e^{2ix}\right)\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)+\frac{1}{2}(in)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \log \left(a \csc^n(x)\right) \, dx = -\frac{1}{2} i n x^2 + n x \log \left(1 - e^{2 i x}\right) + x \log \left(a \csc^n(x)\right) - \frac{1}{2} i n \text{PolyLog}\left(2, e^{2 i x}\right)$$

```
[In] Integrate[Log[a*Csc[x]^n],x]
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*Pol
yLog[2, E^((2*I)*x)]
```

Maple [F]

$$\int \ln \left(a\left(\csc^n (x)\right)\right) dx$$

```
[In] int(ln(a*csc(x)^n),x)
[Out] int(ln(a*csc(x)^n),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(36) = 72.

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.29

$$\begin{aligned}\int \log \left(a \csc^n(x)\right) \, dx = {} & n x \log \left(\frac{1}{\sin (x)}\right) + \frac{1}{2} n x \log (\cos (x) + i \sin (x) + 1) \\ & + \frac{1}{2} n x \log (\cos (x) - i \sin (x) + 1) + \frac{1}{2} n x \log (-\cos (x) + i \sin (x) + 1) \\ & + \frac{1}{2} n x \log (-\cos (x) - i \sin (x) + 1) - \frac{1}{2} i n \text{Li}_2(\cos (x) + i \sin (x)) \\ & + \frac{1}{2} i n \text{Li}_2(\cos (x) - i \sin (x)) + \frac{1}{2} i n \text{Li}_2(-\cos (x) + i \sin (x)) \\ & - \frac{1}{2} i n \text{Li}_2(-\cos (x) - i \sin (x)) + x \log (a)\end{aligned}$$

```
[In] integrate(log(a*csc(x)^n),x, algorithm="fricas")
[Out] n*x*log(1/sin(x)) + 1/2*n*x*log(cos(x) + I*sin(x) + 1) + 1/2*n*x*log(cos(x)
 - I*sin(x) + 1) + 1/2*n*x*log(-cos(x) + I*sin(x) + 1) + 1/2*n*x*log(-cos(x
) - I*sin(x) + 1) - 1/2*I*n*dilog(cos(x) + I*sin(x)) + 1/2*I*n*dilog(cos(x)
 - I*sin(x)) + 1/2*I*n*dilog(-cos(x) + I*sin(x)) - 1/2*I*n*dilog(-cos(x) -
I*sin(x)) + x*log(a)
```

Sympy [F]

$$\int \log (a \csc^n(x)) \, dx = \int \log (a \csc^n (x)) \, dx$$

```
[In] integrate(ln(a*csc(x)**n),x)
```

```
[Out] Integral(log(a*csc(x)**n), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(36) = 72.

Time = 0.46 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.78

$$\int \log (a \csc^n(x)) \, dx$$
$$= \frac{1}{2} \left(-i x^2 + 2i x \arctan (\sin (x), \cos (x) + 1) - 2i x \arctan (\sin (x), -\cos (x) + 1) + x \log \left(\cos (x)^2 + \sin (x)\right.\right.$$
$$+ x \log (a \csc (x)^n)$$

```
[In] integrate(log(a*csc(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -co
s(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n + x*lo
g(a*csc(x)^n)
```

Giac [F]

$$\int \log (a \csc^n(x)) \, dx = \int \log (a \csc (x)^n) \, dx$$

```
[In] integrate(log(a*csc(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \csc ^n(x)\right) \, dx=\int \ln \left(a \left(\frac{1}{\sin \left(x\right)}\right)^n\right) \,d x$$

```
[In] int(log(a*(1/sin(x))^n),x)
[Out] int(log(a*(1/sin(x))^n), x)
```

3.179 $\int \cos(x) \log\left(\frac{1}{2}(1-\cos(2x))\right) \, dx$

Optimal result 1018
Rubi [A] (verified) 1018
Mathematica [A] (verified) 1019
Maple [C] (verified) 1019
Fricas [A] (verification not implemented) 1020
Sympy [F] 1020
Maxima [A] (verification not implemented) 1020
Giac [A] (verification not implemented) 1020
Mupad [F(-1)] 1021

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \cos(x) \log\left(\frac{1}{2}(1-\cos(2x))\right) \, dx = -2\sin(x) + \log\left(\frac{1}{2}(1-\cos(2x))\right) \sin(x)$$

```
[Out] -2*sin(x)+ln(1/2-1/2*cos(2*x))*sin(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2717, 2634, 12}

$$\int \cos(x) \log\left(\frac{1}{2}(1-\cos(2x))\right) \, dx = \sin(x) \log\left(\frac{1}{2}(1-\cos(2x))\right) - 2\sin(x)$$

```
[In] Int[Cos[x]*Log[(1 - Cos[2*x])/2],x]
[Out] -2*Sin[x] + Log[(1 - Cos[2*x])/2]*Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x) - \int 2\cos(x)\,dx \\ &= \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x) - 2\int \cos(x)\,dx \\ &= -2\sin(x) + \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = -2\sin(x) + \log\left(\sin^2(x)\right)\sin(x)$$

```
[In] Integrate[Cos[x]*Log[(1 - Cos[2*x])/2],x]
[Out] -2*Sin[x] + Log[Sin[x]^2]*Sin[x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.89 (sec) , antiderivative size = 111, normalized size of antiderivative = 5.29

method	result	size
default	$-\frac{i\left(e^{ix}\ln\left(\left(-e^{4ix}+2\,e^{2ix}-1\right)e^{-2ix}\right)-2\,e^{ix}-e^{-ix}\ln\left(\left(-e^{4ix}+2\,e^{2ix}-1\right)e^{-2ix}\right)+2\,e^{-ix}-2\ln(2)\left(e^{ix}-e^{-ix}\right)\right)}{2}$	111
risch	Expression too large to display	796

```
[In] int(cos(x)*ln(1/2-1/2*cos(2*x)),x,method=_RETURNVERBOSE)
[Out] -1/2*I*(exp(I*x)*ln((-exp(I*x)^4+2*exp(I*x)^2-1)/exp(I*x)^2)-2*exp(I*x)-exp
(-I*x)*ln((-exp(I*x)^4+2*exp(I*x)^2-1)/exp(I*x)^2)+2/exp(I*x)-2*ln(2)*(exp(
I*x)-1/exp(I*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \log\left(-\cos\left(x\right)^{2}+1\right)\sin\left(x\right)-2\,\sin\left(x\right)$$

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="fricas")
[Out] log(-cos(x)^2 + 1)*sin(x) - 2*sin(x)
```

Sympy [F]

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \int \log\left(\frac{1}{2}-\frac{\cos\left(2x\right)}{2}\right)\cos\left(x\right)\,dx$$

```
[In] integrate(cos(x)*ln(1/2-1/2*cos(2*x)),x)
[Out] Integral(log(1/2 - cos(2*x)/2)*cos(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)\sin\left(x\right)-2\,\sin\left(x\right)$$

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="maxima")
[Out] log(-1/2*cos(2*x) + 1/2)*sin(x) - 2*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx = \log\left(\sin\left(x\right)^{2}\right)\sin\left(x\right)-2\,\sin\left(x\right)$$

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="giac")
[Out] log(sin(x)^2)*sin(x) - 2*sin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(x) \log \left(\frac{1}{2}(1-\cos(2x))\right) \, dx = \int \ln \left(\frac{1}{2}-\frac{\cos (2\,x)}{2}\right)\,\cos (x) \,dx$$

```
[In] int(log(1/2 - cos(2*x)/2)*cos(x),x)
[Out] int(log(1/2 - cos(2*x)/2)*cos(x), x)
```

3.180 $\int \frac{\cot(x)}{\log(e\sin(x))}\,dx$

Optimal result 1022
Rubi [A] (verified) 1022
Mathematica [A] (verified) 1023
Maple [A] (verified) 1023
Fricas [A] (verification not implemented) 1024
Sympy [F] 1024
Maxima [A] (verification not implemented) 1024
Giac [B] (verification not implemented) 1024
Mupad [B] (verification not implemented) 1025

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \log(\log(e\sin(x)))$$

```
[Out] ln(ln(exp(1)*sin(x)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4423, 31}

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \log(\log(\sin(x))+1)$$

```
[In] Int[Cot[x]/Log[E*Sin[x]],x]
```

```
[Out] Log[1 + Log[Sin[x]]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{x+x\log(x)}\,dx, x, \sin(x)\right) \\ &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \log(\sin(x))\right) \\ &= \log(1+\log(\sin(x))) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \log(1+\log(\sin(x)))$$

```
[In] Integrate[Cot[x]/Log[E*Sin[x]],x]
[Out] Log[1 + Log[Sin[x]]]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\ln(\ln(\mathrm{e}\sin(x)))$
default	$\ln(\ln(\mathrm{e}\sin(x)))$
risch	$\ln\left(-\frac{i\pi\,\mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)\mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}(\sin(x))}{2}-\frac{i\pi\,\mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)\mathrm{csgn}(\sin(x))^{2}}{2}-\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}(\sin\right.$

```
[In] int(cot(x)/ln(exp(1)*sin(x)),x,method=_RETURNVERBOSE)
[Out] ln(ln(exp(1)*sin(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \log\left(\log\left(e\sin\left(x\right)\right)\right)$$

```
[In] integrate(cot(x)/log(exp(1)*sin(x)),x, algorithm="fricas")
[Out] log(log(e*sin(x)))
```

Sympy [F]

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \int \frac{\cot\left(x\right)}{\log\left(\sin\left(x\right)\right)+1}\,dx$$

```
[In] integrate(cot(x)/ln(exp(1)*sin(x)),x)
[Out] Integral(cot(x)/(log(sin(x)) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \log\left(\log\left(e\sin\left(x\right)\right)\right)$$

```
[In] integrate(cot(x)/log(exp(1)*sin(x)),x, algorithm="maxima")
[Out] log(log(e*sin(x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(7) = 14.

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 4.00

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \frac{1}{2}\log\left(\frac{1}{4}\pi^2(\text{sgn}(\sin\left(x\right))-1)^2+\left(\log\left(\left|\sin\left(x\right)\right|\right)+1\right)^2\right)$$

```
[In] integrate(cot(x)/log(exp(1)*sin(x)),x, algorithm="giac")
[Out] 1/2*log(1/4*pi^2*(sgn(sin(x)) - 1)^2 + (log(abs(sin(x))) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(e\sin(x))}\,dx = \ln\left(\ln\left(\sin\left(x\right)\right)+1\right)$$

```
[In] int(cot(x)/log(exp(1)*sin(x)),x)
[Out] log(log(sin(x)) + 1)
```

3.181 $\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)} dx$

Optimal result 1026
Rubi [A] (verified) 1026
Mathematica [A] (verified) 1027
Maple [A] (verified) 1028
Fricas [A] (verification not implemented) 1028
Sympy [F] 1028
Maxima [A] (verification not implemented) 1029
Giac [A] (verification not implemented) 1029
Mupad [B] (verification not implemented) 1029

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)} dx = \frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)} - \frac{\log(\sin(x))}{-\log\left(e^{\sin(x)}\right)+\sin(x)}$$

```
[Out] ln(ln(exp(sin(x))))/(-ln(exp(sin(x)))+sin(x))-ln(sin(x))/(-ln(exp(sin(x)))+
sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4423, 2191, 2188, 29}

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)} dx = \frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\sin(x)-\log\left(e^{\sin(x)}\right)} - \frac{\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

```
[In] Int[Cot[x]/Log[E^Sin[x]],x]
```

```
[Out] Log[Log[E^Sin[x]]]/(-Log[E^Sin[x]] + Sin[x]) - Log[Sin[x]]/(-Log[E^Sin[x]]
+ Sin[x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\log\left(e^x\right)}\,dx, x, \sin(x)\right) \\
&= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{\log(e^x)}\,dx, x, \sin(x)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)} \\
&= \frac{\log(\sin(x))}{\log\left(e^{\sin(x)}\right)-\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \log\left(e^{\sin(x)}\right)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)} \\
&= -\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\log\left(e^{\sin(x)}\right)-\sin(x)} + \frac{\log(\sin(x))}{\log\left(e^{\sin(x)}\right)-\sin(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = \frac{\log\left(\log\left(e^{\sin(x)}\right)\right)-\log(\sin(x))}{-\log\left(e^{\sin(x)}\right)+\sin(x)}$$

```
[In] Integrate[Cot[x]/Log[E^Sin[x]],x]
[Out] (Log[Log[E^Sin[x]]] - Log[Sin[x]])/(-Log[E^Sin[x]] + Sin[x])
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(\sin(x))}{\ln(\mathrm{e}^{\sin(x)})-\sin(x)}-\frac{\ln(\ln(\mathrm{e}^{\sin(x)}))}{\ln(\mathrm{e}^{\sin(x)})-\sin(x)}$	35
risch	$\frac{\ln(\mathrm{e}^{ix}+1)}{\ln(\mathrm{e}^{\sin(x)})-\sin(x)}-\frac{\ln(\mathrm{e}^{2ix}+2i(\ln(\mathrm{e}^{\sin(x)})-\sin(x))\mathrm{e}^{ix}-1)}{\ln(\mathrm{e}^{\sin(x)})-\sin(x)}+\frac{\ln(\mathrm{e}^{ix}-1)}{\ln(\mathrm{e}^{\sin(x)})-\sin(x)}$	80

```
[In] int(cot(x)/ln(exp(sin(x))),x,method=_RETURNVERBOSE)
[Out] 1/(ln(exp(sin(x)))-sin(x))*ln(sin(x))-1/(ln(exp(sin(x)))-sin(x))*ln(ln(exp(
sin(x))))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.16

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = -\frac{1}{\sin\left(x\right)}$$

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="fricas")
[Out] -1/sin(x)
```

Sympy [F]

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = \int \frac{\cot\left(x\right)}{\log\left(e^{\sin\left(x\right)}\right)}\,dx$$

```
[In] integrate(cot(x)/ln(exp(sin(x))),x)
[Out] Integral(cot(x)/log(exp(sin(x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.16

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = -\frac{1}{\sin(x)}$$

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="maxima")
[Out] -1/sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.16

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = -\frac{1}{\sin(x)}$$

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="giac")
[Out] -1/sin(x)
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.16

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx = -\frac{1}{\sin(x)}$$

```
[In] int(cot(x)/log(exp(sin(x))),x)
[Out] -1/sin(x)
```

3.182 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal result 1030
Rubi [A] (verified) 1030
Mathematica [A] (verified) 1031
Maple [A] (verified) 1031
Fricas [A] (verification not implemented) 1032
Sympy [A] (verification not implemented) 1032
Maxima [B] (verification not implemented) 1032
Giac [A] (verification not implemented) 1033
Mupad [B] (verification not implemented) 1033

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \log(\cos(x)) \tan(x)$$

```
[Out] -x+tan(x)+ln(cos(x))*tan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3852, 8, 2634, 3554}

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \tan(x) \log(\cos(x))$$

```
[In] Int[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \log(\cos(x))\tan(x) + \int \tan^2(x)\,dx \\
&= \tan(x) + \log(\cos(x))\tan(x) - \int 1\,dx \\
&= -x + \tan(x) + \log(\cos(x))\tan(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \tan(x) + \log(\cos(x))\tan(x)$$

```
[In] Integrate[Log[Cos[x]]*Sec[x]^2,x]
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result
parallelrisch	$-x+\tan\left(x\right)+\ln\left(\cos\left(x\right)\right)\tan\left(x\right)$
norman	$\frac{x-x\left(\tan^2\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)\ln\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)-2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}$
default	$-4i\left(\frac{\frac{e^{2ix}\ln\left(\left(1+e^{2ix}\right)e^{-ix}\right)}{2}-\frac{1}{2}}{1+e^{2ix}}-\frac{\ln\left(1+e^{2ix}\right)}{4}+\frac{\ln(2)}{2+2\,e^{2ix}}\right)$
risch	$-\frac{2i\ln\left(e^{ix}\right)}{1+e^{2ix}}+\frac{-i\ln\left(1+e^{2ix}\right)e^{2ix}+\pi\ \mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}\left(i\left(1+e^{2ix}\right)\right)\mathrm{csgn}\left(i\cos(x)\right)-\pi\ \mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}\left(i\cos(x)\right)^2-\pi\ \mathrm{csgn}\left(i\left(\right.\right.}{1+e^{2ix}}$

```
[In] int(ln(cos(x))*sec(x)^2,x,method=_RETURNVERBOSE)
[Out] -x+tan(x)+ln(cos(x))*tan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
[Out] -(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

Sympy [A] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \log(\cos(x))\tan(x) + \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(ln(cos(x))*sec(x)**2,x)
[Out] -x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(12) = 24.

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 7.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
[Out] -2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)
/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
 + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sec^2(x)\,dx = \log\left(\cos\left(x\right)\right)\tan\left(x\right) - x + \tan\left(x\right)$$

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
[Out] log(cos(x))*tan(x) - x + tan(x)
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \log(\cos(x))\sec^2(x)\,dx = \tan(x) - 2\,x + \ln\left(\cos\left(x\right)\right)\,\tan(x) + \ln\left(\cos\left(x\right)\right)\,1\mathrm{i} - \ln\left(\cos\left(2\,x\right) + 1 + \sin\left(2\,x\right)\,1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int(log(cos(x))/cos(x)^2,x)
[Out] log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(co
s(x))*tan(x)
```

3.183 $\int \cot(x)\log(\sin(x))\,dx$

Optimal result 1034
Rubi [A] (verified) 1034
Mathematica [A] (verified) 1035
Maple [A] (verified) 1035
Fricas [A] (verification not implemented) 1036
Sympy [F(-1)] 1036
Maxima [A] (verification not implemented) 1036
Giac [A] (verification not implemented) 1036
Mupad [B] (verification not implemented) 1037

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \cot(x)\log(\sin(x))\,dx = \frac{1}{2}\log^2(\sin(x))$$

```
[Out] 1/2*ln(sin(x))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556, 4423, 2338}

$$\int \cot(x)\log(\sin(x))\,dx = \frac{1}{2}\log^2(\sin(x))$$

```
[In] Int[Cot[x]*Log[Sin[x]],x]
```

```
[Out] Log[Sin[x]]^2/2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\log^2(\sin(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \cot(x)\log(\sin(x))\,dx = \frac{1}{2}\log^2(\sin(x))$$

```
[In] Integrate[Cot[x]*Log[Sin[x]],x]
[Out] Log[Sin[x]]^2/2
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\ln(\sin(x))^2}{2}$
default	$\frac{\ln(\sin(x))^2}{2}$
risch	$ix\ln(2)+\frac{x\pi\,\mathrm{csgn}(i(\mathrm{e}^{2ix}-1))\,\mathrm{csgn}(i\mathrm{e}^{-ix})\,\mathrm{csgn}(\sin(x))}{2}+\frac{x\pi\,\mathrm{csgn}(i(\mathrm{e}^{2ix}-1))\,\mathrm{csgn}(\sin(x))^2}{2}+\frac{x\pi\,\mathrm{csgn}(i\mathrm{e}^{-ix}$

```
[In] int(cot(x)*ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(sin(x))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cot(x)\log(\sin(x))\,dx = \frac{1}{2}\log\left(\sin\left(x\right)\right)^2$$

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="fricas")
[Out] 1/2*log(sin(x))^2
```

Sympy [F(-1)]

Timed out.

$$\int \cot(x)\log(\sin(x))\,dx = \text{Timed out}$$

```
[In] integrate(cot(x)*ln(sin(x)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cot(x)\log(\sin(x))\,dx = \frac{1}{2}\log\left(\sin\left(x\right)\right)^2$$

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="maxima")
[Out] 1/2*log(sin(x))^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cot(x)\log(\sin(x))\,dx = \frac{1}{2}\log\left(\sin\left(x\right)\right)^2$$

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="giac")
[Out] 1/2*log(sin(x))^2
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cot(x)\log(\sin(x))\,dx = \frac{\ln\left(\sin\left(x\right)\right)^2}{2}$$

```
[In] int(log(sin(x))*cot(x),x)
[Out] log(sin(x))^2/2
```

3.184 $\int \cos(x)\log(\sin(x))\sin^2(x)\,dx$

Optimal result 1038
Rubi [A] (verified) 1038
Mathematica [A] (verified) 1039
Maple [A] (verified) 1039
Fricas [A] (verification not implemented) 1040
Sympy [A] (verification not implemented) 1040
Maxima [A] (verification not implemented) 1041
Giac [A] (verification not implemented) 1041
Mupad [B] (verification not implemented) 1041

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = -\frac{1}{9}\sin^3(x)+\frac{1}{3}\log(\sin(x))\sin^3(x)$$

```
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2644, 30, 2634, 12}

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{1}{3}\sin^3(x)\log(\sin(x))-\frac{\sin^3(x)}{9}$$

```
[In] Int[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] -1/9*Sin[x]^3 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \int \frac{1}{3}\cos(x)\sin^2(x)\,dx \\
&= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\int \cos(x)\sin^2(x)\,dx \\
&= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right) \\
&= -\frac{1}{9}\sin^3(x) + \frac{1}{3}\log(\sin(x))\sin^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{1}{9}(-1+3\log(\sin(x)))\sin^3(x)$$

```
[In] Integrate[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
[Out] ((-1 + 3*Log[Sin[x]])*Sin[x]^3)/9
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{(\sin^3(x))}{9}+\frac{\ln(\sin(x))(\sin^3(x))}{3}$	17
default	$-\frac{(\sin^3(x))}{9}+\frac{\ln(\sin(x))(\sin^3(x))}{3}$	17
parallelrisch	$-\frac{(3\ln(\sin(x))-1)(\sin(3x)-3\sin(x))}{36}$	19
risch	Expression too large to display	577

```
[In] int(cos(x)*ln(sin(x))*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = -\frac{1}{3}\left(\cos\left(x\right)^2-1\right)\log\left(\sin\left(x\right)\right)\sin\left(x\right)+\frac{1}{9}\left(\cos\left(x\right)^2-1\right)\sin\left(x\right)$$

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 - 1)*log(sin(x))*sin(x) + 1/9*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{\log\left(\sin\left(x\right)\right)\sin^3\left(x\right)}{3}-\frac{\sin^3\left(x\right)}{9}$$

```
[In] integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
[Out] log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{1}{3}\log\left(\sin\left(x\right)\right)\sin\left(x\right)^3 - \frac{1}{9}\sin\left(x\right)^3$$

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="maxima")
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{1}{3}\log\left(\sin\left(x\right)\right)\sin\left(x\right)^3 - \frac{1}{9}\sin\left(x\right)^3$$

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="giac")
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int \cos(x)\log(\sin(x))\sin^2(x)\,dx = \frac{\sin\left(x\right)^3\,\left(\ln\left(\sin\left(x\right)\right)-\frac{1}{3}\right)}{3}$$

```
[In] int(log(sin(x))*cos(x)*sin(x)^2,x)
[Out] (sin(x)^3*(log(sin(x)) - 1/3))/3
```

3.185 $\int \cos(a+bx) \log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right) dx$

Optimal result . . . 1042
Rubi [A] (verified) . . . 1042
Mathematica [A] (verified) . . . 1043
Maple [A] (verified) . . . 1043
Fricas [A] (verification not implemented) . . . 1044
Sympy [F] . . . 1044
Maxima [A] (verification not implemented) . . . 1044
Giac [F(-2)] . . . 1045
Mupad [B] (verification not implemented) . . . 1045

Optimal result

Integrand size = 35, antiderivative size = 50

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx = -\frac{\sin(a+bx)}{b}+\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sin(a+bx)}{b}$$

```
[Out] -sin(b*x+a)/b+ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x))*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2717, 2634}

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx = \frac{\sin(a+bx)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sin(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sin[a + b*x]/b) + (Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]]*Sin[a + b*x
])/b
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sin(a+bx)}{b} - \int\cos(a+bx)\,dx\\ &= -\frac{\sin(a+bx)}{b} + \frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int\cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = -\frac{\sin(a+bx)}{b} + \frac{\log\left(\frac{1}{2}\sin(a+bx)\right)\sin(a+bx)}{b}$$

```
[In] Integrate[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
[Out] -(Sin[a + b*x]/b) + (Log[Sin[a + b*x]/2]*Sin[a + b*x])/b
```

Maple [A] (verified)

Time = 7.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\ln\left(\frac{\sin(bx+a)}{2}\right)\sin(bx+a)-\sin(bx+a)}{b}$	30
risch	Expression too large to display	1389

```
[In] int(cos(b*x+a)*ln(cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)),x,method=_RETURNVE
RBOSE)
[Out] 1/b*(ln(1/2*sin(b*x+a))*sin(b*x+a)-sin(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = \frac{2\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}{b}$$

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="fricas")
```

```
[Out] 2*(cos(1/2*b*x + 1/2*a)*log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(
1/2*b*x + 1/2*a) - cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))/b
```

Sympy [F]

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = \int \log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\cos\left(a+bx\right)dx$$

```
[In] integrate(cos(b*x+a)*ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x)
```

```
[Out] Integral(log(sin(a/2 + b*x/2)*cos(a/2 + b*x/2))*cos(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = \frac{\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(bx+a\right)}{b}-\frac{\sin\left(bx+a\right)}{b}$$

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="maxima")
```

```
[Out] log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(b*x + a)/b - sin(b*x + a
)/b
```

Giac [F(-2)]

Exception generated.

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = -\frac{\sin\left(a+b\,x\right)-\ln\left(\frac{\sin(a+b\,x)}{2}\right)\,\sin\left(a+b\,x\right)}{b}$$

```
[In] int(log(cos(a/2 + (b*x)/2)*sin(a/2 + (b*x)/2))*cos(a + b*x),x)
```

```
[Out] -(sin(a + b*x) - log(sin(a + b*x)/2)*sin(a + b*x))/b
```

3.186 $\int \frac{\tan(x)}{\log(\cos(x))} \, dx$

Optimal result 1046
Rubi [A] (verified) 1046
Mathematica [A] (verified) 1047
Maple [A] (verified) 1047
Fricas [A] (verification not implemented) 1048
Sympy [F] 1048
Maxima [A] (verification not implemented) 1048
Giac [A] (verification not implemented) 1048
Mupad [B] (verification not implemented) 1049

Optimal result

Integrand size = 8, antiderivative size = 6

$$\int \frac{\tan(x)}{\log(\cos(x))} \, dx = -\log(\log(\cos(x)))$$

```
[Out] -ln(ln(cos(x)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4424, 2339, 29}

$$\int \frac{\tan(x)}{\log(\cos(x))} \, dx = -\log(\log(\cos(x)))$$

```
[In] Int[Tan[x]/Log[Cos[x]],x]
[Out] -Log[Log[Cos[x]]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{x\log(x)}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(\cos(x))\right)\\ &= -\log(\log(\cos(x)))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\log(\cos(x))}\,dx = -\log(\log(\cos(x)))$$

```
[In] Integrate[Tan[x]/Log[Cos[x]],x]
[Out] -Log[Log[Cos[x]]]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\ln\left(\ln\left(\cos\left(x\right)\right)\right)$
default	$-\ln\left(\ln\left(\cos\left(x\right)\right)\right)$
risch	$-\ln\left(\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}\left(i\left(1+\mathrm{e}^{2ix}\right)\right)\mathrm{csgn}\left(i\cos(x)\right)}{2}-\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}\left(i\cos(x)\right)^{2}}{2}-\frac{i\pi\ \mathrm{csgn}\left(i\left(1+\mathrm{e}^{2ix}\right)\right)\mathrm{csgn}}{2}\right.$

```
[In] int(tan(x)/ln(cos(x)),x,method=_RETURNVERBOSE)
[Out] -ln(ln(cos(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\log(\cos(x))} \, dx = -\log\left(\log\left(\cos\left(x\right)\right)\right)$$

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="fricas")
[Out] -log(log(cos(x)))
```

Sympy [F]

$$\int \frac{\tan(x)}{\log(\cos(x))} \, dx = \int \frac{\tan\left(x\right)}{\log\left(\cos\left(x\right)\right)} \, dx$$

```
[In] integrate(tan(x)/ln(cos(x)),x)
[Out] Integral(tan(x)/log(cos(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\log(\cos(x))} \, dx = -\log\left(\log\left(\cos\left(x\right)\right)\right)$$

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="maxima")
[Out] -log(log(cos(x)))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{\tan(x)}{\log(\cos(x))} \, dx = -\log\left(\left|\log\left(\cos\left(x\right)\right)\right|\right)$$

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="giac")
[Out] -log(abs(log(cos(x))))
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\log(\cos(x))}\,dx = -\ln\left(\ln\left(\cos\left(x\right)\right)\right)$$

```
[In] int(tan(x)/log(cos(x)),x)
[Out] -log(log(cos(x)))
```

3.187 $\int \log(\cos(x)) \tan(x) \, dx$

Optimal result 1050
Rubi [A] (verified) 1050
Mathematica [A] (verified) 1051
Maple [A] (verified) 1051
Fricas [A] (verification not implemented) 1052
Sympy [A] (verification not implemented) 1052
Maxima [A] (verification not implemented) 1052
Giac [A] (verification not implemented) 1052
Mupad [B] (verification not implemented) 1053

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \log(\cos(x)) \tan(x) \, dx = -\frac{1}{2} \log^2(\cos(x))$$

```
[Out] -1/2*ln(cos(x))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556, 4424, 2338}

$$\int \log(\cos(x)) \tan(x) \, dx = -\frac{1}{2} \log^2(\cos(x))$$

```
[In] Int[Log[Cos[x]]*Tan[x],x]
```

```
[Out] -1/2*Log[Cos[x]]^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{2}\log^2(\cos(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{1}{2}\log^2(\cos(x))$$

```
[In] Integrate[Log[Cos[x]]*Tan[x],x]
[Out] -1/2*Log[Cos[x]]^2
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{\ln(\cos(x))^2}{2}$
default	$-\frac{\ln(\cos(x))^2}{2}$
risch	$-ix\ln(2) + \frac{x\pi\ \text{csgn}(i\mathrm{e}^{-ix})\ \text{csgn}(i(1+\mathrm{e}^{2ix}))\ \text{csgn}(i\cos(x))}{2} - \frac{x\pi\ \text{csgn}(i\mathrm{e}^{-ix})\ \text{csgn}(i\cos(x))^2}{2} - \frac{x\pi\ \text{csgn}(i(1+$

```
[In] int(ln(cos(x))*tan(x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(cos(x))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{1}{2}\,\log\left(\cos\left(x\right)\right)^{2}$$

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="fricas")
[Out] -1/2*log(cos(x))^2
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{\log\left(\cos\left(x\right)\right)^{2}}{2}$$

```
[In] integrate(ln(cos(x))*tan(x),x)
[Out] -log(cos(x))**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{1}{2}\,\log\left(\cos\left(x\right)\right)^{2}$$

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="maxima")
[Out] -1/2*log(cos(x))^2
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cos(x))\tan(x)\,dx = -\frac{1}{2}\,\log\left(\cos\left(x\right)\right)^{2}$$

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="giac")
[Out] -1/2*log(cos(x))^2
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cos(x)) \tan(x) \, dx = -\frac{\ln(\cos(x))^2}{2}$$

```
[In] int(log(cos(x))*tan(x),x)
[Out] -log(cos(x))^2/2
```

3.188 $\int \log(\cos(x))\sin(x)\,dx$

Optimal result 1054
Rubi [A] (verified) 1054
Mathematica [A] (verified) 1055
Maple [A] (verified) 1055
Fricas [A] (verification not implemented) 1056
Sympy [A] (verification not implemented) 1056
Maxima [A] (verification not implemented) 1056
Giac [A] (verification not implemented) 1056
Mupad [B] (verification not implemented) 1057

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \log(\cos(x))\sin(x)\,dx = \cos(x) - \cos(x)\log(\cos(x))$$

```
[Out] cos(x)-cos(x)*ln(cos(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2718, 2634}

$$\int \log(\cos(x))\sin(x)\,dx = \cos(x) - \cos(x)\log(\cos(x))$$

```
[In] Int[Log[Cos[x]]*Sin[x],x]
```

```
[Out] Cos[x] - Cos[x]*Log[Cos[x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\cos(x)\log(\cos(x)) - \int \sin(x)\,dx$$
$$= \cos(x) - \cos(x)\log(\cos(x))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sin(x)\,dx = \cos(x) - \cos(x)\log(\cos(x))$$

```
[In] Integrate[Log[Cos[x]]*Sin[x],x]
[Out] Cos[x] - Cos[x]*Log[Cos[x]]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\cos(x) - \cos(x)\ln(\cos(x))$
default	$\cos(x) - \cos(x)\ln(\cos(x))$
parallelrisch	$-\cos(x)\ln(\cos(x)) + \cos(x) + 1$
norman	$\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)\ln\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)-\ln\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)+2}{1+\tan^2\left(\frac{x}{2}\right)}$
risch	$\ln(e^{ix})\cos(x) + \frac{i e^{ix}\,\mathrm{csgn}(i+i e^{2ix})\pi\,\mathrm{csgn}(i e^{-ix})\,\mathrm{csgn}(i\cos(x))}{4} - \frac{i e^{ix}\,\mathrm{csgn}(i+i e^{2ix})\pi\,\mathrm{csgn}(i\cos(x))^2}{4} - \frac{i e^{ix}}{}$

```
[In] int(ln(cos(x))*sin(x),x,method=_RETURNVERBOSE)
[Out] cos(x)-cos(x)*ln(cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x)) \sin(x) \, dx = -\cos(x) \log(\cos(x)) + \cos(x)$$

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="fricas")
[Out] -cos(x)*log(cos(x)) + cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x)) \sin(x) \, dx = -\log(\cos(x)) \cos(x) + \cos(x)$$

```
[In] integrate(ln(cos(x))*sin(x),x)
[Out] -log(cos(x))*cos(x) + cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x)) \sin(x) \, dx = -\cos(x) \log(\cos(x)) + \cos(x)$$

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="maxima")
[Out] -cos(x)*log(cos(x)) + cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(\cos(x)) \sin(x) \, dx = -\cos(x) \log(\cos(x)) + \cos(x)$$

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="giac")
[Out] -cos(x)*log(cos(x)) + cos(x)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \log(\cos(x))\sin(x)\,dx = -\cos(x)\,\left(\ln\left(\cos\left(x\right)\right) - 1\right)$$

```
[In] int(log(cos(x))*sin(x),x)
[Out] -cos(x)*(log(cos(x)) - 1)
```

3.189 $\int \cos(x)\log(\cos(x))\,dx$

Optimal result 1058
Rubi [A] (verified) 1058
Mathematica [B] (verified) 1059
Maple [B] (verified) 1060
Fricas [A] (verification not implemented) 1060
Sympy [B] (verification not implemented) 1061
Maxima [B] (verification not implemented) 1061
Giac [A] (verification not implemented) 1062
Mupad [F(-1)] 1062

Optimal result

Integrand size = 6, antiderivative size = 14

$$\int \cos(x)\log(\cos(x))\,dx = \operatorname{arctanh}(\sin(x)) - \sin(x) + \log(\cos(x))\sin(x)$$

```
[Out] arctanh(sin(x))-sin(x)+ln(cos(x))*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {2717, 2634, 2672, 327, 212}

$$\int \cos(x)\log(\cos(x))\,dx = \operatorname{arctanh}(\sin(x)) - \sin(x) + \sin(x)\log(\cos(x))$$

```
[In] Int[Cos[x]*Log[Cos[x]],x]
[Out] ArcTanh[Sin[x]] - Sin[x] + Log[Cos[x]]*Sin[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \log(\cos(x))\sin(x) + \int \sin(x)\tan(x)\,dx \\
&= \log(\cos(x))\sin(x) + \text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(x)\right) \\
&= -\sin(x) + \log(\cos(x))\sin(x) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right) \\
&= \tanh^{-1}(\sin(x)) - \sin(x) + \log(\cos(x))\sin(x)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \cos(x)\log(\cos(x))\,dx = -\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right) - \sin(x) + \log(\cos(x))\sin(x)$$

```
[In] Integrate[Cos[x]*Log[Cos[x]],x]
```

```
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - Sin[x] + Log[Cos[x]]
*Sin[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 1.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

method	result
parallelrisch	$\ln(\cos(x))\sin(x)+\ln\left(\frac{2}{\cos(x)+1}\right)+\ln(\cos(x))-\sin(x)-2\ln(-\cot(x)+\csc(x)-1)$
default	$-\frac{i\left(e^{ix}\ln\left(\left(1+e^{2ix}\right)e^{-ix}\right)-e^{ix}+4\arctan\left(e^{ix}\right)-e^{-ix}\ln\left(\left(1+e^{2ix}\right)e^{-ix}\right)+e^{-ix}-\ln(2)\left(e^{ix}-e^{-ix}\right)\right)}{2}$
risch	$-\ln\left(e^{ix}\right)\sin(x)-\frac{e^{ix}\pi\ \mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}\left(i+ie^{2ix}\right)\mathrm{csgn}(i\cos(x))}{4}+\frac{e^{ix}\pi\ \mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}(i\cos(x))^2}{4}+\frac{e^{ix}\pi\ \mathrm{csgn}(i+$

```
[In] int(cos(x)*ln(cos(x)),x,method=_RETURNVERBOSE)
[Out] ln(cos(x))*sin(x)+ln(2/(cos(x)+1))+ln(cos(x))-sin(x)-2*ln(-cot(x)+csc(x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \cos(x)\log(\cos(x))\,dx = \log(\cos(x))\sin(x)+\frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1)-\sin(x)$$

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="fricas")
[Out] log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(15) = 30$.

Time = 0.82 (sec) , antiderivative size = 223, normalized size of antiderivative = 15.93

$$\int \cos(x)\log(\cos(x))\,dx = -\frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}$$

```
[In] integrate(cos(x)*ln(cos(x)),x)
```

```
[Out] -log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))*tan(x/2)**2/(tan
(x/2)**2 + 1) + 2*log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))
*tan(x/2)/(tan(x/2)**2 + 1) - log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x
/2)**2 + 1))/(tan(x/2)**2 + 1) + 2*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)*
*2 + 1) + 2*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)*tan(
x/2)**2/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)/(tan(x/2)**2 + 1) - 2*tan(
x/2)/(tan(x/2)**2 + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int \cos(x)\log(\cos(x))\,dx = \frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} + \log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) - \log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="maxima")
```

```
[Out] 2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)/
((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
+ 1)^2 + 1)*(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(
x) + 1) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \cos(x)\log(\cos(x))\,dx = \log(\cos(x))\sin(x) + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1) - \sin(x)$$

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="giac")
[Out] log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(x)\log(\cos(x))\,dx = \int \ln(\cos(x))\,\cos(x)\,dx$$

```
[In] int(log(cos(x))*cos(x),x)
[Out] int(log(cos(x))*cos(x), x)
```

3.190 $\int \cos(x) \log(\sin(x)) \, dx$

Optimal result 1063
Rubi [A] (verified) 1063
Mathematica [A] (verified) 1064
Maple [A] (verified) 1064
Fricas [A] (verification not implemented) 1065
Sympy [A] (verification not implemented) 1065
Maxima [A] (verification not implemented) 1065
Giac [A] (verification not implemented) 1065
Mupad [B] (verification not implemented) 1066

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \cos(x) \log(\sin(x)) \, dx = -\sin(x) + \log(\sin(x)) \sin(x)$$

```
[Out] -sin(x)+ln(sin(x))*sin(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2717, 2634}

$$\int \cos(x) \log(\sin(x)) \, dx = \sin(x) \log(\sin(x)) - \sin(x)$$

```
[In] Int[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \log(\sin(x))\sin(x) - \int \cos(x)\,dx \\ &= -\sin(x) + \log(\sin(x))\sin(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = -\sin(x) + \log(\sin(x))\sin(x)$$

```
[In] Integrate[Cos[x]*Log[Sin[x]],x]
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result
parallelrisch	$(\ln(\sin(x)) - 1)\sin(x)$
derivativedivides	$-\sin(x) + \ln(\sin(x))\sin(x)$
default	$-\sin(x) + \ln(\sin(x))\sin(x)$
norman	$\frac{2\tan\left(\frac{x}{2}\right)\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)-2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$
risch	$-\frac{i e^{-ix}\ln(2)}{2} + \frac{e^{ix}\pi\,\mathrm{csgn}(\sin(x))^3}{4} - \frac{e^{-ix}\pi\,\mathrm{csgn}\left(i e^{2ix}-i\right)\mathrm{csgn}\left(i e^{-ix}\right)\mathrm{csgn}(\sin(x))}{4} + \frac{e^{ix}\pi\,\mathrm{csgn}\left(i e^{2ix}-i\right)\mathrm{csgn}\left(i e^{-}\right.}{4}$

```
[In] int(cos(x)*ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] (ln(sin(x))-1)*sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="fricas")
[Out] log(sin(x))*sin(x) - sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

```
[In] integrate(cos(x)*ln(sin(x)),x)
[Out] log(sin(x))*sin(x) - sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="maxima")
[Out] log(sin(x))*sin(x) - sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\log(\sin(x))\,dx = \log\left(\sin\left(x\right)\right)\sin\left(x\right) - \sin\left(x\right)$$

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="giac")
[Out] log(sin(x))*sin(x) - sin(x)
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos(x)\log(\sin(x))\,dx = \sin(x)\,(\ln(\sin(x)) - 1)$$

```
[In] int(log(sin(x))*cos(x),x)
[Out] sin(x)*(log(sin(x)) - 1)
```

3.191 $\int \log(\sin(x))\sin^2(x)\,dx$

Optimal result 1067
Rubi [A] (verified) 1067
Mathematica [A] (verified) 1069
Maple [B] (verified) 1070
Fricas [B] (verification not implemented) 1070
Sympy [F] 1071
Maxima [B] (verification not implemented) 1071
Giac [F] 1071
Mupad [F(-1)] 1072

Optimal result

Integrand size = 8, antiderivative size = 74

$$\int \log(\sin(x))\sin^2(x)\,dx = \frac{x}{4}+\frac{ix^2}{4}-\frac{1}{2}x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\log(\sin(x))+\frac{1}{4}i\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}\cos(x)\log(\sin(x))\sin(x)$$

```
[Out] 1/4*x+1/4*I*x^2-1/2*x*ln(1-exp(2*I*x))+1/2*x*ln(sin(x))+1/4*I*polylog(2,exp
(2*I*x))+1/4*cos(x)*sin(x)-1/2*cos(x)*ln(sin(x))*sin(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2715, 8, 2634, 12, 6874, 3798, 2221, 2317, 2438}

$$\int \log(\sin(x))\sin^2(x)\,dx = \frac{1}{4}i\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{ix^2}{4}+\frac{x}{4}-\frac{1}{2}x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\log(\sin(x))+\frac{1}{4}\sin(x)\cos(x)-\frac{1}{2}\sin(x)\cos(x)\log(\sin(x))$$

```
[In] Int[Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] x/4 + (I/4)*x^2 - (x*Log[1 - E^((2*I)*x)])/2 + (x*Log[Sin[x]])/2 + (I/4)*Po
lyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (Cos[x]*Log[Sin[x]]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \int \frac{1}{2}\cot(x)(x-\cos(x)\sin(x))\,dx \\
&= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \frac{1}{2}\int \cot(x)(x-\cos(x)\sin(x))\,dx \\
&= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \frac{1}{2}\int \left(-\cos^2(x)+x\cot(x)\right)\,dx \\
&= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) + \frac{1}{2}\int \cos^2(x)\,dx - \frac{1}{2}\int x\cot(x)\,dx \\
&= \frac{ix^2}{4} + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) + i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&\quad + \frac{\int 1\,dx}{4} \\
&= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) \\
&\quad - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) + \frac{1}{2}\int \log\left(1-e^{2ix}\right)\,dx \\
&= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) \\
&\quad - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \frac{1}{4}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}i\text{Li}_2\left(e^{2ix}\right) \\
&\quad + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$
\begin{aligned}
\int \log(\sin(x))\sin^2(x)\,dx = \frac{1}{8}\big(&2x\left(1+ix-2\log\left(1-e^{2ix}\right)+2\log(\sin(x))\right) \\
&+ 2i\,\text{PolyLog}\left(2,e^{2ix}\right) + (1-2\log(\sin(x)))\sin(2x)\big)
\end{aligned}
$$

```
[In] Integrate[Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] (2*x*(1 + I*x - 2*Log[1 - E^((2*I)*x)] + 2*Log[Sin[x]]) + (2*I)*PolyLog[2,
E^((2*I)*x)] + (1 - 2*Log[Sin[x]])*Sin[2*x])/8
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(54) = 108.

Time = 4.48 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.62

method	result
default	$\frac{i\left(\frac{\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)e^{2ix}}{2}-\frac{e^{2ix}}{4}-2\ln\left(e^{ix}\right)\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)-\ln\left(e^{ix}\right)^{2}+2\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right)-2\,\mathrm{dilog}\left(e^{ix}\right)+2\,\mathrm{dilog}\left(e^{ix}+1\right)-\frac{e^{-2ix}\ln(}{}\right.}{4}$
risch	Expression too large to display

```
[In] int(ln(sin(x))*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*(1/2*ln(I*(-exp(I*x)^2+1)/exp(I*x))*exp(2*I*x)-1/4*exp(I*x)^2-2*ln(ex
p(I*x))*ln(I*(-exp(I*x)^2+1)/exp(I*x))-ln(exp(I*x))^2+2*ln(exp(I*x))*ln(exp
(I*x)+1)-2*dilog(exp(I*x))+2*dilog(exp(I*x)+1)-1/2*exp(-2*I*x)*ln(I*(-exp(I
*x)^2+1)/exp(I*x))+1/4/exp(I*x)^2-ln(exp(I*x))-ln(2)*(1/2*exp(I*x)^2-2*ln(e
xp(I*x))-1/2/exp(I*x)^2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(49) = 98.

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \log(\sin(x))\sin^2(x)\,dx = &-\frac{1}{4}\,x\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-\frac{1}{4}\,x\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\&-\frac{1}{4}\,x\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\&-\frac{1}{4}\,x\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\&-\frac{1}{2}\left(\cos\left(x\right)\sin\left(x\right)-x\right)\log\left(\sin\left(x\right)\right)+\frac{1}{4}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x\\&+\frac{1}{4}i\,\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))-\frac{1}{4}i\,\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))\\&-\frac{1}{4}i\,\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))+\frac{1}{4}i\,\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))\end{aligned}$$

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*x*log(cos(x) + I*sin(x) + 1) - 1/4*x*log(cos(x) - I*sin(x) + 1) - 1/4*
x*log(-cos(x) + I*sin(x) + 1) - 1/4*x*log(-cos(x) - I*sin(x) + 1) - 1/2*(co
s(x)*sin(x) - x)*log(sin(x)) + 1/4*cos(x)*sin(x) + 1/4*x + 1/4*I*dilog(cos(
x) + I*sin(x)) - 1/4*I*dilog(cos(x) - I*sin(x)) - 1/4*I*dilog(-cos(x) + I*s
in(x)) + 1/4*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]

$$\int \log(\sin(x))\sin^2(x)\,dx = \int \log(\sin(x))\sin^2(x)\,dx$$

```
[In] integrate(ln(sin(x))*sin(x)**2,x)
```

```
[Out] Integral(log(sin(x))*sin(x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(49) = 98.

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \log(\sin(x))\sin^2(x)\,dx = {} & \frac{1}{4}i\,x^2 - \frac{1}{2}i\,x\arctan(\sin(x),\cos(x)+1) \\ & + \frac{1}{2}i\,x\arctan(\sin(x),-\cos(x)+1) \\ & - \frac{1}{4}\,x\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) \\ & - \frac{1}{4}\,x\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right) \\ & + \frac{1}{4}\,(2\,x-\sin(2\,x))\log(\sin(x)) + \frac{1}{4}\,x \\ & + \frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(e^{(i\,x)}\right) + \frac{1}{8}\,\sin(2\,x)\end{aligned}$$

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*x^2 - 1/2*I*x*arctan2(sin(x), cos(x) + 1) + 1/2*I*x*arctan2(sin(x), -
cos(x) + 1) - 1/4*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/4*x*log(cos
(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 1/4*(2*x - sin(2*x))*log(sin(x)) + 1/4*x
 + 1/2*I*dilog(-e^(I*x)) + 1/2*I*dilog(e^(I*x)) + 1/8*sin(2*x)
```

Giac [F]

$$\int \log(\sin(x))\sin^2(x)\,dx = \int \log(\sin(x))\sin(x)^2\,dx$$

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="giac")
```

```
[Out] integrate(log(sin(x))*sin(x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(\sin(x))\sin^2(x)\,dx = \int \ln\left(\sin\left(x\right)\right)\,{\sin\left(x\right)}^2 \,d x$$

```
[In] int(log(sin(x))*sin(x)^2,x)
```

```
[Out] int(log(sin(x))*sin(x)^2, x)
```

3.192 $\int \log(\sin(x))\sin^3(x)\,dx$

Optimal result . . . 1073
Rubi [A] (verified) . . . 1073
Mathematica [A] (verified) . . . 1075
Maple [A] (verified) . . . 1075
Fricas [A] (verification not implemented) . . . 1076
Sympy [B] (verification not implemented) . . . 1076
Maxima [B] (verification not implemented) . . . 1078
Giac [A] (verification not implemented) . . . 1078
Mupad [F(-1)] . . . 1079

Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{2}{3}\text{arctanh}(\cos(x)) + \frac{2\cos(x)}{3} - \frac{\cos^3(x)}{9} - \cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x))$$

```
[Out] -2/3*arctanh(cos(x))+2/3*cos(x)-1/9*cos(x)^3-cos(x)*ln(sin(x))+1/3*cos(x)^3
*ln(sin(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {2713, 2634, 12, 4451, 470, 327, 212}

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{2}{3}\text{arctanh}(\cos(x)) - \frac{\cos^3(x)}{9} + \frac{2\cos(x)}{3} + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \cos(x)\log(\sin(x))$$

```
[In] Int[Log[Sin[x]]*Sin[x]^3,x]
```

```
[Out] (-2*ArcTanh[Cos[x]])/3 + (2*Cos[x])/3 - Cos[x]^3/9 - Cos[x]*Log[Sin[x]] + (
Cos[x]^3*Log[Sin[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\text{integral} = -\cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \int \frac{1}{6}\cos(x)(-5+\cos(2x))\cot(x)\,dx$$

$$= -\cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \frac{1}{6}\int \cos(x)(-5+\cos(2x))\cot(x)\,dx$$

$$= -\cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) + \frac{1}{6}\text{Subst}\left(\int \frac{2x^2(-3+x^2)}{1-x^2}\,dx, x, \cos(x)\right)$$

$$= -\cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) + \frac{1}{3}\text{Subst}\left(\int \frac{x^2(-3+x^2)}{1-x^2}\,dx, x, \cos(x)\right)$$

$$= -\frac{1}{9}\cos^3(x) - \cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \frac{2}{3}\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right)$$

$$= \frac{2\cos(x)}{3} - \frac{\cos^3(x)}{9} - \cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)$$

$$= -\frac{2}{3}\tanh^{-1}(\cos(x)) + \frac{2\cos(x)}{3} - \frac{\cos^3(x)}{9} - \cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x))$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \log(\sin(x))\sin^3(x)\,dx = \frac{1}{36}\left(24\left(-\log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)\right) + \cos(3x)(-1+3\log(\sin(x))) - 3\cos(x)(-7+9\log(\sin(x)))\right)$$

```
[In] Integrate[Log[Sin[x]]*Sin[x]^3,x]
[Out] (24*(-Log[Cos[x/2]] + Log[Sin[x/2]]) + Cos[3*x]*(-1 + 3*Log[Sin[x]]) - 3*Co
s[x]*(-7 + 9*Log[Sin[x]]))/36
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

method	result
parallelrisch	$\ln\left(2\left(\frac{1}{\cos(2x)+3+4\cos(x)}\right)^{\frac{1}{3}}\right) + \cos(3x)\ln\left(\sin^{\frac{1}{12}}(x)\right) + \cos(x)\ln\left(\frac{1}{\sin(x)^{\frac{3}{4}}}\right) + \ln\left(\sin^{\frac{2}{3}}(x)\right) -$
default	$\frac{e^{3ix}\ln(i(1-e^{2ix})e^{-ix})}{24} - \frac{e^{3ix}}{72} + \frac{7e^{ix}}{24} + \frac{2\ln(e^{ix}-1)}{3} - \frac{2\ln(e^{ix}+1)}{3} - \frac{3e^{ix}\ln(i(1-e^{2ix})e^{-ix})}{8} - \frac{3e^{-ix}\ln(i(1-e^{2i}}{8}$
risch	Expression too large to display

```
[In] int(ln(sin(x))*sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] ln(2*(1/(cos(2*x)+3+4*cos(x)))^(1/3))+cos(3*x)*ln(sin(x)^(1/12))+cos(x)*ln(
1/sin(x)^(3/4))+ln(sin(x)^(2/3))-1/36*cos(3*x)+7/12*cos(x)+1/3
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{1}{9}\cos(x)^3 + \frac{1}{3}\left(\cos(x)^3 - 3\cos(x)\right)\log(\sin(x)) + \frac{2}{3}\cos(x) - \frac{1}{3}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{3}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*lo
g(1/2*cos(x) + 1/2) + 1/3*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(41) = 82.

Time = 3.61 (sec) , antiderivative size = 456, normalized size of antiderivative = 11.40

$$\int \log(\sin(x))\sin^3(x)\,dx = \frac{12\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^6\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{36\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^4\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{6\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^6\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{18\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^4\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{18\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{6\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{12\log(2)\tan^6\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{6\tan^4\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{36\log(2)\tan^4\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{24\tan^2\left(\frac{x}{2}\right)}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9} + \frac{10}{9\tan^6\left(\frac{x}{2}\right)+27\tan^4\left(\frac{x}{2}\right)+27\tan^2\left(\frac{x}{2}\right)+9}$$

```
[In] integrate(ln(sin(x))*sin(x)**3,x)
```

```
[Out] 12*log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)
**4 + 27*tan(x/2)**2 + 9) + 36*log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**4/
(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 6*log(tan(x/2)**2 +
 1)*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 18*
log(tan(x/2)**2 + 1)*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x
/2)**2 + 9) + 18*log(tan(x/2)**2 + 1)*tan(x/2)**2/(9*tan(x/2)**6 + 27*tan(x
/2)**4 + 27*tan(x/2)**2 + 9) + 6*log(tan(x/2)**2 + 1)/(9*tan(x/2)**6 + 27*t
an(x/2)**4 + 27*tan(x/2)**2 + 9) + 12*log(2)*tan(x/2)**6/(9*tan(x/2)**6 + 2
7*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 6*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan
(x/2)**4 + 27*tan(x/2)**2 + 9) + 36*log(2)*tan(x/2)**4/(9*tan(x/2)**6 + 27*
tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 24*tan(x/2)**2/(9*tan(x/2)**6 + 27*tan(
x/2)**4 + 27*tan(x/2)**2 + 9) + 10/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan
(x/2)**2 + 9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(32) = 64.

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 4.48

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{4\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+1\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)}{3\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)} + \frac{2\left(\frac{12\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+5\right)}{9\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)} - \frac{2}{3}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right) + \frac{2}{3}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}\right)$$

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="maxima")
```

```
[Out] -4/3*(3*sin(x)^2/(cos(x) + 1)^2 + 1)*log(2*sin(x)/((sin(x)^2/(cos(x) + 1)^2
 + 1)*(cos(x) + 1)))/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x) + 1)^4
 + sin(x)^6/(cos(x) + 1)^6 + 1) + 2/9*(12*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x
)^4/(cos(x) + 1)^4 + 5)/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x) + 1
)^4 + sin(x)^6/(cos(x) + 1)^6 + 1) - 2/3*log(sin(x)^2/(cos(x) + 1)^2 + 1) +
 2/3*log(sin(x)^2/(cos(x) + 1)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \log(\sin(x))\sin^3(x)\,dx = -\frac{1}{9}\cos(x)^3 + \frac{1}{3}\left(\cos(x)^3 - 3\cos(x)\right)\log(\sin(x)) + \frac{2}{3}\cos(x) - \frac{1}{3}\log(\cos(x)+1) + \frac{1}{3}\log(-\cos(x)+1)$$

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="giac")
```

```
[Out] -1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*lo
g(cos(x) + 1) + 1/3*log(-cos(x) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \log(\sin(x))\sin^3(x)\,dx = \int \ln\left(\sin\left(x\right)\right)\,{\sin\left(x\right)}^3 \,d x$$

```
[In] int(log(sin(x))*sin(x)^3,x)
```

```
[Out] int(log(sin(x))*sin(x)^3, x)
```

3.193 $\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx$

Optimal result 1080
Rubi [A] (verified) 1080
Mathematica [A] (verified) 1082
Maple [F] 1083
Fricas [F] 1083
Sympy [F] 1083
Maxima [B] (verification not implemented) 1083
Giac [F] 1084
Mupad [F(-1)] 1084

Optimal result

Integrand size = 7, antiderivative size = 79

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx=\frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+i\sqrt{x}\,\mathrm{PolyLog}\left(2,e^{2i\sqrt{x}}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{2i\sqrt{x}}\right)$$

```
[Out] 1/3*I*x^(3/2)-x*ln(1-exp(2*I*x^(1/2)))+x*ln(sin(x^(1/2)))-1/2*polylog(3,exp
(2*I*x^(1/2)))+I*polylog(2,exp(2*I*x^(1/2)))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {2628, 12, 3833, 3798, 2221, 2611, 2320, 6724}

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx=i\sqrt{x}\,\mathrm{PolyLog}\left(2,e^{2i\sqrt{x}}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{2i\sqrt{x}}\right)+\frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

```
[In] Int[Log[Sin[Sqrt[x]]],x]
```

```
[Out] (I/3)*x^(3/2) - x*Log[1 - E^((2*I)*Sqrt[x])] + x*Log[Sin[Sqrt[x]]] + I*Sqrt
[x]*PolyLog[2, E^((2*I)*Sqrt[x])] - PolyLog[3, E^((2*I)*Sqrt[x])]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3833

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(\sin\left(\sqrt{x}\right)\right)-\int \frac{1}{2}\sqrt{x}\cot\left(\sqrt{x}\right)\,dx\\
&= x\log\left(\sin\left(\sqrt{x}\right)\right)-\frac{1}{2}\int \sqrt{x}\cot\left(\sqrt{x}\right)\,dx\\
&= x\log\left(\sin\left(\sqrt{x}\right)\right)-\text{Subst}\left(\int x^2\cot(x)\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{3}ix^{3/2}+x\log\left(\sin\left(\sqrt{x}\right)\right)+2i\text{Subst}\left(\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+2\text{Subst}\left(\int x\log\left(1-e^{2ix}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)\\
&\quad+i\sqrt{x}\text{Li}_2\left(e^{2i\sqrt{x}}\right)-i\text{Subst}\left(\int \text{Li}_2\left(e^{2ix}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)\\
&\quad+i\sqrt{x}\text{Li}_2\left(e^{2i\sqrt{x}}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,dx,x,e^{2i\sqrt{x}}\right)\\
&= \frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+i\sqrt{x}\text{Li}_2\left(e^{2i\sqrt{x}}\right)-\frac{1}{2}\text{Li}_3\left(e^{2i\sqrt{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

$$
\begin{aligned}
\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx &= \frac{i\pi^3}{24}-\frac{1}{3}ix^{3/2}-x\log\left(1-e^{-2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)\\
&\quad-i\sqrt{x}\,\text{PolyLog}\left(2,e^{-2i\sqrt{x}}\right)-\frac{1}{2}\,\text{PolyLog}\left(3,e^{-2i\sqrt{x}}\right)
\end{aligned}
$$

```
[In] Integrate[Log[Sin[Sqrt[x]]],x]
[Out] (I/24)*Pi^3 - (I/3)*x^(3/2) - x*Log[1 - E^((-2*I)*Sqrt[x])] + x*Log[Sin[Sqr
t[x]]] - I*Sqrt[x]*PolyLog[2, E^((-2*I)*Sqrt[x])] - PolyLog[3, E^((-2*I)*Sq
rt[x])]/2
```

Maple [F]

$$\int \ln \left(\sin \left(\sqrt{x}\right)\right) dx$$

```
[In] int(ln(sin(x^(1/2))),x)
[Out] int(ln(sin(x^(1/2))),x)
```

Fricas [F]

$$\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx = \int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx$$

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="fricas")
[Out] integral(log(sin(sqrt(x))), x)
```

Sympy [F]

$$\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx = \int \log \left(\sin \left(\sqrt{x}\right)\right) dx$$

```
[In] integrate(ln(sin(x**(1/2))),x)
[Out] Integral(log(sin(sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(49) = 98.

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx = & -i\,x \arctan \left(\sin \left(\sqrt{x}\right), \cos \left(\sqrt{x}\right) + 1\right) \\ & + i\,x \arctan \left(\sin \left(\sqrt{x}\right), -\cos \left(\sqrt{x}\right) + 1\right) \\ & - \frac{1}{2}\,x \log \left(\cos \left(\sqrt{x}\right)^{2} + \sin \left(\sqrt{x}\right)^{2} + 2\,\cos \left(\sqrt{x}\right) + 1\right) \\ & - \frac{1}{2}\,x \log \left(\cos \left(\sqrt{x}\right)^{2} + \sin \left(\sqrt{x}\right)^{2} - 2\,\cos \left(\sqrt{x}\right) + 1\right) \\ & + x \log \left(\sin \left(\sqrt{x}\right)\right) + \frac{1}{3} i\,x^{\frac{3}{2}} + 2i\,\sqrt{x} \mathrm{Li}_2\left(-e^{\left(i\,\sqrt{x}\right)}\right) \\ & + 2i\,\sqrt{x} \mathrm{Li}_2\left(e^{\left(i\,\sqrt{x}\right)}\right) - 2\,\mathrm{Li}_3(-e^{\left(i\,\sqrt{x}\right)}) - 2\,\mathrm{Li}_3(e^{\left(i\,\sqrt{x}\right)})\end{aligned}$$

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="maxima")
```

```
[Out] -I*x*arctan2(sin(sqrt(x)), cos(sqrt(x)) + 1) + I*x*arctan2(sin(sqrt(x)), -c
os(sqrt(x)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 + 2*cos(sqrt(x
)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 - 2*cos(sqrt(x)) + 1) +
 x*log(sin(sqrt(x))) + 1/3*I*x^(3/2) + 2*I*sqrt(x)*dilog(-e^(I*sqrt(x))) +
2*I*sqrt(x)*dilog(e^(I*sqrt(x))) - 2*polylog(3, -e^(I*sqrt(x))) - 2*polylog
(3, e^(I*sqrt(x)))
```

Giac [F]

$$\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx = \int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx$$

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(log(sin(sqrt(x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(\sin \left(\sqrt{x}\right)\right) \, dx = \int \ln \left(\sin \left(\sqrt{x}\right)\right) \, dx$$

```
[In] int(log(sin(x^(1/2))),x)
```

```
[Out] int(log(sin(x^(1/2))), x)
```

3.194 $\int \csc^2(x) \log(\sin(x)) \, dx$

Optimal result 1085
Rubi [A] (verified) 1085
Mathematica [A] (verified) 1086
Maple [A] (verified) 1086
Fricas [A] (verification not implemented) 1087
Sympy [A] (verification not implemented) 1087
Maxima [B] (verification not implemented) 1087
Giac [A] (verification not implemented) 1088
Mupad [B] (verification not implemented) 1088

Optimal result

Integrand size = 8, antiderivative size = 15

$$\int \csc^2(x) \log(\sin(x)) \, dx = -x - \cot(x) - \cot(x) \log(\sin(x))$$

```
[Out] -x-cot(x)-cot(x)*ln(sin(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3852, 8, 2634, 3554}

$$\int \csc^2(x) \log(\sin(x)) \, dx = -x - \cot(x) - \cot(x) \log(\sin(x))$$

```
[In] Int[Csc[x]^2*Log[Sin[x]],x]
```

```
[Out] -x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\cot(x)\log(\sin(x)) + \int \cot^2(x)\,dx \\ &= -\cot(x) - \cot(x)\log(\sin(x)) - \int 1\,dx \\ &= -x - \cot(x) - \cot(x)\log(\sin(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc^2(x)\log(\sin(x))\,dx = -x - \cot(x) - \cot(x)\log(\sin(x))$$

```
[In] Integrate[Csc[x]^2*Log[Sin[x]],x]
[Out] -x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result
parallelrisch	$-x-\cot(x)-\cot(x)\ln(\sin(x))$
norman	$\frac{-\frac{1}{2}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-x\tan\left(\frac{x}{2}\right)+\frac{\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)}{2}}{\tan\left(\frac{x}{2}\right)}$
default	$4i\left(\frac{-\frac{\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)e^{2ix}}{2}-\frac{1}{2}}{e^{2ix}-1}+\frac{\ln(e^{ix}-1)}{4}+\frac{\ln(e^{ix}+1)}{4}+\frac{\ln(2)}{2\,e^{2ix}-2}\right)$
risch	$\frac{2i\ln(e^{ix})}{e^{2ix}-1}-\frac{i\ln(e^{2ix}-1)e^{2ix}-\pi\ \mathrm{csgn}\left(i\left(e^{2ix}-1\right)\right)\mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}(\sin(x))-\pi\ \mathrm{csgn}\left(i\left(e^{2ix}-1\right)\right)\mathrm{csgn}(\sin(x))^2-\mathrm{csgn}(\sin(x))^2}{}$

```
[In] int(csc(x)^2*ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] -x-cot(x)-cot(x)*ln(sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc^2(x)\log(\sin(x))\,dx = -\frac{\cos(x)\log(\sin(x)) + x\sin(x) + \cos(x)}{\sin(x)}$$

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="fricas")
[Out] -(cos(x)*log(sin(x)) + x*sin(x) + cos(x))/sin(x)
```

Sympy [A] (verification not implemented)

Time = 11.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^2(x)\log(\sin(x))\,dx = -x - \log(\sin(x))\cot(x) - \frac{\cos(x)}{\sin(x)}$$

```
[In] integrate(csc(x)**2*ln(sin(x)),x)
[Out] -x - log(sin(x))*cot(x) - cos(x)/sin(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(15) = 30.

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.40

$$\begin{aligned}&\int \csc^2(x)\log(\sin(x))\,dx\\&= -\frac{1}{2}\left(\frac{\cos(x)+1}{\sin(x)} - \frac{\sin(x)}{\cos(x)+1}\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)\\&\quad - \frac{\cos(x)+1}{2\sin(x)} + \frac{\sin(x)}{2(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)\end{aligned}$$

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="maxima")
[Out] -1/2*((cos(x) + 1)/sin(x) - sin(x)/(cos(x) + 1))*log(2*sin(x)/((sin(x)^2/(c
os(x) + 1)^2 + 1)*(cos(x) + 1))) - 1/2*(cos(x) + 1)/sin(x) + 1/2*sin(x)/(co
s(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc^2(x)\log(\sin(x))\,dx = -x - \frac{\log(\sin(x))}{\tan(x)} - \frac{1}{\tan(x)}$$

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="giac")
[Out] -x - log(sin(x))/tan(x) - 1/tan(x)
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.80

$$\int \csc^2(x)\log(\sin(x))\,dx = -2\,x - \ln\left(\mathrm{e}^{x\,2\mathrm{i}} - 1\right)\,1\mathrm{i} - \frac{\ln\left(\frac{\mathrm{e}^{-x\,1\mathrm{i}}\,1\mathrm{i}}{2} - \frac{\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)\,2\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}} - 1} - \frac{2\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}} - 1}$$

```
[In] int(log(sin(x))/sin(x)^2,x)
[Out] - 2*x - log(exp(x*2i) - 1)*1i - (log((exp(-x*1i)*1i)/2 - (exp(x*1i)*1i)/2)*
2i)/(exp(x*2i) - 1) - 2i/(exp(x*2i) - 1)
```

3.195 $\int \log(x)\sinh(a+bx)\,dx$

Optimal result 1089
Rubi [A] (verified) 1089
Mathematica [A] (verified) 1091
Maple [A] (verified) 1091
Fricas [B] (verification not implemented) 1091
Sympy [F] 1092
Maxima [A] (verification not implemented) 1092
Giac [A] (verification not implemented) 1092
Mupad [F(-1)] 1092

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \log(x)\sinh(a+bx)\,dx=-\frac{\cosh(a)\text{Chi}(bx)}{b}+\frac{\cosh(a+bx)\log(x)}{b}-\frac{\sinh(a)\text{Shi}(bx)}{b}$$

```
[Out] -Chi(b*x)*cosh(a)/b+cosh(b*x+a)*ln(x)/b-Shi(b*x)*sinh(a)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2718, 2634, 12, 3384, 3379, 3382}

$$\int \log(x)\sinh(a+bx)\,dx=-\frac{\cosh(a)\text{Chi}(bx)}{b}-\frac{\sinh(a)\text{Shi}(bx)}{b}+\frac{\log(x)\cosh(a+bx)}{b}$$

```
[In] Int[Log[x]*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a]*CoshIntegral[b*x])/b) + (Cosh[a + b*x]*Log[x])/b - (Sinh[a]*Sinh
Integral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
```

```
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\cosh(a+bx)\log(x)}{b} - \int \frac{\cosh(a+bx)}{bx}\,dx \\
&= \frac{\cosh(a+bx)\log(x)}{b} - \frac{\int \frac{\cosh(a+bx)}{x}\,dx}{b} \\
&= \frac{\cosh(a+bx)\log(x)}{b} - \frac{\cosh(a)\int \frac{\cosh(bx)}{x}\,dx}{b} - \frac{\sinh(a)\int \frac{\sinh(bx)}{x}\,dx}{b} \\
&= -\frac{\cosh(a)\text{Chi}(bx)}{b} + \frac{\cosh(a+bx)\log(x)}{b} - \frac{\sinh(a)\text{Shi}(bx)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \log(x)\sinh(a+bx)\,dx = -\frac{\cosh(a)\text{Chi}(bx)-\cosh(a+bx)\log(x)+\sinh(a)\text{Shi}(bx)}{b}$$

```
[In] Integrate[Log[x]*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a]*CoshIntegral[b*x] - Cosh[a + b*x]*Log[x] + Sinh[a]*SinhIntegral[
b*x])/b)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

method	result
risch	$\frac{\ln(x)e^{-bx-a}}{2b}+\frac{e^{bx+a}\ln(x)}{2b}+\frac{e^{-a}\,\text{Ei}_1(bx)}{2b}+\frac{e^{a}\,\text{Ei}_1(-bx)}{2b}$
meijerg	$-\frac{\sinh(a)\sinh(bx)}{b}+\frac{\sinh(a)\ln(x)\sinh(bx)}{b}+\frac{\sinh(a)b^2\left(\frac{9\sinh(bx)}{b^3}-\frac{9\,\text{Shi}(bx)}{b^3}\right)}{9}-\frac{\cosh(a)b\left(-\frac{2}{b^2}+\frac{2\cosh(bx)}{b^2}\right)}{4}+\frac{\cosh(a)b\ln(x}{}$

```
[In] int(ln(x)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*ln(x)*exp(-b*x-a)+1/2*exp(b*x+a)*ln(x)/b+1/2/b*exp(-a)*Ei(1,b*x)+1/2/
b*exp(a)*Ei(1,-b*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(35) = 70$.

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.83

$$\int \log(x)\sinh(a+bx)\,dx = -\frac{(\text{Ei}(bx)+\text{Ei}(-bx))\cosh(bx+a)\cosh(a)-\log(x)\sinh(bx+a)^2+(\text{Ei}(bx)-\text{Ei}(-bx))\cosh(bx+a)\,\text{s}}{}$$

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2 +
 (Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 + 1)*log(x)
+ ((Ei(b*x) + Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) - Ei(-b
*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \log(x)\sinh(a+bx)\,dx = \int \log(x)\sinh(a+bx)\,dx$$

```
[In] integrate(ln(x)*sinh(b*x+a),x)
[Out] Integral(log(x)*sinh(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \log(x)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)\log(x)}{b} - \frac{\mathrm{Ei}(-bx)\,e^{(-a)} + \mathrm{Ei}(bx)\,e^{a}}{2\,b}$$

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="maxima")
[Out] cosh(b*x + a)*log(x)/b - 1/2*(Ei(-b*x)*e^(-a) + Ei(b*x)*e^a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int \log(x)\sinh(a+bx)\,dx = \frac{1}{2}\left(\frac{e^{(bx+a)}}{b} + \frac{e^{(-bx-a)}}{b}\right)\log(x) - \frac{\mathrm{Ei}(-bx)\,e^{(-a)} + \mathrm{Ei}(bx)\,e^{a}}{2\,b}$$

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/2*(e^(b*x + a)/b + e^(-b*x - a)/b)*log(x) - 1/2*(Ei(-b*x)*e^(-a) + Ei(b*x
)*e^a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sinh(a+bx)\,dx = \int \sinh(a+b\,x)\,\ln(x)\,dx$$

```
[In] int(sinh(a + b*x)*log(x),x)
[Out] int(sinh(a + b*x)*log(x), x)
```

3.196 $\int \log(x)\sinh^2(a+bx)\,dx$

Optimal result 1093
Rubi [A] (verified) 1093
Mathematica [A] (verified) 1095
Maple [A] (verified) 1095
Fricas [B] (verification not implemented) 1096
Sympy [F] 1096
Maxima [A] (verification not implemented) 1096
Giac [A] (verification not implemented) 1097
Mupad [F(-1)] 1097

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \log(x)\sinh^2(a+bx)\,dx = \frac{x}{2}-\frac{1}{2}x\log(x)-\frac{\operatorname{Chi}(2bx)\sinh(2a)}{4b}+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\operatorname{Shi}(2bx)}{4b}$$

```
[Out] 1/2*x-1/2*x*ln(x)-1/4*cosh(2*a)*Shi(2*b*x)/b-1/4*Chi(2*b*x)*sinh(2*a)/b+1/2
*cosh(b*x+a)*ln(x)*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2715, 8, 2634, 12, 5382, 3384, 3379, 3382}

$$\int \log(x)\sinh^2(a+bx)\,dx = -\frac{\sinh(2a)\operatorname{Chi}(2bx)}{4b}-\frac{\cosh(2a)\operatorname{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}-\frac{1}{2}x\log(x)$$

```
[In] Int[Log[x]*Sinh[a + b*x]^2,x]
```

```
[Out] x/2 - (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b*x]
*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5382

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[ExpandToSum
[u, x]^m*(a + b*Sinh[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] &&
 LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \int \frac{1}{4}\left(-2 + \frac{\sinh(2(a+bx))}{bx}\right)\,dx$$

$$= -\frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{1}{4}\int\left(-2+\frac{\sinh(2(a+bx))}{bx}\right)dx$$

$$= \frac{x}{2} - \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\int\frac{\sinh(2(a+bx))}{x}dx}{4b}$$

$$= \frac{x}{2} - \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\int\frac{\sinh(2a+2bx)}{x}dx}{4b}$$

$$= \frac{x}{2} - \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\cosh(2a)\int\frac{\sinh(2bx)}{x}dx}{4b} - \frac{\sinh(2a)\int\frac{\cosh(2bx)}{x}dx}{4b}$$

$$= \frac{x}{2} - \frac{1}{2}x\log(x) - \frac{\text{Chi}(2bx)\sinh(2a)}{4b} + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\cosh(2a)\text{Shi}(2bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int\log(x)\sinh^2(a+bx)\,dx$$
$$= -\frac{-2bx + 2bx\log(x) + \text{Chi}(2bx)\sinh(2a) - \log(x)\sinh(2(a+bx)) + \cosh(2a)\text{Shi}(2bx)}{4b}$$

```
[In] Integrate[Log[x]*Sinh[a + b*x]^2,x]
```

```
[Out] -1/4*(-2*b*x + 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[2
*(a + b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/b
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

method	result	siz
risch	$-\frac{\ln(x)x}{2}+\frac{e^{2bx+2a}\ln(x)}{8b}-\frac{e^{-2bx-2a}\ln(x)}{8b}+\frac{e^{2a}\text{Ei}_1(-2bx)}{8b}-\frac{a\ln(bx)}{2b}+\frac{a\ln(-bx)}{2b}-\frac{e^{-2a}\text{Ei}_1(2bx)}{8b}+\frac{x}{2}+\frac{a}{2b}$	99

```
[In] int(ln(x)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x)*x+1/8/b*exp(2*b*x+2*a)*ln(x)-1/8/b*exp(-2*b*x-2*a)*ln(x)+1/8/b*e
xp(2*a)*Ei(1,-2*b*x)-1/2/b*a*ln(b*x)+1/2/b*a*ln(-b*x)-1/8/b*exp(-2*a)*Ei(1,
2*b*x)+1/2*x+1/2*a/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(56) = 112$.

Time = 0.34 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.74

$$\int \log(x)\sinh^2(a+bx)\,dx = \frac{4\cosh(bx+a)\log(x)\sinh(bx+a)^3 + \log(x)\sinh(bx+a)^4 - (\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\cosh(bx+a)^2\sinh(\ldots}{\ldots}$$

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei(
2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) + (4*b*x - (Ei(2*b*x) - Ei(-
2*b*x))*cosh(2*a))*cosh(b*x + a)^2 + (4*b*x - (Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) - 2*(2*b*x - 3*cosh(b*x + a)^2)*log(x) - (Ei(2*b*x) + Ei(-2*b*x))*sin
h(2*a))*sinh(b*x + a)^2 - (4*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 + 1)*log
(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) - (4*b*x - (Ei(2*
b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) + 2*(2*b*x*cosh(b*x + a) - cosh
(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \log(x)\sinh^2(a+bx)\,dx = \int \log(x)\sinh^2(a+bx)\,dx$$

```
[In] integrate(ln(x)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(log(x)*sinh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \log(x)\sinh^2(a+bx)\,dx = -\frac{1}{8}\left(4\,x - \frac{e^{(2\,bx+2\,a)}}{b} + \frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x) + \frac{1}{2}\,x - \frac{\mathrm{Ei}(2\,bx)\,e^{(2\,a)}}{8\,b} + \frac{\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}$$

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)*log(x) + 1/2*x - 1/8*Ei
(2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \log(x)\sinh^2(a+bx)\,dx = -\frac{1}{8}\left(4\,x-\frac{e^{(2\,bx+2\,a)}}{b}+\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log\left(x\right) + \frac{4\,bx-\mathrm{Ei}(2\,bx)\,e^{(2\,a)}+\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}$$

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] -1/8*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)*log(x) + 1/8*(4*b*x - E
i(2*b*x)*e^(2*a) + Ei(-2*b*x)*e^(-2*a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sinh^2(a+bx)\,dx = \int \sinh(a+b\,x)^2\,\ln\left(x\right)\,dx$$

```
[In] int(sinh(a + b*x)^2*log(x),x)
[Out] int(sinh(a + b*x)^2*log(x), x)
```

3.197 $\int \log(x)\sinh^3(a+bx)\,dx$

Optimal result 1098
Rubi [A] (verified) 1098
Mathematica [A] (verified) 1101
Maple [A] (verified) 1101
Fricas [B] (verification not implemented) 1101
Sympy [F] 1102
Maxima [A] (verification not implemented) 1102
Giac [A] (verification not implemented) 1103
Mupad [F(-1)] 1103

Optimal result

Integrand size = 11, antiderivative size = 89

$$\int \log(x)\sinh^3(a+bx)\,dx = \frac{3\cosh(a)\mathrm{Chi}(bx)}{4b} - \frac{\cosh(3a)\mathrm{Chi}(3bx)}{12b} - \frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} + \frac{3\sinh(a)\mathrm{Shi}(bx)}{4b} - \frac{\sinh(3a)\mathrm{Shi}(3bx)}{12b}$$

```
[Out] 3/4*Chi(b*x)*cosh(a)/b-1/12*Chi(3*b*x)*cosh(3*a)/b-cosh(b*x+a)*ln(x)/b+1/3*
cosh(b*x+a)^3*ln(x)/b+3/4*Shi(b*x)*sinh(a)/b-1/12*Shi(3*b*x)*sinh(3*a)/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2713, 2634, 12, 6874, 3384, 3379, 3382, 3393}

$$\int \log(x)\sinh^3(a+bx)\,dx = \frac{3\cosh(a)\mathrm{Chi}(bx)}{4b} - \frac{\cosh(3a)\mathrm{Chi}(3bx)}{12b} + \frac{3\sinh(a)\mathrm{Shi}(bx)}{4b} - \frac{\sinh(3a)\mathrm{Shi}(3bx)}{12b} + \frac{\log(x)\cosh^3(a+bx)}{3b} - \frac{\log(x)\cosh(a+bx)}{b}$$

```
[In] Int[Log[x]*Sinh[a + b*x]^3,x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x])/(4*b) - (Cosh[3*a]*CoshIntegral[3*b*x])/(12*b
) - (Cosh[a + b*x]*Log[x])/b + (Cosh[a + b*x]^3*Log[x])/(3*b) + (3*Sinh[a]*
SinhIntegral[b*x])/(4*b) - (Sinh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} \\
&\quad - \int \frac{\cosh(a+bx)\left(-3+\cosh^2(a+bx)\right)}{3bx}\,dx \\
&= -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} - \frac{\int \frac{\cosh(a+bx)\left(-3+\cosh^2(a+bx)\right)}{x}\,dx}{3b} \\
&= -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} - \frac{\int \left(-\frac{3\cosh(a+bx)}{x} + \frac{\cosh^3(a+bx)}{x}\right)dx}{3b} \\
&= -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} - \frac{\int \frac{\cosh^3(a+bx)}{x}\,dx}{3b} + \frac{\int \frac{\cosh(a+bx)}{x}\,dx}{b} \\
&= -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} - \frac{\int \left(\frac{3\cosh(a+bx)}{4x} + \frac{\cosh(3a+3bx)}{4x}\right)dx}{3b} \\
&\quad + \frac{\cosh(a)\int \frac{\cosh(bx)}{x}\,dx}{b} + \frac{\sinh(a)\int \frac{\sinh(bx)}{x}\,dx}{b} \\
&= \frac{\cosh(a)\mathrm{Chi}(bx)}{b} - \frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} \\
&\quad + \frac{\sinh(a)\mathrm{Shi}(bx)}{b} - \frac{\int \frac{\cosh(3a+3bx)}{x}\,dx}{12b} - \frac{\int \frac{\cosh(a+bx)}{x}\,dx}{4b} \\
&= \frac{\cosh(a)\mathrm{Chi}(bx)}{b} - \frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} \\
&\quad + \frac{\sinh(a)\mathrm{Shi}(bx)}{b} - \frac{\cosh(a)\int \frac{\cosh(bx)}{x}\,dx}{4b} - \frac{\cosh(3a)\int \frac{\cosh(3bx)}{x}\,dx}{12b} \\
&\quad - \frac{\sinh(a)\int \frac{\sinh(bx)}{x}\,dx}{4b} - \frac{\sinh(3a)\int \frac{\sinh(3bx)}{x}\,dx}{12b} \\
&= \frac{3\cosh(a)\mathrm{Chi}(bx)}{4b} - \frac{\cosh(3a)\mathrm{Chi}(3bx)}{12b} - \frac{\cosh(a+bx)\log(x)}{b} \\
&\quad + \frac{\cosh^3(a+bx)\log(x)}{3b} + \frac{3\sinh(a)\mathrm{Shi}(bx)}{4b} - \frac{\sinh(3a)\mathrm{Shi}(3bx)}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

$$\int \log(x)\sinh^3(a+bx)\,dx = \frac{9\cosh(a)\text{Chi}(bx)-\cosh(3a)\text{Chi}(3bx)-9\cosh(a+bx)\log(x)+\cosh(3(a+bx))\log(x)+9\sinh(a)\text{Shi}(bx}{12b}$$

```
[In] Integrate[Log[x]*Sinh[a + b*x]^3,x]
```

```
[Out] (9*Cosh[a]*CoshIntegral[b*x] - Cosh[3*a]*CoshIntegral[3*b*x] - 9*Cosh[a + b
*x]*Log[x] + Cosh[3*(a + b*x)]*Log[x] + 9*Sinh[a]*SinhIntegral[b*x] - Sinh[
3*a]*SinhIntegral[3*b*x])/(12*b)
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.35

method	result
risch	$\frac{e^{-3a}\,\text{Ei}_1(3bx)}{24b}+\frac{e^{3a}\,\text{Ei}_1(-3bx)}{24b}-\frac{3\,e^{-a}\,\text{Ei}_1(bx)}{8b}-\frac{3\,e^{a}\,\text{Ei}_1(-bx)}{8b}-\frac{3\,e^{bx+a}\ln(x)}{8b}+\frac{\ln(x)e^{3bx+3a}}{24b}-\frac{3\ln(x)e^{-bx-a}}{8b}+\ln($

```
[In] int(ln(x)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/b*exp(-3*a)*Ei(1,3*b*x)+1/24/b*exp(3*a)*Ei(1,-3*b*x)-3/8/b*exp(-a)*Ei(
1,b*x)-3/8/b*exp(a)*Ei(1,-b*x)-3/8*exp(b*x+a)*ln(x)/b+1/24/b*ln(x)*exp(3*b*
x+3*a)-3/8/b*ln(x)*exp(-b*x-a)+1/24/b*ln(x)*exp(-3*b*x-3*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. 2(79) = 158.

Time = 0.35 (sec) , antiderivative size = 587, normalized size of antiderivative = 6.60

$$\int \log(x)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 - 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) - Ei(-3*b*x))*co
sh(b*x + a)^3*sinh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^3*sinh(a) -
((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*cosh(
b*x + a)^3 - ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*c
osh(a) - 4*(5*cosh(b*x + a)^3 - 9*cosh(b*x + a))*log(x) + (Ei(3*b*x) - Ei(-
3*b*x))*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*sinh(a))*sinh(b*x + a)^3 - 3*((E
```

```
i(3*b*x) - Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*cos
h(b*x + a)*sinh(a) + ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(
-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 - 18*cosh(b*x + a)^2 - 3
)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 - 9*cosh(b*x + a)^4 - 9*cosh(b
*x + a)^2 + 1)*log(x) - 3*((Ei(3*b*x) - Ei(-3*b*x))*cosh(b*x + a)^2*sinh(3*
a) - 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b*x) + Ei(-3*b
*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*cosh(b*x + a)^2 - 2*(cosh(
b*x + a)^5 - 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*sinh(b*x + a))/(b
*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*si
nh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]

$$\int \log(x)\sinh^3(a+bx)\,dx = \int \log{(x)}\sinh^3{(a+bx)}\,dx$$

```
[In] integrate(ln(x)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(log(x)*sinh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \log(x)\sinh^3(a+bx)\,dx &= \frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log{(x)}\\&\quad-\frac{\mathrm{Ei}(3\,bx)\,e^{(3\,a)}}{24\,b}+\frac{3\,\mathrm{Ei}(-bx)\,e^{(-a)}}{8\,b}\\&\quad-\frac{\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}}{24\,b}+\frac{3\,\mathrm{Ei}(bx)\,e^{a}}{8\,b}\end{aligned}$$

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x -
3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b - 1/24*Ei
(-3*b*x)*e^(-3*a)/b + 3/8*Ei(b*x)*e^a/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

$$\int \log(x)\sinh^3(a+bx)\,dx = \frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x) - \frac{\mathrm{Ei}(3\,bx)\,e^{(3\,a)}-9\,\mathrm{Ei}(-bx)\,e^{(-a)}+\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}-9\,\mathrm{Ei}(bx)\,e^{a}}{24\,b}$$

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="giac")
[Out] 1/24*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x -
3*a)/b)*log(x) - 1/24*(Ei(3*b*x)*e^(3*a) - 9*Ei(-b*x)*e^(-a) + Ei(-3*b*x)*e
^(-3*a) - 9*Ei(b*x)*e^a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \log(x)\sinh^3(a+bx)\,dx = \int \sinh(a+b\,x)^3\,\ln(x)\,dx$$

```
[In] int(sinh(a + b*x)^3*log(x),x)
[Out] int(sinh(a + b*x)^3*log(x), x)
```

3.198 $\int \cosh(a+bx)\log(x)\,dx$

Optimal result 1104
Rubi [A] (verified) 1104
Mathematica [A] (verified) 1106
Maple [A] (verified) 1106
Fricas [B] (verification not implemented) 1106
Sympy [F] 1107
Maxima [A] (verification not implemented) 1107
Giac [A] (verification not implemented) 1107
Mupad [F(-1)] 1107

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \cosh(a+bx)\log(x)\,dx=-\frac{\mathrm{Chi}(bx)\sinh(a)}{b}+\frac{\log(x)\sinh(a+bx)}{b}-\frac{\cosh(a)\mathrm{Shi}(bx)}{b}$$

```
[Out] -cosh(a)*Shi(b*x)/b-Chi(b*x)*sinh(a)/b+ln(x)*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2717, 2634, 12, 3384, 3379, 3382}

$$\int \cosh(a+bx)\log(x)\,dx=-\frac{\sinh(a)\mathrm{Chi}(bx)}{b}-\frac{\cosh(a)\mathrm{Shi}(bx)}{b}+\frac{\log(x)\sinh(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]*Log[x],x]
```

```
[Out] -((CoshIntegral[b*x]*Sinh[a])/b) + (Log[x]*Sinh[a + b*x])/b - (Cosh[a]*Sinh
Integral[b*x])/b
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 2634</u>

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
```

```
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log(x)\sinh(a+bx)}{b} - \int \frac{\sinh(a+bx)}{bx}\,dx \\
&= \frac{\log(x)\sinh(a+bx)}{b} - \frac{\int \frac{\sinh(a+bx)}{x}\,dx}{b} \\
&= \frac{\log(x)\sinh(a+bx)}{b} - \frac{\cosh(a)\int \frac{\sinh(bx)}{x}\,dx}{b} - \frac{\sinh(a)\int \frac{\cosh(bx)}{x}\,dx}{b} \\
&= -\frac{\text{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} - \frac{\cosh(a)\text{Shi}(bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \cosh(a+bx)\log(x)\,dx = -\frac{\text{Chi}(bx)\sinh(a)-\log(x)\sinh(a+bx)+\cosh(a)\text{Shi}(bx)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Log[x],x]
```

```
[Out] -((CoshIntegral[b*x]*Sinh[a] - Log[x]*Sinh[a + b*x] + Cosh[a]*SinhIntegral[
b*x])/b)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

method	result
risch	$\frac{e^{bx+a}\ln(x)}{2b}-\frac{\ln(x)e^{-bx-a}}{2b}+\frac{e^{a}\,\text{Ei}_1(-bx)}{2b}-\frac{e^{-a}\,\text{Ei}_1(bx)}{2b}$
meijerg	$-\frac{\cosh(a)\sinh(bx)}{b}+\frac{\cosh(a)\ln(x)\sinh(bx)}{b}+\frac{\cosh(a)b^2\left(\frac{9\sinh(bx)}{b^3}-\frac{9\ \text{Shi}(bx)}{b^3}\right)}{9}-\frac{\sinh(a)b\left(-\frac{2}{b^2}+\frac{2\cosh(bx)}{b^2}\right)}{4}+\frac{\sinh(a)b\ln(x)(}{}$

```
[In] int(cosh(b*x+a)*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)*ln(x)/b-1/2/b*ln(x)*exp(-b*x-a)+1/2/b*exp(a)*Ei(1,-b*x)-1/2/
b*exp(-a)*Ei(1,b*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(35) = 70.

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.83

$$\int \cosh(a+bx)\log(x)\,dx = -\frac{(\text{Ei}(bx)-\text{Ei}(-bx))\cosh(bx+a)\cosh(a)-\log(x)\sinh(bx+a)^2+(\text{Ei}(bx)+\text{Ei}(-bx))\cosh(bx+a)\sin}{2}$$

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] -1/2*((Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2 +
 (Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 - 1)*log(x)
+ ((Ei(b*x) - Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) + Ei(-b
*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\log(x)\,dx = \int \log\left(x\right)\cosh\left(a+bx\right)\,dx$$

```
[In] integrate(cosh(b*x+a)*ln(x),x)
[Out] Integral(log(x)*cosh(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \cosh(a+bx)\log(x)\,dx = \frac{\log\left(x\right)\sinh\left(bx+a\right)}{b} + \frac{\mathrm{Ei}(-bx)\,e^{(-a)} - \mathrm{Ei}(bx)\,e^{a}}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="maxima")
[Out] log(x)*sinh(b*x + a)/b + 1/2*(Ei(-b*x)*e^(-a) - Ei(b*x)*e^a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \cosh(a+bx)\log(x)\,dx = \frac{1}{2}\left(\frac{e^{(bx+a)}}{b} - \frac{e^{(-bx-a)}}{b}\right)\log\left(x\right) + \frac{\mathrm{Ei}(-bx)\,e^{(-a)} - \mathrm{Ei}(bx)\,e^{a}}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="giac")
[Out] 1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(x) + 1/2*(Ei(-b*x)*e^(-a) - Ei(b*x
)*e^a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\log(x)\,dx = \int \cosh\left(a+b\,x\right)\,\ln\left(x\right)\,dx$$

```
[In] int(cosh(a + b*x)*log(x),x)
[Out] int(cosh(a + b*x)*log(x), x)
```

3.199 $\int \cosh^2(a+bx)\log(x)\,dx$

Optimal result 1108
Rubi [A] (verified) 1108
Mathematica [A] (verified) 1110
Maple [A] (verified) 1110
Fricas [B] (verification not implemented) 1111
Sympy [F] 1111
Maxima [A] (verification not implemented) 1111
Giac [A] (verification not implemented) 1112
Mupad [F(-1)] 1112

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \cosh^2(a+bx)\log(x)\,dx = -\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\mathrm{Chi}(2bx)\sinh(2a)}{4b} + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\mathrm{Shi}(2bx)}{4b}$$

```
[Out] -1/2*x+1/2*x*ln(x)-1/4*cosh(2*a)*Shi(2*b*x)/b-1/4*Chi(2*b*x)*sinh(2*a)/b+1/
2*cosh(b*x+a)*ln(x)*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2715, 8, 2634, 12, 5382, 3384, 3379, 3382}

$$\int \cosh^2(a+bx)\log(x)\,dx = -\frac{\sinh(2a)\mathrm{Chi}(2bx)}{4b}-\frac{\cosh(2a)\mathrm{Shi}(2bx)}{4b} + \frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

```
[In] Int[Cosh[a + b*x]^2*Log[x],x]
```

```
[Out] -1/2*x + (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b
*x]*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5382

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[ExpandToSum
[u, x]^m*(a + b*Sinh[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] &&
 LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \int \frac{1}{4}\left(2 + \frac{\sinh(2(a+bx))}{bx}\right)\,dx$$

$$= \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{1}{4}\int \left(2 + \frac{\sinh(2(a+bx))}{bx}\right)dx$$

$$= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\int \frac{\sinh(2(a+bx))}{x}dx}{4b}$$

$$= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\int \frac{\sinh(2a+2bx)}{x}dx}{4b}$$

$$= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\cosh(2a)\int \frac{\sinh(2bx)}{x}dx}{4b} - \frac{\sinh(2a)\int \frac{\cosh(2bx)}{x}dx}{4b}$$

$$= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\mathrm{Chi}(2bx)\sinh(2a)}{4b} + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\cosh(2a)\mathrm{Shi}(2bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \cosh^2(a+bx)\log(x)\,dx = -\frac{2bx - 2bx\log(x) + \mathrm{Chi}(2bx)\sinh(2a) - \log(x)\sinh(2(a+bx)) + \cosh(2a)\mathrm{Shi}(2bx)}{4b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Log[x],x]
```

```
[Out] -1/4*(2*b*x - 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[2*
(a + b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/b
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

method	result	size
risch	$\frac{\ln(x)x}{2} + \frac{e^{2bx+2a}\ln(x)}{8b} - \frac{e^{-2bx-2a}\ln(x)}{8b} + \frac{e^{2a}\,\mathrm{Ei}_1(-2bx)}{8b} + \frac{a\ln(bx)}{2b} - \frac{a\ln(-bx)}{2b} - \frac{e^{-2a}\,\mathrm{Ei}_1(2bx)}{8b} - \frac{x}{2} - \frac{a}{2b}$	99

```
[In] int(cosh(b*x+a)^2*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)*x+1/8/b*exp(2*b*x+2*a)*ln(x)-1/8/b*exp(-2*b*x-2*a)*ln(x)+1/8/b*ex
p(2*a)*Ei(1,-2*b*x)+1/2/b*a*ln(b*x)-1/2/b*a*ln(-b*x)-1/8/b*exp(-2*a)*Ei(1,2
*b*x)-1/2*x-1/2*a/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(56) = 112.

Time = 0.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.62

$$\int \cosh^2(a+bx)\log(x)\,dx$$
$$= \frac{4\cosh(bx+a)\log(x)\sinh(bx+a)^3 + \log(x)\sinh(bx+a)^4 - (\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\cosh(bx+a)^2\sinh}{}$$

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei(
2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) - (4*b*x + (Ei(2*b*x) - Ei(-
2*b*x))*cosh(2*a))*cosh(b*x + a)^2 - (4*b*x + (Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) - 2*(2*b*x + 3*cosh(b*x + a)^2)*log(x) + (Ei(2*b*x) + Ei(-2*b*x))*sin
h(2*a))*sinh(b*x + a)^2 + (4*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 - 1)*log
(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) + (4*b*x + (Ei(2*
b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) - 2*(2*b*x*cosh(b*x + a) + cosh
(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \cosh^2(a+bx)\log(x)\,dx = \int \log(x)\cosh^2(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)**2*ln(x),x)
```

```
[Out] Integral(log(x)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \cosh^2(a+bx)\log(x)\,dx = \frac{1}{8}\left(4\,x + \frac{e^{(2\,bx+2\,a)}}{b} - \frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x) - \frac{1}{2}\,x - \frac{\mathrm{Ei}(2\,bx)\,e^{(2\,a)}}{8\,b} + \frac{\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}$$

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/8*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)*log(x) - 1/2*x - 1/8*Ei(
2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \cosh^2(a+bx)\log(x)\,dx = \frac{1}{8}\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log\left(x\right) - \frac{4\,bx+\mathrm{Ei}(2\,bx)\,e^{(2\,a)}-\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}$$

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="giac")
[Out] 1/8*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)*log(x) - 1/8*(4*b*x + Ei
(2*b*x)*e^(2*a) - Ei(-2*b*x)*e^(-2*a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^2(a+bx)\log(x)\,dx = \int \cosh(a+b\,x)^2\,\ln\left(x\right)\,dx$$

```
[In] int(cosh(a + b*x)^2*log(x),x)
[Out] int(cosh(a + b*x)^2*log(x), x)
```

3.200 $\int \cosh^3(a+bx)\log(x)\,dx$

Optimal result 1113
Rubi [A] (verified) 1113
Mathematica [A] (verified) 1115
Maple [A] (verified) 1116
Fricas [B] (verification not implemented) 1116
Sympy [F] 1117
Maxima [A] (verification not implemented) 1117
Giac [A] (verification not implemented) 1117
Mupad [F(-1)] 1118

Optimal result

Integrand size = 11, antiderivative size = 88

$$\int \cosh^3(a+bx)\log(x)\,dx = -\frac{3\text{Chi}(bx)\sinh(a)}{4b} - \frac{\text{Chi}(3bx)\sinh(3a)}{12b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{3\cosh(a)\text{Shi}(bx)}{4b} - \frac{\cosh(3a)\text{Shi}(3bx)}{12b}$$

```
[Out] -3/4*cosh(a)*Shi(b*x)/b-1/12*cosh(3*a)*Shi(3*b*x)/b-3/4*Chi(b*x)*sinh(a)/b-
1/12*Chi(3*b*x)*sinh(3*a)/b+ln(x)*sinh(b*x+a)/b+1/3*ln(x)*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2713, 2634, 12, 6874, 3384, 3379, 3382, 3393}

$$\int \cosh^3(a+bx)\log(x)\,dx = -\frac{3\sinh(a)\text{Chi}(bx)}{4b} - \frac{\sinh(3a)\text{Chi}(3bx)}{12b} - \frac{3\cosh(a)\text{Shi}(bx)}{4b} - \frac{\cosh(3a)\text{Shi}(3bx)}{12b} + \frac{\log(x)\sinh^3(a+bx)}{3b} + \frac{\log(x)\sinh(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]^3*Log[x],x]
```

```
[Out] (-3*CoshIntegral[b*x]*Sinh[a])/(4*b) - (CoshIntegral[3*b*x]*Sinh[3*a])/(12*
b) + (Log[x]*Sinh[a + b*x])/b + (Log[x]*Sinh[a + b*x]^3)/(3*b) - (3*Cosh[a]
*SinhIntegral[b*x])/(4*b) - (Cosh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

integral

$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \int \frac{\sinh(a+bx)\left(3+\sinh^2(a+bx)\right)}{3bx}\,dx$$

$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\int \frac{\sinh(a+bx)(3+\sinh^2(a+bx))}{x}\,dx}{3b}$$

$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\int \left(\frac{3\sinh(a+bx)}{x} + \frac{\sinh^3(a+bx)}{x}\right)dx}{3b}$$

$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\int \frac{\sinh^3(a+bx)}{x}\,dx}{3b} - \frac{\int \frac{\sinh(a+bx)}{x}\,dx}{b}$$

$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{i\int \left(\frac{3i\sinh(a+bx)}{4x} - \frac{i\sinh(3a+3bx)}{4x}\right)dx}{3b} - \frac{\cosh(a)\int \frac{\sinh(bx)}{x}\,dx}{b} - \frac{\sinh(a)\int \frac{\cosh(bx)}{x}\,dx}{b}$$

$$= -\frac{\text{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\cosh(a)\text{Shi}(bx)}{b} - \frac{\int \frac{\sinh(3a+3bx)}{x}\,dx}{12b} + \frac{\int \frac{\sinh(a+bx)}{x}\,dx}{4b}$$

$$= -\frac{\text{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\cosh(a)\text{Shi}(bx)}{b} + \frac{\cosh(a)\int \frac{\sinh(bx)}{x}\,dx}{4b} - \frac{\cosh(3a)\int \frac{\sinh(3bx)}{x}\,dx}{12b} + \frac{\sinh(a)\int \frac{\cosh(bx)}{x}\,dx}{4b} - \frac{\sinh(3a)\int \frac{\cosh(3bx)}{x}\,dx}{12b}$$

$$= -\frac{3\text{Chi}(bx)\sinh(a)}{4b} - \frac{\text{Chi}(3bx)\sinh(3a)}{12b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{3\cosh(a)\text{Shi}(bx)}{4b} - \frac{\cosh(3a)\text{Shi}(3bx)}{12b}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \cosh^3(a+bx)\log(x)\,dx =$$

$$-\frac{9\text{Chi}(bx)\sinh(a) + \text{Chi}(3bx)\sinh(3a) - 9\log(x)\sinh(a+bx) - \log(x)\sinh(3(a+bx)) + 9\cosh(a)\text{Shi}(}{12b}$$

```
[In] Integrate[Cosh[a + b*x]^3*Log[x],x]
```

```
[Out] -1/12*(9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] - 9*Log[
x]*Sinh[a + b*x] - Log[x]*Sinh[3*(a + b*x)] + 9*Cosh[a]*SinhIntegral[b*x] +
 Cosh[3*a]*SinhIntegral[3*b*x])/b
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

method	result
risch	$\frac{\ln(x)\mathrm{e}^{3bx+3a}}{24b}-\frac{3\ln(x)\mathrm{e}^{-bx-a}}{8b}-\frac{\ln(x)\mathrm{e}^{-3bx-3a}}{24b}+\frac{\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)}{24b}-\frac{\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)}{24b}-\frac{3\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)}{8b}+\frac{3\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)}{8b}+\frac{3\,\mathrm{e}^{b}}{}$

```
[In] int(cosh(b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/b*ln(x)*exp(3*b*x+3*a)-3/8/b*ln(x)*exp(-b*x-a)-1/24/b*ln(x)*exp(-3*b*x
-3*a)+1/24/b*exp(3*a)*Ei(1,-3*b*x)-1/24/b*exp(-3*a)*Ei(1,3*b*x)-3/8/b*exp(-
a)*Ei(1,b*x)+3/8/b*exp(a)*Ei(1,-b*x)+3/8*exp(b*x+a)*ln(x)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(78) = 156$.

Time = 0.34 (sec) , antiderivative size = 587, normalized size of antiderivative = 6.67

$$\int \cosh^3(a+bx)\log(x)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="fricas")
```

```
[Out] 1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 + 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) + Ei(-3*b*x))*co
sh(b*x + a)^3*sinh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^3*sinh(a) -
((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*cosh(
b*x + a)^3 - ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*c
osh(a) - 4*(5*cosh(b*x + a)^3 + 9*cosh(b*x + a))*log(x) + (Ei(3*b*x) + Ei(-
3*b*x))*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*sinh(a))*sinh(b*x + a)^3 - 3*((E
i(3*b*x) + Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*cos
h(b*x + a)*sinh(a) + ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(
-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 - 3
)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 + 9*cosh(b*x + a)^4 - 9*cosh(b
*x + a)^2 - 1)*log(x) - 3*((Ei(3*b*x) + Ei(-3*b*x))*cosh(b*x + a)^2*sinh(3*
a) + 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b*x) - Ei(-3*b
*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*cosh(b*x + a)^2 - 2*(cosh(
b*x + a)^5 + 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*sinh(b*x + a))/(b
*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*si
nh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]

$$\int \cosh^3(a+bx)\log(x)\,dx = \int \log(x)\cosh^3(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)**3*ln(x),x)
[Out] Integral(log(x)*cosh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.26

$$\int \cosh^3(a+bx)\log(x)\,dx = \frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b} + \frac{9\,e^{(bx+a)}}{b} - \frac{9\,e^{(-bx-a)}}{b} - \frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x) - \frac{\operatorname{Ei}(3\,bx)\,e^{(3\,a)}}{24\,b} + \frac{3\,\operatorname{Ei}(-bx)\,e^{(-a)}}{8\,b} + \frac{\operatorname{Ei}(-3\,bx)\,e^{(-3\,a)}}{24\,b} - \frac{3\,\operatorname{Ei}(bx)\,e^{a}}{8\,b}$$

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="maxima")
[Out] 1/24*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x -
3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b + 1/24*Ei
(-3*b*x)*e^(-3*a)/b - 3/8*Ei(b*x)*e^a/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \cosh^3(a+bx)\log(x)\,dx = \frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b} + \frac{9\,e^{(bx+a)}}{b} - \frac{9\,e^{(-bx-a)}}{b} - \frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x) - \frac{\operatorname{Ei}(3\,bx)\,e^{(3\,a)} - 9\,\operatorname{Ei}(-bx)\,e^{(-a)} - \operatorname{Ei}(-3\,bx)\,e^{(-3\,a)} + 9\,\operatorname{Ei}(bx)\,e^{a}}{24\,b}$$

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="giac")
[Out] 1/24*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x -
3*a)/b)*log(x) - 1/24*(Ei(3*b*x)*e^(3*a) - 9*Ei(-b*x)*e^(-a) - Ei(-3*b*x)*e
^(-3*a) + 9*Ei(b*x)*e^a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^3(a+bx)\log(x)\,dx = \int \cosh(a+b\,x)^3\,\ln(x)\,dx$$

```
[In] int(cosh(a + b*x)^3*log(x),x)
[Out] int(cosh(a + b*x)^3*log(x), x)
```

3.201 $\int \log(a\sinh(x))\,dx$

Optimal result 1119
Rubi [A] (verified) 1119
Mathematica [A] (verified) 1120
Maple [C] (warning: unable to verify) 1121
Fricas [A] (verification not implemented) 1121
Sympy [F] 1122
Maxima [A] (verification not implemented) 1122
Giac [F] 1122
Mupad [F(-1)] 1122

Optimal result

Integrand size = 5, antiderivative size = 39

$$\int \log(a\sinh(x))\,dx = \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) - \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}$$

```
[Out] 1/2*x^2-x*ln(1-exp(2*x))+x*ln(a*sinh(x))-1/2*polylog(2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3797, 2221, 2317, 2438}

$$\int \log(a\sinh(x))\,dx = x\log(a\sinh(x)) - \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2} + \frac{x^2}{2} - x\log\left(1-e^{2x}\right)$$

```
[In] Int[Log[a*Sinh[x]],x]
[Out] x^2/2 - x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]] - PolyLog[2, E^(2*x)]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log(a\sinh(x)) - \int x\coth(x)\,dx \\
&= \frac{x^2}{2} + x\log(a\sinh(x)) + 2\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) + \int \log\left(1-e^{2x}\right)\,dx \\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) - \frac{\text{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \log(a\sinh(x))\,dx = -\frac{x^2}{2} - x\log\left(1-e^{-2x}\right) + x\log(a\sinh(x)) + \frac{1}{2}\,\text{PolyLog}\left(2, e^{-2x}\right)$$

```
[In] Integrate[Log[a*Sinh[x]],x]
[Out] -1/2*x^2 - x*Log[1 - E^(-2*x)] + x*Log[a*Sinh[x]] + PolyLog[2, E^(-2*x)]/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.06 (sec) , antiderivative size = 295, normalized size of antiderivative = 7.56

method	result
risch	$-x\ln\left(\mathrm{e}^{x}\right)+\frac{i\pi\ \mathrm{csgn}\left(i\,\mathrm{e}^{-x}\right)\mathrm{csgn}\left(i\,\mathrm{e}^{-x}\left(-1+\mathrm{e}^{2x}\right)\right)^{2}x}{2}-\frac{i\pi\ \mathrm{csgn}\left(i\,\mathrm{e}^{-x}\left(-1+\mathrm{e}^{2x}\right)\right)\mathrm{csgn}\left(ia\left(-1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}\right)\mathrm{csgn}\left(ia\right)x}{2}+\frac{i\pi\ \mathrm{csgn}}{}$

```
[In] int(ln(a*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -x*ln(exp(x))+1/2*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(-1+exp(2*x)))^2*x-1/
2*I*Pi*csgn(I*exp(-x)*(-1+exp(2*x)))*csgn(I*a*(-1+exp(2*x))*exp(-x))*csgn(I
*a)*x+1/2*I*Pi*csgn(I*(-1+exp(2*x)))*csgn(I*exp(-x)*(-1+exp(2*x)))^2*x-1/2*
I*Pi*csgn(I*exp(-x)*(-1+exp(2*x)))^3*x+1/2*I*Pi*csgn(I*exp(-x)*(-1+exp(2*x)
))*csgn(I*a*(-1+exp(2*x))*exp(-x))^2*x-1/2*I*Pi*csgn(I*a*(-1+exp(2*x))*exp(
-x))^3*x-x*ln(2)+ln(a)*x+1/2*x^2-1/2*I*Pi*csgn(I*(-1+exp(2*x)))*csgn(I*exp(
-x))*csgn(I*exp(-x)*(-1+exp(2*x)))*x+1/2*I*Pi*csgn(I*a*(-1+exp(2*x))*exp(-x
))^2*csgn(I*a)*x+ln(exp(x))*ln(-1+exp(2*x))-dilog(1+exp(x))-ln(exp(x))*ln(1
+exp(x))+dilog(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \log(a\sinh(x))\,dx = \frac{1}{2}x^2 + x\log\left(a\sinh\left(x\right)\right) - x\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - x\log\left(-\cosh\left(x\right) - \sinh\left(x\right) + 1\right) - \mathrm{Li}_2(\cosh\left(x\right) + \sinh\left(x\right)) - \mathrm{Li}_2(-\cosh\left(x\right) - \sinh\left(x\right))$$

```
[In] integrate(log(a*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x*log(a*sinh(x)) - x*log(cosh(x) + sinh(x) + 1) - x*log(-cosh(x)
- sinh(x) + 1) - dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(a\sinh(x))\,dx = \int \log\left(a\sinh\left(x\right)\right)dx$$

```
[In] integrate(ln(a*sinh(x)),x)
[Out] Integral(log(a*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \log(a\sinh(x))\,dx = \frac{1}{2}x^2 + x\log\left(a\sinh\left(x\right)\right) - x\log\left(e^x+1\right) - x\log\left(-e^x+1\right) - \mathrm{Li}_2(-e^x) - \mathrm{Li}_2(e^x)$$

```
[In] integrate(log(a*sinh(x)),x, algorithm="maxima")
[Out] 1/2*x^2 + x*log(a*sinh(x)) - x*log(e^x + 1) - x*log(-e^x + 1) - dilog(-e^x)
 - dilog(e^x)
```

Giac [F]

$$\int \log(a\sinh(x))\,dx = \int \log\left(a\sinh\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*sinh(x)),x, algorithm="giac")
[Out] integrate(log(a*sinh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\sinh(x))\,dx = \int \ln\left(a\sinh(x)\right)\,dx$$

```
[In] int(log(a*sinh(x)),x)
[Out] int(log(a*sinh(x)), x)
```

3.202 $\int \log\left(a\sinh^2(x)\right)\,dx$

Optimal result 1123
Rubi [A] (verified) 1123
Mathematica [A] (verified) 1125
Maple [C] (warning: unable to verify) 1125
Fricas [B] (verification not implemented) 1126
Sympy [F] 1126
Maxima [A] (verification not implemented) 1126
Giac [F] 1127
Mupad [F(-1)] 1127

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \log\left(a\sinh^2(x)\right)\,dx = x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) - \text{PolyLog}\left(2,e^{2x}\right)$$

```
[Out] x^2-2*x*ln(1-exp(2*x))+x*ln(a*sinh(x)^2)-polylog(2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3797, 2221, 2317, 2438}

$$\int \log\left(a\sinh^2(x)\right)\,dx = x\log\left(a\sinh^2(x)\right) - \text{PolyLog}\left(2,e^{2x}\right) + x^2 - 2x\log\left(1-e^{2x}\right)$$

```
[In] Int[Log[a*Sinh[x]^2],x]
```

```
[Out] x^2 - 2*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^2] - PolyLog[2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\sinh^2(x)\right) - \int 2x\coth(x)\,dx \\
&= x\log\left(a\sinh^2(x)\right) - 2\int x\coth(x)\,dx \\
&= x^2 + x\log\left(a\sinh^2(x)\right) + 4\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) + 2\int \log\left(1-e^{2x}\right)\,dx \\
&= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) + \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) - \text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(a \sinh^2(x)\right) \, dx = x\left(-x-2\log \left(1-e^{-2x}\right)+\log \left(a\sinh^2(x)\right)\right)+\text{PolyLog}\left(2,e^{-2x}\right)$$

```
[In] Integrate[Log[a*Sinh[x]^2],x]
[Out] x*(-x - 2*Log[1 - E^(-2*x)] + Log[a*Sinh[x]^2]) + PolyLog[2, E^(-2*x)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.76 (sec) , antiderivative size = 454, normalized size of antiderivative = 12.97

method	result
risch	$\ln \left(a \right) x -\frac{i \pi \,\text{csgn}\left(i a \left(-1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}\right)^{3} x}{2}-2 x \ln \left(2\right)+2 \,\text{dilog}\left({\mathrm e}^{x}\right)+x^{2}+\frac{i \pi \,\text{csgn}\left(i {\mathrm e}^{-2 x} \left(-1+{\mathrm e}^{2 x}\right)^{2}\right) \text{csgn}\left(i a \left(-1+\right.\right.}{2}$

```
[In] int(ln(a*sinh(x)^2),x,method=_RETURNVERBOSE)
[Out] ln(a)*x-1/2*I*Pi*csgn(I*a*(-1+exp(2*x))^2*exp(-2*x))^3*x-2*x*ln(2)+2*dilog(
exp(x))+x^2+1/2*I*Pi*csgn(I*exp(-2*x)*(-1+exp(2*x))^2)*csgn(I*a*(-1+exp(2*x
))^2*exp(-2*x))^2*x-I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2*x+1/2*I*Pi*csgn(
I*exp(x))^2*csgn(I*exp(2*x))*x+I*Pi*csgn(I*(-1+exp(2*x)))*csgn(I*(-1+exp(2*
x))^2)^2*x-1/2*I*Pi*csgn(I*exp(-2*x)*(-1+exp(2*x))^2)*csgn(I*a*(-1+exp(2*x)
)^2*exp(-2*x))*csgn(I*a)*x-1/2*I*Pi*csgn(I*(-1+exp(2*x)))^2*csgn(I*(-1+exp(
2*x))^2)*x-2*dilog(1+exp(x))-1/2*I*Pi*csgn(I*(-1+exp(2*x))^2)^3*x-1/2*I*Pi*
csgn(I*exp(-2*x)*(-1+exp(2*x))^2)^3*x+1/2*I*Pi*csgn(I*exp(-2*x))*csgn(I*exp
(-2*x)*(-1+exp(2*x))^2)^2*x-1/2*I*Pi*csgn(I*(-1+exp(2*x))^2)*csgn(I*exp(-2*
x))*csgn(I*exp(-2*x)*(-1+exp(2*x))^2)*x+1/2*I*Pi*csgn(I*exp(2*x))^3*x+1/2*I
*Pi*csgn(I*(-1+exp(2*x))^2)*csgn(I*exp(-2*x)*(-1+exp(2*x))^2)^2*x+1/2*I*Pi*
csgn(I*a*(-1+exp(2*x))^2*exp(-2*x))^2*csgn(I*a)*x-2*ln(exp(x))*ln(1+exp(x))
-2*x*ln(exp(x))+2*ln(exp(x))*ln(-1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(32) = 64$.

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \log \left(a \sinh^2(x)\right) \, dx = x^2 + x \log \left(\frac{1}{2} a \cosh (x)^2 + \frac{1}{2} a \sinh (x)^2 - \frac{1}{2} a\right) - 2 x \log (\cosh (x) + \sinh (x) + 1) - 2 x \log (-\cosh (x) - \sinh (x) + 1) - 2 \operatorname{Li}_2(\cosh (x) + \sinh (x)) - 2 \operatorname{Li}_2(-\cosh (x) - \sinh (x))$$

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="fricas")
[Out] x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 - 1/2*a) - 2*x*log(cosh(x) +
sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cosh(x) + sinh(x))
 - 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log \left(a \sinh^2(x)\right) \, dx = \int \log \left(a \sinh^2 (x)\right) dx$$

```
[In] integrate(ln(a*sinh(x)**2),x)
[Out] Integral(log(a*sinh(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \log \left(a \sinh^2(x)\right) \, dx = x^2 + x \log \left(a \sinh (x)^2\right) - 2 x \log (e^x + 1) - 2 x \log (-e^x + 1) - 2 \operatorname{Li}_2(-e^x) - 2 \operatorname{Li}_2(e^x)$$

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="maxima")
[Out] x^2 + x*log(a*sinh(x)^2) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-
e^x) - 2*dilog(e^x)
```

Giac [F]

$$\int \log \left(a \sinh^2(x)\right) \, dx = \int \log \left(a \sinh \left(x\right)^2\right) \, dx$$

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="giac")
[Out] integrate(log(a*sinh(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \sinh^2(x)\right) \, dx = \int \ln \left(a \sinh(x)^2\right) \, dx$$

```
[In] int(log(a*sinh(x)^2),x)
[Out] int(log(a*sinh(x)^2), x)
```

3.203 $\int \log(a\sinh^n(x))\,dx$

Optimal result 1128
Rubi [A] (verified) 1128
Mathematica [A] (verified) 1130
Maple [F] 1130
Fricas [A] (verification not implemented) 1130
Sympy [F] 1131
Maxima [A] (verification not implemented) 1131
Giac [F] 1131
Mupad [F(-1)] 1131

Optimal result

Integrand size = 7, antiderivative size = 44

$$\int \log(a\sinh^n(x))\,dx = \frac{nx^2}{2} - nx\log\left(1-e^{2x}\right) + x\log(a\sinh^n(x)) - \frac{1}{2}n\,\text{PolyLog}\left(2,e^{2x}\right)$$

```
[Out] 1/2*n*x^2-n*x*ln(1-exp(2*x))+x*ln(a*sinh(x)^n)-1/2*n*polylog(2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3797, 2221, 2317, 2438}

$$\int \log(a\sinh^n(x))\,dx = x\log(a\sinh^n(x)) - \frac{1}{2}n\,\text{PolyLog}\left(2,e^{2x}\right) + \frac{nx^2}{2} - nx\log\left(1-e^{2x}\right)$$

```
[In] Int[Log[a*Sinh[x]^n],x]
```

```
[Out] (n*x^2)/2 - n*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^n] - (n*PolyLog[2, E^(2*
x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\sinh^n(x)\right) - \int nx\coth(x)\,dx \\
&= x\log\left(a\sinh^n(x)\right) - n\int x\coth(x)\,dx \\
&= \frac{nx^2}{2} + x\log\left(a\sinh^n(x)\right) + (2n)\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= \frac{nx^2}{2} - nx\log\left(1-e^{2x}\right) + x\log\left(a\sinh^n(x)\right) + n\int \log\left(1-e^{2x}\right)\,dx \\
&= \frac{nx^2}{2} - nx\log\left(1-e^{2x}\right) + x\log\left(a\sinh^n(x)\right) + \frac{1}{2}n\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= \frac{nx^2}{2} - nx\log\left(1-e^{2x}\right) + x\log\left(a\sinh^n(x)\right) - \frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \log\left(a\sinh^n(x)\right)\,dx = \frac{1}{2}\left(-x\left(nx+2n\log\left(1-e^{-2x}\right)-2\log\left(a\sinh^n(x)\right)\right)+n\,\text{PolyLog}\left(2,e^{-2x}\right)\right)$$

```
[In] Integrate[Log[a*Sinh[x]^n],x]
[Out] (-(x*(n*x + 2*n*Log[1 - E^(-2*x)] - 2*Log[a*Sinh[x]^n])) + n*PolyLog[2, E^(
-2*x)])/2
```

Maple [F]

$$\int \ln\left(a\left(\sinh^n\left(x\right)\right)\right)dx$$

```
[In] int(ln(a*sinh(x)^n),x)
[Out] int(ln(a*sinh(x)^n),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int \log\left(a\sinh^n(x)\right)\,dx = \frac{1}{2}nx^2 - nx\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right) - nx\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right) + nx\log\left(\sinh\left(x\right)\right) - n\text{Li}_2(\cosh\left(x\right)+\sinh\left(x\right)) - n\text{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right)) + x\log\left(a\right)$$

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="fricas")
[Out] 1/2*n*x^2 - n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(-cosh(x) - sinh(x) + 1
) + n*x*log(sinh(x)) - n*dilog(cosh(x) + sinh(x)) - n*dilog(-cosh(x) - sinh
(x)) + x*log(a)
```

Sympy [F]

$$\int \log \left(a \sinh^n(x)\right)\, dx = \int \log \left(a \sinh^{n}\left(x\right)\right) dx$$

```
[In] integrate(ln(a*sinh(x)**n),x)
```

```
[Out] Integral(log(a*sinh(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \log \left(a \sinh^n(x)\right)\, dx = \frac{1}{2}\left(x^2 - 2\,x\log\left(e^x + 1\right) - 2\,x\log\left(-e^x + 1\right) - 2\,\mathrm{Li}_2(-e^x) - 2\,\mathrm{Li}_2(e^x)\right)n + x\log\left(a\sinh\left(x\right)^n\right)$$

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog(e
^x))*n + x*log(a*sinh(x)^n)
```

Giac [F]

$$\int \log \left(a \sinh^n(x)\right)\, dx = \int \log \left(a \sinh\left(x\right)^{n}\right)\, dx$$

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sinh(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \sinh^n(x)\right)\, dx = \int \ln\left(a\,{\mathrm{sinh}\left(x\right)}^n\right)\, dx$$

```
[In] int(log(a*sinh(x)^n),x)
```

```
[Out] int(log(a*sinh(x)^n), x)
```

3.204 $\int \log(a\cosh(x))\,dx$

Optimal result 1132
Rubi [A] (verified) 1132
Mathematica [A] (verified) 1133
Maple [C] (warning: unable to verify) 1134
Fricas [C] (verification not implemented) 1134
Sympy [F] 1135
Maxima [A] (verification not implemented) 1135
Giac [F] 1135
Mupad [F(-1)] 1135

Optimal result

Integrand size = 5, antiderivative size = 39

$$\int \log(a\cosh(x))\,dx = \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) - \frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)$$

```
[Out] 1/2*x^2-x*ln(1+exp(2*x))+x*ln(a*cosh(x))-1/2*polylog(2,-exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3799, 2221, 2317, 2438}

$$\int \log(a\cosh(x))\,dx = x\log(a\cosh(x)) - \frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right) + \frac{x^2}{2} - x\log\left(e^{2x}+1\right)$$

```
[In] Int[Log[a*Cosh[x]],x]
```

```
[Out] x^2/2 - x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]] - PolyLog[2, -E^(2*x)]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log(a\cosh(x)) - \int x\tanh(x)\,dx \\
&= \frac{x^2}{2} + x\log(a\cosh(x)) - 2\int \frac{e^{2x}x}{1+e^{2x}}\,dx \\
&= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) + \int \log\left(1+e^{2x}\right)\,dx \\
&= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \log(a\cosh(x))\,dx = x\log(a\cosh(x)) + \frac{1}{2}\left(-x\left(x+2\log\left(1+e^{-2x}\right)\right) + \text{PolyLog}\left(2,-e^{-2x}\right)\right)$$

```
[In] Integrate[Log[a*Cosh[x]],x]
[Out] x*Log[a*Cosh[x]] + (-(x*(x + 2*Log[1 + E^(-2*x)])) + PolyLog[2, -E^(-2*x)])
/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.76 (sec) , antiderivative size = 321, normalized size of antiderivative = 8.23

method	result
risch	$-x\ln\left(\mathrm{e}^{x}\right)+\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\right)\mathrm{csgn}\left(ia\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}\right)^{2}x}{2}+\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-x}\right)\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\right)^{2}x}{2}-\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-x}\right)\mathrm{csgn}\left(i(1\right.}{}$

```
[In] int(ln(a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -x*ln(exp(x))+1/2*I*Pi*csgn(I*exp(-x)*(1+exp(2*x)))*csgn(I*a*(1+exp(2*x))*e
xp(-x))^2*x+1/2*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(2*x)))^2*x-1/2*I
*Pi*csgn(I*exp(-x))*csgn(I*(1+exp(2*x)))*csgn(I*exp(-x)*(1+exp(2*x)))*x+1/2
*I*Pi*csgn(I*a*(1+exp(2*x))*exp(-x))^2*csgn(I*a)*x+1/2*I*Pi*csgn(I*(1+exp(2
*x)))*csgn(I*exp(-x)*(1+exp(2*x)))^2*x-1/2*I*Pi*csgn(I*a*(1+exp(2*x))*exp(-
x))^3*x-x*ln(2)+ln(a)*x+1/2*x^2-1/2*I*Pi*csgn(I*exp(-x)*(1+exp(2*x)))*csgn(
I*a*(1+exp(2*x))*exp(-x))*csgn(I*a)*x-1/2*I*Pi*csgn(I*exp(-x)*(1+exp(2*x)))
^3*x+ln(exp(x))*ln(1+exp(2*x))-ln(exp(x))*ln(1+I*exp(x))-ln(exp(x))*ln(1-I*
exp(x))-dilog(1+I*exp(x))-dilog(1-I*exp(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\begin{aligned}\int \log(a\cosh(x))\,dx = {} & \frac{1}{2}\,x^2 + x\log\left(a\cosh\left(x\right)\right) - x\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) \\ & - x\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) \\ & - \mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) - \mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right))\end{aligned}$$

```
[In] integrate(log(a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x*log(a*cosh(x)) - x*log(I*cosh(x) + I*sinh(x) + 1) - x*log(-I*co
sh(x) - I*sinh(x) + 1) - dilog(I*cosh(x) + I*sinh(x)) - dilog(-I*cosh(x) -
I*sinh(x))
```

Sympy [F]

$$\int \log(a\cosh(x))\,dx = \int \log\left(a\cosh\left(x\right)\right)dx$$

```
[In] integrate(ln(a*cosh(x)),x)
[Out] Integral(log(a*cosh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \log(a\cosh(x))\,dx = \frac{1}{2}x^2 + x\log\left(a\cosh\left(x\right)\right) - x\log\left(e^{(2\,x)} + 1\right) - \frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

```
[In] integrate(log(a*cosh(x)),x, algorithm="maxima")
[Out] 1/2*x^2 + x*log(a*cosh(x)) - x*log(e^(2*x) + 1) - 1/2*dilog(-e^(2*x))
```

Giac [F]

$$\int \log(a\cosh(x))\,dx = \int \log\left(a\cosh\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*cosh(x)),x, algorithm="giac")
[Out] integrate(log(a*cosh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\cosh(x))\,dx = \int \ln\left(a\cosh(x)\right)\,dx$$

```
[In] int(log(a*cosh(x)),x)
[Out] int(log(a*cosh(x)), x)
```

3.205 $\int \log\left(a\cosh^2(x)\right)\,dx$

Optimal result 1136
Rubi [A] (verified) 1136
Mathematica [A] (verified) 1138
Maple [C] (warning: unable to verify) 1138
Fricas [C] (verification not implemented) 1139
Sympy [F] 1139
Maxima [A] (verification not implemented) 1139
Giac [F] 1140
Mupad [F(-1)] 1140

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \log\left(a\cosh^2(x)\right)\,dx = x^2 - 2x\log\left(1+e^{2x}\right) + x\log\left(a\cosh^2(x)\right) - \text{PolyLog}\left(2,-e^{2x}\right)$$

```
[Out] x^2-2*x*ln(1+exp(2*x))+x*ln(a*cosh(x)^2)-polylog(2,-exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3799, 2221, 2317, 2438}

$$\int \log\left(a\cosh^2(x)\right)\,dx = x\log\left(a\cosh^2(x)\right) - \text{PolyLog}\left(2,-e^{2x}\right) + x^2 - 2x\log\left(e^{2x}+1\right)$$

```
[In] Int[Log[a*Cosh[x]^2],x]
```

```
[Out] x^2 - 2*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^2] - PolyLog[2, -E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\cosh^2(x)\right)-\int 2x\tanh(x)\,dx\\
&= x\log\left(a\cosh^2(x)\right)-2\int x\tanh(x)\,dx\\
&= x^2+x\log\left(a\cosh^2(x)\right)-4\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= x^2-2x\log\left(1+e^{2x}\right)+x\log\left(a\cosh^2(x)\right)+2\int \log\left(1+e^{2x}\right)\,dx\\
&= x^2-2x\log\left(1+e^{2x}\right)+x\log\left(a\cosh^2(x)\right)+\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= x^2-2x\log\left(1+e^{2x}\right)+x\log\left(a\cosh^2(x)\right)-\text{Li}_2\left(-e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(a\cosh^2(x)\right)\, dx = x\left(-x-2\log \left(1+e^{-2x}\right)+\log \left(a\cosh^2(x)\right)\right)+\text{PolyLog}\left(2,-e^{-2x}\right)$$

```
[In] Integrate[Log[a*Cosh[x]^2],x]
```

```
[Out] x*(-x - 2*Log[1 + E^(-2*x)] + Log[a*Cosh[x]^2]) + PolyLog[2, -E^(-2*x)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.52 (sec) , antiderivative size = 478, normalized size of antiderivative = 13.66

method	result
risch	$\ln(a)\,x+\frac{i\pi\,\mathrm{csgn}(ie^{x})^{2}\,\mathrm{csgn}(ie^{2x})\,x}{2}+i\pi\,\mathrm{csgn}\left(i(1+e^{2x})\right)\mathrm{csgn}\left(i(1+e^{2x})^{2}\right)^{2}x-\frac{i\pi\,\mathrm{csgn}\left(ie^{-2x}(1+e^{2x})^{2}\right)^{3}x}{2}$

```
[In] int(ln(a*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a)*x+1/2*I*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x))*x+I*Pi*csgn(I*(1+exp(2*x
)))*csgn(I*(1+exp(2*x))^2)^2*x-1/2*I*Pi*csgn(I*exp(-2*x)*(1+exp(2*x))^2)^3*
x-I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2*x+1/2*I*Pi*csgn(I*a*(1+exp(2*x))^2
*exp(-2*x))^2*csgn(I*a)*x-1/2*I*Pi*csgn(I*(1+exp(2*x))^2)^3*x+1/2*I*Pi*csgn
(I*(1+exp(2*x))^2)*csgn(I*exp(-2*x)*(1+exp(2*x))^2)^2*x+1/2*I*Pi*csgn(I*exp
(2*x))^3*x-1/2*I*Pi*csgn(I*exp(-2*x))*csgn(I*(1+exp(2*x))^2)*csgn(I*exp(-2*
x)*(1+exp(2*x))^2)*x-2*x*ln(2)-1/2*I*Pi*csgn(I*exp(-2*x)*(1+exp(2*x))^2)*cs
gn(I*a*(1+exp(2*x))^2*exp(-2*x))*csgn(I*a)*x+x^2+1/2*I*Pi*csgn(I*exp(-2*x))
*csgn(I*exp(-2*x)*(1+exp(2*x))^2)^2*x-1/2*I*Pi*csgn(I*a*(1+exp(2*x))^2*exp(
-2*x))^3*x-2*dilog(1+I*exp(x))-2*dilog(1-I*exp(x))-1/2*I*Pi*csgn(I*(1+exp(2
*x)))^2*csgn(I*(1+exp(2*x))^2)*x+1/2*I*Pi*csgn(I*exp(-2*x)*(1+exp(2*x))^2)*
csgn(I*a*(1+exp(2*x))^2*exp(-2*x))^2*x-2*x*ln(exp(x))+2*ln(exp(x))*ln(1+exp
(2*x))-2*ln(exp(x))*ln(1+I*exp(x))-2*ln(exp(x))*ln(1-I*exp(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.20

$$\int \log \left(a\cosh^2(x)\right) \,dx = x^2 + x\log\left(\frac{1}{2}\,a\cosh\left(x\right)^2 + \frac{1}{2}\,a\sinh\left(x\right)^2 + \frac{1}{2}\,a\right) - 2\,x\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) - 2\,x\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) - 2\,\mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) - 2\,\mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right))$$

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 + 1/2*a) - 2*x*log(I*cosh(x)
+ I*sinh(x) + 1) - 2*x*log(-I*cosh(x) - I*sinh(x) + 1) - 2*dilog(I*cosh(x)
+ I*sinh(x)) - 2*dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F]

$$\int \log \left(a\cosh^2(x)\right) \,dx = \int \log \left(a\cosh^{2}\left(x\right)\right)\, dx$$

```
[In] integrate(ln(a*cosh(x)**2),x)
```

```
[Out] Integral(log(a*cosh(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \log \left(a\cosh^2(x)\right) \,dx = x^2 + x\log\left(a\cosh\left(x\right)^2\right) - 2\,x\log\left(e^{(2\,x)} + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] x^2 + x*log(a*cosh(x)^2) - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x))
```

Giac [F]

$$\int \log \left(a \cosh^2(x)\right) \, dx = \int \log \left(a \cosh \left(x\right)^2\right) \, dx$$

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="giac")
[Out] integrate(log(a*cosh(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \cosh^2(x)\right) \, dx = \int \ln \left(a \cosh(x)^2\right) \, dx$$

```
[In] int(log(a*cosh(x)^2),x)
[Out] int(log(a*cosh(x)^2), x)
```

3.206 $\int \log(a\cosh^n(x))\,dx$

Optimal result 1141
Rubi [A] (verified) 1141
Mathematica [A] (verified) 1143
Maple [F] 1143
Fricas [C] (verification not implemented) 1143
Sympy [F] 1144
Maxima [A] (verification not implemented) 1144
Giac [F] 1144
Mupad [F(-1)] 1144

Optimal result

Integrand size = 7, antiderivative size = 44

$$\int \log(a\cosh^n(x))\,dx = \frac{nx^2}{2} - nx\log\left(1+e^{2x}\right) + x\log(a\cosh^n(x)) - \frac{1}{2}n\,\text{PolyLog}\left(2,-e^{2x}\right)$$

```
[Out] 1/2*n*x^2-n*x*ln(1+exp(2*x))+x*ln(a*cosh(x)^n)-1/2*n*polylog(2,-exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3799, 2221, 2317, 2438}

$$\int \log(a\cosh^n(x))\,dx = x\log(a\cosh^n(x)) - \frac{1}{2}n\,\text{PolyLog}\left(2,-e^{2x}\right) + \frac{nx^2}{2} - nx\log\left(e^{2x}+1\right)$$

```
[In] Int[Log[a*Cosh[x]^n],x]
```

```
[Out] (n*x^2)/2 - n*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^n] - (n*PolyLog[2, -E^(2
*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\cosh^n(x)\right)-\int nx\tanh(x)\,dx\\
&= x\log\left(a\cosh^n(x)\right)-n\int x\tanh(x)\,dx\\
&= \frac{nx^2}{2}+x\log\left(a\cosh^n(x)\right)-(2n)\int\frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= \frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+n\int\log\left(1+e^{2x}\right)\,dx\\
&= \frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= \frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \log\left(a\cosh^n(x)\right)\,dx = \frac{1}{2}\left(-x\left(nx+2n\log\left(1+e^{-2x}\right)-2\log\left(a\cosh^n(x)\right)\right)+n\,\text{PolyLog}\left(2,-e^{-2x}\right)\right)$$

```
[In] Integrate[Log[a*Cosh[x]^n],x]
[Out] (-(x*(n*x + 2*n*Log[1 + E^(-2*x)] - 2*Log[a*Cosh[x]^n])) + n*PolyLog[2, -E^
(-2*x)])/2
```

Maple [F]

$$\int \ln\left(a(\cosh^n(x))\right)dx$$

```
[In] int(ln(a*cosh(x)^n),x)
[Out] int(ln(a*cosh(x)^n),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \log\left(a\cosh^n(x)\right)\,dx = {} & \frac{1}{2}nx^2 - nx\log\left(i\cosh(x)+i\sinh(x)+1\right) \\ & - nx\log\left(-i\cosh(x)-i\sinh(x)+1\right) \\ & + nx\log\left(\cosh(x)\right) - n\text{Li}_2(i\cosh(x)+i\sinh(x)) \\ & - n\text{Li}_2(-i\cosh(x)-i\sinh(x)) + x\log(a)\end{aligned}$$

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="fricas")
[Out] 1/2*n*x^2 - n*x*log(I*cosh(x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*sin
h(x) + 1) + n*x*log(cosh(x)) - n*dilog(I*cosh(x) + I*sinh(x)) - n*dilog(-I*
cosh(x) - I*sinh(x)) + x*log(a)
```

Sympy [F]

$$\int \log\left(a\cosh^n(x)\right)\,dx = \int \log\left(a\cosh^n\left(x\right)\right)dx$$

```
[In] integrate(ln(a*cosh(x)**n),x)
[Out] Integral(log(a*cosh(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \log\left(a\cosh^n(x)\right)\,dx = \frac{1}{2}\left(x^2 - 2\,x\log\left(e^{(2\,x)} + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right)\right)n + x\log\left(a\cosh\left(x\right)^n\right)$$

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="maxima")
[Out] 1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*cosh(x)^n)
```

Giac [F]

$$\int \log\left(a\cosh^n(x)\right)\,dx = \int \log\left(a\cosh\left(x\right)^n\right)\,dx$$

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="giac")
[Out] integrate(log(a*cosh(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(a\cosh^n(x)\right)\,dx = \int \ln\left(a\cosh(x)^n\right)\,dx$$

```
[In] int(log(a*cosh(x)^n),x)
[Out] int(log(a*cosh(x)^n), x)
```

3.207 $\int \log(\tanh(x))\,dx$

Optimal result . . . 1145
Rubi [A] (verified) . . . 1145
Mathematica [A] (verified) . . . 1147
Maple [A] (verified) . . . 1147
Fricas [C] (verification not implemented) . . . 1147
Sympy [F] . . . 1148
Maxima [A] (verification not implemented) . . . 1148
Giac [F] . . . 1148
Mupad [B] (verification not implemented) . . . 1149

Optimal result

Integrand size = 3, antiderivative size = 39

$$\int \log(\tanh(x))\,dx = 2x\text{arctanh}\left(e^{2x}\right)+x\log(\tanh(x))+\frac{1}{2}\operatorname{PolyLog}\left(2,-e^{2x}\right)-\frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}$$

```
[Out] 2*x*arctanh(exp(2*x))+x*ln(tanh(x))+1/2*polylog(2,-exp(2*x))-1/2*polylog(2,
exp(2*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {2628, 5569, 4267, 2317, 2438}

$$\int \log(\tanh(x))\,dx = 2x\text{arctanh}\left(e^{2x}\right)+\frac{1}{2}\operatorname{PolyLog}\left(2,-e^{2x}\right)-\frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}+x\log(\tanh(x))$$

```
[In] Int[Log[Tanh[x]],x]
```

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[2,
 E^(2*x)]/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log(\tanh(x)) - \int x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log(\tanh(x)) - 2\int x\text{csch}(2x)\,dx \\
&= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \int \log\left(1-e^{2x}\right)\,dx - \int \log\left(1+e^{2x}\right)\,dx \\
&= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) - \frac{\text{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \log(\tanh(x))\,dx = \frac{1}{2}\log(\tanh(x))\log(1+\tanh(x)) + \frac{1}{2}\text{PolyLog}(2,1-\tanh(x)) + \frac{1}{2}\text{PolyLog}(2,-\tanh(x))$$

```
[In] Integrate[Log[Tanh[x]],x]
[Out] (Log[Tanh[x]]*Log[1 + Tanh[x]])/2 + PolyLog[2, 1 - Tanh[x]]/2 + PolyLog[2,
-Tanh[x]]/2
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{\text{dilog}(\tanh(x))}{2}+\frac{\text{dilog}(\tanh(x)+1)}{2}+\frac{\ln(\tanh(x))\ln(\tanh(x)+1)}{2}$
default	$\frac{\text{dilog}(\tanh(x))}{2}+\frac{\text{dilog}(\tanh(x)+1)}{2}+\frac{\ln(\tanh(x))\ln(\tanh(x)+1)}{2}$
risch	$x\ln\left(-1+e^{2x}\right)+\frac{i\pi\,\text{csgn}\left(i\left(-1+e^{2x}\right)\right)\text{csgn}\left(\frac{i\left(-1+e^{2x}\right)}{1+e^{2x}}\right)^{2}x}{2}+\frac{i\pi\,\text{csgn}\left(\frac{i}{1+e^{2x}}\right)\text{csgn}\left(\frac{i\left(-1+e^{2x}\right)}{1+e^{2x}}\right)^{2}x}{2}-\text{dilo}$

```
[In] int(ln(tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*dilog(tanh(x))+1/2*dilog(tanh(x)+1)+1/2*ln(tanh(x))*ln(tanh(x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.59

$$\begin{aligned}\int \log(\tanh(x))\,dx = {} & x\log\left(\frac{\sinh(x)}{\cosh(x)}\right) - x\log(\cosh(x)+\sinh(x)+1) \\ & + x\log(i\cosh(x)+i\sinh(x)+1) \\ & + x\log(-i\cosh(x)-i\sinh(x)+1) - x\log(-\cosh(x)-\sinh(x)+1) \\ & - \text{Li}_2(\cosh(x)+\sinh(x)) + \text{Li}_2(i\cosh(x)+i\sinh(x)) \\ & + \text{Li}_2(-i\cosh(x)-i\sinh(x)) - \text{Li}_2(-\cosh(x)-\sinh(x))\end{aligned}$$

```
[In] integrate(log(tanh(x)),x, algorithm="fricas")
[Out] x*log(sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x) + I
*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - sinh(x
```

```
) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dilog(-I
*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(\tanh(x))\,dx = \int \log\left(\tanh\left(x\right)\right)dx$$

```
[In] integrate(ln(tanh(x)),x)
[Out] Integral(log(tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \log(\tanh(x))\,dx = x\log\left(e^{(2\,x)}+1\right) - x\log\left(e^{x}+1\right) - x\log\left(-e^{x}+1\right) + x\log\left(\tanh\left(x\right)\right) + \frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2\,x)}\right) - \mathrm{Li}_2(-e^{x}) - \mathrm{Li}_2(e^{x})$$

```
[In] integrate(log(tanh(x)),x, algorithm="maxima")
[Out] x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) + x*log(tanh(x)) + 1/
2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

Giac [F]

$$\int \log(\tanh(x))\,dx = \int \log\left(\tanh\left(x\right)\right)\,dx$$

```
[In] integrate(log(tanh(x)),x, algorithm="giac")
[Out] integrate(log(tanh(x)), x)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \log(\tanh(x))\,dx = x\,\ln(\tanh(x)) - \frac{\mathrm{polylog}(2,\tanh(x))}{2} + \frac{\mathrm{polylog}(2,-\tanh(x))}{2}$$

```
[In] int(log(tanh(x)),x)
[Out] x*log(tanh(x)) - polylog(2, tanh(x))/2 + polylog(2, -tanh(x))/2
```

3.208 $\int \log(a\tanh(x))\,dx$

Optimal result 1150
Rubi [A] (verified) 1150
Mathematica [A] (verified) 1152
Maple [B] (verified) 1152
Fricas [C] (verification not implemented) 1152
Sympy [F] 1153
Maxima [A] (verification not implemented) 1153
Giac [F] 1153
Mupad [F(-1)] 1154

Optimal result

Integrand size = 5, antiderivative size = 41

$$\int \log(a\tanh(x))\,dx = 2x\text{arctanh}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right) - \frac{\text{PolyLog}\left(2,e^{2x}\right)}{2}$$

```
[Out] 2*x*arctanh(exp(2*x))+x*ln(a*tanh(x))+1/2*polylog(2,-exp(2*x))-1/2*polylog(
2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 5569, 4267, 2317, 2438}

$$\int \log(a\tanh(x))\,dx = x\log(a\tanh(x)) + 2x\text{arctanh}\left(e^{2x}\right) + \frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right) - \frac{\text{PolyLog}\left(2,e^{2x}\right)}{2}$$

```
[In] Int[Log[a*Tanh[x]],x]
```

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[
2, E^(2*x)]/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log(a\tanh(x)) - \int x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log(a\tanh(x)) - 2\int x\text{csch}(2x)\,dx \\
&= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \int \log\left(1-e^{2x}\right)\,dx - \int \log\left(1+e^{2x}\right)\,dx \\
&= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) - \frac{\text{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \log(a\tanh(x))\,dx = -\frac{1}{2}\log(1-\tanh(x))\log(a\tanh(x))+\frac{1}{2}\log(a\tanh(x))\log(1+\tanh(x)) + \frac{1}{2}\operatorname{PolyLog}(2,-\tanh(x)) - \frac{\operatorname{PolyLog}(2,\tanh(x))}{2}$$

```
[In] Integrate[Log[a*Tanh[x]],x]
```

```
[Out] -1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]]) + (Log[a*Tanh[x]]*Log[1 + Tanh[x]])/
2 + PolyLog[2, -Tanh[x]]/2 - PolyLog[2, Tanh[x]]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 1.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{\frac{a\left(\operatorname{dilog}\left(\frac{a\tanh(x)+a}{a}\right)+\ln(a\tanh(x))\ln\left(\frac{a\tanh(x)+a}{a}\right)\right)}{2}-\frac{a\left(\operatorname{dilog}\left(-\frac{a\tanh(x)-a}{a}\right)+\ln(a\tanh(x))\ln\left(-\frac{a\tanh(x)-a}{a}\right)\right)}{2}}{a}$
default	$\frac{\frac{a\left(\operatorname{dilog}\left(\frac{a\tanh(x)+a}{a}\right)+\ln(a\tanh(x))\ln\left(\frac{a\tanh(x)+a}{a}\right)\right)}{2}-\frac{a\left(\operatorname{dilog}\left(-\frac{a\tanh(x)-a}{a}\right)+\ln(a\tanh(x))\ln\left(-\frac{a\tanh(x)-a}{a}\right)\right)}{2}}{a}$
risch	$x\ln\left(-1+e^{2x}\right)-\frac{i\pi\operatorname{csgn}\left(i\left(-1+e^{2x}\right)\right)\operatorname{csgn}\left(\frac{i}{1+e^{2x}}\right)\operatorname{csgn}\left(\frac{i\left(-1+e^{2x}\right)}{1+e^{2x}}\right)x}{2}+\frac{i\pi\operatorname{csgn}\left(\frac{i\left(-1+e^{2x}\right)}{1+e^{2x}}\right)\operatorname{csgn}\left(\frac{ia(-1+\cdots}{1+e\cdots}\right.\cdots}{2}$

```
[In] int(ln(a*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*a*(dilog((a*tanh(x)+a)/a)+ln(a*tanh(x))*ln((a*tanh(x)+a)/a))-1/2*a
*(dilog(-(a*tanh(x)-a)/a)+ln(a*tanh(x))*ln(-(a*tanh(x)-a)/a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\int \log(a\tanh(x))\,dx = x\log\left(\frac{a\sinh(x)}{\cosh(x)}\right) - x\log(\cosh(x)+\sinh(x)+1) + x\log(i\cosh(x)+i\sinh(x)+1) + x\log(-i\cosh(x)-i\sinh(x)+1) - x\log(-\cosh(x)-\sinh(x)+1) - \operatorname{Li}_2(\cosh(x)+\sinh(x)) + \operatorname{Li}_2(i\cosh(x)+i\sinh(x)) + \operatorname{Li}_2(-i\cosh(x)-i\sinh(x)) - \operatorname{Li}_2(-\cosh(x)-\sinh(x))$$

```
[In] integrate(log(a*tanh(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x) +
 I*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - sinh
(x) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dilog(
-I*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(a\tanh(x))\,dx = \int \log\left(a\tanh\left(x\right)\right)dx$$

```
[In] integrate(ln(a*tanh(x)),x)
```

```
[Out] Integral(log(a*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \log(a\tanh(x))\,dx = x\log\left(a\tanh\left(x\right)\right) + x\log\left(e^{(2x)} + 1\right) - x\log\left(e^{x} + 1\right) - x\log\left(-e^{x} + 1\right) + \frac{1}{2}\operatorname{Li}_2\left(-e^{(2x)}\right) - \operatorname{Li}_2(-e^{x}) - \operatorname{Li}_2(e^{x})$$

```
[In] integrate(log(a*tanh(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*tanh(x)) + x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) +
1/2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

Giac [F]

$$\int \log(a\tanh(x))\,dx = \int \log\left(a\tanh\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*tanh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*tanh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\tanh(x))\,dx = \int \ln\left(a\tanh(x)\right)\,dx$$

```
[In] int(log(a*tanh(x)),x)
[Out] int(log(a*tanh(x)), x)
```

3.209 $\int \log\left(a\tanh^2(x)\right)\,dx$

Optimal result 1155
Rubi [A] (verified) 1155
Mathematica [A] (verified) 1157
Maple [A] (verified) 1157
Fricas [C] (verification not implemented) 1157
Sympy [F] 1158
Maxima [A] (verification not implemented) 1158
Giac [F] 1158
Mupad [F(-1)] 1159

Optimal result

Integrand size = 7, antiderivative size = 37

$$\int \log\left(a\tanh^2(x)\right)\,dx = 4x\text{arctanh}\left(e^{2x}\right) + x\log\left(a\tanh^2(x)\right) + \text{PolyLog}\left(2,-e^{2x}\right) - \text{PolyLog}\left(2,e^{2x}\right)$$

```
[Out] 4*x*arctanh(exp(2*x))+x*ln(a*tanh(x)^2)+polylog(2,-exp(2*x))-polylog(2,exp(
2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 5569, 4267, 2317, 2438}

$$\int \log\left(a\tanh^2(x)\right)\,dx = x\log\left(a\tanh^2(x)\right) + 4x\text{arctanh}\left(e^{2x}\right) + \text{PolyLog}\left(2,-e^{2x}\right) - \text{PolyLog}\left(2,e^{2x}\right)$$

```
[In] Int[Log[a*Tanh[x]^2],x]
```

```
[Out] 4*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^2] + PolyLog[2, -E^(2*x)] - PolyLog[
2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\tanh^2(x)\right) - \int 2x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\tanh^2(x)\right) - 2\int x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\tanh^2(x)\right) - 4\int x\text{csch}(2x)\,dx \\
&= 4x\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\tanh^2(x)\right) + 2\int \log\left(1-e^{2x}\right)\,dx - 2\int \log\left(1+e^{2x}\right)\,dx \\
&= 4x\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\tanh^2(x)\right) \\
&\quad + \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) - \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= 4x\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\tanh^2(x)\right) + \text{Li}_2\left(-e^{2x}\right) - \text{Li}_2\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \log \left(a \tanh^2(x)\right) \, dx = -\frac{1}{2} \log(1-\tanh(x)) \log \left(a \tanh^2(x)\right) + \frac{1}{2} \log \left(a \tanh^2(x)\right) \log(1+\tanh(x)) + \text{PolyLog}(2,-\tanh(x)) - \text{PolyLog}(2,\tanh(x))$$

```
[In] Integrate[Log[a*Tanh[x]^2],x]
```

```
[Out] -1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]^2]) + (Log[a*Tanh[x]^2]*Log[1 + Tanh[x
]])/2 + PolyLog[2, -Tanh[x]] - PolyLog[2, Tanh[x]]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\ln(\tanh(x)+1)\ln\left(a\left(\tanh^2(x)\right)\right)}{2} + \operatorname{dilog}\left(\tanh\left(x\right)+1\right) - \frac{\ln(\tanh(x)-1)\ln\left(a\left(\tanh^2(x)\right)\right)}{2} + \operatorname{dilog}\left(\tanh\right.$
default	$\frac{\ln(\tanh(x)+1)\ln\left(a\left(\tanh^2(x)\right)\right)}{2} + \operatorname{dilog}\left(\tanh\left(x\right)+1\right) - \frac{\ln(\tanh(x)-1)\ln\left(a\left(\tanh^2(x)\right)\right)}{2} + \operatorname{dilog}\left(\tanh\right.$
risch	$2x\ln\left(-1+\mathrm{e}^{2x}\right) - \frac{i\pi \operatorname{csgn}\left(i\left(-1+\mathrm{e}^{2x}\right)^2\right)^3 x}{2} - \frac{i\pi \operatorname{csgn}\left(\frac{ia\left(-1+\mathrm{e}^{2x}\right)^2}{\left(1+\mathrm{e}^{2x}\right)^2}\right)^3 x}{2} + \frac{i\pi\ \operatorname{csgn}\left(i\left(-1+\mathrm{e}^{2x}\right)^2\right)\operatorname{csgn}\left(\frac{i(-}{(1}\right.}{2}$

```
[In] int(ln(a*tanh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(tanh(x)+1)*ln(a*tanh(x)^2)+dilog(tanh(x)+1)-1/2*ln(tanh(x)-1)*ln(a*t
anh(x)^2)+dilog(tanh(x))+ln(tanh(x)-1)*ln(tanh(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.49

$$\int \log \left(a \tanh^2(x)\right) \, dx = x \log \left(\frac{a \cosh\left(x\right)^2 + a \sinh\left(x\right)^2 - a}{\cosh\left(x\right)^2 + \sinh\left(x\right)^2 + 1}\right) - 2\,x \log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + 2\,x \log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) + 2\,x \log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) - 2\,x \log\left(-\cosh\left(x\right) - \sinh\left(x\right) + 1\right) - 2\,\mathrm{Li}_2(\cosh\left(x\right) + \sinh\left(x\right)) + 2\,\mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) + 2\,\mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right)) - 2\,\mathrm{Li}_2(-\cosh\left(x\right) - \sinh\left(x\right))$$

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="fricas")
[Out] x*log((a*cosh(x)^2 + a*sinh(x)^2 - a)/(cosh(x)^2 + sinh(x)^2 + 1)) - 2*x*lo
g(cosh(x) + sinh(x) + 1) + 2*x*log(I*cosh(x) + I*sinh(x) + 1) + 2*x*log(-I*
cosh(x) - I*sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cosh(x
) + sinh(x)) + 2*dilog(I*cosh(x) + I*sinh(x)) + 2*dilog(-I*cosh(x) - I*sinh
(x)) - 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log\left(a\tanh^2(x)\right)\,dx = \int \log\left(a\tanh^2{(x)}\right)dx$$

```
[In] integrate(ln(a*tanh(x)**2),x)
[Out] Integral(log(a*tanh(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \log\left(a\tanh^2(x)\right)\,dx = x\log\left(a\tanh\left(x\right)^2\right) + 2\,x\log\left(e^{(2\,x)}+1\right) - 2\,x\log\left(e^x+1\right) - 2\,x\log\left(-e^x+1\right) + \mathrm{Li}_2\left(-e^{(2\,x)}\right) - 2\,\mathrm{Li}_2(-e^x) - 2\,\mathrm{Li}_2(e^x)$$

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="maxima")
[Out] x*log(a*tanh(x)^2) + 2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x
 + 1) + dilog(-e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x)
```

Giac [F]

$$\int \log\left(a\tanh^2(x)\right)\,dx = \int \log\left(a\tanh\left(x\right)^2\right)\,dx$$

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="giac")
[Out] integrate(log(a*tanh(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(a\tanh^2(x)\right)\,dx = \int \ln\left(a\tanh(x)^2\right)\,dx$$

```
[In] int(log(a*tanh(x)^2),x)
[Out] int(log(a*tanh(x)^2), x)
```

3.210 $\int \log (a \tanh^n(x)) \, dx$

Optimal result 1160
Rubi [A] (verified) 1160
Mathematica [A] (verified) 1162
Maple [A] (verified) 1162
Fricas [C] (verification not implemented) 1162
Sympy [F] 1163
Maxima [A] (verification not implemented) 1163
Giac [F] 1163
Mupad [F(-1)] 1164

Optimal result

Integrand size = 7, antiderivative size = 46

$$\int \log (a \tanh^n(x)) \, dx = 2nx\text{arctanh}\left(e^{2x}\right) + x\log (a \tanh^n(x)) + \frac{1}{2}n\,\text{PolyLog}\left(2,-e^{2x}\right) - \frac{1}{2}n\,\text{PolyLog}\left(2,e^{2x}\right)$$

```
[Out] 2*n*x*arctanh(exp(2*x))+x*ln(a*tanh(x)^n)+1/2*n*polylog(2,-exp(2*x))-1/2*n*
polylog(2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 5569, 4267, 2317, 2438}

$$\int \log (a \tanh^n(x)) \, dx = x\log (a \tanh^n(x)) + 2nx\text{arctanh}\left(e^{2x}\right) + \frac{1}{2}n\,\text{PolyLog}\left(2,-e^{2x}\right) - \frac{1}{2}n\,\text{PolyLog}\left(2,e^{2x}\right)$$

```
[In] Int[Log[a*Tanh[x]^n],x]
```

```
[Out] 2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^n] + (n*PolyLog[2, -E^(2*x)])/2 -
(n*PolyLog[2, E^(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\tanh^n(x)\right)-\int nx\text{csch}(x)\text{sech}(x)\,dx\\
&= x\log\left(a\tanh^n(x)\right)-n\int x\text{csch}(x)\text{sech}(x)\,dx\\
&= x\log\left(a\tanh^n(x)\right)-(2n)\int x\text{csch}(2x)\,dx\\
&= 2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+n\int\log\left(1-e^{2x}\right)\,dx-n\int\log\left(1+e^{2x}\right)\,dx\\
&= 2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)\\
&\quad-\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= 2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)-\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \log \left(a \tanh^n(x)\right) \, dx = -\frac{1}{2} \log(1-\tanh(x)) \log \left(a \tanh^n(x)\right) + \frac{1}{2} \log \left(a \tanh^n(x)\right) \log(1+\tanh(x)) + \frac{1}{2} n \, \text{PolyLog}(2,-\tanh(x)) - \frac{1}{2} n \, \text{PolyLog}(2,\tanh(x))$$

```
[In] Integrate[Log[a*Tanh[x]^n],x]
```

```
[Out] -1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]^n]) + (Log[a*Tanh[x]^n]*Log[1 + Tanh[x
]])/2 + (n*PolyLog[2, -Tanh[x]])/2 - (n*PolyLog[2, Tanh[x]])/2
```

Maple [A] (verified)

Time = 5.64 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result	s
default	$x\left(\ln \left(a\left(\tanh^{n}\left(x\right)\right)\right)-n \ln \left(\tanh \left(x\right)\right)\right)+n\left(\frac{\operatorname{dilog}\left(\tanh \left(x\right)\right)}{2}+\frac{\operatorname{dilog}\left(\tanh \left(x\right)+1\right)}{2}+\frac{\ln \left(\tanh \left(x\right)\right) \ln \left(\tanh \left(x\right)+1\right)}{2}\right)$	4
risch	Expression too large to display	2

```
[In] int(ln(a*tanh(x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*(ln(a*tanh(x)^n)-n*ln(tanh(x)))+n*(1/2*dilog(tanh(x))+1/2*dilog(tanh(x)+1
)+1/2*ln(tanh(x))*ln(tanh(x)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.52

$$\int \log \left(a \tanh^n(x)\right) \, dx = nx \log \left(\frac{\sinh(x)}{\cosh(x)}\right) - nx \log \left(\cosh(x) + \sinh(x) + 1\right) + nx \log \left(i \cosh(x) + i \sinh(x) + 1\right) + nx \log \left(-i \cosh(x) - i \sinh(x) + 1\right) - nx \log \left(-\cosh(x) - \sinh(x) + 1\right) - n\text{Li}_2(\cosh(x) + \sinh(x)) + n\text{Li}_2(i \cosh(x) + i \sinh(x)) + n\text{Li}_2(-i \cosh(x) - i \sinh(x)) - n\text{Li}_2(-\cosh(x) - \sinh(x)) + x \log(a)$$

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(sinh(x)/cosh(x)) - n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(I*cosh(
x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x) - I*sinh(x) + 1) - n*x*log(-cosh(x
) - sinh(x) + 1) - n*dilog(cosh(x) + sinh(x)) + n*dilog(I*cosh(x) + I*sinh(
x)) + n*dilog(-I*cosh(x) - I*sinh(x)) - n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

Sympy [F]

$$\int \log \left(a \tanh^n(x)\right) \, dx = \int \log \left(a \tanh^{n}\left(x\right)\right)\, dx$$

```
[In] integrate(ln(a*tanh(x)**n),x)
[Out] Integral(log(a*tanh(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \log \left(a \tanh^n(x)\right) \, dx = \frac{1}{2} \left(2 \, x \log \left(e^{(2 \, x)} + 1\right) - 2 \, x \log \left(e^{x} + 1\right) - 2 \, x \log \left(-e^{x} + 1\right) + \mathrm{Li}_2\left(-e^{(2 \, x)}\right) - 2 \, \mathrm{Li}_2(-e^{x}) - 2 \, \mathrm{Li}_2(e^{x})\right) n + x \log \left(a \tanh \left(x\right)^{n}\right)$$

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="maxima")
[Out] 1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(-e
^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*tanh(x)^n)
```

Giac [F]

$$\int \log \left(a \tanh^n(x)\right) \, dx = \int \log \left(a \tanh \left(x\right)^{n}\right) \,d x$$

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="giac")
[Out] integrate(log(a*tanh(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(a\tanh^n(x)\right)\,dx = \int \ln\left(a\tanh(x)^n\right)\,dx$$

```
[In] int(log(a*tanh(x)^n),x)
[Out] int(log(a*tanh(x)^n), x)
```

3.211 $\int \log(\coth(x))\,dx$

Optimal result 1165
Rubi [A] (verified) 1165
Mathematica [A] (verified) 1167
Maple [A] (verified) 1167
Fricas [C] (verification not implemented) 1167
Sympy [F] 1168
Maxima [A] (verification not implemented) 1168
Giac [F] 1168
Mupad [B] (verification not implemented) 1169

Optimal result

Integrand size = 3, antiderivative size = 39

$$\int \log(\coth(x))\,dx = -2x\text{arctanh}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right) + \frac{\text{PolyLog}\left(2,e^{2x}\right)}{2}$$

```
[Out] -2*x*arctanh(exp(2*x))+x*ln(coth(x))-1/2*polylog(2,-exp(2*x))+1/2*polylog(2
,exp(2*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {2628, 5569, 4267, 2317, 2438}

$$\int \log(\coth(x))\,dx = -2x\text{arctanh}\left(e^{2x}\right) - \frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right) + \frac{\text{PolyLog}\left(2,e^{2x}\right)}{2} + x\log(\coth(x))$$

```
[In] Int[Log[Coth[x]],x]
```

```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog[2
, E^(2*x)]/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log(\coth(x)) + \int x\operatorname{csch}(x)\operatorname{sech}(x)\,dx \\
&= x\log(\coth(x)) + 2\int x\operatorname{csch}(2x)\,dx \\
&= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \int \log\left(1-e^{2x}\right)\,dx + \int \log\left(1+e^{2x}\right)\,dx \\
&= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) + \frac{\text{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \log(\coth(x))\,dx = -\frac{1}{2}\log(\coth(x))\log(1-\tanh(x)) + \frac{1}{2}\log(\coth(x))\log(1+\tanh(x)) - \frac{1}{2}\operatorname{PolyLog}(2,-\tanh(x)) + \frac{\operatorname{PolyLog}(2,\tanh(x))}{2}$$

```
[In] Integrate[Log[Coth[x]],x]
```

```
[Out] -1/2*(Log[Coth[x]]*Log[1 - Tanh[x]]) + (Log[Coth[x]]*Log[1 + Tanh[x]])/2 -
PolyLog[2, -Tanh[x]]/2 + PolyLog[2, Tanh[x]]/2
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{\operatorname{dilog}(\coth(x))}{2}+\frac{\operatorname{dilog}(\coth(x)+1)}{2}+\frac{\ln(\coth(x))\ln(\coth(x)+1)}{2}$
default	$\frac{\operatorname{dilog}(\coth(x))}{2}+\frac{\operatorname{dilog}(\coth(x)+1)}{2}+\frac{\ln(\coth(x))\ln(\coth(x)+1)}{2}$
risch	$-x\ln\left(-1+e^{2x}\right)+\frac{i\pi\operatorname{csgn}\left(i\left(1+e^{2x}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+e^{2x}\right)}{-1+e^{2x}}\right)^{2}x}{2}+\frac{i\pi\operatorname{csgn}\left(\frac{i}{-1+e^{2x}}\right)\operatorname{csgn}\left(\frac{i\left(1+e^{2x}\right)}{-1+e^{2x}}\right)^{2}x}{2}+\mathrm{dilo}$

```
[In] int(ln(coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*dilog(coth(x))+1/2*dilog(coth(x)+1)+1/2*ln(coth(x))*ln(coth(x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.59

$$\begin{aligned}\int \log(\coth(x))\,dx = {} & x\log\left(\frac{\cosh(x)}{\sinh(x)}\right) + x\log(\cosh(x)+\sinh(x)+1) \\ & - x\log(i\cosh(x)+i\sinh(x)+1) \\ & - x\log(-i\cosh(x)-i\sinh(x)+1) + x\log(-\cosh(x)-\sinh(x)+1) \\ & + \mathrm{Li}_2(\cosh(x)+\sinh(x)) - \mathrm{Li}_2(i\cosh(x)+i\sinh(x)) \\ & - \mathrm{Li}_2(-i\cosh(x)-i\sinh(x)) + \mathrm{Li}_2(-\cosh(x)-\sinh(x))\end{aligned}$$

```
[In] integrate(log(coth(x)),x, algorithm="fricas")
```

```
[Out] x*log(cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x) + I
*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - sinh(x
```

```
) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dilog(-I
*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(\coth(x))\,dx = \int \log\left(\coth\left(x\right)\right)dx$$

```
[In] integrate(ln(coth(x)),x)
[Out] Integral(log(coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \log(\coth(x))\,dx = -x\log\left(e^{(2x)}+1\right) + x\log\left(e^{x}+1\right) + x\log\left(-e^{x}+1\right) + x\log\left(\coth\left(x\right)\right) - \frac{1}{2}\operatorname{Li}_2\left(-e^{(2x)}\right) + \operatorname{Li}_2\left(-e^{x}\right) + \operatorname{Li}_2\left(e^{x}\right)$$

```
[In] integrate(log(coth(x)),x, algorithm="maxima")
[Out] -x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) + x*log(coth(x)) - 1
/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

Giac [F]

$$\int \log(\coth(x))\,dx = \int \log\left(\coth\left(x\right)\right)\,dx$$

```
[In] integrate(log(coth(x)),x, algorithm="giac")
[Out] integrate(log(coth(x)), x)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

$$\int \log(\coth(x))\,dx$$
$$= \frac{\mathrm{polylog}(2,-\coth(x))}{2} - \frac{\mathrm{polylog}(2,\coth(x))}{2} + \operatorname{atanh}(\coth(x))\,\ln(\coth(x))$$

```
[In] int(log(coth(x)),x)
```

```
[Out] polylog(2, -coth(x))/2 - polylog(2, coth(x))/2 + atanh(coth(x))*log(coth(x)
)
```

3.212 $\int \log(a\coth(x))\,dx$

Optimal result 1170
Rubi [A] (verified) 1170
Mathematica [A] (verified) 1172
Maple [B] (verified) 1172
Fricas [C] (verification not implemented) 1172
Sympy [F] 1173
Maxima [A] (verification not implemented) 1173
Giac [F] 1173
Mupad [F(-1)] 1174

Optimal result

Integrand size = 5, antiderivative size = 41

$$\int \log(a\coth(x))\,dx = -2x\operatorname{arctanh}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\operatorname{PolyLog}\left(2,-e^{2x}\right) + \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}$$

```
[Out] -2*x*arctanh(exp(2*x))+x*ln(a*coth(x))-1/2*polylog(2,-exp(2*x))+1/2*polylog
(2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 5569, 4267, 2317, 2438}

$$\int \log(a\coth(x))\,dx = x\log(a\coth(x)) - 2x\operatorname{arctanh}\left(e^{2x}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2,-e^{2x}\right) + \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}$$

```
[In] Int[Log[a*Coth[x]],x]
```

```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog
[2, E^(2*x)]/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log(a\coth(x)) + \int x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log(a\coth(x)) + 2\int x\text{csch}(2x)\,dx \\
&= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \int \log\left(1-e^{2x}\right)\,dx + \int \log\left(1+e^{2x}\right)\,dx \\
&= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) + \frac{\text{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \log(a\coth(x))\,dx = -\frac{1}{2}\log(a\coth(x))\log(1-\tanh(x))+\frac{1}{2}\log(a\coth(x))\log(1+\tanh(x)) -\frac{1}{2}\operatorname{PolyLog}(2,-\tanh(x))+\frac{\operatorname{PolyLog}(2,\tanh(x))}{2}$$

```
[In] Integrate[Log[a*Coth[x]],x]
```

```
[Out] -1/2*(Log[a*Coth[x]]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]]*Log[1 + Tanh[x]])/
2 - PolyLog[2, -Tanh[x]]/2 + PolyLog[2, Tanh[x]]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 1.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{-\frac{a\left(\operatorname{dilog}\left(-\frac{a\coth(x)-a}{a}\right)+\ln(a\coth(x))\ln\left(-\frac{a\coth(x)-a}{a}\right)\right)}{2}+\frac{a\left(\operatorname{dilog}\left(\frac{a\coth(x)+a}{a}\right)+\ln(a\coth(x))\ln\left(\frac{a\coth(x)+a}{a}\right)\right)}{2}}{a}$
default	$\frac{-\frac{a\left(\operatorname{dilog}\left(-\frac{a\coth(x)-a}{a}\right)+\ln(a\coth(x))\ln\left(-\frac{a\coth(x)-a}{a}\right)\right)}{2}+\frac{a\left(\operatorname{dilog}\left(\frac{a\coth(x)+a}{a}\right)+\ln(a\coth(x))\ln\left(\frac{a\coth(x)+a}{a}\right)\right)}{2}}{a}$
risch	$-x\ln\left(-1+\mathrm{e}^{2x}\right)-\frac{i\pi\operatorname{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)}{-1+\mathrm{e}^{2x}}\right)^{3}x}{2}+\frac{i\pi\ \operatorname{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)}{-1+\mathrm{e}^{2x}}\right)^{2}x}{2}-\frac{i\pi\ \operatorname{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)\right)\operatorname{csg}$

```
[In] int(ln(a*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2*a*(dilog(-(a*coth(x)-a)/a)+ln(a*coth(x))*ln(-(a*coth(x)-a)/a))+1/
2*a*(dilog((a*coth(x)+a)/a)+ln(a*coth(x))*ln((a*coth(x)+a)/a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\int \log(a\coth(x))\,dx = x\log\left(\frac{a\cosh(x)}{\sinh(x)}\right)+x\log(\cosh(x)+\sinh(x)+1) - x\log(i\cosh(x)+i\sinh(x)+1) - x\log(-i\cosh(x)-i\sinh(x)+1)+x\log(-\cosh(x)-\sinh(x)+1) + \operatorname{Li}_2(\cosh(x)+\sinh(x))-\operatorname{Li}_2(i\cosh(x)+i\sinh(x)) - \operatorname{Li}_2(-i\cosh(x)-i\sinh(x))+\operatorname{Li}_2(-\cosh(x)-\sinh(x))$$

```
[In] integrate(log(a*coth(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x) +
 I*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - sinh
(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dilog(
-I*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(a\coth(x))\,dx = \int \log\left(a\coth\left(x\right)\right)dx$$

```
[In] integrate(ln(a*coth(x)),x)
```

```
[Out] Integral(log(a*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \log(a\coth(x))\,dx = x\log\left(a\coth\left(x\right)\right) - x\log\left(e^{(2x)} + 1\right) + x\log\left(e^{x} + 1\right) + x\log\left(-e^{x} + 1\right) - \frac{1}{2}\operatorname{Li}_2\left(-e^{(2x)}\right) + \operatorname{Li}_2\left(-e^{x}\right) + \operatorname{Li}_2\left(e^{x}\right)$$

```
[In] integrate(log(a*coth(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*coth(x)) - x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) -
1/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

Giac [F]

$$\int \log(a\coth(x))\,dx = \int \log\left(a\coth\left(x\right)\right)\,dx$$

```
[In] integrate(log(a*coth(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*coth(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a \coth(x))\, dx = \int \ln\left(a \coth(x)\right)\, dx$$

```
[In] int(log(a*coth(x)),x)
[Out] int(log(a*coth(x)), x)
```

3.213 $\int \log \left(a \coth^2(x)\right) \, dx$

Optimal result 1175
Rubi [A] (verified) 1175
Mathematica [A] (verified) 1177
Maple [A] (verified) 1177
Fricas [C] (verification not implemented) 1177
Sympy [F] 1178
Maxima [A] (verification not implemented) 1178
Giac [F] 1178
Mupad [F(-1)] 1179

Optimal result

Integrand size = 7, antiderivative size = 37

$$\int \log \left(a \coth^2(x)\right) \, dx = -4x\text{arctanh}\left(e^{2x}\right) + x \log \left(a \coth^2(x)\right) - \text{PolyLog}\left(2, -e^{2x}\right) + \text{PolyLog}\left(2, e^{2x}\right)$$

```
[Out] -4*x*arctanh(exp(2*x))+x*ln(a*coth(x)^2)-polylog(2,-exp(2*x))+polylog(2,exp
(2*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 5569, 4267, 2317, 2438}

$$\int \log \left(a \coth^2(x)\right) \, dx = x \log \left(a \coth^2(x)\right) - 4x\text{arctanh}\left(e^{2x}\right) - \text{PolyLog}\left(2, -e^{2x}\right) + \text{PolyLog}\left(2, e^{2x}\right)$$

```
[In] Int[Log[a*Coth[x]^2],x]
```

```
[Out] -4*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^2] - PolyLog[2, -E^(2*x)] + PolyLog
[2, E^(2*x)]
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 2317</u>

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\coth^2(x)\right) - \int -2x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\coth^2(x)\right) + 2\int x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\coth^2(x)\right) + 4\int x\text{csch}(2x)\,dx \\
&= -4x\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\coth^2(x)\right) - 2\int \log\left(1-e^{2x}\right)\,dx + 2\int \log\left(1+e^{2x}\right)\,dx \\
&= -4x\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\coth^2(x)\right) \\
&\quad - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) + \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= -4x\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\coth^2(x)\right) - \text{Li}_2\left(-e^{2x}\right) + \text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \log \left(a \coth^2(x)\right) \, dx = -\frac{1}{2} \log \left(a \coth^2(x)\right) \log(1 - \tanh(x)) + \frac{1}{2} \log \left(a \coth^2(x)\right) \log(1 + \tanh(x)) - \text{PolyLog}(2, -\tanh(x)) + \text{PolyLog}(2, \tanh(x))$$

```
[In] Integrate[Log[a*Coth[x]^2],x]
```

```
[Out] -1/2*(Log[a*Coth[x]^2]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]^2]*Log[1 + Tanh[x
]])/2 - PolyLog[2, -Tanh[x]] + PolyLog[2, Tanh[x]]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result
derivativedivides	$-\frac{\ln(\coth(x)-1)\ln(a(\coth^2(x)))}{2} + \operatorname{dilog}\left(\coth\left(x\right)\right) + \ln\left(\coth\left(x\right) - 1\right)\ln\left(\coth\left(x\right)\right) + \frac{\ln(\coth(x)+}{}$
default	$-\frac{\ln(\coth(x)-1)\ln(a(\coth^2(x)))}{2} + \operatorname{dilog}\left(\coth\left(x\right)\right) + \ln\left(\coth\left(x\right) - 1\right)\ln\left(\coth\left(x\right)\right) + \frac{\ln(\coth(x)+}{}$
risch	$-2x\ln\left(-1+e^{2x}\right) - \frac{i\pi \operatorname{csgn}\left(i\left(1+e^{2x}\right)\right)^2 \operatorname{csgn}\left(i\left(1+e^{2x}\right)^2\right)x}{2} - \frac{i\pi \operatorname{csgn}\left(i\left(1+e^{2x}\right)^2\right)^3 x}{2} + \frac{i\pi\ \operatorname{csgn}\left(i\left(1+e^{2x}\right)^2\right.}{}$

```
[In] int(ln(a*coth(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(coth(x)-1)*ln(a*coth(x)^2)+dilog(coth(x))+ln(coth(x)-1)*ln(coth(x))
+1/2*ln(coth(x)+1)*ln(a*coth(x)^2)+dilog(coth(x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.43

$$\begin{aligned}\int \log \left(a \coth^2(x)\right) \, dx = {} & x \log \left(\frac{a \cosh\left(x\right)^2 + a \sinh\left(x\right)^2 + a}{\cosh\left(x\right)^2 + \sinh\left(x\right)^2 - 1}\right) \\ & + 2\, x \log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - 2\, x \log\left(i\, \cosh\left(x\right) + i\, \sinh\left(x\right) + 1\right) \\ & - 2\, x \log\left(-i\, \cosh\left(x\right) - i\, \sinh\left(x\right) + 1\right) \\ & + 2\, x \log\left(-\cosh\left(x\right) - \sinh\left(x\right) + 1\right) \\ & + 2\, \mathrm{Li}_2(\cosh\left(x\right) + \sinh\left(x\right)) - 2\, \mathrm{Li}_2(i\, \cosh\left(x\right) + i\, \sinh\left(x\right)) \\ & - 2\, \mathrm{Li}_2(-i\, \cosh\left(x\right) - i\, \sinh\left(x\right)) + 2\, \mathrm{Li}_2(-\cosh\left(x\right) - \sinh\left(x\right))\end{aligned}$$

```
[In] integrate(log(a*coth(x)^2),x, algorithm="fricas")
```

```
[Out] x*log((a*cosh(x)^2 + a*sinh(x)^2 + a)/(cosh(x)^2 + sinh(x)^2 - 1)) + 2*x*lo
g(cosh(x) + sinh(x) + 1) - 2*x*log(I*cosh(x) + I*sinh(x) + 1) - 2*x*log(-I*
cosh(x) - I*sinh(x) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cosh(x
) + sinh(x)) - 2*dilog(I*cosh(x) + I*sinh(x)) - 2*dilog(-I*cosh(x) - I*sinh
(x)) + 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log\left(a\coth^2(x)\right)\,dx = \int \log\left(a\coth^2(x)\right)dx$$

```
[In] integrate(ln(a*coth(x)**2),x)
```

```
[Out] Integral(log(a*coth(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\begin{aligned}\int \log\left(a\coth^2(x)\right)\,dx &= x\log\left(a\coth(x)^2\right) - 2\,x\log\left(e^{(2\,x)}+1\right) + 2\,x\log\left(e^x+1\right)\\ &\quad + 2\,x\log\left(-e^x+1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right) + 2\,\mathrm{Li}_2(-e^x) + 2\,\mathrm{Li}_2(e^x)\end{aligned}$$

```
[In] integrate(log(a*coth(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*coth(x)^2) - 2*x*log(e^(2*x) + 1) + 2*x*log(e^x + 1) + 2*x*log(-e^x
 + 1) - dilog(-e^(2*x)) + 2*dilog(-e^x) + 2*dilog(e^x)
```

Giac [F]

$$\int \log\left(a\coth^2(x)\right)\,dx = \int \log\left(a\coth(x)^2\right)\,dx$$

```
[In] integrate(log(a*coth(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*coth(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a \coth^2(x)\right) \, dx = \int \ln \left(a \coth(x)^2\right) \, dx$$

```
[In] int(log(a*coth(x)^2),x)
[Out] int(log(a*coth(x)^2), x)
```

3.214 $\int \log (a \coth^n(x)) \, dx$

Optimal result 1180
Rubi [A] (verified) 1180
Mathematica [A] (verified) 1182
Maple [A] (verified) 1182
Fricas [C] (verification not implemented) 1182
Sympy [F] 1183
Maxima [A] (verification not implemented) 1183
Giac [F] 1183
Mupad [F(-1)] 1184

Optimal result

Integrand size = 7, antiderivative size = 46

$$\int \log (a \coth^n(x)) \, dx = -2nx\text{arctanh}(e^{2x}) + x \log (a \coth^n(x)) - \frac{1}{2}n \,\text{PolyLog}(2, -e^{2x}) + \frac{1}{2}n \,\text{PolyLog}(2, e^{2x})$$

```
[Out] -2*n*x*arctanh(exp(2*x))+x*ln(a*coth(x)^n)-1/2*n*polylog(2,-exp(2*x))+1/2*n
*polylog(2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 5569, 4267, 2317, 2438}

$$\int \log (a \coth^n(x)) \, dx = x \log (a \coth^n(x)) - 2nx\text{arctanh}(e^{2x}) - \frac{1}{2}n \,\text{PolyLog}(2, -e^{2x}) + \frac{1}{2}n \,\text{PolyLog}(2, e^{2x})$$

```
[In] Int[Log[a*Coth[x]^n],x]
```

```
[Out] -2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^n] - (n*PolyLog[2, -E^(2*x)])/2 +
 (n*PolyLog[2, E^(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\coth^n(x)\right)+\int nx\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\coth^n(x)\right)+n\int x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\coth^n(x)\right)+(2n)\int x\text{csch}(2x)\,dx \\
&= -2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right)-n\int\log\left(1-e^{2x}\right)\,dx+n\int\log\left(1+e^{2x}\right)\,dx \\
&= -2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right) \\
&\quad -\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)+\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2x}\right) \\
&= -2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)+\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \log \left(a \coth ^n(x)\right) \, dx=-\frac{1}{2} \log \left(a \coth ^n(x)\right) \log (1-\tanh (x))+\frac{1}{2} \log \left(a \coth ^n(x)\right) \log (1+\tanh (x))-\frac{1}{2} n \operatorname{PolyLog}(2,-\tanh (x))+\frac{1}{2} n \operatorname{PolyLog}(2,\tanh (x))$$

```
[In] Integrate[Log[a*Coth[x]^n],x]
```

```
[Out] -1/2*(Log[a*Coth[x]^n]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]^n]*Log[1 + Tanh[x
]])/2 - (n*PolyLog[2, -Tanh[x]])/2 + (n*PolyLog[2, Tanh[x]])/2
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result	si
default	$x\left(\ln \left(a\left(\coth ^n(x)\right)\right)-n \ln (\coth (x))\right)+n\left(\frac{\operatorname{dilog}(\coth (x))}{2}+\frac{\operatorname{dilog}(\coth (x)+1)}{2}+\frac{\ln (\coth (x)) \ln (\coth (x)+1)}{2}\right)$	43
risch	Expression too large to display	22

```
[In] int(ln(a*coth(x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*(ln(a*coth(x)^n)-n*ln(coth(x)))+n*(1/2*dilog(coth(x))+1/2*dilog(coth(x)+1
)+1/2*ln(coth(x))*ln(coth(x)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.52

$$\int \log \left(a \coth ^n(x)\right) \, dx=n x \log \left(\frac{\cosh (x)}{\sinh (x)}\right)+n x \log (\cosh (x)+\sinh (x)+1)-n x \log (i \cosh (x)+i \sinh (x)+1)-n x \log (-i \cosh (x)-i \sinh (x)+1)+n x \log (-\cosh (x)-\sinh (x)+1)+n \operatorname{Li}_2(\cosh (x)+\sinh (x))-n \operatorname{Li}_2(i \cosh (x)+i \sinh (x))-n \operatorname{Li}_2(-i \cosh (x)-i \sinh (x))+n \operatorname{Li}_2(-\cosh (x)-\sinh (x))+x \log (a)$$

```
[In] integrate(log(a*coth(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(cosh(x)/sinh(x)) + n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(I*cosh(
x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*sinh(x) + 1) + n*x*log(-cosh(x
) - sinh(x) + 1) + n*dilog(cosh(x) + sinh(x)) - n*dilog(I*cosh(x) + I*sinh(
x)) - n*dilog(-I*cosh(x) - I*sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

Sympy [F]

$$\int \log\left(a\coth^n(x)\right)\,dx = \int \log\left(a\coth^n\left(x\right)\right)dx$$

```
[In] integrate(ln(a*coth(x)**n),x)
[Out] Integral(log(a*coth(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \log\left(a\coth^n(x)\right)\,dx =$$
$$-\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)-2\,x\log\left(e^{x}+1\right)-2\,x\log\left(-e^{x}+1\right)+\mathrm{Li}_2\left(-e^{(2\,x)}\right)-2\,\mathrm{Li}_2(-e^{x})-2\,\mathrm{Li}_2(e^{x})\right)n$$
$$+x\log\left(a\coth\left(x\right)^{n}\right)$$

```
[In] integrate(log(a*coth(x)^n),x, algorithm="maxima")
[Out] -1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(-
e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*coth(x)^n)
```

Giac [F]

$$\int \log\left(a\coth^n(x)\right)\,dx = \int \log\left(a\coth\left(x\right)^{n}\right)\,dx$$

```
[In] integrate(log(a*coth(x)^n),x, algorithm="giac")
[Out] integrate(log(a*coth(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(a\coth^n(x)\right)\,dx = \int \ln\left(a\coth(x)^n\right)\,dx$$

```
[In] int(log(a*coth(x)^n),x)
```

```
[Out] int(log(a*coth(x)^n), x)
```

3.215 $\int \log(a\,\mathrm{sech}(x))\,dx$

Optimal result 1185
Rubi [A] (verified) 1185
Mathematica [A] (verified) 1186
Maple [C] (warning: unable to verify) 1187
Fricas [C] (verification not implemented) 1187
Sympy [F] 1188
Maxima [A] (verification not implemented) 1188
Giac [F] 1188
Mupad [F(-1)] 1188

Optimal result

Integrand size = 5, antiderivative size = 38

$$\int \log(a\,\mathrm{sech}(x))\,dx = -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\,\mathrm{sech}(x)) + \frac{1}{2}\,\mathrm{PolyLog}\left(2,-e^{2x}\right)$$

```
[Out] -1/2*x^2+x*ln(1+exp(2*x))+x*ln(a*sech(x))+1/2*polylog(2,-exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3799, 2221, 2317, 2438}

$$\int \log(a\,\mathrm{sech}(x))\,dx = x\log(a\,\mathrm{sech}(x)) + \frac{1}{2}\,\mathrm{PolyLog}\left(2,-e^{2x}\right) - \frac{x^2}{2} + x\log\left(e^{2x}+1\right)$$

```
[In] Int[Log[a*Sech[x]],x]
[Out] -1/2*x^2 + x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]] + PolyLog[2, -E^(2*x)]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log(a\text{sech}(x)) + \int x\tanh(x)\,dx \\
&= -\frac{x^2}{2} + x\log(a\text{sech}(x)) + 2\int \frac{e^{2x}x}{1+e^{2x}}\,dx \\
&= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\text{sech}(x)) - \int \log\left(1+e^{2x}\right)\,dx \\
&= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\text{sech}(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\text{sech}(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \log(a\text{sech}(x))\,dx = \frac{x^2}{2} + x\log\left(1+e^{-2x}\right) + x\log(a\text{sech}(x)) - \frac{1}{2}\text{PolyLog}\left(2,-e^{-2x}\right)$$

```
[In] Integrate[Log[a*Sech[x]],x]
[Out] x^2/2 + x*Log[1 + E^(-2*x)] + x*Log[a*Sech[x]] - PolyLog[2, -E^(-2*x)]/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.59 (sec) , antiderivative size = 314, normalized size of antiderivative = 8.26

method	result
risch	$x\ln\left(\mathrm{e}^{x}\right)+\frac{i\pi\,\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{x}}{1+\mathrm{e}^{2x}}\right)^{2}\mathrm{csgn}(ia)x}{2}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{x}}{1+\mathrm{e}^{2x}}\right)\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{x}}{1+\mathrm{e}^{2x}}\right)^{2}x}{2}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{1+\mathrm{e}^{2x}}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{x}}{1+\mathrm{e}^{2x}}\right)^{2}x}{2}-\frac{i\pi\,\mathrm{csgn}(\ldots$

```
[In] int(ln(a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(exp(x))+1/2*I*Pi*csgn(I*a/(1+exp(2*x))*exp(x))^2*csgn(I*a)*x+1/2*I*Pi*
csgn(I*exp(x)/(1+exp(2*x)))*csgn(I*a/(1+exp(2*x))*exp(x))^2*x+1/2*I*Pi*csgn
(I/(1+exp(2*x)))*csgn(I*exp(x)/(1+exp(2*x)))^2*x-1/2*I*Pi*csgn(I*exp(x))*cs
gn(I/(1+exp(2*x)))*csgn(I*exp(x)/(1+exp(2*x)))*x-1/2*I*Pi*csgn(I*exp(x)/(1+
exp(2*x)))*csgn(I*a/(1+exp(2*x))*exp(x))*csgn(I*a)*x-1/2*I*Pi*csgn(I*exp(x)
/(1+exp(2*x)))^3*x+ln(a)*x+x*ln(2)-1/2*x^2-1/2*I*Pi*csgn(I*a/(1+exp(2*x))*e
xp(x))^3*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I*exp(x)/(1+exp(2*x)))^2*x-ln(exp(x
))*ln(1+exp(2*x))+ln(exp(x))*ln(1+I*exp(x))+ln(exp(x))*ln(1-I*exp(x))+dilog
(1+I*exp(x))+dilog(1-I*exp(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.21

$$\begin{aligned}\int\log(a\mathrm{sech}(x))\,dx=&-\frac{1}{2}\,x^{2}+x\log\left(\frac{2\,(a\cosh{(x)}+a\sinh{(x)})}{\cosh{(x)}^{2}+2\,\cosh{(x)}\sinh{(x)}+\sinh{(x)}^{2}+1}\right)\\&+x\log{(i\,\cosh{(x)}+i\,\sinh{(x)}+1)}+x\log{(-i\,\cosh{(x)}-i\,\sinh{(x)}+1)}\\&+\mathrm{Li}_2(i\,\cosh{(x)}+i\,\sinh{(x)})+\mathrm{Li}_2(-i\,\cosh{(x)}-i\,\sinh{(x)})\end{aligned}$$

```
[In] integrate(log(a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) + x*log(I*cosh(x) + I*sinh(x) + 1) + x*log(-I*cosh(x) - I*
sinh(x) + 1) + dilog(I*cosh(x) + I*sinh(x)) + dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F]

$$\int \log(a\text{sech}(x))\,dx = \int \log\left(a\,\text{sech}\,(x)\right)dx$$

```
[In] integrate(ln(a*sech(x)),x)
[Out] Integral(log(a*sech(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \log(a\text{sech}(x))\,dx = -\frac{1}{2}\,x^2 + x\log\left(a\,\text{sech}\,(x)\right) + x\log\left(e^{(2\,x)} + 1\right) + \frac{1}{2}\,\text{Li}_2\left(-e^{(2\,x)}\right)$$

```
[In] integrate(log(a*sech(x)),x, algorithm="maxima")
[Out] -1/2*x^2 + x*log(a*sech(x)) + x*log(e^(2*x) + 1) + 1/2*dilog(-e^(2*x))
```

Giac [F]

$$\int \log(a\text{sech}(x))\,dx = \int \log\left(a\,\text{sech}\,(x)\right)\,dx$$

```
[In] integrate(log(a*sech(x)),x, algorithm="giac")
[Out] integrate(log(a*sech(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\text{sech}(x))\,dx = -\int \ln\left(\cosh(x)\right) - \ln\left(a\right)\,dx$$

```
[In] int(log(a/cosh(x)),x)
[Out] -int(log(cosh(x)) - log(a), x)
```

3.216 $\int \log\left(a\,\mathrm{sech}^2(x)\right)\,dx$

Optimal result 1189
Rubi [A] (verified) 1189
Mathematica [A] (verified) 1191
Maple [C] (warning: unable to verify) 1191
Fricas [C] (verification not implemented) 1192
Sympy [F] 1192
Maxima [A] (verification not implemented) 1192
Giac [F] 1193
Mupad [F(-1)] 1193

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \log\left(a\,\mathrm{sech}^2(x)\right)\,dx = -x^2 + 2x\log\left(1+e^{2x}\right) + x\log\left(a\,\mathrm{sech}^2(x)\right) + \mathrm{PolyLog}\left(2,-e^{2x}\right)$$

```
[Out] -x^2+2*x*ln(1+exp(2*x))+x*ln(a*sech(x)^2)+polylog(2,-exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3799, 2221, 2317, 2438}

$$\int \log\left(a\,\mathrm{sech}^2(x)\right)\,dx = x\log\left(a\,\mathrm{sech}^2(x)\right) + \mathrm{PolyLog}\left(2,-e^{2x}\right) - x^2 + 2x\log\left(e^{2x}+1\right)$$

```
[In] Int[Log[a*Sech[x]^2],x]
[Out] -x^2 + 2*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^2] + PolyLog[2, -E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\operatorname{sech}^2(x)\right) - \int -2x\tanh(x)\,dx \\
&= x\log\left(a\operatorname{sech}^2(x)\right) + 2\int x\tanh(x)\,dx \\
&= -x^2 + x\log\left(a\operatorname{sech}^2(x)\right) + 4\int \frac{e^{2x}x}{1+e^{2x}}\,dx \\
&= -x^2 + 2x\log\left(1+e^{2x}\right) + x\log\left(a\operatorname{sech}^2(x)\right) - 2\int \log\left(1+e^{2x}\right)\,dx \\
&= -x^2 + 2x\log\left(1+e^{2x}\right) + x\log\left(a\operatorname{sech}^2(x)\right) - \operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= -x^2 + 2x\log\left(1+e^{2x}\right) + x\log\left(a\operatorname{sech}^2(x)\right) + \operatorname{Li}_2\left(-e^{2x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(a \operatorname{sech}^2(x)\right) \, dx = x\left(x+2 \log \left(1+e^{-2 x}\right)+\log \left(a \operatorname{sech}^2(x)\right)\right)-\operatorname{PolyLog}\left(2,-e^{-2 x}\right)$$

```
[In] Integrate[Log[a*Sech[x]^2],x]
[Out] x*(x + 2*Log[1 + E^(-2*x)] + Log[a*Sech[x]^2]) - PolyLog[2, -E^(-2*x)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.15 (sec) , antiderivative size = 480, normalized size of antiderivative = 13.71

method	result
risch	$\ln(a)\, x-\frac{i \pi \operatorname{csgn}\left(\frac{i a\, \mathrm{e}^{2 x}}{\left(1+\mathrm{e}^{2 x}\right)^{2}}\right)^{3} x}{2}+2 x \ln(2)-x^{2}-\frac{i \pi \operatorname{csgn}\left(\frac{i \mathrm{e}^{2 x}}{\left(1+\mathrm{e}^{2 x}\right)^{2}}\right)^{3} x}{2}+2 \operatorname{dilog}(1+i \mathrm{e}^{x})+2 \operatorname{dilog}(1$

```
[In] int(ln(a*sech(x)^2),x,method=_RETURNVERBOSE)
[Out] ln(a)*x-1/2*I*Pi*csgn(I*a/(1+exp(2*x))^2*exp(2*x))^3*x+2*x*ln(2)-x^2-1/2*I*
Pi*csgn(I*exp(2*x)/(1+exp(2*x))^2)^3*x+2*dilog(1+I*exp(x))+2*dilog(1-I*exp(
x))-1/2*I*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x))*x-1/2*I*Pi*csgn(I*exp(2*x)/(
1+exp(2*x))^2)*csgn(I*a/(1+exp(2*x))^2*exp(2*x))*csgn(I*a)*x+1/2*I*Pi*csgn(
I/(1+exp(2*x))^2)*csgn(I*exp(2*x)/(1+exp(2*x))^2)^2*x+1/2*I*Pi*csgn(I*exp(2
*x)/(1+exp(2*x))^2)*csgn(I*a/(1+exp(2*x))^2*exp(2*x))^2*x+1/2*I*Pi*csgn(I*e
xp(2*x))*csgn(I*exp(2*x)/(1+exp(2*x))^2)^2*x+I*Pi*csgn(I*exp(x))*csgn(I*exp
(2*x))^2*x-I*Pi*csgn(I*(1+exp(2*x)))*csgn(I*(1+exp(2*x))^2)^2*x+1/2*I*Pi*cs
gn(I*(1+exp(2*x))^2)^3*x-1/2*I*Pi*csgn(I*exp(2*x))^3*x+1/2*I*Pi*csgn(I*(1+e
xp(2*x)))^2*csgn(I*(1+exp(2*x))^2)*x+1/2*I*Pi*csgn(I*a/(1+exp(2*x))^2*exp(2
*x))^2*csgn(I*a)*x-1/2*I*Pi*csgn(I*exp(2*x))*csgn(I/(1+exp(2*x))^2)*csgn(I*
exp(2*x)/(1+exp(2*x))^2)*x+2*x*ln(exp(x))-2*ln(exp(x))*ln(1+exp(2*x))+2*ln(
exp(x))*ln(1+I*exp(x))+2*ln(exp(x))*ln(1-I*exp(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.03

$$\int \log \left(a\text{sech}^2(x)\right) \, dx = -x^2 + x \log \left(\frac{4 \left(a \cosh \left(x\right) + a \sinh \left(x\right)\right)}{\cosh \left(x\right)^3 + 3 \cosh \left(x\right) \sinh \left(x\right)^2 + \sinh \left(x\right)^3 + \left(3 \cosh \left(x\right)^2 + 1\right) \sinh \left(x\right) + 3 \cosh \left(x\right)} \right) + 2\, x \log \left(i \cosh \left(x\right) + i \sinh \left(x\right) + 1\right) + 2\, x \log \left(-i \cosh \left(x\right) - i \sinh \left(x\right) + 1\right) + 2\, \text{Li}_2(i \cosh \left(x\right) + i \sinh \left(x\right)) + 2\, \text{Li}_2(-i \cosh \left(x\right) - i \sinh \left(x\right))$$

```
[In] integrate(log(a*sech(x)^2),x, algorithm="fricas")
```

```
[Out] -x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + s
inh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + 3*cosh(x))) + 2*x*log(I*cosh(x) + I*
sinh(x) + 1) + 2*x*log(-I*cosh(x) - I*sinh(x) + 1) + 2*dilog(I*cosh(x) + I*
sinh(x)) + 2*dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F]

$$\int \log \left(a\text{sech}^2(x)\right) \, dx = \int \log \left(a \operatorname{sech}^{2}\left(x\right)\right)\, dx$$

```
[In] integrate(ln(a*sech(x)**2),x)
```

```
[Out] Integral(log(a*sech(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \log \left(a\text{sech}^2(x)\right) \, dx = -x^2 + x \log \left(a \operatorname{sech}\left(x\right)^2\right) + 2\, x \log \left(e^{\left(2\, x\right)} + 1\right) + \text{Li}_2\left(-e^{\left(2\, x\right)}\right)$$

```
[In] integrate(log(a*sech(x)^2),x, algorithm="maxima")
```

```
[Out] -x^2 + x*log(a*sech(x)^2) + 2*x*log(e^(2*x) + 1) + dilog(-e^(2*x))
```

Giac [F]

$$\int \log \left(a\text{sech}^2(x)\right) \, dx = \int \log \left(a \,\text{sech}\,(x)^2\right) \, dx$$

```
[In] integrate(log(a*sech(x)^2),x, algorithm="giac")
[Out] integrate(log(a*sech(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a\text{sech}^2(x)\right) \, dx = -\int 2 \,\ln\left(\cosh(x)\right) - \ln\left(a\right) \, dx$$

```
[In] int(log(a/cosh(x)^2),x)
[Out] -int(2*log(cosh(x)) - log(a), x)
```

3.217 $\int \log (a\text{sech}^n(x)) \, dx$

Optimal result 1194
Rubi [A] (verified) 1194
Mathematica [A] (verified) 1196
Maple [F] 1196
Fricas [C] (verification not implemented) 1196
Sympy [F] 1197
Maxima [A] (verification not implemented) 1197
Giac [F] 1197
Mupad [F(-1)] 1197

Optimal result

Integrand size = 7, antiderivative size = 43

$$\int \log (a\text{sech}^n(x)) \, dx = -\frac{nx^2}{2} + nx \log \left(1+e^{2x}\right) + x \log (a\text{sech}^n(x)) + \frac{1}{2} n \operatorname{PolyLog}\left(2,-e^{2x}\right)$$

```
[Out] -1/2*n*x^2+n*x*ln(1+exp(2*x))+x*ln(a*sech(x)^n)+1/2*n*polylog(2,-exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3799, 2221, 2317, 2438}

$$\int \log (a\text{sech}^n(x)) \, dx = x \log (a\text{sech}^n(x)) + \frac{1}{2} n \operatorname{PolyLog}\left(2,-e^{2x}\right) - \frac{nx^2}{2} + nx \log \left(e^{2x}+1\right)$$

```
[In] Int[Log[a*Sech[x]^n],x]
```

```
[Out] -1/2*(n*x^2) + n*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^n] + (n*PolyLog[2, -E
^(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(a\operatorname{sech}^n(x)\right)+\int nx\tanh(x)\,dx\\
&= x\log\left(a\operatorname{sech}^n(x)\right)+n\int x\tanh(x)\,dx\\
&= -\frac{nx^2}{2}+x\log\left(a\operatorname{sech}^n(x)\right)+(2n)\int\frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\operatorname{sech}^n(x)\right)-n\int\log\left(1+e^{2x}\right)\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\operatorname{sech}^n(x)\right)-\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= -\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\operatorname{sech}^n(x)\right)+\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \log \left(a\text{sech}^n(x)\right) \, dx=\frac{nx^2}{2}+nx\log \left(1+e^{-2x}\right)+x\log \left(a\text{sech}^n(x)\right)-\frac{1}{2}n\,\text{PolyLog}\left(2,-e^{-2x}\right)$$

```
[In] Integrate[Log[a*Sech[x]^n],x]
[Out] (n*x^2)/2 + n*x*Log[1 + E^(-2*x)] + x*Log[a*Sech[x]^n] - (n*PolyLog[2, -E^(
-2*x)])/2
```

Maple [F]

$$\int \ln \left(a\,\text{sech}\left(x\right)^{n}\right) dx$$

```
[In] int(ln(a*sech(x)^n),x)
[Out] int(ln(a*sech(x)^n),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \log \left(a\text{sech}^n(x)\right) \, dx=-\frac{1}{2}\,nx^2+nx\log \left(\frac{2\left(\cosh \left(x\right)+\sinh \left(x\right)\right)}{\cosh \left(x\right)^2+2\,\cosh \left(x\right)\sinh \left(x\right)+\sinh \left(x\right)^2+1}\right)+nx\log \left(i\,\cosh \left(x\right)+i\,\sinh \left(x\right)+1\right)+nx\log \left(-i\,\cosh \left(x\right)-i\,\sinh \left(x\right)+1\right)+n\text{Li}_2(i\,\cosh \left(x\right)+i\,\sinh \left(x\right))+n\text{Li}_2(-i\,\cosh \left(x\right)-i\,\sinh \left(x\right))+x\log \left(a\right)$$

```
[In] integrate(log(a*sech(x)^n),x, algorithm="fricas")
[Out] -1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) + n*x*log(I*cosh(x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x)
- I*sinh(x) + 1) + n*dilog(I*cosh(x) + I*sinh(x)) + n*dilog(-I*cosh(x) - I*
sinh(x)) + x*log(a)
```

Sympy [F]

$$\int \log \left(a\text{sech}^n(x)\right)\,dx = \int \log \left(a\,\text{sech}^n\left(x\right)\right)dx$$

```
[In] integrate(ln(a*sech(x)**n),x)
[Out] Integral(log(a*sech(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \log \left(a\text{sech}^n(x)\right)\,dx = -\frac{1}{2}\left(x^2 - 2\,x\log\left(e^{(2\,x)} + 1\right) - \text{Li}_2\left(-e^{(2\,x)}\right)\right)n + x\log\left(a\,\text{sech}\left(x\right)^n\right)$$

```
[In] integrate(log(a*sech(x)^n),x, algorithm="maxima")
[Out] -1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*sech(x)^n)
```

Giac [F]

$$\int \log \left(a\text{sech}^n(x)\right)\,dx = \int \log \left(a\,\text{sech}\left(x\right)^n\right)\,dx$$

```
[In] integrate(log(a*sech(x)^n),x, algorithm="giac")
[Out] integrate(log(a*sech(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a\text{sech}^n(x)\right)\,dx = \int \ln\left(a\left(\frac{1}{\cosh\left(x\right)}\right)^n\right)\,dx$$

```
[In] int(log(a*(1/cosh(x))^n),x)
[Out] int(log(a*(1/cosh(x))^n), x)
```

3.218 $\int \log(a\operatorname{csch}(x))\,dx$

Optimal result 1198
Rubi [A] (verified) 1198
Mathematica [A] (verified) 1199
Maple [C] (warning: unable to verify) 1200
Fricas [B] (verification not implemented) 1200
Sympy [F] 1201
Maxima [A] (verification not implemented) 1201
Giac [F] 1201
Mupad [F(-1)] 1201

Optimal result

Integrand size = 5, antiderivative size = 38

$$\int \log(a\operatorname{csch}(x))\,dx = -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) + \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2}$$

```
[Out] -1/2*x^2+x*ln(1-exp(2*x))+x*ln(a*csch(x))+1/2*polylog(2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2628, 3797, 2221, 2317, 2438}

$$\int \log(a\operatorname{csch}(x))\,dx = x\log(a\operatorname{csch}(x)) + \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)}{2} - \frac{x^2}{2} + x\log\left(1-e^{2x}\right)$$

```
[In] Int[Log[a*Csch[x]],x]
[Out] -1/2*x^2 + x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]] + PolyLog[2, E^(2*x)]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log(a\text{csch}(x)) + \int x\coth(x)\,dx \\
&= -\frac{x^2}{2} + x\log(a\text{csch}(x)) - 2\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\text{csch}(x)) - \int \log\left(1-e^{2x}\right)\,dx \\
&= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\text{csch}(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\text{csch}(x)) + \frac{\text{Li}_2(e^{2x})}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \log(a\text{csch}(x))\,dx = \frac{x^2}{2} + x\log\left(1-e^{-2x}\right) + x\log(a\text{csch}(x)) - \frac{1}{2}\text{PolyLog}\left(2, e^{-2x}\right)$$

```
[In] Integrate[Log[a*Csch[x]],x]
[Out] x^2/2 + x*Log[1 - E^(-2*x)] + x*Log[a*Csch[x]] - PolyLog[2, E^(-2*x)]/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.81 (sec) , antiderivative size = 293, normalized size of antiderivative = 7.71

method	result
risch	$x\ln\left(\mathrm{e}^{x}\right)+\frac{i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{x}}{-1+\mathrm{e}^{2x}}\right)\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{x}}{-1+\mathrm{e}^{2x}}\right)^{2}x}{2}+\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{x}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{x}}{-1+\mathrm{e}^{2x}}\right)^{2}x}{2}-\frac{i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{x}}{-1+\mathrm{e}^{2x}}\right)\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{x}}{-1+\mathrm{e}^{2x}}\right)\mathrm{csgn}(ia}{2}$

```
[In] int(ln(a*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(exp(x))+1/2*I*Pi*csgn(I*exp(x)/(-1+exp(2*x)))*csgn(I*a/(-1+exp(2*x))*e
xp(x))^2*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I*exp(x)/(-1+exp(2*x)))^2*x-1/2*I*P
i*csgn(I*exp(x)/(-1+exp(2*x)))*csgn(I*a/(-1+exp(2*x))*exp(x))*csgn(I*a)*x+1
/2*I*Pi*csgn(I*a/(-1+exp(2*x))*exp(x))^2*csgn(I*a)*x-1/2*I*Pi*csgn(I*a/(-1+
exp(2*x))*exp(x))^3*x-1/2*I*Pi*csgn(I*exp(x))*csgn(I/(-1+exp(2*x)))*csgn(I*
exp(x)/(-1+exp(2*x)))*x+x*ln(2)+ln(a)*x-1/2*x^2+1/2*I*Pi*csgn(I/(-1+exp(2*x
)))*csgn(I*exp(x)/(-1+exp(2*x)))^2*x-1/2*I*Pi*csgn(I*exp(x)/(-1+exp(2*x)))^
3*x-ln(exp(x))*ln(-1+exp(2*x))+dilog(1+exp(x))+ln(exp(x))*ln(1+exp(x))-dilo
g(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(31) = 62.

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\begin{aligned}\int \log(a\mathrm{csch}(x))\,dx = &-\frac{1}{2}x^{2}+x\log\left(\frac{2\,(a\cosh(x)+a\sinh(x))}{\cosh(x)^{2}+2\,\cosh(x)\sinh(x)+\sinh(x)^{2}-1}\right)\\&+x\log(\cosh(x)+\sinh(x)+1)+x\log(-\cosh(x)-\sinh(x)+1)\\&+\mathrm{Li}_2(\cosh(x)+\sinh(x))+\mathrm{Li}_2(-\cosh(x)-\sinh(x))\end{aligned}$$

```
[In] integrate(log(a*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)) + x*log(cosh(x) + sinh(x) + 1) + x*log(-cosh(x) - sinh(x)
+ 1) + dilog(cosh(x) + sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log(a\text{csch}(x))\,dx = \int \log(a\,\text{csch}\,(x))\,dx$$

```
[In] integrate(ln(a*csch(x)),x)
[Out] Integral(log(a*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \log(a\text{csch}(x))\,dx = -\frac{1}{2}x^2 + x\log(a\,\text{csch}\,(x)) + x\log(e^x+1) + x\log(-e^x+1) + \text{Li}_2(-e^x) + \text{Li}_2(e^x)$$

```
[In] integrate(log(a*csch(x)),x, algorithm="maxima")
[Out] -1/2*x^2 + x*log(a*csch(x)) + x*log(e^x + 1) + x*log(-e^x + 1) + dilog(-e^x
) + dilog(e^x)
```

Giac [F]

$$\int \log(a\text{csch}(x))\,dx = \int \log(a\,\text{csch}\,(x))\,dx$$

```
[In] integrate(log(a*csch(x)),x, algorithm="giac")
[Out] integrate(log(a*csch(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(a\text{csch}(x))\,dx = \int \ln\left(\frac{a}{\sinh(x)}\right)\,dx$$

```
[In] int(log(a/sinh(x)),x)
[Out] int(log(a/sinh(x)), x)
```

3.219 $\int \log(a\text{csch}^2(x))\,dx$

Optimal result 1202
Rubi [A] (verified) 1202
Mathematica [A] (verified) 1204
Maple [C] (warning: unable to verify) 1204
Fricas [B] (verification not implemented) 1205
Sympy [F] 1205
Maxima [A] (verification not implemented) 1205
Giac [F] 1206
Mupad [F(-1)] 1206

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \log\left(a\text{csch}^2(x)\right)\,dx = -x^2 + 2x\log\left(1-e^{2x}\right) + x\log\left(a\text{csch}^2(x)\right) + \text{PolyLog}\left(2,e^{2x}\right)$$

```
[Out] -x^2+2*x*ln(1-exp(2*x))+x*ln(a*csch(x)^2)+polylog(2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3797, 2221, 2317, 2438}

$$\int \log\left(a\text{csch}^2(x)\right)\,dx = x\log\left(a\text{csch}^2(x)\right) + \text{PolyLog}\left(2,e^{2x}\right) - x^2 + 2x\log\left(1-e^{2x}\right)$$

```
[In] Int[Log[a*Csch[x]^2],x]
```

```
[Out] -x^2 + 2*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^2] + PolyLog[2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(a\text{csch}^2(x)\right) - \int -2x\coth(x)\,dx \\
&= x\log\left(a\text{csch}^2(x)\right) + 2\int x\coth(x)\,dx \\
&= -x^2 + x\log\left(a\text{csch}^2(x)\right) - 4\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= -x^2 + 2x\log\left(1-e^{2x}\right) + x\log\left(a\text{csch}^2(x)\right) - 2\int \log\left(1-e^{2x}\right)\,dx \\
&= -x^2 + 2x\log\left(1-e^{2x}\right) + x\log\left(a\text{csch}^2(x)\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= -x^2 + 2x\log\left(1-e^{2x}\right) + x\log\left(a\text{csch}^2(x)\right) + \text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(a\text{csch}^2(x)\right) \, dx = x\left(x + 2\log \left(1 - e^{-2x}\right) + \log \left(a\text{csch}^2(x)\right)\right) - \text{PolyLog}\left(2, e^{-2x}\right)$$

```
[In] Integrate[Log[a*Csch[x]^2],x]
[Out] x*(x + 2*Log[1 - E^(-2*x)] + Log[a*Csch[x]^2]) - PolyLog[2, E^(-2*x)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.44 (sec) , antiderivative size = 456, normalized size of antiderivative = 13.03

method	result
risch	$\ln(a)\, x - \frac{i\pi\, \text{csgn}(i\text{e}^{x})^2\, \text{csgn}(i\text{e}^{2x})\, x}{2} - \frac{i\pi\, \text{csgn}\left(\frac{ia\, \text{e}^{2x}}{\left(-1+\text{e}^{2x}\right)^2}\right)^3 x}{2} + \frac{i\pi \text{csgn}\left(i\left(-1+\text{e}^{2x}\right)\right)^2 \text{csgn}\left(i\left(-1+\text{e}^{2x}\right)^2\right) x}{2} + \frac{i\pi\ \text{csgn}\left(\right.}{}$

```
[In] int(ln(a*csch(x)^2),x,method=_RETURNVERBOSE)
[Out] ln(a)*x-1/2*I*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x))*x-1/2*I*Pi*csgn(I*a/(-1+
exp(2*x))^2*exp(2*x))^3*x+1/2*I*Pi*csgn(I*(-1+exp(2*x)))^2*csgn(I*(-1+exp(2
*x))^2)*x+1/2*I*Pi*csgn(I*exp(2*x)/(-1+exp(2*x))^2)*csgn(I*a/(-1+exp(2*x))^
2*exp(2*x))^2*x+1/2*I*Pi*csgn(I*exp(2*x))*csgn(I*exp(2*x)/(-1+exp(2*x))^2)^
2*x-1/2*I*Pi*csgn(I*exp(2*x))^3*x+2*x*ln(2)-2*dilog(exp(x))-x^2+1/2*I*Pi*cs
gn(I/(-1+exp(2*x))^2)*csgn(I*exp(2*x)/(-1+exp(2*x))^2)^2*x-I*Pi*csgn(I*(-1+
exp(2*x)))*csgn(I*(-1+exp(2*x))^2)^2*x-1/2*I*Pi*csgn(I*exp(2*x)/(-1+exp(2*x
))^2)*csgn(I*a/(-1+exp(2*x))^2*exp(2*x))*csgn(I*a)*x+1/2*I*Pi*csgn(I*a/(-1+
exp(2*x))^2*exp(2*x))^2*csgn(I*a)*x+I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2*
x+2*dilog(1+exp(x))-1/2*I*Pi*csgn(I*exp(2*x)/(-1+exp(2*x))^2)^3*x-1/2*I*Pi*
csgn(I*exp(2*x))*csgn(I/(-1+exp(2*x))^2)*csgn(I*exp(2*x)/(-1+exp(2*x))^2)*x
+1/2*I*Pi*csgn(I*(-1+exp(2*x))^2)^3*x+2*ln(exp(x))*ln(1+exp(x))+2*x*ln(exp(
x))-2*ln(exp(x))*ln(-1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(32) = 64.

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

$$\int \log \left(a\text{csch}^2(x)\right) dx = -x^2 + x \log \left(\frac{4 \left(a \cosh(x) + a \sinh(x)\right)}{\cosh(x)^3 + 3 \cosh(x) \sinh(x)^2 + \sinh(x)^3 + 3 \left(\cosh(x)^2 - 1\right) \sinh(x) - \cosh(x)}\right) + 2\,x \log\left(\cosh(x) + \sinh(x) + 1\right) + 2\,x \log\left(-\cosh(x) - \sinh(x) + 1\right) + 2\,\text{Li}_2(\cosh(x) + \sinh(x)) + 2\,\text{Li}_2(-\cosh(x) - \sinh(x))$$

```
[In] integrate(log(a*csch(x)^2),x, algorithm="fricas")
[Out] -x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + s
inh(x)^3 + 3*(cosh(x)^2 - 1)*sinh(x) - cosh(x))) + 2*x*log(cosh(x) + sinh(x
) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cosh(x) + sinh(x)) + 2*d
ilog(-cosh(x) - sinh(x))
```

Sympy [F]

$$\int \log \left(a\text{csch}^2(x)\right) dx = \int \log \left(a \operatorname{csch}^2(x)\right) dx$$

```
[In] integrate(ln(a*csch(x)**2),x)
[Out] Integral(log(a*csch(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \log \left(a\text{csch}^2(x)\right) dx = -x^2 + x \log\left(a \operatorname{csch}(x)^2\right) + 2\,x \log\left(e^x + 1\right) + 2\,x \log\left(-e^x + 1\right) + 2\,\text{Li}_2(-e^x) + 2\,\text{Li}_2(e^x)$$

```
[In] integrate(log(a*csch(x)^2),x, algorithm="maxima")
[Out] -x^2 + x*log(a*csch(x)^2) + 2*x*log(e^x + 1) + 2*x*log(-e^x + 1) + 2*dilog(
-e^x) + 2*dilog(e^x)
```

Giac [F]

$$\int \log \left(a\text{csch}^2(x)\right) \, dx = \int \log \left(a \operatorname{csch}\left(x\right)^2\right) \, dx$$

```
[In] integrate(log(a*csch(x)^2),x, algorithm="giac")
[Out] integrate(log(a*csch(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a\text{csch}^2(x)\right) \, dx = \int \ln \left(\frac{a}{\sinh\left(x\right)^2}\right) \, dx$$

```
[In] int(log(a/sinh(x)^2),x)
[Out] int(log(a/sinh(x)^2), x)
```

3.220 $\int \log \left(a \operatorname{csch}^n(x)\right) dx$

Optimal result 1207
Rubi [A] (verified) 1207
Mathematica [A] (verified) 1209
Maple [F] 1209
Fricas [B] (verification not implemented) 1209
Sympy [F] 1210
Maxima [A] (verification not implemented) 1210
Giac [F] 1210
Mupad [F(-1)] 1210

Optimal result

Integrand size = 7, antiderivative size = 43

$$\int \log \left(a \operatorname{csch}^n(x)\right) dx = -\frac{nx^2}{2} + nx \log \left(1 - e^{2x}\right) + x \log \left(a \operatorname{csch}^n(x)\right) + \frac{1}{2} n \operatorname{PolyLog}\left(2, e^{2x}\right)$$

```
[Out] -1/2*n*x^2+n*x*ln(1-exp(2*x))+x*ln(a*csch(x)^n)+1/2*n*polylog(2,exp(2*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3797, 2221, 2317, 2438}

$$\int \log \left(a \operatorname{csch}^n(x)\right) dx = x \log \left(a \operatorname{csch}^n(x)\right) + \frac{1}{2} n \operatorname{PolyLog}\left(2, e^{2x}\right) - \frac{nx^2}{2} + nx \log \left(1 - e^{2x}\right)$$

```
[In] Int[Log[a*Csch[x]^n],x]
```

```
[Out] -1/2*(n*x^2) + n*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^n] + (n*PolyLog[2, E^
(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi)))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(\operatorname{acsch}^n(x)\right)+\int nx\coth(x)\,dx\\
&= x\log\left(\operatorname{acsch}^n(x)\right)+n\int x\coth(x)\,dx\\
&= -\frac{nx^2}{2}+x\log\left(\operatorname{acsch}^n(x)\right)-(2n)\int\frac{e^{2x}x}{1-e^{2x}}\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(\operatorname{acsch}^n(x)\right)-n\int\log\left(1-e^{2x}\right)\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(\operatorname{acsch}^n(x)\right)-\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(\operatorname{acsch}^n(x)\right)+\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \log \left(a\text{csch}^n(x)\right) \, dx=\frac{nx^2}{2}+nx\log \left(1-e^{-2x}\right)+x\log \left(a\text{csch}^n(x)\right)-\frac{1}{2}n\,\text{PolyLog}\left(2,e^{-2x}\right)$$

```
[In] Integrate[Log[a*Csch[x]^n],x]
[Out] (n*x^2)/2 + n*x*Log[1 - E^(-2*x)] + x*Log[a*Csch[x]^n] - (n*PolyLog[2, E^(-
2*x)])/2
```

Maple [F]

$$\int \ln \left(a\,\text{csch}\,(x)^{n}\right) dx$$

```
[In] int(ln(a*csch(x)^n),x)
[Out] int(ln(a*csch(x)^n),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \log \left(a\text{csch}^n(x)\right) \, dx=-\frac{1}{2}nx^2+nx\log \left(\frac{2\left(\cosh (x)+\sinh (x)\right)}{\cosh (x)^2+2\cosh (x)\sinh (x)+\sinh (x)^2-1}\right)+nx\log \left(\cosh (x)+\sinh (x)+1\right)+nx\log \left(-\cosh (x)-\sinh (x)+1\right)+n\text{Li}_2(\cosh (x)+\sinh (x))+n\text{Li}_2(-\cosh (x)-\sinh (x))+x\log (a)$$

```
[In] integrate(log(a*csch(x)^n),x, algorithm="fricas")
[Out] -1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)) + n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(-cosh(x) - sinh
(x) + 1) + n*dilog(cosh(x) + sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

Sympy [F]

$$\int \log \left(a\text{csch}^n(x)\right) \, dx = \int \log \left(a \operatorname{csch}^{n}{\left(x \right)}\right)\, dx$$

```
[In] integrate(ln(a*csch(x)**n),x)
[Out] Integral(log(a*csch(x)**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \log \left(a\text{csch}^n(x)\right) \, dx = -\frac{1}{2} \, {\left(x^{2} - 2 \, x \log \left(e^{x} + 1\right) - 2 \, x \log \left(-e^{x} + 1\right) - 2 \, {\rm Li}_2\left(-e^{x}\right) - 2 \, {\rm Li}_2\left(e^{x}\right)\right)} n + x \log \left(a \operatorname{csch}\left(x\right)^{n}\right)$$

```
[In] integrate(log(a*csch(x)^n),x, algorithm="maxima")
[Out] -1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog(
e^x))*n + x*log(a*csch(x)^n)
```

Giac [F]

$$\int \log \left(a\text{csch}^n(x)\right) \, dx = \int { \log \left(a \operatorname{csch}\left(x\right)^{n}\right) \,d x }$$

```
[In] integrate(log(a*csch(x)^n),x, algorithm="giac")
[Out] integrate(log(a*csch(x)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(a\text{csch}^n(x)\right) \, dx = \int \ln\left(a\,{\left(\frac{1}{\mathrm{sinh}\left(x\right)}\right)}^n\right) \,d x$$

```
[In] int(log(a*(1/sinh(x))^n),x)
[Out] int(log(a*(1/sinh(x))^n), x)
```

3.221 $\int \cosh(a+bx) \log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right) \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$

Optimal result 1211
Rubi [A] (verified) 1211
Mathematica [A] (verified) 1212
Maple [A] (verified) 1212
Fricas [B] (verification not implemented) 1213
Sympy [F] 1213
Maxima [B] (verification not implemented) 1213
Giac [B] (verification not implemented) 1214
Mupad [B] (verification not implemented) 1214

Optimal result

Integrand size = 35, antiderivative size = 50

$$\int \cosh(a+bx) \log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right) \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = -\frac{\sinh(a+bx)}{b}+\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sinh(a+bx)}{b}$$

```
[Out] -sinh(b*x+a)/b+ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x))*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2717, 2634}

$$\int \cosh(a+bx) \log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right) \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = \frac{\sinh(a+bx)\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sinh(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sinh[a + b*x]/b) + (Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]]*Sinh[a +
 b*x])/b
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sinh(a+bx)}{b} - \int \cosh(a+bx)\,dx$$
$$= -\frac{\sinh(a+bx)}{b} + \frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sinh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx$$
$$= -\frac{\sinh(a+bx)}{b} + \frac{\log\left(\frac{1}{2}\sinh(a+bx)\right)\sinh(a+bx)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
[Out] -(Sinh[a + b*x]/b) + (Log[Sinh[a + b*x]/2]*Sinh[a + b*x])/b
```

Maple [A] (verified)

Time = 9.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\ln\left(\frac{\sinh(bx+a)}{2}\right)\sinh(bx+a)-\sinh(bx+a)}{b}$	30
risch	Expression too large to display	1098

```
[In] int(cosh(b*x+a)*ln(cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)),x,method=_RETUR
NVERBOSE)
[Out] 1/b*(ln(1/2*sinh(b*x+a))*sinh(b*x+a)-sinh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(42) = 84.

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 5.16

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = -\frac{\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^4+4\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^3\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+6\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2+4}{}$$

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="fricas")
```

```
[Out] -1/2*(cosh(1/2*b*x + 1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1/
2*a) + 6*cosh(1/2*b*x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x +
 1/2*a)*sinh(1/2*b*x + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - (cosh(1/2*b*x +
 1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1/2*a) + 6*cosh(1/2*b*
x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x + 1/2*a)*sinh(1/2*b*x
 + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - 1)*log(cosh(1/2*b*x + 1/2*a)*sinh(1
/2*b*x + 1/2*a)) - 1)/(b*cosh(1/2*b*x + 1/2*a)^2 + 2*b*cosh(1/2*b*x + 1/2*a
)*sinh(1/2*b*x + 1/2*a) + b*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F]

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = \int \log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\cosh\left(a+bx\right)dx$$

```
[In] integrate(cosh(b*x+a)*ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x)
```

```
[Out] Integral(log(sinh(a/2 + b*x/2)*cosh(a/2 + b*x/2))*cosh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(42) = 84.

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.24

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx = \frac{\log\left(\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sinh\left(bx+a\right)}{b} - \frac{b\left(\frac{2\,(bx+a)}{b}+\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)-b\left(\frac{2\,(bx+a)}{b}-\frac{e^{(bx+a)}}{b}+\frac{e^{(-bx-a)}}{b}\right)}{4\,b}$$

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="maxima")
```

```
[Out] log(cosh(1/2*b*x + 1/2*a)*sinh(1/2*b*x + 1/2*a))*sinh(b*x + a)/b - 1/4*(b*(
2*(b*x + a)/b + e^(b*x + a)/b - e^(-b*x - a)/b) - b*(2*(b*x + a)/b - e^(b*x
 + a)/b + e^(-b*x - a)/b))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.

Time = 0.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.88

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$$
$$=\frac{1}{2}\left(\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)\log\left(\frac{1}{4}\left(e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)\left(e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)\right)-\frac{e^{(bx+a)}-e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(1/4*(e^(1/2*b*x + 1/2*a) + e^(-1/2
*b*x - 1/2*a))*(e^(1/2*b*x + 1/2*a) - e^(-1/2*b*x - 1/2*a))) - 1/2*(e^(b*x
+ a) - e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx=\frac{\ln\left(\frac{\sinh(a+b\,x)}{2}\right)\,\sinh(a+b\,x)}{b}-\frac{\sinh(a+b\,x)}{b}$$

```
[In] int(log(cosh(a/2 + (b*x)/2)*sinh(a/2 + (b*x)/2))*cosh(a + b*x),x)
```

```
[Out] (log(sinh(a + b*x)/2)*sinh(a + b*x))/b - sinh(a + b*x)/b
```

3.222 $\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx$

Optimal result 1215
Rubi [A] (verified) 1215
Mathematica [A] (verified) 1216
Maple [A] (verified) 1216
Fricas [B] (verification not implemented) 1217
Sympy [A] (verification not implemented) 1217
Maxima [A] (verification not implemented) 1217
Giac [B] (verification not implemented) 1218
Mupad [B] (verification not implemented) 1218

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx = -2\cosh(x)+\cosh(x)\log\left(\cosh^2(x)\right)$$

```
[Out] -2*cosh(x)+cosh(x)*ln(cosh(x)^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2718, 2634, 12}

$$\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx = \cosh(x)\log\left(\cosh^2(x)\right)-2\cosh(x)$$

```
[In] Int[Log[Cosh[x]^2]*Sinh[x],x]
```

```
[Out] -2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(x)\log\left(\cosh^2(x)\right) - \int 2\sinh(x)\,dx\\ &= \cosh(x)\log\left(\cosh^2(x)\right) - 2\int \sinh(x)\,dx\\ &= -2\cosh(x) + \cosh(x)\log\left(\cosh^2(x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx = -2\cosh(x) + \cosh(x)\log\left(\cosh^2(x)\right)$$

```
[In] Integrate[Log[Cosh[x]^2]*Sinh[x],x]
[Out] -2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Maple [A] (verified)

Time = 141.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-2\cosh(x) + \cosh(x)\ln\left(\cosh^2(x)\right)$
default	$-2\cosh(x) + \cosh(x)\ln\left(\cosh^2(x)\right)$
risch	$-(1+e^{2x})e^{-x}\ln(e^x) + \frac{\left(-4-4e^{2x}-4\ln(2)+4\ln(1+e^{2x})+4\ln(1+e^{2x})e^{2x}+i\pi\,\text{csgn}\left(i(1+e^{2x})^2\right)\text{csgn}\left(ie^{-2x}(1\right.\right.}{}$

```
[In] int(ln(cosh(x)^2)*sinh(x),x,method=_RETURNVERBOSE)
[Out] -2*cosh(x)+cosh(x)*ln(cosh(x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(13) = 26.

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 4.77

$$\int \log \left(\cosh^2(x)\right) \sinh(x)\, dx = -\frac{2 \cosh \left(x\right)^{2} - \left(\cosh \left(x\right)^{2} + 2 \cosh \left(x\right) \sinh \left(x\right) + \sinh \left(x\right)^{2} + 1\right) \log \left(\frac{1}{2} \cosh \left(x\right)^{2} + \frac{1}{2} \sinh \left(x\right)^{2} + \frac{1}{2}\right) + 4}{2 \left(\cosh \left(x\right) + \sinh \left(x\right)\right)}$$

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="fricas")
[Out] -1/2*(2*cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(1/2
*cosh(x)^2 + 1/2*sinh(x)^2 + 1/2) + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + 2)/(c
osh(x) + sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \log \left(\cosh^2(x)\right) \sinh(x)\, dx = \log \left(\cosh^{2}\left(x\right)\right) \cosh \left(x\right) - 2 \cosh \left(x\right)$$

```
[In] integrate(ln(cosh(x)**2)*sinh(x),x)
[Out] log(cosh(x)**2)*cosh(x) - 2*cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \log \left(\cosh^2(x)\right) \sinh(x)\, dx = 2 \cosh \left(x\right) \log \left(\cosh \left(x\right)\right) - 2 \cosh \left(x\right)$$

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="maxima")
[Out] 2*cosh(x)*log(cosh(x)) - 2*cosh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.85

$$\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx = \left(e^{(2x)}+1\right)e^{(-x)}\log\left(\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}\right) - \left(e^{(2x)}+1\right)e^{(-x)}$$

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="giac")
[Out] (e^(2*x) + 1)*e^(-x)*log(1/2*(e^(2*x) + 1)*e^(-x)) - (e^(2*x) + 1)*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx = 2\cosh(x)\ \left(\ln\left(\cosh(x)\right)-1\right)$$

```
[In] int(log(cosh(x)^2)*sinh(x),x)
[Out] 2*cosh(x)*(log(cosh(x)) - 1)
```

3.223 $\int \frac{\log(x)}{\sqrt{x}} \, dx$

Optimal result 1219
Rubi [A] (verified) 1219
Mathematica [A] (verified) 1220
Maple [A] (verified) 1220
Fricas [A] (verification not implemented) 1220
Sympy [B] (verification not implemented) 1221
Maxima [A] (verification not implemented) 1221
Giac [A] (verification not implemented) 1221
Mupad [B] (verification not implemented) 1222

Optimal result

Integrand size = 8, antiderivative size = 17

$$\int \frac{\log(x)}{\sqrt{x}} \, dx = -4\sqrt{x} + 2\sqrt{x}\log(x)$$

```
[Out] -4*x^(1/2)+2*ln(x)*x^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2341}

$$\int \frac{\log(x)}{\sqrt{x}} \, dx = 2\sqrt{x}\log(x) - 4\sqrt{x}$$

```
[In] Int[Log[x]/Sqrt[x],x]
[Out] -4*Sqrt[x] + 2*Sqrt[x]*Log[x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -4\sqrt{x} + 2\sqrt{x}\log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\sqrt{x}(-2+\log(x))$$

```
[In] Integrate[Log[x]/Sqrt[x],x]
[Out] 2*Sqrt[x]*(-2 + Log[x])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-4\sqrt{x}+2\ln\left(x\right)\sqrt{x}$	14
default	$-4\sqrt{x}+2\ln\left(x\right)\sqrt{x}$	14
risch	$-4\sqrt{x}+2\ln\left(x\right)\sqrt{x}$	14

```
[In] int(ln(x)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -4*x^(1/2)+2*ln(x)*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\,\sqrt{x}(\log\left(x\right)-2)$$

```
[In] integrate(log(x)/x^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x)*(log(x) - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(15) = 30$.

Time = 0.81 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.53

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = \begin{cases} -2\sqrt{x}\log\left(\frac{1}{x}\right) + 2\sqrt{x}\log(x) - 8\sqrt{x} & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ 2\sqrt{x}\log(x) - 4\sqrt{x} & \text{for } |x| < 1 \\ -2\sqrt{x}\log\left(\frac{1}{x}\right) - 4\sqrt{x} & \text{for } \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{3}{2}, \frac{3}{2} \\ \frac{1}{2}, \frac{1}{2} & 0 \end{matrix} \middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{3}{2}, \frac{3}{2}, 1 & \\ & \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(x)/x**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(x)*log(1/x) + 2*sqrt(x)*log(x) - 8*sqrt(x), (Abs(x) < 1)
 & (1/Abs(x) < 1)), (2*sqrt(x)*log(x) - 4*sqrt(x), Abs(x) < 1), (-2*sqrt(x)
*log(1/x) - 4*sqrt(x), 1/Abs(x) < 1), (-meijerg(((1,), (3/2, 3/2)), ((1/2,
1/2), (0,)), x) + meijerg(((3/2, 3/2, 1), ()), ((), (1/2, 1/2, 0)), x), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\sqrt{x}\log(x) - 4\sqrt{x}$$

```
[In] integrate(log(x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*log(x) - 4*sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\sqrt{x}\log(x) - 4\sqrt{x}$$

```
[In] integrate(log(x)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*log(x) - 4*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \frac{\log(x)}{\sqrt{x}}\,dx = 2\sqrt{x}\,(\ln(x) - 2)$$

```
[In] int(log(x)/x^(1/2),x)
[Out] 2*x^(1/2)*(log(x) - 2)
```

3.224 $\int x \log(2-3x^2)\,dx$

Optimal result 1223
Rubi [A] (verified) 1223
Mathematica [A] (verified) 1224
Maple [A] (verified) 1224
Fricas [A] (verification not implemented) 1225
Sympy [A] (verification not implemented) 1225
Maxima [A] (verification not implemented) 1225
Giac [A] (verification not implemented) 1226
Mupad [B] (verification not implemented) 1226

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int x \log(2-3x^2)\,dx = -\frac{x^2}{2} - \frac{1}{6}(2-3x^2)\log(2-3x^2)$$

```
[Out] -1/2*x^2-1/6*(-3*x^2+2)*ln(-3*x^2+2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2504, 2436, 2332}

$$\int x \log(2-3x^2)\,dx = -\frac{x^2}{2} - \frac{1}{6}(2-3x^2)\log(2-3x^2)$$

```
[In] Int[x*Log[2 - 3*x^2],x]
```

```
[Out] -1/2*x^2 - ((2 - 3*x^2)*Log[2 - 3*x^2])/6
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \log(2-3x)\,dx, x, x^2\right)\\ &= -\left(\frac{1}{6}\text{Subst}\left(\int \log(x)\,dx, x, 2-3x^2\right)\right)\\ &= -\frac{x^2}{2}-\frac{1}{6}\left(2-3x^2\right)\log\left(2-3x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x\log\left(2-3x^2\right)\,dx=\frac{1}{6}\left(-3x^2+\left(-2+3x^2\right)\log\left(2-3x^2\right)\right)$$

```
[In] Integrate[x*Log[2 - 3*x^2],x]
[Out] (-3*x^2 + (-2 + 3*x^2)*Log[2 - 3*x^2])/6
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{(-3x^2+2)\ln(-3x^2+2)}{6}-\frac{x^2}{2}+\frac{1}{3}$	25
default	$-\frac{(-3x^2+2)\ln(-3x^2+2)}{6}-\frac{x^2}{2}+\frac{1}{3}$	25
norman	$-\frac{x^2}{2}+\frac{\ln(-3x^2+2)x^2}{2}-\frac{\ln(-3x^2+2)}{3}$	30
risch	$\frac{\ln(-3x^2+2)x^2}{2}-\frac{x^2}{2}-\frac{\ln(3x^2-2)}{3}$	30
parts	$\frac{\ln(-3x^2+2)x^2}{2}-\frac{x^2}{2}-\frac{\ln(3x^2-2)}{3}$	30
parallelrisch	$\frac{\ln(-3x^2+2)x^2}{2}-\frac{1}{3}-\frac{x^2}{2}-\frac{\ln(-3x^2+2)}{3}$	31

```
[In] int(x*ln(-3*x^2+2),x,method=_RETURNVERBOSE)
[Out] -1/6*(-3*x^2+2)*ln(-3*x^2+2)-1/2*x^2+1/3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x\log\left(2-3x^2\right)\,dx = -\frac{1}{2}\,x^2+\frac{1}{6}\left(3\,x^2-2\right)\log\left(-3\,x^2+2\right)$$

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="fricas")
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x\log\left(2-3x^2\right)\,dx = \frac{x^2\log\left(2-3x^2\right)}{2}-\frac{x^2}{2}-\frac{\log\left(3x^2-2\right)}{3}$$

```
[In] integrate(x*ln(-3*x**2+2),x)
[Out] x**2*log(2 - 3*x**2)/2 - x**2/2 - log(3*x**2 - 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x\log\left(2-3x^2\right)\,dx = -\frac{1}{2}\,x^2+\frac{1}{6}\left(3\,x^2-2\right)\log\left(-3\,x^2+2\right)+\frac{1}{3}$$

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="maxima")
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x \log \left(2-3 x^{2}\right) \, dx=-\frac{1}{2} \, x^{2}+\frac{1}{6} \, \left(3 \, x^{2}-2\right) \log \left(-3 \, x^{2}+2\right)+\frac{1}{3}$$

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="giac")
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x \log \left(2-3 x^{2}\right) \, dx=x^{2} \left(\frac{\ln \left(2-3 \, x^{2}\right)}{2}-\frac{1}{2}\right)-\frac{\ln \left(x^{2}-\frac{2}{3}\right)}{3}$$

```
[In] int(x*log(2 - 3*x^2),x)
[Out] x^2*(log(2 - 3*x^2)/2 - 1/2) - log(x^2 - 2/3)/3
```

3.225 $\int \frac{1}{x\sqrt{1-\log^2(x)}} \, dx$

Optimal result . . . 1227
Rubi [A] (verified) . . . 1227
Mathematica [B] (verified) . . . 1228
Maple [A] (verified) . . . 1228
Fricas [B] (verification not implemented) . . . 1228
Sympy [F] . . . 1229
Maxima [A] (verification not implemented) . . . 1229
Giac [A] (verification not implemented) . . . 1229
Mupad [B] (verification not implemented) . . . 1229

Optimal result

Integrand size = 16, antiderivative size = 3

$$\int \frac{1}{x\sqrt{1-\log^2(x)}} \, dx = \arcsin(\log(x))$$

```
[Out] arcsin(ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {222}

$$\int \frac{1}{x\sqrt{1-\log^2(x)}} \, dx = \arcsin(\log(x))$$

```
[In] Int[1/(x*Sqrt[1 - Log[x]^2]),x]
```

```
[Out] ArcSin[Log[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, \log(x)\right) \\ &= \sin^{-1}(\log(x)) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. $2(3) = 6$.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 7.33

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = -2\arctan\left(\frac{\sqrt{1-\log^2(x)}}{1+\log(x)}\right)$$

```
[In] Integrate[1/(x*Sqrt[1 - Log[x]^2]),x]
[Out] -2*ArcTan[Sqrt[1 - Log[x]^2]/(1 + Log[x])]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arcsin(\ln(x))$	4
default	$\arcsin(\ln(x))$	4

```
[In] int(1/x/(1-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(3) = 6$.

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 6.67

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = -2\arctan\left(\frac{\sqrt{-\log(x)^2+1}-1}{\log(x)}\right)$$

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(-log(x)^2 + 1) - 1)/log(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{-\left(\log\left(x\right)-1\right)\left(\log\left(x\right)+1\right)}}\,dx$$

```
[In] integrate(1/x/(1-ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(log(x) - 1)*(log(x) + 1))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = \arcsin\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = \arcsin\left(\log\left(x\right)\right)$$

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="giac")
[Out] arcsin(log(x))
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx = \mathrm{asin}(\ln\left(x\right))$$

```
[In] int(1/(x*(1 - log(x)^2)^(1/2)),x)
[Out] asin(log(x))
```

3.226 $\int 16x^3 \log^2(x) \, dx$

Optimal result 1230
Rubi [A] (verified) 1230
Mathematica [A] (verified) 1231
Maple [A] (verified) 1231
Fricas [A] (verification not implemented) 1232
Sympy [A] (verification not implemented) 1232
Maxima [A] (verification not implemented) 1232
Giac [A] (verification not implemented) 1232
Mupad [B] (verification not implemented) 1233

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int 16x^3 \log^2(x) \, dx = \frac{x^4}{2} - 2x^4 \log(x) + 4x^4 \log^2(x)$$

```
[Out] 1/2*x^4-2*x^4*ln(x)+4*x^4*ln(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 2342, 2341}

$$\int 16x^3 \log^2(x) \, dx = \frac{x^4}{2} + 4x^4 \log^2(x) - 2x^4 \log(x)$$

```
[In] Int[16*x^3*Log[x]^2,x]
```

```
[Out] x^4/2 - 2*x^4*Log[x] + 4*x^4*Log[x]^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 16\int x^3\log^2(x)\,dx\\ &= 4x^4\log^2(x)-8\int x^3\log(x)\,dx\\ &= \frac{x^4}{2}-2x^4\log(x)+4x^4\log^2(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int 16x^3\log^2(x)\,dx=16\left(\frac{x^4}{32}-\frac{1}{8}x^4\log(x)+\frac{1}{4}x^4\log^2(x)\right)$$

```
[In] Integrate[16*x^3*Log[x]^2,x]
[Out] 16*(x^4/32 - (x^4*Log[x])/8 + (x^4*Log[x]^2)/4)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{x^4}{2}-2x^4\ln(x)+4x^4\ln(x)^2$	23
norman	$\frac{x^4}{2}-2x^4\ln(x)+4x^4\ln(x)^2$	23
risch	$\frac{x^4}{2}-2x^4\ln(x)+4x^4\ln(x)^2$	23
parallelrisch	$\frac{x^4}{2}-2x^4\ln(x)+4x^4\ln(x)^2$	23
parts	$\frac{x^4}{2}-2x^4\ln(x)+4x^4\ln(x)^2$	23

```
[In] int(16*x^3*ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^4-2*x^4*ln(x)+4*x^4*ln(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int 16x^3 \log^2(x) \, dx = 4 \, x^4 \log(x)^2 - 2 \, x^4 \log(x) + \frac{1}{2} \, x^4$$

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="fricas")
[Out] 4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int 16x^3 \log^2(x) \, dx = 4x^4 \log(x)^2 - 2x^4 \log(x) + \frac{x^4}{2}$$

```
[In] integrate(16*x**3*ln(x)**2,x)
[Out] 4*x**4*log(x)**2 - 2*x**4*log(x) + x**4/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int 16x^3 \log^2(x) \, dx = \frac{1}{2} \left(8 \, \log(x)^2 - 4 \, \log(x) + 1\right)x^4$$

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="maxima")
[Out] 1/2*(8*log(x)^2 - 4*log(x) + 1)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int 16x^3 \log^2(x) \, dx = 4 \, x^4 \log(x)^2 - 2 \, x^4 \log(x) + \frac{1}{2} \, x^4$$

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="giac")
[Out] 4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int 16x^3 \log^2(x)\,dx = \frac{x^4 \left(8\ln(x)^2 - 4\ln(x) + 1\right)}{2}$$

```
[In] int(16*x^3*log(x)^2,x)
[Out] (x^4*(8*log(x)^2 - 4*log(x) + 1))/2
```

3.227 $\int \log \left(\sqrt{a+bx}\right) \, dx$

Optimal result 1234
Rubi [A] (verified) 1234
Mathematica [A] (verified) 1235
Maple [A] (verified) 1235
Fricas [A] (verification not implemented) 1236
Sympy [A] (verification not implemented) 1236
Maxima [A] (verification not implemented) 1236
Giac [A] (verification not implemented) 1236
Mupad [B] (verification not implemented) 1237

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \log \left(\sqrt{a+bx}\right) \, dx = -\frac{x}{2}+\frac{(a+bx) \log \left(\sqrt{a+bx}\right)}{b}$$

```
[Out] -1/2*x+1/2*(b*x+a)*ln(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2332}

$$\int \log \left(\sqrt{a+bx}\right) \, dx = \frac{(a+bx) \log \left(\sqrt{a+bx}\right)}{b}-\frac{x}{2}$$

```
[In] Int[Log[Sqrt[a + b*x]],x]
```

```
[Out] -1/2*x + ((a + b*x)*Log[Sqrt[a + b*x]])/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \log\left(\sqrt{x}\right)\, dx, x, a+bx\right)}{b}$$
$$= -\frac{x}{2} + \frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \log\left(\sqrt{a+bx}\right)\, dx = \frac{1}{2}\left(-x + \frac{(a+bx)\log(a+bx)}{b}\right)$$

```
[In] Integrate[Log[Sqrt[a + b*x]],x]
[Out] (-x + ((a + b*x)*Log[a + b*x])/b)/2
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(bx+a)\ln(bx+a)-bx-a}{2b}$	26
default	$\frac{(bx+a)\ln(bx+a)-bx-a}{2b}$	26
norman	$-\frac{x}{2}+\frac{x\ln(bx+a)}{2}+\frac{a\ln(bx+a)}{2b}$	26
risch	$-\frac{x}{2}+\frac{x\ln(bx+a)}{2}+\frac{a\ln(bx+a)}{2b}$	26
parallelrisch	$\frac{\ln(bx+a)xb-bx+a\ln(bx+a)+a}{2b}$	29
parts	$\frac{x\ln(bx+a)}{2}-\frac{b\left(\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}\right)}{2}$	32

```
[In] int(1/2*ln(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2/b*((b*x+a)*ln(b*x+a)-b*x-a)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \log \left(\sqrt{a+bx}\right) \, dx = -\frac{bx-(bx+a)\log(bx+a)}{2\,b}$$

```
[In] integrate(1/2*log(b*x+a),x, algorithm="fricas")
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \log \left(\sqrt{a+bx}\right) \, dx = -b\left(-\frac{a\log(a+bx)}{2b^2}+\frac{x}{2b}\right)+\frac{x\log(a+bx)}{2}$$

```
[In] integrate(1/2*ln(b*x+a),x)
[Out] -b*(-a*log(a + b*x)/(2*b**2) + x/(2*b)) + x*log(a + b*x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \log \left(\sqrt{a+bx}\right) \, dx = -\frac{bx-(bx+a)\log(bx+a)+a}{2\,b}$$

```
[In] integrate(1/2*log(b*x+a),x, algorithm="maxima")
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \log \left(\sqrt{a+bx}\right) \, dx = -\frac{bx-(bx+a)\log(bx+a)+a}{2\,b}$$

```
[In] integrate(1/2*log(b*x+a),x, algorithm="giac")
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log \left(\sqrt{a+bx}\right) \, dx = \frac{x \ln (a+b\,x)}{2} - \frac{x}{2} + \frac{a \ln (a+b\,x)}{2\,b}$$

```
[In] int(log(a + b*x)/2,x)
[Out] (x*log(a + b*x))/2 - x/2 + (a*log(a + b*x))/(2*b)
```

3.228 $\int x \log \left(\sqrt{2+x}\right) \, dx$

Optimal result 1238
Rubi [A] (verified) 1238
Mathematica [A] (verified) 1239
Maple [A] (verified) 1239
Fricas [A] (verification not implemented) 1240
Sympy [A] (verification not implemented) 1240
Maxima [A] (verification not implemented) 1240
Giac [A] (verification not implemented) 1240
Mupad [B] (verification not implemented) 1241

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int x \log \left(\sqrt{2+x}\right) \, dx = \frac{x}{2}-\frac{x^2}{8}+\frac{1}{2}x^2 \log \left(\sqrt{2+x}\right)-\log(2+x)$$

```
[Out] 1/2*x-1/8*x^2-ln(2+x)+1/4*x^2*ln(2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2442, 45}

$$\int x \log \left(\sqrt{2+x}\right) \, dx = -\frac{x^2}{8}+\frac{1}{2}x^2 \log \left(\sqrt{x+2}\right)+\frac{x}{2}-\log(x+2)$$

```
[In] Int[x*Log[Sqrt[2 + x]],x]
[Out] x/2 - x^2/8 + (x^2*Log[Sqrt[2 + x]])/2 - Log[2 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\log\left(\sqrt{2+x}\right) - \frac{1}{4}\int \frac{x^2}{2+x}\,dx \\ &= \frac{1}{2}x^2\log\left(\sqrt{2+x}\right) - \frac{1}{4}\int \left(-2+x+\frac{4}{2+x}\right)dx \\ &= \frac{x}{2} - \frac{x^2}{8} + \frac{1}{2}x^2\log\left(\sqrt{2+x}\right) - \log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x\log\left(\sqrt{2+x}\right)\,dx = \frac{1}{2}\left(x - \frac{x^2}{4} - 2\log(2+x) + \frac{1}{2}x^2\log(2+x)\right)$$

```
[In] Integrate[x*Log[Sqrt[2 + x]],x]
[Out] (x - x^2/4 - 2*Log[2 + x] + (x^2*Log[2 + x])/2)/2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{x}{2}-\frac{x^2}{8}-\ln\left(x+2\right)+\frac{x^2\ln(x+2)}{4}$	25
risch	$\frac{x}{2}-\frac{x^2}{8}-\ln\left(x+2\right)+\frac{x^2\ln(x+2)}{4}$	25
parts	$\frac{x}{2}-\frac{x^2}{8}-\ln\left(x+2\right)+\frac{x^2\ln(x+2)}{4}$	25
parallelrisch	$\frac{x^2\ln(x+2)}{4}-\frac{x^2}{8}+\frac{x}{2}-\ln\left(x+2\right)-1$	26
derivativedivides	$-\ln\left(x+2\right)\left(x+2\right)+x+2+\frac{(x+2)^2\ln(x+2)}{4}-\frac{(x+2)^2}{8}$	31
default	$-\ln\left(x+2\right)\left(x+2\right)+x+2+\frac{(x+2)^2\ln(x+2)}{4}-\frac{(x+2)^2}{8}$	31

```
[In] int(1/2*x*ln(x+2),x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/8*x^2-ln(x+2)+1/4*x^2*ln(x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int x \log \left(\sqrt{2+x}\right) \, dx = -\frac{1}{8} \, x^{2} + \frac{1}{4} \, {\left(x^{2} - 4\right)} \log \left(x + 2\right) + \frac{1}{2} \, x$$

```
[In] integrate(1/2*x*log(2+x),x, algorithm="fricas")
[Out] -1/8*x^2 + 1/4*(x^2 - 4)*log(x + 2) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x \log \left(\sqrt{2+x}\right) \, dx = \frac{x^{2} \log \left(x + 2\right)}{4} - \frac{x^{2}}{8} + \frac{x}{2} - \log \left(x + 2\right)$$

```
[In] integrate(1/2*x*ln(2+x),x)
[Out] x**2*log(x + 2)/4 - x**2/8 + x/2 - log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int x \log \left(\sqrt{2+x}\right) \, dx = \frac{1}{4} \, x^{2} \log \left(x + 2\right) - \frac{1}{8} \, x^{2} + \frac{1}{2} \, x - \log \left(x + 2\right)$$

```
[In] integrate(1/2*x*log(2+x),x, algorithm="maxima")
[Out] 1/4*x^2*log(x + 2) - 1/8*x^2 + 1/2*x - log(x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x \log \left(\sqrt{2+x}\right) \, dx = \frac{1}{4} \, {\left(x + 2\right)}^{2} \log \left(x + 2\right) - \frac{1}{8} \, {\left(x + 2\right)}^{2} - {\left(x + 2\right)} \log \left(x + 2\right) + x + 2$$

```
[In] integrate(1/2*x*log(2+x),x, algorithm="giac")
[Out] 1/4*(x + 2)^2*log(x + 2) - 1/8*(x + 2)^2 - (x + 2)*log(x + 2) + x + 2
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int x\log\left(\sqrt{2+x}\right)\,dx=\frac{x}{2}-\frac{x^2}{8}+\frac{\ln\left(x+2\right)\,\left(x^2-4\right)}{4}$$

```
[In] int((x*log(x + 2))/2,x)
```

```
[Out] x/2 - x^2/8 + (log(x + 2)*(x^2 - 4))/4
```

3.229 $\int x \log\left(\sqrt[3]{1+3x}\right) \, dx$

Optimal result 1242
Rubi [A] (verified) 1242
Mathematica [A] (verified) 1243
Maple [A] (verified) 1243
Fricas [A] (verification not implemented) 1244
Sympy [A] (verification not implemented) 1244
Maxima [A] (verification not implemented) 1244
Giac [A] (verification not implemented) 1245
Mupad [B] (verification not implemented) 1245

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int x \log\left(\sqrt[3]{1+3x}\right) \, dx = \frac{x}{18} - \frac{x^2}{12} + \frac{1}{2}x^2 \log\left(\sqrt[3]{1+3x}\right) - \frac{1}{54}\log(1+3x)$$

```
[Out] 1/18*x-1/12*x^2+1/6*x^2*ln(1+3*x)-1/54*ln(1+3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2442, 45}

$$\int x \log\left(\sqrt[3]{1+3x}\right) \, dx = -\frac{x^2}{12} + \frac{1}{2}x^2 \log\left(\sqrt[3]{3x+1}\right) + \frac{x}{18} - \frac{1}{54}\log(3x+1)$$

```
[In] Int[x*Log[(1 + 3*x)^(1/3)],x]
```

```
[Out] x/18 - x^2/12 + (x^2*Log[(1 + 3*x)^(1/3)])/2 - Log[1 + 3*x]/54
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{2}\int\frac{x^2}{1+3x}\,dx\\ &= \frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{2}\int\left(-\frac{1}{9}+\frac{x}{3}+\frac{1}{9(1+3x)}\right)dx\\ &= \frac{x}{18}-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{54}\log(1+3x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int x\log\left(\sqrt[3]{1+3x}\right)dx=\frac{1}{3}\left(\frac{x}{6}-\frac{x^2}{4}-\frac{1}{18}\log(1+3x)+\frac{1}{2}x^2\log(1+3x)\right)$$

```
[In] Integrate[x*Log[(1 + 3*x)^(1/3)],x]
[Out] (x/6 - x^2/4 - Log[1 + 3*x]/18 + (x^2*Log[1 + 3*x])/2)/3
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

method	result	size
meijerg	$\frac{x(-9x+6)}{108}-\frac{(-27x^2+3)\ln(1+3x)}{162}$	25
norman	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(1+3x)}{6}-\frac{\ln(1+3x)}{54}$	29
risch	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(1+3x)}{6}-\frac{\ln(1+3x)}{54}$	29
parallelrisch	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(1+3x)}{6}-\frac{\ln(1+3x)}{54}$	29
parts	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(1+3x)}{6}-\frac{\ln(1+3x)}{54}$	29
derivativedivides	$-\frac{\ln(1+3x)(1+3x)}{27}+\frac{1}{27}+\frac{x}{9}+\frac{(1+3x)^2\ln(1+3x)}{54}-\frac{(1+3x)^2}{108}$	43
default	$-\frac{\ln(1+3x)(1+3x)}{27}+\frac{1}{27}+\frac{x}{9}+\frac{(1+3x)^2\ln(1+3x)}{54}-\frac{(1+3x)^2}{108}$	43

```
[In] int(1/3*x*ln(1+3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*x*(-9*x+6)-1/162*(-27*x^2+3)*ln(1+3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x\log\left(\sqrt[3]{1+3x}\right)\,dx=-\frac{1}{12}\,x^2+\frac{1}{54}\left(9\,x^2-1\right)\log\left(3\,x+1\right)+\frac{1}{18}\,x$$

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="fricas")
[Out] -1/12*x^2 + 1/54*(9*x^2 - 1)*log(3*x + 1) + 1/18*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

$$\int x\log\left(\sqrt[3]{1+3x}\right)\,dx=\frac{x^2\log\left(3x+1\right)}{6}-\frac{x^2}{12}+\frac{x}{18}-\frac{\log\left(3x+1\right)}{54}$$

```
[In] integrate(1/3*x*ln(1+3*x),x)
[Out] x**2*log(3*x + 1)/6 - x**2/12 + x/18 - log(3*x + 1)/54
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int x\log\left(\sqrt[3]{1+3x}\right)\,dx=\frac{1}{6}\,x^2\log\left(3\,x+1\right)-\frac{1}{12}\,x^2+\frac{1}{18}\,x-\frac{1}{54}\,\log\left(3\,x+1\right)$$

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="maxima")
[Out] 1/6*x^2*log(3*x + 1) - 1/12*x^2 + 1/18*x - 1/54*log(3*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int x\log\left(\sqrt[3]{1+3x}\right)\,dx = \frac{1}{54}\,(3\,x+1)^2\log\left(3\,x+1\right) - \frac{1}{108}\,(3\,x+1)^2 - \frac{1}{27}\,(3\,x+1)\log\left(3\,x+1\right) + \frac{1}{9}\,x + \frac{1}{27}$$

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="giac")
```

```
[Out] 1/54*(3*x + 1)^2*log(3*x + 1) - 1/108*(3*x + 1)^2 - 1/27*(3*x + 1)*log(3*x
+ 1) + 1/9*x + 1/27
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

$$\int x\log\left(\sqrt[3]{1+3x}\right)\,dx = \frac{x}{18} + \frac{\ln\left(3\,x+1\right)\,\left(x^2-\frac{1}{9}\right)}{6} - \frac{x^2}{12}$$

```
[In] int((x*log(3*x + 1))/3,x)
```

```
[Out] x/18 + (log(3*x + 1)*(x^2 - 1/9))/6 - x^2/12
```

3.230 $\int x \log(x+x^3)\,dx$

Optimal result 1246
Rubi [A] (verified) 1246
Mathematica [A] (verified) 1247
Maple [A] (verified) 1247
Fricas [A] (verification not implemented) 1248
Sympy [A] (verification not implemented) 1248
Maxima [A] (verification not implemented) 1248
Giac [A] (verification not implemented) 1249
Mupad [B] (verification not implemented) 1249

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int x\log\left(x+x^3\right)\,dx=-\frac{3x^2}{4}+\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{2}x^2\log\left(x+x^3\right)$$

```
[Out] -3/4*x^2+1/2*ln(x^2+1)+1/2*x^2*ln(x^3+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2605, 455, 45}

$$\int x\log\left(x+x^3\right)\,dx=-\frac{3x^2}{4}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\log\left(x^3+x\right)$$

```
[In] Int[x*Log[x + x^3],x]
```

```
[Out] (-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\log\left(x+x^3\right)-\frac{1}{2}\int\frac{x(1+3x^2)}{1+x^2}\,dx\\ &= \frac{1}{2}x^2\log\left(x+x^3\right)-\frac{1}{4}\text{Subst}\left(\int\frac{1+3x}{1+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}x^2\log\left(x+x^3\right)-\frac{1}{4}\text{Subst}\left(\int\left(3-\frac{2}{1+x}\right)\,dx,x,x^2\right)\\ &= -\frac{3x^2}{4}+\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{2}x^2\log\left(x+x^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x\log\left(x+x^3\right)\,dx=-\frac{3x^2}{4}+\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{2}x^2\log\left(x+x^3\right)$$

```
[In] Integrate[x*Log[x + x^3],x]
[Out] (-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{3x^2}{4}+\frac{\ln(x^2+1)}{2}+\frac{x^2\ln(x^3+x)}{2}$	26
risch	$-\frac{3x^2}{4}+\frac{\ln(x^2+1)}{2}+\frac{x^2\ln(x^3+x)}{2}$	26
parts	$-\frac{3x^2}{4}+\frac{\ln(x^2+1)}{2}+\frac{x^2\ln(x^3+x)}{2}$	26
parallelrisch	$\frac{x^2\ln(x^3+x)}{2}+\frac{3}{4}-\frac{3x^2}{4}-\frac{\ln(x)}{2}+\frac{\ln(x^3+x)}{2}$	31

```
[In] int(x*ln(x^3+x),x,method=_RETURNVERBOSE)
[Out] -3/4*x^2+1/2*ln(x^2+1)+1/2*x^2*ln(x^3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x\log\left(x+x^3\right)\,dx=\frac{1}{2}\,x^2\log\left(x^3+x\right)-\frac{3}{4}\,x^2+\frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x*log(x^3+x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x\log\left(x+x^3\right)\,dx=\frac{x^2\log\left(x^3+x\right)}{2}-\frac{3x^2}{4}+\frac{\log\left(x^2+1\right)}{2}$$

```
[In] integrate(x*ln(x**3+x),x)
[Out] x**2*log(x**3 + x)/2 - 3*x**2/4 + log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x\log\left(x+x^3\right)\,dx=\frac{1}{2}\,x^2\log\left(x^3+x\right)-\frac{3}{4}\,x^2+\frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x*log(x^3+x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x \log \left(x+x^3\right) \, dx=\frac{1}{2} \, x^2 \log \left(x^3+x\right)-\frac{3}{4} \, x^2+\frac{1}{2} \, \log \left(x^2+1\right)$$

```
[In] integrate(x*log(x^3+x),x, algorithm="giac")
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x \log \left(x+x^3\right) \, dx=\frac{\ln \left(x^2+1\right)}{2}+\frac{x^2\,\ln \left(x^3+x\right)}{2}-\frac{3\,x^2}{4}$$

```
[In] int(x*log(x + x^3),x)
[Out] log(x^2 + 1)/2 + (x^2*log(x + x^3))/2 - (3*x^2)/4
```

3.231 $\int \log\left(x+\sqrt{1+x^2}\right)\,dx$

Optimal result 1250
Rubi [A] (verified) 1250
Mathematica [A] (verified) 1251
Maple [A] (verified) 1251
Fricas [A] (verification not implemented) 1251
Sympy [A] (verification not implemented) 1252
Maxima [F] 1252
Giac [A] (verification not implemented) 1252
Mupad [B] (verification not implemented) 1252

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \log\left(x+\sqrt{1+x^2}\right)\,dx = -\sqrt{1+x^2}+x\log\left(x+\sqrt{1+x^2}\right)$$

```
[Out] x*ln(x+(x^2+1)^(1/2))-(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2614, 267}

$$\int \log\left(x+\sqrt{1+x^2}\right)\,dx = x\log\left(\sqrt{x^2+1}+x\right)-\sqrt{x^2+1}$$

```
[In] Int[Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2614

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
```

```
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(x+\sqrt{1+x^2}\right)-\int\frac{x}{\sqrt{1+x^2}}\,dx\\ &= -\sqrt{1+x^2}+x\log\left(x+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int\log\left(x+\sqrt{1+x^2}\right)\,dx=-\sqrt{1+x^2}+x\log\left(x+\sqrt{1+x^2}\right)$$

```
[In] Integrate[Log[x + Sqrt[1 + x^2]],x]
[Out] -Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$x\ln\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$	23
parts	$x\ln\left(x+\sqrt{x^2+1}\right)+\frac{x^2\sqrt{x^2+1}}{3}-\frac{2\sqrt{x^2+1}}{3}-\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	44

```
[In] int(ln(x+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*ln(x+(x^2+1)^(1/2))-(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int\log\left(x+\sqrt{1+x^2}\right)\,dx=x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log \left(x+\sqrt{1+x^2}\right) dx = x\log \left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

```
[In] integrate(ln(x+(x**2+1)**(1/2)),x)
[Out] x*log(x + sqrt(x**2 + 1)) - sqrt(x**2 + 1)
```

Maxima [F]

$$\int \log \left(x+\sqrt{1+x^2}\right) dx = \int \log \left(x+\sqrt{x^2+1}\right) dx$$

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] x*log(x + sqrt(x^2 + 1)) - x + arctan(x) - integrate(x/(x^3 + (x^2 + 1)^(3/
2) + x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x+\sqrt{1+x^2}\right) dx = x\log \left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="giac")
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x+\sqrt{1+x^2}\right) dx = x\,\ln \left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

```
[In] int(log(x + (x^2 + 1)^(1/2)),x)
[Out] x*log(x + (x^2 + 1)^(1/2)) - (x^2 + 1)^(1/2)
```

3.232 $\int \log \left(x+\sqrt{-1+x^2}\right) \, dx$

Optimal result . . . 1253
Rubi [A] (verified) . . . 1253
Mathematica [A] (verified) . . . 1254
Maple [A] (verified) . . . 1254
Fricas [A] (verification not implemented) . . . 1254
Sympy [A] (verification not implemented) . . . 1255
Maxima [F] . . . 1255
Giac [A] (verification not implemented) . . . 1255
Mupad [B] (verification not implemented) . . . 1255

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \log \left(x+\sqrt{-1+x^2}\right) \, dx=-\sqrt{-1+x^2}+x \log \left(x+\sqrt{-1+x^2}\right)$$

```
[Out] x*ln(x+(x^2-1)^(1/2))-(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2614, 267}

$$\int \log \left(x+\sqrt{-1+x^2}\right) \, dx=x \log \left(\sqrt{x^2-1}+x\right)-\sqrt{x^2-1}$$

```
[In] Int[Log[x + Sqrt[-1 + x^2]],x]
```

```
[Out] -Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2614

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
```

```
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(x+\sqrt{-1+x^2}\right)-\int\frac{x}{\sqrt{-1+x^2}}\,dx\\ &= -\sqrt{-1+x^2}+x\log\left(x+\sqrt{-1+x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int\log\left(x+\sqrt{-1+x^2}\right)\,dx=-\sqrt{-1+x^2}+x\log\left(x+\sqrt{-1+x^2}\right)$$

```
[In] Integrate[Log[x + Sqrt[-1 + x^2]],x]
[Out] -Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$x\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$	23
parts	$x\ln\left(x+\sqrt{x^2-1}\right)-\frac{x^2\sqrt{x^2-1}}{3}-\frac{2\sqrt{x^2-1}}{3}+\frac{\left(x^2-1\right)^{\frac{3}{2}}}{3}$	44

```
[In] int(ln(x+(x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*ln(x+(x^2-1)^(1/2))-(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int\log\left(x+\sqrt{-1+x^2}\right)\,dx=x\log\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="fricas")
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log \left(x+\sqrt{-1+x^2}\right) dx = x \log \left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

```
[In] integrate(ln(x+(x**2-1)**(1/2)),x)
[Out] x*log(x + sqrt(x**2 - 1)) - sqrt(x**2 - 1)
```

Maxima [F]

$$\int \log \left(x+\sqrt{-1+x^2}\right) dx = \int \log \left(x+\sqrt{x^2-1}\right) dx$$

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="maxima")
[Out] x*log(sqrt(x + 1)*sqrt(x - 1) + x) - x + integrate(x/(x^3 + (x^2 - 1)*e^(1/
2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x+\sqrt{-1+x^2}\right) dx = x \log \left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="giac")
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x+\sqrt{-1+x^2}\right) dx = x \ln \left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

```
[In] int(log(x + (x^2 - 1)^(1/2)),x)
[Out] x*log(x + (x^2 - 1)^(1/2)) - (x^2 - 1)^(1/2)
```

3.233 $\int \log\left(x-\sqrt{-1+x^2}\right)\,dx$

Optimal result 1256
Rubi [A] (verified) 1256
Mathematica [A] (verified) 1257
Maple [A] (verified) 1257
Fricas [A] (verification not implemented) 1257
Sympy [A] (verification not implemented) 1258
Maxima [F] 1258
Giac [A] (verification not implemented) 1258
Mupad [B] (verification not implemented) 1258

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \log\left(x-\sqrt{-1+x^2}\right)\,dx=\sqrt{-1+x^2}+x\log\left(x-\sqrt{-1+x^2}\right)$$

```
[Out] x*ln(x-(x^2-1)^(1/2))+(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2614, 267}

$$\int \log\left(x-\sqrt{-1+x^2}\right)\,dx=\sqrt{x^2-1}+x\log\left(x-\sqrt{x^2-1}\right)$$

```
[In] Int[Log[x - Sqrt[-1 + x^2]],x]
```

```
[Out] Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2614

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
```

```
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = x\log\left(x-\sqrt{-1+x^2}\right)+\int\frac{x}{\sqrt{-1+x^2}}\,dx$$
$$=\sqrt{-1+x^2}+x\log\left(x-\sqrt{-1+x^2}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int\log\left(x-\sqrt{-1+x^2}\right)\,dx=\sqrt{-1+x^2}+x\log\left(x-\sqrt{-1+x^2}\right)$$

```
[In] Integrate[Log[x - Sqrt[-1 + x^2]],x]
[Out] Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$x\ln\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$	23
parts	$x\ln\left(x-\sqrt{x^2-1}\right)+\frac{x^2\sqrt{x^2-1}}{3}+\frac{2\sqrt{x^2-1}}{3}-\frac{(x^2-1)^{\frac{3}{2}}}{3}$	46

```
[In] int(ln(x-(x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*ln(x-(x^2-1)^(1/2))+(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int\log\left(x-\sqrt{-1+x^2}\right)\,dx=x\log\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="fricas")
[Out] x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log \left(x-\sqrt{-1+x^2}\right) dx = x \log \left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

```
[In] integrate(ln(x-(x**2-1)**(1/2)),x)
[Out] x*log(x - sqrt(x**2 - 1)) + sqrt(x**2 - 1)
```

Maxima [F]

$$\int \log \left(x-\sqrt{-1+x^2}\right) dx = \int \log \left(x-\sqrt{x^2-1}\right) dx$$

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="maxima")
[Out] x*log(-sqrt(x + 1)*sqrt(x - 1) + x) - x - integrate(-x/(x^3 - (x^2 - 1)*e^(
1/2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x-\sqrt{-1+x^2}\right) dx = x \log \left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="giac")
[Out] x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x-\sqrt{-1+x^2}\right) dx = x \ln \left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

```
[In] int(log(x - (x^2 - 1)^(1/2)),x)
[Out] x*log(x - (x^2 - 1)^(1/2)) + (x^2 - 1)^(1/2)
```

3.234 $\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx$

Optimal result 1259
Rubi [A] (verified) 1259
Mathematica [A] (verified) 1261
Maple [A] (verified) 1261
Fricas [A] (verification not implemented) 1261
Sympy [F] 1262
Maxima [F] 1262
Giac [A] (verification not implemented) 1262
Mupad [B] (verification not implemented) 1262

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx = -\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)$$

```
[Out] 1/2*arcsinh(x^(1/2))+x*ln(x^(1/2)+(1+x)^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2628, 12, 1978, 52, 56, 221}

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{1}{2}\sqrt{x}\sqrt{x+1}+x\log\left(\sqrt{x}+\sqrt{x+1}\right)$$

```
[In] Int[Log[Sqrt[x] + Sqrt[1 + x]],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 +
x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\int \frac{1}{2}\sqrt{\frac{x}{1+x}}\,dx\\
&= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{1}{2}\int \sqrt{\frac{x}{1+x}}\,dx\\
&= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)+\frac{1}{4}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)+x\log\left(\sqrt{x}+\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx=-\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)$$

```
[In] Integrate[Log[Sqrt[x] + Sqrt[1 + x]],x]
[Out] -1/2*(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

method	result	size
default	$x\ln\left(\sqrt{x}+\sqrt{x+1}\right)-\frac{\sqrt{x}\,\sqrt{x+1}}{2}+\frac{\sqrt{x(x+1)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)}{4\sqrt{x}\,\sqrt{x+1}}$	52
parts	$x\ln\left(\sqrt{x}+\sqrt{x+1}\right)-\frac{\sqrt{x}\,(x+1)^{\frac{3}{2}}}{4}-\frac{\sqrt{x}\,\sqrt{x+1}}{4}+\frac{\sqrt{x(x+1)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)}{4\sqrt{x}\,\sqrt{x+1}}+\frac{x^{\frac{3}{2}}\sqrt{x+1}}{4}$	72

```
[In] int(ln(x^(1/2)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*ln(x^(1/2)+(x+1)^(1/2))-1/2*x^(1/2)*(x+1)^(1/2)+1/4*(x*(x+1))^(1/2)/x^(1/2)/(x+1)^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx=\frac{1}{2}\,(2\,x+1)\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x+1}\sqrt{x}$$

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

Sympy [F]

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx = \int \log\left(\sqrt{x}+\sqrt{x+1}\right)\,dx$$

```
[In] integrate(ln(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Integral(log(sqrt(x) + sqrt(x + 1)), x)
```

Maxima [F]

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx = \int \log\left(\sqrt{x+1}+\sqrt{x}\right)\,dx$$

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*x - integrate(1/2*x/(x^2 + (x^(3/2) + sq
rt(x))*sqrt(x + 1) + x), x) + 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx = x\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\sqrt{x^2+x} - \frac{1}{4}\log\left(\left|-2\,x+2\sqrt{x^2+x}-1\right|\right)$$

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sqr
t(x^2 + x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx = \operatorname{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)-\frac{\sqrt{x}\,\sqrt{x+1}}{2}+x\,\ln\left(\sqrt{x+1}+\sqrt{x}\right)$$

```
[In] int(log((x + 1)^(1/2) + x^(1/2)),x)
```

```
[Out] atanh(x^(1/2)/((x + 1)^(1/2) - 1)) - (x^(1/2)*(x + 1)^(1/2))/2 + x*log((x +
 1)^(1/2) + x^(1/2))
```

3.235 $\int \sqrt[3]{x} \log(x) \, dx$

Optimal result 1263
Rubi [A] (verified) 1263
Mathematica [A] (verified) 1264
Maple [A] (verified) 1264
Fricas [A] (verification not implemented) 1264
Sympy [B] (verification not implemented) 1265
Maxima [A] (verification not implemented) 1265
Giac [A] (verification not implemented) 1265
Mupad [B] (verification not implemented) 1266

Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \sqrt[3]{x} \log(x) \, dx = -\frac{9x^{4/3}}{16} + \frac{3}{4}x^{4/3} \log(x)$$

```
[Out] -9/16*x^(4/3)+3/4*x^(4/3)*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$\int \sqrt[3]{x} \log(x) \, dx = \frac{3}{4}x^{4/3} \log(x) - \frac{9x^{4/3}}{16}$$

```
[In] Int[x^(1/3)*Log[x],x]
```

```
[Out] (-9*x^(4/3))/16 + (3*x^(4/3)*Log[x])/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{9x^{4/3}}{16} + \frac{3}{4}x^{4/3} \log(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt[3]{x}\log(x)\,dx = \frac{3}{16}x^{4/3}(-3+4\log(x))$$

```
[In] Integrate[x^(1/3)*Log[x],x]
[Out] (3*x^(4/3)*(-3 + 4*Log[x]))/16
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{9x^{\frac{4}{3}}}{16}+\frac{3x^{\frac{4}{3}}\ln(x)}{4}$	14
default	$-\frac{9x^{\frac{4}{3}}}{16}+\frac{3x^{\frac{4}{3}}\ln(x)}{4}$	14
risch	$-\frac{9x^{\frac{4}{3}}}{16}+\frac{3x^{\frac{4}{3}}\ln(x)}{4}$	14

```
[In] int(x^(1/3)*ln(x),x,method=_RETURNVERBOSE)
[Out] -9/16*x^(4/3)+3/4*x^(4/3)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \sqrt[3]{x}\log(x)\,dx = \frac{3}{16}\,(4\,x\log{(x)} - 3\,x)x^{\frac{1}{3}}$$

```
[In] integrate(x^(1/3)*log(x),x, algorithm="fricas")
[Out] 3/16*(4*x*log(x) - 3*x)*x^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(19) = 38$.

Time = 1.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \sqrt[3]{x}\log(x)\,dx = \begin{cases} -\frac{3x^{\frac{4}{3}}\log\left(\frac{1}{x}\right)}{4} + \frac{3x^{\frac{4}{3}}\log(x)}{4} - \frac{9x^{\frac{4}{3}}}{8} & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \frac{3x^{\frac{4}{3}}\log(x)}{4} - \frac{9x^{\frac{4}{3}}}{16} & \text{for } |x| < 1 \\ -\frac{3x^{\frac{4}{3}}\log\left(\frac{1}{x}\right)}{4} - \frac{9x^{\frac{4}{3}}}{16} & \text{for } \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{7}{3}, \frac{7}{3} \\ \frac{4}{3}, \frac{4}{3} & 0 \end{matrix} \middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{7}{3}, \frac{7}{3}, 1 & \\ & \frac{4}{3}, \frac{4}{3}, 0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(1/3)*ln(x),x)
```

```
[Out] Piecewise((-3*x**(4/3)*log(1/x)/4 + 3*x**(4/3)*log(x)/4 - 9*x**(4/3)/8, (Ab
s(x) < 1) & (1/Abs(x) < 1)), (3*x**(4/3)*log(x)/4 - 9*x**(4/3)/16, Abs(x) <
 1), (-3*x**(4/3)*log(1/x)/4 - 9*x**(4/3)/16, 1/Abs(x) < 1), (-meijerg(((1,
), (7/3, 7/3)), ((4/3, 4/3), (0,)), x) + meijerg(((7/3, 7/3, 1), ()), ((),
(4/3, 4/3, 0)), x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{x}\log(x)\,dx = \frac{3}{4}\,x^{\frac{4}{3}}\log(x) - \frac{9}{16}\,x^{\frac{4}{3}}$$

```
[In] integrate(x^(1/3)*log(x),x, algorithm="maxima")
```

```
[Out] 3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{x}\log(x)\,dx = \frac{3}{4}\,x^{\frac{4}{3}}\log(x) - \frac{9}{16}\,x^{\frac{4}{3}}$$

```
[In] integrate(x^(1/3)*log(x),x, algorithm="giac")
```

```
[Out] 3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \sqrt[3]{x} \log(x) \, dx = \frac{3 \, x^{4/3} \left(\ln(x) - \frac{3}{4}\right)}{4}$$

```
[In] int(x^(1/3)*log(x),x)
[Out] (3*x^(4/3)*(log(x) - 3/4))/4
```

3.236 $\int 2^{\log(x)} \, dx$

Optimal result 1267
Rubi [A] (verified) 1267
Mathematica [A] (verified) 1268
Maple [A] (verified) 1268
Fricas [A] (verification not implemented) 1269
Sympy [A] (verification not implemented) 1269
Maxima [A] (verification not implemented) 1269
Giac [A] (verification not implemented) 1270
Mupad [B] (verification not implemented) 1270

Optimal result

Integrand size = 4, antiderivative size = 13

$$\int 2^{\log(x)} \, dx = \frac{x^{1+\log(2)}}{1+\log(2)}$$

```
[Out] x^(1+ln(2))/(1+ln(2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2306, 30}

$$\int 2^{\log(x)} \, dx = \frac{x^{1+\log(2)}}{1+\log(2)}$$

```
[In] Int[2^Log[x],x]
```

```
[Out] x^(1 + Log[2])/(1 + Log[2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2306

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{\log(2)}\,dx\\ &= \frac{x^{1+\log(2)}}{1+\log(2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int 2^{\log(x)}\,dx = \frac{2^{\log(x)}x}{1+\log(2)}$$

```
[In] Integrate[2^Log[x],x]
[Out] (2^Log[x]*x)/(1 + Log[2])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x 2^{\ln(x)}}{1+\ln(2)}$	13
risch	$\frac{x\,x^{\ln(2)}}{1+\ln(2)}$	13
parallelrisch	$\frac{x 2^{\ln(x)}}{1+\ln(2)}$	13
norman	$\frac{x\,e^{\ln(2)\ln(x)}}{1+\ln(2)}$	15

```
[In] int(2^ln(x),x,method=_RETURNVERBOSE)
[Out] x/(1+ln(2))*2^ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int 2^{\log(x)}\,dx = \frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

```
[In] integrate(2^log(x),x, algorithm="fricas")
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int 2^{\log(x)}\,dx = \frac{2^{\log(x)}x}{\log(2)+1}$$

```
[In] integrate(2**ln(x),x)
[Out] 2**log(x)*x/(log(2) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int 2^{\log(x)}\,dx = \frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(x)}}{\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

```
[In] integrate(2^log(x),x, algorithm="maxima")
[Out] 2^((1/log(2) + 1)*log(x))/((1/log(2) + 1)*log(2))
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int 2^{\log(x)}\,dx = \frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

```
[In] integrate(2^log(x),x, algorithm="giac")
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int 2^{\log(x)}\,dx = \frac{x^{\ln(2)+1}}{\ln(2)+1}$$

```
[In] int(2^log(x),x)
[Out] x^(log(2) + 1)/(log(2) + 1)
```

3.237 $\int \frac{1-\log(x)}{x^2} dx$

Optimal result 1271
Rubi [A] (verified) 1271
Mathematica [A] (verified) 1272
Maple [A] (verified) 1272
Fricas [A] (verification not implemented) 1272
Sympy [A] (verification not implemented) 1273
Maxima [B] (verification not implemented) 1273
Giac [A] (verification not implemented) 1273
Mupad [B] (verification not implemented) 1273

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{1-\log(x)}{x^2} dx = \frac{\log(x)}{x}$$

```
[Out] ln(x)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2340}

$$\int \frac{1-\log(x)}{x^2} dx = \frac{\log(x)}{x}$$

```
[In] Int[(1 - Log[x])/x^2,x]
```

```
[Out] Log[x]/x
```

Rule 2340

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[b*(d*x)^(m + 1)*(Log[c*x^n]/(d*(m + 1))), x] /; FreeQ[{a, b, c, d, m,
n}, x] && NeQ[m, -1] && EqQ[a*(m + 1) - b*n, 0]
```

Rubi steps

$$\text{integral} = \frac{\log(x)}{x}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1-\log(x)}{x^2}\,dx=\frac{\log(x)}{x}$$

```
[In] Integrate[(1 - Log[x])/x^2,x]
[Out] Log[x]/x
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\ln(x)}{x}$	7
norman	$\frac{\ln(x)}{x}$	7
risch	$\frac{\ln(x)}{x}$	7
parallelrisch	$\frac{\ln(x)}{x}$	7
parts	$\frac{\ln(x)}{x}$	7

```
[In] int((1-ln(x))/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1-\log(x)}{x^2}\,dx=\frac{\log\left(x\right)}{x}$$

```
[In] integrate((1-log(x))/x^2,x, algorithm="fricas")
[Out] log(x)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log{(x)}}{x}$$

```
[In] integrate((1-ln(x))/x**2,x)
[Out] log(x)/x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log{(x)}+1}{x} - \frac{1}{x}$$

```
[In] integrate((1-log(x))/x^2,x, algorithm="maxima")
[Out] (log(x) + 1)/x - 1/x
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log{(x)}}{x}$$

```
[In] integrate((1-log(x))/x^2,x, algorithm="giac")
[Out] log(x)/x
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\ln{(x)}}{x}$$

```
[In] int(-(log(x) - 1)/x^2,x)
[Out] log(x)/x
```

3.238 $\int \log\left(1+x+\sqrt{1+x}\right)\,dx$

Optimal result 1274
Rubi [A] (verified) 1274
Mathematica [A] (verified) 1275
Maple [A] (verified) 1275
Fricas [A] (verification not implemented) 1275
Sympy [B] (verification not implemented) 1276
Maxima [A] (verification not implemented) 1276
Giac [A] (verification not implemented) 1277
Mupad [B] (verification not implemented) 1277

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \log\left(1+x+\sqrt{1+x}\right)\,dx=-x+\sqrt{1+x}+\frac{1}{2}\log(1+x)+x\log\left(1+x+\sqrt{1+x}\right)$$

```
[Out] -x+1/2*ln(1+x)+x*ln(1+x+(1+x)^(1/2))+(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2628}

$$\int \log\left(1+x+\sqrt{1+x}\right)\,dx=-x+\sqrt{x+1}+x\log\left(x+\sqrt{x+1}+1\right)+\frac{1}{2}\log(x+1)$$

```
[In] Int[Log[1 + x + Sqrt[1 + x]],x]
[Out] -x + Sqrt[1 + x] + Log[1 + x]/2 + x*Log[1 + x + Sqrt[1 + x]]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=x\log\left(1+x+\sqrt{1+x}\right)-\int \frac{x\left(1+\frac{1}{2\sqrt{1+x}}\right)}{1+x+\sqrt{1+x}}\,dx\\&=x\log\left(1+x+\sqrt{1+x}\right)-2\text{Subst}\left(\int \left(-\frac{1}{2}-\frac{1}{2x}+x\right)\,dx,x,\sqrt{1+x}\right)\\&=-x+\sqrt{1+x}+\frac{1}{2}\log(1+x)+x\log\left(1+x+\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \log \left(1+x+\sqrt{1+x}\right) \, dx=-x+\sqrt{1+x}-\log \left(1+\sqrt{1+x}\right)+(1+x) \log \left(1+x+\sqrt{1+x}\right)$$

```
[In] Integrate[Log[1 + x + Sqrt[1 + x]],x]
[Out] -x + Sqrt[1 + x] - Log[1 + Sqrt[1 + x]] + (1 + x)*Log[1 + x + Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result	size
parts	$x \ln \left(1+x+\sqrt{x+1}\right)+\sqrt{x+1}+\frac{\ln(x+1)}{2}-x-1$	28
derivativedivides	$(x+1) \ln \left(1+x+\sqrt{x+1}\right)-x-1+\sqrt{x+1}-\ln \left(\sqrt{x+1}+1\right)$	34
default	$(x+1) \ln \left(1+x+\sqrt{x+1}\right)-x-1+\sqrt{x+1}-\ln \left(\sqrt{x+1}+1\right)$	34

```
[In] int(ln(1+x+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*ln(1+x+(x+1)^(1/2))+(x+1)^(1/2)+1/2*ln(x+1)-x-1
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \log \left(1+x+\sqrt{1+x}\right) \, dx=(x-1) \log \left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}+\log \left(\sqrt{x+1}+1\right)+2 \log \left(\sqrt{x+1}\right)$$

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="fricas")
[Out] (x - 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) + log(sqrt(x + 1) + 1) +
 2*log(sqrt(x + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(27) = 54$.

Time = 0.41 (sec) , antiderivative size = 184, normalized size of antiderivative = 5.75

$$\int \log\left(1+x+\sqrt{1+x}\right)\,dx = \frac{x\sqrt{x+1}\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} - \frac{x\sqrt{x+1}}{\sqrt{x+1}+1} + \frac{x\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} - \frac{\sqrt{x+1}\log\left(\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} + \frac{\sqrt{x+1}\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} + \frac{\sqrt{x+1}}{\sqrt{x+1}+1} - \frac{\log\left(\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} + \frac{\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1} + \frac{1}{\sqrt{x+1}+1}$$

```
[In] integrate(ln(1+x+(1+x)**(1/2)),x)
```

```
[Out] x*sqrt(x + 1)*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - x*sqrt(x + 1)/(s
qrt(x + 1) + 1) + x*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - sqrt(x + 1
)*log(sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) + sqrt(x + 1)*log(x + sqrt(x + 1)
+ 1)/(sqrt(x + 1) + 1) + sqrt(x + 1)/(sqrt(x + 1) + 1) - log(sqrt(x + 1) +
1)/(sqrt(x + 1) + 1) + log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) + 1/(sqrt
(x + 1) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \log\left(1+x+\sqrt{1+x}\right)\,dx = (x+1)\log\left(x+\sqrt{x+1}+1\right) - x + \sqrt{x+1} - \log\left(\sqrt{x+1}+1\right) - 1$$

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] (x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1) -
 1
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \log \left(1+x+\sqrt{1+x}\right) \, dx = (x+1)\log \left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}-\log \left(\sqrt{x+1}+1\right) - 1$$

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="giac")
[Out] (x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1) -
 1
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \log \left(1+x+\sqrt{1+x}\right) \, dx = \ln \left(\sqrt{x+1}\right)-x+\sqrt{x+1}+x\,\ln \left(x+\sqrt{x+1}+1\right)$$

```
[In] int(log(x + (x + 1)^(1/2) + 1),x)
[Out] log((x + 1)^(1/2)) - x + (x + 1)^(1/2) + x*log(x + (x + 1)^(1/2) + 1)
```

3.239 $\int \log(x+x^3)\,dx$

Optimal result 1278
Rubi [A] (verified) 1278
Mathematica [A] (verified) 1279
Maple [A] (verified) 1279
Fricas [A] (verification not implemented) 1280
Sympy [A] (verification not implemented) 1280
Maxima [A] (verification not implemented) 1280
Giac [A] (verification not implemented) 1280
Mupad [B] (verification not implemented) 1281

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \log\left(x+x^3\right)\,dx=-3x+2\arctan(x)+x\log\left(x+x^3\right)$$

```
[Out] -3*x+2*arctan(x)+x*ln(x^3+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2603, 396, 209}

$$\int \log\left(x+x^3\right)\,dx=2\arctan(x)+x\log\left(x^3+x\right)-3x$$

```
[In] Int[Log[x + x^3],x]
```

```
[Out] -3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(x+x^3\right)-\int \frac{1+3x^2}{1+x^2}\,dx\\ &= -3x+x\log\left(x+x^3\right)+2\int \frac{1}{1+x^2}\,dx\\ &= -3x+2\tan^{-1}(x)+x\log\left(x+x^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log\left(x+x^3\right)\,dx=-3x+2\arctan(x)+x\log\left(x+x^3\right)$$

```
[In] Integrate[Log[x + x^3],x]
[Out] -3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-3x+2\arctan\left(x\right)+x\ln\left(x^{3}+x\right)$	17
risch	$-3x+2\arctan\left(x\right)+x\ln\left(x^{3}+x\right)$	17
parts	$-3x+2\arctan\left(x\right)+x\ln\left(x^{3}+x\right)$	17
parallelrisch	$-i\ln\left(x\right)-2i\ln\left(x-i\right)+i\ln\left(x^{3}+x\right)+x\ln\left(x^{3}+x\right)-3x$	35

```
[In] int(ln(x^3+x),x,method=_RETURNVERBOSE)
[Out] -3*x+2*arctan(x)+x*ln(x^3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(x+x^3\right) \, dx = x \log \left(x^3+x\right) - 3\,x + 2\,\arctan\left(x\right)$$

```
[In] integrate(log(x^3+x),x, algorithm="fricas")
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \log \left(x+x^3\right) \, dx = x \log \left(x^3+x\right) - 3x + 2\,\mathrm{atan}\left(x\right)$$

```
[In] integrate(ln(x**3+x),x)
[Out] x*log(x**3 + x) - 3*x + 2*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(x+x^3\right) \, dx = x \log \left(x^3+x\right) - 3\,x + 2\,\arctan\left(x\right)$$

```
[In] integrate(log(x^3+x),x, algorithm="maxima")
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(x+x^3\right) \, dx = x \log \left(x^3+x\right) - 3\,x + 2\,\arctan\left(x\right)$$

```
[In] integrate(log(x^3+x),x, algorithm="giac")
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(x+x^3\right) \, dx=2\,\mathrm{atan}(x)-3\,x+x\,\ln \left(x^3+x\right)$$

```
[In] int(log(x + x^3),x)
[Out] 2*atan(x) - 3*x + x*log(x + x^3)
```

3.240 $\int 2^{\log(-8+7x)}\,dx$

Optimal result . . . 1282
Rubi [A] (verified) . . . 1282
Mathematica [A] (verified) . . . 1283
Maple [A] (verified) . . . 1283
Fricas [A] (verification not implemented) . . . 1284
Sympy [B] (verification not implemented) . . . 1284
Maxima [A] (verification not implemented) . . . 1284
Giac [A] (verification not implemented) . . . 1285
Mupad [B] (verification not implemented) . . . 1285

Optimal result

Integrand size = 8, antiderivative size = 20

$$\int 2^{\log(-8+7x)}\,dx=\frac{(-8+7x)^{1+\log(2)}}{7(1+\log(2))}$$

```
[Out] 1/7*(-8+7*x)^(1+ln(2))/(1+ln(2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2306, 32}

$$\int 2^{\log(-8+7x)}\,dx=\frac{(7x-8)^{1+\log(2)}}{7(1+\log(2))}$$

```
[In] Int[2^Log[-8 + 7*x],x]
```

```
[Out] (-8 + 7*x)^(1 + Log[2])/(7*(1 + Log[2]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2306

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\text{integral} = \int (-8+7x)^{\log(2)}\,dx$$
$$= \frac{(-8+7x)^{1+\log(2)}}{7(1+\log(2))}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int 2^{\log(-8+7x)}\,dx = \frac{2^{\log(-8+7x)}(-8+7x)}{7+\log(128)}$$

```
[In] Integrate[2^Log[-8 + 7*x],x]
[Out] (2^Log[-8 + 7*x]*(-8 + 7*x))/(7 + Log[128])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

method	result	size
gosper	$\frac{2^{\ln(-8+7x)}(-8+7x)}{7\ln(2)+7}$	22
risch	$\frac{(-8+7x)(-8+7x)^{\ln(2)}}{7\ln(2)+7}$	22
parallelrisch	$\frac{7x2^{\ln(-8+7x)}-8\,2^{\ln(-8+7x)}}{7\ln(2)+7}$	31
norman	$\frac{x\,\mathrm{e}^{\ln(-8+7x)\ln(2)}}{1+\ln(2)}-\frac{8\,\mathrm{e}^{\ln(-8+7x)\ln(2)}}{7(1+\ln(2))}$	38

```
[In] int(2^ln(-8+7*x),x,method=_RETURNVERBOSE)
[Out] 1/7*(-8+7*x)/(1+ln(2))*2^ln(-8+7*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int 2^{\log(-8+7x)}\,dx = \frac{(7\,x-8)e^{(\log(2)\log(7\,x-8))}}{7\,(\log(2)+1)}$$

```
[In] integrate(2^log(-8+7*x),x, algorithm="fricas")
[Out] 1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int 2^{\log(-8+7x)}\,dx = \frac{7\cdot 2^{\log(7x-8)}x}{7\log(2)+7} - \frac{8\cdot 2^{\log(7x-8)}}{7\log(2)+7}$$

```
[In] integrate(2**ln(-8+7*x),x)
[Out] 7*2**log(7*x - 8)*x/(7*log(2) + 7) - 8*2**log(7*x - 8)/(7*log(2) + 7)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int 2^{\log(-8+7x)}\,dx = \frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(7\,x-8)}}{7\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

```
[In] integrate(2^log(-8+7*x),x, algorithm="maxima")
[Out] 1/7*2^((1/log(2) + 1)*log(7*x - 8))/((1/log(2) + 1)*log(2))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int 2^{\log(-8+7x)}\,dx = \frac{(7\,x-8)e^{(\log(2)\log(7\,x-8))}}{7\,(\log(2)+1)}$$

```
[In] integrate(2^log(-8+7*x),x, algorithm="giac")
[Out] 1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int 2^{\log(-8+7x)}\,dx = \frac{(7\,x-8)^{\ln(2)+1}}{7\,(\ln(2)+1)}$$

```
[In] int(2^log(7*x - 8),x)
[Out] (7*x - 8)^(log(2) + 1)/(7*(log(2) + 1))
```

3.241 $\int \log \left(\frac{-11+5x}{5+76x}\right) \, dx$

Optimal result 1286
Rubi [A] (verified) 1286
Mathematica [A] (verified) 1287
Maple [A] (verified) 1287
Fricas [A] (verification not implemented) 1288
Sympy [A] (verification not implemented) 1288
Maxima [A] (verification not implemented) 1288
Giac [B] (verification not implemented) 1288
Mupad [B] (verification not implemented) 1289

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \log \left(\frac{-11+5x}{5+76x}\right) \, dx = -\frac{1}{5}(11-5x) \log \left(-\frac{11-5x}{5+76x}\right) - \frac{861}{380} \log(5+76x)$$

```
[Out] -1/5*(11-5*x)*ln((-11+5*x)/(5+76*x))-861/380*ln(5+76*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2535, 31}

$$\int \log \left(\frac{-11+5x}{5+76x}\right) \, dx = -\frac{1}{5}(11-5x) \log \left(-\frac{11-5x}{76x+5}\right) - \frac{861}{380} \log(76x+5)$$

```
[In] Int[Log[(-11 + 5*x)/(5 + 76*x)],x]
[Out] -1/5*((11 - 5*x)*Log[-((11 - 5*x)/(5 + 76*x))]) - (861*Log[5 + 76*x])/380
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
```

```
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{5+76x}\right)-\frac{861}{5}\int\frac{1}{5+76x}\,dx\\ &= -\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{5+76x}\right)-\frac{861}{380}\log(5+76x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int\log\left(\frac{-11+5x}{5+76x}\right)\,dx=\left(-\frac{11}{5}+x\right)\log\left(\frac{-11+5x}{5+76x}\right)-\frac{861}{380}\log(5+76x)$$

```
[In] Integrate[Log[(-11 + 5*x)/(5 + 76*x)],x]
[Out] (-11/5 + x)*Log[(-11 + 5*x)/(5 + 76*x)] - (861*Log[5 + 76*x])/380
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result	size
risch	$x\ln\left(\frac{-11+5x}{5+76x}\right)-\frac{11\ln(-11+5x)}{5}-\frac{5\ln(5+76x)}{76}$	34
parts	$x\ln\left(\frac{-11+5x}{5+76x}\right)-\frac{11\ln(-11+5x)}{5}-\frac{5\ln(5+76x)}{76}$	34
parallelrisch	$x\ln\left(\frac{-11+5x}{5+76x}\right)-\frac{861\ln\left(x-\frac{11}{5}\right)}{380}+\frac{5\ln\left(\frac{-11+5x}{5+76x}\right)}{76}$	40
derivativedivides	$\frac{861\ln\left(-\frac{861}{5+76x}\right)}{380}+\frac{\ln\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)(5+76x)}{5}$	44
default	$\frac{861\ln\left(-\frac{861}{5+76x}\right)}{380}+\frac{\ln\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)(5+76x)}{5}$	44

```
[In] int(ln((-11+5*x)/(5+76*x)),x,method=_RETURNVERBOSE)
[Out] x*ln((-11+5*x)/(5+76*x))-11/5*ln(-11+5*x)-5/76*ln(5+76*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(\frac{-11+5x}{5+76x}\right)\,dx = x\log\left(\frac{5\,x-11}{76\,x+5}\right) - \frac{5}{76}\,\log\left(76\,x+5\right) - \frac{11}{5}\,\log\left(5\,x-11\right)$$

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="fricas")
[Out] x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \log \left(\frac{-11+5x}{5+76x}\right)\,dx = x\log\left(\frac{5x-11}{76x+5}\right) - \frac{11\log\left(x-\frac{11}{5}\right)}{5} - \frac{5\log\left(x+\frac{5}{76}\right)}{76}$$

```
[In] integrate(ln((-11+5*x)/(5+76*x)),x)
[Out] x*log((5*x - 11)/(76*x + 5)) - 11*log(x - 11/5)/5 - 5*log(x + 5/76)/76
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \log \left(\frac{-11+5x}{5+76x}\right)\,dx = x\log\left(\frac{5\,x-11}{76\,x+5}\right) - \frac{5}{76}\,\log\left(76\,x+5\right) - \frac{11}{5}\,\log\left(5\,x-11\right)$$

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="maxima")
[Out] x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(30) = 60.

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.97

$$\int \log \left(\frac{-11+5x}{5+76x}\right)\,dx = -\frac{861\,\log\left(\frac{\frac{5\left(\frac{5\,(5\,x-11)}{76\,x+5}+11\right)}{\frac{76\,(5\,x-11)}{76\,x+5}-5}+11}{\frac{76\left(\frac{5\,(5\,x-11)}{76\,x+5}+11\right)}{\frac{76\,(5\,x-11)}{76\,x+5}-5}-5}\right)}{76\left(\frac{76\,(5\,x-11)}{76\,x+5}-5\right)} - \frac{861}{380}\,\log\left(\frac{|5\,x-11|}{|76\,x+5|}\right) + \frac{861}{380}\,\log\left(\left|\frac{76\,(5\,x-11)}{76\,x+5}-5\right|\right)$$

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="giac")
```

```
[Out] -861/76*log((5*(5*(5*x - 11)/(76*x + 5) + 11)/(76*(5*x - 11)/(76*x + 5) - 5
) + 11)/(76*(5*(5*x - 11)/(76*x + 5) + 11)/(76*(5*x - 11)/(76*x + 5) - 5) -
 5))/(76*(5*x - 11)/(76*x + 5) - 5) - 861/380*log(abs(5*x - 11)/abs(76*x +
5)) + 861/380*log(abs(76*(5*x - 11)/(76*x + 5) - 5))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \log\left(\frac{-11+5x}{5+76x}\right)\,dx = x\,\ln\left(\frac{5\,x-11}{76\,x+5}\right)-\frac{5\,\ln\left(x+\frac{5}{76}\right)}{76}-\frac{11\,\ln\left(x-\frac{11}{5}\right)}{5}$$

```
[In] int(log((5*x - 11)/(76*x + 5)),x)
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - (5*log(x + 5/76))/76 - (11*log(x - 11/5))/5
```

3.242 $\int \log \left(\frac{1}{13+x}\right) \, dx$

Optimal result 1290
Rubi [A] (verified) 1290
Mathematica [A] (verified) 1291
Maple [A] (verified) 1291
Fricas [A] (verification not implemented) 1292
Sympy [A] (verification not implemented) 1292
Maxima [A] (verification not implemented) 1292
Giac [A] (verification not implemented) 1292
Mupad [B] (verification not implemented) 1293

Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \log \left(\frac{1}{13+x}\right) \, dx = x+(13+x) \log \left(\frac{1}{13+x}\right)$$

```
[Out] x+(13+x)*ln(1/(13+x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2436, 2332}

$$\int \log \left(\frac{1}{13+x}\right) \, dx = x+(x+13) \log \left(\frac{1}{x+13}\right)$$

```
[In] Int[Log[(13 + x)^(-1)],x]
```

```
[Out] x + (13 + x)*Log[(13 + x)^(-1)]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \log\left(\frac{1}{x}\right)\,dx, x, 13+x\right)$$
$$= x + (13+x)\log\left(\frac{1}{13+x}\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log\left(\frac{1}{13+x}\right)\,dx = x + (13+x)\log\left(\frac{1}{13+x}\right)$$

```
[In] Integrate[Log[(13 + x)^(-1)],x]
[Out] x + (13 + x)*Log[(13 + x)^(-1)]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$(13+x)\ln\left(\frac{1}{13+x}\right)+13+x$	14
default	$(13+x)\ln\left(\frac{1}{13+x}\right)+13+x$	14
risch	$x\ln\left(\frac{1}{13+x}\right)+x-13\ln\left(13+x\right)$	17
parts	$x\ln\left(\frac{1}{13+x}\right)+x-13\ln\left(13+x\right)$	17
norman	$x+x\ln\left(\frac{1}{13+x}\right)+13\ln\left(\frac{1}{13+x}\right)$	19
parallelrisch	$-13+x\ln\left(\frac{1}{13+x}\right)+x+13\ln\left(\frac{1}{13+x}\right)$	20

```
[In] int(ln(1/(13+x)),x,method=_RETURNVERBOSE)
[Out] (13+x)*ln(1/(13+x))+13+x
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1}{13+x}\right) \, dx = (x+13)\log \left(\frac{1}{x+13}\right) + x$$

```
[In] integrate(log(1/(13+x)),x, algorithm="fricas")
[Out] (x + 13)*log(1/(x + 13)) + x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \log \left(\frac{1}{13+x}\right) \, dx = x\log \left(\frac{1}{x+13}\right) + x - 13\log (x+13)$$

```
[In] integrate(ln(1/(13+x)),x)
[Out] x*log(1/(x + 13)) + x - 13*log(x + 13)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1}{13+x}\right) \, dx = -(x+13)\log (x+13) + x + 13$$

```
[In] integrate(log(1/(13+x)),x, algorithm="maxima")
[Out] -(x + 13)*log(x + 13) + x + 13
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1}{13+x}\right) \, dx = -(x+13)\log (x+13) + x + 13$$

```
[In] integrate(log(1/(13+x)),x, algorithm="giac")
[Out] -(x + 13)*log(x + 13) + x + 13
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1}{13+x}\right) dx = \left(\ln \left(\frac{1}{x+13}\right)+1\right) (x+13)$$

```
[In] int(log(1/(x + 13)),x)
[Out] (log(1/(x + 13)) + 1)*(x + 13)
```

3.243 $\int x \log\left(\frac{1+x}{x^2}\right) dx$

Optimal result 1294
Rubi [A] (verified) 1294
Mathematica [A] (verified) 1295
Maple [A] (verified) 1295
Fricas [A] (verification not implemented) 1296
Sympy [A] (verification not implemented) 1296
Maxima [A] (verification not implemented) 1297
Giac [A] (verification not implemented) 1297
Mupad [B] (verification not implemented) 1297

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int x \log\left(\frac{1+x}{x^2}\right) dx = \frac{x}{2} + \frac{x^2}{4} - \frac{1}{2}\log(1+x) + \frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)$$

```
[Out] 1/2*x+1/4*x^2-1/2*ln(1+x)+1/2*x^2*ln((1+x)/x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2581, 30, 45}

$$\int x \log\left(\frac{1+x}{x^2}\right) dx = \frac{x^2}{4} + \frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right) + \frac{x}{2} - \frac{1}{2}\log(x+1)$$

```
[In] Int[x*Log[(1 + x)/x^2],x]
```

```
[Out] x/2 + x^2/4 - Log[1 + x]/2 + (x^2*Log[(1 + x)/x^2])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)-\frac{1}{2}\int\frac{x^2}{1+x}\,dx+\int x\,dx\\ &= \frac{x^2}{2}+\frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)-\frac{1}{2}\int\left(-1+x+\frac{1}{1+x}\right)\,dx\\ &= \frac{x}{2}+\frac{x^2}{4}-\frac{1}{2}\log(1+x)+\frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{1}{4}\left(-2\log(1+x)+x\left(2+x+2x\log\left(\frac{1+x}{x^2}\right)\right)\right)$$

```
[In] Integrate[x*Log[(1 + x)/x^2],x]
[Out] (-2*Log[1 + x] + x*(2 + x + 2*x*Log[(1 + x)/x^2]))/4
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{2}+\frac{x^{2}}{4}-\frac{\ln(x+1)}{2}+\frac{x^{2}\ln\left(\frac{x+1}{x^{2}}\right)}{2}$	29
parts	$\frac{x}{2}+\frac{x^{2}}{4}-\frac{\ln(x+1)}{2}+\frac{x^{2}\ln\left(\frac{x+1}{x^{2}}\right)}{2}$	29
parallelrisch	$\frac{x^{2}\ln\left(\frac{x+1}{x^{2}}\right)}{2}-\frac{1}{2}+\frac{x^{2}}{4}-\ln\left(x\right)+\frac{x}{2}-\frac{\ln\left(\frac{x+1}{x^{2}}\right)}{2}$	38
derivativedivides	$\frac{x^{2}\ln\left(\frac{1+\frac{1}{x}}{x}\right)}{2}+\frac{x^{2}}{4}+\frac{x}{2}+\frac{\ln\left(\frac{1}{x}\right)}{2}-\frac{\ln\left(1+\frac{1}{x}\right)}{2}$	39
default	$\frac{x^{2}\ln\left(\frac{1+\frac{1}{x}}{x}\right)}{2}+\frac{x^{2}}{4}+\frac{x}{2}+\frac{\ln\left(\frac{1}{x}\right)}{2}-\frac{\ln\left(1+\frac{1}{x}\right)}{2}$	39

```
[In] int(x*ln((x+1)/x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/4*x^2-1/2*ln(x+1)+1/2*x^2*ln((x+1)/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x\log\left(\frac{1+x}{x^{2}}\right)\,dx=\frac{1}{2}\,x^{2}\log\left(\frac{x+1}{x^{2}}\right)+\frac{1}{4}\,x^{2}+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(x+1\right)$$

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="fricas")
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int x\log\left(\frac{1+x}{x^{2}}\right)\,dx=\frac{x^{2}\log\left(\frac{x+1}{x^{2}}\right)}{2}+\frac{x^{2}}{4}+\frac{x}{2}-\frac{\log\left(x+1\right)}{2}$$

```
[In] integrate(x*ln((1+x)/x**2),x)
[Out] x**2*log((x + 1)/x**2)/2 + x**2/4 + x/2 - log(x + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{1}{2}\,x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}\,x^2+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(x+1\right)$$

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="maxima")
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{1}{2}\,x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}\,x^2+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(|x+1|\right)$$

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="giac")
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x\log\left(\frac{1+x}{x^2}\right)\,dx=\frac{x}{2}-\frac{\ln\left(x\,\left(x+1\right)\right)}{3}-\frac{\ln\left(\frac{x+1}{x^2}\right)}{6}+\frac{x^2\,\ln\left(\frac{x+1}{x^2}\right)}{2}+\frac{x^2}{4}$$

```
[In] int(x*log((x + 1)/x^2),x)
[Out] x/2 - log(x*(x + 1))/3 - log((x + 1)/x^2)/6 + (x^2*log((x + 1)/x^2))/2 + x^
2/4
```

3.244 $\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx$

Optimal result 1298
Rubi [A] (verified) 1298
Mathematica [A] (verified) 1299
Maple [A] (verified) 1299
Fricas [A] (verification not implemented) 1300
Sympy [A] (verification not implemented) 1300
Maxima [A] (verification not implemented) 1301
Giac [A] (verification not implemented) 1301
Mupad [B] (verification not implemented) 1301

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx = \frac{343x}{500} - \frac{49x^2}{200} + \frac{7x^3}{60} + \frac{x^4}{16} - \frac{2401\log(7+5x)}{2500} + \frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)$$

```
[Out] 343/500*x-49/200*x^2+7/60*x^3+1/16*x^4-2401/2500*ln(7+5*x)+1/4*x^4*ln((7+5*
x)/x^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2581, 30, 45}

$$\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx = \frac{x^4}{16} + \frac{7x^3}{60} - \frac{49x^2}{200} + \frac{1}{4}x^4\log\left(\frac{5x+7}{x^2}\right) + \frac{343x}{500} - \frac{2401\log(5x+7)}{2500}$$

```
[In] Int[x^3*Log[(7 + 5*x)/x^2],x]
```

```
[Out] (343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/2500
 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)+\frac{\int x^3\,dx}{2}-\frac{5}{4}\int\frac{x^4}{7+5x}\,dx\\
&= \frac{x^4}{8}+\frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)-\frac{5}{4}\int\left(-\frac{343}{625}+\frac{49x}{125}-\frac{7x^2}{25}+\frac{x^3}{5}+\frac{2401}{625(7+5x)}\right)dx\\
&= \frac{343x}{500}-\frac{49x^2}{200}+\frac{7x^3}{60}+\frac{x^4}{16}-\frac{2401\log(7+5x)}{2500}+\frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^3\log\left(\frac{7+5x}{x^2}\right)dx=\frac{343x}{500}-\frac{49x^2}{200}+\frac{7x^3}{60}+\frac{x^4}{16}-\frac{2401\log(7+5x)}{2500}+\frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)$$

```
[In] Integrate[x^3*Log[(7 + 5*x)/x^2],x]
[Out] (343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/2500
 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{343x}{500}-\frac{49x^{2}}{200}+\frac{7x^{3}}{60}+\frac{x^{4}}{16}-\frac{2401\ln(7+5x)}{2500}+\frac{x^{4}\ln\left(\frac{7+5x}{x^{2}}\right)}{4}$	43
parts	$\frac{343x}{500}-\frac{49x^{2}}{200}+\frac{7x^{3}}{60}+\frac{x^{4}}{16}-\frac{2401\ln(7+5x)}{2500}+\frac{x^{4}\ln\left(\frac{7+5x}{x^{2}}\right)}{4}$	43
parallelrisch	$\frac{x^{4}\ln\left(\frac{7+5x}{x^{2}}\right)}{4}+\frac{x^{4}}{16}+\frac{7x^{3}}{60}-\frac{2401}{2500}-\frac{49x^{2}}{200}-\frac{2401\ln(x)}{1250}+\frac{343x}{500}-\frac{2401\ln\left(\frac{7+5x}{x^{2}}\right)}{2500}$	52
derivativedivides	$\frac{x^{4}\ln\left(\frac{\frac{7}{x}+5}{x}\right)}{4}+\frac{x^{4}}{16}+\frac{7x^{3}}{60}-\frac{49x^{2}}{200}+\frac{343x}{500}+\frac{2401\ln(\frac{1}{x})}{2500}-\frac{2401\ln(\frac{7}{x}+5)}{2500}$	53
default	$\frac{x^{4}\ln\left(\frac{\frac{7}{x}+5}{x}\right)}{4}+\frac{x^{4}}{16}+\frac{7x^{3}}{60}-\frac{49x^{2}}{200}+\frac{343x}{500}+\frac{2401\ln(\frac{1}{x})}{2500}-\frac{2401\ln(\frac{7}{x}+5)}{2500}$	53

```
[In] int(x^3*ln((7+5*x)/x^2),x,method=_RETURNVERBOSE)
[Out] 343/500*x-49/200*x^2+7/60*x^3+1/16*x^4-2401/2500*ln(7+5*x)+1/4*x^4*ln((7+5*
x)/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int x^{3}\log\left(\frac{7+5x}{x^{2}}\right)\,dx=\frac{1}{4}\,x^{4}\log\left(\frac{5\,x+7}{x^{2}}\right)+\frac{1}{16}\,x^{4}+\frac{7}{60}\,x^{3}-\frac{49}{200}\,x^{2}+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(5\,x+7\right)$$

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="fricas")
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(5*x + 7)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x^{3}\log\left(\frac{7+5x}{x^{2}}\right)\,dx=\frac{x^{4}\log\left(\frac{5x+7}{x^{2}}\right)}{4}+\frac{x^{4}}{16}+\frac{7x^{3}}{60}-\frac{49x^{2}}{200}+\frac{343x}{500}-\frac{2401\log\left(5x+7\right)}{2500}$$

```
[In] integrate(x**3*ln((7+5*x)/x**2),x)
[Out] x**4*log((5*x + 7)/x**2)/4 + x**4/16 + 7*x**3/60 - 49*x**2/200 + 343*x/500
- 2401*log(5*x + 7)/2500
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx = \frac{1}{4} x^4 \log\left(\frac{5x+7}{x^2}\right) + \frac{1}{16} x^4 + \frac{7}{60} x^3 - \frac{49}{200} x^2 + \frac{343}{500} x - \frac{2401}{2500} \log(5x+7)$$

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="maxima")
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(5*x + 7)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx = \frac{1}{4} x^4 \log\left(\frac{5x+7}{x^2}\right) + \frac{1}{16} x^4 + \frac{7}{60} x^3 - \frac{49}{200} x^2 + \frac{343}{500} x - \frac{2401}{2500} \log(|5x+7|)$$

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="giac")
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(abs(5*x + 7))
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx = \frac{343x}{500} - \frac{2401 \ln(x(5x+7))}{3750} - \frac{2401 \ln\left(\frac{5x+7}{x^2}\right)}{7500} + \frac{x^4 \ln\left(\frac{5x+7}{x^2}\right)}{4} - \frac{49x^2}{200} + \frac{7x^3}{60} + \frac{x^4}{16}$$

```
[In] int(x^3*log((5*x + 7)/x^2),x)
[Out] (343*x)/500 - (2401*log(x*(5*x + 7)))/3750 - (2401*log((5*x + 7)/x^2))/7500
 + (x^4*log((5*x + 7)/x^2))/4 - (49*x^2)/200 + (7*x^3)/60 + x^4/16
```

3.245 $\int (a+bx)\log(a+bx)\,dx$

Optimal result 1302
Rubi [A] (verified) 1302
Mathematica [A] (verified) 1303
Maple [A] (verified) 1303
Fricas [A] (verification not implemented) 1304
Sympy [A] (verification not implemented) 1304
Maxima [A] (verification not implemented) 1304
Giac [A] (verification not implemented) 1304
Mupad [B] (verification not implemented) 1305

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int (a+bx)\log(a+bx)\,dx=-\frac{(a+bx)^2}{4b}+\frac{(a+bx)^2\log(a+bx)}{2b}$$

```
[Out] -1/4*(b*x+a)^2/b+1/2*(b*x+a)^2*ln(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2437, 2341}

$$\int (a+bx)\log(a+bx)\,dx=\frac{(a+bx)^2\log(a+bx)}{2b}-\frac{(a+bx)^2}{4b}$$

```
[In] Int[(a + b*x)*Log[a + b*x],x]
```

```
[Out] -1/4*(a + b*x)^2/b + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int x \log(x)\,dx, x, a+bx)}{b}$$
$$= -\frac{(a+bx)^2}{4b} + \frac{(a+bx)^2\log(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int (a+bx)\log(a+bx)\,dx = -\frac{1}{4}x(2a+bx) + \frac{(a+bx)^2\log(a+bx)}{2b}$$

```
[In] Integrate[(a + b*x)*Log[a + b*x],x]
[Out] -1/4*(x*(2*a + b*x)) + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{(bx+a)^2\ln(bx+a)}{2}-\frac{(bx+a)^2}{4}}{b}$	30
default	$\frac{\frac{(bx+a)^2\ln(bx+a)}{2}-\frac{(bx+a)^2}{4}}{b}$	30
risch	$\left(\frac{1}{2}b\,x^2+ax\right)\ln\left(bx+a\right)-\frac{b\,x^2}{4}-\frac{ax}{2}+\frac{a^2\ln(bx+a)}{2b}$	43
norman	$ax\ln\left(bx+a\right)-\frac{ax}{2}-\frac{b\,x^2}{4}+\frac{a^2\ln(bx+a)}{2b}+\frac{b\,x^2\ln(bx+a)}{2}$	47
parts	$\frac{b\,x^2\ln(bx+a)}{2}+ax\ln\left(bx+a\right)-\frac{b\left(\frac{\frac{1}{2}b\,x^2+ax}{b}-\frac{a^2\ln(bx+a)}{b^2}\right)}{2}$	55
parallelrisch	$\frac{2x^2\ln(bx+a)b^2-b^2x^2+4x\ln(bx+a)ab-2abx+2a^2\ln(bx+a)+2a^2}{4b}$	61

```
[In] int((b*x+a)*ln(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*(b*x+a)^2*ln(b*x+a)-1/4*(b*x+a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int (a+bx)\log(a+bx)\,dx = -\frac{b^2x^2 + 2\,abx - 2\,(b^2x^2 + 2\,abx + a^2)\log\left(bx+a\right)}{4\,b}$$

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="fricas")
[Out] -1/4*(b^2*x^2 + 2*a*b*x - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int (a+bx)\log(a+bx)\,dx = \frac{a^2\log\left(a+bx\right)}{2b} - \frac{ax}{2} - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log\left(a+bx\right)$$

```
[In] integrate((b*x+a)*ln(b*x+a),x)
[Out] a**2*log(a + b*x)/(2*b) - a*x/2 - b*x**2/4 + (a*x + b*x**2/2)*log(a + b*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int (a+bx)\log(a+bx)\,dx = \frac{1}{4}\,b\left(\frac{2\,a^2\log\left(bx+a\right)}{b^2} - \frac{bx^2+2\,ax}{b}\right) + \frac{1}{2}\left(bx^2+2\,ax\right)\log\left(bx+a\right)$$

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="maxima")
[Out] 1/4*b*(2*a^2*log(b*x + a)/b^2 - (b*x^2 + 2*a*x)/b) + 1/2*(b*x^2 + 2*a*x)*lo
g(b*x + a)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a+bx)\log(a+bx)\,dx = \frac{(bx+a)^2\log\left(bx+a\right)}{2\,b} - \frac{(bx+a)^2}{4\,b}$$

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="giac")
[Out] 1/2*(b*x + a)^2*log(b*x + a)/b - 1/4*(b*x + a)^2/b
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int (a+bx)\log(a+bx)\,dx = \frac{a^2\,\ln(a+b\,x)}{2\,b} - \frac{b\,x^2}{4} - \frac{a\,x}{2} + a\,x\,\ln(a+b\,x) + \frac{b\,x^2\,\ln(a+b\,x)}{2}$$

```
[In] int(log(a + b*x)*(a + b*x),x)
[Out] (a^2*log(a + b*x))/(2*b) - (b*x^2)/4 - (a*x)/2 + a*x*log(a + b*x) + (b*x^2*
log(a + b*x))/2
```

3.246 $\int (a+bx)^2 \log(a+bx)\,dx$

Optimal result 1306
Rubi [A] (verified) 1306
Mathematica [A] (verified) 1307
Maple [A] (verified) 1307
Fricas [B] (verification not implemented) 1308
Sympy [B] (verification not implemented) 1308
Maxima [B] (verification not implemented) 1308
Giac [A] (verification not implemented) 1309
Mupad [B] (verification not implemented) 1309

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int (a+bx)^2 \log(a+bx)\,dx = -\frac{(a+bx)^3}{9b}+\frac{(a+bx)^3\log(a+bx)}{3b}$$

```
[Out] -1/9*(b*x+a)^3/b+1/3*(b*x+a)^3*ln(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2437, 2341}

$$\int (a+bx)^2 \log(a+bx)\,dx = \frac{(a+bx)^3\log(a+bx)}{3b}-\frac{(a+bx)^3}{9b}$$

```
[In] Int[(a + b*x)^2*Log[a + b*x],x]
```

```
[Out] -1/9*(a + b*x)^3/b + ((a + b*x)^3*Log[a + b*x])/(3*b)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^2\log(x)\,dx, x, a+bx\right)}{b}\\ &= -\frac{(a+bx)^3}{9b}+\frac{(a+bx)^3\log(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int (a+bx)^2\log(a+bx)\,dx = -\frac{1}{9}x\left(3a^2+3abx+b^2x^2\right)+\frac{(a+bx)^3\log(a+bx)}{3b}$$

```
[In] Integrate[(a + b*x)^2*Log[a + b*x],x]
[Out] -1/9*(x*(3*a^2 + 3*a*b*x + b^2*x^2)) + ((a + b*x)^3*Log[a + b*x])/(3*b)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{(bx+a)^3\ln(bx+a)}{3}-\frac{(bx+a)^3}{9}}{b}$	30
default	$\frac{\frac{(bx+a)^3\ln(bx+a)}{3}-\frac{(bx+a)^3}{9}}{b}$	30
risch	$-\frac{x^3b^2}{9}-\frac{ab\,x^2}{3}-\frac{a^2x}{3}-\frac{a^3}{9b}+\frac{(bx+a)^3\ln(bx+a)}{3b}$	49
parts	$\frac{x^3b^2\ln(bx+a)}{3}+ab\,x^2\ln(bx+a)+a^2x\ln(bx+a)+\frac{a^3\ln(bx+a)}{3b}-\frac{(bx+a)^3}{9b}$	65
norman	$a^2x\ln(bx+a)+ab\,x^2\ln(bx+a)-\frac{a^2x}{3}-\frac{x^3b^2}{9}-\frac{ab\,x^2}{3}+\frac{a^3\ln(bx+a)}{3b}+\frac{x^3b^2\ln(bx+a)}{3}$	74
parallelrisch	$\frac{3x^3\ln(bx+a)b^3-b^3x^3+9x^2\ln(bx+a)a\,b^2-3a\,b^2x^2+9x\ln(bx+a)a^2b-3a^2bx+3\ln(bx+a)a^3+3a^3}{9b}$	89

```
[In] int((b*x+a)^2*ln(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*(b*x+a)^3*ln(b*x+a)-1/9*(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(31) = 62.

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int (a+bx)^2\log(a+bx)\,dx = -\frac{b^3x^3+3\,ab^2x^2+3\,a^2bx-3\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\log(bx+a)}{9\,b}$$

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x - 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int (a+bx)^2\log(a+bx)\,dx = \frac{a^3\log(a+bx)}{3b}-\frac{a^2x}{3}-\frac{abx^2}{3}-\frac{b^2x^3}{9}+\left(a^2x+abx^2+\frac{b^2x^3}{3}\right)\log(a+bx)$$

```
[In] integrate((b*x+a)**2*ln(b*x+a),x)
```

```
[Out] a**3*log(a + b*x)/(3*b) - a**2*x/3 - a*b*x**2/3 - b**2*x**3/9 + (a**2*x + a
*b*x**2 + b**2*x**3/3)*log(a + b*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(31) = 62.

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.11

$$\int (a+bx)^2\log(a+bx)\,dx = \frac{1}{9}\left(\frac{3\,a^3\log(bx+a)}{b^2}-\frac{b^2x^3+3\,abx^2+3\,a^2x}{b}\right)b+\frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\log(bx+a)$$

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/9*(3*a^3*log(b*x + a)/b^2 - (b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/b)*b + 1/3*(b
^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(b*x + a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a+bx)^2 \log(a+bx)\, dx = \frac{(bx+a)^3 \log{(bx+a)}}{3\,b} - \frac{(bx+a)^3}{9\,b}$$

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3*log(b*x + a)/b - 1/9*(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int (a+bx)^2 \log(a+bx)\, dx = \frac{a^3\,\ln{(a+b\,x)}}{3\,b} - \frac{b^2\,x^3}{9} - \frac{a^2\,x}{3} + \frac{b^2\,x^3\,\ln{(a+b\,x)}}{3} - \frac{a\,b\,x^2}{3} + a^2\,x\,\ln{(a+b\,x)} + a\,b\,x^2\,\ln{(a+b\,x)}$$

```
[In] int(log(a + b*x)*(a + b*x)^2,x)
```

```
[Out] (a^3*log(a + b*x))/(3*b) - (b^2*x^3)/9 - (a^2*x)/3 + (b^2*x^3*log(a + b*x))
/3 - (a*b*x^2)/3 + a^2*x*log(a + b*x) + a*b*x^2*log(a + b*x)
```

3.247 $\int \frac{\log(a+bx)}{a+bx}\,dx$

Optimal result 1310
Rubi [A] (verified) 1310
Mathematica [A] (verified) 1311
Maple [A] (verified) 1311
Fricas [A] (verification not implemented) 1312
Sympy [A] (verification not implemented) 1312
Maxima [A] (verification not implemented) 1312
Giac [A] (verification not implemented) 1312
Mupad [B] (verification not implemented) 1313

Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\log^2(a+bx)}{2b}$$

```
[Out] 1/2*ln(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2437, 2338}

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\log^2(a+bx)}{2b}$$

```
[In] Int[Log[a + b*x]/(a + b*x),x]
```

```
[Out] Log[a + b*x]^2/(2*b)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\log^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\log^2(a+bx)}{2b}$$

```
[In] Integrate[Log[a + b*x]/(a + b*x),x]
[Out] Log[a + b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln(bx+a)^2}{2b}$	14
default	$\frac{\ln(bx+a)^2}{2b}$	14
norman	$\frac{\ln(bx+a)^2}{2b}$	14
risch	$\frac{\ln(bx+a)^2}{2b}$	14
parallelrisch	$\frac{\ln(bx+a)^2}{2b}$	14
parts	$\frac{\ln(bx+a)^2}{2b}$	14

```
[In] int(ln(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\log\left(bx+a\right)^2}{2\,b}$$

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="fricas")
[Out] 1/2*log(b*x + a)^2/b
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\log\left(a+bx\right)^2}{2b}$$

```
[In] integrate(ln(b*x+a)/(b*x+a),x)
[Out] log(a + b*x)**2/(2*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\log\left(bx+a\right)^2}{2\,b}$$

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="maxima")
[Out] 1/2*log(b*x + a)^2/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\log\left(bx+a\right)^2}{2\,b}$$

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="giac")
[Out] 1/2*log(b*x + a)^2/b
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\log(a+bx)}{a+bx}\,dx = \frac{\ln\left(a+b\,x\right)^2}{2\,b}$$

```
[In] int(log(a + b*x)/(a + b*x),x)
[Out] log(a + b*x)^2/(2*b)
```

3.248 $\int \frac{\log(a+bx)}{(a+bx)^2} dx$

Optimal result 1314
Rubi [A] (verified) 1314
Mathematica [A] (verified) 1315
Maple [A] (verified) 1315
Fricas [A] (verification not implemented) 1316
Sympy [A] (verification not implemented) 1316
Maxima [A] (verification not implemented) 1316
Giac [A] (verification not implemented) 1316
Mupad [B] (verification not implemented) 1317

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{\log(a+bx)}{(a+bx)^2} dx = -\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)-ln(b*x+a)/b/(b*x+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2437, 2341}

$$\int \frac{\log(a+bx)}{(a+bx)^2} dx = -\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}$$

```
[In] Int[Log[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -(1/(b*(a + b*x))) - Log[a + b*x]/(b*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log(x)}{x^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -\frac{1+\log(a+bx)}{ab+b^2x}$$

```
[In] Integrate[Log[a + b*x]/(a + b*x)^2,x]
[Out] -((1 + Log[a + b*x])/(a*b + b^2*x))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
norman	$\frac{\frac{x}{a}-\frac{\ln(bx+a)}{b}}{bx+a}$	26
parallelrisch	$\frac{-\ln(bx+a)b^2-b^2}{(bx+a)b^3}$	29
derivativedivides	$\frac{-\frac{\ln(bx+a)}{bx+a}-\frac{1}{bx+a}}{b}$	30
default	$\frac{-\frac{\ln(bx+a)}{bx+a}-\frac{1}{bx+a}}{b}$	30
risch	$-\frac{1}{b(bx+a)}-\frac{\ln(bx+a)}{b(bx+a)}$	32
parts	$-\frac{1}{b(bx+a)}-\frac{\ln(bx+a)}{b(bx+a)}$	32

```
[In] int(ln(b*x+a)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] (x/a-ln(b*x+a)/b)/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -\frac{\log\left(bx+a\right)+1}{b^2x+ab}$$

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
[Out] -(log(b*x + a) + 1)/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -\frac{\log\left(a+bx\right)}{ab+b^2x} - \frac{1}{ab+b^2x}$$

```
[In] integrate(ln(b*x+a)/(b*x+a)**2,x)
[Out] -log(a + b*x)/(a*b + b**2*x) - 1/(a*b + b**2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -\frac{\log\left(bx+a\right)}{(bx+a)b} - \frac{1}{(bx+a)b}$$

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
[Out] -log(b*x + a)/((b*x + a)*b) - 1/((b*x + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -b\left(\frac{\log\left(bx+a\right)}{(bx+a)b^2} + \frac{1}{(bx+a)b^2}\right)$$

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="giac")
[Out] -b*(log(b*x + a)/((b*x + a)*b^2) + 1/((b*x + a)*b^2))
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = -\frac{a+a\,\ln\left(a+b\,x\right)}{a\,b\,\left(a+b\,x\right)}$$

```
[In] int(log(a + b*x)/(a + b*x)^2,x)
[Out] -(a + a*log(a + b*x))/(a*b*(a + b*x))
```

3.249 $\int (a+bx)^n \log(a+bx)\, dx$

Optimal result 1318
Rubi [A] (verified) 1318
Mathematica [A] (verified) 1319
Maple [A] (verified) 1319
Fricas [A] (verification not implemented) 1320
Sympy [B] (verification not implemented) 1320
Maxima [A] (verification not implemented) 1321
Giac [F] 1321
Mupad [B] (verification not implemented) 1321

Optimal result

Integrand size = 14, antiderivative size = 44

$$\int (a+bx)^n \log(a+bx)\, dx = -\frac{(a+bx)^{1+n}}{b(1+n)^2} + \frac{(a+bx)^{1+n}\log(a+bx)}{b(1+n)}$$

```
[Out] -(b*x+a)^(1+n)/b/(1+n)^2+(b*x+a)^(1+n)*ln(b*x+a)/b/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2437, 2341}

$$\int (a+bx)^n \log(a+bx)\, dx = \frac{(a+bx)^{n+1}\log(a+bx)}{b(n+1)} - \frac{(a+bx)^{n+1}}{b(n+1)^2}$$

```
[In] Int[(a + b*x)^n*Log[a + b*x],x]
```

```
[Out] -((a + b*x)^(1 + n)/(b*(1 + n)^2)) + ((a + b*x)^(1 + n)*Log[a + b*x])/(b*(1
 + n))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^n \log(x)\,dx, x, a+bx\right)}{b}$$
$$= -\frac{(a+bx)^{1+n}}{b(1+n)^2} + \frac{(a+bx)^{1+n}\log(a+bx)}{b(1+n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int (a+bx)^n \log(a+bx)\,dx = \frac{(a+bx)^{1+n}(-1+(1+n)\log(a+bx))}{b(1+n)^2}$$

```
[In] Integrate[(a + b*x)^n*Log[a + b*x],x]
[Out] ((a + b*x)^(1 + n)*(-1 + (1 + n)*Log[a + b*x]))/(b*(1 + n)^2)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

method	result	size
risch	$\frac{(bnx\ln(bx+a)+an\ln(bx+a)+\ln(bx+a)xb+a\ln(bx+a)-bx-a)(bx+a)^n}{(1+n)^2 b}$	61
norman	$\frac{x\ln(bx+a)\mathrm{e}^{n\ln(bx+a)}}{1+n}+\frac{a\ln(bx+a)\mathrm{e}^{n\ln(bx+a)}}{b(1+n)}-\frac{x\,\mathrm{e}^{n\ln(bx+a)}}{n^2+2n+1}-\frac{a\,\mathrm{e}^{n\ln(bx+a)}}{b(n^2+2n+1)}$	96
parallelrisch	$\frac{x(bx+a)^n\ln(bx+a)bn+x(bx+a)^n\ln(bx+a)b+(bx+a)^n\ln(bx+a)an-x(bx+a)^n b+(bx+a)^n\ln(bx+a)a-a(bx+a)^n}{(1+n)^2 b}$	96

```
[In] int((b*x+a)^n*ln(b*x+a),x,method=_RETURNVERBOSE)
[Out] (b*n*x*ln(b*x+a)+a*n*ln(b*x+a)+ln(b*x+a)*x*b+a*ln(b*x+a)-b*x-a)/(1+n)^2/b*(
b*x+a)^n
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int (a+bx)^n \log(a+bx)\,dx = -\frac{(bx-(an+(bn+b)x+a)\log{(bx+a)}+a)(bx+a)^n}{bn^2+2\,bn+b}$$

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x - (a*n + (b*n + b)*x + a)*log(b*x + a) + a)*(b*x + a)^n/(b*n^2 + 2*b*
n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(34) = 68.

Time = 0.43 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.20

$$\int (a+bx)^n \log(a+bx)\,dx$$
$$= \begin{cases} \frac{x\log{(a)}}{a} & \text{for} \\ a^n x\log{(a)} & \text{for} \\ \frac{\log{(a+bx)}^2}{2b} & \text{for} \\ \frac{an(a+bx)^n\log{(a+bx)}}{bn^2+2bn+b} + \frac{a(a+bx)^n\log{(a+bx)}}{bn^2+2bn+b} - \frac{a(a+bx)^n}{bn^2+2bn+b} + \frac{bnx(a+bx)^n\log{(a+bx)}}{bn^2+2bn+b} + \frac{bx(a+bx)^n\log{(a+bx)}}{bn^2+2bn+b} - \frac{bx(a+bx)^n}{bn^2+2bn+b} & \text{oth} \end{cases}$$

```
[In] integrate((b*x+a)**n*ln(b*x+a),x)
```

```
[Out] Piecewise((x*log(a)/a, Eq(b, 0) & Eq(n, -1)), (a**n*x*log(a), Eq(b, 0)), (l
og(a + b*x)**2/(2*b), Eq(n, -1)), (a*n*(a + b*x)**n*log(a + b*x)/(b*n**2 +
2*b*n + b) + a*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n + b) - a*(a + b*x)
**n/(b*n**2 + 2*b*n + b) + b*n*x*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n
+ b) + b*x*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n + b) - b*x*(a + b*x)**
n/(b*n**2 + 2*b*n + b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx)^n \log(a+bx)\,dx = \frac{(bx+a)^{n+1}\log(bx+a)}{b(n+1)} - \frac{(bx+a)^{n+1}}{b(n+1)^2}$$

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="maxima")
[Out] (b*x + a)^(n + 1)*log(b*x + a)/(b*(n + 1)) - (b*x + a)^(n + 1)/(b*(n + 1)^2
)
```

Giac [F]

$$\int (a+bx)^n \log(a+bx)\,dx = \int (bx+a)^n \log(bx+a)\,dx$$

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="giac")
[Out] integrate((b*x + a)^n*log(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int (a+bx)^n \log(a+bx)\,dx = \begin{cases} \frac{\ln(a+b\,x)^2}{2\,b} & \text{if } n=-1 \\ \frac{\left(\ln(a+b\,x)-\frac{1}{n+1}\right)\,(a+b\,x)^{n+1}}{b\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

```
[In] int(log(a + b*x)*(a + b*x)^n,x)
[Out] piecewise(n == -1, log(a + b*x)^2/(2*b), n ~= -1, ((log(a + b*x) - 1/(n + 1
))*(a + b*x)^(n + 1))/(b*(n + 1)))
```

3.250 $\int \frac{1}{ax+bx\log(cx^n)} dx$

Optimal result 1322
Rubi [A] (verified) 1322
Mathematica [A] (verified) 1323
Maple [A] (verified) 1323
Fricas [A] (verification not implemented) 1323
Sympy [B] (verification not implemented) 1324
Maxima [A] (verification not implemented) 1324
Giac [B] (verification not implemented) 1324
Mupad [B] (verification not implemented) 1325

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{ax+bx\log(cx^n)}\,dx=\frac{\log(a+b\log(cx^n))}{bn}$$

```
[Out] ln(a+b*ln(c*x^n))/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\int \frac{1}{ax+bx\log(cx^n)}\,dx=\frac{\log(a+b\log(cx^n))}{bn}$$

```
[In] Int[(a*x + b*x*Log[c*x^n])^(-1),x]
```

```
[Out] Log[a + b*Log[c*x^n]]/(b*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,\log(cx^n)\right)}{n}\\&=\frac{\log(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax + bx\log(cx^n)} \, dx = \frac{\log(a + b\log(cx^n))}{bn}$$

```
[In] Integrate[(a*x + b*x*Log[c*x^n])^(-1),x]
[Out] Log[a + b*Log[c*x^n]]/(b*n)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
default	$\frac{\ln(a+b\ln(c\,x^n))}{bn}$
parallelrisch	$\frac{\ln(a+b\ln(c\,x^n))}{bn}$
norman	$\frac{\ln\left(b\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+a\right)}{bn}$
risch	$\frac{\ln\left(\ln(x^n)-\frac{ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\ \mathrm{csgn}(ic\,x^n)-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\ \mathrm{csgn}(ix^n)\ \mathrm{csgn}(ic\,x^n)^2+ib\pi\,\mathrm{csgn}(ic\,x^n)^3-2b\ln(c)-2a}{2b}\right)}{bn}$

```
[In] int(1/(a*x+b*x*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{ax + bx\log(cx^n)} \, dx = \frac{\log(bn\log(x) + b\log(c) + a)}{bn}$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="fricas")
[Out] log(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{ax + bx\log(cx^n)}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \frac{\log(x)}{a+b\log(c)} & \text{for } n = 0 \\ \frac{\log\left(\frac{a}{b}+\log(cx^n)\right)}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)),x)
[Out] Piecewise((log(x)/a, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)/(a + b*log(
c)), Eq(n, 0)), (log(a/b + log(c*x**n))/(b*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{1}{ax + bx\log(cx^n)}\,dx = \frac{\log\left(\frac{b\log(c)+b\log(x^n)+a}{b}\right)}{bn}$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="maxima")
[Out] log((b*log(c) + b*log(x^n) + a)/b)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{ax + bx\log(cx^n)}\,dx = \frac{\log\left(\frac{1}{4}\left(\pi bn(\operatorname{sgn}(x) - 1) + \pi b(\operatorname{sgn}(c) - 1)\right)^2 + \left(bn\log(|x|) + b\log(|c|) + a\right)^2\right)}{2\,bn}$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="giac")
[Out] 1/2*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x))
+ b*log(abs(c)) + a)^2)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx\log(cx^n)}\,dx=\frac{\ln\left(a+b\,\ln\left(c\,x^n\right)\right)}{b\,n}$$

```
[In] int(1/(a*x + b*x*log(c*x^n)),x)
[Out] log(a + b*log(c*x^n))/(b*n)
```

3.251 $\int \frac{1}{ax+bx\log^2(cx^n)}\,dx$

Optimal result 1326
Rubi [A] (verified) 1326
Mathematica [A] (verified) 1327
Maple [A] (verified) 1327
Fricas [A] (verification not implemented) 1327
Sympy [B] (verification not implemented) 1328
Maxima [F] 1328
Giac [A] (verification not implemented) 1328
Mupad [B] (verification not implemented) 1329

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

```
[Out] arctan(ln(c*x^n)*b^(1/2)/a^(1/2))/n/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {211}

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

```
[In] Int[(a*x + b*x*Log[c*x^n]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result
default	$\frac{\arctan\left(\frac{b\ln(c\,x^n)}{\sqrt{ab}}\right)}{n\sqrt{ab}}$
risch	$-\frac{\ln\left(\ln(x^n)+\frac{-i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\sqrt{-ab}+i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2\sqrt{-ab}+i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2\sqrt{-ab}-i\pi\,\mathrm{csgn}(ic\,x^n)^3\sqrt{-ab}+}{2\sqrt{-ab}}\right.}{2\sqrt{-ab}\,n}$

```
[In] int(1/(a*x+b*x*ln(c*x^n)^2),x,method=_RETURNVERBOSE)
[Out] 1/n/(a*b)^(1/2)*arctan(b*ln(c*x^n)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.78

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx$$
$$=\left[-\frac{\sqrt{-ab}\log\left(\frac{bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2-2\sqrt{-ab}(n\log(x)+\log(c))-a}{bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2+a}\right)}{2\,abn},\ \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}(n\log(x)+\log(c))}{a}\right)}{abn}\right]$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 - 2
*sqrt(-a*b)*(n*log(x) + log(c)) - a)/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x)
+ b*log(c)^2 + a))/(a*b*n), sqrt(a*b)*arctan(sqrt(a*b)*(n*log(x) + log(c))/
a)/(a*b*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(29) = 58$.

Time = 2.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.09

$$\int \frac{1}{ax + bx\log^2(cx^n)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^2} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a+b\log(c)^2} & \text{for } n = 0 \\ -\frac{1}{bn\log(cx^n)} & \text{for } a = 0 \\ \frac{\log\left(-\sqrt{-\frac{a}{b}}+\log(cx^n)\right)}{2bn\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\log(cx^n)\right)}{2bn\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**2),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a
, Eq(b, 0)), (log(x)/(a + b*log(c)**2), Eq(n, 0)), (-1/(b*n*log(c*x**n)), E
q(a, 0)), (log(-sqrt(-a/b) + log(c*x**n))/(2*b*n*sqrt(-a/b)) - log(sqrt(-a/
b) + log(c*x**n))/(2*b*n*sqrt(-a/b)), True))
```

Maxima [F]

$$\int \frac{1}{ax + bx\log^2(cx^n)}\,dx = \int \frac{1}{bx\log(cx^n)^2 + ax}\,dx$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^2 + a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{ax + bx\log^2(cx^n)}\,dx = \frac{\arctan\left(\frac{bn\log(x)+b\log(c)}{\sqrt{ab}}\right)}{\sqrt{ab}n}$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] arctan((b*n*log(x) + b*log(c))/sqrt(a*b))/(sqrt(a*b)*n)
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{1}{ax+bx\log^2(cx^n)}\,dx=-\frac{\ln\left(\frac{1}{b\,x}+\frac{\ln(c\,x^n)}{\sqrt{-a}\,\sqrt{b}\,x}\right)-\ln\left(\frac{1}{b\,x}-\frac{\ln(c\,x^n)}{\sqrt{-a}\,\sqrt{b}\,x}\right)}{2\,\sqrt{-a}\,\sqrt{b}\,n}$$

```
[In] int(1/(a*x + b*x*log(c*x^n)^2),x)
```

```
[Out] -(log(1/(b*x) + log(c*x^n)/((-a)^(1/2)*b^(1/2)*x)) - log(1/(b*x) - log(c*x^
n)/((-a)^(1/2)*b^(1/2)*x)))/(2*(-a)^(1/2)*b^(1/2)*n)
```

3.252 $\int \frac{1}{ax+bx\log^3(cx^n)}\,dx$

Optimal result 1330
Rubi [A] (verified) 1330
Mathematica [A] (verified) 1332
Maple [C] (warning: unable to verify) 1333
Fricas [A] (verification not implemented) 1333
Sympy [A] (verification not implemented) 1334
Maxima [F] 1335
Giac [B] (verification not implemented) 1335
Mupad [B] (verification not implemented) 1336

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}n}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}n}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*ln(c*x^n))/a^(2/3)/b^(1/3)/n-1/6*ln(a^(2/3)-a^(1/3)*
b^(1/3)*ln(c*x^n)+b^(2/3)*ln(c*x^n)^2)/a^(2/3)/b^(1/3)/n-1/3*arctan(1/3*(a^
(1/3)-2*b^(1/3)*ln(c*x^n))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/n*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {206, 31, 648, 631, 210, 642}

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}n}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}n}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n}$$

```
[In] Int[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Log[c*x^n])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/
3)*b^(1/3)*n)) + Log[a^(1/3) + b^(1/3)*Log[c*x^n]]/(3*a^(2/3)*b^(1/3)*n) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + b^(2/3)*Log[c*x^n]^2]/(6*a^(2/3)
*b^(1/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \log(cx^n)\right)}{3a^{2/3}n} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{3a^{2/3}n}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{2\sqrt[3]{a}n} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}n}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}n} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}n}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}n} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}n}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}n}$$

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*Log[c*x^n])/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*Log[c*x^n]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n]
+ b^(2/3)*Log[c*x^n]^2])/(a^(2/3)*b^(1/3)*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.97 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(27a^2b\,n^3\_Z^3-1\right)} \_R\ln\left(\ln\left(x^n\right)+3an\_R-\frac{i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)}{2}+\frac{i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2}{2}\right.$
default	$\frac{\frac{\ln\left(\ln(c\,x^n)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\ln(c\,x^n)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln(c\,x^n)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2\ln(c\,x^n)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{n}$

```
[In] int(1/(a*x+b*x*ln(c*x^n)^3),x,method=_RETURNVERBOSE)
```

```
[Out] sum(_R*ln(ln(x^n)+3*a*n*_R-1/2*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2
*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*
Pi*csgn(I*c*x^n)^3+ln(c)),_R=RootOf(27*_Z^3*a^2*b*n^3-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.33

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abn^3\log(x)^3+6\,abn^2\log(c)\log(x)^2+6\,abn\log(c)^2\log(x)+2\,ab\log(c)^3-a^2+3\sqrt{\frac{1}{3}}\left(2\,abn^2\log(x)^2+4\,abn\log(\right.}{bn^3\log(x)^3+3\,bn^2\log(c)\log(x)^2+3}\right.}{}\right.$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*n^3*log(x)^3 + 6*a*
b*n^2*log(c)*log(x)^2 + 6*a*b*n*log(c)^2*log(x) + 2*a*b*log(c)^3 - a^2 + 3*
sqrt(1/3)*(2*a*b*n^2*log(x)^2 + 4*a*b*n*log(c)*log(x) + 2*a*b*log(c)^2 + (a
^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) -
 3*(a^2*b)^(1/3)*(a*n*log(x) + a*log(c)))/(b*n^3*log(x)^3 + 3*b*n^2*log(c)*
log(x)^2 + 3*b*n*log(c)^2*log(x) + b*log(c)^3 + a)) - (a^2*b)^(2/3)*log(a*b
*n^2*log(x)^2 + 2*a*b*n*log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)*(n*log
(x) + log(c)) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x) + a*b*l
og(c) + (a^2*b)^(2/3)))/(a^2*b*n), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/
b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*n^2*log(x)^2 + 2*a*b*n*
log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)*(n*log(x) + log(c)) + (a^2*b)^
(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x) + a*b*log(c) + (a^2*b)^(2/3)))/
(a^2*b*n)]
```

Sympy [A] (verification not implemented)

Time = 25.99 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22

$$\int \frac{1}{ax + bx\log^3(cx^n)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^3} \\ -\frac{1}{2bn\log(cx^n)^2} \\ \frac{\log(x)}{a} \\ \frac{\log(x)}{a+b\log(c)^3} \\ -\frac{\sqrt[3]{-\frac{a}{b}}\log\left(-\sqrt[3]{-\frac{a}{b}}+\log(cx^n)\right)}{3an} + \frac{\sqrt[3]{-\frac{a}{b}}\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\log(cx^n)+4\log(cx^n)^2\right)}{6an} + \frac{\sqrt{3}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\log(cx^n)}{3\sqrt[3]{-\frac{a}{b}}}\right)}{3an} \end{cases}$$

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**3),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(2*b*
n*log(c*x**n)**2), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b*log(c)**
3), Eq(n, 0)), (-(-a/b)**(1/3)*log(-(-a/b)**(1/3) + log(c*x**n))/(3*a*n) +
(-a/b)**(1/3)*log(4*(-a/b)**(2/3) + 4*(-a/b)**(1/3)*log(c*x**n) + 4*log(c*x
**n)**2)/(6*a*n) + sqrt(3)*(-a/b)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*log(c*x
**n)/(3*(-a/b)**(1/3)))/(3*a*n), True))
```

Maxima [F]

$$\int \frac{1}{ax + bx\log^3(cx^n)}\,dx = \int \frac{1}{bx\log(cx^n)^3 + ax}\,dx$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^3 + a*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(109) = 218$.

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.66

$$\int \frac{1}{ax + bx\log^3(cx^n)}\,dx$$
$$= \frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\pi b(\mathrm{sgn}(c)-1) - 2\,bn\log(x) - 2\,b\log(|c|) - 2\,(ab^2)^{\frac{1}{3}}}{2\sqrt{3}bn\log(x) + \pi b(\mathrm{sgn}(c)-1) + 2\sqrt{3}b\log(|c|) - 2\sqrt{3}\,(ab^2)^{\frac{1}{3}}}\right)$$
$$+ \frac{1}{6}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\log\left(\frac{1}{4}\left(\pi bn(\mathrm{sgn}(x)-1) + \pi b(\mathrm{sgn}(c)-1)\right)^2 + \left(bn\log(|x|) + b\log(|c|) + \left(ab^2\right)^{\frac{1}{3}}\right)^2\right)$$
$$- \frac{1}{6}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\log\left(\left(\sqrt{3}\pi b(\mathrm{sgn}(c)-1) - 2\,bn\log(x) - 2\,b\log(|c|) - 2\left(ab^2\right)^{\frac{1}{3}}\right)^2 + \left(2\sqrt{3}bn\log(x) + \pi b(\mathrm{sgn}(c)-1) + 2\sqrt{3}b\log(|c|) - 2\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2\right)$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*n^3))^(1/3)*arctan((sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*n
*log(x) - 2*b*log(abs(c)) - 2*(a*b^2)^(1/3))/(2*sqrt(3)*b*n*log(x) + pi*b*(
sgn(c) - 1) + 2*sqrt(3)*b*log(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))) + 1/6*(1/
(a^2*b*n^3))^(1/3)*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b
*n*log(abs(x)) + b*log(abs(c)) + (a*b^2)^(1/3))^2) - 1/6*(1/(a^2*b*n^3))^(1
/3)*log((sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*n*log(x) - 2*b*log(abs(c)) - 2*(a*
b^2)^(1/3))^2 + (2*sqrt(3)*b*n*log(x) + pi*b*(sgn(c) - 1) + 2*sqrt(3)*b*log
(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))^2)
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx=\frac{\ln\left(\frac{3\,a^{1/3}\,n}{b^{4/3}\,x^2}+\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}\right)}{3\,a^{2/3}\,b^{1/3}\,n}+\frac{\ln\left(\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}+\frac{3\,a^{1/3}\,n\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{4/3}\,x^2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,n}-\frac{\ln\left(\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}-\frac{3\,a^{1/3}\,n\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{4/3}\,x^2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,n}$$

```
[In] int(1/(a*x + b*x*log(c*x^n)^3),x)
```

```
[Out] log((3*a^(1/3)*n)/(b^(4/3)*x^2) + (3*n*log(c*x^n))/(b*x^2))/(3*a^(2/3)*b^(1
/3)*n) + (log((3*n*log(c*x^n))/(b*x^2) + (3*a^(1/3)*n*(3^(1/2)*1i - 1))/(2*
b^(4/3)*x^2))*(3^(1/2)*1i - 1))/(6*a^(2/3)*b^(1/3)*n) - (log((3*n*log(c*x^n
))/(b*x^2) - (3*a^(1/3)*n*(3^(1/2)*1i + 1))/(2*b^(4/3)*x^2))*(3^(1/2)*1i +
1))/(6*a^(2/3)*b^(1/3)*n)
```

3.253 $\int \frac{1}{ax+bx\log^4(cx^n)}\,dx$

Optimal result 1337
Rubi [A] (verified) 1337
Mathematica [A] (verified) 1340
Maple [C] (warning: unable to verify) 1341
Fricas [C] (verification not implemented) 1341
Sympy [A] (verification not implemented) 1342
Maxima [F] 1342
Giac [A] (verification not implemented) 1342
Mupad [B] (verification not implemented) 1343

Optimal result

Integrand size = 17, antiderivative size = 227

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

```
[Out] -1/4*arctan(1-b^(1/4)*ln(c*x^n)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/n*2^(1/2)+
1/4*arctan(1+b^(1/4)*ln(c*x^n)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/n*2^(1/2)-1
/8*ln(-a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/2)+a^(1/2)+ln(c*x^n)^2*b^(1/2))/a^(3/
4)/b^(1/4)/n*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/2)+a^(1/2)+ln(c*
x^n)^2*b^(1/2))/a^(3/4)/b^(1/4)/n*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used

= {217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

```
[In] Int[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1
/4)*n) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(2*Sqrt[2]*a^(3/4
)*b^(1/4)*n) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[b]*L
og[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Log[c*x^n] + Sqrt[b]*Log[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \log\left(cx^n\right)\right)}{2\sqrt{a}n} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \log\left(cx^n\right)\right)}{2\sqrt{a}n}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \log\left(cx^n\right)\right)}{4\sqrt{a}\sqrt{b}n}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \log\left(cx^n\right)\right)}{4\sqrt{a}\sqrt{b}n}$$

$$- \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \log\left(cx^n\right)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

$$- \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \log\left(cx^n\right)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} - \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.74

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Log[c*x^n])/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[
c*x^n] + Sqrt[b]*Log[c*x^n]^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[
c*x^n] + Sqrt[b]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.49

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(256a^3b\,n^4\_Z^4+1\right)} \_R\ln\left(\ln\left(x^n\right)+4an\_R-\frac{i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)}{2}+\frac{i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2}{2}\right.$
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\ln(c\,x^n)^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln(c\,x^n)\sqrt{2}+\sqrt{\frac{a}{b}}}{\ln(c\,x^n)^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln(c\,x^n)\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\ \ln(c\,x^n)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\ \ln(c\,x^n)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\right)}{8na}$

[In] int(1/(a*x+b*x*ln(c*x^n)^4),x,method=_RETURNVERBOSE)

[Out] sum(_R*ln(ln(x^n)+4*a*n*_R-1/2*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*csgn(I*c*x^n)^3+ln(c)),_R=RootOf(256*_Z^4*a^3*b*n^4+1))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.70

$$\begin{aligned}\int \frac{1}{ax+bx\log^4(cx^n)}\,dx &= \frac{1}{4}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(an\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}+n\log(x)+\log(c)\right)\\ &\quad+\frac{1}{4}i\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(i\,an\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}+n\log(x)+\log(c)\right)\\ &\quad-\frac{1}{4}i\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(-i\,an\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}+n\log(x)+\log(c)\right)\\ &\quad-\frac{1}{4}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(-an\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}+n\log(x)+\log(c)\right)\end{aligned}$$

[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="fricas")

[Out] 1/4*(-1/(a^3*b*n^4))^(1/4)*log(a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c)) + 1/4*I*(-1/(a^3*b*n^4))^(1/4)*log(I*a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c)) - 1/4*I*(-1/(a^3*b*n^4))^(1/4)*log(-I*a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c)) - 1/4*(-1/(a^3*b*n^4))^(1/4)*log(-a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c))

Sympy [A] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.59

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = \begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^4} & \text{for } a=0 \wedge b=0 \wedge n= \\ -\frac{1}{3bn\log(cx^n)^3} & \text{for } a=0 \\ \frac{\log(x)}{a} & \text{for } b=0 \\ \frac{\log(x)}{a+b\log(c)^4} & \text{for } n=0 \\ -\frac{\sqrt[4]{-\frac{a}{b}}\log\left(-\sqrt[4]{-\frac{a}{b}}+\log(cx^n)\right)}{4an} + \frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt[4]{-\frac{a}{b}}+\log(cx^n)\right)}{4an} + \frac{\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\log(cx^n)}{\sqrt[4]{-\frac{a}{b}}}\right)}{2an} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**4),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(3*b*
n*log(c*x**n)**3), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b*log(c)**
4), Eq(n, 0)), (-(-a/b)**(1/4)*log(-(-a/b)**(1/4) + log(c*x**n))/(4*a*n) +
(-a/b)**(1/4)*log((-a/b)**(1/4) + log(c*x**n))/(4*a*n) + (-a/b)**(1/4)*atan
(log(c*x**n)/(-a/b)**(1/4))/(2*a*n), True))
```

Maxima [F]

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = \int \frac{1}{bx\log(cx^n)^4+ax}\,dx$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^4 + a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.75

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = -\frac{1}{2}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\pi b(\mathrm{sgn}(c)-1)+2\left(-ab^3\right)^{\frac{1}{4}}}{2\left(bn\log\left(x\right)+b\log\left(|c|\right)\right)}\right) + \frac{1}{8}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi bn(\mathrm{sgn}(x)-1)+\pi b(\mathrm{sgn}(c)-1)\right)^2 + \left(bn\log\left(|x|\right)+b\log\left(|c|\right)+\left(-ab^3\right)^{\frac{1}{4}}\right)^2\right) - \frac{1}{8}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi bn(\mathrm{sgn}(x)-1)+\pi b(\mathrm{sgn}(c)-1)\right)^2 + \left(bn\log\left(|x|\right)+b\log\left(|c|\right)-\left(-ab^3\right)^{\frac{1}{4}}\right)^2\right)$$

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="giac")
```

```
[Out] -1/2*(-1/(a^3*b*n^4))^(1/4)*arctan(1/2*(pi*b*(sgn(c) - 1) + 2*(-a*b^3)^(1/4
))/(b*n*log(x) + b*log(abs(c)))) + 1/8*(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi*b
*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)) +
 (-a*b^3)^(1/4))^2) - 1/8*(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi*b*n*(sgn(x) -
1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)) - (-a*b^3)^(1/
4))^2)
```

Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.42

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = -\frac{\ln\left(\left(-a\right)^{1/4}+b^{1/4}\,\ln\left(c\,x^n\right)\right)-\ln\left(\left(-a\right)^{1/4}-b^{1/4}\,\ln\left(c\,x^n\right)\right)+\ln\left(\left(-a\right)^{1/4}-b^{1/4}\,\ln\left(c\,x^n\right)\,1\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\right.}{4\,\left(-a\right)^{3/4}\,b^{1/4}\,n}$$

```
[In] int(1/(a*x + b*x*log(c*x^n)^4),x)
```

```
[Out] -(log((-a)^(1/4) + b^(1/4)*log(c*x^n)) - log((-a)^(1/4) - b^(1/4)*log(c*x^n
)) + log((-a)^(1/4) - b^(1/4)*log(c*x^n)*1i)*1i - log((-a)^(1/4) + b^(1/4)*
log(c*x^n)*1i)*1i)/(4*(-a)^(3/4)*b^(1/4)*n)
```

3.254 $\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx$

Optimal result 1344
Rubi [A] (verified) 1344
Mathematica [A] (verified) 1345
Maple [A] (verified) 1345
Fricas [A] (verification not implemented) 1346
Sympy [B] (verification not implemented) 1346
Maxima [A] (verification not implemented) 1347
Giac [A] (verification not implemented) 1347
Mupad [B] (verification not implemented) 1347

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx=\frac{\log(x)}{a}-\frac{b\log\left(b+a\log\left(cx^n\right)\right)}{a^2n}$$

```
[Out] ln(x)/a-b*ln(b+a*ln(c*x^n))/a^2/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx=\frac{\log(x)}{a}-\frac{b\log\left(a\log\left(cx^n\right)+b\right)}{a^2n}$$

```
[In] Int[(a*x + (b*x)/Log[c*x^n])^(-1),x]
```

```
[Out] Log[x]/a - (b*Log[b + a*Log[c*x^n]])/(a^2*n)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{b+ax}\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a} - \frac{b}{a(b+ax)}\right)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{\log(x)}{a} - \frac{b\log\left(b+a\log\left(cx^n\right)\right)}{a^2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$
\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx = \frac{\log\left(cx^n\right)}{an} - \frac{b\log\left(b+a\log\left(cx^n\right)\right)}{a^2n}
$$

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n])^(-1),x]
[Out] Log[c*x^n]/(a*n) - (b*Log[b + a*Log[c*x^n]])/(a^2*n)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

method	result
parallelrisch	$-\frac{-\ln(x)an+b\ln(b+a\ln(c\,x^n))}{a^2n}$
norman	$\frac{\ln(x)}{a}-\frac{b\ln\left(a\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+b\right)}{a^2n}$
default	$\frac{\frac{\ln(c\,x^n)}{a}-\frac{b\ln(b+a\ln(c\,x^n))}{a^2}}{n}$
risch	$\frac{\ln(x)}{a}-\frac{b\ln\left(\ln(x^n)+\frac{-i\pi a\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+i\pi a\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+i\pi a\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-i\pi a\,\mathrm{csgn}(ic\,x^n)^3+2}{2a}\right.}{a^2n}$

```
[In] int(1/(a*x+b*x/ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -(-ln(x)*a*n+b*ln(b+a*ln(c*x^n)))/a^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx=\frac{an\log(x)-b\log(an\log(x)+a\log(c)+b)}{a^2n}$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="fricas")
[Out] (a*n*log(x) - b*log(a*n*log(x) + a*log(c) + b))/(a^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(22) = 44$.

Time = 1.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx = \begin{cases} \frac{\log(c)\log(x)}{b} & \text{for}\ a=0\wedge n=0 \\ \frac{\begin{cases} 0 & \text{for}\ \frac{1}{|cx^n|}<1\wedge |cx^n|<1 \\ \frac{\log(cx^n)^2}{2n} & \text{for}\ |cx^n|<1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for}\ \frac{1}{|cx^n|}<1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix}\middle| cx^n\right)}{n}+\frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for}\ a=0 \\ \frac{\log(c)\log(x)}{a\log(c)+b} & \text{for}\ n=0 \\ \frac{\log(cx^n)}{an}-\frac{b\log\left(\log(cx^n)+\frac{b}{a}\right)}{a^2n} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)),x)
[Out] Piecewise((log(c)*log(x)/b, Eq(a, 0) & Eq(n, 0)), (Piecewise((0, (Abs(c*x**
n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**2/(2*n), Abs(c*x**n) < 1), (l
og(1/(c*x**n))**2/(2*n), 1/Abs(c*x**n) < 1), (meijerg(((), (1, 1, 1)), ((0,
 0, 0), ()), c*x**n)/n + meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), c*x**n)/
n, True))/b, Eq(a, 0)), (log(c)*log(x)/(a*log(c) + b), Eq(n, 0)), (log(c*x*
*n)/(a*n) - b*log(log(c*x**n) + b/a)/(a**2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{ax + \frac{bx}{\log(cx^n)}}\,dx = \frac{\log(x)}{a} - \frac{b\log\left(\frac{a\log(c)+a\log(x^n)+b}{a}\right)}{a^2n}$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="maxima")
```

```
[Out] log(x)/a - b*log((a*log(c) + a*log(x^n) + b)/a)/(a^2*n)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{1}{ax + \frac{bx}{\log(cx^n)}}\,dx = \frac{\log(x)}{a} - \frac{b\log\left(\frac{1}{4}\left(\pi an(\operatorname{sgn}(x)-1) + \pi a(\operatorname{sgn}(c)-1)\right)^2 + \left(an\log(|x|) + a\log(|c|) + b\right)^2\right)}{2\,a^2n}$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="giac")
```

```
[Out] log(x)/a - 1/2*b*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 + (a*n
*log(abs(x)) + a*log(abs(c)) + b)^2)/(a^2*n)
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax + \frac{bx}{\log(cx^n)}}\,dx = \frac{\ln(x)}{a} - \frac{b\,\ln\left(b + a\,\ln\left(c\,x^n\right)\right)}{a^2\,n}$$

```
[In] int(1/(a*x + (b*x)/log(c*x^n)),x)
```

```
[Out] log(x)/a - (b*log(b + a*log(c*x^n)))/(a^2*n)
```

3.255 $\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$

Optimal result 1348
Rubi [A] (verified) 1348
Mathematica [A] (verified) 1349
Maple [A] (verified) 1349
Fricas [A] (verification not implemented) 1350
Sympy [B] (verification not implemented) 1350
Maxima [F] 1351
Giac [A] (verification not implemented) 1351
Mupad [B] (verification not implemented) 1351

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}+\frac{\log(x)}{a}$$

```
[Out] ln(x)/a-arctan(ln(c*x^n)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {327, 211}

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx = \frac{\log(x)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}$$

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[x]/a
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\log(x)}{a} - \frac{b\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \log\left(cx^n\right)\right)}{an}\\ &= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n} + \frac{\log(x)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax + \frac{bx}{\log^2(cx^n)}}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n} + \frac{\log(x)}{a}$$

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[x]/a
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

method	result
default	$\frac{\frac{\ln(c\,x^n)}{a} - \frac{b\arctan\left(\frac{a\ln(c\,x^n)}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{n}$
risch	$\frac{\ln(x)}{a} + \frac{\sqrt{-ab}\,\ln\left(\ln(x^n) - \frac{i\pi a\,\text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n) - i\pi a\,\text{csgn}(ic)\,\text{csgn}(ic\,x^n)^2 - i\pi a\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)^2 + i\pi a\,\text{csgn}(ic\,x^n)^3 - 2\ln(}{2a}\right.}{2a^2n}$

```
[In] int(1/(a*x+b*x/ln(c*x^n)^2),x,method=_RETURNVERBOSE)
[Out] 1/n*(ln(c*x^n)/a-1/a*b/(a*b)^(1/2)*arctan(a*ln(c*x^n)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.58

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$$
$$=\left[\frac{2\,n\log\left(x\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{an^2\log(x)^2+2\,an\log(c)\log(x)+a\log(c)^2-2\,(an\log(x)+a\log(c))\sqrt{-\frac{b}{a}}-b}{an^2\log(x)^2+2\,an\log(c)\log(x)+a\log(c)^2+b}\right)}{2\,an},\ \frac{n\log\left(x\right)-\sqrt{\frac{b}{a}}\arctan\left(\right.}{an}\right.$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*n*log(x) + sqrt(-b/a)*log((a*n^2*log(x)^2 + 2*a*n*log(c)*log(x) + a
*log(c)^2 - 2*(a*n*log(x) + a*log(c))*sqrt(-b/a) - b)/(a*n^2*log(x)^2 + 2*a
*n*log(c)*log(x) + a*log(c)^2 + b)))/(a*n), (n*log(x) - sqrt(b/a)*arctan((a
*n*log(x) + a*log(c))*sqrt(b/a)/b))/(a*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(34) = 68.

Time = 3.63 (sec) , antiderivative size = 204, normalized size of antiderivative = 5.10

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$$
$$=\begin{cases} \tilde{\infty}\log\left(c\right)^2\log\left(x\right) & \text{for}\ a=0\ \wedge \\ \frac{\begin{cases} -\frac{\log\left(\frac{x^{-n}}{c}\right)^3}{3n}+\frac{\log\left(cx^n\right)^3}{3n} & \text{for}\ \frac{1}{\left|cx^n\right|}<1\wedge\left|cx^n\right|<1 \\ \frac{\log\left(cx^n\right)^3}{3n} & \text{for}\ \left|cx^n\right|<1 \\ -\frac{\log\left(\frac{x^{-n}}{c}\right)^3}{3n} & \text{for}\ \frac{1}{\left|cx^n\right|}<1 \\ -\frac{2G_{4,4}^{4,0}\left(\begin{matrix} & 1,1,1,1 \\ 0,0,0,0 & \end{matrix}\middle|\,cx^n\right)}{n}+\frac{2G_{4,4}^{0,4}\left(\begin{matrix}1,1,1,1 & \\ & 0,0,0,0\end{matrix}\middle|\,cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for}\ a=0 \\ \frac{\log\left(c\right)^2\log\left(x\right)}{a\log\left(c\right)^2+b} & \text{for}\ n=0 \\ \frac{\log\left(x\right)}{a} & \text{for}\ b=0 \\ \frac{\log\left(cx^n\right)}{an}-\frac{b\log\left(-\sqrt{-\frac{b}{a}}+\log\left(cx^n\right)\right)}{2a^2n\sqrt{-\frac{b}{a}}}+\frac{b\log\left(\sqrt{-\frac{b}{a}}+\log\left(cx^n\right)\right)}{2a^2n\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**2),x)
```

```
[Out] Piecewise((zoo*log(c)**2*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (Piecewis
e((-log(1/(c*x**n))**3/(3*n) + log(c*x**n)**3/(3*n), (Abs(c*x**n) < 1) & (1
/Abs(c*x**n) < 1)), (log(c*x**n)**3/(3*n), Abs(c*x**n) < 1), (-log(1/(c*x**
n))**3/(3*n), 1/Abs(c*x**n) < 1), (-2*meijerg(((), (1, 1, 1, 1)), ((0, 0, 0
, 0), ()), c*x**n)/n + 2*meijerg(((1, 1, 1, 1), ()), ((), (0, 0, 0, 0)), c*
x**n)/n, True))/b, Eq(a, 0)), (log(c)**2*log(x)/(a*log(c)**2 + b), Eq(n, 0)
), (log(x)/a, Eq(b, 0)), (log(c*x**n)/(a*n) - b*log(-sqrt(-b/a) + log(c*x**
n))/(2*a**2*n*sqrt(-b/a)) + b*log(sqrt(-b/a) + log(c*x**n))/(2*a**2*n*sqrt(
-b/a)), True))
```

Maxima [F]

$$\int \frac{1}{ax + \frac{bx}{\log^2(cx^n)}}\,dx = \int \frac{1}{ax + \frac{bx}{\log(cx^n)^2}}\,dx$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(2*a^2*x*log(c)*log(x^n) + a^2*x*log(x^n)^2 + (a^2*log(c)^2
+ a*b)*x), x) + log(x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax + \frac{bx}{\log^2(cx^n)}}\,dx = \frac{\log(x)}{a} - \frac{b \arctan\left(\frac{an\log(x)+a\log(c)}{\sqrt{ab}}\right)}{\sqrt{ab}an}$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] log(x)/a - b*arctan((a*n*log(x) + a*log(c))/sqrt(a*b))/(sqrt(a*b)*a*n)
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{ax + \frac{bx}{\log^2(cx^n)}}\,dx = \frac{\ln(x)}{a} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{a^2\,n\,\ln(c\,x^n)}{\sqrt{b}\,\sqrt{a^3\,n^2}}\right)}{\sqrt{a^3\,n^2}}$$

```
[In] int(1/(a*x + (b*x)/log(c*x^n)^2),x)
```

```
[Out] log(x)/a - (b^(1/2)*atan((a^2*n*log(c*x^n))/(b^(1/2)*(a^3*n^2)^(1/2))))/(a^
3*n^2)^(1/2)
```

3.256 $\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}} dx$

Optimal result 1352
Rubi [A] (verified) 1352
Mathematica [A] (verified) 1355
Maple [C] (warning: unable to verify) 1355
Fricas [A] (verification not implemented) 1356
Sympy [A] (verification not implemented) 1356
Maxima [F] 1357
Giac [B] (verification not implemented) 1357
Mupad [B] (verification not implemented) 1358

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}} dx = \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{4/3}n} + \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}\log^2(cx^n)\right)}{6a^{4/3}n}$$

```
[Out] ln(x)/a-1/3*b^(1/3)*ln(b^(1/3)+a^(1/3)*ln(c*x^n))/a^(4/3)/n+1/6*b^(1/3)*ln(
b^(2/3)-a^(1/3)*b^(1/3)*ln(c*x^n)+a^(2/3)*ln(c*x^n)^2)/a^(4/3)/n+1/3*b^(1/3
)*arctan(1/3*(b^(1/3)-2*a^(1/3)*ln(c*x^n))/b^(1/3)*3^(1/2))/a^(4/3)/n*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {327, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}} dx = \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{4/3}n} + \frac{\sqrt[3]{b}\log\left(a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\right)}{6a^{4/3}n} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}\log(cx^n)+\sqrt[3]{b}\right)}{3a^{4/3}n} + \frac{\log(x)}{a}$$

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

```
[Out] (b^(1/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Log[c*x^n])/(Sqrt[3]*b^(1/3))])/(Sqrt[
3]*a^(4/3)*n) + Log[x]/a - (b^(1/3)*Log[b^(1/3) + a^(1/3)*Log[c*x^n]])/(3*a
^(4/3)*n) + (b^(1/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log
[c*x^n]^2])/(6*a^(4/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{b+ax^3}\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{\log(x)}{a} - \frac{b\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, \log\left(cx^n\right)\right)}{an}\\
&= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx, x, \log\left(cx^n\right)\right)}{3an}\\
&\quad - \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \log\left(cx^n\right)\right)}{3an}\\
&= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log\left(cx^n\right)\right)}{3a^{4/3}n}\\
&\quad + \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \log\left(cx^n\right)\right)}{6a^{4/3}n}\\
&\quad - \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \log\left(cx^n\right)\right)}{2an}\\
&= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log\left(cx^n\right)\right)}{3a^{4/3}n}\\
&\quad + \frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+a^{2/3}\log^2\left(cx^n\right)\right)}{6a^{4/3}n}\\
&\quad - \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{a}\log\left(cx^n\right)}{\sqrt[3]{b}}\right)}{a^{4/3}n}\\
&= \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\log\left(cx^n\right)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}n} + \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log\left(cx^n\right)\right)}{3a^{4/3}n}\\
&\quad + \frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+a^{2/3}\log^2\left(cx^n\right)\right)}{6a^{4/3}n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$$
$$= \frac{2\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right)+6\sqrt[3]{a}n\log(x)+\sqrt[3]{b}\left(-2\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log(cx^n)\right)+\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.\right.}{6a^{4/3}n}$$

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*a^(1/3)*Log[c*x^n])/b^(1/3))/Sqrt[3]] + 6
*a^(1/3)*n*Log[x] + b^(1/3)*(-2*Log[b^(1/3) + a^(1/3)*Log[c*x^n]] + Log[b^(
2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log[c*x^n]^2]))/(6*a^(4/3)*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.98 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

method	result
risch	$\frac{\ln(x)}{a}+\left(\sum_{\_R=\mathrm{RootOf}(27n^3a^4\_Z^3+b)}\_R\ln\left(\ln(x^n)-3an\_R-\frac{i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)}{2}+\frac{i\pi\,\mathrm{csgn}(ic)\,\mathrm{cs}}{2}\right.\right.$
default	$\frac{\frac{\ln(cx^n)}{a}-\frac{\left(\frac{\ln\left(\ln(cx^n)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\ln(cx^n)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\ln(cx^n)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\ln(cx^n)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)b}{a}}{n}$

```
[In] int(1/(a*x+b*x/ln(c*x^n)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*ln(x)+sum(_R*ln(ln(x^n)-3*a*n*_R-1/2*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)+1/2*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n
)^2-1/2*I*Pi*csgn(I*c*x^n)^3+ln(c)),_R=RootOf(27*_Z^3*a^4*n^3+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$$
$$= \frac{6\,n\log(x) + 2\sqrt{3}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(\sqrt{3}an\log(x)+\sqrt{3}a\log(c)\right)\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right) - \left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(n^2\log(x)^2 + 2\,n\log(c)\,\mathrm{l}\right.}{6\,an}$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(6*n*log(x) + 2*sqrt(3)*(-b/a)^(1/3)*arctan(1/3*(2*(sqrt(3)*a*n*log(x)
+ sqrt(3)*a*log(c))*(-b/a)^(2/3) - sqrt(3)*b)/b) - (-b/a)^(1/3)*log(n^2*log
(x)^2 + 2*n*log(c)*log(x) + log(c)^2 + (n*log(x) + log(c))*(-b/a)^(1/3) + (
-b/a)^(2/3)) + 2*(-b/a)^(1/3)*log(n*log(x) - (-b/a)^(1/3) + log(c)))/(a*n)
```

Sympy [A] (verification not implemented)

Time = 26.84 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.64

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$$
$$= \begin{cases} \tilde{\infty}\log(c)^3\log(x) \\ \frac{\begin{cases} 0 & \text{for}\ \frac{1}{|cx^n|}<1 \wedge |cx^n|<1 \\ \frac{\log(cx^n)^4}{4n} & \text{for}\ |cx^n|<1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^4}{4n} & \text{for}\ \frac{1}{|cx^n|}<1 \\ \frac{6G_{5,5}^{5,0}\left(\begin{matrix} & 1,1,1,1,1 \\ 0,0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{6G_{5,5}^{0,5}\left(\begin{matrix} 1,1,1,1,1 & \\ & 0,0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} \\ \frac{\log(c)^3\log(x)}{a\log(c)^3+b} \\ \frac{\log(x)}{a} \\ \frac{\sqrt[3]{-\frac{b}{a}}\log\left(-\sqrt[3]{-\frac{b}{a}}+\log(cx^n)\right)}{3an} - \frac{\sqrt[3]{-\frac{b}{a}}\log\left(4\left(-\frac{b}{a}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{b}{a}}\log(cx^n)+4\log(cx^n)^2\right)}{6an} - \frac{\sqrt{3}\sqrt[3]{-\frac{b}{a}}\,\mathrm{atan}\left(\frac{\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\log(cx^n)}{3\sqrt[3]{-\frac{b}{a}}}}{}\right)}{3an} \end{cases}$$

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**3),x)
```

```
[Out] Piecewise((zoo*log(c)**3*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (Piecewis
e((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**4/(4*n), Abs(
c*x**n) < 1), (log(1/(c*x**n))**4/(4*n), 1/Abs(c*x**n) < 1), (6*meijerg(((()
, (1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0), ()), c*x**n)/n + 6*meijerg(((1, 1, 1,
 1, 1), ()), ((), (0, 0, 0, 0, 0)), c*x**n)/n, True))/b, Eq(a, 0)), (log(c)
**3*log(x)/(a*log(c)**3 + b), Eq(n, 0)), (log(x)/a, Eq(b, 0)), ((-b/a)**(1/
3)*log(-(-b/a)**(1/3) + log(c*x**n))/(3*a*n) - (-b/a)**(1/3)*log(4*(-b/a)**
(2/3) + 4*(-b/a)**(1/3)*log(c*x**n) + 4*log(c*x**n)**2)/(6*a*n) - sqrt(3)*(
-b/a)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*log(c*x**n)/(3*(-b/a)**(1/3)))/(3*a
*n) + log(c*x**n)/(a*n), True))
```

Maxima [F]

$$\int \frac{1}{ax + \frac{bx}{\log^3(cx^n)}}\, dx = \int \frac{1}{ax + \frac{bx}{\log(cx^n)^3}}\, dx$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(3*a^2*x*log(c)^2*log(x^n) + 3*a^2*x*log(c)*log(x^n)^2 + a^2
*x*log(x^n)^3 + (a^2*log(c)^3 + a*b)*x), x) + log(x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(115) = 230$.

Time = 0.37 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

$$\int \frac{1}{ax + \frac{bx}{\log^3(cx^n)}}\, dx = \frac{\log(x)}{a} + \frac{2\sqrt{3}\left(-\frac{bn^6}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\pi a(\mathrm{sgn}(c)-1)-2\,an\log(x)-2\,a\log(|c|)+2\,(-a^2b)^{\frac{1}{3}}}{2\sqrt{3}an\log(x)+\pi a(\mathrm{sgn}(c)-1)+2\sqrt{3}a\log(|c|)+2\sqrt{3}(-a^2b)^{\frac{1}{3}}}\right) + \left(-\frac{bn^6}{a}\right)^{\frac{1}{3}} \log\left(\frac{1}{4}\left(\pi an(\mathrm{sgn}(x) -\right.\right.}{}$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="giac")
```

```
[Out] log(x)/a + 1/6*(2*sqrt(3)*(-b*n^6/a)^(1/3)*arctan((sqrt(3)*pi*a*(sgn(c) - 1
) - 2*a*n*log(x) - 2*a*log(abs(c)) + 2*(-a^2*b)^(1/3))/(2*sqrt(3)*a*n*log(x
) + pi*a*(sgn(c) - 1) + 2*sqrt(3)*a*log(abs(c)) + 2*sqrt(3)*(-a^2*b)^(1/3))
) + (-b*n^6/a)^(1/3)*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 +
(a*n*log(abs(x)) + a*log(abs(c)) - (-a^2*b)^(1/3))^2) - (-b*n^6/a)^(1/3)*lo
g((sqrt(3)*pi*a*(sgn(c) - 1) - 2*a*n*log(x) - 2*a*log(abs(c)) + 2*(-a^2*b)^
(1/3))^2 + (2*sqrt(3)*a*n*log(x) + pi*a*(sgn(c) - 1) + 2*sqrt(3)*a*log(abs(
c)) + 2*sqrt(3)*(-a^2*b)^(1/3))^2))/(a*n^3)
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.17

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx=\frac{\ln(x)}{a}+\frac{(-b)^{1/3}\,\ln\left(\frac{3\,(-b)^{4/3}\,n}{a^{7/3}\,x^2}+\frac{3\,b\,n\,\ln(c\,x^n)}{a^2\,x^2}\right)}{3\,a^{4/3}\,n}+\frac{(-b)^{1/3}\,\ln\left(\frac{3\,b\,n\,\ln(c\,x^n)}{a^2\,x^2}+\frac{3\,(-b)^{4/3}\,n\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{7/3}\,x^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,n}-\frac{(-b)^{1/3}\,\ln\left(\frac{3\,b\,n\,\ln(c\,x^n)}{a^2\,x^2}-\frac{3\,(-b)^{4/3}\,n\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{7/3}\,x^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,n}$$

```
[In] int(1/(a*x + (b*x)/log(c*x^n)^3),x)
```

```
[Out] log(x)/a + ((-b)^(1/3)*log((3*(-b)^(4/3)*n)/(a^(7/3)*x^2) + (3*b*n*log(c*x^
n))/(a^2*x^2)))/(3*a^(4/3)*n) + ((-b)^(1/3)*log((3*b*n*log(c*x^n))/(a^2*x^2
) + (3*(-b)^(4/3)*n*((3^(1/2)*1i)/2 - 1/2))/(a^(7/3)*x^2))*((3^(1/2)*1i)/2
- 1/2))/(3*a^(4/3)*n) - ((-b)^(1/3)*log((3*b*n*log(c*x^n))/(a^2*x^2) - (3*(
-b)^(4/3)*n*((3^(1/2)*1i)/2 + 1/2))/(a^(7/3)*x^2))*((3^(1/2)*1i)/2 + 1/2))/
(3*a^(4/3)*n)
```

3.257 $\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$

Optimal result 1359
Rubi [A] (verified) 1359
Mathematica [A] (verified) 1363
Maple [C] (warning: unable to verify) 1363
Fricas [C] (verification not implemented) 1364
Sympy [A] (verification not implemented) 1364
Maxima [F] 1365
Giac [A] (verification not implemented) 1365
Mupad [B] (verification not implemented) 1366

Optimal result

Integrand size = 17, antiderivative size = 233

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n}-\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n}+\frac{\log(x)}{a}+\frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{4\sqrt{2}a^{5/4}n}-\frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{4\sqrt{2}a^{5/4}n}$$

```
[Out] ln(x)/a-1/4*b^(1/4)*arctan(-1+a^(1/4)*ln(c*x^n)*2^(1/2)/b^(1/4))/a^(5/4)/n*
2^(1/2)-1/4*b^(1/4)*arctan(1+a^(1/4)*ln(c*x^n)*2^(1/2)/b^(1/4))/a^(5/4)/n*2
^(1/2)+1/8*b^(1/4)*ln(-a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/2)+ln(c*x^n)^2*a^(1/2
)+b^(1/2))/a^(5/4)/n*2^(1/2)-1/8*b^(1/4)*ln(a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/
2)+ln(c*x^n)^2*a^(1/2)+b^(1/2))/a^(5/4)/n*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used

$= \{327, 217, 1179, 642, 1176, 631, 210\}$

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n} - \frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}a^{5/4}n} + \frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n} - \frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n} + \frac{\log(x)}{a}$$

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```

```
[Out] (b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[2]*a^(5/
4)*n) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[
2]*a^(5/4)*n) + Log[x]/a + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*L
og[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(5/4)*n) - (b^(1/4)*Log[Sqr
t[b] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[
2]*a^(5/4)*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{b+ax^4}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{\log(x)}{a} - \frac{b\text{Subst}\left(\int \frac{1}{b+ax^4}\,dx, x, \log\left(cx^n\right)\right)}{an}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt{b}\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}x^2}{b+ax^4}\,dx, x, \log\left(cx^n\right)\right)}{2an} - \frac{\sqrt{b}\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{a}x^2}{b+ax^4}\,dx, x, \log\left(cx^n\right)\right)}{2an}$$

$$= \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}+2x}{-\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}-x^2}\,dx, x, \log(cx^n)\right)}{4\sqrt{2}a^{5/4}n}$$

$$+ \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}-2x}{-\frac{\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}-x^2}\,dx, x, \log(cx^n)\right)}{4\sqrt{2}a^{5/4}n}$$

$$- \frac{\sqrt{b}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+x^2}\,dx, x, \log(cx^n)\right)}{4a^{3/2}n}$$

$$- \frac{\sqrt{b}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+x^2}\,dx, x, \log(cx^n)\right)}{4a^{3/2}n}$$

$$= \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{4\sqrt{2}a^{5/4}n}$$

$$- \frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{4\sqrt{2}a^{5/4}n}$$

$$- \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n}$$

$$+ \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n}$$

$$= \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n}$$

$$+ \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{4\sqrt{2}a^{5/4}n}$$

$$- \frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{4\sqrt{2}a^{5/4}n}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.89

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \frac{2\sqrt{2}\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)-2\sqrt{2}\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)+8\sqrt[4]{a}n\log(x)+\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{\right.}}{8a^{5/4}n}$$

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```

```
[Out] (2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] - 2*Sqr
t[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] + 8*a^(1/4)*n
*Log[x] + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n]
+ Sqrt[a]*Log[c*x^n]^2] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(
1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(8*a^(5/4)*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.51

method	result
risch	$\frac{\ln(x)}{a}+\left(\sum_{\_R=\mathrm{RootOf}(256n^4a^5\_Z^4+b)}\_R\ln\left(\ln(x^n)-4an\_R-\frac{i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)}{2}+\frac{i\pi\,\mathrm{csgn}(ic)\,\mathrm{c}}{2}\right.\right.$
default	$\frac{\frac{\ln(cx^n)}{a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\ln(cx^n)^2+\left(\frac{b}{a}\right)^{\frac{1}{4}}\ln(cx^n)\sqrt{2}+\sqrt{\frac{b}{a}}}{\ln(cx^n)^2-\left(\frac{b}{a}\right)^{\frac{1}{4}}\ln(cx^n)\sqrt{2}+\sqrt{\frac{b}{a}}}\right)+2\arctan\left(\frac{\sqrt{2}\ln(cx^n)}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\ln(cx^n)}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)\right)}{8a}}{n}$

```
[In] int(1/(a*x+b*x/ln(c*x^n)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*ln(x)+sum(_R*ln(ln(x^n)-4*a*n*_R-1/2*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)+1/2*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n
)^2-1/2*I*Pi*csgn(I*c*x^n)^3+ln(c)),_R=RootOf(256*_Z^4*a^5*n^4+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.67

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = -\frac{a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}+n\log(x)+\log(c)\right)+i\,a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(i\,an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}+n\log(x)+\log(c)\right)$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(a*(-b/(a^5*n^4))^(1/4)*log(a*n*(-b/(a^5*n^4))^(1/4) + n*log(x) + log(
c)) + I*a*(-b/(a^5*n^4))^(1/4)*log(I*a*n*(-b/(a^5*n^4))^(1/4) + n*log(x) +
log(c)) - I*a*(-b/(a^5*n^4))^(1/4)*log(-I*a*n*(-b/(a^5*n^4))^(1/4) + n*log(
x) + log(c)) - a*(-b/(a^5*n^4))^(1/4)*log(-a*n*(-b/(a^5*n^4))^(1/4) + n*log
(x) + log(c)) - 4*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 15.02 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.98

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \begin{cases} \tilde{\infty}\log(c)^4\log(x) \\ \frac{\begin{cases} -\frac{\log\left(\frac{x^{-n}}{c}\right)^5}{5n}+\frac{\log(cx^n)^5}{5n} & \text{for } \frac{1}{|cx^n|}<1 \wedge |cx^n \\ \frac{\log(cx^n)^5}{5n} & \text{for } |cx^n|<1 \\ -\frac{\log\left(\frac{x^{-n}}{c}\right)^5}{5n} & \text{for } \frac{1}{|cx^n|}<1 \\ -\frac{24G_{6,6}^{6,0}\left(\begin{matrix} & 1,1,1,1,1,1 \\ 0,0,0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n}+\frac{24G_{6,6}^{0,6}\left(\begin{matrix} 1,1,1,1,1,1 & \\ & 0,0,0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} \\ \frac{\log(c)^4\log(x)}{a\log(c)^4+b} \\ \frac{\log(x)}{a} \\ \frac{\sqrt[4]{-\frac{b}{a}}\log\left(-\sqrt[4]{-\frac{b}{a}}+\log(cx^n)\right)}{4an}-\frac{\sqrt[4]{-\frac{b}{a}}\log\left(\sqrt[4]{-\frac{b}{a}}+\log(cx^n)\right)}{4an}-\frac{\sqrt[4]{-\frac{b}{a}}\operatorname{atan}\left(\frac{\log(cx^n)}{\sqrt[4]{-\frac{b}{a}}}\right)}{2an}+\frac{\log(cx^n)}{an} \end{cases}$$

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**4),x)
```

```
[Out] Piecewise((zoo*log(c)**4*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (Piecewis
e((-log(1/(c*x**n))**5/(5*n) + log(c*x**n)**5/(5*n), (Abs(c*x**n) < 1) & (1
/Abs(c*x**n) < 1)), (log(c*x**n)**5/(5*n), Abs(c*x**n) < 1), (-log(1/(c*x**
n))**5/(5*n), 1/Abs(c*x**n) < 1), (-24*meijerg(((), (1, 1, 1, 1, 1, 1)), ((
0, 0, 0, 0, 0, 0), ()), c*x**n)/n + 24*meijerg(((1, 1, 1, 1, 1, 1), ()), ((
), (0, 0, 0, 0, 0, 0)), c*x**n)/n, True))/b, Eq(a, 0)), (log(c)**4*log(x)/(
a*log(c)**4 + b), Eq(n, 0)), (log(x)/a, Eq(b, 0)), ((-b/a)**(1/4)*log(-(-b/
a)**(1/4) + log(c*x**n))/(4*a*n) - (-b/a)**(1/4)*log((-b/a)**(1/4) + log(c*
x**n))/(4*a*n) - (-b/a)**(1/4)*atan(log(c*x**n)/(-b/a)**(1/4))/(2*a*n) + lo
g(c*x**n)/(a*n), True))
```

Maxima [F]

$$\int \frac{1}{ax + \frac{bx}{\log^4(cx^n)}}\,dx = \int \frac{1}{ax + \frac{bx}{\log(cx^n)^4}}\,dx$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(4*a^2*x*log(c)^3*log(x^n) + 6*a^2*x*log(c)^2*log(x^n)^2 + 4
*a^2*x*log(c)*log(x^n)^3 + a^2*x*log(x^n)^4 + (a^2*log(c)^4 + a*b)*x), x) +
 log(x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.76

$$\int \frac{1}{ax + \frac{bx}{\log^4(cx^n)}}\,dx = \frac{\log(x)}{a} - \frac{4\left(-\frac{bn^{12}}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{\pi a(\mathrm{sgn}(c)-1)-2\left(-a^3b\right)^{\frac{1}{4}}}{2\left(an\log(x)+a\log(|c|)\right)}\right) + \left(-\frac{bn^{12}}{a}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi an(\mathrm{sgn}(x)-1)+\pi a(\mathrm{sgn}(c)-1)\right)^2 + \right.}{}$$

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="giac")
```

```
[Out] log(x)/a - 1/8*(4*(-b*n^12/a)^(1/4)*arctan(1/2*(pi*a*(sgn(c) - 1) - 2*(-a^3
*b)^(1/4))/(a*n*log(x) + a*log(abs(c)))) + (-b*n^12/a)^(1/4)*log(1/4*(pi*a*
n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 + (a*n*log(abs(x)) + a*log(abs(c)) +
(-a^3*b)^(1/4))^2) - (-b*n^12/a)^(1/4)*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*
(sgn(c) - 1))^2 + (a*n*log(abs(x)) + a*log(abs(c)) - (-a^3*b)^(1/4))^2))/(a
*n^4)
```

Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.76

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$$
$$=\frac{\ln(x)}{a}+\frac{(-b)^{1/4}\left(\ln\left(-\frac{(-b)^{5/2}}{a^{11/2}\,x^3}-\frac{(-b)^{9/4}\,\ln(c\,x^n)\,1\mathrm{i}}{a^{21/4}\,x^3}\right)\,1\mathrm{i}-\ln\left(-\frac{(-b)^{5/2}}{a^{11/2}\,x^3}+\frac{(-b)^{9/4}\,\ln(c\,x^n)\,1\mathrm{i}}{a^{21/4}\,x^3}\right)\,1\mathrm{i}\right)}{4\,a^{5/4}\,n}$$
$$-\frac{(-b)^{1/4}\,\ln\left(\frac{(-b)^{5/2}+a^{1/4}\,(-b)^{9/4}\,\ln(c\,x^n)}{x^3}\right)}{4\,a^{5/4}\,n}+\frac{(-b)^{1/4}\,\ln\left(\frac{(-b)^{5/2}-a^{1/4}\,(-b)^{9/4}\,\ln(c\,x^n)}{x^3}\right)}{4\,a^{5/4}\,n}$$

```
[In] int(1/(a*x + (b*x)/log(c*x^n)^4),x)
```

```
[Out] log(x)/a + ((-b)^(1/4)*(log(- (-b)^(5/2)/(a^(11/2)*x^3) - ((-b)^(9/4)*log(c
*x^n)*1i)/(a^(21/4)*x^3))*1i - log(((-b)^(9/4)*log(c*x^n)*1i)/(a^(21/4)*x^3
) - (-b)^(5/2)/(a^(11/2)*x^3))*1i))/(4*a^(5/4)*n) - ((-b)^(1/4)*log(((-b)^(
5/2) + a^(1/4)*(-b)^(9/4)*log(c*x^n))/x^3))/(4*a^(5/4)*n) + ((-b)^(1/4)*log
(((-b)^(5/2) - a^(1/4)*(-b)^(9/4)*log(c*x^n))/x^3))/(4*a^(5/4)*n)
```

3.258 $\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx$

Optimal result 1367
Rubi [A] (verified) 1367
Mathematica [A] (verified) 1368
Maple [A] (verified) 1368
Fricas [A] (verification not implemented) 1369
Sympy [A] (verification not implemented) 1369
Maxima [F] 1369
Giac [F] 1369
Mupad [B] (verification not implemented) 1370

Optimal result

Integrand size = 18, antiderivative size = 22

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx=\frac{2\arctan\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan(1/3*(1+2*ln(7*x))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {632, 210}

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx=\frac{2\arctan\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
[Out] (2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \log(7x)\right)\\ &= -\left(2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\log(7x)\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx = \frac{2\arctan\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
[Out] (2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{(1+2\ln(7x))\sqrt{3}}{3}\right)\sqrt{3}}{3}$	20
default	$\frac{2\arctan\left(\frac{(1+2\ln(7x))\sqrt{3}}{3}\right)\sqrt{3}}{3}$	20
risch	$\frac{i\sqrt{3}\,\ln\left(\ln(7x)+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(\ln(7x)+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{3}$	40

```
[In] int(1/(x+x*ln(7*x)+x*ln(7*x)^2),x,method=_RETURNVERBOSE)
[Out] 2/3*arctan(1/3*(1+2*ln(7*x))*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\log\left(7\,x\right)+\frac{1}{3}\,\sqrt{3}\right)$$

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(2/3*sqrt(3)*log(7*x) + 1/3*sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx = \operatorname{RootSum}\left(3z^2+1,\left(i\mapsto i\log\left(\frac{3i}{2}+\log\left(7x\right)+\frac{1}{2}\right)\right)\right)$$

```
[In] integrate(1/(x+x*ln(7*x)+x*ln(7*x)**2),x)
[Out] RootSum(3*_z**2 + 1, Lambda(_i, _i*log(3*_i/2 + log(7*x) + 1/2)))
```

Maxima [F]

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx = \int \frac{1}{x\log\left(7\,x\right)^2+x\log\left(7\,x\right)+x}\,dx$$

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="maxima")
[Out] integrate(1/(x*log(7*x)^2 + x*log(7*x) + x), x)
```

Giac [F]

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx = \int \frac{1}{x\log\left(7\,x\right)^2+x\log\left(7\,x\right)+x}\,dx$$

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="giac")
[Out] integrate(1/(x*log(7*x)^2 + x*log(7*x) + x), x)
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x + x\log(7x) + x\log^2(7x)}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,\ln(7\,x)+1)}{3}\right)}{3}$$

```
[In] int(1/(x + x*log(7*x) + x*log(7*x)^2),x)
[Out] (2*3^(1/2)*atan((3^(1/2)*(2*log(7*x) + 1))/3))/3
```

3.259 $\int \frac{-1+\log(3x)}{x(1-\log(3x)+\log^2(3x))} dx$

Optimal result 1371
Rubi [A] (verified) 1371
Mathematica [A] (verified) 1372
Maple [A] (verified) 1373
Fricas [A] (verification not implemented) 1373
Sympy [A] (verification not implemented) 1373
Maxima [A] (verification not implemented) 1374
Giac [F] 1374
Mupad [B] (verification not implemented) 1374

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)} dx = \frac{\arctan\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)+1/3*arctan(1/3*(1-2*ln(3*x))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {648, 632, 210, 642}

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)} dx = \frac{\arctan\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)$$

```
[In] Int[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
```

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, \log(3x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \log(3x)\right)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \log(3x)\right)\\
&= \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\log(3x)\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = -\frac{\arctan\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

```
[In] Integrate[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
[Out] -(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]^
2]/2
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^2\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$	38
default	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^2\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$	38
risch	$\frac{\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{2}+\frac{i\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2}-\frac{i\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$	70

```
[In] int((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)-1/3*3^(1/2)*arctan(1/3*(-1+2*ln(3*x))*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log\left(3x\right)-\frac{1}{3}\sqrt{3}\right) + \frac{1}{2}\log\left(\log\left(3x\right)^2-\log\left(3x\right)+1\right)$$

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)^
2 - log(3*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = \operatorname{RootSum}\left(3z^2-3z+1,\left(i\mapsto i\log\left(-3i+\log\left(3x\right)+1\right)\right)\right)$$

```
[In] integrate((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)**2),x)
[Out] RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\log(3x)-1)\right) + \frac{1}{2}\log\left(\log(3x)^2-\log(3x)+1\right)$$

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*log(3*x) - 1)) + 1/2*log(log(3*x)^2 - lo
g(3*x) + 1)
```

Giac [F]

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = \int \frac{\log(3x)-1}{\left(\log(3x)^2-\log(3x)+1\right)x}\,dx$$

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="giac")
[Out] integrate((log(3*x) - 1)/((log(3*x)^2 - log(3*x) + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx = \frac{\ln\left(\ln(3x)^2-\ln(3x)+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\ln(3x)-1)}{3}\right)}{3}$$

```
[In] int((log(3*x) - 1)/(x*(log(3*x)^2 - log(3*x) + 1)),x)
[Out] log(log(3*x)^2 - log(3*x) + 1)/2 - (3^(1/2)*atan((3^(1/2)*(2*log(3*x) - 1))
/3))/3
```

3.260 $\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx$

Optimal result 1375
Rubi [A] (verified) 1375
Mathematica [A] (verified) 1376
Maple [A] (verified) 1377
Fricas [A] (verification not implemented) 1377
Sympy [A] (verification not implemented) 1377
Maxima [F] 1378
Giac [F] 1378
Mupad [B] (verification not implemented) 1378

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx=\frac{\arctan\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)+1/3*arctan(1/3*(1-2*ln(3*x))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {648, 632, 210, 642}

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx=\frac{\arctan\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)$$

```
[In] Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
```

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, \log(3x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \log(3x)\right)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \log(3x)\right)\\
&= \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\log(3x)\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$
\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = -\frac{\arctan\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)
$$

```
[In] Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
[Out] -(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]^
2]/2
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^{2}\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$	38
default	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^{2}\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$	38
risch	$\frac{\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{2}+\frac{i\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2}-\frac{i\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$	70

```
[In] int((-1+ln(3*x)^2)/(x+x*ln(3*x)^3),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)-1/3*3^(1/2)*arctan(1/3*(-1+2*ln(3*x))*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log(3x)-\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\log\left(\log(3x)^2-\log(3x)+1\right)$$

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)^
2 - log(3*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = \operatorname{RootSum}\left(3z^2-3z+1,\left(i\mapsto i\log\left(-3i+\log(3x)+1\right)\right)\right)$$

```
[In] integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)**3),x)
[Out] RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

Maxima [F]

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = \int \frac{\log(3x)^2-1}{x\log(3x)^3+x}\,dx$$

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="maxima")
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^3 + x), x)
```

Giac [F]

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = \int \frac{\log(3x)^2-1}{x\log(3x)^3+x}\,dx$$

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="giac")
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^3 + x), x)
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx = \frac{\ln\left(\ln(3x)^2-\ln(3x)+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\ln(3x)-1)}{3}\right)}{3}$$

```
[In] int((log(3*x)^2 - 1)/(x + x*log(3*x)^3),x)
[Out] log(log(3*x)^2 - log(3*x) + 1)/2 - (3^(1/2)*atan((3^(1/2)*(2*log(3*x) - 1))
/3))/3
```

3.261 $\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx$

Optimal result 1379
Rubi [A] (verified) 1379
Mathematica [A] (verified) 1381
Maple [A] (verified) 1381
Fricas [A] (verification not implemented) 1381
Sympy [A] (verification not implemented) 1382
Maxima [F] 1382
Giac [F] 1382
Mupad [B] (verification not implemented) 1382

Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = -\sqrt{3}\arctan\left(\frac{1+2\log(3x)}{\sqrt{3}}\right)+\log(x) - \frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)$$

```
[Out] ln(x)-1/2*ln(1+ln(3*x)+ln(3*x)^2)-arctan(1/3*(1+2*ln(3*x))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = -\sqrt{3}\arctan\left(\frac{2\log(3x)+1}{\sqrt{3}}\right) - \frac{1}{2}\log\left(\log^2(3x)+\log(3x)+1\right)+\log(x)$$

```
[In] Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[x] - Log[1 + Log[3*x] + L
og[3*x]^2]/2
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1+x^2}{1+x+x^2}\,dx, x, \log(3x)\right)\\
&= \text{Subst}\left(\int \left(1-\frac{2+x}{1+x+x^2}\right)\,dx, x, \log(3x)\right)\\
&= \log(x)-\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, \log(3x)\right)\\
&= \log(x)-\frac{1}{2}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \log(3x)\right)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \log(3x)\right)\\
&= \log(x)-\frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)+3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\log(3x)\right)\\
&= -\sqrt{3}\tan^{-1}\left(\frac{1+2\log(3x)}{\sqrt{3}}\right)+\log(x)-\frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = -\sqrt{3}\arctan\left(\frac{1+2\log(3x)}{\sqrt{3}}\right)+\log(3x)-\frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)$$

```
[In] Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[3*x] - Log[1 + Log[3*x] +
 Log[3*x]^2]/2
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\ln(3x)-\frac{\ln\left(1+\ln(3x)+\ln(3x)^2\right)}{2}-\arctan\left(\frac{(1+2\ln(3x))\sqrt{3}}{3}\right)\sqrt{3}$
default	$\ln(3x)-\frac{\ln\left(1+\ln(3x)+\ln(3x)^2\right)}{2}-\arctan\left(\frac{(1+2\ln(3x))\sqrt{3}}{3}\right)\sqrt{3}$
risch	$\ln(x)-\frac{\ln\left(\ln(3x)+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{2}+\frac{i\ln\left(\ln(3x)+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{2}-\frac{\ln\left(\ln(3x)+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2}-\frac{i\ln\left(\ln(3x)+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{2}$

```
[In] int((-1+ln(3*x)^2)/(x+x*ln(3*x)+x*ln(3*x)^2),x,method=_RETURNVERBOSE)
[Out] ln(3*x)-1/2*ln(1+ln(3*x)+ln(3*x)^2)-arctan(1/3*(1+2*ln(3*x))*3^(1/2))*3^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = -\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log(3x)+\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\log\left(\log(3x)^2+\log(3x)+1\right)+\log(3x)$$

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="fricas"
)
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) + 1/3*sqrt(3)) - 1/2*log(log(3*x)^2 +
log(3*x) + 1) + log(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.45

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx$$
$$= \log(x) + \operatorname{RootSum}\left(z^2+z+1, \left(i\mapsto i\log\left(-i+\log(3x)\right)\right)\right)$$

```
[In] integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)+x*ln(3*x)**2),x)
[Out] log(x) + RootSum(_z**2 + _z + 1, Lambda(_i, _i*log(-_i + log(3*x))))
```

Maxima [F]

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = \int \frac{\log(3x)^2-1}{x\log(3x)^2+x\log(3x)+x}\,dx$$

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="maxima"
)
[Out] -integrate((log(3) + log(x) + 2)/(x*(2*log(3) + 1)*log(x) + x*log(x)^2 + (l
og(3)^2 + log(3) + 1)*x), x) + log(x)
```

Giac [F]

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = \int \frac{\log(3x)^2-1}{x\log(3x)^2+x\log(3x)+x}\,dx$$

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="giac")
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^2 + x*log(3*x) + x), x)
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx = \ln(x) - \frac{\ln\left(\ln(3x)^2+\ln(3x)+1\right)}{2} - \sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\ln(3x)+1)}{3}\right)$$

```
[In] int((log(3*x)^2 - 1)/(x + x*log(3*x) + x*log(3*x)^2),x)
[Out] log(x) - log(log(3*x) + log(3*x)^2 + 1)/2 - 3^(1/2)*atan((3^(1/2)*(2*log(3*
x) + 1))/3)
```

3.262 $\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx$

Optimal result 1383
Rubi [A] (verified) 1383
Mathematica [A] (verified) 1384
Maple [A] (verified) 1384
Fricas [A] (verification not implemented) 1385
Sympy [A] (verification not implemented) 1385
Maxima [A] (verification not implemented) 1385
Giac [A] (verification not implemented) 1385
Mupad [B] (verification not implemented) 1386

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{1}{32x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4} - \frac{\log^2\left(\frac{1}{x}\right)}{4x^4}$$

```
[Out] -1/32/x^4+1/8*ln(1/x)/x^4-1/4*ln(1/x)^2/x^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2342, 2341}

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{1}{32x^4} - \frac{\log^2\left(\frac{1}{x}\right)}{4x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4}$$

```
[In] Int[Log[x^(-1)]^2/x^5,x]
```

```
[Out] -1/32*1/x^4 + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log^2\left(\frac{1}{x}\right)}{4x^4} - \frac{1}{2}\int \frac{\log\left(\frac{1}{x}\right)}{x^5}\,dx \\ &= -\frac{1}{32x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4} - \frac{\log^2\left(\frac{1}{x}\right)}{4x^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{1}{32x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4} - \frac{\log^2\left(\frac{1}{x}\right)}{4x^4}$$

```
[In] Integrate[Log[x^(-1)]^2/x^5,x]
[Out] -1/32*1/x^4 + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
norman	$\frac{-\frac{1}{32}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4}+\frac{\ln\left(\frac{1}{x}\right)}{8}}{x^{4}}$	21
parallelrisch	$\frac{-1-8\ln\left(\frac{1}{x}\right)^{2}+4\ln\left(\frac{1}{x}\right)}{32x^{4}}$	22
derivativedivides	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27
default	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27
risch	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27
parts	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27

```
[In] int(ln(1/x)^2/x^5,x,method=_RETURNVERBOSE)
[Out] (-1/32-1/4*ln(1/x)^2+1/8*ln(1/x))/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{8\,\log\left(\frac{1}{x}\right)^2 - 4\,\log\left(\frac{1}{x}\right) + 1}{32\,x^4}$$

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="fricas")
[Out] -1/32*(8*log(1/x)^2 - 4*log(1/x) + 1)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{\log\left(\frac{1}{x}\right)^2}{4x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4} - \frac{1}{32x^4}$$

```
[In] integrate(ln(1/x)**2/x**5,x)
[Out] -log(1/x)**2/(4*x**4) + log(1/x)/(8*x**4) - 1/(32*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{8\,\log\left(x\right)^2 + 4\,\log\left(x\right) + 1}{32\,x^4}$$

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="maxima")
[Out] -1/32*(8*log(x)^2 + 4*log(x) + 1)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{\log\left(x\right)^2}{4\,x^4} - \frac{\log\left(x\right)}{8\,x^4} - \frac{1}{32\,x^4}$$

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="giac")
[Out] -1/4*log(x)^2/x^4 - 1/8*log(x)/x^4 - 1/32/x^4
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{\frac{\ln\left(\frac{1}{x}\right)^2}{4} - \frac{\ln\left(\frac{1}{x}\right)}{8} + \frac{1}{32}}{x^4}$$

```
[In] int(log(1/x)^2/x^5,x)
[Out] -(log(1/x)^2/4 - log(1/x)/8 + 1/32)/x^4
```

3.263 $\int \frac{1}{\sqrt{-\log(ax^2)}} dx$

Optimal result 1387
Rubi [A] (verified) 1387
Mathematica [A] (verified) 1388
Maple [F] 1388
Fricas [F(-2)] 1389
Sympy [F] 1389
Maxima [F] 1389
Giac [F] 1389
Mupad [F(-1)] 1390

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{1}{\sqrt{-\log(ax^2)}} dx = -\frac{\sqrt{\frac{\pi}{2}}x\operatorname{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

```
[Out] -1/2*x*erf(1/2*(-ln(a*x^2))^(1/2)*2^(1/2))*2^(1/2)*Pi^(1/2)/(a*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2337, 2211, 2236}

$$\int \frac{1}{\sqrt{-\log(ax^2)}} dx = -\frac{\sqrt{\frac{\pi}{2}}x\operatorname{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

```
[In] Int[1/Sqrt[-Log[a*x^2]],x]
[Out] -((Sqrt[Pi/2]*x*Erf[Sqrt[-Log[a*x^2]]/Sqrt[2]])/Sqrt[a*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{x\text{Subst}\left(\int \frac{e^{x/2}}{\sqrt{-x}}\,dx, x, \log\left(ax^2\right)\right)}{2\sqrt{ax^2}}$$
$$= -\frac{x\text{Subst}\left(\int e^{-\frac{x^2}{2}}\,dx, x, \sqrt{-\log\left(ax^2\right)}\right)}{\sqrt{ax^2}}$$
$$= -\frac{\sqrt{\frac{\pi}{2}}x\text{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}x\text{erfi}\left(\frac{\sqrt{\log(ax^2)}}{\sqrt{2}}\right)\sqrt{\log\left(ax^2\right)}}{\sqrt{ax^2}\sqrt{-\log\left(ax^2\right)}}$$

```
[In] Integrate[1/Sqrt[-Log[a*x^2]],x]
[Out] (Sqrt[Pi/2]*x*Erfi[Sqrt[Log[a*x^2]]/Sqrt[2]]*Sqrt[Log[a*x^2]])/(Sqrt[a*x^2]
*Sqrt[-Log[a*x^2]])
```

Maple [F]

$$\int \frac{1}{\sqrt{-\ln\left(x^2a\right)}}dx$$

```
[In] int(1/(-ln(x^2*a))^(1/2),x)
[Out] int(1/(-ln(x^2*a))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx = \int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$$

```
[In] integrate(1/(-ln(a*x**2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx = \int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$$

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-log(a*x^2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx = \int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$$

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-log(a*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\log (ax^2)}}\,dx = \int \frac{1}{\sqrt{-\ln (a\,x^2)}}\,dx$$

```
[In] int(1/(-log(a*x^2))^(1/2),x)
[Out] int(1/(-log(a*x^2))^(1/2), x)
```

3.264 $\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$

Optimal result 1391
Rubi [A] (verified) 1391
Mathematica [A] (verified) 1392
Maple [F] 1393
Fricas [F(-2)] 1393
Sympy [F] 1393
Maxima [F] 1393
Giac [F] 1394
Mupad [F(-1)] 1394

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}x\operatorname{erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

```
[Out] 1/2*x*erfi(1/2*(-ln(a/x^2))^(1/2)*2^(1/2))*2^(1/2)*Pi^(1/2)*(a/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2337, 2211, 2235}

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \sqrt{\frac{\pi}{2}}x\sqrt{\frac{a}{x^2}}\operatorname{erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

```
[In] Int[1/Sqrt[-Log[a/x^2]],x]
[Out] Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erfi[Sqrt[-Log[a/x^2]]/Sqrt[2]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int\frac{e^{-x/2}}{\sqrt{-x}}\,dx,x,\log\left(\frac{a}{x^2}\right)\right)\right)\\ &= \left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int e^{\frac{x^2}{2}}\,dx,x,\sqrt{-\log\left(\frac{a}{x^2}\right)}\right)\\ &= \sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}x\text{erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int\frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}x\text{erf}\left(\frac{\sqrt{\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)\sqrt{\log\left(\frac{a}{x^2}\right)}}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}$$

```
[In] Integrate[1/Sqrt[-Log[a/x^2]],x]
[Out] -((Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erf[Sqrt[Log[a/x^2]]/Sqrt[2]]*Sqrt[Log[a/x^2]])
/Sqrt[-Log[a/x^2]])
```

Maple [F]

$$\int \frac{1}{\sqrt{-\ln\left(\frac{a}{x^2}\right)}}dx$$

```
[In] int(1/(-ln(1/x^2*a))^(1/2),x)
[Out] int(1/(-ln(1/x^2*a))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

```
[In] integrate(1/(-ln(a/x**2))**(1/2),x)
[Out] Integral(1/sqrt(-log(a/x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-log(a/x^2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-log(a/x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = \int \frac{1}{\sqrt{-\ln\left(\frac{a}{x^2}\right)}}\,dx$$

```
[In] int(1/(-log(a/x^2))^(1/2),x)
[Out] int(1/(-log(a/x^2))^(1/2), x)
```

3.265 $\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx$

Optimal result . . . 1395
Rubi [A] (verified) . . . 1395
Mathematica [A] (verified) . . . 1396
Maple [F] . . . 1396
Fricas [F(-2)] . . . 1397
Sympy [F] . . . 1397
Maxima [F] . . . 1397
Giac [A] (verification not implemented) . . . 1397
Mupad [F(-1)] . . . 1398

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = -\frac{\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[Out] -x*erf((-ln(a*x^n))^(1/2)/n^(1/2))*Pi^(1/2)/((a*x^n)^(1/n))/n^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2337, 2211, 2236}

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = -\frac{\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[In] Int[1/Sqrt[-Log[a*x^n]],x]
[Out] -((Sqrt[Pi]*x*Erf[Sqrt[-Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*(a*x^n)^n^(-1)))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x(ax^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{-x}}\,dx, x, \log\left(ax^n\right)\right)}{n}\\ &= -\frac{\left(2x(ax^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{x^2}{n}}\,dx, x, \sqrt{-\log\left(ax^n\right)}\right)}{n}\\ &= -\frac{\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{-\log\left(ax^n\right)}}\,dx = \frac{\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)\sqrt{\log\left(ax^n\right)}}{\sqrt{n}\sqrt{-\log\left(ax^n\right)}}$$

```
[In] Integrate[1/Sqrt[-Log[a*x^n]],x]
[Out] (Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]]*Sqrt[Log[a*x^n]])/(Sqrt[n]*(a*x^
n)^n^(-1)*Sqrt[-Log[a*x^n]])
```

Maple [F]

$$\int \frac{1}{\sqrt{-\ln\left(a\,x^{n}\right)}}dx$$

```
[In] int(1/(-ln(a*x^n))^(1/2),x)
[Out] int(1/(-ln(a*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = \int \frac{1}{\sqrt{-\log(ax^n)}}\,dx$$

```
[In] integrate(1/(-ln(a*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = \int \frac{1}{\sqrt{-\log(ax^n)}}\,dx$$

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-log(a*x^n)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{-n\log(x)-\log(a)}}{\sqrt{n}}\right)}{a^{\left(\frac{1}{n}\right)}\sqrt{n}}$$

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*erf(-sqrt(-n*log(x) - log(a))/sqrt(n))/(a^(1/n)*sqrt(n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = \int \frac{1}{\sqrt{-\ln(a\,x^n)}}\,dx$$

```
[In] int(1/(-log(a*x^n))^(1/2),x)
[Out] int(1/(-log(a*x^n))^(1/2), x)
```

3.266 $\int \frac{\log(1+\sqrt{x}-x)}{x} \, dx$

Optimal result . . . 1399
Rubi [A] (verified) . . . 1399
Mathematica [A] (verified) . . . 1401
Maple [A] (verified) . . . 1402
Fricas [F] . . . 1402
Sympy [F] . . . 1402
Maxima [F] . . . 1403
Giac [F] . . . 1403
Mupad [F(-1)] . . . 1403

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{\log \left(1+\sqrt{x}-x\right)}{x} \, dx = -2 \log \left(\frac{1}{2} \left(1+\sqrt{5}\right)\right) \log \left(1+\sqrt{5}-2 \sqrt{x}\right) - 2 \log \left(1-\frac{2 \sqrt{x}}{1-\sqrt{5}}\right) \log \left(\sqrt{x}\right) + 2 \log \left(1+\sqrt{x}-x\right) \log \left(\sqrt{x}\right) + 2 \operatorname{PolyLog}\left(2,1-\frac{2 \sqrt{x}}{1+\sqrt{5}}\right) - 2 \operatorname{PolyLog}\left(2,\frac{2 \sqrt{x}}{1-\sqrt{5}}\right)$$

```
[Out] -2*ln(1/2+1/2*5^(1/2))*ln(1+5^(1/2)-2*x^(1/2))+ln(x)*ln(1-x+x^(1/2))-ln(x)*
ln(1-2*x^(1/2)/(-5^(1/2)+1))-2*polylog(2,2*x^(1/2)/(-5^(1/2)+1))+2*polylog(
2,1-2*x^(1/2)/(5^(1/2)+1))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2610, 2604, 2404, 2354, 2438, 2353, 2352}

$$\int \frac{\log \left(1+\sqrt{x}-x\right)}{x} \, dx = 2 \operatorname{PolyLog}\left(2,1-\frac{2 \sqrt{x}}{1+\sqrt{5}}\right) - 2 \operatorname{PolyLog}\left(2,\frac{2 \sqrt{x}}{1-\sqrt{5}}\right) - 2 \log \left(\frac{1}{2} \left(1+\sqrt{5}\right)\right) \log \left(-2 \sqrt{x}+\sqrt{5}+1\right) - 2 \log \left(1-\frac{2 \sqrt{x}}{1-\sqrt{5}}\right) \log \left(\sqrt{x}\right) + 2 \log \left(-x+\sqrt{x}+1\right) \log \left(\sqrt{x}\right)$$

```
[In] Int[Log[1 + Sqrt[x] - x]/x,x]
```

```
[Out] -2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] - 2*Log[1 - (2*Sqrt[x]
)/(1 - Sqrt[5])]*Log[Sqrt[x]] + 2*Log[1 + Sqrt[x] - x]*Log[Sqrt[x]] + 2*Pol
yLog[2, 1 - (2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (2*Sqrt[x])/(1 - Sqrt
[5])]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
(a + b*Log[(-c)*(d/e)])*(Log[d + e*x]/e), x] + Dist[b, Int[Log[(-e)*(x/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[(-c)*(d/e), 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2610

```
Int[((a_.) + Log[u_]*(b_.))*(RFx_), x_Symbol] :> With[{lst = SubstForFracti
onalPowerOfLinear[RFx*(a + b*Log[u]), x]}, Dist[lst[[2]]*lst[[4]], Subst[In
t[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x] /;  !FalseQ[lst]] /; FreeQ[{a
, b}, x] && RationalFunctionQ[RFx, x]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{\log(1+x-x^2)}{x}\,dx, x, \sqrt{x}\right)$$
$$= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) - 2\text{Subst}\left(\int \frac{(1-2x)\log(x)}{1+x-x^2}\,dx, x, \sqrt{x}\right)$$
$$= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) - 2\text{Subst}\left(\int \left(-\frac{2\log(x)}{1-\sqrt{5}-2x} - \frac{2\log(x)}{1+\sqrt{5}-2x}\right)\,dx, x, \sqrt{x}\right)$$
$$= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) + 4\text{Subst}\left(\int \frac{\log(x)}{1-\sqrt{5}-2x}\,dx, x, \sqrt{x}\right) + 4\text{Subst}\left(\int \frac{\log(x)}{1+\sqrt{5}-2x}\,dx, x, \sqrt{x}\right)$$
$$= -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right) - 2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right) + 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) + 2\text{Subst}\left(\int \frac{\log\left(1-\frac{2x}{1-\sqrt{5}}\right)}{x}\,dx, x, \sqrt{x}\right) + 4\text{Subst}\left(\int \frac{\log\left(\frac{2x}{1+\sqrt{5}}\right)}{1+\sqrt{5}-2x}\,dx, x, \sqrt{x}\right)$$
$$= -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right) - 2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right) + 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) + 2\text{Li}_2\left(1-\frac{2\sqrt{x}}{1+\sqrt{5}}\right) - 2\text{Li}_2\left(\frac{2\sqrt{x}}{1-\sqrt{5}}\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx = -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right) + \left(\log\left(-1+\sqrt{5}\right) - \log\left(-1+\sqrt{5}+2\sqrt{x}\right)\right)\log(x) + \log\left(1+\sqrt{x}-x\right)\log(x) + 2\,\text{PolyLog}\left(2, \frac{1+\sqrt{5}-2\sqrt{x}}{1+\sqrt{5}}\right) - 2\,\text{PolyLog}\left(2, -\frac{2\sqrt{x}}{-1+\sqrt{5}}\right)$$

```
[In] Integrate[Log[1 + Sqrt[x] - x]/x,x]
[Out] -2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] + (Log[-1 + Sqrt[5]] -
 Log[-1 + Sqrt[5] + 2*Sqrt[x]])*Log[x] + Log[1 + Sqrt[x] - x]*Log[x] + 2*Po
lyLog[2, (1 + Sqrt[5] - 2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (-2*Sqrt[x
])/(-1 + Sqrt[5])]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\ln(x)\ln\left(1-x+\sqrt{x}\right)-\ln(x)\ln\left(\frac{1+\sqrt{5}-2\sqrt{x}}{\sqrt{5}+1}\right)-\ln(x)\ln\left(\frac{-1+\sqrt{5}+2\sqrt{x}}{\sqrt{5}-1}\right)-2\operatorname{dilog}\left(\frac{1+\sqrt{5}-}{\sqrt{5}+}\right.$
default	$\ln(x)\ln\left(1-x+\sqrt{x}\right)-\ln(x)\ln\left(\frac{1+\sqrt{5}-2\sqrt{x}}{\sqrt{5}+1}\right)-\ln(x)\ln\left(\frac{-1+\sqrt{5}+2\sqrt{x}}{\sqrt{5}-1}\right)-2\operatorname{dilog}\left(\frac{1+\sqrt{5}-}{\sqrt{5}+}\right.$
parts	$\ln(x)\ln\left(1-x+\sqrt{x}\right)-\ln(x)\ln\left(\frac{1+\sqrt{5}-2\sqrt{x}}{\sqrt{5}+1}\right)-\ln(x)\ln\left(\frac{-1+\sqrt{5}+2\sqrt{x}}{\sqrt{5}-1}\right)-2\operatorname{dilog}\left(\frac{1+\sqrt{5}-}{\sqrt{5}+}\right.$

```
[In] int(ln(1-x+x^(1/2))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*ln(1-x+x^(1/2))-ln(x)*ln((1+5^(1/2)-2*x^(1/2))/(5^(1/2)+1))-ln(x)*ln(
(-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))-2*dilog((1+5^(1/2)-2*x^(1/2))/(5^(1/2)+
1))-2*dilog((-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))
```

Fricas [F]

$$\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx=\int \frac{\log\left(-x+\sqrt{x}+1\right)}{x}\,dx$$

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="fricas")
[Out] integral(log(-x + sqrt(x) + 1)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx=\int \frac{\log\left(\sqrt{x}-x+1\right)}{x}\,dx$$

```
[In] integrate(ln(1-x+x**(1/2))/x,x)
[Out] Integral(log(sqrt(x) - x + 1)/x, x)
```

Maxima [F]

$$\int \frac{\log \left(1+\sqrt{x}-x\right)}{x}\,dx=\int \frac{\log \left(-x+\sqrt{x}+1\right)}{x}\,dx$$

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="maxima")
[Out] integrate(log(-x + sqrt(x) + 1)/x, x)
```

Giac [F]

$$\int \frac{\log \left(1+\sqrt{x}-x\right)}{x}\,dx=\int \frac{\log \left(-x+\sqrt{x}+1\right)}{x}\,dx$$

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="giac")
[Out] integrate(log(-x + sqrt(x) + 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(1+\sqrt{x}-x\right)}{x}\,dx=\int \frac{\ln \left(\sqrt{x}-x+1\right)}{x}\,dx$$

```
[In] int(log(x^(1/2) - x + 1)/x,x)
[Out] int(log(x^(1/2) - x + 1)/x, x)
```

3.267 $\int \frac{x\log(c+dx)}{a+bx}\,dx$

Optimal result 1404
Rubi [A] (verified) 1404
Mathematica [A] (verified) 1406
Maple [A] (verified) 1407
Fricas [F] 1407
Sympy [F] 1407
Maxima [A] (verification not implemented) 1408
Giac [F] 1408
Mupad [F(-1)] 1408

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{x\log(c+dx)}{a+bx}\,dx = -\frac{x}{b}+\frac{(c+dx)\log(c+dx)}{bd} - \frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2} - \frac{a\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2}$$

```
[Out] -x/b+(d*x+c)*ln(d*x+c)/b/d-a*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^2-a*poly
log(2,b*(d*x+c)/(-a*d+b*c))/b^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {45, 2463, 2436, 2332, 2441, 2440, 2438}

$$\int \frac{x\log(c+dx)}{a+bx}\,dx = -\frac{a\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2} - \frac{a\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b^2} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{x}{b}$$

```
[In] Int[(x*Log[c + d*x])/(a + b*x),x]
```

```
[Out] -(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) - (a*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/b^2 - (a*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{\log(c+dx)}{b} - \frac{a\log(c+dx)}{b(a+bx)}\right) dx \\ &= \frac{\int \log(c+dx)\, dx}{b} - \frac{a\int \frac{\log(c+dx)}{a+bx}\, dx}{b}\end{aligned}$$

$$= -\frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2} + \frac{\text{Subst}(\int \log(x)\,dx, x, c+dx)}{bd} + \frac{(ad)\int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{b^2}$$

$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2} + \frac{a\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{b^2}$$

$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2} - \frac{a\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{x\log(c+dx)}{a+bx}\,dx = \frac{-bdx + \left(bc + bdx - ad\log\left(\frac{d(a+bx)}{-bc+ad}\right)\right)\log(c+dx) - ad\,\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{b^2d}$$

```
[In] Integrate[(x*Log[c + d*x])/(a + b*x),x]
```

```
[Out] (-(b*d*x) + (b*c + b*d*x - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)])*Log[c + d
*x] - a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*d)
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{d((dx+c)\ln(dx+c)-dx-c)}{b}-\frac{a\,d^{2}\left(\frac{\operatorname{dilog}\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b}\right)}{b}}{d^{2}}$
default	$\frac{\frac{d((dx+c)\ln(dx+c)-dx-c)}{b}-\frac{a\,d^{2}\left(\frac{\operatorname{dilog}\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b}\right)}{b}}{d^{2}}$
risch	$\frac{x\ln(dx+c)}{b}+\frac{\ln(dx+c)c}{db}-\frac{x}{b}-\frac{c}{db}-\frac{a\operatorname{dilog}\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b^{2}}-\frac{a\ln(dx+c)\ln\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b^{2}}$
parts	$\frac{x\ln(dx+c)}{b}-\frac{\ln(dx+c)a\ln(bx+a)}{b^{2}}-\frac{d\left(\frac{bx+a}{bd}-\frac{c\ln(ad-cb-d(bx+a))}{d^{2}}+\frac{a\left(-\frac{\operatorname{dilog}\left(\frac{-ad+cb+d(bx+a)}{-ad+cb}\right)}{d}-\frac{\ln(bx+a)\ln\left(\frac{-a\cdots}{\cdots}\right)}{d\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots$

[In] int(x*ln(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/d^2*(d/b*((d*x+c)*ln(d*x+c)-d*x-c)-a*d^2/b*(dilog((a*d-c*b+b*(d*x+c))/(a*d-b*c))/b+ln(d*x+c)*ln((a*d-c*b+b*(d*x+c))/(a*d-b*c))/b))

Fricas [F]

$$\int \frac{x\log(c+dx)}{a+bx}\,dx=\int \frac{x\log\left(dx+c\right)}{bx+a}\,dx$$

[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="fricas")

[Out] integral(x*log(d*x + c)/(b*x + a), x)

Sympy [F]

$$\int \frac{x\log(c+dx)}{a+bx}\,dx=\int \frac{x\log\left(c+dx\right)}{a+bx}\,dx$$

[In] integrate(x*ln(d*x+c)/(b*x+a),x)

[Out] Integral(x*log(c + d*x)/(a + b*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \frac{x\log(c+dx)}{a+bx}\,dx = d\left(\frac{\left(\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)a}{b^2d}-\frac{x}{bd}+\frac{c\log\left(dx+c\right)}{bd^2}\right) + \left(\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}\right)\log\left(dx+c\right)$$

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] d*((log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*a/(b^2*d) - x/(b*d) + c*log(d*x + c)/(b*d^2)) + (x/b - a*log(
b*x + a)/b^2)*log(d*x + c)
```

Giac [F]

$$\int \frac{x\log(c+dx)}{a+bx}\,dx = \int \frac{x\log\left(dx+c\right)}{bx+a}\,dx$$

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*log(d*x + c)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\log(c+dx)}{a+bx}\,dx = \int \frac{x\,\ln\left(c+d\,x\right)}{a+b\,x}\,dx$$

```
[In] int((x*log(c + d*x))/(a + b*x),x)
```

```
[Out] int((x*log(c + d*x))/(a + b*x), x)
```

3.268 $\int \frac{\log(x)}{-1+x} dx$

Optimal result 1409
Rubi [A] (verified) 1409
Mathematica [A] (verified) 1410
Maple [A] (verified) 1410
Fricas [A] (verification not implemented) 1410
Sympy [C] (verification not implemented) 1411
Maxima [A] (verification not implemented) 1411
Giac [F] 1411
Mupad [B] (verification not implemented) 1411

Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \frac{\log(x)}{-1+x} dx = -\operatorname{PolyLog}(2, 1-x)$$

```
[Out] -polylog(2,1-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2352}

$$\int \frac{\log(x)}{-1+x} dx = -\operatorname{PolyLog}(2, 1-x)$$

```
[In] Int[Log[x]/(-1 + x),x]
```

```
[Out] -PolyLog[2, 1 - x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\text{integral} = -\text{Li}_2(1-x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{-1+x}\,dx = -\operatorname{PolyLog}(2,1-x)$$

```
[In] Integrate[Log[x]/(-1 + x),x]
[Out] -PolyLog[2, 1 - x]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

method	result	size
default	$-\operatorname{dilog}(x)$	5
risch	$-\operatorname{dilog}(x)$	5
parts	$-\operatorname{dilog}(x)$	5

```
[In] int(ln(x)/(-1+x),x,method=_RETURNVERBOSE)
[Out] -dilog(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\log(x)}{-1+x}\,dx = -\mathrm{Li}_2(-x+1)$$

```
[In] integrate(log(x)/(-1+x),x, algorithm="fricas")
[Out] -dilog(-x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)}{-1+x}\,dx = -\operatorname{Li}_2\left((x-1)\,e^{i\pi}\right)$$

```
[In] integrate(ln(x)/(-1+x),x)
[Out] -polylog(2, (x - 1)*exp_polar(I*pi))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\log(x)}{-1+x}\,dx = \log(x)\log(-x+1) + \operatorname{Li}_2(x)$$

```
[In] integrate(log(x)/(-1+x),x, algorithm="maxima")
[Out] log(x)*log(-x + 1) + dilog(x)
```

Giac [F]

$$\int \frac{\log(x)}{-1+x}\,dx = \int \frac{\log(x)}{x-1}\,dx$$

```
[In] integrate(log(x)/(-1+x),x, algorithm="giac")
[Out] integrate(log(x)/(x - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{\log(x)}{-1+x}\,dx = -\operatorname{Li}_2(x)$$

```
[In] int(log(x)/(x - 1),x)
[Out] -dilog(x)
```

3.269 $\int \frac{x \log(1-a-bx)}{a+bx}\,dx$

Optimal result 1412
Rubi [A] (verified) 1412
Mathematica [A] (verified) 1414
Maple [A] (verified) 1414
Fricas [F] 1414
Sympy [F] 1415
Maxima [B] (verification not implemented) 1415
Giac [F] 1415
Mupad [B] (verification not implemented) 1416

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \frac{x \log(1-a-bx)}{a+bx}\,dx = -\frac{x}{b} - \frac{(1-a-bx)\log(1-a-bx)}{b^2} + \frac{a\,\mathrm{PolyLog}(2,a+bx)}{b^2}$$

```
[Out] -x/b-(-b*x-a+1)*ln(-b*x-a+1)/b^2+a*polylog(2,b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {45, 2463, 2436, 2332, 2440, 2438}

$$\int \frac{x \log(1-a-bx)}{a+bx}\,dx = \frac{a\,\mathrm{PolyLog}(2,a+bx)}{b^2} - \frac{(-a-bx+1)\log(-a-bx+1)}{b^2} - \frac{x}{b}$$

```
[In] Int[(x*Log[1 - a - b*x])/(a + b*x),x]
[Out] -(x/b) - ((1 - a - b*x)*Log[1 - a - b*x])/b^2 + (a*PolyLog[2, a + b*x])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log(1-a-bx)}{b} - \frac{a\log(1-a-bx)}{b(a+bx)}\right) dx \\
&= \frac{\int \log(1-a-bx)\, dx}{b} - \frac{a\int \frac{\log(1-a-bx)}{a+bx}\, dx}{b} \\
&= -\frac{\text{Subst}(\int \log(x)\, dx, x, 1-a-bx)}{b^2} - \frac{a\text{Subst}\left(\int \frac{\log(1-x)}{x}\, dx, x, a+bx\right)}{b^2} \\
&= -\frac{x}{b} - \frac{(1-a-bx)\log(1-a-bx)}{b^2} + \frac{a\text{Li}_2(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx = \frac{-bx+(-1+a+bx)\log(1-a-bx)+a\,\mathrm{PolyLog}(2,a+bx)}{b^2}$$

```
[In] Integrate[(x*Log[1 - a - b*x])/(a + b*x),x]
[Out] (-(b*x) + (-1 + a + b*x)*Log[1 - a - b*x] + a*PolyLog[2, a + b*x])/b^2
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{-(-bx-a+1)\ln(-bx-a+1)-bx-a+1+a\,\mathrm{dilog}(-bx-a+1)}{b^2}$	47
default	$\frac{-(-bx-a+1)\ln(-bx-a+1)-bx-a+1+a\,\mathrm{dilog}(-bx-a+1)}{b^2}$	47
parts	$\frac{x\ln(-bx-a+1)}{b}-\frac{\ln(-bx-a+1)a\ln(bx+a)}{b^2}+\frac{-bx-a+(a-1)\ln(bx+a-1)}{b^2}-\frac{a\,\mathrm{dilog}(bx+a)}{b^2}$	74
risch	$\frac{x\ln(-bx-a+1)}{b}+\frac{a\,\mathrm{dilog}(-bx-a+1)}{b^2}+\frac{\ln(-bx-a+1)a}{b^2}-\frac{x}{b}-\frac{\ln(-bx-a+1)}{b^2}-\frac{a}{b^2}+\frac{1}{b^2}$	77

```
[In] int(x*ln(-b*x-a+1)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b^2*(-(-b*x-a+1)*ln(-b*x-a+1)-b*x-a+1+a*dilog(-b*x-a+1))
```

Fricas [F]

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx = \int \frac{x\log(-bx-a+1)}{bx+a}\,dx$$

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="fricas")
[Out] integral(x*log(-b*x - a + 1)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx = \int \frac{x\log(-a-bx+1)}{a+bx}\,dx$$

```
[In] integrate(x*ln(-b*x-a+1)/(b*x+a),x)
[Out] Integral(x*log(-a - b*x + 1)/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(38) = 76.

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx = b\left(\frac{(\log(bx+a)\log(-bx-a+1)+\mathrm{Li}_2(bx+a))a}{b^3}-\frac{x}{b^2}+\frac{(a-1)\log(bx+a-1)}{b^3}\right) + \left(\frac{x}{b}-\frac{a\log(bx+a)}{b^2}\right)\log(-bx-a+1)$$

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="maxima")
[Out] b*((log(b*x + a)*log(-b*x - a + 1) + dilog(b*x + a))*a/b^3 - x/b^2 + (a - 1
)*log(b*x + a - 1)/b^3) + (x/b - a*log(b*x + a)/b^2)*log(-b*x - a + 1)
```

Giac [F]

$$\int \frac{x\log(1-a-bx)}{a+bx}\,dx = \int \frac{x\log(-bx-a+1)}{bx+a}\,dx$$

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="giac")
[Out] integrate(x*log(-b*x - a + 1)/(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{x \log(1-a-bx)}{a+bx} \, dx = -\frac{\ln(1-b\,x-a)+b\,(x-x\,\ln(1-b\,x-a))-a\,\mathrm{Li}_2(1-b\,x-a)-a\,\ln(1-b\,x-a)}{b^2}$$

```
[In] int((x*log(1 - b*x - a))/(a + b*x),x)
[Out] -(log(1 - b*x - a) + b*(x - x*log(1 - b*x - a)) - a*dilog(1 - b*x - a) - a*
log(1 - b*x - a))/b^2
```

3.270 $\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx$

Optimal result . . . 1417
Rubi [A] (verified) . . . 1417
Mathematica [A] (verified) . . . 1418
Maple [A] (verified) . . . 1419
Fricas [F] . . . 1419
Sympy [C] (verification not implemented) . . . 1420
Maxima [A] (verification not implemented) . . . 1421
Giac [F] . . . 1421
Mupad [F(-1)] . . . 1421

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx = \frac{\log^2(x)}{2} + \log(x)\log\left(1+\frac{cx}{b}\right) + \operatorname{PolyLog}\left(2,-\frac{cx}{b}\right)$$

```
[Out] 1/2*ln(x)^2+ln(x)*ln(1+c*x/b)+polylog(2,-c*x/b)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2404, 2338, 2354, 2438}

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx = \operatorname{PolyLog}\left(2,-\frac{cx}{b}\right) + \log(x)\log\left(\frac{cx}{b}+1\right) + \frac{\log^2(x)}{2}$$

```
[In] Int[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
[Out] Log[x]^2/2 + Log[x]*Log[1 + (c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
```

```
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log(x)}{x}+\frac{c\log(x)}{b+cx}\right)dx \\
&= c\int \frac{\log(x)}{b+cx}\,dx+\int \frac{\log(x)}{x}\,dx \\
&= \frac{\log^2(x)}{2}+\log(x)\log\left(1+\frac{cx}{b}\right)-\int \frac{\log\left(1+\frac{cx}{b}\right)}{x}\,dx \\
&= \frac{\log^2(x)}{2}+\log(x)\log\left(1+\frac{cx}{b}\right)+\text{Li}_2\left(-\frac{cx}{b}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx=\frac{\log^2(x)}{2}+\log(x)\log\left(\frac{b+cx}{b}\right)+\text{PolyLog}\left(2,-\frac{cx}{b}\right)$$

```
[In] Integrate[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
[Out] Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{\ln(x)^2}{2}+\ln(x)\ln\left(\frac{xc+b}{b}\right)+\operatorname{dilog}\left(\frac{xc+b}{b}\right)$	31
default	$\frac{\ln(x)^2}{2}+c\left(\frac{\operatorname{dilog}\left(\frac{xc+b}{b}\right)}{c}+\frac{\ln(x)\ln\left(\frac{xc+b}{b}\right)}{c}\right)$	41
parts	$\frac{\ln(x)^2}{2}+c\left(\frac{\operatorname{dilog}\left(\frac{xc+b}{b}\right)}{c}+\frac{\ln(x)\ln\left(\frac{xc+b}{b}\right)}{c}\right)$	41

```
[In] int((2*c*x+b)*ln(x)/x/(c*x+b),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)^2+ln(x)*ln((c*x+b)/b)+dilog((c*x+b)/b)
```

Fricas [F]

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx=\int \frac{(2\,cx+b)\log(x)}{(cx+b)x}\,dx$$

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="fricas")
[Out] integral((2*c*x + b)*log(x)/(c*x^2 + b*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.67 (sec) , antiderivative size = 228, normalized size of antiderivative = 7.60

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx$$
$$= b \left(\begin{cases} -\frac{1}{cx} & \text{for } b = 0 \\ \frac{\begin{cases} \operatorname{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(c)\log(x) + \operatorname{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{for } |x| < 1 \\ -\log(c)\log\left(\frac{1}{x}\right) + \operatorname{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(c) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(c) + \operatorname{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases} \right)$$
$$- b \left(\begin{cases} \frac{1}{cx} & \text{for } b = 0 \\ \frac{\log\left(\frac{b}{x}+c\right)}{b} & \text{otherwise} \end{cases} \right) \log(x)$$
$$- 2c \left(\begin{cases} \frac{x}{b} & \text{for } c = 0 \\ \frac{\begin{cases} -\operatorname{Li}_2\left(\frac{cxe^{i\pi}}{b}\right) & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(b)\log(x) - \operatorname{Li}_2\left(\frac{cxe^{i\pi}}{b}\right) & \text{for } |x| < 1 \\ -\log(b)\log\left(\frac{1}{x}\right) - \operatorname{Li}_2\left(\frac{cxe^{i\pi}}{b}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(b) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(b) - \operatorname{Li}_2\left(\frac{cxe^{i\pi}}{b}\right) & \text{otherwise} \end{cases}}{c} & \text{otherwise} \end{cases} \right)$$
$$+ 2c \left(\begin{cases} \frac{x}{b} & \text{for } c = 0 \\ \frac{\log(b+cx)}{c} & \text{otherwise} \end{cases} \right) \log(x)$$

```
[In] integrate((2*c*x+b)*ln(x)/x/(c*x+b),x)
```

```
[Out] b*Piecewise((-1/(c*x), Eq(b, 0)), (Piecewise((polylog(2, b*exp_polar(I*pi)/
(c*x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(c)*log(x) + polylog(2, b*exp_p
olar(I*pi)/(c*x)), Abs(x) < 1), (-log(c)*log(1/x) + polylog(2, b*exp_polar(
I*pi)/(c*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(c
) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(c) + polylog(2, b*exp_polar(
I*pi)/(c*x)), True))/b, True)) - b*Piecewise((1/(c*x), Eq(b, 0)), (log(b/x
```

```
+ c)/b, True))*log(x) - 2*c*Piecewise((x/b, Eq(c, 0)), (Piecewise((-polylog
(2, c*x*exp_polar(I*pi)/b), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(b)*log(x)
- polylog(2, c*x*exp_polar(I*pi)/b), Abs(x) < 1), (-log(b)*log(1/x) - polyl
og(2, c*x*exp_polar(I*pi)/b), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0
), ()), x)*log(b) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(b) - polylog
(2, c*x*exp_polar(I*pi)/b), True))/c, True)) + 2*c*Piecewise((x/b, Eq(c, 0)
), (log(b + c*x)/c, True))*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx = (\log(cx+b)+\log(x))\log(x) - \log(cx+b)\log(x) + \log\left(\frac{cx}{b}+1\right)\log(x) - \frac{1}{2}\log(x)^2 + \text{Li}_2\left(-\frac{cx}{b}\right)$$

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="maxima")
[Out] (log(c*x + b) + log(x))*log(x) - log(c*x + b)*log(x) + log(c*x/b + 1)*log(x
) - 1/2*log(x)^2 + dilog(-c*x/b)
```

Giac [F]

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx = \int \frac{(2\,cx+b)\log(x)}{(cx+b)x}\,dx$$

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="giac")
[Out] integrate((2*c*x + b)*log(x)/((c*x + b)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx = \int \frac{\ln(x)\,(b+2\,c\,x)}{x\,(b+c\,x)}\,dx$$

```
[In] int((log(x)*(b + 2*c*x))/(x*(b + c*x)),x)
[Out] int((log(x)*(b + 2*c*x))/(x*(b + c*x)), x)
```

3.271 $\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx$

Optimal result 1422
Rubi [A] (verified) 1422
Mathematica [A] (verified) 1423
Maple [A] (verified) 1423
Fricas [A] (verification not implemented) 1423
Sympy [F] 1424
Maxima [A] (verification not implemented) 1424
Giac [A] (verification not implemented) 1424
Mupad [B] (verification not implemented) 1424

Optimal result

Integrand size = 14, antiderivative size = 7

$$\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx = -\cos(x\log(x))$$

```
[Out] -cos(x*ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4607}

$$\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx = -\cos(x\log(x))$$

```
[In] Int[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
[Out] -Cos[x*Log[x]]
```

Rule 4607

```
Int[Log[(b_.)*(x_)]*Sin[Log[(b_.)*(x_)]*(a_.)*(x_)], x_Symbol] :> Simp[-Cos
[a*x*Log[b*x]]/a, x] - Int[Sin[a*x*Log[b*x]], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \sin(x\log(x))\,dx + \int \log(x)\sin(x\log(x))\,dx \\ &= -\cos(x\log(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx = -\cos(x\log(x))$$

```
[In] Integrate[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
[Out] -Cos[x*Log[x]]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\cos\left(\ln\left(x\right)x\right)$	8
default	$-\cos\left(\ln\left(x\right)x\right)$	8
parallelrisch	$-\cos\left(2x\ln\left(\sqrt{x}\right)\right)-1$	13
norman	$-\frac{2}{1+\tan^{2}\left(\frac{\ln(x)x}{2}\right)}$	15
risch	$-\frac{x^{ix}}{2}-\frac{x^{-ix}}{2}$	20

```
[In] int(sin(ln(x)*x)+ln(x)*sin(ln(x)*x),x,method=_RETURNVERBOSE)
[Out] -cos(ln(x)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx = -\cos\left(x\log\left(x\right)\right)$$

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="fricas")
[Out] -cos(x*log(x))
```

Sympy [F]

$$\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx = \int (\log{(x)} + 1)\sin{(x\log{(x)})}\,dx$$

```
[In] integrate(sin(x*ln(x))+ln(x)*sin(x*ln(x)),x)
[Out] Integral((log(x) + 1)*sin(x*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx = -\cos{(x\log{(x)})}$$

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="maxima")
[Out] -cos(x*log(x))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx = -\cos{(x\log{(x)})}$$

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="giac")
[Out] -cos(x*log(x))
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx = -\cos{(x\,\ln{(x)})}$$

```
[In] int(sin(x*log(x)) + sin(x*log(x))*log(x),x)
[Out] -cos(x*log(x))
```

3.272 $\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx$

Optimal result 1425
Rubi [A] (verified) 1425
Mathematica [A] (verified) 1427
Maple [A] (verified) 1428
Fricas [A] (verification not implemented) 1428
Sympy [A] (verification not implemented) 1428
Maxima [A] (verification not implemented) 1429
Giac [A] (verification not implemented) 1429
Mupad [B] (verification not implemented) 1429

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = -\frac{1}{x}+\arctan(1-x)-\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}-\frac{1}{2}\log\left(2-2x+x^2\right)$$

```
[Out] -1/x-arctan(-1+x)-ln((1-(1-x)^2)/(1+(-1+x)^2))/x+1/2*ln(2-x)+1/2*ln(x)-1/2*
ln(x^2-2*x+2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2605, 12, 6860, 648, 631, 210, 642}

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = \arctan(1-x)-\frac{1}{2}\log\left(x^2-2x+2\right)-\frac{1}{x}-\frac{\log\left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}$$

```
[In] Int[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
```

```
[Out] -x^(-1) + ArcTan[1 - x] - Log[(1 - (1 - x)^2)/(1 + (-1 + x)^2)]/x + Log[2 -
 x]/2 + Log[x]/2 - Log[2 - 2*x + x^2]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \int \frac{4(1-x)}{(2-x)x^2(2-2x+x^2)}\,dx$$

$$= -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + 4\int \frac{1-x}{(2-x)x^2(2-2x+x^2)}\,dx$$

$$= -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + 4\int \left(\frac{1}{8(-2+x)} + \frac{1}{4x^2} + \frac{1}{8x} - \frac{x}{4(2-2x+x^2)}\right)dx$$

$$= -\frac{1}{x} - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \int \frac{x}{2-2x+x^2}\,dx$$

$$= -\frac{1}{x} - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{1}{2}\int \frac{-2+2x}{2-2x+x^2}\,dx - \int \frac{1}{2-2x+x^2}\,dx$$

$$= -\frac{1}{x} - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{1}{2}\log\left(2-2x+x^2\right) - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-x\right)$$

$$= -\frac{1}{x} + \tan^{-1}(1-x) - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{1}{2}\log\left(2-2x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = -\frac{1}{x} + \arctan(1-x) + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{\log\left(-\frac{(-2+x)x}{2-2x+x^2}\right)}{x} - \frac{1}{2}\log\left(2-2x+x^2\right)$$

```
[In] Integrate[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
[Out] -x^(-1) + ArcTan[1 - x] + Log[2 - x]/2 + Log[x]/2 - Log[-(((-2 + x)*x)/(2 -
 2*x + x^2))]/x - Log[2 - 2*x + x^2]/2
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\ln\left(\frac{x(2-x)}{x^2-2x+2}\right)}{x}-\frac{1}{x}+\frac{\ln(x)}{2}-\frac{\ln(x^2-2x+2)}{2}-\arctan\left(-1+x\right)+\frac{\ln(-2+x)}{2}$
parts	$-\frac{\ln\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x}-\frac{1}{x}+\frac{\ln(x)}{2}-\frac{\ln(x^2-2x+2)}{2}-\arctan\left(-1+x\right)+\frac{\ln(-2+x)}{2}$
risch	$-\frac{\ln\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x}+\frac{i\ln(x-1-i)x-i\ln(x-1+i)x-\ln(x-1-i)x-\ln(x-1+i)x+\ln\left(x^2-2x\right)x-2}{2x}$
parallelrisch	$-\frac{12+6i\ln(x-1+i)x-6i\ln(x-1-i)x-14\ln(x)x-14\ln(-2+x)x+14\ln(x-1-i)x+14\ln(x-1+i)x+8\ln\left(-\frac{x(-2+x)}{x^2-2x+2}\right)x+3x+12}{12x}$

[In] int(ln((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x,method=_RETURNVERBOSE)

[Out] -1/x*ln(x*(2-x)/(x^2-2*x+2))-1/x+1/2*ln(x)-1/2*ln(x^2-2*x+2)-arctan(-1+x)+1
/2*ln(-2+x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = -\frac{2\,x\arctan\left(x-1\right)+x\log\left(x^2-2\,x+2\right)-x\log\left(x^2-2\,x\right)+2\,\log\left(-\frac{x^2-2\,x}{x^2-2\,x+2}\right)+2}{2\,x}$$

[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="fricas")

[Out] -1/2*(2*x*arctan(x - 1) + x*log(x^2 - 2*x + 2) - x*log(x^2 - 2*x) + 2*log(-
(x^2 - 2*x)/(x^2 - 2*x + 2)) + 2)/x

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = \frac{\log\left(x^2-2x\right)}{2}-\frac{\log\left(x^2-2x+2\right)}{2}-\operatorname{atan}\left(x-1\right)-\frac{\log\left(\frac{1-(x-1)^2}{(x-1)^2+1}\right)}{x}-\frac{1}{x}$$

[In] integrate(ln((1-(-1+x)**2)/(1+(-1+x)**2))/x**2,x)

[Out] log(x**2 - 2*x)/2 - log(x**2 - 2*x + 2)/2 - atan(x - 1) - log((1 - (x - 1)*
*2)/((x - 1)**2 + 1))/x - 1/x

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = -\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x} - \frac{1}{x} - \arctan\left(x-1\right) - \frac{1}{2}\log\left(x^2-2\,x+2\right) + \frac{1}{2}\log\left(x-2\right) + \frac{1}{2}\log\left(x\right)$$

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="maxima")
```

```
[Out] -log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x^
2 - 2*x + 2) + 1/2*log(x - 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = -\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x} - \frac{1}{x} - \arctan\left(x-1\right) - \frac{1}{2}\log\left(x^2-2\,x+2\right) + \frac{1}{2}\log\left(|x-2|\right) + \frac{1}{2}\log\left(|x|\right)$$

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="giac")
```

```
[Out] -log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x^
2 - 2*x + 2) + 1/2*log(abs(x - 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = \frac{\ln\left(x\,\left(x-2\right)\right)}{2} - \mathrm{atan}(x-1) - \frac{\ln\left(x^2-2\,x+2\right)}{2} - \frac{\ln\left(2\,x-x^2\right)}{x} + \frac{\ln\left(x^2-2\,x+2\right)}{x} - \frac{1}{x}$$

```
[In] int(log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x^2,x)
```

```
[Out] log(x*(x - 2))/2 - atan(x - 1) - log(x^2 - 2*x + 2)/2 - log(2*x - x^2)/x +
log(x^2 - 2*x + 2)/x - 1/x
```

3.273 $\int \log\left(\sqrt{x}+x\right)\,dx$

Optimal result 1430
Rubi [A] (verified) 1430
Mathematica [A] (verified) 1431
Maple [A] (verified) 1431
Fricas [A] (verification not implemented) 1432
Sympy [A] (verification not implemented) 1432
Maxima [A] (verification not implemented) 1432
Giac [A] (verification not implemented) 1432
Mupad [B] (verification not implemented) 1433

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int \log\left(\sqrt{x}+x\right)\,dx=\sqrt{x}-x-\log\left(1+\sqrt{x}\right)+x\log\left(\sqrt{x}+x\right)$$

```
[Out] -x-ln(1+x^(1/2))+x*ln(x+x^(1/2))+x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {2628, 383, 78}

$$\int \log\left(\sqrt{x}+x\right)\,dx=-x+\sqrt{x}+x\log\left(x+\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)$$

```
[In] Int[Log[Sqrt[x] + x],x]
```

```
[Out] Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
```

```
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(\sqrt{x}+x\right)-\int \frac{1+2\sqrt{x}}{2+2\sqrt{x}}\,dx \\
&= x\log\left(\sqrt{x}+x\right)-2\text{Subst}\left(\int \frac{x(1+2x)}{2+2x}\,dx,x,\sqrt{x}\right) \\
&= x\log\left(\sqrt{x}+x\right)-2\text{Subst}\left(\int \left(-\frac{1}{2}+x+\frac{1}{2(1+x)}\right)\,dx,x,\sqrt{x}\right) \\
&= \sqrt{x}-x-\log\left(1+\sqrt{x}\right)+x\log\left(\sqrt{x}+x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \log\left(\sqrt{x}+x\right)\,dx=\sqrt{x}-x-\log\left(1+\sqrt{x}\right)+x\log\left(\sqrt{x}+x\right)$$

```
[In] Integrate[Log[Sqrt[x] + x],x]
[Out] Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-x-\ln\left(1+\sqrt{x}\right)+x\ln\left(x+\sqrt{x}\right)+\sqrt{x}$	24
default	$-x-\ln\left(1+\sqrt{x}\right)+x\ln\left(x+\sqrt{x}\right)+\sqrt{x}$	24
parts	$-x-\ln\left(1+\sqrt{x}\right)+x\ln\left(x+\sqrt{x}\right)+\sqrt{x}$	24

```
[In] int(ln(x+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -x-ln(1+x^(1/2))+x*ln(x+x^(1/2))+x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \log \left(\sqrt{x}+x\right) \, dx = (x+1) \log \left(x+\sqrt{x}\right) - x + \sqrt{x} - 2 \log \left(\sqrt{x}+1\right) - \log \left(\sqrt{x}\right)$$

```
[In] integrate(log(x+x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*log(x + sqrt(x)) - x + sqrt(x) - 2*log(sqrt(x) + 1) - log(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \log \left(\sqrt{x}+x\right) \, dx = \sqrt{x} + x \log \left(\sqrt{x}+x\right) - x + \log \left(-\frac{1}{\sqrt{x}}\right) - \log \left(-1-\frac{1}{\sqrt{x}}\right)$$

```
[In] integrate(ln(x+x**(1/2)),x)
[Out] sqrt(x) + x*log(sqrt(x) + x) - x + log(-1/sqrt(x)) - log(-1 - 1/sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \log \left(\sqrt{x}+x\right) \, dx = x \log \left(x+\sqrt{x}\right) - x + \sqrt{x} - \log \left(\sqrt{x}+1\right)$$

```
[In] integrate(log(x+x^(1/2)),x, algorithm="maxima")
[Out] x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \log \left(\sqrt{x}+x\right) \, dx = x \log \left(x+\sqrt{x}\right) - x + \sqrt{x} - \log \left(\sqrt{x}+1\right)$$

```
[In] integrate(log(x+x^(1/2)),x, algorithm="giac")
[Out] x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \log \left(\sqrt{x}+x\right) \, dx=\sqrt{x}-\ln \left(\sqrt{x}+1\right)-x+x \, \ln \left(x+\sqrt{x}\right)$$

```
[In] int(log(x + x^(1/2)),x)
[Out] x^(1/2) - log(x^(1/2) + 1) - x + x*log(x + x^(1/2))
```

3.274 $\int \log\left(-\frac{x}{1+x}\right)\,dx$

Optimal result 1434
Rubi [A] (verified) 1434
Mathematica [A] (verified) 1435
Maple [A] (verified) 1435
Fricas [A] (verification not implemented) 1436
Sympy [A] (verification not implemented) 1436
Maxima [A] (verification not implemented) 1436
Giac [B] (verification not implemented) 1436
Mupad [B] (verification not implemented) 1437

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \log\left(-\frac{x}{1+x}\right)\,dx = x\log\left(-\frac{x}{1+x}\right) - \log(1+x)$$

```
[Out] x*ln(-x/(1+x))-ln(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2536, 31}

$$\int \log\left(-\frac{x}{1+x}\right)\,dx = x\log\left(-\frac{x}{x+1}\right) - \log(x+1)$$

```
[In] Int[Log[-(x/(1 + x))],x]
```

```
[Out] x*Log[-(x/(1 + x))] - Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
```

```
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = x\log\left(-\frac{x}{1+x}\right) - \int \frac{1}{1+x}\,dx$$
$$= x\log\left(-\frac{x}{1+x}\right) - \log(1+x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log\left(-\frac{x}{1+x}\right)\,dx = x\log\left(-\frac{x}{1+x}\right) - \log(1+x)$$

```
[In] Integrate[Log[-(x/(1 + x))],x]
[Out] x*Log[-(x/(1 + x))] - Log[1 + x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
risch	$x\ln\left(-\frac{x}{x+1}\right)-\ln\left(x+1\right)$	19
parts	$x\ln\left(-\frac{x}{x+1}\right)-\ln\left(x+1\right)$	19
parallelrisch	$x\ln\left(-\frac{x}{x+1}\right)-\ln\left(x\right)+\ln\left(-\frac{x}{x+1}\right)$	26
derivativedivides	$\ln\left(\frac{1}{x+1}\right)-\ln\left(-1+\frac{1}{x+1}\right)\left(-1+\frac{1}{x+1}\right)\left(x+1\right)$	28
default	$\ln\left(\frac{1}{x+1}\right)-\ln\left(-1+\frac{1}{x+1}\right)\left(-1+\frac{1}{x+1}\right)\left(x+1\right)$	28

```
[In] int(ln(-x/(x+1)),x,method=_RETURNVERBOSE)
[Out] x*ln(-x/(x+1))-ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log\left(-\frac{x}{1+x}\right)\,dx = x\log\left(-\frac{x}{x+1}\right) - \log\left(x+1\right)$$

```
[In] integrate(log(-x/(1+x)),x, algorithm="fricas")
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \log\left(-\frac{x}{1+x}\right)\,dx = x\log\left(-\frac{x}{x+1}\right) - \log\left(x+1\right)$$

```
[In] integrate(ln(-x/(1+x)),x)
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log\left(-\frac{x}{1+x}\right)\,dx = x\log\left(-\frac{x}{x+1}\right) - \log\left(x+1\right)$$

```
[In] integrate(log(-x/(1+x)),x, algorithm="maxima")
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(18) = 36$.

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.44

$$\int \log\left(-\frac{x}{1+x}\right)\,dx = -\frac{\log\left(-\frac{x}{(x+1)\left(\frac{x}{x+1}-1\right)\left(\frac{x}{(x+1)\left(\frac{x}{x+1}-1\right)}-1\right)}\right)}{\frac{x}{x+1}-1} - \log\left(\frac{|x|}{|x+1|}\right) + \log\left(\left|-\frac{x}{x+1}+1\right|\right)$$

```
[In] integrate(log(-x/(1+x)),x, algorithm="giac")
```

```
[Out] -log(-x/((x + 1)*(x/(x + 1) - 1)*(x/((x + 1)*(x/(x + 1) - 1)) - 1)))/(x/(x
+ 1) - 1) - log(abs(x)/abs(x + 1)) + log(abs(-x/(x + 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \log \left(-\frac{x}{1+x}\right) \,dx = x \ln \left(-\frac{x}{x+1}\right) - \ln \left(x+1\right)$$

```
[In] int(log(-x/(x + 1)),x)
```

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

3.275 $\int \log \left(\frac{-1+x}{1+x}\right) dx$

Optimal result 1438
Rubi [A] (verified) 1438
Mathematica [A] (verified) 1439
Maple [A] (verified) 1439
Fricas [A] (verification not implemented) 1440
Sympy [A] (verification not implemented) 1440
Maxima [A] (verification not implemented) 1440
Giac [B] (verification not implemented) 1440
Mupad [B] (verification not implemented) 1441

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \log \left(\frac{-1+x}{1+x}\right) dx = -\left((1-x)\log \left(-\frac{1-x}{1+x}\right)\right) - 2\log(1+x)$$

```
[Out] -(1-x)*ln((-1+x)/(1+x))-2*ln(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2535, 31}

$$\int \log \left(\frac{-1+x}{1+x}\right) dx = -\left((1-x)\log \left(-\frac{1-x}{x+1}\right)\right) - 2\log(x+1)$$

```
[In] Int[Log[(-1 + x)/(1 + x)],x]
```

```
[Out] -((1 - x)*Log[-((1 - x)/(1 + x))]) - 2*Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
```

```
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\left((1-x)\log\left(-\frac{1-x}{1+x}\right)\right) - 2\int \frac{1}{1+x}\,dx$$
$$= -\left((1-x)\log\left(-\frac{1-x}{1+x}\right)\right) - 2\log(1+x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \log\left(\frac{-1+x}{1+x}\right)\,dx = (-1+x)\log\left(\frac{-1+x}{1+x}\right) - 2\log(1+x)$$

```
[In] Integrate[Log[(-1 + x)/(1 + x)],x]
[Out] (-1 + x)*Log[(-1 + x)/(1 + x)] - 2*Log[1 + x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
risch	$x\ln\left(\frac{-1+x}{x+1}\right)-\ln\left(x^{2}-1\right)$	22
parts	$x\ln\left(\frac{-1+x}{x+1}\right)-\ln\left(\left(-1+x\right)\left(x+1\right)\right)$	24
parallelrisch	$x\ln\left(\frac{-1+x}{x+1}\right)-2\ln\left(-1+x\right)+\ln\left(\frac{-1+x}{x+1}\right)$	30
derivativedivides	$2\ln\left(-\frac{2}{x+1}\right)+\ln\left(1-\frac{2}{x+1}\right)\left(1-\frac{2}{x+1}\right)\left(x+1\right)$	35
default	$2\ln\left(-\frac{2}{x+1}\right)+\ln\left(1-\frac{2}{x+1}\right)\left(1-\frac{2}{x+1}\right)\left(x+1\right)$	35

```
[In] int(ln((-1+x)/(x+1)),x,method=_RETURNVERBOSE)
[Out] x*ln((-1+x)/(x+1))-ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \log \left(\frac{-1+x}{1+x}\right) \, dx = x \log \left(\frac{x-1}{x+1}\right) - \log \left(x^{2}-1\right)$$

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="fricas")
[Out] x*log((x - 1)/(x + 1)) - log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \log \left(\frac{-1+x}{1+x}\right) \, dx = x \log \left(\frac{x-1}{x+1}\right) - \log \left(x^{2}-1\right)$$

```
[In] integrate(ln((-1+x)/(1+x)),x)
[Out] x*log((x - 1)/(x + 1)) - log(x**2 - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \log \left(\frac{-1+x}{1+x}\right) \, dx = x \log \left(\frac{x-1}{x+1}\right) - \log \left(x+1\right) - \log \left(x-1\right)$$

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="maxima")
[Out] x*log((x - 1)/(x + 1)) - log(x + 1) - log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(21) = 42.

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.81

$$\int \log \left(\frac{-1+x}{1+x}\right) \, dx = -\frac{2 \log \left(\frac{\frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1}+1}{\frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1}-1}\right)}{\frac{x-1}{x+1}-1} - 2 \log \left(\frac{|x-1|}{|x+1|}\right) + 2 \log \left(\left|\frac{x-1}{x+1}-1\right|\right)$$

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="giac")
```

```
[Out] -2*log((((x - 1)/(x + 1) + 1)/((x - 1)/(x + 1) - 1) + 1)/(((x - 1)/(x + 1)
+ 1)/((x - 1)/(x + 1) - 1) - 1))/((x - 1)/(x + 1) - 1) - 2*log(abs(x - 1)/a
bs(x + 1)) + 2*log(abs((x - 1)/(x + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \log \left(\frac{-1+x}{1+x}\right) dx = x \ln \left(\frac{x-1}{x+1}\right) - \ln \left(x^2-1\right)$$

```
[In] int(log((x - 1)/(x + 1)),x)
```

```
[Out] x*log((x - 1)/(x + 1)) - log(x^2 - 1)
```

3.276 $\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2} dx$

Optimal result 1442
Rubi [A] (verified) 1442
Mathematica [C] (verified) 1444
Maple [A] (verified) 1444
Fricas [A] (verification not implemented) 1445
Sympy [A] (verification not implemented) 1445
Maxima [A] (verification not implemented) 1445
Giac [A] (verification not implemented) 1446
Mupad [B] (verification not implemented) 1446

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2} dx = -\frac{1}{1+x} - \arctan(x) + \frac{1}{2}\log\left(1-x^2\right) - \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} - \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -1/(1+x)-arctan(x)+1/2*ln(-x^2+1)-ln((-x^2+1)/(x^2+1))/(1+x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2605, 12, 2099, 266, 649, 209}

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2} dx = -\arctan(x) + \frac{1}{2}\log\left(1-x^2\right) - \frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1} - \frac{1}{2}\log\left(x^2+1\right) - \frac{1}{x+1}$$

```
[In] Int[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) - ArcTan[x] + Log[1 - x^2]/2 - Log[(1 - x^2)/(1 + x^2)]/(1 +
x) - Log[1 + x^2]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} + \int \frac{4x}{-1-x+x^4+x^5}\,dx \\
&= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} + 4\int \frac{x}{-1-x+x^4+x^5}\,dx \\
&= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} + 4\int \left(\frac{1}{4(1+x)^2} + \frac{x}{4\left(-1+x^2\right)} + \frac{-1-x}{4\left(1+x^2\right)}\right)dx \\
&= -\frac{1}{1+x} - \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} + \int \frac{x}{-1+x^2}\,dx + \int \frac{-1-x}{1+x^2}\,dx
\end{aligned}$$

$$= -\frac{1}{1+x} + \frac{1}{2}\log\left(1-x^2\right) - \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} - \int \frac{1}{1+x^2}\,dx - \int \frac{x}{1+x^2}\,dx$$

$$= -\frac{1}{1+x} - \tan^{-1}(x) + \frac{1}{2}\log\left(1-x^2\right) - \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x} - \frac{1}{2}\log\left(1+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = \frac{1}{2}\left((-1+i)\log(i-x) - (1+i)\log(i+x) + \log\left(1-x^2\right) - \frac{2\left(1+\log\left(\frac{1-x^2}{1+x^2}\right)\right)}{1+x}\right)$$

```
[In] Integrate[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

```
[Out] ((-1 + I)*Log[I - x] - (1 + I)*Log[I + x] + Log[1 - x^2] - (2*(1 + Log[(1 -
 x^2)/(1 + x^2)]))/(1 + x))/2
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

method	result
parts	$-\frac{\ln\left(\frac{-x^2+1}{x^2+1}\right)}{x+1}+\frac{\ln(-1+x)}{2}-\frac{\ln(x^2+1)}{2}-\arctan\left(x\right)-\frac{1}{x+1}+\frac{\ln(x+1)}{2}$
parallelrisch	$\frac{-i\ln(x+i)-i\ln(x+i)x-1+i\ln(x-i)+i\ln(x-i)x+x\ln\left(-\frac{x^2-1}{x^2+1}\right)+x-\ln\left(-\frac{x^2-1}{x^2+1}\right)}{2+2x}$
risch	$-\frac{\ln\left(\frac{-x^2+1}{x^2+1}\right)}{x+1}+\frac{i\ln(x-i)x-i\ln(x+i)x+i\ln(x-i)-i\ln(x+i)-\ln(x-i)x-\ln(x+i)x+\ln(x^2-1)x-\ln(x-i)-\ln(x+i)+\ln(x^2-1)}{2+2x}$

```
[In] int(ln((-x^2+1)/(x^2+1))/(x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln((-x^2+1)/(x^2+1))/(x+1)+1/2*ln(-1+x)-1/2*ln(x^2+1)-arctan(x)-1/(x+1)+1/
2*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = -\frac{2\,(x+1)\arctan\left(x\right)+(x+1)\log\left(x^2+1\right)-(x+1)\log\left(x^2-1\right)+2\,\log\left(-\frac{x^2-1}{x^2+1}\right)+2}{2\,(x+1)}$$

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(x + 1)*arctan(x) + (x + 1)*log(x^2 + 1) - (x + 1)*log(x^2 - 1) + 2
*log(-(x^2 - 1)/(x^2 + 1)) + 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = \frac{\log\left(x^2-1\right)}{2}-\frac{\log\left(x^2+1\right)}{2}-\operatorname{atan}\left(x\right)-\frac{4}{4x+4}-\frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}$$

```
[In] integrate(ln((-x**2+1)/(x**2+1))/(1+x)**2,x)
```

```
[Out] log(x**2 - 1)/2 - log(x**2 + 1)/2 - atan(x) - 4/(4*x + 4) - log((1 - x**2)/
(x**2 + 1))/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx = -\frac{\log\left(-\frac{x^2-1}{x^2+1}\right)}{x+1}-\frac{1}{x+1}-\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x+1\right)+\frac{1}{2}\,\log\left(x-1\right)$$

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 +
1) + 1/2*log(x + 1) + 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2} \, dx = -\frac{\log \left(-\frac{x^2-1}{x^2+1}\right)}{x+1} - \frac{1}{x+1} - \arctan(x) - \frac{1}{2} \log \left(x^2+1\right) + \frac{1}{2} \log \left(|x+1|\right) + \frac{1}{2} \log \left(|x-1|\right)$$

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="giac")
```

```
[Out] -log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 +
1) + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{\log \left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2} \, dx = \frac{\ln \left(x^2-1\right)}{2} - \frac{\ln \left(x^2+1\right)}{2} - \mathrm{atan}(x) - \frac{1}{x+1} + \frac{\ln \left(x^2+1\right)}{x+1} - \frac{\ln \left(1-x^2\right)}{x+1}$$

```
[In] int(log(-(x^2 - 1)/(x^2 + 1))/(x + 1)^2,x)
```

```
[Out] log(x^2 - 1)/2 - log(x^2 + 1)/2 - atan(x) - 1/(x + 1) + log(x^2 + 1)/(x + 1
) - log(1 - x^2)/(x + 1)
```

3.277 $\int \frac{\log\left(c(1+x^2)^n\right)}{1+x^2}\,dx$

Optimal result 1447
Rubi [A] (verified) 1447
Mathematica [A] (verified) 1449
Maple [B] (verified) 1449
Fricas [F] 1450
Sympy [F] 1450
Maxima [F] 1450
Giac [F] 1450
Mupad [F(-1)] 1451

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \frac{\log\left(c(1+x^2)^n\right)}{1+x^2}\,dx = in\arctan(x)^2 + 2n\arctan(x)\log\left(\frac{2}{1+ix}\right) + \arctan(x)\log\left(c(1+x^2)^n\right) + in\,\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)$$

```
[Out] I*n*arctan(x)^2+2*n*arctan(x)*ln(2/(1+I*x))+arctan(x)*ln(c*(x^2+1)^n)+I*n*p
olylog(2,1-2/(1+I*x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {209, 2520, 5040, 4964, 2449, 2352}

$$\int \frac{\log\left(c(1+x^2)^n\right)}{1+x^2}\,dx = \arctan(x)\log\left(c(x^2+1)^n\right) + in\arctan(x)^2 + 2n\arctan(x)\log\left(\frac{2}{1+ix}\right) + in\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

```
[In] Int[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

```
[Out] I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[2/(1 + I*x)] + ArcTan[x]*Log[c*(1 + x^2
)^n] + I*n*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)-(2n)\int\frac{x\tan^{-1}(x)}{1+x^2}\,dx\\
&= in\tan^{-1}(x)^2+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+(2n)\int\frac{\tan^{-1}(x)}{i-x}\,dx\\
&= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)-(2n)\int\frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx
\end{aligned}
$$

$$= in\tan^{-1}(x)^2 + 2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right) + \tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right) + (2in)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+ix}\right)$$

$$= in\tan^{-1}(x)^2 + 2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right) + \tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right) + in\text{Li}_2\left(1-\frac{2}{1+ix}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c\left(1+x^2\right)^n\right)}{1+x^2}\,dx = in\arctan(x)^2 + 2n\arctan(x)\log\left(\frac{2i}{i-x}\right) + \arctan(x)\log\left(c\left(1+x^2\right)^n\right) + in\,\text{PolyLog}\left(2, \frac{i+x}{-i+x}\right)$$

```
[In] Integrate[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

```
[Out] I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[(2*I)/(I - x)] + ArcTan[x]*Log[c*(1 + x
^2)^n] + I*n*PolyLog[2, (I + x)/(-I + x)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(56) = 112.

Time = 1.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.15

method	result
parts	$\arctan(x)\ln\left(c(x^2+1)^n\right) - 2n\left(\frac{\arctan(x)\ln(x^2+1)}{2} + \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \text{dilog}\left(-\frac{i(x+i)}{2}\right) - \ln(x-i)\ln(\right.}{4}\right.$
risch	$\ln\left((x^2+1)^n\right)\arctan(x) - n\arctan(x)\ln(x^2+1) - \frac{in\ln(x-i)\ln(x^2+1)}{2} + \frac{in\ln(x-i)^2}{4} + \frac{in\,\text{dilog}\left(-\frac{i(x+i)}{2}\right.}{2}$

```
[In] int(ln(c*(x^2+1)^n)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x)*ln(c*(x^2+1)^n)-2*n*(1/2*arctan(x)*ln(x^2+1)+1/4*I*(ln(x-I)*ln(x^
2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x+I))-ln(x-I)*ln(-1/2*I*(x+I)))-1/4*I*(ln(
x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilog(1/2*I*(x-I))-ln(x+I)*ln(1/2*I*(x-I))))
```

Fricas [F]

$$\int \frac{\log \left(c(1+x^2)^n\right)}{1+x^2}\,dx=\int \frac{\log \left((x^2+1)^n c\right)}{x^2+1}\,dx$$

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="fricas")
[Out] integral(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log \left(c(1+x^2)^n\right)}{1+x^2}\,dx=\int \frac{\log \left(c(x^2+1)^n\right)}{x^2+1}\,dx$$

```
[In] integrate(ln(c*(x**2+1)**n)/(x**2+1),x)
[Out] Integral(log(c*(x**2 + 1)**n)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\log \left(c(1+x^2)^n\right)}{1+x^2}\,dx=\int \frac{\log \left((x^2+1)^n c\right)}{x^2+1}\,dx$$

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="maxima")
[Out] integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log \left(c(1+x^2)^n\right)}{1+x^2}\,dx=\int \frac{\log \left((x^2+1)^n c\right)}{x^2+1}\,dx$$

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="giac")
[Out] integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(1+x^2)^n\right)}{1+x^2}\,dx=\int \frac{\ln \left(c\,{\left(x^2+1\right)}^n\right)}{x^2+1}\,dx$$

```
[In] int(log(c*(x^2 + 1)^n)/(x^2 + 1),x)
[Out] int(log(c*(x^2 + 1)^n)/(x^2 + 1), x)
```

3.278 $\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2} dx$

Optimal result 1452
Rubi [A] (verified) 1452
Mathematica [B] (verified) 1454
Maple [B] (verified) 1455
Fricas [F] 1455
Sympy [F] 1455
Maxima [F] 1456
Giac [F] 1456
Mupad [F(-1)] 1456

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2} dx = i\arctan(x)^2 - 2\arctan(x)\log\left(2-\frac{2}{1-ix}\right) + \arctan(x)\log\left(\frac{x^2}{1+x^2}\right) + i\,\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)$$

```
[Out] I*arctan(x)^2-2*arctan(x)*ln(2-2/(1-I*x))+arctan(x)*ln(x^2/(x^2+1))+I*polyl
og(2,-1+2/(1-I*x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {209, 2606, 12, 5044, 4988, 2497}

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2} dx = \arctan(x)\log\left(\frac{x^2}{x^2+1}\right) + i\arctan(x)^2 - 2\arctan(x)\log\left(2-\frac{2}{1-ix}\right) + i\,\text{PolyLog}\left(2,\frac{2}{1-ix}-1\right)$$

```
[In] Int[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] I*ArcTan[x]^2 - 2*ArcTan[x]*Log[2 - 2/(1 - I*x)] + ArcTan[x]*Log[x^2/(1 + x
^2)] + I*PolyLog[2, -1 + 2/(1 - I*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2606

```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Dist[n, Int[SimplifyIn
tegrand[u*(D[RFx, x]/RFx), x], x], x]] /; FreeQ[{c, d, e, n}, x] && Rationa
lFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-\int\frac{2\tan^{-1}(x)}{x(1+x^2)}\,dx\\ &= \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-2\int\frac{\tan^{-1}(x)}{x(1+x^2)}\,dx\end{aligned}$$

$$= i\tan^{-1}(x)^2 + \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right) - 2i\int \frac{\tan^{-1}(x)}{x(i+x)}\,dx$$

$$= i\tan^{-1}(x)^2 - 2\tan^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) + \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right) + 2\int \frac{\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx$$

$$= i\tan^{-1}(x)^2 - 2\tan^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) + \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right) + i\mathrm{Li}_2\left(-1+\frac{2}{1-ix}\right)$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 239 vs. 2(61) = 122.

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.92

$$\begin{aligned}\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx &= -\frac{1}{4}i\log^2(i-x) + i\log(i-x)\log(-ix) - \frac{1}{2}i\log(i-x)\log\left(-\frac{1}{2}i(i+x)\right) \\ &+ \frac{1}{2}i\log\left(-\frac{1}{2}i(i-x)\right)\log(i+x) - i\log(ix)\log(i+x) + \frac{1}{4}i\log^2(i+x) \\ &- \frac{1}{2}i\log(i-x)\log\left(\frac{x^2}{1+x^2}\right) + \frac{1}{2}i\log(i+x)\log\left(\frac{x^2}{1+x^2}\right) \\ &- \frac{1}{2}i\,\mathrm{PolyLog}\left(2,-\frac{1}{2}i(i-x)\right) + i\,\mathrm{PolyLog}(2,-i(i-x)) \\ &+ \frac{1}{2}i\,\mathrm{PolyLog}\left(2,-\frac{1}{2}i(i+x)\right) - i\,\mathrm{PolyLog}(2,-i(i+x))\end{aligned}$$

```
[In] Integrate[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] (-1/4*I)*Log[I - x]^2 + I*Log[I - x]*Log[(-I)*x] - (I/2)*Log[I - x]*Log[(-1
/2*I)*(I + x)] + (I/2)*Log[(-1/2*I)*(I - x)]*Log[I + x] - I*Log[I*x]*Log[I
+ x] + (I/4)*Log[I + x]^2 - (I/2)*Log[I - x]*Log[x^2/(1 + x^2)] + (I/2)*Log
[I + x]*Log[x^2/(1 + x^2)] - (I/2)*PolyLog[2, (-1/2*I)*(I - x)] + I*PolyLog
[2, (-I)*(I - x)] + (I/2)*PolyLog[2, (-1/2*I)*(I + x)] - I*PolyLog[2, (-I)*
(I + x)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(57) = 114.

Time = 2.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.39

method	result
default	$-\frac{i\left(\ln(x-i)\ln\left(\frac{x^2}{x^2+1}\right)-2\,\mathrm{dilog}(-ix)-2\ln(x-i)\ln(-ix)+\mathrm{dilog}\left(-\frac{i(x+i)}{2}\right)+\ln(x-i)\ln\left(-\frac{i(x+i)}{2}\right)+\frac{\ln(x-i)^2}{2}\right)}{2}+\frac{i\left(\ln(x+i)\ln\left(\frac{x}{x^2}\right.\right.}{}$
risch	$-\frac{i\ln(x-i)\ln\left(\frac{x^2}{x^2+1}\right)}{2}+i\,\mathrm{dilog}\left(-ix\right)+i\ln\left(x-i\right)\ln\left(-ix\right)-\frac{i\,\mathrm{dilog}\left(-\frac{i(x+i)}{2}\right)}{2}-\frac{i\ln(x-i)\ln\left(-\frac{i(x+i)}{2}\right)}{2}-\frac{i\ln(}{}$
parts	$\arctan\left(x\right)\ln\left(\frac{x^2}{x^2+1}\right)-2\arctan\left(x\right)\ln\left(x\right)+\arctan\left(x\right)\ln\left(x^2+1\right)-i\ln\left(x\right)\ln\left(ix+1\right)+i\ln\left(x\right)$

```
[In] int(ln(x^2/(x^2+1))/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(ln(x-I)*ln(x^2/(x^2+1))-2*dilog(-I*x)-2*ln(x-I)*ln(-I*x)+dilog(-1/2
*I*(x+I))+ln(x-I)*ln(-1/2*I*(x+I))+1/2*ln(x-I)^2)+1/2*I*(ln(x+I)*ln(x^2/(x^
2+1))-2*dilog(I*x)-2*ln(x+I)*ln(I*x)+dilog(1/2*I*(x-I))+ln(x+I)*ln(1/2*I*(x
-I))+1/2*ln(x+I)^2)
```

Fricas [F]

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = \int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = \int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

```
[In] integrate(ln(x**2/(x**2+1))/(x**2+1),x)
```

```
[Out] Integral(log(x**2/(x**2 + 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = \int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="maxima")
[Out] integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = \int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="giac")
[Out] integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx = \int \frac{\ln\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

```
[In] int(log(x^2/(x^2 + 1))/(x^2 + 1),x)
[Out] int(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

3.279 $\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2} dx$

Optimal result 1457
Rubi [A] (verified) 1457
Mathematica [B] (verified) 1459
Maple [C] (verified) 1460
Fricas [F] 1460
Sympy [F] 1461
Maxima [F] 1461
Giac [F] 1461
Mupad [F(-1)] 1461

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2} dx = \frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{i\,\mathrm{PolyLog}\left(2,-1+\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] I*arctan(x*b^(1/2)/a^(1/2))^2/a^(1/2)/b^(1/2)+arctan(x*b^(1/2)/a^(1/2))*ln(
c*x^2/(b*x^2+a))/a^(1/2)/b^(1/2)-2*arctan(x*b^(1/2)/a^(1/2))*ln(2-2*a^(1/2)
/(a^(1/2)-I*x*b^(1/2)))/a^(1/2)/b^(1/2)+I*polylog(2,-1+2*a^(1/2)/(a^(1/2)-I
*x*b^(1/2)))/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {211, 2606, 12, 5044, 4988, 2497}

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} + \frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} - \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}-1\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Int[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

```
[Out] (I*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (ArcTan[(Sqrt[b]*x)/S
qrt[a]]*Log[(c*x^2)/(a + b*x^2)])/(Sqrt[a]*Sqrt[b]) - (2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]]*Log[2 - (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) +
 (I*PolyLog[2, -1 + (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2606

```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Dist[n, Int[SimplifyIn
tegrand[u*(D[RFx, x]/RFx), x], x], x]] /; FreeQ[{c, d, e, n}, x] && Rationa
lFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \int \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}x\,(a+bx^2)}\,dx\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \frac{\left(2\sqrt{a}\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{x(a+bx^2)}\,dx}{\sqrt{b}}\\
&= \frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \frac{(2i)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{x\left(i+\frac{\sqrt{b}x}{\sqrt{a}}\right)}\,dx}{\sqrt{a}\sqrt{b}}\\
&= \frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}\\
&\quad - \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\int \frac{\log\left(2-\frac{2}{1-\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}\,dx}{a}\\
&= \frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}\\
&\quad - \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{i\,\text{Li}_2\left(-1+\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 373 vs. $2(165) = 330$.

Time = 0.16 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.26

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \frac{-4\log\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right)\log\left(\sqrt{-a}-\sqrt{b}x\right)+\log^2\left(\sqrt{-a}-\sqrt{b}x\right)+4\log\left(\frac{a\sqrt{b}x}{(-a)^{3/2}}\right)\log\left(\sqrt{-a}+\sqrt{b}x\right)-\log^2\left(\sqrt{-a}\right.}{}$$

```
[In] Integrate[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

```
[Out] (-4*Log[(Sqrt[b]*x)/Sqrt[-a]]*Log[Sqrt[-a] - Sqrt[b]*x] + Log[Sqrt[-a] - Sq
rt[b]*x]^2 + 4*Log[(a*Sqrt[b]*x)/(-a)^(3/2)]*Log[Sqrt[-a] + Sqrt[b]*x] - Lo
g[Sqrt[-a] + Sqrt[b]*x]^2 + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(a - Sqrt[-a]*S
qrt[b]*x)/(2*a)] - 2*Log[Sqrt[-a] + Sqrt[b]*x]*Log[(a + Sqrt[-a]*Sqrt[b]*x)
```

```
/(2*a)] + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] - 2*Log[Sqrt
[-a] + Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] + 4*PolyLog[2, 1 + (Sqrt[b]*x)/S
qrt[-a]] - 2*PolyLog[2, (a - Sqrt[-a]*Sqrt[b]*x)/(2*a)] + 2*PolyLog[2, (a +
 Sqrt[-a]*Sqrt[b]*x)/(2*a)] - 4*PolyLog[2, 1 + (a*Sqrt[b]*x)/(-a)^(3/2)])/(
4*Sqrt[-a]*Sqrt[b])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.73

method	result
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+a\right)} \frac{2\ln(x-\_\alpha)\ln\left(\frac{c x^2}{b x^2+a}\right)+b\left(\frac{\ln(x-\_\alpha)^2}{\_\alpha b}+\frac{2\_\alpha\ln(x-\_\alpha)\ln\left(\frac{x+\_\alpha}{2\_\alpha}\right)}{a}+\frac{2\_\alpha\,\mathrm{dilog}\left(\frac{x+\_\alpha}{2\_\alpha}\right)}{a}\right)-4\,\mathrm{dilog}\left(\frac{x}{\_\alpha}\right)-4\ln(x-\_\alpha)}{\_\alpha}}{4b}$
risch	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^2+a\right)} \frac{2\ln(x-\_\alpha)\ln\left(\frac{c x^2}{b x^2+a}\right)+b\left(\frac{\ln(x-\_\alpha)^2}{\_\alpha b}+\frac{2\_\alpha\ln(x-\_\alpha)\ln\left(\frac{x+\_\alpha}{2\_\alpha}\right)}{a}+\frac{2\_\alpha\,\mathrm{dilog}\left(\frac{x+\_\alpha}{2\_\alpha}\right)}{a}\right)-4\,\mathrm{dilog}\left(\frac{x}{\_\alpha}\right)-4\ln(x-\_\alpha)}{\_\alpha}}{4b}$

```
[In] int(ln(c*x^2/(b*x^2+a))/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*sum(1/_alpha*(2*ln(x-_alpha)*ln(c*x^2/(b*x^2+a))+b*(1/_alpha/b*ln(x-_
alpha)^2+2*_alpha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/a*dilog
(1/2*(x+_alpha)/_alpha))-4*dilog(x/_alpha)-4*ln(x-_alpha)*ln(x/_alpha)),_al
pha=RootOf(_Z^2*b+a))
```

Fricas [F]

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}\,dx$$

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$$

```
[In] integrate(ln(c*x**2/(b*x**2+a))/(b*x**2+a),x)
[Out] Integral(log(c*x**2/(a + b*x**2))/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}\,dx$$

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}\,dx$$

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="giac")
[Out] integrate(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \int \frac{\ln\left(\frac{c\,x^2}{b\,x^2+a}\right)}{b\,x^2+a}\,dx$$

```
[In] int(log((c*x^2)/(a + b*x^2))/(a + b*x^2),x)
[Out] int(log((c*x^2)/(a + b*x^2))/(a + b*x^2), x)
```

3.280 $\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result 1462
Rubi [A] (verified) 1462
Mathematica [B] (verified) 1463
Maple [F] 1463
Fricas [A] (verification not implemented) 1463
Sympy [F(-1)] 1464
Maxima [F] 1464
Giac [F] 1464
Mupad [F(-1)] 1465

Optimal result

Integrand size = 39, antiderivative size = 29

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2598}

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[In] Int[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\text{integral} = \frac{\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. 2(29) = 58.

Time = 0.53 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{4\text{arctanh}(ax)\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{PolyLog}\left(2,-e^{-2\text{arctanh}(ax)}\right)-2\left(\text{arctanh}(ax)\left(\log\left(1+e^{-2\text{arctanh}(ax)}\right)-\right.\right.}{}$$

```
[In] Integrate[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (4*ArcTanh[a*x]*Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(-
2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] - Log[1 -
I/E^ArcTanh[a*x]] + Log[1 + I/E^ArcTanh[a*x]]) - PolyLog[2, (-I)/E^ArcTanh[
a*x]] + PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

Maple [F]

$$\int \frac{\ln\left(1+\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-x^2a^2+1}dx$$

```
[In] int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{Li}_2\left(-\frac{ax-\sqrt{ax+1}\sqrt{ax-1}+1}{ax+1}+1\right)}{a}$$

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
fricas")
```

```
[Out] dilog(-(a*x - sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/(a*x + 1) + 1)/a
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(1+I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\log\left(\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
maxima")
```

```
[Out] -1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*lo
g(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x + 1
))*log(sqrt(a*x + 1) + I*sqrt(-a*x + 1)))/a - integrate(-1/2*sqrt(a*x + 1)*
(log(a*x + 1) - log(-a*x + 1))/((a^2*x^2 - 1)*sqrt(a*x + 1) - (-I*a^2*x^2 +
 I)*sqrt(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\log\left(\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
giac")
```

```
[Out] integrate(-log(I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{\ln \left(1+\frac{\sqrt{1-a\,x}\,1\mathrm{i}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

```
[In] int(-log(((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2) + 1)/(a^2*x^2 - 1),x)
[Out] int(-log(((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2) + 1)/(a^2*x^2 - 1), x)
```

3.281 $\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result 1466
Rubi [A] (verified) 1466
Mathematica [B] (verified) 1467
Maple [F] 1467
Fricas [A] (verification not implemented) 1467
Sympy [F(-1)] 1468
Maxima [F] 1468
Giac [F] 1468
Mupad [F(-1)] 1469

Optimal result

Integrand size = 39, antiderivative size = 29

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\frac{\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2598}

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\frac{\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[In] Int[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\text{integral}=\frac{\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. 2(29) = 58.

Time = 0.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{4\text{arctanh}(ax)\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{PolyLog}\left(2,-e^{-2\text{arctanh}(ax)}\right)-2\big(\text{arctanh}(ax)\left(\log\left(1+e^{-2\text{arctanh}(ax)}\right)+\right.}{}$$

```
[In] Integrate[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (4*ArcTanh[a*x]*Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(-
2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] + Log[1 -
I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[
a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

Maple [F]

$$\int \frac{\ln\left(1-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-x^2a^2+1}\,dx$$

```
[In] int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{Li}_2\left(-\frac{ax+\sqrt{ax+1}\sqrt{ax-1}+1}{ax+1}+1\right)}{a}$$

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
fricas")
```

```
[Out] dilog(-(a*x + sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/(a*x + 1) + 1)/a
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1 - \frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1 - a^2x^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(1-I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(1 - \frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1 - a^2x^2}\,dx = \int -\frac{\log\left(-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}} + 1\right)}{a^2x^2 - 1}\,dx$$

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
maxima")
```

```
[Out] -1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*lo
g(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x + 1
))*log(sqrt(a*x + 1) - I*sqrt(-a*x + 1)))/a + integrate(1/2*sqrt(a*x + 1)*(
log(a*x + 1) - log(-a*x + 1))/((a^2*x^2 - 1)*sqrt(a*x + 1) + (-I*a^2*x^2 +
I)*sqrt(-a*x + 1)), x)
```

Giac [F]

$$\int \frac{\log\left(1 - \frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1 - a^2x^2}\,dx = \int -\frac{\log\left(-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}} + 1\right)}{a^2x^2 - 1}\,dx$$

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
giac")
```

```
[Out] integrate(-log(-I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{\ln\left(1-\frac{\sqrt{1-a\,x}\,1\mathrm{i}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

```
[In] int(-log(1 - ((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
[Out] int(-log(1 - ((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.282 $\int \log\left(e^{a+bx}\right)\,dx$

Optimal result 1470
Rubi [A] (verified) 1470
Mathematica [A] (verified) 1471
Maple [A] (verified) 1471
Fricas [A] (verification not implemented) 1472
Sympy [A] (verification not implemented) 1472
Maxima [A] (verification not implemented) 1472
Giac [A] (verification not implemented) 1472
Mupad [B] (verification not implemented) 1473

Optimal result

Integrand size = 8, antiderivative size = 17

$$\int \log\left(e^{a+bx}\right)\,dx = \frac{\log^2\left(e^{a+bx}\right)}{2b}$$

```
[Out] 1/2*ln(exp(b*x+a))^2/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2188, 30}

$$\int \log\left(e^{a+bx}\right)\,dx = \frac{\log^2\left(e^{a+bx}\right)}{2b}$$

```
[In] Int[Log[E^(a + b*x)],x]
```

```
[Out] Log[E^(a + b*x)]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x\,dx, x, \log\left(e^{a+bx}\right)\right)}{b}\\
&= \frac{\log^2\left(e^{a+bx}\right)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \log\left(e^{a+bx}\right)\,dx = \frac{\log^2\left(e^{a+bx}\right)}{2b}$$

```
[In] Integrate[Log[E^(a + b*x)],x]
[Out] Log[E^(a + b*x)]^2/(2*b)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)^2}{2b}$	15
default	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)^2}{2b}$	15
norman	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)^2}{2b}$	15
risch	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{b\,x^2}{2}$	17
parallelrisch	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{b\,x^2}{2}$	17
parts	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{b\,x^2}{2}$	17

```
[In] int(ln(exp(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(exp(b*x+a))^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx}\right) \, dx = \frac{1}{2} \, bx^2 + ax$$

```
[In] integrate(log(exp(b*x+a)),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int \log \left(e^{a+bx}\right) \, dx = ax + \frac{bx^2}{2}$$

```
[In] integrate(ln(exp(b*x+a)),x)
[Out] a*x + b*x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx}\right) \, dx = \frac{1}{2} \, bx^2 + ax$$

```
[In] integrate(log(exp(b*x+a)),x, algorithm="maxima")
[Out] 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx}\right) \, dx = \frac{1}{2} \, bx^2 + ax$$

```
[In] integrate(log(exp(b*x+a)),x, algorithm="giac")
[Out] 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \log \left(e^{a+bx}\right) \, dx = x \ln \left(\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\right) - \frac{b\,x^2}{2}$$

```
[In] int(log(exp(a + b*x)),x)
[Out] x*log(exp(b*x)*exp(a)) - (b*x^2)/2
```

3.283 $\int \log\left(e^{a+bx^n}\right)\,dx$

Optimal result 1474
Rubi [A] (verified) 1474
Mathematica [A] (verified) 1475
Maple [A] (verified) 1475
Fricas [A] (verification not implemented) 1476
Sympy [B] (verification not implemented) 1476
Maxima [A] (verification not implemented) 1476
Giac [A] (verification not implemented) 1477
Mupad [B] (verification not implemented) 1477

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \log\left(e^{a+bx^n}\right)\,dx = -\frac{bnx^{1+n}}{1+n} + x\log\left(e^{a+bx^n}\right)$$

```
[Out] -b*n*x^(1+n)/(1+n)+x*ln(exp(a+b*x^n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2628, 12, 30}

$$\int \log\left(e^{a+bx^n}\right)\,dx = x\log\left(e^{a+bx^n}\right) - \frac{bnx^{n+1}}{n+1}$$

```
[In] Int[Log[E^(a + b*x^n)],x]
```

```
[Out] -((b*n*x^(1 + n))/(1 + n)) + x*Log[E^(a + b*x^n)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(e^{a+bx^n}\right)-\int bnx^n\,dx\\ &= x\log\left(e^{a+bx^n}\right)-(bn)\int x^n\,dx\\ &= -\frac{bnx^{1+n}}{1+n}+x\log\left(e^{a+bx^n}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int\log\left(e^{a+bx^n}\right)\,dx=x\left(-\frac{bnx^n}{1+n}+\log\left(e^{a+bx^n}\right)\right)$$

```
[In] Integrate[Log[E^(a + b*x^n)],x]
[Out] x*(-((b*n*x^n)/(1 + n)) + Log[E^(a + b*x^n)])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
risch	$x\ln\left(\mathrm{e}^{a+b\,x^{n}}\right)-\frac{bnx\,x^{n}}{1+n}$	26
default	$-\frac{bn\,x^{1+n}}{1+n}+x\ln\left(\mathrm{e}^{a+b\,x^{n}}\right)$	27
parts	$-\frac{bn\,x^{1+n}}{1+n}+x\ln\left(\mathrm{e}^{a+b\,x^{n}}\right)$	27
parallelrisch	$-\frac{bn\,x^{n}x-\ln\left(\mathrm{e}^{a+b\,x^{n}}\right)xn-x\ln\left(\mathrm{e}^{a+b\,x^{n}}\right)}{1+n}$	41

```
[In] int(ln(exp(a+b*x^n)),x,method=_RETURNVERBOSE)
[Out] x*ln(exp(a+b*x^n))-b*n/(1+n)*x*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \log \left(e^{a+bx^n}\right) \, dx = \frac{bxx^n + (an+a)x}{n+1}$$

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="fricas")
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(22) = 44.

Time = 0.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \log \left(e^{a+bx^n}\right) \, dx = \begin{cases} -\frac{bnxx^n}{n+1} + \frac{nx\log\left(e^a e^{bx^n}\right)}{n+1} + \frac{x\log\left(e^a e^{bx^n}\right)}{n+1} & \text{for } n \neq -1 \\ b\log(x) + x\log\left(e^a e^{\frac{b}{x}}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(exp(a+b*x**n)),x)
[Out] Piecewise((-b*n*x*x**n/(n + 1) + n*x*log(exp(a)*exp(b*x**n))/(n + 1) + x*lo
g(exp(a)*exp(b*x**n))/(n + 1), Ne(n, -1)), (b*log(x) + x*log(exp(a)*exp(b/x
)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx^n}\right) \, dx = ax + \frac{bx^{n+1}}{n+1}$$

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="maxima")
[Out] a*x + b*x^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \log \left(e^{a+bx^n}\right) dx = ax + \frac{bx^{n+1}}{n+1}$$

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="giac")
[Out] a*x + b*x^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \log \left(e^{a+bx^n}\right) dx = \begin{cases} x \ln \left(\mathrm{e}^{a+\frac{b}{x}}\right) + b \ln (x) & \text{if} \ \ n = -1 \\ x \ln \left(\mathrm{e}^{a+b\,x^n}\right) - \frac{b\,n\,x^{n+1}}{n+1} & \text{if} \ \ n \neq -1 \end{cases}$$

```
[In] int(log(exp(a + b*x^n)),x)
[Out] piecewise(n == -1, x*log(exp(a + b/x)) + b*log(x), n ~= -1, x*log(exp(a + b
*x^n)) - (b*n*x^(n + 1))/(n + 1))
```

3.284 $\int e^x \log(a+be^x)\,dx$

Optimal result 1478
Rubi [A] (verified) 1478
Mathematica [A] (verified) 1480
Maple [A] (verified) 1480
Fricas [A] (verification not implemented) 1480
Sympy [F(-1)] 1481
Maxima [A] (verification not implemented) 1481
Giac [A] (verification not implemented) 1481
Mupad [B] (verification not implemented) 1481

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int e^x \log(a+be^x)\,dx = -e^x + \frac{(a+be^x)\log(a+be^x)}{b}$$

```
[Out] -exp(x)+(a+b*exp(x))*ln(a+b*exp(x))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2225, 2634, 12, 2280, 45}

$$\int e^x \log(a+be^x)\,dx = e^x\log(a+be^x) + \frac{a\log(a+be^x)}{b} - e^x$$

```
[In] Int[E^x*Log[a + b*E^x],x]
```

```
[Out] -E^x + (a*Log[a + b*E^x])/b + E^x*Log[a + b*E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= e^x \log(a+be^x) - \int \frac{be^{2x}}{a+be^x}\,dx \\
&= e^x \log(a+be^x) - b\int \frac{e^{2x}}{a+be^x}\,dx \\
&= e^x \log(a+be^x) - b\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, e^x\right) \\
&= e^x \log(a+be^x) - b\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)\,dx, x, e^x\right) \\
&= -e^x + \frac{a\log(a+be^x)}{b} + e^x \log(a+be^x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^x \log\left(a+be^x\right)\,dx=-e^x+\frac{\left(a+be^x\right)\log\left(a+be^x\right)}{b}$$

```
[In] Integrate[E^x*Log[a + b*E^x],x]
[Out] -E^x + ((a + b*E^x)*Log[a + b*E^x])/b
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{\left(a+b\,{\rm e}^{x}\right)\ln\left(a+b\,{\rm e}^{x}\right)-b\,{\rm e}^{x}-a}{b}$	28
default	$\frac{\left(a+b\,{\rm e}^{x}\right)\ln\left(a+b\,{\rm e}^{x}\right)-b\,{\rm e}^{x}-a}{b}$	28
norman	${\rm e}^{x}\ln\left(a+b\,{\rm e}^{x}\right)+\frac{a\ln\left(a+b\,{\rm e}^{x}\right)}{b}-{\rm e}^{x}$	28
risch	${\rm e}^{x}\ln\left(a+b\,{\rm e}^{x}\right)-{\rm e}^{x}+\frac{a\ln\left({\rm e}^{x}+\frac{a}{b}\right)}{b}$	30
parallelrisch	$\frac{\ln\left(a+b\,{\rm e}^{x}\right){\rm e}^{x}b-b\,{\rm e}^{x}+\ln\left(a+b\,{\rm e}^{x}\right)a+a}{b}$	32

```
[In] int(exp(x)*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
[Out] 1/b*((a+b*exp(x))*ln(a+b*exp(x))-b*exp(x)-a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^x \log\left(a+be^x\right)\,dx=-\frac{be^x-\left(be^x+a\right)\log\left(be^x+a\right)}{b}$$

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="fricas")
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int e^x \log(a + be^x) \, dx = \text{Timed out}$$

```
[In] integrate(exp(x)*ln(a+b*exp(x)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^x \log(a + be^x) \, dx = -\frac{be^x - (be^x + a) \log(be^x + a) + a}{b}$$

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="maxima")
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^x \log(a + be^x) \, dx = -\frac{be^x - (be^x + a) \log(be^x + a) + a}{b}$$

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="giac")
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int e^x \log(a + be^x) \, dx = \mathrm{e}^x \ln\left(a + b\,\mathrm{e}^x\right) - \mathrm{e}^x + \frac{a \ln\left(a + b\,\mathrm{e}^x\right)}{b}$$

```
[In] int(exp(x)*log(a + b*exp(x)),x)
[Out] exp(x)*log(a + b*exp(x)) - exp(x) + (a*log(a + b*exp(x)))/b
```

3.285 $\int e^{a+bx} \log(x)\,dx$

Optimal result 1482
Rubi [A] (verified) 1482
Mathematica [A] (verified) 1483
Maple [A] (verified) 1483
Fricas [A] (verification not implemented) 1484
Sympy [A] (verification not implemented) 1484
Maxima [A] (verification not implemented) 1484
Giac [A] (verification not implemented) 1485
Mupad [B] (verification not implemented) 1485

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^{a+bx} \log(x)\,dx = -\frac{e^a \operatorname{ExpIntegralEi}(bx)}{b} + \frac{e^{a+bx} \log(x)}{b}$$

```
[Out] -exp(a)*Ei(b*x)/b+exp(b*x+a)*ln(x)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2225, 2634, 12, 2209}

$$\int e^{a+bx} \log(x)\,dx = \frac{\log(x)e^{a+bx}}{b} - \frac{e^a \operatorname{ExpIntegralEi}(bx)}{b}$$

```
[In] Int[E^(a + b*x)*Log[x],x]
```

```
[Out] -((E^a*ExpIntegralEi[b*x])/b) + (E^(a + b*x)*Log[x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{a+bx}\log(x)}{b} - \int \frac{e^{a+bx}}{bx}\,dx\\ &= \frac{e^{a+bx}\log(x)}{b} - \frac{\int \frac{e^{a+bx}}{x}\,dx}{b}\\ &= -\frac{e^a\text{Ei}(bx)}{b} + \frac{e^{a+bx}\log(x)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int e^{a+bx}\log(x)\,dx = \frac{e^a(-\text{ExpIntegralEi}(bx) + e^{bx}\log(x))}{b}$$

```
[In] Integrate[E^(a + b*x)*Log[x],x]
[Out] (E^a*(-ExpIntegralEi[b*x] + E^(b*x)*Log[x]))/b
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{e^{bx+a}\ln(x)}{b} + \frac{e^{a}\,\text{Ei}_1(-bx)}{b}$	26

```
[In] int(exp(b*x+a)*ln(x),x,method=_RETURNVERBOSE)
[Out] exp(b*x+a)*ln(x)/b+1/b*exp(a)*Ei(1,-b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\log(x)\,dx = -\frac{\mathrm{Ei}(bx)\,e^{a} - e^{(bx+a)}\log{(x)}}{b}$$

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="fricas")
[Out] -(Ei(b*x)*e^a - e^(b*x + a)*log(x))/b
```

Sympy [A] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\log(x)\,dx = \left(\begin{cases} x & \text{for}\ b = 0 \\ \frac{e^{bx}}{b} & \text{otherwise} \end{cases}\right) e^{a}\log{(x)} - \left(\begin{cases} x & \text{for}\ b = 0 \\ \frac{\mathrm{Ei}\,(bx)}{b} & \text{otherwise} \end{cases}\right) e^{a}$$

```
[In] integrate(exp(b*x+a)*ln(x),x)
[Out] Piecewise((x, Eq(b, 0)), (exp(b*x)/b, True))*exp(a)*log(x) - Piecewise((x,
Eq(b, 0)), (Ei(b*x)/b, True))*exp(a)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\log(x)\,dx = -\frac{\mathrm{Ei}(bx)\,e^{a}}{b} + \frac{e^{(bx+a)}\log{(x)}}{b}$$

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="maxima")
[Out] -Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\log(x)\,dx = -\frac{\mathrm{Ei}(bx)\,e^{a}}{b} + \frac{e^{(bx+a)}\log(x)}{b}$$

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="giac")
[Out] -Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int e^{a+bx}\log(x)\,dx = -\frac{\mathrm{e}^{a}\left(\mathrm{ei}(b\,x)-\mathrm{e}^{b\,x}\,\ln(x)\right)}{b}$$

```
[In] int(exp(a + b*x)*log(x),x)
[Out] -(exp(a)*(ei(b*x) - exp(b*x)*log(x)))/b
```

3.286 $\int \frac{x^2}{x+\log(x)}\,dx$

Optimal result 1486
Rubi [N/A] 1486
Mathematica [N/A] 1487
Maple [N/A] 1487
Fricas [N/A] 1487
Sympy [N/A] 1487
Maxima [N/A] 1488
Giac [N/A] 1488
Mupad [N/A] 1488

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x^2}{x+\log(x)}\,dx = \text{Int}\left(\frac{x^2}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate(x^2/(x+ln(x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log(x)}\,dx$$

```
[In] Int[x^2/(x + Log[x]),x]
```

```
[Out] Defer[Int][x^2/(x + Log[x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{x+\log(x)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.59 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x + \log(x)} \, dx = \int \frac{x^2}{x + \log(x)} \, dx$$

```
[In] Integrate[x^2/(x + Log[x]),x]
[Out] Integrate[x^2/(x + Log[x]), x]
```

Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{x + \ln (x)} dx$$

```
[In] int(x^2/(x+ln(x)),x)
[Out] int(x^2/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x + \log(x)} \, dx = \int \frac{x^2}{x + \log (x)} \, dx$$

```
[In] integrate(x^2/(x+log(x)),x, algorithm="fricas")
[Out] integral(x^2/(x + log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{x + \log(x)} \, dx = \int \frac{x^2}{x + \log (x)} \, dx$$

```
[In] integrate(x**2/(x+ln(x)),x)
[Out] Integral(x**2/(x + log(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log{(x)}}\,dx$$

```
[In] integrate(x^2/(x+log(x)),x, algorithm="maxima")
[Out] integrate(x^2/(x + log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log{(x)}}\,dx$$

```
[In] integrate(x^2/(x+log(x)),x, algorithm="giac")
[Out] integrate(x^2/(x + log(x)), x)
```

Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\ln{(x)}}\,dx$$

```
[In] int(x^2/(x + log(x)),x)
[Out] int(x^2/(x + log(x)), x)
```

3.287 $\int \frac{x}{x+\log(x)}\,dx$

Optimal result 1489
Rubi [N/A] 1489
Mathematica [N/A] 1490
Maple [N/A] 1490
Fricas [N/A] 1490
Sympy [N/A] 1490
Maxima [N/A] 1491
Giac [N/A] 1491
Mupad [N/A] 1491

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{x}{x+\log(x)}\,dx = \text{Int}\left(\frac{x}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate(x/(x+ln(x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{x+\log(x)}\,dx = \int \frac{x}{x+\log(x)}\,dx$$

```
[In] Int[x/(x + Log[x]),x]
```

```
[Out] Defer[Int][x/(x + Log[x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{x+\log(x)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx=\int \frac{x}{x+\log(x)}\,dx$$

```
[In] Integrate[x/(x + Log[x]),x]
[Out] Integrate[x/(x + Log[x]), x]
```

Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{x+\ln\left(x\right)}dx$$

```
[In] int(x/(x+ln(x)),x)
[Out] int(x/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx=\int \frac{x}{x+\log\left(x\right)}\,dx$$

```
[In] integrate(x/(x+log(x)),x, algorithm="fricas")
[Out] integral(x/(x + log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{x}{x+\log(x)}\,dx=\int \frac{x}{x+\log\left(x\right)}\,dx$$

```
[In] integrate(x/(x+ln(x)),x)
[Out] Integral(x/(x + log(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx = \int \frac{x}{x+\log{(x)}}\,dx$$

```
[In] integrate(x/(x+log(x)),x, algorithm="maxima")
[Out] integrate(x/(x + log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx = \int \frac{x}{x+\log{(x)}}\,dx$$

```
[In] integrate(x/(x+log(x)),x, algorithm="giac")
[Out] integrate(x/(x + log(x)), x)
```

Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{x+\log(x)}\,dx = \int \frac{x}{x+\ln{(x)}}\,dx$$

```
[In] int(x/(x + log(x)),x)
[Out] int(x/(x + log(x)), x)
```

3.288 $\int \frac{1}{x+\log(x)}\,dx$

Optimal result 1492
Rubi [N/A] 1492
Mathematica [N/A] 1493
Maple [N/A] 1493
Fricas [N/A] 1493
Sympy [N/A] 1493
Maxima [N/A] 1494
Giac [N/A] 1494
Mupad [N/A] 1494

Optimal result

Integrand size = 6, antiderivative size = 6

$$\int \frac{1}{x+\log(x)}\,dx = \operatorname{Int}\left(\frac{1}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate(1/(x+ln(x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x+\log(x)}\,dx = \int \frac{1}{x+\log(x)}\,dx$$

```
[In] Int[(x + Log[x])^(-1),x]
```

```
[Out] Defer[Int][(x + Log[x])^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x+\log(x)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log(x)}\,dx$$

```
[In] Integrate[(x + Log[x])^(-1),x]
[Out] Integrate[(x + Log[x])^(-1), x]
```

Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+\ln\left(x\right)}dx$$

```
[In] int(1/(x+ln(x)),x)
[Out] int(1/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log\left(x\right)}\,dx$$

```
[In] integrate(1/(x+log(x)),x, algorithm="fricas")
[Out] integral(1/(x + log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log\left(x\right)}\,dx$$

```
[In] integrate(1/(x+ln(x)),x)
[Out] Integral(1/(x + log(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx = \int \frac{1}{x+\log(x)}\,dx$$

```
[In] integrate(1/(x+log(x)),x, algorithm="maxima")
[Out] integrate(1/(x + log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx = \int \frac{1}{x+\log(x)}\,dx$$

```
[In] integrate(1/(x+log(x)),x, algorithm="giac")
[Out] integrate(1/(x + log(x)), x)
```

Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{x+\log(x)}\,dx = \int \frac{1}{x+\ln(x)}\,dx$$

```
[In] int(1/(x + log(x)),x)
[Out] int(1/(x + log(x)), x)
```

3.289 $\int \frac{1}{x(x+\log(x))} dx$

Optimal result 1495
Rubi [N/A] 1495
Mathematica [N/A] 1496
Maple [N/A] 1496
Fricas [N/A] 1496
Sympy [N/A] 1496
Maxima [N/A] 1497
Giac [N/A] 1497
Mupad [N/A] 1497

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x(x+\log(x))} dx = \operatorname{Int}\left(\frac{1}{x(x+\log(x))}, x\right)$$

```
[Out] CannotIntegrate(1/x/(x+ln(x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(x+\log(x))} dx = \int \frac{1}{x(x+\log(x))} dx$$

```
[In] Int[1/(x*(x + Log[x])),x]
```

```
[Out] Defer[Int][1/(x*(x + Log[x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(x+\log(x))} dx$$

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx=\int \frac{1}{x(x+\log(x))}\,dx$$

```
[In] Integrate[1/(x*(x + Log[x])),x]
[Out] Integrate[1/(x*(x + Log[x])), x]
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(x+\ln\,(x))}dx$$

```
[In] int(1/x/(x+ln(x)),x)
[Out] int(1/x/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx=\int \frac{1}{(x+\log\,(x))x}\,dx$$

```
[In] integrate(1/x/(x+log(x)),x, algorithm="fricas")
[Out] integral(1/(x^2 + x*log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(x+\log(x))}\,dx=\int \frac{1}{x\,(x+\log\,(x))}\,dx$$

```
[In] integrate(1/x/(x+ln(x)),x)
[Out] Integral(1/(x*(x + log(x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx = \int \frac{1}{(x+\log(x))x}\,dx$$

```
[In] integrate(1/x/(x+log(x)),x, algorithm="maxima")
[Out] integrate(1/((x + log(x))*x), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx = \int \frac{1}{(x+\log(x))x}\,dx$$

```
[In] integrate(1/x/(x+log(x)),x, algorithm="giac")
[Out] integrate(1/((x + log(x))*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(x+\log(x))}\,dx = \int \frac{1}{x\,(x+\ln(x))}\,dx$$

```
[In] int(1/(x*(x + log(x))),x)
[Out] int(1/(x*(x + log(x))), x)
```

3.290 $\int \frac{1}{x^2(x+\log(x))} dx$

Optimal result . . . 1498
Rubi [N/A] . . . 1498
Mathematica [N/A] . . . 1499
Maple [N/A] . . . 1499
Fricas [N/A] . . . 1499
Sympy [N/A] . . . 1499
Maxima [N/A] . . . 1500
Giac [N/A] . . . 1500
Mupad [N/A] . . . 1500

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2(x+\log(x))} dx = \text{Int}\left(\frac{1}{x^2(x+\log(x))}, x\right)$$

```
[Out] CannotIntegrate(1/x^2/(x+ln(x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2(x+\log(x))} dx = \int \frac{1}{x^2(x+\log(x))} dx$$

```
[In] Int[1/(x^2*(x + Log[x])),x]
[Out] Defer[Int][1/(x^2*(x + Log[x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2(x+\log(x))} dx$$

Mathematica [N/A]

Not integrable

Time = 17.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(x+\log(x))}\,dx=\int \frac{1}{x^2(x+\log(x))}\,dx$$

```
[In] Integrate[1/(x^2*(x + Log[x])),x]
[Out] Integrate[1/(x^2*(x + Log[x])), x]
```

Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(x+\ln \left(x\right)\right)}dx$$

```
[In] int(1/x^2/(x+ln(x)),x)
[Out] int(1/x^2/(x+ln(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^2(x+\log(x))}\,dx=\int \frac{1}{(x+\log\left(x\right))x^{2}}\,dx$$

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="fricas")
[Out] integral(1/(x^3 + x^2*log(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(x+\log(x))}\,dx=\int \frac{1}{x^{2} \left(x+\log{\left(x \right)}\right)}\,dx$$

```
[In] integrate(1/x**2/(x+ln(x)),x)
[Out] Integral(1/(x**2*(x + log(x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(x+\log(x))}\,dx = \int \frac{1}{(x+\log{(x)})x^2}\,dx$$

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="maxima")
[Out] integrate(1/((x + log(x))*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(x+\log(x))}\,dx = \int \frac{1}{(x+\log{(x)})x^2}\,dx$$

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="giac")
[Out] integrate(1/((x + log(x))*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(x+\log(x))}\,dx = \int \frac{1}{x^2\ (x+\ln{(x)})}\,dx$$

```
[In] int(1/(x^2*(x + log(x))),x)
[Out] int(1/(x^2*(x + log(x))), x)
```

3.291 $\int \frac{\log(x)}{x+4x\log^2(x)}\,dx$

Optimal result 1501
Rubi [A] (verified) 1501
Mathematica [A] (verified) 1502
Maple [A] (verified) 1502
Fricas [A] (verification not implemented) 1503
Sympy [A] (verification not implemented) 1503
Maxima [A] (verification not implemented) 1503
Giac [A] (verification not implemented) 1503
Mupad [B] (verification not implemented) 1504

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{1}{8}\log\left(1+4\log^2(x)\right)$$

```
[Out] 1/8*ln(1+4*ln(x)^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {209, 266}

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{1}{8}\log\left(4\log^2(x)+1\right)$$

```
[In] Int[Log[x]/(x + 4*x*Log[x]^2),x]
```

```
[Out] Log[1 + 4*Log[x]^2]/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x}{1+4x^2}\,dx, x, \log(x)\right)$$
$$= \frac{1}{8}\log\left(1+4\log^2(x)\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{1}{8}\log\left(1+4\log^2(x)\right)$$

```
[In] Integrate[Log[x]/(x + 4*x*Log[x]^2),x]
[Out] Log[1 + 4*Log[x]^2]/8
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{\ln\left(\ln(x)^2+\frac{1}{4}\right)}{8}$	10
parallelrisch	$\frac{\ln\left(\ln(x)^2+\frac{1}{4}\right)}{8}$	10
default	$\frac{\ln\left(1+4\ln(x)^2\right)}{8}$	12
norman	$\frac{\ln\left(1+4\ln(x)^2\right)}{8}$	12

```
[In] int(ln(x)/(x+4*x*ln(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(ln(x)^2+1/4)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{1}{8}\log\left(4\log\left(x\right)^2+1\right)$$

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="fricas")
[Out] 1/8*log(4*log(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{\log\left(\log\left(x\right)^2+\frac{1}{4}\right)}{8}$$

```
[In] integrate(ln(x)/(x+4*x*ln(x)**2),x)
[Out] log(log(x)**2 + 1/4)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{1}{8}\log\left(\log\left(x\right)^2+\frac{1}{4}\right)$$

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="maxima")
[Out] 1/8*log(log(x)^2 + 1/4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\log(x)}{x+4x\log^2(x)}\,dx = \frac{1}{8}\log\left(4\log\left(x\right)^2+1\right)$$

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="giac")
[Out] 1/8*log(4*log(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\log(x)}{x + 4x\log^2(x)}\,dx = \frac{\ln\left(4\ln\left(x\right)^2 + 1\right)}{8}$$

```
[In] int(log(x)/(x + 4*x*log(x)^2),x)
[Out] log(4*log(x)^2 + 1)/8
```

3.292 $\int \frac{1-\log(x)}{x(x+\log(x))} \, dx$

Optimal result 1505
Rubi [A] (verified) 1505
Mathematica [A] (verified) 1506
Maple [A] (verified) 1506
Fricas [A] (verification not implemented) 1507
Sympy [A] (verification not implemented) 1507
Maxima [A] (verification not implemented) 1507
Giac [A] (verification not implemented) 1507
Mupad [B] (verification not implemented) 1508

Optimal result

Integrand size = 16, antiderivative size = 9

$$\int \frac{1-\log(x)}{x(x+\log(x))} \, dx = \log\left(1+\frac{\log(x)}{x}\right)$$

```
[Out] ln(1+ln(x)/x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6844, 31}

$$\int \frac{1-\log(x)}{x(x+\log(x))} \, dx = \log\left(\frac{\log(x)}{x}+1\right)$$

```
[In] Int[(1 - Log[x])/(x*(x + Log[x])),x]
```

```
[Out] Log[1 + Log[x]/x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6844

```
Int[(u_)*(v_)^(r_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol]
:> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Dist[(-c)*q, Subst[I
nt[(a + b*x^q)^m, x], x, v^(m*p + r + 1)*w], x] /; FreeQ[c, x]] /; FreeQ[{a
, b, m, p, q, r}, x] && EqQ[p + q*(m*p + r + 1), 0] && IntegerQ[q] && Integ
```

```
erQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\log(x)}{x}\right)\\ &= \log\left(1+\frac{\log(x)}{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = -\log(x)+\log(x+\log(x))$$

```
[In] Integrate[(1 - Log[x])/(x*(x + Log[x])),x]
[Out] -Log[x] + Log[x + Log[x]]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

method	result	size
default	$-\ln(x)+\ln(x+\ln(x))$	11
norman	$-\ln(x)+\ln(x+\ln(x))$	11
risch	$-\ln(x)+\ln(x+\ln(x))$	11
parallelrisch	$-\ln(x)+\ln(x+\ln(x))$	11

```
[In] int((1-ln(x))/x/(x+ln(x)),x,method=_RETURNVERBOSE)
[Out] -ln(x)+ln(x+ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = \log\left(x+\log\left(x\right)\right) - \log\left(x\right)$$

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="fricas")
[Out] log(x + log(x)) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = -\log\left(x\right) + \log\left(x+\log\left(x\right)\right)$$

```
[In] integrate((1-ln(x))/x/(x+ln(x)),x)
[Out] -log(x) + log(x + log(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = \log\left(x+\log\left(x\right)\right) - \log\left(x\right)$$

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="maxima")
[Out] log(x + log(x)) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = -\log\left(x\right) + \log\left(-x-\log\left(x\right)\right)$$

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="giac")
[Out] -log(x) + log(-x - log(x))
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = \ln\left(x+\ln\left(x\right)\right)-\ln\left(x\right)$$

```
[In] int(-(log(x) - 1)/(x*(x + log(x))),x)
[Out] log(x + log(x)) - log(x)
```

3.293 $\int \frac{1+x}{\log(x)(x+\log(x))}\,dx$

Optimal result . . . 1509
Rubi [A] (verified) . . . 1509
Mathematica [A] (verified) . . . 1511
Maple [A] (verified) . . . 1511
Fricas [A] (verification not implemented) . . . 1511
Sympy [A] (verification not implemented) . . . 1512
Maxima [F] . . . 1512
Giac [F] . . . 1512
Mupad [B] (verification not implemented) . . . 1512

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = \log(\log(x)) - \log(x+\log(x)) + \text{LogIntegral}(x)$$

```
[Out] Li(x)+ln(ln(x))-ln(x+ln(x))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6874, 2395, 2335, 2339, 29, 6816}

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = \text{LogIntegral}(x) + \log(\log(x)) - \log(x+\log(x))$$

```
[In] Int[(1 + x)/(Log[x]*(x + Log[x])),x]
```

```
[Out] Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1+x}{x\log(x)}+\frac{-1-x}{x(x+\log(x))}\right)\,dx\\
&= \int \frac{1+x}{x\log(x)}\,dx+\int \frac{-1-x}{x(x+\log(x))}\,dx\\
&= -\log(x+\log(x))+\int \left(\frac{1}{\log(x)}+\frac{1}{x\log(x)}\right)\,dx\\
&= -\log(x+\log(x))+\int \frac{1}{\log(x)}\,dx+\int \frac{1}{x\log(x)}\,dx\\
&= -\log(x+\log(x))+\mathrm{li}(x)+\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,\log(x)\right)\\
&= \log(\log(x))-\log(x+\log(x))+\mathrm{li}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = \log(\log(x)) - \log(x+\log(x)) + \text{LogIntegral}(x)$$

```
[In] Integrate[(1 + x)/(Log[x]*(x + Log[x])),x]
[Out] Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

method	result	size
default	$-\operatorname{Ei}_1(-\ln(x)) + \ln(\ln(x)) - \ln(x+\ln(x))$	20
risch	$-\operatorname{Ei}_1(-\ln(x)) + \ln(\ln(x)) - \ln(x+\ln(x))$	20

```
[In] int((x+1)/ln(x)/(x+ln(x)),x,method=_RETURNVERBOSE)
[Out] -Ei(1,-ln(x))+ln(ln(x))-ln(x+ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = -\log(x+\log(x)) + \log(\log(x)) + \text{log\_integral}(x)$$

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="fricas")
[Out] -log(x + log(x)) + log(log(x)) + log_integral(x)
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = -\log\left(x+\log\left(x\right)\right)+\log\left(\log\left(x\right)\right)+\mathrm{Ei}\left(\log\left(x\right)\right)$$

```
[In] integrate((1+x)/ln(x)/(x+ln(x)),x)
[Out] -log(x + log(x)) + log(log(x)) + Ei(log(x))
```

Maxima [F]

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = \int \frac{x+1}{\left(x+\log\left(x\right)\right)\log\left(x\right)}\,dx$$

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="maxima")
[Out] integrate((x + 1)/(x*log(x)), x) - log(x + log(x))
```

Giac [F]

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = \int \frac{x+1}{\left(x+\log\left(x\right)\right)\log\left(x\right)}\,dx$$

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="giac")
[Out] integrate((x + 1)/((x + log(x))*log(x)), x)
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{\log(x)(x+\log(x))}\,dx = \ln\left(\ln\left(x\right)\right)-\ln\left(x+\ln\left(x\right)\right)+\mathrm{logint}(x)$$

```
[In] int((x + 1)/(log(x)*(x + log(x))),x)
[Out] log(log(x)) - log(x + log(x)) + logint(x)
```

3.294 $\int \log\left(2+\sqrt{\frac{1+x}{x}}\right) dx$

Optimal result . . . 1513
Rubi [A] (verified) . . . 1513
Mathematica [A] (verified) . . . 1514
Maple [A] (verified) . . . 1515
Fricas [A] (verification not implemented) . . . 1515
Sympy [A] (verification not implemented) . . . 1516
Maxima [A] (verification not implemented) . . . 1516
Giac [A] (verification not implemented) . . . 1516
Mupad [B] (verification not implemented) . . . 1517

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \log\left(2+\sqrt{\frac{1+x}{x}}\right) dx = -\frac{1}{6}\log\left(1-\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\log\left(1+\sqrt{1+\frac{1}{x}}\right) -\frac{1}{3}\log\left(2+\sqrt{1+\frac{1}{x}}\right)+x\log\left(2+\sqrt{\frac{1+x}{x}}\right)$$

```
[Out] -1/6*ln(1-(1+1/x)^(1/2))+1/2*ln(1+(1+1/x)^(1/2))-1/3*ln(2+(1+1/x)^(1/2))+x*
ln(2+((1+x)/x)^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2628, 12, 2083}

$$\int \log\left(2+\sqrt{\frac{1+x}{x}}\right) dx = -\frac{1}{6}\log\left(1-\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}+1\right) -\frac{1}{3}\log\left(\sqrt{\frac{1}{x}+1}+2\right)+x\log\left(\sqrt{\frac{x+1}{x}}+2\right)$$

```
[In] Int[Log[2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -1/6*Log[1 - Sqrt[1 + x^(-1)]] + Log[1 + Sqrt[1 + x^(-1)]]/2 - Log[2 + Sqrt
[1 + x^(-1)]]/3 + x*Log[2 + Sqrt[(1 + x)/x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(2+\sqrt{\frac{1+x}{x}}\right)-\int\frac{1}{2\left(-1-x-2x\sqrt{\frac{1+x}{x}}\right)}\,dx\\
&= x\log\left(2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{2}\int\frac{1}{-1-x-2x\sqrt{\frac{1+x}{x}}}\,dx\\
&= x\log\left(2+\sqrt{\frac{1+x}{x}}\right)+\text{Subst}\left(\int\frac{1}{2+x-2x^2-x^3}\,dx,x,\sqrt{\frac{1+x}{x}}\right)\\
&= x\log\left(2+\sqrt{\frac{1+x}{x}}\right)+\text{Subst}\left(\int\left(-\frac{1}{6(-1+x)}+\frac{1}{2(1+x)}-\frac{1}{3(2+x)}\right)\,dx,x,\sqrt{\frac{1+x}{x}}\right)\\
&= -\frac{1}{6}\log\left(1-\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\log\left(1+\sqrt{1+\frac{1}{x}}\right)\\
&\quad-\frac{1}{3}\log\left(2+\sqrt{1+\frac{1}{x}}\right)+x\log\left(2+\sqrt{\frac{1+x}{x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int\log\left(2+\sqrt{\frac{1+x}{x}}\right)\,dx=\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\left(1+2\sqrt{1+\frac{1}{x}}\right)\right)-\text{arctanh}\left(3+2\sqrt{1+\frac{1}{x}}\right)+x\log\left(2+\sqrt{1+\frac{1}{x}}\right)$$

```
[In] Integrate[Log[2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] ArcTanh[(1 + 2*Sqrt[1 + x^(-1)])/3]/3 - ArcTanh[3 + 2*Sqrt[1 + x^(-1)]] + x
*Log[2 + Sqrt[1 + x^(-1)]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

method	result	size
default	$x \ln\left(2+\sqrt{\frac{x+1}{x}}\right)-\frac{\sqrt{9}\, \ln\left(\frac{4 \sqrt{9}\, \sqrt{x^{2}+x}+15 x+3}{9 x-3}\right) \sqrt{x (x+1)}+3 \sqrt{\frac{x+1}{x}}\, x \ln(-3 x+1)-6 \ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right) \sqrt{x (x+1)}}{18 \sqrt{\frac{x+1}{x}}\, x}$	107
parts	$x \ln\left(2+\sqrt{\frac{x+1}{x}}\right)-\frac{\sqrt{9}\, \ln\left(\frac{4 \sqrt{9}\, \sqrt{x^{2}+x}+15 x+3}{9 x-3}\right) \sqrt{x (x+1)}+3 \sqrt{\frac{x+1}{x}}\, x \ln(-3 x+1)-6 \ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right) \sqrt{x (x+1)}}{18 \sqrt{\frac{x+1}{x}}\, x}$	107

```
[In] int(ln(2+((x+1)/x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(2+((x+1)/x)^(1/2))-1/18/((x+1)/x)^(1/2)/x*(9^(1/2)*ln(1/3*(4*9^(1/2)*(
x^2+x)^(1/2)+15*x+3)/(3*x-1))*(x*(x+1))^(1/2)+3*((x+1)/x)^(1/2)*x*ln(-3*x+1
)-6*ln(1/2+x+(x^2+x)^(1/2))*(x*(x+1))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \log\left(2+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{1}{3}\,(3\,x-1)\log\left(\sqrt{\frac{x+1}{x}}+2\right) + \frac{1}{2}\,\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{6}\,\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x - 1)*log(sqrt((x + 1)/x) + 2) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/6
*log(sqrt((x + 1)/x) - 1)
```

Sympy [A] (verification not implemented)

Time = 45.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) dx = x\log \left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{\log \left(\sqrt{1+\frac{1}{x}}-1\right)}{6}+\frac{\log \left(\sqrt{1+\frac{1}{x}}+1\right)}{2}-\frac{\log \left(\sqrt{1+\frac{1}{x}}+2\right)}{3}$$

```
[In] integrate(ln(2+((1+x)/x)**(1/2)),x)
[Out] x*log(sqrt((x + 1)/x) + 2) - log(sqrt(1 + 1/x) - 1)/6 + log(sqrt(1 + 1/x) +
 1)/2 - log(sqrt(1 + 1/x) + 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) dx = \frac{\log \left(\sqrt{\frac{x+1}{x}}+2\right)}{\frac{x+1}{x}-1}-\frac{1}{3}\log \left(\sqrt{\frac{x+1}{x}}+2\right)+\frac{1}{2}\log \left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{6}\log \left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt((x + 1)/x) + 2)/((x + 1)/x - 1) - 1/3*log(sqrt((x + 1)/x) + 2) + 1
/2*log(sqrt((x + 1)/x) + 1) - 1/6*log(sqrt((x + 1)/x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) dx = x\log \left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{\log \left(\left|-x+\sqrt{x^2+x}+1\right|\right)}{6\,\mathrm{sgn}\left(x\right)}-\frac{\log \left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)}{3\,\mathrm{sgn}\left(x\right)}+\frac{\log \left(\left|-3\,x+3\,\sqrt{x^2+x}-1\right|\right)}{6\,\mathrm{sgn}\left(x\right)}-\frac{1}{6}\log \left(\left|3\,x-1\right|\right)$$

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) + 2) - 1/6*log(abs(-x + sqrt(x^2 + x) + 1))/sgn(x) -
1/3*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))/sgn(x) + 1/6*log(abs(-3*x + 3*sqrt
(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(3*x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \log \left(2+\sqrt{\frac{1+x}{x}}\right) dx = \frac{\ln \left(-5\sqrt{\frac{x+1}{x}}-5\right)}{2} - \frac{\ln \left(\frac{\sqrt{\frac{x+1}{x}}}{9}-\frac{1}{9}\right)}{6} - \frac{\ln \left(-\frac{5\sqrt{\frac{x+1}{x}}}{9}-\frac{10}{9}\right)}{3} + x\ln \left(\sqrt{\frac{x+1}{x}}+2\right)$$

```
[In] int(log(((x + 1)/x)^(1/2) + 2),x)
```

```
[Out] log(- 5*((x + 1)/x)^(1/2) - 5)/2 - log(((x + 1)/x)^(1/2)/9 - 1/9)/6 - log(-
 (5*((x + 1)/x)^(1/2))/9 - 10/9)/3 + x*log(((x + 1)/x)^(1/2) + 2)
```

3.295 $\int \log \left(1+\sqrt{\frac{1+x}{x}}\right) dx$

Optimal result 1518
Rubi [A] (verified) 1518
Mathematica [A] (verified) 1520
Maple [A] (verified) 1520
Fricas [A] (verification not implemented) 1520
Sympy [A] (verification not implemented) 1521
Maxima [A] (verification not implemented) 1521
Giac [A] (verification not implemented) 1521
Mupad [B] (verification not implemented) 1522

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \log \left(1+\sqrt{\frac{1+x}{x}}\right) dx=-\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)}+\frac{1}{2}\text{arctanh}\left(\sqrt{\frac{1+x}{x}}\right)+x\log \left(1+\sqrt{\frac{1+x}{x}}\right)$$

```
[Out] 1/2*arctanh(((1+x)/x)^(1/2))+x*ln(1+((1+x)/x)^(1/2))-1/2/(1+(1+1/x)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2628, 12, 46, 213}

$$\int \log \left(1+\sqrt{\frac{1+x}{x}}\right) dx=\frac{1}{2}\text{arctanh}\left(\sqrt{\frac{x+1}{x}}\right)-\frac{1}{2\left(\sqrt{\frac{1}{x}+1}+1\right)}+x\log \left(\sqrt{\frac{x+1}{x}}+1\right)$$

```
[In] Int[Log[1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -1/2*1/(1 + Sqrt[1 + x^(-1)]) + ArcTanh[Sqrt[(1 + x)/x]]/2 + x*Log[1 + Sqrt
[(1 + x)/x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\int\frac{1}{2\left(-1-x-x\sqrt{\frac{1+x}{x}}\right)}\,dx\\
&= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{2}\int\frac{1}{-1-x-x\sqrt{\frac{1+x}{x}}}\,dx\\
&= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int\frac{1}{(-1+x)(1+x)^2}\,dx,x,\sqrt{\frac{1+x}{x}}\right)\\
&= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int\left(-\frac{1}{2(1+x)^2}+\frac{1}{2\left(-1+x^2\right)}\right)\,dx,x,\sqrt{\frac{1+x}{x}}\right)\\
&= -\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)}+x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\sqrt{\frac{1+x}{x}}\right)\\
&= -\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)}+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1+x}{x}}\right)+x\log\left(1+\sqrt{\frac{1+x}{x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{1}{4}\left(2x-2\sqrt{1+\frac{1}{x}}x+4x\log\left(1+\sqrt{1+\frac{1}{x}}\right)+\log\left(1+\left(2+2\sqrt{1+\frac{1}{x}}\right)x\right)\right)$$

```
[In] Integrate[Log[1 + Sqrt[(1 + x)/x]],x]
[Out] (2*x - 2*Sqrt[1 + x^(-1)]*x + 4*x*Log[1 + Sqrt[1 + x^(-1)]] + Log[1 + (2 + 2*Sqrt[1 + x^(-1)])*x])/4
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.60

method	result	size
default	$x\ln\left(1+\sqrt{\frac{x+1}{x}}\right)+\frac{2\sqrt{\frac{x+1}{x}}\,x^{2}+\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)\sqrt{x(x+1)}-2\sqrt{x(x+1)}\,\sqrt{x^{2}+x}}{4\sqrt{\frac{x+1}{x}}\,x}$	80
parts	$x\ln\left(1+\sqrt{\frac{x+1}{x}}\right)+\frac{2\sqrt{\frac{x+1}{x}}\,x^{2}+\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)\sqrt{x(x+1)}-2\sqrt{x(x+1)}\,\sqrt{x^{2}+x}}{4\sqrt{\frac{x+1}{x}}\,x}$	80

```
[In] int(ln(1+((x+1)/x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*ln(1+((x+1)/x)^(1/2))+1/4*(2*((x+1)/x)^(1/2)*x^2+ln(1/2+x+(x^2+x)^(1/2))*(x*(x+1))^(1/2)-2*(x*(x+1))^(1/2)*(x^2+x)^(1/2))/((x+1)/x)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{1}{4}\,(4\,x+1)\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\,x\sqrt{\frac{x+1}{x}}+\frac{1}{2}\,x-\frac{1}{4}\,\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="fricas")
[Out] 1/4*(4*x + 1)*log(sqrt((x + 1)/x) + 1) - 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1/4*log(sqrt((x + 1)/x) - 1)
```

Sympy [A] (verification not implemented)

Time = 47.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log \left(1+\sqrt{\frac{1+x}{x}}\right) dx = x\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{4} + \frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{4} - \frac{1}{2\left(\sqrt{1+\frac{1}{x}}+1\right)}$$

```
[In] integrate(ln(1+((1+x)/x)**(1/2)),x)
[Out] x*log(sqrt((x + 1)/x) + 1) - log(sqrt(1 + 1/x) - 1)/4 + log(sqrt(1 + 1/x) +
 1)/4 - 1/(2*(sqrt(1 + 1/x) + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \log \left(1+\sqrt{\frac{1+x}{x}}\right) dx = \frac{\log\left(\sqrt{\frac{x+1}{x}}+1\right)}{\frac{x+1}{x}-1} - \frac{1}{2\left(\sqrt{\frac{x+1}{x}}+1\right)} + \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt((x + 1)/x) + 1)/((x + 1)/x - 1) - 1/2/(sqrt((x + 1)/x) + 1) + 1/4*
log(sqrt((x + 1)/x) + 1) - 1/4*log(sqrt((x + 1)/x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log \left(1+\sqrt{\frac{1+x}{x}}\right) dx = x\log\left(\sqrt{\frac{x+1}{x}}+1\right) + \frac{1}{2}x - \frac{\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)}{4\,\mathrm{sgn}\left(x\right)} - \frac{\sqrt{x^2+x}}{2\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="giac")
[Out] x*log(sqrt((x + 1)/x) + 1) + 1/2*x - 1/4*log(abs(-2*x + 2*sqrt(x^2 + x) - 1
))/sgn(x) - 1/2*sqrt(x^2 + x)/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{x}{2}+\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right)}{2}+x\,\ln\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{x\,\sqrt{\frac{1}{x}+1}}{2}$$

```
[In] int(log(((x + 1)/x)^(1/2) + 1),x)
```

```
[Out] x/2 + atanh((1/x + 1)^(1/2))/2 + x*log(((x + 1)/x)^(1/2) + 1) - (x*(1/x + 1
)^(1/2))/2
```

3.296 $\int \log \left(\sqrt{\frac{1+x}{x}}\right) \, dx$

Optimal result 1523
Rubi [A] (verified) 1523
Mathematica [A] (verified) 1524
Maple [A] (verified) 1525
Fricas [A] (verification not implemented) 1525
Sympy [A] (verification not implemented) 1525
Maxima [A] (verification not implemented) 1526
Giac [B] (verification not implemented) 1526
Mupad [B] (verification not implemented) 1526

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \log \left(\sqrt{\frac{1+x}{x}}\right) \, dx = x \log \left(\sqrt{1+\frac{1}{x}}\right) + \frac{1}{2} \log(1+x)$$

```
[Out] 1/2*ln(1+x)+1/2*x*ln(1+1/x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2503, 2498, 269, 31}

$$\int \log \left(\sqrt{\frac{1+x}{x}}\right) \, dx = x \log \left(\sqrt{\frac{1}{x}+1}\right) + \frac{1}{2} \log(x+1)$$

```
[In] Int[Log[Sqrt[(1 + x)/x]],x]
```

```
[Out] x*Log[Sqrt[1 + x^(-1)]] + Log[1 + x]/2
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 269</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2503

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \log\left(\sqrt{1+\frac{1}{x}}\right) dx \\
&= x\log\left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\int \frac{1}{\left(1+\frac{1}{x}\right)x} dx \\
&= x\log\left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\int \frac{1}{1+x} dx \\
&= x\log\left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\log(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right) dx = \frac{1}{2}\left(x\log\left(1+\frac{1}{x}\right)+\log(1+x)\right)$$

```
[In] Integrate[Log[Sqrt[(1 + x)/x]],x]
[Out] (x*Log[1 + x^(-1)] + Log[1 + x])/2
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{x\ln\left(\frac{x+1}{x}\right)}{2}+\frac{\ln(x+1)}{2}$	19
parts	$\frac{x\ln\left(\frac{x+1}{x}\right)}{2}+\frac{\ln(x+1)}{2}$	19
derivativedivides	$-\frac{\ln\left(\frac{1}{x}\right)}{2}+\frac{\ln\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)x}{2}$	22
default	$-\frac{\ln\left(\frac{1}{x}\right)}{2}+\frac{\ln\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)x}{2}$	22
parallelrisch	$\frac{x\ln\left(\frac{x+1}{x}\right)}{2}+\frac{\ln(x)}{2}+\frac{\ln\left(\frac{x+1}{x}\right)}{2}$	27

```
[In] int(1/2*ln((x+1)/x),x,method=_RETURNVERBOSE)
[Out] 1/2*x*ln((x+1)/x)+1/2*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right)\,dx=\frac{1}{2}\,x\log\left(\frac{x+1}{x}\right)+\frac{1}{2}\,\log\left(x+1\right)$$

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="fricas")
[Out] 1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right)\,dx=\frac{x\log\left(\frac{x+1}{x}\right)}{2}+\frac{\log\left(2x+2\right)}{2}$$

```
[In] integrate(1/2*ln((1+x)/x),x)
[Out] x*log((x + 1)/x)/2 + log(2*x + 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{1}{2}\,x\log\left(\frac{x+1}{x}\right) + \frac{1}{2}\,\log\left(x+1\right)$$

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="maxima")
[Out] 1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(16) = 32$.

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.24

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{\log\left(\frac{x+1}{x}\right)}{2\left(\frac{x+1}{x}-1\right)} + \frac{1}{2}\,\log\left(\frac{|x+1|}{|x|}\right) - \frac{1}{2}\,\log\left(\left|\frac{x+1}{x}-1\right|\right)$$

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="giac")
[Out] 1/2*log((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(abs(x + 1)/abs(x)) - 1/2*log(a
bs((x + 1)/x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{\ln\left(x+1\right)}{2} + \frac{x\,\ln\left(\frac{x+1}{x}\right)}{2}$$

```
[In] int(log((x + 1)/x)/2,x)
[Out] log(x + 1)/2 + (x*log((x + 1)/x))/2
```

3.297 $\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) dx$

Optimal result 1527
Rubi [A] (verified) 1527
Mathematica [A] (verified) 1528
Maple [A] (verified) 1529
Fricas [A] (verification not implemented) 1529
Sympy [A] (verification not implemented) 1530
Maxima [A] (verification not implemented) 1530
Giac [A] (verification not implemented) 1530
Mupad [B] (verification not implemented) 1531

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) dx = -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)} - \frac{1}{2}\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\right) + x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)$$

```
[Out] -1/2*arctanh((1+1/x)^(1/2))+x*ln(-1+((1+x)/x)^(1/2))-1/2/(1-(1+1/x)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2628, 46, 213}

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) dx = -\frac{1}{2}\text{arctanh}\left(\sqrt{\frac{1}{x}+1}\right) - \frac{1}{2\left(1-\sqrt{\frac{1}{x}+1}\right)} + x\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] Int[Log[-1 + Sqrt[(1 + x)/x]],x]
[Out] -1/2*1/(1 - Sqrt[1 + x^(-1)]) - ArcTanh[Sqrt[1 + x^(-1)]]/2 + x*Log[-1 + Sq
rt[(1 + x)/x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)-\int\frac{1}{-2+\left(-2+2\sqrt{1+\frac{1}{x}}\right)x}\,dx\\
&= x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int\frac{1}{(-1+x)^2(1+x)}\,dx,x,\sqrt{1+\frac{1}{x}}\right)\\
&= x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int\left(\frac{1}{2(-1+x)^2}-\frac{1}{2\left(-1+x^2\right)}\right)\,dx,x,\sqrt{1+\frac{1}{x}}\right)\\
&= -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)}+x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\sqrt{1+\frac{1}{x}}\right)\\
&= -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)}-\frac{1}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)+x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int\log\left(-1+\sqrt{\frac{1+x}{x}}\right)\,dx=\frac{1}{2}\left(1+\sqrt{1+\frac{1}{x}}\right)x+x\log\left(-1+\sqrt{1+\frac{1}{x}}\right)-\frac{1}{4}\log\left(1+\left(2+2\sqrt{1+\frac{1}{x}}\right)x\right)$$

```
[In] Integrate[Log[-1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] ((1 + Sqrt[1 + x^(-1)])*x)/2 + x*Log[-1 + Sqrt[1 + x^(-1)]] - Log[1 + (2 +
2*Sqrt[1 + x^(-1)])*x]/4
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.60

method	result	size
default	$x \ln\left(-1+\sqrt{\frac{x+1}{x}}\right)-\frac{-2\sqrt{\frac{x+1}{x}}\, x^{2}+\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)\sqrt{x(x+1)}-2\sqrt{x(x+1)}\,\sqrt{x^{2}+x}}{4\sqrt{\frac{x+1}{x}}\, x}$	80
parts	$x \ln\left(-1+\sqrt{\frac{x+1}{x}}\right)-\frac{-2\sqrt{\frac{x+1}{x}}\, x^{2}+\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)\sqrt{x(x+1)}-2\sqrt{x(x+1)}\,\sqrt{x^{2}+x}}{4\sqrt{\frac{x+1}{x}}\, x}$	80

```
[In] int(ln(-1+((x+1)/x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(-1+((x+1)/x)^(1/2))-1/4*(-2*((x+1)/x)^(1/2)*x^2+ln(1/2+x+(x^2+x)^(1/2)
)*(x*(x+1))^(1/2)-2*(x*(x+1))^(1/2)*(x^2+x)^(1/2))/((x+1)/x)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)\, dx = \frac{1}{4}\,(4\,x+1)\log\left(\sqrt{\frac{x+1}{x}}-1\right) + \frac{1}{2}\,x\sqrt{\frac{x+1}{x}}+\frac{1}{2}\,x-\frac{1}{4}\,\log\left(\sqrt{\frac{x+1}{x}}+1\right)$$

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + 1)*log(sqrt((x + 1)/x) - 1) + 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1/
4*log(sqrt((x + 1)/x) + 1)
```

Sympy [A] (verification not implemented)

Time = 47.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) dx = x \log \left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{\log \left(\sqrt{1+\frac{1}{x}}-1\right)}{4} -\frac{\log \left(\sqrt{1+\frac{1}{x}}+1\right)}{4}+\frac{1}{2\left(\sqrt{1+\frac{1}{x}}-1\right)}$$

```
[In] integrate(ln(-1+((1+x)/x)**(1/2)),x)
[Out] x*log(sqrt((x + 1)/x) - 1) + log(sqrt(1 + 1/x) - 1)/4 - log(sqrt(1 + 1/x) +
 1)/4 + 1/(2*(sqrt(1 + 1/x) - 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) dx = \frac{\log \left(\sqrt{\frac{x+1}{x}}-1\right)}{\frac{x+1}{x}-1}+\frac{1}{2\left(\sqrt{\frac{x+1}{x}}-1\right)} -\frac{1}{4} \log \left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{4} \log \left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt((x + 1)/x) - 1)/((x + 1)/x - 1) + 1/2/(sqrt((x + 1)/x) - 1) - 1/4*
log(sqrt((x + 1)/x) + 1) + 1/4*log(sqrt((x + 1)/x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) dx = x \log \left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{1}{2} x +\frac{\log \left(\left|-2 x+2 \sqrt{x^2+x}-1\right|\right)}{4 \operatorname{sgn}(x)}+\frac{\sqrt{x^2+x}}{2 \operatorname{sgn}(x)}$$

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="giac")
[Out] x*log(sqrt((x + 1)/x) - 1) + 1/2*x + 1/4*log(abs(-2*x + 2*sqrt(x^2 + x) - 1
))/sgn(x) + 1/2*sqrt(x^2 + x)/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \log \left(-1+\sqrt{\frac{1+x}{x}}\right) dx = \frac{x}{2} - \frac{\operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right)}{2} + x \ln \left(\sqrt{\frac{x+1}{x}}-1\right) + \frac{x\sqrt{\frac{1}{x}+1}}{2}$$

```
[In] int(log(((x + 1)/x)^(1/2) - 1),x)
[Out] x/2 - atanh((1/x + 1)^(1/2))/2 + x*log(((x + 1)/x)^(1/2) - 1) + (x*(1/x + 1
)^(1/2))/2
```

3.298 $\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx$

Optimal result 1532
Rubi [A] (verified) 1532
Mathematica [A] (verified) 1534
Maple [A] (verified) 1534
Fricas [A] (verification not implemented) 1535
Sympy [A] (verification not implemented) 1535
Maxima [A] (verification not implemented) 1535
Giac [A] (verification not implemented) 1536
Mupad [B] (verification not implemented) 1536

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx=\frac{1}{2}\log\left(1-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{6}\log\left(1+\sqrt{1+\frac{1}{x}}\right)+x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)$$

```
[Out] 1/2*ln(1-(1+1/x)^(1/2))-1/3*ln(2-(1+1/x)^(1/2))-1/6*ln(1+(1+1/x)^(1/2))+x*l
n(-2+((1+x)/x)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2628, 720, 31, 647}

$$\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx=\frac{1}{2}\log\left(1-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{3}\log\left(2-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{6}\log\left(\sqrt{\frac{1}{x}+1}+1\right)+x\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

```
[In] Int[Log[-2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] Log[1 - Sqrt[1 + x^(-1)]]/2 - Log[2 - Sqrt[1 + x^(-1)]]/3 - Log[1 + Sqrt[1
+ x^(-1)]]/6 + x*Log[-2 + Sqrt[(1 + x)/x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\int\frac{1}{-2+\left(-2+4\sqrt{1+\frac{1}{x}}\right)x}\,dx\\
&= x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int\frac{1}{(-2+x)\left(-1+x^2\right)}\,dx,x,\sqrt{1+\frac{1}{x}}\right)\\
&= x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{3}\text{Subst}\left(\int\frac{1}{-2+x}\,dx,x,\sqrt{1+\frac{1}{x}}\right)\\
&\quad-\frac{1}{3}\text{Subst}\left(\int\frac{-2-x}{-1+x^2}\,dx,x,\sqrt{1+\frac{1}{x}}\right)\\
&= -\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)+x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)\\
&\quad-\frac{1}{6}\text{Subst}\left(\int\frac{1}{1+x}\,dx,x,\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{-1+x}\,dx,x,\sqrt{1+\frac{1}{x}}\right)
\end{aligned}$$

$$
= \frac{1}{2}\log\left(1-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)
- \frac{1}{6}\log\left(1+\sqrt{1+\frac{1}{x}}\right)+x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$
\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx = \frac{1}{6}\left(-6\text{arctanh}\left(3-2\sqrt{1+\frac{1}{x}}\right)+\log\left(2-\sqrt{1+\frac{1}{x}}\right)+6x\log\left(-2+\sqrt{1+\frac{1}{x}}\right)-\log\left(1+\sqrt{1+\frac{1}{x}}\right)\right)
$$

```
[In] Integrate[Log[-2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] (-6*ArcTanh[3 - 2*Sqrt[1 + x^(-1)]] + Log[2 - Sqrt[1 + x^(-1)]] + 6*x*Log[-
2 + Sqrt[1 + x^(-1)]] - Log[1 + Sqrt[1 + x^(-1)]])/6
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.57

method	result	size
default	$x\ln\left(-2+\sqrt{\frac{x+1}{x}}\right)-\frac{3\sqrt{\frac{x+1}{x}}\,x\ln(-3x+1)-\sqrt{9}\,\ln\left(\frac{4\sqrt{9}\,\sqrt{x^2+x}+15x+3}{9x-3}\right)\sqrt{x(x+1)}+6\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\sqrt{x(x+1)}}{18\sqrt{\frac{x+1}{x}}\,x}$	108
parts	$x\ln\left(-2+\sqrt{\frac{x+1}{x}}\right)-\frac{3\sqrt{\frac{x+1}{x}}\,x\ln(-3x+1)-\sqrt{9}\,\ln\left(\frac{4\sqrt{9}\,\sqrt{x^2+x}+15x+3}{9x-3}\right)\sqrt{x(x+1)}+6\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\sqrt{x(x+1)}}{18\sqrt{\frac{x+1}{x}}\,x}$	108

```
[In] int(ln(-2+((x+1)/x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(-2+((x+1)/x)^(1/2))-1/18/((x+1)/x)^(1/2)/x*(3*((x+1)/x)^(1/2)*x*ln(-3*
x+1)-9^(1/2)*ln(1/3*(4*9^(1/2)*(x^2+x)^(1/2)+15*x+3)/(3*x-1))*(x*(x+1))^(1/
2)+6*ln(1/2+x+(x^2+x)^(1/2))*(x*(x+1))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right) dx = \frac{1}{3}(3x-1)\log \left(\sqrt{\frac{x+1}{x}}-2\right) - \frac{1}{6}\log \left(\sqrt{\frac{x+1}{x}}+1\right) + \frac{1}{2}\log \left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="fricas")
[Out] 1/3*(3*x - 1)*log(sqrt((x + 1)/x) - 2) - 1/6*log(sqrt((x + 1)/x) + 1) + 1/2
*log(sqrt((x + 1)/x) - 1)
```

Sympy [A] (verification not implemented)

Time = 46.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right) dx = x\log \left(\sqrt{\frac{x+1}{x}}-2\right) - \frac{\log \left(\sqrt{1+\frac{1}{x}}-2\right)}{3} + \frac{\log \left(\sqrt{1+\frac{1}{x}}-1\right)}{2} - \frac{\log \left(\sqrt{1+\frac{1}{x}}+1\right)}{6}$$

```
[In] integrate(ln(-2+((1+x)/x)**(1/2)),x)
[Out] x*log(sqrt((x + 1)/x) - 2) - log(sqrt(1 + 1/x) - 2)/3 + log(sqrt(1 + 1/x) -
 1)/2 - log(sqrt(1 + 1/x) + 1)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right) dx = \frac{\log \left(\sqrt{\frac{x+1}{x}}-2\right)}{\frac{x+1}{x}-1} - \frac{1}{6}\log \left(\sqrt{\frac{x+1}{x}}+1\right) + \frac{1}{2}\log \left(\sqrt{\frac{x+1}{x}}-1\right) - \frac{1}{3}\log \left(\sqrt{\frac{x+1}{x}}-2\right)$$

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt((x + 1)/x) - 2)/((x + 1)/x - 1) - 1/6*log(sqrt((x + 1)/x) + 1) + 1
/2*log(sqrt((x + 1)/x) - 1) - 1/3*log(sqrt((x + 1)/x) - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right) dx = x \log \left(\sqrt{\frac{x+1}{x}}-2\right)+\frac{\log \left(\left|-x+\sqrt{x^2+x}+1\right|\right)}{6 \operatorname{sgn}(x)}+\frac{\log \left(\left|-2 x+2 \sqrt{x^2+x}-1\right|\right)}{3 \operatorname{sgn}(x)}-\frac{\log \left(\left|-3 x+3 \sqrt{x^2+x}-1\right|\right)}{6 \operatorname{sgn}(x)}-\frac{1}{6} \log (|3 x-1|)$$

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) - 2) + 1/6*log(abs(-x + sqrt(x^2 + x) + 1))/sgn(x) +
1/3*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(-3*x + 3*sqrt
(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(3*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right) dx = \frac{\ln \left(5-5 \sqrt{\frac{x+1}{x}}\right)}{2}-\frac{\ln \left(\frac{\sqrt{\frac{x+1}{x}}}{9}+\frac{1}{9}\right)}{6}-\frac{\ln \left(\frac{10}{9}-\frac{5 \sqrt{\frac{x+1}{x}}}{9}\right)}{3}+x \ln \left(\sqrt{\frac{x+1}{x}}-2\right)$$

```
[In] int(log(((x + 1)/x)^(1/2) - 2),x)
```

```
[Out] log(5 - 5*((x + 1)/x)^(1/2))/2 - log(((x + 1)/x)^(1/2)/9 + 1/9)/6 - log(10/
9 - (5*((x + 1)/x)^(1/2))/9)/3 + x*log(((x + 1)/x)^(1/2) - 2)
```

3.299 $\int (x^{ax}+x^{ax}\log(x))\,dx$

Optimal result . . . 1537
Rubi [A] (verified) . . . 1537
Mathematica [A] (verified) . . . 1538
Maple [A] (verified) . . . 1538
Fricas [A] (verification not implemented) . . . 1538
Sympy [A] (verification not implemented) . . . 1539
Maxima [A] (verification not implemented) . . . 1539
Giac [F] . . . 1539
Mupad [B] (verification not implemented) . . . 1539

Optimal result

Integrand size = 14, antiderivative size = 9

$$\int (x^{ax}+x^{ax}\log(x))\,dx=\frac{x^{ax}}{a}$$

```
[Out] x^(a*x)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2633}

$$\int (x^{ax}+x^{ax}\log(x))\,dx=\frac{x^{ax}}{a}$$

```
[In] Int[x^(a*x) + x^(a*x)*Log[x],x]
```

```
[Out] x^(a*x)/a
```

Rule 2633

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int x^{ax}\,dx+\int x^{ax}\log(x)\,dx\\&=\frac{x^{ax}}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (x^{ax} + x^{ax}\log(x))\ dx = \frac{x^{ax}}{a}$$

```
[In] Integrate[x^(a*x) + x^(a*x)*Log[x],x]
[Out] x^(a*x)/a
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{x^{ax}}{a}$	10
parallelrisch	$\frac{x^{ax}}{a}$	10
norman	$\frac{e^{ax\ln(x)}}{a}$	11

```
[In] int(x^(a*x)+x^(a*x)*ln(x),x,method=_RETURNVERBOSE)
[Out] x^(a*x)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (x^{ax} + x^{ax}\log(x))\ dx = \frac{x^{ax}}{a}$$

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="fricas")
[Out] x^(a*x)/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int (x^{ax} + x^{ax} \log(x)) \, dx = \begin{cases} \frac{x^{ax}}{a} & \text{for } a \neq 0 \\ x \log(x) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(a*x)+x**(a*x)*ln(x),x)
[Out] Piecewise((x**(a*x)/a, Ne(a, 0)), (x*log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (x^{ax} + x^{ax} \log(x)) \, dx = \frac{x^{ax}}{a}$$

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="maxima")
[Out] x^(a*x)/a
```

Giac [F]

$$\int (x^{ax} + x^{ax} \log(x)) \, dx = \int x^{ax} \log(x) + x^{ax} \, dx$$

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="giac")
[Out] integrate(x^(a*x)*log(x) + x^(a*x), x)
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (x^{ax} + x^{ax} \log(x)) \, dx = \frac{x^{a\,x}}{a}$$

```
[In] int(x^(a*x) + x^(a*x)*log(x),x)
[Out] x^(a*x)/a
```

3.300 $\int \log^m(x)^p \, dx$

Optimal result 1540
Rubi [A] (verified) 1540
Mathematica [A] (verified) 1541
Maple [F] 1541
Fricas [C] (verification not implemented) 1542
Sympy [F] 1542
Maxima [F] 1542
Giac [F] 1542
Mupad [F(-1)] 1543

Optimal result

Integrand size = 6, antiderivative size = 26

$$\int \log^m(x)^p \, dx = \Gamma(1 + mp, -\log(x))(-\log(x))^{-mp} \log^m(x)^p$$

```
[Out] GAMMA(m*p+1,-ln(x))*(ln(x)^m)^p/((-ln(x))^(m*p))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6852, 2336, 2212}

$$\int \log^m(x)^p \, dx = (-\log(x))^{-mp} \log^m(x)^p \Gamma(mp + 1, -\log(x))$$

```
[In] Int[(Log[x]^m)^p,x]
[Out] (Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
```

```
c, p}, x] && IntegerQ[1/n]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\log^{-mp}(x)\log^m(x)^p\right)\int \log^{mp}(x)\,dx\\ &= \left(\log^{-mp}(x)\log^m(x)^p\right)\text{Subst}\left(\int e^x x^{mp}\,dx, x, \log(x)\right)\\ &= \Gamma(1+mp,-\log(x))(-\log(x))^{-mp}\log^m(x)^p\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \log^m(x)^p\,dx = \Gamma(1+mp,-\log(x))(-\log(x))^{-mp}\log^m(x)^p$$

```
[In] Integrate[(Log[x]^m)^p,x]
[Out] (Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Maple [F]

$$\int (\ln(x)^m)^p\,dx$$

```
[In] int((ln(x)^m)^p,x)
[Out] int((ln(x)^m)^p,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \log^m(x)^p\,dx = e^{(-i\pi mp)}\Gamma(mp+1,-\log(x))$$

```
[In] integrate((log(x)^m)^p,x, algorithm="fricas")
[Out] e^(-I*pi*m*p)*gamma(m*p + 1, -log(x))
```

Sympy [F]

$$\int \log^m(x)^p\,dx = \int \left(\log(x)^m\right)^p\,dx$$

```
[In] integrate((ln(x)**m)**p,x)
[Out] Integral((log(x)**m)**p, x)
```

Maxima [F]

$$\int \log^m(x)^p\,dx = \int (\log(x)^m)^p\,dx$$

```
[In] integrate((log(x)^m)^p,x, algorithm="maxima")
[Out] integrate((log(x)^m)^p, x)
```

Giac [F]

$$\int \log^m(x)^p\,dx = \int (\log(x)^m)^p\,dx$$

```
[In] integrate((log(x)^m)^p,x, algorithm="giac")
[Out] integrate((log(x)^m)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^m(x)^p\,dx = \int (\ln(x)^m)^p\,dx$$

```
[In] int((log(x)^m)^p,x)
```

```
[Out] int((log(x)^m)^p, x)
```

3.301 $\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal result 1544
Rubi [A] (verified) 1544
Mathematica [A] (verified) 1545
Maple [F] 1546
Fricas [F(-2)] 1546
Sympy [F] 1546
Maxima [B] (verification not implemented) 1546
Giac [A] (verification not implemented) 1547
Mupad [F(-1)] 1547

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx=-\frac{(2a+b)e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{x\sqrt{a+b\log(x)}}{b}$$

```
[Out] -1/2*(2*a+b)*erfi((a+b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a/b)+x*(a
+b*ln(x))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2399, 2336, 2211, 2235}

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx=\frac{x\sqrt{a+b\log(x)}}{b}-\frac{\sqrt{\pi}(2a+b)e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}$$

```
[In] Int[Log[x]/Sqrt[a + b*Log[x]],x]
```

```
[Out] -1/2*((2*a + b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(b^(3/2)*E^(a/b)
) + (x*Sqrt[a + b*Log[x]])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2399

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\sqrt{a+b\log(x)}}{b}+\frac{(-2a-b)\int\frac{1}{\sqrt{a+b\log(x)}}\,dx}{2b}\\
&= \frac{x\sqrt{a+b\log(x)}}{b}+\frac{(-2a-b)\text{Subst}\left(\int\frac{e^x}{\sqrt{a+bx}}\,dx,x,\log(x)\right)}{2b}\\
&= \frac{x\sqrt{a+b\log(x)}}{b}-\frac{(2a+b)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\log(x)}\right)}{b^2}\\
&= -\frac{(2a+b)e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{x\sqrt{a+b\log(x)}}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int\frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx=\frac{2x(a+b\log(x))-(2a+b)e^{-\frac{a}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\log(x)}{b}\right)\sqrt{-\frac{a+b\log(x)}{b}}}{2b\sqrt{a+b\log(x)}}$$

```
[In] Integrate[Log[x]/Sqrt[a + b*Log[x]],x]
```

```
[Out] (2*x*(a + b*Log[x]) - ((2*a + b)*Gamma[1/2, -((a + b*Log[x])/b)]*Sqrt[-((a
+ b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

Maple [F]

$$\int \frac{\ln(x)}{\sqrt{a+b\ln(x)}}dx$$

```
[In] int(ln(x)/(a+b*ln(x))^(1/2),x)
[Out] int(ln(x)/(a+b*ln(x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx$$

```
[In] integrate(ln(x)/(a+b*ln(x))**(1/2),x)
[Out] Integral(log(x)/sqrt(a + b*log(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.80

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = -\frac{\frac{2\sqrt{\pi}a\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}+\frac{\sqrt{\pi}b\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-2\sqrt{b\log(x)+a}\,be^{\left(\frac{b\log(x)+a}{b}-\frac{a}{b}\right)}}{2\,b^2}$$

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="maxima")
[Out] -1/2*(2*sqrt(pi)*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) +
 sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*sqrt
(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}} + \frac{\sqrt{\pi}a\,\mathrm{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}b} + \frac{\sqrt{b\log(x)+a}x}{b}$$

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(pi)*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(p
i)*a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*log
(x) + a)*x/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx = \int \frac{\ln(x)}{\sqrt{a+b\,\ln(x)}}\,dx$$

```
[In] int(log(x)/(a + b*log(x))^(1/2),x)
```

```
[Out] int(log(x)/(a + b*log(x))^(1/2), x)
```

3.302 $\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal result 1548
Rubi [A] (verified) 1548
Mathematica [A] (verified) 1549
Maple [F] 1550
Fricas [F(-2)] 1550
Sympy [F] 1550
Maxima [A] (verification not implemented) 1550
Giac [A] (verification not implemented) 1551
Mupad [F(-1)] 1551

Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx=-\frac{(2a-b)e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

```
[Out] -1/2*(2*a-b)*exp(a/b)*erf((a-b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)-x*(a-
b*ln(x))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2399, 2336, 2211, 2236}

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx=-\frac{\sqrt{\pi}(2a-b)e^{a/b}\text{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

```
[In] Int[Log[x]/Sqrt[a - b*Log[x]],x]
```

```
[Out] -1/2*((2*a - b)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]])/b^(3/2) -
 (x*Sqrt[a - b*Log[x]])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2399

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x\sqrt{a-b\log(x)}}{b}-\frac{(-2a+b)\int\frac{1}{\sqrt{a-b\log(x)}}\,dx}{2b}\\
&= -\frac{x\sqrt{a-b\log(x)}}{b}-\frac{(-2a+b)\text{Subst}\left(\int\frac{e^x}{\sqrt{a-bx}}\,dx,x,\log(x)\right)}{2b}\\
&= -\frac{x\sqrt{a-b\log(x)}}{b}-\frac{(2a-b)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a-b\log(x)}\right)}{b^2}\\
&= -\frac{(2a-b)e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int\frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx=\frac{-\left((-2a+b)e^{a/b}\Gamma\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)\sqrt{\frac{a}{b}-\log(x)}\right)-2x(a-b\log(x))}{2b\sqrt{a-b\log(x)}}$$

```
[In] Integrate[Log[x]/Sqrt[a - b*Log[x]],x]
[Out] (-((-2*a + b)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]) - 2*x*(a
 - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

Maple [F]

$$\int \frac{\ln(x)}{\sqrt{a-b\ln(x)}}dx$$

```
[In] int(ln(x)/(a-b*ln(x))^(1/2),x)
[Out] int(ln(x)/(a-b*ln(x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = \int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx$$

```
[In] integrate(ln(x)/(a-b*ln(x))**(1/2),x)
[Out] Integral(log(x)/sqrt(a - b*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = -\frac{2\sqrt{\pi}a\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} - \sqrt{\pi}b^{\frac{3}{2}}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} + 2\sqrt{-b\log(x)+a}\,be^{\left(\frac{b\log(x)-a}{b}+\frac{a}{b}\right)}}{2\,b^2}$$

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="maxima")
[Out] -1/2*(2*sqrt(pi)*a*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - sqrt(
pi)*b^(3/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) + 2*sqrt(-b*log(x) + a
)*b*e^((b*log(x) - a)/b + a/b))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = \frac{\sqrt{\pi}a\,\mathrm{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}} - \frac{\sqrt{-b\log(x)+a}x}{b}$$

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) - 1/2*sqrt(pi)
*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*x/
b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx = \int \frac{\ln(x)}{\sqrt{a-b\,\ln(x)}}\,dx$$

```
[In] int(log(x)/(a - b*log(x))^(1/2),x)
```

```
[Out] int(log(x)/(a - b*log(x))^(1/2), x)
```

3.303 $\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal result 1552
Rubi [A] (verified) 1552
Mathematica [A] (verified) 1553
Maple [F] 1554
Fricas [F(-2)] 1554
Sympy [F] 1554
Maxima [B] (verification not implemented) 1554
Giac [B] (verification not implemented) 1555
Mupad [F(-1)] 1555

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx=\frac{(2Ab-(2a+b)B)e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

```
[Out] 1/2*(2*A*b-(2*a+b)*B)*erfi((a+b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/exp(
a/b)+B*x*(a+b*ln(x))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2399, 2336, 2211, 2235}

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx=\frac{\sqrt{\pi}e^{-\frac{a}{b}}(2Ab-B(2a+b))\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

```
[In] Int[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
```

```
[Out] ((2*A*b - (2*a + b)*B)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(2*b^(3/2
)*E^(a/b)) + (B*x*Sqrt[a + b*Log[x]])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2399

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx\sqrt{a+b\log(x)}}{b}+\frac{(2Ab-(2a+b)B)\int\frac{1}{\sqrt{a+b\log(x)}}\,dx}{2b}\\
&= \frac{Bx\sqrt{a+b\log(x)}}{b}+\frac{(2Ab-(2a+b)B)\text{Subst}\left(\int\frac{e^x}{\sqrt{a+bx}}\,dx,x,\log(x)\right)}{2b}\\
&= \frac{Bx\sqrt{a+b\log(x)}}{b}+\frac{(2Ab-(2a+b)B)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\log(x)}\right)}{b^2}\\
&= \frac{(2Ab-(2a+b)B)e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int\frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx=\frac{2Bx(a+b\log(x))+(2Ab-(2a+b)B)e^{-\frac{a}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\log(x)}{b}\right)\sqrt{-\frac{a+b\log(x)}{b}}}{2b\sqrt{a+b\log(x)}}$$

```
[In] Integrate[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
[Out] (2*B*x*(a + b*Log[x]) + ((2*A*b - (2*a + b)*B)*Gamma[1/2, -((a + b*Log[x])/
b)]*Sqrt[-((a + b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

Maple [F]

$$\int \frac{A+B\ln(x)}{\sqrt{a+b\ln(x)}}dx$$

```
[In] int((A+B*ln(x))/(a+b*ln(x))^(1/2),x)
[Out] int((A+B*ln(x))/(a+b*ln(x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx = \int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx$$

```
[In] integrate((A+B*ln(x))/(a+b*ln(x))**(1/2),x)
[Out] Integral((A + B*log(x))/sqrt(a + b*log(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx$$
$$= \frac{\frac{2\sqrt{\pi}A\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}} - \frac{2\sqrt{\pi}Ba\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{b\sqrt{-\frac{1}{b}}} - \frac{\left(\frac{\sqrt{\pi}b\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}} - 2\sqrt{b\log(x)+a}\,be^{\left(\frac{b\log\cdots}{}\right.}}{b}}{2b}$$

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(pi)*A*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) -
2*sqrt(pi)*B*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/(b*sqrt(-1/b)) -
 (sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*sqr
t(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))*B/b)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.87

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx = -\frac{\sqrt{\pi}A\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}} + \frac{\sqrt{\pi}B\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}} + \frac{\sqrt{\pi}Ba\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}b} + \frac{\sqrt{b\log(x)+a}Bx}{b}$$

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(pi)*A*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + 1/2*sqr
t(pi)*B*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(pi)*B*
a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*log(x)
 + a)*B*x/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx = \int \frac{A+B\,\ln(x)}{\sqrt{a+b\,\ln(x)}}\,dx$$

```
[In] int((A + B*log(x))/(a + b*log(x))^(1/2),x)
```

```
[Out] int((A + B*log(x))/(a + b*log(x))^(1/2), x)
```

3.304 $\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal result . . . 1556
Rubi [A] (verified) . . . 1556
Mathematica [A] (verified) . . . 1557
Maple [F] . . . 1558
Fricas [F(-2)] . . . 1558
Sympy [F] . . . 1558
Maxima [B] (verification not implemented) . . . 1558
Giac [A] (verification not implemented) . . . 1559
Mupad [F(-1)] . . . 1559

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx=-\frac{(2Ab+2aB-bB)e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

```
[Out] -1/2*(2*A*b+2*B*a-B*b)*exp(a/b)*erf((a-b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(
3/2)-B*x*(a-b*ln(x))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2399, 2336, 2211, 2236}

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx=-\frac{\sqrt{\pi}e^{a/b}(2aB+2Ab-bB)\,\mathrm{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

```
[In] Int[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
```

```
[Out] -1/2*((2*A*b + 2*a*B - b*B)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]
])/b^(3/2) - (B*x*Sqrt[a - b*Log[x]])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2399

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{Bx\sqrt{a-b\log(x)}}{b}+\frac{(2Ab+2aB-bB)\int\frac{1}{\sqrt{a-b\log(x)}}\,dx}{2b}\\
&= -\frac{Bx\sqrt{a-b\log(x)}}{b}+\frac{(2Ab+2aB-bB)\text{Subst}\left(\int\frac{e^x}{\sqrt{a-bx}}\,dx,x,\log(x)\right)}{2b}\\
&= -\frac{Bx\sqrt{a-b\log(x)}}{b}-\frac{(2Ab+2aB-bB)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a-b\log(x)}\right)}{b^2}\\
&= -\frac{(2Ab+2aB-bB)e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

$$\int\frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = \frac{(2Ab+2aB-bB)e^{a/b}\Gamma\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)\sqrt{\frac{a}{b}-\log(x)}-2Bx(a-b\log(x))}{2b\sqrt{a-b\log(x)}}$$

```
[In] Integrate[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
[Out] ((2*A*b + 2*a*B - b*B)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]
- 2*B*x*(a - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

Maple [F]

$$\int \frac{A+B\ln(x)}{\sqrt{a-b\ln(x)}}dx$$

```
[In] int((A+B*ln(x))/(a-b*ln(x))^(1/2),x)
[Out] int((A+B*ln(x))/(a-b*ln(x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = \int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx$$

```
[In] integrate((A+B*ln(x))/(a-b*ln(x))**(1/2),x)
[Out] Integral((A + B*log(x))/sqrt(a - b*log(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(58) = 116.

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = -\frac{\frac{2\sqrt{\pi}Ba\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}}+2\sqrt{\pi}A\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}-\frac{\left(\sqrt{\pi}b^{\frac{3}{2}}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}-2\sqrt{-b\log(x)+a}\,be^{\left(\frac{b\log(x)-a}{b}\right.}\cdots}{b}}{2b}$$

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="maxima")
[Out] -1/2*(2*sqrt(pi)*B*a*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) + 2*s
qrt(pi)*A*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - (sqrt(pi)*b^(3
/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - 2*sqrt(-b*log(x) + a)*b*e^((
b*log(x) - a)/b + a/b))*B/b)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = \frac{\sqrt{\pi}Ba\,\mathrm{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}} + \frac{\sqrt{\pi}A\,\mathrm{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}} - \frac{\sqrt{\pi}B\,\mathrm{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}} - \frac{\sqrt{-b\log\left(x\right)+a}Bx}{b}$$

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*B*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) + sqrt(pi)*A
*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - 1/2*sqrt(pi)*B*erf(-sq
rt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*B*x/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx = \int \frac{A+B\,\ln\left(x\right)}{\sqrt{a-b\,\ln\left(x\right)}}\,dx$$

```
[In] int((A + B*log(x))/(a - b*log(x))^(1/2),x)
```

```
[Out] int((A + B*log(x))/(a - b*log(x))^(1/2), x)
```

3.305 $\int x^2 \log(\log(x)\sin(x))\,dx$

Optimal result 1560
Rubi [A] (verified) 1560
Mathematica [A] (verified) 1563
Maple [C] (warning: unable to verify) 1564
Fricas [B] (verification not implemented) 1564
Sympy [F] 1566
Maxima [A] (verification not implemented) 1566
Giac [F] 1566
Mupad [F(-1)] 1567

Optimal result

Integrand size = 10, antiderivative size = 98

$$\int x^2 \log(\log(x)\sin(x))\,dx = \frac{ix^4}{12} - \frac{1}{3}\,\text{ExpIntegralEi}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{2}x\,\text{PolyLog}\left(3,e^{2ix}\right) - \frac{1}{4}i\,\text{PolyLog}\left(4,e^{2ix}\right)$$

```
[Out] 1/12*I*x^4-1/3*Ei(3*ln(x))-1/3*x^3*ln(1-exp(2*I*x))+1/3*x^3*ln(ln(x)*sin(x)
)+1/2*I*x^2*polylog(2,exp(2*I*x))-1/2*x*polylog(3,exp(2*I*x))-1/4*I*polylog
(4,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {30, 2635, 12, 6820, 14, 3798, 2221, 2611, 6744, 2320, 6724, 2346, 2209}

$$\int x^2 \log(\log(x)\sin(x))\,dx = -\frac{1}{3}\,\text{ExpIntegralEi}(3\log(x)) + \frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{2}x\,\text{PolyLog}\left(3,e^{2ix}\right) - \frac{1}{4}i\,\text{PolyLog}\left(4,e^{2ix}\right) + \frac{ix^4}{12} - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x))$$

```
[In] Int[x^2*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])/3 + (x^
3*Log[Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (x*PolyLog[3,
 E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \int \frac{x^2(1+x\cot(x)\log(x))}{3\log(x)}\,dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int \frac{x^2(1+x\cot(x)\log(x))}{\log(x)}\,dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int x^2\left(x\cot(x)+\frac{1}{\log(x)}\right)\,dx
\end{aligned}$$

$$= \frac{1}{3}x^3 \log(\log(x)\sin(x)) - \frac{1}{3}\int \left(x^3 \cot(x) + \frac{x^2}{\log(x)}\right) dx$$

$$= \frac{1}{3}x^3 \log(\log(x)\sin(x)) - \frac{1}{3}\int x^3 \cot(x)\, dx - \frac{1}{3}\int \frac{x^2}{\log(x)}\, dx$$

$$= \frac{ix^4}{12} + \frac{1}{3}x^3 \log(\log(x)\sin(x)) + \frac{2}{3}i\int \frac{e^{2ix}x^3}{1-e^{2ix}}\, dx - \frac{1}{3}\text{Subst}\left(\int \frac{e^{3x}}{x}\, dx, x, \log(x)\right)$$

$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3 \log\left(1-e^{2ix}\right) + \frac{1}{3}x^3 \log(\log(x)\sin(x)) + \int x^2 \log\left(1-e^{2ix}\right)\, dx$$

$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3 \log\left(1-e^{2ix}\right) + \frac{1}{3}x^3 \log(\log(x)\sin(x)) + \frac{1}{2}ix^2 \text{Li}_2\left(e^{2ix}\right) - i\int x\text{Li}_2\left(e^{2ix}\right)\, dx$$

$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3 \log\left(1-e^{2ix}\right) + \frac{1}{3}x^3 \log(\log(x)\sin(x)) + \frac{1}{2}ix^2 \text{Li}_2\left(e^{2ix}\right) - \frac{1}{2}x\text{Li}_3\left(e^{2ix}\right) + \frac{1}{2}\int \text{Li}_3\left(e^{2ix}\right)\, dx$$

$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3 \log\left(1-e^{2ix}\right) + \frac{1}{3}x^3 \log(\log(x)\sin(x)) + \frac{1}{2}ix^2 \text{Li}_2\left(e^{2ix}\right) - \frac{1}{2}x\text{Li}_3\left(e^{2ix}\right) - \frac{1}{4}i\text{Subst}\left(\int \frac{\text{Li}_3(x)}{x}\, dx, x, e^{2ix}\right)$$

$$= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3 \log\left(1-e^{2ix}\right) + \frac{1}{3}x^3 \log(\log(x)\sin(x)) + \frac{1}{2}ix^2 \text{Li}_2\left(e^{2ix}\right) - \frac{1}{2}x\text{Li}_3\left(e^{2ix}\right) - \frac{1}{4}i\text{Li}_4\left(e^{2ix}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int x^2 \log(\log(x)\sin(x))\, dx = \frac{1}{192}i\left(\pi^4 - 16x^4 + 64i\,\text{ExpIntegralEi}(3\log(x)) + 64ix^3 \log\left(1-e^{-2ix}\right) - 64ix^3 \log(\log(x)\sin(x)) - 96x^2\,\text{PolyLog}\left(2, e^{-2ix}\right) + 96ix\,\text{PolyLog}\left(3, e^{-2ix}\right) + 48\,\text{PolyLog}\left(4, e^{-2ix}\right)\right)$$

```
[In] Integrate[x^2*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/192)*(Pi^4 - 16*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*x^3*Log[1
- E^((-2*I)*x)] - (64*I)*x^3*Log[Log[x]*Sin[x]] - 96*x^2*PolyLog[2, E^((-2*
I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^((-2*I)*x)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.60 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.35

method	result
risch	$-\frac{x^3 \ln(\mathrm{e}^{ix})}{3} + \frac{\left(-i\pi \operatorname{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right) \operatorname{csgn}\left(i\ln(x)\right) \operatorname{csgn}\left(i\ln(x)\left(\mathrm{e}^{2ix}-1\right)\right)+i\pi \operatorname{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)\operatorname{csgn}\left(i\ln(x)\left(\mathrm{e}^{2ix}-1\right)\right)^2+i\pi \operatorname{csgn}\left(i\mathrm{e}^{-}\right.\right.}{}$

```
[In] int(x^2*ln(ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3*ln(exp(I*x))+1/6*(-I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(
I*ln(x)*(exp(2*I*x)-1))+I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x
)-1))^2+I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(
x))+I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(I*ln(x))*csgn(I*l
n(x)*(exp(2*I*x)-1))^2-I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3+I*Pi*csgn(I*ln(x
)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(ln(x)*sin(x))^3-I*Pi*csgn(
ln(x)*sin(x))*csgn(I*ln(x)*sin(x))^2+I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*s
in(x))-I*Pi*csgn(I*ln(x)*sin(x))^3+I*Pi*csgn(I*ln(x)*sin(x))^2-I*Pi-2*ln(2)
)*x^3+1/3*x^3*ln(exp(2*I*x)-1)-1/3*x^3*ln(exp(I*x)+1)+I*x^2*polylog(2,-exp(
I*x))-2*x*polylog(3,-exp(I*x))-2*I*polylog(4,-exp(I*x))-1/3*x^3*ln(1-exp(I*
x))+I*x^2*polylog(2,exp(I*x))-2*x*polylog(3,exp(I*x))-2*I*polylog(4,exp(I*x
))+1/3*x^3*ln(ln(x))+1/3*Ei(1,-3*ln(x))+1/12*I*x^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(65) = 130.

Time = 0.33 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.39

$$\int x^2 \log(\log(x)\sin(x))\,dx = \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{6}x^3\log(\cos(x) + i\sin(x) + 1) - \frac{1}{6}x^3\log(\cos(x) - i\sin(x) + 1) - \frac{1}{6}x^3\log(-\cos(x) + i\sin(x) + 1) - \frac{1}{6}x^3\log(-\cos(x) - i\sin(x) + 1) + \frac{1}{2}i\,x^2\mathrm{Li}_2(\cos(x) + i\sin(x)) - \frac{1}{2}i\,x^2\mathrm{Li}_2(\cos(x) - i\sin(x)) - \frac{1}{2}i\,x^2\mathrm{Li}_2(-\cos(x) + i\sin(x)) + \frac{1}{2}i\,x^2\mathrm{Li}_2(-\cos(x) - i\sin(x)) - x\mathrm{polylog}(3, \cos(x) + i\sin(x)) - x\mathrm{polylog}(3, \cos(x) - i\sin(x)) - x\mathrm{polylog}(3, -\cos(x) + i\sin(x)) - x\mathrm{polylog}(3, -\cos(x) - i\sin(x)) - \frac{1}{3}\ \mathrm{log\_integral}\left(x^3\right) - i\,\mathrm{polylog}(4, \cos(x) + i\sin(x)) + i\,\mathrm{polylog}(4, \cos(x) - i\sin(x)) + i\,\mathrm{polylog}(4, -\cos(x) + i\sin(x)) - i\,\mathrm{polylog}(4, -\cos(x) - i\sin(x))$$

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x) + 1) - 1/6*x^3*l
og(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x) + 1) - 1/6*x^3*l
og(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*sin(x)) - 1/2*I*x^2
*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I*sin(x)) + 1/2*I*x^2
*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*sin(x)) - x*polylog(3,
 cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)) - x*polylog(3, -cos(
x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4, cos(x) + I*sin(x)) +
I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos(x) + I*sin(x)) - I*poly
log(4, -cos(x) - I*sin(x))
```

Sympy [F]

$$\int x^2 \log(\log(x) \sin(x))\, dx = \int x^2 \log(\log(x) \sin(x))\, dx$$

```
[In] integrate(x**2*ln(ln(x)*sin(x)),x)
[Out] Integral(x**2*log(log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int x^2 \log(\log(x) \sin(x))\, dx = -\frac{1}{6} (-i\pi + 2 \log(2)) x^3 - \frac{1}{4} i x^4 + \frac{1}{3} x^3 \log(\log(x)) + i x^2 \mathrm{Li}_2(-e^{(ix)}) + i x^2 \mathrm{Li}_2(e^{(ix)}) - 2 x \mathrm{Li}_3(-e^{(ix)}) - 2 x \mathrm{Li}_3(e^{(ix)}) - \frac{1}{3} \mathrm{Ei}(3 \log(x)) - 2i \mathrm{Li}_4(-e^{(ix)}) - 2i \mathrm{Li}_4(e^{(ix)})$$

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="maxima")
[Out] -1/6*(-I*pi + 2*log(2))*x^3 - 1/4*I*x^4 + 1/3*x^3*log(log(x)) + I*x^2*dilog
(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*x*polylog(
3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*polylog(4,
e^(I*x))
```

Giac [F]

$$\int x^2 \log(\log(x) \sin(x))\, dx = \int x^2 \log(\log(x) \sin(x))\, dx$$

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="giac")
[Out] integrate(x^2*log(log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log(\log(x)\sin(x))\,dx = \int x^2\,\ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

```
[In] int(x^2*log(log(x)*sin(x)),x)
[Out] int(x^2*log(log(x)*sin(x)), x)
```

3.306 $\int x \log(\log(x)\sin(x))\,dx$

Optimal result 1568
Rubi [A] (verified) 1568
Mathematica [A] (verified) 1571
Maple [C] (warning: unable to verify) 1571
Fricas [B] (verification not implemented) 1572
Sympy [F] 1573
Maxima [A] (verification not implemented) 1573
Giac [F] 1573
Mupad [F(-1)] 1574

Optimal result

Integrand size = 8, antiderivative size = 80

$$\int x\log(\log(x)\sin(x))\,dx=\frac{ix^3}{6}-\frac{1}{2}\text{ExpIntegralEi}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log(\log(x)\sin(x))+\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2ix}\right)$$

```
[Out] 1/6*I*x^3-1/2*Ei(2*ln(x))-1/2*x^2*ln(1-exp(2*I*x))+1/2*x^2*ln(ln(x)*sin(x))
+1/2*I*x*polylog(2,exp(2*I*x))-1/4*polylog(3,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {30, 2635, 12, 6820, 14, 3798, 2221, 2611, 2320, 6724, 2346, 2209}

$$\int x\log(\log(x)\sin(x))\,dx=-\frac{1}{2}\text{ExpIntegralEi}(2\log(x))+\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2ix}\right)+\frac{ix^3}{6}-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log(\log(x)\sin(x))$$

```
[In] Int[x*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2 + (x^2
*Log[Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^((2
*I)*x)]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \int \frac{x(1+x\cot(x)\log(x))}{2\log(x)}\,dx \\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \frac{x(1+x\cot(x)\log(x))}{\log(x)}\,dx \\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int x\left(x\cot(x)+\frac{1}{\log(x)}\right)\,dx \\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \left(x^2\cot(x)+\frac{x}{\log(x)}\right)\,dx \\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int x^2\cot(x)\,dx - \frac{1}{2}\int \frac{x}{\log(x)}\,dx \\
&= \frac{ix^3}{6} + \frac{1}{2}x^2\log(\log(x)\sin(x)) + i\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx - \frac{1}{2}\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log(x)\right)
\end{aligned}$$

$$= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2 \log\left(1 - e^{2ix}\right) + \frac{1}{2}x^2 \log(\log(x)\sin(x)) + \int x \log\left(1 - e^{2ix}\right) \, dx$$

$$= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2 \log\left(1 - e^{2ix}\right) + \frac{1}{2}x^2 \log(\log(x)\sin(x)) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right) - \frac{1}{2}i \int \text{Li}_2\left(e^{2ix}\right) \, dx$$

$$= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2 \log\left(1 - e^{2ix}\right) + \frac{1}{2}x^2 \log(\log(x)\sin(x)) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x} \, dx, x, e^{2ix}\right)$$

$$= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2 \log\left(1 - e^{2ix}\right) + \frac{1}{2}x^2 \log(\log(x)\sin(x)) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right) - \frac{1}{4}\text{Li}_3\left(e^{2ix}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int x \log(\log(x)\sin(x)) \, dx = \frac{1}{48}\left(i\pi^3 - 8ix^3 - 24\,\text{ExpIntegralEi}(2\log(x)) - 24x^2 \log\left(1 - e^{-2ix}\right) + 24x^2 \log(\log(x)\sin(x)) - 24ix\,\text{PolyLog}\left(2, e^{-2ix}\right) - 12\,\text{PolyLog}\left(3, e^{-2ix}\right)\right)$$

```
[In] Integrate[x*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I*Pi^3 - (8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((-2*I)
*x)] + 24*x^2*Log[Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*x)] - 12*P
olyLog[3, E^((-2*I)*x)])/48
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.38 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.98

method	result
risch	$-\frac{x^2 \ln(e^{ix})}{2} + \frac{\left(-i\pi\ \text{csgn}\left(i\left(e^{2ix}-1\right)\right)\text{csgn}(i\ln(x))\,\text{csgn}\left(i\ln(x)\left(e^{2ix}-1\right)\right)+i\pi\ \text{csgn}\left(i\left(e^{2ix}-1\right)\right)\text{csgn}\left(i\ln(x)\left(e^{2ix}-1\right)\right)^2+i\pi\ \text{csgn}(i$

```
[In] int(x*ln(ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*ln(exp(I*x))+1/4*(-I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(
I*ln(x)*(exp(2*I*x)-1))+I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x
)-1))^2+I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(
x))+I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(I*ln(x))*csgn(I*l
```

```
n(x)*(exp(2*I*x)-1))^2-I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3+I*Pi*csgn(I*ln(x
)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(ln(x)*sin(x))^3-I*Pi*csgn(
ln(x)*sin(x))*csgn(I*ln(x)*sin(x))^2+I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*s
in(x))-I*Pi*csgn(I*ln(x)*sin(x))^3+I*Pi*csgn(I*ln(x)*sin(x))^2-I*Pi-2*ln(2)
)*x^2+1/2*x^2*ln(exp(2*I*x)-1)-1/2*x^2*ln(exp(I*x)+1)+I*x*polylog(2,-exp(I*
x))-polylog(3,-exp(I*x))-1/2*x^2*ln(1-exp(I*x))+I*x*polylog(2,exp(I*x))-pol
ylog(3,exp(I*x))+1/2*ln(ln(x))*x^2+1/2*Ei(1,-2*ln(x))+1/6*I*x^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(54) = 108.

Time = 0.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.18

$$\begin{aligned}\int x\log(\log(x)\sin(x))\,dx &= \frac{1}{2}x^2\log\left(\log\left(x\right)\sin\left(x\right)\right) - \frac{1}{4}x^2\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\&\quad-\frac{1}{4}x^2\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\&\quad-\frac{1}{4}x^2\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\&\quad-\frac{1}{4}x^2\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\&\quad+\frac{1}{2}i\,x\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right)) - \frac{1}{2}i\,x\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))\\&\quad-\frac{1}{2}i\,x\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right)) + \frac{1}{2}i\,x\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))\\&\quad-\frac{1}{2}\ \mathrm{log\_integral}\left(x^2\right) - \frac{1}{2}\,\mathrm{polylog}(3,\cos\left(x\right)+i\,\sin\left(x\right))\\&\quad-\frac{1}{2}\,\mathrm{polylog}(3,\cos\left(x\right)-i\,\sin\left(x\right))\\&\quad-\frac{1}{2}\,\mathrm{polylog}(3,-\cos\left(x\right)+i\,\sin\left(x\right))\\&\quad-\frac{1}{2}\,\mathrm{polylog}(3,-\cos\left(x\right)-i\,\sin\left(x\right))\end{aligned}$$

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x) + 1) - 1/4*x^2*l
og(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x) + 1) - 1/4*x^2*l
og(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin(x)) - 1/2*I*x*dil
og(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)) + 1/2*I*x*dilog(-
cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylog(3, cos(x) + I*sin(
x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(3, -cos(x) + I*sin(x)
) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

Sympy [F]

$$\int x\log(\log(x)\sin(x))\,dx = \int x\log\left(\log\left(x\right)\sin\left(x\right)\right)\,dx$$

```
[In] integrate(x*ln(ln(x)*sin(x)),x)
[Out] Integral(x*log(log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int x\log(\log(x)\sin(x))\,dx = -\frac{1}{4}\,(-i\,\pi + 2\,\log\left(2\right))x^2 - \frac{1}{3}i\,x^3 + \frac{1}{2}\,x^2\log\left(\log\left(x\right)\right) + i\,x\mathrm{Li}_2\left(-e^{(i\,x)}\right) + i\,x\mathrm{Li}_2\left(e^{(i\,x)}\right) - \frac{1}{2}\,\mathrm{Ei}(2\,\log\left(x\right)) - \mathrm{Li}_3(-e^{(i\,x)}) - \mathrm{Li}_3(e^{(i\,x)})$$

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="maxima")
[Out] -1/4*(-I*pi + 2*log(2))*x^2 - 1/3*I*x^3 + 1/2*x^2*log(log(x)) + I*x*dilog(-
e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^(I*x)) - p
olylog(3, e^(I*x))
```

Giac [F]

$$\int x\log(\log(x)\sin(x))\,dx = \int x\log\left(\log\left(x\right)\sin\left(x\right)\right)\,dx$$

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="giac")
[Out] integrate(x*log(log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log(\log(x)\sin(x))\,dx = \int x\,\ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

```
[In] int(x*log(log(x)*sin(x)),x)
[Out] int(x*log(log(x)*sin(x)), x)
```

3.307 $\int \log(\log(x)\sin(x))\,dx$

Optimal result 1575
Rubi [A] (verified) 1575
Mathematica [A] (verified) 1577
Maple [C] (warning: unable to verify) 1577
Fricas [B] (verification not implemented) 1578
Sympy [F] 1578
Maxima [A] (verification not implemented) 1578
Giac [F] 1579
Mupad [F(-1)] 1579

Optimal result

Integrand size = 6, antiderivative size = 52

$$\int \log(\log(x)\sin(x))\,dx = \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \text{LogIntegral}(x) + \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] 1/2*I*x^2-Li(x)-x*ln(1-exp(2*I*x))+x*ln(ln(x)*sin(x))+1/2*I*polylog(2,exp(2
*I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2629, 3798, 2221, 2317, 2438, 2335}

$$\int \log(\log(x)\sin(x))\,dx = -\text{LogIntegral}(x) + \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right) + \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x))$$

```
[In] Int[Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[Log[x]*Sin[x]] - LogIntegral[x]
+ (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2629

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log(\log(x)\sin(x)) - \int\left(x\cot(x)+\frac{1}{\log(x)}\right)dx\\
&= x\log(\log(x)\sin(x)) - \int x\cot(x)\,dx - \int\frac{1}{\log(x)}\,dx\\
&= \frac{ix^2}{2} + x\log(\log(x)\sin(x)) - \text{li}(x) + 2i\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \text{li}(x) + \int\log\left(1-e^{2ix}\right)\,dx\\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \text{li}(x) - \frac{1}{2}i\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)\\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \text{li}(x) + \frac{1}{2}i\,\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \log(\log(x)\sin(x))\,dx = -x\log\left(1-e^{2ix}\right)+x\log(\log(x)\sin(x)) - \text{LogIntegral}(x)+\frac{1}{2}i\left(x^2+\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

```
[In] Integrate[Log[Log[x]*Sin[x]],x]
[Out] -(x*Log[1 - E^((2*I)*x)]) + x*Log[Log[x]*Sin[x]] - LogIntegral[x] + (I/2)*(
x^2 + PolyLog[2, E^((2*I)*x)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 368, normalized size of antiderivative = 7.08

method	result
risch	$-x\ln\left(e^{ix}\right)+\frac{ix^2}{2}+\frac{i\pi\,\text{csgn}(\ln(x)\sin(x))\,\text{csgn}(i\ln(x)\sin(x))x}{2}-\frac{i\pi x}{2}+\frac{i\pi\,\text{csgn}\left(ie^{-ix}\right)\text{csgn}(\ln(x)\sin(x))^2x}{2}+\frac{i\pi\,\text{csgn}(\ln$

```
[In] int(ln(ln(x)*sin(x)),x,method=_RETURNVERBOSE)
[Out] -x*ln(exp(I*x))+1/2*I*x^2+1/2*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*sin(x))*
x-1/2*I*Pi*x+1/2*I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2*x+1/2*I*Pi*csg
n(ln(x)*sin(x))^3*x-I*ln(exp(I*x))*ln(exp(2*I*x)-1)-x*ln(2)+1/2*I*Pi*csgn(I
*exp(-I*x))*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))*x+I*ln(exp(I*x)
)*ln(exp(I*x)+1)-1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3*x-1/2*I*Pi*csgn(I*
(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-1))*x-1/2*I*Pi*csgn(
ln(x)*sin(x))*csgn(I*ln(x)*sin(x))^2*x+1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1)
)*csgn(ln(x)*sin(x))^2*x+I*dilog(exp(I*x)+1)-I*dilog(exp(I*x))+1/2*I*Pi*csg
n(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x+1/2*I*Pi*csgn(I*(exp(2*I*x)-1))
*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x+1/2*I*Pi*csgn(I*ln(x)*sin(x))^2*x-1/2*I*P
i*csgn(I*ln(x)*sin(x))^3*x+ln(ln(x))*x+Ei(1,-ln(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(37) = 74.

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.10

$$\int \log(\log(x)\sin(x))\,dx = x\log\left(\log\left(x\right)\sin\left(x\right)\right) - \frac{1}{2}\,x\log\left(\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(-\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(-\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) + \frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) + i\,\sin\left(x\right)) - \frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) - i\,\sin\left(x\right)) - \frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) + i\,\sin\left(x\right)) + \frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) - i\,\sin\left(x\right)) - \mathrm{log\_integral}\left(x\right)$$

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] x*log(log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x)
- I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) - I
*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*sin(
x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x)) - l
og_integral(x)
```

Sympy [F]

$$\int \log(\log(x)\sin(x))\,dx = \int \log\left(\log\left(x\right)\sin\left(x\right)\right)\,dx$$

```
[In] integrate(ln(ln(x)*sin(x)),x)
```

```
[Out] Integral(log(log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \log(\log(x)\sin(x))\,dx = \frac{1}{2}\,(i\,\pi - 2\,\log\left(2\right))x - \frac{1}{2}i\,x^2 + x\log\left(\log\left(x\right)\right) - \mathrm{Ei}(\log\left(x\right)) + i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)$$

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(I*pi - 2*log(2))*x - 1/2*I*x^2 + x*log(log(x)) - Ei(log(x)) + I*dilog(
-e^(I*x)) + I*dilog(e^(I*x))
```

Giac [F]

$$\int \log(\log(x)\sin(x))\,dx = \int \log\left(\log\left(x\right)\sin\left(x\right)\right)\,dx$$

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="giac")
[Out] integrate(log(log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log(\log(x)\sin(x))\,dx = \int \ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

```
[In] int(log(log(x)*sin(x)),x)
[Out] int(log(log(x)*sin(x)), x)
```

3.308 $\int \frac{\log(\log(x)\sin(x))}{x}\,dx$

Optimal result 1580
Rubi [N/A] 1580
Mathematica [N/A] 1581
Maple [N/A] 1581
Fricas [N/A] 1581
Sympy [N/A] 1581
Maxima [N/A] 1582
Giac [N/A] 1582
Mupad [N/A] 1582

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \text{Int}\left(\frac{\log(\log(x)\sin(x))}{x},x\right)$$

```
[Out] CannotIntegrate(ln(ln(x)*sin(x))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

```
[In] Int[Log[Log[x]*Sin[x]]/x,x]
[Out] Defer[Int][Log[Log[x]*Sin[x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

```
[In] Integrate[Log[Log[x]*Sin[x]]/x,x]
[Out] Integrate[Log[Log[x]*Sin[x]]/x, x]
```

Maple [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(\ln\left(x\right)\sin\left(x\right)\right)}{x}dx$$

```
[In] int(ln(ln(x)*sin(x))/x,x)
[Out] int(ln(ln(x)*sin(x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="fricas")
[Out] integral(log(log(x)*sin(x))/x, x)
```

Sympy [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

```
[In] integrate(ln(ln(x)*sin(x))/x,x)
[Out] Integral(log(log(x)*sin(x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 101, normalized size of antiderivative = 10.10

$$\int \frac{\log(\log(x)\sin(x))}{x} \, dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x} \, dx$$

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="maxima")
```

```
[Out] -(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x) +
 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x)) +
integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - integrat
e(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x} \, dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x} \, dx$$

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="giac")
```

```
[Out] integrate(log(log(x)*sin(x))/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x} \, dx = \int \frac{\ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)}{x} \, dx$$

```
[In] int(log(log(x)*sin(x))/x,x)
```

```
[Out] int(log(log(x)*sin(x))/x, x)
```

3.309 $\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$

Optimal result 1583
Rubi [N/A] 1583
Mathematica [N/A] 1584
Maple [N/A] 1584
Fricas [N/A] 1584
Sympy [N/A] 1585
Maxima [N/A] 1585
Giac [N/A] 1585
Mupad [N/A] 1586

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \text{ExpIntegralEi}(-\log(x)) - \frac{\log(\log(x)\sin(x))}{x} + \text{Int}\left(\frac{\cot(x)}{x}, x\right)$$

```
[Out] Ei(-ln(x))-ln(ln(x)*sin(x))/x+Unintegrable(cot(x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$$

```
[In] Int[Log[Log[x]*Sin[x]]/x^2,x]
```

```
[Out] ExpIntegralEi[-Log[x]] - Log[Log[x]*Sin[x]]/x + Defer[Int][Cot[x]/x, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log(\log(x)\sin(x))}{x} - \int \frac{-1 - x\cot(x)\log(x)}{x^2\log(x)}\,dx \\
&= -\frac{\log(\log(x)\sin(x))}{x} - \int \left(-\frac{\cot(x)}{x} - \frac{1}{x^2\log(x)}\right)dx \\
&= -\frac{\log(\log(x)\sin(x))}{x} + \int \frac{\cot(x)}{x}\,dx + \int \frac{1}{x^2\log(x)}\,dx
\end{aligned}$$

$$= -\frac{\log(\log(x)\sin(x))}{x} + \int \frac{\cot(x)}{x}\,dx + \text{Subst}\left(\int \frac{e^{-x}}{x}\,dx, x, \log(x)\right)$$

$$= \text{Ei}(-\log(x)) - \frac{\log(\log(x)\sin(x))}{x} + \int \frac{\cot(x)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$$

```
[In] Integrate[Log[Log[x]*Sin[x]]/x^2,x]
[Out] Integrate[Log[Log[x]*Sin[x]]/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(\ln\left(x\right)\sin\left(x\right)\right)}{x^2}dx$$

```
[In] int(ln(ln(x)*sin(x))/x^2,x)
[Out] int(ln(ln(x)*sin(x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="fricas")
[Out] integral(log(log(x)*sin(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 20.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(ln(ln(x)*sin(x))/x**2,x)
[Out] Integral(log(log(x)*sin(x))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 12.10

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="maxima")
[Out] 1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*cos
(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)^2
 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*log(2) - log(cos(x)^2 + sin(x)^2 +
2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*log(log(x)))/x
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="giac")
[Out] integrate(log(log(x)*sin(x))/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx = \int \frac{\ln(\ln(x)\ \sin(x))}{x^2}\,dx$$

```
[In] int(log(log(x)*sin(x))/x^2,x)
[Out] int(log(log(x)*sin(x))/x^2, x)
```

3.310 $\int x^2 \log\left(e^x \log(x) \sin(x)\right)\,dx$

Optimal result 1587
Rubi [A] (verified) 1587
Mathematica [A] (verified) 1590
Maple [C] (warning: unable to verify) 1591
Fricas [B] (verification not implemented) 1592
Sympy [F] 1593
Maxima [A] (verification not implemented) 1593
Giac [F] 1593
Mupad [F(-1)] 1594

Optimal result

Integrand size = 13, antiderivative size = 103

$$\begin{aligned}\int x^2 \log\left(e^x \log(x) \sin(x)\right)\,dx =& \left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}\text{ExpIntegralEi}(3\log(x))\\&-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)\\&+\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)\\&-\frac{1}{2}x\,\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{4}i\,\text{PolyLog}\left(4,e^{2ix}\right)\end{aligned}$$

```
[Out] (-1/12+1/12*I)*x^4-1/3*Ei(3*ln(x))-1/3*x^3*ln(1-exp(2*I*x))+1/3*x^3*ln(exp(
x)*ln(x)*sin(x))+1/2*I*x^2*polylog(2,exp(2*I*x))-1/2*x*polylog(3,exp(2*I*x)
)-1/4*I*polylog(4,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {30, 2635, 12, 14, 3798, 2221, 2611, 6744, 2320, 6724, 2346, 2209}

$$\begin{aligned}\int x^2 \log\left(e^x \log(x) \sin(x)\right)\,dx =& -\frac{1}{3}\text{ExpIntegralEi}(3\log(x))\\&+\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}x\,\text{PolyLog}\left(3,e^{2ix}\right)\\&-\frac{1}{4}i\,\text{PolyLog}\left(4,e^{2ix}\right)+\left(-\frac{1}{12}+\frac{i}{12}\right)x^4\\&-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)\end{aligned}$$

```
[In] Int[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/12 + I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])
/3 + (x^3*Log[E^x*Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (
x*PolyLog[3, E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
```

```
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \int \frac{1}{3}x^3\left(1+\cot(x)+\frac{1}{x\log(x)}\right)\,dx \\
&= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int x^3\left(1+\cot(x)+\frac{1}{x\log(x)}\right)\,dx \\
&= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int \left(x^3(1+\cot(x))+\frac{x^2}{\log(x)}\right)\,dx
\end{aligned}
$$

$$= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int x^3(1+\cot(x))\,dx - \frac{1}{3}\int \frac{x^2}{\log(x)}\,dx$$

$$= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int \left(x^3+x^3\cot(x)\right)\,dx - \frac{1}{3}\text{Subst}\left(\int \frac{e^{3x}}{x}\,dx, x, \log(x)\right)$$

$$= -\frac{x^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int x^3\cot(x)\,dx$$

$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{2}{3}i\int \frac{e^{2ix}x^3}{1-e^{2ix}}\,dx$$

$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \int x^2\log\left(1-e^{2ix}\right)\,dx$$

$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right) - i\int x\text{Li}_2\left(e^{2ix}\right)\,dx$$

$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right) - \frac{1}{2}x\text{Li}_3\left(e^{2ix}\right) + \frac{1}{2}\int \text{Li}_3\left(e^{2ix}\right)\,dx$$

$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right) - \frac{1}{2}x\text{Li}_3\left(e^{2ix}\right) - \frac{1}{4}i\text{Subst}\left(\int \frac{\text{Li}_3(x)}{x}\,dx, x, e^{2ix}\right)$$

$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right) - \frac{1}{2}x\text{Li}_3\left(e^{2ix}\right) - \frac{1}{4}i\text{Li}_4\left(e^{2ix}\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int x^2\log\left(e^x\log(x)\sin(x)\right)\,dx = \frac{1}{192}i\Big(\pi^4 - (16-16i)x^4 + 64i\,\text{ExpIntegralEi}(3\log(x)) + 64ix^3\log\left(1-e^{-2ix}\right) - 64ix^3\log\left(e^x\log(x)\sin(x)\right) - 96x^2\,\text{PolyLog}\left(2,e^{-2ix}\right) + 96ix\,\text{PolyLog}\left(3,e^{-2ix}\right) + 48\,\text{PolyLog}\left(4,e^{-2ix}\right)\Big)$$

```
[In] Integrate[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (I/192)*(Pi^4 - (16 - 16*I)*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*x
^3*Log[1 - E^((-2*I)*x)] - (64*I)*x^3*Log[E^x*Log[x]*Sin[x]] - 96*x^2*PolyL
og[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^(
(-2*I)*x)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.03 (sec) , antiderivative size = 643, normalized size of antiderivative = 6.24

method	result	size
risch	Expression too large to display	643

```
[In] int(x^2*ln(exp(x)*ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3*ln(exp(I*x))+1/6*(-I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^
3-I*Pi*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^3+I*Pi*csgn(I*ln(x)*(exp((1+
I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^2+I*Pi*csgn(I*
exp(x))*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))+I*Pi*c
sgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^2+I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(x
)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))+I*Pi*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2
*I*x)-1))^2-I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I
*x)-1))+I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(I*
exp(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2-I*Pi*csgn(I*ln(x)*(exp(
2*I*x)-1))^3-I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x
)))^2+I*Pi*csgn(ln(x)*sin(x))^3+I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2
-I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x)-exp((1-I
)*x))*ln(x))-I*Pi+I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x)-1))^
2-2*ln(2))*x^3+1/3*x^3*ln(exp(2*I*x)-1)-1/3*x^3*ln(exp(I*x)+1)+1/12*I*x^4-2
*x*polylog(3,-exp(I*x))-2*I*polylog(4,exp(I*x))-1/3*x^3*ln(1-exp(I*x))+I*x^
2*polylog(2,-exp(I*x))-2*x*polylog(3,exp(I*x))-2*I*polylog(4,-exp(I*x))+1/3
*x^3*ln(exp(x))-1/12*x^4+1/3*x^3*ln(ln(x))+1/3*Ei(1,-3*ln(x))+I*x^2*polylog
(2,exp(I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(67) = 134.

Time = 0.32 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.34

$$\begin{aligned}\int x^2\log\left(e^x\log(x)\sin(x)\right)\,dx =& -\frac{1}{12}\,x^4+\frac{1}{3}\,x^3\log\left(e^x\log\left(x\right)\sin\left(x\right)\right)\\&-\frac{1}{6}\,x^3\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\&-\frac{1}{6}\,x^3\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\&-\frac{1}{6}\,x^3\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)\\&-\frac{1}{6}\,x^3\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)\\&+\frac{1}{2}i\,x^2\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))-\frac{1}{2}i\,x^2\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))\\&-\frac{1}{2}i\,x^2\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))\\&+\frac{1}{2}i\,x^2\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))\\&-x\mathrm{polylog}(3,\cos\left(x\right)+i\,\sin\left(x\right))\\&-x\mathrm{polylog}(3,\cos\left(x\right)-i\,\sin\left(x\right))\\&-x\mathrm{polylog}(3,-\cos\left(x\right)+i\,\sin\left(x\right))\\&-x\mathrm{polylog}(3,-\cos\left(x\right)-i\,\sin\left(x\right))\\&-\frac{1}{3}\,\mathrm{log\_integral}\left(x^3\right)-i\,\mathrm{polylog}(4,\cos\left(x\right)+i\,\sin\left(x\right))\\&+i\,\mathrm{polylog}(4,\cos\left(x\right)-i\,\sin\left(x\right))\\&+i\,\mathrm{polylog}(4,-\cos\left(x\right)+i\,\sin\left(x\right))\\&-i\,\mathrm{polylog}(4,-\cos\left(x\right)-i\,\sin\left(x\right))\end{aligned}$$

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/12*x^4 + 1/3*x^3*log(e^x*log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x)
+ 1) - 1/6*x^3*log(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x)
+ 1) - 1/6*x^3*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*sin
(x)) - 1/2*I*x^2*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I*sin
(x)) + 1/2*I*x^2*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*sin(x)
) - x*polylog(3, cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)) - x*
polylog(3, -cos(x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4, cos(x
) + I*sin(x)) + I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos(x) + I*
sin(x)) - I*polylog(4, -cos(x) - I*sin(x))
```

Sympy [F]

$$\int x^2 \log \left(e^x \log(x) \sin(x)\right)\, dx = \int x^2 \log \left(e^x \log \left(x\right) \sin \left(x\right)\right) dx$$

```
[In] integrate(x**2*ln(exp(x)*ln(x)*sin(x)),x)
[Out] Integral(x**2*log(exp(x)*log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91

$$\begin{aligned}\int x^2 \log \left(e^x \log(x) \sin(x)\right)\, dx = & -\frac{1}{6}\left(-i\,\pi + 2\,\log\left(2\right)\right)x^3 - \left(\frac{1}{4}i - \frac{1}{4}\right)\,x^4 \\ & + \frac{1}{3}\,x^3 \log\left(\log\left(x\right)\right) + i\,x^2 \mathrm{Li}_2\left(-e^{(i\,x)}\right) \\ & + i\,x^2 \mathrm{Li}_2\left(e^{(i\,x)}\right) - 2\,x\mathrm{Li}_3\left(-e^{(i\,x)}\right) - 2\,x\mathrm{Li}_3\left(e^{(i\,x)}\right) \\ & - \frac{1}{3}\,\mathrm{Ei}\left(3\,\log\left(x\right)\right) - 2i\,\mathrm{Li}_4\left(-e^{(i\,x)}\right) - 2i\,\mathrm{Li}_4\left(e^{(i\,x)}\right)\end{aligned}$$

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
[Out] -1/6*(-I*pi + 2*log(2))*x^3 - (1/4*I - 1/4)*x^4 + 1/3*x^3*log(log(x)) + I*x
^2*dilog(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*x*
polylog(3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*pol
ylog(4, e^(I*x))
```

Giac [F]

$$\int x^2 \log \left(e^x \log(x) \sin(x)\right)\, dx = \int x^2 \log \left(e^x \log \left(x\right) \sin \left(x\right)\right)\, dx$$

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
[Out] integrate(x^2*log(e^x*log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log \left(e^x \log(x) \sin(x)\right) \, dx = \int x^2 \, \ln \left(\mathrm{e}^x \, \ln \left(x\right) \, \sin \left(x\right)\right) \, dx$$

```
[In] int(x^2*log(exp(x)*log(x)*sin(x)),x)
[Out] int(x^2*log(exp(x)*log(x)*sin(x)), x)
```

3.311 $\int x \log(e^x \log(x) \sin(x)) \, dx$

Optimal result . . . 1595
Rubi [A] (verified) . . . 1595
Mathematica [A] (verified) . . . 1598
Maple [C] (warning: unable to verify) . . . 1598
Fricas [B] (verification not implemented) . . . 1599
Sympy [F] . . . 1600
Maxima [A] (verification not implemented) . . . 1600
Giac [F] . . . 1601
Mupad [F(-1)] . . . 1601

Optimal result

Integrand size = 11, antiderivative size = 85

$$\int x \log\left(e^x \log(x) \sin(x)\right) \, dx = \left(-\frac{1}{6}+\frac{i}{6}\right) x^3 - \frac{1}{2} \text{ExpIntegralEi}(2 \log(x)) - \frac{1}{2} x^2 \log\left(1-e^{2 i x}\right) + \frac{1}{2} x^2 \log\left(e^x \log(x) \sin(x)\right) + \frac{1}{2} i x \,\text{PolyLog}\left(2, e^{2 i x}\right) - \frac{1}{4} \text{PolyLog}\left(3, e^{2 i x}\right)$$

```
[Out] (-1/6+1/6*I)*x^3-1/2*Ei(2*ln(x))-1/2*x^2*ln(1-exp(2*I*x))+1/2*x^2*ln(exp(x)
*ln(x)*sin(x))+1/2*I*x*polylog(2,exp(2*I*x))-1/4*polylog(3,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {30, 2635, 12, 14, 3798, 2221, 2611, 2320, 6724, 2346, 2209}

$$\int x \log\left(e^x \log(x) \sin(x)\right) \, dx = -\frac{1}{2} \text{ExpIntegralEi}(2 \log(x)) + \frac{1}{2} i x \,\text{PolyLog}\left(2, e^{2 i x}\right) - \frac{1}{4} \text{PolyLog}\left(3, e^{2 i x}\right) + \left(-\frac{1}{6}+\frac{i}{6}\right) x^3 - \frac{1}{2} x^2 \log\left(1-e^{2 i x}\right) + \frac{1}{2} x^2 \log\left(e^x \log(x) \sin(x)\right)$$

```
[In] Int[x*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/6 + I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2
 + (x^2*Log[E^x*Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyL
og[3, E^((2*I)*x)]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\int\frac{1}{2}x^2\left(1+\cot(x)+\frac{1}{x\log(x)}\right)\,dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2\left(1+\cot(x)+\frac{1}{x\log(x)}\right)\,dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int\left(x^2(1+\cot(x))+\frac{x}{\log(x)}\right)\,dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2(1+\cot(x))\,dx-\frac{1}{2}\int\frac{x}{\log(x)}\,dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int\left(x^2+x^2\cot(x)\right)\,dx-\frac{1}{2}\text{Subst}\left(\int\frac{e^{2x}}{x}\,dx,x,\log(x)\right)\\
&= -\frac{x^3}{6}-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2\cot(x)\,dx\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+i\int\frac{e^{2ix}x^2}{1-e^{2ix}}\,dx
\end{aligned}
$$

$$= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3 - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right) + \int x\log\left(1-e^{2ix}\right)dx$$

$$= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3 - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right) - \frac{1}{2}i\int \text{Li}_2\left(e^{2ix}\right)dx$$

$$= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3 - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}dx, x, e^{2ix}\right)$$

$$= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3 - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right) - \frac{1}{4}\text{Li}_3\left(e^{2ix}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int x\log\left(e^x\log(x)\sin(x)\right)dx = \frac{1}{48}\left(i\pi^3 - (8+8i)x^3 - 24\,\text{ExpIntegralEi}(2\log(x)) - 24x^2\log\left(1-e^{-2ix}\right) + 24x^2\log\left(e^x\log(x)\sin(x)\right) - 24ix\,\text{PolyLog}\left(2,e^{-2ix}\right) - 12\,\text{PolyLog}\left(3,e^{-2ix}\right)\right)$$

```
[In] Integrate[x*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (I*Pi^3 - (8 + 8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((-
2*I)*x)] + 24*x^2*Log[E^x*Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*x)
] - 12*PolyLog[3, E^((-2*I)*x)])/48
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.65 (sec) , antiderivative size = 615, normalized size of antiderivative = 7.24

method	result	size
risch	Expression too large to display	615

```
[In] int(x*ln(exp(x)*ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*ln(exp(I*x))+1/2*x^2*ln(exp(2*I*x)-1)-1/2*x^2*ln(exp(I*x)+1)+I*x*p
olylog(2,-exp(I*x))-polylog(3,-exp(I*x))-1/2*x^2*ln(1-exp(I*x))+I*x*polylog
(2,exp(I*x))-polylog(3,exp(I*x))+1/2*ln(exp(x))*x^2-1/6*x^3+1/2*ln(ln(x))*x
^2+1/2*Ei(1,-2*ln(x))+1/4*(-I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^
3-I*Pi*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^3+I*Pi*csgn(I*ln(x)*(exp((1+
I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^2+I*Pi*csgn(I*
exp(x))*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))+I*Pi*c
sgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^2+I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(x
)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))+I*Pi*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2
*I*x)-1))^2-I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I
*x)-1))+I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2+I*Pi*csgn(I*
exp(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2-I*Pi*csgn(I*ln(x)*(exp(
2*I*x)-1))^3-I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x
)))^2+I*Pi*csgn(ln(x)*sin(x))^3+I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2
-I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x)-exp((1-I
)*x))*ln(x))-I*Pi+I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x)-1))^
2-2*ln(2))*x^2+1/6*I*x^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(56) = 112$.

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.13

$$\begin{aligned}\int x\log\left(e^x\log(x)\sin(x)\right)\,dx =& -\frac{1}{6}x^3+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)\\&-\frac{1}{4}x^2\log\left(\cos(x)+i\sin(x)+1\right)\\&-\frac{1}{4}x^2\log\left(\cos(x)-i\sin(x)+1\right)\\&-\frac{1}{4}x^2\log\left(-\cos(x)+i\sin(x)+1\right)\\&-\frac{1}{4}x^2\log\left(-\cos(x)-i\sin(x)+1\right)\\&+\frac{1}{2}ix\mathrm{Li}_2(\cos(x)+i\sin(x))-\frac{1}{2}ix\mathrm{Li}_2(\cos(x)-i\sin(x))\\&-\frac{1}{2}ix\mathrm{Li}_2(-\cos(x)+i\sin(x))+\frac{1}{2}ix\mathrm{Li}_2(-\cos(x)-i\sin(x))\\&-\frac{1}{2}\,\mathrm{log\_integral}\left(x^2\right)-\frac{1}{2}\mathrm{polylog}(3,\cos(x)+i\sin(x))\\&-\frac{1}{2}\mathrm{polylog}(3,\cos(x)-i\sin(x))\\&-\frac{1}{2}\mathrm{polylog}(3,-\cos(x)+i\sin(x))\\&-\frac{1}{2}\mathrm{polylog}(3,-\cos(x)-i\sin(x))\end{aligned}$$

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/6*x^3 + 1/2*x^2*log(e^x*log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x) +
 1) - 1/4*x^2*log(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x) +
 1) - 1/4*x^2*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin(x)
) - 1/2*I*x*dilog(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)) +
1/2*I*x*dilog(-cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylog(3,
cos(x) + I*sin(x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(3, -co
s(x) + I*sin(x)) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

Sympy [F]

$$\int x\log\left(e^{x}\log(x)\sin(x)\right)\,dx=\int x\log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)dx$$

```
[In] integrate(x*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Integral(x*log(exp(x)*log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\begin{aligned}\int x\log\left(e^{x}\log(x)\sin(x)\right)\,dx=&-\frac{1}{4}\left(-i\,\pi+2\,\log\left(2\right)\right)x^{2}-\left(\frac{1}{3}i-\frac{1}{3}\right)x^{3}\\&+\frac{1}{2}\,x^{2}\log\left(\log\left(x\right)\right)+i\,x\mathrm{Li}_{2}\left(-e^{(i\,x)}\right)+i\,x\mathrm{Li}_{2}\left(e^{(i\,x)}\right)\\&-\frac{1}{2}\,\mathrm{Ei}(2\,\log\left(x\right))-\mathrm{Li}_{3}(-e^{(i\,x)})-\mathrm{Li}_{3}(e^{(i\,x)})\end{aligned}$$

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(-I*pi + 2*log(2))*x^2 - (1/3*I - 1/3)*x^3 + 1/2*x^2*log(log(x)) + I*x
*dilog(-e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^(I
*x)) - polylog(3, e^(I*x))
```

Giac [F]

$$\int x\log\left(e^x\log(x)\sin(x)\right)\,dx=\int x\log\left(e^x\log\left(x\right)\sin\left(x\right)\right)\,dx$$

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
[Out] integrate(x*log(e^x*log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log\left(e^x\log(x)\sin(x)\right)\,dx=\int x\,\ln\left(\mathrm{e}^x\,\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

```
[In] int(x*log(exp(x)*log(x)*sin(x)),x)
[Out] int(x*log(exp(x)*log(x)*sin(x)), x)
```

3.312 $\int \log(e^x \log(x)\sin(x))\, dx$

Optimal result 1602
Rubi [A] (verified) 1602
Mathematica [A] (verified) 1604
Maple [C] (warning: unable to verify) 1604
Fricas [B] (verification not implemented) 1605
Sympy [F] 1605
Maxima [A] (verification not implemented) 1606
Giac [F] 1606
Mupad [F(-1)] 1606

Optimal result

Integrand size = 9, antiderivative size = 57

$$\int \log(e^x \log(x)\sin(x))\, dx = \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log(e^x\log(x)\sin(x)) - \text{LogIntegral}(x) + \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] (-1/2+1/2*I)*x^2-Li(x)-x*ln(1-exp(2*I*x))+x*ln(exp(x)*ln(x)*sin(x))+1/2*I*p
olylog(2,exp(2*I*x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2629, 3798, 2221, 2317, 2438, 2335}

$$\int \log(e^x \log(x)\sin(x))\, dx = -\text{LogIntegral}(x) + \frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right) + \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log(e^x\log(x)\sin(x))$$

```
[In] Int[Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/2 + I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[E^x*Log[x]*Sin[x]] - LogI
ntegral[x] + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2629

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(e^x\log(x)\sin(x)\right)-\int\left(x+x\cot(x)+\frac{1}{\log(x)}\right)dx\\
&= -\frac{x^2}{2}+x\log\left(e^x\log(x)\sin(x)\right)-\int x\cot(x)\,dx-\int\frac{1}{\log(x)}\,dx\\
&= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2+x\log\left(e^x\log(x)\sin(x)\right)-\text{li}(x)+2i\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2-x\log\left(1-e^{2ix}\right)+x\log\left(e^x\log(x)\sin(x)\right)-\text{li}(x)+\int\log\left(1-e^{2ix}\right)\,dx\\
&= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2-x\log\left(1-e^{2ix}\right)+x\log\left(e^x\log(x)\sin(x)\right)\\
&\quad-\text{li}(x)-\frac{1}{2}i\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)
\end{aligned}$$

$$= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \text{li}(x) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \log\left(e^x\log(x)\sin(x)\right)\,dx = \frac{1}{2}\left((-1+i)x^2 - 2x\log\left(1-e^{2ix}\right) + 2x\log\left(e^x\log(x)\sin(x)\right) - 2\,\text{LogIntegral}(x) + i\,\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]],x]
[Out] ((-1 + I)*x^2 - 2*x*Log[1 - E^((2*I)*x)] + 2*x*Log[E^x*Log[x]*Sin[x]] - 2*L
ogIntegral[x] + I*PolyLog[2, E^((2*I)*x)])/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 583, normalized size of antiderivative = 10.23

method	result	size
risch	Expression too large to display	583

```
[In] int(ln(exp(x)*ln(x)*sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*I*Pi*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x+1/2*ln(exp(x))^2-1/
2*I*Pi*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^3*x-1/2*I*Pi*csgn(I*ln(x)*(e
xp((1+I)*x)-exp((1-I)*x)))^3*x+1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)
*(exp(2*I*x)-1))^2*x+1/2*I*Pi*csgn(I*exp(x))*csgn(ln(x)*sin(x))*csgn(I*ln(x
)*(exp((1+I)*x)-exp((1-I)*x)))*x+1/2*I*Pi*csgn((exp((1+I)*x)-exp((1-I)*x))*
ln(x))^2*x-x*ln(2)-1/2*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn(
(exp((1+I)*x)-exp((1-I)*x))*ln(x))*x+1/2*I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*
sin(x))^2*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)
))^2*x+1/2*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x
)-exp((1-I)*x))*ln(x))^2*x+1/2*I*Pi*csgn(ln(x)*sin(x))^3*x+I*ln(exp(I*x))*l
n(exp(I*x)+1)+Ei(1,-ln(x))+1/2*I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(x)*(exp(2*I
*x)-1))*csgn(ln(x)*sin(x))*x-1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3*x+1/2*
I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2*x-1/2*I*Pi*x-I*ln(ex
p(I*x))*ln(exp(2*I*x)-1)-1/2*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I
)*x)-exp((1-I)*x)))^2*x-1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(
I*ln(x)*(exp(2*I*x)-1))*x+1/2*I*x^2+ln(ln(x))*x+I*dilog(exp(I*x)+1)-I*dilog
(exp(I*x))-x*ln(exp(I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(39) = 78.

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.04

$$\begin{aligned}\int \log\left(e^x \log(x)\sin(x)\right)\,dx = &-\frac{1}{2}x^2 + x\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{2}x\log\left(\cos(x) + i\sin(x) + 1\right)\\ &- \frac{1}{2}x\log\left(\cos(x) - i\sin(x) + 1\right)\\ &- \frac{1}{2}x\log\left(-\cos(x) + i\sin(x) + 1\right)\\ &- \frac{1}{2}x\log\left(-\cos(x) - i\sin(x) + 1\right) + \frac{1}{2}i\,\mathrm{Li}_2(\cos(x) + i\sin(x))\\ &- \frac{1}{2}i\,\mathrm{Li}_2(\cos(x) - i\sin(x)) - \frac{1}{2}i\,\mathrm{Li}_2(-\cos(x) + i\sin(x))\\ &+ \frac{1}{2}i\,\mathrm{Li}_2(-\cos(x) - i\sin(x)) - \mathrm{log\_integral}(x)\end{aligned}$$

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(e^x*log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/
2*x*log(cos(x) - I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*
log(-cos(x) - I*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(
cos(x) - I*sin(x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x)
- I*sin(x)) - log_integral(x)
```

Sympy [F]

$$\int \log\left(e^x \log(x)\sin(x)\right)\,dx = \int \log\left(e^x\log(x)\sin(x)\right)dx$$

```
[In] integrate(ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Integral(log(exp(x)*log(x)*sin(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \log \left(e^x \log(x)\sin(x)\right)\,dx = \frac{1}{2}\,(i\,\pi - 2\,\log\left(2\right))x - \left(\frac{1}{2}i - \frac{1}{2}\right)\,x^2 + x\log\left(\log\left(x\right)\right) - \mathrm{Ei}(\log\left(x\right)) + i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)$$

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
[Out] 1/2*(I*pi - 2*log(2))*x - (1/2*I - 1/2)*x^2 + x*log(log(x)) - Ei(log(x)) +
I*dilog(-e^(I*x)) + I*dilog(e^(I*x))
```

Giac [F]

$$\int \log \left(e^x \log(x)\sin(x)\right)\,dx = \int \log \left(e^x \log\left(x\right)\sin\left(x\right)\right)\,dx$$

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
[Out] integrate(log(e^x*log(x)*sin(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(e^x \log(x)\sin(x)\right)\,dx = \int \ln\left(\mathrm{e}^x\,\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

```
[In] int(log(exp(x)*log(x)*sin(x)),x)
[Out] int(log(exp(x)*log(x)*sin(x)), x)
```

3.313 $\int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx$

Optimal result 1607
Rubi [N/A] 1607
Mathematica [N/A] 1608
Maple [N/A] 1608
Fricas [N/A] 1608
Sympy [N/A] 1608
Maxima [N/A] 1609
Giac [N/A] 1609
Mupad [N/A] 1609

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx = \operatorname{Int}\left(\frac{\log(e^x \log(x) \sin(x))}{x}, x\right)$$

```
[Out] CannotIntegrate(ln(exp(x)*ln(x)*sin(x))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx = \int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx$$

```
[In] Int[Log[E^x*Log[x]*Sin[x]]/x,x]
[Out] Defer[Int][Log[E^x*Log[x]*Sin[x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\log (e^x \log(x) \sin(x))}{x} \, dx = \int \frac{\log (e^x \log(x) \sin(x))}{x} \, dx$$

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]]/x,x]
[Out] Integrate[Log[E^x*Log[x]*Sin[x]]/x, x]
```

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\ln (\mathrm{e}^x \ln (x) \sin (x))}{x} dx$$

```
[In] int(ln(exp(x)*ln(x)*sin(x))/x,x)
[Out] int(ln(exp(x)*ln(x)*sin(x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log (e^x \log(x) \sin(x))}{x} \, dx = \int \frac{\log (e^x \log (x) \sin (x))}{x} \, dx$$

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="fricas")
[Out] integral(log(e^x*log(x)*sin(x))/x, x)
```

Sympy [N/A]

Not integrable

Time = 13.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log (e^x \log(x) \sin(x))}{x} \, dx = \int \frac{\log (e^x \log (x) \sin (x))}{x} \, dx$$

```
[In] integrate(ln(exp(x)*ln(x)*sin(x))/x,x)
[Out] Integral(log(exp(x)*log(x)*sin(x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 7.85

$$\int \frac{\log(e^x \log(x)\sin(x))}{x}\,dx = \int \frac{\log\left(e^x \log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="maxima")
```

```
[Out] -(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x) +
 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x)) +
x + integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - inte
grate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log(e^x \log(x)\sin(x))}{x}\,dx = \int \frac{\log\left(e^x \log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="giac")
```

```
[Out] integrate(log(e^x*log(x)*sin(x))/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log(e^x \log(x)\sin(x))}{x}\,dx = \int \frac{\ln\left(\mathrm{e}^x\ \ln\left(x\right)\ \sin\left(x\right)\right)}{x}\,dx$$

```
[In] int(log(exp(x)*log(x)*sin(x))/x,x)
```

```
[Out] int(log(exp(x)*log(x)*sin(x))/x, x)
```

3.314 $\int \frac{\log(e^x \log(x) \sin(x))}{x^2} dx$

Optimal result . . . 1610
Rubi [N/A] . . . 1610
Mathematica [N/A] . . . 1611
Maple [N/A] . . . 1611
Fricas [N/A] . . . 1611
Sympy [N/A] . . . 1612
Maxima [N/A] . . . 1612
Giac [N/A] . . . 1612
Mupad [N/A] . . . 1613

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\log\left(e^x \log(x) \sin(x)\right)}{x^2} dx = \text{ExpIntegralEi}(-\log(x)) + \log(x) - \frac{\log\left(e^x \log(x) \sin(x)\right)}{x} + \text{Int}\left(\frac{\cot(x)}{x}, x\right)$$

```
[Out] Ei(-ln(x))+ln(x)-ln(exp(x)*ln(x)*sin(x))/x+Unintegrable(cot(x)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(e^x \log(x) \sin(x)\right)}{x^2} dx = \int \frac{\log\left(e^x \log(x) \sin(x)\right)}{x^2} dx$$

```
[In] Int[Log[E^x*Log[x]*Sin[x]]/x^2,x]
```

```
[Out] ExpIntegralEi[-Log[x]] + Log[x] - Log[E^x*Log[x]*Sin[x]]/x + Defer[Int][Cot
[x]/x, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\log\left(e^x \log(x) \sin(x)\right)}{x} + \int \frac{1 + \cot(x) + \frac{1}{x \log(x)}}{x} dx \\ &= -\frac{\log\left(e^x \log(x) \sin(x)\right)}{x} + \int \left(\frac{1 + \cot(x)}{x} + \frac{1}{x^2 \log(x)}\right) dx \end{aligned}$$

$$= -\frac{\log\left(e^{x}\log(x)\sin(x)\right)}{x} + \int \frac{1+\cot(x)}{x}\,dx + \int \frac{1}{x^{2}\log(x)}\,dx$$

$$= -\frac{\log\left(e^{x}\log(x)\sin(x)\right)}{x} + \int \left(\frac{1}{x} + \frac{\cot(x)}{x}\right)\,dx + \text{Subst}\left(\int \frac{e^{-x}}{x}\,dx, x, \log(x)\right)$$

$$= \text{Ei}(-\log(x)) + \log(x) - \frac{\log\left(e^{x}\log(x)\sin(x)\right)}{x} + \int \frac{\cot(x)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(e^{x}\log(x)\sin(x)\right)}{x^{2}}\,dx = \int \frac{\log\left(e^{x}\log(x)\sin(x)\right)}{x^{2}}\,dx$$

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]]/x^2,x]
[Out] Integrate[Log[E^x*Log[x]*Sin[x]]/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\ln\left(\mathrm{e}^{x}\ln\left(x\right)\sin\left(x\right)\right)}{x^{2}}dx$$

```
[In] int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
[Out] int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log\left(e^{x}\log(x)\sin(x)\right)}{x^{2}}\,dx = \int \frac{\log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)}{x^{2}}\,dx$$

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="fricas")
[Out] integral(log(e^x*log(x)*sin(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 59.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x^2}\,dx = \int \frac{\log \left(e^x \log\left(x\right) \sin\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(ln(exp(x)*ln(x)*sin(x))/x**2,x)
[Out] Integral(log(exp(x)*log(x)*sin(x))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 9.69

$$\int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x^2}\,dx = \int \frac{\log \left(e^x \log\left(x\right) \sin\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="maxima")
[Out] 1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*cos
(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)^2
 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*x*log(x) + 2*log(2) - log(cos(x)^2
+ sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*lo
g(log(x)))/x
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log \left(e^x \log(x) \sin(x)\right)}{x^2}\,dx = \int \frac{\log \left(e^x \log\left(x\right) \sin\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="giac")
[Out] integrate(log(e^x*log(x)*sin(x))/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log(e^x \log(x)\sin(x))}{x^2} \, dx = \int \frac{\ln\left(\mathrm{e}^x\ \ln(x)\ \sin(x)\right)}{x^2} \, dx$$

```
[In] int(log(exp(x)*log(x)*sin(x))/x^2,x)
[Out] int(log(exp(x)*log(x)*sin(x))/x^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 1615

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).___name___ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```