Computer Algebra
Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.3-Inverse-tangent/152-5.3.6-Exponentials-of-inverse-tangent

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [385]. This is test number [152].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (385)	0.00 (0)
Mathematica	95.58 (368)	4.42 (17)
Fricas	74.29 (286)	25.71 (99)
Maple	52.73 (203)	47.27 (182)
Mupad	38.18 (147)	61.82 (238)
Maxima	34.81 (134)	65.19 (251)
Giac	30.39 (117)	69.61 (268)
Sympy	23.64 (91)	76.36 (294)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

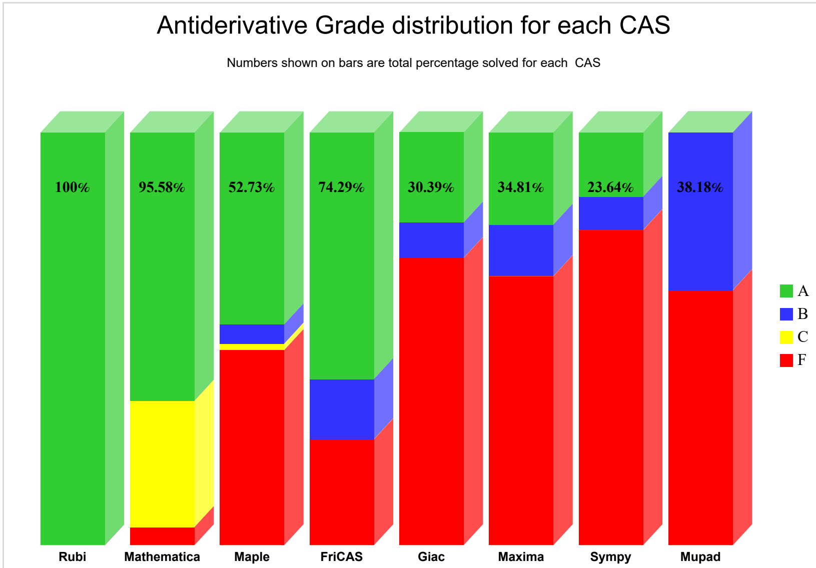
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

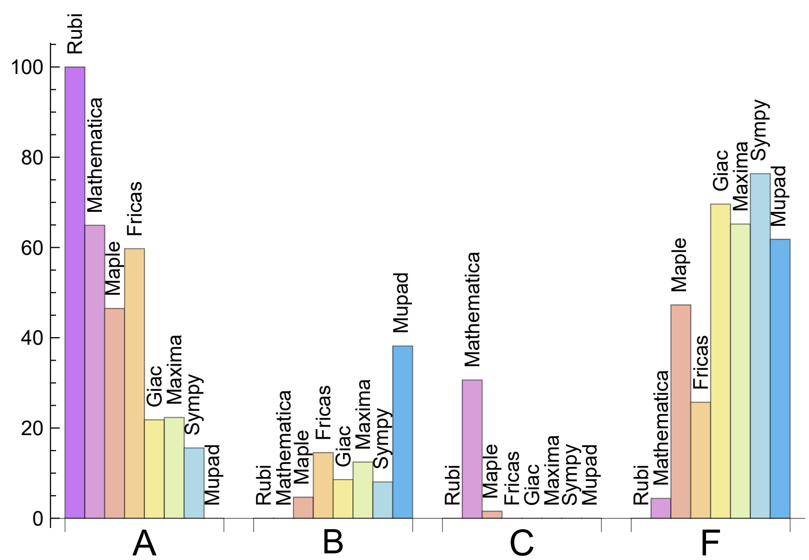
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	64.935	0.000	30.649	4.416
Fricas	59.740	14.545	0.000	25.714
Maple	46.494	4.675	1.558	47.273
Maxima	22.338	12.468	0.000	65.195
Giac	21.818	8.571	0.000	69.610
Sympy	15.584	8.052	0.000	76.364
Mupad	0.000	38.182	0.000	61.818

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	17	100.00	0.00	0.00
Fricas	99	100.00	0.00	0.00
Maple	182	100.00	0.00	0.00
Mupad	238	0.00	100.00	0.00
Maxima	251	95.62	0.00	4.38
Giac	268	55.22	2.24	42.54
Sympy	294	81.63	18.37	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Mathematica	0.06
Rubi	0.09
Maxima	0.23
Fricas	0.27
Giac	0.29
Mupad	0.61
Maple	2.02
Sympy	4.41

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	68.97	1.09	52.00	0.95
Mathematica	85.10	0.80	76.50	0.83
Giac	114.21	1.40	68.00	1.14
Maple	115.70	1.38	68.00	0.97
Sympy	124.20	1.76	54.00	1.16
Rubi	142.49	1.01	95.00	1.00
Fricas	176.45	1.31	118.50	1.00
Maxima	221.49	1.80	64.50	1.25

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

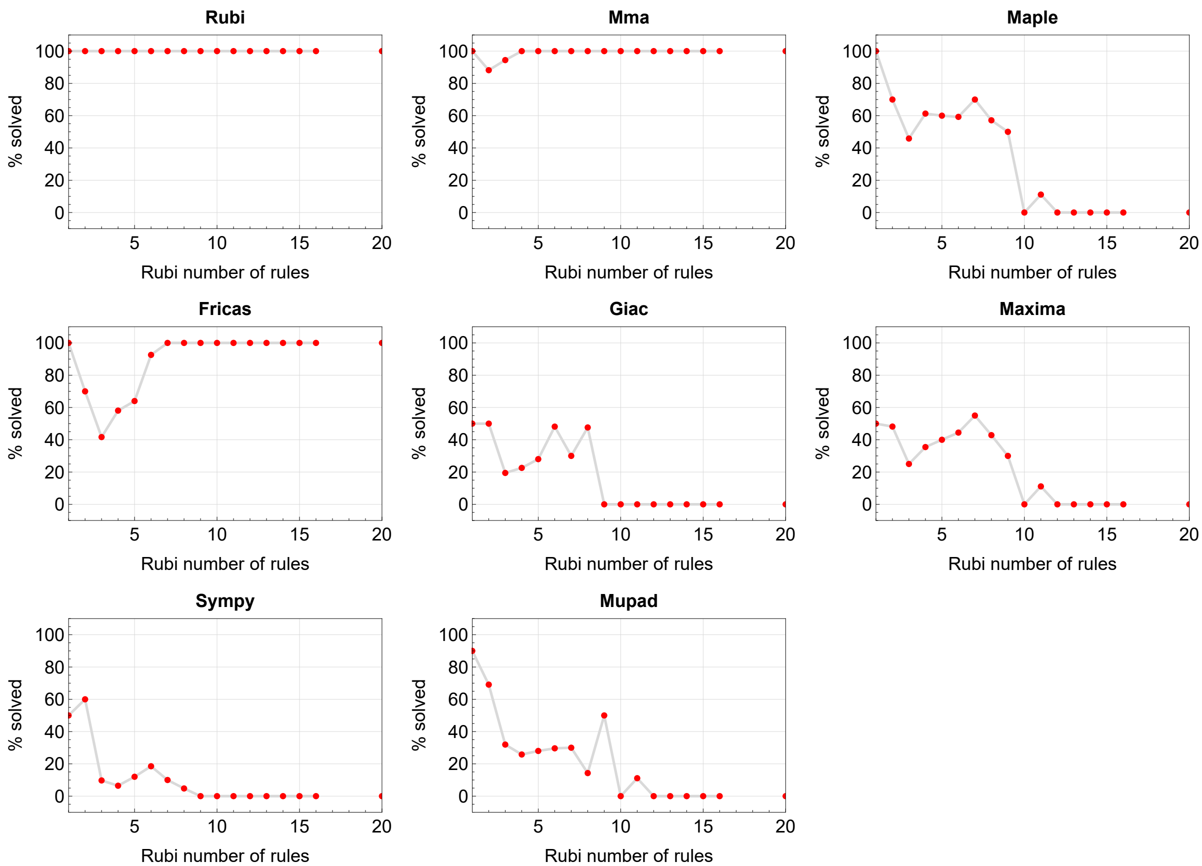


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

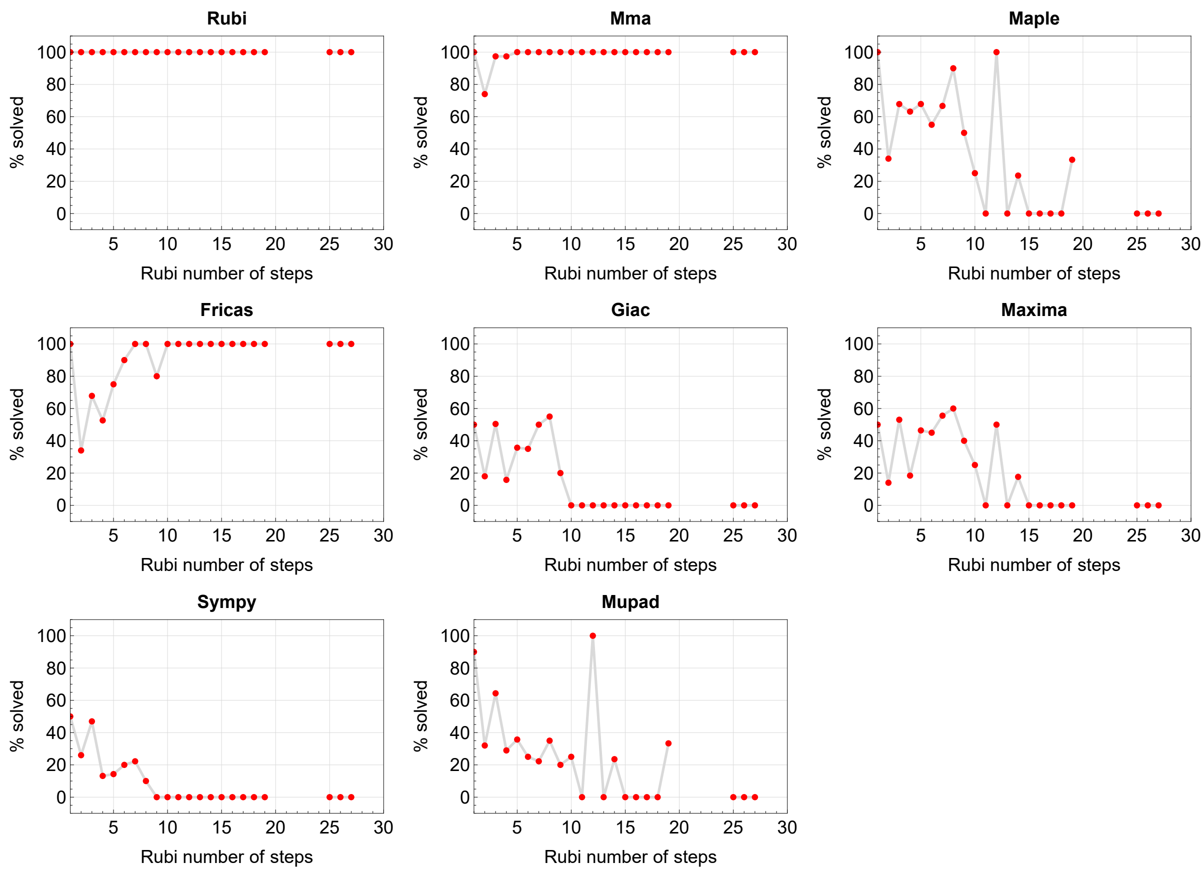


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

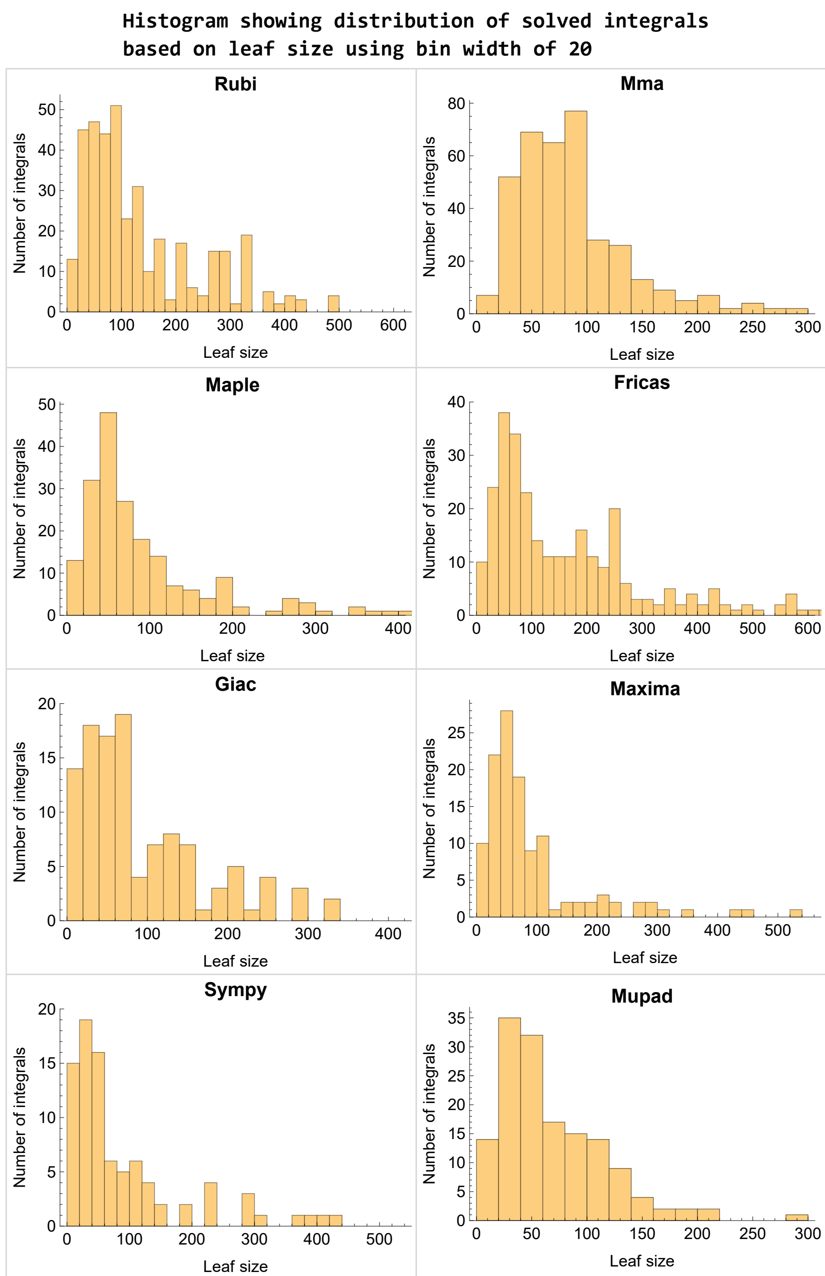


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

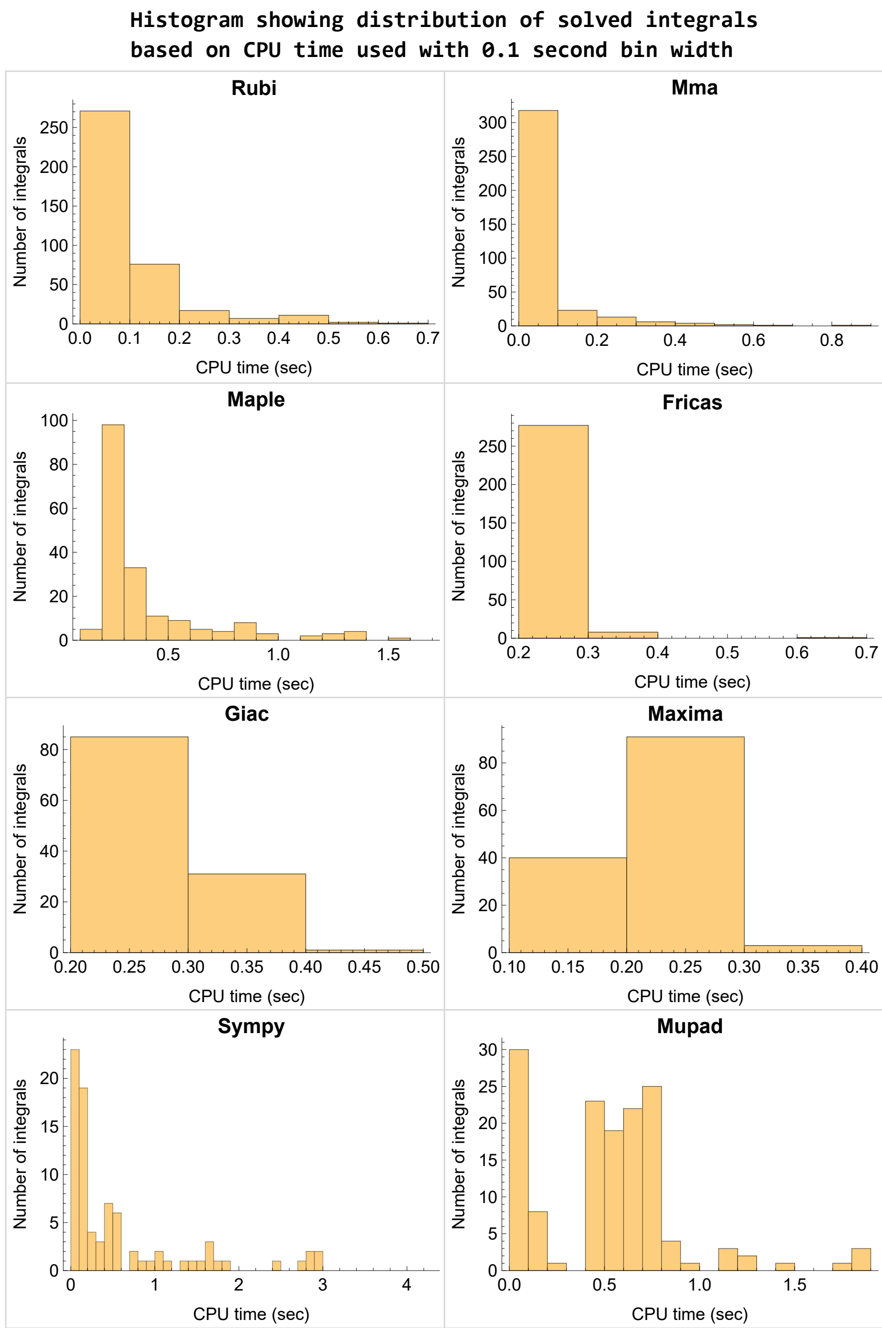


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

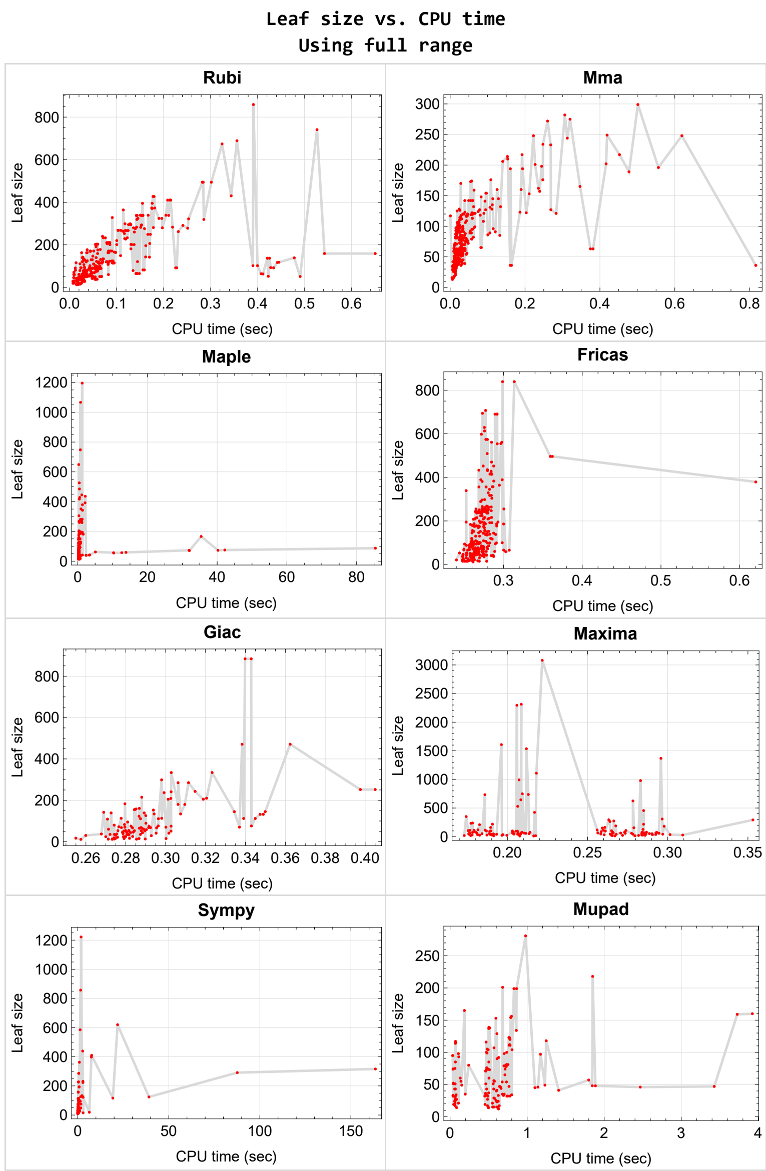


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {140, 141, 142, 143}

Maple {212}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

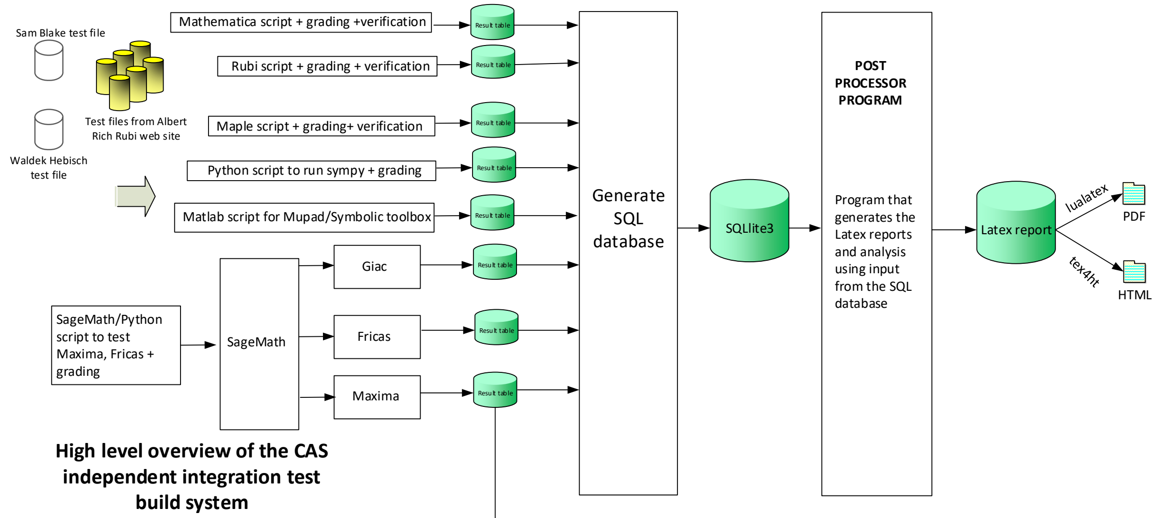
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 134, 135, 136, 137, 138, 139, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade { }

C grade { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 140, 141, 142, 143, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 248, 249, 250, 251, 252, 263, 264, 265, 266, 277, 278, 279, 280, 291, 292, 293, 294, 302, 311, 343, 347 }

F normal fail { 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 236, 364, 366, 367, 368, 369, 370 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 140, 141, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 248, 249, 250, 251, 252, 256, 257, 258, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 301, 303, 305, 306, 308, 310, 312, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 343, 347, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade { 6, 23, 39, 56, 171, 177, 178, 179, 185, 194, 212, 302, 304, 307, 309, 311, 313, 318 }

C grade { 134, 135, 136, 137, 138, 139 }

F normal fail { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 259, 260, 261, 262, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 162, 163, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 189, 190, 191, 192, 193, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 216, 217, 218, 219, 221, 222, 223, 226, 227, 228, 229, 231, 232, 233, 248, 249, 250, 251, 252, 256, 257, 258, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 321, 322, 325, 326, 327, 328, 329, 330, 331, 334, 335, 336, 343, 347, 372, 373, 374, 377, 378 }

B grade { 6, 7, 23, 24, 39, 40, 56, 57, 65, 75, 93, 102, 111, 112, 167, 168, 169, 170, 185, 186, 187, 188, 194, 195, 196, 197, 212, 213, 214, 215, 220, 224, 225, 230, 234, 235, 310, 311, 312, 313, 314, 315, 316, 317, 318, 323, 324, 332, 333, 375, 376, 379, 380, 381, 384, 385 }

C grade { }

F normal fail { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 259, 260, 261, 262, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 382, 383 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 166, 191, 192, 193, 198, 199, 200, 201, 202, 203, 211, 248, 263, 277, 291, 301, 303, 304, 306, 307, 308, 315, 316, 317, 318, 322, 323, 325, 326, 333, 334, 335, 343, 372, 383, 384 }

B grade { 53, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 204, 205, 206, 207, 208, 209, 210, 302, 305, 309, 319, 320, 321, 327, 336, 375, 376, 377, 385 }

C grade { }

F normal fail { 40, 41, 42, 43, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 195, 196, 197, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 310, 311, 312, 313, 328, 329, 331, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374 }

F(-1) timedout fail { }

F(-2) exception fail { 194, 215, 314, 324, 330, 332, 378, 379, 380, 381, 382 }

Giac

A grade { 2, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 44, 45, 49, 50, 51, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 189, 190, 191, 192, 193, 194, 195, 202, 207, 208, 209, 210, 248, 263, 277, 291, 301, 302, 303, 305, 306, 308, 309, 311, 313, 316, 318, 319, 321, 322, 323, 324, 325, 326, 331, 334, 343, 377 }

B grade { 6, 7, 8, 9, 10, 39, 46, 47, 48, 169, 170, 184, 185, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 211, 212, 320, 327, 329, 336, 375, 376, 378, 379 }

C grade { }

F normal fail { 20, 21, 22, 23, 24, 25, 26, 41, 43, 53, 54, 55, 56, 57, 58, 59, 60, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 141, 142, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 186, 187, 188, 213, 214, 215, 236, 239, 240, 241, 244, 245, 246, 247, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 298, 299, 300, 304, 307, 310, 312, 314, 317, 328, 330, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 350, 351, 352, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 380, 381, 382, 383, 384 }

F(-1) timedout fail { 237, 238, 242, 243, 269, 297 }

F(-2) exception fail { 1, 3, 19, 35, 36, 40, 42, 52, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 128, 129, 130, 131, 132, 133, 140, 143, 144, 145, 146, 147, 148, 149, 152, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 253, 254, 267, 268, 281, 282, 295, 296, 315, 332, 333, 335, 348, 349, 353, 360, 385 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 198, 199, 200, 201, 202, 203, 204, 205, 206, 248, 249, 250, 251, 252, 256, 257, 258, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 334, 335, 336, 343, 347, 372, 373, 376, 377, 378, 379, 380, 381, 384, 385 }

C grade { }

F normal fail { }

F(-1) timedout fail { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 169, 170, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 259, 260, 261, 262, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 310, 311, 312, 313, 314, 315, 316, 317, 318, 332, 333, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 382, 383 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 33, 34, 44, 45, 46, 47, 48, 49, 50, 51, 141, 166, 171, 172, 173, 174, 175, 198, 199, 200, 201, 202, 248, 263, 277, 291, 301, 303, 305, 306, 308, 319, 321, 323, 324, 326, 377, 378 }

B grade { 4, 5, 136, 137, 162, 163, 164, 165, 176, 177, 178, 179, 203, 204, 205, 206, 249, 250, 251, 252, 264, 265, 266, 278, 279, 292, 293, 343, 375, 376, 379 }

C grade { }

F normal fail { 19, 20, 21, 22, 23, 24, 25, 26, 35, 36, 37, 38, 39, 40, 41, 42, 43, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 125, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 142, 143, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 167, 168, 169, 170, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 210, 211, 212, 218, 219, 226, 227, 228, 229, 230, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 259, 260, 261, 262, 267, 268, 269, 270, 271, 273, 274, 275, 276, 282, 283, 284, 287, 288, 289, 290, 296, 297, 298, 302, 304, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 322, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 348, 349, 350, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 373, 381, 382, 383 }

F(-1) timedout fail { 79, 80, 81, 84, 85, 86, 87, 113, 114, 120, 121, 122, 123, 124, 126, 127, 144, 145, 149, 207, 208, 209, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 231, 232, 234, 235, 258, 272, 280, 281, 285, 286, 294, 295, 299, 300, 347, 351, 370, 374, 380, 384, 385 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	64	84	100	67	114	0	98
N.S.	1	1.00	0.57	0.74	0.88	0.59	1.01	0.00	0.87
time (sec)	N/A	0.064	0.039	0.270	0.190	0.250	0.561	0.000	0.107

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	56	77	81	59	104	70	85
N.S.	1	1.00	0.62	0.86	0.90	0.66	1.16	0.78	0.94
time (sec)	N/A	0.049	0.031	0.249	0.204	0.263	0.557	0.280	0.503

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	46	67	62	51	90	0	71
N.S.	1	1.00	0.61	0.89	0.83	0.68	1.20	0.00	0.95
time (sec)	N/A	0.036	0.025	0.217	0.207	0.264	0.522	0.000	0.462

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	59	42	43	78	53	51
N.S.	1	1.00	0.90	1.40	1.00	1.02	1.86	1.26	1.21
time (sec)	N/A	0.015	0.023	0.204	0.206	0.278	0.541	0.276	0.045

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	41	25	37	61	41	32
N.S.	1	1.00	0.90	1.41	0.86	1.28	2.10	1.41	1.10
time (sec)	N/A	0.007	0.012	0.190	0.183	0.259	0.452	0.295	0.039

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	29	48	18	58	48	68	32
N.S.	1	1.00	1.16	1.92	0.72	2.32	1.92	2.72	1.28
time (sec)	N/A	0.026	0.012	0.209	0.177	0.268	1.629	0.288	0.040

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	47	34	29	66	26	75	33
N.S.	1	1.00	1.24	0.89	0.76	1.74	0.68	1.97	0.87
time (sec)	N/A	0.026	0.022	0.225	0.190	0.254	1.149	0.283	0.039

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	53	48	83	48	153	52
N.S.	1	1.00	0.90	0.84	0.76	1.32	0.76	2.43	0.83
time (sec)	N/A	0.036	0.031	0.255	0.191	0.261	1.628	0.294	0.043

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	70	68	67	92	75	161	74
N.S.	1	1.00	0.78	0.76	0.74	1.02	0.83	1.79	0.82
time (sec)	N/A	0.049	0.036	0.247	0.209	0.261	1.758	0.287	0.039

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	76	77	86	101	122	237	95
N.S.	1	1.00	0.67	0.68	0.76	0.89	1.08	2.10	0.84
time (sec)	N/A	0.062	0.040	0.274	0.203	0.259	2.957	0.299	0.033

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	46	56	46	41	46	43
N.S.	1	1.00	1.00	0.96	1.17	0.96	0.85	0.96	0.90
time (sec)	N/A	0.026	0.017	0.245	0.284	0.249	0.081	0.280	0.475

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	47	37	31	37	36
N.S.	1	1.00	1.00	0.95	1.21	0.95	0.79	0.95	0.92
time (sec)	N/A	0.022	0.011	0.235	0.286	0.248	0.071	0.268	0.461

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	29	38	29	22	29	27
N.S.	1	1.00	1.00	1.00	1.31	1.00	0.76	1.00	0.93
time (sec)	N/A	0.015	0.009	0.238	0.293	0.261	0.055	0.293	0.065

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	30	20	28	21	12	15	19
N.S.	1	1.00	1.58	1.05	1.47	1.11	0.63	0.79	1.00
time (sec)	N/A	0.007	0.010	0.236	0.284	0.258	0.071	0.279	0.047

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	21	15	17	12	14
N.S.	1	1.00	1.00	1.00	1.62	1.15	1.31	0.92	1.08
time (sec)	N/A	0.015	0.006	0.223	0.282	0.254	0.071	0.271	0.500

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	26	31	26	32	21	17
N.S.	1	1.00	1.00	1.00	1.19	1.00	1.23	0.81	0.65
time (sec)	N/A	0.017	0.009	0.273	0.260	0.255	0.082	0.276	0.066

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	42	39	42	31	27
N.S.	1	1.00	1.00	1.06	1.17	1.08	1.17	0.86	0.75
time (sec)	N/A	0.019	0.009	0.246	0.278	0.262	0.106	0.275	0.083

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	48	51	47	54	39	34
N.S.	1	1.00	1.00	1.00	1.06	0.98	1.12	0.81	0.71
time (sec)	N/A	0.021	0.011	0.260	0.290	0.249	0.127	0.279	0.078

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	80	124	114	88	0	0	137
N.S.	1	1.00	0.58	0.91	0.83	0.64	0.00	0.00	1.00
time (sec)	N/A	0.425	0.042	0.420	0.186	0.260	0.000	0.000	0.502

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	111	95	80	0	0	114
N.S.	1	1.00	0.62	1.09	0.93	0.78	0.00	0.00	1.12
time (sec)	N/A	0.400	0.039	0.395	0.205	0.275	0.000	0.000	0.069

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	54	106	76	72	0	0	104
N.S.	1	1.00	0.59	1.15	0.83	0.78	0.00	0.00	1.13
time (sec)	N/A	0.226	0.043	0.342	0.209	0.263	0.000	0.000	0.495

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	93	57	60	0	0	72
N.S.	1	1.00	0.70	1.55	0.95	1.00	0.00	0.00	1.20
time (sec)	N/A	0.031	0.030	0.349	0.211	0.278	0.000	0.000	0.466

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	55	101	46	100	0	0	73
N.S.	1	1.00	1.08	1.98	0.90	1.96	0.00	0.00	1.43
time (sec)	N/A	0.491	0.032	0.274	0.204	0.270	0.000	0.000	0.511

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	80	60	109	0	0	75
N.S.	1	1.00	0.97	1.27	0.95	1.73	0.00	0.00	1.19
time (sec)	N/A	0.411	0.037	0.336	0.211	0.265	0.000	0.000	0.063

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	79	105	81	130	0	0	99
N.S.	1	1.00	0.86	1.14	0.88	1.41	0.00	0.00	1.08
time (sec)	N/A	0.434	0.058	0.313	0.180	0.265	0.000	0.000	0.480

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	89	116	100	139	0	0	116
N.S.	1	1.00	0.76	0.99	0.85	1.19	0.00	0.00	0.99
time (sec)	N/A	0.442	0.057	0.372	0.187	0.279	0.000	0.000	0.468

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	67	77	70	56	60	60
N.S.	1	1.00	1.00	1.03	1.18	1.08	0.86	0.92	0.92
time (sec)	N/A	0.033	0.033	0.260	0.266	0.274	0.122	0.271	0.505

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	58	67	62	44	51	51
N.S.	1	1.00	1.00	1.09	1.26	1.17	0.83	0.96	0.96
time (sec)	N/A	0.029	0.028	0.266	0.293	0.267	0.106	0.279	0.066

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	50	60	53	36	43	43
N.S.	1	1.00	1.00	1.11	1.33	1.18	0.80	0.96	0.96
time (sec)	N/A	0.020	0.021	0.237	0.294	0.253	0.085	0.288	0.068

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	42	33	44	43	22	25	32
N.S.	1	1.00	1.35	1.06	1.42	1.39	0.71	0.81	1.03
time (sec)	N/A	0.010	0.018	0.234	0.286	0.253	0.089	0.283	0.490

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	22	18	10	13	14
N.S.	1	1.00	1.00	0.94	1.38	1.12	0.62	0.81	0.88
time (sec)	N/A	0.016	0.009	0.242	0.272	0.257	0.101	0.290	0.084

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	44	53	60	44	35	37
N.S.	1	1.00	1.00	1.16	1.39	1.58	1.16	0.92	0.97
time (sec)	N/A	0.021	0.020	0.272	0.289	0.273	0.151	0.287	0.501

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	53	69	77	58	46	43
N.S.	1	1.00	1.00	1.02	1.33	1.48	1.12	0.88	0.83
time (sec)	N/A	0.025	0.034	0.277	0.285	0.267	0.165	0.284	0.567

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	61	77	86	70	54	55
N.S.	1	1.00	1.00	0.98	1.24	1.39	1.13	0.87	0.89
time (sec)	N/A	0.028	0.028	0.275	0.284	0.256	0.179	0.282	0.141

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	56	77	76	59	0	0	85
N.S.	1	1.00	0.62	0.86	0.84	0.66	0.00	0.00	0.94
time (sec)	N/A	0.046	0.031	0.270	0.293	0.303	0.000	0.000	0.069

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	46	67	59	51	0	0	71
N.S.	1	1.00	0.61	0.89	0.79	0.68	0.00	0.00	0.95
time (sec)	N/A	0.034	0.023	0.254	0.277	0.267	0.000	0.000	0.463

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	59	42	43	0	53	51
N.S.	1	1.00	0.90	1.40	1.00	1.02	0.00	1.26	1.21
time (sec)	N/A	0.014	0.022	0.237	0.297	0.263	0.000	0.284	0.464

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	48	25	37	0	41	32
N.S.	1	1.00	0.90	1.66	0.86	1.28	0.00	1.41	1.10
time (sec)	N/A	0.008	0.015	0.252	0.265	0.287	0.000	0.302	0.451

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	29	121	26	58	0	68	32
N.S.	1	1.00	1.16	4.84	1.04	2.32	0.00	2.72	1.28
time (sec)	N/A	0.027	0.014	0.234	0.284	0.264	0.000	0.291	0.039

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	47	34	0	66	0	0	33
N.S.	1	1.00	1.24	0.89	0.00	1.74	0.00	0.00	0.87
time (sec)	N/A	0.026	0.022	0.243	0.000	0.265	0.000	0.000	0.039

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	60	0	83	0	0	52
N.S.	1	1.00	0.90	0.95	0.00	1.32	0.00	0.00	0.83
time (sec)	N/A	0.037	0.033	0.246	0.000	0.274	0.000	0.000	0.042

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	70	68	0	92	0	0	74
N.S.	1	1.00	0.78	0.76	0.00	1.02	0.00	0.00	0.82
time (sec)	N/A	0.048	0.039	0.253	0.000	0.257	0.000	0.000	0.039

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	76	77	0	101	0	0	95
N.S.	1	1.00	0.67	0.68	0.00	0.89	0.00	0.00	0.84
time (sec)	N/A	0.062	0.042	0.277	0.000	0.270	0.000	0.000	0.034

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	48	44	46	41	68	43
N.S.	1	1.00	1.00	0.98	0.90	0.94	0.84	1.39	0.88
time (sec)	N/A	0.026	0.019	0.194	0.176	0.266	0.068	0.292	0.060

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	40	35	37	31	58	36
N.S.	1	1.00	1.00	1.00	0.88	0.92	0.78	1.45	0.90
time (sec)	N/A	0.022	0.011	0.205	0.189	0.248	0.067	0.290	0.459

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	28	29	22	52	27
N.S.	1	1.00	1.00	1.03	0.93	0.97	0.73	1.73	0.90
time (sec)	N/A	0.017	0.011	0.198	0.187	0.260	0.051	0.290	0.486

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	30	19	16	21	14	65	19
N.S.	1	1.00	1.50	0.95	0.80	1.05	0.70	3.25	0.95
time (sec)	N/A	0.008	0.011	0.216	0.218	0.247	0.059	0.287	0.454

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	12	15	17	44	14
N.S.	1	1.00	1.00	1.00	0.86	1.07	1.21	3.14	1.00
time (sec)	N/A	0.016	0.007	0.189	0.216	0.279	0.072	0.282	0.497

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	34	26	32	34	17
N.S.	1	1.00	1.00	0.93	1.26	0.96	1.19	1.26	0.63
time (sec)	N/A	0.018	0.010	0.217	0.207	0.249	0.084	0.281	0.486

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	50	39	42	54	26
N.S.	1	1.00	1.00	0.92	1.35	1.05	1.14	1.46	0.70
time (sec)	N/A	0.019	0.010	0.238	0.213	0.260	0.100	0.280	0.077

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	57	47	54	67	33
N.S.	1	1.00	1.00	0.90	1.16	0.96	1.10	1.37	0.67
time (sec)	N/A	0.022	0.012	0.263	0.203	0.267	0.122	0.295	0.481

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	80	124	216	88	0	0	138
N.S.	1	1.00	0.58	0.91	1.58	0.64	0.00	0.00	1.01
time (sec)	N/A	0.421	0.043	0.371	0.285	0.253	0.000	0.000	0.514

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	111	181	80	0	0	115
N.S.	1	1.00	0.62	1.09	1.77	0.78	0.00	0.00	1.13
time (sec)	N/A	0.390	0.040	0.363	0.298	0.259	0.000	0.000	0.075

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	60	106	112	72	0	0	105
N.S.	1	1.00	0.65	1.15	1.22	0.78	0.00	0.00	1.14
time (sec)	N/A	0.228	0.040	0.327	0.260	0.280	0.000	0.000	0.481

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	93	65	60	0	0	73
N.S.	1	1.00	0.70	1.55	1.08	1.00	0.00	0.00	1.22
time (sec)	N/A	0.031	0.027	0.328	0.266	0.289	0.000	0.000	0.473

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	55	649	0	100	0	0	74
N.S.	1	1.00	1.06	12.48	0.00	1.92	0.00	0.00	1.42
time (sec)	N/A	0.423	0.034	0.293	0.000	0.298	0.000	0.000	0.494

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	61	82	0	109	0	0	76
N.S.	1	1.00	0.95	1.28	0.00	1.70	0.00	0.00	1.19
time (sec)	N/A	0.407	0.040	0.355	0.000	0.275	0.000	0.000	0.476

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	79	108	0	130	0	0	100
N.S.	1	1.00	0.85	1.16	0.00	1.40	0.00	0.00	1.08
time (sec)	N/A	0.428	0.057	0.395	0.000	0.270	0.000	0.000	0.472

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	89	116	0	139	0	0	117
N.S.	1	1.00	0.75	0.98	0.00	1.18	0.00	0.00	0.99
time (sec)	N/A	0.445	0.053	0.460	0.000	0.276	0.000	0.000	0.071

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	95	125	0	146	0	0	139
N.S.	1	1.00	0.68	0.90	0.00	1.05	0.00	0.00	1.00
time (sec)	N/A	0.478	0.060	0.425	0.000	0.268	0.000	0.000	0.504

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	339	339	82	0	0	244	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.167	0.030	0.000	0.000	0.269	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	63	0	0	236	0	0	0
N.S.	1	1.00	0.21	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.129	0.016	0.000	0.000	0.266	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	41	0	0	209	0	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.103	0.022	0.000	0.000	0.264	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	97	0	0	243	0	0	0
N.S.	1	1.00	0.36	0.00	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.125	0.027	0.000	0.000	0.275	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	71	0	0	151	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.024	0.013	0.000	0.000	0.279	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	175	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.030	0.015	0.000	0.000	0.263	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	93	0	0	184	0	0	0
N.S.	1	1.00	0.55	0.00	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.044	0.020	0.000	0.000	0.258	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	192	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.057	0.023	0.000	0.000	0.276	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	111	0	0	200	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.070	0.028	0.000	0.000	0.269	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	337	337	148	0	0	254	0	0	0
N.S.	1	1.00	0.44	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.155	0.083	0.000	0.000	0.278	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	339	339	82	0	0	247	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.151	0.025	0.000	0.000	0.273	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	61	0	0	239	0	0	0
N.S.	1	1.00	0.21	0.00	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.125	0.018	0.000	0.000	0.272	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	41	0	0	215	0	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.103	0.025	0.000	0.000	0.266	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	96	0	0	243	0	0	0
N.S.	1	1.00	0.36	0.00	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.122	0.024	0.000	0.000	0.278	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	68	0	0	157	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.024	0.011	0.000	0.000	0.276	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	179	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.030	0.014	0.000	0.000	0.260	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	93	0	0	187	0	0	0
N.S.	1	1.00	0.55	0.00	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.044	0.019	0.000	0.000	0.277	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	195	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.056	0.024	0.000	0.000	0.272	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	373	373	96	0	0	251	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.181	0.034	0.000	0.000	0.269	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	371	86	0	0	244	0	0	0
N.S.	1	1.00	0.23	0.00	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.175	0.028	0.000	0.000	0.266	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	324	324	72	0	0	236	0	0	0
N.S.	1	1.00	0.22	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.145	0.026	0.000	0.000	0.280	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	41	0	0	209	0	0	0
N.S.	1	1.00	0.14	0.00	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.129	0.032	0.000	0.000	0.274	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	112	0	0	267	0	0	0
N.S.	1	1.00	0.38	0.00	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.153	0.031	0.000	0.000	0.279	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	87	0	0	152	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.027	0.016	0.000	0.000	0.281	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	99	0	0	176	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.038	0.020	0.000	0.000	0.263	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	106	0	0	184	0	0	0
N.S.	1	1.00	0.52	0.00	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.060	0.024	0.000	0.000	0.275	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	233	118	0	0	192	0	0	0
N.S.	1	1.00	0.51	0.00	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.072	0.030	0.000	0.000	0.271	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	337	337	127	0	0	255	0	0	0
N.S.	1	1.00	0.38	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.155	0.079	0.000	0.000	0.300	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	339	339	73	0	0	247	0	0	0
N.S.	1	1.00	0.22	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.151	0.029	0.000	0.000	0.284	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	63	0	0	238	0	0	0
N.S.	1	1.00	0.21	0.00	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.130	0.012	0.000	0.000	0.268	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	41	0	0	213	0	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.103	0.024	0.000	0.000	0.265	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	96	0	0	243	0	0	0
N.S.	1	1.00	0.36	0.00	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.119	0.018	0.000	0.000	0.264	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	69	0	0	156	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.024	0.011	0.000	0.000	0.261	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	178	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.031	0.015	0.000	0.000	0.266	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	92	0	0	187	0	0	0
N.S.	1	1.00	0.54	0.00	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.044	0.022	0.000	0.000	0.263	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	195	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.055	0.026	0.000	0.000	0.270	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	337	337	127	0	0	251	0	0	0
N.S.	1	1.00	0.38	0.00	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.153	0.095	0.000	0.000	0.274	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	339	339	73	0	0	243	0	0	0
N.S.	1	1.00	0.22	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.156	0.025	0.000	0.000	0.273	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	63	0	0	236	0	0	0
N.S.	1	1.00	0.21	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.130	0.013	0.000	0.000	0.279	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	39	0	0	209	0	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.109	0.033	0.000	0.000	0.280	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	97	0	0	243	0	0	0
N.S.	1	1.00	0.36	0.00	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.119	0.019	0.000	0.000	0.276	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	69	0	0	152	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.024	0.012	0.000	0.000	0.276	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	81	0	0	176	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.030	0.016	0.000	0.000	0.264	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	93	0	0	184	0	0	0
N.S.	1	1.00	0.55	0.00	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.045	0.019	0.000	0.000	0.275	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	99	0	0	192	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.057	0.027	0.000	0.000	0.276	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	373	373	100	0	0	304	0	0	0
N.S.	1	1.00	0.27	0.00	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.175	0.034	0.000	0.000	0.285	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	371	91	0	0	296	0	0	0
N.S.	1	1.00	0.25	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.175	0.027	0.000	0.000	0.273	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	324	324	63	0	0	289	0	0	0
N.S.	1	1.00	0.19	0.00	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.145	0.022	0.000	0.000	0.271	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	39	0	0	261	0	0	0
N.S.	1	1.00	0.13	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.118	0.039	0.000	0.000	0.276	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	106	0	0	329	0	0	0
N.S.	1	1.00	0.36	0.00	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.153	0.041	0.000	0.000	0.281	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	69	0	0	212	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.028	0.013	0.000	0.000	0.282	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	81	0	0	238	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.036	0.018	0.000	0.000	0.270	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	93	0	0	246	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.057	0.022	0.000	0.000	0.280	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	233	99	0	0	254	0	0	0
N.S.	1	1.00	0.42	0.00	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.076	0.027	0.000	0.000	0.276	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	73	0	0	208	0	0	0
N.S.	1	1.00	0.23	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.286	0.030	0.000	0.000	0.262	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	278	57	0	0	195	0	0	0
N.S.	1	1.00	0.21	0.00	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.251	0.016	0.000	0.000	0.293	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	34	0	0	195	0	0	0
N.S.	1	1.00	0.13	0.00	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.231	0.016	0.000	0.000	0.252	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	430	430	90	0	0	339	0	0	0
N.S.	1	1.00	0.21	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.344	0.024	0.000	0.000	0.253	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	64	0	0	211	0	0	0
N.S.	1	1.00	0.25	0.00	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.117	0.010	0.000	0.000	0.275	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	72	0	0	234	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.136	0.013	0.000	0.000	0.272	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	81	0	0	243	0	0	0
N.S.	1	1.00	0.25	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.143	0.015	0.000	0.000	0.268	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	73	0	0	117	0	0	0
N.S.	1	1.00	0.41	0.00	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.037	0.026	0.000	0.000	0.286	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	54	0	0	116	0	0	0
N.S.	1	1.00	0.39	0.00	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.027	0.018	0.000	0.000	0.260	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	34	0	0	104	0	0	0
N.S.	1	1.00	0.29	0.00	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.014	0.017	0.000	0.000	0.261	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	90	0	0	145	0	0	0
N.S.	1	1.00	0.55	0.00	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.026	0.019	0.000	0.000	0.260	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	59	0	0	120	0	0	0
N.S.	1	1.00	0.53	0.00	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.022	0.010	0.000	0.000	0.260	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	69	0	0	138	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.029	0.012	0.000	0.000	0.290	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	741	741	83	0	0	435	0	0	0
N.S.	1	1.00	0.11	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.526	0.030	0.000	0.000	0.280	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	689	689	63	0	0	428	0	0	0
N.S.	1	1.00	0.09	0.00	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.356	0.017	0.000	0.000	0.284	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	674	674	41	0	0	383	0	0	0
N.S.	1	1.00	0.06	0.00	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.324	0.022	0.000	0.000	0.288	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	859	859	97	0	0	509	0	0	0
N.S.	1	1.00	0.11	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.391	0.026	0.000	0.000	0.279	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	71	0	0	345	0	0	0
N.S.	1	1.00	0.22	0.00	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.091	0.015	0.000	0.000	0.282	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	364	364	84	0	0	381	0	0	0
N.S.	1	1.00	0.23	0.00	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.114	0.017	0.000	0.000	0.273	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	94	748	0	0	0	0	0
N.S.	1	1.00	0.82	6.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.034	0.757	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	58	417	0	0	0	0	0
N.S.	1	1.00	1.16	8.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.018	0.452	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	175	0	0	126	0	0
N.S.	1	1.00	0.74	4.49	0.00	0.00	3.23	0.00	0.00
time (sec)	N/A	0.016	0.006	0.296	0.000	0.000	1.629	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	158	0	0	133	0	0
N.S.	1	1.00	0.74	4.05	0.00	0.00	3.41	0.00	0.00
time (sec)	N/A	0.017	0.010	0.296	0.000	0.000	2.403	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	58	428	0	0	0	0	0
N.S.	1	1.00	1.16	8.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.018	0.569	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	1196	0	0	0	0	0
N.S.	1	1.00	0.82	10.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.032	1.280	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	113	146	0	0	0	0	0
N.S.	1	1.00	0.71	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.650	0.059	0.234	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	85	71	0	0	94	0	0
N.S.	1	1.00	1.08	0.90	0.00	0.00	1.19	0.00	0.00
time (sec)	N/A	0.031	0.038	0.205	0.000	0.000	1.419	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	85	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	113	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.542	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.017	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.017	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	210	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.083	0.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	116	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.043	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	105	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	53	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.009	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	106	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.019	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	82	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	0.013	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	114	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	119	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.058	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	217	197	749	177	1222	205	0
N.S.	1	1.00	0.79	0.71	2.71	0.64	4.43	0.74	0.00
time (sec)	N/A	0.140	0.452	0.806	0.209	0.291	1.857	0.301	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	176	150	529	139	857	155	0
N.S.	1	1.00	0.88	0.75	2.63	0.69	4.26	0.77	0.00
time (sec)	N/A	0.132	0.247	0.591	0.206	0.278	1.596	0.284	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	135	113	351	106	585	113	0
N.S.	1	1.00	0.79	0.66	2.05	0.62	3.42	0.66	0.00
time (sec)	N/A	0.087	0.113	0.545	0.174	0.259	1.330	0.298	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	108	87	209	79	362	75	0
N.S.	1	1.00	0.98	0.79	1.90	0.72	3.29	0.68	0.00
time (sec)	N/A	0.051	0.089	0.414	0.183	0.274	0.916	0.303	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	28	69	62	60	36	51	97
N.S.	1	1.00	0.54	1.33	1.19	1.15	0.69	0.98	1.87
time (sec)	N/A	0.025	0.013	0.375	0.176	0.268	0.773	0.303	1.173

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	142	107	233	144	0	112	118
N.S.	1	1.00	1.60	1.20	2.62	1.62	0.00	1.26	1.33
time (sec)	N/A	0.058	0.055	0.304	0.177	0.287	0.000	0.345	1.248

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	120	93	239	224	0	145	218
N.S.	1	1.00	0.92	0.72	1.84	1.72	0.00	1.12	1.68
time (sec)	N/A	0.046	0.061	0.681	0.178	0.271	0.000	0.350	1.850

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	154	187	424	452	0	471	0
N.S.	1	1.00	0.77	0.93	2.11	2.25	0.00	2.34	0.00
time (sec)	N/A	0.100	0.095	0.435	0.217	0.274	0.000	0.362	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	234	281	644	690	0	884	0
N.S.	1	1.00	0.83	0.99	2.28	2.44	0.00	3.12	0.00
time (sec)	N/A	0.143	0.248	0.796	0.208	0.289	0.000	0.343	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	160	149	105	110	123	201
N.S.	1	1.00	1.00	1.74	1.62	1.14	1.20	1.34	2.18
time (sec)	N/A	0.063	0.053	0.319	0.259	0.267	0.252	0.287	0.682

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	109	116	77	75	83	153
N.S.	1	1.00	1.00	1.51	1.61	1.07	1.04	1.15	2.12
time (sec)	N/A	0.046	0.048	0.298	0.256	0.264	0.177	0.279	0.595

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	70	87	53	46	53	107
N.S.	1	1.00	1.00	1.30	1.61	0.98	0.85	0.98	1.98
time (sec)	N/A	0.037	0.026	0.285	0.258	0.253	0.148	0.288	0.570

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	41	64	35	29	35	60
N.S.	1	1.00	1.00	1.11	1.73	0.95	0.78	0.95	1.62
time (sec)	N/A	0.023	0.017	0.280	0.256	0.275	0.116	0.274	0.132

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	32	21	46	22	14	16	21
N.S.	1	1.00	1.60	1.05	2.30	1.10	0.70	0.80	1.05
time (sec)	N/A	0.009	0.009	0.269	0.258	0.263	0.074	0.282	0.550

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	47	78	27	100	33	32
N.S.	1	1.00	0.82	1.24	2.05	0.71	2.63	0.87	0.84
time (sec)	N/A	0.026	0.016	0.285	0.287	0.262	0.398	0.282	0.776

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	39	96	126	40	156	61	98
N.S.	1	1.00	0.71	1.75	2.29	0.73	2.84	1.11	1.78
time (sec)	N/A	0.032	0.022	0.319	0.285	0.273	0.310	0.289	0.702

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	63	201	188	69	228	89	154
N.S.	1	1.00	0.83	2.64	2.47	0.91	3.00	1.17	2.03
time (sec)	N/A	0.036	0.026	0.336	0.268	0.266	0.442	0.290	0.787

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	267	263	94	286	126	199
N.S.	1	1.00	0.95	2.87	2.83	1.01	3.08	1.35	2.14
time (sec)	N/A	0.042	0.040	0.352	0.264	0.251	0.558	0.290	0.829

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	249	436	3081	264	0	334	0
N.S.	1	1.00	0.77	1.35	9.51	0.81	0.00	1.03	0.00
time (sec)	N/A	0.190	0.419	2.174	0.222	0.281	0.000	0.323	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	201	342	2295	216	0	285	0
N.S.	1	1.00	0.81	1.37	9.22	0.87	0.00	1.14	0.00
time (sec)	N/A	0.166	0.227	1.382	0.206	0.285	0.000	0.311	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	160	259	1608	174	0	243	0
N.S.	1	1.00	0.70	1.14	7.08	0.77	0.00	1.07	0.00
time (sec)	N/A	0.119	0.190	1.214	0.196	0.275	0.000	0.315	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	132	186	1108	136	0	209	0
N.S.	1	1.00	0.81	1.14	6.80	0.83	0.00	1.28	0.00
time (sec)	N/A	0.083	0.135	0.822	0.218	0.280	0.000	0.303	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	45	120	736	99	0	180	0
N.S.	1	1.00	0.48	1.28	7.83	1.05	0.00	1.91	0.00
time (sec)	N/A	0.031	0.033	0.811	0.213	0.282	0.000	0.310	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	196	485	733	356	0	252	0
N.S.	1	1.00	1.46	3.62	5.47	2.66	0.00	1.88	0.00
time (sec)	N/A	0.071	0.556	0.503	0.186	0.278	0.000	0.398	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	145	181	992	389	0	0	0
N.S.	1	1.00	0.82	1.03	5.64	2.21	0.00	0.00	0.00
time (sec)	N/A	0.060	0.114	1.577	0.207	0.298	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	194	269	1536	574	0	0	0
N.S.	1	1.00	0.73	1.02	5.82	2.17	0.00	0.00	0.00
time (sec)	N/A	0.114	0.161	1.182	0.212	0.278	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	282	379	2313	839	0	0	0
N.S.	1	1.00	0.83	1.12	6.84	2.48	0.00	0.00	0.00
time (sec)	N/A	0.211	0.306	1.343	0.209	0.314	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	248	197	456	177	0	205	0
N.S.	1	1.00	0.90	0.71	1.65	0.64	0.00	0.74	0.00
time (sec)	N/A	0.159	0.619	0.840	0.285	0.268	0.000	0.319	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	202	150	308	139	0	155	0
N.S.	1	1.00	1.00	0.75	1.53	0.69	0.00	0.77	0.00
time (sec)	N/A	0.137	0.417	0.675	0.297	0.282	0.000	0.284	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	162	113	161	106	0	113	0
N.S.	1	1.00	0.95	0.66	0.94	0.62	0.00	0.66	0.00
time (sec)	N/A	0.090	0.236	0.597	0.261	0.277	0.000	0.278	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	131	87	97	79	0	75	0
N.S.	1	1.00	1.19	0.79	0.88	0.72	0.00	0.68	0.00
time (sec)	N/A	0.051	0.107	0.518	0.261	0.279	0.000	0.292	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	28	69	35	60	0	51	0
N.S.	1	1.00	0.54	1.33	0.67	1.15	0.00	0.98	0.00
time (sec)	N/A	0.025	0.014	0.466	0.265	0.275	0.000	0.281	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	142	260	0	144	0	112	0
N.S.	1	1.00	1.60	2.92	0.00	1.62	0.00	1.26	0.00
time (sec)	N/A	0.047	0.061	0.456	0.000	0.274	0.000	0.339	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	119	93	0	224	0	145	0
N.S.	1	1.00	0.92	0.72	0.00	1.72	0.00	1.12	0.00
time (sec)	N/A	0.048	0.056	0.612	0.000	0.273	0.000	0.334	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	154	187	0	452	0	471	0
N.S.	1	1.00	0.77	0.93	0.00	2.25	0.00	2.34	0.00
time (sec)	N/A	0.086	0.097	0.602	0.000	0.288	0.000	0.338	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	282	233	281	0	690	0	884	0
N.S.	1	1.00	0.82	0.99	0.00	2.44	0.00	3.12	0.00
time (sec)	N/A	0.151	0.269	0.978	0.000	0.292	0.000	0.340	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	95	125	105	105	114	215	165
N.S.	1	1.00	0.96	1.26	1.06	1.06	1.15	2.17	1.67
time (sec)	N/A	0.065	0.058	0.260	0.175	0.259	0.232	0.288	0.186

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	85	73	77	76	158	129
N.S.	1	1.00	1.00	1.10	0.95	1.00	0.99	2.05	1.68
time (sec)	N/A	0.045	0.046	0.238	0.178	0.256	0.187	0.285	0.608

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	59	53	53	49	109	90
N.S.	1	1.00	0.93	1.00	0.90	0.90	0.83	1.85	1.53
time (sec)	N/A	0.036	0.026	0.222	0.176	0.268	0.151	0.271	0.610

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	36	35	29	72	51
N.S.	1	1.00	1.00	0.98	0.90	0.88	0.72	1.80	1.28
time (sec)	N/A	0.024	0.017	0.225	0.181	0.262	0.112	0.276	0.559

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	32	22	19	22	15	37	21
N.S.	1	1.00	1.39	0.96	0.83	0.96	0.65	1.61	0.91
time (sec)	N/A	0.010	0.008	0.221	0.173	0.255	0.079	0.275	0.064

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	42	47	27	99	68	34
N.S.	1	1.00	0.83	1.02	1.15	0.66	2.41	1.66	0.83
time (sec)	N/A	0.029	0.017	0.252	0.176	0.261	0.408	0.278	0.800

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	42	69	110	40	158	95	100
N.S.	1	1.00	0.68	1.11	1.77	0.65	2.55	1.53	1.61
time (sec)	N/A	0.032	0.021	0.303	0.177	0.261	0.310	0.281	0.735

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	81	66	109	160	69	226	142	156
N.S.	1	0.98	0.80	1.31	1.93	0.83	2.72	1.71	1.88
time (sec)	N/A	0.038	0.028	0.290	0.189	0.269	0.441	0.269	0.797

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	102	91	150	218	94	286	183	199
N.S.	1	0.98	0.88	1.44	2.10	0.90	2.75	1.76	1.91
time (sec)	N/A	0.043	0.036	0.296	0.191	0.266	0.573	0.280	0.863

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	299	445	1368	264	0	334	0
N.S.	1	1.00	0.92	1.37	4.22	0.81	0.00	1.03	0.00
time (sec)	N/A	0.196	0.501	1.166	0.296	0.284	0.000	0.303	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	244	351	979	216	0	285	0
N.S.	1	1.00	0.98	1.41	3.93	0.87	0.00	1.14	0.00
time (sec)	N/A	0.172	0.313	0.924	0.283	0.276	0.000	0.306	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	198	268	624	174	0	241	0
N.S.	1	1.00	0.86	1.17	2.72	0.76	0.00	1.05	0.00
time (sec)	N/A	0.117	0.244	0.914	0.278	0.269	0.000	0.303	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	157	195	293	136	0	210	0
N.S.	1	1.00	0.96	1.20	1.80	0.83	0.00	1.29	0.00
time (sec)	N/A	0.082	0.239	0.832	0.263	0.278	0.000	0.321	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	45	129	103	99	0	180	0
N.S.	1	1.00	0.48	1.37	1.10	1.05	0.00	1.91	0.00
time (sec)	N/A	0.032	0.035	0.852	0.265	0.266	0.000	0.306	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	134	134	189	1067	0	356	0	252	0
N.S.	1	1.00	1.41	7.96	0.00	2.66	0.00	1.88	0.00
time (sec)	N/A	0.072	0.478	0.814	0.000	0.269	0.000	0.405	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	145	194	0	389	0	0	0
N.S.	1	1.00	0.81	1.09	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.065	0.129	1.335	0.000	0.271	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	194	282	0	574	0	0	0
N.S.	1	1.00	0.73	1.07	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.113	0.193	1.275	0.000	0.279	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	275	392	0	839	0	0	0
N.S.	1	1.00	0.81	1.16	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.205	0.320	2.114	0.000	0.299	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	494	494	121	0	0	554	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.302	0.064	0.000	0.000	0.292	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	81	0	0	415	0	0	0
N.S.	1	1.00	0.20	0.00	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.215	0.032	0.000	0.000	0.283	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	338	338	45	0	0	255	0	0	0
N.S.	1	1.00	0.13	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.144	0.012	0.000	0.000	0.265	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	395	395	124	0	0	414	0	0	0
N.S.	1	1.00	0.31	0.00	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.176	0.075	0.000	0.000	0.285	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	205	110	0	0	598	0	0	0
N.S.	1	1.00	0.54	0.00	0.00	2.92	0.00	0.00	0.00
time (sec)	N/A	0.080	0.025	0.000	0.000	0.272	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	494	494	121	0	0	561	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.283	0.067	0.000	0.000	0.298	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	79	0	0	431	0	0	0
N.S.	1	1.00	0.19	0.00	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.215	0.033	0.000	0.000	0.280	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	338	338	45	0	0	268	0	0	0
N.S.	1	1.00	0.13	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.142	0.012	0.000	0.000	0.279	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	427	427	122	0	0	690	0	0	0
N.S.	1	1.00	0.29	0.00	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.180	0.058	0.000	0.000	0.292	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	106	0	0	694	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	3.29	0.00	0.00	0.00
time (sec)	N/A	0.082	0.020	0.000	0.000	0.273	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	494	494	99	0	0	561	0	0	0
N.S.	1	1.00	0.20	0.00	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.284	0.066	0.000	0.000	0.285	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	84	0	0	421	0	0	0
N.S.	1	1.00	0.20	0.00	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.213	0.029	0.000	0.000	0.284	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	338	338	45	0	0	266	0	0	0
N.S.	1	1.00	0.13	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.143	0.012	0.000	0.000	0.273	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	395	395	126	0	0	470	0	0	0
N.S.	1	1.00	0.32	0.00	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.155	0.026	0.000	0.000	0.285	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	107	0	0	707	0	0	0
N.S.	1	1.00	0.51	0.00	0.00	3.37	0.00	0.00	0.00
time (sec)	N/A	0.079	0.022	0.000	0.000	0.277	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	494	494	98	0	0	555	0	0	0
N.S.	1	1.00	0.20	0.00	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.283	0.065	0.000	0.000	0.297	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	84	0	0	433	0	0	0
N.S.	1	1.00	0.20	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.209	0.028	0.000	0.000	0.269	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	338	338	43	0	0	255	0	0	0
N.S.	1	1.00	0.13	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.143	0.012	0.000	0.000	0.273	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	427	427	128	0	0	629	0	0	0
N.S.	1	1.00	0.30	0.00	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.177	0.024	0.000	0.000	0.276	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	107	0	0	613	0	0	0
N.S.	1	1.00	0.51	0.00	0.00	2.91	0.00	0.00	0.00
time (sec)	N/A	0.086	0.023	0.000	0.000	0.276	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	260	272	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.260	0.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	160	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.125	0.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	128	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	60	0	0	0	0	0	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	191	191	170	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	125	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	173	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	102	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.025	0.000	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.011	0.000	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	45	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.009	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	35	13	12	12	12	12	12
N.S.	1	1.00	2.69	1.00	0.92	0.92	0.92	0.92	0.92
time (sec)	N/A	0.018	0.007	0.493	0.267	0.261	0.407	0.285	0.628

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	39	0	39	95	0	44
N.S.	1	1.00	1.20	0.78	0.00	0.78	1.90	0.00	0.88
time (sec)	N/A	0.037	0.015	2.411	0.000	0.261	1.038	0.000	0.662

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	89	55	0	66	223	0	74
N.S.	1	1.00	1.07	0.66	0.00	0.80	2.69	0.00	0.89
time (sec)	N/A	0.060	0.106	10.336	0.000	0.266	2.872	0.000	0.714

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	123	71	0	93	398	0	104
N.S.	1	1.00	1.06	0.61	0.00	0.80	3.43	0.00	0.90
time (sec)	N/A	0.083	0.187	32.036	0.000	0.263	7.512	0.000	0.804

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	153	87	0	120	620	0	134
N.S.	1	1.00	1.03	0.58	0.00	0.81	4.16	0.00	0.90
time (sec)	N/A	0.108	0.211	85.375	0.000	0.284	21.896	0.000	0.860

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	37	0	42	0	0	33
N.S.	1	1.00	1.00	1.06	0.00	1.20	0.00	0.00	0.94
time (sec)	N/A	0.027	0.012	0.207	0.000	0.262	0.000	0.000	0.693

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	54	0	70	0	0	78
N.S.	1	1.00	0.83	0.75	0.00	0.97	0.00	0.00	1.08
time (sec)	N/A	0.055	0.025	0.202	0.000	0.273	0.000	0.000	0.729

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	79	70	0	97	0	0	120
N.S.	1	1.00	0.73	0.65	0.00	0.90	0.00	0.00	1.11
time (sec)	N/A	0.082	0.027	0.200	0.000	0.261	0.000	0.000	0.778

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.010	0.000	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	37	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.009	0.020	0.000	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	34	16	15	15	15	15	15
N.S.	1	1.00	1.89	0.89	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.020	0.007	0.520	0.279	0.269	0.418	0.300	0.606

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	55	40	0	40	99	0	46
N.S.	1	1.00	1.04	0.75	0.00	0.75	1.87	0.00	0.87
time (sec)	N/A	0.041	0.015	2.392	0.000	0.260	1.057	0.000	0.654

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	86	57	0	68	231	0	79
N.S.	1	1.00	0.98	0.65	0.00	0.77	2.62	0.00	0.90
time (sec)	N/A	0.064	0.101	10.279	0.000	0.267	2.865	0.000	0.699

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	122	73	0	95	410	0	111
N.S.	1	1.00	0.99	0.59	0.00	0.77	3.33	0.00	0.90
time (sec)	N/A	0.091	0.203	31.910	0.000	0.290	7.639	0.000	0.778

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	39	0	44	0	0	35
N.S.	1	1.00	1.00	1.05	0.00	1.19	0.00	0.00	0.95
time (sec)	N/A	0.029	0.014	0.223	0.000	0.265	0.000	0.000	0.677

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	62	56	0	72	0	0	80
N.S.	1	1.00	0.82	0.74	0.00	0.95	0.00	0.00	1.05
time (sec)	N/A	0.059	0.029	0.215	0.000	0.264	0.000	0.000	0.241

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	81	72	0	99	0	0	122
N.S.	1	1.00	0.71	0.63	0.00	0.87	0.00	0.00	1.07
time (sec)	N/A	0.093	0.028	0.211	0.000	0.279	0.000	0.000	0.770

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.020	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.013	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.011	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	45	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	36	16	23	15	15	15	15
N.S.	1	1.00	2.25	1.00	1.44	0.94	0.94	0.94	0.94
time (sec)	N/A	0.020	0.008	0.658	0.200	0.261	2.946	0.287	0.624

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	60	41	0	41	116	0	47
N.S.	1	1.00	1.11	0.76	0.00	0.76	2.15	0.00	0.87
time (sec)	N/A	0.041	0.017	3.331	0.000	0.284	19.288	0.000	0.642

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	91	57	0	68	291	0	80
N.S.	1	1.00	1.02	0.64	0.00	0.76	3.27	0.00	0.90
time (sec)	N/A	0.067	0.121	12.617	0.000	0.257	87.600	0.000	0.723

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	127	73	0	95	0	0	112
N.S.	1	1.00	1.02	0.59	0.00	0.77	0.00	0.00	0.90
time (sec)	N/A	0.090	0.269	40.229	0.000	0.268	0.000	0.000	0.772

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	39	0	44	0	0	35
N.S.	1	1.00	0.97	1.03	0.00	1.16	0.00	0.00	0.92
time (sec)	N/A	0.028	0.017	0.200	0.000	0.270	0.000	0.000	0.702

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	56	0	72	0	0	81
N.S.	1	1.00	0.81	0.73	0.00	0.94	0.00	0.00	1.05
time (sec)	N/A	0.057	0.031	0.212	0.000	0.265	0.000	0.000	0.733

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	72	0	99	0	0	123
N.S.	1	1.00	0.70	0.63	0.00	0.86	0.00	0.00	1.07
time (sec)	N/A	0.091	0.039	0.212	0.000	0.291	0.000	0.000	0.763

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.019	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.014	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.011	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	37	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.009	0.021	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	34	18	23	15	19	15	15
N.S.	1	1.00	1.89	1.00	1.28	0.83	1.06	0.83	0.83
time (sec)	N/A	0.021	0.007	0.730	0.197	0.250	6.375	0.275	0.622

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	55	42	0	40	124	0	47
N.S.	1	1.00	1.02	0.78	0.00	0.74	2.30	0.00	0.87
time (sec)	N/A	0.045	0.017	3.461	0.000	0.258	39.108	0.000	0.643

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	85	59	0	68	316	0	79
N.S.	1	1.00	0.96	0.66	0.00	0.76	3.55	0.00	0.89
time (sec)	N/A	0.066	0.133	13.843	0.000	0.267	163.452	0.000	0.710

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	121	75	0	95	0	0	111
N.S.	1	1.00	0.98	0.60	0.00	0.77	0.00	0.00	0.90
time (sec)	N/A	0.092	0.283	42.191	0.000	0.263	0.000	0.000	0.765

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	41	0	44	0	0	35
N.S.	1	1.00	0.97	1.08	0.00	1.16	0.00	0.00	0.92
time (sec)	N/A	0.029	0.016	0.223	0.000	0.278	0.000	0.000	0.197

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	72	0	0	81
N.S.	1	1.00	0.81	0.75	0.00	0.94	0.00	0.00	1.05
time (sec)	N/A	0.060	0.030	0.194	0.000	0.275	0.000	0.000	0.752

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	74	0	99	0	0	123
N.S.	1	1.00	0.70	0.64	0.00	0.86	0.00	0.00	1.07
time (sec)	N/A	0.095	0.034	0.219	0.000	0.260	0.000	0.000	0.769

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	34	63	53	36	30	49
N.S.	1	1.00	0.84	0.68	1.26	1.06	0.72	0.60	0.98
time (sec)	N/A	0.035	0.018	0.305	0.271	0.244	0.136	0.260	0.151

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	178	112	86	0	24	92
N.S.	1	1.00	0.66	2.44	1.53	1.18	0.00	0.33	1.26
time (sec)	N/A	0.030	0.012	0.333	0.179	0.281	0.000	0.279	0.631

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	28	43	31	19	24	28
N.S.	1	1.00	1.00	0.93	1.43	1.03	0.63	0.80	0.93
time (sec)	N/A	0.030	0.014	0.264	0.264	0.252	0.084	0.270	0.581

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	52	87	40	54	0	0	55
N.S.	1	1.00	1.27	2.12	0.98	1.32	0.00	0.00	1.34
time (sec)	N/A	0.026	0.022	0.260	0.190	0.265	0.000	0.000	0.576

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	24	15	8	11	15
N.S.	1	1.00	1.00	0.93	1.60	1.00	0.53	0.73	1.00
time (sec)	N/A	0.022	0.005	0.224	0.286	0.248	0.023	0.257	0.552

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	14	12	15	10	12	15
N.S.	1	1.00	1.00	0.88	0.75	0.94	0.62	0.75	0.94
time (sec)	N/A	0.023	0.005	0.250	0.208	0.252	0.025	0.273	0.569

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	56	149	33	54	0	0	56
N.S.	1	1.00	1.37	3.63	0.80	1.32	0.00	0.00	1.37
time (sec)	N/A	0.026	0.045	0.273	0.290	0.274	0.000	0.000	0.569

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	30	41	31	19	24	29
N.S.	1	1.00	1.00	0.94	1.28	0.97	0.59	0.75	0.91
time (sec)	N/A	0.030	0.013	0.234	0.193	0.269	0.088	0.279	0.568

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	82	305	107	86	0	24	93
N.S.	1	1.00	1.12	4.18	1.47	1.18	0.00	0.33	1.27
time (sec)	N/A	0.030	0.062	0.349	0.274	0.272	0.000	0.280	0.113

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	69	82	0	364	0	0	0
N.S.	1	1.00	0.53	0.63	0.00	2.78	0.00	0.00	0.00
time (sec)	N/A	0.066	0.030	0.257	0.000	0.295	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	71	526	0	186	0	132	0
N.S.	1	1.00	0.74	5.48	0.00	1.94	0.00	1.38	0.00
time (sec)	N/A	0.063	0.018	0.447	0.000	0.300	0.000	0.349	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	55	61	0	357	0	0	0
N.S.	1	1.00	0.65	0.73	0.00	4.25	0.00	0.00	0.00
time (sec)	N/A	0.060	0.023	0.253	0.000	0.287	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	91	204	0	152	0	70	0
N.S.	1	1.00	1.44	3.24	0.00	2.41	0.00	1.11	0.00
time (sec)	N/A	0.046	0.028	0.355	0.000	0.271	0.000	0.300	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	253	0	0	0
N.S.	1	1.00	1.00	0.90	0.00	6.02	0.00	0.00	0.00
time (sec)	N/A	0.048	0.011	0.234	0.000	0.281	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	15	253	0	0	0
N.S.	1	1.00	1.00	0.91	0.35	5.88	0.00	0.00	0.00
time (sec)	N/A	0.049	0.012	0.254	0.191	0.281	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	117	87	40	152	0	70	0
N.S.	1	1.00	1.86	1.38	0.63	2.41	0.00	1.11	0.00
time (sec)	N/A	0.045	0.046	0.284	0.268	0.262	0.000	0.337	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	60	66	35	357	0	0	0
N.S.	1	1.00	0.70	0.77	0.41	4.15	0.00	0.00	0.00
time (sec)	N/A	0.058	0.024	0.248	0.197	0.286	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	132	188	76	186	0	132	0
N.S.	1	1.00	1.38	1.96	0.79	1.94	0.00	1.38	0.00
time (sec)	N/A	0.061	0.063	0.382	0.273	0.291	0.000	0.347	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	20	59	35	39	18	21
N.S.	1	1.00	0.69	0.57	1.69	1.00	1.11	0.51	0.60
time (sec)	N/A	0.030	0.013	0.255	0.295	0.258	0.135	0.274	0.109

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	47	45	95	75	0	111	41
N.S.	1	1.00	0.70	0.67	1.42	1.12	0.00	1.66	0.61
time (sec)	N/A	0.030	0.014	0.313	0.206	0.268	0.000	0.287	0.607

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	15	35	21	20	12	24
N.S.	1	1.00	0.95	0.79	1.84	1.11	1.05	0.63	1.26
time (sec)	N/A	0.025	0.011	0.250	0.301	0.249	0.091	0.273	0.625

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	48	31	45	51	0	67	33
N.S.	1	1.00	0.72	0.46	0.67	0.76	0.00	1.00	0.49
time (sec)	N/A	0.029	0.011	0.250	0.208	0.264	0.000	0.283	0.626

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	21	24	28	49	32	35	23
N.S.	1	1.00	0.75	0.86	1.00	1.75	1.14	1.25	0.82
time (sec)	N/A	0.032	0.015	0.244	0.309	0.274	0.101	0.272	0.607

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	21	24	0	49	34	35	25
N.S.	1	1.00	0.72	0.83	0.00	1.69	1.17	1.21	0.86
time (sec)	N/A	0.032	0.015	0.256	0.000	0.260	0.098	0.281	0.596

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	48	46	58	51	0	67	31
N.S.	1	1.00	0.72	0.69	0.87	0.76	0.00	1.00	0.46
time (sec)	N/A	0.030	0.015	0.279	0.273	0.258	0.000	0.288	0.068

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	15	13	21	22	13	24
N.S.	1	1.00	0.95	0.79	0.68	1.11	1.16	0.68	1.26
time (sec)	N/A	0.024	0.011	0.229	0.216	0.240	0.081	0.274	0.059

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	47	46	99	75	0	111	40
N.S.	1	1.00	0.70	0.69	1.48	1.12	0.00	1.66	0.60
time (sec)	N/A	0.029	0.013	0.342	0.203	0.271	0.000	0.296	0.600

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	56	47	0	101	0	0	48
N.S.	1	1.00	0.59	0.49	0.00	1.06	0.00	0.00	0.51
time (sec)	N/A	0.062	0.024	0.262	0.000	0.262	0.000	0.000	1.886

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	77	57	0	66	0	134	46
N.S.	1	1.00	1.12	0.83	0.00	0.96	0.00	1.94	0.67
time (sec)	N/A	0.052	0.029	0.375	0.000	0.307	0.000	0.294	1.141

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	42	0	71	0	0	41
N.S.	1	1.00	0.98	0.86	0.00	1.45	0.00	0.00	0.84
time (sec)	N/A	0.055	0.022	0.243	0.000	0.250	0.000	0.000	1.409

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	78	46	0	47	0	76	32
N.S.	1	1.00	1.44	0.85	0.00	0.87	0.00	1.41	0.59
time (sec)	N/A	0.040	0.024	0.311	0.000	0.265	0.000	0.296	0.747

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	51	58	0	317	0	0	0
N.S.	1	1.00	0.58	0.66	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	0.063	0.029	0.251	0.000	0.282	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	60	86	52	317	0	0	0
N.S.	1	1.00	0.67	0.97	0.58	3.56	0.00	0.00	0.00
time (sec)	N/A	0.064	0.029	0.251	0.184	0.284	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	78	56	59	47	0	76	32
N.S.	1	1.00	1.44	1.04	1.09	0.87	0.00	1.41	0.59
time (sec)	N/A	0.042	0.028	0.260	0.181	0.275	0.000	0.343	0.738

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	42	29	71	0	0	49
N.S.	1	1.00	0.98	0.86	0.59	1.45	0.00	0.00	1.00
time (sec)	N/A	0.054	0.022	0.252	0.178	0.261	0.000	0.000	1.231

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	77	58	119	66	0	134	45
N.S.	1	1.00	1.12	0.84	1.72	0.96	0.00	1.94	0.65
time (sec)	N/A	0.049	0.034	0.377	0.182	0.300	0.000	0.307	1.101

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	90	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	88	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	206	141	0	0	0	0	0	0
N.S.	1	1.57	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	121	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	0.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	109	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	42	18	17	17	26	17	17
N.S.	1	1.00	2.33	1.00	0.94	0.94	1.44	0.94	0.94
time (sec)	N/A	0.023	0.007	0.521	0.265	0.267	0.439	0.255	0.639

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	122	120	0	0	0	0	0	0
N.S.	1	1.88	1.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.034	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	166	142	0	0	0	0	0	0
N.S.	1	1.84	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.039	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	233	174	0	0	0	0	0	0
N.S.	1	1.85	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	165	166	0	298	0	0	281
N.S.	1	1.00	0.91	0.92	0.00	1.65	0.00	0.00	1.55
time (sec)	N/A	0.130	0.347	35.408	0.000	0.276	0.000	0.000	0.980

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	118	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.043	0.000	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	283	283	217	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.192	0.000	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	214	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.153	0.000	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	322	248	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.222	0.000	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	206	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	176	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.001	0.000	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	120	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	142	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	0.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	281	281	159	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	0.064	0.000	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.081	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	123	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	96	0	0	0	0	0	0
N.S.	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.135	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	115	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	41	76	42	0	0	54
N.S.	1	1.00	0.74	0.77	1.43	0.79	0.00	0.00	1.02
time (sec)	N/A	0.049	0.024	0.817	0.214	0.287	0.000	0.000	0.732

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	41	0	44	0	0	54
N.S.	1	1.00	0.74	0.77	0.00	0.83	0.00	0.00	1.02
time (sec)	N/A	0.047	0.021	1.363	0.000	0.263	0.000	0.000	0.677

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	55	62	0	78	0	0	0
N.S.	1	1.00	0.92	1.03	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.083	0.019	5.089	0.000	0.271	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	292	379	439	299	0
N.S.	1	1.00	0.95	0.89	7.68	9.97	11.55	7.87	0.00
time (sec)	N/A	0.059	0.816	0.783	0.353	0.621	2.711	0.298	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	155	169	194	139	160
N.S.	1	1.00	0.95	0.89	4.08	4.45	5.11	3.66	4.21
time (sec)	N/A	0.057	0.164	0.452	0.279	0.276	0.811	0.273	3.923

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	62	49	54	45	47
N.S.	1	1.00	0.95	0.89	1.63	1.29	1.42	1.18	1.24
time (sec)	N/A	0.055	0.029	0.276	0.265	0.263	0.204	0.278	0.765

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	0	49	53	80	65
N.S.	1	1.00	0.95	0.89	0.00	1.29	1.39	2.11	1.71
time (sec)	N/A	0.056	0.029	0.292	0.000	0.257	0.213	0.274	0.712

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	0	169	192	139	159
N.S.	1	1.00	0.95	0.89	0.00	4.45	5.05	3.66	4.18
time (sec)	N/A	0.054	0.160	0.587	0.000	0.268	0.712	0.289	3.727

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	63	57	0	496	0	0	46
N.S.	1	1.00	0.97	0.88	0.00	7.63	0.00	0.00	0.71
time (sec)	N/A	0.141	0.375	0.375	0.000	0.359	0.000	0.000	2.468

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	57	0	192	0	0	48
N.S.	1	1.00	1.00	0.88	0.00	2.95	0.00	0.00	0.74
time (sec)	N/A	0.142	0.082	0.323	0.000	0.285	0.000	0.000	1.846

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	74	87	0	0	0	0	0
N.S.	1	1.00	0.52	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	0.041	0.243	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	75	86	54	0	0	0	0
N.S.	1	1.00	0.52	0.60	0.38	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	0.046	0.283	0.189	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	50	93	192	0	0	57
N.S.	1	1.00	1.00	0.77	1.43	2.95	0.00	0.00	0.88
time (sec)	N/A	0.144	0.083	0.314	0.189	0.283	0.000	0.000	1.800

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	63	50	274	496	0	0	47
N.S.	1	1.00	0.97	0.77	4.22	7.63	0.00	0.00	0.72
time (sec)	N/A	0.148	0.382	0.368	0.266	0.362	0.000	0.000	3.428

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [131] had the largest ratio of [1.25000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	14	0.286
2	A	5	4	1.00	14	0.286
3	A	7	5	1.00	14	0.357
4	A	3	3	1.00	12	0.250
5	A	3	3	1.00	10	0.300
6	A	6	6	1.00	14	0.429
7	A	5	5	1.00	14	0.357
8	A	6	6	1.00	14	0.429
9	A	7	6	1.00	14	0.429
10	A	8	6	1.00	14	0.429
11	A	3	2	1.00	14	0.143
12	A	3	2	1.00	14	0.143
13	A	3	2	1.00	12	0.167
14	A	3	2	1.00	10	0.200
15	A	3	2	1.00	14	0.143
16	A	3	2	1.00	14	0.143
17	A	3	2	1.00	14	0.143
18	A	3	2	1.00	14	0.143
19	A	14	11	1.00	14	0.786
20	A	10	9	1.00	14	0.643
21	A	9	7	1.00	12	0.583
22	A	5	5	1.00	10	0.500
23	A	8	7	1.00	14	0.500
24	A	8	7	1.00	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	12	8	1.00	14	0.571
26	A	14	9	1.00	14	0.643
27	A	3	2	1.00	14	0.143
28	A	3	2	1.00	14	0.143
29	A	3	2	1.00	12	0.167
30	A	3	2	1.00	10	0.200
31	A	3	2	1.00	14	0.143
32	A	3	2	1.00	14	0.143
33	A	3	2	1.00	14	0.143
34	A	3	2	1.00	14	0.143
35	A	5	4	1.00	14	0.286
36	A	7	5	1.00	14	0.357
37	A	3	3	1.00	12	0.250
38	A	3	3	1.00	10	0.300
39	A	6	6	1.00	14	0.429
40	A	5	5	1.00	14	0.357
41	A	6	6	1.00	14	0.429
42	A	7	6	1.00	14	0.429
43	A	8	6	1.00	14	0.429
44	A	3	2	1.00	14	0.143
45	A	3	2	1.00	14	0.143
46	A	3	2	1.00	12	0.167
47	A	3	2	1.00	10	0.200
48	A	3	2	1.00	14	0.143
49	A	3	2	1.00	14	0.143
50	A	3	2	1.00	14	0.143
51	A	3	2	1.00	14	0.143
52	A	14	11	1.00	14	0.786
53	A	10	9	1.00	14	0.643
54	A	9	7	1.00	12	0.583
55	A	5	5	1.00	10	0.500
56	A	8	7	1.00	14	0.500
57	A	8	7	1.00	14	0.500
58	A	12	8	1.00	14	0.571
59	A	14	9	1.00	14	0.643

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	19	9	1.00	14	0.643
61	A	15	12	1.00	16	0.750
62	A	14	11	1.00	14	0.786
63	A	13	10	1.00	12	0.833
64	A	17	14	1.00	16	0.875
65	A	6	6	1.00	16	0.375
66	A	7	7	1.00	16	0.438
67	A	9	8	1.00	16	0.500
68	A	10	8	1.00	16	0.500
69	A	11	8	1.00	16	0.500
70	A	15	12	1.00	16	0.750
71	A	15	12	1.00	16	0.750
72	A	14	11	1.00	14	0.786
73	A	13	10	1.00	12	0.833
74	A	17	14	1.00	16	0.875
75	A	6	6	1.00	16	0.375
76	A	7	7	1.00	16	0.438
77	A	9	8	1.00	16	0.500
78	A	10	8	1.00	16	0.500
79	A	16	13	1.00	16	0.812
80	A	16	12	1.00	16	0.750
81	A	15	11	1.00	14	0.786
82	A	14	11	1.00	12	0.917
83	A	19	16	1.00	16	1.000
84	A	7	6	1.00	16	0.375
85	A	8	7	1.00	16	0.438
86	A	10	9	1.00	16	0.562
87	A	11	9	1.00	16	0.562
88	A	15	12	1.00	16	0.750
89	A	15	12	1.00	16	0.750
90	A	14	11	1.00	14	0.786
91	A	13	10	1.00	12	0.833
92	A	17	14	1.00	16	0.875
93	A	6	6	1.00	16	0.375
94	A	7	7	1.00	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	9	8	1.00	16	0.500
96	A	10	8	1.00	16	0.500
97	A	15	12	1.00	16	0.750
98	A	15	12	1.00	16	0.750
99	A	14	11	1.00	14	0.786
100	A	13	10	1.00	12	0.833
101	A	17	14	1.00	16	0.875
102	A	6	6	1.00	16	0.375
103	A	7	7	1.00	16	0.438
104	A	9	8	1.00	16	0.500
105	A	10	8	1.00	16	0.500
106	A	16	13	1.00	16	0.812
107	A	16	12	1.00	16	0.750
108	A	15	11	1.00	14	0.786
109	A	14	11	1.00	12	0.917
110	A	19	16	1.00	16	1.000
111	A	7	6	1.00	16	0.375
112	A	8	7	1.00	16	0.438
113	A	10	9	1.00	16	0.562
114	A	11	9	1.00	16	0.562
115	A	16	12	1.00	14	0.857
116	A	15	11	1.00	12	0.917
117	A	14	10	1.00	10	1.000
118	A	25	13	1.00	14	0.929
119	A	13	9	1.00	14	0.643
120	A	14	10	1.00	14	0.714
121	A	16	11	1.00	14	0.786
122	A	5	5	1.00	14	0.357
123	A	4	4	1.00	12	0.333
124	A	3	3	1.00	10	0.300
125	A	4	4	1.00	14	0.286
126	A	3	3	1.00	14	0.214
127	A	4	4	1.00	14	0.286
128	A	27	13	1.00	16	0.812
129	A	26	12	1.00	14	0.857

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	25	11	1.00	12	0.917
131	A	39	20	1.00	16	1.250
132	A	16	13	1.00	16	0.812
133	A	17	14	1.00	16	0.875
134	A	4	4	1.00	14	0.286
135	A	4	4	1.00	14	0.286
136	A	3	3	1.00	14	0.214
137	A	3	3	1.00	14	0.214
138	A	4	4	1.00	14	0.286
139	A	4	4	1.00	14	0.286
140	A	9	5	1.00	14	0.357
141	A	4	3	1.00	14	0.214
142	A	4	3	1.00	14	0.214
143	A	9	5	1.00	14	0.357
144	A	2	2	1.00	16	0.125
145	A	2	2	1.00	16	0.125
146	A	2	2	1.00	16	0.125
147	A	2	2	1.00	16	0.125
148	A	2	2	1.00	16	0.125
149	A	2	2	1.00	16	0.125
150	A	2	2	1.00	12	0.167
151	A	2	2	1.00	12	0.167
152	A	2	2	1.00	16	0.125
153	A	2	2	1.00	15	0.133
154	A	4	4	1.00	15	0.267
155	A	4	4	1.00	15	0.267
156	A	3	3	1.00	13	0.231
157	A	2	2	1.00	11	0.182
158	A	4	4	1.00	15	0.267
159	A	2	2	1.00	15	0.133
160	A	3	3	1.00	15	0.200
161	A	5	5	1.00	15	0.333
162	A	8	8	1.00	16	0.500
163	A	7	7	1.00	16	0.438
164	A	7	7	1.00	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	6	6	1.00	14	0.429
166	A	5	5	1.00	12	0.417
167	A	8	8	1.00	16	0.500
168	A	4	4	1.00	16	0.250
169	A	5	5	1.00	16	0.312
170	A	7	6	1.00	16	0.375
171	A	3	2	1.00	16	0.125
172	A	3	2	1.00	16	0.125
173	A	3	2	1.00	16	0.125
174	A	3	2	1.00	14	0.143
175	A	3	2	1.00	12	0.167
176	A	3	2	1.00	16	0.125
177	A	3	2	1.00	16	0.125
178	A	3	2	1.00	16	0.125
179	A	3	2	1.00	16	0.125
180	A	9	8	1.00	16	0.500
181	A	8	8	1.00	16	0.500
182	A	8	7	1.00	16	0.438
183	A	7	6	1.00	14	0.429
184	A	6	6	1.00	12	0.500
185	A	8	8	1.00	16	0.500
186	A	5	4	1.00	16	0.250
187	A	6	5	1.00	16	0.312
188	A	8	7	1.00	16	0.438
189	A	8	8	1.00	16	0.500
190	A	7	7	1.00	16	0.438
191	A	7	7	1.00	16	0.438
192	A	6	6	1.00	14	0.429
193	A	5	5	1.00	12	0.417
194	A	8	8	1.00	16	0.500
195	A	4	4	1.00	16	0.250
196	A	5	5	1.00	16	0.312
197	A	7	6	1.00	16	0.375
198	A	3	2	1.00	16	0.125
199	A	3	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	3	2	1.00	16	0.125
201	A	3	2	1.00	14	0.143
202	A	3	2	1.00	12	0.167
203	A	3	2	1.00	16	0.125
204	A	3	2	1.00	16	0.125
205	A	3	2	0.98	16	0.125
206	A	3	2	0.98	16	0.125
207	A	9	8	1.00	16	0.500
208	A	8	8	1.00	16	0.500
209	A	8	7	1.00	16	0.438
210	A	7	6	1.00	14	0.429
211	A	6	6	1.00	12	0.500
212	A	8	8	1.00	16	0.500
213	A	5	4	1.00	16	0.250
214	A	6	5	1.00	16	0.312
215	A	8	7	1.00	16	0.438
216	A	15	12	1.00	18	0.667
217	A	14	11	1.00	16	0.688
218	A	13	10	1.00	14	0.714
219	A	15	12	1.00	18	0.667
220	A	6	6	1.00	18	0.333
221	A	15	12	1.00	18	0.667
222	A	14	11	1.00	16	0.688
223	A	13	10	1.00	14	0.714
224	A	18	15	1.00	18	0.833
225	A	6	6	1.00	18	0.333
226	A	15	12	1.00	18	0.667
227	A	14	11	1.00	16	0.688
228	A	13	10	1.00	14	0.714
229	A	14	11	1.00	18	0.611
230	A	5	5	1.00	18	0.278
231	A	15	12	1.00	18	0.667
232	A	14	11	1.00	16	0.688
233	A	13	10	1.00	14	0.714
234	A	18	15	1.00	18	0.833

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	6	6	1.00	18	0.333
236	A	4	3	1.00	14	0.214
237	A	4	4	1.00	14	0.286
238	A	4	4	1.00	14	0.286
239	A	3	3	1.00	12	0.250
240	A	2	2	1.00	10	0.200
241	A	5	5	1.00	14	0.357
242	A	2	2	1.00	14	0.143
243	A	3	3	1.00	14	0.214
244	A	3	3	1.00	19	0.158
245	A	2	2	1.00	19	0.105
246	A	2	2	1.00	17	0.118
247	A	2	2	1.00	6	0.333
248	A	1	1	1.00	19	0.053
249	A	2	2	1.00	19	0.105
250	A	3	2	1.00	19	0.105
251	A	4	2	1.00	19	0.105
252	A	5	2	1.00	19	0.105
253	A	3	3	1.00	21	0.143
254	A	3	3	1.00	21	0.143
255	A	3	3	1.00	21	0.143
256	A	1	1	1.00	21	0.048
257	A	2	2	1.00	21	0.095
258	A	3	2	1.00	21	0.095
259	A	3	3	1.00	21	0.143
260	A	2	2	1.00	21	0.095
261	A	2	2	1.00	19	0.105
262	A	2	2	1.00	8	0.250
263	A	1	1	1.00	21	0.048
264	A	2	2	1.00	21	0.095
265	A	3	2	1.00	21	0.095
266	A	4	2	1.00	21	0.095
267	A	3	3	1.00	23	0.130
268	A	3	3	1.00	23	0.130
269	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	1	1	1.00	23	0.043
271	A	2	2	1.00	23	0.087
272	A	3	2	1.00	23	0.087
273	A	3	3	1.00	21	0.143
274	A	2	2	1.00	21	0.095
275	A	2	2	1.00	19	0.105
276	A	2	2	1.00	8	0.250
277	A	1	1	1.00	21	0.048
278	A	2	2	1.00	21	0.095
279	A	3	2	1.00	21	0.095
280	A	4	2	1.00	21	0.095
281	A	3	3	1.00	23	0.130
282	A	3	3	1.00	23	0.130
283	A	3	3	1.00	23	0.130
284	A	1	1	1.00	23	0.043
285	A	2	2	1.00	23	0.087
286	A	3	2	1.00	23	0.087
287	A	3	3	1.00	21	0.143
288	A	2	2	1.00	21	0.095
289	A	2	2	1.00	19	0.105
290	A	2	2	1.00	8	0.250
291	A	1	1	1.00	21	0.048
292	A	2	2	1.00	21	0.095
293	A	3	2	1.00	21	0.095
294	A	4	2	1.00	21	0.095
295	A	3	3	1.00	23	0.130
296	A	3	3	1.00	23	0.130
297	A	3	3	1.00	23	0.130
298	A	1	1	1.00	23	0.043
299	A	2	2	1.00	23	0.087
300	A	3	2	1.00	23	0.087
301	A	3	2	1.00	24	0.083
302	A	5	4	1.00	24	0.167
303	A	3	2	1.00	24	0.083
304	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	2	2	1.00	24	0.083
306	A	2	2	1.00	24	0.083
307	A	4	4	1.00	24	0.167
308	A	3	2	1.00	24	0.083
309	A	5	4	1.00	24	0.167
310	A	4	3	1.00	25	0.120
311	A	5	5	1.00	25	0.200
312	A	4	3	1.00	25	0.120
313	A	4	4	1.00	25	0.160
314	A	3	3	1.00	25	0.120
315	A	3	3	1.00	25	0.120
316	A	4	4	1.00	25	0.160
317	A	4	3	1.00	25	0.120
318	A	5	5	1.00	25	0.200
319	A	3	2	1.00	24	0.083
320	A	3	3	1.00	24	0.125
321	A	2	2	1.00	24	0.083
322	A	3	3	1.00	24	0.125
323	A	4	3	1.00	24	0.125
324	A	4	3	1.00	24	0.125
325	A	3	3	1.00	24	0.125
326	A	2	2	1.00	24	0.083
327	A	3	3	1.00	24	0.125
328	A	4	3	1.00	25	0.120
329	A	3	3	1.00	25	0.120
330	A	3	3	1.00	25	0.120
331	A	3	3	1.00	25	0.120
332	A	5	4	1.00	25	0.160
333	A	5	4	1.00	25	0.160
334	A	3	3	1.00	25	0.120
335	A	3	3	1.00	25	0.120
336	A	3	3	1.00	25	0.120
337	A	2	2	1.00	21	0.095
338	A	2	2	1.00	19	0.105
339	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	4	4	1.57	24	0.167
341	A	4	4	1.00	24	0.167
342	A	3	3	1.00	22	0.136
343	A	1	1	1.00	21	0.048
344	A	3	3	1.88	24	0.125
345	A	5	5	1.84	24	0.208
346	A	6	6	1.85	24	0.250
347	A	4	2	1.00	21	0.095
348	A	3	3	1.00	23	0.130
349	A	3	3	1.00	23	0.130
350	A	3	3	1.00	23	0.130
351	A	5	5	1.00	26	0.192
352	A	5	5	1.00	26	0.192
353	A	5	5	1.00	26	0.192
354	A	5	5	1.00	26	0.192
355	A	4	4	1.00	24	0.167
356	A	3	3	1.00	23	0.130
357	A	3	3	1.00	26	0.115
358	A	4	4	1.00	26	0.154
359	A	6	6	1.00	26	0.231
360	A	3	3	1.00	23	0.130
361	A	3	3	1.00	23	0.130
362	A	3	3	1.00	23	0.130
363	A	3	3	1.00	23	0.130
364	A	2	2	1.00	22	0.091
365	A	2	2	1.00	24	0.083
366	A	2	2	1.00	24	0.083
367	A	2	2	1.00	24	0.083
368	A	3	3	1.00	26	0.115
369	A	3	3	1.00	26	0.115
370	A	3	3	1.00	26	0.115
371	A	3	3	1.00	21	0.143
372	A	3	3	1.00	24	0.125
373	A	3	3	1.00	24	0.125
374	A	1	1	1.00	35	0.029

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	2	2	1.00	26	0.077
376	A	2	2	1.00	26	0.077
377	A	2	2	1.00	26	0.077
378	A	2	2	1.00	26	0.077
379	A	2	2	1.00	26	0.077
380	A	3	3	1.00	28	0.107
381	A	3	3	1.00	28	0.107
382	A	4	3	1.00	28	0.107
383	A	4	3	1.00	28	0.107
384	A	3	3	1.00	28	0.107
385	A	3	3	1.00	28	0.107

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int e^{i\arctan(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 113

$$\int e^{i\arctan(ax)}x^4\,dx = -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3}+\frac{x^3\sqrt{1+a^2x^2}}{4a^2}+\frac{ix^4\sqrt{1+a^2x^2}}{5a}+\frac{(64i-45ax)\sqrt{1+a^2x^2}}{120a^5}+\frac{3\operatorname{arcsinh}(ax)}{8a^5}$$

```
[Out] 3/8*arcsinh(a*x)/a^5-4/15*I*x^2*(a^2*x^2+1)^(1/2)/a^3+1/4*x^3*(a^2*x^2+1)^(
1/2)/a^2+1/5*I*x^4*(a^2*x^2+1)^(1/2)/a+1/120*(64*I-45*a*x)*(a^2*x^2+1)^(1/2
)/a^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5168, 847, 794, 221}

$$\int e^{i\arctan(ax)}x^4\,dx = \frac{3\operatorname{arcsinh}(ax)}{8a^5}+\frac{ix^4\sqrt{a^2x^2+1}}{5a}+\frac{x^3\sqrt{a^2x^2+1}}{4a^2}+\frac{(-45ax+64i)\sqrt{a^2x^2+1}}{120a^5}-\frac{4ix^2\sqrt{a^2x^2+1}}{15a^3}$$

```
[In] Int[E^(I*ArcTan[a*x])*x^4,x]
```

```
[Out] (((-4*I)/15)*x^2*Sqrt[1 + a^2*x^2])/a^3 + (x^3*Sqrt[1 + a^2*x^2])/(4*a^2) +
 ((I/5)*x^4*Sqrt[1 + a^2*x^2])/a + ((64*I - 45*a*x)*Sqrt[1 + a^2*x^2])/(120
*a^5) + (3*ArcSinh[a*x])/(8*a^5)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4(1+iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{\int \frac{x^3(-4ia+5a^2x)}{\sqrt{1+a^2x^2}}\,dx}{5a^2} \\
&= \frac{x^3\sqrt{1+a^2x^2}}{4a^2} + \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{\int \frac{x^2(-15a^2-16ia^3x)}{\sqrt{1+a^2x^2}}\,dx}{20a^4} \\
&= -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3} + \frac{x^3\sqrt{1+a^2x^2}}{4a^2} + \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{\int \frac{x(32ia^3-45a^4x)}{\sqrt{1+a^2x^2}}\,dx}{60a^6} \\
&= -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3} + \frac{x^3\sqrt{1+a^2x^2}}{4a^2} + \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{(64i-45ax)\sqrt{1+a^2x^2}}{120a^5} + \frac{3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a^4} \\
&= -\frac{4ix^2\sqrt{1+a^2x^2}}{15a^3} + \frac{x^3\sqrt{1+a^2x^2}}{4a^2} + \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{(64i-45ax)\sqrt{1+a^2x^2}}{120a^5} + \frac{3\operatorname{arcsinh}(ax)}{8a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.57

$$\int e^{i\arctan(ax)}x^4\,dx = \frac{\sqrt{1+a^2x^2}(64i-45ax-32ia^2x^2+30a^3x^3+24ia^4x^4)+45\text{arcsinh}(ax)}{120a^5}$$

[In] Integrate[E^(I*ArcTan[a*x])*x^4,x]

[Out] (Sqrt[1 + a^2*x^2]*(64*I - 45*a*x - (32*I)*a^2*x^2 + 30*a^3*x^3 + (24*I)*a^4*x^4) + 45*ArcSinh[a*x])/(120*a^5)

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{i(24a^4x^4-30ia^3x^3-32a^2x^2+45iax+64)\sqrt{a^2x^2+1}}{120a^5}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^4\sqrt{a^2}}$	84
meijerg	$\frac{-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{5}{2}}\left(-10a^2x^2+15\right)\sqrt{a^2x^2+1}}{20a^4}+\frac{3\sqrt{\pi}\left(a^2\right)^{\frac{5}{2}}\text{arcsinh}(ax)}{4a^5}}{2a^4\sqrt{\pi}\sqrt{a^2}}+\frac{i\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(6a^4x^4-8a^2x^2+16\right)\sqrt{a^2x^2+1}}{15}\right)}{2a^5\sqrt{\pi}}$	117
default	$\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}\right)}{4a^2}+ia\left(\frac{x^4\sqrt{a^2x^2+1}}{5a^2}-\frac{4\left(\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4}\right)}{5a^2}\right)$	142

[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x,method=_RETURNVERBOSE)

[Out] 1/120*I*(24*a^4*x^4-30*I*a^3*x^3-32*a^2*x^2+45*I*a*x+64)*(a^2*x^2+1)^(1/2)/a^5+3/8/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

$$\int e^{i\arctan(ax)}x^4\,dx = \frac{(24i\,a^4x^4+30\,a^3x^3-32i\,a^2x^2-45\,ax+64i)\sqrt{a^2x^2+1}-45\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{120\,a^5}$$

[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="fricas")

[Out] 1/120*((24*I*a^4*x^4 + 30*a^3*x^3 - 32*I*a^2*x^2 - 45*a*x + 64*I)*sqrt(a^2*x^2 + 1) - 45*log(-a*x + sqrt(a^2*x^2 + 1)))/a^5

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int e^{i\arctan(ax)}x^4\,dx = \begin{cases} \sqrt{a^2x^2+1}\left(\frac{ix^4}{5a}+\frac{x^3}{4a^2}-\frac{4ix^2}{15a^3}-\frac{3x}{8a^4}+\frac{8i}{15a^5}\right)+\frac{3\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{8a^4\sqrt{a^2}} & \text{for } a^2\neq 0 \\ \frac{iax^6}{6}+\frac{x^5}{5} & \text{otherwise} \end{cases}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**4,x)
```

```
[Out] Piecewise((sqrt(a**2*x**2 + 1)*(I*x**4/(5*a) + x**3/(4*a**2) - 4*I*x**2/(15
*a**3) - 3*x/(8*a**4) + 8*I/(15*a**5)) + 3*log(2*a**2*x + 2*sqrt(a**2*x**2
+ 1)*sqrt(a**2))/(8*a**4*sqrt(a**2)), Ne(a**2, 0)), (I*a*x**6/6 + x**5/5, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int e^{i\arctan(ax)}x^4\,dx = \frac{i\sqrt{a^2x^2+1}x^4}{5a}+\frac{\sqrt{a^2x^2+1}x^3}{4a^2}-\frac{4i\sqrt{a^2x^2+1}x^2}{15a^3}-\frac{3\sqrt{a^2x^2+1}x}{8a^4}+\frac{3\,\mathrm{arsinh}\,(ax)}{8a^5}+\frac{8i\sqrt{a^2x^2+1}}{15a^5}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*I*sqrt(a^2*x^2 + 1)*x^4/a + 1/4*sqrt(a^2*x^2 + 1)*x^3/a^2 - 4/15*I*sqrt
(a^2*x^2 + 1)*x^2/a^3 - 3/8*sqrt(a^2*x^2 + 1)*x/a^4 + 3/8*arcsinh(a*x)/a^5
+ 8/15*I*sqrt(a^2*x^2 + 1)/a^5
```

Giac [F(-2)]

Exception generated.

$$\int e^{i\arctan(ax)}x^4\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int e^{i\arctan(ax)}x^4\,dx=\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{x^3\,{\left(a^2\right)}^{3/2}}{4\,a^4}-\frac{3\,x\,\sqrt{a^2}}{8\,a^4}+\frac{a\,8{}\mathrm{i}}{15\,{\left(a^2\right)}^{5/2}}-\frac{a^3\,x^2\,4{}\mathrm{i}}{15\,{\left(a^2\right)}^{5/2}}+\frac{a^5\,x^4\,1{}\mathrm{i}}{5\,{\left(a^2\right)}^{5/2}}\right)}{\sqrt{a^2}}+\frac{3\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{8\,a^4\,\sqrt{a^2}}$$

```
[In] int((x^4*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*((a*8i)/(15*(a^2)^(5/2)) - (a^3*x^2*4i)/(15*(a^2)^(5/2
)) + (x^3*(a^2)^(3/2))/(4*a^4) + (a^5*x^4*1i)/(5*(a^2)^(5/2)) - (3*x*(a^2)^
(1/2))/(8*a^4)))/(a^2)^(1/2) + (3*asinh(x*(a^2)^(1/2)))/(8*a^4*(a^2)^(1/2))
```

3.2 $\int e^{i \arctan(ax)} x^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int e^{i \arctan(ax)} x^3 \, dx = \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16+9iax)\sqrt{1+a^2x^2}}{24a^4} + \frac{3i\operatorname{arcsinh}(ax)}{8a^4}$$

```
[Out] 3/8*I*arcsinh(a*x)/a^4+1/3*x^2*(a^2*x^2+1)^(1/2)/a^2+1/4*I*x^3*(a^2*x^2+1)^
(1/2)/a-1/24*(16+9*I*a*x)*(a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5168, 847, 794, 221}

$$\int e^{i \arctan(ax)} x^3 \, dx = \frac{3i\operatorname{arcsinh}(ax)}{8a^4} + \frac{x^2\sqrt{a^2x^2+1}}{3a^2} + \frac{ix^3\sqrt{a^2x^2+1}}{4a} - \frac{(16+9iax)\sqrt{a^2x^2+1}}{24a^4}$$

```
[In] Int[E^(I*ArcTan[a*x])*x^3,x]
```

```
[Out] (x^2*Sqrt[1 + a^2*x^2])/(3*a^2) + ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - ((16 +
(9*I)*a*x)*Sqrt[1 + a^2*x^2])/(24*a^4) + (((3*I)/8)*ArcSinh[a*x])/a^4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3(1+iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x^2(-3ia+4a^2x)}{\sqrt{1+a^2x^2}}\,dx}{4a^2} \\
&= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x(-8a^2-9ia^3x)}{\sqrt{1+a^2x^2}}\,dx}{12a^4} \\
&= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16+9iax)\sqrt{1+a^2x^2}}{24a^4} + \frac{(3i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a^3} \\
&= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16+9iax)\sqrt{1+a^2x^2}}{24a^4} + \frac{3i\operatorname{arcsinh}(ax)}{8a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int e^{i\arctan(ax)}x^3\,dx=\frac{\sqrt{1+a^2x^2}(-16-9iax+8a^2x^2+6ia^3x^3)+9i\text{arcsinh}(ax)}{24a^4}$$

```
[In] Integrate[E^(I*ArcTan[a*x])*x^3,x]
[Out] (Sqrt[1 + a^2*x^2]*(-16 - (9*I)*a*x + 8*a^2*x^2 + (6*I)*a^3*x^3) + (9*I)*Ar
cSinh[a*x])/(24*a^4)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{i(6a^3x^3-8ia^2x^2-9ax+16i)\sqrt{a^2x^2+1}}{24a^4}+\frac{3i\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^3\sqrt{a^2}}$	77
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-4a^2x^2+8\right)\sqrt{a^2x^2+1}}{6}}{2a^4\sqrt{\pi}}+\frac{i\left(-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{5}{2}}\left(-10a^2x^2+15\right)\sqrt{a^2x^2+1}}{20a^4}+\frac{3\sqrt{\pi}\left(a^2\right)^{\frac{5}{2}}\text{arcsinh}(ax)}{4a^5}\right)}{2a^3\sqrt{\pi}\sqrt{a^2}}$	109
default	$\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4}+ia\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}\right)}{4a^2}\right)$	117

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x,method=_RETURNVERBOSE)
[Out] 1/24*I*(6*a^3*x^3-8*I*a^2*x^2-9*a*x+16*I)*(a^2*x^2+1)^(1/2)/a^4+3/8*I/a^3*l
n(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int e^{i\arctan(ax)}x^3\,dx=\frac{(6i\,a^3x^3+8\,a^2x^2-9i\,ax-16)\sqrt{a^2x^2+1}-9i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{24\,a^4}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="fricas")
[Out] 1/24*((6*I*a^3*x^3 + 8*a^2*x^2 - 9*I*a*x - 16)*sqrt(a^2*x^2 + 1) - 9*I*log(
-a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int e^{i\arctan(ax)}x^3\,dx = \begin{cases} \sqrt{a^2x^2+1}\left(\frac{ix^3}{4a}+\frac{x^2}{3a^2}-\frac{3ix}{8a^3}-\frac{2}{3a^4}\right)+\frac{3i\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{8a^3\sqrt{a^2}} & \text{for } a^2\neq 0 \\ \frac{iax^5}{5}+\frac{x^4}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**3,x)
```

```
[Out] Piecewise((sqrt(a**2*x**2 + 1)*(I*x**3/(4*a) + x**2/(3*a**2) - 3*I*x/(8*a**
3) - 2/(3*a**4)) + 3*I*log(2*a**2*x + 2*sqrt(a**2*x**2 + 1)*sqrt(a**2))/(8*
a**3*sqrt(a**2)), Ne(a**2, 0)), (I*a*x**5/5 + x**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int e^{i\arctan(ax)}x^3\,dx = \frac{i\sqrt{a^2x^2+1}x^3}{4a}+\frac{\sqrt{a^2x^2+1}x^2}{3a^2}-\frac{3i\sqrt{a^2x^2+1}x}{8a^3}+\frac{3i\operatorname{arsinh}(ax)}{8a^4}-\frac{2\sqrt{a^2x^2+1}}{3a^4}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*I*sqrt(a^2*x^2 + 1)*x^3/a + 1/3*sqrt(a^2*x^2 + 1)*x^2/a^2 - 3/8*I*sqrt(
a^2*x^2 + 1)*x/a^3 + 3/8*I*arcsinh(a*x)/a^4 - 2/3*sqrt(a^2*x^2 + 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int e^{i\arctan(ax)}x^3\,dx = -\frac{1}{24}\sqrt{a^2x^2+1}\left(\left(2x\left(-\frac{3ix}{a}-\frac{4}{a^2}\right)+\frac{9i}{a^3}\right)x+\frac{16}{a^4}\right)-\frac{3i\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{8a^3|a|}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] -1/24*sqrt(a^2*x^2 + 1)*((2*x*(-3*I*x/a - 4/a^2) + 9*I/a^3)*x + 16/a^4) - 3
/8*I*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^3*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int e^{i\arctan(ax)}x^3\,dx$$
$$=\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)\,3\mathrm{i}}{8\,a^3\,\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{2}{3\,{\left(a^2\right)}^{3/2}}-\frac{a^2\,x^2}{3\,{\left(a^2\right)}^{3/2}}-\frac{x^3\,{\left(a^2\right)}^{3/2}\,1\mathrm{i}}{4\,a^3}+\frac{x\,\sqrt{a^2}\,3\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}}$$

```
[In] int((x^3*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] (asinh(x*(a^2)^(1/2))*3i)/(8*a^3*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*(2/(3*
(a^2)^(3/2)) - (a^2*x^2)/(3*(a^2)^(3/2)) - (x^3*(a^2)^(3/2)*1i)/(4*a^3) + (
x*(a^2)^(1/2)*3i)/(8*a^3)))/(a^2)^(1/2)
```

3.3 $\int e^{i \arctan(ax)} x^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int e^{i \arctan(ax)} x^2 \, dx = -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} + \frac{i(1+a^2x^2)^{3/2}}{3a^3} - \frac{\operatorname{arcsinh}(ax)}{2a^3}$$

```
[Out] 1/3*I*(a^2*x^2+1)^(3/2)/a^3-1/2*arcsinh(a*x)/a^3-I*(a^2*x^2+1)^(1/2)/a^3+1/
2*x*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5168, 811, 655, 201, 221}

$$\int e^{i \arctan(ax)} x^2 \, dx = -\frac{\operatorname{arcsinh}(ax)}{2a^3} + \frac{x\sqrt{a^2x^2+1}}{2a^2} + \frac{i(a^2x^2+1)^{3/2}}{3a^3} - \frac{i\sqrt{a^2x^2+1}}{a^3}$$

```
[In] Int[E^(I*ArcTan[a*x])*x^2,x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2])/a^3 + (x*Sqrt[1 + a^2*x^2])/(2*a^2) + ((I/3)*(1 +
a^2*x^2)^(3/2))/a^3 - ArcSinh[a*x]/(2*a^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2(1+iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\int \frac{1+iax}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int (1+iax)\sqrt{1+a^2x^2}\,dx}{a^2} \\
&= -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{i(1+a^2x^2)^{3/2}}{3a^3} - \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int \sqrt{1+a^2x^2}\,dx}{a^2} \\
&= -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} + \frac{i(1+a^2x^2)^{3/2}}{3a^3} - \frac{\operatorname{arcsinh}(ax)}{a^3} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2} \\
&= -\frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} + \frac{i(1+a^2x^2)^{3/2}}{3a^3} - \frac{\operatorname{arcsinh}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int e^{i\arctan(ax)}x^2\,dx = \frac{(-4i+3ax+2ia^2x^2)\sqrt{1+a^2x^2}-3\text{arcsinh}(ax)}{6a^3}$$

```
[In] Integrate[E^(I*ArcTan[a*x])*x^2,x]
```

```
[Out] ((-4*I + 3*a*x + (2*I)*a^2*x^2)*Sqrt[1 + a^2*x^2] - 3*ArcSinh[a*x])/(6*a^3)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{i(2a^2x^2-3iax-4)\sqrt{a^2x^2+1}}{6a^3}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}$	67
default	$\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}+ia\left(\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4}\right)$	92
meijerg	$\frac{\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{3}{2}}\sqrt{a^2x^2+1}}{a^2}-\frac{\sqrt{\pi}\left(a^2\right)^{\frac{3}{2}}\text{arcsinh}(ax)}{a^3}}{2a^2\sqrt{\pi}\sqrt{a^2}}+\frac{i\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-4a^2x^2+8\right)\sqrt{a^2x^2+1}}{6}\right)}{2a^3\sqrt{\pi}}$	98

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*(2*a^2*x^2-3*I*a*x-4)*(a^2*x^2+1)^(1/2)/a^3-1/2/a^2*ln(a^2*x/(a^2)^(1
/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int e^{i\arctan(ax)}x^2\,dx = \frac{\sqrt{a^2x^2+1}(2i\,a^2x^2+3\,ax-4i)+3\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{6\,a^3}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(a^2*x^2 + 1)*(2*I*a^2*x^2 + 3*a*x - 4*I) + 3*log(-a*x + sqrt(a^2*
x^2 + 1)))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int e^{i\arctan(ax)}x^2\,dx = \begin{cases} \sqrt{a^2x^2+1}\left(\frac{ix^2}{3a}+\frac{x}{2a^2}-\frac{2i}{3a^3}\right)-\frac{\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{2a^2\sqrt{a^2}} & \text{for } a^2\neq 0 \\ \frac{iax^4}{4}+\frac{x^3}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**2,x)
[Out] Piecewise((sqrt(a**2*x**2 + 1)*(I*x**2/(3*a) + x/(2*a**2) - 2*I/(3*a**3)) -
 log(2*a**2*x + 2*sqrt(a**2*x**2 + 1)*sqrt(a**2))/(2*a**2*sqrt(a**2)), Ne(a
**2, 0)), (I*a*x**4/4 + x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int e^{i\arctan(ax)}x^2\,dx = \frac{i\sqrt{a^2x^2+1}x^2}{3a}+\frac{\sqrt{a^2x^2+1}x}{2a^2}-\frac{\operatorname{arsinh}(ax)}{2a^3}-\frac{2i\sqrt{a^2x^2+1}}{3a^3}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="maxima")
[Out] 1/3*I*sqrt(a^2*x^2 + 1)*x^2/a + 1/2*sqrt(a^2*x^2 + 1)*x/a^2 - 1/2*arcsinh(a
*x)/a^3 - 2/3*I*sqrt(a^2*x^2 + 1)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int e^{i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int e^{i\arctan(ax)}x^2\,dx=\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{x\,\sqrt{a^2}}{2\,a^2}-\frac{a\,2\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}+\frac{a^3\,x^2\,1\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}\right)}{\sqrt{a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}}$$

```
[In] int((x^2*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*((a^3*x^2*1i)/(3*(a^2)^(3/2)) - (a*2i)/(3*(a^2)^(3/2))
 + (x*(a^2)^(1/2))/(2*a^2)))/(a^2)^(1/2) - asinh(x*(a^2)^(1/2))/(2*a^2*(a^2
)^(1/2))
```

3.4 $\int e^{i \arctan(ax)} x \, dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int e^{i \arctan(ax)} x \, dx = \frac{(2+iax)\sqrt{1+a^2x^2}}{2a^2} - \frac{i\text{arcsinh}(ax)}{2a^2}$$

```
[Out] -1/2*I*arcsinh(a*x)/a^2+1/2*(2+I*a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5168, 794, 221}

$$\int e^{i \arctan(ax)} x \, dx = \frac{(2+iax)\sqrt{a^2x^2+1}}{2a^2} - \frac{i\text{arcsinh}(ax)}{2a^2}$$

```
[In] Int[E^(I*ArcTan[a*x])*x,x]
```

```
[Out] ((2 + I*a*x)*Sqrt[1 + a^2*x^2])/(2*a^2) - ((I/2)*ArcSinh[a*x])/a^2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(1+iax)}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{(2+iax)\sqrt{1+a^2x^2}}{2a^2} - \frac{i\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a}\\ &= \frac{(2+iax)\sqrt{1+a^2x^2}}{2a^2} - \frac{i\text{arcsinh}(ax)}{2a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int e^{i\arctan(ax)}x\,dx = \frac{(2+iax)\sqrt{1+a^2x^2} - i\text{arcsinh}(ax)}{2a^2}$$

```
[In] Integrate[E^(I*ArcTan[a*x])*x,x]
[Out] ((2 + I*a*x)*Sqrt[1 + a^2*x^2] - I*ArcSinh[a*x])/(2*a^2)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

method	result	size
risch	$\frac{i(ax-2i)\sqrt{a^2x^2+1}}{2a^2} - \frac{i\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a\sqrt{a^2}}$	59
default	$\frac{\sqrt{a^2x^2+1}}{a^2} + ia\left(\frac{x\sqrt{a^2x^2+1}}{2a^2} - \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}\right)$	72
meijerg	$\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{a^2x^2+1}}{2a^2\sqrt{\pi}} + \frac{i\left(\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{3}{2}}\sqrt{a^2x^2+1}}{a^2} - \frac{\sqrt{\pi}\left(a^2\right)^{\frac{3}{2}}\text{arcsinh}(ax)}{a^3}\right)}{2a\sqrt{\pi}\,\sqrt{a^2}}$	88

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(a*x-2*I)*(a^2*x^2+1)^(1/2)/a^2-1/2*I/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2
+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int e^{i\arctan(ax)}x\,dx = \frac{\sqrt{a^2x^2+1}(i\,ax+2)+i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2*x^2 + 1)*(I*a*x + 2) + I*log(-a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(34) = 68$.

Time = 0.54 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int e^{i\arctan(ax)}x\,dx = \begin{cases} \sqrt{a^2x^2+1}\left(\frac{ix}{2a}+\frac{1}{a^2}\right)-\frac{i\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{2a\sqrt{a^2}} & \text{for } a^2\neq 0 \\ \frac{iax^3}{3}+\frac{x^2}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x,x)
```

```
[Out] Piecewise((sqrt(a**2*x**2 + 1)*(I*x/(2*a) + a**(-2)) - I*log(2*a**2*x + 2*s
qrt(a**2*x**2 + 1)*sqrt(a**2))/(2*a*sqrt(a**2)), Ne(a**2, 0)), (I*a*x**3/3
+ x**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{i\arctan(ax)}x\,dx = \frac{i\,\sqrt{a^2x^2+1}x}{2\,a}-\frac{i\,\operatorname{arsinh}\left(ax\right)}{2\,a^2}+\frac{\sqrt{a^2x^2+1}}{a^2}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(a^2*x^2 + 1)*x/a - 1/2*I*arcsinh(a*x)/a^2 + sqrt(a^2*x^2 + 1)/a^
2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int e^{i\arctan(ax)}x\,dx = -\frac{1}{2}\sqrt{a^2x^2+1}\left(-\frac{i\,x}{a}-\frac{2}{a^2}\right)+\frac{i\,\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{2\,a|a|}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a^2*x^2 + 1)*(-I*x/a - 2/a^2) + 1/2*I*log(-x*abs(a) + sqrt(a^2*x^
2 + 1))/(a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int e^{i\arctan(ax)}x\,dx = \frac{\left(\frac{1}{\sqrt{a^2}}+\frac{x\,\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)\,\sqrt{a^2\,x^2+1}-\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{2\,a}}{\sqrt{a^2}}$$

```
[In] int((x*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

```
[Out] ((1/(a^2)^(1/2) + (x*(a^2)^(1/2)*1i)/(2*a))*(a^2*x^2 + 1)^(1/2) - (asinh(x*
(a^2)^(1/2))*1i)/(2*a))/(a^2)^(1/2)
```

3.5 $\int e^{i \arctan(ax)} dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int e^{i \arctan(ax)} dx = \frac{i\sqrt{1+a^2x^2}}{a} + \frac{\operatorname{arcsinh}(ax)}{a}$$

```
[Out] arcsinh(a*x)/a+I*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5167, 655, 221}

$$\int e^{i \arctan(ax)} dx = \frac{\operatorname{arcsinh}(ax)}{a} + \frac{i\sqrt{a^2x^2+1}}{a}$$

```
[In] Int[E^(I*ArcTan[a*x]),x]
[Out] (I*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 5167

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+iax}{\sqrt{1+a^2x^2}}\,dx \\ &= \frac{i\sqrt{1+a^2x^2}}{a} + \int \frac{1}{\sqrt{1+a^2x^2}}\,dx \\ &= \frac{i\sqrt{1+a^2x^2}}{a} + \frac{\operatorname{arcsinh}(ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{i\arctan(ax)}\,dx = \frac{i\sqrt{1+a^2x^2}+\operatorname{arcsinh}(ax)}{a}$$

```
[In] Integrate[E^(I*ArcTan[a*x]),x]
[Out] (I*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/a
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

method	result	size
meijerg	$\frac{\operatorname{arcsinh}(ax)}{a}+\frac{i\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{a^2x^2+1}\right)}{2a\sqrt{\pi}}$	41
default	$\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}+\frac{i\sqrt{a^2x^2+1}}{a}$	48
risch	$\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}+\frac{i\sqrt{a^2x^2+1}}{a}$	48

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(a*x)/a+1/2*I/a/Pi^(1/2)*(-2*Pi^(1/2)+2*Pi^(1/2)*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int e^{i\arctan(ax)}\,dx = \frac{i\sqrt{a^2x^2+1} - \log\left(-ax+\sqrt{a^2x^2+1}\right)}{a}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(a^2*x^2 + 1) - log(-a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(20) = 40.

Time = 0.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.10

$$\int e^{i\arctan(ax)}\,dx = \begin{cases} \frac{\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{i\sqrt{a^2x^2+1}}{a} & \text{for } a^2 \neq 0 \\ \frac{iax^2}{2} + x & \text{otherwise} \end{cases}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2),x)
[Out] Piecewise((log(2*a**2*x + 2*sqrt(a**2*x**2 + 1)*sqrt(a**2))/sqrt(a**2) + I*
sqrt(a**2*x**2 + 1)/a, Ne(a**2, 0)), (I*a*x**2/2 + x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int e^{i\arctan(ax)}\,dx = \frac{\operatorname{arsinh}(ax)}{a} + \frac{i\sqrt{a^2x^2+1}}{a}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(a*x)/a + I*sqrt(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int e^{i\arctan(ax)}\,dx = -\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|} + \frac{i\sqrt{a^2x^2+1}}{a}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) + I*sqrt(a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int e^{i\arctan(ax)}\,dx = \frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a}$$

```
[In] int((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2),x)
[Out] ((a^2*x^2 + 1)^(1/2)*1i)/a + asinh(x*(a^2)^(1/2))/(a^2)^(1/2)
```

3.6 $\int \frac{e^{i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = i\operatorname{arcsinh}(ax) - \operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5168, 858, 221, 272, 65, 214}

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = -\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right) + i\operatorname{arcsinh}(ax)$$

```
[In] Int[E^(I*ArcTan[a*x])/x,x]
```

```
[Out] I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+iax}{x\sqrt{1+a^2x^2}}\,dx \\
&= (ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx + \int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= i\operatorname{arcsinh}(ax) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= i\operatorname{arcsinh}(ax) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a^2} \\
&= i\operatorname{arcsinh}(ax) - \operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = i\text{arcsinh}(ax) + \log(x) - \log\left(1+\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[E^(I*ArcTan[a*x])/x,x]
[Out] I*ArcSinh[a*x] + Log[x] - Log[1 + Sqrt[1 + a^2*x^2]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

method	result	size
default	$-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{ia\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}$	48
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)+(-2\ln(2)+2\ln(x)+\ln(a^2))\sqrt{\pi}}{2\sqrt{\pi}}+i\operatorname{arcsinh}(ax)$	53

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] -arctanh(1/(a^2*x^2+1)^(1/2))+I*a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(
a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(21) = 42.

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = -\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - i\log\left(-ax+\sqrt{a^2x^2+1}\right) + \log\left(-ax+\sqrt{a^2x^2+1}-1\right)$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
[Out] -log(-a*x + sqrt(a^2*x^2 + 1) + 1) - I*log(-a*x + sqrt(a^2*x^2 + 1)) + log(
-a*x + sqrt(a^2*x^2 + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = ia\left(\begin{cases}\frac{\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{\sqrt{a^2}} & \text{for } a^2\neq 0\\ x & \text{otherwise}\end{cases}\right)-\operatorname{asinh}\left(\frac{1}{ax}\right)$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] I*a*Piecewise((log(2*a**2*x + 2*sqrt(a**2*x**2 + 1)*sqrt(a**2))/sqrt(a**2),
 Ne(a**2, 0)), (x, True)) - asinh(1/(a*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = i\operatorname{arsinh}(ax)-\operatorname{arsinh}\left(\frac{1}{a|x|}\right)$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] I*arcsinh(a*x) - arcsinh(1/(a*abs(x)))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.72

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = -\frac{i\,a\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}-\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right) + \log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -I*a*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - log(abs(-x*abs(a) + sqrt(a
^2*x^2 + 1) + 1)) + log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = -\operatorname{atanh}\left(\sqrt{a^2\,x^2+1}\right)+\frac{a\,\operatorname{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{\sqrt{a^2}}$$

```
[In] int((a*x*1i + 1)/(x*(a^2*x^2 + 1)^(1/2)),x)
[Out] (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2) - atanh((a^2*x^2 + 1)^(1/2))
```

3.7 $\int \frac{e^{i \arctan(ax)}}{x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{e^{i \arctan(ax)}}{x^2} \, dx = -\frac{\sqrt{1+a^2x^2}}{x} - ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -I*a*arctanh((a^2*x^2+1)^(1/2))-(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5168, 821, 272, 65, 214}

$$\int \frac{e^{i \arctan(ax)}}{x^2} \, dx = -\frac{\sqrt{a^2x^2+1}}{x} - ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right)$$

```
[In] Int[E^(I*ArcTan[a*x])/x^2,x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) - I*a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+iax}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + (ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{1}{2}(ia)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{i\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1+a^2x^2}}{x} - ia\,\text{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{1+a^2x^2}}{x} + ia\log(x) - ia\log\left(1+\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[E^(I*ArcTan[a*x])/x^2,x]
[Out] -(Sqrt[1 + a^2*x^2]/x) + I*a*Log[x] - I*a*Log[1 + Sqrt[1 + a^2*x^2]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\sqrt{a^2x^2+1}}{x} - ia\ \operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)$	34
risch	$-\frac{\sqrt{a^2x^2+1}}{x} - ia\ \operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)$	34
meijerg	$-\frac{\sqrt{a^2x^2+1}}{x} + \frac{ia\left(-2\sqrt{\pi}\ \ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)+(-2\ln(2)+2\ln(x)+\ln(a^2))\sqrt{\pi}\right)}{2\sqrt{\pi}}$	64

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(a^2*x^2+1)^(1/2)/x-I*a*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = \frac{-i\,ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+i\,ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-ax-\sqrt{a^2x^2+1}}{x}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
[Out] (-I*a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + I*a*x*log(-a*x + sqrt(a^2*x^2 +
 1) - 1) - a*x - sqrt(a^2*x^2 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = -a\sqrt{1+\frac{1}{a^2x^2}} - ia\operatorname{asinh}\left(\frac{1}{ax}\right)$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**2,x)
[Out] -a*sqrt(1 + 1/(a**2*x**2)) - I*a*asinh(1/(a*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = -i\,a\operatorname{arsinh}\left(\frac{1}{a|x|}\right) - \frac{\sqrt{a^2x^2+1}}{x}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
[Out] -I*a*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)/x
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = -i\,a\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right) + i\,a\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right) + \frac{2\,|a|}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
[Out] -I*a*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) + I*a*log(abs(-x*abs(a) +
sqrt(a^2*x^2 + 1) - 1)) + 2*abs(a)/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{a^2\,x^2+1}}{x} - a\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,1\mathrm{i}$$

```
[In] int((a*x*1i + 1)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
[Out] - a*atanh((a^2*x^2 + 1)^(1/2))*1i - (a^2*x^2 + 1)^(1/2)/x
```

3.8 $\int \frac{e^{i \arctan(ax)}}{x^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{e^{i \arctan(ax)}}{x^3} \, dx = -\frac{\sqrt{1+a^2x^2}}{2x^2} - \frac{ia\sqrt{1+a^2x^2}}{x} + \frac{1}{2}a^2 \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] 1/2*a^2*arctanh((a^2*x^2+1)^(1/2))-1/2*(a^2*x^2+1)^(1/2)/x^2-I*a*(a^2*x^2+1
)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5168, 849, 821, 272, 65, 214}

$$\int \frac{e^{i \arctan(ax)}}{x^3} \, dx = \frac{1}{2}a^2 \text{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{ia\sqrt{a^2x^2+1}}{x} - \frac{\sqrt{a^2x^2+1}}{2x^2}$$

```
[In] Int[E^(I*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^2]/x^2 - (I*a*Sqrt[1 + a^2*x^2])/x + (a^2*ArcTanh[Sqrt[
1 + a^2*x^2]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+iax}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} - \frac{1}{2}\int \frac{-2ia+a^2x}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} - \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{2}a^2\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} - \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{4}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right)
\end{aligned}
$$

$$= -\frac{\sqrt{1+a^2x^2}}{2x^2} - \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)$$

$$= -\frac{\sqrt{1+a^2x^2}}{2x^2} - \frac{ia\sqrt{1+a^2x^2}}{x} + \frac{1}{2}a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx = \frac{1}{2}\left(\frac{(-1-2iax)\sqrt{1+a^2x^2}}{x^2} - a^2\log(x) + a^2\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

```
[In] Integrate[E^(I*ArcTan[a*x])/x^3,x]
[Out] (((-1 - (2*I)*a*x)*Sqrt[1 + a^2*x^2])/x^2 - a^2*Log[x] + a^2*Log[1 + Sqrt[1
 + a^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{a^2\,\text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}-\frac{ia\sqrt{a^2x^2+1}}{x}$	53
risch	$-\frac{i\left(2a^3x^3-ia^2x^2+2ax-i\right)}{2x^2\sqrt{a^2x^2+1}}+\frac{a^2\,\text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}$	60
meijerg	$\frac{a^2\left(\frac{\sqrt{\pi}\left(4a^2x^2+8\right)}{8a^2x^2}-\frac{\sqrt{\pi}\sqrt{a^2x^2+1}}{a^2x^2}+\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)-\frac{\left(1-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{x^2a^2}\right)}{2\sqrt{\pi}}-\frac{ia\sqrt{a^2x^2+1}}{x}$	122

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(a^2*x^2+1)^(1/2)/x^2+1/2*a^2*arctanh(1/(a^2*x^2+1)^(1/2))-I*a*(a^2*x^
2+1)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx = \frac{a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-2i\,a^2x^2+\sqrt{a^2x^2+1}(-2i\,ax-1)}{2\,x^2}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^2*x^2*log(-a*x + sqrt(a^
2*x^2 + 1) - 1) - 2*I*a^2*x^2 + sqrt(a^2*x^2 + 1)*(-2*I*a*x - 1))/x^2
```

Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx = -ia^2\sqrt{1+\frac{1}{a^2x^2}}+\frac{a^2\operatorname{asinh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{2x}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] -I*a**2*sqrt(1 + 1/(a**2*x**2)) + a**2*asinh(1/(a*x))/2 - a*sqrt(1 + 1/(a**
2*x**2))/(2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx = \frac{1}{2}\,a^2\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{i\,\sqrt{a^2x^2+1}a}{x}-\frac{\sqrt{a^2x^2+1}}{2\,x^2}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*arcsinh(1/(a*abs(x))) - I*sqrt(a^2*x^2 + 1)*a/x - 1/2*sqrt(a^2*x^2
+ 1)/x^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.43

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{1}{2}\,a^2\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)-\frac{1}{2}\,a^2\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$
$$+\frac{\left(x|a|-\sqrt{a^2x^2+1}\right)^3a^2+2i\left(x|a|-\sqrt{a^2x^2+1}\right)^2a|a|+\left(x|a|-\sqrt{a^2x^2+1}\right)a^2-2i\,a|a|}{\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)^2}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) - 1/2*a^2*log(abs(-x*ab
s(a) + sqrt(a^2*x^2 + 1) - 1)) + ((x*abs(a) - sqrt(a^2*x^2 + 1))^3*a^2 + 2*
I*(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a*abs(a) + (x*abs(a) - sqrt(a^2*x^2 + 1)
)*a^2 - 2*I*a*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx = \frac{a^2\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)}{2}-\frac{\sqrt{a^2\,x^2+1}}{2\,x^2}-\frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{x}$$

```
[In] int((a*x*1i + 1)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] (a^2*atanh((a^2*x^2 + 1)^(1/2)))/2 - (a^2*x^2 + 1)^(1/2)/(2*x^2) - (a*(a^2*
x^2 + 1)^(1/2)*1i)/x
```

3.9 $\int \frac{e^{i\arctan(ax)}}{x^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{i\arctan(ax)}}{x^4} dx = -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} + \frac{1}{2}ia^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] 1/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))-1/3*(a^2*x^2+1)^(1/2)/x^3-1/2*I*a*(a^2
*x^2+1)^(1/2)/x^2+2/3*a^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5168, 849, 821, 272, 65, 214}

$$\int \frac{e^{i\arctan(ax)}}{x^4} dx = \frac{2a^2\sqrt{a^2x^2+1}}{3x} - \frac{ia\sqrt{a^2x^2+1}}{2x^2} - \frac{\sqrt{a^2x^2+1}}{3x^3} + \frac{1}{2}ia^3\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)$$

```
[In] Int[E^(I*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*Sqrt[1 + a^2*x^2]/x^3 - ((I/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (2*a^2*Sqrt[
1 + a^2*x^2])/(3*x) + (I/2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+iax}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{1}{3}\int \frac{-3ia+2a^2x}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{1}{6}\int \frac{-4a^2-3ia^3x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} - \frac{1}{2}\left(ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx
\end{aligned}$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} - \frac{1}{4}\left(ia^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} - \frac{1}{2}(ia)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} + \frac{1}{2}ia^3\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{1}{6}\left(\frac{\sqrt{1+a^2x^2}(-2-3iax+4a^2x^2)}{x^3} - 3ia^3\log(x) + 3ia^3\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

```
[In] Integrate[E^(I*ArcTan[a*x])/x^4,x]
[Out] ((Sqrt[1 + a^2*x^2]*(-2 - (3*I)*a*x + 4*a^2*x^2))/x^3 - (3*I)*a^3*Log[x] +
(3*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result	s
risch	$\frac{4a^4x^4-3ia^3x^3+2a^2x^2-3iax-2}{6x^3\sqrt{a^2x^2+1}} + \frac{ia^3\,\text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}$	6
default	$-\frac{\sqrt{a^2x^2+1}}{3x^3} + \frac{2a^2\sqrt{a^2x^2+1}}{3x} + ia\left(-\frac{\sqrt{a^2x^2+1}}{2x^2} + \frac{a^2\,\text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}\right)$	7
meijerg	$-\frac{(-2a^2x^2+1)\sqrt{a^2x^2+1}}{3x^3} + \frac{ia^3\left(\frac{\sqrt{\pi}\,(4a^2x^2+8)}{8a^2x^2} - \frac{\sqrt{\pi}\,\sqrt{a^2x^2+1}}{a^2x^2} + \sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right) - \frac{\left(1-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{x^2a^2}\right)}{2\sqrt{\pi}}$	1

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/6*(4*a^4*x^4-3*I*a^3*x^3+2*a^2*x^2-3*I*a*x-2)/x^3/(a^2*x^2+1)^(1/2)+1/2*I
*a^3*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+4\,a^3x^3+\left(4\,a^2x^2-3i\,ax-2\right)}{6\,x^3}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*I*a^3*x^3*log(-a*x +
 sqrt(a^2*x^2 + 1) - 1) + 4*a^3*x^3 + (4*a^2*x^2 - 3*I*a*x - 2)*sqrt(a^2*x^
2 + 1))/x^3
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{2a^3\sqrt{1+\frac{1}{a^2x^2}}}{3}+\frac{ia^3\operatorname{asinh}\left(\frac{1}{ax}\right)}{2}-\frac{ia^2\sqrt{1+\frac{1}{a^2x^2}}}{2x}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{3x^2}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] 2*a**3*sqrt(1 + 1/(a**2*x**2))/3 + I*a**3*asinh(1/(a*x))/2 - I*a**2*sqrt(1
+ 1/(a**2*x**2))/(2*x) - a*sqrt(1 + 1/(a**2*x**2))/(3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{1}{2}i\,a^3\operatorname{arsinh}\left(\frac{1}{a|x|}\right)+\frac{2\,\sqrt{a^2x^2+1}a^2}{3\,x}-\frac{i\,\sqrt{a^2x^2+1}a}{2\,x^2}-\frac{\sqrt{a^2x^2+1}}{3\,x^3}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*I*a^3*arcsinh(1/(a*abs(x))) + 2/3*sqrt(a^2*x^2 + 1)*a^2/x - 1/2*I*sqrt(
a^2*x^2 + 1)*a/x^2 - 1/3*sqrt(a^2*x^2 + 1)/x^3
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(70) = 140.

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.79

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx$$
$$= \frac{1}{2}i\,a^3\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)-\frac{1}{2}i\,a^3\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$
$$-\frac{-3i\left(x|a|-\sqrt{a^2x^2+1}\right)^5a^3-12\left(x|a|-\sqrt{a^2x^2+1}\right)^2a^2|a|+3\left(i\,x|a|-i\sqrt{a^2x^2+1}\right)a^3+4\,a^2|a|}{3\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)^3}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*I*a^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) - 1/2*I*a^3*log(abs(-
x*abs(a) + sqrt(a^2*x^2 + 1) - 1)) - 1/3*(-3*I*(x*abs(a) - sqrt(a^2*x^2 + 1
))^5*a^3 - 12*(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a^2*abs(a) + 3*(I*x*abs(a) -
 I*sqrt(a^2*x^2 + 1))*a^3 + 4*a^2*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2
 - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{a^3\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2}-\frac{\sqrt{a^2\,x^2+1}}{3\,x^3}+\frac{2\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x}-\frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{2\,x^2}$$

```
[In] int((a*x*1i + 1)/(x^4*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] (a^3*atan((a^2*x^2 + 1)^(1/2)*1i))/2 - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (a*(a^
2*x^2 + 1)^(1/2)*1i)/(2*x^2) + (2*a^2*(a^2*x^2 + 1)^(1/2))/(3*x)
```

3.10 $\int \frac{e^{i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 113

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} - \frac{3}{8}a^4\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -3/8*a^4*arctanh((a^2*x^2+1)^(1/2))-1/4*(a^2*x^2+1)^(1/2)/x^4-1/3*I*a*(a^2*
x^2+1)^(1/2)/x^3+3/8*a^2*(a^2*x^2+1)^(1/2)/x^2+2/3*I*a^3*(a^2*x^2+1)^(1/2)/
x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5168, 849, 821, 272, 65, 214}

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = \frac{3a^2\sqrt{a^2x^2+1}}{8x^2} - \frac{\sqrt{a^2x^2+1}}{4x^4} - \frac{ia\sqrt{a^2x^2+1}}{3x^3} - \frac{3}{8}a^4\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right) + \frac{2ia^3\sqrt{a^2x^2+1}}{3x}$$

```
[In] Int[E^(I*ArcTan[a*x])/x^5,x]
```

```
[Out] -1/4*Sqrt[1 + a^2*x^2]/x^4 - ((I/3)*a*Sqrt[1 + a^2*x^2])/x^3 + (3*a^2*Sqrt[
1 + a^2*x^2])/(8*x^2) + (((2*I)/3)*a^3*Sqrt[1 + a^2*x^2])/x - (3*a^4*ArcTan
h[Sqrt[1 + a^2*x^2]])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{1+iax}{x^5\sqrt{1+a^2x^2}}\,dx$$

$$
\begin{aligned}
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{1}{4}\int \frac{-4ia+3a^2x}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{1}{12}\int \frac{-9a^2-8ia^3x}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{1}{24}\int \frac{16ia^3-9a^4x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{8}(3a^4)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} \\
&\quad + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{16}(3a^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} \\
&\quad + \frac{1}{8}(3a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} + \frac{2ia^3\sqrt{1+a^2x^2}}{3x} - \frac{3}{8}a^4\text{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$
\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = \frac{1}{24}\left(\frac{\sqrt{1+a^2x^2}(-6-8iax+9a^2x^2+16ia^3x^3)}{x^4} + 9a^4\log(x) - 9a^4\log\left(1+\sqrt{1+a^2x^2}\right)\right)
$$

```
[In] Integrate[E^(I*ArcTan[a*x])/x^5,x]
[Out] ((Sqrt[1 + a^2*x^2]*(-6 - (8*I)*a*x + 9*a^2*x^2 + (16*I)*a^3*x^3))/x^4 + 9*
a^4*Log[x] - 9*a^4*Log[1 + Sqrt[1 + a^2*x^2]])/24
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

method	result
risch	$\frac{i\left(16a^{5}x^{5}-9ia^{4}x^{4}+8a^{3}x^{3}-3ia^{2}x^{2}-8ax+6i\right)}{24x^{4}\sqrt{a^{2}x^{2}+1}}-\frac{3a^{4}\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)}{8}$
default	$-\frac{\sqrt{a^{2}x^{2}+1}}{4x^{4}}-\frac{3a^{2}\left(-\frac{\sqrt{a^{2}x^{2}+1}}{2x^{2}}+\frac{a^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)}{2}\right)}{4}+ia\left(-\frac{\sqrt{a^{2}x^{2}+1}}{3x^{3}}+\frac{2a^{2}\sqrt{a^{2}x^{2}+1}}{3x}\right)$
meijerg	$\frac{a^{4}\left(\frac{\sqrt{\pi}\left(-7a^{4}x^{4}-8a^{2}x^{2}+8\right)}{16a^{4}x^{4}}-\frac{\sqrt{\pi}\left(-12a^{2}x^{2}+8\right)\sqrt{a^{2}x^{2}+1}}{16a^{4}x^{4}}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{a^{2}x^{2}+1}}{2}\right)}{4}+\frac{3\left(\frac{7}{6}-2\ln\left(2\right)+2\ln\left(x\right)+\ln\left(a^{2}\right)\right)\sqrt{\pi}}{8}-\frac{\sqrt{\pi}}{2x^{4}a^{4}}+\frac{\sqrt{\pi}}{2x^{2}a^{2}}\right)}{2\sqrt{\pi}}-$

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x,method=_RETURNVERBOSE)
[Out] 1/24*I*(16*a^5*x^5-9*I*a^4*x^4+8*a^3*x^3-3*I*a^2*x^2-8*a*x+6*I)/x^4/(a^2*x^
2+1)^(1/2)-3/8*a^4*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{e^{i\arctan(ax)}}{x^{5}}\,dx = -\frac{9\,a^{4}x^{4}\log\left(-ax+\sqrt{a^{2}x^{2}+1}+1\right)-9\,a^{4}x^{4}\log\left(-ax+\sqrt{a^{2}x^{2}+1}-1\right)-16i\,a^{4}x^{4}-\left(16i\,a^{3}x^{3}+9\,a^{2}x^{2}\right.}{24\,x^{4}}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/24*(9*a^4*x^4*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 9*a^4*x^4*log(-a*x + s
qrt(a^2*x^2 + 1) - 1) - 16*I*a^4*x^4 - (16*I*a^3*x^3 + 9*a^2*x^2 - 8*I*a*x
- 6)*sqrt(a^2*x^2 + 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08

$$\int \frac{e^{i\arctan(ax)}}{x^{5}}\,dx = \frac{2ia^{4}\sqrt{1+\frac{1}{a^{2}x^{2}}}}{3}-\frac{3a^{4}\operatorname{asinh}\left(\frac{1}{ax}\right)}{8}+\frac{3a^{3}}{8x\sqrt{1+\frac{1}{a^{2}x^{2}}}}-\frac{ia^{2}\sqrt{1+\frac{1}{a^{2}x^{2}}}}{3x^{2}}+\frac{a}{8x^{3}\sqrt{1+\frac{1}{a^{2}x^{2}}}}-\frac{1}{4ax^{5}\sqrt{1+\frac{1}{a^{2}x^{2}}}}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] 2*I*a**4*sqrt(1 + 1/(a**2*x**2))/3 - 3*a**4*asinh(1/(a*x))/8 + 3*a**3/(8*x*
sqrt(1 + 1/(a**2*x**2))) - I*a**2*sqrt(1 + 1/(a**2*x**2))/(3*x**2) + a/(8*x
**3*sqrt(1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(1 + 1/(a**2*x**2)))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = -\frac{3}{8}\,a^4\operatorname{arsinh}\left(\frac{1}{a|x|}\right) + \frac{2i\,\sqrt{a^2x^2+1}a^3}{3\,x} + \frac{3\,\sqrt{a^2x^2+1}a^2}{8\,x^2} - \frac{i\,\sqrt{a^2x^2+1}a}{3\,x^3} - \frac{\sqrt{a^2x^2+1}}{4\,x^4}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -3/8*a^4*arcsinh(1/(a*abs(x))) + 2/3*I*sqrt(a^2*x^2 + 1)*a^3/x + 3/8*sqrt(a
^2*x^2 + 1)*a^2/x^2 - 1/3*I*sqrt(a^2*x^2 + 1)*a/x^3 - 1/4*sqrt(a^2*x^2 + 1)
/x^4
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(89) = 178.

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.10

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = -\frac{3}{8}\,a^4\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right) + \frac{3}{8}\,a^4\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right) - \frac{9\left(x|a|-\sqrt{a^2x^2+1}\right)^7a^4 - 33\left(x|a|-\sqrt{a^2x^2+1}\right)^5a^4 - 48i\left(x|a|-\sqrt{a^2x^2+1}\right)^4a^3|a| - 33\left(x|a|-\sqrt{a^2\right.}{12\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-\right.}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/8*a^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) + 3/8*a^4*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 + 1) - 1)) - 1/12*(9*(x*abs(a) - sqrt(a^2*x^2 + 1))^7*
a^4 - 33*(x*abs(a) - sqrt(a^2*x^2 + 1))^5*a^4 - 48*I*(x*abs(a) - sqrt(a^2*x
^2 + 1))^4*a^3*abs(a) - 33*(x*abs(a) - sqrt(a^2*x^2 + 1))^3*a^4 + 64*I*(x*a
bs(a) - sqrt(a^2*x^2 + 1))^2*a^3*abs(a) + 9*(x*abs(a) - sqrt(a^2*x^2 + 1))*
a^4 - 16*I*a^3*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = \frac{a^4\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,3\mathrm{i}}{8} - \frac{\sqrt{a^2\,x^2+1}}{4\,x^4} - \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{3\,x^3} + \frac{3\,a^2\,\sqrt{a^2\,x^2+1}}{8\,x^2} + \frac{a^3\,\sqrt{a^2\,x^2+1}\,2\mathrm{i}}{3\,x}$$

```
[In] int((a*x*1i + 1)/(x^5*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] (a^4*atan((a^2*x^2 + 1)^(1/2)*1i)*3i)/8 - (a^2*x^2 + 1)^(1/2)/(4*x^4) - (a*
(a^2*x^2 + 1)^(1/2)*1i)/(3*x^3) + (3*a^2*(a^2*x^2 + 1)^(1/2))/(8*x^2) + (a^
3*(a^2*x^2 + 1)^(1/2)*2i)/(3*x)
```

3.11 $\int e^{2i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int e^{2i\arctan(ax)}x^3\,dx=-\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i+ax)}{a^4}$$

```
[Out] -2*I*x/a^3+x^2/a^2+2/3*I*x^3/a-1/4*x^4-2*ln(I+a*x)/a^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int e^{2i\arctan(ax)}x^3\,dx=-\frac{2\log(ax+i)}{a^4}-\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((-2*I)*x)/a^3 + x^2/a^2 + (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I + a*x])/a^4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(1+iax)}{1-iax}\,dx \\
&= \int \left(-\frac{2i}{a^3}+\frac{2x}{a^2}+\frac{2ix^2}{a}-x^3-\frac{2}{a^3(i+ax)}\right)dx \\
&= -\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i+ax)}{a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x^3\,dx = -\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i+ax)}{a^4}$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x^3,x]
[Out] ((-2*I)*x)/a^3 + x^2/a^2 + (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I + a*x])/a^4
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$\frac{-3a^4x^4+8ia^3x^3+12a^2x^2-24iax-24\ln(ax+i)}{12a^4}$	46
risch	$-\frac{x^4}{4}+\frac{2ix^3}{3a}+\frac{x^2}{a^2}-\frac{2ix}{a^3}-\frac{\ln(a^2x^2+1)}{a^4}+\frac{2i\arctan(ax)}{a^4}$	55
default	$\frac{-\frac{1}{4}a^3x^4+\frac{2}{3}ia^2x^3+a\,x^2-2ix}{a^3}+\frac{-\frac{\ln\left(a^2x^2+1\right)}{a}+\frac{2i\arctan(ax)}{a}}{a^3}$	63
meijerg	$\frac{a^2x^2-\ln(a^2x^2+1)}{2a^4}+\frac{i\left(-\frac{2x\left(a^2\right)^{\frac{5}{2}}\left(-5a^2x^2+15\right)}{15a^4}+\frac{2\left(a^2\right)^{\frac{5}{2}}\arctan(ax)}{a^5}\right)}{a^3\sqrt{a^2}}-\frac{-\frac{x^2a^2\left(-3a^2x^2+6\right)}{6}+\ln(a^2x^2+1)}{2a^4}$	108

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^3,x,method=_RETURNVERBOSE)
[Out] 1/12*(-3*a^4*x^4+8*I*x^3*a^3+12*a^2*x^2-24*I*a*x-24*ln(I+a*x))/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int e^{2i\arctan(ax)}x^3\,dx = -\frac{3\,a^4x^4 - 8i\,a^3x^3 - 12\,a^2x^2 + 24i\,ax + 24\,\log\left(\frac{ax+i}{a}\right)}{12\,a^4}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 - 8*I*a^3*x^3 - 12*a^2*x^2 + 24*I*a*x + 24*log((a*x + I)/a
))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int e^{2i\arctan(ax)}x^3\,dx = -\frac{x^4}{4} + \frac{2ix^3}{3a} + \frac{x^2}{a^2} - \frac{2ix}{a^3} - \frac{2\log(ax+i)}{a^4}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**3,x)
```

```
[Out] -x**4/4 + 2*I*x**3/(3*a) + x**2/a**2 - 2*I*x/a**3 - 2*log(a*x + I)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int e^{2i\arctan(ax)}x^3\,dx = -\frac{3\,a^3x^4 - 8i\,a^2x^3 - 12\,ax^2 + 24i\,x}{12\,a^3} + \frac{2i\,\arctan(ax)}{a^4} - \frac{\log(a^2x^2+1)}{a^4}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^4 - 8*I*a^2*x^3 - 12*a*x^2 + 24*I*x)/a^3 + 2*I*arctan(a*x)/a
^4 - log(a^2*x^2 + 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int e^{2i\arctan(ax)}x^3\,dx = -\frac{3\,a^4x^4 - 8i\,a^3x^3 - 12\,a^2x^2 + 24i\,ax}{12\,a^4} - \frac{2\,\log\left(ax+i\right)}{a^4}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^4*x^4 - 8*I*a^3*x^3 - 12*a^2*x^2 + 24*I*a*x)/a^4 - 2*log(a*x + I
)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int e^{2i\arctan(ax)}x^3\,dx = \frac{x^2}{a^2} - \frac{x^4}{4} - \frac{2\,\ln\left(x+\frac{1\mathrm{i}}{a}\right)}{a^4} - \frac{x\,2\mathrm{i}}{a^3} + \frac{x^3\,2\mathrm{i}}{3\,a}$$

```
[In] int((x^3*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
```

```
[Out] (x^3*2i)/(3*a) - (x*2i)/a^3 - x^4/4 - (2*log(x + 1i/a))/a^4 + x^2/a^2
```

3.12 $\int e^{2i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int e^{2i\arctan(ax)}x^2\,dx=\frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}-\frac{2i\log(i+ax)}{a^3}$$

```
[Out] 2*x/a^2+I*x^2/a-1/3*x^3-2*I*ln(I+a*x)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int e^{2i\arctan(ax)}x^2\,dx=-\frac{2i\log(ax+i)}{a^3}+\frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + (I*x^2)/a - x^3/3 - ((2*I)*Log[I + a*x])/a^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2(1+iax)}{1-iax}\,dx\\ &= \int \left(\frac{2}{a^2}+\frac{2ix}{a}-x^2-\frac{2i}{a^2(i+ax)}\right)dx\\ &= \frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}-\frac{2i\log(i+ax)}{a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x^2\,dx = \frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}-\frac{2i\log(i+ax)}{a^3}$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x^2,x]
[Out] (2*x)/a^2 + (I*x^2)/a - x^3/3 - ((2*I)*Log[I + a*x])/a^3
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{a^3x^3-3ia^2x^2+6i\ln(ax+i)-6ax}{3a^3}$	37
risch	$\frac{2x}{a^2}-\frac{x^3}{3}+\frac{ix^2}{a}-\frac{i\ln(a^2x^2+1)}{a^3}-\frac{2\arctan(ax)}{a^3}$	47
default	$\frac{2x-\frac{1}{3}a^2x^3+ia\,x^2}{a^2}+\frac{-\frac{i\ln\left(a^2x^2+1\right)}{a}-\frac{2\arctan(ax)}{a}}{a^2}$	55
meijerg	$\frac{\frac{2x\left(a^2\right)^{\frac{3}{2}}}{a^2}-\frac{2\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{a^3}}{2a^2\sqrt{a^2}}+\frac{i\left(a^2x^2-\ln\left(a^2x^2+1\right)\right)}{a^3}-\frac{-\frac{2x\left(a^2\right)^{\frac{5}{2}}\left(-5a^2x^2+15\right)}{15a^4}+\frac{2\left(a^2\right)^{\frac{5}{2}}\arctan(ax)}{a^5}}{2a^2\sqrt{a^2}}$	110

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^2,x,method=_RETURNVERBOSE)
[Out] -1/3*(a^3*x^3-3*I*a^2*x^2+6*I*ln(I+a*x)-6*a*x)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2i\arctan(ax)}x^2\,dx = -\frac{a^3x^3 - 3i\,a^2x^2 - 6\,ax + 6i\,\log\left(\frac{ax+i}{a}\right)}{3\,a^3}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="fricas")
[Out] -1/3*(a^3*x^3 - 3*I*a^2*x^2 - 6*a*x + 6*I*log((a*x + I)/a))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int e^{2i\arctan(ax)}x^2\,dx = -\frac{x^3}{3} + \frac{ix^2}{a} + \frac{2x}{a^2} - \frac{2i\log(ax+i)}{a^3}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**2,x)
[Out] -x**3/3 + I*x**2/a + 2*x/a**2 - 2*I*log(a*x + I)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int e^{2i\arctan(ax)}x^2\,dx = -\frac{a^2x^3 - 3i\,ax^2 - 6\,x}{3\,a^2} - \frac{2\,\arctan(ax)}{a^3} - \frac{i\,\log(a^2x^2+1)}{a^3}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="maxima")
[Out] -1/3*(a^2*x^3 - 3*I*a*x^2 - 6*x)/a^2 - 2*arctan(a*x)/a^3 - I*log(a^2*x^2 +
1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2i\arctan(ax)}x^2\,dx = -\frac{a^3x^3 - 3i\,a^2x^2 - 6\,ax}{3\,a^3} - \frac{2i\,\log(ax+i)}{a^3}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="giac")
[Out] -1/3*(a^3*x^3 - 3*I*a^2*x^2 - 6*a*x)/a^3 - 2*I*log(a*x + I)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int e^{2i\arctan(ax)}x^2\,dx = \frac{2\,x}{a^2} - \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a^3} - \frac{x^3}{3} + \frac{x^2\,1\mathrm{i}}{a}$$

```
[In] int((x^2*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
[Out] (2*x)/a^2 - (log(x + 1i/a)*2i)/a^3 - x^3/3 + (x^2*1i)/a
```

3.13 $\int e^{2i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int e^{2i\arctan(ax)}x\,dx = \frac{2ix}{a} - \frac{x^2}{2} + \frac{2\log(i+ax)}{a^2}$$

```
[Out] 2*I*x/a-1/2*x^2+2*ln(I+a*x)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5170, 78}

$$\int e^{2i\arctan(ax)}x\,dx = \frac{2\log(ax+i)}{a^2} + \frac{2ix}{a} - \frac{x^2}{2}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((2*I)*x)/a - x^2/2 + (2*Log[I + a*x])/a^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x(1+iax)}{1-iax}\,dx \\ &= \int \left(\frac{2i}{a}-x+\frac{2}{a(i+ax)}\right)dx \\ &= \frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i+ax)}{a^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x\,dx = \frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i+ax)}{a^2}$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x,x]
[Out] ((2*I)*x)/a - x^2/2 + (2*Log[I + a*x])/a^2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-a^2x^2+4iax+4\ln(ax+i)}{2a^2}$	29
risch	$-\frac{x^2}{2}+\frac{2ix}{a}+\frac{\ln(a^2x^2+1)}{a^2}-\frac{2i\arctan(ax)}{a^2}$	38
default	$\frac{-\frac{1}{2}a\,x^2+2ix}{a}+\frac{\frac{\ln\left(a^2x^2+1\right)}{a}-\frac{2i\arctan(ax)}{a}}{a}$	46
meijerg	$\frac{\ln(a^2x^2+1)}{2a^2}+\frac{i\left(\frac{2x\left(a^2\right)^{\frac{3}{2}}}{a^2}-\frac{2\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{a^3}\right)}{a\sqrt{a^2}}-\frac{a^2x^2-\ln(a^2x^2+1)}{2a^2}$	79

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*(-a^2*x^2+4*I*a*x+4*ln(I+a*x))/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x\,dx = -\frac{a^2x^2 - 4i\,ax - 4\,\log\left(\frac{ax+i}{a}\right)}{2\,a^2}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="fricas")
[Out] -1/2*(a^2*x^2 - 4*I*a*x - 4*log((a*x + I)/a))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int e^{2i\arctan(ax)}x\,dx = -\frac{x^2}{2} + \frac{2ix}{a} + \frac{2\log(ax+i)}{a^2}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x,x)
[Out] -x**2/2 + 2*I*x/a + 2*log(a*x + I)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int e^{2i\arctan(ax)}x\,dx = -\frac{ax^2 - 4i\,x}{2\,a} - \frac{2i\,\arctan(ax)}{a^2} + \frac{\log(a^2x^2+1)}{a^2}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="maxima")
[Out] -1/2*(a*x^2 - 4*I*x)/a - 2*I*arctan(a*x)/a^2 + log(a^2*x^2 + 1)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x\,dx = -\frac{a^2x^2 - 4i\,ax}{2\,a^2} + \frac{2\,\log(ax+i)}{a^2}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="giac")
[Out] -1/2*(a^2*x^2 - 4*I*a*x)/a^2 + 2*log(a*x + I)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int e^{2i\arctan(ax)}x\,dx = \frac{2\,\ln\left(x+\frac{1\mathrm{i}}{a}\right)}{a^2} - \frac{x^2}{2} + \frac{x\,2\mathrm{i}}{a}$$

```
[In] int((x*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
[Out] (2*log(x + 1i/a))/a^2 + (x*2i)/a - x^2/2
```

3.14 $\int e^{2i \arctan(ax)} dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int e^{2i \arctan(ax)} dx = -x + \frac{2i \log(i + ax)}{a}$$

```
[Out] -x+2*I*ln(I+a*x)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5169, 45}

$$\int e^{2i \arctan(ax)} dx = -x + \frac{2i \log(ax + i)}{a}$$

```
[In] Int[E^((2*I)*ArcTan[a*x]),x]
[Out] -x + ((2*I)*Log[I + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+iax}{1-iax}\,dx\\ &= \int \left(-1+\frac{2i}{i+ax}\right)dx\\ &= -x+\frac{2i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int e^{2i\arctan(ax)}\,dx = -x+\frac{2\arctan(ax)}{a}+\frac{i\log(1+a^2x^2)}{a}$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x]),x]
[Out] -x + (2*ArcTan[a*x])/a + (I*Log[1 + a^2*x^2])/a
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{2i\ln(ax+i)-ax}{a}$	20
default	$-x+\frac{i\ln(a^2x^2+1)}{a}+\frac{2\arctan(ax)}{a}$	30
risch	$-x+\frac{i\ln(a^2x^2+1)}{a}+\frac{2\arctan(ax)}{a}$	30
meijerg	$\frac{\arctan(ax)}{a}+\frac{i\ln(a^2x^2+1)}{a}-\frac{\frac{2x(a^2)^{\frac{3}{2}}}{a^2}-\frac{2(a^2)^{\frac{3}{2}}\arctan(ax)}{a^3}}{2\sqrt{a^2}}$	59

```
[In] int((1+I*a*x)^2/(a^2*x^2+1),x,method=_RETURNVERBOSE)
[Out] (2*I*ln(I+a*x)-a*x)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int e^{2i\arctan(ax)}\,dx = -\frac{ax - 2i\log\left(\frac{ax+i}{a}\right)}{a}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="fricas")
[Out] -(a*x - 2*I*log((a*x + I)/a))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{2i\log(ax+i)}{a}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1),x)
[Out] -x + 2*I*log(a*x + I)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{2\arctan(ax)}{a} + \frac{i\log(a^2x^2+1)}{a}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="maxima")
[Out] -x + 2*arctan(a*x)/a + I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{2i\log(ax+i)}{a}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="giac")
[Out] -x + 2*I*log(a*x + I)/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{\ln\left(x + \frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a}$$

```
[In] int((a*x*1i + 1)^2/(a^2*x^2 + 1),x)
[Out] (log(x + 1i/a)*2i)/a - x
```

3.15 $\int \frac{e^{2i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \log(x) - 2\log(i + ax)$$

```
[Out] ln(x)-2*ln(I+a*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \log(x) - 2\log(ax + i)$$

```
[In] Int[E^((2*I)*ArcTan[a*x])/x,x]
```

```
[Out] Log[x] - 2*Log[I + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+iax}{x(1-iax)}\,dx\\ &= \int \left(\frac{1}{x}-\frac{2a}{i+ax}\right)dx\\ &= \log(x)-2\log(i+ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \log(x)-2\log(i+ax)$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x,x]
[Out] Log[x] - 2*Log[I + a*x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\ln(x)-2\ln(ax+i)$	13
risch	$\ln(-x)-\ln(a^2x^2+1)+2i\arctan(ax)$	25
meijerg	$-\ln(a^2x^2+1)+\ln(x)+\frac{\ln(a^2)}{2}+2i\arctan(ax)$	29
default	$\ln(x)+2a\left(-\frac{\ln(a^2x^2+1)}{2a}+\frac{i\arctan(ax)}{a}\right)$	33

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)-2*ln(I+a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \log(x) - 2\log\left(\frac{ax+i}{a}\right)$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="fricas")
[Out] log(x) - 2*log((a*x + I)/a)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \log(3ax) - 2\log(3ax + 3i)$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x,x)
[Out] log(3*a*x) - 2*log(3*a*x + 3*I)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = 2i\arctan(ax) - \log\left(a^2x^2 + 1\right) + \log(x)$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="maxima")
[Out] 2*I*arctan(a*x) - log(a^2*x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = -2\log(ax+i) + \log(|x|)$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="giac")
[Out] -2*log(a*x + I) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \ln(x) - 2\ln\left(x + \frac{1\mathrm{i}}{a}\right)$$

```
[In] int((a*x*1i + 1)^2/(x*(a^2*x^2 + 1)),x)
[Out] log(x) - 2*log(x + 1i/a)
```

3.16 $\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -\frac{1}{x} + 2ia\log(x) - 2ia\log(i+ax)$$

```
[Out] -1/x+2*I*a*ln(x)-2*I*a*ln(I+a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = 2ia\log(x) - 2ia\log(ax+i) - \frac{1}{x}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (2*I)*a*Log[x] - (2*I)*a*Log[I + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+iax}{x^2(1-iax)}\,dx\\ &= \int \left(\frac{1}{x^2}+\frac{2ia}{x}-\frac{2ia^2}{i+ax}\right)dx\\ &= -\frac{1}{x}+2ia\log(x)-2ia\log(i+ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -\frac{1}{x}+2ia\log(x)-2ia\log(i+ax)$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x^2,x]
[Out] -x^(-1) + (2*I)*a*Log[x] - (2*I)*a*Log[I + a*x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{2ia\ln(x)x-2ia\ln(ax+i)x-1}{x}$	26
risch	$-\frac{1}{x}-2a\arctan(ax)-ia\ln(a^2x^2+1)+2ia\ln(x)$	34
default	$-\frac{1}{x}+2ia\ln(x)-2a^2\left(\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}\right)$	43
meijerg	$\frac{a^2\left(-\frac{2}{x\sqrt{a^2}}-\frac{2a\arctan(ax)}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}+ia(-\ln(a^2x^2+1)+2\ln(x)+\ln(a^2))-a\arctan(ax)$	67

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x^2,x,method=_RETURNVERBOSE)
[Out] (2*I*a*ln(x)*x-2*I*a*ln(I+a*x)*x-1)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = \frac{2i\,ax\log(x) - 2i\,ax\log\left(\frac{ax+i}{a}\right) - 1}{x}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="fricas")
[Out] (2*I*a*x*log(x) - 2*I*a*x*log((a*x + I)/a) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -2a\left(-i\log\left(4a^2x\right) + i\log\left(4a^2x + 4ia\right)\right) - \frac{1}{x}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x**2,x)
[Out] -2*a*(-I*log(4*a**2*x) + I*log(4*a**2*x + 4*I*a)) - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -2\,a\arctan(ax) - i\,a\log\left(a^2x^2 + 1\right) + 2i\,a\log(x) - \frac{1}{x}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="maxima")
[Out] -2*a*arctan(a*x) - I*a*log(a^2*x^2 + 1) + 2*I*a*log(x) - 1/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -2i\,a\log(ax+i) + 2i\,a\log(|x|) - \frac{1}{x}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="giac")
[Out] -2*I*a*log(a*x + I) + 2*I*a*log(abs(x)) - 1/x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -4\,a\,\mathrm{atan}(2\,a\,x + 1\mathrm{i}) - \frac{1}{x}$$

```
[In] int((a*x*1i + 1)^2/(x^2*(a^2*x^2 + 1)),x)
[Out] - 4*a*atan(2*a*x + 1i) - 1/x
```

3.17 $\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2}-\frac{2ia}{x}-2a^2\log(x)+2a^2\log(i+ax)$$

```
[Out] -1/2/x^2-2*I*a/x-2*a^2*ln(x)+2*a^2*ln(I+a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -2a^2\log(x)+2a^2\log(ax+i)-\frac{2ia}{x}-\frac{1}{2x^2}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*1/x^2 - ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+iax}{x^3(1-iax)}\,dx\\ &= \int \left(\frac{1}{x^3}+\frac{2ia}{x^2}-\frac{2a^2}{x}+\frac{2a^3}{i+ax}\right)dx\\ &= -\frac{1}{2x^2}-\frac{2ia}{x}-2a^2\log(x)+2a^2\log(i+ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2}-\frac{2ia}{x}-2a^2\log(x)+2a^2\log(i+ax)$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x^3,x]
[Out] -1/2*1/x^2 - ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I + a*x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$-\frac{4a^2\ln(x)x^2-4a^2\ln(ax+i)x^2+4iax+1}{2x^2}$	38
risch	$\frac{-2iax-\frac{1}{2}}{x^2}-2a^2\ln(x)-2ia^2\arctan(ax)+a^2\ln(a^2x^2+1)$	44
default	$-\frac{1}{2x^2}-\frac{2ia}{x}-2a^2\ln(x)-2a^3\left(-\frac{\ln(a^2x^2+1)}{2a}+\frac{i\arctan(ax)}{a}\right)$	52
meijerg	$\frac{a^2\left(\ln(a^2x^2+1)-2\ln(x)-\ln(a^2)-\frac{1}{a^2x^2}\right)}{2}+\frac{ia^3\left(-\frac{2}{x\sqrt{a^2}}-\frac{2a\arctan(ax)}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{a^2\left(-\ln(a^2x^2+1)+2\ln(x)+\ln(a^2)\right)}{2}$	96

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(4*a^2*ln(x)*x^2-4*a^2*ln(I+a*x)*x^2+4*I*a*x+1)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -\frac{4\,a^2x^2\log\left(x\right) - 4\,a^2x^2\log\left(\frac{ax+i}{a}\right) + 4i\,ax + 1}{2\,x^2}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="fricas")
[Out] -1/2*(4*a^2*x^2*log(x) - 4*a^2*x^2*log((a*x + I)/a) + 4*I*a*x + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -2a^2\left(\log\left(4a^3x\right) - \log\left(4a^3x + 4ia^2\right)\right) - \frac{4iax+1}{2x^2}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x**3,x)
[Out] -2*a**2*(log(4*a**3*x) - log(4*a**3*x + 4*I*a**2)) - (4*I*a*x + 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -2i\,a^2\arctan\left(ax\right) + a^2\log\left(a^2x^2+1\right) - 2\,a^2\log\left(x\right) - \frac{4i\,ax+1}{2\,x^2}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="maxima")
[Out] -2*I*a^2*arctan(a*x) + a^2*log(a^2*x^2 + 1) - 2*a^2*log(x) - 1/2*(4*I*a*x +
 1)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = 2\,a^2\log\left(ax+i\right) - 2\,a^2\log\left(|x|\right) - \frac{4i\,ax+1}{2\,x^2}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="giac")
[Out] 2*a^2*log(a*x + I) - 2*a^2*log(abs(x)) - 1/2*(4*I*a*x + 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -a^2\,\mathrm{atan}(2\,a\,x+1\mathrm{i})\;4\mathrm{i}-\frac{\frac{1}{2}+a\,x\,2\mathrm{i}}{x^2}$$

```
[In] int((a*x*1i + 1)^2/(x^3*(a^2*x^2 + 1)),x)
[Out] - a^2*atan(2*a*x + 1i)*4i - (a*x*2i + 1/2)/x^2
```

3.18 $\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3} - \frac{ia}{x^2} + \frac{2a^2}{x} - 2ia^3\log(x) + 2ia^3\log(i+ax)$$

```
[Out] -1/3/x^3-I*a/x^2+2*a^2/x-2*I*a^3*ln(x)+2*I*a^3*ln(I+a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = -2ia^3\log(x) + 2ia^3\log(ax+i) + \frac{2a^2}{x} - \frac{ia}{x^2} - \frac{1}{3x^3}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*1/x^3 - (I*a)/x^2 + (2*a^2)/x - (2*I)*a^3*Log[x] + (2*I)*a^3*Log[I + a
*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+iax}{x^4(1-iax)}\,dx\\ &= \int \left(\frac{1}{x^4}+\frac{2ia}{x^3}-\frac{2a^2}{x^2}-\frac{2ia^3}{x}+\frac{2ia^4}{i+ax}\right)dx\\ &= -\frac{1}{3x^3}-\frac{ia}{x^2}+\frac{2a^2}{x}-2ia^3\log(x)+2ia^3\log(i+ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3}-\frac{ia}{x^2}+\frac{2a^2}{x}-2ia^3\log(x)+2ia^3\log(i+ax)$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/x^4,x]
[Out] -1/3*1/x^3 - (I*a)/x^2 + (2*a^2)/x - (2*I)*a^3*Log[x] + (2*I)*a^3*Log[I + a
*x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

method	result
parallelrisch	$-\frac{6ia^3\ln(x)x^3-6ia^3\ln(ax+i)x^3+1-6a^2x^2+3iax}{3x^3}$
risch	$\frac{2a^2x^2-iax-\frac{1}{3}}{x^3}-2ia^3\ln(-x)+2a^3\arctan(ax)+ia^3\ln(a^2x^2+1)$
default	$-\frac{1}{3x^3}-2ia^3\ln(x)-\frac{ia}{x^2}+\frac{2a^2}{x}+2a^4\left(\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}\right)$
meijerg	$\frac{a^4\left(\frac{2a^2}{x(a^2)^{\frac{3}{2}}}-\frac{2}{3x^3(a^2)^{\frac{3}{2}}}+\frac{2a^3\arctan(ax)}{(a^2)^{\frac{3}{2}}}\right)}{2\sqrt{a^2}}+ia^3\left(\ln(a^2x^2+1)-2\ln(x)-\ln(a^2)-\frac{1}{a^2x^2}\right)-\frac{a^4\left(-\frac{2}{x\sqrt{a^2}}-\ldots\right.}{2\sqrt{\ldots}}$

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*(6*I*a^3*ln(x)*x^3-6*I*a^3*ln(I+a*x)*x^3+1-6*a^2*x^2+3*I*a*x)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = \frac{-6i\,a^3x^3\log(x) + 6i\,a^3x^3\log\left(\frac{ax+i}{a}\right) + 6\,a^2x^2 - 3i\,ax - 1}{3\,x^3}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(-6*I*a^3*x^3*log(x) + 6*I*a^3*x^3*log((a*x + I)/a) + 6*a^2*x^2 - 3*I*a
*x - 1)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = -2a^3\left(i\log\left(4a^4x\right) - i\log\left(4a^4x + 4ia^3\right)\right) - \frac{-6a^2x^2 + 3iax + 1}{3x^3}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/x**4,x)
```

```
[Out] -2*a**3*(I*log(4*a**4*x) - I*log(4*a**4*x + 4*I*a**3)) - (-6*a**2*x**2 + 3*
I*a*x + 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = 2\,a^3\arctan(ax) + i\,a^3\log\left(a^2x^2 + 1\right) - 2i\,a^3\log(x) + \frac{6\,a^2x^2 - 3i\,ax - 1}{3\,x^3}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] 2*a^3*arctan(a*x) + I*a^3*log(a^2*x^2 + 1) - 2*I*a^3*log(x) + 1/3*(6*a^2*x^
2 - 3*I*a*x - 1)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = 2i\,a^3\log\left(ax+i\right) - 2i\,a^3\log\left(|x|\right) + \frac{6\,a^2x^2 - 3i\,ax - 1}{3\,x^3}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*I*a^3*log(a*x + I) - 2*I*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*I*a*x - 1)/
x^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = 4\,a^3\,\mathrm{atan}(2\,a\,x + 1\mathrm{i}) - \frac{-2\,a^2\,x^2 + a\,x\,1\mathrm{i} + \frac{1}{3}}{x^3}$$

```
[In] int((a*x*1i + 1)^2/(x^4*(a^2*x^2 + 1)),x)
```

```
[Out] 4*a^3*atan(2*a*x + 1i) - (a*x*1i - 2*a^2*x^2 + 1/3)/x^3
```

3.19 $\int e^{3i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 137

$$\int e^{3i\arctan(ax)}x^3\,dx = \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}}+\frac{27\sqrt{1+a^2x^2}}{4a^4}-\frac{x^2\sqrt{1+a^2x^2}}{a^2}-\frac{ix^3\sqrt{1+a^2x^2}}{4a}-\frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4}-\frac{51i\operatorname{arcsinh}(ax)}{8a^4}$$

```
[Out] -51/8*I*arcsinh(a*x)/a^4+(1+I*a*x)^3/a^4/(a^2*x^2+1)^(1/2)+27/4*(a^2*x^2+1)
^(1/2)/a^4-x^2*(a^2*x^2+1)^(1/2)/a^2-1/4*I*x^3*(a^2*x^2+1)^(1/2)/a-9/8*I*(2
*I-3*a*x)*(a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5168, 1647, 1607, 12, 866, 1649, 1829, 27, 757, 655, 221}

$$\int e^{3i\arctan(ax)}x^3\,dx = -\frac{51i\operatorname{arcsinh}(ax)}{8a^4}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{9i(-3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1+iax)^3}{a^4\sqrt{a^2x^2+1}}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] (1 + I*a*x)^3/(a^4*Sqrt[1 + a^2*x^2]) + (27*Sqrt[1 + a^2*x^2])/(4*a^4) - (x
^2*Sqrt[1 + a^2*x^2])/a^2 - ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - (((9*I)/8)*(2
*I - 3*a*x)*Sqrt[1 + a^2*x^2])/a^4 - (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
```

```
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= -\left((ia)\int \frac{\sqrt{1+a^2x^2}\left(\frac{ix^3}{a}-x^4\right)}{(1-iax)^2}\,dx\right) \\
&= -\left((ia)\int \frac{\left(\frac{i}{a}-x\right)x^3\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right) \\
&= a^2\int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{a^2(1-iax)^3}\,dx \\
&= \int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{(1-iax)^3}\,dx \\
&= \int \frac{x^3(1+iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx
\end{aligned}$$

$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \int \frac{(1+iax)^2\left(\frac{3i}{a^3}-\frac{x}{a^2}-\frac{ix^2}{a}\right)}{\sqrt{1+a^2x^2}}\,dx$$

$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{\frac{12i}{a}-28x-27iax^2+12a^2x^3}{\sqrt{1+a^2x^2}}\,dx}{4a^2}$$

$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{36ia-108a^2x-81ia^3x^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4}$$

$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int -\frac{9ia(-2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{12a^4}$$

$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{(3i)\int \frac{(-2i+3ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a^3}$$

$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4} + \frac{(3i)\int \frac{-17a^2-18ia^3x}{\sqrt{1+a^2x^2}}\,dx}{8a^5}$$

$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4} - \frac{(51i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a^3}$$

$$= \frac{(1+iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i-3ax)\sqrt{1+a^2x^2}}{8a^4} - \frac{51i\operatorname{arcsinh}(ax)}{8a^4}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int e^{3i\arctan(ax)}x^3\,dx = \sqrt{1+a^2x^2}\left(\frac{6}{a^4}+\frac{19ix}{8a^3}-\frac{x^2}{a^2}-\frac{ix^3}{4a}+\frac{4i}{a^4(i+ax)}\right)-\frac{51i\operatorname{arcsinh}(ax)}{8a^4}$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(6/a^4 + (((19*I)/8)*x)/a^3 - x^2/a^2 - ((I/4)*x^3)/a + (
4*I)/(a^4*(I + a*x))) - (((51*I)/8)*ArcSinh[a*x])/a^4
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{i(2a^3x^3-8ia^2x^2-19ax+48i)\sqrt{a^2x^2+1}}{8a^4}-\frac{i\left(\frac{51\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}-\frac{32\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a^2\left(x+\frac{i}{a}\right)}\right)}{8a^3}$
meijerg	$\frac{-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(4a^2x^2+8\right)}{4\sqrt{a^2x^2+1}}}{a^4\sqrt{\pi}}+\frac{3i\left(\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{5}{2}}\left(5a^2x^2+15\right)}{10a^4\sqrt{a^2x^2+1}}-\frac{3\sqrt{\pi}\left(a^2\right)^{\frac{5}{2}}\operatorname{arcsinh}(ax)}{2a^5}\right)}{a^3\sqrt{\pi}\sqrt{a^2}}-\frac{3\left(\frac{8\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-2a^4x^4+8a^2x^2+16\right)}{6\sqrt{a^2x^2+1}}\right)}{a^4\sqrt{\pi}}-\frac{i\left(-\frac{\sqrt{}}{}\right.}{}$
default	$\frac{x^2}{a^2\sqrt{a^2x^2+1}}+\frac{2}{a^4\sqrt{a^2x^2+1}}-3a^2\left(\frac{x^4}{3a^2\sqrt{a^2x^2+1}}-\frac{4\left(\frac{x^2}{a^2\sqrt{a^2x^2+1}}+\frac{2}{a^4\sqrt{a^2x^2+1}}\right)}{3a^2}\right)-ia^3\left(\frac{x^5}{4a^2\sqrt{a^2x^2+1}}-\frac{5\left(\frac{x}{2a^2\sqrt{a}}\right.}{}\right.$

[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x,method=_RETURNVERBOSE)

[Out] -1/8*I*(2*a^3*x^3-8*I*a^2*x^2-19*a*x+48*I)*(a^2*x^2+1)^(1/2)/a^4-1/8*I/a^3*(51*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-32/a^2/(x+I/a)*((x+I/a)^2*a^2-2*I*a*(x+I/a))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.64

$$\int e^{3i\arctan(ax)}x^3\,dx = \frac{32i\,ax-51\,(-i\,ax+1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+(-2i\,a^4x^4-6\,a^3x^3+11i\,a^2x^2+29\,ax+80i)\sqrt{a^2x^2+1}}{8\,(a^5x+i\,a^4)}$$

[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="fricas")

[Out] 1/8*(32*I*a*x - 51*(-I*a*x + 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + (-2*I*a^4*x^4 - 6*a^3*x^3 + 11*I*a^2*x^2 + 29*a*x + 80*I)*sqrt(a^2*x^2 + 1) - 32)/(a^5*x + I*a^4)

Sympy [F]

$$\int e^{3i\arctan(ax)}x^3\,dx = -i\left(\int \frac{ix^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax^4}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^6}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^5}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**3,x)
```

```
[Out] -I*(Integral(I*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x**4/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1
)), x) + Integral(a**3*x**6/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2
 + 1)), x) + Integral(-3*I*a**2*x**5/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(
a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int e^{3i\arctan(ax)}x^3\,dx = -\frac{i\,ax^5}{4\,\sqrt{a^2x^2+1}} - \frac{x^4}{\sqrt{a^2x^2+1}} + \frac{17i\,x^3}{8\,\sqrt{a^2x^2+1}a} + \frac{5\,x^2}{\sqrt{a^2x^2+1}a^2} + \frac{51i\,x}{8\,\sqrt{a^2x^2+1}a^3} - \frac{51i\,\operatorname{arsinh}(ax)}{8\,a^4} + \frac{10}{\sqrt{a^2x^2+1}a^4}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*a*x^5/sqrt(a^2*x^2 + 1) - x^4/sqrt(a^2*x^2 + 1) + 17/8*I*x^3/(sqrt(a
^2*x^2 + 1)*a) + 5*x^2/(sqrt(a^2*x^2 + 1)*a^2) + 51/8*I*x/(sqrt(a^2*x^2 + 1
)*a^3) - 51/8*I*arcsinh(a*x)/a^4 + 10/(sqrt(a^2*x^2 + 1)*a^4)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00

$$\int e^{3i\arctan(ax)}x^3\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{4}{{\left(a^2\right)}^{3/2}}+\frac{2\,\sqrt{a^2}}{a^4}-\frac{x^2\,\sqrt{a^2}}{a^2}-\frac{x^3\,{\left(a^2\right)}^{3/2}\,1{}\mathrm{i}}{4\,a^3}+\frac{x\,\sqrt{a^2}\,19{}\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}} - \frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,51{}\mathrm{i}}{8\,a^3\,\sqrt{a^2}} + \frac{\sqrt{a^2\,x^2+1}\,4{}\mathrm{i}}{a^3\,\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1{}\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((x^3*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*(4/(a^2)^(3/2) + (2*(a^2)^(1/2))/a^4 - (x^2*(a^2)^(1/2
))/a^2 - (x^3*(a^2)^(3/2)*1i)/(4*a^3) + (x*(a^2)^(1/2)*19i)/(8*a^3)))/(a^2)
^(1/2) - (asinh(x*(a^2)^(1/2))*51i)/(8*a^3*(a^2)^(1/2)) + ((a^2*x^2 + 1)^(1
/2)*4i)/(a^3*(((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.20 $\int e^{3i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int e^{3i\arctan(ax)}x^2\,dx=\frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}}+\frac{(28i-3ax)\sqrt{1+a^2x^2}}{6a^3}+\frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3}+\frac{11\text{arcsinh}(ax)}{2a^3}$$

```
[Out] 11/2*arcsinh(a*x)/a^3+I*(1+I*a*x)^3/a^3/(a^2*x^2+1)^(1/2)+1/6*(28*I-3*a*x)*
(a^2*x^2+1)^(1/2)/a^3+1/3*I*(3+I*a*x)^2*(a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5168, 1647, 1607, 12, 866, 1649, 1668, 794, 221}

$$\int e^{3i\arctan(ax)}x^2\,dx=\frac{11\text{arcsinh}(ax)}{2a^3}+\frac{i(1+iax)^3}{a^3\sqrt{a^2x^2+1}}+\frac{i(3+iax)^2\sqrt{a^2x^2+1}}{3a^3}+\frac{(-3ax+28i)\sqrt{a^2x^2+1}}{6a^3}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (I*(1 + I*a*x)^3)/(a^3*Sqrt[1 + a^2*x^2]) + ((28*I - 3*a*x)*Sqrt[1 + a^2*x^
2])/(6*a^3) + ((I/3)*(3 + I*a*x)^2*Sqrt[1 + a^2*x^2])/a^3 + (11*ArcSinh[a*x
])/(2*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx\\
&= -\left((ia)\int \frac{\sqrt{1+a^2x^2}\left(\frac{ix^2}{a}-x^3\right)}{(1-iax)^2}\,dx\right)\\
&= -\left((ia)\int \frac{\left(\frac{i}{a}-x\right)x^2\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right)\\
&= a^2\int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{a^2(1-iax)^3}\,dx\\
&= \int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{(1-iax)^3}\,dx\\
&= \int \frac{x^2(1+iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx\\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}}-\int \frac{\left(-\frac{3}{a^2}-\frac{ix}{a}\right)(1+iax)^2}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}}+\frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3}+\frac{1}{3}\int \frac{\left(-\frac{3}{a^2}-\frac{ix}{a}\right)(-5-3iax)}{\sqrt{1+a^2x^2}}\,dx
\end{aligned}$$

$$= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} + \frac{(28i-3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{11\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2}$$

$$= \frac{i(1+iax)^3}{a^3\sqrt{1+a^2x^2}} + \frac{(28i-3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{i(3+iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{11\operatorname{arcsinh}(ax)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int e^{3i\arctan(ax)}x^2\,dx = \frac{\frac{\sqrt{1+a^2x^2}(-52+19iax-7a^2x^2-2ia^3x^3)}{i+ax} + 33\operatorname{arcsinh}(ax)}{6a^3}$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-52 + (19*I)*a*x - 7*a^2*x^2 - (2*I)*a^3*x^3))/(I + a*
x) + 33*ArcSinh[a*x])/(6*a^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{i(2a^2x^2-9iax-28)\sqrt{a^2x^2+1}}{6a^3} + \frac{11\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}} - \frac{4\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a^4\left(x+\frac{i}{a}\right)}$
meijerg	$\frac{-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{3}{2}}}{a^2\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\left(a^2\right)^{\frac{3}{2}}\operatorname{arcsinh}(ax)}{a^3}}{a^2\sqrt{\pi}\sqrt{a^2}} + \frac{3i\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(4a^2x^2+8\right)}{4\sqrt{a^2x^2+1}}\right)}{a^3\sqrt{\pi}} - \frac{3\left(\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{5}{2}}\left(5a^2x^2+15\right)}{10a^4\sqrt{a^2x^2+1}}-\frac{3\sqrt{\pi}\left(a^2\right)^{\frac{5}{2}}\operatorname{arcsinh}(ax)}{2a^5}\right)}{a^2\sqrt{\pi}\sqrt{a^2}} - \cdots$
default	$-\frac{x}{a^2\sqrt{a^2x^2+1}} + \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}} - 3a^2\left(\frac{x^3}{2a^2\sqrt{a^2x^2+1}} - \frac{3\left(-\frac{x}{a^2\sqrt{a^2x^2+1}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}\right)}{2a^2}\right) - ia^3\left(\frac{\cdots}{3a^2\cdots}\right.$

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*(2*a^2*x^2-9*I*a*x-28)*(a^2*x^2+1)^(1/2)/a^3+11/2/a^2*ln(a^2*x/(a^2)
^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-4/a^4/(x+I/a)*((x+I/a)^2*a^2-2*I*a*(x
+I/a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int e^{3i\arctan(ax)}x^2\,dx = -\frac{24\,ax + 33\,(ax+i)\log\left(-ax+\sqrt{a^2x^2+1}\right) - \left(-2i\,a^3x^3 - 7\,a^2x^2 + 19i\,ax - 52\right)\sqrt{a^2x^2+1} + 24i}{6\,(a^4x + i\,a^3)}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(24*a*x + 33*(a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1)) - (-2*I*a^3*x^3 -
 7*a^2*x^2 + 19*I*a*x - 52)*sqrt(a^2*x^2 + 1) + 24*I)/(a^4*x + I*a^3)
```

Sympy [F]

$$\int e^{3i\arctan(ax)}x^2\,dx = -i\left(\int\frac{ix^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3ax^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx + \int\frac{a^3x^5}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3ia^2x^4}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**2,x)
```

```
[Out] -I*(Integral(I*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1
)), x) + Integral(a**3*x**5/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2
 + 1)), x) + Integral(-3*I*a**2*x**4/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(
a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int e^{3i\arctan(ax)}x^2\,dx = -\frac{i\,ax^4}{3\,\sqrt{a^2x^2+1}} - \frac{3\,x^3}{2\,\sqrt{a^2x^2+1}} + \frac{13i\,x^2}{3\,\sqrt{a^2x^2+1}a} - \frac{11\,x}{2\,\sqrt{a^2x^2+1}a^2} + \frac{11\,\operatorname{arsinh}(ax)}{2\,a^3} + \frac{26i}{3\,\sqrt{a^2x^2+1}a^3}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*I*a*x^4/sqrt(a^2*x^2 + 1) - 3/2*x^3/sqrt(a^2*x^2 + 1) + 13/3*I*x^2/(sq
rt(a^2*x^2 + 1)*a) - 11/2*x/(sqrt(a^2*x^2 + 1)*a^2) + 11/2*arcsinh(a*x)/a^3
 + 26/3*I/(sqrt(a^2*x^2 + 1)*a^3)
```

Giac [F]

$$\int e^{3i\arctan(ax)}x^2\,dx = \int \frac{(i\,ax+1)^3x^2}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int e^{3i\arctan(ax)}x^2\,dx = \frac{11\,\operatorname{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,x\,\sqrt{a^2}}{2\,a^2} - \frac{a\,14\mathrm{i}}{3\,(a^2)^{3/2}} + \frac{a^3\,x^2\,1\mathrm{i}}{3\,(a^2)^{3/2}}\right)}{\sqrt{a^2}} - \frac{4\,\sqrt{a^2\,x^2+1}}{a^2\,\left(x\,\sqrt{a^2} + \frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((x^2*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] (11*asinh(x*(a^2)^(1/2)))/(2*a^2*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*((a^3*
x^2*1i)/(3*(a^2)^(3/2)) - (a*14i)/(3*(a^2)^(3/2)) + (3*x*(a^2)^(1/2))/(2*a^
2)))/(a^2)^(1/2) - (4*(a^2*x^2 + 1)^(1/2))/(a^2*(((a^2)^(1/2)*1i)/a + x*(a^
2)^(1/2))*(a^2)^(1/2))
```

3.21 $\int e^{3i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 92

$$\int e^{3i\arctan(ax)}x\,dx=-\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3(1+a^2x^2)^{3/2}}{2a^2(1-iax)}-\frac{(1+a^2x^2)^{5/2}}{a^2(1-iax)^3}+\frac{9i\text{arcsinh}(ax)}{2a^2}$$

```
[Out] -3/2*(a^2*x^2+1)^(3/2)/a^2/(1-I*a*x)-(a^2*x^2+1)^(5/2)/a^2/(1-I*a*x)^3+9/2*
I*arcsinh(a*x)/a^2-9/2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5168, 1647, 1607, 12, 807, 679, 221}

$$\int e^{3i\arctan(ax)}x\,dx=\frac{9i\text{arcsinh}(ax)}{2a^2}-\frac{(a^2x^2+1)^{5/2}}{a^2(1-iax)^3}-\frac{3(a^2x^2+1)^{3/2}}{2a^2(1-iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])*x,x]
```

```
[Out] (-9*Sqrt[1 + a^2*x^2])/(2*a^2) - (3*(1 + a^2*x^2)^(3/2))/(2*a^2*(1 - I*a*x)
) - (1 + a^2*x^2)^(5/2)/(a^2*(1 - I*a*x)^3) + (((9*I)/2)*ArcSinh[a*x])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= -\left((ia)\int \frac{\left(\frac{ix}{a}-x^2\right)\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right) \\
&= -\left((ia)\int \frac{\left(\frac{i}{a}-x\right)x\sqrt{1+a^2x^2}}{(1-iax)^2}\,dx\right)
\end{aligned}$$

$$= a^2 \int \frac{x(1+a^2x^2)^{3/2}}{a^2(1-iax)^3}\,dx$$

$$= \int \frac{x(1+a^2x^2)^{3/2}}{(1-iax)^3}\,dx$$

$$= -\frac{(1+a^2x^2)^{5/2}}{a^2(1-iax)^3} + \frac{(3i)\int \frac{(1+a^2x^2)^{3/2}}{(1-iax)^2}\,dx}{a}$$

$$= -\frac{3(1+a^2x^2)^{3/2}}{2a^2(1-iax)} - \frac{(1+a^2x^2)^{5/2}}{a^2(1-iax)^3} + \frac{(9i)\int \frac{\sqrt{1+a^2x^2}}{1-iax}\,dx}{2a}$$

$$= -\frac{9\sqrt{1+a^2x^2}}{2a^2} - \frac{3(1+a^2x^2)^{3/2}}{2a^2(1-iax)} - \frac{(1+a^2x^2)^{5/2}}{a^2(1-iax)^3} + \frac{(9i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a}$$

$$= -\frac{9\sqrt{1+a^2x^2}}{2a^2} - \frac{3(1+a^2x^2)^{3/2}}{2a^2(1-iax)} - \frac{(1+a^2x^2)^{5/2}}{a^2(1-iax)^3} + \frac{9i\text{arcsinh}(ax)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

$$\int e^{3i\arctan(ax)}x\,dx = -\frac{i\left(\frac{\sqrt{1+a^2x^2}(14-5iax+a^2x^2)}{i+ax} - 9\text{arcsinh}(ax)\right)}{2a^2}$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x,x]
[Out] ((-1/2*I)*((Sqrt[1 + a^2*x^2]*(14 - (5*I)*a*x + a^2*x^2))/(I + a*x) - 9*Arc
Sinh[a*x]))/a^2
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{i(ax-6i)\sqrt{a^2x^2+1}}{2a^2}+\frac{i\left(\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}-\frac{8\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a^2\left(x+\frac{i}{a}\right)}\right)}{2a}$
meijerg	$\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}}{a^2\sqrt{\pi}}+\frac{3i\left(-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{3}{2}}}{a^2\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\left(a^2\right)^{\frac{3}{2}}\operatorname{arcsinh}(ax)}{a^3}\right)}{a\sqrt{\pi}\sqrt{a^2}}-\frac{3\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(4a^2x^2+8\right)}{4\sqrt{a^2x^2+1}}\right)}{a^2\sqrt{\pi}}-\frac{i\left(\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{5}{2}}\left(5a^2x^2+15\right)}{10a^4\sqrt{a^2x^2+1}}-\frac{3\sqrt{\pi}}{}\right.}{a\sqrt{\pi}\sqrt{a^2}}$
default	$-\frac{1}{a^2\sqrt{a^2x^2+1}}-3a^2\left(\frac{x^2}{a^2\sqrt{a^2x^2+1}}+\frac{2}{a^4\sqrt{a^2x^2+1}}\right)-ia^3\left(\frac{x^3}{2a^2\sqrt{a^2x^2+1}}-\frac{3\left(-\frac{x}{a^2\sqrt{a^2x^2+1}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}\right)}{2a^2}\right)$

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(a*x-6*I)*(a^2*x^2+1)^(1/2)/a^2+1/2*I/a*(9*ln(a^2*x/(a^2)^(1/2)+(a^2
*x^2+1)^(1/2))/(a^2)^(1/2)-8/a^2/(x+I/a)*((x+I/a)^2*a^2-2*I*a*(x+I/a))^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.78

$$\int e^{3i\arctan(ax)}x\,dx = \frac{-8i\,ax-9\,(i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}(-i\,a^2x^2-5\,ax-14i)+8}{2\,(a^3x+i\,a^2)}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(-8*I*a*x - 9*(I*a*x - 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2
+ 1)*(-I*a^2*x^2 - 5*a*x - 14*I) + 8)/(a^3*x + I*a^2)
```

Sympy [F]

$$\int e^{3i\arctan(ax)}x\,dx = -i\left(\int\frac{ix}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx+\int\left(-\frac{3ax^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx+\int\frac{a^3x^4}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx+\int\left(-\frac{3ia^2x^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x,x)
```

```
[Out] -I*(Integral(I*x/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x)
+ Integral(-3*a*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
 x) + Integral(a**3*x**4/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 +
1)), x) + Integral(-3*I*a**2*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**
2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int e^{3i\arctan(ax)}x\,dx = -\frac{i\,ax^3}{2\sqrt{a^2x^2+1}} - \frac{3\,x^2}{\sqrt{a^2x^2+1}} - \frac{9i\,x}{2\sqrt{a^2x^2+1}a} + \frac{9i\,\operatorname{arsinh}(ax)}{2\,a^2} - \frac{7}{\sqrt{a^2x^2+1}a^2}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*I*a*x^3/sqrt(a^2*x^2 + 1) - 3*x^2/sqrt(a^2*x^2 + 1) - 9/2*I*x/(sqrt(a^
2*x^2 + 1)*a) + 9/2*I*arcsinh(a*x)/a^2 - 7/(sqrt(a^2*x^2 + 1)*a^2)
```

Giac [F]

$$\int e^{3i\arctan(ax)}x\,dx = \int \frac{(i\,ax+1)^3x}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13

$$\int e^{3i\arctan(ax)}x\,dx = -\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,\sqrt{a^2}}{a^2}+\frac{x\,\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)}{\sqrt{a^2}} + \frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,9\mathrm{i}}{2\,a\,\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{a\,\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((x*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] (asinh(x*(a^2)^(1/2))*9i)/(2*a*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*((3*(a^2
)^(1/2))/a^2 + (x*(a^2)^(1/2)*1i)/(2*a)))/(a^2)^(1/2) - ((a^2*x^2 + 1)^(1/2
)*4i)/(a*(((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.22 $\int e^{3i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int e^{3i\arctan(ax)}\,dx=-\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}}-\frac{3i\sqrt{1+a^2x^2}}{a}-\frac{3\text{arcsinh}(ax)}{a}$$

```
[Out] -3*arcsinh(a*x)/a-2*I*(1+I*a*x)^2/a/(a^2*x^2+1)^(1/2)-3*I*(a^2*x^2+1)^(1/2)
/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5167, 867, 683, 655, 221}

$$\int e^{3i\arctan(ax)}\,dx=-\frac{2i(1+iax)^2}{a\sqrt{a^2x^2+1}}-\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\text{arcsinh}(ax)}{a}$$

```
[In] Int[E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*(1 + I*a*x)^2)/(a*Sqrt[1 + a^2*x^2]) - ((3*I)*Sqrt[1 + a^2*x^2])/a
- (3*ArcSinh[a*x])/a
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 5167

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \frac{(1+iax)^3}{(1+a^2x^2)^{3/2}}\,dx \\
&= -\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}} - 3\int \frac{1+iax}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}} - \frac{3i\sqrt{1+a^2x^2}}{a} - 3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2i(1+iax)^2}{a\sqrt{1+a^2x^2}} - \frac{3i\sqrt{1+a^2x^2}}{a} - \frac{3\operatorname{arcsinh}(ax)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int e^{3i\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\left(-i+\frac{4}{i+ax}\right)}{a} - \frac{3\text{arcsinh}(ax)}{a}$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x]),x]
[Out] (Sqrt[1 + a^2*x^2]*(-I + 4/(I + a*x)))/a - (3*ArcSinh[a*x])/a
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.55

method	result	size
risch	$-\frac{i\sqrt{a^2x^2+1}}{a}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}+\frac{4\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a^2\left(x+\frac{i}{a}\right)}$	93
default	$\frac{x}{\sqrt{a^2x^2+1}}-3a^2\left(-\frac{x}{a^2\sqrt{a^2x^2+1}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}\right)-ia^3\left(\frac{x^2}{a^2\sqrt{a^2x^2+1}}+\frac{2}{a^4\sqrt{a^2x^2+1}}\right)-\frac{3i}{a\sqrt{a^2x^2+1}}$	128
meijerg	$\frac{x}{\sqrt{a^2x^2+1}}+\frac{3i\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}\right)}{a\sqrt{\pi}}-\frac{3\left(-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{3}{2}}}{a^2\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\left(a^2\right)^{\frac{3}{2}}\text{arcsinh}(ax)}{a^3}\right)}{\sqrt{\pi}\sqrt{a^2}}-\frac{i\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(4a^2x^2+8\right)}{4\sqrt{a^2x^2+1}}\right)}{a\sqrt{\pi}}$	137

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -I*(a^2*x^2+1)^(1/2)/a-3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2
)+4/a^2/(x+I/a)*((x+I/a)^2*a^2-2*I*a*(x+I/a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int e^{3i\arctan(ax)}\,dx = \frac{4\,ax+3\,(ax+i)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}(-i\,ax+5)+4i}{a^2x+i\,a}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] (4*a*x + 3*(a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)*(-I*
a*x + 5) + 4*I)/(a^2*x + I*a)
```

Sympy [F]

$$\int e^{3i\arctan(ax)}\,dx = -i\left(\int \frac{i}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2),x)
```

```
[Out] -I*(Integral(I/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x) +
Integral(-3*a*x/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x) +
 Integral(a**3*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) + Integral(-3*I*a**2*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**
2 + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int e^{3i\arctan(ax)}\,dx = -\frac{i\,ax^2}{\sqrt{a^2x^2+1}} + \frac{4\,x}{\sqrt{a^2x^2+1}} - \frac{3\,\operatorname{arsinh}(ax)}{a} - \frac{5i}{\sqrt{a^2x^2+1}a}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*a*x^2/sqrt(a^2*x^2 + 1) + 4*x/sqrt(a^2*x^2 + 1) - 3*arcsinh(a*x)/a - 5*I
/(sqrt(a^2*x^2 + 1)*a)
```

Giac [F]

$$\int e^{3i\arctan(ax)}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int e^{3i\arctan(ax)}\,dx = -\frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a} - \frac{3\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{4\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a*x*1i + 1)^3/(a^2*x^2 + 1)^(3/2),x)
```

```
[Out] (4*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
- (3*asinh(x*(a^2)^(1/2)))/(a^2)^(1/2) - ((a^2*x^2 + 1)^(1/2)*1i)/a
```

3.23 $\int \frac{e^{3i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = \frac{4i\sqrt{1+a^2x^2}}{i+ax} - i\,\mathrm{arcsinh}(ax) - \mathrm{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))+4*I*(a^2*x^2+1)^(1/2)/(I+a*x)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5168, 6874, 221, 272, 65, 214, 665}

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = -\mathrm{arctanh}\left(\sqrt{a^2x^2+1}\right) + \frac{4i\sqrt{a^2x^2+1}}{ax+i} - i\,\mathrm{arcsinh}(ax)$$

```
[In] Int[E^((3*I)*ArcTan[a*x])/x,x]
```

```
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I + a*x) - I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2
*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^2}{x(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(-\frac{ia}{\sqrt{1+a^2x^2}}+\frac{1}{x\sqrt{1+a^2x^2}}-\frac{4a}{(i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= -\left((ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\right)-(4a)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= \frac{4i\sqrt{1+a^2x^2}}{i+ax}-i\text{arcsinh}(ax)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)
\end{aligned}
$$

$$= \frac{4i\sqrt{1+a^2x^2}}{i+ax} - i\text{arcsinh}(ax) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a^2}$$

$$= \frac{4i\sqrt{1+a^2x^2}}{i+ax} - i\text{arcsinh}(ax) - \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = \frac{4i\sqrt{1+a^2x^2}}{i+ax} - i\text{arcsinh}(ax) + \log(x) - \log\left(1+\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x,x]
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I + a*x) - I*ArcSinh[a*x] + Log[x] - Log[1 + Sqr
t[1 + a^2*x^2]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(44) = 88.

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.98

method	result
default	$\frac{4}{\sqrt{a^2x^2+1}} - \text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right) + \frac{3iax}{\sqrt{a^2x^2+1}} - ia^3\left(-\frac{x}{a^2\sqrt{a^2x^2+1}} + \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}\right)$
meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)+\frac{\left(2-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{2}}{\sqrt{\pi}} + \frac{3iax}{\sqrt{a^2x^2+1}} - \frac{3\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}\right)}{\sqrt{\pi}} - \frac{ia\left(-\frac{\sqrt{\pi}\,x\left(a^2\right)}{a^2\sqrt{a^2x^2-}}\right.}{}$

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] 4/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))+3*I*a*x/(a^2*x^2+1)^(1/2)-
I*a^3*(-x/a^2/(a^2*x^2+1)^(1/2)+1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2
))/(a^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(41) = 82.

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.96

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx$$
$$= \frac{4i\,ax - (ax+i)\log\left(-ax+\sqrt{a^2x^2+1}+1\right) + (i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right) + (ax+i)\log\left(-ax+\sqrt{a^2x^2+1}\right.}{ax+i}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (4*I*a*x - (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + (I*a*x - 1)*log(-a
*x + sqrt(a^2*x^2 + 1)) + (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + 4*I
*sqrt(a^2*x^2 + 1) - 4)/(a*x + I)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = -i\left(\int \frac{i}{a^2x^3\sqrt{a^2x^2+1}+x\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax}{a^2x^3\sqrt{a^2x^2+1}+x\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^3\sqrt{a^2x^2+1}+x\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^3\sqrt{a^2x^2+1}+x\sqrt{a^2x^2+1}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] -I*(Integral(I/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(a**2*x**2 + 1)), x)
+ Integral(-3*a*x/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(a**2*x**2 + 1)),
x) + Integral(a**3*x**3/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(a**2*x**2 +
 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(
a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = \frac{4i\,ax}{\sqrt{a^2x^2+1}} + \frac{4}{\sqrt{a^2x^2+1}} - i\ \mathrm{arsinh}\,(ax) - \mathrm{arsinh}\left(\frac{1}{a|x|}\right)$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*I*a*x/sqrt(a^2*x^2 + 1) + 4/sqrt(a^2*x^2 + 1) - I*arcsinh(a*x) - arcsinh(
1/(a*abs(x)))
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = -\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right) - \frac{a\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\ 1\mathrm{i}}{\sqrt{a^2}} + \frac{a\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a*x*1i + 1)^3/(x*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] (a*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2
)) - (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2) - atanh((a^2*x^2 + 1)^(1/2))
```

3.24 $\int \frac{e^{3i\arctan(ax)}}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{e^{3i\arctan(ax)}}{x^2} dx = -\frac{\sqrt{1+a^2x^2}}{x} - \frac{4a\sqrt{1+a^2x^2}}{i+ax} - 3ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -3*I*a*arctanh((a^2*x^2+1)^(1/2))-(a^2*x^2+1)^(1/2)/x-4*a*(a^2*x^2+1)^(1/2)
/(I+a*x)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5168, 6874, 270, 272, 65, 214, 665}

$$\int \frac{e^{3i\arctan(ax)}}{x^2} dx = -3ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{4a\sqrt{a^2x^2+1}}{ax+i} - \frac{\sqrt{a^2x^2+1}}{x}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) - (4*a*Sqrt[1 + a^2*x^2])/(I + a*x) - (3*I)*a*ArcTan
h[Sqrt[1 + a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^2}{x^2(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^2\sqrt{1+a^2x^2}}+\frac{3ia}{x\sqrt{1+a^2x^2}}-\frac{4ia^2}{(i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= (3ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx-\left(4ia^2\right)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x}-\frac{4a\sqrt{1+a^2x^2}}{i+ax}+\frac{1}{2}(3ia)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)
\end{aligned}
$$

$$= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{4a\sqrt{1+a^2x^2}}{i+ax} + \frac{(3i)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a}$$

$$= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{4a\sqrt{1+a^2x^2}}{i+ax} - 3ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = \sqrt{1+a^2x^2}\left(-\frac{1}{x} - \frac{4a}{i+ax}\right) + 3ia\log(x) - 3ia\log\left(1+\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-x^(-1) - (4*a)/(I + a*x)) + (3*I)*a*Log[x] - (3*I)*a*Lo
g[1 + Sqrt[1 + a^2*x^2]]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

method	result
default	$-\frac{1}{x\sqrt{a^2x^2+1}} - \frac{5a^2x}{\sqrt{a^2x^2+1}} + \frac{ia}{\sqrt{a^2x^2+1}} + 3ia\left(\frac{1}{\sqrt{a^2x^2+1}} - \text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)$
risch	$-\frac{\sqrt{a^2x^2+1}}{x} + ia\left(-3\ \text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right) + \frac{4i\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a\left(x+\frac{i}{a}\right)}\right)$
meijerg	$-\frac{2a^2x^2+1}{x\sqrt{a^2x^2+1}} + \frac{3ia\left(-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}-\sqrt{\pi}\ \ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)+\frac{\left(2-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{2}\right)}{\sqrt{\pi}} - \frac{3a^2x}{\sqrt{a^2x^2+1}} - \frac{ia\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{a^2x^2-}}\right.}{\sqrt{\pi}}$

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x/(a^2*x^2+1)^(1/2)-5/(a^2*x^2+1)^(1/2)*a^2*x+I*a/(a^2*x^2+1)^(1/2)+3*I*
a*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(53) = 106$.

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.73

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = -\frac{5\,a^2x^2 + 5i\,ax + 3\,(i\,a^2x^2 - ax)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) + 3\,(-i\,a^2x^2 + ax)\log\left(-ax + \sqrt{a^2x^2+1}\right.}{ax^2 + i\,x}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a^2*x^2 + 5*I*a*x + 3*(I*a^2*x^2 - a*x)*log(-a*x + sqrt(a^2*x^2 + 1) +
1) + 3*(-I*a^2*x^2 + a*x)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + sqrt(a^2*x^2
+ 1)*(5*a*x + I))/(a*x^2 + I*x)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = -i\left(\int \frac{i}{a^2x^4\sqrt{a^2x^2+1} + x^2\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax}{a^2x^4\sqrt{a^2x^2+1} + x^2\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^4\sqrt{a^2x^2+1} + x^2\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^4\sqrt{a^2x^2+1} + x^2\sqrt{a^2x^2+1}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] -I*(Integral(I/(a**2*x**4*sqrt(a**2*x**2 + 1) + x**2*sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x/(a**2*x**4*sqrt(a**2*x**2 + 1) + x**2*sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**3/(a**2*x**4*sqrt(a**2*x**2 + 1) + x**2*sqrt(a*
*2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**4*sqrt(a**2*x**2 + 1)
+ x**2*sqrt(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = -\frac{5\,a^2x}{\sqrt{a^2x^2+1}} - 3i\,a\,\mathrm{arsinh}\left(\frac{1}{a|x|}\right) + \frac{4i\,a}{\sqrt{a^2x^2+1}} - \frac{1}{\sqrt{a^2x^2+1}x}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
[Out] -5*a^2*x/sqrt(a^2*x^2 + 1) - 3*I*a*arcsinh(1/(a*abs(x))) + 4*I*a/sqrt(a^2*x
^2 + 1) - 1/(sqrt(a^2*x^2 + 1)*x)
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = -a\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,3\mathrm{i} - \frac{\sqrt{a^2\,x^2+1}}{x} - \frac{4\,a^2\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a*x*1i + 1)^3/(x^2*(a^2*x^2 + 1)^(3/2)),x)
[Out] - a*atanh((a^2*x^2 + 1)^(1/2))*3i - (a^2*x^2 + 1)^(1/2)/x - (4*a^2*(a^2*x^2
 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.25 $\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx=-\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}+\frac{9}{2}a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] 9/2*a^2*arctanh((a^2*x^2+1)^(1/2))-1/2*(a^2*x^2+1)^(1/2)/x^2-3*I*a*(a^2*x^2
+1)^(1/2)/x-4*I*a^2*(a^2*x^2+1)^(1/2)/(I+a*x)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5168, 6874, 272, 44, 65, 214, 270, 665}

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx=\frac{9}{2}a^2\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{4ia^2\sqrt{a^2x^2+1}}{ax+i}-\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^2]/x^2 - ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sqr
t[1 + a^2*x^2])/(I + a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^2}{x^3(1-iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \left(\frac{1}{x^3\sqrt{1+a^2x^2}}+\frac{3ia}{x^2\sqrt{1+a^2x^2}}-\frac{4a^2}{x\sqrt{1+a^2x^2}}+\frac{4a^3}{(i+ax)\sqrt{1+a^2x^2}}\right)dx\\
&= (3ia)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx-\left(4a^2\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&\quad+\left(4a^3\right)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&\quad-\left(2a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}\\
&\quad-4\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)-\frac{1}{4}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}\\
&\quad+4a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4ia^2\sqrt{1+a^2x^2}}{i+ax}+\frac{9}{2}a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx=\sqrt{1+a^2x^2}\left(-\frac{1}{2x^2}-\frac{3ia}{x}-\frac{4ia^2}{i+ax}\right)-\frac{9}{2}a^2\log(x)+\frac{9}{2}a^2\log\left(1+\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x^3,x]
[Out] Sqrt[1 + a^2*x^2]*(-1/2*1/x^2 - ((3*I)*a)/x - ((4*I)*a^2)/(I + a*x)) - (9*a
^2*Log[x])/2 + (9*a^2*Log[1 + Sqrt[1 + a^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.14

method	result
default	$-\frac{1}{2x^{2}\sqrt{a^{2}x^{2}+1}}-\frac{9a^{2}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)\right)}{2}-\frac{i a^{3}x}{\sqrt{a^{2}x^{2}+1}}+3ia\left(-\frac{1}{x\sqrt{a^{2}x^{2}+1}}-\frac{2a^{2}x}{\sqrt{a^{2}x^{2}+1}}\right)$
risch	$-\frac{i\left(6a^{3}x^{3}-ia^{2}x^{2}+6ax-i\right)}{2x^{2}\sqrt{a^{2}x^{2}+1}}-\frac{a^{2}\left(-9\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)+\frac{8i\sqrt{\left(x+\frac{i}{a}\right)^{2}a^{2}-2ia\left(x+\frac{i}{a}\right)}}{a\left(x+\frac{i}{a}\right)}\right)}{2}$
meijerg	$\frac{a^{2}\left(\frac{\sqrt{\pi}\left(20a^{2}x^{2}+8\right)}{16a^{2}x^{2}}-\frac{\sqrt{\pi}\left(24a^{2}x^{2}+8\right)}{16a^{2}x^{2}\sqrt{a^{2}x^{2}+1}}+\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{a^{2}x^{2}+1}}{2}\right)}{2}-\frac{3\left(\frac{5}{3}-2\ln(2)+2\ln(x)+\ln\left(a^{2}\right)\right)\sqrt{\pi}}{4}-\frac{\sqrt{\pi}}{2x^{2}a^{2}}\right)}{\sqrt{\pi}}-\frac{3ia\left(2a^{2}x^{2}+1\right)}{x\sqrt{a^{2}x^{2}+1}}-\frac{3a^{2}}{}$

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2/(a^2*x^2+1)^(1/2)-9/2*a^2*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+
1)^(1/2)))-I*a^3*x/(a^2*x^2+1)^(1/2)+3*I*a*(-1/x/(a^2*x^2+1)^(1/2)-2/(a^2*x
^2+1)^(1/2)*a^2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.41

$$\int \frac{e^{3i\arctan(ax)}}{x^{3}}\,dx$$
$$=\frac{-14i\,a^{3}x^{3}+14\,a^{2}x^{2}+9\left(a^{3}x^{3}+i\,a^{2}x^{2}\right)\log\left(-ax+\sqrt{a^{2}x^{2}+1}+1\right)-9\left(a^{3}x^{3}+i\,a^{2}x^{2}\right)\log\left(-ax+\sqrt{a^{2}x^{2}}\right.}{2\left(ax^{3}+i\,x^{2}\right)}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(-14*I*a^3*x^3 + 14*a^2*x^2 + 9*(a^3*x^3 + I*a^2*x^2)*log(-a*x + sqrt(a
^2*x^2 + 1) + 1) - 9*(a^3*x^3 + I*a^2*x^2)*log(-a*x + sqrt(a^2*x^2 + 1) - 1
) + sqrt(a^2*x^2 + 1)*(-14*I*a^2*x^2 + 5*a*x - I))/(a*x^3 + I*x^2)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = -i\left(\int \frac{i}{a^2x^5\sqrt{a^2x^2+1}+x^3\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax}{a^2x^5\sqrt{a^2x^2+1}+x^3\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^5\sqrt{a^2x^2+1}+x^3\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^5\sqrt{a^2x^2+1}+x^3\sqrt{a^2x^2+1}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] -I*(Integral(I/(a**2*x**5*sqrt(a**2*x**2 + 1) + x**3*sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x/(a**2*x**5*sqrt(a**2*x**2 + 1) + x**3*sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**3/(a**2*x**5*sqrt(a**2*x**2 + 1) + x**3*sqrt(a*
*2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**5*sqrt(a**2*x**2 + 1)
+ x**3*sqrt(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = -\frac{7i\,a^3x}{\sqrt{a^2x^2+1}} + \frac{9}{2}\,a^2\,\text{arsinh}\left(\frac{1}{a|x|}\right) - \frac{9\,a^2}{2\,\sqrt{a^2x^2+1}} - \frac{3i\,a}{\sqrt{a^2x^2+1}x} - \frac{1}{2\,\sqrt{a^2x^2+1}x^2}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -7*I*a^3*x/sqrt(a^2*x^2 + 1) + 9/2*a^2*arcsinh(1/(a*abs(x))) - 9/2*a^2/sqrt
(a^2*x^2 + 1) - 3*I*a/(sqrt(a^2*x^2 + 1)*x) - 1/2/(sqrt(a^2*x^2 + 1)*x^2)
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")

[Out] undef

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = -\frac{a^2\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,9\mathrm{i}}{2} - \frac{\sqrt{a^2\,x^2+1}}{2\,x^2} - \frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{x} - \frac{a^3\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

[In] int((a*x*1i + 1)^3/(x^3*(a^2*x^2 + 1)^(3/2)),x)

[Out] - (a^2*atan((a^2*x^2 + 1)^(1/2)*1i)*9i)/2 - (a^2*x^2 + 1)^(1/2)/(2*x^2) - (a*(a^2*x^2 + 1)^(1/2)*3i)/x - (a^3*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))

3.26 $\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{3ia\sqrt{1+a^2x^2}}{2x^2}+\frac{14a^2\sqrt{1+a^2x^2}}{3x}+\frac{4a^3\sqrt{1+a^2x^2}}{i+ax}+\frac{11}{2}ia^3\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] 11/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))-1/3*(a^2*x^2+1)^(1/2)/x^3-3/2*I*a*(a^
2*x^2+1)^(1/2)/x^2+14/3*a^2*(a^2*x^2+1)^(1/2)/x+4*a^3*(a^2*x^2+1)^(1/2)/(I+
a*x)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5168, 6874, 277, 270, 272, 44, 65, 214, 665}

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{14a^2\sqrt{a^2x^2+1}}{3x}-\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{11}{2}ia^3\text{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{4a^3\sqrt{a^2x^2+1}}{ax+i}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*Sqrt[1 + a^2*x^2]/x^3 - (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2*
Sqrt[1 + a^2*x^2])/(3*x) + (4*a^3*Sqrt[1 + a^2*x^2])/(I + a*x) + ((11*I)/2)
*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^2}{x^4(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^4\sqrt{1+a^2x^2}}+\frac{3ia}{x^3\sqrt{1+a^2x^2}}-\frac{4a^2}{x^2\sqrt{1+a^2x^2}}-\frac{4ia^3}{x\sqrt{1+a^2x^2}}+\frac{4ia^4}{(i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= (3ia)\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx-\left(4a^2\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx-\left(4ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&\quad +\left(4ia^4\right)\int \frac{1}{(i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{4a^2\sqrt{1+a^2x^2}}{x}+\frac{4a^3\sqrt{1+a^2x^2}}{i+ax} \\
&\quad +\frac{1}{2}(3ia)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&\quad -\frac{1}{3}\left(2a^2\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx-\left(2ia^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{3ia\sqrt{1+a^2x^2}}{2x^2}+\frac{14a^2\sqrt{1+a^2x^2}}{3x} \\
&\quad +\frac{4a^3\sqrt{1+a^2x^2}}{i+ax}-(4ia)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right) \\
&\quad -\frac{1}{4}\left(3ia^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{3ia\sqrt{1+a^2x^2}}{2x^2}+\frac{14a^2\sqrt{1+a^2x^2}}{3x}+\frac{4a^3\sqrt{1+a^2x^2}}{i+ax} \\
&\quad +4ia^3\text{arctanh}\left(\sqrt{1+a^2x^2}\right)-\frac{1}{2}(3ia)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{3ia\sqrt{1+a^2x^2}}{2x^2}+\frac{14a^2\sqrt{1+a^2x^2}}{3x} \\
&\quad +\frac{4a^3\sqrt{1+a^2x^2}}{i+ax}+\frac{11}{2}ia^3\text{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{1}{6}\left(\frac{\sqrt{1+a^2x^2}(-2i+7ax+19ia^2x^2+52a^3x^3)}{x^3(i+ax)} - 33ia^3\log(x) + 33ia^3\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-2*I + 7*a*x + (19*I)*a^2*x^2 + 52*a^3*x^3))/(x^3*(I +
 a*x)) - (33*I)*a^3*Log[x] + (33*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99

method	result
risch	$\frac{28a^4x^4-9ia^3x^3+26a^2x^2-9iax-2}{6x^3\sqrt{a^2x^2+1}}-\frac{ia^3\left(-11\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{8i\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a\left(x+\frac{i}{a}\right)}\right)}{2}$
default	$-\frac{1}{3x^3\sqrt{a^2x^2+1}}-\frac{13a^2\left(-\frac{1}{x\sqrt{a^2x^2+1}}-\frac{2a^2x}{\sqrt{a^2x^2+1}}\right)}{3}-ia^3\left(\frac{1}{\sqrt{a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)+3ia\left(-\frac{1}{2x^2\sqrt{a^2x^2+1}}\right.$
meijerg	$-\frac{-8a^4x^4-4a^2x^2+1}{3x^3\sqrt{a^2x^2+1}}+\frac{3ia^3\left(\frac{\sqrt{\pi}\left(20a^2x^2+8\right)}{16a^2x^2}-\frac{\sqrt{\pi}\left(24a^2x^2+8\right)}{16a^2x^2\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)}{2}-\frac{3\left(\frac{5}{3}-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{4}-\frac{\sqrt{\pi}}{2x^2a^2}\right)}{\sqrt{\pi}}$

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(28*a^4*x^4-9*I*a^3*x^3+26*a^2*x^2-9*I*a*x-2)/x^3/(a^2*x^2+1)^(1/2)-1/2
*I*a^3*(-11*arctanh(1/(a^2*x^2+1)^(1/2))+8*I/a/(x+I/a)*((x+I/a)^2*a^2-2*I*a
*(x+I/a))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.19

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{52\,a^4x^4 + 52i\,a^3x^3 - 33\left(-i\,a^4x^4 + a^3x^3\right)\log\left(-ax + \sqrt{a^2x^2+1}+1\right) - 33\left(i\,a^4x^4 - a^3x^3\right)\log\left(-ax + \sqrt{a}\right.}{6\left(ax^4 + i\,x^3\right)}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(52*a^4*x^4 + 52*I*a^3*x^3 - 33*(-I*a^4*x^4 + a^3*x^3)*log(-a*x + sqrt(
a^2*x^2 + 1) + 1) - 33*(I*a^4*x^4 - a^3*x^3)*log(-a*x + sqrt(a^2*x^2 + 1) -
 1) + (52*a^3*x^3 + 19*I*a^2*x^2 + 7*a*x - 2*I)*sqrt(a^2*x^2 + 1))/(a*x^4 +
 I*x^3)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = -i\left(\int \frac{i}{a^2x^6\sqrt{a^2x^2+1} + x^4\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax}{a^2x^6\sqrt{a^2x^2+1} + x^4\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^6\sqrt{a^2x^2+1} + x^4\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^6\sqrt{a^2x^2+1} + x^4\sqrt{a^2x^2+1}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] -I*(Integral(I/(a**2*x**6*sqrt(a**2*x**2 + 1) + x**4*sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x/(a**2*x**6*sqrt(a**2*x**2 + 1) + x**4*sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**3/(a**2*x**6*sqrt(a**2*x**2 + 1) + x**4*sqrt(a*
*2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**6*sqrt(a**2*x**2 + 1)
+ x**4*sqrt(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{26\,a^4x}{3\,\sqrt{a^2x^2+1}} + \frac{11}{2}i\,a^3\operatorname{arsinh}\left(\frac{1}{a|x|}\right) - \frac{11i\,a^3}{2\,\sqrt{a^2x^2+1}} + \frac{13\,a^2}{3\,\sqrt{a^2x^2+1}x} - \frac{3i\,a}{2\,\sqrt{a^2x^2+1}x^2} - \frac{1}{3\,\sqrt{a^2x^2+1}x^3}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 26/3*a^4*x/sqrt(a^2*x^2 + 1) + 11/2*I*a^3*arcsinh(1/(a*abs(x))) - 11/2*I*a^
3/sqrt(a^2*x^2 + 1) + 13/3*a^2/(sqrt(a^2*x^2 + 1)*x) - 3/2*I*a/(sqrt(a^2*x^
2 + 1)*x^2) - 1/3/(sqrt(a^2*x^2 + 1)*x^3)
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{11\,a^3\operatorname{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2} - \frac{\sqrt{a^2\,x^2+1}}{3\,x^3} - \frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{2\,x^2} + \frac{14\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x} + \frac{4\,a^4\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a*x*1i + 1)^3/(x^4*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] (11*a^3*atan((a^2*x^2 + 1)^(1/2)*1i))/2 - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (a*
(a^2*x^2 + 1)^(1/2)*3i)/(2*x^2) + (14*a^2*(a^2*x^2 + 1)^(1/2))/(3*x) + (4*a
^4*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.27 $\int e^{4i \arctan(ax)} x^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int e^{4i \arctan(ax)} x^3 \, dx = \frac{12ix}{a^3} - \frac{4x^2}{a^2} - \frac{4ix^3}{3a} + \frac{x^4}{4} + \frac{4i}{a^4(i+ax)} + \frac{16\log(i+ax)}{a^4}$$

```
[Out] 12*I*x/a^3-4*x^2/a^2-4/3*I*x^3/a+1/4*x^4+4*I/a^4/(I+a*x)+16*ln(I+a*x)/a^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5170, 90}

$$\int e^{4i \arctan(ax)} x^3 \, dx = \frac{4i}{a^4(ax+i)} + \frac{16\log(ax+i)}{a^4} + \frac{12ix}{a^3} - \frac{4x^2}{a^2} - \frac{4ix^3}{3a} + \frac{x^4}{4}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((12*I)*x)/a^3 - (4*x^2)/a^2 - (((4*I)/3)*x^3)/a + x^4/4 + (4*I)/(a^4*(I +
a*x)) + (16*Log[I + a*x])/a^4
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3(1+iax)^2}{(1-iax)^2}\,dx\\ &= \int \left(\frac{12i}{a^3}-\frac{8x}{a^2}-\frac{4ix^2}{a}+x^3-\frac{4i}{a^3(i+ax)^2}+\frac{16}{a^3(i+ax)}\right)dx\\ &= \frac{12ix}{a^3}-\frac{4x^2}{a^2}-\frac{4ix^3}{3a}+\frac{x^4}{4}+\frac{4i}{a^4(i+ax)}+\frac{16\log(i+ax)}{a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int e^{4i\arctan(ax)}x^3\,dx = \frac{12ix}{a^3}-\frac{4x^2}{a^2}-\frac{4ix^3}{3a}+\frac{x^4}{4}+\frac{4i}{a^4(i+ax)}+\frac{16\log(i+ax)}{a^4}$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x^3,x]
[Out] ((12*I)*x)/a^3 - (4*x^2)/a^2 - (((4*I)/3)*x^3)/a + x^4/4 + (4*I)/(a^4*(I +
a*x)) + (16*Log[I + a*x])/a^4
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

method	result
default	$-\frac{-\frac{1}{4}a^3x^4+\frac{4}{3}ia^2x^3+4a\,x^2-12ix}{a^3}-\frac{4\left(-\frac{i}{a(ax+i)}-\frac{4\ln(ax+i)}{a}\right)}{a^3}$
risch	$\frac{x^4}{4}-\frac{4ix^3}{3a}-\frac{4x^2}{a^2}+\frac{12ix}{a^3}+\frac{4i}{a^4(ax+i)}+\frac{8\ln(a^2x^2+1)}{a^4}-\frac{16i\arctan(ax)}{a^4}$
parallelrisch	$-\frac{-3x^6a^6+16ix^5a^5+45a^4x^4-128ia^3x^3-192a^2\ln(ax+i)x^2+96a^2x^2-192iax-192\ln(ax+i)}{12a^4(a^2x^2+1)}$
meijerg	$\frac{-\frac{a^2x^2}{a^2x^2+1}+\ln(a^2x^2+1)}{2a^4}+\frac{2i\left(\frac{x(a^2)^{\frac{5}{2}}(10a^2x^2+15)}{5a^4(a^2x^2+1)}-\frac{3(a^2)^{\frac{5}{2}}\arctan(ax)}{a^5}\right)}{a^3\sqrt{a^2}}-\frac{3\left(\frac{x^2a^2(3a^2x^2+6)}{3a^2x^2+3}-2\ln(a^2x^2+1)\right)}{a^4}-\frac{2i\left(-\ldots\right.}{\ldots}$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x,method=_RETURNVERBOSE)
[Out] -1/a^3*(-1/4*a^3*x^4+4/3*I*a^2*x^3+4*a*x^2-12*I*x)-4/a^3*(-I/a/(I+a*x)-4/a*
ln(I+a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int e^{4i\arctan(ax)}x^3\,dx = \frac{3\,a^5x^5 - 13i\,a^4x^4 - 32\,a^3x^3 + 96i\,a^2x^2 - 144\,ax + 192\,(ax+i)\log\left(\frac{ax+i}{a}\right) + 48i}{12\,(a^5x + i\,a^4)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^5*x^5 - 13*I*a^4*x^4 - 32*a^3*x^3 + 96*I*a^2*x^2 - 144*a*x + 192*
(a*x + I)*log((a*x + I)/a) + 48*I)/(a^5*x + I*a^4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int e^{4i\arctan(ax)}x^3\,dx = \frac{x^4}{4} + \frac{4i}{a^5x + ia^4} - \frac{4ix^3}{3a} - \frac{4x^2}{a^2} + \frac{12ix}{a^3} + \frac{16\log(ax+i)}{a^4}$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**3,x)
```

```
[Out] x**4/4 + 4*I/(a**5*x + I*a**4) - 4*I*x**3/(3*a) - 4*x**2/a**2 + 12*I*x/a**3
 + 16*log(a*x + I)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int e^{4i\arctan(ax)}x^3\,dx = -\frac{4\,(-i\,ax-1)}{a^6x^2+a^4} + \frac{3\,a^3x^4 - 16i\,a^2x^3 - 48\,ax^2 + 144i\,x}{12\,a^3} - \frac{16i\,\arctan(ax)}{a^4} + \frac{8\,\log(a^2x^2+1)}{a^4}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -4*(-I*a*x - 1)/(a^6*x^2 + a^4) + 1/12*(3*a^3*x^4 - 16*I*a^2*x^3 - 48*a*x^2
 + 144*I*x)/a^3 - 16*I*arctan(a*x)/a^4 + 8*log(a^2*x^2 + 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{4i\arctan(ax)}x^3\,dx = \frac{16\log(ax+i)}{a^4} + \frac{4i}{(ax+i)a^4} + \frac{3a^8x^4 - 16i\,a^7x^3 - 48\,a^6x^2 + 144i\,a^5x}{12\,a^8}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="giac")
```

```
[Out] 16*log(a*x + I)/a^4 + 4*I/((a*x + I)*a^4) + 1/12*(3*a^8*x^4 - 16*I*a^7*x^3
- 48*a^6*x^2 + 144*I*a^5*x)/a^8
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{4i\arctan(ax)}x^3\,dx = \frac{16\ln\left(x+\frac{1i}{a}\right)}{a^4} + \frac{x^4}{4} - \frac{4\,x^2}{a^2} + \frac{4i}{a^5\,\left(x+\frac{1i}{a}\right)} + \frac{x\,12i}{a^3} - \frac{x^3\,4i}{3\,a}$$

```
[In] int((x^3*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

```
[Out] 4i/(a^5*(x + 1i/a)) + (16*log(x + 1i/a))/a^4 + (x*12i)/a^3 + x^4/4 - (x^3*4
i)/(3*a) - (4*x^2)/a^2
```

3.28 $\int e^{4i \arctan(ax)} x^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int e^{4i \arctan(ax)} x^2 \, dx = -\frac{8x}{a^2} - \frac{2ix^2}{a} + \frac{x^3}{3} - \frac{4}{a^3(i+ax)} + \frac{12i \log(i+ax)}{a^3}$$

```
[Out] -8*x/a^2-2*I*x^2/a+1/3*x^3-4/a^3/(I+a*x)+12*I*ln(I+a*x)/a^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 90}

$$\int e^{4i \arctan(ax)} x^2 \, dx = -\frac{4}{a^3(ax+i)} + \frac{12i \log(ax+i)}{a^3} - \frac{8x}{a^2} - \frac{2ix^2}{a} + \frac{x^3}{3}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])*x^2,x]
```

```
[Out] (-8*x)/a^2 - ((2*I)*x^2)/a + x^3/3 - 4/(a^3*(I + a*x)) + ((12*I)*Log[I + a*
x])/a^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2(1+iax)^2}{(1-iax)^2}\,dx\\ &= \int \left(-\frac{8}{a^2}-\frac{4ix}{a}+x^2+\frac{4}{a^2(i+ax)^2}+\frac{12i}{a^2(i+ax)}\right)dx\\ &= -\frac{8x}{a^2}-\frac{2ix^2}{a}+\frac{x^3}{3}-\frac{4}{a^3(i+ax)}+\frac{12i\log(i+ax)}{a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int e^{4i\arctan(ax)}x^2\,dx = -\frac{8x}{a^2}-\frac{2ix^2}{a}+\frac{x^3}{3}-\frac{4}{a^3(i+ax)}+\frac{12i\log(i+ax)}{a^3}$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x^2,x]
[Out] (-8*x)/a^2 - ((2*I)*x^2)/a + x^3/3 - 4/(a^3*(I + a*x)) + ((12*I)*Log[I + a*
x])/a^3
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

method	result
default	$-\frac{8x-\frac{1}{3}a^2x^3+2iax^2}{a^2}+\frac{-\frac{4}{a(ax+i)}+\frac{12i\ln(ax+i)}{a}}{a^2}$
risch	$-\frac{8x}{a^2}+\frac{x^3}{3}-\frac{2ix^2}{a}-\frac{4}{a^3(ax+i)}+\frac{6i\ln\left(a^2x^2+1\right)}{a^3}+\frac{12\arctan(ax)}{a^3}$
parallelrisch	$\frac{a^5x^5-6ia^4x^4+36i\ln(ax+i)x^2a^2-23a^3x^3-18ia^2x^2+36i\ln(ax+i)-36ax}{3a^3(a^2x^2+1)}$
meijerg	$\frac{-\frac{x\left(a^2\right)^{\frac{3}{2}}}{a^2\left(a^2x^2+1\right)}+\frac{\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{a^3}}{2a^2\sqrt{a^2}}+\frac{2i\left(-\frac{a^2x^2}{a^2x^2+1}+\ln\left(a^2x^2+1\right)\right)}{a^3}-\frac{3\left(\frac{x\left(a^2\right)^{\frac{5}{2}}\left(10a^2x^2+15\right)}{5a^4\left(a^2x^2+1\right)}-\frac{3\left(a^2\right)^{\frac{5}{2}}\arctan(ax)}{a^5}\right)}{a^2\sqrt{a^2}}-$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x,method=_RETURNVERBOSE)
[Out] -1/a^2*(8*x-1/3*a^2*x^3+2*I*a*x^2)+4/a^2*(-1/a/(I+a*x)+3*I*ln(I+a*x)/a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int e^{4i\arctan(ax)}x^2\,dx = \frac{a^4x^4 - 5i\,a^3x^3 - 18\,a^2x^2 - 24i\,ax - 36\,(-i\,ax + 1)\log\left(\frac{ax+i}{a}\right) - 12}{3\,(a^4x + i\,a^3)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*x^4 - 5*I*a^3*x^3 - 18*a^2*x^2 - 24*I*a*x - 36*(-I*a*x + 1)*log((a
*x + I)/a) - 12)/(a^4*x + I*a^3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int e^{4i\arctan(ax)}x^2\,dx = \frac{x^3}{3} - \frac{4}{a^4x + ia^3} - \frac{2ix^2}{a} - \frac{8x}{a^2} + \frac{12i\log(ax + i)}{a^3}$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**2,x)
```

```
[Out] x**3/3 - 4/(a**4*x + I*a**3) - 2*I*x**2/a - 8*x/a**2 + 12*I*log(a*x + I)/a*
*3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int e^{4i\arctan(ax)}x^2\,dx = -\frac{4\,(ax - i)}{a^5x^2 + a^3} + \frac{a^2x^3 - 6i\,ax^2 - 24\,x}{3\,a^2} + \frac{12\,\arctan(ax)}{a^3} + \frac{6i\,\log(a^2x^2 + 1)}{a^3}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="maxima")
```

```
[Out] -4*(a*x - I)/(a^5*x^2 + a^3) + 1/3*(a^2*x^3 - 6*I*a*x^2 - 24*x)/a^2 + 12*ar
ctan(a*x)/a^3 + 6*I*log(a^2*x^2 + 1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int e^{4i\arctan(ax)}x^2\,dx = \frac{12i\,\log(ax+i)}{a^3} - \frac{4}{(ax+i)a^3} + \frac{a^6x^3 - 6i\,a^5x^2 - 24\,a^4x}{3\,a^6}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="giac")
```

```
[Out] 12*I*log(a*x + I)/a^3 - 4/((a*x + I)*a^3) + 1/3*(a^6*x^3 - 6*I*a^5*x^2 - 24
*a^4*x)/a^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int e^{4i\arctan(ax)}x^2\,dx = \frac{x^3}{3} + \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,12\mathrm{i}}{a^3} - \frac{8\,x}{a^2} - \frac{4}{a^4\,\left(x+\frac{1\mathrm{i}}{a}\right)} - \frac{x^2\,2\mathrm{i}}{a}$$

```
[In] int((x^2*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

```
[Out] (log(x + 1i/a)*12i)/a^3 - 4/(a^4*(x + 1i/a)) - (8*x)/a^2 + x^3/3 - (x^2*2i)
/a
```

3.29 $\int e^{4i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int e^{4i\arctan(ax)}x\,dx=-\frac{4ix}{a}+\frac{x^2}{2}-\frac{4i}{a^2(i+ax)}-\frac{8\log(i+ax)}{a^2}$$

```
[Out] -4*I*x/a+1/2*x^2-4*I/a^2/(I+a*x)-8*ln(I+a*x)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5170, 78}

$$\int e^{4i\arctan(ax)}x\,dx=-\frac{4i}{a^2(ax+i)}-\frac{8\log(ax+i)}{a^2}-\frac{4ix}{a}+\frac{x^2}{2}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])*x,x]
```

```
[Out] ((-4*I)*x)/a + x^2/2 - (4*I)/(a^2*(I + a*x)) - (8*Log[I + a*x])/a^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(1+iax)^2}{(1-iax)^2}\,dx \\
&= \int \left(-\frac{4i}{a}+x+\frac{4i}{a(i+ax)^2}-\frac{8}{a(i+ax)}\right)dx \\
&= -\frac{4ix}{a}+\frac{x^2}{2}-\frac{4i}{a^2(i+ax)}-\frac{8\log(i+ax)}{a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int e^{4i\arctan(ax)}x\,dx = -\frac{4ix}{a}+\frac{x^2}{2}-\frac{4i}{a^2(i+ax)}-\frac{8\log(i+ax)}{a^2}$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x,x]
[Out] ((-4*I)*x)/a + x^2/2 - (4*I)/(a^2*(I + a*x)) - (8*Log[I + a*x])/a^2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

method	result
default	$-\frac{-\frac{1}{2}a\,x^2+4ix}{a}+\frac{-\frac{4i}{a(ax+i)}-\frac{8\ln(ax+i)}{a}}{a}$
risch	$\frac{x^2}{2}-\frac{4ix}{a}-\frac{4i}{a^2(ax+i)}-\frac{4\ln(a^2x^2+1)}{a^2}+\frac{8i\arctan(ax)}{a^2}$
parallelrisch	$-\frac{-a^4x^4+8ia^3x^3+16a^2\ln(ax+i)x^2-9a^2x^2+16iax+16\ln(ax+i)}{2a^2(a^2x^2+1)}$
meijerg	$\frac{x^2}{2a^2x^2+2}+\frac{2i\left(-\frac{x(a^2)^{\frac{3}{2}}}{a^2(a^2x^2+1)}+\frac{(a^2)^{\frac{3}{2}}\arctan(ax)}{a^3}\right)}{a\sqrt{a^2}}-\frac{3\left(-\frac{a^2x^2}{a^2x^2+1}+\ln(a^2x^2+1)\right)}{a^2}-\frac{2i\left(\frac{x(a^2)^{\frac{5}{2}}(10a^2x^2+15)}{5a^4(a^2x^2+1)}-\frac{3(a^2)^{\frac{5}{2}}\text{arct}}{a^5}\right.}{a\sqrt{a^2}}$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x,x,method=_RETURNVERBOSE)
[Out] -1/a*(-1/2*a*x^2+4*I*x)+4/a*(-I/a/(I+a*x)-2/a*ln(I+a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int e^{4i\arctan(ax)}x\,dx = \frac{a^3x^3 - 7i\,a^2x^2 + 8\,ax - 16\,(ax+i)\log\left(\frac{ax+i}{a}\right) - 8i}{2\,(a^3x + i\,a^2)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="fricas")
[Out] 1/2*(a^3*x^3 - 7*I*a^2*x^2 + 8*a*x - 16*(a*x + I)*log((a*x + I)/a) - 8*I)/(
a^3*x + I*a^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int e^{4i\arctan(ax)}x\,dx = \frac{x^2}{2} - \frac{4i}{a^3x + ia^2} - \frac{4ix}{a} - \frac{8\log(ax+i)}{a^2}$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x,x)
[Out] x**2/2 - 4*I/(a**3*x + I*a**2) - 4*I*x/a - 8*log(a*x + I)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int e^{4i\arctan(ax)}x\,dx = -\frac{4\,(i\,ax+1)}{a^4x^2 + a^2} + \frac{ax^2 - 8i\,x}{2\,a} + \frac{8i\,\arctan(ax)}{a^2} - \frac{4\,\log(a^2x^2+1)}{a^2}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="maxima")
[Out] -4*(I*a*x + 1)/(a^4*x^2 + a^2) + 1/2*(a*x^2 - 8*I*x)/a + 8*I*arctan(a*x)/a^
2 - 4*log(a^2*x^2 + 1)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{4i\arctan(ax)}x\,dx = -\frac{8\,\log\left(ax+i\right)}{a^2} + \frac{a^4x^2 - 8i\,a^3x}{2\,a^4} - \frac{4i}{(ax+i)a^2}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="giac")
[Out] -8*log(a*x + I)/a^2 + 1/2*(a^4*x^2 - 8*I*a^3*x)/a^4 - 4*I/((a*x + I)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{4i\arctan(ax)}x\,dx = \frac{x^2}{2} - \frac{8\,\ln\left(x+\frac{1\mathrm{i}}{a}\right)}{a^2} - \frac{4\mathrm{i}}{a^3\,\left(x+\frac{1\mathrm{i}}{a}\right)} - \frac{x\,4\mathrm{i}}{a}$$

```
[In] int((x*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
[Out] x^2/2 - (8*log(x + 1i/a))/a^2 - (x*4i)/a - 4i/(a^3*(x + 1i/a))
```

3.30 $\int e^{4i \arctan(ax)} dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int e^{4i \arctan(ax)} dx = x + \frac{4}{a(i+ax)} - \frac{4i \log(i+ax)}{a}$$

```
[Out] x+4/a/(I+a*x)-4*I*ln(I+a*x)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5169, 45}

$$\int e^{4i \arctan(ax)} dx = \frac{4}{a(ax+i)} - \frac{4i \log(ax+i)}{a} + x$$

```
[In] Int[E^((4*I)*ArcTan[a*x]),x]
```

```
[Out] x + 4/(a*(I + a*x)) - ((4*I)*Log[I + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^2}{(1-iax)^2}\,dx \\
&= \int \left(1-\frac{4}{(i+ax)^2}-\frac{4i}{i+ax}\right)dx \\
&= x+\frac{4}{a(i+ax)}-\frac{4i\log(i+ax)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int e^{4i\arctan(ax)}\,dx = x+\frac{4}{a(i+ax)}-\frac{4\arctan(ax)}{a}-\frac{2i\log\left(1+a^2x^2\right)}{a}$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x]),x]
[Out] x + 4/(a*(I + a*x)) - (4*ArcTan[a*x])/a - ((2*I)*Log[1 + a^2*x^2])/a
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result
default	$x-4a\left(-\frac{1}{a^2(ax+i)}+\frac{i\ln(ax+i)}{a^2}\right)$
risch	$x+\frac{4}{a(ax+i)}-\frac{2i\ln\left(a^2x^2+1\right)}{a}-\frac{4\arctan(ax)}{a}$
parallelrisch	$-\frac{4i\ln(ax+i)x^2a^2-a^3x^3-4ia^2x^2+4i\ln(ax+i)-5ax}{(a^2x^2+1)a}$
meijerg	$\frac{\frac{2x\sqrt{a^2}}{2a^2x^2+2}+\frac{\sqrt{a^2}\arctan(ax)}{a}}{2\sqrt{a^2}}+\frac{2iax^2}{a^2x^2+1}-\frac{3\left(-\frac{x\left(a^2\right)^{\frac{3}{2}}}{a^2\left(a^2x^2+1\right)}+\frac{\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{a^3}\right)}{\sqrt{a^2}}-\frac{2i\left(-\frac{a^2x^2}{a^2x^2+1}+\ln\left(a^2x^2+1\right)\right)}{a}+\frac{x\left(a^2\right)^{\frac{5}{2}}(}{5a^4(}$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] x-4*a*(-1/a^2/(I+a*x)+I/a^2*ln(I+a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int e^{4i\arctan(ax)}\,dx = \frac{a^2x^2 + i\,ax - 4\,(i\,ax - 1)\log\left(\frac{ax+i}{a}\right) + 4}{a^2x + i\,a}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="fricas")
[Out] (a^2*x^2 + I*a*x - 4*(I*a*x - 1)*log((a*x + I)/a) + 4)/(a^2*x + I*a)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int e^{4i\arctan(ax)}\,dx = x + \frac{4}{a^2x + ia} - \frac{4i\log(ax + i)}{a}$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2,x)
[Out] x + 4/(a**2*x + I*a) - 4*I*log(a*x + I)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int e^{4i\arctan(ax)}\,dx = x + \frac{4\,(ax - i)}{a^3x^2 + a} - \frac{4\,\arctan(ax)}{a} - \frac{2i\,\log(a^2x^2 + 1)}{a}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="maxima")
[Out] x + 4*(a*x - I)/(a^3*x^2 + a) - 4*arctan(a*x)/a - 2*I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int e^{4i\arctan(ax)}\,dx = x - \frac{4i\,\log(ax + i)}{a} + \frac{4}{(ax + i)a}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="giac")
[Out] x - 4*I*log(a*x + I)/a + 4/((a*x + I)*a)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int e^{4i\arctan(ax)}\,dx = x + \frac{4}{a^2\,\left(x + \frac{1\mathrm{i}}{a}\right)} - \frac{\ln\left(x + \frac{1\mathrm{i}}{a}\right)\,4\mathrm{i}}{a}$$

```
[In] int((a*x*1i + 1)^4/(a^2*x^2 + 1)^2,x)
[Out] x + 4/(a^2*(x + 1i/a)) - (log(x + 1i/a)*4i)/a
```

3.31 $\int \frac{e^{4i\arctan(ax)}}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{e^{4i\arctan(ax)}}{x} dx = \frac{4i}{i+ax} + \log(x)$$

```
[Out] 4*I/(I+a*x)+ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5170, 90}

$$\int \frac{e^{4i\arctan(ax)}}{x} dx = \log(x) + \frac{4i}{ax+i}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])/x,x]
```

```
[Out] (4*I)/(I + a*x) + Log[x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\frac{(1+iax)^2}{x(1-iax)^2}\,dx\\&=\int\left(\frac{1}{x}-\frac{4ia}{(i+ax)^2}\right)dx\\&=\frac{4i}{i+ax}+\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int\frac{e^{4i\arctan(ax)}}{x}\,dx=\frac{4i}{i+ax}+\log(x)$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x,x]
[Out] (4*I)/(I + a*x) + Log[x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{4i}{ax+i}+\ln(x)$
risch	$\frac{4i}{ax+i}+\ln(-x)$
norman	$\frac{-4a^2x^2+4iax}{a^2x^2+1}+\ln(x)$
parallelrisch	$\frac{a^2\ln(x)x^2-4a^2x^2+4iax+\ln(x)}{a^2x^2+1}$
meijerg	$-\frac{a^2x^2}{2a^2x^2+2}+\frac{1}{2}+\ln(x)+\frac{\ln(a^2)}{2}+\frac{2ia\left(\frac{2x\sqrt{a^2}}{2a^2x^2+2}+\frac{\sqrt{a^2}\arctan(ax)}{a}\right)}{\sqrt{a^2}}-\frac{7a^2x^2}{2(a^2x^2+1)}-\frac{2ia\left(-\frac{x(a^2)^{\frac{3}{2}}}{a^2(a^2x^2+1)}+\frac{(a^2)^{\frac{3}{2}}\text{a}}{a}\right.}{\sqrt{a^2}}$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x,x,method=_RETURNVERBOSE)
[Out] 4*I/(I+a*x)+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \frac{(ax+i)\log(x)+4i}{ax+i}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="fricas")
[Out] ((a*x + I)*log(x) + 4*I)/(a*x + I)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \log(x) + \frac{4i}{ax+i}$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x,x)
[Out] log(x) + 4*I/(a*x + I)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = -\frac{4(-i\,ax-1)}{a^2x^2+1} + \log(x)$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="maxima")
[Out] -4*(-I*a*x - 1)/(a^2*x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \frac{4i}{ax+i} + \log(|x|)$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="giac")
[Out] 4*I/(a*x + I) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \ln(x) + \frac{4\mathrm{i}}{a\,x + 1\mathrm{i}}$$

```
[In] int((a*x*1i + 1)^4/(x*(a^2*x^2 + 1)^2),x)
[Out] log(x) + 4i/(a*x + 1i)
```

3.32 $\int \frac{e^{4 i \arctan(a x)}}{x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{e^{4 i \arctan(a x)}}{x^2} \, dx=-\frac{1}{x}-\frac{4 a}{i+a x}+4 i a \log(x)-4 i a \log(i+a x)$$

```
[Out] -1/x-4*a/(I+a*x)+4*I*a*ln(x)-4*I*a*ln(I+a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 90}

$$\int \frac{e^{4 i \arctan(a x)}}{x^2} \, dx=-\frac{4 a}{a x+i}+4 i a \log(x)-4 i a \log(a x+i)-\frac{1}{x}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])/x^2,x]
```

```
[Out] -x^(-1) - (4*a)/(I + a*x) + (4*I)*a*Log[x] - (4*I)*a*Log[I + a*x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

rQ[(I*n - 1)/2]

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(1+iax)^2}{x^2(1-iax)^2}\,dx \\ &= \int \left(\frac{1}{x^2}+\frac{4ia}{x}+\frac{4a^2}{(i+ax)^2}-\frac{4ia^2}{i+ax}\right)dx \\ &= -\frac{1}{x}-\frac{4a}{i+ax}+4ia\log(x)-4ia\log(i+ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -\frac{1}{x}-\frac{4a}{i+ax}+4ia\log(x)-4ia\log(i+ax)$$

[In] Integrate[E^((4*I)*ArcTan[a*x])/x^2,x]

[Out] -x^(-1) - (4*a)/(I + a*x) + (4*I)*a*Log[x] - (4*I)*a*Log[I + a*x]

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

method	result
default	$-\frac{1}{x}+4ia\ln(x)-4a^2\left(\frac{1}{a(ax+i)}+\frac{i\ln(ax+i)}{a}\right)$
risch	$\frac{-5ax-i}{(ax+i)x}+4ia\ln(x)-4a\arctan(ax)-2ia\ln(a^2x^2+1)$
parallelrisch	$\frac{4ia^3\ln(x)x^3-4ia^3\ln(ax+i)x^3-4ia^3x^3-1+4ia\ln(x)x-4ia\ln(ax+i)x-5a^2x^2}{(a^2x^2+1)x}$
meijerg	$\frac{a^2\left(-\frac{2(3a^2x^2+2)}{x\sqrt{a^2}(2a^2x^2+2)}-\frac{3a\arctan(ax)}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}+2ia\left(-\frac{2a^2x^2}{2a^2x^2+2}-\ln(a^2x^2+1)+1+2\ln(x)+\ln(a^2)\right)-\frac{3a^2(}{}$

[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x,method=_RETURNVERBOSE)

[Out] -1/x+4*I*a*ln(x)-4*a^2*(1/a/(I+a*x)+I*ln(I+a*x)/a)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -\frac{5\,ax + 4\,(-i\,a^2x^2 + ax)\log{(x)} + 4\,(i\,a^2x^2 - ax)\log{\left(\frac{ax+i}{a}\right)} + i}{ax^2 + i\,x}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a*x + 4*(-I*a^2*x^2 + a*x)*log(x) + 4*(I*a^2*x^2 - a*x)*log((a*x + I)/a
) + I)/(a*x^2 + I*x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = 4a\big(i\log{\left(8a^2x\right)} - i\log{\left(8a^2x + 8ia\right)}\big) + \frac{-5ax - i}{ax^2 + ix}$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**2,x)
```

```
[Out] 4*a*(I*log(8*a**2*x) - I*log(8*a**2*x + 8*I*a)) + (-5*a*x - I)/(a*x**2 + I*
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -4\,a\arctan{(ax)} - 2i\,a\log{\left(a^2x^2 + 1\right)} + 4i\,a\log{(x)} - \frac{5\,a^2x^2 - 4i\,ax + 1}{a^2x^3 + x}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4*a*arctan(a*x) - 2*I*a*log(a^2*x^2 + 1) + 4*I*a*log(x) - (5*a^2*x^2 - 4*I
*a*x + 1)/(a^2*x^3 + x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -4i\,a\log\left(ax+i\right)+4i\,a\log\left(|x|\right)-\frac{5\,ax+i}{ax^2+i\,x}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="giac")
[Out] -4*I*a*log(a*x + I) + 4*I*a*log(abs(x)) - (5*a*x + I)/(a*x^2 + I*x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -8\,a\,\mathrm{atan}(2\,a\,x+1\mathrm{i})-\frac{5\,x+\frac{1\mathrm{i}}{a}}{x^2+\frac{x\,1\mathrm{i}}{a}}$$

```
[In] int((a*x*1i + 1)^4/(x^2*(a^2*x^2 + 1)^2),x)
[Out] - 8*a*atan(2*a*x + 1i) - (5*x + 1i/a)/((x*1i)/a + x^2)
```

3.33 $\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2} - \frac{4ia}{x} - \frac{4ia^2}{i+ax} - 8a^2\log(x) + 8a^2\log(i+ax)$$

```
[Out] -1/2/x^2-4*I*a/x-4*I*a^2/(I+a*x)-8*a^2*ln(x)+8*a^2*ln(I+a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5170, 90}

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = -\frac{4ia^2}{ax+i} - 8a^2\log(x) + 8a^2\log(ax+i) - \frac{4ia}{x} - \frac{1}{2x^2}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*1/x^2 - ((4*I)*a)/x - ((4*I)*a^2)/(I + a*x) - 8*a^2*Log[x] + 8*a^2*Log
[I + a*x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(1+iax)^2}{x^3(1-iax)^2}\,dx\\ &= \int \left(\frac{1}{x^3}+\frac{4ia}{x^2}-\frac{8a^2}{x}+\frac{4ia^3}{(i+ax)^2}+\frac{8a^3}{i+ax}\right)dx\\ &= -\frac{1}{2x^2}-\frac{4ia}{x}-\frac{4ia^2}{i+ax}-8a^2\log(x)+8a^2\log(i+ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2}-\frac{4ia}{x}-\frac{4ia^2}{i+ax}-8a^2\log(x)+8a^2\log(i+ax)$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x^3,x]
[Out] -1/2*1/x^2 - ((4*I)*a)/x - ((4*I)*a^2)/(I + a*x) - 8*a^2*Log[x] + 8*a^2*Log
[I + a*x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

method	result
default	$-\frac{1}{2x^2}-\frac{4ia}{x}-8a^2\ln(x)-4a^3\left(\frac{i}{a(ax+i)}-\frac{2\ln(ax+i)}{a}\right)$
risch	$\frac{-8ia^2x^2+\frac{7}{2}ax-\frac{1}{2}i}{(ax+i)x^2}-8ia^2\arctan(ax)+4a^2\ln(a^2x^2+1)-8a^2\ln(x)$
parallelrisch	$-\frac{16\ln(x)x^4a^4-16\ln(ax+i)x^4a^4-9a^4x^4+16ia^3x^3+1+16a^2\ln(x)x^2-16a^2\ln(ax+i)x^2+8iax}{2(a^2x^2+1)x^2}$
meijerg	$\frac{a^2\left(\frac{3a^2x^2}{3a^2x^2+3}+2\ln(a^2x^2+1)-1-4\ln(x)-2\ln(a^2)-\frac{1}{a^2x^2}\right)}{2}+\frac{2ia^3\left(-\frac{2(3a^2x^2+2)}{x\sqrt{a^2}(2a^2x^2+2)}-\frac{3a\arctan(ax)}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-3a^2\left(-\frac{2a^2x^2}{2a^2x^2+2}\right.$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-4*I*a/x-8*a^2*ln(x)-4*a^3*(I/a/(I+a*x)-2/a*ln(I+a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = \frac{-16i\,a^2x^2 + 7\,ax - 16\,(a^3x^3 + i\,a^2x^2)\log(x) + 16\,(a^3x^3 + i\,a^2x^2)\log\left(\frac{ax+i}{a}\right) - i}{2\,(ax^3 + i\,x^2)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(-16*I*a^2*x^2 + 7*a*x - 16*(a^3*x^3 + I*a^2*x^2)*log(x) + 16*(a^3*x^3
+ I*a^2*x^2)*log((a*x + I)/a) - I)/(a*x^3 + I*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = 8a^2\left(-\log\left(16a^3x\right) + \log\left(16a^3x + 16ia^2\right)\right) + \frac{-16ia^2x^2 + 7ax - i}{2ax^3 + 2ix^2}$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**3,x)
```

```
[Out] 8*a**2*(-log(16*a**3*x) + log(16*a**3*x + 16*I*a**2)) + (-16*I*a**2*x**2 +
7*a*x - I)/(2*a*x**3 + 2*I*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = -8i\,a^2\arctan(ax) + 4\,a^2\log\left(a^2x^2 + 1\right) - 8\,a^2\log(x) + \frac{-16i\,a^3x^3 - 9\,a^2x^2 - 8i\,ax - 1}{2\,(a^2x^4 + x^2)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -8*I*a^2*arctan(a*x) + 4*a^2*log(a^2*x^2 + 1) - 8*a^2*log(x) + 1/2*(-16*I*a
^3*x^3 - 9*a^2*x^2 - 8*I*a*x - 1)/(a^2*x^4 + x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = 8\,a^2\log\left(ax+i\right) - 8\,a^2\log\left(|x|\right) - \frac{16i\,a^2x^2 - 7\,ax + i}{2\,(ax+i)x^2}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="giac")
```

```
[Out] 8*a^2*log(a*x + I) - 8*a^2*log(abs(x)) - 1/2*(16*I*a^2*x^2 - 7*a*x + I)/((a
*x + I)*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = -a^2\,\mathrm{atan}(2\,a\,x + 1\mathrm{i})\ 16\mathrm{i} + \frac{8\,a^2\,x^2 + \frac{a\,x\,7\mathrm{i}}{2} + \frac{1}{2}}{x^2\,\left(-1 + a\,x\,1\mathrm{i}\right)}$$

```
[In] int((a*x*1i + 1)^4/(x^3*(a^2*x^2 + 1)^2),x)
```

```
[Out] ((a*x*7i)/2 + 8*a^2*x^2 + 1/2)/(x^2*(a*x*1i - 1)) - a^2*atan(2*a*x + 1i)*16
i
```

3.34 $\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3} - \frac{2ia}{x^2} + \frac{8a^2}{x} + \frac{4a^3}{i+ax} - 12ia^3\log(x) + 12ia^3\log(i+ax)$$

```
[Out] -1/3/x^3-2*I*a/x^2+8*a^2/x+4*a^3/(I+a*x)-12*I*a^3*ln(x)+12*I*a^3*ln(I+a*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 90}

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = \frac{4a^3}{ax+i} - 12ia^3\log(x) + 12ia^3\log(ax+i) + \frac{8a^2}{x} - \frac{2ia}{x^2} - \frac{1}{3x^3}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*1/x^3 - ((2*I)*a)/x^2 + (8*a^2)/x + (4*a^3)/(I + a*x) - (12*I)*a^3*Log
[x] + (12*I)*a^3*Log[I + a*x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^2}{x^4(1-iax)^2}\,dx \\
&= \int \left(\frac{1}{x^4}+\frac{4ia}{x^3}-\frac{8a^2}{x^2}-\frac{12ia^3}{x}-\frac{4a^4}{(i+ax)^2}+\frac{12ia^4}{i+ax}\right)dx \\
&= -\frac{1}{3x^3}-\frac{2ia}{x^2}+\frac{8a^2}{x}+\frac{4a^3}{i+ax}-12ia^3\log(x)+12ia^3\log(i+ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3}-\frac{2ia}{x^2}+\frac{8a^2}{x}+\frac{4a^3}{i+ax}-12ia^3\log(x)+12ia^3\log(i+ax)$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*1/x^3 - ((2*I)*a)/x^2 + (8*a^2)/x + (4*a^3)/(I + a*x) - (12*I)*a^3*Log
[x] + (12*I)*a^3*Log[I + a*x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
default	$-\frac{1}{3x^3}-12ia^3\ln(x)-\frac{2ia}{x^2}+\frac{8a^2}{x}+4a^4\left(\frac{1}{a(ax+i)}+\frac{3i\ln(ax+i)}{a}\right)$
risch	$\frac{12a^3x^3+6ia^2x^2+\frac{5}{3}ax-\frac{1}{3}i}{(ax+i)x^3}+12a^3\arctan(ax)+6ia^3\ln(a^2x^2+1)-12ia^3\ln(-x)$
parallelrisch	$-\frac{36i\ln(x)x^5a^5-36i\ln(ax+i)x^5a^5-18ix^5a^5+36ia^3\ln(x)x^3-36ia^3\ln(ax+i)x^3+1-36a^4x^4-23a^2x^2+6iax}{3(a^2x^2+1)x^3}$
meijerg	$\frac{a^4\left(-\frac{2\left(-15a^4x^4-10a^2x^2+2\right)}{3x^3\left(a^2\right)^{\frac{3}{2}}\left(2a^2x^2+2\right)}+\frac{5a^3\arctan(ax)}{\left(a^2\right)^{\frac{3}{2}}}\right)}{2\sqrt{a^2}}+2ia^3\left(\frac{3a^2x^2}{3a^2x^2+3}+2\ln(a^2x^2+1)-1-4\ln(x)-2\ln(a^2)\right.$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3-12*I*a^3*ln(x)-2*I*a/x^2+8*a^2/x+4*a^4*(1/a/(I+a*x)+3*I*ln(I+a*x)/
a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.39

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = \frac{36\,a^3x^3 + 18i\,a^2x^2 + 5\,ax - 36\,(i\,a^4x^4 - a^3x^3)\log(x) - 36\,(-i\,a^4x^4 + a^3x^3)\log\left(\frac{ax+i}{a}\right) - i}{3\,(ax^4 + i\,x^3)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(36*a^3*x^3 + 18*I*a^2*x^2 + 5*a*x - 36*(I*a^4*x^4 - a^3*x^3)*log(x) -
36*(-I*a^4*x^4 + a^3*x^3)*log((a*x + I)/a) - I)/(a*x^4 + I*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = 12a^3\left(-i\log\left(24a^4x\right) + i\log\left(24a^4x + 24ia^3\right)\right) + \frac{36a^3x^3 + 18ia^2x^2 + 5ax - i}{3ax^4 + 3ix^3}$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**4,x)
```

```
[Out] 12*a**3*(-I*log(24*a**4*x) + I*log(24*a**4*x + 24*I*a**3)) + (36*a**3*x**3
+ 18*I*a**2*x**2 + 5*a*x - I)/(3*a*x**4 + 3*I*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = 12\,a^3\arctan(ax) + 6i\,a^3\log\left(a^2x^2 + 1\right) - 12i\,a^3\log(x) + \frac{36\,a^4x^4 - 18i\,a^3x^3 + 23\,a^2x^2 - 6i\,ax - 1}{3\,(a^2x^5 + x^3)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] 12*a^3*arctan(a*x) + 6*I*a^3*log(a^2*x^2 + 1) - 12*I*a^3*log(x) + 1/3*(36*a
^4*x^4 - 18*I*a^3*x^3 + 23*a^2*x^2 - 6*I*a*x - 1)/(a^2*x^5 + x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = 12i\,a^3\log(ax+i) - 12i\,a^3\log(|x|) + \frac{36\,a^3x^3 + 18i\,a^2x^2 + 5\,ax - i}{3\,(ax+i)x^3}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="giac")
```

```
[Out] 12*I*a^3*log(a*x + I) - 12*I*a^3*log(abs(x)) + 1/3*(36*a^3*x^3 + 18*I*a^2*x
^2 + 5*a*x - I)/((a*x + I)*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = 24\,a^3\,\mathrm{atan}(2\,a\,x+1\mathrm{i}) + \frac{\frac{5\,x}{3} + 12\,a^2\,x^3 + a\,x^2\,6\mathrm{i} - \frac{1\mathrm{i}}{3\,a}}{x^4 + \frac{x^3\,1\mathrm{i}}{a}}$$

```
[In] int((a*x*1i + 1)^4/(x^4*(a^2*x^2 + 1)^2),x)
```

```
[Out] 24*a^3*atan(2*a*x + 1i) + ((5*x)/3 + a*x^2*6i - 1i/(3*a) + 12*a^2*x^3)/(x^4
 + (x^3*1i)/a)
```

3.35 $\int e^{-i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int e^{-i\arctan(ax)}x^3\,dx=\frac{x^2\sqrt{1+a^2x^2}}{3a^2}-\frac{ix^3\sqrt{1+a^2x^2}}{4a}-\frac{(16-9iax)\sqrt{1+a^2x^2}}{24a^4}-\frac{3i\text{arcsinh}(ax)}{8a^4}$$

```
[Out] -3/8*I*arcsinh(a*x)/a^4+1/3*x^2*(a^2*x^2+1)^(1/2)/a^2-1/4*I*x^3*(a^2*x^2+1)
^(1/2)/a-1/24*(16-9*I*a*x)*(a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5168, 847, 794, 221}

$$\int e^{-i\arctan(ax)}x^3\,dx=-\frac{3i\text{arcsinh}(ax)}{8a^4}+\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{(16-9iax)\sqrt{a^2x^2+1}}{24a^4}$$

```
[In] Int[x^3/E^(I*ArcTan[a*x]),x]
```

```
[Out] (x^2*Sqrt[1 + a^2*x^2])/(3*a^2) - ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - ((16 -
(9*I)*a*x)*Sqrt[1 + a^2*x^2])/(24*a^4) - (((3*I)/8)*ArcSinh[a*x])/a^4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(1-iax)}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x^2(3ia+4a^2x)}{\sqrt{1+a^2x^2}}\,dx}{4a^2}\\
&= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} + \frac{\int \frac{x(-8a^2+9ia^3x)}{\sqrt{1+a^2x^2}}\,dx}{12a^4}\\
&= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16-9iax)\sqrt{1+a^2x^2}}{24a^4} - \frac{(3i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a^3}\\
&= \frac{x^2\sqrt{1+a^2x^2}}{3a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(16-9iax)\sqrt{1+a^2x^2}}{24a^4} - \frac{3i\operatorname{arcsinh}(ax)}{8a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int e^{-i\arctan(ax)}x^3\,dx=\frac{\sqrt{1+a^2x^2}(-16+9iax+8a^2x^2-6ia^3x^3)-9i\text{arcsinh}(ax)}{24a^4}$$

```
[In] Integrate[x^3/E^(I*ArcTan[a*x]),x]
[Out] (Sqrt[1 + a^2*x^2]*(-16 + (9*I)*a*x + 8*a^2*x^2 - (6*I)*a^3*x^3) - (9*I)*Ar
cSinh[a*x])/(24*a^4)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{i(6a^3x^3+8ia^2x^2-9ax-16i)\sqrt{a^2x^2+1}}{24a^4}-\frac{3i\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^3\sqrt{a^2}}$
default	$\frac{(a^2x^2+1)^{\frac{3}{2}}}{3a^4}+\frac{i\left(\frac{\sqrt{a^2x^2+1}\,x}{2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}\right)}{a^3}-\frac{i\left(\frac{x(a^2x^2+1)^{\frac{3}{2}}}{4a^2}-\frac{\frac{\sqrt{a^2x^2+1}\,x}{2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}}{4a^2}\right)}{a}-\sqrt{(x-\frac{i}{a})^2a^2\cdots}$

```
[In] int(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/24*I*(6*a^3*x^3+8*I*a^2*x^2-9*a*x-16*I)*(a^2*x^2+1)^(1/2)/a^4-3/8*I/a^3*
ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int e^{-i\arctan(ax)}x^3\,dx=\frac{(-6i\,a^3x^3+8\,a^2x^2+9i\,ax-16)\sqrt{a^2x^2+1}+9i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{24\,a^4}$$

```
[In] integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/24*((-6*I*a^3*x^3 + 8*a^2*x^2 + 9*I*a*x - 16)*sqrt(a^2*x^2 + 1) + 9*I*log
(-a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [F]

$$\int e^{-i\arctan(ax)}x^3\,dx = -i\int \frac{x^3\sqrt{a^2x^2+1}}{ax-i}\,dx$$

```
[In] integrate(x**3/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
[Out] -I*Integral(x**3*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int e^{-i\arctan(ax)}x^3\,dx = -\frac{i\,(a^2x^2+1)^{\frac{3}{2}}x}{4\,a^3} + \frac{5i\,\sqrt{a^2x^2+1}x}{8\,a^3} + \frac{(a^2x^2+1)^{\frac{3}{2}}}{3\,a^4} - \frac{3i\,\operatorname{arsinh}(ax)}{8\,a^4} - \frac{\sqrt{a^2x^2+1}}{a^4}$$

```
[In] integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*I*(a^2*x^2 + 1)^(3/2)*x/a^3 + 5/8*I*sqrt(a^2*x^2 + 1)*x/a^3 + 1/3*(a^2
*x^2 + 1)^(3/2)/a^4 - 3/8*I*arcsinh(a*x)/a^4 - sqrt(a^2*x^2 + 1)/a^4
```

Giac [F(-2)]

Exception generated.

$$\int e^{-i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int e^{-i\arctan(ax)}x^3\,dx = -\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)\,3\mathrm{i}}{8\,a^3\,\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{2}{3\,{\left(a^2\right)}^{3/2}}-\frac{a^2\,x^2}{3\,{\left(a^2\right)}^{3/2}}+\frac{x^3\,{\left(a^2\right)}^{3/2}\,1\mathrm{i}}{4\,a^3}-\frac{x\,\sqrt{a^2}\,3\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}}$$

```
[In] int((x^3*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

```
[Out] - (asinh(x*(a^2)^(1/2))*3i)/(8*a^3*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*(2/(
3*(a^2)^(3/2)) - (a^2*x^2)/(3*(a^2)^(3/2)) + (x^3*(a^2)^(3/2)*1i)/(4*a^3) -
 (x*(a^2)^(1/2)*3i)/(8*a^3)))/(a^2)^(1/2)
```

3.36 $\int e^{-i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int e^{-i\arctan(ax)}x^2\,dx=\frac{i\sqrt{1+a^2x^2}}{a^3}+\frac{x\sqrt{1+a^2x^2}}{2a^2}-\frac{i(1+a^2x^2)^{3/2}}{3a^3}-\frac{\operatorname{arcsinh}(ax)}{2a^3}$$

```
[Out] -1/3*I*(a^2*x^2+1)^(3/2)/a^3-1/2*arcsinh(a*x)/a^3+I*(a^2*x^2+1)^(1/2)/a^3+1
/2*x*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5168, 811, 655, 201, 221}

$$\int e^{-i\arctan(ax)}x^2\,dx=-\frac{\operatorname{arcsinh}(ax)}{2a^3}+\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{i(a^2x^2+1)^{3/2}}{3a^3}+\frac{i\sqrt{a^2x^2+1}}{a^3}$$

```
[In] Int[x^2/E^(I*ArcTan[a*x]),x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2])/a^3 + (x*Sqrt[1 + a^2*x^2])/(2*a^2) - ((I/3)*(1 + a^2
*x^2)^(3/2))/a^3 - ArcSinh[a*x]/(2*a^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(1-iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\int \frac{1-iax}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int (1-iax)\sqrt{1+a^2x^2}\,dx}{a^2} \\
&= \frac{i\sqrt{1+a^2x^2}}{a^3} - \frac{i(1+a^2x^2)^{3/2}}{3a^3} - \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{a^2} + \frac{\int \sqrt{1+a^2x^2}\,dx}{a^2} \\
&= \frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} - \frac{i(1+a^2x^2)^{3/2}}{3a^3} - \frac{\operatorname{arcsinh}(ax)}{a^3} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2} \\
&= \frac{i\sqrt{1+a^2x^2}}{a^3} + \frac{x\sqrt{1+a^2x^2}}{2a^2} - \frac{i(1+a^2x^2)^{3/2}}{3a^3} - \frac{\operatorname{arcsinh}(ax)}{2a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int e^{-i\arctan(ax)}x^2\,dx = \frac{(4i+3ax-2ia^2x^2)\sqrt{1+a^2x^2}-3\text{arcsinh}(ax)}{6a^3}$$

```
[In] Integrate[x^2/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((4*I + 3*a*x - (2*I)*a^2*x^2)*Sqrt[1 + a^2*x^2] - 3*ArcSinh[a*x])/(6*a^3)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{i(2a^2x^2+3iax-4)\sqrt{a^2x^2+1}}{6a^3}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}$
default	$\frac{\frac{\sqrt{a^2x^2+1}\,x}{2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}}{a^2}-\frac{i(a^2x^2+1)^{\frac{3}{2}}}{3a^3}+\frac{i\left(\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}+\frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}\right)}{\sqrt{a^2}}\right)}{a^3}$

```
[In] int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*(2*a^2*x^2+3*I*a*x-4)*(a^2*x^2+1)^(1/2)/a^3-1/2/a^2*ln(a^2*x/(a^2)^(
1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int e^{-i\arctan(ax)}x^2\,dx = \frac{\sqrt{a^2x^2+1}(-2i\,a^2x^2+3\,ax+4i)+3\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{6\,a^3}$$

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(a^2*x^2 + 1)*(-2*I*a^2*x^2 + 3*a*x + 4*I) + 3*log(-a*x + sqrt(a^2
*x^2 + 1)))/a^3
```

Sympy [F]

$$\int e^{-i\arctan(ax)}x^2\,dx = -i\int \frac{x^2\sqrt{a^2x^2+1}}{ax-i}\,dx$$

```
[In] integrate(x**2/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

```
[Out] -I*Integral(x**2*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int e^{-i\arctan(ax)}x^2\,dx = \frac{\sqrt{a^2x^2+1}x}{2\,a^2} - \frac{i\,(a^2x^2+1)^{\frac{3}{2}}}{3\,a^3} - \frac{\operatorname{arsinh}(ax)}{2\,a^3} + \frac{i\,\sqrt{a^2x^2+1}}{a^3}$$

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a^2*x^2 + 1)*x/a^2 - 1/3*I*(a^2*x^2 + 1)^(3/2)/a^3 - 1/2*arcsinh(a
*x)/a^3 + I*sqrt(a^2*x^2 + 1)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int e^{-i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int e^{-i\arctan(ax)}x^2\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{x\,\sqrt{a^2}}{2\,a^2}+\frac{a\,2\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}-\frac{a^3\,x^2\,1\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}\right)}{\sqrt{a^2}} - \frac{\operatorname{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}}$$

```
[In] int((x^2*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*((a*2i)/(3*(a^2)^(3/2)) - (a^3*x^2*1i)/(3*(a^2)^(3/2))
 + (x*(a^2)^(1/2))/(2*a^2)))/(a^2)^(1/2) - asinh(x*(a^2)^(1/2))/(2*a^2*(a^2
)^(1/2))
```

3.37 $\int e^{-i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int e^{-i\arctan(ax)}x\,dx=\frac{(2-iax)\sqrt{1+a^2x^2}}{2a^2}+\frac{i\text{arcsinh}(ax)}{2a^2}$$

```
[Out] 1/2*I*arcsinh(a*x)/a^2+1/2*(2-I*a*x)*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5168, 794, 221}

$$\int e^{-i\arctan(ax)}x\,dx=\frac{i\text{arcsinh}(ax)}{2a^2}+\frac{\sqrt{a^2x^2+1}(2-iax)}{2a^2}$$

```
[In] Int[x/E^(I*ArcTan[a*x]),x]
```

```
[Out] ((2 - I*a*x)*Sqrt[1 + a^2*x^2])/(2*a^2) + ((I/2)*ArcSinh[a*x])/a^2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(1-iax)}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{(2-iax)\sqrt{1+a^2x^2}}{2a^2}+\frac{i\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a}\\ &= \frac{(2-iax)\sqrt{1+a^2x^2}}{2a^2}+\frac{i\text{arcsinh}(ax)}{2a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int e^{-i\arctan(ax)}x\,dx=\frac{(2-iax)\sqrt{1+a^2x^2}+i\text{arcsinh}(ax)}{2a^2}$$

```
[In] Integrate[x/E^(I*ArcTan[a*x]),x]
[Out] ((2 - I*a*x)*Sqrt[1 + a^2*x^2] + I*ArcSinh[a*x])/(2*a^2)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

method	result	size
risch	$-\frac{i(ax+2i)\sqrt{a^2x^2+1}}{2a^2}+\frac{i\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a\sqrt{a^2}}$	59
default	$-\frac{i\left(\frac{\sqrt{a^2x^2+1}\,x}{2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}\right)}{a}+\frac{\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}+\frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}\right)}{\sqrt{a^2}}}{a^2}$	150

```
[In] int(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*(a*x+2*I)*(a^2*x^2+1)^(1/2)/a^2+1/2*I/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int e^{-i\arctan(ax)}x\,dx = \frac{\sqrt{a^2x^2+1}(-i\,ax+2) - i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2}$$

```
[In] integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(a^2*x^2 + 1)*(-I*a*x + 2) - I*log(-a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [F]

$$\int e^{-i\arctan(ax)}x\,dx = -i\int \frac{x\sqrt{a^2x^2+1}}{ax-i}\,dx$$

```
[In] integrate(x/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
[Out] -I*Integral(x*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{-i\arctan(ax)}x\,dx = -\frac{i\,\sqrt{a^2x^2+1}x}{2\,a} + \frac{i\,\operatorname{arsinh}(ax)}{2\,a^2} + \frac{\sqrt{a^2x^2+1}}{a^2}$$

```
[In] integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*I*sqrt(a^2*x^2 + 1)*x/a + 1/2*I*arcsinh(a*x)/a^2 + sqrt(a^2*x^2 + 1)/a
^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int e^{-i\arctan(ax)}x\,dx = -\frac{1}{2}\,\sqrt{a^2x^2+1}\left(\frac{i\,x}{a}-\frac{2}{a^2}\right) - \frac{i\,\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{2\,a|a|}$$

```
[In] integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(a^2*x^2 + 1)*(I*x/a - 2/a^2) - 1/2*I*log(-x*abs(a) + sqrt(a^2*x^2
 + 1))/(a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int e^{-i\arctan(ax)}x\,dx = \frac{\left(\frac{1}{\sqrt{a^2}} - \frac{x\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)\sqrt{a^2\,x^2+1} + \frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{2\,a}}{\sqrt{a^2}}$$

```
[In] int((x*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

```
[Out] ((1/(a^2)^(1/2) - (x*(a^2)^(1/2)*1i)/(2*a))*(a^2*x^2 + 1)^(1/2) + (asinh(x*
(a^2)^(1/2))*1i)/(2*a))/(a^2)^(1/2)
```

3.38 $\int e^{-i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int e^{-i\arctan(ax)}\,dx = -\frac{i\sqrt{1+a^2x^2}}{a} + \frac{\operatorname{arcsinh}(ax)}{a}$$

```
[Out] arcsinh(a*x)/a-I*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5167, 655, 221}

$$\int e^{-i\arctan(ax)}\,dx = \frac{\operatorname{arcsinh}(ax)}{a} - \frac{i\sqrt{a^2x^2+1}}{a}$$

```
[In] Int[E^((-I)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 5167

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1 - iax}{\sqrt{1 + a^2x^2}}\,dx \\ &= -\frac{i\sqrt{1 + a^2x^2}}{a} + \int \frac{1}{\sqrt{1 + a^2x^2}}\,dx \\ &= -\frac{i\sqrt{1 + a^2x^2}}{a} + \frac{\operatorname{arcsinh}(ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{-i\arctan(ax)}\,dx = \frac{-i\sqrt{1 + a^2x^2} + \operatorname{arcsinh}(ax)}{a}$$

```
[In] Integrate[E^((-I)*ArcTan[a*x]),x]
[Out] ((-I)*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/a
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

method	result	size
risch	$-\frac{i\sqrt{a^2x^2+1}}{a}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}$	48
default	$-\frac{i\left(\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}+\frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}\right)}{\sqrt{a^2}}\right)}{a}$	100

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -I*(a^2*x^2+1)^(1/2)/a+ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int e^{-i\arctan(ax)}\,dx = \frac{-i\sqrt{a^2x^2+1} - \log\left(-ax+\sqrt{a^2x^2+1}\right)}{a}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (-I*sqrt(a^2*x^2 + 1) - log(-a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [F]

$$\int e^{-i\arctan(ax)}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax-i}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int e^{-i\arctan(ax)}\,dx = \frac{\operatorname{arsinh}(ax)}{a} - \frac{i\sqrt{a^2x^2+1}}{a}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(a*x)/a - I*sqrt(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int e^{-i\arctan(ax)}\,dx = -\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|} - \frac{i\sqrt{a^2x^2+1}}{a}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - I*sqrt(a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int e^{-i\arctan(ax)}\,dx = \frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a}$$

```
[In] int((a^2*x^2 + 1)^(1/2)/(a*x*1i + 1),x)
[Out] asinh(x*(a^2)^(1/2))/(a^2)^(1/2) - ((a^2*x^2 + 1)^(1/2)*1i)/a
```

3.39 $\int \frac{e^{-i \arctan(ax)}}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{e^{-i \arctan(ax)}}{x} \, dx = -i\text{arcsinh}(ax) - \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5168, 858, 221, 272, 65, 214}

$$\int \frac{e^{-i \arctan(ax)}}{x} \, dx = -\text{arctanh}\left(\sqrt{a^2x^2+1}\right) - i\text{arcsinh}(ax)$$

```
[In] Int[1/(E^(I*ArcTan[a*x])*x),x]
```

```
[Out] (-I)*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1-iax}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\left((ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\right)+\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\
&= -i\text{arcsinh}(ax)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -i\text{arcsinh}(ax)+\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)}{a^2}\\
&= -i\text{arcsinh}(ax)-\text{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -i\text{arcsinh}(ax) + \log(x) - \log\left(1+\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x),x]
[Out] (-I)*ArcSinh[a*x] + Log[x] - Log[1 + Sqrt[1 + a^2*x^2]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(22) = 44.

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.84

method	result
default	$\sqrt{a^2x^2+1} - \text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right) - \sqrt{\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x-\frac{i}{a}\right)} - \frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}} + \sqrt{\left(x-\frac{i}{a}\right)^2 a^2 + 2ia\left(x\right.}\right.}{\sqrt{a^2}}$

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))-((x-I/a)^2*a^2+2*I*a*(x-I/a)
)^(1/2)-I*a*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^
(1/2))/(a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -\log\left(-ax+\sqrt{a^2x^2+1}+1\right) + i\log\left(-ax+\sqrt{a^2x^2+1}\right) + \log\left(-ax+\sqrt{a^2x^2+1}-1\right)$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
[Out] -log(-a*x + sqrt(a^2*x^2 + 1) + 1) + I*log(-a*x + sqrt(a^2*x^2 + 1)) + log(
-a*x + sqrt(a^2*x^2 + 1) - 1)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^2-ix}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x,x)
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**2 - I*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -i\,a\left(\frac{\operatorname{arsinh}(ax)}{a} - \frac{i\,\operatorname{arsinh}\left(\frac{1}{a|x|}\right)}{a}\right)$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
[Out] -I*a*(arcsinh(a*x)/a - I*arcsinh(1/(a*abs(x)))/a)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(21) = 42$.

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.72

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = \frac{i\,a\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{|a|} - \log\left(\left|-x|a| + \sqrt{a^2x^2+1} + 1\right|\right) + \log\left(\left|-x|a| + \sqrt{a^2x^2+1} - 1\right|\right)$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
[Out] I*a*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - log(abs(-x*abs(a) + sqrt(a^
2*x^2 + 1) + 1)) + log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right) - \frac{a\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{\sqrt{a^2}}$$

```
[In] int((a^2*x^2 + 1)^(1/2)/(x*(a*x*1i + 1)),x)
[Out] - atanh((a^2*x^2 + 1)^(1/2)) - (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2)
```

3.40 $\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{1+a^2x^2}}{x} + ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] I*a*arctanh((a^2*x^2+1)^(1/2))-(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5168, 821, 272, 65, 214}

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{a^2x^2+1}}{x} + ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right)$$

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + I*a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1-iax}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x} - (ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{1}{2}(ia)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{x} - \frac{i\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1+a^2x^2}}{x} + ia\,\text{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{1+a^2x^2}}{x} - ia\log(x) + ia\log\left(1+\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^2),x]
[Out] -(Sqrt[1 + a^2*x^2]/x) - I*a*Log[x] + I*a*Log[1 + Sqrt[1 + a^2*x^2]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{\sqrt{a^2x^2+1}}{x}+ia\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)$
default	$-\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{x}+2a^2\left(\frac{\sqrt{a^2x^2+1}\,x}{2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}\right)-ia\left(\sqrt{a^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)+ia$

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(a^2*x^2+1)^(1/2)/x+I*a*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = \frac{i\,ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-i\,ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-ax-\sqrt{a^2x^2+1}}{x}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
[Out] (I*a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - I*a*x*log(-a*x + sqrt(a^2*x^2 +
1) - 1) - a*x - sqrt(a^2*x^2 + 1))/x
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^3-ix^2}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**2,x)
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**3 - I*x**2), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^2}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{a^2\,x^2+1}}{x} + a\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,1\mathrm{i}$$

```
[In] int((a^2*x^2 + 1)^(1/2)/(x^2*(a*x*1i + 1)),x)
[Out] a*atanh((a^2*x^2 + 1)^(1/2))*1i - (a^2*x^2 + 1)^(1/2)/x
```

3.41 $\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = -\frac{\sqrt{1+a^2x^2}}{2x^2}+\frac{ia\sqrt{1+a^2x^2}}{x}+\frac{1}{2}a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] 1/2*a^2*arctanh((a^2*x^2+1)^(1/2))-1/2*(a^2*x^2+1)^(1/2)/x^2+I*a*(a^2*x^2+1
)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5168, 849, 821, 272, 65, 214}

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \frac{1}{2}a^2\text{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}$$

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^3),x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^2]/x^2 + (I*a*Sqrt[1 + a^2*x^2])/x + (a^2*ArcTanh[Sqrt[
1 + a^2*x^2]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1-iax}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{1}{2}\int \frac{2ia+a^2x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}+\frac{ia\sqrt{1+a^2x^2}}{x}-\frac{1}{2}a^2\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2}+\frac{ia\sqrt{1+a^2x^2}}{x}-\frac{1}{4}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)
\end{aligned}
$$

$$= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)$$

$$= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{ia\sqrt{1+a^2x^2}}{x} + \frac{1}{2}a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \frac{1}{2}\left(\frac{(-1+2iax)\sqrt{1+a^2x^2}}{x^2} - a^2\log(x) + a^2\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^3),x]
[Out] (((-1 + (2*I)*a*x)*Sqrt[1 + a^2*x^2])/x^2 - a^2*Log[x] + a^2*Log[1 + Sqrt[1
 + a^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
risch	$\frac{i(2a^3x^3+ia^2x^2+2ax+i)}{2x^2\sqrt{a^2x^2+1}} + \frac{a^2\,\text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}$
default	$-\frac{(a^2x^2+1)^{\frac{3}{2}}}{2x^2} - \frac{a^2\left(\sqrt{a^2x^2+1}-\text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)}{2} - ia\left(-\frac{(a^2x^2+1)^{\frac{3}{2}}}{x} + 2a^2\left(\frac{\sqrt{a^2x^2+1}\,x}{2} + \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}\right.\right.$

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2*I*(2*a^3*x^3+I*a^2*x^2+2*a*x+I)/x^2/(a^2*x^2+1)^(1/2)+1/2*a^2*arctanh(1
/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+2i\,a^2x^2+\sqrt{a^2x^2+1}(2i\,ax-1)}{2\,x^2}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^2*x^2*log(-a*x + sqrt(a^
2*x^2 + 1) - 1) + 2*I*a^2*x^2 + sqrt(a^2*x^2 + 1)*(2*I*a*x - 1))/x^2
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^4-ix^3}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**4 - I*x**3), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^3}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^3), x)
```

Giac [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^3}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \frac{a^2\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)}{2} - \frac{\sqrt{a^2\,x^2+1}}{2\,x^2} + \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{x}$$

```
[In] int((a^2*x^2 + 1)^(1/2)/(x^3*(a*x*1i + 1)),x)
[Out] (a^2*atanh((a^2*x^2 + 1)^(1/2)))/2 - (a^2*x^2 + 1)^(1/2)/(2*x^2) + (a*(a^2*
x^2 + 1)^(1/2)*1i)/x
```

3.42 $\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{ia\sqrt{1+a^2x^2}}{2x^2}+\frac{2a^2\sqrt{1+a^2x^2}}{3x}-\frac{1}{2}ia^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -1/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))-1/3*(a^2*x^2+1)^(1/2)/x^3+1/2*I*a*(a^
2*x^2+1)^(1/2)/x^2+2/3*a^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5168, 849, 821, 272, 65, 214}

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \frac{2a^2\sqrt{a^2x^2+1}}{3x}+\frac{ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{2}ia^3\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)$$

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^4),x]
```

```
[Out] -1/3*Sqrt[1 + a^2*x^2]/x^3 + ((I/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (2*a^2*Sqrt[
1 + a^2*x^2])/(3*x) - (I/2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1-iax}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{1}{3}\int \frac{3ia+2a^2x}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{1}{6}\int \frac{-4a^2+3ia^3x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} + \frac{1}{2}\left(ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx
\end{aligned}$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} + \frac{1}{4}\left(ia^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} + \frac{1}{2}(ia)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} - \frac{1}{2}ia^3\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \frac{1}{6}\left(\frac{\sqrt{1+a^2x^2}(-2+3iax+4a^2x^2)}{x^3} + 3ia^3\log(x) - 3ia^3\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^4),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-2 + (3*I)*a*x + 4*a^2*x^2))/x^3 + (3*I)*a^3*Log[x] -
(3*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result
risch	$\frac{4a^4x^4+3ia^3x^3+2a^2x^2+3iax-2}{6x^3\sqrt{a^2x^2+1}} - \frac{ia^3\,\text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}$
default	$-\frac{(a^2x^2+1)^{\frac{3}{2}}}{3x^3} - a^2\left(-\frac{(a^2x^2+1)^{\frac{3}{2}}}{x} + 2a^2\left(\frac{\sqrt{a^2x^2+1}\,x}{2} + \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}\right)\right) + ia^3\left(\sqrt{a^2x^2+1} - \text{arctanh}\right.$

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(4*a^4*x^4+3*I*a^3*x^3+2*a^2*x^2+3*I*a*x-2)/x^3/(a^2*x^2+1)^(1/2)-1/2*I
*a^3*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \frac{-3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+4\,a^3x^3+(4\,a^2x^2+3i\,ax-2$$

$$6\,x^3$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(-3*I*a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + 3*I*a^3*x^3*log(-a*x
+ sqrt(a^2*x^2 + 1) - 1) + 4*a^3*x^3 + (4*a^2*x^2 + 3*I*a*x - 2)*sqrt(a^2*x
^2 + 1))/x^3
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^5-ix^4}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**5 - I*x**4), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^4}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \frac{2\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x} - \frac{\sqrt{a^2\,x^2+1}}{3\,x^3} - \frac{a^3\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2} + \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{2\,x^2}$$

```
[In] int((a^2*x^2 + 1)^(1/2)/(x^4*(a*x*1i + 1)),x)
```

```
[Out] (a*(a^2*x^2 + 1)^(1/2)*1i)/(2*x^2) - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (a^3*ata
n((a^2*x^2 + 1)^(1/2)*1i))/2 + (2*a^2*(a^2*x^2 + 1)^(1/2))/(3*x)
```

3.43 $\int \frac{e^{-i\arctan(ax)}}{x^5} dx$

Optimal result

Integrand size = 14, antiderivative size = 113

$$\int \frac{e^{-i\arctan(ax)}}{x^5} dx = -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{2ia^3\sqrt{1+a^2x^2}}{3x} - \frac{3}{8}a^4\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -3/8*a^4*arctanh((a^2*x^2+1)^(1/2))-1/4*(a^2*x^2+1)^(1/2)/x^4+1/3*I*a*(a^2*
x^2+1)^(1/2)/x^3+3/8*a^2*(a^2*x^2+1)^(1/2)/x^2-2/3*I*a^3*(a^2*x^2+1)^(1/2)/
x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5168, 849, 821, 272, 65, 214}

$$\int \frac{e^{-i\arctan(ax)}}{x^5} dx = \frac{3a^2\sqrt{a^2x^2+1}}{8x^2} - \frac{\sqrt{a^2x^2+1}}{4x^4} + \frac{ia\sqrt{a^2x^2+1}}{3x^3} - \frac{3}{8}a^4\text{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{2ia^3\sqrt{a^2x^2+1}}{3x}$$

```
[In] Int[1/(E^(I*ArcTan[a*x])*x^5),x]
```

```
[Out] -1/4*Sqrt[1 + a^2*x^2]/x^4 + ((I/3)*a*Sqrt[1 + a^2*x^2])/x^3 + (3*a^2*Sqrt[
1 + a^2*x^2])/(8*x^2) - (((2*I)/3)*a^3*Sqrt[1 + a^2*x^2])/x - (3*a^4*ArcTan
h[Sqrt[1 + a^2*x^2]])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{1 - iax}{x^5\sqrt{1 + a^2x^2}}\,dx$$

$$
\begin{aligned}
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} - \frac{1}{4}\int \frac{4ia+3a^2x}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{1}{12}\int \frac{-9a^2+8ia^3x}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{1}{24}\int \frac{-16ia^3-9a^4x}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{8}(3a^4)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} \\
&\quad - \frac{2ia^3\sqrt{1+a^2x^2}}{3x} + \frac{1}{16}(3a^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{2ia^3\sqrt{1+a^2x^2}}{3x} \\
&\quad + \frac{1}{8}(3a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{3x^3} + \frac{3a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{2ia^3\sqrt{1+a^2x^2}}{3x} - \frac{3}{8}a^4\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$
\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = \frac{1}{24}\left(\frac{\sqrt{1+a^2x^2}(-6+8iax+9a^2x^2-16ia^3x^3)}{x^4} + 9a^4\log(x) - 9a^4\log\left(1+\sqrt{1+a^2x^2}\right)\right)
$$

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*x^5),x]
[Out] ((Sqrt[1 + a^2*x^2]*(-6 + (8*I)*a*x + 9*a^2*x^2 - (16*I)*a^3*x^3))/x^4 + 9*
a^4*Log[x] - 9*a^4*Log[1 + Sqrt[1 + a^2*x^2]])/24
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{i(16a^5x^5+9ia^4x^4+8a^3x^3+3ia^2x^2-8ax-6i)}{24x^4\sqrt{a^2x^2+1}}-\frac{3a^4\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{8}$
default	$-\frac{(a^2x^2+1)^{\frac{3}{2}}}{4x^4}-\frac{5a^2\left(-\frac{(a^2x^2+1)^{\frac{3}{2}}}{2x^2}+\frac{a^2\left(\sqrt{a^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)}{2}\right)}{4}+a^4\left(\sqrt{a^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)$

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*I*(16*a^5*x^5+9*I*a^4*x^4+8*a^3*x^3+3*I*a^2*x^2-8*a*x-6*I)/x^4/(a^2*x
^2+1)^(1/2)-3/8*a^4*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = -\frac{9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+16i\,a^4x^4-(-16i\,a^3x^3+9\,a^2x}{24\,x^4}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(9*a^4*x^4*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 9*a^4*x^4*log(-a*x + s
qrt(a^2*x^2 + 1) - 1) + 16*I*a^4*x^4 - (-16*I*a^3*x^3 + 9*a^2*x^2 + 8*I*a*x
 - 6)*sqrt(a^2*x^2 + 1))/x^4
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^6-ix^5}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x**6 - I*x**5), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^5}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^5), x)
```

Giac [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^5}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = \frac{a^4\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,3\mathrm{i}}{8} - \frac{\sqrt{a^2\,x^2+1}}{4\,x^4} + \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{3\,x^3} + \frac{3\,a^2\,\sqrt{a^2\,x^2+1}}{8\,x^2} - \frac{a^3\,\sqrt{a^2\,x^2+1}\,2\mathrm{i}}{3\,x}$$

```
[In] int((a^2*x^2 + 1)^(1/2)/(x^5*(a*x*1i + 1)),x)
[Out] (a^4*atan((a^2*x^2 + 1)^(1/2)*1i)*3i)/8 - (a^2*x^2 + 1)^(1/2)/(4*x^4) + (a*
(a^2*x^2 + 1)^(1/2)*1i)/(3*x^3) + (3*a^2*(a^2*x^2 + 1)^(1/2))/(8*x^2) - (a^
3*(a^2*x^2 + 1)^(1/2)*2i)/(3*x)
```

3.44 $\int e^{-2i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int e^{-2i\arctan(ax)}x^3\,dx=\frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i-ax)}{a^4}$$

```
[Out] 2*I*x/a^3+x^2/a^2-2/3*I*x^3/a-1/4*x^4-2*ln(I-a*x)/a^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int e^{-2i\arctan(ax)}x^3\,dx=-\frac{2\log(-ax+i)}{a^4}+\frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}$$

```
[In] Int[x^3/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] ((2*I)*x)/a^3 + x^2/a^2 - (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I - a*x])/a^4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3(1-iax)}{1+iax}\,dx\\ &= \int \left(\frac{2i}{a^3}+\frac{2x}{a^2}-\frac{2ix^2}{a}-x^3-\frac{2}{a^3(-i+ax)}\right)dx\\ &= \frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i-ax)}{a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(ax)}x^3\,dx = \frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i-ax)}{a^4}$$

```
[In] Integrate[x^3/E^((2*I)*ArcTan[a*x]),x]
[Out] ((2*I)*x)/a^3 + x^2/a^2 - (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I - a*x])/a^4
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\frac{1}{4}a^3x^4+\frac{2}{3}ia^2x^3-a\,x^2-2ix}{a^3}-\frac{2\ln(-ax+i)}{a^4}$	48
risch	$-\frac{x^4}{4}-\frac{2ix^3}{3a}+\frac{x^2}{a^2}+\frac{2ix}{a^3}-\frac{\ln(a^2x^2+1)}{a^4}-\frac{2i\arctan(ax)}{a^4}$	55
parallelrisch	$-\frac{-3a^5x^5-5ia^4x^4+4a^3x^3+24i+12ia^2x^2-24\ln(ax-i)xa+24i\ln(ax-i)}{12a^4(-ax+i)}$	73
meijerg	$-\frac{-\frac{ixa\left(-3a^4x^4-5ia^3x^3+10a^2x^2+30iax+60\right)}{12(iax+1)}+5\ln(iax+1)}{a^4}+\frac{-\frac{iax\left(2a^2x^2+6iax+12\right)}{4(iax+1)}+3\ln(iax+1)}{a^4}$	108

```
[In] int(x^3/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/a^3*(1/4*a^3*x^4+2/3*I*a^2*x^3-a*x^2-2*I*x)-2*ln(I-a*x)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int e^{-2i\arctan(ax)}x^3\,dx = -\frac{3\,a^4x^4 + 8i\,a^3x^3 - 12\,a^2x^2 - 24i\,ax + 24\,\log\left(\frac{ax-i}{a}\right)}{12\,a^4}$$

```
[In] integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 + 8*I*a^3*x^3 - 12*a^2*x^2 - 24*I*a*x + 24*log((a*x - I)/a
))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int e^{-2i\arctan(ax)}x^3\,dx = -\frac{x^4}{4} - \frac{2ix^3}{3a} + \frac{x^2}{a^2} + \frac{2ix}{a^3} - \frac{2\log(ax-i)}{a^4}$$

```
[In] integrate(x**3/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -x**4/4 - 2*I*x**3/(3*a) + x**2/a**2 + 2*I*x/a**3 - 2*log(a*x - I)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(ax)}x^3\,dx = -\frac{i\left(-3i\,a^3x^4 + 8\,a^2x^3 + 12i\,ax^2 - 24\,x\right)}{12\,a^3} - \frac{2\,\log(i\,ax+1)}{a^4}$$

```
[In] integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/12*I*(-3*I*a^3*x^4 + 8*a^2*x^3 + 12*I*a*x^2 - 24*x)/a^3 - 2*log(I*a*x +
1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.39

$$\int e^{-2i\arctan(ax)}x^3\,dx = \frac{(i\,ax+1)^4\left(\frac{20}{i\,ax+1}-\frac{54}{(i\,ax+1)^2}+\frac{84}{(i\,ax+1)^3}-3\right)}{12\,a^4}+\frac{2\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a^4}$$

```
[In] integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*(I*a*x + 1)^4*(20/(I*a*x + 1) - 54/(I*a*x + 1)^2 + 84/(I*a*x + 1)^3 -
3)/a^4 + 2*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int e^{-2i\arctan(ax)}x^3\,dx = \frac{x^2}{a^2}-\frac{x^4}{4}-\frac{2\,\ln\left(x-\frac{1\mathrm{i}}{a}\right)}{a^4}+\frac{x\,2\mathrm{i}}{a^3}-\frac{x^3\,2\mathrm{i}}{3\,a}$$

```
[In] int((x^3*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
```

```
[Out] (x*2i)/a^3 - (2*log(x - 1i/a))/a^4 - x^4/4 - (x^3*2i)/(3*a) + x^2/a^2
```

3.45 $\int e^{-2i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int e^{-2i\arctan(ax)}x^2\,dx=\frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}+\frac{2i\log(i-ax)}{a^3}$$

```
[Out] 2*x/a^2-I*x^2/a-1/3*x^3+2*I*ln(I-a*x)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int e^{-2i\arctan(ax)}x^2\,dx=\frac{2i\log(-ax+i)}{a^3}+\frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}$$

```
[In] Int[x^2/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] (2*x)/a^2 - (I*x^2)/a - x^3/3 + ((2*I)*Log[I - a*x])/a^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2(1-iax)}{1+iax}\,dx\\ &= \int \left(\frac{2}{a^2}-\frac{2ix}{a}-x^2+\frac{2i}{a^2(-i+ax)}\right)dx\\ &= \frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}+\frac{2i\log(i-ax)}{a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(ax)}x^2\,dx = \frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}+\frac{2i\log(i-ax)}{a^3}$$

```
[In] Integrate[x^2/E^((2*I)*ArcTan[a*x]),x]
[Out] (2*x)/a^2 - (I*x^2)/a - x^3/3 + ((2*I)*Log[I - a*x])/a^3
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\frac{1}{3}a^2x^3+ia\,x^2-2x}{a^2}+\frac{2i\ln(-ax+i)}{a^3}$	40
risch	$-\frac{x^3}{3}-\frac{ix^2}{a}+\frac{2x}{a^2}+\frac{i\ln(a^2x^2+1)}{a^3}-\frac{2\arctan(ax)}{a^3}$	47
parallelrisch	$-\frac{-a^4x^4-2ia^3x^3+6+6i\ln(ax-i)xa+3a^2x^2+6\ln(ax-i)}{3a^3(-ax+i)}$	63
meijerg	$-\frac{i\left(\frac{ixa\left(-5ia^3x^3+10a^2x^2+30iax+60\right)}{15iax+15}-4\ln(iax+1)\right)}{a^3}+\frac{i\left(\frac{iax(3iax+6)}{3iax+3}-2\ln(iax+1)\right)}{a^3}$	95

```
[In] int(x^2/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/a^2*(1/3*a^2*x^3+I*a*x^2-2*x)+2*I*ln(I-a*x)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{-2i\arctan(ax)}x^2\,dx = -\frac{a^3x^3 + 3i\,a^2x^2 - 6\,ax - 6i\,\log\left(\frac{ax-i}{a}\right)}{3\,a^3}$$

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
[Out] -1/3*(a^3*x^3 + 3*I*a^2*x^2 - 6*a*x - 6*I*log((a*x - I)/a))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int e^{-2i\arctan(ax)}x^2\,dx = -\frac{x^3}{3} - \frac{ix^2}{a} + \frac{2x}{a^2} + \frac{2i\log\left(ax-i\right)}{a^3}$$

```
[In] integrate(x**2/(1+I*a*x)**2*(a**2*x**2+1),x)
[Out] -x**3/3 - I*x**2/a + 2*x/a**2 + 2*I*log(a*x - I)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{-2i\arctan(ax)}x^2\,dx = -\frac{a^2x^3 + 3i\,ax^2 - 6\,x}{3\,a^2} + \frac{2i\,\log\left(i\,ax+1\right)}{a^3}$$

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
[Out] -1/3*(a^2*x^3 + 3*I*a*x^2 - 6*x)/a^2 + 2*I*log(I*a*x + 1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{-2i\arctan(ax)}x^2\,dx = \frac{i\,(i\,ax+1)^3\left(\frac{6}{i\,ax+1} - \frac{15}{(i\,ax+1)^2} - 1\right)}{3\,a^3} - \frac{2i\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a^3}$$

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
[Out] 1/3*I*(I*a*x + 1)^3*(6/(I*a*x + 1) - 15/(I*a*x + 1)^2 - 1)/a^3 - 2*I*log(1/
(sqrt(a^2*x^2 + 1)*abs(a)))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(ax)}x^2\,dx = \frac{\ln\left(x-\frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a^3}+\frac{2\,x}{a^2}-\frac{x^3}{3}-\frac{x^2\,1\mathrm{i}}{a}$$

```
[In] int((x^2*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
[Out] (log(x - 1i/a)*2i)/a^3 + (2*x)/a^2 - x^3/3 - (x^2*1i)/a
```

3.46 $\int e^{-2i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int e^{-2i\arctan(ax)}x\,dx=-\frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i-ax)}{a^2}$$

```
[Out] -2*I*x/a-1/2*x^2+2*ln(I-a*x)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5170, 78}

$$\int e^{-2i\arctan(ax)}x\,dx=\frac{2\log(-ax+i)}{a^2}-\frac{2ix}{a}-\frac{x^2}{2}$$

```
[In] Int[x/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*x)/a - x^2/2 + (2*Log[I - a*x])/a^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(1-iax)}{1+iax}\,dx\\ &= \int \left(-\frac{2i}{a}-x+\frac{2}{a(-i+ax)}\right)dx\\ &= -\frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i-ax)}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(ax)}x\,dx = -\frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i-ax)}{a^2}$$

```
[In] Integrate[x/E^((2*I)*ArcTan[a*x]),x]
[Out] ((-2*I)*x)/a - x^2/2 + (2*Log[I - a*x])/a^2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{\frac{1}{2}a\,x^2+2ix}{a}+\frac{2\ln(-ax+i)}{a^2}$	31
risch	$-\frac{x^2}{2}-\frac{2ix}{a}+\frac{\ln(a^2x^2+1)}{a^2}+\frac{2i\arctan(ax)}{a^2}$	38
parallelrisch	$\frac{a^3x^3+3ia^2x^2-4\ln(ax-i)xa+4i\ln(ax-i)+4ax}{2a^2(-ax+i)}$	57
meijerg	$\frac{-\frac{iax\left(2a^2x^2+6iax+12\right)}{4(iax+1)}+3\ln(iax+1)}{a^2}-\frac{-\frac{iax}{iax+1}+\ln(iax+1)}{a^2}$	74

```
[In] int(x/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/a*(1/2*a*x^2+2*I*x)+2*ln(I-a*x)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int e^{-2i\arctan(ax)}x\,dx = -\frac{a^2x^2 + 4i\,ax - 4\,\log\left(\frac{ax-i}{a}\right)}{2\,a^2}$$

```
[In] integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
[Out] -1/2*(a^2*x^2 + 4*I*a*x - 4*log((a*x - I)/a))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int e^{-2i\arctan(ax)}x\,dx = -\frac{x^2}{2} - \frac{2ix}{a} + \frac{2\log(ax - i)}{a^2}$$

```
[In] integrate(x/(1+I*a*x)**2*(a**2*x**2+1),x)
[Out] -x**2/2 - 2*I*x/a + 2*log(a*x - I)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int e^{-2i\arctan(ax)}x\,dx = \frac{i\,(i\,ax^2 - 4\,x)}{2\,a} + \frac{2\,\log(i\,ax + 1)}{a^2}$$

```
[In] integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
[Out] 1/2*I*(I*a*x^2 - 4*x)/a + 2*log(I*a*x + 1)/a^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(24) = 48.

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int e^{-2i\arctan(ax)}x\,dx = -\frac{i\left(\frac{(i\,ax+1)^2\left(-\frac{6i}{i\,ax+1}+i\right)}{a} - \frac{4i\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a}\right)}{2\,a}$$

```
[In] integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
[Out] -1/2*I*((I*a*x + 1)^2*(-6*I/(I*a*x + 1) + I)/a - 4*I*log(1/(sqrt(a^2*x^2 +
1)*abs(a)))/a)/a
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(ax)}x\,dx=\frac{2\,\ln\left(x-\frac{1\mathrm{i}}{a}\right)}{a^2}-\frac{x^2}{2}-\frac{x\,2\mathrm{i}}{a}$$

```
[In] int((x*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
[Out] (2*log(x - 1i/a))/a^2 - (x*2i)/a - x^2/2
```

3.47 $\int e^{-2i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int e^{-2i\arctan(ax)}\,dx = -x - \frac{2i\log(i-ax)}{a}$$

```
[Out] -x-2*I*ln(I-a*x)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5169, 45}

$$\int e^{-2i\arctan(ax)}\,dx = -x - \frac{2i\log(-ax+i)}{a}$$

```
[In] Int[E^((-2*I)*ArcTan[a*x]),x]
```

```
[Out] -x - ((2*I)*Log[I - a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-iax}{1+iax}\,dx\\ &= \int \left(-1-\frac{2i}{-i+ax}\right)dx\\ &= -x-\frac{2i\log(i-ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int e^{-2i\arctan(ax)}\,dx = -x+\frac{2\arctan(ax)}{a}-\frac{i\log(1+a^2x^2)}{a}$$

```
[In] Integrate[E^((-2*I)*ArcTan[a*x]),x]
[Out] -x + (2*ArcTan[a*x])/a - (I*Log[1 + a^2*x^2])/a
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$-x-\frac{2i\ln(-ax+i)}{a}$	19
risch	$-x-\frac{i\ln(a^2x^2+1)}{a}+\frac{2\arctan(ax)}{a}$	30
parallelrisch	$\frac{2i\ln(ax-i)xa+a^2x^2+1+2\ln(ax-i)}{a(-ax+i)}$	44
meijerg	$\frac{i\left(\frac{iax(3iax+6)}{3iax+3}-2\ln(iax+1)\right)}{a}+\frac{x}{iax+1}$	51

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -x-2*I*ln(I-a*x)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int e^{-2i\arctan(ax)}\,dx = -\frac{ax + 2i\,\log\left(\frac{ax-i}{a}\right)}{a}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
[Out] -(a*x + 2*I*log((a*x - I)/a))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int e^{-2i\arctan(ax)}\,dx = -x - \frac{2i\log(ax - i)}{a}$$

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1),x)
[Out] -x - 2*I*log(a*x - I)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int e^{-2i\arctan(ax)}\,dx = -x - \frac{2i\,\log(i\,ax + 1)}{a}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
[Out] -x - 2*I*log(I*a*x + 1)/a
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int e^{-2i\arctan(ax)}\,dx = a^2\left(\frac{i\,(i\,ax + 1)}{a^3} + \frac{2i\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a^3} - \frac{i}{(i\,ax + 1)a^3}\right) + \frac{i}{(i\,ax + 1)a}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
[Out] a^2*(I*(I*a*x + 1)/a^3 + 2*I*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^3 - I/((I*
a*x + 1)*a^3)) + I/((I*a*x + 1)*a)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int e^{-2i\arctan(ax)}\,dx = -x - \frac{\ln\left(x - \frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a}$$

```
[In] int((a^2*x^2 + 1)/(a*x*1i + 1)^2,x)
[Out] - x - (log(x - 1i/a)*2i)/a
```

3.48 $\int \frac{e^{-2i \arctan(ax)}}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{e^{-2i \arctan(ax)}}{x} dx = \log(x) - 2\log(i - ax)$$

```
[Out] ln(x)-2*ln(I-a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int \frac{e^{-2i \arctan(ax)}}{x} dx = \log(x) - 2\log(-ax + i)$$

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x),x]
```

```
[Out] Log[x] - 2*Log[I - a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-iax}{x(1+iax)}\,dx\\ &= \int \left(\frac{1}{x}-\frac{2a}{-i+ax}\right)dx\\ &= \log(x)-2\log(i-ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx=\log(x)-2\log(i-ax)$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x),x]
[Out] Log[x] - 2*Log[I - a*x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$\ln(x)-2\ln(-ax+i)$	14
parallelrisch	$\frac{\ln(x)a-2\ln(ax-i)a}{a}$	20
risch	$\ln(x)-\ln(a^2x^2+1)-2i\arctan(ax)$	23
meijerg	$\frac{iax}{iax+1}-2\ln(iax+1)-\frac{2iax}{2iax+2}+1+\ln(x)+\ln(ia)$	48

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)-2*ln(I-a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = \log(x) - 2\,\log\left(\frac{ax-i}{a}\right)$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="fricas")
[Out] log(x) - 2*log((a*x - I)/a)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = \log(3ax) - 2\log(3ax - 3i)$$

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x,x)
[Out] log(3*a*x) - 2*log(3*a*x - 3*I)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = -2\,\log(i\,ax+1) + \log(x)$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="maxima")
[Out] -2*log(I*a*x + 1) + log(x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.14

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = i\,a\left(-\frac{i\,\log\left(\frac{i}{i\,ax+1} - i\right)}{a} - \frac{i\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a}\right)$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="giac")
[Out] I*a*(-I*log(I/(I*a*x + 1) - I)/a - I*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = \ln(x) - 2\,\ln\left(x - \frac{1\mathrm{i}}{a}\right)$$

```
[In] int((a^2*x^2 + 1)/(x*(a*x*1i + 1)^2),x)
[Out] log(x) - 2*log(x - 1i/a)
```

3.49 $\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -\frac{1}{x} - 2ia\log(x) + 2ia\log(i - ax)$$

```
[Out] -1/x-2*I*a*ln(x)+2*I*a*ln(I-a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -2ia\log(x) + 2ia\log(-ax + i) - \frac{1}{x}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] -x^(-1) - (2*I)*a*Log[x] + (2*I)*a*Log[I - a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-iax}{x^2(1+iax)}\,dx\\ &= \int \left(\frac{1}{x^2}-\frac{2ia}{x}+\frac{2ia^2}{-i+ax}\right)dx\\ &= -\frac{1}{x}-2ia\log(x)+2ia\log(i-ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -\frac{1}{x}-2ia\log(x)+2ia\log(i-ax)$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x^2),x]
[Out] -x^(-1) - (2*I)*a*Log[x] + (2*I)*a*Log[I - a*x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{x}-2ia\ln(x)+2ia\ln(-ax+i)$	25
risch	$-\frac{1}{x}-2ia\ln(x)-2a\arctan(ax)+ia\ln(a^2x^2+1)$	34
parallelrisch	$-\frac{2ia^2\ln(x)x-2ia^2\ln(ax-i)x+a}{ax}$	34
meijerg	$\frac{a^2x}{iax+1}+ia\left(\frac{3iax}{3iax+3}+2\ln(iax+1)-1-2\ln(x)-2\ln(ia)+\frac{i}{xa}\right)$	66

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x-2*I*a*ln(x)+2*I*a*ln(I-a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = \frac{-2i\,ax\log(x) + 2i\,ax\log\left(\frac{ax-i}{a}\right) - 1}{x}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="fricas")
[Out] (-2*I*a*x*log(x) + 2*I*a*x*log((a*x - I)/a) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -2a\left(i\log\left(4a^2x\right) - i\log\left(4a^2x - 4ia\right)\right) - \frac{1}{x}$$

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**2,x)
[Out] -2*a*(I*log(4*a**2*x) - I*log(4*a**2*x - 4*I*a)) - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = 2i\,a\log(i\,ax+1) - 2i\,a\log(x) - \frac{ax-i}{ax^2 - i\,x}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="maxima")
[Out] 2*I*a*log(I*a*x + 1) - 2*I*a*log(x) - (a*x - I)/(a*x^2 - I*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -2i\,a\log\left(\frac{i}{i\,ax+1} - i\right) - \frac{a}{\frac{i}{i\,ax+1} - i}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="giac")
[Out] -2*I*a*log(I/(I*a*x + 1) - I) - a/(I/(I*a*x + 1) - I)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -4\,a\,\mathrm{atan}(2\,a\,x-\mathrm{i})-\frac{1}{x}$$

```
[In] int((a^2*x^2 + 1)/(x^2*(a*x*1i + 1)^2),x)
[Out] - 4*a*atan(2*a*x - 1i) - 1/x
```

3.50 $\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2}+\frac{2ia}{x}-2a^2\log(x)+2a^2\log(i-ax)$$

```
[Out] -1/2/x^2+2*I*a/x-2*a^2*ln(x)+2*a^2*ln(I-a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -2a^2\log(x)+2a^2\log(-ax+i)+\frac{2ia}{x}-\frac{1}{2x^2}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x^3),x]
```

```
[Out] -1/2*1/x^2 + ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I - a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1-iax}{x^3(1+iax)}\,dx \\
&= \int \left(\frac{1}{x^3}-\frac{2ia}{x^2}-\frac{2a^2}{x}+\frac{2a^3}{-i+ax}\right)dx \\
&= -\frac{1}{2x^2}+\frac{2ia}{x}-2a^2\log(x)+2a^2\log(i-ax)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2}+\frac{2ia}{x}-2a^2\log(x)+2a^2\log(i-ax)$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x^3),x]
[Out] -1/2*1/x^2 + ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I - a*x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result
default	$-\frac{1}{2x^2}+\frac{2ia}{x}-2a^2\ln(x)+2a^2\ln(-ax+i)$
risch	$\frac{2iax-\frac{1}{2}}{x^2}-2a^2\ln(-x)+2ia^2\arctan(ax)+a^2\ln(a^2x^2+1)$
parallelrisch	$-\frac{-4\ln(x)x^3a^3+4\ln(ax-i)x^3a^3+4i\ln(x)x^2a^2-4i\ln(ax-i)x^2a^2+i+4a^3x^3+3ax}{2(-ax+i)x^2}$
meijerg	$a^2\left(-\frac{2iax}{2iax+2}-\ln(iax+1)+1+\ln(x)+\ln(ia)\right)-a^2\left(-\frac{4iax}{4iax+4}-3\ln(iax+1)+1+3\ln(\right.$

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2/x^2+2*I*a/x-2*a^2*ln(x)+2*a^2*ln(I-a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -\frac{4\,a^2x^2\log\left(x\right) - 4\,a^2x^2\log\left(\frac{ax-i}{a}\right) - 4i\,ax + 1}{2\,x^2}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="fricas")
[Out] -1/2*(4*a^2*x^2*log(x) - 4*a^2*x^2*log((a*x - I)/a) - 4*I*a*x + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -2a^2\left(\log\left(4a^3x\right) - \log\left(4a^3x - 4ia^2\right)\right) - \frac{-4iax + 1}{2x^2}$$

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**3,x)
[Out] -2*a**2*(log(4*a**3*x) - log(4*a**3*x - 4*I*a**2)) - (-4*I*a*x + 1)/(2*x**2
)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = 2\,a^2\log\left(i\,ax + 1\right) - 2\,a^2\log\left(x\right) - \frac{4\,a^2x^2 - 3i\,ax + 1}{2i\,ax^3 + 2\,x^2}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="maxima")
[Out] 2*a^2*log(I*a*x + 1) - 2*a^2*log(x) - (4*a^2*x^2 - 3*I*a*x + 1)/(2*I*a*x^3
+ 2*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -2\,a^2\log\left(\frac{i}{i\,ax+1}-i\right)+\frac{5\,a^2-\frac{6\,a^2}{i\,ax+1}}{2\left(\frac{i}{i\,ax+1}-i\right)^2}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] -2*a^2*log(I/(I*a*x + 1) - I) + 1/2*(5*a^2 - 6*a^2/(I*a*x + 1))/(I/(I*a*x +
 1) - I)^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = a^2\,\mathrm{atan}(2\,a\,x-\mathrm{i})\,4\mathrm{i}+\frac{-\frac{1}{2}+a\,x\,2\mathrm{i}}{x^2}$$

```
[In] int((a^2*x^2 + 1)/(x^3*(a*x*1i + 1)^2),x)
```

```
[Out] a^2*atan(2*a*x - 1i)*4i + (a*x*2i - 1/2)/x^2
```

3.51 $\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3} + \frac{ia}{x^2} + \frac{2a^2}{x} + 2ia^3\log(x) - 2ia^3\log(i-ax)$$

```
[Out] -1/3/x^3+I*a/x^2+2*a^2/x+2*I*a^3*ln(x)-2*I*a^3*ln(I-a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5170, 78}

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = 2ia^3\log(x) - 2ia^3\log(-ax+i) + \frac{2a^2}{x} + \frac{ia}{x^2} - \frac{1}{3x^3}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] -1/3*1/x^3 + (I*a)/x^2 + (2*a^2)/x + (2*I)*a^3*Log[x] - (2*I)*a^3*Log[I - a
*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-iax}{x^4(1+iax)}\,dx\\ &= \int \left(\frac{1}{x^4}-\frac{2ia}{x^3}-\frac{2a^2}{x^2}+\frac{2ia^3}{x}-\frac{2ia^4}{-i+ax}\right)dx\\ &= -\frac{1}{3x^3}+\frac{ia}{x^2}+\frac{2a^2}{x}+2ia^3\log(x)-2ia^3\log(i-ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3}+\frac{ia}{x^2}+\frac{2a^2}{x}+2ia^3\log(x)-2ia^3\log(i-ax)$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*x^4),x]
[Out] -1/3*1/x^3 + (I*a)/x^2 + (2*a^2)/x + (2*I)*a^3*Log[x] - (2*I)*a^3*Log[I - a
*x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
default	$-\frac{1}{3x^3}+\frac{ia}{x^2}+\frac{2a^2}{x}+2ia^3\ln(x)-2ia^3\ln(-ax+i)$
risch	$\frac{2a^2x^2+iax-\frac{1}{3}}{x^3}+2a^3\arctan(ax)-ia^3\ln(a^2x^2+1)+2ia^3\ln(-x)$
parallelrisch	$-\frac{6i\ln(x)x^4a^5-6i\ln(ax-i)x^4a^5+6\ln(x)x^3a^4-6\ln(ax-i)x^3a^4+6a^4x^3-3ia^3x^2+2a^2x+ia}{3a(-ax+i)x^3}$
meijerg	$ia^3\left(\frac{3iax}{3iax+3}+2\ln(iax+1)-1-2\ln(x)-2\ln(ia)+\frac{i}{xa}\right)-ia^3\left(\frac{5iax}{5iax+5}+4\ln(iax+1)-1\right.$

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3/x^3+I*a/x^2+2*a^2/x+2*I*a^3*ln(x)-2*I*a^3*ln(I-a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = \frac{6i\,a^3x^3\log(x) - 6i\,a^3x^3\log\left(\frac{ax-i}{a}\right) + 6\,a^2x^2 + 3i\,ax - 1}{3\,x^3}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*I*a^3*x^3*log(x) - 6*I*a^3*x^3*log((a*x - I)/a) + 6*a^2*x^2 + 3*I*a*
x - 1)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = -2a^3\left(-i\log\left(4a^4x\right) + i\log\left(4a^4x - 4ia^3\right)\right) - \frac{-6a^2x^2 - 3iax + 1}{3x^3}$$

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**4,x)
```

```
[Out] -2*a**3*(-I*log(4*a**4*x) + I*log(4*a**4*x - 4*I*a**3)) - (-6*a**2*x**2 - 3
*I*a*x + 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = -2i\,a^3\log(i\,ax + 1) + 2i\,a^3\log(x) + \frac{6i\,a^3x^3 + 3\,a^2x^2 + 2i\,ax - 1}{3i\,ax^4 + 3\,x^3}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -2*I*a^3*log(I*a*x + 1) + 2*I*a^3*log(x) + (6*I*a^3*x^3 + 3*a^2*x^2 + 2*I*a
*x - 1)/(3*I*a*x^4 + 3*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = 2i\,a^3\log\left(\frac{i}{i\,ax+1}-i\right)-\frac{10\,a^3-\frac{24\,a^3}{i\,ax+1}+\frac{15\,a^3}{(i\,ax+1)^2}}{3\left(\frac{i}{i\,ax+1}-i\right)^3}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*I*a^3*log(I/(I*a*x + 1) - I) - 1/3*(10*a^3 - 24*a^3/(I*a*x + 1) + 15*a^3/
(I*a*x + 1)^2)/(I/(I*a*x + 1) - I)^3
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = 4\,a^3\,\mathrm{atan}(2\,a\,x-\mathrm{i})+\frac{2\,a^2\,x^2+a\,x\,1\mathrm{i}-\frac{1}{3}}{x^3}$$

```
[In] int((a^2*x^2 + 1)/(x^4*(a*x*1i + 1)^2),x)
```

```
[Out] 4*a^3*atan(2*a*x - 1i) + (a*x*1i + 2*a^2*x^2 - 1/3)/x^3
```

3.52 $\int e^{-3i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 137

$$\int e^{-3i\arctan(ax)}x^3\,dx=\frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}}+\frac{27\sqrt{1+a^2x^2}}{4a^4}-\frac{x^2\sqrt{1+a^2x^2}}{a^2}+\frac{ix^3\sqrt{1+a^2x^2}}{4a}-\frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4}+\frac{51i\text{arcsinh}(ax)}{8a^4}$$

```
[Out] 51/8*I*arcsinh(a*x)/a^4+(1-I*a*x)^3/a^4/(a^2*x^2+1)^(1/2)+27/4*(a^2*x^2+1)^
(1/2)/a^4-x^2*(a^2*x^2+1)^(1/2)/a^2+1/4*I*x^3*(a^2*x^2+1)^(1/2)/a-9/8*I*(2*
I+3*a*x)*(a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5168, 1647, 1607, 12, 866, 1649, 1829, 27, 757, 655, 221}

$$\int e^{-3i\arctan(ax)}x^3\,dx=\frac{51i\text{arcsinh}(ax)}{8a^4}-\frac{x^2\sqrt{a^2x^2+1}}{a^2}+\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{9i(3ax+2i)\sqrt{a^2x^2+1}}{8a^4}+\frac{27\sqrt{a^2x^2+1}}{4a^4}+\frac{(1-iax)^3}{a^4\sqrt{a^2x^2+1}}$$

```
[In] Int[x^3/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (1 - I*a*x)^3/(a^4*Sqrt[1 + a^2*x^2]) + (27*Sqrt[1 + a^2*x^2])/(4*a^4) - (x
^2*Sqrt[1 + a^2*x^2])/a^2 + ((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - (((9*I)/8)*(2
*I + 3*a*x)*Sqrt[1 + a^2*x^2])/a^4 + (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
```

```
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= (ia)\int \frac{\sqrt{1+a^2x^2}\left(-\frac{ix^3}{a}-x^4\right)}{(1+iax)^2}\,dx \\
&= (ia)\int \frac{\left(-\frac{i}{a}-x\right)x^3\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx \\
&= a^2\int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{a^2(1+iax)^3}\,dx \\
&= \int \frac{x^3\left(1+a^2x^2\right)^{3/2}}{(1+iax)^3}\,dx \\
&= \int \frac{x^3(1-iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx
\end{aligned}$$

$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \int \frac{(1-iax)^2\left(-\frac{3i}{a^3}-\frac{x}{a^2}+\frac{ix^2}{a}\right)}{\sqrt{1+a^2x^2}}dx$$

$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{-\frac{12i}{a}-28x+27iax^2+12a^2x^3}{\sqrt{1+a^2x^2}}dx}{4a^2}$$

$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{-36ia-108a^2x+81ia^3x^2}{\sqrt{1+a^2x^2}}dx}{12a^4}$$

$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{\int \frac{9ia(2i+3ax)^2}{\sqrt{1+a^2x^2}}dx}{12a^4}$$

$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{(3i)\int \frac{(2i+3ax)^2}{\sqrt{1+a^2x^2}}dx}{4a^3}$$

$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4} - \frac{(3i)\int \frac{-17a^2+18ia^3x}{\sqrt{1+a^2x^2}}dx}{8a^5}$$

$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4} + \frac{(51i)\int \frac{1}{\sqrt{1+a^2x^2}}dx}{8a^3}$$

$$= \frac{(1-iax)^3}{a^4\sqrt{1+a^2x^2}} + \frac{27\sqrt{1+a^2x^2}}{4a^4} - \frac{x^2\sqrt{1+a^2x^2}}{a^2} + \frac{ix^3\sqrt{1+a^2x^2}}{4a} - \frac{9i(2i+3ax)\sqrt{1+a^2x^2}}{8a^4} + \frac{51i\operatorname{arcsinh}(ax)}{8a^4}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int e^{-3i\arctan(ax)}x^3\,dx = \sqrt{1+a^2x^2}\left(\frac{6}{a^4}-\frac{19ix}{8a^3}-\frac{x^2}{a^2}+\frac{ix^3}{4a}-\frac{4i}{a^4(-i+ax)}\right)+\frac{51i\operatorname{arcsinh}(ax)}{8a^4}$$

```
[In] Integrate[x^3/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(6/a^4 - (((19*I)/8)*x)/a^3 - x^2/a^2 + ((I/4)*x^3)/a - (
4*I)/(a^4*(-I + a*x))) + (((51*I)/8)*ArcSinh[a*x])/a^4
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result
risch	$\frac{i(2a^3x^3+8ia^2x^2-19ax-48i)\sqrt{a^2x^2+1}}{8a^4}+\frac{i\left(\frac{51\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}-\frac{32\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^2\left(x-\frac{i}{a}\right)}\right)}{8a^3}$
default	$\frac{i\left(\frac{x\left(a^2x^2+1\right)^{\frac{3}{2}}}{4}+\frac{3\sqrt{a^2x^2+1}\,x}{8}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8\sqrt{a^2}}\right)}{a^3}+\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3}-2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2}+3ia\left(\right.\right.}{}$

```
[In] int(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*(2*a^3*x^3+8*I*a^2*x^2-19*a*x-48*I)*(a^2*x^2+1)^(1/2)/a^4+1/8*I/a^3*(
51*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-32/a^2/(x-I/a)*((x-I
/a)^2*a^2+2*I*a*(x-I/a))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.64

$$\int e^{-3i\arctan(ax)}x^3\,dx$$
$$=\frac{-32i\,ax-51\,(i\,ax+1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+(2i\,a^4x^4-6\,a^3x^3-11i\,a^2x^2+29\,ax-80i)\sqrt{a^2x^2+1}-}{8\,(a^5x-i\,a^4)}$$

```
[In] integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(-32*I*a*x - 51*(I*a*x + 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + (2*I*a^4*x^
4 - 6*a^3*x^3 - 11*I*a^2*x^2 + 29*a*x - 80*I)*sqrt(a^2*x^2 + 1) - 32)/(a^5*
x - I*a^4)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}x^3\,dx=i\left(\int\frac{x^3\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int\frac{a^2x^5\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

```
[In] integrate(x**3/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] I*(Integral(x**3*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I
), x) + Integral(a**2*x**5*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 -
 3*a*x + I), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.58

$$\int e^{-3i\arctan(ax)}x^3\,dx = \frac{(a^2x^2+1)^{\frac{3}{2}}}{a^6x^2-2i\,a^5x-a^4}+\frac{3\,(a^2x^2+1)^{\frac{3}{2}}}{2i\,a^5x+2\,a^4}+\frac{6\,\sqrt{a^2x^2+1}}{i\,a^5x+a^4}+\frac{i\,(a^2x^2+1)^{\frac{3}{2}}x}{4\,a^3}+\frac{3i\,\sqrt{a^2x^2+1}x}{8\,a^3}-\frac{3i\,\sqrt{-a^2x^2+4i\,ax+3}x}{2\,a^3}-\frac{(a^2x^2+1)^{\frac{3}{2}}}{a^4}+\frac{3i\,\arcsin(i\,ax+2)}{2\,a^4}+\frac{63i\,\operatorname{arsinh}(ax)}{8\,a^4}+\frac{9\,\sqrt{a^2x^2+1}}{2\,a^4}-\frac{3\,\sqrt{-a^2x^2+4i\,ax+3}}{a^4}$$

```
[In] integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] (a^2*x^2 + 1)^(3/2)/(a^6*x^2 - 2*I*a^5*x - a^4) + 3*(a^2*x^2 + 1)^(3/2)/(2*
I*a^5*x + 2*a^4) + 6*sqrt(a^2*x^2 + 1)/(I*a^5*x + a^4) + 1/4*I*(a^2*x^2 + 1
)^(3/2)*x/a^3 + 3/8*I*sqrt(a^2*x^2 + 1)*x/a^3 - 3/2*I*sqrt(-a^2*x^2 + 4*I*a
*x + 3)*x/a^3 - (a^2*x^2 + 1)^(3/2)/a^4 + 3/2*I*arcsin(I*a*x + 2)/a^4 + 63/
8*I*arcsinh(a*x)/a^4 + 9/2*sqrt(a^2*x^2 + 1)/a^4 - 3*sqrt(-a^2*x^2 + 4*I*a*
x + 3)/a^4
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int e^{-3i\arctan(ax)}x^3\,dx=\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{4}{{\left(a^2\right)}^{3/2}}+\frac{2\,\sqrt{a^2}}{a^4}-\frac{x^2\,\sqrt{a^2}}{a^2}+\frac{x^3\,{\left(a^2\right)}^{3/2}\,1{}\mathrm{i}}{4\,a^3}-\frac{x\,\sqrt{a^2}\,19{}\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}}+\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,51{}\mathrm{i}}{8\,a^3\,\sqrt{a^2}}+\frac{\sqrt{a^2\,x^2+1}\,4{}\mathrm{i}}{a^3\,\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1{}\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((x^3*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*(4/(a^2)^(3/2) + (2*(a^2)^(1/2))/a^4 - (x^2*(a^2)^(1/2
))/a^2 + (x^3*(a^2)^(3/2)*1i)/(4*a^3) - (x*(a^2)^(1/2)*19i)/(8*a^3)))/(a^2)
^(1/2) + (asinh(x*(a^2)^(1/2))*51i)/(8*a^3*(a^2)^(1/2)) + ((a^2*x^2 + 1)^(1
/2)*4i)/(a^3*(((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.53 $\int e^{-3i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int e^{-3i\arctan(ax)}x^2\,dx=-\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}}-\frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3}-\frac{(28i+3ax)\sqrt{1+a^2x^2}}{6a^3}+\frac{11\text{arcsinh}(ax)}{2a^3}$$

```
[Out] 11/2*arcsinh(a*x)/a^3-I*(1-I*a*x)^3/a^3/(a^2*x^2+1)^(1/2)-1/3*I*(3-I*a*x)^2
*(a^2*x^2+1)^(1/2)/a^3-1/6*(28*I+3*a*x)*(a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5168, 1647, 1607, 12, 866, 1649, 1668, 794, 221}

$$\int e^{-3i\arctan(ax)}x^2\,dx=\frac{11\text{arcsinh}(ax)}{2a^3}-\frac{i(1-iax)^3}{a^3\sqrt{a^2x^2+1}}-\frac{i(3-iax)^2\sqrt{a^2x^2+1}}{3a^3}-\frac{(3ax+28i)\sqrt{a^2x^2+1}}{6a^3}$$

```
[In] Int[x^2/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*(1 - I*a*x)^3)/(a^3*Sqrt[1 + a^2*x^2]) - ((I/3)*(3 - I*a*x)^2*Sqrt[1
+ a^2*x^2])/a^3 - ((28*I + 3*a*x)*Sqrt[1 + a^2*x^2])/(6*a^3) + (11*ArcSinh[
a*x])/(2*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= (ia)\int \frac{\sqrt{1+a^2x^2}\left(-\frac{ix^2}{a}-x^3\right)}{(1+iax)^2}\,dx \\
&= (ia)\int \frac{\left(-\frac{i}{a}-x\right)x^2\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx \\
&= a^2\int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{a^2(1+iax)^3}\,dx \\
&= \int \frac{x^2\left(1+a^2x^2\right)^{3/2}}{(1+iax)^3}\,dx \\
&= \int \frac{x^2(1-iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx \\
&= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \int \frac{\left(-\frac{3}{a^2}+\frac{ix}{a}\right)(1-iax)^2}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3} + \frac{1}{3}\int \frac{\left(-\frac{3}{a^2}+\frac{ix}{a}\right)(-5+3iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3} - \frac{(28i+3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{11\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a^2}
\end{aligned}
$$

$$= -\frac{i(1-iax)^3}{a^3\sqrt{1+a^2x^2}} - \frac{i(3-iax)^2\sqrt{1+a^2x^2}}{3a^3} - \frac{(28i+3ax)\sqrt{1+a^2x^2}}{6a^3} + \frac{11\text{arcsinh}(ax)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int e^{-3i\arctan(ax)}x^2\,dx = \frac{\frac{\sqrt{1+a^2x^2}(-52-19iax-7a^2x^2+2ia^3x^3)}{-i+ax} + 33\text{arcsinh}(ax)}{6a^3}$$

```
[In] Integrate[x^2/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(-52 - (19*I)*a*x - 7*a^2*x^2 + (2*I)*a^3*x^3))/(-I + a
*x) + 33*ArcSinh[a*x])/(6*a^3)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
risch	$\frac{i(2a^2x^2+9iax-28)\sqrt{a^2x^2+1}}{6a^3} + \frac{11\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}} - \frac{4\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^4\left(x-\frac{i}{a}\right)}$
default	$-\frac{2\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2}+3ia\left(\frac{\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{3}+ia\left(\frac{\left(2\left(x-\frac{i}{a}\right)a^2+2ia\right)\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{4a^2}+\frac{\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}\right.}{}\right.\right.\right.}{a^4}$

```
[In] int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*(2*a^2*x^2+9*I*a*x-28)*(a^2*x^2+1)^(1/2)/a^3+11/2/a^2*ln(a^2*x/(a^2)^
(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)-4/a^4/(x-I/a)*((x-I/a)^2*a^2+2*I*a*(x-
I/a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int e^{-3i\arctan(ax)}x^2\,dx =$$

$$-\frac{24\,ax + 33\,(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}\right) - (2i\,a^3x^3 - 7\,a^2x^2 - 19i\,ax - 52)\sqrt{a^2x^2+1} - 24i}{6\,(a^4x - i\,a^3)}$$

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(24*a*x + 33*(a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1)) - (2*I*a^3*x^3 -
7*a^2*x^2 - 19*I*a*x - 52)*sqrt(a^2*x^2 + 1) - 24*I)/(a^4*x - I*a^3)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}x^2\,dx = i\left(\int\frac{x^2\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int\frac{a^2x^4\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

```
[In] integrate(x**2/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] I*(Integral(x**2*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I
), x) + Integral(a**2*x**4*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 -
 3*a*x + I), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(80) = 160$.

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.77

$$\int e^{-3i\arctan(ax)}x^2\,dx = -\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^5x^2-2i\,a^4x-a^3}-\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{i\,a^4x+a^3}-\frac{6i\sqrt{a^2x^2+1}}{i\,a^4x+a^3}-\frac{\sqrt{-a^2x^2+4i\,ax+3}\,x}{2\,a^2}+\frac{i\left(a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^3}+\frac{\arcsin\left(i\,ax+2\right)}{2\,a^3}+\frac{6\,\operatorname{arsinh}\left(ax\right)}{a^3}-\frac{3i\sqrt{a^2x^2+1}}{a^3}+\frac{i\sqrt{-a^2x^2+4i\,ax+3}}{a^3}$$

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(a^2*x^2 + 1)^(3/2)/(a^5*x^2 - 2*I*a^4*x - a^3) - I*(a^2*x^2 + 1)^(3/2)/
(I*a^4*x + a^3) - 6*I*sqrt(a^2*x^2 + 1)/(I*a^4*x + a^3) - 1/2*sqrt(-a^2*x^2
 + 4*I*a*x + 3)*x/a^2 + 1/3*I*(a^2*x^2 + 1)^(3/2)/a^3 + 1/2*arcsin(I*a*x +
2)/a^3 + 6*arcsinh(a*x)/a^3 - 3*I*sqrt(a^2*x^2 + 1)/a^3 + I*sqrt(-a^2*x^2 +
 4*I*a*x + 3)/a^3
```

Giac [F]

$$\int e^{-3i\arctan(ax)}x^2\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}x^2}{(i\,ax+1)^3}\,dx$$

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13

$$\int e^{-3i\arctan(ax)}x^2\,dx = \frac{11\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,x\,\sqrt{a^2}}{2\,a^2}+\frac{a\,14\mathrm{i}}{3\,(a^2)^{3/2}}-\frac{a^3\,x^2\,1\mathrm{i}}{3\,(a^2)^{3/2}}\right)}{\sqrt{a^2}} + \frac{4\,\sqrt{a^2\,x^2+1}}{a^2\,\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((x^2*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

```
[Out] (11*asinh(x*(a^2)^(1/2)))/(2*a^2*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*((a*14
i)/(3*(a^2)^(3/2)) - (a^3*x^2*1i)/(3*(a^2)^(3/2)) + (3*x*(a^2)^(1/2))/(2*a^
2)))/(a^2)^(1/2) + (4*(a^2*x^2 + 1)^(1/2))/(a^2*(((a^2)^(1/2)*1i)/a - x*(a^
2)^(1/2))*(a^2)^(1/2))
```

3.54 $\int e^{-3i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 92

$$\int e^{-3i\arctan(ax)}x\,dx=-\frac{9\sqrt{1+a^2x^2}}{2a^2}-\frac{3(1+a^2x^2)^{3/2}}{2a^2(1+iax)}-\frac{(1+a^2x^2)^{5/2}}{a^2(1+iax)^3}-\frac{9i\text{arcsinh}(ax)}{2a^2}$$

```
[Out] -3/2*(a^2*x^2+1)^(3/2)/a^2/(1+I*a*x)-(a^2*x^2+1)^(5/2)/a^2/(1+I*a*x)^3-9/2*
I*arcsinh(a*x)/a^2-9/2*(a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5168, 1647, 1607, 12, 807, 679, 221}

$$\int e^{-3i\arctan(ax)}x\,dx=-\frac{9i\text{arcsinh}(ax)}{2a^2}-\frac{(a^2x^2+1)^{5/2}}{a^2(1+iax)^3}-\frac{3(a^2x^2+1)^{3/2}}{2a^2(1+iax)}-\frac{9\sqrt{a^2x^2+1}}{2a^2}$$

```
[In] Int[x/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (-9*Sqrt[1 + a^2*x^2])/(2*a^2) - (3*(1 + a^2*x^2)^(3/2))/(2*a^2*(1 + I*a*x)
) - (1 + a^2*x^2)^(5/2)/(a^2*(1 + I*a*x)^3) - (((9*I)/2)*ArcSinh[a*x])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= (ia)\int \frac{\left(-\frac{ix}{a}-x^2\right)\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx \\
&= (ia)\int \frac{\left(-\frac{i}{a}-x\right)x\sqrt{1+a^2x^2}}{(1+iax)^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= a^2 \int \frac{x(1+a^2x^2)^{3/2}}{a^2(1+iax)^3}\,dx \\
&= \int \frac{x(1+a^2x^2)^{3/2}}{(1+iax)^3}\,dx \\
&= -\frac{(1+a^2x^2)^{5/2}}{a^2(1+iax)^3} - \frac{(3i)\int \frac{(1+a^2x^2)^{3/2}}{(1+iax)^2}\,dx}{a} \\
&= -\frac{3(1+a^2x^2)^{3/2}}{2a^2(1+iax)} - \frac{(1+a^2x^2)^{5/2}}{a^2(1+iax)^3} - \frac{(9i)\int \frac{\sqrt{1+a^2x^2}}{1+iax}\,dx}{2a} \\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2} - \frac{3(1+a^2x^2)^{3/2}}{2a^2(1+iax)} - \frac{(1+a^2x^2)^{5/2}}{a^2(1+iax)^3} - \frac{(9i)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{2a} \\
&= -\frac{9\sqrt{1+a^2x^2}}{2a^2} - \frac{3(1+a^2x^2)^{3/2}}{2a^2(1+iax)} - \frac{(1+a^2x^2)^{5/2}}{a^2(1+iax)^3} - \frac{9i\text{arcsinh}(ax)}{2a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$
\int e^{-3i\arctan(ax)}x\,dx = \sqrt{1+a^2x^2}\left(-\frac{3}{a^2}+\frac{ix}{2a}+\frac{4i}{a^2(-i+ax)}\right) - \frac{9i\text{arcsinh}(ax)}{2a^2}
$$

```
[In] Integrate[x/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-3/a^2 + ((I/2)*x)/a + (4*I)/(a^2*(-I + a*x))) - (((9*I)
/2)*ArcSinh[a*x])/a^2
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

method	result
risch	$\frac{i(ax+6i)\sqrt{a^2x^2+1}}{2a^2} - \frac{i\left(\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}} - \frac{8\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^2\left(x-\frac{i}{a}\right)}\right)}{2a}$
default	$-\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3} - 2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2} + 3ia\left(\frac{\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{3} + ia\left(\frac{\left(2\left(x-\frac{i}{a}\right)a^2+2ia\right)\sqrt{\left(x-\frac{i}{a}\right.}}{4a^2}\right.\right.\right.}{a^4}$

```
[In] int(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(a*x+6*I)*(a^2*x^2+1)^(1/2)/a^2-1/2*I/a*(9*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2+1)^(1/2))/(a^2)^(1/2)-8/a^2/(x-I/a)*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.78

$$\int e^{-3i\arctan(ax)}x\,dx = \frac{8i\,ax-9\left(-i\,ax-1\right)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}(i\,a^2x^2-5\,ax+14i)+8}{2\left(a^3x-i\,a^2\right)}$$

```
[In] integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(8*I*a*x - 9*(-I*a*x - 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2
+ 1)*(I*a^2*x^2 - 5*a*x + 14*I) + 8)/(a^3*x - I*a^2)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}x\,dx = i\left(\int\frac{x\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int\frac{a^2x^3\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

```
[In] integrate(x/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

```
[Out] I*(Integral(x*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I),
x) + Integral(a**2*x**3*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*
a*x + I), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int e^{-3i\arctan(ax)}x\,dx = -\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^4x^2-2i\,a^3x-a^2}-\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{2i\,a^3x+2\,a^2}-\frac{6\,\sqrt{a^2x^2+1}}{i\,a^3x+a^2}-\frac{9i\,\operatorname{arsinh}\left(ax\right)}{2\,a^2}-\frac{3\,\sqrt{a^2x^2+1}}{2\,a^2}$$

```
[In] integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 + 1)^(3/2)/(a^4*x^2 - 2*I*a^3*x - a^2) - (a^2*x^2 + 1)^(3/2)/(2*I
*a^3*x + 2*a^2) - 6*sqrt(a^2*x^2 + 1)/(I*a^3*x + a^2) - 9/2*I*arcsinh(a*x)/
a^2 - 3/2*sqrt(a^2*x^2 + 1)/a^2
```

Giac [F]

$$\int e^{-3i\arctan(ax)}x\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}x}{(i\,ax+1)^3}\,dx$$

```
[In] integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.14

$$\int e^{-3i\arctan(ax)}x\,dx = -\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,\sqrt{a^2}}{a^2}-\frac{x\,\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)}{\sqrt{a^2}} - \frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,9\mathrm{i}}{2\,a\,\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{a\,\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((x*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

```
[Out] - ((a^2*x^2 + 1)^(1/2)*((3*(a^2)^(1/2))/a^2 - (x*(a^2)^(1/2)*1i)/(2*a)))/(a
^2)^(1/2) - (asinh(x*(a^2)^(1/2))*9i)/(2*a*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1
/2)*4i)/(a*(((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.55 $\int e^{-3i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int e^{-3i\arctan(ax)}\,dx=\frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}}+\frac{3i\sqrt{1+a^2x^2}}{a}-\frac{3\operatorname{arcsinh}(ax)}{a}$$

```
[Out] -3*arcsinh(a*x)/a+2*I*(1-I*a*x)^2/a/(a^2*x^2+1)^(1/2)+3*I*(a^2*x^2+1)^(1/2)
/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5167, 867, 683, 655, 221}

$$\int e^{-3i\arctan(ax)}\,dx=\frac{2i(1-iax)^2}{a\sqrt{a^2x^2+1}}+\frac{3i\sqrt{a^2x^2+1}}{a}-\frac{3\operatorname{arcsinh}(ax)}{a}$$

```
[In] Int[E^((-3*I)*ArcTan[a*x]),x]
```

```
[Out] ((2*I)*(1 - I*a*x)^2)/(a*Sqrt[1 + a^2*x^2]) + ((3*I)*Sqrt[1 + a^2*x^2])/a -
 (3*ArcSinh[a*x])/a
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 5167

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx\\
&= \int \frac{(1-iax)^3}{\left(1+a^2x^2\right)^{3/2}}\,dx\\
&= \frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}} - 3\int \frac{1-iax}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}} + \frac{3i\sqrt{1+a^2x^2}}{a} - 3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{2i(1-iax)^2}{a\sqrt{1+a^2x^2}} + \frac{3i\sqrt{1+a^2x^2}}{a} - \frac{3\operatorname{arcsinh}(ax)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int e^{-3i\arctan(ax)}\,dx=\frac{\sqrt{1+a^2x^2}\left(i+\frac{4}{-i+ax}\right)}{a}-\frac{3\text{arcsinh}(ax)}{a}$$

```
[In] Integrate[E^((-3*I)*ArcTan[a*x]),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*(I + 4/(-I + a*x)))/a - (3*ArcSinh[a*x])/a
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.55

method	result
risch	$\frac{i\sqrt{a^2x^2+1}}{a}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}+\frac{4\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^2\left(x-\frac{i}{a}\right)}$
default	$\frac{i\left(\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3}-2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2}+3ia\left(\frac{\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{3}+ia\left(\frac{\left(2\left(x-\frac{i}{a}\right)a^2+2ia\right)\sqrt{\left(x-\frac{i}{a}\right)^2}}{4a^2}\right.\right.\right.\right.}{a^3}$

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(a^2*x^2+1)^(1/2)/a-3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
+4/a^2/(x-I/a)*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int e^{-3i\arctan(ax)}\,dx=\frac{4\,ax+3\,(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}(i\,ax+5)-4i}{a^2x-i\,a}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + 3*(a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)*(I*a
*x + 5) - 4*I)/(a^2*x - I*a)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I), x)
 + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*
x + I), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int e^{-3i\arctan(ax)}\,dx = \frac{i\,(a^2x^2+1)^{\frac{3}{2}}}{a^3x^2-2i\,a^2x-a} - \frac{3\,\mathrm{arsinh}\,(ax)}{a} + \frac{6i\,\sqrt{a^2x^2+1}}{i\,a^2x+a}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] I*(a^2*x^2 + 1)^(3/2)/(a^3*x^2 - 2*I*a^2*x - a) - 3*arcsinh(a*x)/a + 6*I*sq
rt(a^2*x^2 + 1)/(I*a^2*x + a)
```

Giac [F]

$$\int e^{-3i\arctan(ax)}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int e^{-3i\arctan(ax)}\,dx = \frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a} - \frac{3\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{\sqrt{a^2}} - \frac{4\,\sqrt{a^2\,x^2+1}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a^2*x^2 + 1)^(3/2)/(a*x*1i + 1)^3,x)
[Out] ((a^2*x^2 + 1)^(1/2)*1i)/a - (3*asinh(x*(a^2)^(1/2)))/(a^2)^(1/2) - (4*(a^2
*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.56 $\int \frac{e^{-3i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = \frac{4i\sqrt{1+a^2x^2}}{i-ax} + i\text{arcsinh}(ax) - \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))+4*I*(a^2*x^2+1)^(1/2)/(I-a*x)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5168, 6874, 221, 272, 65, 214, 665}

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = -\text{arctanh}\left(\sqrt{a^2x^2+1}\right) + \frac{4i\sqrt{a^2x^2+1}}{-ax+i} + i\text{arcsinh}(ax)$$

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x),x]
```

```
[Out] ((4*I)*Sqrt[1 + a^2*x^2])/(I - a*x) + I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2
*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^2}{x(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{ia}{\sqrt{1+a^2x^2}}+\frac{1}{x\sqrt{1+a^2x^2}}-\frac{4a}{(-i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= (ia)\int \frac{1}{\sqrt{1+a^2x^2}}\,dx-(4a)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= \frac{4i\sqrt{1+a^2x^2}}{i-ax}+i\text{arcsinh}(ax)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)
\end{aligned}$$

$$= \frac{4i\sqrt{1+a^2x^2}}{i-ax} + i\text{arcsinh}(ax) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a^2}$$
$$= \frac{4i\sqrt{1+a^2x^2}}{i-ax} + i\text{arcsinh}(ax) - \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = -\frac{4i\sqrt{1+a^2x^2}}{-i+ax} + i\text{arcsinh}(ax) + \log(x) - \log\left(1+\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x),x]
[Out] ((-4*I)*Sqrt[1 + a^2*x^2])/(-I + a*x) + I*ArcSinh[a*x] + Log[x] - Log[1 + S
qrt[1 + a^2*x^2]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 649, normalized size of antiderivative = 12.48

method	result
default	$\frac{(a^2x^2+1)^{\frac{3}{2}}}{3} + \sqrt{a^2x^2+1} - \text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right) + \frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3} - 2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2} + 3ia \cdots\right.}{\cdots}$

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/3*(a^2*x^2+1)^(3/2)+(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))+1/a^2*
(I/a/(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-2*I*a*(-I/a/(x-I/a)^2*((
x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)+3*I*a*(1/3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^
(3/2)+I*a*(1/4*(2*(x-I/a)*a^2+2*I*a)/a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2
)+1/2*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))
/(a^2)^(1/2))))-1/3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)-I*a*(1/4*(2*(x-I/a
)*a^2+2*I*a)/a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)+1/2*ln((I*a+(x-I/a)*a^
2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2))+I/a*(-I/a/
(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)+3*I*a*(1/3*((x-I/a)^2*a^2+2*I
*a*(x-I/a))^(3/2)+I*a*(1/4*(2*(x-I/a)*a^2+2*I*a)/a^2*((x-I/a)^2*a^2+2*I*a*(
```

```
x-I/a))^(1/2)+1/2*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-
I/a))^(1/2))/(a^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(41) = 82$.

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.92

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = \frac{-4i\,ax-(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+(-i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+(ax-i)\log\left(-ax\right.}{ax-i}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (-4*I*a*x - (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + (-I*a*x - 1)*log(
-a*x + sqrt(a^2*x^2 + 1)) + (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) - 4
*I*sqrt(a^2*x^2 + 1) - 4)/(a*x - I)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^4-3ia^2x^3-3ax^2+ix}\,dx+\int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^4-3ia^2x^3-3ax^2+ix}\,dx\right)$$

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**4 - 3*I*a**2*x**3 - 3*a*x**2 + I*x
), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**4 - 3*I*a**2*x**3 -
 3*a*x**2 + I*x), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3 x}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = -\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right) + \frac{a\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{\sqrt{a^2}} + \frac{a\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a^2*x^2 + 1)^(3/2)/(x*(a*x*1i + 1)^3),x)
```

```
[Out] (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2) - atanh((a^2*x^2 + 1)^(1/2)) + (a*(
a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.57 $\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4a\sqrt{1+a^2x^2}}{i-ax} + 3ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] 3*I*a*arctanh((a^2*x^2+1)^(1/2))-(a^2*x^2+1)^(1/2)/x+4*a*(a^2*x^2+1)^(1/2)/
(I-a*x)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5168, 6874, 270, 272, 65, 214, 665}

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = 3ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right) + \frac{4a\sqrt{a^2x^2+1}}{-ax+i} - \frac{\sqrt{a^2x^2+1}}{x}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/x) + (4*a*Sqrt[1 + a^2*x^2])/(I - a*x) + (3*I)*a*ArcTan
h[Sqrt[1 + a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^2}{x^2(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^2\sqrt{1+a^2x^2}}-\frac{3ia}{x\sqrt{1+a^2x^2}}+\frac{4ia^2}{(-i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= -\left((3ia)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\right)+\left(4ia^2\right)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{x}+\frac{4a\sqrt{1+a^2x^2}}{i-ax}-\frac{1}{2}(3ia)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4a\sqrt{1+a^2x^2}}{i-ax} - \frac{(3i)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a}$$

$$= -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4a\sqrt{1+a^2x^2}}{i-ax} + 3ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = \sqrt{1+a^2x^2}\left(-\frac{1}{x} - \frac{4a}{-i+ax}\right) - 3ia\log(x) + 3ia\log\left(1+\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^2),x]
```

```
[Out] Sqrt[1 + a^2*x^2]*(-x^(-1) - (4*a)/(-I + a*x)) - (3*I)*a*Log[x] + (3*I)*a*L
og[1 + Sqrt[1 + a^2*x^2]]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{\sqrt{a^2x^2+1}}{x} + ia\left(3\,\text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right) + \frac{4i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a\left(x-\frac{i}{a}\right)}\right)$
default	$-\frac{\left(a^2x^2+1\right)^{\frac{5}{2}}}{x} + 4a^2\left(\frac{x\left(a^2x^2+1\right)^{\frac{3}{2}}}{4} + \frac{3\sqrt{a^2x^2+1}\,x}{8} + \frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8\sqrt{a^2}}\right) - 3ia\left(\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{3} + \sqrt{a^2x^2+1}\right.$

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(a^2*x^2+1)^(1/2)/x+I*a*(3*arctanh(1/(a^2*x^2+1)^(1/2))+4*I/a/(x-I/a)*((x-
I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.70

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = -\frac{5\,a^2x^2 - 5i\,ax + 3\left(-i\,a^2x^2 - ax\right)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) + 3\left(i\,a^2x^2 + ax\right)\log\left(-ax + \sqrt{a^2x^2+1} - \right.}{ax^2 - i\,x}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a^2*x^2 - 5*I*a*x + 3*(-I*a^2*x^2 - a*x)*log(-a*x + sqrt(a^2*x^2 + 1) +
 1) + 3*(I*a^2*x^2 + a*x)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + sqrt(a^2*x^2
+ 1)*(5*a*x - I))/(a*x^2 - I*x)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^5 - 3ia^2x^4 - 3ax^3 + ix^2}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^5 - 3ia^2x^4 - 3ax^3 + ix^2}\,dx\right)$$

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**5 - 3*I*a**2*x**4 - 3*a*x**3 + I*x
**2), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**5 - 3*I*a**2*x**
4 - 3*a*x**3 + I*x**2), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(i\,ax+1\right)^3x^2}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^2), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^2}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = a\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,3\mathrm{i} - \frac{\sqrt{a^2\,x^2+1}}{x} + \frac{4\,a^2\,\sqrt{a^2\,x^2+1}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a^2*x^2 + 1)^(3/2)/(x^2*(a*x*1i + 1)^3),x)
```

```
[Out] a*atanh((a^2*x^2 + 1)^(1/2))*3i - (a^2*x^2 + 1)^(1/2)/x + (4*a^2*(a^2*x^2 +
 1)^(1/2))/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.58 $\int \frac{e^{-3i \arctan(ax)}}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 93

$$\int \frac{e^{-3i \arctan(ax)}}{x^3} dx = -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} + \frac{9}{2}a^2 \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] 9/2*a^2*arctanh((a^2*x^2+1)^(1/2))-1/2*(a^2*x^2+1)^(1/2)/x^2+3*I*a*(a^2*x^2
+1)^(1/2)/x-4*I*a^2*(a^2*x^2+1)^(1/2)/(I-a*x)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5168, 6874, 272, 44, 65, 214, 270, 665}

$$\int \frac{e^{-3i \arctan(ax)}}{x^3} dx = \frac{9}{2}a^2 \text{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{4ia^2\sqrt{a^2x^2+1}}{-ax+i} + \frac{3ia\sqrt{a^2x^2+1}}{x} - \frac{\sqrt{a^2x^2+1}}{2x^2}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^3),x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^2]/x^2 + ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sqr
t[1 + a^2*x^2])/(I - a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^2}{x^3(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^3\sqrt{1+a^2x^2}} - \frac{3ia}{x^2\sqrt{1+a^2x^2}} - \frac{4a^2}{x\sqrt{1+a^2x^2}} + \frac{4a^3}{(-i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= -\left((3ia)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx\right) - (4a^2)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&\quad + (4a^3)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx + \int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx \\
&= \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&\quad - (2a^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} \\
&\quad - 4\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right) - \frac{1}{4}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} \\
&\quad + 4a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} - \frac{4ia^2\sqrt{1+a^2x^2}}{i-ax} + \frac{9}{2}a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$
\begin{aligned}
\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx &= \sqrt{1+a^2x^2}\left(-\frac{1}{2x^2} + \frac{3ia}{x} + \frac{4ia^2}{-i+ax}\right) \\
&\quad - \frac{9}{2}a^2\log(x) + \frac{9}{2}a^2\log\left(1+\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^3),x]
[Out] Sqrt[1 + a^2*x^2]*(-1/2*1/x^2 + ((3*I)*a)/x + ((4*I)*a^2)/(-I + a*x)) - (9*
a^2*Log[x])/2 + (9*a^2*Log[1 + Sqrt[1 + a^2*x^2]])/2
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

method	result
risch	$\frac{i(6a^3x^3+ia^2x^2+6ax+i)}{2x^2\sqrt{a^2x^2+1}}-\frac{a^2\left(-9\,\mathrm{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{8i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a\left(x-\frac{i}{a}\right)}\right)}{2}$
default	$-\frac{(a^2x^2+1)^{\frac{5}{2}}}{2x^2}-\frac{9a^2\left(\frac{(a^2x^2+1)^{\frac{3}{2}}}{3}+\sqrt{a^2x^2+1}-\mathrm{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)}{2}-3ia\left(-\frac{(a^2x^2+1)^{\frac{5}{2}}}{x}+4a^2\left(\frac{x(a^2x^2+1)^{\frac{3}{2}}}{4}+\frac{3\sqrt{}}{}\right.\right.$

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2*I*(6*a^3*x^3+I*a^2*x^2+6*a*x+I)/x^2/(a^2*x^2+1)^(1/2)-1/2*a^2*(-9*arcta
nh(1/(a^2*x^2+1)^(1/2))-8*I/a/(x-I/a)*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = \frac{14i\,a^3x^3+14\,a^2x^2+9\,(a^3x^3-i\,a^2x^2)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-9\,(a^3x^3-i\,a^2x^2)\log\left(-ax+\sqrt{a^2x^2+\cdots}\right.}{2\,(ax^3-i\,x^2)}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
[Out] 1/2*(14*I*a^3*x^3 + 14*a^2*x^2 + 9*(a^3*x^3 - I*a^2*x^2)*log(-a*x + sqrt(a^
2*x^2 + 1) + 1) - 9*(a^3*x^3 - I*a^2*x^2)*log(-a*x + sqrt(a^2*x^2 + 1) - 1)
 + sqrt(a^2*x^2 + 1)*(14*I*a^2*x^2 + 5*a*x + I))/(a*x^3 - I*x^2)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^6-3ia^2x^5-3ax^4+ix^3}\,dx+\int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^6-3ia^2x^5-3ax^4+ix^3}\,dx\right)$$

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**6 - 3*I*a**2*x**5 - 3*a*x**4 + I*x
**3), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**6 - 3*I*a**2*x**
5 - 3*a*x**4 + I*x**3), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^3}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^3), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^3}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = -\frac{a^2\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,9\mathrm{i}}{2} - \frac{\sqrt{a^2\,x^2+1}}{2\,x^2} + \frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{x} - \frac{a^3\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a^2*x^2 + 1)^(3/2)/(x^3*(a*x*1i + 1)^3),x)
```

```
[Out] (a*(a^2*x^2 + 1)^(1/2)*3i)/x - (a^2*x^2 + 1)^(1/2)/(2*x^2) - (a^2*atan((a^2
*x^2 + 1)^(1/2)*1i)*9i)/2 - (a^3*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)
/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.59 $\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 118

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{3ia\sqrt{1+a^2x^2}}{2x^2}+\frac{14a^2\sqrt{1+a^2x^2}}{3x} -\frac{4a^3\sqrt{1+a^2x^2}}{i-ax}-\frac{11}{2}ia^3\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -11/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))-1/3*(a^2*x^2+1)^(1/2)/x^3+3/2*I*a*(a
^2*x^2+1)^(1/2)/x^2+14/3*a^2*(a^2*x^2+1)^(1/2)/x-4*a^3*(a^2*x^2+1)^(1/2)/(I
-a*x)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5168, 6874, 277, 270, 272, 44, 65, 214, 665}

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \frac{14a^2\sqrt{a^2x^2+1}}{3x}+\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3} -\frac{11}{2}ia^3\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{4a^3\sqrt{a^2x^2+1}}{-ax+i}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] -1/3*Sqrt[1 + a^2*x^2]/x^3 + (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2*
Sqrt[1 + a^2*x^2])/(3*x) - (4*a^3*Sqrt[1 + a^2*x^2])/(I - a*x) - ((11*I)/2)
*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^2}{x^4(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^4\sqrt{1+a^2x^2}}-\frac{3ia}{x^3\sqrt{1+a^2x^2}}-\frac{4a^2}{x^2\sqrt{1+a^2x^2}}+\frac{4ia^3}{x\sqrt{1+a^2x^2}}-\frac{4ia^4}{(-i+ax)\sqrt{1+a^2x^2}}\right)dx \\
&= -\left((3ia)\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx\right) \\
&\quad -\left(4a^2\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx+\left(4ia^3\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&\quad -\left(4ia^4\right)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{4a^2\sqrt{1+a^2x^2}}{x}-\frac{4a^3\sqrt{1+a^2x^2}}{i-ax} \\
&\quad -\frac{1}{2}(3ia)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&\quad -\frac{1}{3}\left(2a^2\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx+\left(2ia^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{3ia\sqrt{1+a^2x^2}}{2x^2}+\frac{14a^2\sqrt{1+a^2x^2}}{3x} \\
&\quad -\frac{4a^3\sqrt{1+a^2x^2}}{i-ax}+(4ia)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right) \\
&\quad +\frac{1}{4}\left(3ia^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{3x^3}+\frac{3ia\sqrt{1+a^2x^2}}{2x^2}+\frac{14a^2\sqrt{1+a^2x^2}}{3x}-\frac{4a^3\sqrt{1+a^2x^2}}{i-ax} \\
&\quad -4ia^3\text{arctanh}\left(\sqrt{1+a^2x^2}\right)+\frac{1}{2}(3ia)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)
\end{aligned}$$

$$= -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} - \frac{4a^3\sqrt{1+a^2x^2}}{i-ax} - \frac{11}{2}ia^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \frac{1}{6}\left(\frac{\sqrt{1+a^2x^2}(2i+7ax-19ia^2x^2+52a^3x^3)}{x^3(-i+ax)} + 33ia^3\log(x) - 33ia^3\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((Sqrt[1 + a^2*x^2]*(2*I + 7*a*x - (19*I)*a^2*x^2 + 52*a^3*x^3))/(x^3*(-I +
 a*x)) + (33*I)*a^3*Log[x] - (33*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

method	result
risch	$\frac{28a^4x^4+9ia^3x^3+26a^2x^2+9iax-2}{6x^3\sqrt{a^2x^2+1}} + \frac{ia^3\left(-11\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right) - \frac{8i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a\left(x-\frac{i}{a}\right)}\right)}{2}$
default	$-\frac{(a^2x^2+1)^{\frac{5}{2}}}{3x^3} - \frac{16a^2\left(-\frac{(a^2x^2+1)^{\frac{5}{2}}}{x} + 4a^2\left(\frac{x(a^2x^2+1)^{\frac{3}{2}}}{4} + \frac{3\sqrt{a^2x^2+1}\,x}{8} + \frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8\sqrt{a^2}}\right)\right)}{3} - 3ia\left(-\frac{(a^2x^2+1)^{\frac{5}{2}}}{2x^2} + \right.$

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(28*a^4*x^4+9*I*a^3*x^3+26*a^2*x^2+9*I*a*x-2)/x^3/(a^2*x^2+1)^(1/2)+1/2
*I*a^3*(-11*arctanh(1/(a^2*x^2+1)^(1/2))-8*I/a/(x-I/a)*((x-I/a)^2*a^2+2*I*a
*(x-I/a))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.18

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \frac{52\,a^4x^4 - 52i\,a^3x^3 - 33\,(i\,a^4x^4 + a^3x^3)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) - 33\,(-i\,a^4x^4 - a^3x^3)\log\left(-ax + \sqrt{a}\right.}{6\,(ax^4 - i\,x^3)}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(52*a^4*x^4 - 52*I*a^3*x^3 - 33*(I*a^4*x^4 + a^3*x^3)*log(-a*x + sqrt(a
^2*x^2 + 1) + 1) - 33*(-I*a^4*x^4 - a^3*x^3)*log(-a*x + sqrt(a^2*x^2 + 1) -
 1) + (52*a^3*x^3 - 19*I*a^2*x^2 + 7*a*x + 2*I)*sqrt(a^2*x^2 + 1))/(a*x^4 -
 I*x^3)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^7 - 3ia^2x^6 - 3ax^5 + ix^4}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^7 - 3ia^2x^6 - 3ax^5 + ix^4}\,dx\right)$$

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**7 - 3*I*a**2*x**6 - 3*a*x**5 + I*x
**4), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**7 - 3*I*a**2*x**
6 - 3*a*x**5 + I*x**4), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^4}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^4), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(i\,ax+1\right)^3x^4}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \frac{14\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x} - \frac{\sqrt{a^2\,x^2+1}}{3\,x^3} + \frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{2\,x^2} - \frac{11\,a^3\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2} - \frac{4\,a^4\,\sqrt{a^2\,x^2+1}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a^2*x^2 + 1)^(3/2)/(x^4*(a*x*1i + 1)^3),x)
```

```
[Out] (a*(a^2*x^2 + 1)^(1/2)*3i)/(2*x^2) - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (11*a^3*
atan((a^2*x^2 + 1)^(1/2)*1i))/2 + (14*a^2*(a^2*x^2 + 1)^(1/2))/(3*x) - (4*a
^4*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.60 $\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 139

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{x^3}+\frac{19a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{6ia^3\sqrt{1+a^2x^2}}{x}+\frac{4ia^4\sqrt{1+a^2x^2}}{i-ax}-\frac{51}{8}a^4\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -51/8*a^4*arctanh((a^2*x^2+1)^(1/2))-1/4*(a^2*x^2+1)^(1/2)/x^4+I*a*(a^2*x^2
+1)^(1/2)/x^3+19/8*a^2*(a^2*x^2+1)^(1/2)/x^2-6*I*a^3*(a^2*x^2+1)^(1/2)/x+4*
I*a^4*(a^2*x^2+1)^(1/2)/(I-a*x)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5168, 6874, 272, 44, 65, 214, 277, 270, 665}

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \frac{19a^2\sqrt{a^2x^2+1}}{8x^2}-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{ia\sqrt{a^2x^2+1}}{x^3}-\frac{51}{8}a^4\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{4ia^4\sqrt{a^2x^2+1}}{-ax+i}-\frac{6ia^3\sqrt{a^2x^2+1}}{x}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*x^5),x]
```

```
[Out] -1/4*Sqrt[1 + a^2*x^2]/x^4 + (I*a*Sqrt[1 + a^2*x^2])/x^3 + (19*a^2*Sqrt[1 +
 a^2*x^2])/(8*x^2) - ((6*I)*a^3*Sqrt[1 + a^2*x^2])/x + ((4*I)*a^4*Sqrt[1 +
a^2*x^2])/(I - a*x) - (51*a^4*ArcTanh[Sqrt[1 + a^2*x^2]])/8
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^2}{x^5(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^5\sqrt{1+a^2x^2}}-\frac{3ia}{x^4\sqrt{1+a^2x^2}}-\frac{4a^2}{x^3\sqrt{1+a^2x^2}}+\frac{4ia^3}{x^2\sqrt{1+a^2x^2}}+\frac{4a^4}{x\sqrt{1+a^2x^2}} - \frac{4a^5}{(-i+ax)\sqrt{1+a^2x^2}}\right) dx \\
&= -\left((3ia)\int \frac{1}{x^4\sqrt{1+a^2x^2}}\,dx\right)-\left(4a^2\right)\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx+\left(4ia^3\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx \\
&\quad +\left(4a^4\right)\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx-\left(4a^5\right)\int \frac{1}{(-i+ax)\sqrt{1+a^2x^2}}\,dx+\int \frac{1}{x^5\sqrt{1+a^2x^2}}\,dx \\
&= \frac{ia\sqrt{1+a^2x^2}}{x^3}-\frac{4ia^3\sqrt{1+a^2x^2}}{x}+\frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} \\
&\quad +\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+a^2x}}\,dx,x,x^2\right)-\left(2a^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&\quad +\left(2ia^3\right)\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx+\left(2a^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{x^3}+\frac{2a^2\sqrt{1+a^2x^2}}{x^2}-\frac{6ia^3\sqrt{1+a^2x^2}}{x} \\
&\quad +\frac{4ia^4\sqrt{1+a^2x^2}}{i-ax}-\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&\quad +\left(4a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right) \\
&\quad +a^4\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{x^3}+\frac{19a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{6ia^3\sqrt{1+a^2x^2}}{x}+\frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} \\
&\quad -4a^4\text{arctanh}\left(\sqrt{1+a^2x^2}\right)+\left(2a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right) \\
&\quad +\frac{1}{16}\left(3a^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{x^3}+\frac{19a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{6ia^3\sqrt{1+a^2x^2}}{x}+\frac{4ia^4\sqrt{1+a^2x^2}}{i-ax} \\
&\quad -6a^4\text{arctanh}\left(\sqrt{1+a^2x^2}\right)+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{x^3}+\frac{19a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{6ia^3\sqrt{1+a^2x^2}}{x} \\
&\quad +\frac{4ia^4\sqrt{1+a^2x^2}}{i-ax}-\frac{51}{8}a^4\text{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$
\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx=\frac{1}{8}\left(\frac{\sqrt{1+a^2x^2}\left(2i+6ax-11ia^2x^2-29a^3x^3-80ia^4x^4\right)}{x^4(-i+ax)}+51a^4\log(x)-51a^4\log\left(1+\sqrt{1+a^2x^2}\right)\right)
$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*x^5),x]
[Out] ((Sqrt[1 + a^2*x^2]*(2*I + 6*a*x - (11*I)*a^2*x^2 - 29*a^3*x^3 - (80*I)*a^4
*x^4))/(x^4*(-I + a*x)) + 51*a^4*Log[x] - 51*a^4*Log[1 + Sqrt[1 + a^2*x^2]]
)/8
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{i\left(48a^5x^5+19ia^4x^4+40a^3x^3+17ia^2x^2-8ax-2i\right)}{8x^4\sqrt{a^2x^2+1}}+\frac{a^4\left(-51\ \text{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{32i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a\left(x-\frac{i}{a}\right)}\right)}{8}$	125
default	Expression too large to display	937

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*(48*a^5*x^5+19*I*a^4*x^4+40*a^3*x^3+17*I*a^2*x^2-8*a*x-2*I)/x^4/(a^2
*x^2+1)^(1/2)+1/8*a^4*(-51*arctanh(1/(a^2*x^2+1)^(1/2))-32*I/a/(x-I/a)*((x-
I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \frac{-80i\,a^5x^5 - 80\,a^4x^4 - 51\,(a^5x^5 - i\,a^4x^4)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) + 51\,(a^5x^5 - i\,a^4x^4)\log\left(-ax + \sqrt{a}\right.}{8\,(ax^5 - i\,x^4)}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(-80*I*a^5*x^5 - 80*a^4*x^4 - 51*(a^5*x^5 - I*a^4*x^4)*log(-a*x + sqrt(
a^2*x^2 + 1) + 1) + 51*(a^5*x^5 - I*a^4*x^4)*log(-a*x + sqrt(a^2*x^2 + 1) -
 1) + (-80*I*a^4*x^4 - 29*a^3*x^3 - 11*I*a^2*x^2 + 6*a*x + 2*I)*sqrt(a^2*x^
2 + 1))/(a*x^5 - I*x^4)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^8 - 3ia^2x^7 - 3ax^6 + ix^5}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^8 - 3ia^2x^7 - 3ax^6 + ix^5}\,dx\right)$$

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**8 - 3*I*a**2*x**7 - 3*a*x**6 + I*x
**5), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**8 - 3*I*a**2*x**
7 - 3*a*x**6 + I*x**5), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^5}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^5), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^5}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \frac{a^4\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,51\mathrm{i}}{8} - \frac{\sqrt{a^2\,x^2+1}}{4\,x^4} + \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{x^3} + \frac{19\,a^2\,\sqrt{a^2\,x^2+1}}{8\,x^2} - \frac{a^3\,\sqrt{a^2\,x^2+1}\,6\mathrm{i}}{x} + \frac{a^5\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

```
[In] int((a^2*x^2 + 1)^(3/2)/(x^5*(a*x*1i + 1)^3),x)
```

```
[Out] (a^4*atan((a^2*x^2 + 1)^(1/2)*1i)*51i)/8 - (a^2*x^2 + 1)^(1/2)/(4*x^4) + (a
*(a^2*x^2 + 1)^(1/2)*1i)/x^3 + (19*a^2*(a^2*x^2 + 1)^(1/2))/(8*x^2) - (a^3*
(a^2*x^2 + 1)^(1/2)*6i)/x + (a^5*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)
/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.61 $\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 339

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}-\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3}+\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}+\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}+\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

```
[Out] -3/8*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^3-1/12*I*(1-I*a*x)^(3/4)*(1+I*a*x)
^(5/4)/a^3+1/3*x*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)/a^2+3/16*I*arctan(1-(1-I*a
*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-3/16*I*arctan(1+(1-I*a*x)^(1
/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-3/32*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2
)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)+3/32*I*ln(1+
(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^
3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5170, 92, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} - \frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3} + \frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}$$

```
[In] Int[E^((I/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] (((-3*I)/8)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 - ((I/12)*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(5/4))/a^3 + (x*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/(3*a
^2) + (((3*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])
/(Sqrt[2]*a^3) - (((3*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(Sqrt[2]*a^3) - (((3*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*
a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((3
*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx\\
&= \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{\int \frac{\left(-1-\frac{iax}{2}\right)\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{3a^2}\\
&= -\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{3\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{8a^2}\\
&= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3}\\
&\quad + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{3\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{16a^2}\\
&= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3}\\
&\quad + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{(3i)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{4a^3}
\end{aligned}
$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{(3i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^3}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{(3i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3} - \frac{(3i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{(3i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} - \frac{(3i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} - \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{(3i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} + \frac{(3i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}$$

$$= -\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.24

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx$$
$$=\frac{(1-iax)^{3/4}\left(\sqrt[4]{1+iax}(-i+5ax+4ia^2x^2)-6i\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)\right)}{12a^3}$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*((1 + I*a*x)^(1/4)*(-I + 5*a*x + (4*I)*a^2*x^2) - (6*I)*
2^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (1 - I*a*x)/2]))/(12*a^3)
```

Maple [F]

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^2dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.72

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx =$$
$$-\frac{12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(\frac{8}{3}i\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(-\frac{8}{3}i\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3\sqrt{-}}{}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(9/64*I/a^6)*log(8/3*I*a^3*sqrt(9/64*I/a^6) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(9/64*I/a^6)*log(-8/3*I*a^3*sqrt(9/6
4*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(-9/64*I/a^6)*
log(8/3*I*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12
*a^3*sqrt(-9/64*I/a^6)*log(-8/3*I*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) - (8*a^3*x^3 - 2*I*a^2*x^2 - a*x - 11*I)*sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \int x^2\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x**2,x)
[Out] Integral(x**2*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \int x^2\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \int x^2\,\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\,dx$$

```
[In] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

3.62 $\int e^{\frac{1}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 295

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}-\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

```
[Out] 1/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^2+1/2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)
/a^2-1/8*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+1/8*
arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+1/16*ln(1-(1-
I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2
^(1/2)-1/16*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules

used = {5170, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} + \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}+\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}$$

```
[In] Int[E^((I/2)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) + ((1 - I*a*x)^(3/4)*(1 + I*a
*x)^(5/4))/(2*a^2) - ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)]/(4*Sqrt[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/(4*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqrt[2]*a^2) - Log[1 + Sqrt[1 - I*a
*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqr
t[2]*a^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx$$

$$= \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{i\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{4a}$$

$$= \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{i\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{8a}$$

$$= \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2}$$

$$= \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$

$$= \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}$$

$$= \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

$$
\begin{aligned}
&=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}\\
&+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}-\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}\\
&+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}\\
&-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}\\
&=\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}\\
&-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}\\
&+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}-\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.21

$$
\int e^{\frac{1}{2}i\arctan(ax)}x\,dx
=\frac{(1-iax)^{3/4}\left(3(1+iax)^{5/4}+2\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)\right)}{6a^2}
$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x])*x,x]
[Out] ((1 - I*a*x)^(3/4)*(3*(1 + I*a*x)^(5/4) + 2*2^(1/4)*Hypergeometric2F1[-1/4,
 3/4, 7/4, (1 - I*a*x)/2]))/(6*a^2)
```

Maple [F]

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x dx$$

[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)

[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.80

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = -\frac{2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(4\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(-4\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{i}{16\,a^4}}\cdots}{\cdots}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="fricas")

[Out] -1/4*(2*a^2*sqrt(1/16*I/a^4)*log(4*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(1/16*I/a^4)*log(-4*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-1/16*I/a^4)*log(4*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-1/16*I/a^4)*log(-4*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (2*a^2*x^2 - I*a*x + 3)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^2

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \int x\sqrt{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x,x)

[Out] Integral(x*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \int x\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="maxima")
[Out] integrate(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \int x\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\,dx$$

```
[In] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

3.63 $\int e^{\frac{1}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 268

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx=\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}-\frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

```
[Out] I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a-1/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a*2^(1/2)+1/2*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)
^(1/4))/a*2^(1/2)+1/4*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a
*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)-1/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+
I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules

used = {5169, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = -\frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}$$

```
[In] Int[E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - (I*ArcTan[1 - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*ArcTan[1 + (Sqrt[2]*(1 - I*
a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((I/2)*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2
]*a) - ((I/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x
)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{1}{2}\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{(2i)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx,x,\sqrt[4]{1-iax}\right)}{a}\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{(2i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}\\
&\quad+\frac{i\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a}\\
&\quad+\frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a}+\frac{i\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}\\
&\quad+\frac{i\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}\\
&= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}\\
&\quad-\frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}+\frac{i\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}\\
&\quad-\frac{i\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}
\end{aligned}
$$

$$= \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = -\frac{8ie^{\frac{5}{2}i\arctan(ax)}\text{Hypergeometric2F1}\left(\frac{5}{4},2,\frac{9}{4},-e^{2i\arctan(ax)}\right)}{5a}$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x]),x]
[Out] (((-8*I)/5)*E^(((5*I)/2)*ArcTan[a*x])*Hypergeometric2F1[5/4, 2, 9/4, -E^((2
*I)*ArcTan[a*x])])/a
```

Maple [F]

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.78

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \frac{a\sqrt{\frac{i}{a^2}}\log\left(i\,a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - a\sqrt{\frac{i}{a^2}}\log\left(-i\,a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + a\sqrt{-\frac{i}{a^2}}\log\left(i\,a\sqrt{-\frac{i}{a^2}}+\sqrt{\frac{i}{}}\right.}{2\,a}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(I/a^2)*log(I*a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)
)) - a*sqrt(I/a^2)*log(-I*a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + a*sqrt(-I/a^2)*log(I*a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))) - a*sqrt(-I/a^2)*log(-I*a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I))) + 2*(a*x + I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \int\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt((I*a*x + 1)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \int\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx=\int\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

3.64 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 267

$$\begin{aligned}\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = &-2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)\\ &-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\ &-\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\end{aligned}$$

```
[Out] -2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a
*x)^(1/4))-1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)
/(1+I*a*x)^(1/2))*2^(1/2)+1/2*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+
(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)+arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(
1+I*a*x)^(1/4))*2^(1/2)-arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2
^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules

used = {5170, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 95, 218, 212, 209}

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 1\right)}{\sqrt{2}} + \frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 1\right)}{\sqrt{2}}$$

```
[In] Int[E^((I/2)*ArcTan[a*x])/x,x]
```

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2
]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^
(1/4)] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\,dx \\
&= (ia)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\left(4\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)\right) + 4\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\left(2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) \\
&\quad - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 4\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) \\
&= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad + 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)
\end{aligned}
$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.36

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = -\frac{2(1-iax)^{3/4}\left(\sqrt[4]{2}(1+iax)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\right.\right.}{3(1+iax)^{3/4}}$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x,x]
```

```
[Out] (-2*(1 - I*a*x)^(3/4)*(2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4
, 7/4, (1 - I*a*x)/2] + 2*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x
)]))/(3*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = {} & \frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ & -\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ & +\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ & -\frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ & -\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) \\ & +i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)\end{aligned}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/
2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1/2
*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/2
*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - lo
g(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)
```

```
/(a*x + I)) + I) + I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sqr
t(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x,x)
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x, x)
```

3.65 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - ia\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/
4))-I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5170, 96, 95, 218, 212, 209}

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = -ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - ia\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) - I*a*ArcTan[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)] - I*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1+iax}}{x^2\sqrt[4]{1-iax}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+\frac{1}{2}(ia)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}+(2ia)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - (ia)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (ia)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{i(1-iax)^{3/4}\left(-3i+3ax+2ax\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{3x(1+iax)^{3/4}}$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^2,x]
[Out] ((-1/3*I)*(1 - I*a*x)^(3/4)*(-3*I + 3*a*x + 2*a*x*Hypergeometric2F1[3/4, 1,
 7/4, (I + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^2}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(64) = 128.

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.64

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \frac{-i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{2\,x}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + a*x*log(sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)) + I) - a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I)) - I) + I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(-I*a*x +
 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^2}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^2}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^2}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^2,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^2, x)
```

3.66 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} + \frac{1}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/4*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-1/2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5
/4)/x^2+1/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/4*a^2*arctanh((1+
I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5170, 98, 96, 95, 218, 212, 209}

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \frac{1}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} - \frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((-1/4*I)*a*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - ((1 - I*a*x)^(3/4)*(1
+ I*a*x)^(5/4))/(2*x^2) + (a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])
/4 + (a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1+iax}}{x^3\sqrt[4]{1-iax}}\,dx \\
&= -\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} + \frac{1}{4}(ia)\int \frac{\sqrt[4]{1+iax}}{x^2\sqrt[4]{1-iax}}\,dx \\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} - \frac{1}{8}a^2\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} \\
&\quad - \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} \\
&\quad + \frac{1}{4}a^2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2} \\
&\quad + \frac{1}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$
\begin{aligned}
&\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx \\
&= \frac{(1-iax)^{3/4}\left(-6-15iax+9a^2x^2+2a^2x^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{12x^2(1+iax)^{3/4}}
\end{aligned}
$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^3,x]
[Out] ((1 - I*a*x)^(3/4)*(-6 - (15*I)*a*x + 9*a^2*x^2 + 2*a^2*x^2*Hypergeometric2
F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(12*x^2*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^3}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.33

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \frac{a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{8\,x^2}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/8*(a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*a^2*x^2*log(s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - I*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - I) - a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1
) - 2*(3*a^2*x^2 + I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**3, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^3, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}}}{x^3}\,dx$$

[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^3,x)

[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^3, x)

3.67 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{3}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{3}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/3*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^3-5/12*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)
^(1/4)/x^2+11/24*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x+3/8*I*a^3*arctan((1+
I*a*x)^(1/4)/(1-I*a*x)^(1/4))+3/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5170, 101, 156, 12, 95, 218, 212, 209}

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \frac{3}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{3}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} - \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}$$

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^3 - (((5*I)/12)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^2 + (11*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(24*x) + ((3*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((3*I
)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1+iax}}{x^4\sqrt[4]{1-iax}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{1}{3}\int \frac{\frac{5ia}{2}-2a^2x}{x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}-\frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}-\frac{1}{6}\int \frac{\frac{11a^2}{4}+\frac{5}{2}ia^3x}{x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}-\frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} \\
&\quad +\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{1}{6}\int -\frac{9ia^3}{8x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}-\frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} \\
&\quad +\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{1}{16}\left(3ia^3\right)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}-\frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} \\
&\quad +\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{1}{4}\left(3ia^3\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{1}{8}(3ia^3)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{8}(3ia^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} - \frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} + \frac{3}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{3}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \frac{(1-iax)^{3/4}\left(-8-18iax+21a^2x^2+11ia^3x^3+6ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3(1+iax)^{3/4}}$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-8 - (18*I)*a*x + 21*a^2*x^2 + (11*I)*a^3*x^3 + (6*I)*a
^3*x^3*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I
*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \frac{9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9i\,a^3x^3\log\left(\right.}{48\,x^3}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/48*(9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*a^3*x^3*
log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*a^3*x^3*log(sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)) - I) - 9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I)) - 1) - 2*(11*I*a^3*x^3 - a^2*x^2 + 2*I*a*x + 8)*sqrt(I*sqrt(a^2*x^2 +
 1)/(a*x + I)))/x^3
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**4, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^4}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^4, x)
```

3.68 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 202

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x} - \frac{11}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{11}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^4-7/24*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)
^(1/4)/x^3+29/96*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^2+83/192*I*a^3*(1-I*
a*x)^(3/4)*(1+I*a*x)^(1/4)/x-11/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/
4))-11/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5170, 101, 156, 12, 95, 218, 212, 209}

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{11}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{11}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} - \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3}$$

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] -1/4*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^4 - (((7*I)/24)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^3 + (29*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(96*x^2) + (((83*I)/192)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - (11*
a^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1+iax}}{x^5\sqrt[4]{1-iax}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}+\frac{1}{4}\int \frac{\frac{7ia}{2}-3a^2x}{x^4\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}-\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3}-\frac{1}{12}\int \frac{\frac{29a^2}{4}+7ia^3x}{x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}-\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} \\
&\quad +\frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2}+\frac{1}{24}\int \frac{-\frac{83ia^3}{8}+\frac{29a^4x}{4}}{x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}-\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3}+\frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} \\
&\quad +\frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x}-\frac{1}{24}\int -\frac{33a^4}{16x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}-\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3}+\frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} \\
&\quad +\frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x}+\frac{1}{128}\left(11a^4\right)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx
\end{aligned}
$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x} + \frac{1}{32}(11a^4)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x} - \frac{1}{64}(11a^4)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{1}{64}(11a^4)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} - \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} + \frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2} + \frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x} - \frac{11}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{11}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{(1-iax)^{3/4}\left(48+104iax-114a^2x^2-141ia^3x^3+83a^4x^4+22a^4x^4\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{192x^4(1+iax)^{3/4}}$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^5,x]
[Out] -1/192*((1 - I*a*x)^(3/4)*(48 + (104*I)*a*x - 114*a^2*x^2 - (141*I)*a^3*x^3
 + 83*a^4*x^4 + 22*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*
x)]))/(x^4*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^5}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-33\,a^4}{384\,x^4}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 33*I*a^4*
x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 33*I*a^4*x^4*log(sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) - 1) - 2*(83*a^4*x^4 + 25*I*a^3*x^3 + 2*a^2*x^2 - 8*I*a*x - 48)
*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^5}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**5, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^5}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}}}{x^5}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^5, x)
```

3.69 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 240

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2} - \frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x} - \frac{31}{128}ia^5\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{31}{128}ia^5\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/5*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^5-9/40*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)
^(1/4)/x^4+11/48*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^3+269/960*I*a^3*(1-I
*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^2-611/1920*a^4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4
)/x-31/128*I*a^5*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-31/128*I*a^5*arcta
nh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

= {5170, 101, 156, 12, 95, 218, 212, 209}

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = -\frac{31}{128}ia^5\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{31}{128}ia^5\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x} + \frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} - \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4}$$

```
[In] Int[E^((I/2)*ArcTan[a*x])/x^6,x]
```

```
[Out] -1/5*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^5 - (((9*I)/40)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^4 + (11*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(48*x^3) + (((269*I)/960)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^2 - (
611*a^4*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(1920*x) - ((31*I)/128)*a^5*Ar
cTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((31*I)/128)*a^5*ArcTanh[(1 + I
*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1+iax}}{x^6\sqrt[4]{1-iax}}\,dx\\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}+\frac{1}{5}\int \frac{\frac{9ia}{2}-4a^2x}{x^5\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx\\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}-\frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4}-\frac{1}{20}\int \frac{\frac{55a^2}{4}+\frac{27}{2}ia^3x}{x^4\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx
\end{aligned}
$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{1}{60}\int \frac{-\frac{269ia^3}{8}+\frac{55a^4x}{2}}{x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2} - \frac{1}{120}\int \frac{-\frac{611a^4}{16}-\frac{269}{8}ia^5x}{x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2} - \frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x} + \frac{1}{120}\int \frac{465ia^5}{32x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2} - \frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x} + \frac{1}{256}\left(31ia^5\right)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2} - \frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x} + \frac{1}{64}\left(31ia^5\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2} - \frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x} - \frac{1}{128}\left(31ia^5\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{1}{128}\left(31ia^5\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5} - \frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4} + \frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3} + \frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2} - \frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x} - \frac{31}{128}ia^5\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{31}{128}ia^5\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.46

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \frac{(1-iax)^{3/4}\left(-384-816iax+872a^2x^2+978ia^3x^3-1149a^4x^4-611ia^5x^5-310ia^5x^5\,\text{Hypergeometric2F}\right.}{1920x^5(1+iax)^{3/4}}$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x])/x^6,x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-384 - (816*I)*a*x + 872*a^2*x^2 + (978*I)*a^3*x^3 - 11
49*a^4*x^4 - (611*I)*a^5*x^5 - (310*I)*a^5*x^5*Hypergeometric2F1[3/4, 1, 7/
4, (I + a*x)/(I - a*x)]))/(1920*x^5*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^6}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.83

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \frac{-465i\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+465\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-465\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+465}{3840\,x}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/3840*(-465*I*a^5*x^5*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 465*a
^5*x^5*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 465*a^5*x^5*log(sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 465*I*a^5*x^5*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - 1) - 2*(-611*I*a^5*x^5 + 73*a^4*x^4 - 98*I*a^3*x^3 - 8*a
^2*x^2 + 48*I*a*x + 384)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^5
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^6}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**6,x)
[Out] Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**6, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^6}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
[Out] integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^6, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}}}{x^6}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^6,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^6, x)
```

3.70 $\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 337

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4}+\frac{123\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}-\frac{123\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}+\frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}-\frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

```
[Out] -41/64*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^4+1/4*x^2*(1-I*a*x)^(1/4)*(1+I*a*x
)^(7/4)/a^2-1/32*(1-I*a*x)^(1/4)*(1+I*a*x)^(7/4)*(11+4*I*a*x)/a^4+123/128*a
rctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)-123/128*arctan
(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+123/256*ln(1-(1-I*a
*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1
/2)-123/256*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5170, 102, 152, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx=\frac{123\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}-\frac{123\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4}-\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4}-\frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4}+\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] (-41*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(64*a^4) + (x^2*(1 - I*a*x)^(1/4)
*(1 + I*a*x)^(7/4))/(4*a^2) - ((1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4)*(11 + (4
*I)*a*x))/(32*a^4) + (123*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(64*Sqrt[2]*a^4) - (123*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
- (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx \\
&= \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}+\frac{\int \frac{x(1+iax)^{3/4}\left(-2-\frac{3iax}{2}\right)}{(1-iax)^{3/4}}\,dx}{4a^2} \\
&= \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4}+\frac{(41i)\int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx}{64a^3} \\
&= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} \\
&\quad -\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4}+\frac{(123i)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{128a^3}
\end{aligned}
$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{32a^4}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{32a^4}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4} - \frac{123\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} - \frac{123\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} - \frac{123\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} + \frac{123\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} + \frac{123\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{123\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{123\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}$$

$$= -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{32a^4} + \frac{123\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{123\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.44

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \frac{\sqrt[4]{1-iax}\left(a^2x^2(1+iax)^{3/4} + ia^3x^3(1+iax)^{3/4} - 24\,2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right) + 8\right.}{4a^4}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(a^2*x^2*(1 + I*a*x)^(3/4) + I*a^3*x^3*(1 + I*a*x)^(3/4)
 - 24*2^(3/4)*Hypergeometric2F1[-11/4, 1/4, 5/4, (1 - I*a*x)/2] + 8*2^(3/4)
*Hypergeometric2F1[-7/4, 1/4, 5/4, (1 - I*a*x)/2] + 2*2^(3/4)*Hypergeometri
c2F1[-3/4, 1/4, 5/4, (1 - I*a*x)/2]))/(4*a^4)
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^3 dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.75

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx$$
$$= \frac{32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(\frac{64}{123}i\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(-\frac{64}{123}i\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/64*(32*a^4*sqrt(15129/4096*I/a^8)*log(64/123*I*a^4*sqrt(15129/4096*I/a^8)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(15129/4096*I/a^8)*log
(-64/123*I*a^4*sqrt(15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) - 32*a^4*sqrt(-15129/4096*I/a^8)*log(64/123*I*a^4*sqrt(-15129/4096*I/a^8)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 32*a^4*sqrt(-15129/4096*I/a^8)*lo
g(-64/123*I*a^4*sqrt(-15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + (16*I*a^3*x^3 + 24*a^2*x^2 - 30*I*a*x - 63)*sqrt(a^2*x^2 + 1)*sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)))/a^4
```

Sympy [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**3,x)
```

```
[Out] Integral(x**3*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 23, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

```
[In] int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

3.71 $\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 339

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx = -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3}-\frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3}+\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}+\frac{17i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{17i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}-\frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

```
[Out] -17/24*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^3-1/4*I*(1-I*a*x)^(1/4)*(1+I*a*x
)^(7/4)/a^3+1/3*x*(1-I*a*x)^(1/4)*(1+I*a*x)^(7/4)/a^2+17/16*I*arctan(1-(1-I
*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-17/16*I*arctan(1+(1-I*a*x)
^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)+17/32*I*ln(1-(1-I*a*x)^(1/4)*2^
(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)-17/32*I*
ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2
))/a^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5170, 92, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx = \frac{17i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{17i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} - \frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} + \frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3} - \frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] (((-17*I)/24)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a^3 - ((I/4)*(1 - I*a*x)
^(1/4)*(1 + I*a*x)^(7/4))/a^3 + (x*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4))/(3*
a^2) + (((17*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(Sqrt[2]*a^3) - (((17*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
+ I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) -
(((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)
^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx \\
&= \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} + \frac{\int \frac{(1+iax)^{3/4}\left(-1-\frac{3iax}{2}\right)}{(1-iax)^{3/4}}\,dx}{3a^2} \\
&= -\frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{17\int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx}{24a^2} \\
&= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} \\
&\quad + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{17\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{16a^2} \\
&= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} \\
&\quad + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{(17i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{4a^3}
\end{aligned}
$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{(17i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^3}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{(17i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3} - \frac{(17i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} - \frac{(17i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} - \frac{(17i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} + \frac{(17i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} + \frac{(17i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} + \frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{(17i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} + \frac{(17i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}$$

$$= -\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3} - \frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3} + \frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2} + \frac{17i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{17i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} + \frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.24

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx = \frac{\sqrt[4]{1-iax}\left((1+iax)^{3/4}(-3i+7ax+4ia^2x^2) - 34i2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right)\right)}{12a^3}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^2,x]
[Out] ((1 - I*a*x)^(1/4)*((1 + I*a*x)^(3/4)*(-3*I + 7*a*x + (4*I)*a^2*x^2) - (34*
I)*2^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, (1 - I*a*x)/2]))/(12*a^3)
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^2 dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.73

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx =$$

$$-\frac{12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(\frac{8}{17}\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(-\frac{8}{17}\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\ldots}}{}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(289/64*I/a^6)*log(8/17*a^3*sqrt(289/64*I/a^6) + sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(289/64*I/a^6)*log(-8/17*a^3*sqrt
(289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-289/64
*I/a^6)*log(8/17*a^3*sqrt(-289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) + 12*a^3*sqrt(-289/64*I/a^6)*log(-8/17*a^3*sqrt(-289/64*I/a^6) + sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(8*I*a^2*x^2 + 14*a*x -
 23*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=\int x^2\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**2,x)
```

```
[Out] Integral(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=\int x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=\int x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

```
[In] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
[Out] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

3.72 $\int e^{\frac{3}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 295

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx=\frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-\frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}+\frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

```
[Out] 3/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^2+1/2*(1-I*a*x)^(1/4)*(1+I*a*x)^(7/4)
/a^2-9/8*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+9/8*
arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-9/16*ln(1-(1-
I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2
^(1/2)+9/16*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5170, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx=-\frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}+\frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}-\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}+\frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x,x]
```

```
[Out] (3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*a^2) + ((1 - I*a*x)^(1/4)*(1 + I
*a*x)^(7/4))/(2*a^2) - (9*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(4*Sqrt[2]*a^2) + (9*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x]
 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (9*Log
[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx \\
&= \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} - \frac{(3i)\int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx}{4a} \\
&= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} - \frac{(9i)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{8a} \\
&= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} + \frac{9\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2} \\
&= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} + \frac{9\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2} \\
&= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} \\
&\quad + \frac{9\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} + \frac{9\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} \\
&+ \frac{9\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} + \frac{9\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} \\
&- \frac{9\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} \\
&- \frac{9\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} \\
&= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} \\
&- \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} \\
&+ \frac{9\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} \\
&- \frac{9\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} \\
&= \frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} \\
&- \frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} + \frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} \\
&- \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.21

$$
\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \frac{\sqrt[4]{1-iax}\left((1+iax)^{7/4}+6\ 2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right)\right)}{2a^2}
$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x,x]
```

[Out] ((1 - I*a*x)^(1/4)*((1 + I*a*x)^(7/4) + 6*2^(3/4)*Hypergeometric2F1[-3/4, 1
/4, 5/4, (1 - I*a*x)/2]))/(2*a^2)

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x dx$$

[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)

[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.81

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx =$$

$$-\frac{2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(\frac{4}{9}i\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(-\frac{4}{9}i\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{-\cdots}}{\cdots}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="fricas")

[Out] -1/4*(2*a^2*sqrt(81/16*I/a^4)*log(4/9*I*a^2*sqrt(81/16*I/a^4) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(81/16*I/a^4)*log(-4/9*I*a^2*sqrt(81/
16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-81/16*I/a^4)
*log(4/9*I*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
2*a^2*sqrt(-81/16*I/a^4)*log(-4/9*I*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(2*I*a*x + 5)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)))/a^2

Sympy [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \int x\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x,x)

[Out] Integral(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 23, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \int x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

```
[In] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

3.73 $\int e^{\frac{3}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 268

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx=\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}-\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}+\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

```
[Out] I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a-3/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a*2^(1/2)+3/2*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)
^(1/4))/a*2^(1/2)-3/4*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a
*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)+3/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+
I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules

used = {5169, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{\frac{3}{2}i\arctan(ax)}dx = -\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a} + \frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((3*I)*ArcTan[1 + (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - (((3*I)/2)*Log[1 + Sq
rt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)])/(Sqrt[2]*a) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sq
rt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx$$

$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{3}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(6i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$

$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(6i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(3i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{(3i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(3i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{(3i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} - \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{(3i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{(3i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}$$

$$= \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = -\frac{8ie^{\frac{7}{2}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(\frac{7}{4},2,\frac{11}{4},-e^{2i\arctan(ax)}\right)}{7a}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x]),x]
[Out] (((-8*I)/7)*E^(((7*I)/2)*ArcTan[a*x])*Hypergeometric2F1[7/4, 2, 11/4, -E^((
2*I)*ArcTan[a*x])])/a
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.80

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \frac{a\sqrt{\frac{9i}{a^2}}\log\left(\frac{1}{3}a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{9i}{a^2}}\log\left(-\frac{1}{3}a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{-\frac{9i}{a^2}}\log\left(\frac{1}{3}a\sqrt{-\frac{9i}{a^2}}+\sqrt{i\right.}}{2a}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")

[Out] 1/2*(a*sqrt(9*I/a^2)*log(1/3*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(9*I/a^2)*log(-1/3*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(-9*I/a^2)*log(1/3*a*sqrt(-9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + a*sqrt(-9*I/a^2)*log(-1/3*a*sqrt(-9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*I*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a

Sympy [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)

[Out] Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(3/2), x)

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \int\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")

[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 23, a substitution variable should perhaps be purged
.Warni

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \int \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)

[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)

3.74 $\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 267

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}$$

```
[Out] 2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*
x)^(1/4))+1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/
(1+I*a*x)^(1/2))*2^(1/2)-1/2*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(
1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)+arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1
+I*a*x)^(1/4))*2^(1/2)-arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^
(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules

used = {5170, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 95, 304, 209, 212}

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) -\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) +\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I
*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(
1/4)] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^{3/4}}{x(1-iax)^{3/4}}\,dx \\
&= (ia)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx + \int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\left(4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)\right) + 4\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\left(2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) \\
&\quad + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) \\
&= 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad - 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} \\
&\quad - \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) \\
&\quad - \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} \\
&\quad - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) \\
&\quad + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) \\
&\quad - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$
\begin{aligned}
\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx &= -22^{3/4}\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right) \\
&\quad - \frac{4\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-\frac{1-iax}{-1-iax}\right)}{\sqrt[4]{1+iax}}
\end{aligned}
$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] -2*2^(3/4)*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - I*a*x)/2
] - (4*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - I*a*x)/(-1 -
 I*a*x))])/(1 + I*a*x)^(1/4)
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}dx &= \frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ &\quad -\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ &\quad -\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ &\quad +\frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ &\quad -\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) \\ &\quad -i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)\end{aligned}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
[Out] 1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 + 1/2*sqrt(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
```

```
))) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*log(sqrt(I*sqrt(a^2*
x^2 + 1)/(a*x + I)) + I) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) +
 log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2} i \arctan(ax)}}{x}\, dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\, dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x,x)
```

```
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2} i \arctan(ax)}}{x}\, dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\, dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2} i \arctan(ax)}}{x}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 23, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2} i \arctan(ax)}}{x}\, dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x}\, dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x, x)
```

3.75 $\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} + 3ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 3ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x+3*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))-3*I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5170, 96, 95, 304, 209, 212}

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = 3ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 3ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) + (3*I)*a*ArcTan[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)] - (3*I)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^{3/4}}{x^2(1-iax)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}+\frac{1}{2}(3ia)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx
\end{aligned}
$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} + (6ia)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - (3ia)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + (3ia)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} + 3ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 3ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{i\sqrt[4]{1-iax}\left(-i+ax+6ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{x\sqrt[4]{1+iax}}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^2,x]
[Out] ((-I)*(1 - I*a*x)^(1/4)*(-I + a*x + 6*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I
 + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(64) = 128.

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.71

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx$$

$$= \frac{-3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - 3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) + 3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) + 3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{a}}\right.}{2\,x}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")

[Out] 1/2*(-3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 3*a*x*log(sqrt (I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 3*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*s qrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x

Sympy [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**2,x)

[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**2, x)

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^2, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W arning, replacing 0 by 23, a substitution variable should perhaps be purged .Warni

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^2}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^2,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^2, x)
```

3.76 $\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2} - \frac{9}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{9}{4}a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -3/4*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-1/2*(1-I*a*x)^(1/4)*(1+I*a*x)^(7
/4)/x^2-9/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+9/4*a^2*arctanh((1+
I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5170, 98, 96, 95, 304, 209, 212}

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{9}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{9}{4}a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (((-3*I)/4)*a*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x - ((1 - I*a*x)^(1/4)*(
1 + I*a*x)^(7/4))/(2*x^2) - (9*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/
4)])/4 + (9*a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^{3/4}}{x^3(1-iax)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}+\frac{1}{4}(3ia)\int \frac{(1+iax)^{3/4}}{x^2(1-iax)^{3/4}}\,dx\\
&= -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}-\frac{1}{8}\left(9a^2\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}\\
&\quad -\frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}\\
&\quad +\frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&\quad -\frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\
&= -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}\\
&\quad -\frac{9}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{9}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{\sqrt[4]{1-iax}\left(-2-7iax+5a^2x^2+18a^2x^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{4x^2\sqrt[4]{1+iax}}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-2 - (7*I)*a*x + 5*a^2*x^2 + 18*a^2*x^2*Hypergeometric2
F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - 9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) + 9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) - 9\,a^2x^2\log\left(\right.}{8\,x^2}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*I*a^2*x^2*l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*I*a^2*x^2*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) - 9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)) - 1) - 2*sqrt(a^2*x^2 + 1)*(5*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I)))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
```

```
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 23, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^3}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^3,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^3, x)
```

3.77 $\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{17}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{17}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/3*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^3-7/12*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)
^(3/4)/x^2+23/24*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-17/8*I*a^3*arctan((1
+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+17/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)
^(1/4))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5170, 101, 156, 12, 95, 304, 209, 212}

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{17}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{17}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x^3 - (((7*I)/12)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^2 + (23*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(24*x) - ((17*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((17
*I)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^{3/4}}{x^4(1-iax)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{1}{3}\int \frac{\frac{7ia}{2}-2a^2x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} - \frac{1}{6}\int \frac{\frac{23a^2}{4}+\frac{7}{2}ia^3x}{x^2(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}\\
&\quad + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{1}{6}\int -\frac{51ia^3}{8x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}\\
&\quad + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{1}{16}\left(17ia^3\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}\\
&\quad + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{1}{4}\left(17ia^3\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{1}{8}(17ia^3)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{1}{8}(17ia^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{17}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{17}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \frac{\sqrt[4]{1-iax}\left(-8-22iax+37a^2x^2+23ia^3x^3+102ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3\sqrt[4]{1+iax}}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^4,x]
[Out] ((1 - I*a*x)^(1/4)*(-8 - (22*I)*a*x + 37*a^2*x^2 + (23*I)*a^3*x^3 + (102*I)
*a^3*x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 +
 I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.10

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \frac{51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-51i\,a^3x^3\,\mathrm{l}}{48\,x^3}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")

[Out] 1/48*(51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 51*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 51*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) + 2*(23*a^2*x^2 - 14*I*a*x - 8)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^3

Sympy [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**4,x)

[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**4, x)

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")

[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^4, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 23, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^4, x)
```

3.78 $\int \frac{e^{\frac{3}{2} i \arctan(ax)}}{x^5} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 202

$$\int \frac{e^{\frac{3}{2} i \arctan(ax)}}{x^5} \, dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x} + \frac{123}{64}a^4 \arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{123}{64}a^4 \operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^4-3/8*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)^
(3/4)/x^3+15/32*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^2+63/64*I*a^3*(1-I*a*
x)^(1/4)*(1+I*a*x)^(3/4)/x+123/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4
))-123/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5170, 101, 156, 12, 95, 304, 209, 212}

$$\int \frac{e^{\frac{3}{2} i \arctan(ax)}}{x^5} \, dx = \frac{123}{64}a^4 \arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{123}{64}a^4 \operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] -1/4*((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x^4 - (((3*I)/8)*a*(1 - I*a*x)^(
1/4)*(1 + I*a*x)^(3/4))/x^3 + (15*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/
(32*x^2) + (((63*I)/64)*a^3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x + (123*a
^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1+iax)^{3/4}}{x^5(1-iax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{\frac{9ia}{2} - 3a^2x}{x^4(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} - \frac{1}{12}\int \frac{\frac{45a^2}{4} + 9ia^3x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{1}{24}\int \frac{-\frac{189ia^3}{8} + \frac{45a^4x}{4}}{x^2(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x} - \frac{1}{24}\int -\frac{369a^4}{16x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x} + \frac{1}{128}\left(123a^4\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x} + \frac{1}{32}(123a^4)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x} - \frac{1}{64}(123a^4)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{64}(123a^4)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x} + \frac{123}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{123}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{\sqrt[4]{1-iax}\left(16+40iax-54a^2x^2-93ia^3x^3+63a^4x^4+246a^4x^4\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{64x^4\sqrt[4]{1+iax}}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^5,x]
[Out] -1/64*((1 - I*a*x)^(1/4)*(16 + (40*I)*a*x - 54*a^2*x^2 - (93*I)*a^3*x^3 + 6
3*a^4*x^4 + 246*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]
))/(x^4*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx =$$
$$-\frac{123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-12}{128\,x^4}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
[Out] -1/128*(123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 123*I*a^
4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 123*I*a^4*x^4*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)) - 1) + 2*(-63*I*a^3*x^3 - 30*a^2*x^2 + 24*I*a*x + 16)*sqrt(
a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 23, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5} dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^5} dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^5,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^5, x)
```

3.79 $\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 373

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx=\frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}-\frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2}-\frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4}-\frac{475\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}+\frac{475\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}+\frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}-\frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

```
[Out] 475/64*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^4-4*I*x^3*(1+I*a*x)^(5/4)/a/(1-I*a
*x)^(1/4)-17/4*x^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)/a^2-1/96*I*(521*I-452*a*
x)*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)/a^4-475/128*arctan(1-(1-I*a*x)^(1/4)*2^(
1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+475/128*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a^4*2^(1/2)+475/256*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)
^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)-475/256*ln(1+(1-I*a*x)^
(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5170, 99, 158, 152, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = -\frac{475\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}+\frac{475\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{i(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} + \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4} - \frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] (475*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(64*a^4) - ((4*I)*x^3*(1 + I*a*x)
^(5/4))/(a*(1 - I*a*x)^(1/4)) - (17*x^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4)
)/(4*a^2) - ((I/96)*(521*I - 452*a*x)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/
a^4 - (475*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(64*S
qrt[2]*a^4) + (475*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(64*Sqrt[2]*a^4) + (475*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (475*Log[1 +
Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)])/(128*Sqrt[2]*a^4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x^3(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$

$$= -\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} + \frac{(4i)\int \frac{x^2\sqrt[4]{1+iax}\left(3+\frac{17iax}{4}\right)}{\sqrt[4]{1-iax}}\,dx}{a}$$

$$= -\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} + \frac{i\int \frac{x\sqrt[4]{1+iax}\left(-\frac{17ia}{2}+\frac{113a^2x}{8}\right)}{\sqrt[4]{1-iax}}\,dx}{a^3}$$

$$= -\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} - \frac{(475i)\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{64a^3}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} - \frac{(475i)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{128a^3}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} + \frac{475\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{32a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} + \frac{475\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{32a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} - \frac{475\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4} + \frac{475\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} + \frac{475\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} + \frac{475\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} + \frac{475\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} + \frac{475\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} + \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} + \frac{475\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{475\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}$$

$$= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} - \frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2} - \frac{i(521i-452ax)(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4} - \frac{475\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{475\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.26

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \frac{-\sqrt[4]{1+iax}(-i+ax)^2\left(59-5iax+6a^2x^2\right)+380\sqrt[4]{2}(1-iax)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)}{24a^4\sqrt[4]{1-iax}}$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^3,x]
```

```
[Out] (-((1 + I*a*x)^(1/4)*(-I + a*x)^2*(59 - (5*I)*a*x + 6*a^2*x^2)) + 380*2^(1/
4)*(1 - I*a*x)*Hypergeometric2F1[-5/4, 3/4, 7/4, (1 - I*a*x)/2])/(24*a^4*(1
 - I*a*x)^(1/4))
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}x^3dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.67

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = -\frac{96\,a^4\sqrt{\frac{225625i}{4096\,a^8}}\log\left(\frac{64}{475}\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\,a^4\sqrt{\frac{225625i}{4096\,a^8}}\log\left(-\frac{64}{475}\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-}{}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/192*(96*a^4*sqrt(225625/4096*I/a^8)*log(64/475*a^4*sqrt(225625/4096*I/a^
8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(225625/4096*I/a^8)*
log(-64/475*a^4*sqrt(225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + 96*a^4*sqrt(-225625/4096*I/a^8)*log(64/475*a^4*sqrt(-225625/4096*I/a^
8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(-225625/4096*I/a^8)
*log(-64/475*a^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))) + (48*a^4*x^4 - 136*I*a^3*x^3 - 226*a^2*x^2 + 521*I*a*x - 2467)*sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^4
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="maxima")
[Out] integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

```
[In] int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.80 $\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 371

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}+\frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3}+\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}+\frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3}-\frac{55i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{55i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}-\frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

```
[Out] 55/8*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^3+11/4*I*(1-I*a*x)^(3/4)*(1+I*a*x)
^(5/4)/a^3+2*I*(1+I*a*x)^(9/4)/a^3/(1-I*a*x)^(1/4)+1/3*I*(1-I*a*x)^(3/4)*(1
+I*a*x)^(9/4)/a^3-55/16*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))
/a^3*2^(1/2)+55/16*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*
2^(1/2)+55/32*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2
)/(1+I*a*x)^(1/2))/a^3*2^(1/2)-55/32*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*
x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5170, 91, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx= & -\frac{55i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{55i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}\\ & +\frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3}+\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}\\ & +\frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3}+\frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}\\ & +\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}\\ & -\frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}\end{aligned}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] (((55*I)/8)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 + (((11*I)/4)*(1 - I*a
*x)^(3/4)*(1 + I*a*x)^(5/4))/a^3 + ((2*I)*(1 + I*a*x)^(9/4))/(a^3*(1 - I*a*
x)^(1/4)) + ((I/3)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(9/4))/a^3 - (((55*I)/8)*A
rcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (
((55*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt
[2]*a^3) + (((55*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*
(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((55*I)/16)*Log[1 +
 Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)])/(Sqrt[2]*a^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$

$$= \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} - \frac{(2i)\int \frac{(1+iax)^{5/4}\left(\frac{5ia}{2}-\frac{a^2x}{2}\right)}{\sqrt[4]{1-iax}}\,dx}{a^3}$$

$$= \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{11\int \frac{(1+iax)^{5/4}}{\sqrt[4]{1-iax}}\,dx}{2a^2}$$

$$= \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{55\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{8a^2}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{55\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{16a^2}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{(55i)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{4a^3}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{(55i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^3}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} - \frac{(55i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3} + \frac{(55i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3}$$

$$= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} + \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} + \frac{(55i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} + \frac{(55i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} + \frac{(55i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} + \frac{(55i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

$$
\begin{aligned}
&= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} \\
&+ \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} \\
&+ \frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} \\
&+ \frac{(55i)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} \\
&- \frac{(55i)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} \\
&= \frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3} + \frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3} \\
&+ \frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}} + \frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3} \\
&- \frac{55i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} + \frac{55i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} \\
&+ \frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.23

$$
\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \frac{-\sqrt[4]{1+iax}(-i+ax)^2(7i+ax)+44\sqrt[4]{2}(i+ax)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)}{3a^3\sqrt[4]{1-iax}}
$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^2,x]
```

```
[Out] (-((1 + I*a*x)^(1/4)*(-I + a*x)^2*(7*I + a*x)) + 44*2^(1/4)*(I + a*x)*Hyper
geometric2F1[-5/4, 3/4, 7/4, (1 - I*a*x)/2])/(3*a^3*(1 - I*a*x)^(1/4))
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^2 dx$$

[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)

[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.66

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \frac{12\,a^3\sqrt{\frac{3025i}{64\,a^6}}\log\left(\frac{8}{55}i\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{3025i}{64\,a^6}}\log\left(-\frac{8}{55}i\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3\sqrt{\cdots}}{\cdots}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="fricas")

[Out] 1/24*(12*a^3*sqrt(3025/64*I/a^6)*log(8/55*I*a^3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(3025/64*I/a^6)*log(-8/55*I*a^3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(-3025/64*I/a^6)*log(8/55*I*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-3025/64*I/a^6)*log(-8/55*I*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (8*a^3*x^3 - 26*I*a^2*x^2 - 61*a*x - 287*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \text{Timed out}$$

[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**2,x)

[Out] Timed out

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \int x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \int x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

```
[In] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

```
[Out] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.81 $\int e^{\frac{5}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 324

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}-\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}+\frac{25\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{25\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}+\frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

```
[Out] -25/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^2-5/2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/
4)/a^2-2*(1+I*a*x)^(9/4)/a^2/(1-I*a*x)^(1/4)+25/8*arctan(1-(1-I*a*x)^(1/4)*
2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-25/8*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a^2*2^(1/2)-25/16*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(
1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2^(1/2)+25/16*ln(1+(1-I*a*x)^(1/4
)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5170, 79, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = \frac{25\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{25\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}-\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2} + \frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x,x]
```

```
[Out] (-25*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) - (5*(1 - I*a*x)^(3/4)*(1
 + I*a*x)^(5/4))/(2*a^2) - (2*(1 + I*a*x)^(9/4))/(a^2*(1 - I*a*x)^(1/4)) +
(25*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqrt[2]*a
^2) - (25*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqr
t[2]*a^2) - (25*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (25*Log[1 + Sqrt[1 - I*a*x
]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt
[2]*a^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$

$$= -\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{(5i)\int \frac{(1+iax)^{5/4}}{\sqrt[4]{1-iax}}\,dx}{a}$$

$$= -\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{(25i)\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx}{4a}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{(25i)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{8a}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{25\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} - \frac{25\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} - \frac{25\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} - \frac{25\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} - \frac{25\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} - \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} - \frac{25\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} + \frac{25\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}$$

$$= -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} - \frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}} + \frac{25\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{25\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.22

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = \frac{2\left(-3(1+iax)^{9/4}+20i\sqrt[4]{2}(i+ax)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)\right)}{3a^2\sqrt[4]{1-iax}}$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x,x]
[Out] (2*(-3*(1 + I*a*x)^(9/4) + (20*I)*2^(1/4)*(I + a*x)*Hypergeometric2F1[-5/4,
 3/4, 7/4, (1 - I*a*x)/2]))/(3*a^2*(1 - I*a*x)^(1/4))
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.73

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = \frac{2\,a^2\sqrt{\frac{625i}{16\,a^4}}\log\left(\frac{4}{25}\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{625i}{16\,a^4}}\log\left(-\frac{4}{25}\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{625}{16\,a}}}{}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="fricas")
[Out] 1/4*(2*a^2*sqrt(625/16*I/a^4)*log(4/25*a^2*sqrt(625/16*I/a^4) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(625/16*I/a^4)*log(-4/25*a^2*sqrt(625
/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-625/16*I/a^
4)*log(4/25*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
- 2*a^2*sqrt(-625/16*I/a^4)*log(-4/25*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I))) - (2*a^2*x^2 - 9*I*a*x + 43)*sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = \int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="maxima")
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = \int x\,\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

```
[In] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.82 $\int e^{\frac{5}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 299

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx=-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}+\frac{5i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{5i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}+\frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

```
[Out] -5*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a-4*I*(1+I*a*x)^(5/4)/a/(1-I*a*x)^(1/4
)+5/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a*2^(1/2)-5/2*I*a
rctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a*2^(1/2)-5/4*I*ln(1-(1-I*
a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/
2)+5/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*
a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5169, 49, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx=\frac{5i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{5i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((-5*I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - ((4*I)*(1 + I*a*x)^(5/4))/
(a*(1 - I*a*x)^(1/4)) + ((5*I)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(Sqrt[2]*a) - ((5*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/
(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - (((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (
((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1
/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx$$

$$= -\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - 5\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx$$

$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{5}{2}\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{(10i)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$

$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{(10i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} + \frac{(5i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} - \frac{(5i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{(5i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} - \frac{(5i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} - \frac{(5i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{(5i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} - \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{(5i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{(5i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}$$

$$= -\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}} + \frac{5i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{5i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.14

$$\int e^{\frac{5}{2} i \arctan(ax)}\,dx = -\frac{8ie^{\frac{9}{2} i \arctan(ax)}\,\text{Hypergeometric2F1}\left(2,\frac{9}{4},\frac{13}{4},-e^{2i\arctan(ax)}\right)}{9a}$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (((-8*I)/9)*E^(((9*I)/2)*ArcTan[a*x])*Hypergeometric2F1[2, 9/4, 13/4, -E^((
2*I)*ArcTan[a*x])])/a
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.70

$$\int e^{\frac{5}{2} i \arctan(ax)}\,dx =$$

$$-\frac{a\sqrt{\frac{25i}{a^2}}\log\left(\frac{1}{5}i\,a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{25i}{a^2}}\log\left(-\frac{1}{5}i\,a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{25i}{a^2}}\log\left(\frac{1}{5}i\,a\sqrt{-\cdots}\right.}{2a}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*sqrt(25*I/a^2)*log(1/5*I*a*sqrt(25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1
)/(a*x + I))) - a*sqrt(25*I/a^2)*log(-1/5*I*a*sqrt(25*I/a^2) + sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I))) + a*sqrt(-25*I/a^2)*log(1/5*I*a*sqrt(-25*I/a^2) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(-25*I/a^2)*log(-1/5*I*a*sqrt(
-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*(a*x + 9*I)*sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx=\int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx=\int\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx=\int\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.83 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 293

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}$$

```
[Out] 8*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2
)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-1/2*ln(1+(1-I*a*
x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-a
rctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^(1/2)+arctan(1+(1-I*a*x)
^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules

used = {5170, 100, 21, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 95, 218, 212, 209}

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + \frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 1\right)}{\sqrt{2}} - \frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 1\right)}{\sqrt{2}}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] (8*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] -
 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Log[1 + Sqrt[1 - I*a*x]/S
qrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - L
og[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1+iax)^{5/4}}{x(1-iax)^{5/4}}\,dx$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + \frac{(4i)\int \frac{-\frac{ia}{4}-\frac{a^2x}{4}}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{a}$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + \int \frac{(1-iax)^{3/4}}{x(1+iax)^{3/4}}\,dx$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - (ia)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} + 4\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right) + 4\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 4\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\arctan\left(1 + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.38

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = \frac{4\left(3+3iax+3\sqrt[4]{2}(1+iax)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{4},\frac{3}{4},\frac{1}{2}(1-iax)\right)+(-1+iax)\,\text{Hypergeometri}\right.}{3\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x,x]
```

```
[Out] (4*(3 + (3*I)*a*x + 3*2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[-1/4, -1/
4, 3/4, (1 - I*a*x)/2] + (-1 + I*a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a
*x)/(I - a*x)]))/(3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)

[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = &-\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)\\ &+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)\\ &-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)\\ &+\frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+8\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\\ &-\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)\\ &+i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)\end{aligned}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1
/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/
2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1/
2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 8
*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + I*log(sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) -
 1)
```

Sympy [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx=\int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x,x)
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx=\int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx=\int \frac{\left(\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x, x)
```

3.84 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} - 5ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 5ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] 10*I*a*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-(1+I*a*x)^(5/4)/x/(1-I*a*x)^(1/4)-5*
I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-5*I*a*arctanh((1+I*a*x)^(1/4)/(
1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5170, 96, 95, 218, 212, 209}

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = -5ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 5ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} + \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] ((10*I)*a*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - (1 + I*a*x)^(5/4)/(x*(1 -
I*a*x)^(1/4)) - (5*I)*a*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - (5*I)
*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1+iax)^{5/4}}{x^2(1-iax)^{5/4}}\,dx$$

$$= -\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} + \frac{1}{2}(5ia)\int \frac{\sqrt[4]{1+iax}}{x(1-iax)^{5/4}}\,dx$$

$$= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} + \frac{1}{2}(5ia)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} + (10ia)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} - (5ia)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (5ia)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} - 5ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 5ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \frac{-3(1-8iax+9a^2x^2) - 10ax(i+ax)\,\text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, \frac{i+ax}{i-ax}\right)}{3x\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^2,x]
```

```
[Out] (-3*(1 - (8*I)*a*x + 9*a^2*x^2) - 10*a*x*(I + a*x)*Hypergeometric2F1[3/4, 1
, 7/4, (I + a*x)/(I - a*x)])/(3*x*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.26

$$\int \frac{e^{\frac{5}{2} i \arctan(ax)}}{x^2}\,dx = \frac{-5i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+5\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-5\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+5i\,ax\log\left(\sqrt{\frac{i\sqrt{}}{}}\right.}{2\,x}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-5*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 5*a*x*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 5*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) - I) + 5*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(
-9*I*a*x + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2} i \arctan(ax)}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{5}{2} i \arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^2}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^2, x)
```

3.85 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 163

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}} - \frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}} - \frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}} + \frac{25}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{25}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -25/2*a^2*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-5/4*I*a*(1+I*a*x)^(5/4)/x/(1-I*a*
x)^(1/4)-1/2*(1+I*a*x)^(9/4)/x^2/(1-I*a*x)^(1/4)+25/4*a^2*arctan((1+I*a*x)^
(1/4)/(1-I*a*x)^(1/4))+25/4*a^2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5170, 98, 96, 95, 218, 212, 209}

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \frac{25}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{25}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}} - \frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}} - \frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (-25*a^2*(1 + I*a*x)^(1/4))/(2*(1 - I*a*x)^(1/4)) - (((5*I)/4)*a*(1 + I*a*x
)^(5/4))/(x*(1 - I*a*x)^(1/4)) - (1 + I*a*x)^(9/4)/(2*x^2*(1 - I*a*x)^(1/4)
) + (25*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4 + (25*a^2*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^{5/4}}{x^3(1-iax)^{5/4}}\,dx \\
&= -\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}+\frac{1}{4}(5ia)\int \frac{(1+iax)^{5/4}}{x^2(1-iax)^{5/4}}\,dx \\
&= -\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1+iax}}{x(1-iax)^{5/4}}\,dx \\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}-\frac{1}{8}\left(25a^2\right)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}} \\
&\quad -\frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}} \\
&\quad +\frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad +\frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}}-\frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}}-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}} \\
&\quad +\frac{25}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{25}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \frac{-6-33iax-102a^2x^2-129ia^3x^3+50a^2x^2(1-iax)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)}{12x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^3,x]
```

```
[Out] (-6 - (33*I)*a*x - 102*a^2*x^2 - (129*I)*a^3*x^3 + 50*a^2*x^2*(1 - I*a*x)*H
ypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)])/(12*x^2*(1 - I*a*x)^(1/
4)*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^3}\,dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.08

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \frac{25\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+25i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-25i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-25\,a^2x^2\,\mathrm{l}}{8\,x^2}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(25*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 25*I*a^2*x^2
*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 25*I*a^2*x^2*log(sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) - I) - 25*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)) - 1) - 2*(43*a^2*x^2 + 9*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I)))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^3}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^3}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^3,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^3, x)
```

3.86 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 203

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} + \frac{55}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{55}{8}ia^3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -287/24*I*a^3*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-1/3*(1+I*a*x)^(1/4)/x^3/(1-I*
a*x)^(1/4)-13/12*I*a*(1+I*a*x)^(1/4)/x^2/(1-I*a*x)^(1/4)+61/24*a^2*(1+I*a*x
)^(1/4)/x/(1-I*a*x)^(1/4)+55/8*I*a^3*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)
)+55/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5170, 100, 156, 160, 12, 95, 218, 212, 209}

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{55}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{55}{8}ia^3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] (((-287*I)/24)*a^3*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) - (1 + I*a*x)^(1/4)
/(3*x^3*(1 - I*a*x)^(1/4)) - (((13*I)/12)*a*(1 + I*a*x)^(1/4))/(x^2*(1 - I*
```

```
a*x)^(1/4)) + (61*a^2*(1 + I*a*x)^(1/4))/(24*x*(1 - I*a*x)^(1/4)) + ((55*I)
/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + ((55*I)/8)*a^3*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
```

```
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^{5/4}}{x^4(1-iax)^{5/4}}\,dx \\
&= -\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{1}{3}\int \frac{-\frac{13ia}{2}+6a^2x}{x^3(1-iax)^{5/4}(1+iax)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{1}{6}\int \frac{-\frac{61a^2}{4}-13ia^3x}{x^2(1-iax)^{5/4}(1+iax)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{1}{6}\int \frac{\frac{165ia^3}{8}-\frac{61a^4x}{4}}{x(1-iax)^{5/4}(1+iax)^{3/4}}\,dx \\
&= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} \\
&\quad + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{i\int \frac{165a^4}{16x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{3a}
\end{aligned}$$

$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{1}{16}(55ia^3)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} - \frac{1}{4}(55ia^3)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} + \frac{1}{8}(55ia^3)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{8}(55ia^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}} - \frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}} + \frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}} + \frac{55}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{55}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.52

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{-8-34iax+87a^2x^2-226ia^3x^3+287a^4x^4+110a^3x^3(i+ax)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)}{24x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^4,x]
```

```
[Out] (-8 - (34*I)*a*x + 87*a^2*x^2 - (226*I)*a^3*x^3 + 287*a^4*x^4 + 110*a^3*x^3
*(I + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)])/(24*x^3*(1
- I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}dx$$

[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)

[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.91

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{165i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-165\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+165\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-165i\,a}{48\,x^3}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")

[Out] 1/48*(165*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 165*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 165*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 165*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(287*I*a^3*x^3 - 61*a^2*x^2 + 26*I*a*x + 8)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^3

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \text{Timed out}$$

[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**4,x)

[Out] Timed out

Maxima [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")

[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^4, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^4}\,dx$$

[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^4,x)

[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^4, x)

3.87 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 233

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} - \frac{475}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{475}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] 2467/192*a^4*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-1/4*(1+I*a*x)^(1/4)/x^4/(1-I*a
*x)^(1/4)-17/24*I*a*(1+I*a*x)^(1/4)/x^3/(1-I*a*x)^(1/4)+113/96*a^2*(1+I*a*x
)^(1/4)/x^2/(1-I*a*x)^(1/4)+521/192*I*a^3*(1+I*a*x)^(1/4)/x/(1-I*a*x)^(1/4)
-475/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-475/64*a^4*arctanh((1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5170, 100, 156, 160, 12, 95, 218, 212, 209}

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{475}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{475}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}} + \frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}} + \frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}} - \frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}} - \frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] (2467*a^4*(1 + I*a*x)^(1/4))/(192*(1 - I*a*x)^(1/4)) - (1 + I*a*x)^(1/4)/(4
*x^4*(1 - I*a*x)^(1/4)) - (((17*I)/24)*a*(1 + I*a*x)^(1/4))/(x^3*(1 - I*a*x
)^(1/4)) + (113*a^2*(1 + I*a*x)^(1/4))/(96*x^2*(1 - I*a*x)^(1/4)) + (((521*
I)/192)*a^3*(1 + I*a*x)^(1/4))/(x*(1 - I*a*x)^(1/4)) - (475*a^4*ArcTan[(1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (475*a^4*ArcTanh[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^{5/4}}{x^5(1-iax)^{5/4}}\,dx \\
&= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{1}{4}\int \frac{-\frac{17ia}{2}+8a^2x}{x^4(1-iax)^{5/4}(1+iax)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}+\frac{1}{12}\int \frac{-\frac{113a^2}{4}-\frac{51}{2}ia^3x}{x^3(1-iax)^{5/4}(1+iax)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}+\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}-\frac{1}{24}\int \frac{\frac{521ia^3}{8}-\frac{113a^4x}{2}}{x^2(1-iax)^{5/4}(1+iax)^{3/4}}\,dx
\end{aligned}$$

$$=-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}+\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}+\frac{1}{24}\int\frac{\frac{1425a^4}{16}+\frac{521}{8}ia^5x}{x(1-iax)^{5/4}(1+iax)^{3/4}}\,dx$$

$$=\frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}+\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}+\frac{i\int-\frac{1425ia^5}{32x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{12a}$$

$$=\frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}+\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}+\frac{1}{128}\left(475a^4\right)\int\frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$=\frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}+\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}+\frac{1}{32}\left(475a^4\right)\text{Subst}\left(\int\frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$=\frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}+\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}-\frac{1}{64}\left(475a^4\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{64}\left(475a^4\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$=\frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}+\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}-\frac{475}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.51

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \frac{-48-184iax+362a^2x^2+747ia^3x^3+1946a^4x^4+2467ia^5x^5+950ia^4x^4(i+ax)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\right.}{192x^4\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^5,x]
```

```
[Out] (-48 - (184*I)*a*x + 362*a^2*x^2 + (747*I)*a^3*x^3 + 1946*a^4*x^4 + (2467*I
)*a^5*x^5 + (950*I)*a^4*x^4*(I + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a
*x)/(I - a*x)])/(192*x^4*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.82

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{1425\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+1425i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-1425i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)}{384\,x}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(1425*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 1425*I*
a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 1425*I*a^4*x^4*log(s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 1425*a^4*x^4*log(sqrt(I*sqrt(a^2*
x^2 + 1)/(a*x + I)) - 1) - 2*(2467*a^4*x^4 + 521*I*a^3*x^3 + 226*a^2*x^2 -
136*I*a*x - 48)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^5}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^5, x)
```

3.88 $\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 337

$$\begin{aligned}\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx &= -\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}\\&-\frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4}\\&-\frac{11\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}+\frac{11\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}\\&-\frac{11\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}\\&+\frac{11\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}\end{aligned}$$

```
[Out] -11/64*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^4+1/4*x^2*(1-I*a*x)^(5/4)*(1+I*a*x
)^(3/4)/a^2-1/96*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)*(25-4*I*a*x)/a^4-11/128*ar
ctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+11/128*arctan(1
+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)-11/256*ln(1-(1-I*a*x)
^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)
+11/256*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a
*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5170, 102, 152, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = -\frac{11\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}+\frac{11\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} - \frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{11\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4} + \frac{11\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}$$

```
[In] Int[x^3/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (-11*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(64*a^4) + (x^2*(1 - I*a*x)^(5/4)
*(1 + I*a*x)^(3/4))/(4*a^2) - ((1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)*(25 - (4
*I)*a*x))/(96*a^4) - (11*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)
^(1/4)])/(64*Sqrt[2]*a^4) + (11*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
 I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) - (11*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a
*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) + (
11*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x^3\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$

$$= \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} + \frac{\int \frac{x\left(-2+\frac{iax}{2}\right)\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{4a^2}$$

$$= \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} - \frac{(11i)\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{64a^3}$$

$$= -\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} - \frac{(11i)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{128a^3}$$

$$= -\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} + \frac{11\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{32a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} + \frac{11\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{32a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} + \frac{11\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4} + \frac{11\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} + \frac{11\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} + \frac{11\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} - \frac{11\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{11\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} - \frac{11\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} + \frac{11\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} + \frac{11\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{11\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}$$

$$= -\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} + \frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}(25-4iax)}{96a^4} - \frac{11\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{11\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{11\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} + \frac{11\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.38

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \frac{(1-iax)^{5/4}\left(5a^2x^2(1+iax)^{3/4}+4\,2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)-12\,2^{3/4}\,\text{Hypergeom}\right.}{20a^4}$$

```
[In] Integrate[x^3/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(5/4)*(5*a^2*x^2*(1 + I*a*x)^(3/4) + 4*2^(3/4)*Hypergeometric2
F1[-7/4, 5/4, 9/4, (1 - I*a*x)/2] - 12*2^(3/4)*Hypergeometric2F1[-3/4, 5/4,
 9/4, (1 - I*a*x)/2] + 5*2^(3/4)*Hypergeometric2F1[1/4, 5/4, 9/4, (1 - I*a*
x)/2]))/(20*a^4)
```

Maple [F]

$$\int \frac{x^3}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

```
[In] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
[Out] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.76

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx =$$

$$-\frac{96\,a^4\sqrt{\frac{121i}{4096\,a^8}}\log\left(\frac{64}{11}i\,a^4\sqrt{\frac{121i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\,a^4\sqrt{\frac{121i}{4096\,a^8}}\log\left(-\frac{64}{11}i\,a^4\sqrt{\frac{121i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-}{}$$

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] -1/192*(96*a^4*sqrt(121/4096*I/a^8)*log(64/11*I*a^4*sqrt(121/4096*I/a^8) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(121/4096*I/a^8)*log(-64/
11*I*a^4*sqrt(121/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a
^4*sqrt(-121/4096*I/a^8)*log(64/11*I*a^4*sqrt(-121/4096*I/a^8) + sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I))) + 96*a^4*sqrt(-121/4096*I/a^8)*log(-64/11*I*a^4*
sqrt(-121/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (-48*I*a^3*x
^3 + 56*a^2*x^2 + 58*I*a*x - 83)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)))/a^4
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
[Out] Integral(x**3/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(x^3/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

[In] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)

[Out] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)

3.89 $\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 339

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx = \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3}+\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3}+\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}+\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}-\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

```
[Out] 3/8*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^3+1/12*I*(1-I*a*x)^(5/4)*(1+I*a*x)^
(3/4)/a^3+1/3*x*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^2+3/16*I*arctan(1-(1-I*a*
x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-3/16*I*arctan(1+(1-I*a*x)^(1/
4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)+3/32*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)
/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)-3/32*I*ln(1+(
1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3
*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5170, 92, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx=\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{i(1+iax)^{3/4}(1-iax)^{5/4}}{12a^3}+\frac{3i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}-\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}+\frac{x(1+iax)^{3/4}(1-iax)^{5/4}}{3a^2}$$

```
[In] Int[x^2/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] (((3*I)/8)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a^3 + ((I/12)*(1 - I*a*x)^(
5/4)*(1 + I*a*x)^(3/4))/a^3 + (x*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(3*a^
2) + (((3*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/
(Sqrt[2]*a^3) - (((3*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*
x)^(1/4)])/(Sqrt[2]*a^3) + (((3*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a
*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((3*
I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx \\
&= \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} + \frac{\int \frac{\left(-1+\frac{iax}{2}\right)\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{3a^2} \\
&= \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{3\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{8a^2} \\
&= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} \\
&\quad + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{3\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{16a^2} \\
&= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} \\
&\quad + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{4a^3}
\end{aligned}
$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^3}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3} - \frac{(3i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} - \frac{(3i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} - \frac{(3i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} + \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} + \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{(3i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} + \frac{(3i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}$$

$$= \frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} + \frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3} + \frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2} + \frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.22

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx = \frac{(1-iax)^{5/4}\left(5(1+iax)^{3/4}(i+4ax)-9i2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)\right)}{60a^3}$$

```
[In] Integrate[x^2/E^((I/2)*ArcTan[a*x]),x]
[Out] ((1 - I*a*x)^(5/4)*(5*(1 + I*a*x)^(3/4)*(I + 4*a*x) - (9*I)*2^(3/4)*Hyperge
ometric2F1[1/4, 5/4, 9/4, (1 - I*a*x)/2]))/(60*a^3)
```

Maple [F]

$$\int \frac{x^2}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

```
[In] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
[Out] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.73

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx = -\frac{12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(\frac{8}{3}\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(-\frac{8}{3}\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{-}$$

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] -1/24*(12*a^3*sqrt(9/64*I/a^6)*log(8/3*a^3*sqrt(9/64*I/a^6) + sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(9/64*I/a^6)*log(-8/3*a^3*sqrt(9/64*I/
a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-9/64*I/a^6)*log(
8/3*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*s
qrt(-9/64*I/a^6)*log(-8/3*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I))) - sqrt(a^2*x^2 + 1)*(-8*I*a^2*x^2 + 10*a*x + 11*I)*sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx=\int\frac{x^2}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
[Out] Integral(x**2/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx=\int\frac{x^2}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx=\int\frac{x^2}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

```
[In] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

3.90 $\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 295

$$\begin{aligned}\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx &= \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}\\ &+\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}\\ &+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}\\ &-\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}\end{aligned}$$

```
[Out] 1/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^2+1/2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)
/a^2+1/8*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-1/8*
arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+1/16*ln(1-(1-
I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2
^(1/2)-1/16*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5170, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}+\frac{(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}$$

```
[In] Int[x/E^((I/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*a^2) + ((1 - I*a*x)^(5/4)*(1 + I*a
*x)^(3/4))/(2*a^2) + ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/
4)]/(4*Sqrt[2]*a^2) - ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/(4*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqrt[2]*a^2) - Log[1 + Sqrt[1 - I*a
*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(8*Sqr
t[2]*a^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$

$$= \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{i\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{4a}$$

$$= \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{i\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{8a}$$

$$= \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2}$$

$$= \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$

$$= \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}$$

$$= \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

$$= \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}$$

$$= \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} + \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.21

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \frac{(1-iax)^{5/4}\left(5(1+iax)^{3/4} - 2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)\right)}{10a^2}$$

```
[In] Integrate[x/E^((I/2)*ArcTan[a*x]),x]
[Out] ((1 - I*a*x)^(5/4)*(5*(1 + I*a*x)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 5/
4, 9/4, (1 - I*a*x)/2]))/(10*a^2)
```

Maple [F]

$$\int \frac{x}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

[In] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)

[Out] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.81

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \frac{2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(4i\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(-4i\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{-\frac{i}{16\,a^4}}\,\mathrm{l}}{}$$

[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 1/4*(2*a^2*sqrt(1/16*I/a^4)*log(4*I*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(1/16*I/a^4)*log(-4*I*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-1/16*I/a^4)*log(4*I*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-1/16*I/a^4)*log(-4*I*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + sqrt(a^2*x^2 + 1)*(-2*I*a*x + 3)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^2

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

[In] integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)

[Out] Integral(x/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

```
[In] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

3.91 $\int e^{-\frac{1}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 268

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

```
[Out] -I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a-1/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)
/(1+I*a*x)^(1/4))/a*2^(1/2)+1/2*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x
)^(1/4))/a*2^(1/2)-1/4*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*
a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)+1/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1
+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules

used = {5169, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = -\frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a} + \frac{i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}$$

```
[In] Int[E^((-1/2*I)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*ArcTan[1 + (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - ((I/2)*Log[1 + Sqrt[1 - I*
a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqr
t[2]*a) + ((I/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*
a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(2i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$

$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{(2i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{i\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} + \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} - \frac{i\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{i\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{i\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{i\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}$$

$$= -\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = -\frac{8ie^{\frac{3}{2}i\arctan(ax)}\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2i\arctan(ax)}\right)}{3a}$$

```
[In] Integrate[E^((-1/2*I)*ArcTan[a*x]),x]
[Out] (((-8*I)/3)*E^(((3*I)/2)*ArcTan[a*x])*Hypergeometric2F1[3/4, 2, 7/4, -E^((2
*I)*ArcTan[a*x])])/a
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.79

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \frac{a\sqrt{\frac{i}{a^2}}\log\left(a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - a\sqrt{\frac{i}{a^2}}\log\left(-a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - a\sqrt{-\frac{i}{a^2}}\log\left(a\sqrt{-\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)}{2a}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(I/a^2)*log(a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 - a*sqrt(I/a^2)*log(-a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
- a*sqrt(-I/a^2)*log(a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
+ a*sqrt(-I/a^2)*log(-a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 - 2*I*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt((I*a*x + 1)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \int \frac{1}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx=\int\frac{1}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

```
[In] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

3.92 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 267

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}$$

```
[Out] 2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*
x)^(1/4))-1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/
(1+I*a*x)^(1/2))*2^(1/2)+1/2*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(
1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1
+I*a*x)^(1/4))*2^(1/2)+arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^
(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5170, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}$$

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x),x]
```

```
[Out] 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I
*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(
1/4)] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
```

```
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\,dx \\
&= -\left((ia)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\right)+\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= 4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx,x,\sqrt[4]{1-iax}\right)+4\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\left(2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) \\
&\quad +2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad + 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} \\
&\quad + \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) \\
&\quad + \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} \\
&\quad + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) \\
&\quad - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) \\
&\quad + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = 2\ 2^{3/4}\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right) - \frac{4\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-\frac{1-iax}{-1-iax}\right)}{\sqrt[4]{1+iax}}$$

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x),x]
[Out] 2*2^(3/4)*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - I*a*x)/2]
 - (4*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - I*a*x)/(-1 -
I*a*x))])/(1 + I*a*x)^(1/4)
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x}\,dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = -\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + \frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + \frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) + i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) - i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) + \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
[Out] -1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
+ 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) - 1/2*sqrt(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*log(sqrt(I*sqrt(a^2
*x^2 + 1)/(a*x + I)) + I) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I)
+ log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x,x)
[Out] Integral(1/(x*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(1/(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\,\sqrt{\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}}}\,dx$$

```
[In] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

3.93 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/
4))+I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5170, 96, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = -ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] -(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) - I*a*ArcTan[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)] + I*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[4]{1-iax}}{x^2\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - \frac{1}{2}(ia)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - (2ia)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} + (ia)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (ia)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \frac{i\sqrt[4]{1-iax}\left(i-ax+2ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{x\sqrt[4]{1+iax}}$$

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^2),x]
[Out] (I*(1 - I*a*x)^(1/4)*(I - a*x + 2*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I + a
*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^2}\,dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.70

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx$$
$$= \frac{i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-\right.}{2\,x}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + a*x*log(sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)) + I) - a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
)) - I) - I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*sqrt(a^2*x
^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\,\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

```
[In] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

3.94 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] 1/4*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-1/2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/
4)/x^2-1/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/4*a^2*arctanh((1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5170, 98, 96, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{1}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} + \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x}$$

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] ((I/4)*a*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x - ((1 - I*a*x)^(5/4)*(1 + I
*a*x)^(3/4))/(2*x^2) - (a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
+ (a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1-iax}}{x^3\sqrt[4]{1+iax}}\,dx \\
&= -\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{4}(ia)\int \frac{\sqrt[4]{1-iax}}{x^2\sqrt[4]{1+iax}}\,dx \\
&= \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{8}a^2\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} \\
&\quad - \frac{1}{2}a^2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} \\
&\quad + \frac{1}{4}a^2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{1}{4}a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} \\
&\quad - \frac{1}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$
\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \frac{\sqrt[4]{1-iax}\left(-2+iax-3a^2x^2+2a^2x^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{4x^2\sqrt[4]{1+iax}}
$$

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^3),x]
[Out] ((1 - I*a*x)^(1/4)*(-2 + I*a*x - 3*a^2*x^2 + 2*a^2*x^2*Hypergeometric2F1[1/
4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^3}dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.35

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \frac{a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-a^2x^2\log\left(\sqrt{\frac{i\cdots}{}}\right.}{8\,x^2}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/8*(a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*a^2*x^2*log(s
qrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + I*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - I) - a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1
) - 2*sqrt(a^2*x^2 + 1)*(-3*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
/x^2
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
[Out] Integral(1/(x**3*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(1/(x^3*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W arning, replacing 0 by -46, a substitution variable should perhaps be purge d.Warn

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\,\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

[In] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)

[Out] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)

3.95 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{3}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{3}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/3*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^3+5/12*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)
^(3/4)/x^2+11/24*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x+3/8*I*a^3*arctan((1+
I*a*x)^(1/4)/(1-I*a*x)^(1/4))-3/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5170, 101, 156, 12, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \frac{3}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{3}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}$$

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] -1/3*((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x^3 + (((5*I)/12)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^2 + (11*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(24*x) + ((3*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((3*I
)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1-iax}}{x^4\sqrt[4]{1+iax}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}+\frac{1}{3}\int \frac{-\frac{5ia}{2}-2a^2x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}+\frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2}-\frac{1}{6}\int \frac{\frac{11a^2}{4}-\frac{5}{2}ia^3x}{x^2(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}+\frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} \\
&\quad +\frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x}+\frac{1}{6}\int \frac{9ia^3}{8x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}+\frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} \\
&\quad +\frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x}+\frac{1}{16}\left(3ia^3\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}+\frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} \\
&\quad +\frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x}+\frac{1}{4}\left(3ia^3\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{1}{8}(3ia^3)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{8}(3ia^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} + \frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{3}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{3}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.54

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \frac{\sqrt[4]{1-iax}\left(-8+2iax+a^2x^2+11ia^3x^3-18ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3\sqrt[4]{1+iax}}$$

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^4),x]
[Out] ((1 - I*a*x)^(1/4)*(-8 + (2*I)*a*x + a^2*x^2 + (11*I)*a^3*x^3 - (18*I)*a^3*
x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I*a*
x)^(1/4))
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^4}dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.10

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \frac{-9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+9i\,a^3x^3\log}{48\,x^3}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/48*(-9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*a^3*x^3
*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*a^3*x^3*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) + 9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I)) - 1) + 2*(11*a^2*x^2 + 10*I*a*x - 8)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
[Out] Integral(1/(x**4*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(1/(x^4*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

[In] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)

[Out] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)

3.96 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 202

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}+\frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2}-\frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x} + \frac{11}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^4+7/24*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)
^(3/4)/x^3+29/96*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^2-83/192*I*a^3*(1-I*
a*x)^(1/4)*(1+I*a*x)^(3/4)/x+11/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/
4))-11/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5170, 101, 156, 12, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \frac{11}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x}+\frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} - \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}+\frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3}$$

```
[In] Int[1/(E^((I/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] -1/4*((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x^4 + (((7*I)/24)*a*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/x^3 + (29*a^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(96*x^2) - (((83*I)/192)*a^3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x + (11*
a^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1-iax}}{x^5\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{-\frac{7ia}{2}-3a^2x}{x^4(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} - \frac{1}{12}\int \frac{\frac{29a^2}{4}-7ia^3x}{x^3(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3}\\
&\quad + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} + \frac{1}{24}\int \frac{\frac{83ia^3}{8}+\frac{29a^4x}{4}}{x^2(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2}\\
&\quad - \frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x} - \frac{1}{24}\int -\frac{33a^4}{16x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx\\
&= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2}\\
&\quad - \frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x} + \frac{1}{128}\left(11a^4\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx
\end{aligned}
$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x} + \frac{1}{32}(11a^4)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x} - \frac{1}{64}(11a^4)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{64}(11a^4)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} + \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3} + \frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2} - \frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x} + \frac{11}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{11}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \frac{\sqrt[4]{1-iax}\left(-48+8iax+2a^2x^2-25ia^3x^3+83a^4x^4-66a^4x^4\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{192x^4\sqrt[4]{1+iax}}$$

```
[In] Integrate[1/(E^((I/2)*ArcTan[a*x])*x^5),x]
[Out] ((1 - I*a*x)^(1/4)*(-48 + (8*I)*a*x + 2*a^2*x^2 - (25*I)*a^3*x^3 + 83*a^4*x
^4 - 66*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(192*
x^4*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^5}dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-33\,a}{384\,x^4}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/384*(33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 33*I*a^4*
x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 33*I*a^4*x^4*log(sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) - 1) + 2*(83*I*a^3*x^3 - 58*a^2*x^2 - 56*I*a*x + 48)*sqrt(a^2*x
^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
[Out] Integral(1/(x**5*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(1/(x^5*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

```
[In] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

3.97 $\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 337

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}+\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} - \frac{123\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}+\frac{123\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

```
[Out] -41/64*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^4+1/4*x^2*(1-I*a*x)^(7/4)*(1+I*a*x
)^(1/4)/a^2-1/32*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/4)*(11-4*I*a*x)/a^4-123/128*a
rctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+123/128*arctan
(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)+123/256*ln(1-(1-I*a
*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1
/2)-123/256*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5170, 102, 152, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = -\frac{123\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}+\frac{123\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4}-\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4} - \frac{123\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4}+\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}$$

```
[In] Int[x^3/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (-41*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(64*a^4) + (x^2*(1 - I*a*x)^(7/4)
*(1 + I*a*x)^(1/4))/(4*a^2) - ((1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4)*(11 - (4
*I)*a*x))/(32*a^4) - (123*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(64*Sqrt[2]*a^4) + (123*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
- (123*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x^3(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$

$$= \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} + \frac{\int \frac{x(1-iax)^{3/4}\left(-2+\frac{3iax}{2}\right)}{(1+iax)^{3/4}}\,dx}{4a^2}$$

$$= \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} - \frac{(41i)\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx}{64a^3}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} - \frac{(123i)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{128a^3}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} + \frac{123\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{32a^4}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} + \frac{123\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{32a^4}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} - \frac{123\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4} + \frac{123\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} + \frac{123\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} + \frac{123\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} + \frac{123\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} + \frac{123\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} + \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} + \frac{123\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{123\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}$$

$$= -\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4} + \frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}(11-4iax)}{32a^4} - \frac{123\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{123\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{123\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.38

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \frac{(1-iax)^{7/4}\left(7a^2x^2\sqrt[4]{1+iax}+12\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)-20\sqrt[4]{2}\,\text{Hypergeometric}\right.}{28a^4}$$

```
[In] Integrate[x^3/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((1 - I*a*x)^(7/4)*(7*a^2*x^2*(1 + I*a*x)^(1/4) + 12*2^(1/4)*Hypergeometric
2F1[-5/4, 7/4, 11/4, (1 - I*a*x)/2] - 20*2^(1/4)*Hypergeometric2F1[-1/4, 7/
4, 11/4, (1 - I*a*x)/2] + 7*2^(1/4)*Hypergeometric2F1[3/4, 7/4, 11/4, (1 -
I*a*x)/2]))/(28*a^4)
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.74

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx =$$

$$-\frac{32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(\frac{64}{123}\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(-\frac{64}{123}\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)}{}$$

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(32*a^4*sqrt(15129/4096*I/a^8)*log(64/123*a^4*sqrt(15129/4096*I/a^8)
+ sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(15129/4096*I/a^8)*log(
-64/123*a^4*sqrt(15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
 32*a^4*sqrt(-15129/4096*I/a^8)*log(64/123*a^4*sqrt(-15129/4096*I/a^8) + sq
rt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(-15129/4096*I/a^8)*log(-64
/123*a^4*sqrt(-15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (
16*a^4*x^4 + 40*I*a^3*x^3 - 54*a^2*x^2 - 93*I*a*x + 63)*sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)))/a^4
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(x**3/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

3.98 $\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 339

$$\begin{aligned}\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = {} & \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3}+\frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} \\ & +\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}+\frac{17i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} \\ & -\frac{17i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} \\ & -\frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} \\ & +\frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}\end{aligned}$$

```
[Out] 17/24*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^3+1/4*I*(1-I*a*x)^(7/4)*(1+I*a*x)
^(1/4)/a^3+1/3*x*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/4)/a^2+17/16*I*arctan(1-(1-I*
a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-17/16*I*arctan(1+(1-I*a*x)^
(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3*2^(1/2)-17/32*I*ln(1-(1-I*a*x)^(1/4)*2^(
1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)+17/32*I*l
n(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)
)/a^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5170, 92, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx=\frac{17i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{17i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{i\sqrt[4]{1+iax}(1-iax)^{7/4}}{4a^3}+\frac{17i\sqrt[4]{1+iax}(1-iax)^{3/4}}{24a^3}-\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}+\frac{17i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}+\frac{x\sqrt[4]{1+iax}(1-iax)^{7/4}}{3a^2}$$

```
[In] Int[x^2/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (((17*I)/24)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a^3 + ((I/4)*(1 - I*a*x)^
(7/4)*(1 + I*a*x)^(1/4))/a^3 + (x*(1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4))/(3*a
^2) + (((17*I)/8)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]
)/(Sqrt[2]*a^3) - (((17*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)])/(Sqrt[2]*a^3) - (((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 +
 I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (
((17*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^
(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx \\
&= \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} + \frac{\int \frac{(1-iax)^{3/4}\left(-1+\frac{3iax}{2}\right)}{(1+iax)^{3/4}}\,dx}{3a^2} \\
&= \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{17\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx}{24a^2} \\
&= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} \\
&\quad + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{17\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{16a^2} \\
&= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} \\
&\quad + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} - \frac{(17i)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{4a^3}
\end{aligned}
$$

$$=\frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3}+\frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3}+\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{(17i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^3}$$

$$=\frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3}+\frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3}+\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}+\frac{(17i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3}-\frac{(17i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3}$$

$$=\frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3}+\frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3}+\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{(17i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3}-\frac{(17i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3}-\frac{(17i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}-\frac{(17i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

$$=\frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3}+\frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3}+\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}+\frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}-\frac{(17i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{(17i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}$$

$$= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3} + \frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3} + \frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2} + \frac{17i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{17i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} + \frac{17i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.22

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = \frac{(1-iax)^{7/4}\left(7\sqrt[4]{1+iax}(3i+4ax)-17i\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)\right)}{84a^3}$$

```
[In] Integrate[x^2/E^(((3*I)/2)*ArcTan[a*x]),x]
[Out] ((1 - I*a*x)^(7/4)*(7*(1 + I*a*x)^(1/4)*(3*I + 4*a*x) - (17*I)*2^(1/4)*Hype
rgeometric2F1[3/4, 7/4, 11/4, (1 - I*a*x)/2]))/(84*a^3)
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
[Out] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.72

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = -\frac{12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(\frac{8}{17}i\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(-\frac{8}{17}i\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3\cdots}{\cdots}$$

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^3*sqrt(289/64*I/a^6)*log(8/17*I*a^3*sqrt(289/64*I/a^6) + sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(289/64*I/a^6)*log(-8/17*I*a^3*
sqrt(289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(-28
9/64*I/a^6)*log(8/17*I*a^3*sqrt(-289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I))) - 12*a^3*sqrt(-289/64*I/a^6)*log(-8/17*I*a^3*sqrt(-289/64*I/a^6)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (8*a^3*x^3 + 22*I*a^2*x^2 - 37*a*
x - 23*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(x**2/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx=\int \frac{x^2}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
[Out] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

3.99 $\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 295

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx=\frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2}+\frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}+\frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

```
[Out] 3/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^2+1/2*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/4)
/a^2+9/8*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-9/8*
arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)-9/16*ln(1-(1-
I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2
^(1/2)+9/16*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1
+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5170, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx = \frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} + \frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2} + \frac{3\sqrt[4]{1+iax}(1-iax)^{3/4}}{4a^2} - \frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2} + \frac{9\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}$$

```
[In] Int[x/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/(4*a^2) + ((1 - I*a*x)^(7/4)*(1 + I
*a*x)^(1/4))/(2*a^2) + (9*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)])/(4*Sqrt[2]*a^2) - (9*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x]
 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (9*Log
[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$

$$= \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{(3i)\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx}{4a}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{(9i)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx}{8a}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} + \frac{9\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} - \frac{9\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}$$

$$= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} - \frac{9\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} - \frac{9\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} - \frac{9\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} - \frac{9\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

$$
\begin{aligned}
&= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} \\
&\quad - \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} \\
&\quad - \frac{9\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} \\
&\quad + \frac{9\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} \\
&= \frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2} + \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2} \\
&\quad + \frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} \\
&\quad - \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{9\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.21

$$
\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx = \frac{(1-iax)^{7/4}\left(7\sqrt[4]{1+iax} - 3\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)\right)}{14a^2}
$$

```
[In] Integrate[x/E^(((3*I)/2)*ArcTan[a*x]),x]
[Out] ((1 - I*a*x)^(7/4)*(7*(1 + I*a*x)^(1/4) - 3*2^(1/4)*Hypergeometric2F1[3/4,
7/4, 11/4, (1 - I*a*x)/2]))/(14*a^2)
```

Maple [F]

$$\int \frac{x}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

[In] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)

[Out] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.80

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx = \frac{2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(\frac{4}{9}\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(-\frac{4}{9}\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{81i}{16\,a^4}}\,\mathrm{lo}}{}$$

[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")

[Out] 1/4*(2*a^2*sqrt(81/16*I/a^4)*log(4/9*a^2*sqrt(81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(81/16*I/a^4)*log(-4/9*a^2*sqrt(81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-81/16*I/a^4)*log(4/9*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-81/16*I/a^4)*log(-4/9*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (2*a^2*x^2 + 7*I*a*x - 5)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^2

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)

[Out] Integral(x/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx=\int\frac{x}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")

[Out] integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx=\text{Exception raised: TypeError}$$

[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]Warning, replacing 0 by -46, a substitution variable should perhaps be purged.Warn

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx=\int\frac{x}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2}}\,dx$$

[In] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)

[Out] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)

3.100 $\int e^{-\frac{3}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 268

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}-\frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

```
[Out] -I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a-3/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)
/(1+I*a*x)^(1/4))/a*2^(1/2)+3/2*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x
)^(1/4))/a*2^(1/2)+3/4*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*
a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)-3/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1
+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules

used = {5169, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = -\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a} - \frac{3i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}$$

```
[In] Int[E^(((-3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((-I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + ((3*I)*ArcTan[1 + (Sqrt
[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (((3*I)/2)*Log[1 +
 Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)])/(Sqrt[2]*a) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$

$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{3}{2}\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(6i)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$

$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(6i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{(3i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} + \frac{(3i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{(3i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{(3i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} + \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} + \frac{(3i)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{(3i)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}$$

$$= -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a} - \frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} + \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{3i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.15

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = -\frac{8ie^{\frac{1}{2}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},2,\frac{5}{4},-e^{2i\arctan(ax)}\right)}{a}$$

```
[In] Integrate[E^(((-3*I)/2)*ArcTan[a*x]),x]
[Out] ((-8*I)*E^((I/2)*ArcTan[a*x])*Hypergeometric2F1[1/4, 2, 5/4, -E^((2*I)*ArcT
an[a*x])])/a
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.78

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx$$
$$=\frac{a\sqrt{\frac{9i}{a^2}}\log\left(\frac{1}{3}i\,a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{9i}{a^2}}\log\left(-\frac{1}{3}i\,a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{9i}{a^2}}\log\left(\frac{1}{3}i\,a\sqrt{-\frac{9i}{a^2}}+\right.}{2\,a}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(9*I/a^2)*log(1/3*I*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I))) - a*sqrt(9*I/a^2)*log(-1/3*I*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) + a*sqrt(-9*I/a^2)*log(1/3*I*a*sqrt(-9*I/a^2) + sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(-9*I/a^2)*log(-1/3*I*a*sqrt(-9*I/a^2
) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*(a*x + I)*sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)))/a
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx=\int\frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(-3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx=\int\frac{1}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = \int \frac{1}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

3.101 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 267

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx &= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)\\ &\quad+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\ &\quad+\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\\ &\quad-\frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\end{aligned}$$

```
[Out] -2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a
*x)^(1/4))+1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)
/(1+I*a*x)^(1/2))*2^(1/2)-1/2*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+
(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(
1+I*a*x)^(1/4))*2^(1/2)+arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2
^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5170, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 95, 218, 212, 209}

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}$$

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2
]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] - 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^
(1/4)] + Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
```

```
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{3/4}}{x(1+iax)^{3/4}}\,dx \\
&= -\left((ia)\int \frac{1}{\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx\right)+\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= 4\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-iax}\right)+4\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\left(2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right) \\
&\quad -2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+4\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)
\end{aligned}
$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx$$
$$=\frac{2(1-iax)^{3/4}\left(\sqrt[4]{2}(1+iax)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)-2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+}{i-}\right.\right.}{3(1+iax)^{3/4}}$$

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] (2*(1 - I*a*x)^(3/4)*(2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4,
 7/4, (1 - I*a*x)/2] - 2*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)
]))/(3*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x}dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = -\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + \frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + \frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) + i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) + \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1
/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1/
2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1/
2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1
)/(a*x + I)) + I) + I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sq
rt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x,x)
```

```
[Out] Integral(1/(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
[Out] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

3.102 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} + 3ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 3ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x+3*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))+3*I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5170, 96, 95, 218, 212, 209}

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = 3ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 3ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] -(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) + (3*I)*a*ArcTan[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)] + (3*I)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1-iax)^{3/4}}{x^2(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - \frac{1}{2}(3ia)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - (6ia)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} + (3ia)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + (3ia)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} + 3ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 3ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \frac{i(1-iax)^{3/4}\left(i-ax+2ax\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{x(1+iax)^{3/4}}$$

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^2),x]
[Out] (I*(1 - I*a*x)^(3/4)*(I - a*x + 2*a*x*Hypergeometric2F1[3/4, 1, 7/4, (I + a
*x)/(I - a*x)]))/(x*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^2}dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(64) = 128.

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.65

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx$$

$$= \frac{3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - 3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) + 3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) - 3i\,ax\log\left(\sqrt{\frac{i\sqrt{a}}{a}}\right.}{2\,x}$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")

[Out] 1/2*(3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 3*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 3*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(-I*a*x + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**2,x)

[Out] Integral(1/(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
[Out] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

3.103 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} + \frac{9}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{9}{4}a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] 3/4*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-1/2*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/
4)/x^2+9/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+9/4*a^2*arctanh((1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5170, 98, 96, 95, 218, 212, 209}

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{9}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{9}{4}a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}$$

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] (((3*I)/4)*a*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - ((1 - I*a*x)^(7/4)*(1
 + I*a*x)^(1/4))/(2*x^2) + (9*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4
)])/4 + (9*a^2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{3/4}}{x^3(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} - \frac{1}{4}(3ia)\int \frac{(1-iax)^{3/4}}{x^2(1+iax)^{3/4}}\,dx \\
&= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} - \frac{1}{8}(9a^2)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} \\
&\quad - \frac{1}{2}(9a^2)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} \\
&\quad + \frac{1}{4}(9a^2)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad + \frac{1}{4}(9a^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} \\
&\quad + \frac{9}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{9}{4}a^2\,\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{(1-iax)^{3/4}\left(-2+3iax-5a^2x^2+6a^2x^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{4x^2(1+iax)^{3/4}}$$

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] ((1 - I*a*x)^(3/4)*(-2 + (3*I)*a*x - 5*a^2*x^2 + 6*a^2*x^2*Hypergeometric2F
1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^3} dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.33

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3} dx = \frac{9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9\,a^2x^2\log}{8\,x^2}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 9*I*a^2*x^2*l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 9*I*a^2*x^2*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - I) - 9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)) - 1) + 2*(5*a^2*x^2 + 7*I*a*x - 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
)/x^2
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3} dx = \int \frac{1}{x^3\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
[Out] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

3.104 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}+\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{17}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{17}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/3*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^3+7/12*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)
^(1/4)/x^2+23/24*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-17/8*I*a^3*arctan((1
+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-17/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)
^(1/4))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5170, 101, 156, 12, 95, 218, 212, 209}

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{17}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{17}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}$$

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] -1/3*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^3 + (((7*I)/12)*a*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(1/4))/x^2 + (23*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(24*x) - ((17*I)/8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((17
*I)/8)*a^3*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{3/4}}{x^4(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{1}{3}\int \frac{-\frac{7ia}{2}-2a^2x}{x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}-\frac{1}{6}\int \frac{\frac{23a^2}{4}-\frac{7}{2}ia^3x}{x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} \\
&\quad +\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{1}{6}\int \frac{51ia^3}{8x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} \\
&\quad +\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{1}{16}\left(17ia^3\right)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} \\
&\quad +\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}+\frac{1}{4}\left(17ia^3\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} \\
&\quad -\frac{1}{8}(17ia^3)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{1}{8}(17ia^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3} + \frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2} + \frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x} \\
&\quad - \frac{17}{8}ia^3 \arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{17}{8}ia^3 \operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.55

$$
\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \frac{(1-iax)^{3/4}\left(-8+6iax+9a^2x^2+23ia^3x^3-34ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3(1+iax)^{3/4}}
$$

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^4),x]
[Out] ((1 - I*a*x)^(3/4)*(-8 + (6*I)*a*x + 9*a^2*x^2 + (23*I)*a^3*x^3 - (34*I)*a^
3*x^3*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I*
a*x)^(3/4))
```

Maple [F]

$$
\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^4}dx
$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \frac{-51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+51i\,a^3}{48\,x^3}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(-51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 51*a^3*x
^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 51*a^3*x^3*log(sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) - I) + 51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I)) - 1) - 2*(23*I*a^3*x^3 - 37*a^2*x^2 - 22*I*a*x + 8)*sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Integral(1/(x**4*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

3.105 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 202

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}+\frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3}+\frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2}-\frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x}-\frac{123}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -1/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^4+3/8*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)^
(1/4)/x^3+15/32*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^2-63/64*I*a^3*(1-I*a*
x)^(3/4)*(1+I*a*x)^(1/4)/x-123/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4
))-123/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5170, 101, 156, 12, 95, 218, 212, 209}

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{123}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x}+\frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2}-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}+\frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3}$$

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] -1/4*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^4 + (((3*I)/8)*a*(1 - I*a*x)^(
3/4)*(1 + I*a*x)^(1/4))/x^3 + (15*a^2*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/
(32*x^2) - (((63*I)/64)*a^3*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - (123*a
^4*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 +
I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{3/4}}{x^5(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{1}{4}\int \frac{-\frac{9ia}{2}-3a^2x}{x^4\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} - \frac{1}{12}\int \frac{\frac{45a^2}{4}-9ia^3x}{x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} \\
&\quad + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} + \frac{1}{24}\int \frac{\frac{189ia^3}{8}+\frac{45a^4x}{4}}{x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} \\
&\quad - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x} - \frac{1}{24}\int -\frac{369a^4}{16x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx \\
&= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} \\
&\quad - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x} + \frac{1}{128}\left(123a^4\right)\int \frac{1}{x\sqrt[4]{1-iax}(1+iax)^{3/4}}\,dx
\end{aligned}$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x} + \frac{1}{32}(123a^4)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x} - \frac{1}{64}(123a^4)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{1}{64}(123a^4)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4} + \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3} + \frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2} - \frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x} - \frac{123}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{123}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \frac{(1-iax)^{3/4}\left(-16+8iax+6a^2x^2-33ia^3x^3+63a^4x^4-82a^4x^4\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{64x^4(1+iax)^{3/4}}$$

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^5),x]
[Out] ((1 - I*a*x)^(3/4)*(-16 + (8*I)*a*x + 6*a^2*x^2 - (33*I)*a^3*x^3 + 63*a^4*x
^4 - 82*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(64*x
^4*(1 + I*a*x)^(3/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^5}\,dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-}{128\,x^4}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/128*(123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 123*I*a^
4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 123*I*a^4*x^4*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I)) - 1) + 2*(63*a^4*x^4 + 93*I*a^3*x^3 - 54*a^2*x^2 - 40*I*a*x
 + 16)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
```

```
[Out] Integral(1/(x**5*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

3.106 $\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 373

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx = \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} + \frac{475\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{475\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

```
[Out] 4*I*x^3*(1-I*a*x)^(5/4)/a/(1+I*a*x)^(1/4)+475/64*(1-I*a*x)^(1/4)*(1+I*a*x)^
(3/4)/a^4-17/4*x^2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^2-1/96*I*(1-I*a*x)^(5/
4)*(1+I*a*x)^(3/4)*(521*I+452*a*x)/a^4+475/128*arctan(1-(1-I*a*x)^(1/4)*2^(
1/2)/(1+I*a*x)^(1/4))/a^4*2^(1/2)-475/128*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a^4*2^(1/2)+475/256*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)
^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)-475/256*ln(1+(1-I*a*x)^
(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^4*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5170, 99, 158, 152, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx=\frac{475\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}-\frac{475\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{96a^4}+\frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4}-\frac{475\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{128\sqrt{2}a^4}-\frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}+\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}$$

```
[In] Int[x^3/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((4*I)*x^3*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) + (475*(1 - I*a*x)^(1/4
)*(1 + I*a*x)^(3/4))/(64*a^4) - (17*x^2*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)
)/(4*a^2) - ((I/96)*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4)*(521*I + 452*a*x))/
a^4 + (475*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(64*S
qrt[2]*a^4) - (475*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
])/(64*Sqrt[2]*a^4) + (475*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (475*Log[1 +
Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)])/(128*Sqrt[2]*a^4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x^3(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{(4i)\int \frac{x^2\sqrt[4]{1-iax}\left(3-\frac{17iax}{4}\right)}{\sqrt[4]{1+iax}}\,dx}{a}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i\int \frac{x\sqrt[4]{1-iax}\left(\frac{17ia}{2}+\frac{113a^2x}{8}\right)}{\sqrt[4]{1+iax}}\,dx}{a^3}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} + \frac{(475i)\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{64a^3}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} + \frac{(475i)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{128a^3}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} - \frac{475\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{32a^4}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} - \frac{475\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{32a^4}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} - \frac{475\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4} - \frac{475\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64a^4}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} - \frac{475\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} - \frac{475\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128a^4} + \frac{475\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} + \frac{475\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} + \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{475\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{475\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}$$

$$= \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4} - \frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{i(1-iax)^{5/4}(1+iax)^{3/4}(521i+452ax)}{96a^4} + \frac{475\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} - \frac{475\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} + \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4} - \frac{475\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{128\sqrt{2}a^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.27

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx =$$

$$-\frac{\sqrt[4]{1-iax}(i+ax)^2\left(3(59+5iax+6a^2x^2)-95\,2^{3/4}\sqrt[4]{1+iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-iax)\right)\right)}{72a^4\sqrt[4]{1+iax}}$$

```
[In] Integrate[x^3/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] -1/72*((1 - I*a*x)^(1/4)*(I + a*x)^2*(3*(59 + (5*I)*a*x + 6*a^2*x^2) - 95*2
^(3/4)*(1 + I*a*x)^(1/4)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - I*a*x)/2]))
/(a^4*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.82

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx$$

$$= \frac{96\left(a^5x-i\,a^4\right)\sqrt{\frac{225625i}{4096\,a^8}}\log\left(\frac{64}{475}i\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\left(a^5x-i\,a^4\right)\sqrt{\frac{225625i}{4096\,a^8}}\log\left(-\frac{64}{475}i\,a^4\sqrt{\frac{225625i}{4096\,a^8}}\right.}{}$$

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(96*(a^5*x - I*a^4)*sqrt(225625/4096*I/a^8)*log(64/475*I*a^4*sqrt(225
625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*(a^5*x - I*a^4)
*sqrt(225625/4096*I/a^8)*log(-64/475*I*a^4*sqrt(225625/4096*I/a^8) + sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*(a^5*x - I*a^4)*sqrt(-225625/4096*I/a^8
)*log(64/475*I*a^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
```

```
 + I))) + 96*(a^5*x - I*a^4)*sqrt(-225625/4096*I/a^8)*log(-64/475*I*a^4*sqr
t(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (48*I*a^4*x^
4 - 136*a^3*x^3 - 226*I*a^2*x^2 + 521*a*x - 2467*I)*sqrt(a^2*x^2 + 1)*sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(a^5*x - I*a^4)
```

Sympy [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx=\int\frac{x^3}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Integral(x**3/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx=\int\frac{x^3}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 81, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx=\int \frac{x^3}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}\,dx$$

```
[In] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.107 $\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 371

$$\begin{aligned}\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx =& -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}-\frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3}\\&-\frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3}-\frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3}\\&-\frac{55i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{55i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}\\&-\frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}\\&+\frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}\end{aligned}$$

```
[Out] -2*I*(1-I*a*x)^(9/4)/a^3/(1+I*a*x)^(1/4)-55/8*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(
3/4)/a^3-11/4*I*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^3-1/3*I*(1-I*a*x)^(9/4)*(
1+I*a*x)^(3/4)/a^3-55/16*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)
)/a^3*2^(1/2)+55/16*I*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a^3
*2^(1/2)-55/32*I*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/
2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)+55/32*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a
*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5170, 91, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = -\frac{55i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{55i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} - \frac{i(1+iax)^{3/4}(1-iax)^{9/4}}{3a^3}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{11i(1+iax)^{3/4}(1-iax)^{5/4}}{4a^3}-\frac{55i(1+iax)^{3/4}\sqrt[4]{1-iax}}{8a^3} - \frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3} + \frac{55i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{16\sqrt{2}a^3}$$

```
[In] Int[x^2/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((-2*I)*(1 - I*a*x)^(9/4))/(a^3*(1 + I*a*x)^(1/4)) - (((55*I)/8)*(1 - I*a*x
)^(1/4)*(1 + I*a*x)^(3/4))/a^3 - (((11*I)/4)*(1 - I*a*x)^(5/4)*(1 + I*a*x)^
(3/4))/a^3 - ((I/3)*(1 - I*a*x)^(9/4)*(1 + I*a*x)^(3/4))/a^3 - (((55*I)/8)*
ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) +
(((55*I)/8)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqr
t[2]*a^3) - (((55*I)/16)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a^3) + (((55*I)/16)*Log[1
+ Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)
^(1/4)])/(Sqrt[2]*a^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx \\
&= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}+\frac{(2i)\int \frac{(1-iax)^{5/4}\left(-\frac{5ia}{2}-\frac{a^2x}{2}\right)}{\sqrt[4]{1+iax}}\,dx}{a^3} \\
&= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}-\frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3}+\frac{11\int \frac{(1-iax)^{5/4}}{\sqrt[4]{1+iax}}\,dx}{2a^2} \\
&= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}-\frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} \\
&\quad -\frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3}+\frac{55\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{8a^2}
\end{aligned}
$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} + \frac{55\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{16a^2}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} + \frac{(55i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{4a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} + \frac{(55i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} + \frac{(55i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3} + \frac{(55i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^3}$$

$$= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} + \frac{(55i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} + \frac{(55i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16a^3} - \frac{(55i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} - \frac{(55i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}$$

$$
\begin{aligned}
&= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} \\
&\quad - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} \\
&\quad - \frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} + \frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} \\
&\quad + \frac{(55i)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} \\
&\quad - \frac{(55i)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} \\
&= -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}} - \frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3} \\
&\quad - \frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3} - \frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3} \\
&\quad - \frac{55i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} + \frac{55i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3} \\
&\quad - \frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3} + \frac{55i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{16\sqrt{2}a^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.25

$$
\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = -\frac{\sqrt[4]{1-iax}(i+ax)^2\left(-21i+3ax+11i2^{3/4}\sqrt[4]{1+iax}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-iax)\right)\right)}{9a^3\sqrt[4]{1+iax}}
$$

```
[In] Integrate[x^2/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] -1/9*((1 - I*a*x)^(1/4)*(I + a*x)^2*(-21*I + 3*a*x + (11*I)*2^(3/4)*(1 + I*
a*x)^(1/4)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - I*a*x)/2]))/(a^3*(1 + I*a
*x)^(1/4))
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

[In] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)

[Out] int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.80

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx$$
$$= \frac{12\left(a^4x - i\,a^3\right)\sqrt{\frac{3025i}{64\,a^6}}\log\left(\frac{8}{55}\,a^3\sqrt{\frac{3025i}{64\,a^6}} + \sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - 12\left(a^4x - i\,a^3\right)\sqrt{\frac{3025i}{64\,a^6}}\log\left(-\frac{8}{55}\,a^3\sqrt{\frac{3025i}{64\,a^6}} + \sqrt{\frac{i\sqrt{a}}{a}}\right.}{}$$

[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")

[Out] 1/24*(12*(a^4*x - I*a^3)*sqrt(3025/64*I/a^6)*log(8/55*a^3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*(a^4*x - I*a^3)*sqrt(3025/64*I/a^6)*log(-8/55*a^3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*(a^4*x - I*a^3)*sqrt(-3025/64*I/a^6)*log(8/55*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*(a^4*x - I*a^3)*sqrt(-3025/64*I/a^6)*log(-8/55*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (8*I*a^3*x^3 - 26*a^2*x^2 - 61*I*a*x - 287)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(a^4*x - I*a^3)

Sympy [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)

[Out] Integral(x**2/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2), x)

Maxima [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}\,dx$$

```
[In] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.108 $\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 324

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx=-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}-\frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}-\frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}-\frac{25\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{25\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}+\frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

```
[Out] -2*(1-I*a*x)^(9/4)/a^2/(1+I*a*x)^(1/4)-25/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)
/a^2-5/2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^2-25/8*arctan(1-(1-I*a*x)^(1/4)*
2^(1/2)/(1+I*a*x)^(1/4))/a^2*2^(1/2)+25/8*arctan(1+(1-I*a*x)^(1/4)*2^(1/2)/
(1+I*a*x)^(1/4))/a^2*2^(1/2)-25/16*ln(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(
1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2^(1/2)+25/16*ln(1+(1-I*a*x)^(1/4
)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5170, 79, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = -\frac{25\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{25\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}-\frac{5(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2} - \frac{25(1+iax)^{3/4}\sqrt[4]{1-iax}}{4a^2}-\frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2} + \frac{25\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{8\sqrt{2}a^2}$$

```
[In] Int[x/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (-2*(1 - I*a*x)^(9/4))/(a^2*(1 + I*a*x)^(1/4)) - (25*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/(4*a^2) - (5*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(2*a^2) - (25*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (25*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2) + (25*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx \\
&= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{(5i)\int \frac{(1-iax)^{5/4}}{\sqrt[4]{1+iax}}\,dx}{a} \\
&= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{(25i)\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx}{4a} \\
&= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} \\
&\quad - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{(25i)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{8a} \\
&= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} \\
&\quad - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{2a^2} \\
&= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} \\
&\quad - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a^2}
\end{aligned}
$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2} + \frac{25\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4a^2}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{25\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} + \frac{25\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8a^2} - \frac{25\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} - \frac{25\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{25\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{25\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}$$

$$= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}} - \frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2} - \frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} - \frac{25\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} + \frac{25\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{25\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.19

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = \frac{2(1-iax)^{9/4}\left(-\frac{9}{\sqrt[4]{1+iax}}+5\,2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-iax)\right)\right)}{9a^2}$$

```
[In] Integrate[x/E^(((5*I)/2)*ArcTan[a*x]),x]
[Out] (2*(1 - I*a*x)^(9/4)*(-9/(1 + I*a*x)^(1/4) + 5*2^(3/4)*Hypergeometric2F1[1/
4, 9/4, 13/4, (1 - I*a*x)/2]))/(9*a^2)
```

Maple [F]

$$\int \frac{x}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
[Out] int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.89

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = -\frac{2\left(a^3x-i\,a^2\right)\sqrt{\frac{625i}{16\,a^4}}\log\left(\frac{4}{25}i\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\left(a^3x-i\,a^2\right)\sqrt{\frac{625i}{16\,a^4}}\log\left(-\frac{4}{25}i\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i}{}}\right.}{}$$

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
[Out] -1/4*(2*(a^3*x - I*a^2)*sqrt(625/16*I/a^4)*log(4/25*I*a^2*sqrt(625/16*I/a^4
) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*(a^3*x - I*a^2)*sqrt(625/16*I/
a^4)*log(-4/25*I*a^2*sqrt(625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) - 2*(a^3*x - I*a^2)*sqrt(-625/16*I/a^4)*log(4/25*I*a^2*sqrt(-625/16*I/a
^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*(a^3*x - I*a^2)*sqrt(-625/16
*I/a^4)*log(-4/25*I*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I))) - sqrt(a^2*x^2 + 1)*(2*I*a^2*x^2 - 9*a*x + 43*I)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)))/(a^3*x - I*a^2)
```

Sympy [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Integral(x/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 81, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = \int \frac{x}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}\,dx$$

```
[In] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.109 $\int e^{-\frac{5}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 299

$$\begin{aligned}\int e^{-\frac{5}{2}i\arctan(ax)}\,dx =& \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\\&+\frac{5i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{5i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}\\&+\frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}\\&-\frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}\end{aligned}$$

```
[Out] 4*I*(1-I*a*x)^(5/4)/a/(1+I*a*x)^(1/4)+5*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a
+5/2*I*arctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a*2^(1/2)-5/2*I*ar
ctan(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))/a*2^(1/2)+5/4*I*ln(1-(1-I*a
*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))/a*2^(1/2
)-5/4*I*ln(1+(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a
*x)^(1/2))/a*2^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5169, 49, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx = \frac{5i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{5i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i(1+iax)^{3/4}\sqrt[4]{1-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}-\frac{5i\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}a}$$

```
[In] Int[E^(((-5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] ((4*I)*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) + ((5*I)*(1 - I*a*x)^(1/4)*
(1 + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)])/(Sqrt[2]*a) - ((5*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(
1 + I*a*x)^(1/4)])/(Sqrt[2]*a) + (((5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1
+ I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a) - ((
(5*I)/2)*Log[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$

$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - 5\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx$$

$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{5}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{(10i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right)}{a}$$

$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{(10i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} - \frac{(5i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a} - \frac{(5i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{a}$$

$$
\begin{aligned}
&= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} \\
&- \frac{(5i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} \\
&- \frac{(5i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2a} \\
&+ \frac{(5i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} \\
&+ \frac{(5i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} \\
&+ \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} \\
&- \frac{(5i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} \\
&+ \frac{(5i)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} + \frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a} \\
&+ \frac{5i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} - \frac{5i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a} \\
&+ \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a} - \frac{5i\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}a}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.13

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx = \frac{8ie^{-\frac{1}{2}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},2,\frac{3}{4},-e^{2i\arctan(ax)}\right)}{a}$$

```
[In] Integrate[E^(((-5*I)/2)*ArcTan[a*x]),x]
[Out] ((8*I)*Hypergeometric2F1[-1/4, 2, 3/4, -E^((2*I)*ArcTan[a*x])])/(a*E^((I/2)
*ArcTan[a*x]))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.87

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx = -\frac{(a^2x-ia)\sqrt{\frac{25i}{a^2}}\log\left(\frac{1}{5}a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-(a^2x-ia)\sqrt{\frac{25i}{a^2}}\log\left(-\frac{1}{5}a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-(a^2x\cdots}{}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
[Out] -1/2*((a^2*x - I*a)*sqrt(25*I/a^2)*log(1/5*a*sqrt(25*I/a^2) + sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I))) - (a^2*x - I*a)*sqrt(25*I/a^2)*log(-1/5*a*sqrt(25*I
/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (a^2*x - I*a)*sqrt(-25*I/a^2
)*log(1/5*a*sqrt(-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (a^2*x
 - I*a)*sqrt(-25*I/a^2)*log(-1/5*a*sqrt(-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 +
1)/(a*x + I))) + 2*sqrt(a^2*x^2 + 1)*(-I*a*x - 9)*sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I)))/(a^2*x - I*a)
```

Sympy [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx = \int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(-5/2), x)
```

Maxima [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx = \int \frac{1}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 81, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx = \int \frac{1}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}\,dx$$

```
[In] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.110 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 293

$$\begin{aligned}\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx &= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\ &\quad + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) \\ &\quad - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} \\ &\quad - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\end{aligned}$$

```
[Out] 8*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/2*ln(1-(1-I*a*x)^(1/4)*2^(1/2
)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)-1/2*ln(1+(1-I*a*
x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2))*2^(1/2)+a
rctan(1-(1-I*a*x)^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^(1/2)-arctan(1+(1-I*a*x)
^(1/4)*2^(1/2)/(1+I*a*x)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5170, 100, 21, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) -\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) +\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}} -\frac{\log\left(\frac{\sqrt{1-iax}}{\sqrt{1+iax}}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{\sqrt{2}}$$

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] (8*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) + 2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)] -
 2*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + Log[1 + Sqrt[1 - I*a*x]/S
qrt[1 + I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] - L
og[1 + Sqrt[1 - I*a*x]/Sqrt[1 + I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1-iax)^{5/4}}{x(1+iax)^{5/4}}\,dx$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(4i)\int \frac{\frac{ia}{4}-\frac{a^2x}{4}}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{a}$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \int \frac{(1+iax)^{3/4}}{x(1-iax)^{3/4}}\,dx$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + (ia)\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx + \int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - 4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-iax}\right) + 4\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$
$$+ 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$
$$- 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$
$$+ \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}$$
$$- \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$
$$- \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)$$

$$= \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \frac{\sqrt[4]{1-iax}\left(20-20\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)+2^{3/4}(1-iax)\sqrt[4]{1+iax}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4}\right.\right.}{5\sqrt[4]{1+iax}}$$

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(20 - 20*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a
*x)] + 2^(3/4)*(1 - I*a*x)*(1 + I*a*x)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/
4, (1 - I*a*x)/2]))/(5*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x}dx$$

[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)

[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.12

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \frac{\sqrt{4i}(ax-i)\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\sqrt{4i}(ax-i)\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\sqrt{-4i}(ax-i)\log\left(\right.}{}$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")

[Out] 1/2*(sqrt(4*I)*(a*x - I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - sqrt(4*I)*(a*x - I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - sqrt(-4*I)*(a*x - I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + sqrt(-4*I)*(a*x - I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*(a*x - I)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 2*(-I*a*x - 1)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 2*(I*a*x + 1)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 2*(a*x - I)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 16*I*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(a*x - I)

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x,x)

[Out] Integral(1/(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)), x)

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")

[Out] integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 81, a substitution variable should perhaps be purged
.Warni

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

[In] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)

[Out] int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)

3.111 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - 5ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 5ia\,\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -10*I*a*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(5/4)/x/(1+I*a*x)^(1/4)-5
*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+5*I*a*arctanh((1+I*a*x)^(1/4)/
(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5170, 96, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = -5ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 5ia\,\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - \frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^2),x]
```

```
[Out] ((-10*I)*a*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) - (1 - I*a*x)^(5/4)/(x*(1 +
 I*a*x)^(1/4)) - (5*I)*a*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] + (5*I
)*a*ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1-iax)^{5/4}}{x^2(1+iax)^{5/4}}\,dx$$

$$= -\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - \frac{1}{2}(5ia)\int \frac{\sqrt[4]{1-iax}}{x(1+iax)^{5/4}}\,dx$$

$$= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - \frac{1}{2}(5ia)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - (10ia)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} + (5ia)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - (5ia)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - 5ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 5ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \frac{i\sqrt[4]{1-iax}\left(i-9ax+10ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{x\sqrt[4]{1+iax}}$$

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^2),x]
[Out] (I*(1 - I*a*x)^(1/4)*(I - 9*a*x + 10*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I
+ a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^2}\,dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(83) = 166$.

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.75

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{2\sqrt{a^2x^2+1}(9\,ax-i)\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} + 5\left(-i\,a^2x^2-ax\right)\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - 5\left(a^2x^2-i\,ax\right)\log\left(\sqrt{\frac{i\sqrt{\cdots}}{\cdots}}\cdots\right)\cdots}{2\left(ax^2-i\,x\right)}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(a^2*x^2 + 1)*(9*a*x - I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) +
 5*(-I*a^2*x^2 - a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 5*(a^2
*x^2 - I*a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 5*(a^2*x^2 - I
*a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 5*(I*a^2*x^2 + a*x)*lo
g(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1))/(a*x^2 - I*x)
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 81, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

```
[In] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

```
[Out] int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

3.112 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 163

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{25}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{25}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] -25/2*a^2*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+5/4*I*a*(1-I*a*x)^(5/4)/x/(1+I*a*
x)^(1/4)-1/2*(1-I*a*x)^(9/4)/x^2/(1+I*a*x)^(1/4)-25/4*a^2*arctan((1+I*a*x)^
(1/4)/(1-I*a*x)^(1/4))+25/4*a^2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5170, 98, 96, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{25}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{25}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}}$$

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^3),x]
```

```
[Out] (-25*a^2*(1 - I*a*x)^(1/4))/(2*(1 + I*a*x)^(1/4)) + (((5*I)/4)*a*(1 - I*a*x
)^(5/4))/(x*(1 + I*a*x)^(1/4)) - (1 - I*a*x)^(9/4)/(2*x^2*(1 + I*a*x)^(1/4)
) - (25*a^2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4 + (25*a^2*ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/4
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{5/4}}{x^3(1+iax)^{5/4}}\,dx \\
&= -\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{4}(5ia)\int \frac{(1-iax)^{5/4}}{x^2(1+iax)^{5/4}}\,dx \\
&= \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1-iax}}{x(1+iax)^{5/4}}\,dx \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{1}{8}\left(25a^2\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} \\
&\quad - \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} \\
&\quad + \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad - \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} \\
&\quad - \frac{25}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{25}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.50

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \frac{\sqrt[4]{1-iax}\left(-2+9iax-43a^2x^2+50a^2x^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{4x^2\sqrt[4]{1+iax}}$$

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^3),x]
[Out] ((1 - I*a*x)^(1/4)*(-2 + (9*I)*a*x - 43*a^2*x^2 + 50*a^2*x^2*Hypergeometric
2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(111) = 222.

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.46

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{2\sqrt{a^2x^2+1}(-43i\,a^2x^2-9\,ax-2i)\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-25\left(a^3x^3-i\,a^2x^2\right)\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+25\left(i\,a^3x^3+\right.}{8}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
[Out] -1/8*(2*sqrt(a^2*x^2 + 1)*(-43*I*a^2*x^2 - 9*a*x - 2*I)*sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - 25*(a^3*x^3 - I*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)) + 1) + 25*(I*a^3*x^3 + a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I)) + I) + 25*(-I*a^3*x^3 - a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I)) - I) + 25*(a^3*x^3 - I*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
 - 1))/(a*x^3 - I*x^2)
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
[Out] Integral(1/(x**3*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
[Out] integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 81, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

```
[In] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
[Out] int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

3.113 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 203

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{55}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{55}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] 287/24*I*a^3*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-1/3*(1-I*a*x)^(1/4)/x^3/(1+I*a
*x)^(1/4)+13/12*I*a*(1-I*a*x)^(1/4)/x^2/(1+I*a*x)^(1/4)+61/24*a^2*(1-I*a*x)
^(1/4)/x/(1+I*a*x)^(1/4)+55/8*I*a^3*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
-55/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5170, 100, 156, 160, 12, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{55}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{55}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}}$$

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] (((287*I)/24)*a^3*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) - (1 - I*a*x)^(1/4)/
(3*x^3*(1 + I*a*x)^(1/4)) + (((13*I)/12)*a*(1 - I*a*x)^(1/4))/(x^2*(1 + I*a
```

```
*x)^(1/4)) + (61*a^2*(1 - I*a*x)^(1/4))/(24*x*(1 + I*a*x)^(1/4)) + ((55*I)/
8)*a^3*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ((55*I)/8)*a^3*ArcTanh
[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
```

```
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1-iax)^{5/4}}{x^4(1+iax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} - \frac{1}{3}\int \frac{\frac{13ia}{2}+6a^2x}{x^3(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{1}{6}\int \frac{-\frac{61a^2}{4}+13ia^3x}{x^2(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} - \frac{1}{6}\int \frac{-\frac{165ia^3}{8}-\frac{61a^4x}{4}}{x(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$

$$= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{i\int \frac{165a^4}{16x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{3a}$$

$$
\begin{aligned}
&= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} \\
&\quad + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{1}{16}(55ia^3)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx \\
&= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} \\
&\quad + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{1}{4}(55ia^3)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} \\
&\quad + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} - \frac{1}{8}(55ia^3)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&\quad + \frac{1}{8}(55ia^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) \\
&= \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} \\
&\quad + \frac{55}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{55}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.46

$$
\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{\sqrt[4]{1-iax}\left(-8+26iax+61a^2x^2+287ia^3x^3-330ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3\sqrt[4]{1+iax}}
$$

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^4),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-8 + (26*I)*a*x + 61*a^2*x^2 + (287*I)*a^3*x^3 - (330*I
)*a^3*x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1
+ I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^4}dx$$

[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)

[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.21

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{2\,(287\,a^3x^3 - 61i\,a^2x^2 + 26\,ax + 8i)\sqrt{a^2x^2+1}\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} - 165\,(i\,a^4x^4 + a^3x^3)\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - 16}{}$$

[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")

[Out] 1/48*(2*(287*a^3*x^3 - 61*I*a^2*x^2 + 26*a*x + 8*I)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 165*(I*a^4*x^4 + a^3*x^3)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 165*(a^4*x^4 - I*a^3*x^3)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 165*(a^4*x^4 - I*a^3*x^3)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 165*(-I*a^4*x^4 - a^3*x^3)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1))/(a*x^4 - I*x^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \text{Timed out}$$

[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**4,x)

[Out] Timed out

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
[Out] integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 81, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

```
[In] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
[Out] int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

3.114 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 233

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{475}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{475}{64}a^4\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

```
[Out] 2467/192*a^4*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-1/4*(1-I*a*x)^(1/4)/x^4/(1+I*a
*x)^(1/4)+17/24*I*a*(1-I*a*x)^(1/4)/x^3/(1+I*a*x)^(1/4)+113/96*a^2*(1-I*a*x
)^(1/4)/x^2/(1+I*a*x)^(1/4)-521/192*I*a^3*(1-I*a*x)^(1/4)/x/(1+I*a*x)^(1/4)
+475/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-475/64*a^4*arctanh((1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5170, 100, 156, 160, 12, 95, 304, 209, 212}

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \frac{475}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{475}{64}a^4\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}}$$

```
[In] Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] (2467*a^4*(1 - I*a*x)^(1/4))/(192*(1 + I*a*x)^(1/4)) - (1 - I*a*x)^(1/4)/(4
*x^4*(1 + I*a*x)^(1/4)) + (((17*I)/24)*a*(1 - I*a*x)^(1/4))/(x^3*(1 + I*a*x
)^(1/4)) + (113*a^2*(1 - I*a*x)^(1/4))/(96*x^2*(1 + I*a*x)^(1/4)) - (((521*
I)/192)*a^3*(1 - I*a*x)^(1/4))/(x*(1 + I*a*x)^(1/4)) + (475*a^4*ArcTan[(1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/64 - (475*a^4*ArcTanh[(1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{5/4}}{x^5(1+iax)^{5/4}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} - \frac{1}{4}\int \frac{\frac{17ia}{2}+8a^2x}{x^4(1-iax)^{3/4}(1+iax)^{5/4}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{1}{12}\int \frac{-\frac{113a^2}{4}+\frac{51}{2}ia^3x}{x^3(1-iax)^{3/4}(1+iax)^{5/4}}\,dx \\
&= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{1}{24}\int \frac{-\frac{521ia^3}{8}-\frac{113a^4x}{2}}{x^2(1-iax)^{3/4}(1+iax)^{5/4}}\,dx
\end{aligned}
$$

$$= -\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{1}{24}\int \frac{\frac{1425a^4}{16} - \frac{521}{8}ia^5x}{x(1-iax)^{3/4}(1+iax)^{5/4}}\,dx$$

$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} - \frac{i\int \frac{1425ia^5}{32x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx}{12a}$$

$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{1}{128}\left(475a^4\right)\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{1+iax}}\,dx$$

$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{1}{32}\left(475a^4\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} - \frac{1}{64}\left(475a^4\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{64}\left(475a^4\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

$$= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}} + \frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}} + \frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}} - \frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}} + \frac{475}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{475}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.42

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \frac{\sqrt[4]{1-iax}\left(-48+136iax+226a^2x^2-521ia^3x^3+2467a^4x^4-2850a^4x^4\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right.}{192x^4\sqrt[4]{1+iax}}$$

```
[In] Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^5),x]
```

```
[Out] ((1 - I*a*x)^(1/4)*(-48 + (136*I)*a*x + 226*a^2*x^2 - (521*I)*a^3*x^3 + 246
7*a^4*x^4 - 2850*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)
]))/(192*x^4*(1 + I*a*x)^(1/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^5}\,dx$$

```
[In] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.09

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{2\left(2467i\,a^4x^4+521\,a^3x^3+226i\,a^2x^2-136\,ax-48i\right)\sqrt{a^2x^2+1}\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1425\left(a^5x^5-i\,a^4x^4\right)\log\left(\right.}{}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(2*(2467*I*a^4*x^4 + 521*a^3*x^3 + 226*I*a^2*x^2 - 136*a*x - 48*I)*s
qrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1425*(a^5*x^5 - I*a^
4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 1425*(-I*a^5*x^5 - a^
4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 1425*(I*a^5*x^5 + a^4
*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 1425*(a^5*x^5 - I*a^4*
x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1))/(a*x^5 - I*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
[Out] integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 81, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

```
[In] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
[Out] int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

3.115 $\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 319

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x + \frac{19}{162}i\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - \frac{19}{162}i\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - \frac{19}{81}i\arctan\left(\right.$$

```
[Out] -19/54*I*(1-I*x)^(5/6)*(1+I*x)^(1/6)-1/18*I*(1-I*x)^(5/6)*(1+I*x)^(7/6)+1/3
*(1-I*x)^(5/6)*(1+I*x)^(7/6)*x-19/81*I*arctan((1-I*x)^(1/6)/(1+I*x)^(1/6))-
19/162*I*arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)-3^(1/2))-19/162*I*arctan(2*(1
-I*x)^(1/6)/(1+I*x)^(1/6)+3^(1/2))-19/324*I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3
)-(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)+19/324*I*ln(1+(1-I*x)^(1/3)/
(1+I*x)^(1/3)+(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5170, 92, 81, 52, 65, 338, 301, 648, 632, 210, 642, 209}

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \frac{19}{162}i\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - \frac{19}{162}i\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - \frac{19}{81}i\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} - \frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{19i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{108\sqrt{3}} + \frac{19i\log\left(\frac{\sqrt[3]{1-}}{\sqrt[3]{1-}}\right.}{}$$

```
[In] Int[E^((I/3)*ArcTan[x])*x^2,x]
```

```
[Out] ((-19*I)/54)*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) - (I/18)*(1 - I*x)^(5/6)*(1 +
I*x)^(7/6) + ((1 - I*x)^(5/6)*(1 + I*x)^(7/6)*x)/3 + ((19*I)/162)*ArcTan[Sq
rt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ((19*I)/162)*ArcTan[Sqrt[3]
+ (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ((19*I)/81)*ArcTan[(1 - I*x)^(1/6)
/(1 + I*x)^(1/6)] - (((19*I)/108)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) -
 (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/Sqrt[3] + (((19*I)/108)*Log[1
+ (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/
6)])/Sqrt[3]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[6]{1+ix}x^2}{\sqrt[6]{1-ix}}\,dx\\
&= \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x + \frac{1}{3}\int \frac{\left(-1-\frac{ix}{3}\right)\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx\\
&= -\frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{54}\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx\\
&= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix}\\
&\quad -\frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x-\frac{19}{162}\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}}\,dx\\
&= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}\\
&\quad + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{27}i\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{1-ix}\right)\\
&= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}\\
&\quad + \frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x - \frac{19}{27}i\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\
&= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}\\
&\quad +\frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x-\frac{19}{81}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\frac{19}{81}i\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\right.\\
&= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}\\
&\quad +\frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x-\frac{19}{81}i\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\frac{19}{324}i\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right.\\
&= -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}\\
&\quad +\frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x-\frac{19}{81}i\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\frac{19i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{108\sqrt{3}}+\frac{19i}{}
\end{aligned}$$

$$= -\frac{19}{54} i(1-ix)^{5/6} \sqrt[6]{1+ix} - \frac{1}{18} i(1-ix)^{5/6}(1+ix)^{7/6}$$

$$+\frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x+\frac{19}{162}i\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\frac{19}{162}i\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\frac{19}{81}i$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.23

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \frac{1}{90}(1-ix)^{5/6}\left(5\sqrt[6]{1+ix}(-i+7x+6ix^2) - 38i\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

```
[In] Integrate[E^((I/3)*ArcTan[x])*x^2,x]
[Out] ((1 - I*x)^(5/6)*(5*(1 + I*x)^(1/6)*(-I + 7*x + (6*I)*x^2) - (38*I)*2^(1/6)
*Hypergeometric2F1[-1/6, 5/6, 11/6, 1/2 - (I/2)*x]))/90
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} x^2 dx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.65

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = -\frac{19}{324}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) - \frac{19}{324}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) - \frac{19}{324}\left(i\sqrt{3}+1\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) - \frac{19}{324}\left(i\sqrt{3}-1\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + \frac{1}{54}\left(18x^3-3i\,x^2-x-22i\right)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} - \frac{19}{162}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i\right) + \frac{19}{162}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-i\right)$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="fricas")
```

```
[Out] -19/324*(-I*sqrt(3) + 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
+ 1/2*I) - 19/324*(-I*sqrt(3) - 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x +
I))^(1/3) - 1/2*I) - 19/324*(I*sqrt(3) + 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2
+ 1)/(x + I))^(1/3) + 1/2*I) - 19/324*(I*sqrt(3) - 1)*log(-1/2*sqrt(3) + (I
*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2*I) + 1/54*(18*x^3 - 3*I*x^2 - x - 22*I)
*(I*sqrt(x^2 + 1)/(x + I))^(1/3) - 19/162*log((I*sqrt(x^2 + 1)/(x + I))^(1/
3) + I) + 19/162*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - I)
```

Sympy [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \int x^2\sqrt[3]{\frac{i(x-i)}{\sqrt{x^2+1}}}\,dx$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)*x**2,x)
```

```
[Out] Integral(x**2*(I*(x - I)/sqrt(x**2 + 1))**(1/3), x)
```

Maxima [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \int x^2\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Giac [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \int x^2\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="giac")
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \int x^2\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}\,dx$$

```
[In] int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3),x)
[Out] int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3), x)
```

3.116 $\int e^{\frac{1}{3}i\arctan(x)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 278

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}$$
$$-\frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{18}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{9}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{\log\left(1+\frac{\sqrt[3]{1-}}{\sqrt[3]{1+}}\right.}{1}$$

```
[Out] 1/6*(1-I*x)^(5/6)*(1+I*x)^(1/6)+1/2*(1-I*x)^(5/6)*(1+I*x)^(7/6)+1/9*arctan(
(1-I*x)^(1/6)/(1+I*x)^(1/6))+1/18*arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)-3^(1
/2))+1/18*arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)+3^(1/2))+1/36*ln(1+(1-I*x)^(
1/3)/(1+I*x)^(1/3)-(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)-1/36*ln(1+(
1-I*x)^(1/3)/(1+I*x)^(1/3)+(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5170, 81, 52, 65, 338, 301, 648, 632, 210, 642, 209}

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = -\frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{18}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{9}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}+\frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{12\sqrt{3}}$$

```
[In] Int[E^((I/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/6 + ((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/2 -
 ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/18 + ArcTan[Sqrt[3]
+ (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/18 + ArcTan[(1 - I*x)^(1/6)/(1 + I*x
)^(1/6)]/9 + Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(
1/6))/(1 + I*x)^(1/6)]/(12*Sqrt[3]) - Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/
3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]/(12*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[6]{1+ixx}}{\sqrt[6]{1-ix}}\,dx$$

$$= \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} - \frac{1}{6}i\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx$$

$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} - \frac{1}{18}i\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}}\,dx$$

$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{1-ix}\right)$$

$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$

$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{9}\text{Subst}\Bigg(\int$$

$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{9}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{36}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{36}\text{Subst}\Bigg(\int \frac{1}{1+\sqrt{3}x+}$$

$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} + \frac{1}{9}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{12\sqrt{3}} - \frac{\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{12\sqrt{3}}$$

$$= \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6} - \frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{18}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{9}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{\log\left(1-\right.}{}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.21

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = \frac{1}{10}(1-ix)^{5/6}\left(5(1+ix)^{7/6}+2\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

```
[In] Integrate[E^((I/3)*ArcTan[x])*x,x]
[Out] ((1 - I*x)^(5/6)*(5*(1 + I*x)^(7/6) + 2*2^(1/6)*Hypergeometric2F1[-1/6, 5/6
, 11/6, 1/2 - (I/2)*x]))/10
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} x dx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.70

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = -\frac{1}{36}\left(\sqrt{3}+i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) - \frac{1}{36}\left(\sqrt{3}-i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + \frac{1}{36}\left(\sqrt{3}-i\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) + \frac{1}{36}\left(\sqrt{3}+i\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + \frac{1}{6}\left(3x^2-ix+4\right)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} - \frac{1}{18}i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i\right) + \frac{1}{18}i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-i\right)$$

```
[In] integrate((((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="fricas")
```

```
[Out] -1/36*(sqrt(3) + I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2
*I) - 1/36*(sqrt(3) - I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
- 1/2*I) + 1/36*(sqrt(3) - I)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^
(1/3) + 1/2*I) + 1/36*(sqrt(3) + I)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x
+ I))^(1/3) - 1/2*I) + 1/6*(3*x^2 - I*x + 4)*(I*sqrt(x^2 + 1)/(x + I))^(1/3
) - 1/18*I*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) + 1/18*I*log((I*sqrt(x^
2 + 1)/(x + I))^(1/3) - I)
```

Sympy [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = \int x\sqrt[3]{\frac{i(x-i)}{\sqrt{x^2+1}}}\,dx$$

```
[In] integrate((((1+I*x)/(x**2+1)**(1/2))**(1/3)*x,x)
```

```
[Out] Integral(x*(I*(x - I)/sqrt(x**2 + 1))**(1/3), x)
```

Maxima [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = \int x\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="maxima")
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Giac [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = \int x\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="giac")
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = \int x\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}\,dx$$

```
[In] int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3),x)
[Out] int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3), x)
```

3.117 $\int e^{\frac{1}{3}i\arctan(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 262

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{3}i\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{3}i\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{2}{3}i\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}} - \frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}}$$

```
[Out] I*(1-I*x)^(5/6)*(1+I*x)^(1/6)+2/3*I*arctan((1-I*x)^(1/6)/(1+I*x)^(1/6))+1/3
*I*arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)-3^(1/2))+1/3*I*arctan(2*(1-I*x)^(1/
6)/(1+I*x)^(1/6)+3^(1/2))+1/6*I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3)-(1-I*x)^(1
/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)-1/6*I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3)+(
1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5169, 52, 65, 338, 301, 648, 632, 210, 642, 209}

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx=-\frac{1}{3}i\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{3}i\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{2}{3}i\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}-\frac{i\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)}{2\sqrt{3}}$$

```
[In] Int[E^((I/3)*ArcTan[x]),x]
```

```
[Out] I*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) - (I/3)*ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/
6))/(1 + I*x)^(1/6)] + (I/3)*ArcTan[Sqrt[3] + (2*(1 - I*x)^(1/6))/(1 + I*x)
^(1/6)] + ((2*I)/3)*ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)] + ((I/2)*Log[1
+ (1 - I*x)^(1/3)/(1 + I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/
6)])/Sqrt[3] - ((I/2)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1
 - I*x)^(1/6))/(1 + I*x)^(1/6)])/Sqrt[3]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\,dx \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{1}{3}\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}}\,dx \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix} + 2i\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{1-ix}\right) \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix} + 2i\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix} + \frac{2}{3}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) \\
&\quad + \frac{2}{3}i\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) \\
&\quad + \frac{2}{3}i\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) \\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix} \\
&\quad + \frac{2}{3}i\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{6}i\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) \\
&\quad + \frac{1}{6}i\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) \\
&\quad + \frac{i\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}} - \frac{i\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}}
\end{aligned}
$$

$$
\begin{aligned}
&= i(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{2}{3}i\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\
&\quad+\frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}}-\frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}}\\
&\quad-\frac{1}{3}i\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\
&\quad-\frac{1}{3}i\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\
&= i(1-ix)^{5/6}\sqrt[6]{1+ix}-\frac{1}{3}i\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\
&\quad+\frac{1}{3}i\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{2}{3}i\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\
&\quad+\frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}}-\frac{i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)}{2\sqrt{3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.13

$$
\int e^{\frac{1}{3}i\arctan(x)}\,dx=-\frac{12}{7}ie^{\frac{7}{3}i\arctan(x)}\,\text{Hypergeometric2F1}\left(\frac{7}{6},2,\frac{13}{6},-e^{2i\arctan(x)}\right)
$$

```
[In] Integrate[E^((I/3)*ArcTan[x]),x]
[Out] ((-12*I)/7)*E^(((7*I)/3)*ArcTan[x])*Hypergeometric2F1[7/6, 2, 13/6, -E^((2*
I)*ArcTan[x])]
```

Maple [F]

$$
\int\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}dx
$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3),x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.74

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \frac{1}{6}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) + \frac{1}{6}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + \frac{1}{6}\left(i\sqrt{3}+1\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) + \frac{1}{6}\left(i\sqrt{3}-1\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + (x+i)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{3}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i\right) - \frac{1}{3}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-i\right)$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*(-I*sqrt(3) + 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/
2*I) + 1/6*(-I*sqrt(3) - 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/
3) - 1/2*I) + 1/6*(I*sqrt(3) + 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x +
I))^(1/3) + 1/2*I) + 1/6*(I*sqrt(3) - 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1
)/(x + I))^(1/3) - 1/2*I) + (x + I)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/3*l
og((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) - 1/3*log((I*sqrt(x^2 + 1)/(x + I))
^(1/3) - I)
```

Sympy [F]

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \int \sqrt[3]{\frac{ix+1}{\sqrt{x^2+1}}}\,dx$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3),x)
```

```
[Out] Integral(((I*x + 1)/sqrt(x**2 + 1))**(1/3), x)
```

Maxima [F]

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \int \left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Giac [F]

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \int \left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \int \left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}\,dx$$

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3),x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3), x)
```

3.118 $\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 430

$$\begin{aligned}\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx &= \arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\ &\quad+\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)-\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)\\ &\quad-2\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-2\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)\\ &\quad-\frac{1}{2}\sqrt{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\ &\quad+\frac{1}{2}\sqrt{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\ &\quad+\frac{1}{2}\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)-\frac{1}{2}\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)\end{aligned}$$

```
[Out] -2*arctan((1-I*x)^(1/6)/(1+I*x)^(1/6))-arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)
-3^(1/2))-arctan(2*(1-I*x)^(1/6)/(1+I*x)^(1/6)+3^(1/2))-2*arctanh((1+I*x)^(
1/6)/(1-I*x)^(1/6))+1/2*ln(1-(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I
*x)^(1/3))-1/2*ln(1+(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3)
)+arctan(1/3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)-arctan(1/3*
(1+2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)-1/2*ln(1+(1-I*x)^(1/3)/(
1+I*x)^(1/3)-(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)+1/2*ln(1+(1-I*x)^
(1/3)/(1+I*x)^(1/3)+(1-I*x)^(1/6)*3^(1/2)/(1+I*x)^(1/6))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5170, 132, 65, 338, 301, 648, 632, 210, 642, 209, 95, 216, 212}

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) +\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)-\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) -2\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-2\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) -\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right) +\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right) +\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)$$

```
[In] Int[E^((I/3)*ArcTan[x])/x,x]
```

```
[Out] ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - ArcTan[Sqrt[3] + (2
*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] + Sqrt[3]*ArcTan[(1 - (2*(1 + I*x)^(1/6)
)/(1 - I*x)^(1/6))/Sqrt[3]] - Sqrt[3]*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 -
I*x)^(1/6))/Sqrt[3]] - 2*ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)] - 2*ArcTan
h[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] - (Sqrt[3]*Log[1 + (1 - I*x)^(1/3)/(1 +
I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/2 + (Sqrt[3]*Log[1
 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1
/6)])/2 + Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*
x)^(1/3)]/2 - Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1
- I*x)^(1/3)]/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
```

```
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x}\,dx\\ &= i\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}}\,dx+\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,dx\end{aligned}$$

$$=-\left(6\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx,x,\sqrt[6]{1-ix}\right)\right)+6\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx,x,\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$

$$=-\left(2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)\right)-2\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx,x,\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$
$$-2\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx,x,\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)-6\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$

$$=-2\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$
$$-\frac{1}{2}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$
$$-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$
$$-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)-2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$
$$-2\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$
$$-2\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$

$$=-2\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-2\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)+\frac{1}{2}\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)$$
$$-\frac{1}{2}\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$
$$-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$
$$+3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$
$$+3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$
$$-\frac{1}{2}\sqrt{3}\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$
$$+\frac{1}{2}\sqrt{3}\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$

$$= \sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - \sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - 2\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - 2\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{2}\sqrt{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{2}\sqrt{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{2}\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) - \frac{1}{2}\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) + \text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, \sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)$$

$$= \arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - \arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - \sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right) - 2\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - 2\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{2}\sqrt{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{2}\sqrt{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{2}\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) - \frac{1}{2}\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.21

$$\int\frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = -\frac{3(1-ix)^{5/6}\left(\sqrt[6]{2}(1+ix)^{5/6}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}-\frac{ix}{2}\right)+2\,\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{i+x}{i-x}\right)\right)}{5(1+ix)^{5/6}}$$

```
[In] Integrate[E^((I/3)*ArcTan[x])/x,x]
```

```
[Out] (-3*(1 - I*x)^(5/6)*(2^(1/6)*(1 + I*x)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11
/6, 1/2 - (I/2)*x] + 2*Hypergeometric2F1[5/6, 1, 11/6, (I + x)/(I - x)]))/(
5*(1 + I*x)^(5/6))
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}dx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x)
```

```
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.79

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \frac{1}{2}\left(\sqrt{3}+i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) + \frac{1}{2}\left(\sqrt{3}-i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + \frac{1}{2}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right) + \frac{1}{2}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}\right) + \frac{1}{2}\left(i\sqrt{3}-1\right)\log\left(-\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right) + \frac{1}{2}\left(i\sqrt{3}+1\right)\log\left(-\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}\right) - \frac{1}{2}\left(\sqrt{3}-i\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) - \frac{1}{2}\left(\sqrt{3}+i\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) - \log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right) + i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i\right) - i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-i\right) + \log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right)$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(3) + I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2*I
) + 1/2*(sqrt(3) - I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1
/2*I) + 1/2*(-I*sqrt(3) - 1)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^
(1/3) + 1/2) + 1/2*(-I*sqrt(3) + 1)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x
 + I))^(1/3) - 1/2) + 1/2*(I*sqrt(3) - 1)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2
+ 1)/(x + I))^(1/3) + 1/2) + 1/2*(I*sqrt(3) + 1)*log(-1/2*I*sqrt(3) + (I*sq
rt(x^2 + 1)/(x + I))^(1/3) - 1/2) - 1/2*(sqrt(3) - I)*log(-1/2*sqrt(3) + (I
*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2*I) - 1/2*(sqrt(3) + I)*log(-1/2*sqrt(3)
```

```
 + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2*I) - log((I*sqrt(x^2 + 1)/(x + I))
^(1/3) + 1) + I*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) - I*log((I*sqrt(x^
2 + 1)/(x + I))^(1/3) - I) + log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \int \frac{\sqrt[3]{\frac{i(x-i)}{\sqrt{x^2+1}}}}{x}\,dx$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x,x)
[Out] Integral((I*(x - I)/sqrt(x**2 + 1))**(1/3)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x, x)
```

Giac [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}}{x}\,dx$$

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x, x)
```

3.119 $\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + \frac{i\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{i\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}i\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) - \frac{1}{6}i\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)$$

```
[Out] -(1-I*x)^(5/6)*(1+I*x)^(1/6)/x-2/3*I*arctanh((1+I*x)^(1/6)/(1-I*x)^(1/6))+1
/6*I*ln(1-(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3))-1/6*I*ln
(1+(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3))+1/3*I*arctan(1/
3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)-1/3*I*arctan(1/3*(1+2*
(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5170, 96, 95, 216, 648, 632, 210, 642, 212}

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \frac{i\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{i\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}i\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + \frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}} - \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}} + 1\right) - \frac{1}{6}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}} + \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}} + 1\right)$$

```
[In] Int[E^((I/3)*ArcTan[x])/x^2,x]
```

```
[Out] -(((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/x) + (I*ArcTan[(1 - (2*(1 + I*x)^(1/6))
/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] - (I*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1
 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] - ((2*I)/3)*ArcTanh[(1 + I*x)^(1/6)/(1 - I
*x)^(1/6)] + (I/6)*Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3
)/(1 - I*x)^(1/3)] - (I/6)*Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I
*x)^(1/3)/(1 - I*x)^(1/3)]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^2}\,dx$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + \frac{1}{3}i\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,dx$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + 2i\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} - \frac{2}{3}i\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{2}{3}i\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{2}{3}i\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} - \frac{2}{3}i\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{6}i\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{2}i\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{2}i\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} - \frac{2}{3}i\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) - \frac{1}{6}i\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) + i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x} + \frac{i\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{i\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}i\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{6}i\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) - \frac{1}{6}i\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.25

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = -\frac{i(1-ix)^{5/6}\left(-5i+5x+2x\,\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{i+x}{i-x}\right)\right)}{5(1+ix)^{5/6}x}$$

```
[In] Integrate[E^((I/3)*ArcTan[x])/x^2,x]
[Out] ((-1/5*I)*(1 - I*x)^(5/6)*(-5*I + 5*x + 2*x*Hypergeometric2F1[5/6, 1, 11/6,
 (I + x)/(I - x)]))/((1 + I*x)^(5/6)*x)
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^2}dx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.83

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \frac{\left(\sqrt{3}x - i\,x\right)\log\left(\frac{1}{2}i\sqrt{3} + \left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} + \frac{1}{2}\right) + \left(\sqrt{3}x + i\,x\right)\log\left(\frac{1}{2}i\sqrt{3} + \left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} - \frac{1}{2}\right) - \left(\sqrt{3}x + i\,x\right)\mathrm{lo}}{}$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*((sqrt(3)*x - I*x)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
+ 1/2) + (sqrt(3)*x + I*x)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1
/3) - 1/2) - (sqrt(3)*x + I*x)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I
))^(1/3) + 1/2) - (sqrt(3)*x - I*x)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(
x + I))^(1/3) - 1/2) - 2*I*x*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) + 2*I
*x*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1) - 6*(-I*x + 1)*(I*sqrt(x^2 + 1)
/(x + I))^(1/3))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\sqrt[3]{\frac{i(x-i)}{\sqrt{x^2+1}}}}{x^2}\,dx$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**2,x)
```

```
[Out] Integral((I*(x - I)/sqrt(x**2 + 1))**(1/3)/x**2, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^2, x)
```

Giac [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

```
[In] integrate((((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}}{x^2}\,dx$$

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^2,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^2, x)
```

3.120 $\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 280

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}-\frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)-\frac{1}{36}\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)+\frac{1}{36}\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)$$

```
[Out] -1/2*(1-I*x)^(5/6)*(1+I*x)^(7/6)/x^2-1/6*I*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x+1/
9*arctanh((1+I*x)^(1/6)/(1-I*x)^(1/6))-1/36*ln(1-(1+I*x)^(1/6)/(1-I*x)^(1/6
)+(1+I*x)^(1/3)/(1-I*x)^(1/3))+1/36*ln(1+(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x
)^(1/3)/(1-I*x)^(1/3))-1/18*arctan(1/3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^
(1/2))*3^(1/2)+1/18*arctan(1/3*(1+2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules

used = {5170, 98, 96, 95, 216, 648, 632, 210, 642, 212}

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}} - \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}} + 1\right) + \frac{1}{36}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}} + \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}} + 1\right)$$

```
[In] Int[E^((I/3)*ArcTan[x])/x^3,x]
```

```
[Out] -1/2*((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/x^2 - ((I/6)*(1 - I*x)^(5/6)*(1 + I*
x)^(1/6))/x - ArcTan[(1 - (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]/(6*
Sqrt[3]) + ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]/(6*Sqr
t[3]) + ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)]/9 - Log[1 - (1 + I*x)^(1/6
)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/36 + Log[1 + (1 + I*x)
^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/36
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^3}\,dx$$

$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} + \frac{1}{6}i\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^2}\,dx$$

$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{1}{18}\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,dx$$

$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$

$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{}{1}\right.$$

$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} + \frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{36}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{36}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx\right.$$

$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} + \frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{36}\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) + \frac{1}{36}\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) -$$

$$= -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{36}\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) + \frac{1}{36}\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.26

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \frac{(1-ix)^{5/6}\left(5(-3-7ix+4x^2)+2x^2\,\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{i+x}{i-x}\right)\right)}{30(1+ix)^{5/6}x^2}$$

```
[In] Integrate[E^((I/3)*ArcTan[x])/x^3,x]
[Out] ((1 - I*x)^(5/6)*(5*(-3 - (7*I)*x + 4*x^2) + 2*x^2*Hypergeometric2F1[5/6, 1
, 11/6, (I + x)/(I - x)]))/(30*(1 + I*x)^(5/6)*x^2)
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}dx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.84

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \frac{2\,x^2\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right)-2\,x^2\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right)+\left(i\sqrt{3}x^2+x^2\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right)+}{}$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="fricas")
[Out] 1/36*(2*x^2*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) - 2*x^2*log((I*sqrt(x^
2 + 1)/(x + I))^(1/3) - 1) + (I*sqrt(3)*x^2 + x^2)*log(1/2*I*sqrt(3) + (I*s
qrt(x^2 + 1)/(x + I))^(1/3) + 1/2) + (I*sqrt(3)*x^2 - x^2)*log(1/2*I*sqrt(3
) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) + (-I*sqrt(3)*x^2 + x^2)*log(-1/
2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) + (-I*sqrt(3)*x^2 - x^
2)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) - 6*(4*x^2 +
 I*x + 3)*(I*sqrt(x^2 + 1)/(x + I))^(1/3))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^3, x)
```

Giac [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}}{x^3}\,dx$$

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^3,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^3, x)
```

3.121 $\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 319

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19i\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19i\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19}{81}i\,\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{19}{324}i\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) + \frac{19}{324}i\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)$$

```
[Out] -1/3*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x^3-7/18*I*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x^2
+11/27*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x+19/81*I*arctanh((1+I*x)^(1/6)/(1-I*x)^
(1/6))-19/324*I*ln(1-(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3
))+19/324*I*ln(1+(1+I*x)^(1/6)/(1-I*x)^(1/6)+(1+I*x)^(1/3)/(1-I*x)^(1/3))-1
9/162*I*arctan(1/3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)+19/16
2*I*arctan(1/3*(1+2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules

used = {5170, 101, 156, 12, 95, 216, 648, 632, 210, 642, 212}

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx$$
$$= -\frac{19i\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19i\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19}{81}i\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)$$
$$- \frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x}$$
$$- \frac{19}{324}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}} - \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}} + 1\right) + \frac{19}{324}i\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}} + \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}} + 1\right)$$

```
[In] Int[E^((I/3)*ArcTan[x])/x^4,x]
```

```
[Out] -1/3*((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/x^3 - (((7*I)/18)*(1 - I*x)^(5/6)*(1
 + I*x)^(1/6))/x^2 + (11*(1 - I*x)^(5/6)*(1 + I*x)^(1/6))/(27*x) - (((19*I)
/54)*ArcTan[(1 - (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt[3] + (
((19*I)/54)*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]])/Sqrt
[3] + ((19*I)/81)*ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] - ((19*I)/324)*L
og[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)] +
 ((19*I)/324)*Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1
- I*x)^(1/3)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^4}\,dx \\
&= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} + \frac{1}{3}\int \frac{\frac{7i}{3}-2x}{\sqrt[6]{1-ix}(1+ix)^{5/6}x^3}\,dx \\
&= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} - \frac{1}{6}\int \frac{\frac{22}{9}+\frac{7ix}{3}}{\sqrt[6]{1-ix}(1+ix)^{5/6}x^2}\,dx \\
&= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} \\
&\quad + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{1}{6}\int -\frac{19i}{27\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,dx \\
&= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} \\
&\quad + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19}{162}i\int \frac{1}{\sqrt[6]{1-ix}(1+ix)^{5/6}x}\,dx \\
&= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} \\
&\quad + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19}{27}i\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) \\
&= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} \\
&\quad + \frac{19}{81}i\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{19}{81}i\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{19}{81}i\text{Subst}\Bigg(\\
&= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} \\
&\quad + \frac{19}{81}i\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{19}{324}i\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{19}{324}i\text{Subst}\Bigg(\int \frac{1+2}{1+x+}
\end{aligned}$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} + \frac{19}{81}i\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{19}{324}i\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) + \frac{19}{324}i\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right.$$

$$= -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3} - \frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2} + \frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19i\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19i\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19}{81}i\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{19}{324}i\log\left(1-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right) + \frac{19}{324}i\log\left(1+\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right.$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.25

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \frac{(1-ix)^{5/6}\left(5(-18-39ix+43x^2+22ix^3)+38ix^3\,\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{i+x}{i-x}\right)\right)}{270(1+ix)^{5/6}x^3}$$

```
[In] Integrate[E^((I/3)*ArcTan[x])/x^4,x]
[Out] ((1 - I*x)^(5/6)*(5*(-18 - (39*I)*x + 43*x^2 + (22*I)*x^3) + (38*I)*x^3*Hyp
ergeometric2F1[5/6, 1, 11/6, (I + x)/(I - x)]))/(270*(1 + I*x)^(5/6)*x^3)
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \frac{38i\,x^3\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right)-38i\,x^3\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right)-19\left(\sqrt{3}x^3-i\,x^3\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}\right.}{}$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="fricas")
```

```
[Out] 1/324*(38*I*x^3*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) - 38*I*x^3*log((I*
sqrt(x^2 + 1)/(x + I))^(1/3) - 1) - 19*(sqrt(3)*x^3 - I*x^3)*log(1/2*I*sqrt
(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) - 19*(sqrt(3)*x^3 + I*x^3)*log
(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) + 19*(sqrt(3)*x^3 +
 I*x^3)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) + 19*(s
qrt(3)*x^3 - I*x^3)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) -
1/2) - 6*(22*I*x^3 - x^2 + 3*I*x + 18)*(I*sqrt(x^2 + 1)/(x + I))^(1/3))/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^4, x)
```

Giac [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}}{x^4}\,dx$$

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^4,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^4, x)
```

3.122 $\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 177

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = -\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3} +\frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x+\frac{22i\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{27\sqrt{3}}+\frac{11}{27}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{11}{81}i\log(1+ix)$$

```
[Out] -11/27*I*(1-I*x)^(2/3)*(1+I*x)^(1/3)-1/9*I*(1-I*x)^(2/3)*(1+I*x)^(4/3)+1/3*
(1-I*x)^(2/3)*(1+I*x)^(4/3)*x+11/27*I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))+11/
81*I*ln(1+I*x)+22/81*I*arctan(1/3*3^(1/2)-2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3
^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5170, 92, 81, 52, 62}

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = \frac{22i\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{27\sqrt{3}}+\frac{1}{3}(1-ix)^{2/3}x(1+ix)^{4/3} -\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3}-\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{11}{27}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{11}{81}i\log(1+ix)$$

```
[In] Int[E^(((2*I)/3)*ArcTan[x])*x^2,x]
```

```
[Out] ((-11*I)/27)*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) - (I/9)*(1 - I*x)^(2/3)*(1 + I
*x)^(4/3) + ((1 - I*x)^(2/3)*(1 + I*x)^(4/3)*x)/3 + (((22*I)/27)*ArcTan[1/S
```

```
qrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] + ((11*I)/
27)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)] + ((11*I)/81)*Log[1 + I*x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[3]{1+ix}x^2}{\sqrt[3]{1-ix}}\,dx$$

$$= \frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x + \frac{1}{3}\int \frac{\left(-1-\frac{2ix}{3}\right)\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx$$

$$= -\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3} + \frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x - \frac{11}{27}\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx$$

$$= -\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix} - \frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3} + \frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x - \frac{22}{81}\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}}\,dx$$

$$= -\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix} - \frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3} + \frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x + \frac{22i\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{27\sqrt{3}} + \frac{11}{27}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right) + \frac{11}{81}i\log(1+$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = \frac{1}{18}(1-ix)^{2/3}\left(2\sqrt[3]{1+ix}(-i+4x+3ix^2) - 11i\sqrt[3]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])*x^2,x]
[Out] ((1 - I*x)^(2/3)*(2*(1 + I*x)^(1/3)*(-I + 4*x + (3*I)*x^2) - (11*I)*2^(1/3)
*Hypergeometric2F1[-1/3, 2/3, 5/3, 1/2 - (I/2)*x]))/18
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} x^2 dx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.66

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = -\frac{11}{81}\left(\sqrt{3}+i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right) + \frac{11}{81}\left(\sqrt{3}-i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right) + \frac{1}{27}\left(9\,x^3-3i\,x^2-2\,x-14i\right)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} + \frac{22}{81}i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+1\right)$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="fricas")
```

```
[Out] -11/81*(sqrt(3) + I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) -
1/2) + 11/81*(sqrt(3) - I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt
(3) - 1/2) + 1/27*(9*x^3 - 3*I*x^2 - 2*x - 14*I)*(I*sqrt(x^2 + 1)/(x + I))^
(2/3) + 22/81*I*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)*x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = \int x^2\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Giac [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = \int x^2\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="giac")
[Out] integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = \int x^2\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}\,dx$$

```
[In] int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3),x)
[Out] int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3), x)
```

3.123 $\int e^{\frac{2}{3}i\arctan(x)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 140

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx=\frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}-\frac{2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{9}\log(1+ix)$$

```
[Out] 1/3*(1-I*x)^(2/3)*(1+I*x)^(1/3)+1/2*(1-I*x)^(2/3)*(1+I*x)^(4/3)-1/3*ln(1+(1
-I*x)^(1/3)/(1+I*x)^(1/3))-1/9*ln(1+I*x)-2/9*arctan(1/3*3^(1/2)-2/3*(1-I*x)
^(1/3)/(1+I*x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5170, 81, 52, 62}

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx=-\frac{2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}+\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}+\frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{9}\log(1+ix)$$

```
[In] Int[E^(((2*I)/3)*ArcTan[x])*x,x]
```

```
[Out] ((1 - I*x)^(2/3)*(1 + I*x)^(1/3))/3 + ((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/2 -
 (2*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/(3*S
qrt[3]) - Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)]/3 - Log[1 + I*x]/9
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[3]{1+ix}x}{\sqrt[3]{1-ix}}\,dx \\
&= \frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3} - \frac{1}{3}i\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx \\
&= \frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix} + \frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3} - \frac{2}{9}i\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}}\,dx \\
&= \frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix} + \frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3} - \frac{2\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}} \\
&\quad - \frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right) - \frac{1}{9}\log(1+ix)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.39

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx=\frac{1}{2}(1-ix)^{2/3}\left((1+ix)^{4/3}+\sqrt[3]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])*x,x]
[Out] ((1 - I*x)^(2/3)*((1 + I*x)^(4/3) + 2^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/
3, 1/2 - (I/2)*x]))/2
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} xdx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\begin{aligned}\int e^{\frac{2}{3}i\arctan(x)}x\,dx=&-\frac{1}{9}\left(i\sqrt{3}-1\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)\\&-\frac{1}{9}\left(-i\sqrt{3}-1\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)\\&+\frac{1}{6}\left(3x^2-2ix+5\right)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{2}{9}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+1\right)\end{aligned}$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="fricas")
[Out] -1/9*(I*sqrt(3) - 1)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) -
1/2) - 1/9*(-I*sqrt(3) - 1)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqr
t(3) - 1/2) + 1/6*(3*x^2 - 2*I*x + 5)*(I*sqrt(x^2 + 1)/(x + I))^(2/3) - 2/9
*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)*x,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx = \int x\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="maxima")
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Giac [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx = \int x\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="giac")
[Out] integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx = \int x\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}\,dx$$

```
[In] int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3),x)
[Out] int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3), x)
```

3.124 $\int e^{\frac{2}{3}i\arctan(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 116

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = i(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{2i\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}} - i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{3}i\log(1+ix)$$

```
[Out] I*(1-I*x)^(2/3)*(1+I*x)^(1/3)-I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))-1/3*I*ln(
1+I*x)-2/3*I*arctan(1/3*3^(1/2)-2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3^(1/2))*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5169, 52, 62}

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = -\frac{2i\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}+i(1-ix)^{2/3}\sqrt[3]{1+ix} - i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{3}i\log(1+ix)$$

```
[In] Int[E^(((2*I)/3)*ArcTan[x]),x]
```

```
[Out] I*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) - ((2*I)*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^
(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] - I*Log[1 + (1 - I*x)^(1/3)/(1 +
 I*x)^(1/3)] - (I/3)*Log[1 + I*x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\,dx \\
&= i(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{2}{3}\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}}\,dx \\
&= i(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{2i\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}-i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{3}i\log(1+ix)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = -\frac{3}{2}ie^{\frac{8}{3}i\arctan(x)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},2,\frac{7}{3},-e^{2i\arctan(x)}\right)$$

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x]),x]
[Out] ((-3*I)/2)*E^(((8*I)/3)*ArcTan[x])*Hypergeometric2F1[4/3, 2, 7/3, -E^((2*I)
*ArcTan[x])]
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} dx$$

[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3),x)

[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = \frac{1}{3}\left(\sqrt{3}+i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right) - \frac{1}{3}\left(\sqrt{3}-i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right) + (x+i)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} - \frac{2}{3}i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+1\right)$$

[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="fricas")

[Out] 1/3*(sqrt(3) + I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) - 1/2
) - 1/3*(sqrt(3) - I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt(3) -
 1/2) + (x + I)*(I*sqrt(x^2 + 1)/(x + I))^(2/3) - 2/3*I*log((I*sqrt(x^2 + 1
)/(x + I))^(2/3) + 1)

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = \text{Timed out}$$

[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3),x)

[Out] Timed out

Maxima [F]

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = \int \left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Giac [F]

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = \int \left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = \int \left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}\,dx$$

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3),x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3), x)
```

3.125 $\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 163

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx=\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{1}{2}\log(1+ix)-\frac{\log(x)}{2}$$

```
[Out] 3/2*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))+3/2*ln((1-I*x)^(1/3)-(1+I*x)^(1/3))+1
/2*ln(1+I*x)-1/2*ln(x)+arctan(1/3*3^(1/2)-2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3
^(1/2))*3^(1/2)+arctan(1/3*3^(1/2)+2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3^(1/2))
*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5170, 132, 62, 93}

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx=\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{1}{2}\log(1+ix)-\frac{\log(x)}{2}$$

```
[In] Int[E^(((2*I)/3)*ArcTan[x])/x,x]
```

```
[Out] Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))] +
 Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))]
+ (3*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)])/2 + (3*Log[(1 - I*x)^(1/3) -
 (1 + I*x)^(1/3)])/2 + Log[1 + I*x]/2 - Log[x]/2
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x}\,dx\\ &= i\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}}\,dx + \int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}x}\,dx\end{aligned}$$

$$= \sqrt{3}\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right) + \sqrt{3}\arctan\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right) + \frac{3}{2}\log\left(1 + \frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right) + \frac{3}{2}\log\left(\sqrt[3]{1-ix} - \sqrt[3]{1+ix}\right) + \frac{1}{2}\log(1+ix) - \frac{\log(x)}{2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = -\frac{3(1-ix)^{2/3}\left(\sqrt[3]{2}(1+ix)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}-\frac{ix}{2}\right) + 2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{i+x}{i-x}\right)\right)}{4(1+ix)^{2/3}}$$

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])/x,x]
[Out] (-3*(1 - I*x)^(2/3)*(2^(1/3)*(1 + I*x)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/
3, 1/2 - (I/2)*x] + 2*Hypergeometric2F1[2/3, 1, 5/3, (I + x)/(I - x)]))/(4*
(1 + I*x)^(2/3))
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.89

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \frac{1}{2}\left(i\sqrt{3}-1\right)\log\left(\frac{\sqrt{3}(ix-1)+x+2i\sqrt{x^2+1}\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i}{2\,(x+i)}\right) + \frac{1}{2}\left(-i\sqrt{3}-1\right)\log\left(\frac{\sqrt{3}(-ix+1)+x+2i\sqrt{x^2+1}\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i}{2\,(x+i)}\right) + \log\left(-\frac{x-i\sqrt{x^2+1}\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i}{x+i}\right)$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(I*sqrt(3) - 1)*log(1/2*(sqrt(3)*(I*x - 1) + x + 2*I*sqrt(x^2 + 1)*(I*s
qrt(x^2 + 1)/(x + I))^(1/3) + I)/(x + I)) + 1/2*(-I*sqrt(3) - 1)*log(1/2*(s
qrt(3)*(-I*x + 1) + x + 2*I*sqrt(x^2 + 1)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) +
 I)/(x + I)) + log(-(x - I*sqrt(x^2 + 1)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) +
I)/(x + I))
```

Sympy [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{i(x-i)}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x,x)
```

```
[Out] Integral((I*(x - I)/sqrt(x**2 + 1))**(2/3)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x, x)
```

Giac [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

```
[In] integrate((((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}}{x}\,dx$$

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x, x)
```

3.126 $\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = -\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}+\frac{2i\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}} + i\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{1}{3}i\log(x)$$

```
[Out] -(1-I*x)^(2/3)*(1+I*x)^(1/3)/x+I*ln((1-I*x)^(1/3)-(1+I*x)^(1/3))-1/3*I*ln(x
)+2/3*I*arctan(1/3*3^(1/2)+2/3*(1-I*x)^(1/3)/(1+I*x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5170, 96, 93}

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \frac{2i\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}-\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x} + i\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{1}{3}i\log(x)$$

```
[In] Int[E^(((2*I)/3)*ArcTan[x])/x^2,x]
```

```
[Out] -(((1 - I*x)^(2/3)*(1 + I*x)^(1/3))/x) + ((2*I)*ArcTan[1/Sqrt[3] + (2*(1 -
I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))])/Sqrt[3] + I*Log[(1 - I*x)^(1/3) - (
1 + I*x)^(1/3)] - (I/3)*Log[x]
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x^2}\,dx\\
&= -\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}+\frac{2}{3}i\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}x}\,dx\\
&= -\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}+\frac{2i\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}}\\
&\quad +i\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{1}{3}i\log(x)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx=-\frac{i(1-ix)^{2/3}\left(-i+x+x\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{i+x}{i-x}\right)\right)}{(1+ix)^{2/3}x}$$

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])/x^2,x]
[Out] ((-I)*(1 - I*x)^(2/3)*(-I + x + x*Hypergeometric2F1[2/3, 1, 5/3, (I + x)/(I
 - x)]))/((1 + I*x)^(2/3)*x)
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}dx$$

[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x)

[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \frac{\left(\sqrt{3}x - i\,x\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} + \frac{1}{2}i\sqrt{3} + \frac{1}{2}\right) - \left(\sqrt{3}x + i\,x\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} - \frac{1}{2}i\sqrt{3} + \frac{1}{2}\right) + 2i\,x\log\left(\left(\frac{i\cdots}{}\right.\right.}{3\,x}$$

[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="fricas")

[Out] 1/3*((sqrt(3)*x - I*x)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) + 1/2) - (sqrt(3)*x + I*x)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt(3) + 1/2) + 2*I*x*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1) - 3*(-I*x + 1)*(I*sqrt(x^2 + 1)/(x + I))^(2/3))/x

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \text{Timed out}$$

[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^2, x)
```

Giac [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}}{x^2}\,dx$$

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^2,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^2, x)
```

3.127 $\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 142

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2} - \frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x} - \frac{2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right) + \frac{\log(x)}{9}$$

```
[Out] -1/2*(1-I*x)^(2/3)*(1+I*x)^(4/3)/x^2-1/3*I*(1-I*x)^(2/3)*(1+I*x)^(1/3)/x-1/
3*ln((1-I*x)^(1/3)-(1+I*x)^(1/3))+1/9*ln(x)-2/9*arctan(1/3*3^(1/2)+2/3*(1-I
*x)^(1/3)/(1+I*x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5170, 98, 96, 93}

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = -\frac{2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}} - \frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2} - \frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x} - \frac{1}{3}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right) + \frac{\log(x)}{9}$$

```
[In] Int[E^(((2*I)/3)*ArcTan[x])/x^3,x]
```

```
[Out] -1/2*((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/x^2 - ((I/3)*(1 - I*x)^(2/3)*(1 + I*
x)^(1/3))/x - (2*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^
(1/3))])/(3*Sqrt[3]) - Log[(1 - I*x)^(1/3) - (1 + I*x)^(1/3)]/3 + Log[x]/9
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x^3}\,dx \\
&= -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2} + \frac{1}{3}i\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x^2}\,dx \\
&= -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2} - \frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x} - \frac{2}{9}\int \frac{1}{\sqrt[3]{1-ix}(1+ix)^{2/3}x}\,dx
\end{aligned}
$$

$$= -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2} - \frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x} - \frac{2\arctan\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(\sqrt[3]{1-ix} - \sqrt[3]{1+ix}\right) + \frac{\log(x)}{9}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \frac{(1-ix)^{2/3}\left(-3-8ix+5x^2+2x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{i+x}{i-x}\right)\right)}{6(1+ix)^{2/3}x^2}$$

```
[In] Integrate[E^(((2*I)/3)*ArcTan[x])/x^3,x]
[Out] ((1 - I*x)^(2/3)*(-3 - (8*I)*x + 5*x^2 + 2*x^2*Hypergeometric2F1[2/3, 1, 5/
3, (I + x)/(I - x)]))/(6*(1 + I*x)^(2/3)*x^2)
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

```
[In] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x)
[Out] int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = -\frac{4\,x^2\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-1\right)+2\left(-i\sqrt{3}x^2-x^2\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}+\frac{1}{2}\right)+2\left(i\sqrt{3}x^2-x^2\right)\log\left(\right.}{18\,x^2}$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="fricas")
[Out] -1/18*(4*x^2*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1) + 2*(-I*sqrt(3)*x^2 -
 x^2)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) + 1/2) + 2*(I*sqr
t(3)*x^2 - x^2)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt(3) + 1/2)
+ 3*(5*x^2 + 2*I*x + 3)*(I*sqrt(x^2 + 1)/(x + I))^(2/3))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="maxima")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^3, x)
```

Giac [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

```
[In] integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="giac")
[Out] integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}}{x^3}\,dx$$

```
[In] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^3,x)
[Out] int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^3, x)
```

3.128 $\int e^{\frac{1}{4} i \arctan(ax)} x^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 741

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{11i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{128a^3} + \frac{11i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{128a^3} - \frac{11i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{128a^3} - \frac{11i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{128a^3} - \frac{11i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3} + \frac{11i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3} - \frac{11i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3} + \frac{11i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}$$

```
[Out] -11/32*I*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/a^3-1/24*I*(1-I*a*x)^(7/8)*(1+I*a*
x)^(9/8)/a^3+1/3*x*(1-I*a*x)^(7/8)*(1+I*a*x)^(9/8)/a^2+11/128*I*arctan((-2*
(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^
(1/2))^(1/2)/a^3-11/128*I*arctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1
/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a^3-11/256*I*ln(1+(1-I*a*x
```

```
)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*
(2-2^(1/2))^(1/2)/a^3+11/256*I*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+(1-I*a*
x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2^(1/2))^(1/2)/a^3+11/128*I*
arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(
1/2))*(2+2^(1/2))^(1/2)/a^3-11/128*I*arctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1
/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)/a^3-11/256*I*ln
(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a
*x)^(1/8))*(2+2^(1/2))^(1/2)/a^3+11/256*I*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1
/4)+(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2+2^(1/2))^(1/2)/a^
3
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules

used = {5170, 92, 81, 52, 65, 338, 305, 1136, 1183, 648, 632, 210, 642}

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \frac{11i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{128a^3} + \frac{11i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{128a^3} - \frac{11i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{128a^3} - \frac{11i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{128a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} - \frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{11i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}} + 1\right)}{256a^3} + \frac{11i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}} + 1\right)}{256a^3} - \frac{11i\sqrt{2+\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}} + 1\right)}{256a^3} + \frac{11i\sqrt{2+\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}} + 1\right)}{256a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}$$

```
[In] Int[E^((I/4)*ArcTan[a*x])*x^2,x]
```

```
[Out] (((-11*I)/32)*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a^3 - ((I/24)*(1 - I*a*x
)^(7/8)*(1 + I*a*x)^(9/8))/a^3 + (x*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(9/8))/(3
*a^2) + (((11*I)/128)*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 -
 I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/a^3 + (((11*I)/128)*S
qrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a
```

```
*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a^3 - (((11*I)/128)*Sqrt[2 + Sqrt[2]]*ArcTan
[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqr
t[2]]])/a^3 - (((11*I)/128)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (
2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a^3 - (((11*I)/
256)*Sqrt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[
2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 + (((11*I)/256)*Sqr
t[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt
[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 - (((11*I)/256)*Sqrt[2 + Sq
rt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1
- I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a^3 + (((11*I)/256)*Sqrt[2 + Sqrt[2]]*L
og[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)
^(1/8))/(1 + I*a*x)^(1/8)])/a^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
```

```
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx \\
&= \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{\int \frac{\left(-1-\frac{iax}{4}\right)\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx}{3a^2} \\
&= -\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{11\int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx}{32a^2} \\
&= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} \\
&\quad + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{11\int \frac{1}{\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx}{128a^2} \\
&= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} \\
&\quad + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-iax}\right)}{16a^3} \\
&= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} \\
&\quad + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16a^3}
\end{aligned}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{32\sqrt{2}a^3} + \frac{(11i)\text{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{32\sqrt{2}a^3}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{(11i)\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{32\sqrt{2}a^3} - \frac{(11i)\text{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{32\sqrt{2}a^3}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} - \frac{(11i)\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64\sqrt{2\left(2-\sqrt{2}\right)}a^3} - \frac{(11i)\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64\sqrt{2\left(2-\sqrt{2}\right)}a^3} + \frac{(11i)\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64\sqrt{2\left(2+\sqrt{2}\right)}a^3} + \frac{(11i)\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64\sqrt{2\left(2+\sqrt{2}\right)}a^3}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}$$

$$- \frac{\left(11i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{128a^3}$$

$$- \frac{\left(11i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{128a^3}$$

$$- \frac{\left(11i\sqrt{2-\sqrt{2}}\right)\text{Subst}\left(\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}$$

$$+ \frac{\left(11i\sqrt{2-\sqrt{2}}\right)\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}$$

$$- \frac{\left(11i\sqrt{2+\sqrt{2}}\right)\text{Subst}\left(\int \frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}$$

$$+ \frac{\left(11i\sqrt{2+\sqrt{2}}\right)\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}$$

$$- \frac{\left(11i\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{128a^3}$$

$$- \frac{\left(11i\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{128a^3}$$

$$= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} + \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}$$
$$- \frac{11i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}$$
$$+ \frac{11i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}$$
$$- \frac{11i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}$$
$$+ \frac{11i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}$$
$$+ \frac{\left(11i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx, x, -\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^3}$$
$$+ \frac{\left(11i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx, x, \sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^3}$$
$$+ \frac{\left(11i\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\right)\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx, x, -\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^3}$$
$$+ \frac{\left(11i\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\right)\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx, x, \sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^3}$$

$$
\begin{aligned}
&= -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3} - \frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} \\
&+ \frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} + \frac{11i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{128a^3} \\
&+ \frac{11i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{128a^3} \\
&- \frac{11i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{128a^3} \\
&- \frac{11i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{128a^3} \\
&- \frac{11i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3} \\
&+ \frac{11i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3} \\
&- \frac{11i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3} \\
&+ \frac{11i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{256a^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.11

$$
\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx
$$

$$
= \frac{(1-iax)^{7/8}\left(7\sqrt[8]{1+iax}(-i+9ax+8ia^2x^2)-66i\sqrt[8]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-iax)\right)\right)}{168a^3}
$$

[In] Integrate[E^((I/4)*ArcTan[a*x])*x^2,x]

[Out] ((1 - I*a*x)^(7/8)*(7*(1 + I*a*x)^(1/8)*(-I + 9*a*x + (8*I)*a^2*x^2) - (66*I)*2^(1/8)*Hypergeometric2F1[-1/8, 7/8, 15/8, (1 - I*a*x)/2]))/(168*a^3)

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}} x^2 dx$$

[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x)

[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.59

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \frac{96i\,a^3\left(\frac{14641i}{268435456\,a^{12}}\right)^{\frac{1}{4}}\log\left(\frac{128}{11}\,a^3\left(\frac{14641i}{268435456\,a^{12}}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)-96\,a^3\left(\frac{14641i}{268435456\,a^{12}}\right)^{\frac{1}{4}}\log\left(\frac{128}{11}i\,a^3\left(\frac{14641i}{268435456}\right.\right.}{}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="fricas")

[Out] 1/96*(96*I*a^3*(14641/268435456*I/a^12)^(1/4)*log(128/11*a^3*(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*a^3*(14641/268435456*I/a^12)^(1/4)*log(128/11*I*a^3*(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 96*a^3*(14641/268435456*I/a^12)^(1/4)*log(-128/11*I*a^3*(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*I*a^3*(14641/268435456*I/a^12)^(1/4)*log(-128/11*a^3*(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 96*I*a^3*(-14641/268435456*I/a^12)^(1/4)*log(128/11*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*a^3*(-14641/268435456*I/a^12)^(1/4)*log(128/11*I*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 96*a^3*(-14641/268435456*I/a^12)^(1/4)*log(-128/11*I*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*I*a^3*(-14641/268435456*I/a^12)^(1/4)*log(-128/11*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + (32*a^3*x^3 - 4*I*a^2*x^2 - a*x - 37*I)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/a^3

Sympy [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \int x^2\sqrt[4]{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x**2,x)
[Out] Integral(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \int x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \int x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}\,dx$$

```
[In] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
[Out] int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

3.129 $\int e^{\frac{1}{4}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 689

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2}+\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} - \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} - \frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} + \frac{\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} - \frac{\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2}$$

```
[Out] 1/8*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/a^2+1/2*(1-I*a*x)^(7/8)*(1+I*a*x)^(9/8)
/a^2-1/32*arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-
2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a^2+1/32*arctan((2*(1-I*a*x)^(1/8)/(1+I*a
*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a^2+1/64*
ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I
*a*x)^(1/8))*(2-2^(1/2))^(1/2)/a^2-1/64*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4
)+(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2^(1/2))^(1/2)/a^2-
```

```
1/32*arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2))*(2+2^(1/2))^(1/2)/a^2+1/32*arctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(
1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)/a^2+1/64*ln(1+
(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)
^(1/8))*(2+2^(1/2))^(1/2)/a^2-1/64*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+(1-
I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2+2^(1/2))^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules

used = {5170, 81, 52, 65, 338, 305, 1136, 1183, 648, 632, 210, 642}

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = -\frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} - \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{32a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}} + 1\right)}{64a^2} - \frac{\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}} + 1\right)}{64a^2} + \frac{\sqrt{2+\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}} + 1\right)}{64a^2} - \frac{\sqrt{2+\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}} + 1\right)}{64a^2}$$

```
[In] Int[E^((I/4)*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/(8*a^2) + ((1 - I*a*x)^(7/8)*(1 + I*a
*x)^(9/8))/(2*a^2) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 -
 I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) - (Sqrt[2 -
Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8
))/Sqrt[2 - Sqrt[2]]])/(32*a^2) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[
2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2)
 + (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1
```

```
+ I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(32*a^2) + (Sqrt[2 - Sqrt[2]]*Log[1 + (
1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8)])/(64*a^2) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)
/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8
)])/(64*a^2) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/
4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/(64*a^2) - (
Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + S
qrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/(64*a^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
```

```
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx$$

$$= \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{i\int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx}{8a}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{i\int \frac{1}{\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx}{32a}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-iax}\right)}{4a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{4a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8\sqrt{2}a^2} - \frac{\text{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8\sqrt{2}a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8\sqrt{2}a^2} + \frac{\text{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8\sqrt{2}a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16\sqrt{2\left(2-\sqrt{2}\right)}a^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16\sqrt{2\left(2-\sqrt{2}\right)}a^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16\sqrt{2\left(2+\sqrt{2}\right)}a^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16\sqrt{2\left(2+\sqrt{2}\right)}a^2}$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} + \frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{32a^2} + \frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{32a^2} + \frac{\sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} - \frac{\sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} + \frac{\sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} - \frac{\sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} + \frac{\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{32a^2} + \frac{\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{32a^2}$$

$$
\begin{aligned}
&= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2}+\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}\\
&+\frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2}\\
&-\frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2}\\
&+\frac{\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2}\\
&-\frac{\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2}\\
&-\frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx,x,-\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16a^2}\\
&-\frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx,x,\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16a^2}\\
&-\frac{\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\,\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx,x,-\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16a^2}\\
&-\frac{\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\,\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx,x,\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{16a^2}
\end{aligned}
$$

$$= \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2} + \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} - \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} - \frac{\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} + \frac{\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2} - \frac{\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{64a^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.09

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \frac{(1-iax)^{7/8}\left(7(1+iax)^{9/8}+2\sqrt[8]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-iax)\right)\right)}{14a^2}$$

[In] Integrate[E^((I/4)*ArcTan[a*x])*x,x]

[Out] ((1 - I*a*x)^(7/8)*(7*(1 + I*a*x)^(9/8) + 2*2^(1/8)*Hypergeometric2F1[-1/8, 7/8, 15/8, (1 - I*a*x)/2]))/(14*a^2)

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}} xdx$$

[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x)

[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.62

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx =$$

$$-\frac{8\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}\log\left(32\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+8i\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}\log\left(32i\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}+\left(\frac{i}{}\right.\right.}{}$$

[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="fricas")

[Out] -1/8*(8*a^2*(1/1048576*I/a^8)^(1/4)*log(32*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*I*a^2*(1/1048576*I/a^8)^(1/4)*log(32*I*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*I*a^2*(1/1048576*I/a^8)^(1/4)*log(-32*I*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*a^2*(1/1048576*I/a^8)^(1/4)*log(-32*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*a^2*(-1/1048576*I/a^8)^(1/4)*log(32*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*I*a^2*(-1/1048576*I/a^8)^(1/4)*log(32*I*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*I*a^2*(-1/1048576*I/a^8)^(1/4)*log(-32*I*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*a^2*(-1/1048576*I/a^8)^(1/4)*log(-32*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - (4*a^2*x^2 - I*a*x + 5)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/a^2

Sympy [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \int x\sqrt[4]{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x,x)
[Out] Integral(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="maxima")
[Out] integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \int x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}\,dx$$

```
[In] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
[Out] int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

3.130 $\int e^{\frac{1}{4} i \arctan(ax)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 674

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx=\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}-\frac{i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a}-\frac{i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a}+\frac{i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a}+\frac{i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a}+\frac{i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}-\frac{i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}+\frac{i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}-\frac{i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}$$

```
[Out] I*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/a-1/4*I*arctan((-2*(1-I*a*x)^(1/8)/(1+I*a
*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a+1/4*I*a
rctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/
2))*(2-2^(1/2))^(1/2)/a+1/8*I*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x
)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2^(1/2))^(1/2)/a-1/8*I*ln(1+(
1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^
(1/8))*(2-2^(1/2))^(1/2)/a-1/4*I*arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)
+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)/a+1/4*I*arctan((2*
(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^
```

```
(1/2))^(1/2)/a+1/8*I*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(
2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2+2^(1/2))^(1/2)/a-1/8*I*ln(1+(1-I*a*x)^
(1/4)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2
+2^(1/2))^(1/2)/a
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5169, 52, 65, 338, 305, 1136, 1183, 648, 632, 210, 642}

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx = -\frac{i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a} - \frac{i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a} + \frac{i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a} + \frac{i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a} + \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a} - \frac{i\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a} + \frac{i\sqrt{2+\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a} - \frac{i\sqrt{2+\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{8a}$$

```
[In] Int[E^((I/4)*ArcTan[a*x]),x]
```

```
[Out] (I*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a - ((I/4)*Sqrt[2 + Sqrt[2]]*ArcTan
[(Sqrt[2 - Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqr
t[2]]])/a - ((I/4)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*
a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a + ((I/4)*Sqrt[2 + Sqrt
[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/S
qrt[2 + Sqrt[2]]])/a + ((I/4)*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] +
 (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/a + ((I/8)*Sq
rt[2 - Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqr
t[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a - ((I/8)*Sqrt[2 - Sqrt[2]]*L
og[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)
^(1/8))/(1 + I*a*x)^(1/8)])/a + ((I/8)*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x
)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*
x)^(1/8)])/a - ((I/8)*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*
x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/a
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx$$

$$= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{1}{4}\int \frac{1}{\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx$$

$$= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{(2i)\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-iax}\right)}{a}$$

$$= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{(2i)\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{a}$$

$$= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a} - \frac{i\text{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a}$$

$$= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} - \frac{i\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a} + \frac{i\text{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{\sqrt{2}a}$$

$$= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}a} + \frac{i\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}a} - \frac{i\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}a} - \frac{i\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}a}$$

$$
\begin{aligned}
&= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} \\
&+ \frac{\left(i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{4a} \\
&+ \frac{\left(i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{4a} \\
&+ \frac{\left(i\sqrt{2-\sqrt{2}}\right)\text{Subst}\left(\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a} \\
&- \frac{\left(i\sqrt{2-\sqrt{2}}\right)\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a} \\
&+ \frac{\left(i\sqrt{2+\sqrt{2}}\right)\text{Subst}\left(\int \frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a} \\
&- \frac{\left(i\sqrt{2+\sqrt{2}}\right)\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a} \\
&+ \frac{\left(i\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{4a} \\
&+ \frac{\left(i\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{4a}
\end{aligned}
$$

$$= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} + \frac{i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}$$

$$- \frac{i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}$$

$$+ \frac{i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}$$

$$- \frac{i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}$$

$$- \frac{\left(i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx, x, -\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2a}$$

$$- \frac{\left(i\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right)\text{Subst}\left(\int\frac{1}{-2+\sqrt{2}-x^2}\,dx, x, \sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2a}$$

$$- \frac{\left(i\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\right)\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx, x, -\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2a}$$

$$- \frac{\left(i\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\right)\text{Subst}\left(\int\frac{1}{-2-\sqrt{2}-x^2}\,dx, x, \sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{2a}$$

$$= \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} - \frac{i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a}$$
$$- \frac{i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a}$$
$$+ \frac{i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a}$$
$$+ \frac{i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a}$$
$$+ \frac{i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}$$
$$- \frac{i\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}$$
$$+ \frac{i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}$$
$$- \frac{i\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)}{8a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.06

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx = -\frac{16ie^{\frac{9}{4}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(\frac{9}{8},2,\frac{17}{8},-e^{2i\arctan(ax)}\right)}{9a}$$

```
[In] Integrate[E^((I/4)*ArcTan[a*x]),x]
[Out] (((-16*I)/9)*E^(((9*I)/4)*ArcTan[a*x])*Hypergeometric2F1[9/8, 2, 17/8, -E^(
(2*I)*ArcTan[a*x])])/a
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}} dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.57

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx = \frac{-i\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log\left(4\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log\left(4i\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)-a\left(\frac{i}{256\,a^4}\right.}{}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="fricas")
[Out] (-I*a*(1/256*I/a^4)^(1/4)*log(4*a*(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1
)/(a*x + I))^(1/4)) + a*(1/256*I/a^4)^(1/4)*log(4*I*a*(1/256*I/a^4)^(1/4) +
 (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - a*(1/256*I/a^4)^(1/4)*log(-4*I*a*
(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*a*(1/256*I
/a^4)^(1/4)*log(-4*a*(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^
(1/4)) - I*a*(-1/256*I/a^4)^(1/4)*log(4*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^
2*x^2 + 1)/(a*x + I))^(1/4)) + a*(-1/256*I/a^4)^(1/4)*log(4*I*a*(-1/256*I/a
^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - a*(-1/256*I/a^4)^(1/4)
*log(-4*I*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) +
 I*a*(-1/256*I/a^4)^(1/4)*log(-4*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 +
 1)/(a*x + I))^(1/4)) + (a*x + I)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/a
```

Sympy [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx = \int \sqrt[4]{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4),x)
[Out] Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{4} i \arctan(ax)} \, dx = \int \left(\frac{i \, ax + 1}{\sqrt{a^2 x^2 + 1}} \right)^{\frac{1}{4}} \, dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{4} i \arctan(ax)} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4} i \arctan(ax)} \, dx = \int \left(\frac{1 + a \, x \, 1\mathrm{i}}{\sqrt{a^2 \, x^2 + 1}} \right)^{1/4} \, dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

3.131 $\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 859

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = -2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)+\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)$$
$$+\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)$$
$$-\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)$$
$$-\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)$$
$$+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-2\operatorname{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$-\frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+\frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+\frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}}$$
$$-\frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}}$$

```
[Out] -2*arctan((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))-2*arctanh((1+I*a*x)^(1/8)/(1-I*a
*x)^(1/8))+1/2*ln(1+(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-(1+I*a*x)^(1/8)*2^(1/2)
```

```
/(1-I*a*x)^(1/8))*2^(1/2)-1/2*ln(1+(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)+(1+I*a*x
)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)+arctan(1-(1+I*a*x)^(1/8)*2^(1/2)/(
1-I*a*x)^(1/8))*2^(1/2)-arctan(1+(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2
^(1/2)+arctan((-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(
1/2))^(1/2))*(2-2^(1/2))^(1/2)-arctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2
+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/2*ln(1+(1-I*a*x)^(1
/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/8)*(2-2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2
^(1/2))^(1/2)+1/2*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/8)*(2-2
^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2-2^(1/2))^(1/2)+arctan((-2*(1-I*a*x)^(1/8)
/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-ar
ctan((2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2
))*(2+2^(1/2))^(1/2)-1/2*ln(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(1/
8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2+2^(1/2))^(1/2)+1/2*ln(1+(1-I*a*x)^
(1/4)/(1+I*a*x)^(1/4)+(1-I*a*x)^(1/8)*(2+2^(1/2))^(1/2)/(1+I*a*x)^(1/8))*(2
+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5170, 132, 65, 338, 305, 1136, 1183, 648, 632, 210, 642, 95, 220, 218, 212, 209, 217, 1179, 1176, 631}

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = -2\arctan\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right) + \sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2+\sqrt{2}}}\right)$$
$$+\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt{2-\sqrt{2}}}\right)$$
$$-\sqrt{2+\sqrt{2}}\arctan\left(\frac{\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)$$
$$-\sqrt{2-\sqrt{2}}\arctan\left(\frac{\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$
$$+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)$$
$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right) - 2\operatorname{arctanh}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)$$
$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1\right)$$
$$+\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1\right)$$
$$-\frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1\right)$$
$$+\frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1\right)$$
$$+\frac{\log\left(\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\sqrt{2}}$$
$$-\frac{\log\left(\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\sqrt{2}}$$

```
[In] Int[E^((I/4)*ArcTan[a*x])/x,x]
```

```
[Out] -2*ArcTan[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)] + Sqrt[2 + Sqrt[2]]*ArcTan[(
Sqrt[2 - Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[
2]]] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 -
Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - Sq
rt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*
x)^(1/8))/Sqrt[2 - Sqrt[2]]] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8
))/(1 - I*a*x)^(1/8)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 -
 I*a*x)^(1/8)] - 2*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)] - (Sqrt[2 -
 Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*
(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 -
I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 +
 I*a*x)^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x
)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + (Sq
rt[2 + Sqrt[2]]*Log[1 + (1 - I*a*x)^(1/4)/(1 + I*a*x)^(1/4) + (Sqrt[2 + Sqr
t[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2 + Log[1 - (Sqrt[2]*(1 + I*a*
x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/Sqrt[2]
- Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)
/(1 - I*a*x)^(1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[8]{1+iax}}{x\sqrt[8]{1-iax}}\,dx \\
&= (ia)\int \frac{1}{\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx + \int \frac{1}{x\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx \\
&= -\left(8\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-iax}\right)\right) + 8\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) \\
&= -\left(4\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right) \\
&\quad - 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 8\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)
\end{aligned}
$$

$$= -\left(2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$- 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$- \left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+ \left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$+ \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}$$
$$+ \left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$- \left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$- \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$- \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

$$
\begin{aligned}
&= -2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) \\
&\quad + \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}} \\
&\quad - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) \\
&\quad + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) \\
&\quad + \sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad + \sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad - \sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad - \sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) \\
&\quad - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) \\
&\quad + \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}} \\
&\quad - \frac{1}{2}\sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad + \frac{1}{2}\sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad + \frac{1}{2}\left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad + \frac{1}{2}\left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad - \frac{1}{2}\sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad + \frac{1}{2}\sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad - \frac{1}{2}\left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right) \\
&\quad - \frac{1}{2}\left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, \frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)
\end{aligned}
$$

$$= -2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2}\arctan\left(1 - \frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$- \sqrt{2}\arctan\left(1 + \frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$
$$- \frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1 + \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+ \frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1 + \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$- \frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(1 + \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+ \frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(1 + \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+ \frac{\log\left(1 - \frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}} + \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}} - \frac{\log\left(1 + \frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}} + \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}}$$
$$+ \left(2-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx, x, -\sqrt{2+\sqrt{2}} + \frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+ \left(2-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx, x, \sqrt{2+\sqrt{2}} + \frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+ \left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx, x, -\sqrt{2-\sqrt{2}} + \frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$
$$+ \left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx, x, \sqrt{2-\sqrt{2}} + \frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$

$$= -2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}} - \frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$+ \sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}} - \frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$- \sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}} + \frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$- \sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}} + \frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2}\arctan\left(1 - \frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

$$- \sqrt{2}\arctan\left(1 + \frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

$$- \frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1 + \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$

$$+ \frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1 + \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$

$$- \frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(1 + \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$

$$+ \frac{1}{2}\sqrt{2+\sqrt{2}}\log\left(1 + \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}\right)$$

$$+ \frac{\log\left(1 - \frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}} + \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}} - \frac{\log\left(1 + \frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}} + \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.11

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx =$$

$$-\frac{4(1-iax)^{7/8}\left(\sqrt[8]{2}(1+iax)^{7/8}\,\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-iax)\right) + 2\,\text{Hypergeometric2F1}\left(\frac{7}{8},1,\right.\right.}{7(1+iax)^{7/8}}$$

```
[In] Integrate[E^((I/4)*ArcTan[a*x])/x,x]
```

```
[Out] (-4*(1 - I*a*x)^(7/8)*(2^(1/8)*(1 + I*a*x)^(7/8)*Hypergeometric2F1[7/8, 7/8
, 15/8, (1 - I*a*x)/2] + 2*Hypergeometric2F1[7/8, 1, 15/8, (I + a*x)/(I - a
*x)]))/(7*(1 + I*a*x)^(7/8))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

```
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.59

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = -\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$+\frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$+i^{\frac{1}{4}}\log\left(i^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+i\,i^{\frac{1}{4}}\log\left(i\,i^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$-i\,i^{\frac{1}{4}}\log\left(-i\,i^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$-i^{\frac{1}{4}}\log\left(-i^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$+(-i)^{\frac{1}{4}}\log\left((-i)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$+i\,(-i)^{\frac{1}{4}}\log\left(i\,(-i)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$-i\,(-i)^{\frac{1}{4}}\log\left(-i\,(-i)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$-(-i)^{\frac{1}{4}}\log\left(-(-i)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)$$
$$-\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+1\right)-i\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+i\right)$$
$$+i\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-i\right)+\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-1\right)$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) +
 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))
- 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
) + 1/2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1
/4)) + I^(1/4)*log(I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*I^(
1/4)*log(I*I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I*I^(1/4)*log
(-I*I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I^(1/4)*log(-I^(1/4)
 + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + (-I)^(1/4)*log((-I)^(1/4) + (I*
sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*(-I)^(1/4)*log(I*(-I)^(1/4) + (I*sq
rt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I*(-I)^(1/4)*log(-I*(-I)^(1/4) + (I*sqr
t(a^2*x^2 + 1)/(a*x + I))^(1/4)) - (-I)^(1/4)*log(-(-I)^(1/4) + (I*sqrt(a^2
*x^2 + 1)/(a*x + I))^(1/4)) - log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1
) - I*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) + I*log((I*sqrt(a^2*x^
2 + 1)/(a*x + I))^(1/4) - I) + log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) -
1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x,x)
```

```
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}}{x}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x, x)
```

3.132 $\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 328

$$\begin{aligned}\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx =& -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) \\ &+ \frac{ia\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}} - \frac{ia\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}} \\ &- \frac{1}{2}i a\operatorname{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{ia\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{4\sqrt{2}} \\ &- \frac{ia\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{4\sqrt{2}}\end{aligned}$$

```
[Out] -(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/x-1/2*I*a*arctan((1+I*a*x)^(1/8)/(1-I*a*x)
^(1/8))-1/2*I*a*arctanh((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))+1/4*I*a*arctan(1-(
1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)-1/4*I*a*arctan(1+(1+I*a*x)^
(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)+1/8*I*a*ln(1+(1+I*a*x)^(1/4)/(1-I*a*
x)^(1/4)-(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)-1/8*I*a*ln(1+(1+I
*a*x)^(1/4)/(1-I*a*x)^(1/4)+(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5170, 96, 95, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = -\frac{1}{2}ia\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)+\frac{ia\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}} -\frac{ia\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}}-\frac{1}{2}ia\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}+\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}} -\frac{ia\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{4\sqrt{2}}$$

```
[In] Int[E^((I/4)*ArcTan[a*x])/x^2,x]
```

```
[Out] -(((1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/x) - (I/2)*a*ArcTan[(1 + I*a*x)^(1/
8)/(1 - I*a*x)^(1/8)] + ((I/2)*a*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1
- I*a*x)^(1/8)])/Sqrt[2] - ((I/2)*a*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/
(1 - I*a*x)^(1/8)])/Sqrt[2] - (I/2)*a*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)
^(1/8)] + ((I/4)*a*Log[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) +
(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/Sqrt[2] - ((I/4)*a*Log[1 + (Sqrt[2]*(
1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])
/Sqrt[2]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[8]{1+iax}}{x^2\sqrt[8]{1-iax}}\,dx\\
&= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} + \frac{1}{4}(ia)\int \frac{1}{x\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx\\
&= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} + (2ia)\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\\
&= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - (ia)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\\
&\quad - (ia)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)
\end{aligned}$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}(ia)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}(ia)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}(ia)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}(ia)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}ia\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{4}(ia)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{4}(ia)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{(ia)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{4\sqrt{2}} + \frac{(ia)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{4\sqrt{2}}$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{1}{2}ia\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{ia\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{4\sqrt{2}} - \frac{ia\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{4\sqrt{2}} - \frac{(ia)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}} + \frac{(ia)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}}$$

$$= -\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} - \frac{1}{2}ia\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{ia\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}} - \frac{ia\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}} - \frac{1}{2}ia\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{ia\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{4\sqrt{2}} - \frac{ia\log\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{4\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.22

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = -\frac{i(1-iax)^{7/8}\left(-7i+7ax+2ax\,\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{i+ax}{i-ax}\right)\right)}{7x(1+iax)^{7/8}}$$

```
[In] Integrate[E^((I/4)*ArcTan[a*x])/x^2,x]
[Out] ((-1/7*I)*(1 - I*a*x)^(7/8)*(-7*I + 7*a*x + 2*a*x*Hypergeometric2F1[7/8, 1,
 15/8, (I + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(7/8))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}\,dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \frac{-i\,ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+1\right)+ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+i\right)-ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-i\right)+i\,ax\log\left(\left(\frac{i\sqrt{a^2}}{ax}\right.\right.}{}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(-I*a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1) + a*x*log((I*sqr
t(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) - a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x +
I))^(1/4) - I) + I*a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - 1) + sqr
t(I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I*sqrt(I*a^2))/a)
 - sqrt(I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - I*sqrt(I*a^
2))/a) + sqrt(-I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I*sq
rt(-I*a^2))/a) - sqrt(-I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4
) - I*sqrt(-I*a^2))/a) - 4*(-I*a*x + 1)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/
4))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^2}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x**2,x)
```

```
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4)/x**2, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}}{x^2}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^2,x)
```

```
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^2, x)
```

3.133 $\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 364

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{16\sqrt{2}} + \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{16\sqrt{2}} + \frac{1}{16}a^2\text{arctan}$$

```
[Out] -1/8*I*a*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/x-1/2*(1-I*a*x)^(7/8)*(1+I*a*x)^(9
/8)/x^2+1/16*a^2*arctan((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))+1/16*a^2*arctanh((
1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))-1/32*a^2*arctan(1-(1+I*a*x)^(1/8)*2^(1/2)/(
1-I*a*x)^(1/8))*2^(1/2)+1/32*a^2*arctan(1+(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)
^(1/8))*2^(1/2)-1/64*a^2*ln(1+(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-(1+I*a*x)^(1/
8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)+1/64*a^2*ln(1+(1+I*a*x)^(1/4)/(1-I*a*x)
^(1/4)+(1+I*a*x)^(1/8)*2^(1/2)/(1-I*a*x)^(1/8))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5170, 98, 96, 95, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \frac{1}{16}a^2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{16\sqrt{2}} + \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{16\sqrt{2}} + \frac{1}{16}a^2\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}} + \frac{a^2\log\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+1\right)}{32\sqrt{2}} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} - \frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x}$$

```
[In] Int[E^((I/4)*ArcTan[a*x])/x^3,x]
```

```
[Out] ((-1/8*I)*a*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/x - ((1 - I*a*x)^(7/8)*(1
+ I*a*x)^(9/8))/(2*x^2) + (a^2*ArcTan[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)])
/16 - (a^2*ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)])/(16*S
qrt[2]) + (a^2*ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)])/(
16*Sqrt[2]) + (a^2*ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)])/16 - (a^2*
Log[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(
1 - I*a*x)^(1/4)])/(32*Sqrt[2]) + (a^2*Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/
(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)])/(32*Sqrt[2])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[8]{1+iax}}{x^3\sqrt[8]{1-iax}}\,dx$$

$$= -\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{8}(ia)\int \frac{\sqrt[8]{1+iax}}{x^2\sqrt[8]{1-iax}}\,dx$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} - \frac{1}{32}a^2\int \frac{1}{x\sqrt[8]{1-iax}(1+iax)^{7/8}}\,dx$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} - \frac{1}{4}a^2\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{8}a^2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{8}a^2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{16}a^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{16}a^2\text{Subst}$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{16}a^2\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{32}a^2\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1}}{\sqrt[8]{1}}\right.$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \frac{1}{16}a^2\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{a^2\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}{32\sqrt{2}}$$

$$= -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2} + \frac{1}{16}a^2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{16\sqrt{2}} + \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{16\sqrt{2}} + \frac{1}{16}a$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \frac{(1-iax)^{7/8}\left(7(-4-9iax+5a^2x^2)+2a^2x^2\,\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{i+ax}{i-ax}\right)\right)}{56x^2(1+iax)^{7/8}}$$

```
[In] Integrate[E^((I/4)*ArcTan[a*x])/x^3,x]
[Out] ((1 - I*a*x)^(7/8)*(7*(-4 - (9*I)*a*x + 5*a^2*x^2) + 2*a^2*x^2*Hypergeometr
ic2F1[7/8, 1, 15/8, (I + a*x)/(I - a*x)]))/(56*x^2*(1 + I*a*x)^(7/8))
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \frac{a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+1\right)+i\,a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+i\right)-i\,a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-i\right)-a^2x^2\log}{}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="fricas")
[Out] 1/32*(a^2*x^2*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1) + I*a^2*x^2*lo
g((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) - I*a^2*x^2*log((I*sqrt(a^2*x^
2 + 1)/(a*x + I))^(1/4) - I) - a^2*x^2*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^
(1/4) - 1) + sqrt(I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
 + sqrt(I*a^4))/a^2) - sqrt(I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2 + 1)/(a*x +
```

```
 I))^(1/4) - sqrt(I*a^4))/a^2) + sqrt(-I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2
+ 1)/(a*x + I))^(1/4) + sqrt(-I*a^4))/a^2) - sqrt(-I*a^4)*x^2*log((a^2*(I*s
qrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - sqrt(-I*a^4))/a^2) - 4*(5*a^2*x^2 + I*a
*x + 4)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x**3,x)
[Out] Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4)/x**3, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="maxima")
[Out] integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}}{x^3}\,dx$$

```
[In] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^3,x)
[Out] int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^3, x)
```

3.134 $\int e^{6i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 114

$$\int e^{6i\arctan(ax)}x^m\,dx=-\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2}+\frac{4ix^{1+m}\left(i(1+m)^2+a(3+3m+m^2)\,x\right)}{(1+m)(1-iax)^2}+\frac{2(3+4m+2m^2)\,x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,iax)}{1+m}$$

```
[Out] -x^(1+m)*(1+I*a*x)^2/(1+m)/(1-I*a*x)^2+4*I*x^(1+m)*(I*(1+m)^2+a*(m^2+3*m+3)
*x)/(1+m)/(1-I*a*x)^2+2*(2*m^2+4*m+3)*x^(1+m)*hypergeom([1, 1+m],[2+m],I*a*
x)/(1+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5170, 102, 150, 66}

$$\int e^{6i\arctan(ax)}x^m\,dx=\frac{2(2m^2+4m+3)\,x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,iax)}{m+1}+\frac{4ix^{m+1}\left(a(m^2+3m+3)\,x+i(m+1)^2\right)}{(m+1)(1-iax)^2}-\frac{(1+iax)^2x^{m+1}}{(m+1)(1-iax)^2}$$

```
[In] Int[E^((6*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] -((x^(1 + m)*(1 + I*a*x)^2)/((1 + m)*(1 - I*a*x)^2)) + ((4*I)*x^(1 + m)*(I*
(1 + m)^2 + a*(3 + 3*m + m^2)*x))/((1 + m)*(1 - I*a*x)^2) + (2*(3 + 4*m + 2
*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*x])/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] && !LtQ[n, -2]))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^m(1+iax)^3}{(1-iax)^3}\,dx \\
&= -\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2} + \frac{i\int \frac{x^m(1+iax)(-2ia(1+m)+2a^2(3+m)x)}{(1-iax)^3}\,dx}{a(1+m)} \\
&= -\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2} + \frac{4ix^{1+m}(i(1+m)^2+a(3+3m+m^2)\,x)}{(1+m)(1-iax)^2} \\
&\quad + \left(2\left(3+4m+2m^2\right)\right)\int \frac{x^m}{1-iax}\,dx
\end{aligned}$$

$$
= -\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2} + \frac{4ix^{1+m}(i(1+m)^2 + a(3+3m+m^2)\,x)}{(1+m)(1-iax)^2} + \frac{2(3+4m+2m^2)\,x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,iax)}{1+m}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$
\int e^{6i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(5-10iax-a^2x^2+4m(2-3iax)+m^2(4-4iax)+2(3+4m+2m^2)\,(i+ax)^2\,\text{Hypergeometric2F1}}{(1+m)(i+ax)^2}
$$

```
[In] Integrate[E^((6*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(5 - (10*I)*a*x - a^2*x^2 + 4*m*(2 - (3*I)*a*x) + m^2*(4 - (4*I)
*a*x) + 2*(3 + 4*m + 2*m^2)*(I + a*x)^2*Hypergeometric2F1[1, 1 + m, 2 + m,
I*a*x]))/((1 + m)*(I + a*x)^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.76 (sec) , antiderivative size = 748, normalized size of antiderivative = 6.56

method	result
meijerg	$\frac{(a^2)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{x^{1+m}(a^2)^{\frac{1}{2}+\frac{m}{2}}(-a^2m^2x^2+2a^2m\,x^2+3a^2x^2-m^2+4m+5)}{2(1+m)(a^2x^2+1)^2}+\frac{4x^{1+m}(a^2)^{\frac{1}{2}+\frac{m}{2}}(\frac{1}{16}m^3-\frac{3}{16}m^2-\frac{1}{16}m+\frac{3}{16})\,\text{LerchPhi}(-a^2x^2,1,}{1+m}\right.}{4}$

```
[In] int((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a^2)^(-1/2-1/2*m)*(1/2/(1+m)*x^(1+m)*(a^2)^(1/2+1/2*m)*(-a^2*m^2*x^2+2
*a^2*m*x^2+3*a^2*x^2-m^2+4*m+5)/(a^2*x^2+1)^2+4/(1+m)*x^(1+m)*(a^2)^(1/2+1/
2*m)*(1/16*m^3-3/16*m^2-1/16*m+3/16)*LerchPhi(-a^2*x^2,1,1/2+1/2*m))+3/2*I/
a*(a^2)^(-1/2*m)*(1/2*x^m*(a^2)^(1/2*m)*(a^2*m*x^2+m-2)/(a^2*x^2+1)^2-1/4*x
^m*(a^2)^(1/2*m)*(m-2)*m*LerchPhi(-a^2*x^2,1,1/2*m))-15/4*(a^2)^(-1/2-1/2*m
)*(1/2*x^(1+m)*(a^2)^(3/2+1/2*m)*(a^2*m*x^2+a^2*x^2+m-1)/(a^2*x^2+1)^2/a^2-
1/4*x^(1+m)*(a^2)^(3/2+1/2*m)*(1+m)*(m-1)/a^2*LerchPhi(-a^2*x^2,1,1/2+1/2*m
))-5*I/a*(a^2)^(-1/2*m)*(-1/2*x^m*(a^2)^(1/2*m)*(a^2*m*x^2+4*a^2*x^2+m+2)/(
a^2*x^2+1)^2+1/4*x^m*(a^2)^(1/2*m)*m*(2+m)*LerchPhi(-a^2*x^2,1,1/2*m))+15/4
*(a^2)^(-1/2-1/2*m)*(-1/2*x^(1+m)*(a^2)^(1/2*m+5/2)*(a^2*m*x^2+5*a^2*x^2+m+
3)/a^4/(a^2*x^2+1)^2+1/4*x^(1+m)*(a^2)^(1/2*m+5/2)*(m^2+4*m+3)/a^4*LerchPhi
(-a^2*x^2,1,1/2+1/2*m))+3/2*I*(a^2)^(-1/2*m)/a*(1/2*x^m*(a^2)^(1/2*m)*(8*a^
4*x^4+a^2*m^2*x^2+8*a^2*m*x^2+16*a^2*x^2+m^2+6*m+8)/(a^2*x^2+1)^2/m-1/4*x^m
```

```
*(a^2)^(1/2*m)*(m^2+6*m+8)*LerchPhi(-a^2*x^2,1,1/2*m))-1/4*(a^2)^(-1/2-1/2*
m)*(1/2*x^(1+m)*(a^2)^(7/2+1/2*m)*(8*a^4*x^4+a^2*m^2*x^2+10*a^2*m*x^2+25*a^
2*x^2+m^2+8*m+15)/(a^2*x^2+1)^2/(1+m)/a^6-1/4*x^(1+m)*(a^2)^(7/2+1/2*m)*(m^
2+8*m+15)/a^6*LerchPhi(-a^2*x^2,1,1/2+1/2*m))
```

Fricas [F]

$$\int e^{6i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^6x^m}{(a^2x^2+1)^3}\,dx$$

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="fricas")
[Out] integral(-(a^3*x^3 - 3*I*a^2*x^2 - 3*a*x + I)*x^m/(a^3*x^3 + 3*I*a^2*x^2 -
3*a*x - I), x)
```

Sympy [F]

$$\begin{aligned}\int e^{6i\arctan(ax)}x^m\,dx = &-\int\left(-\frac{x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx\\ &-\int\frac{15a^2x^2x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx\\ &-\int\left(-\frac{15a^4x^4x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx\\ &-\int\frac{a^6x^6x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx\\ &-\int\left(-\frac{6iaxx^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx\\ &-\int\frac{20ia^3x^3x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx\\ &-\int\left(-\frac{6ia^5x^5x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx\end{aligned}$$

```
[In] integrate((1+I*a*x)**6/(a**2*x**2+1)**3*x**m,x)
[Out] -Integral(-x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integral(
15*a**2*x**2*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integra
l(-15*a**4*x**4*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Inte
gral(a**6*x**6*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integ
ral(-6*I*a*x*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integra
l(20*I*a**3*x**3*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Int
egral(-6*I*a**5*x**5*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
```

Maxima [F]

$$\int e^{6i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^6x^m}{(a^2x^2+1)^3}\,dx$$

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="maxima")
[Out] integrate((I*a*x + 1)^6*x^m/(a^2*x^2 + 1)^3, x)
```

Giac [F]

$$\int e^{6i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^6x^m}{(a^2x^2+1)^3}\,dx$$

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="giac")
[Out] integrate((I*a*x + 1)^6*x^m/(a^2*x^2 + 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{6i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(1+a\,x\,1\mathrm{i})^6}{(a^2\,x^2+1)^3}\,dx$$

```
[In] int((x^m*(a*x*1i + 1)^6)/(a^2*x^2 + 1)^3,x)
[Out] int((x^m*(a*x*1i + 1)^6)/(a^2*x^2 + 1)^3, x)
```

3.135 $\int e^{4i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int e^{4i\arctan(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1-iax}-4x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,iax)$$

```
[Out] x^(1+m)/(1+m)+4*x^(1+m)/(1-I*a*x)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],I*a*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5170, 91, 81, 66}

$$\int e^{4i\arctan(ax)}x^m\,dx=-4x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,iax)+\frac{4x^{m+1}}{1-iax}+\frac{x^{m+1}}{m+1}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - I*a*x) - 4*x^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, I*a*x]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^m(1+iax)^2}{(1-iax)^2}\,dx \\
&= \frac{4x^{1+m}}{1-iax} + \frac{\int \frac{x^m(-a^2(3+4m)-ia^3x)}{1-iax}\,dx}{a^2} \\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-iax} - (4(1+m))\int \frac{x^m}{1-iax}\,dx \\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-iax} - 4x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,iax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int e^{4i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(5i+4im+ax-4(1+m)(i+ax)\,\text{Hypergeometric2F1}(1,1+m,2+m,iax))}{(1+m)(i+ax)}$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])*x^m,x]
[Out] (x^(1 + m)*(5*I + (4*I)*m + a*x - 4*(1 + m)*(I + a*x)*Hypergeometric2F1[1,
1 + m, 2 + m, I*a*x]))/((1 + m)*(I + a*x))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.45 (sec) , antiderivative size = 417, normalized size of antiderivative = 8.34

method	result
meijerg	$\frac{\left(a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}\left(a^2\right)^{\frac{1}{2}+\frac{m}{2}}}{2a^2x^2+2}+\frac{2x^{1+m}\left(a^2\right)^{\frac{1}{2}+\frac{m}{2}}\left(-\frac{m^2}{4}+\frac{1}{4}\right)\mathrm{LerchPhi}\left(-a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)}{1+m}\right)}{2}+\frac{2i\left(a^2\right)^{-\frac{m}{2}}\left(\frac{x^m\left(a^2\right)^{\frac{m}{2}}(-2-m)}{(2+m)\left(a^2x^2+1\right)}+\frac{x^m(\ldots}{\ldots}\right.}{a}$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2)^(-1/2-1/2*m)*(2*x^(1+m)*(a^2)^(1/2+1/2*m)/(2*a^2*x^2+2)+2/(1+m)*x
^(1+m)*(a^2)^(1/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(-a^2*x^2,1,1/2+1/2*m))+2*I
/a*(a^2)^(-1/2*m)*(1/(2+m)*x^m*(a^2)^(1/2*m)*(-2-m)/(a^2*x^2+1)+1/2*x^m*(a^
2)^(1/2*m)*m*LerchPhi(-a^2*x^2,1,1/2*m))-3*(a^2)^(-1/2-1/2*m)*(1/(3+m)*x^(1
+m)*(a^2)^(3/2+1/2*m)*(-3-m)/a^2/(a^2*x^2+1)+1/2*x^(1+m)*(a^2)^(3/2+1/2*m)*
(1+m)/a^2*LerchPhi(-a^2*x^2,1,1/2+1/2*m))-2*I/a*(a^2)^(-1/2*m)*(x^m*(a^2)^(
1/2*m)*(2*a^2*x^2+m+2)/(a^2*x^2+1)/m-1/2*x^m*(a^2)^(1/2*m)*(2+m)*LerchPhi(-
a^2*x^2,1,1/2*m))+1/2*(a^2)^(-1/2-1/2*m)*(x^(1+m)*(a^2)^(1/2*m+5/2)*(2*a^2*
x^2+m+3)/(a^2*x^2+1)/a^4/(1+m)-1/2*x^(1+m)*(a^2)^(1/2*m+5/2)*(3+m)/a^4*Lerc
hPhi(-a^2*x^2,1,1/2+1/2*m))
```

Fricas [F]

$$\int e^{4i\arctan(ax)}x^m\,dx=\int\frac{(i\,ax+1)^4x^m}{(a^2x^2+1)^2}\,dx$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - 2*I*a*x - 1)*x^m/(a^2*x^2 + 2*I*a*x - 1), x)
```

Sympy [F]

$$\int e^{4i\arctan(ax)}x^m\,dx=\int\frac{x^m(ax-i)^4}{(a^2x^2+1)^2}\,dx$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**m,x)
```

```
[Out] Integral(x**m*(a*x - I)**4/(a**2*x**2 + 1)**2, x)
```

Maxima [F]

$$\int e^{4i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^4x^m}{(a^2x^2+1)^2}\,dx$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="maxima")
[Out] integrate((I*a*x + 1)^4*x^m/(a^2*x^2 + 1)^2, x)
```

Giac [F]

$$\int e^{4i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^4x^m}{(a^2x^2+1)^2}\,dx$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="giac")
[Out] integrate((I*a*x + 1)^4*x^m/(a^2*x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{4i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(1+a\,x\,1\mathrm{i})^4}{(a^2\,x^2+1)^2}\,dx$$

```
[In] int((x^m*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
[Out] int((x^m*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2, x)
```

3.136 $\int e^{2i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int e^{2i\arctan(ax)}x^m\,dx=-\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,iax)}{1+m}$$

```
[Out] -x^(1+m)/(1+m)+2*x^(1+m)*hypergeom([1, 1+m],[2+m],I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5170, 81, 66}

$$\int e^{2i\arctan(ax)}x^m\,dx=-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,iax)}{m+1}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*
x])/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x^m(1+iax)}{1-iax}\,dx \\ &= -\frac{x^{1+m}}{1+m} + 2\int \frac{x^m}{1-iax}\,dx \\ &= -\frac{x^{1+m}}{1+m} + \frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,iax)}{1+m} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int e^{2i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(-1+2\,\text{Hypergeometric2F1}(1,1+m,2+m,iax))}{1+m}$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])*x^m,x]
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*x]))/(1 + m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.49

method	result
meijerg	$\frac{x^{1+m}\left(\frac{1}{2}+\frac{m}{2}\right)\text{LerchPhi}\left(-a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)}{1+m}+\frac{i(a^2)^{-\frac{m}{2}}\left(\frac{2x^m\left(a^2\right)^{\frac{m}{2}}}{m}+\frac{x^m\left(a^2\right)^{\frac{m}{2}}(-2-m)\,\text{LerchPhi}\left(-a^2x^2,1,\frac{m}{2}\right)}{2+m}\right)}{a}-\frac{(a^2)^{-\frac{1}{2}-\frac{m}{2}}\left(\right.}{}$

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^m,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x^(1+m)*(1/2+1/2*m)*LerchPhi(-a^2*x^2,1,1/2+1/2*m)+I/a*(a^2)^(-1/2*
m)*(2*x^m*(a^2)^(1/2*m)/m+1/(2+m)*x^m*(a^2)^(1/2*m)*(-2-m)*LerchPhi(-a^2*x^
2,1,1/2*m))-1/2*(a^2)^(-1/2-1/2*m)*(2*x^(1+m)*(a^2)^(3/2+1/2*m)/(1+m)/a^2+1
/(3+m)*x^(1+m)*(a^2)^(3/2+1/2*m)*(-3-m)/a^2*LerchPhi(-a^2*x^2,1,1/2+1/2*m))
```

Fricas [F]

$$\int e^{2i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^2x^m}{a^2x^2+1}\,dx$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a*x - I)*x^m/(a*x + I), x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(27) = 54.

Time = 1.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.23

$$\begin{aligned}\int e^{2i\arctan(ax)}x^m\,dx = &\frac{iamx^{m+2}\Phi\left(axe^{\frac{i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)} \\ &+ \frac{2iax^{m+2}\Phi\left(axe^{\frac{i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)} \\ &+ \frac{mx^{m+1}\Phi\left(axe^{\frac{i\pi}{2}},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)} \\ &+ \frac{x^{m+1}\Phi\left(axe^{\frac{i\pi}{2}},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)}\end{aligned}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**m,x)
```

```
[Out] I*a*m*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi/2), 1, m + 2)*gamma(m + 2)/gam
ma(m + 3) + 2*I*a*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi/2), 1, m + 2)*gamm
a(m + 2)/gamma(m + 3) + m*x**(m + 1)*lerchphi(a*x*exp_polar(I*pi/2), 1, m +
 1)*gamma(m + 1)/gamma(m + 2) + x**(m + 1)*lerchphi(a*x*exp_polar(I*pi/2),
1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

Maxima [F]

$$\int e^{2i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^2x^m}{a^2x^2+1}\,dx$$

[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="maxima")

[Out] integrate((I*a*x + 1)^2*x^m/(a^2*x^2 + 1), x)

Giac [F]

$$\int e^{2i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^2x^m}{a^2x^2+1}\,dx$$

[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="giac")

[Out] integrate((I*a*x + 1)^2*x^m/(a^2*x^2 + 1), x)

Mupad [F(-1)]

Timed out.

$$\int e^{2i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(1+a\,x\,1\mathrm{i})^2}{a^2\,x^2+1}\,dx$$

[In] int((x^m*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)

[Out] int((x^m*(a*x*1i + 1)^2)/(a^2*x^2 + 1), x)

3.137 $\int e^{-2i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int e^{-2i\arctan(ax)}x^m\,dx=-\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax)}{1+m}$$

```
[Out] -x^(1+m)/(1+m)+2*x^(1+m)*hypergeom([1, 1+m],[2+m],-I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5170, 81, 66}

$$\int e^{-2i\arctan(ax)}x^m\,dx=-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,-iax)}{m+1}$$

```
[In] Int[x^m/E^((2*I)*ArcTan[a*x]),x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)
*a*x])/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^m(1-iax)}{1+iax}\,dx \\
&= -\frac{x^{1+m}}{1+m} + 2\int \frac{x^m}{1+iax}\,dx \\
&= -\frac{x^{1+m}}{1+m} + \frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax)}{1+m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int e^{-2i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(-1+2\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax))}{1+m}$$

```
[In] Integrate[x^m/E^((2*I)*ArcTan[a*x]),x]
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*a*x]))/(1 + m)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.05

method	result
meijerg	$\frac{i(ia)^{-m}\left(\frac{x^m(ia)^m\left(-a^2m\,x^2-iamx-2iax-m^2-3m-2\right)}{(1+m)m(iax+1)}+x^m(ia)^m(2+m)\,\text{LerchPhi}(-iax,1,m)\right)}{a} - \frac{i(ia)^{-m}\left(\frac{x^m(ia)^m(-1-m)}{(1+m)(iax+1)}+x^m(ia\right.}{a}$

```
[In] int(x^m/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] I*(I*a)^(-m)/a*(x^m*(I*a)^m*(-a^2*m*x^2-I*a*m*x-m^2-2*I*a*x-3*m-2)/(1+m)/m/
(1+I*a*x)+x^m*(I*a)^m*(2+m)*LerchPhi(-I*a*x,1,m))-I*(I*a)^(-m)/a*(1/(1+m)*x
^m*(I*a)^m*(-1-m)/(1+I*a*x)+x^m*(I*a)^m*m*LerchPhi(-I*a*x,1,m))
```

Fricas [F]

$$\int e^{-2i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)x^m}{(i\,ax+1)^2}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a*x + I)*x^m/(a*x - I), x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(29) = 58$.

Time = 2.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int e^{-2i\arctan(ax)}x^m\,dx = -\frac{iamx^{m+2}\Phi\left(axe^{\frac{3i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)} - \frac{2iax^{m+2}\Phi\left(axe^{\frac{3i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)} + \frac{mx^{m+1}\Phi\left(axe^{\frac{3i\pi}{2}},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)} + \frac{x^{m+1}\Phi\left(axe^{\frac{3i\pi}{2}},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)}$$

```
[In] integrate(x**m/(1+I*a*x)**2*(a**2*x**2+1),x)
```

```
[Out] -I*a*m*x**(m + 2)*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 2)*gamma(m + 2)/
gamma(m + 3) - 2*I*a*x**(m + 2)*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 2)
*gamma(m + 2)/gamma(m + 3) + m*x**(m + 1)*lerchphi(a*x*exp_polar(3*I*pi/2),
 1, m + 1)*gamma(m + 1)/gamma(m + 2) + x**(m + 1)*lerchphi(a*x*exp_polar(3*
I*pi/2), 1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

Maxima [F]

$$\int e^{-2i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)x^m}{(i\,ax+1)^2}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
[Out] integrate((a^2*x^2 + 1)*x^m/(I*a*x + 1)^2, x)
```

Giac [F]

$$\int e^{-2i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)x^m}{(i\,ax+1)^2}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
[Out] integrate((a^2*x^2 + 1)*x^m/(I*a*x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(a^2\,x^2+1)}{(1+a\,x\,1\mathrm{i})^2}\,dx$$

```
[In] int((x^m*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
[Out] int((x^m*(a^2*x^2 + 1))/(a*x*1i + 1)^2, x)
```

3.138 $\int e^{-4i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int e^{-4i\arctan(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1+iax}-4x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax)$$

```
[Out] x^(1+m)/(1+m)+4*x^(1+m)/(1+I*a*x)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],-I*a*x
)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5170, 91, 81, 66}

$$\int e^{-4i\arctan(ax)}x^m\,dx=-4x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,-iax)+\frac{4x^{m+1}}{1+iax}+\frac{x^{m+1}}{m+1}$$

```
[In] Int[x^m/E^((4*I)*ArcTan[a*x]),x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 + I*a*x) - 4*x^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, (-I)*a*x]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^m(1-iax)^2}{(1+iax)^2}\,dx\\
&= \frac{4x^{1+m}}{1+iax} + \frac{\int \frac{x^m\left(-a^2(3+4m)+ia^3x\right)}{1+iax}\,dx}{a^2}\\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1+iax} - (4(1+m))\int \frac{x^m}{1+iax}\,dx\\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1+iax} - 4x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$
\int e^{-4i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(-5i-4im+ax-4(1+m)(-i+ax)\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax))}{(1+m)(-i+ax)}
$$

[In] Integrate[x^m/E^((4*I)*ArcTan[a*x]),x]

[Out] (x^(1 + m)*(-5*I - (4*I)*m + a*x - 4*(1 + m)*(-I + a*x)*Hypergeometric2F1[1
, 1 + m, 2 + m, (-I)*a*x]))/((1 + m)*(-I + a*x))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.57 (sec) , antiderivative size = 428, normalized size of antiderivative = 8.56

method	result
meijerg	$-\frac{i(ia)^{-m}\left(\frac{x^{m}(ia)^{m}\left(6a^{4}x^{4}m+6ia^{3}x^{3}m+a^{2}x^{2}m^{4}+24ia^{3}x^{3}+11a^{2}x^{2}m^{3}-2iax\,m^{4}+46a^{2}m^{2}x^{2}-21iax\,m^{3}+90a^{2}m\,x^{2}-79iax\,m^{2}+72a^{2}x^{2}-126}{(1+m)m(iax+1)^{3}}}{6a}$

[In] int(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x,method=_RETURNVERBOSE)

[Out] -1/6*I*(I*a)^(-m)/a*(x^m*(I*a)^m*(a^2*x^2*m^4+6*a^4*x^4*m+11*a^2*x^2*m^3-2*
I*a*x*m^4+6*I*a^3*x^3*m+46*a^2*m^2*x^2-72*I*a*x-m^4-79*I*a*x*m^2+90*a^2*m*x
^2+24*I*a^3*x^3-10*m^3+72*a^2*x^2-21*I*a*x*m^3-35*m^2-126*I*a*m*x-50*m-24)/
(1+m)/m/(1+I*a*x)^3+x^m*(I*a)^m*(m^3+9*m^2+26*m+24)*LerchPhi(-I*a*x,1,m))+1
/3*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(-a^2*m^2*x^2-4*a^2*m*x^2+2*I*a*x*m^2-6*a^2
*x^2+7*I*a*m*x+m^2+6*I*a*x+3*m+2)/(1+I*a*x)^3+x^m*(I*a)^m*m*(m^2+3*m+2)*Ler
chPhi(-I*a*x,1,m))-1/6*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(-a^2*m^2*x^2+2*a^2*m*x
^2+2*I*a*x*m^2-5*I*a*m*x+m^2-3*m+2)/(1+I*a*x)^3+x^m*(I*a)^m*(m^2-3*m+2)*m*L
erchPhi(-I*a*x,1,m))

Fricas [F]

$$\int e^{-4i\arctan(ax)}x^{m}\,dx=\int\frac{\left(a^{2}x^{2}+1\right)^{2}x^{m}}{\left(i\,ax+1\right)^{4}}\,dx$$

[In] integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="fricas")

[Out] integral((a^2*x^2 + 2*I*a*x - 1)*x^m/(a^2*x^2 - 2*I*a*x - 1), x)

Sympy [F]

$$\int e^{-4i\arctan(ax)}x^m\,dx = \int \frac{x^m(a^2x^2+1)^2}{(ax-i)^4}\,dx$$

```
[In] integrate(x**m/(1+I*a*x)**4*(a**2*x**2+1)**2,x)
[Out] Integral(x**m*(a**2*x**2 + 1)**2/(a*x - I)**4, x)
```

Maxima [F]

$$\int e^{-4i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^2x^m}{(i\,ax+1)^4}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="maxima")
[Out] integrate((a^2*x^2 + 1)^2*x^m/(I*a*x + 1)^4, x)
```

Giac [F]

$$\int e^{-4i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^2x^m}{(i\,ax+1)^4}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="giac")
[Out] integrate((a^2*x^2 + 1)^2*x^m/(I*a*x + 1)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-4i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(a^2\,x^2+1)^2}{(1+a\,x\,1\mathrm{i})^4}\,dx$$

```
[In] int((x^m*(a^2*x^2 + 1)^2)/(a*x*1i + 1)^4,x)
[Out] int((x^m*(a^2*x^2 + 1)^2)/(a*x*1i + 1)^4, x)
```

3.139 $\int e^{-6i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 115

$$\int e^{-6i\arctan(ax)}x^m\,dx = -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2}+\frac{4ix^{1+m}\left(i(1+m)^2-a(3+3m+m^2)\,x\right)}{(1+m)(1+iax)^2} + \frac{2(3+4m+2m^2)\,x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax)}{1+m}$$

```
[Out] -x^(1+m)*(1-I*a*x)^2/(1+m)/(1+I*a*x)^2+4*I*x^(1+m)*(I*(1+m)^2-a*(m^2+3*m+3)
*x)/(1+m)/(1+I*a*x)^2+2*(2*m^2+4*m+3)*x^(1+m)*hypergeom([1, 1+m],[2+m],-I*a
*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5170, 102, 150, 66}

$$\int e^{-6i\arctan(ax)}x^m\,dx = \frac{2(2m^2+4m+3)\,x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,-iax)}{m+1} + \frac{4ix^{m+1}\left(-a(m^2+3m+3)\,x+i(m+1)^2\right)}{(m+1)(1+iax)^2}-\frac{(1-iax)^2x^{m+1}}{(m+1)(1+iax)^2}$$

```
[In] Int[x^m/E^((6*I)*ArcTan[a*x]),x]
```

```
[Out] -((x^(1 + m)*(1 - I*a*x)^2)/((1 + m)*(1 + I*a*x)^2)) + ((4*I)*x^(1 + m)*(I*
(1 + m)^2 - a*(3 + 3*m + m^2)*x))/((1 + m)*(1 + I*a*x)^2) + (2*(3 + 4*m + 2
*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*a*x])/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] && !LtQ[n, -2]))
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x^m(1-iax)^3}{(1+iax)^3}\,dx$$

$$= -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2} - \frac{i\int \frac{x^m(1-iax)\left(2ia(1+m)+2a^2(3+m)x\right)}{(1+iax)^3}\,dx}{a(1+m)}$$

$$= -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2} + \frac{4ix^{1+m}(i(1+m)^2 - a(3+3m+m^2)\,x)}{(1+m)(1+iax)^2} + \left(2\left(3+4m+2m^2\right)\right)\int \frac{x^m}{1+iax}\,dx$$

$$= -\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2} + \frac{4ix^{1+m}(i(1+m)^2 - a(3+3m+m^2)\,x)}{(1+m)(1+iax)^2} + \frac{2(3+4m+2m^2)\,x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax)}{1+m}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int e^{-6i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(5+10iax-a^2x^2+4m(2+3iax)+m^2(4+4iax)+2(3+4m+2m^2)\,(-i+ax)^2\,\text{Hypergeometric}}{(1+m)(-i+ax)^2}$$

```
[In] Integrate[x^m/E^((6*I)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*(5 + (10*I)*a*x - a^2*x^2 + 4*m*(2 + (3*I)*a*x) + m^2*(4 + (4*I)
*a*x) + 2*(3 + 4*m + 2*m^2)*(-I + a*x)^2*Hypergeometric2F1[1, 1 + m, 2 + m,
 (-I)*a*x]))/((1 + m)*(-I + a*x)^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 1.28 (sec) , antiderivative size = 1196, normalized size of antiderivative = 10.40

method	result	size
meijerg	Expression too large to display	1196

```
[In] int(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*I*(I*a)^(-m)/a*(x^m*(I*a)^m*(-720-175*m^4-1764*m+14400*a^2*m*x^2-m^6-
21*m^5+7200*a^2*x^2+1112*a^2*x^2*m^4+4911*a^2*x^2*m^3-735*m^3-1624*m^2+1172
2*a^2*m^2*x^2-4200*a^4*x^4*m+6*a^2*x^2*m^6-120*a^6*x^6*m+129*a^2*x^2*m^5-72
0*I*a^5*x^5+7200*I*a^3*x^3-3600*I*a*x+764*I*a^3*x^3*m^4-4*I*a*x*m^6-120*I*a
^5*x^5*m+3483*I*a^3*x^3*m^3-85*I*a*x*m^5+8802*I*a^3*x^3*m^2-720*I*a*x*m^4+1
2000*I*a^3*x^3*m-3095*I*a*x*m^3-7076*I*a*x*m^2-8100*I*a*m*x-932*a^4*x^4*m^3
-2556*a^4*x^4*m^2+4*I*a^3*x^3*m^6+87*I*a^3*x^3*m^5-a^4*x^4*m^6-22*a^4*x^4*m
^5-197*a^4*x^4*m^4-3600*a^4*x^4)/(1+m)/m/(1+I*a*x)^5+x^m*(I*a)^m*(m^5+20*m^
4+155*m^3+580*m^2+1044*m+720)*LerchPhi(-I*a*x,1,m))-1/40*I*(I*a)^(-m)/a*(-x
^m*(I*a)^m*(24+m^4+50*m-392*a^2*m*x^2-240*a^2*x^2-4*I*a^3*x^3*m^4-43*I*a^3*
```

```
x^3*m^3-171*I*a^3*x^3*m^2+4*I*a*x*m^4-312*I*a^3*x^3*m+41*I*a*x*m^3+149*I*a*
x*m^2+226*I*a*m*x-6*a^2*x^2*m^4-63*a^2*x^2*m^3+10*m^3+35*m^2-239*a^2*m^2*x^
2+96*a^4*x^4*m-240*I*a^3*x^3+120*I*a*x+11*a^4*x^4*m^3+46*a^4*x^4*m^2+a^4*x^
4*m^4+120*a^4*x^4)/(1+I*a*x)^5+x^m*(I*a)^m*m*(m^4+10*m^3+35*m^2+50*m+24)*Le
rchPhi(-I*a*x,1,m))+1/40*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(a^4*x^4*m^4+a^4*x^4*
m^3-4*I*a*m*x-4*a^4*x^4*m^2+18*I*a^3*x^3*m-6*a^2*x^2*m^4-4*a^4*x^4*m-21*I*a
*x*m^2-3*a^2*x^2*m^3+I*a*x*m^3+20*I*a*x+31*a^2*m^2*x^2+19*I*a^3*x^3*m^2+m^4
+18*a^2*m*x^2+4*I*a*x*m^4-40*a^2*x^2-4*I*a^3*x^3*m^4-5*m^2-3*I*a^3*x^3*m^3+
4)/(1+I*a*x)^5+x^m*(I*a)^m*(m^2-3*m+2)*m*(m^2+3*m+2)*LerchPhi(-I*a*x,1,m))-
1/120*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(a^4*x^4*m^4-9*a^4*x^4*m^3-154*I*a*m*x+2
6*a^4*x^4*m^2+108*I*a^3*x^3*m-6*a^2*x^2*m^4-24*a^4*x^4*m+129*I*a*x*m^2+57*a
^2*x^2*m^3-39*I*a*x*m^3-111*I*a^3*x^3*m^2-179*a^2*m^2*x^2+4*I*a*x*m^4+m^4+1
88*a^2*m*x^2-4*I*a^3*x^3*m^4-10*m^3+37*I*a^3*x^3*m^3+35*m^2-50*m+24)/(1+I*a
*x)^5+x^m*(I*a)^m*(m^4-10*m^3+35*m^2-50*m+24)*m*LerchPhi(-I*a*x,1,m))
```

Fricas [F]

$$\int e^{-6i\arctan(ax)}x^m\,dx = \int \frac{\left(a^2x^2+1\right)^3x^m}{\left(i\,ax+1\right)^6}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*x^3 + 3*I*a^2*x^2 - 3*a*x - I)*x^m/(a^3*x^3 - 3*I*a^2*x^2 -
3*a*x + I), x)
```

Sympy [F]

$$\begin{aligned}\int e^{-6i\arctan(ax)}x^m\,dx = &-\int \frac{x^m}{a^6x^6-6ia^5x^5-15a^4x^4+20ia^3x^3+15a^2x^2-6iax-1}\,dx\\&-\int \frac{3a^2x^2x^m}{a^6x^6-6ia^5x^5-15a^4x^4+20ia^3x^3+15a^2x^2-6iax-1}\,dx\\&-\int \frac{3a^4x^4x^m}{a^6x^6-6ia^5x^5-15a^4x^4+20ia^3x^3+15a^2x^2-6iax-1}\,dx\\&-\int \frac{a^6x^6x^m}{a^6x^6-6ia^5x^5-15a^4x^4+20ia^3x^3+15a^2x^2-6iax-1}\,dx\end{aligned}$$

```
[In] integrate(x**m/(1+I*a*x)**6*(a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**m/(a**6*x**6 - 6*I*a**5*x**5 - 15*a**4*x**4 + 20*I*a**3*x**3 +
 15*a**2*x**2 - 6*I*a*x - 1), x) - Integral(3*a**2*x**2*x**m/(a**6*x**6 - 6
*I*a**5*x**5 - 15*a**4*x**4 + 20*I*a**3*x**3 + 15*a**2*x**2 - 6*I*a*x - 1),
 x) - Integral(3*a**4*x**4*x**m/(a**6*x**6 - 6*I*a**5*x**5 - 15*a**4*x**4 +
```

```
 20*I*a**3*x**3 + 15*a**2*x**2 - 6*I*a*x - 1), x) - Integral(a**6*x**6*x**m
/(a**6*x**6 - 6*I*a**5*x**5 - 15*a**4*x**4 + 20*I*a**3*x**3 + 15*a**2*x**2
- 6*I*a*x - 1), x)
```

Maxima [F]

$$\int e^{-6i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^3x^m}{(i\,ax+1)^6}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="maxima")
[Out] integrate((a^2*x^2 + 1)^3*x^m/(I*a*x + 1)^6, x)
```

Giac [F]

$$\int e^{-6i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^3x^m}{(i\,ax+1)^6}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="giac")
[Out] integrate((a^2*x^2 + 1)^3*x^m/(I*a*x + 1)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-6i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(a^2\,x^2+1)^3}{(1+a\,x\,1\mathrm{i})^6}\,dx$$

```
[In] int((x^m*(a^2*x^2 + 1)^3)/(a*x*1i + 1)^6,x)
[Out] int((x^m*(a^2*x^2 + 1)^3)/(a*x*1i + 1)^6, x)
```

3.140 $\int e^{3i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 159

$$\int e^{3i\arctan(ax)}x^m\,dx = -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} - \frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m} + \frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} + \frac{4iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)-I*a*x^(2+
m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)+4*x^(1+m)*hypergeom([
3/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+4*I*a*x^(2+m)*hypergeom([3/2, 1
+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used

= {5168, 6874, 371, 864, 822}

$$\int e^{3i\arctan(ax)}x^m\,dx = -\frac{3x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} + \frac{4x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} - \frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2} + \frac{4iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])*x^m,x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
+ m) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^
2)])/(2 + m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -(
a^2*x^2)])/(1 + m) + ((4*I)*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (
4 + m)/2, -(a^2*x^2)])/(2 + m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^m(1+iax)^2}{(1-iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(-\frac{3x^m}{\sqrt{1+a^2x^2}}-\frac{iax^{1+m}}{\sqrt{1+a^2x^2}}+\frac{4x^m}{(1-iax)\sqrt{1+a^2x^2}}\right)dx \\
&= -\left(3\int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx\right)+4\int \frac{x^m}{(1-iax)\sqrt{1+a^2x^2}}\,dx-(ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad -\frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}+4\int \frac{x^m(1+iax)}{(1+a^2x^2)^{3/2}}\,dx \\
&= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad -\frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m} \\
&\quad +4\int \frac{x^m}{(1+a^2x^2)^{3/2}}\,dx+(4ia)\int \frac{x^{1+m}}{(1+a^2x^2)^{3/2}}\,dx \\
&= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad -\frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m} \\
&\quad +\frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad +\frac{4iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

$$\int e^{3i\arctan(ax)}x^m\,dx = -\frac{ix^{1+m}\sqrt{1-iax}\sqrt{-i+ax}\left(\text{AppellF1}\left(1+m,-\frac{1}{2},\frac{1}{2},2+m,-iax,iax\right)-2\,\text{AppellF1}\left(1+m,-\frac{1}{2},\frac{3}{2},2+\right.\right.}{(1+m)\sqrt{1+iax}\sqrt{i+ax}}$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])*x^m,x]
[Out] ((-I)*x^(1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x]*(AppellF1[1 + m, -1/2, 1/2,
2 + m, (-I)*a*x, I*a*x] - 2*AppellF1[1 + m, -1/2, 3/2, 2 + m, (-I)*a*x, I*a
*x]))/((1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.92

method	result
meijerg	$\frac{x^{1+m}\,\text{hypergeom}\left(\left[\frac{3}{2},\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-a^2x^2\right)}{1+m}+\frac{3ia\,x^{2+m}\,\text{hypergeom}\left(\left[\frac{3}{2},1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-a^2x^2\right)}{2+m}-\frac{3a^2x^{3+m}\,\text{hypergeom}\left(\left[\frac{3}{2},\frac{3}{2}+\right.\right.}{3+m}$

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x,method=_RETURNVERBOSE)
[Out] x^(1+m)*hypergeom([3/2,1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+3*I*a/(2+m)*x
^(2+m)*hypergeom([3/2,1+1/2*m],[2+1/2*m],-a^2*x^2)-3*a^2/(3+m)*x^(3+m)*hype
rgeom([3/2,3/2+1/2*m],[1/2*m+5/2],-a^2*x^2)-I*a^3/(4+m)*x^(4+m)*hypergeom([
3/2,2+1/2*m],[1/2*m+3],-a^2*x^2)
```

Fricas [F]

$$\int e^{3i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^3x^m}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*(-I*a*x - 1)*x^m/(a^2*x^2 + 2*I*a*x - 1), x)
```

Sympy [F]

$$\int e^{3i\arctan(ax)}x^m\,dx = -i\left(\int \frac{ix^m}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3axx^m}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3x^m}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3ia^2x^2x^m}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**m,x)
```

```
[Out] -I*(Integral(I*x**m/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) + Integral(-3*a*x*x**m/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**3*x**m/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a*
*2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2*x**m/(a**2*x**2*sqrt(a**2*x**2
+ 1) + sqrt(a**2*x**2 + 1)), x))
```

Maxima [F]

$$\int e^{3i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^3x^m}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((I*a*x + 1)^3*x^m/(a^2*x^2 + 1)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(ax)}x^m\,dx=\int \frac{x^m\,(1+a\,x\,1\mathrm{i})^3}{(a^2\,x^2+1)^{3/2}}\,dx$$

```
[In] int((x^m*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
[Out] int((x^m*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2), x)
```

3.141 $\int e^{i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int e^{i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m}+\frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}$$

```
[Out] x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*x^(2+m)*
hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5168, 822, 371}

$$\int e^{i\arctan(ax)}x^m\,dx=\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

```
[In] Int[E^(I*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 + m
) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]
)/(2 + m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^m(1+iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= (ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx + \int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad + \frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int e^{i\arctan(ax)}x^m\,dx = \frac{ix^{1+m}\sqrt{1-iax}\sqrt{-i+ax}\,\text{AppellF1}\left(1+m,-\frac{1}{2},\frac{1}{2},2+m,-iax,iax\right)}{(1+m)\sqrt{1+iax}\sqrt{i+ax}}$$

```
[In] Integrate[E^(I*ArcTan[a*x])*x^m,x]
[Out] (I*x^(1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x]*AppellF1[1 + m, -1/2, 1/2, 2 +
m, (-I)*a*x, I*a*x])/((1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

method	result	size
meijerg	$\frac{x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-a^{2}x^{2}\right)}{1+m}+\frac{i a\, x^{2+m} \operatorname{hypergeom}\left(\left[\frac{1}{2},1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],-a^{2}x^{2}\right)}{2+m}$	71

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+m)*hypergeom([1/2,1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*x^(2+m)*h
ypergeom([1/2,1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Fricas [F]

$$\int e^{i\arctan(ax)}x^m\,dx=\int\frac{(i\,ax+1)x^m}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(I*sqrt(a^2*x^2 + 1)*x^m/(a*x + I), x)
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int e^{i\arctan(ax)}x^m\,dx=\frac{iax^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\middle|a^2x^2e^{i\pi}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**m,x)
```

```
[Out] I*a*x**(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*e
xp_polar(I*pi))/(2*gamma(m/2 + 2)) + x**(m + 1)*gamma(m/2 + 1/2)*hyper((1/2
, m/2 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(I*pi))/(2*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int e^{i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)x^m}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="maxima")
[Out] integrate((I*a*x + 1)*x^m/sqrt(a^2*x^2 + 1), x)
```

Giac [F]

$$\int e^{i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)x^m}{\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="giac")
[Out] integrate((I*a*x + 1)*x^m/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(1+a\,x\,1\mathrm{i})}{\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^m*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
[Out] int((x^m*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2), x)
```

3.142 $\int e^{-i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int e^{-i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m}-\frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}$$

```
[Out] x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)-I*a*x^(2+m)*
hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5168, 822, 371}

$$\int e^{-i\arctan(ax)}x^m\,dx=\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

```
[In] Int[x^m/E^(I*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 + m
) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]
)/(2 + m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^m(1-iax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\left((ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx\right)+\int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad -\frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int e^{-i\arctan(ax)}x^m\,dx=-\frac{ix^{1+m}\sqrt{1+iax}\sqrt{i+ax}\,\text{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,-iax,iax\right)}{(1+m)\sqrt{1-iax}\sqrt{-i+ax}}$$

```
[In] Integrate[x^m/E^(I*ArcTan[a*x]),x]
[Out] ((-I)*x^(1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x]*AppellF1[1 + m, 1/2, -1/2, 2
+ m, (-I)*a*x, I*a*x])/((1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x])
```

Maple [F]

$$\int \frac{x^m\sqrt{a^2x^2+1}}{iax+1}dx$$

```
[In] int(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
[Out] int(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int e^{-i\arctan(ax)}x^m\,dx = \int \frac{\sqrt{a^2x^2+1}x^m}{i\,ax+1}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-I*sqrt(a^2*x^2 + 1)*x^m/(a*x - I), x)
```

Sympy [F]

$$\int e^{-i\arctan(ax)}x^m\,dx = -i\int \frac{x^m\sqrt{a^2x^2+1}}{ax-i}\,dx$$

```
[In] integrate(x**m/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
[Out] -I*Integral(x**m*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [F]

$$\int e^{-i\arctan(ax)}x^m\,dx = \int \frac{\sqrt{a^2x^2+1}x^m}{i\,ax+1}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*x^2 + 1)*x^m/(I*a*x + 1), x)
```

Giac [F]

$$\int e^{-i\arctan(ax)}x^m\,dx = \int \frac{\sqrt{a^2x^2+1}x^m}{i\,ax+1}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2*x^2 + 1)*x^m/(I*a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(ax)}x^m\,dx = \int \frac{x^m\,\sqrt{a^2\,x^2+1}}{1+a\,x\,1\mathrm{i}}\,dx$$

```
[In] int((x^m*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
[Out] int((x^m*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1), x)
```

3.143 $\int e^{-3i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 159

$$\int e^{-3i\arctan(ax)}x^m\,dx = -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} + \frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m} + \frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} - \frac{4iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*x^(2+
m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)+4*x^(1+m)*hypergeom([
3/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)-4*I*a*x^(2+m)*hypergeom([3/2, 1
+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used

= {5168, 6874, 371, 864, 822}

$$\int e^{-3i\arctan(ax)}x^m\,dx = -\frac{3x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} + \frac{4x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} + \frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2} - \frac{4iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

```
[In] Int[x^m/E^((3*I)*ArcTan[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
+ m) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^
2)])/(2 + m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -(
a^2*x^2)])/(1 + m) - ((4*I)*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (
4 + m)/2, -(a^2*x^2)])/(2 + m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 5168

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^m(1-iax)^2}{(1+iax)\sqrt{1+a^2x^2}}\,dx \\
&= \int \left(-\frac{3x^m}{\sqrt{1+a^2x^2}}+\frac{iax^{1+m}}{\sqrt{1+a^2x^2}}+\frac{4x^m}{(1+iax)\sqrt{1+a^2x^2}}\right)dx \\
&= -\left(3\int \frac{x^m}{\sqrt{1+a^2x^2}}\,dx\right)+4\int \frac{x^m}{(1+iax)\sqrt{1+a^2x^2}}\,dx+(ia)\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad +\frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}+4\int \frac{x^m(1-iax)}{(1+a^2x^2)^{3/2}}\,dx \\
&= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad +\frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m} \\
&\quad +4\int \frac{x^m}{(1+a^2x^2)^{3/2}}\,dx-(4ia)\int \frac{x^{1+m}}{(1+a^2x^2)^{3/2}}\,dx \\
&= -\frac{3x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad +\frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m} \\
&\quad +\frac{4x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} \\
&\quad -\frac{4iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

$$\int e^{-3i\arctan(ax)}x^m\,dx = \frac{ix^{1+m}\sqrt{1+iax}\sqrt{i+ax}\left(\text{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,-iax,iax\right)-2\,\text{AppellF1}\left(1+m,\frac{3}{2},-\frac{1}{2},2+m,-\right.\right.}{(1+m)\sqrt{1-iax}\sqrt{-i+ax}}$$

```
[In] Integrate[x^m/E^((3*I)*ArcTan[a*x]),x]
[Out] (I*x^(1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x]*(AppellF1[1 + m, 1/2, -1/2, 2 +
m, (-I)*a*x, I*a*x] - 2*AppellF1[1 + m, 3/2, -1/2, 2 + m, (-I)*a*x, I*a*x])
)/((1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x])
```

Maple [F]

$$\int \frac{x^m(a^2x^2+1)^{\frac{3}{2}}}{(iax+1)^3}dx$$

```
[In] int(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
[Out] int(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

Fricas [F]

$$\int e^{-3i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}x^m}{(i\,ax+1)^3}\,dx$$

```
[In] integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*(I*a*x - 1)*x^m/(a^2*x^2 - 2*I*a*x - 1), x)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}x^m\,dx = i\left(\int \frac{x^m\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx + \int \frac{a^2x^2x^m\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

```
[In] integrate(x**m/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
[Out] I*(Integral(x**m*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I
), x) + Integral(a**2*x**2*x**m*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x
**2 - 3*a*x + I), x))
```

Maxima [F]

$$\int e^{-3i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}x^m}{(i\,ax+1)^3}\,dx$$

[In] integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate((a^2*x^2 + 1)^(3/2)*x^m/(I*a*x + 1)^3, x)

Giac [F(-2)]

Exception generated.

$$\int e^{-3i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(a^2\,x^2+1)^{3/2}}{(1+a\,x\,1\mathrm{i})^3}\,dx$$

[In] int((x^m*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)

[Out] int((x^m*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3, x)

3.144 $\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,\frac{5}{4},-\frac{5}{4},2+m,iax,-iax\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,-5/4,5/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5170, 138}

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,\frac{5}{4},-\frac{5}{4},m+2,iax,-iax\right)}{m+1}$$

```
[In] Int[E^(((5*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 5/4, -5/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^m(1+iax)^{5/4}}{(1-iax)^{5/4}}\,dx\\ &= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,\frac{5}{4},-\frac{5}{4},2+m,iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx = \int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx$$

```
[In] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^m,x]
[Out] Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^m, x]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^m dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="fricas")
[Out] integral(-(a*x - I)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x + I), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**m,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

```
[In] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.145 $\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,\frac{3}{4},-\frac{3}{4},2+m,iax,-iax\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,-3/4,3/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5170, 138}

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,\frac{3}{4},-\frac{3}{4},m+2,iax,-iax\right)}{m+1}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3/4, -3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^m(1+iax)^{3/4}}{(1-iax)^{3/4}}\,dx\\ &= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,\frac{3}{4},-\frac{3}{4},2+m,iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx=\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^m,x]
[Out] Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^m, x]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}x^m dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx=\int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="fricas")
[Out] integral(I*sqrt(a^2*x^2 + 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x +
 I), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**m,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by 23, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

```
[In] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
[Out] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

3.146 $\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,\frac{1}{4},-\frac{1}{4},2+m,iax,-iax\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,-1/4,1/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5170, 138}

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,\frac{1}{4},-\frac{1}{4},m+2,iax,-iax\right)}{m+1}$$

```
[In] Int[E^((I/2)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/4, -1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^m \sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\,dx\\ &= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,\frac{1}{4},-\frac{1}{4},2+m,iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx$$

```
[In] Integrate[E^((I/2)*ArcTan[a*x])*x^m,x]
[Out] Integrate[E^((I/2)*ArcTan[a*x])*x^m, x]
```

Maple [F]

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^m dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \int x^m\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="fricas")
[Out] integral(x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)), x)
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \int x^m\sqrt{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x**m,x)
[Out] Integral(x**m*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \int x^m\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \int x^m\,\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\,dx$$

```
[In] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

3.147 $\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,-\frac{1}{4},\frac{1}{4},2+m,iax,-iax\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/4,-1/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5170, 138}

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{4},\frac{1}{4},m+2,iax,-iax\right)}{m+1}$$

```
[In] Int[x^m/E^((I/2)*ArcTan[a*x]),x]
[Out] (x^(1 + m)*AppellF1[1 + m, -1/4, 1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^m \sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\,dx\\ &= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,-\frac{1}{4},\frac{1}{4},2+m,iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx=\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx$$

```
[In] Integrate[x^m/E^((I/2)*ArcTan[a*x]),x]
[Out] Integrate[x^m/E^((I/2)*ArcTan[a*x]), x]
```

Maple [F]

$$\int \frac{x^m}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

```
[In] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
[Out] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx=\int \frac{x^m}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] integral(-I*sqrt(a^2*x^2 + 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x
- I), x)
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
[Out] Integral(x**m/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x^m/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}}}\,dx$$

```
[In] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

3.148 $\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx = \frac{x^{1+m}\operatorname{AppellF1}\left(1+m,-\frac{3}{4},\frac{3}{4},2+m,iax,-iax\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,3/4,-3/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5170, 138}

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx = \frac{x^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{3}{4},\frac{3}{4},m+2,iax,-iax\right)}{m+1}$$

```
[In] Int[x^m/E^(((3*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -3/4, 3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x^m(1-iax)^{3/4}}{(1+iax)^{3/4}}\,dx$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,-\frac{3}{4},\frac{3}{4},2+m,iax,-iax\right)}{1+m}$$

Mathematica [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx = \int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx$$

```
[In] Integrate[x^m/E^(((3*I)/2)*ArcTan[a*x]),x]
[Out] Integrate[x^m/E^(((3*I)/2)*ArcTan[a*x]), x]
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
[Out] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
[Out] integral(-(a*x + I)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x - I), x)
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx=\int\frac{x^m}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
[Out] Integral(x**m/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx=\int\frac{x^m}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate(x^m/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -46, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx=\int\frac{x^m}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2}}\,dx$$

```
[In] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
[Out] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

3.149 $\int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,-\frac{5}{4},\frac{5}{4},2+m,iax,-iax\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,5/4,-5/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5170, 138}

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{5}{4},\frac{5}{4},m+2,iax,-iax\right)}{m+1}$$

```
[In] Int[x^m/E^(((5*I)/2)*ArcTan[a*x]),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -5/4, 5/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x^m(1-iax)^{5/4}}{(1+iax)^{5/4}}\,dx$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,-\frac{5}{4},\frac{5}{4},2+m,iax,-iax\right)}{1+m}$$

Mathematica [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx = \int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx$$

```
[In] Integrate[x^m/E^(((5*I)/2)*ArcTan[a*x]),x]
[Out] Integrate[x^m/E^(((5*I)/2)*ArcTan[a*x]), x]
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
[Out] int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*x^2 + 1)*(I*a*x - 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))/(a^2*x^2 - 2*I*a*x - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx = \text{Timed out}$$

```
[In] integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(x^m/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -8, a substitution variable should perhaps be purged
.Warni
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}\,dx$$

```
[In] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
[Out] int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

3.150 $\int e^{\frac{2\arctan(x)}{3}} x^m \, dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int e^{\frac{2\arctan(x)}{3}} x^m \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(1+m,-\frac{i}{3},\frac{i}{3},2+m,ix,-ix\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/3*I,-1/3*I,2+m,-I*x,I*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5170, 138}

$$\int e^{\frac{2\arctan(x)}{3}} x^m \, dx = \frac{x^{m+1} \operatorname{AppellF1}\left(m+1,-\frac{i}{3},\frac{i}{3},m+2,ix,-ix\right)}{m+1}$$

```
[In] Int[E^((2*ArcTan[x])/3)*x^m,x]
[Out] (x^(1 + m)*AppellF1[1 + m, -1/3*I, I/3, 2 + m, I*x, (-I)*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int (1-ix)^{\frac{i}{3}}(1+ix)^{-\frac{i}{3}}x^m\,dx$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,-\frac{i}{3},\frac{i}{3},2+m,ix,-ix\right)}{1+m}$$

Mathematica [F]

$$\int e^{\frac{2\arctan(x)}{3}}x^m\,dx = \int e^{\frac{2\arctan(x)}{3}}x^m\,dx$$

```
[In] Integrate[E^((2*ArcTan[x])/3)*x^m,x]
[Out] Integrate[E^((2*ArcTan[x])/3)*x^m, x]
```

Maple [F]

$$\int \mathrm{e}^{\frac{2\arctan(x)}{3}}x^m dx$$

```
[In] int(exp(2/3*arctan(x))*x^m,x)
[Out] int(exp(2/3*arctan(x))*x^m,x)
```

Fricas [F]

$$\int e^{\frac{2\arctan(x)}{3}}x^m\,dx = \int x^m e^{\left(\frac{2}{3}\,\arctan(x)\right)}\,dx$$

```
[In] integrate(exp(2/3*arctan(x))*x^m,x, algorithm="fricas")
[Out] integral(x^m*e^(2/3*arctan(x)), x)
```

Sympy [F]

$$\int e^{\frac{2\arctan(x)}{3}} x^m \, dx = \int x^m e^{\frac{2\,\mathrm{atan}\,(x)}{3}} \, dx$$

[In] integrate(exp(2/3*atan(x))*x**m,x)

[Out] Integral(x**m*exp(2*atan(x)/3), x)

Maxima [F]

$$\int e^{\frac{2\arctan(x)}{3}} x^m \, dx = \int x^m e^{\left(\frac{2}{3}\arctan(x)\right)} \, dx$$

[In] integrate(exp(2/3*arctan(x))*x^m,x, algorithm="maxima")

[Out] integrate(x^m*e^(2/3*arctan(x)), x)

Giac [F]

$$\int e^{\frac{2\arctan(x)}{3}} x^m \, dx = \int x^m e^{\left(\frac{2}{3}\arctan(x)\right)} \, dx$$

[In] integrate(exp(2/3*arctan(x))*x^m,x, algorithm="giac")

[Out] integrate(x^m*e^(2/3*arctan(x)), x)

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2\arctan(x)}{3}} x^m \, dx = \int x^m \, \mathrm{e}^{\frac{2\,\mathrm{atan}(x)}{3}} \, dx$$

[In] int(x^m*exp((2*atan(x))/3),x)

[Out] int(x^m*exp((2*atan(x))/3), x)

3.151 $\int e^{\frac{\arctan(x)}{3}} x^m \, dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int e^{\frac{\arctan(x)}{3}} x^m \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(1+m,-\frac{i}{6},\frac{i}{6},2+m,ix,-ix\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/6*I,-1/6*I,2+m,-I*x,I*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5170, 138}

$$\int e^{\frac{\arctan(x)}{3}} x^m \, dx = \frac{x^{m+1} \operatorname{AppellF1}\left(m+1,-\frac{i}{6},\frac{i}{6},m+2,ix,-ix\right)}{m+1}$$

```
[In] Int[E^(ArcTan[x]/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -1/6*I, I/6, 2 + m, I*x, (-I)*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int (1-ix)^{\frac{i}{6}}(1+ix)^{-\frac{i}{6}}x^m\,dx$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,-\frac{i}{6},\frac{i}{6},2+m,ix,-ix\right)}{1+m}$$

Mathematica [F]

$$\int e^{\frac{\arctan(x)}{3}}x^m\,dx = \int e^{\frac{\arctan(x)}{3}}x^m\,dx$$

```
[In] Integrate[E^(ArcTan[x]/3)*x^m,x]
[Out] Integrate[E^(ArcTan[x]/3)*x^m, x]
```

Maple [F]

$$\int \mathrm{e}^{\frac{\arctan(x)}{3}}x^m dx$$

```
[In] int(exp(1/3*arctan(x))*x^m,x)
[Out] int(exp(1/3*arctan(x))*x^m,x)
```

Fricas [F]

$$\int e^{\frac{\arctan(x)}{3}}x^m\,dx = \int x^m e^{\left(\frac{1}{3}\arctan(x)\right)}\,dx$$

```
[In] integrate(exp(1/3*arctan(x))*x^m,x, algorithm="fricas")
[Out] integral(x^m*e^(1/3*arctan(x)), x)
```

Sympy [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m\,dx = \int x^m e^{\frac{\operatorname{atan}(x)}{3}}\,dx$$

```
[In] integrate(exp(1/3*atan(x))*x**m,x)
[Out] Integral(x**m*exp(atan(x)/3), x)
```

Maxima [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m\,dx = \int x^m e^{\left(\frac{1}{3}\arctan(x)\right)}\,dx$$

```
[In] integrate(exp(1/3*arctan(x))*x^m,x, algorithm="maxima")
[Out] integrate(x^m*e^(1/3*arctan(x)), x)
```

Giac [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m\,dx = \int x^m e^{\left(\frac{1}{3}\arctan(x)\right)}\,dx$$

```
[In] integrate(exp(1/3*arctan(x))*x^m,x, algorithm="giac")
[Out] integrate(x^m*e^(1/3*arctan(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{\arctan(x)}{3}} x^m\,dx = \int x^m\,\mathrm{e}^{\frac{\operatorname{atan}(x)}{3}}\,dx$$

```
[In] int(x^m*exp(atan(x)/3),x)
[Out] int(x^m*exp(atan(x)/3), x)
```

3.152 $\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,\frac{1}{8},-\frac{1}{8},2+m,iax,-iax\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,-1/8,1/8,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5170, 138}

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx=\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,\frac{1}{8},-\frac{1}{8},m+2,iax,-iax\right)}{m+1}$$

```
[In] Int[E^((I/4)*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/8, -1/8, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{x^m \sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\,dx$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,\frac{1}{8},-\frac{1}{8},2+m,iax,-iax\right)}{1+m}$$

Mathematica [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx$$

```
[In] Integrate[E^((I/4)*ArcTan[a*x])*x^m,x]
[Out] Integrate[E^((I/4)*ArcTan[a*x])*x^m, x]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}} x^m dx$$

```
[In] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
[Out] int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="fricas")
[Out] integral(x^m*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4), x)
```

Sympy [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int x^m\sqrt[4]{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x**m,x)
[Out] Integral(x**m*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]W
arning, replacing 0 by -28, a substitution variable should perhaps be purge
d.Warn
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}\,dx$$

```
[In] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
[Out] int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

3.153 $\int e^{in \arctan(ax)} x^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int e^{in \arctan(ax)} x^m \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(1+m, \frac{n}{2}, -\frac{n}{2}, 2+m, iax, -iax\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(1+m,-1/2*n,1/2*n,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5170, 138}

$$\int e^{in \arctan(ax)} x^m \, dx = \frac{x^{m+1} \operatorname{AppellF1}\left(m+1, \frac{n}{2}, -\frac{n}{2}, m+2, iax, -iax\right)}{m+1}$$

```
[In] Int[E^(I*n*ArcTan[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, n/2, -1/2*n, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
```

```
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int x^m(1-iax)^{-n/2}(1+iax)^{n/2}\,dx$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,\frac{n}{2},-\frac{n}{2},2+m,iax,-iax\right)}{1+m}$$

Mathematica [F]

$$\int e^{in\arctan(ax)}x^m\,dx = \int e^{in\arctan(ax)}x^m\,dx$$

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^m,x]
[Out] Integrate[E^(I*n*ArcTan[a*x])*x^m, x]
```

Maple [F]

$$\int e^{in\arctan(ax)}x^m dx$$

```
[In] int(exp(I*n*arctan(a*x))*x^m,x)
[Out] int(exp(I*n*arctan(a*x))*x^m,x)
```

Fricas [F]

$$\int e^{in\arctan(ax)}x^m\,dx = \int x^m e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="fricas")
[Out] integral(x^m/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F]

$$\int e^{in\arctan(ax)}x^m\,dx=\int x^m e^{in\,\mathrm{atan}\,(ax)}\,dx$$

```
[In] integrate(exp(I*n*atan(a*x))*x**m,x)
[Out] Integral(x**m*exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in\arctan(ax)}x^m\,dx=\int x^m e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="maxima")
[Out] integrate(x^m*e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in\arctan(ax)}x^m\,dx=\int x^m e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x^m\,dx=\int x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

```
[In] int(x^m*exp(n*atan(a*x)*1i),x)
[Out] int(x^m*exp(n*atan(a*x)*1i), x)
```

3.154 $\int e^{in\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 171

$$\int e^{in\arctan(ax)}x^3\,dx = \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}(6+n^2+2ianx)}{24a^4} - \frac{2^{-2+\frac{n}{2}}n(8+n^2)(1-iax)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{3a^4(2-n)}$$

```
[Out] 1/4*x^2*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^2-1/24*(1-I*a*x)^(1-1/2*n
)*(1+I*a*x)^(1+1/2*n)*(6+n^2+2*I*a*n*x)/a^4-1/3*2^(-2+1/2*n)*n*(n^2+8)*(1-I
*a*x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*a*x)/a^4/(2
-n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5170, 102, 152, 71}

$$\int e^{in\arctan(ax)}x^3\,dx = -\frac{2^{\frac{n}{2}-2}n(n^2+8)(1-iax)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{3a^4(2-n)} - \frac{(1+iax)^{\frac{n+2}{2}}(2ianx+n^2+6)(1-iax)^{1-\frac{n}{2}}}{24a^4} + \frac{x^2(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{4a^2}$$

```
[In] Int[E^(I*n*ArcTan[a*x])*x^3,x]
```

```
[Out] (x^2*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(4*a^2) - ((1 - I*a*x)^
(1 - n/2)*(1 + I*a*x)^((2 + n)/2)*(6 + n^2 + (2*I)*a*n*x))/(24*a^4) - (2^(-
2 + n/2)*n*(8 + n^2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*
n, 2 - n/2, (1 - I*a*x)/2])/(3*a^4*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int x^3(1-iax)^{-n/2}(1+iax)^{n/2}\,dx$$

$$= \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} + \frac{\int x(1-iax)^{-n/2}(1+iax)^{n/2}(-2-ianx)\,dx}{4a^2}$$

$$= \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}(6+n^2+2ianx)}{24a^4} + \frac{(in(8+n^2))\int(1-iax)^{-n/2}(1+iax)^{n/2}\,dx}{24a^3}$$

$$= \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}(6+n^2+2ianx)}{24a^4} - \frac{2^{-2+\frac{n}{2}}n(8+n^2)(1-iax)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{3a^4(2-n)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.23

$$\int e^{in\arctan(ax)}x^3\,dx = \frac{(1-iax)^{-n/2}(i+ax)\left(-i2^{3+\frac{n}{2}}n\,\text{Hypergeometric2F1}\left(-2-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)+i2^{3+\frac{n}{2}}(-1+n\right.}{}$$

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^3,x]
```

```
[Out] ((I + a*x)*((-I)*2^(3 + n/2)*n*Hypergeometric2F1[-2 - n/2, 1 - n/2, 2 - n/2
, (1 - I*a*x)/2] + I*2^(3 + n/2)*(-1 + n)*Hypergeometric2F1[-1 - n/2, 1 - n
/2, 2 - n/2, (1 - I*a*x)/2] + (-2 + n)*(a^2*x^2*(1 + I*a*x)^(n/2)*(-I + a*x
) - I*2^(1 + n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2
])))/(4*a^4*(-2 + n)*(1 - I*a*x)^(n/2))
```

Maple [F]

$$\int e^{in\arctan(ax)}x^3dx$$

```
[In] int(exp(I*n*arctan(a*x))*x^3,x)
```

```
[Out] int(exp(I*n*arctan(a*x))*x^3,x)
```

Fricas [F]

$$\int e^{in\arctan(ax)}x^3\,dx = \int x^3e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F]

$$\int e^{in\arctan(ax)}x^3\,dx = \int x^3e^{in\,\mathrm{atan}\,(ax)}\,dx$$

```
[In] integrate(exp(I*n*atan(a*x))*x**3,x)
[Out] Integral(x**3*exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in\arctan(ax)}x^3\,dx = \int x^3e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="maxima")
[Out] integrate(x^3*e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in\arctan(ax)}x^3\,dx = \int x^3e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x^3\,dx = \int x^3\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

```
[In] int(x^3*exp(n*atan(a*x)*1i),x)
[Out] int(x^3*exp(n*atan(a*x)*1i), x)
```

3.155 $\int e^{in \arctan(ax)} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 159

$$\int e^{in \arctan(ax)} x^2 \, dx = -\frac{in(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6a^3} + \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} - \frac{i2^{n/2}(2+n^2)(1-iax)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{3a^3(2-n)}$$

```
[Out] -1/6*I*n*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^3+1/3*x*(1-I*a*x)^(1-1/2
*n)*(1+I*a*x)^(1+1/2*n)/a^2-1/3*I*2^(1/2*n)*(n^2+2)*(1-I*a*x)^(1-1/2*n)*hyp
ergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*a*x)/a^3/(2-n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5170, 92, 81, 71}

$$\int e^{in \arctan(ax)} x^2 \, dx = -\frac{i2^{n/2}(n^2+2)(1-iax)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{3a^3(2-n)} - \frac{in(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{6a^3} + \frac{x(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{3a^2}$$

```
[In] Int[E^(I*n*ArcTan[a*x])*x^2,x]
```

```
[Out] ((-1/6*I)*n*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/a^3 + (x*(1 - I*
a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(3*a^2) - ((I/3)*2^(n/2)*(2 + n^2)*
(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*
x)/2])/(a^3*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^2(1-iax)^{-n/2}(1+iax)^{n/2}\,dx \\
&= \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} + \frac{\int (1-iax)^{-n/2}(1+iax)^{n/2}(-1-ianx)\,dx}{3a^2} \\
&= -\frac{in(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6a^3} + \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} \\
&\quad - \frac{(2+n^2)\int (1-iax)^{-n/2}(1+iax)^{n/2}\,dx}{6a^2}
\end{aligned}
$$

$$
= -\frac{in(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6a^3} + \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} - \frac{i2^{n/2}(2+n^2)\,(1-iax)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{3a^3(2-n)}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$
\int e^{in\arctan(ax)}x^2\,dx = \frac{(1-iax)^{-n/2}(i+ax)\left((-2+n)(1+iax)^{n/2}(-i+ax)(-in+2ax)+2^{1+\frac{n}{2}}(2+n^2)\,\text{Hypergeometric2F1}\left(\right.\right.}{6a^3(-2+n)}
$$

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^2,x]
[Out] ((I + a*x)*((-2 + n)*(1 + I*a*x)^(n/2)*(-I + a*x)*((-I)*n + 2*a*x) + 2^(1 +
 n/2)*(2 + n^2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2])
)/(6*a^3*(-2 + n)*(1 - I*a*x)^(n/2))
```

Maple [F]

$$
\int e^{in\arctan(ax)}x^2dx
$$

```
[In] int(exp(I*n*arctan(a*x))*x^2,x)
[Out] int(exp(I*n*arctan(a*x))*x^2,x)
```

Fricas [F]

$$
\int e^{in\arctan(ax)}x^2\,dx = \int x^2e^{(i\,n\arctan(ax))}\,dx
$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="fricas")
[Out] integral(x^2/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F]

$$\int e^{in\arctan(ax)}x^2\,dx = \int x^2e^{in\,\mathrm{atan}\,(ax)}\,dx$$

```
[In] integrate(exp(I*n*atan(a*x))*x**2,x)
[Out] Integral(x**2*exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in\arctan(ax)}x^2\,dx = \int x^2e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="maxima")
[Out] integrate(x^2*e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in\arctan(ax)}x^2\,dx = \int x^2e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x^2\,dx = \int x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

```
[In] int(x^2*exp(n*atan(a*x)*1i),x)
[Out] int(x^2*exp(n*atan(a*x)*1i), x)
```

3.156 $\int e^{in \arctan(ax)} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int e^{in \arctan(ax)} x \, dx = \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2a^2} + \frac{2^{n/2} n (1-iax)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2}, -\frac{n}{2}, 2-\frac{n}{2}, \frac{1}{2}(1-iax)\right)}{a^2(2-n)}$$

```
[Out] 1/2*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^2+2^(1/2*n)*n*(1-I*a*x)^(1-1/
2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*a*x)/a^2/(2-n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5170, 81, 71}

$$\int e^{in \arctan(ax)} x \, dx = \frac{2^{n/2} n (1-iax)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2}, -\frac{n}{2}, 2-\frac{n}{2}, \frac{1}{2}(1-iax)\right)}{a^2(2-n)} + \frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2a^2}$$

```
[In] Int[E^(I*n*ArcTan[a*x])*x,x]
```

```
[Out] ((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(2*a^2) + (2^(n/2)*n*(1 - I
*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2])
/(a^2*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x(1-iax)^{-n/2}(1+iax)^{n/2}\,dx \\
&= \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2a^2} - \frac{(in)\int(1-iax)^{-n/2}(1+iax)^{n/2}\,dx}{2a} \\
&= \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2a^2} \\
&\quad + \frac{2^{n/2}n(1-iax)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{a^2(2-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int e^{in\arctan(ax)}x\,dx = \frac{(1-iax)^{-n/2}(i+ax)\left((-2+n)(1+iax)^{n/2}(-i+ax)+i2^{1+\frac{n}{2}}n\,\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(\right.\right.}{2a^2(-2+n)}$$

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x,x]
```

```
[Out] ((I + a*x)*((-2 + n)*(1 + I*a*x)^(n/2)*(-I + a*x) + I*2^(1 + n/2)*n*Hyperge
ometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2]))/(2*a^2*(-2 + n)*(1 -
I*a*x)^(n/2))
```

Maple [F]

$$\int e^{in\arctan(ax)}x dx$$

```
[In] int(exp(I*n*arctan(a*x))*x,x)
[Out] int(exp(I*n*arctan(a*x))*x,x)
```

Fricas [F]

$$\int e^{in\arctan(ax)}x\,dx = \int xe^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x,x, algorithm="fricas")
[Out] integral(x/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F]

$$\int e^{in\arctan(ax)}x\,dx = \int xe^{in\operatorname{atan}(ax)}\,dx$$

```
[In] integrate(exp(I*n*atan(a*x))*x,x)
[Out] Integral(x*exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in\arctan(ax)}x\,dx = \int xe^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x,x, algorithm="maxima")
[Out] integrate(x*e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in\arctan(ax)}x\,dx = \int xe^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x\,dx = \int x\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

```
[In] int(x*exp(n*atan(a*x)*1i),x)
[Out] int(x*exp(n*atan(a*x)*1i), x)
```

3.157 $\int e^{in \arctan(ax)} dx$

Optimal result

Integrand size = 11, antiderivative size = 71

$$\int e^{in \arctan(ax)} dx = \frac{i2^{1+\frac{n}{2}}(1-iax)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{a(2-n)}$$

```
[Out] I*2^(1+1/2*n)*(1-I*a*x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2
-1/2*I*a*x)/a/(2-n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5169, 71}

$$\int e^{in \arctan(ax)} dx = \frac{i2^{\frac{n}{2}+1}(1-iax)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{a(2-n)}$$

```
[In] Int[E^(I*n*ArcTan[a*x]),x]
```

```
[Out] (I*2^(1 + n/2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 -
 n/2, (1 - I*a*x)/2])/(a*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] && !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int (1-iax)^{-n/2}(1+iax)^{n/2}\,dx$$
$$= \frac{i2^{1+\frac{n}{2}}(1-iax)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{a(2-n)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int e^{in\arctan(ax)}\,dx = -\frac{4ie^{i(2+n)\arctan(ax)}\,\text{Hypergeometric2F1}\left(2,1+\frac{n}{2},2+\frac{n}{2},-e^{2i\arctan(ax)}\right)}{a(2+n)}$$

```
[In] Integrate[E^(I*n*ArcTan[a*x]),x]
[Out] ((-4*I)*E^(I*(2 + n)*ArcTan[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E
^((2*I)*ArcTan[a*x])])/(a*(2 + n))
```

Maple [F]

$$\int \mathrm{e}^{in\arctan(ax)}dx$$

```
[In] int(exp(I*n*arctan(a*x)),x)
[Out] int(exp(I*n*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{in\arctan(ax)}\,dx = \int e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x)),x, algorithm="fricas")
[Out] integral(1/((-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F]

$$\int e^{in\arctan(ax)}\,dx=\int e^{in\,\mathrm{atan}\,(ax)}\,dx$$

```
[In] integrate(exp(I*n*atan(a*x)),x)
[Out] Integral(exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in\arctan(ax)}\,dx=\int e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in\arctan(ax)}\,dx=\int e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}\,dx=\int \mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

```
[In] int(exp(n*atan(a*x)*1i),x)
[Out] int(exp(n*atan(a*x)*1i), x)
```

3.158 $\int \frac{e^{in\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\,\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{n} - \frac{2^{1+\frac{n}{2}}(1-iax)^{-n/2}\,\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{n}$$

```
[Out] 2*(1+I*a*x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(1-I*a*x)/(1+I*a*x))/n/
((1-I*a*x)^(1/2*n))-2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],1/2-1/
2*I*a*x)/n/((1-I*a*x)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5170, 132, 71, 133}

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\,\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{1-iax}{iax+1}\right)}{n} - \frac{2^{\frac{n}{2}+1}(1-iax)^{-n/2}\,\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{n}$$

```
[In] Int[E^(I*n*ArcTan[a*x])/x,x]
```

```
[Out] (2*(1 + I*a*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (1 - I*a*x)/(1 +
 I*a*x)])/(n*(1 - I*a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-1/2*n, -1
/2*n, 1 - n/2, (1 - I*a*x)/2])/(n*(1 - I*a*x)^(n/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x}\,dx \\
&= -\left((ia)\int (1-iax)^{-1-\frac{n}{2}}(1+iax)^{n/2}\,dx\right)+\int \frac{(1-iax)^{-1-\frac{n}{2}}(1+iax)^{n/2}}{x}\,dx \\
&= \frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\,\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{n} \\
&\quad -\frac{2^{1+\frac{n}{2}}(1-iax)^{-n/2}\,\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \frac{2(1-iax)^{-n/2}\left((1+iax)^{n/2}\,\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{i+ax}{i-ax}\right)-2^{n/2}\,\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2}\right.\right.}{n}$$

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x,x]
[Out] (2*((1 + I*a*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (I + a*x)/(I -
a*x)] - 2^(n/2)*Hypergeometric2F1[-1/2*n, -1/2*n, 1 - n/2, (1 - I*a*x)/2]))
/(n*(1 - I*a*x)^(n/2))
```

Maple [F]

$$\int \frac{e^{in\arctan(ax)}}{x}dx$$

```
[In] int(exp(I*n*arctan(a*x))/x,x)
[Out] int(exp(I*n*arctan(a*x))/x,x)
```

Fricas [F]

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x,x, algorithm="fricas")
[Out] integral(1/(x*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F]

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{in\,\mathrm{atan}\,(ax)}}{x}\,dx$$

```
[In] integrate(exp(I*n*atan(a*x))/x,x)
[Out] Integral(exp(I*n*atan(a*x))/x, x)
```

Maxima [F]

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x,x, algorithm="maxima")
[Out] integrate(e^(I*n*arctan(a*x))/x, x)
```

Giac [F]

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{x}\,dx$$

```
[In] int(exp(n*atan(a*x)*1i)/x,x)
[Out] int(exp(n*atan(a*x)*1i)/x, x)
```

3.159 $\int \frac{e^{in\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = -\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{2-n}$$

```
[Out] -4*I*a*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1
/2*n],(1-I*a*x)/(1+I*a*x))/(2-n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5170, 133}

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = -\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{iax+1}\right)}{2-n}$$

```
[In] Int[E^(I*n*ArcTan[a*x])/x^2,x]
```

```
[Out] ((-4*I)*a*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[
2, 1 - n/2, 2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^2}\,dx$$
$$= -\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{2-n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx$$
$$= -\frac{2ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{-1+\frac{n}{2}}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},-\frac{1-iax}{-1-iax}\right)}{1-\frac{n}{2}}$$

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x^2,x]
[Out] ((-2*I)*a*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^(-1 + n/2)*Hypergeometric2F1[2,
 1 - n/2, 2 - n/2, -((1 - I*a*x)/(-1 - I*a*x))])/(1 - n/2)
```

Maple [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}dx$$

```
[In] int(exp(I*n*arctan(a*x))/x^2,x)
[Out] int(exp(I*n*arctan(a*x))/x^2,x)
```

Fricas [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^2}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="fricas")
[Out] integral(1/(x^2*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{e^{in\,\mathrm{atan}\,(ax)}}{x^2}\,dx$$

```
[In] integrate(exp(I*n*atan(a*x))/x**2,x)
[Out] Integral(exp(I*n*atan(a*x))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^2}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="maxima")
[Out] integrate(e^(I*n*arctan(a*x))/x^2, x)
```

Giac [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^2}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{x^2}\,dx$$

```
[In] int(exp(n*atan(a*x)*1i)/x^2,x)
[Out] int(exp(n*atan(a*x)*1i)/x^2, x)
```

3.160 $\int \frac{e^{in\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2x^2} + \frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{2-n}$$

```
[Out] -1/2*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/x^2+2*a^2*n*(1-I*a*x)^(1-1/2*n
)*(1+I*a*x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(1-I*a*x)/(1+I*a*x)
)/(2-n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5170, 98, 133}

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{iax+1}\right)}{2-n} - \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{2x^2}$$

```
[In] Int[E^(I*n*ArcTan[a*x])/x^3,x]
```

```
[Out] -1/2*((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/x^2 + (2*a^2*n*(1 - I*
a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n
/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^3}\,dx \\
&= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2x^2}+\frac{1}{2}(ian)\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^2}\,dx \\
&= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2x^2} \\
&\quad +\frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{2-n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.95

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx$$
$$= \frac{(1-iax)^{-n/2}(1+iax)^{n/2}(i+ax)\left(-((-2+n)(-i+ax)^2)+4a^2nx^2\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\right.\right.}{2(-2+n)x^2(-i+ax)}$$

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x^3,x]
```

```
[Out] ((1 + I*a*x)^(n/2)*(I + a*x)*(-((-2 + n)*(-I + a*x)^2) + 4*a^2*n*x^2*Hyperg
eometric2F1[2, 1 - n/2, 2 - n/2, (I + a*x)/(I - a*x)]))/(2*(-2 + n)*x^2*(1
- I*a*x)^(n/2)*(-I + a*x))
```

Maple [F]

$$\int \frac{e^{in\arctan(ax)}}{x^3}dx$$

```
[In] int(exp(I*n*arctan(a*x))/x^3,x)
```

```
[Out] int(exp(I*n*arctan(a*x))/x^3,x)
```

Fricas [F]

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^3}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F]

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \int \frac{e^{in\operatorname{atan}(ax)}}{x^3}\,dx$$

```
[In] integrate(exp(I*n*atan(a*x))/x**3,x)
```

```
[Out] Integral(exp(I*n*atan(a*x))/x**3, x)
```

Maxima [F]

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^3}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="maxima")
[Out] integrate(e^(I*n*arctan(a*x))/x^3, x)
```

Giac [F]

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^3}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{x^3}\,dx$$

```
[In] int(exp(n*atan(a*x)*1i)/x^3,x)
[Out] int(exp(n*atan(a*x)*1i)/x^3, x)
```

3.161 $\int \frac{e^{in \arctan(ax)}}{x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 171

$$\int \frac{e^{in \arctan(ax)}}{x^4} \, dx = -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2} + \frac{2ia^3(2+n^2)\,(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{3(2-n)}$$

```
[Out] -1/3*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/x^3-1/6*I*a*n*(1-I*a*x)^(1-1/2
*n)*(1+I*a*x)^(1+1/2*n)/x^2+2/3*I*a^3*(n^2+2)*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)
^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(1-I*a*x)/(1+I*a*x))/(2-n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5170, 105, 156, 12, 133}

$$\int \frac{e^{in \arctan(ax)}}{x^4} \, dx = \frac{2ia^3(n^2+2)\,(1+iax)^{\frac{n-2}{2}}(1-iax)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{iax+1}\right)}{3(2-n)} - \frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{3x^3} - \frac{ian(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{6x^2}$$

```
[In] Int[E^(I*n*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/3*((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/x^3 - ((I/6)*a*n*(1 -
I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/x^2 + (((2*I)/3)*a^3*(2 + n^2)*(1
 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2,
2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 5170

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^4}\,dx\\
&= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{1}{3}\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}\left(-ian+a^2x\right)}{x^3}\,dx\\
&= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2}\\
&\quad - \frac{1}{6}\int \frac{a^2(2+n^2)\,(1-iax)^{-n/2}(1+iax)^{n/2}}{x^2}\,dx\\
&= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2}\\
&\quad - \frac{1}{6}\left(a^2\left(2+n^2\right)\right)\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^2}\,dx\\
&= -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2}\\
&\quad + \frac{2ia^3(2+n^2)\,(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{3(2-n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

$$
\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = -\frac{(1-iax)^{-n/2}(1+iax)^{\frac{1}{2}(-2+n)}(i+ax)\left(-((-2+n)(-i+ax)^2(-2i+anx))+4a^3(2+n^2)\,x^3\,\text{Hypergeor}\right.}{6(-2+n)x^3}
$$

```
[In] Integrate[E^(I*n*ArcTan[a*x])/x^4,x]
```

```
[Out] -1/6*((1 + I*a*x)^((-2 + n)/2)*(I + a*x)*(-((-2 + n)*(-I + a*x)^2*(-2*I + a
*n*x)) + 4*a^3*(2 + n^2)*x^3*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (I + a*
x)/(I - a*x)]))/((-2 + n)*x^3*(1 - I*a*x)^(n/2))
```

Maple [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}dx$$

[In] int(exp(I*n*arctan(a*x))/x^4,x)

[Out] int(exp(I*n*arctan(a*x))/x^4,x)

Fricas [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^4}\,dx$$

[In] integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="fricas")

[Out] integral(1/(x^4*(-(a*x + I)/(a*x - I))^(1/2*n)), x)

Sympy [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{e^{in\,\mathrm{atan}\,(ax)}}{x^4}\,dx$$

[In] integrate(exp(I*n*atan(a*x))/x**4,x)

[Out] Integral(exp(I*n*atan(a*x))/x**4, x)

Maxima [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^4}\,dx$$

[In] integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="maxima")

[Out] integrate(e^(I*n*arctan(a*x))/x^4, x)

Giac [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^4}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{x^4}\,dx$$

```
[In] int(exp(n*atan(a*x)*1i)/x^4,x)
```

```
[Out] int(exp(n*atan(a*x)*1i)/x^4, x)
```

3.162 $\int e^{i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 276

$$\begin{aligned}\int e^{i\arctan(a+bx)}x^4\,dx &= \frac{(3i+12a-24ia^2-16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5}\\ &\quad-\frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3}+\frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2}\\ &\quad+\frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(19i+114a-86ia^2-96a^3-2(13-14ia-36a^2)\,bx\right)}{120b^5}\\ &\quad+\frac{(3-12ia-24a^2+16ia^3+8a^4)\operatorname{arcsinh}(a+bx)}{8b^5}\end{aligned}$$

```
[Out] 1/8*(3-12*I*a-24*a^2+16*I*a^3+8*a^4)*arcsinh(b*x+a)/b^5-1/20*(I+8*a)*x^2*(1
+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^3+1/5*x^3*(1+I*a+I*b*x)^(3/2)*(1-I*
a-I*b*x)^(1/2)/b^2+1/120*(1+I*a+I*b*x)^(3/2)*(19*I+114*a-86*I*a^2-96*a^3-2*
(13-14*I*a-36*a^2)*b*x)*(1-I*a-I*b*x)^(1/2)/b^5+1/8*(3*I+12*a-24*I*a^2-16*a
^3+8*I*a^4)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{5203, 102, 158, 152, 52, 55, 633, 221\}$

$$\int e^{i\arctan(a+bx)}x^4\,dx$$
$$= \frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(-96a^3-2(-36a^2-14ia+13)\,bx-86ia^2+114a+19i\right)}{120b^5}$$
$$+\frac{(8a^4+16ia^3-24a^2-12ia+3)\operatorname{arcsinh}(a+bx)}{8b^5}$$
$$+\frac{(8ia^4-16a^3-24ia^2+12a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{8b^5}$$
$$-\frac{(8a+i)x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{20b^3}+\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}$$

```
[In] Int[E^(I*ArcTan[a + b*x])*x^4,x]
```

```
[Out] ((3*I + 12*a - (24*I)*a^2 - 16*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt[
1 + I*a + I*b*x])/(8*b^5) - ((I + 8*a)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a +
 I*b*x)^(3/2))/(20*b^3) + (x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2
))/(5*b^2) + (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(19*I + 114*a -
 (86*I)*a^2 - 96*a^3 - 2*(13 - (14*I)*a - 36*a^2)*b*x))/(120*b^5) + ((3 - (
12*I)*a - 24*a^2 + (16*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^5)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$

$$= \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{\int \frac{x^2\sqrt{1+ia+ibx}\left(-3(1+a^2)-(i+8a)bx\right)}{\sqrt{1-ia-ibx}}\,dx}{5b^2}$$

$$= -\frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{\int \frac{x\sqrt{1+ia+ibx}(-2(i-a)(i+a)(i+8a)b-(13-14ia-36a^2)b^2x)}{\sqrt{1-ia-ibx}}dx}{20b^4}$$

$$= -\frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}(19i+114a-86ia^2-96a^3-2(13-14ia-36a^2)bx)}{120b^5} + \frac{(3-12ia-24a^2+16ia^3+8a^4)\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}dx}{8b^4}$$

$$= \frac{(3i+12a-24ia^2-16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}(19i+114a-86ia^2-96a^3-2(13-14ia-36a^2)bx)}{120b^5} + \frac{(3-12ia-24a^2+16ia^3+8a^4)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}dx}{8b^4}$$

$$= \frac{(3i+12a-24ia^2-16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}(19i+114a-86ia^2-96a^3-2(13-14ia-36a^2)bx)}{120b^5} + \frac{(3-12ia-24a^2+16ia^3+8a^4)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}dx}{8b^4}$$

$$= \frac{(3i+12a-24ia^2-16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}(19i+114a-86ia^2-96a^3-2(13-14ia-36a^2)bx)}{120b^5} + \frac{(3-12ia-24a^2+16ia^3+8a^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}dx, x, 2ab+2b^2x\right)}{16b^6}$$

$$
\begin{aligned}
&= \frac{(3i+12a-24ia^2-16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} \\
&\quad - \frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} + \frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} \\
&\quad + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(19i+114a-86ia^2-96a^3-2(13-14ia-36a^2)\,bx\right)}{120b^5} \\
&\quad + \frac{(3-12ia-24a^2+16ia^3+8a^4)\,\text{arcsinh}(a+bx)}{8b^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.79

$$
\int e^{i\arctan(a+bx)}x^4\,dx
$$

$$
= \frac{i\sqrt{1+a^2+2abx+b^2x^2}(64+24a^4+45ibx-32b^2x^2-30ib^3x^3+24b^4x^4+a^3(250i-24bx)+2a^2(-166 \cdots}{120b^5}
$$

$$
+ \frac{\sqrt[4]{-1}(3-12ia-24a^2+16ia^3+8a^4)\sqrt{-ib}\,\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4b^{11/2}}
$$

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x^4,x]
```

```
[Out] ((I/120)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(64 + 24*a^4 + (45*I)*b*x - 32*b
^2*x^2 - (30*I)*b^3*x^3 + 24*b^4*x^4 + a^3*(250*I - 24*b*x) + 2*a^2*(-166 -
 (65*I)*b*x + 12*b^2*x^2) + a*(-275*I + 116*b*x + (70*I)*b^2*x^2 - 24*b^3*x
^3)))/b^5 + ((-1)^(1/4)*(3 - (12*I)*a - 24*a^2 + (16*I)*a^3 + 8*a^4)*Sqrt[(
-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]]
)/(4*b^(11/2))
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.71

method	result
risch	$\frac{i(24x^4b^4-24a\,b^3x^3-30ib^3x^3+24a^2b^2x^2+70ia\,b^2x^2-24a^3bx-130ia^2bx+24a^4+250ia^3-32b^2x^2+116abx+45bxi-332a^2-275ia+64)\cdots}{120b^5}$
default	Expression too large to display

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*I*(24*x^4*b^4-30*I*b^3*x^3-24*a*b^3*x^3+70*I*a*b^2*x^2+24*a^2*b^2*x^2
-130*I*a^2*b*x-24*a^3*b*x+250*I*a^3+24*a^4-32*b^2*x^2+45*I*b*x+116*a*b*x-27
5*I*a-332*a^2+64)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^5+1/8*(3-12*I*a-24*a^2+16
```

```
*I*a^3+8*a^4)/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.64

$$\int e^{i\arctan(a+bx)}x^4\,dx$$
$$= \frac{186i\,a^5 - 1345\,a^4 - 1730i\,a^3 + 1320\,a^2 - 120\,(8\,a^4 + 16i\,a^3 - 24\,a^2 - 12i\,a + 3)\log\left(-bx - a + \sqrt{b^2x^2 + 2}\right.}{}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/960*(186*I*a^5 - 1345*a^4 - 1730*I*a^3 + 1320*a^2 - 120*(8*a^4 + 16*I*a^3
 - 24*a^2 - 12*I*a + 3)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 8*(-24*I*b^4*x^4 + 6*(4*I*a - 5)*b^3*x^3 + 2*(-12*I*a^2 + 35*a + 16*I)*b^2
*x^2 - 24*I*a^4 + 250*a^3 + (24*I*a^3 - 130*a^2 - 116*I*a + 45)*b*x + 332*I
*a^2 - 275*a - 64*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 300*I*a)/b^5
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1222 vs. 2(236) = 472.

Time = 1.86 (sec) , antiderivative size = 1222, normalized size of antiderivative = 4.43

$$\int e^{i\arctan(a+bx)}x^4\,dx = \text{Too large to display}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**4,x)
```

```
[Out] Piecewise(((-a*(-3*a*(-5*a*(-7*a*(-4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b
))/(3*b) - (3*a**2 + 3)*(-4*I*a/5 + 1)/(4*b**2))/(2*b) - (2*a**2 + 2)*(-7*a
*(-4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b**2))/b - (a**2 + 1)*(-5*
a*(-7*a*(-4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b) - (3*a**2 + 3)*(
-4*I*a/5 + 1)/(4*b**2))/(2*b**2))*log(2*a*b + 2*b**2*x + 2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2) + sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1)*(I*x**4/(5*b) + x**3*(-4*I*a/5 + 1)/(4*b**2) + x**2*(-7*a*(-4*I*a/5
 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b**2) + x*(-5*a*(-7*a*(-4*I*a/5 + 1)
/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b) - (3*a**2 + 3)*(-4*I*a/5 + 1)/(4*b**2)
)/(2*b**2) + (-3*a*(-5*a*(-7*a*(-4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))
/(3*b) - (3*a**2 + 3)*(-4*I*a/5 + 1)/(4*b**2))/(2*b) - (2*a**2 + 2)*(-7*a*(
-4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b**2))/b**2), Ne(b**2, 0)),
```

```
((I*(a**8*sqrt(a**2 + 2*a*b*x + 1) + 4*a**6*sqrt(a**2 + 2*a*b*x + 1) + 6*a*
*4*sqrt(a**2 + 2*a*b*x + 1) + 4*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-4*a**2 -
4)*(a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 + 2*a*b*x + 1)**(9/2)/9 + (a**2 +
2*a*b*x + 1)**(5/2)*(6*a**4 + 12*a**2 + 6)/5 + (a**2 + 2*a*b*x + 1)**(3/2)*
(-4*a**6 - 12*a**4 - 12*a**2 - 4)/3 + sqrt(a**2 + 2*a*b*x + 1))/(8*a**3*b**
4) + (a**8*sqrt(a**2 + 2*a*b*x + 1) + 4*a**6*sqrt(a**2 + 2*a*b*x + 1) + 6*a
**4*sqrt(a**2 + 2*a*b*x + 1) + 4*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-4*a**2 -
 4)*(a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 + 2*a*b*x + 1)**(9/2)/9 + (a**2 +
 2*a*b*x + 1)**(5/2)*(6*a**4 + 12*a**2 + 6)/5 + (a**2 + 2*a*b*x + 1)**(3/2)
*(-4*a**6 - 12*a**4 - 12*a**2 - 4)/3 + sqrt(a**2 + 2*a*b*x + 1))/(8*a**4*b*
*4) + I*(-a**10*sqrt(a**2 + 2*a*b*x + 1) - 5*a**8*sqrt(a**2 + 2*a*b*x + 1)
- 10*a**6*sqrt(a**2 + 2*a*b*x + 1) - 10*a**4*sqrt(a**2 + 2*a*b*x + 1) - 5*a
**2*sqrt(a**2 + 2*a*b*x + 1) + (-5*a**2 - 5)*(a**2 + 2*a*b*x + 1)**(9/2)/9
+ (a**2 + 2*a*b*x + 1)**(11/2)/11 + (a**2 + 2*a*b*x + 1)**(7/2)*(10*a**4 +
20*a**2 + 10)/7 + (a**2 + 2*a*b*x + 1)**(5/2)*(-10*a**6 - 30*a**4 - 30*a**2
 - 10)/5 + (a**2 + 2*a*b*x + 1)**(3/2)*(5*a**8 + 20*a**6 + 30*a**4 + 20*a**
2 + 5)/3 - sqrt(a**2 + 2*a*b*x + 1))/(16*a**5*b**4))/(2*a*b), Ne(a*b, 0)),
((I*a*x**5/5 + I*b*x**6/6 + x**5/5)/sqrt(a**2 + 1), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(200) = 400$.

Time = 0.21 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.71

$$\int e^{i\arctan(a+bx)}x^4\,dx = \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}\,x^4}{5\,b} - \frac{9i\sqrt{b^2x^2+2\,abx+a^2+1}\,ax^3}{20\,b^2}$$
$$- \frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,(-i\,a-1)x^3}{4\,b^2}$$
$$+ \frac{21i\sqrt{b^2x^2+2\,abx+a^2+1}\,a^2x^2}{20\,b^3}$$
$$- \frac{7\sqrt{b^2x^2+2\,abx+a^2+1}\,a(i\,a+1)x^2}{12\,b^3}$$
$$- \frac{63i\,a^5\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^5}$$
$$+ \frac{35\,a^4(i\,a+1)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^5}$$
$$- \frac{21i\sqrt{b^2x^2+2\,abx+a^2+1}\,a^3x}{8\,b^4}$$
$$+ \frac{35\sqrt{b^2x^2+2\,abx+a^2+1}\,a^2(i\,a+1)x}{24\,b^4}$$
$$- \frac{4\sqrt{b^2x^2+2\,abx+a^2+1}\,(i\,a^2+i)x^2}{15\,b^3}$$
$$+ \frac{35i\,(a^2+1)a^3\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{4\,b^5}$$
$$- \frac{15\,(a^2+1)a^2(i\,a+1)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{4\,b^5}$$
$$+ \frac{63i\sqrt{b^2x^2+2\,abx+a^2+1}\,a^4}{8\,b^5}$$
$$- \frac{35\sqrt{b^2x^2+2\,abx+a^2+1}\,a^3(i\,a+1)}{8\,b^5}$$
$$+ \frac{161i\sqrt{b^2x^2+2\,abx+a^2+1}\,(a^2+1)ax}{120\,b^4}$$
$$- \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}\,(a^2+1)(i\,a+1)x}{8\,b^4}$$
$$- \frac{15i\,(a^2+1)^2a\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^5}$$
$$- \frac{3\,(a^2+1)^2(-i\,a-1)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^5}$$
$$- \frac{49i\sqrt{b^2x^2+2\,abx+a^2+1}\,(a^2+1)a^2}{8\,b^5}$$
$$+ \frac{55\sqrt{b^2x^2+2\,abx+a^2+1}\,(a^2+1)a(i\,a+1)}{24\,b^5}$$
$$+ \frac{8i\sqrt{b^2x^2+2\,abx+a^2+1}\,(a^2+1)^2}{15\,b^5}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^4/b - 9/20*I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*a*x^3/b^2 - 1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1)*
x^3/b^2 + 21/20*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x^2/b^3 - 7/12*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*x^2/b^3 - 63/8*I*a^5*arcsinh(2*(b
^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 35/8*a^4*(I*a + 1)*ar
csinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 21/8*I*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*x/b^4 + 35/24*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*a^2*(I*a + 1)*x/b^4 - 4/15*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*a^2
+ I)*x^2/b^3 + 35/4*I*(a^2 + 1)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2
 + 4*(a^2 + 1)*b^2))/b^5 - 15/4*(a^2 + 1)*a^2*(I*a + 1)*arcsinh(2*(b^2*x +
a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 63/8*I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*a^4/b^5 - 35/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*(I*a + 1)
/b^5 + 161/120*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a*x/b^4 - 3/8*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*(I*a + 1)*x/b^4 - 15/8*I*(a^2 +
 1)^2*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 3
/8*(a^2 + 1)^2*(-I*a - 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2
+ 1)*b^2))/b^5 - 49/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a^2/b^5
 + 55/24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a*(I*a + 1)/b^5 + 8/15
*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.74

$$\int e^{i\arctan(a+bx)}x^4\,dx =$$

$$-\frac{1}{120}\sqrt{(bx+a)^2+1}\left(\left(2\left(3\,x\left(-\frac{4i\,x}{b}-\frac{-4i\,ab^7+5\,b^7}{b^9}\right)-\frac{12i\,a^2b^6-35\,ab^6-16i\,b^6}{b^9}\right)x-\frac{-24i\,a^3b^5-\cdots}{\cdots}\right.\right.$$

$$-\frac{(8\,a^4+16i\,a^3-24\,a^2-12i\,a+3)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{8\,b^4|b|}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="giac")
```

```
[Out] -1/120*sqrt((b*x + a)^2 + 1)*((2*(3*x*(-4*I*x/b - (-4*I*a*b^7 + 5*b^7)/b^9)
 - (12*I*a^2*b^6 - 35*a*b^6 - 16*I*b^6)/b^9)*x - (-24*I*a^3*b^5 + 130*a^2*b
^5 + 116*I*a*b^5 - 45*b^5)/b^9)*x - (24*I*a^4*b^4 - 250*a^3*b^4 - 332*I*a^2
*b^4 + 275*a*b^4 + 64*I*b^4)/b^9) - 1/8*(8*a^4 + 16*I*a^3 - 24*a^2 - 12*I*a
 + 3)*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{i\arctan(a+bx)}x^4\,dx=\int\frac{x^4\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}{\sqrt{(a+b\,x)^2+1}}\,dx$$

```
[In] int((x^4*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
[Out] int((x^4*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

3.163 $\int e^{i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 201

$$\int e^{i\arctan(a+bx)}x^3\,dx = -\frac{(3-12ia-12a^2+8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(7-10ia-18a^2+2(i+6a)bx\right)}{24b^4} + \frac{(3i+12a-12ia^2-8a^3)\operatorname{arcsinh}(a+bx)}{8b^4}$$

```
[Out] 1/8*(3*I+12*a-12*I*a^2-8*a^3)*arcsinh(b*x+a)/b^4+1/4*x^2*(1+I*a+I*b*x)^(3/2
)*(1-I*a-I*b*x)^(1/2)/b^2-1/24*(1+I*a+I*b*x)^(3/2)*(7-10*I*a-18*a^2+2*(I+6*
a)*b*x)*(1-I*a-I*b*x)^(1/2)/b^4-1/8*(3-12*I*a-12*a^2+8*I*a^3)*(1-I*a-I*b*x)
^(1/2)*(1+I*a+I*b*x)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5203, 102, 152, 52, 55, 633, 221}

$$\int e^{i\arctan(a+bx)}x^3\,dx = -\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(-18a^2+2(6a+i)bx-10ia+7\right)}{24b^4} + \frac{(-8a^3-12ia^2+12a+3i)\operatorname{arcsinh}(a+bx)}{8b^4} - \frac{(8ia^3-12a^2-12ia+3)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{8b^4} + \frac{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b^2}$$

```
[In] Int[E^(I*ArcTan[a + b*x])*x^3,x]
```

```
[Out] -1/8*((3 - (12*I)*a - 12*a^2 + (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*
a + I*b*x])/b^4 + (x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b^
2) - (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(7 - (10*I)*a - 18*a^2
+ 2*(I + 6*a)*b*x))/(24*b^4) + ((3*I + 12*a - (12*I)*a^2 - 8*a^3)*ArcSinh[a
 + b*x])/(8*b^4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx \\
&= \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} + \frac{\int \frac{x\sqrt{1+ia+ibx}(-2(1+a^2)-(i+6a)bx)}{\sqrt{1-ia-ibx}}\,dx}{4b^2} \\
&= \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} \\
&\quad - \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\,(7-10ia-18a^2+2(i+6a)bx)}{24b^4} \\
&\quad + \frac{(3i+12a-12ia^2-8a^3)\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{8b^3} \\
&= -\frac{(3-12ia-12a^2+8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} \\
&\quad + \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} \\
&\quad - \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\,(7-10ia-18a^2+2(i+6a)bx)}{24b^4} \\
&\quad + \frac{(3i+12a-12ia^2-8a^3)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{8b^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(3-12ia-12a^2+8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} \\
&+ \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} \\
&- \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\,(7-10ia-18a^2+2(i+6a)bx)}{24b^4} \\
&+ \frac{(3i+12a-12ia^2-8a^3)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{8b^3} \\
&= -\frac{(3-12ia-12a^2+8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} \\
&+ \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} \\
&- \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\,(7-10ia-18a^2+2(i+6a)bx)}{24b^4} \\
&+ \frac{(3i+12a-12ia^2-8a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{16b^5} \\
&= -\frac{(3-12ia-12a^2+8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} \\
&+ \frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} \\
&- \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\,(7-10ia-18a^2+2(i+6a)bx)}{24b^4} \\
&+ \frac{(3i+12a-12ia^2-8a^3)\,\text{arcsinh}(a+bx)}{8b^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

$$
\int e^{i\arctan(a+bx)}x^3\,dx = \frac{\sqrt{b}\sqrt{1+a^2+2abx+b^2x^2}(-16-6ia^3-9ibx+8b^2x^2+6ib^3x^3+a^2(44+6ibx)+a(39i-20bx-6ib^2x^2))}{24b^{9/2}}
$$

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (Sqrt[b]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-16 - (6*I)*a^3 - (9*I)*b*x + 8
*b^2*x^2 + (6*I)*b^3*x^3 + a^2*(44 + (6*I)*b*x) + a*(39*I - 20*b*x - (6*I)*
b^2*x^2)) - 6*(-1)^(1/4)*(-3*I - 12*a + (12*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*Ar
cSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(24*b^(
9/2))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{i\left(-6b^3x^3+6a\,b^2x^2+8ib^2x^2-6a^2bx-20iabx+6a^3+44ia^2+9bx-39a-16i\right)\sqrt{b^2x^2+2abx+a^2+1}}{24b^4}-\frac{\left(8a^3+12ia^2-12a-3i\right)\ln\left(\frac{b^2x+\cdots}{\sqrt{b\cdots}}\right)}{8b^3\sqrt{\cdots}}$
default	$ib\left(\frac{x^3\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}-\frac{7a\left(\frac{x^2\sqrt{b^2x^2+2abx+a^2+1}}{3b^2}-\frac{5a\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2\cdots}\right)}{b\sqrt{b^2}}\cdots\right)}{2b}\cdots\right)}{3b}\cdots\right)}{\cdots}\cdots\right.$

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*I*(-6*b^3*x^3+8*I*b^2*x^2+6*a*b^2*x^2-20*I*a*b*x-6*a^2*b*x+44*I*a^2+6
*a^3+9*b*x-16*I-39*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^4-1/8*(12*I*a^2+8*a^3
-3*I-12*a)/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.69

$$\int e^{i\arctan(a+bx)}x^3\,dx = \frac{-45i\,a^4+224\,a^3+192i\,a^2+24\left(8\,a^3+12i\,a^2-12\,a-3i\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-8\left(\cdots\right.}{\cdots}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/192*(-45*I*a^4 + 224*a^3 + 192*I*a^2 + 24*(8*a^3 + 12*I*a^2 - 12*a - 3*I)
*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 8*(-6*I*b^3*x^3 + 2*(3
*I*a - 4)*b^2*x^2 + 6*I*a^3 + (-6*I*a^2 + 20*a + 9*I)*b*x - 44*a^2 - 39*I*a
 + 16)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 72*a)/b^4
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(173) = 346$.

Time = 1.60 (sec) , antiderivative size = 857, normalized size of antiderivative = 4.26

$$\int e^{i\arctan(a+bx)}x^3\,dx$$

$$= \begin{cases} \frac{\left(-\frac{a\left(-\frac{3a\left(-\frac{5a\left(-\frac{3ia}{4}+1\right)}{3b}-\frac{i(3a^2+3)}{4b}\right)}{2b}-\frac{(2a^2+2)\left(-\frac{3ia}{4}+1\right)}{3b^2}\right)}{b}-\frac{(a^2+1)\left(-\frac{5a\left(-\frac{3ia}{4}+1\right)}{3b}-\frac{i(3a^2+3)}{4b}\right)}{2b^2}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{\vphantom{b}}\right.}{\sqrt{b^2}} \\ \frac{i\left(-a^6\sqrt{a^2+2abx+1}-3a^4\sqrt{a^2+2abx+1}-3a^2\sqrt{a^2+2abx+1}+\frac{(-3a^2-3)(a^2+2abx+1)^{\frac{5}{2}}}{5}+\frac{(a^2+2abx+1)^{\frac{7}{2}}}{7}+\frac{(a^2+2abx+1)^{\frac{3}{2}}\cdot(3a^4+6a^2+3)}{3}-\sqrt{a^2+2abx+1}\right)}{4a^2b^3} \\ \frac{\frac{iax^4}{4}+\frac{ibx^5}{5}+\frac{x^4}{4}}{\sqrt{a^2+1}} \end{cases}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**3,x)
```

```
[Out] Piecewise(((-a*(-3*a*(-5*a*(-3*I*a/4 + 1)/(3*b) - I*(3*a**2 + 3)/(4*b))/(2*
b) - (2*a**2 + 2)*(-3*I*a/4 + 1)/(3*b**2))/b - (a**2 + 1)*(-5*a*(-3*I*a/4 +
 1)/(3*b) - I*(3*a**2 + 3)/(4*b))/(2*b**2))*log(2*a*b + 2*b**2*x + 2*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2) + sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)*(I*x**3/(4*b) + x**2*(-3*I*a/4 + 1)/(3*b**2) + x*(-5*a*(-
3*I*a/4 + 1)/(3*b) - I*(3*a**2 + 3)/(4*b))/(2*b**2) + (-3*a*(-5*a*(-3*I*a/4
 + 1)/(3*b) - I*(3*a**2 + 3)/(4*b))/(2*b) - (2*a**2 + 2)*(-3*I*a/4 + 1)/(3*
b**2))/b**2), Ne(b**2, 0)), ((I*(-a**6*sqrt(a**2 + 2*a*b*x + 1) - 3*a**4*sq
rt(a**2 + 2*a*b*x + 1) - 3*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-3*a**2 - 3)*(a
**2 + 2*a*b*x + 1)**(5/2)/5 + (a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 + 2*a*b
*x + 1)**(3/2)*(3*a**4 + 6*a**2 + 3)/3 - sqrt(a**2 + 2*a*b*x + 1))/(4*a**2*
b**3) + (-a**6*sqrt(a**2 + 2*a*b*x + 1) - 3*a**4*sqrt(a**2 + 2*a*b*x + 1) -
 3*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-3*a**2 - 3)*(a**2 + 2*a*b*x + 1)**(5/2
)/5 + (a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 + 2*a*b*x + 1)**(3/2)*(3*a**4 +
 6*a**2 + 3)/3 - sqrt(a**2 + 2*a*b*x + 1))/(4*a**3*b**3) + I*(a**8*sqrt(a**
```

```
2 + 2*a*b*x + 1) + 4*a**6*sqrt(a**2 + 2*a*b*x + 1) + 6*a**4*sqrt(a**2 + 2*a
*b*x + 1) + 4*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-4*a**2 - 4)*(a**2 + 2*a*b*x
 + 1)**(7/2)/7 + (a**2 + 2*a*b*x + 1)**(9/2)/9 + (a**2 + 2*a*b*x + 1)**(5/2
)*(6*a**4 + 12*a**2 + 6)/5 + (a**2 + 2*a*b*x + 1)**(3/2)*(-4*a**6 - 12*a**4
 - 12*a**2 - 4)/3 + sqrt(a**2 + 2*a*b*x + 1))/(8*a**4*b**3))/(2*a*b), Ne(a*
b, 0)), ((I*a*x**4/4 + I*b*x**5/5 + x**4/4)/sqrt(a**2 + 1), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(145) = 290.

Time = 0.21 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.63

$$\begin{aligned}\int e^{i\arctan(a+bx)}x^3\,dx = {} & \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}x^3}{4\,b} - \frac{7i\sqrt{b^2x^2+2\,abx+a^2+1}ax^2}{12\,b^2} \\ & - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(-i\,a-1)x^2}{3\,b^2} \\ & + \frac{35i\,a^4\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^4} \\ & - \frac{5\,a^3(i\,a+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^4} \\ & + \frac{35i\sqrt{b^2x^2+2\,abx+a^2+1}a^2x}{24\,b^3} \\ & - \frac{5\sqrt{b^2x^2+2\,abx+a^2+1}a(i\,a+1)x}{6\,b^3} \\ & - \frac{15i\,(a^2+1)a^2\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{4\,b^4} \\ & + \frac{3\,(a^2+1)a(i\,a+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^4} \\ & - \frac{35i\sqrt{b^2x^2+2\,abx+a^2+1}a^3}{8\,b^4} \\ & + \frac{5\sqrt{b^2x^2+2\,abx+a^2+1}a^2(i\,a+1)}{2\,b^4} \\ & - \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}(i\,a^2+i)x}{8\,b^3} \\ & + \frac{3i\,(a^2+1)^2\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^4} \\ & + \frac{55i\sqrt{b^2x^2+2\,abx+a^2+1}(a^2+1)a}{24\,b^4} \\ & - \frac{2\sqrt{b^2x^2+2\,abx+a^2+1}(a^2+1)(i\,a+1)}{3\,b^4}\end{aligned}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^3/b - 7/12*I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*a*x^2/b^2 - 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1)*
x^2/b^2 + 35/8*I*a^4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2))/b^4 - 5/2*a^3*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2))/b^4 + 35/24*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^3 -
5/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*x/b^3 - 15/4*I*(a^2 + 1)*
a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 3/2*(
a^2 + 1)*a*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2))/b^4 - 35/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/b^4 + 5/2*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*a^2*(I*a + 1)/b^4 - 3/8*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(I*a^2 + I)*x/b^3 + 3/8*I*(a^2 + 1)^2*arcsinh(2*(b^2*x + a*b)/sqrt
(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 55/24*I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*(a^2 + 1)*a/b^4 - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*(I*a
 + 1)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.77

$$\int e^{i\arctan(a+bx)}x^3\,dx = -\frac{1}{24}\sqrt{(bx+a)^2+1}\left(\left(2\,x\left(-\frac{3i\,x}{b}-\frac{-3i\,ab^5+4\,b^5}{b^7}\right)-\frac{6i\,a^2b^4-20\,ab^4-9i\,b^4}{b^7}\right)x-\frac{-6i\,a^3b^3+44\,a^2b^3 \cdots}{b^7}\right. + \frac{(8\,a^3+12i\,a^2-12\,a-3i)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{8\,b^3|b|}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] -1/24*sqrt((b*x + a)^2 + 1)*((2*x*(-3*I*x/b - (-3*I*a*b^5 + 4*b^5)/b^7) - (
6*I*a^2*b^4 - 20*a*b^4 - 9*I*b^4)/b^7)*x - (-6*I*a^3*b^3 + 44*a^2*b^3 + 39*
I*a*b^3 - 16*b^3)/b^7) + 1/8*(8*a^3 + 12*I*a^2 - 12*a - 3*I)*log(-a*b - (x*
abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{i\arctan(a+bx)}x^3\,dx = \int \frac{x^3\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}{\sqrt{(a+b\,x)^2+1}}\,dx$$

```
[In] int((x^3*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
[Out] int((x^3*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

3.164 $\int e^{i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 171

$$\int e^{i\arctan(a+bx)}x^2\,dx = -\frac{(i+2a-2ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2} - \frac{(1-2ia-2a^2)\operatorname{arcsinh}(a+bx)}{2b^3}$$

```
[Out] -1/2*(1-2*I*a-2*a^2)*arcsinh(b*x+a)/b^3-1/6*(I+4*a)*(1+I*a+I*b*x)^(3/2)*(1-
I*a-I*b*x)^(1/2)/b^3+1/3*x*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^2-1/2*
(I+2*a-2*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5203, 92, 81, 52, 55, 633, 221}

$$\int e^{i\arctan(a+bx)}x^2\,dx = -\frac{(-2a^2-2ia+1)\operatorname{arcsinh}(a+bx)}{2b^3} - \frac{(-2ia^2+2a+i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^3} - \frac{(4a+i)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{6b^3} + \frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}$$

```
[In] Int[E^(I*ArcTan[a + b*x])*x^2,x]
```

```
[Out] -1/2*((I + 2*a - (2*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b^
3 - ((I + 4*a)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(6*b^3) + (x*
Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(3*b^2) - ((1 - (2*I)*a - 2*
a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx$$

$$= \frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2} + \frac{\int \frac{\sqrt{1+ia+ibx}(-1-a^2-(i+4a)bx)}{\sqrt{1-ia-ibx}}\,dx}{3b^2}$$

$$= -\frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2} - \frac{(1-2ia-2a^2)\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{2b^2}$$

$$= -\frac{(i+2a-2ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2} - \frac{(1-2ia-2a^2)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2b^2}$$

$$= -\frac{(i+2a-2ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2} - \frac{(1-2ia-2a^2)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{2b^2}$$

$$= -\frac{(i+2a-2ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2} - \frac{(1-2ia-2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{4b^4}$$

$$= -\frac{(i+2a-2ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} + \frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2} - \frac{(1-2ia-2a^2)\,\text{arcsinh}(a+bx)}{2b^3}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79

$$\int e^{i\arctan(a+bx)}x^2\,dx = \frac{\sqrt{1+a^2+2abx+b^2x^2}(-4i+2ia^2+3bx+2ib^2x^2+a(-9-2ibx))}{6b^3} + \frac{\sqrt[4]{-1}(-1+2ia+2a^2)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{7/2}}$$

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-4*I + (2*I)*a^2 + 3*b*x + (2*I)*b^2*x^
2 + a*(-9 - (2*I)*b*x)))/(6*b^3) + ((-1)^(1/4)*(-1 + (2*I)*a + 2*a^2)*Sqrt[
(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]
])/b^(7/2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

method	result
risch	$\frac{i(2b^2x^2-2abx-3bxi+2a^2+9ia-4)\sqrt{b^2x^2+2abx+a^2+1}}{6b^3}+\frac{(2a^2+2ia-1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b^2\sqrt{b^2}}$
default	$ib\left(\frac{x^2\sqrt{b^2x^2+2abx+a^2+1}}{3b^2}-\frac{5a\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b}-\frac{(a^2+1)\ln}{}\right.}{3b}\right.$

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*(2*b^2*x^2-3*I*b*x-2*a*b*x+9*I*a+2*a^2-4)*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)/b^3+1/2*(2*I*a+2*a^2-1)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+
a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int e^{i\arctan(a+bx)}x^2\,dx = \frac{7i\,a^3-21\,a^2-12\,(2\,a^2+2i\,a-1)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-4\,\sqrt{b^2x^2+2\,abx+a^2+1}(-2}{24\,b^3}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/24*(7*I*a^3 - 21*a^2 - 12*(2*a^2 + 2*I*a - 1)*log(-b*x - a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)) - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-2*I*b^2*x^2
+ (2*I*a - 3)*b*x - 2*I*a^2 + 9*a + 4*I) - 9*I*a)/b^3
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(133) = 266.

Time = 1.33 (sec) , antiderivative size = 585, normalized size of antiderivative = 3.42

$$\int e^{i\arctan(a+bx)}x^2\,dx = \begin{cases} \frac{\left(-\frac{a\left(-\frac{3a\left(-\frac{2ia}{3}+1\right)}{2b}-\frac{i\left(2a^2+2\right)}{3b}\right)}{b}-\frac{\left(a^2+1\right)\left(-\frac{2ia}{3}+1\right)}{2b^2}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}}+\left(\frac{ix^2}{3b}+\frac{x\left(-\frac{2ia}{3}+1\right)}{2b^2}+\frac{-\frac{3a\left(-\frac{2ia}{3}+\right.}{2b}}{} \right. \\ \frac{\frac{i\left(a^4\sqrt{a^2+2abx+1}+2a^2\sqrt{a^2+2abx+1}+\frac{\left(-2a^2-2\right)\left(a^2+2abx+1\right)^{\frac{3}{2}}}{3}+\frac{\left(a^2+2abx+1\right)^{\frac{5}{2}}}{5}+\sqrt{a^2+2abx+1}\right)}{2ab^2}+\frac{a^4\sqrt{a^2+2abx+1}+2a^2\sqrt{a^2+2abx+1}+\frac{\left(-2a^2-2\right)\left(a^2\right.}{3}}{2a^2b^2}}{} \\ \frac{\frac{iax^3}{3}+\frac{ibx^4}{4}+\frac{x^3}{3}}{\sqrt{a^2+1}} \end{cases}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**2,x)
```

```
[Out] Piecewise((((-a*(-3*a*(-2*I*a/3 + 1)/(2*b) - I*(2*a**2 + 2)/(3*b))/b - (a**2
 + 1)*(-2*I*a/3 + 1)/(2*b**2))*log(2*a*b + 2*b**2*x + 2*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2) + (I*x**2/(3*b) + x*(-2*I*a/3 + 1)
/(2*b**2) + (-3*a*(-2*I*a/3 + 1)/(2*b) - I*(2*a**2 + 2)/(3*b))/b**2)*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1), Ne(b**2, 0)), ((I*(a**4*sqrt(a**2 + 2*a*b*x
 + 1) + 2*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-2*a**2 - 2)*(a**2 + 2*a*b*x + 1
)**(3/2)/3 + (a**2 + 2*a*b*x + 1)**(5/2)/5 + sqrt(a**2 + 2*a*b*x + 1))/(2*a
```

```
*b**2) + (a**4*sqrt(a**2 + 2*a*b*x + 1) + 2*a**2*sqrt(a**2 + 2*a*b*x + 1) +
 (-2*a**2 - 2)*(a**2 + 2*a*b*x + 1)**(3/2)/3 + (a**2 + 2*a*b*x + 1)**(5/2)/
5 + sqrt(a**2 + 2*a*b*x + 1))/(2*a**2*b**2) + I*(-a**6*sqrt(a**2 + 2*a*b*x
+ 1) - 3*a**4*sqrt(a**2 + 2*a*b*x + 1) - 3*a**2*sqrt(a**2 + 2*a*b*x + 1) +
(-3*a**2 - 3)*(a**2 + 2*a*b*x + 1)**(5/2)/5 + (a**2 + 2*a*b*x + 1)**(7/2)/7
 + (a**2 + 2*a*b*x + 1)**(3/2)*(3*a**4 + 6*a**2 + 3)/3 - sqrt(a**2 + 2*a*b*
x + 1))/(4*a**3*b**2))/(2*a*b), Ne(a*b, 0)), ((I*a*x**3/3 + I*b*x**4/4 + x*
*3/3)/sqrt(a**2 + 1), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(119) = 238$.

Time = 0.17 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.05

$$\begin{aligned}\int e^{i\arctan(a+bx)}x^2\,dx &= \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}x^2}{3\,b} - \frac{5i\,a^3\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3}\\&\quad+\frac{3\,a^2(i\,a+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3}\\&\quad-\frac{5i\sqrt{b^2x^2+2\,abx+a^2+1}ax}{6\,b^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}(-i\,a-1)x}{2\,b^2}\\&\quad+\frac{3i\,(a^2+1)a\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3}\\&\quad-\frac{(a^2+1)(i\,a+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3}\\&\quad+\frac{5i\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{2\,b^3}-\frac{3\sqrt{b^2x^2+2\,abx+a^2+1}a(i\,a+1)}{2\,b^3}\\&\quad-\frac{2\sqrt{b^2x^2+2\,abx+a^2+1}(i\,a^2+i)}{3\,b^3}\end{aligned}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^2/b - 5/2*I*a^3*arcsinh(2*(b^2*x
+ a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 3/2*a^2*(I*a + 1)*arcsinh(
2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 5/6*I*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*a*x/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*
a - 1)*x/b^2 + 3/2*I*(a^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
4*(a^2 + 1)*b^2))/b^3 - 1/2*(a^2 + 1)*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 5/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*a^2/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)/b^3 - 2/3*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*a^2 + I)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int e^{i\arctan(a+bx)}x^2\,dx = -\frac{1}{6}\sqrt{(bx+a)^2+1}\left(x\left(-\frac{2i\,x}{b}-\frac{-2i\,ab^3+3\,b^3}{b^5}\right)-\frac{2i\,a^2b^2-9\,ab^2-4i\,b^2}{b^5}\right) - \frac{(2\,a^2+2i\,a-1)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2\,b^2|b|}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="giac")
[Out] -1/6*sqrt((b*x + a)^2 + 1)*(x*(-2*I*x/b - (-2*I*a*b^3 + 3*b^3)/b^5) - (2*I*
a^2*b^2 - 9*a*b^2 - 4*I*b^2)/b^5) - 1/2*(2*a^2 + 2*I*a - 1)*log(-a*b - (x*a
bs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{i\arctan(a+bx)}x^2\,dx = \int \frac{x^2\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}{\sqrt{(a+b\,x)^2+1}}\,dx$$

```
[In] int((x^2*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
[Out] int((x^2*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

3.165 $\int e^{i \arctan(a+bx)} x \, dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int e^{i \arctan(a+bx)} x \, dx = \frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\text{arcsinh}(a+bx)}{2b^2}$$

```
[Out] -1/2*(I+2*a)*arcsinh(b*x+a)/b^2+1/2*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)
/b^2+1/2*(1-2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5203, 81, 52, 55, 633, 221}

$$\int e^{i \arctan(a+bx)} x \, dx = -\frac{(2a+i)\text{arcsinh}(a+bx)}{2b^2} + \frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2} + \frac{(1-2ia)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^2}$$

```
[In] Int[E^(I*ArcTan[a + b*x])*x,x]
```

```
[Out] ((1 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) + (Sqrt
[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b^2) - ((I + 2*a)*ArcSinh[a +
 b*x])/(2*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx \\
&= \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{2b} \\
&= \frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} \\
&\quad + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} \\
&\quad - \frac{(i+2a)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{2b} \\
&= \frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} \\
&\quad - \frac{(i+2a)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{4b^3} \\
&= \frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} \\
&\quad + \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(i+2a)\text{arcsinh}(a+bx)}{2b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int e^{i\arctan(a+bx)}x\,dx &= \frac{(2-ia+ibx)\sqrt{1+a^2+2abx+b^2x^2}}{2b^2} \\
&\quad + \frac{(-1)^{3/4}(i+2a)\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{\sqrt{-ib}b^{3/2}}
\end{aligned}
$$

```
[In] Integrate[E^(I*ArcTan[a + b*x])*x,x]
```

```
[Out] ((2 - I*a + I*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + ((-1)^(3/4)
*(I + 2*a)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)
*b]])/(Sqrt[(-I)*b]*b^(3/2))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{i(-bx+a+2i)\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{(i+2a)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b\sqrt{b^2}}$
default	$ib\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b}-\frac{(a^2+1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2aba\right.}}{2b^2\sqrt{b^2}}\right.$

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(-b*x+a+2*I)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^2-1/2*(I+2*a)/b*ln((b^2
*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int e^{i\arctan(a+bx)}x\,dx = \frac{-3i\,a^2+4\,(2\,a+i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-4\,\sqrt{b^2x^2+2\,abx+a^2+1}(-i\,bx+i\,a-2)+}{8\,b^2}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(-3*I*a^2 + 4*(2*a + I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*b*x + I*a - 2) + 4*a)/b^2
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(85) = 170.

Time = 0.92 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.29

$$\int e^{i\arctan(a+bx)}x\,dx = \begin{cases} \left(\frac{ix}{2b}+\frac{-\frac{ia}{2}+1}{b^2}\right)\sqrt{a^2+2abx+b^2x^2+1}+\frac{\left(-\frac{a\left(-\frac{ia}{2}+1\right)}{b}-\frac{i\left(a^2+1\right)}{2b}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}} \\ \frac{\frac{i\left(-a^2\sqrt{a^2+2abx+1}+\frac{\left(a^2+2abx+1\right)^{\frac{3}{2}}}{3}-\sqrt{a^2+2abx+1}\right)}{b}+\frac{-a^2\sqrt{a^2+2abx+1}+\frac{\left(a^2+2abx+1\right)^{\frac{3}{2}}}{3}-\sqrt{a^2+2abx+1}}{ab}+\frac{i\left(a^4\sqrt{a^2+2abx+1}+2a^2\sqrt{a^2+2abx+1}+\frac{(-2}{}\right.}{}}{2ab} \\ \frac{\frac{iax^2}{2}+\frac{ibx^3}{3}+\frac{x^2}{2}}{\sqrt{a^2+1}} \end{cases}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x,x)
```

```
[Out] Piecewise(((I*x/(2*b) + (-I*a/2 + 1)/b**2)*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1) + (-a*(-I*a/2 + 1)/b - I*(a**2 + 1)/(2*b))*log(2*a*b + 2*b**2*x + 2*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2), Ne(b**2, 0)), ((
I*(-a**2*sqrt(a**2 + 2*a*b*x + 1) + (a**2 + 2*a*b*x + 1)**(3/2)/3 - sqrt(a*
*2 + 2*a*b*x + 1))/b + (-a**2*sqrt(a**2 + 2*a*b*x + 1) + (a**2 + 2*a*b*x +
```

```
1)**(3/2)/3 - sqrt(a**2 + 2*a*b*x + 1))/(a*b) + I*(a**4*sqrt(a**2 + 2*a*b*x
 + 1) + 2*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-2*a**2 - 2)*(a**2 + 2*a*b*x + 1
)**(3/2)/3 + (a**2 + 2*a*b*x + 1)**(5/2)/5 + sqrt(a**2 + 2*a*b*x + 1))/(2*a
**2*b))/(2*a*b), Ne(a*b, 0)), ((I*a*x**2/2 + I*b*x**3/3 + x**2/2)/sqrt(a**2
 + 1), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(76) = 152.

Time = 0.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.90

$$\int e^{i\arctan(a+bx)}x\,dx = \frac{3i\,a^2\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2} - \frac{a(i\,a+1)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^2} + \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}x}{2\,b} - \frac{(i\,a^2+i)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2} - \frac{3i\sqrt{b^2x^2+2\,abx+a^2+1}a}{2\,b^2} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(i\,a+1)}{b^2}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 3/2*I*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 -
 a*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^
2 + 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b - 1/2*(I*a^2 + I)*arcsinh(2
*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 - 3/2*I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a/b^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*a + 1)/b
^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int e^{i\arctan(a+bx)}x\,dx = -\frac{1}{2}\sqrt{(bx+a)^2+1}\left(-\frac{i\,x}{b}+\frac{i\,ab-2\,b}{b^3}\right) + \frac{(2\,a+i)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2\,b|b|}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(-I*x/b + (I*a*b - 2*b)/b^3) + 1/2*(2*a + I)*log
(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{i\arctan(a+bx)}x\,dx=\int \frac{x\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}{\sqrt{{(a+b\,x)}^2+1}}\,dx$$

```
[In] int((x*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
[Out] int((x*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

3.166 $\int e^{i \arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int e^{i \arctan(a+bx)}\,dx = \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \frac{\operatorname{arcsinh}(a+bx)}{b}$$

```
[Out] arcsinh(b*x+a)/b+I*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5201, 52, 55, 633, 221}

$$\int e^{i \arctan(a+bx)}\,dx = \frac{\operatorname{arcsinh}(a+bx)}{b} + \frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

```
[In] Int[E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (I*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[a + b*x]/b
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx \\
&= \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx \\
&= \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx \\
&= \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b^2} \\
&= \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \frac{\operatorname{arcsinh}(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

$$\int e^{i\arctan(a+bx)}\,dx=\frac{i\sqrt{1+(a+bx)^2}+\operatorname{arcsinh}(a+bx)}{b}$$

```
[In] Integrate[E^(I*ArcTan[a + b*x]),x]
[Out] (I*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x])/b
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result
risch	$\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{b}+\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}$
default	$\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}+\frac{ia\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}+ib\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x}\right.}{b\sqrt{b}}\right.$

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int e^{i\arctan(a+bx)}\,dx=\frac{i\,a+2i\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,b}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(I*a + 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*log(-b*x - a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{i\arctan(a+bx)}\,dx = \begin{cases} \frac{i\sqrt{(a+bx)^2+1}+\operatorname{asinh}(a+bx)}{b} & \text{for } b\neq 0 \\ \frac{x(ia+1)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2),x)
[Out] Piecewise((((I*sqrt((a + b*x)**2 + 1) + asinh(a + b*x))/b, Ne(b, 0)), (x*(I*
a + 1)/sqrt(a**2 + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

$$\int e^{i\arctan(a+bx)}\,dx = \frac{\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b} + \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
[Out] arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b + I*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int e^{i\arctan(a+bx)}\,dx = -\frac{\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|} + \frac{i\sqrt{(bx+a)^2+1}}{b}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] -log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/abs(b) + I*sqrt((b*x
 + a)^2 + 1)/b
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.87

$$\int e^{i\arctan(a+bx)}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,1\mathrm{i}}{b}+\frac{\mathrm{asinh}(a+b\,x)}{b}+\frac{a\,\mathrm{asinh}(a+b\,x)\ 1\mathrm{i}}{b} - \frac{a\,b^2\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)\ 1\mathrm{i}}{\left(b^2\right)^{3/2}}$$

[In] int((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2),x)

[Out] ((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)*1i)/b + asinh(a + b*x)/b + (a*asinh(a + b*x)*1i)/b - (a*b^2*log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*x)/(b^2)^(1/2))*1i)/(b^2)^(3/2)

3.167 $\int \frac{e^{i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = i\text{arcsinh}(a+bx) - \frac{2\sqrt{i-a}\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i+a}}$$

```
[Out] I*arcsinh(b*x+a)-2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I
*a-I*b*x)^(1/2))*(I-a)^(1/2)/(I+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5203, 132, 55, 633, 221, 12, 95, 214}

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = i\text{arcsinh}(a+bx) - \frac{2\sqrt{-a+i}\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{a+i}}$$

```
[In] Int[E^(I*ArcTan[a + b*x])/x,x]
```

```
[Out] I*ArcSinh[a + b*x] - (2*Sqrt[I - a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b
*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I + a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+ia+ibx}}{x\sqrt{1-ia-ibx}}\,dx \\
&= (ib)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx + \int \frac{1+ia}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx \\
&= (1+ia)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx + (ib)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx \\
&= (2(1+ia))\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right) \\
&\quad + \frac{i\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b} \\
&= i\text{arcsinh}(a+bx) - \frac{2\sqrt{i-a}\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i+a}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$
\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = \frac{2(-1)^{3/4}\sqrt{-ib}\,\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{\sqrt{b}} - \frac{2\sqrt{-1-ia}\,\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1+ia}}
$$

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*(-1)^(3/4)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a +
b*x)])/Sqrt[(-I)*b]])/Sqrt[b] - (2*Sqrt[-1 - I*a]*ArcTanh[(Sqrt[-1 - I*a]*S
qrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 +
 I*a]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{ib\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\frac{(ia+1)\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{\sqrt{a^2+1}}$	107

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] I*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(
1+I*a)/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(59) = 118.

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.62

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = \sqrt{-\frac{a-i}{a+i}}\log\left(-bx+(i\,a-1)\sqrt{-\frac{a-i}{a+i}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right) - \sqrt{-\frac{a-i}{a+i}}\log\left(-bx+(-i\,a+1)\sqrt{-\frac{a-i}{a+i}} + \sqrt{b^2x^2+2\,abx+a^2+1}\right) - i\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(-(a - I)/(a + I))*log(-b*x + (I*a - 1)*sqrt(-(a - I)/(a + I)) + sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(-(a - I)/(a + I))*log(-b*x + (-I*a + 1)
*sqrt(-(a - I)/(a + I)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - I*log(-b*x -
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
```

Sympy [F]

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = i\left(\int \frac{b}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx + \int \left(-\frac{i}{x\sqrt{a^2+2abx+b^2x^2+1}}\right)dx + \int \frac{a}{x\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] I*(Integral(b/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(-I/(x*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a/(x*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(59) = 118$.

Time = 0.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.62

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = -\frac{i\,a\,\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{\sqrt{a^2+1}} - \frac{\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{\sqrt{a^2+1}} + i\,\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -I*a*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)))/sqrt(a^2 + 1) - arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 +
 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 + 1) + I*arcsinh(2*(b^2
*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = -\frac{(-i\,a-1)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}} - \frac{i\,b\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -(-I*a - 1)*log(abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1)
)/abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/sqrt(a^2 +
1) - I*b*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.33

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = \operatorname{asinh}(a+b\,x)\ 1\mathrm{i} - \frac{\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}} - \frac{a\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)\ 1\mathrm{i}}{\sqrt{a^2+1}}$$

```
[In] int((a*1i + b*x*1i + 1)/(x*((a + b*x)^2 + 1)^(1/2)),x)
```

```
[Out] asinh(a + b*x)*1i - log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2
 + 2*a*b*x + 1)^(1/2))/x)/(a^2 + 1)^(1/2) - (a*log(a*b + (a^2 + 1)/x + ((a^
2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x)*1i)/(a^2 + 1)^(1/2)
```

3.168 $\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1-ia)x} + \frac{2ib\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}(i+a)^{3/2}}$$

```
[Out] 2*I*b*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/
2))/(I+a)^(3/2)/(I-a)^(1/2)-(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1-I*a)
/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5203, 96, 95, 214}

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = \frac{2ib\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}(a+i)^{3/2}} - \frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}$$

```
[In] Int[E^(I*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*x)) + ((2*I)*b*A
rcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*
x])])/(Sqrt[I - a]*(I + a)^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+ia+ibx}}{x^2\sqrt{1-ia-ibx}}\,dx \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1-ia)x} - \frac{b\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{i+a} \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1-ia)x} - \frac{(2b)\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{i+a} \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1-ia)x} + \frac{2ib\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}(i+a)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = -i\left(\frac{\sqrt{1+a^2+2abx+b^2x^2}}{ix+ax}+\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}(-1+ia)^{3/2}}\right)$$

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (-I)*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/(I*x + a*x) + (2*b*ArcTanh[(Sqrt[-1
 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/
(Sqrt[-1 - I*a]*(-1 + I*a)^(3/2)))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{(i+a)x}+\frac{b\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(i+a)\sqrt{a^2+1}}$
default	$-\frac{ib\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{\sqrt{a^2+1}}+(ia+1)\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)x}+\frac{ab\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2}}{x}\right.}{(a^2+1)^{\frac{3}{2}}}\right.$

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -I*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(I+a)/x+1/(I+a)*b/(a^2+1)^(1/2)*ln((2*a^2+
2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(86) = 172.

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.72

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx =$$

$$-\frac{(a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b^2x^2+2\,abx+a^2+1}\,b+(a^3+i\,a^2+a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}}{b}\right)-(a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-}}}{(a+i)x}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -((a + I)*sqrt(b^2/(a^4 + 2*I*a^3 + 2*I*a - 1))*x*log(-(b^2*x - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b + (a^3 + I*a^2 + a + I)*sqrt(b^2/(a^4 + 2*I*a^3 +
2*I*a - 1)))/b) - (a + I)*sqrt(b^2/(a^4 + 2*I*a^3 + 2*I*a - 1))*x*log(-(b^2
*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b - (a^3 + I*a^2 + a + I)*sqrt(b^2/(
a^4 + 2*I*a^3 + 2*I*a - 1)))/b) + I*b*x + I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1))/((a + I)*x)
```

Sympy [F]

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = i\left(\int\left(-\frac{i}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\right)dx + \int\frac{a}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\,dx + \int\frac{b}{x\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] I*(Integral(-I/(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a
/(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b/(x*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(86) = 172.

Time = 0.18 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.84

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = \frac{a(ia+1)b\,\text{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{(a^2+1)^{\frac{3}{2}}} - \frac{i\,b\,\text{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{\sqrt{a^2+1}} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(-i\,a-1)}{(a^2+1)x}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] a*(I*a + 1)*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) +
 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - I*b*arcsinh(2*a*b*x/(sqrt(-4*a^2
*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 + 1) + s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1)/((a^2 + 1)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = \frac{b\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}(a+i)} - \frac{2\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2-a^2-1\right)(i\,a-1)}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(2*x*a
bs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a + I))
 - 2*((x*abs(b) - sqrt((b*x + a)^2 + 1))*a*b + a^2*abs(b) + abs(b))/(((x*ab
s(b) - sqrt((b*x + a)^2 + 1))^2 - a^2 - 1)*(I*a - 1))
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.68

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = \frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)}{\left(a^2+1\right)^{3/2}} - \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,\left(a^2+1\right)} - \frac{b\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)\,1\mathrm{i}}{\sqrt{a^2+1}} + \frac{a^2\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)\,1\mathrm{i}}{\left(a^2+1\right)^{3/2}} - \frac{a\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,1\mathrm{i}}{x\,\left(a^2+1\right)}$$

```
[In] int((a*1i + b*x*1i + 1)/(x^2*((a + b*x)^2 + 1)^(1/2)),x)
```

```
[Out] (a^2*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x +
1)^(1/2)))*1i)/(a^2 + 1)^(3/2) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/(x*(a^
2 + 1)) - (b*log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*
b*x + 1)^(1/2))/x)*1i)/(a^2 + 1)^(1/2) - (a*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(
1/2)*1i)/(x*(a^2 + 1)) + (a*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2
 + b^2*x^2 + 2*a*b*x + 1)^(1/2))))/(a^2 + 1)^(3/2)
```

3.169 $\int \frac{e^{i \arctan(a+bx)}}{x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 201

$$\int \frac{e^{i \arctan(a+bx)}}{x^3} \, dx = -\frac{(1+2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)(i+a)^2x} - \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2(1+a^2)x^2} + \frac{(1+2ia)b^2\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}(i+a)^{5/2}}$$

```
[Out] (1+2*I*a)*b^2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*
b*x)^(1/2))/(I-a)^(3/2)/(I+a)^(5/2)-1/2*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(
1/2)/(a^2+1)/x^2-1/2*(1+2*I*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I
-a)/(I+a)^2/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5203, 98, 96, 95, 214}

$$\int \frac{e^{i \arctan(a+bx)}}{x^3} \, dx = -\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2(a^2+1)x^2} + \frac{(1+2ia)b^2\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{5/2}} - \frac{(1+2ia)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(-a+i)(a+i)^2x}$$

```
[In] Int[E^(I*ArcTan[a + b*x])/x^3,x]
```

```
[Out] -1/2*((1 + (2*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((I - a)
*(I + a)^2*x) - (Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*(1 + a^2
)*x^2) + ((1 + (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sq
rt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(3/2)*(I + a)^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{1+ia+ibx}}{x^3\sqrt{1-ia-ibx}}\,dx$$

$$=-\frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2}+\frac{((i-2a)b)\int\frac{\sqrt{1+ia+ibx}}{x^2\sqrt{1-ia-ibx}}\,dx}{2\left(1+a^2\right)}$$

$$=-\frac{(i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(1-ia)\left(1+a^2\right)x}-\frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2}-\frac{((i-2a)b^2)\int\frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2(i+a)\left(1+a^2\right)}$$

$$=-\frac{(i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(1-ia)\left(1+a^2\right)x}-\frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2}-\frac{((i-2a)b^2)\,\text{Subst}\left(\int\frac{1}{-1-ia-(-1+ia)x^2}\,dx,x,\frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i+a)\left(1+a^2\right)}$$

$$=-\frac{(i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(1-ia)\left(1+a^2\right)x}-\frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2\left(1+a^2\right)x^2}+\frac{(1+2ia)b^2\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}(i+a)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.77

$$\int\frac{e^{i\arctan(a+bx)}}{x^3}\,dx=\frac{-\frac{i\left(1+a^2+2ibx-abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{x^2}+\frac{2(-i+2a)b^2\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}\sqrt{-1+ia}}}{2(-i+a)(i+a)^2}$$

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x^3,x]
```

```
[Out] (((-I)*(1 + a^2 + (2*I)*b*x - a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2
 + (2*(-I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqr
t[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*Sqrt[-1 + I*a]))/(2*(-
I + a)*(I + a)^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{i\left(-a\,b^{3}x^{3}+2ib^{3}x^{3}-a^{2}b^{2}x^{2}+4ia\,b^{2}x^{2}+a^{3}bx+2ia^{2}bx+a^{4}+b^{2}x^{2}+abx+2bxi+2a^{2}+1\right)}{2x^{2}(i+a)^{2}(a-i)\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}-\frac{b^{2}(-i+2a)\ln\left(\frac{2a^{2}+2+2abx+2\sqrt{a^{2}+1}\,\sqrt{b^{2}x^{2}}}{x}\right.}{2(a^{2}+1)^{\frac{3}{2}}(i+a)}$
default	$ib\left(-\frac{\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{(a^{2}+1)x}+\frac{ab\ln\left(\frac{2a^{2}+2+2abx+2\sqrt{a^{2}+1}\,\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{x}\right)}{(a^{2}+1)^{\frac{3}{2}}}\right)+(ia+1)\left(-\frac{\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{2(a^{2}+1)x^{2}}-\frac{3ab}{}\right.$

[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*I*(-a*b^3*x^3-a^2*b^2*x^2+a^3*b*x+2*I*b^3*x^3+a^4+b^2*x^2+4*I*a*b^2*x^2+a*b*x+2*I*a^2*b*x+2*a^2+2*I*b*x+1)/x^2/(I+a)^2/(a-I)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*b^2*(-I+2*a)/(a^2+1)^(3/2)/(I+a)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(135) = 270$.

Time = 0.27 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.25

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx$$
$$=\frac{(i\,a+2)b^{2}x^{2}+\sqrt{\frac{(4\,a^{2}-4i\,a-1)b^{4}}{a^{8}+2i\,a^{7}+2\,a^{6}+6i\,a^{5}+6i\,a^{3}-2\,a^{2}+2i\,a-1}}(a^{3}+i\,a^{2}+a+i)x^{2}\log\left(-\frac{(2\,a-i)b^{3}x-\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}(2\,a-i}{}\right.}{}$$

[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")

[Out] 1/2*((I*a + 2)*b^2*x^2 + sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))*(a^3 + I*a^2 + a + I)*x^2*log(-((2*a - I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - I)*b^2 + (a^5 + I*a^4 + 2*a^3 + 2*I*a^2 + a + I)*sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1)))/((2*a - I)*b^2)) - sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))*(a^3 + I*a^2 + a + I)*x^2*log(-((2*a - I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - I)*b^2 - (a^5 + I*a^4 + 2*a^3 + 2*I*a^2 + a + I)*sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1)))/((2*a - I)*b^2)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((I*a + 2)*b*x - I*a^2 - I))/((a^3 + I*a^2 + a + I)*x^2)

Sympy [F]

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = i\left(\int \left(-\frac{i}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\right)dx + \int \frac{a}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\,dx + \int \frac{b}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**3,x)
```

```
[Out] I*(Integral(-I/(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a
/(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b/(x**2*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(135) = 270.

Time = 0.22 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.11

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = -\frac{3a^2(ia+1)b^2 \operatorname{arsinh}\left(\frac{2abx}{\sqrt{-4a^2b^2+4(a^2+1)b^2}|x|} + \frac{2a^2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}|x|} + \frac{2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}|x|}\right)}{2(a^2+1)^{\frac{5}{2}}} + \frac{iab^2 \operatorname{arsinh}\left(\frac{2abx}{\sqrt{-4a^2b^2+4(a^2+1)b^2}|x|} + \frac{2a^2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}|x|} + \frac{2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}|x|}\right)}{(a^2+1)^{\frac{3}{2}}} - \frac{(-ia-1)b^2 \operatorname{arsinh}\left(\frac{2abx}{\sqrt{-4a^2b^2+4(a^2+1)b^2}|x|} + \frac{2a^2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}|x|} + \frac{2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}|x|}\right)}{2(a^2+1)^{\frac{3}{2}}} + \frac{3\sqrt{b^2x^2+2abx+a^2+1}a(ia+1)b}{2(a^2+1)^2x} - \frac{i\sqrt{b^2x^2+2abx+a^2+1}b}{(a^2+1)x} - \frac{\sqrt{b^2x^2+2abx+a^2+1}(ia+1)}{2(a^2+1)x^2}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*a^2*(I*a + 1)*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*
abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^
2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + I*a*b^2*arcsinh(2*a*b*x
/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2
```

```
 + 1)^(3/2) - 1/2*(-I*a - 1)*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2
+ 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) + 3/2*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*b/((a^2 + 1)^2*x) - I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)*b/((a^2 + 1)*x) - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(
I*a + 1)/((a^2 + 1)*x^2)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(135) = 270$.

Time = 0.36 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.34

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = -\frac{(2\,ab^2 - i\,b^2)\log\left(\left|\frac{2\,x|b| - 2\,\sqrt{(bx+a)^2+1} - 2\,\sqrt{a^2+1}}{2\,x|b| - 2\,\sqrt{(bx+a)^2+1} + 2\,\sqrt{a^2+1}}\right|\right)}{2\,(a^3 + i\,a^2 + a + i)\sqrt{a^2+1}} - \frac{4\left(-i\,x|b| + i\,\sqrt{(bx+a)^2+1}\right)a^4b^2 - 2i\left(x|b| - \sqrt{(bx+a)^2+1}\right)^2 a^3b|b| - 2i\,a^5b|b| + 2\left(x|b| - \sqrt{(bx\right.}{}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b^2 - I*b^2)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqr
t(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/((
a^3 + I*a^2 + a + I)*sqrt(a^2 + 1)) - (4*(-I*x*abs(b) + I*sqrt((b*x + a)^2
+ 1))*a^4*b^2 - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^3*b*abs(b) - 2*I
*a^5*b*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a*b^2 - 2*(x*abs(b)
- sqrt((b*x + a)^2 + 1))*a^3*b^2 + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a
^2*b*abs(b) - 2*a^4*b*abs(b) - I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*b^2 +
 5*(-I*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*a^2*b^2 - 2*I*(x*abs(b) - sqrt((
b*x + a)^2 + 1))^2*a*b*abs(b) - 4*I*a^3*b*abs(b) - 2*(x*abs(b) - sqrt((b*x
+ a)^2 + 1))*a*b^2 + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b*abs(b) - 4*a^
2*b*abs(b) - (I*x*abs(b) - I*sqrt((b*x + a)^2 + 1))*b^2 - 2*I*a*b*abs(b) -
2*b*abs(b))/((a^3 + I*a^2 + a + I)*((x*abs(b) - sqrt((b*x + a)^2 + 1))^2 -
a^2 - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = \int \frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{x^3\,\sqrt{(a+b\,x)^2+1}}\,dx$$

```
[In] int((a*1i + b*x*1i + 1)/(x^3*((a + b*x)^2 + 1)^(1/2)),x)
[Out] int((a*1i + b*x*1i + 1)/(x^3*((a + b*x)^2 + 1)^(1/2)), x)
```

3.170 $\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 283

$$\begin{aligned}\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx = &-\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3}\\ &-\frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2}\\ &+\frac{\left(4+9ia-2a^2\right)b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)^2x}\\ &+\frac{\left(2a-i\left(1-2a^2\right)\right)b^3\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}(i+a)^{7/2}}\end{aligned}$$

```
[Out] (2*a-I*(-2*a^2+1))*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/
(1-I*a-I*b*x)^(1/2))/(I-a)^(5/2)/(I+a)^(7/2)-1/3*(1-I*a-I*b*x)^(1/2)*(1+I*a
+I*b*x)^(1/2)/(1-I*a)/x^3-1/6*(3*I-2*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)
^(1/2)/(1-I*a)/(a^2+1)/x^2+1/6*(4+9*I*a-2*a^2)*b^2*(1-I*a-I*b*x)^(1/2)*(1+I
*a+I*b*x)^(1/2)/(1-I*a)/(a^2+1)^2/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

$= \{5203, 101, 156, 12, 95, 214\}$

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx = \frac{(2a-i(1-2a^2))\,b^3\mathrm{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{7/2}} + \frac{(-2a^2+9ia+4)\,b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1-ia)\,(a^2+1)^2\,x} - \frac{(-2a+3i)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1-ia)\,(a^2+1)\,x^2} - \frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1-ia)x^3}$$

```
[In] Int[E^(I*ArcTan[a + b*x])/x^4,x]
```

```
[Out] -1/3*(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*x^3) - ((3*I
- 2*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(6*(1 - I*a)*(1 + a^2
)*x^2) + ((4 + (9*I)*a - 2*a^2)*b^2*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*
b*x])/(6*(1 - I*a)*(1 + a^2)^2*x) + ((2*a - I*(1 - 2*a^2))*b^3*ArcTanh[(Sqr
t[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I -
 a)^(5/2)*(I + a)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+ia+ibx}}{x^4\sqrt{1-ia-ibx}}\,dx \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} + \frac{\int \frac{(3i-2a)b-2b^2x}{x^3\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{3(1-ia)} \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} \\
&\quad - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} - \frac{\int \frac{\left(4+9ia-2a^2\right)b^2+(3i-2a)b^3x}{x^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{6(1-ia)\left(1+a^2\right)} \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} \\
&\quad + \frac{\left(4+9ia-2a^2\right)b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)^2x} + \frac{\int -\frac{3\left(i-2a-2ia^2\right)b^3}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{6(1-ia)\left(1+a^2\right)^2} \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)x^2} \\
&\quad + \frac{\left(4+9ia-2a^2\right)b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\left(1+a^2\right)^2x} \\
&\quad + \frac{\left(\left(1+2ia-2a^2\right)b^3\right)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2(i-a)^2(i+a)^3}
\end{aligned}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)(1+a^2)x^2} + \frac{(4+9ia-2a^2)b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)(1+a^2)^2x} + \frac{((1+2ia-2a^2)b^3)\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i-a)^2(i+a)^3}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} - \frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)(1+a^2)x^2} + \frac{(4+9ia-2a^2)b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)(1+a^2)^2x} + \frac{(2a-i(1-2a^2))\,b^3\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}(i+a)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.83

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx$$

$$= \frac{\frac{2(1-ia)(-i+a)(-i+a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{x^3} + \frac{(1+4ia)b(-i+a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{x^2} + 3i(1+2ia-2a^2)\,b^2\left(\frac{\sqrt{1+a^2+2abx+b^2x^2}}{ix+ax}\right.}{6(1+a^2)^2}$$

```
[In] Integrate[E^(I*ArcTan[a + b*x])/x^4,x]
```

```
[Out] ((2*(1 - I*a)*(-I + a)*(-I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^
3 + ((1 + (4*I)*a)*b*(-I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2
+ (3*I)*(1 + (2*I)*a - 2*a^2)*b^2*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/(I*x +
 a*x) + (2*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I
*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*(-1 + I*a)^(3/2))))/(6*(1 + a^
2)^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{i\left(2a^2b^4x^4-9ia\,b^4x^4+2a^3b^3x^3-15ia^2b^3x^3-3ia^3b^2x^2-4x^4b^4+2a^5bx+3ia^4bx-10a\,b^3x^3+3ib^3x^3+2a^6-2a^2b^2x^2-3ia\,b^2x^2+4a^3bx+\right.}{6x^3(a-i)^2(i+a)^3\sqrt{b^2x^2+2abx+a^2+1}}$
default	$ib\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)x^2}-\frac{3ab\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)x}+\frac{ab\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(a^2+1)^{\frac{3}{2}}}\right)}{2(a^2+1)}+\frac{b^2\ln\left(\frac{2a^2+2+2abx\right.}{}\right.$

```
[In] int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*(3*I*a^4*b*x+2*a^2*b^4*x^4-3*I*a*b^2*x^2+2*a^3*b^3*x^3-9*I*a*b^4*x^4
-4*x^4*b^4-15*I*a^2*b^3*x^3-3*I*a^3*b^2*x^2+2*a^5*b*x-10*a*b^3*x^3+6*I*a^2*
b*x+2*a^6-2*a^2*b^2*x^2+3*I*b^3*x^3+4*a^3*b*x+6*a^4-2*b^2*x^2+3*I*b*x+2*a*b
*x+6*a^2+2)/x^3/(a-I)^2/(I+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*b^3*(-2*I
*a+2*a^2-1)/(a^2+1)^(5/2)/(I+a)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^
2+2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(198) = 396.

Time = 0.29 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.44

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx$$

$$= \frac{(-2i\,a^2-9\,a+4i)b^3x^3-3\sqrt{\frac{(4\,a^4-8i\,a^3-8\,a^2+4i\,a+1)b^6}{a^{12}+2i\,a^{11}+4\,a^{10}+10i\,a^9+5\,a^8+20i\,a^7+20i\,a^5-5\,a^4+10i\,a^3-4\,a^2+2i\,a-1}}(a^5+i\,a^4+2\,a^3+2i\,\ldots}{}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((-2*I*a^2 - 9*a + 4*I)*b^3*x^3 - 3*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I
*a + 1)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*
a^5 - 5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1))*(a^5 + I*a^4 + 2*a^3 + 2*I*a^2
```

```
 + a + I)*x^3*log(-((2*a^2 - 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(2*a^2 - 2*I*a - 1)*b^3 + (a^7 + I*a^6 + 3*a^5 + 3*I*a^4 + 3*a^3 + 3
*I*a^2 + a + I)*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I*a + 1)*b^6/(a^12 + 2*I*
a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*a^5 - 5*a^4 + 10*I*a^3 -
 4*a^2 + 2*I*a - 1)))/((2*a^2 - 2*I*a - 1)*b^3)) + 3*sqrt((4*a^4 - 8*I*a^3
- 8*a^2 + 4*I*a + 1)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*
I*a^7 + 20*I*a^5 - 5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1))*(a^5 + I*a^4 + 2*
a^3 + 2*I*a^2 + a + I)*x^3*log(-((2*a^2 - 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*(2*a^2 - 2*I*a - 1)*b^3 - (a^7 + I*a^6 + 3*a^5 + 3*I*a^
4 + 3*a^3 + 3*I*a^2 + a + I)*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I*a + 1)*b^6
/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*a^5 - 5*a^4
 + 10*I*a^3 - 4*a^2 + 2*I*a - 1)))/((2*a^2 - 2*I*a - 1)*b^3)) + ((-2*I*a^2
- 9*a + 4*I)*b^2*x^2 - 2*I*a^4 + (2*I*a^3 + 3*a^2 + 2*I*a + 3)*b*x - 4*I*a^
2 - 2*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((a^5 + I*a^4 + 2*a^3 + 2*I*a^2
 + a + I)*x^3)
```

Sympy [F]

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx = i\left(\int \left(-\frac{i}{x^4\sqrt{a^2+2abx+b^2x^2+1}}\right)dx + \int \frac{a}{x^4\sqrt{a^2+2abx+b^2x^2+1}}\,dx + \int \frac{b}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**4,x)
```

```
[Out] I*(Integral(-I/(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a
/(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b/(x**3*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(198) = 396$.

Time = 0.21 (sec) , antiderivative size = 644, normalized size of antiderivative = 2.28

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx = \frac{5\,a^3(i\,a+1)b^3\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{7}{2}}} - \frac{3i\,a^2b^3\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{5}{2}}} - \frac{3\,a(i\,a+1)b^3\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{5}{2}}} + \frac{i\,b^3\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{3}{2}}} - \frac{5\,\sqrt{b^2x^2+2\,abx+a^2+1}a^2(i\,a+1)b^2}{2\,(a^2+1)^3x} + \frac{3i\,\sqrt{b^2x^2+2\,abx+a^2+1}ab^2}{2\,(a^2+1)^2x} - \frac{2\,\sqrt{b^2x^2+2\,abx+a^2+1}(-i\,a-1)b^2}{3\,(a^2+1)^2x} + \frac{5\,\sqrt{b^2x^2+2\,abx+a^2+1}a(i\,a+1)b}{6\,(a^2+1)^2x^2} - \frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}b}{2\,(a^2+1)x^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(i\,a+1)}{3\,(a^2+1)x^3}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/2*a^3*(I*a + 1)*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*a
bs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2
*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 3/2*I*a^2*b^3*arcsinh(2*
a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2
+ 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))
/(a^2 + 1)^(5/2) - 3/2*a*(I*a + 1)*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x))
 + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + 1/2*I*b
^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2)*abs(x)))/(a^2 + 1)^(3/2) - 5/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*(
I*a + 1)*b^2/((a^2 + 1)^3*x) + 3/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^
2/((a^2 + 1)^2*x) - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1)*b^2/((
a^2 + 1)^2*x) + 5/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*b/((a^2 +
 1)^2*x^2) - 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b/((a^2 + 1)*x^2) - 1/
3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*a + 1)/((a^2 + 1)*x^3)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(198) = 396$.

Time = 0.34 (sec) , antiderivative size = 884, normalized size of antiderivative = 3.12

$$\int \frac{e^{i\arctan(a+bx)}}{x^4} dx = \text{Too large to display}$$

```
[In] integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*b^3 - 2*I*a*b^3 - b^3)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 +
 1) - 2*sqrt(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^
2 + 1)))/((a^5 + I*a^4 + 2*a^3 + 2*I*a^2 + a + I)*sqrt(a^2 + 1)) + 1/3*(8*I
*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^5*b^3 + 24*(I*x*abs(b) - I*sqrt((b*
x + a)^2 + 1))*a^7*b^3 + 24*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^6*b^2*
abs(b) + 8*I*a^8*b^2*abs(b) + 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*a^2*b^
3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^4*b^3 + 18*(x*abs(b) - sqrt((
b*x + a)^2 + 1))*a^6*b^3 - 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^5*b^2*
abs(b) + 12*a^7*b^2*abs(b) - 6*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*a*b^3
 + 32*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^3*b^3 + 54*(I*x*abs(b) - I*s
qrt((b*x + a)^2 + 1))*a^5*b^3 + 60*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a
^4*b^2*abs(b) + 20*I*a^6*b^2*abs(b) - 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^
5*b^3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^2*b^3 + 39*(x*abs(b) - sq
rt((b*x + a)^2 + 1))*a^4*b^3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^3*
b^2*abs(b) + 36*a^5*b^2*abs(b) + 24*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*
a*b^3 + 36*(I*x*abs(b) - I*sqrt((b*x + a)^2 + 1))*a^3*b^3 + 48*I*(x*abs(b)
- sqrt((b*x + a)^2 + 1))^2*a^2*b^2*abs(b) + 12*I*a^4*b^2*abs(b) + 24*(x*abs
(b) - sqrt((b*x + a)^2 + 1))*a^2*b^3 - 12*(x*abs(b) - sqrt((b*x + a)^2 + 1)
)^2*a*b^2*abs(b) + 36*a^3*b^2*abs(b) + 6*(I*x*abs(b) - I*sqrt((b*x + a)^2 +
 1))*a*b^3 + 12*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b^2*abs(b) - 4*I*a^2
*b^2*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*b^3 + 12*a*b^2*abs(b) -
4*I*b^2*abs(b))/((a^5 + I*a^4 + 2*a^3 + 2*I*a^2 + a + I)*((x*abs(b) - sqrt(
(b*x + a)^2 + 1))^2 - a^2 - 1)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i\arctan(a+bx)}}{x^4} dx = \int \frac{1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i}}{x^4\,\sqrt{(a+b\,x)^2+1}}\,dx$$

```
[In] int((a*1i + b*x*1i + 1)/(x^4*((a + b*x)^2 + 1)^(1/2)),x)
```

```
[Out] int((a*1i + b*x*1i + 1)/(x^4*((a + b*x)^2 + 1)^(1/2)), x)
```

3.171 $\int e^{2i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int e^{2i\arctan(a+bx)}x^4\,dx=-\frac{2(1-ia)^3x}{b^4}+\frac{i(i+a)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}+\frac{2i(i+a)^4\log(i+a+bx)}{b^5}$$

```
[Out] -2*(1-I*a)^3*x/b^4+I*(I+a)^2*x^2/b^3+2/3*(1-I*a)*x^3/b^2+1/2*I*x^4/b-1/5*x^
5+2*I*(I+a)^4*ln(I+a+b*x)/b^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int e^{2i\arctan(a+bx)}x^4\,dx=\frac{2i(a+i)^4\log(a+bx+i)}{b^5}-\frac{2(1-ia)^3x}{b^4}+\frac{i(a+i)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}$$

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] (-2*(1 - I*a)^3*x)/b^4 + (I*(I + a)^2*x^2)/b^3 + (2*(1 - I*a)*x^3)/(3*b^2)
+ ((I/2)*x^4)/b - x^5/5 + ((2*I)*(I + a)^4*Log[I + a + b*x])/b^5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^4(1+ia+ibx)}{1-ia-ibx}\,dx\\ &= \int \left(\frac{2(-1+ia)^3}{b^4}+\frac{2i(i+a)^2x}{b^3}+\frac{2(1-ia)x^2}{b^2}+\frac{2ix^3}{b}-x^4+\frac{2i(i+a)^4}{b^4(i+a+bx)}\right)dx\\ &= -\frac{2(1-ia)^3x}{b^4}+\frac{i(i+a)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}+\frac{2i(i+a)^4\log(i+a+bx)}{b^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{2(1-ia)^3x}{b^4}+\frac{i(i+a)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}+\frac{2i(i+a)^4\log(i+a+bx)}{b^5}$$

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x^4,x]
[Out] (-2*(1 - I*a)^3*x)/b^4 + (I*(I + a)^2*x^2)/b^3 + (2*(1 - I*a)*x^3)/(3*b^2)
+ ((I/2)*x^4)/b - x^5/5 + ((2*I)*(I + a)^4*Log[I + a + b*x])/b^5
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(78) = 156.

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.74

method	result
parallelrisch	$\frac{-6x^5b^5+60i\ln(bx+a+i)+15ix^4b^4+60i\ln(bx+a+i)a^4-240\ln(bx+a+i)a^3-20ix^3ab^3-60ixa^3b+20b^3x^3+30ix^2a^2b^2-60ab^2x^2-}{30b^5}$
default	$-\frac{i\left(-\frac{1}{5}ib^4x^5-\frac{1}{2}b^3x^4+\frac{2}{3}ib^2x^3+\frac{2}{3}ab^2x^3-2iabx^2-a^2bx^2+6ia^2x+2a^3x+x^2b-2ix-6ax\right)}{b^4}+\frac{\left(2ia^4b-8a^3b-12ia^2b+8ab+2ib\right)\ln(b}{2b^2}$
risch	$-\frac{x^5}{5}+\frac{8i\arctan(bx+a)a^3}{b^5}+\frac{2x^3}{3b^2}+\frac{ia^2x^2}{b^3}-\frac{2ax^2}{b^3}+\frac{i\ln(b^2x^2+2abx+a^2+1)a^4}{b^5}+\frac{6a^2x}{b^4}-\frac{8i\arctan(bx+a)a}{b^5}+\frac{ix^4}{2b}$

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(-6*x^5*b^5+60*I*ln(I+a+b*x)+15*I*x^4*b^4+60*I*ln(I+a+b*x)*a^4-240*ln(
I+a+b*x)*a^3-20*I*x^3*a*b^3-60*I*x*a^3*b+20*b^3*x^3+30*I*x^2*a^2*b^2-60*a*b
^2*x^2+240*ln(I+a+b*x)*a-360*I*ln(I+a+b*x)*a^2+180*I*x*a*b+180*a^2*b*x-30*I
*x^2*b^2-60*b*x)/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.14

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{6\,b^5x^5-15i\,b^4x^4+20\,(i\,a-1)b^3x^3+30\,(-i\,a^2+2\,a+i)b^2x^2+60\,(i\,a^3-3\,a^2-3i\,a+1)bx+60\,(-i\,a^4}{30\,b^5}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 - 15*I*b^4*x^4 + 20*(I*a - 1)*b^3*x^3 + 30*(-I*a^2 + 2*a +
 I)*b^2*x^2 + 60*(I*a^3 - 3*a^2 - 3*I*a + 1)*b*x + 60*(-I*a^4 + 4*a^3 + 6*I
*a^2 - 4*a - I)*log((b*x + a + I)/b))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{x^5}{5}-x^3\cdot\left(\frac{2ia}{3b^2}-\frac{2}{3b^2}\right)-x^2\left(-\frac{ia^2}{b^3}+\frac{2a}{b^3}+\frac{i}{b^3}\right) -x\left(\frac{2ia^3}{b^4}-\frac{6a^2}{b^4}-\frac{6ia}{b^4}+\frac{2}{b^4}\right)+\frac{ix^4}{2b}+\frac{2i(a+i)^4\log(a+bx+i)}{b^5}$$

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**4,x)
```

```
[Out] -x**5/5 - x**3*(2*I*a/(3*b**2) - 2/(3*b**2)) - x**2*(-I*a**2/b**3 + 2*a/b**
3 + I/b**3) - x*(2*I*a**3/b**4 - 6*a**2/b**4 - 6*I*a/b**4 + 2/b**4) + I*x**
4/(2*b) + 2*I*(a + I)**4*log(a + b*x + I)/b**5
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(70) = 140$.

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{6\,b^4x^5 - 15i\,b^3x^4 + 20\,(i\,a-1)b^2x^3 + 30\,(-i\,a^2+2\,a+i)bx^2 + 60\,(i\,a^3-3\,a^2-3i\,a+1)x}{30\,b^4} + \frac{2\,(a^4+4i\,a^3-6\,a^2-4i\,a+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5} + \frac{(i\,a^4-4\,a^3-6i\,a^2+4\,a+i)\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^5}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 - 15*I*b^3*x^4 + 20*(I*a - 1)*b^2*x^3 + 30*(-I*a^2 + 2*a +
 I)*b*x^2 + 60*(I*a^3 - 3*a^2 - 3*I*a + 1)*x)/b^4 + 2*(a^4 + 4*I*a^3 - 6*a^
2 - 4*I*a + 1)*arctan((b^2*x + a*b)/b)/b^5 + (I*a^4 - 4*a^3 - 6*I*a^2 + 4*a
 + I)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{2\,(-i\,a^4+4\,a^3+6i\,a^2-4\,a-i)\log\left(bx+a+i\right)}{b^5} - \frac{6\,b^5x^5 - 15i\,b^4x^4 + 20i\,ab^3x^3 - 30i\,a^2b^2x^2 - 20\,b^3x^3 + 60i\,a^3bx + 60\,ab^2x^2 - 180\,a^2bx + 30i\,b^2x^2 - 180}{30\,b^5}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="giac")
```

```
[Out] -2*(-I*a^4 + 4*a^3 + 6*I*a^2 - 4*a - I)*log(b*x + a + I)/b^5 - 1/30*(6*b^5*
x^5 - 15*I*b^4*x^4 + 20*I*a*b^3*x^3 - 30*I*a^2*b^2*x^2 - 20*b^3*x^3 + 60*I*
a^3*b*x + 60*a*b^2*x^2 - 180*a^2*b*x + 30*I*b^2*x^2 - 180*I*a*b*x + 60*b*x)
/b^5
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.18

$$\int e^{2i\arctan(a+bx)}x^4\,dx = \ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(\frac{8\,a-8\,a^3}{b^5}+\frac{(2\,a^4-12\,a^2+2)\,1\mathrm{i}}{b^5}\right) - x^4\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{4\,b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{4\,b}\right)-\frac{x^5}{5} + \frac{x^2\,(-1+a\,1\mathrm{i})^2\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)}{2\,b^2} - \frac{x^3\,(-1+a\,1\mathrm{i})\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)\,1\mathrm{i}}{3\,b} + \frac{x\,(-1+a\,1\mathrm{i})^3\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)\,1\mathrm{i}}{b^3}$$

```
[In] int((x^4*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

```
[Out] log(x + (a + 1i)/b)*((8*a - 8*a^3)/b^5 + ((2*a^4 - 12*a^2 + 2)*1i)/b^5) - x
^4*(((a*1i - 1)*1i)/(4*b) - ((a*1i + 1)*1i)/(4*b)) - x^5/5 + (x^2*(a*1i - 1
)^2*(((a*1i - 1)*1i)/b - ((a*1i + 1)*1i)/b))/(2*b^2) - (x^3*(a*1i - 1)*(((a
*1i - 1)*1i)/b - ((a*1i + 1)*1i)/b)*1i)/(3*b) + (x*(a*1i - 1)^3*(((a*1i - 1
)*1i)/b - ((a*1i + 1)*1i)/b)*1i)/b^3
```

3.172 $\int e^{2i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int e^{2i\arctan(a+bx)}x^3\,dx=\frac{2i(i+a)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1-ia)^3\log(i+a+bx)}{b^4}$$

```
[Out] 2*I*(I+a)^2*x/b^3+(1-I*a)*x^2/b^2+2/3*I*x^3/b-1/4*x^4-2*(1-I*a)^3*ln(I+a+b*
x)/b^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int e^{2i\arctan(a+bx)}x^3\,dx=-\frac{2(1-ia)^3\log(a+bx+i)}{b^4}+\frac{2i(a+i)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}$$

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] ((2*I)*(I + a)^2*x)/b^3 + ((1 - I*a)*x^2)/b^2 + (((2*I)/3)*x^3)/b - x^4/4 -
 (2*(1 - I*a)^3*Log[I + a + b*x])/b^4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3(1+ia+ibx)}{1-ia-ibx}\,dx \\
&= \int \left(\frac{2i(i+a)^2}{b^3}+\frac{2(1-ia)x}{b^2}+\frac{2ix^2}{b}-x^3+\frac{2(-1+ia)^3}{b^3(i+a+bx)}\right)dx \\
&= \frac{2i(i+a)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1-ia)^3\log(i+a+bx)}{b^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$
\int e^{2i\arctan(a+bx)}x^3\,dx = \frac{2i(i+a)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1-ia)^3\log(i+a+bx)}{b^4}
$$

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] ((2*I)*(I + a)^2*x)/b^3 + ((1 - I*a)*x^2)/b^2 + (((2*I)/3)*x^3)/b - x^4/4 -
 (2*(1 - I*a)^3*Log[I + a + b*x])/b^4
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.51

method	result
parallelrisch	$-\frac{3x^4b^4-8ib^3x^3+12ia\,b^2x^2-72\ln(bx+a+i)a^2+24i\ln(bx+a+i)a^3-24ia^2bx-12b^2x^2+24\ln(bx+a+i)-72i\ln(bx+a+i)a+24bxi}{12b^4}$
default	$\frac{i\left(\frac{1}{4}ib^3x^4+\frac{2}{3}b^2x^3-ib\,x^2-ab\,x^2+4iax+2a^2x-2x\right)}{b^3}+\frac{\left(-2ia^3b+6a^2b+6iab-2b\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(-2ia^4+4a^3+2i+4a-\left(-2ia\right.\right.}{b^3}$
risch	$-\frac{x^4}{4}+\frac{2ix^3}{3b}+\frac{x^2}{b^2}-\frac{ia\,x^2}{b^2}-\frac{4ax}{b^3}+\frac{2ia^2x}{b^3}-\frac{2ix}{b^3}+\frac{3\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{b^4}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^3}{b^4}-\frac{\ln(}{}$

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(3*x^4*b^4-8*I*x^3*b^3+12*I*a*b^2*x^2-72*ln(I+a+b*x)*a^2+24*I*ln(I+a+
b*x)*a^3-24*I*x*a^2*b-12*b^2*x^2+24*ln(I+a+b*x)-72*I*ln(I+a+b*x)*a+24*I*x*b
+48*a*b*x)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -\frac{3\,b^4x^4 - 8i\,b^3x^3 + 12\,(i\,a-1)b^2x^2 + 24\,(-i\,a^2+2\,a+i)bx + 24\,(i\,a^3-3\,a^2-3i\,a+1)\log\left(\frac{bx+a+i}{b}\right)}{12\,b^4}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 - 8*I*b^3*x^3 + 12*(I*a - 1)*b^2*x^2 + 24*(-I*a^2 + 2*a +
I)*b*x + 24*(I*a^3 - 3*a^2 - 3*I*a + 1)*log((b*x + a + I)/b))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -\frac{x^4}{4} - x^2\left(\frac{ia}{b^2}-\frac{1}{b^2}\right) - x\left(-\frac{2ia^2}{b^3}+\frac{4a}{b^3}+\frac{2i}{b^3}\right) + \frac{2ix^3}{3b} - \frac{2i(a+i)^3\log(a+bx+i)}{b^4}$$

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**3,x)
```

```
[Out] -x**4/4 - x**2*(I*a/b**2 - 1/b**2) - x*(-2*I*a**2/b**3 + 4*a/b**3 + 2*I/b**
3) + 2*I*x**3/(3*b) - 2*I*(a + I)**3*log(a + b*x + I)/b**4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(56) = 112.

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -\frac{3\,b^3x^4 - 8i\,b^2x^3 + 12\,(i\,a-1)bx^2 + 24\,(-i\,a^2+2\,a+i)x}{12\,b^3} - \frac{2\,(a^3+3i\,a^2-3\,a-i)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4} + \frac{(-i\,a^3+3\,a^2+3i\,a-1)\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^4}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^3*x^4 - 8*I*b^2*x^3 + 12*(I*a - 1)*b*x^2 + 24*(-I*a^2 + 2*a + I)
*x)/b^3 - 2*(a^3 + 3*I*a^2 - 3*a - I)*arctan((b^2*x + a*b)/b)/b^4 + (-I*a^3
 + 3*a^2 + 3*I*a - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -\frac{2\left(i\,a^3-3\,a^2-3i\,a+1\right)\log\left(bx+a+i\right)}{b^4} - \frac{3\,b^4x^4-8i\,b^3x^3+12i\,ab^2x^2-24i\,a^2bx-12\,b^2x^2+48\,abx+24i\,bx}{12\,b^4}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] -2*(I*a^3 - 3*a^2 - 3*I*a + 1)*log(b*x + a + I)/b^4 - 1/12*(3*b^4*x^4 - 8*I
*b^3*x^3 + 12*I*a*b^2*x^2 - 24*I*a^2*b*x - 12*b^2*x^2 + 48*a*b*x + 24*I*b*x
)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.12

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -x^3\left(\frac{(-1+a\,1\mathrm{i})\ 1\mathrm{i}}{3\,b}-\frac{(1+a\,1\mathrm{i})\ 1\mathrm{i}}{3\,b}\right)-\frac{x^4}{4} + \ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(\frac{6\,a^2-2}{b^4}+\frac{(6\,a-2\,a^3)\ 1\mathrm{i}}{b^4}\right) - \frac{x^2\,(-1+a\,1\mathrm{i})\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)\ 1\mathrm{i}}{2\,b} + \frac{x\,(-1+a\,1\mathrm{i})^2\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)}{b^2}$$

```
[In] int((x^3*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

```
[Out] log(x + (a + 1i)/b)*(((6*a - 2*a^3)*1i)/b^4 + (6*a^2 - 2)/b^4) - x^4/4 - x^
3*(((a*1i - 1)*1i)/(3*b) - ((a*1i + 1)*1i)/(3*b)) - (x^2*(a*1i - 1)*(((a*1i
 - 1)*1i)/b - ((a*1i + 1)*1i)/b)*1i)/(2*b) + (x*(a*1i - 1)^2*(((a*1i - 1)*1
i)/b - ((a*1i + 1)*1i)/b))/b^2
```

3.173 $\int e^{2i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int e^{2i\arctan(a+bx)}x^2\,dx=\frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}+\frac{2i(i+a)^2\log(i+a+bx)}{b^3}$$

```
[Out] 2*(1-I*a)*x/b^2+I*x^2/b-1/3*x^3+2*I*(I+a)^2*ln(I+a+b*x)/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int e^{2i\arctan(a+bx)}x^2\,dx=\frac{2i(a+i)^2\log(a+bx+i)}{b^3}+\frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}$$

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (2*(1 - I*a)*x)/b^2 + (I*x^2)/b - x^3/3 + ((2*I)*(I + a)^2*Log[I + a + b*x]
)/b^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2(1+ia+ibx)}{1-ia-ibx}\,dx\\&= \int \left(-\frac{2i(i+a)}{b^2}+\frac{2ix}{b}-x^2+\frac{2i(i+a)^2}{b^2(i+a+bx)}\right)dx\\&= \frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}+\frac{2i(i+a)^2\log(i+a+bx)}{b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(a+bx)}x^2\,dx = \frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}+\frac{2i(i+a)^2\log(i+a+bx)}{b^3}$$

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x^2,x]
[Out] (2*(1 - I*a)*x)/b^2 + (I*x^2)/b - x^3/3 + ((2*I)*(I + a)^2*Log[I + a + b*x]
)/b^3
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

method	result
parallelrisch	$\frac{-b^3x^3+3ix^2b^2-12\ln(bx+a+i)a+6i\ln(bx+a+i)a^2-6iabx-6i\ln(bx+a+i)+6bx}{3b^3}$
default	$\frac{i\left(\frac{1}{3}ib^2x^3+x^2b-2ix-2ax\right)}{b^2}+\frac{\frac{\left(2ia^2b-4ab-2ib\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(2ia^3+2ia-2a^2-2-\frac{\left(2ia^2b-4ab-2ib\right)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^2}$
risch	$-\frac{x^3}{3}+\frac{ix^2}{b}+\frac{2x}{b^2}-\frac{2iax}{b^2}-\frac{2\ln\left(b^2x^2+2abx+a^2+1\right)a}{b^3}+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{b^3}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)}{b^3}+$ 4i ar

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x,method=_RETURNVERBOSE)
[Out] 1/3*(-b^3*x^3+3*I*x^2*b^2-12*ln(I+a+b*x)*a+6*I*ln(I+a+b*x)*a^2-6*I*x*a*b-6*
I*ln(I+a+b*x)+6*b*x)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{2i\arctan(a+bx)}x^2\,dx = -\frac{b^3x^3 - 3i\,b^2x^2 + 6\,(i\,a-1)bx + 6\,(-i\,a^2 + 2\,a + i)\log\left(\frac{bx+a+i}{b}\right)}{3\,b^3}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 3*I*b^2*x^2 + 6*(I*a - 1)*b*x + 6*(-I*a^2 + 2*a + I)*log((b
*x + a + I)/b))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int e^{2i\arctan(a+bx)}x^2\,dx = -\frac{x^3}{3} - x\left(\frac{2ia}{b^2} - \frac{2}{b^2}\right) + \frac{ix^2}{b} + \frac{2i(a+i)^2\log(a+bx+i)}{b^3}$$

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**2,x)
```

```
[Out] -x**3/3 - x*(2*I*a/b**2 - 2/b**2) + I*x**2/b + 2*I*(a + I)**2*log(a + b*x +
 I)/b**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(42) = 84.

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int e^{2i\arctan(a+bx)}x^2\,dx = -\frac{b^2x^3 - 3i\,bx^2 + 6\,(i\,a-1)x}{3\,b^2} + \frac{2\,(a^2 + 2i\,a - 1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3} + \frac{(i\,a^2 - 2\,a - i)\log(b^2x^2 + 2\,abx + a^2 + 1)}{b^3}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^2*x^3 - 3*I*b*x^2 + 6*(I*a - 1)*x)/b^2 + 2*(a^2 + 2*I*a - 1)*arctan
((b^2*x + a*b)/b)/b^3 + (I*a^2 - 2*a - I)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/
b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{2i\arctan(a+bx)}x^2\,dx = -\frac{2\left(-i\,a^2+2\,a+i\right)\log\left(bx+a+i\right)}{b^3} - \frac{b^3x^3-3i\,b^2x^2+6i\,abx-6\,bx}{3\,b^3}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] -2*(-I*a^2 + 2*a + I)*log(b*x + a + I)/b^3 - 1/3*(b^3*x^3 - 3*I*b^2*x^2 + 6
*I*a*b*x - 6*b*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.98

$$\int e^{2i\arctan(a+bx)}x^2\,dx = -\ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(\frac{4\,a}{b^3}-\frac{\left(2\,a^2-2\right)\,1\mathrm{i}}{b^3}\right) - x^2\left(\frac{\left(-1+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}-\frac{\left(1+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}\right)-\frac{x^3}{3} - \frac{x\left(-1+a\,1\mathrm{i}\right)\left(\frac{\left(-1+a\,1\mathrm{i}\right)\,1\mathrm{i}}{b}-\frac{\left(1+a\,1\mathrm{i}\right)\,1\mathrm{i}}{b}\right)\,1\mathrm{i}}{b}$$

```
[In] int((x^2*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

```
[Out] - log(x + (a + 1i)/b)*((4*a)/b^3 - ((2*a^2 - 2)*1i)/b^3) - x^2*(((a*1i - 1)
*1i)/(2*b) - ((a*1i + 1)*1i)/(2*b)) - x^3/3 - (x*(a*1i - 1)*(((a*1i - 1)*1i
)/b - ((a*1i + 1)*1i)/b)*1i)/b
```

3.174 $\int e^{2i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int e^{2i\arctan(a+bx)}x\,dx=\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1-ia)\log(i+a+bx)}{b^2}$$

```
[Out] 2*I*x/b-1/2*x^2+2*(1-I*a)*ln(I+a+b*x)/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5203, 78}

$$\int e^{2i\arctan(a+bx)}x\,dx=\frac{2(1-ia)\log(a+bx+i)}{b^2}+\frac{2ix}{b}-\frac{x^2}{2}$$

```
[In] Int[E^((2*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((2*I)*x)/b - x^2/2 + (2*(1 - I*a)*Log[I + a + b*x])/b^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(1+ia+ibx)}{1-ia-ibx}\,dx\\ &= \int \left(\frac{2i}{b}-x+\frac{2(1-ia)}{b(i+a+bx)}\right)dx\\ &= \frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1-ia)\log(i+a+bx)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(a+bx)}x\,dx=\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1-ia)\log(i+a+bx)}{b^2}$$

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])*x,x]
[Out] ((2*I)*x)/b - x^2/2 + (2*(1 - I*a)*Log[I + a + b*x])/b^2
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$-\frac{b^2x^2-4\ln(bx+a+i)+4i\ln(bx+a+i)a-4bxi}{2b^2}$	41
risch	$-\frac{x^2}{2}+\frac{2ix}{b}+\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{b^2}-\frac{2i\arctan(bx+a)}{b^2}-\frac{ia\ln\left(b^2x^2+2abx+a^2+1\right)}{b^2}-\frac{2a\arctan(bx+a)}{b^2}$	85
default	$\frac{-\frac{1}{2}x^2b+2ix}{b}+\frac{\frac{(-2iab+2b)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(-2ia^2-2i-\frac{(-2iab+2b)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b}$	99

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x,method=_RETURNVERBOSE)
[Out] -1/2*(b^2*x^2-4*ln(I+a+b*x)+4*I*ln(I+a+b*x)*a-4*I*x*b)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int e^{2i\arctan(a+bx)}x\,dx = -\frac{b^2x^2 - 4i\,bx + 4\,(i\,a-1)\log\left(\frac{bx+a+i}{b}\right)}{2\,b^2}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 - 4*I*b*x + 4*(I*a - 1)*log((b*x + a + I)/b))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int e^{2i\arctan(a+bx)}x\,dx = -\frac{x^2}{2} + \frac{2ix}{b} - \frac{2i(a+i)\log(a+bx+i)}{b^2}$$

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x,x)
[Out] -x**2/2 + 2*I*x/b - 2*I*(a + I)*log(a + b*x + I)/b**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(29) = 58.

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int e^{2i\arctan(a+bx)}x\,dx = -\frac{bx^2 - 4i\,x}{2\,b} - \frac{2\,(a+i)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2} + \frac{(-i\,a+1)\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{b^2}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="maxima")
[Out] -1/2*(b*x^2 - 4*I*x)/b - 2*(a + I)*arctan((b^2*x + a*b)/b)/b^2 + (-I*a + 1)
*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int e^{2i\arctan(a+bx)}x\,dx = -\frac{2\,(i\,a-1)\log\left(bx+a+i\right)}{b^2} - \frac{b^2x^2-4i\,bx}{2\,b^2}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="giac")
```

```
[Out] -2*(I*a - 1)*log(b*x + a + I)/b^2 - 1/2*(b^2*x^2 - 4*I*b*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2i\arctan(a+bx)}x\,dx = -\ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(-\frac{2}{b^2}+\frac{a\,2\mathrm{i}}{b^2}\right) - x\left(\frac{(-1+a\,1\mathrm{i})\;1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\;1\mathrm{i}}{b}\right)-\frac{x^2}{2}$$

```
[In] int((x*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

```
[Out] - log(x + (a + 1i)/b)*((a*2i)/b^2 - 2/b^2) - x*(((a*1i - 1)*1i)/b - ((a*1i
+ 1)*1i)/b) - x^2/2
```

3.175 $\int e^{2i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2i\log(i+a+bx)}{b}$$

```
[Out] -x+2*I*ln(I+a+b*x)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5201, 45}

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2i\log(a+bx+i)}{b}$$

```
[In] Int[E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x + ((2*I)*Log[I + a + b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
```

```
c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1+ia+ibx}{1-ia-ibx}\,dx \\ &= \int \left(-1+\frac{2i}{i+a+bx}\right)dx \\ &= -x+\frac{2i\log(i+a+bx)}{b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int e^{2i\arctan(a+bx)}\,dx = -x+\frac{2\arctan(a+bx)}{b}+\frac{i\log\left(1+(a+bx)^2\right)}{b}$$

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x]),x]
[Out] -x + (2*ArcTan[a + b*x])/b + (I*Log[1 + (a + b*x)^2])/b
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{2i\ln(bx+a+i)-bx}{b}$	21
risch	$-x+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)}{b}+\frac{2\arctan(bx+a)}{b}$	40
default	$-x+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)}{b}+\frac{2\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}$	51

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] (2*I*ln(I+a+b*x)-b*x)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int e^{2i\arctan(a+bx)}\,dx = -\frac{bx - 2i\,\log\left(\frac{bx+a+i}{b}\right)}{b}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="fricas")
[Out] -(b*x - 2*I*log((b*x + a + I)/b))/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2i\log\left(a+bx+i\right)}{b}$$

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2),x)
[Out] -x + 2*I*log(a + b*x + I)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2\,\arctan\left(\frac{b^2x+ab}{b}\right)}{b} + \frac{i\,\log\left(b^2x^2+2\,abx+a^2+1\right)}{b}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="maxima")
[Out] -x + 2*arctan((b^2*x + a*b)/b)/b + I*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2i\,\log\left(bx + a + i\right)}{b}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="giac")
[Out] -x + 2*I*log(b*x + a + I)/b
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{\ln\left(x + \frac{a+1\mathrm{i}}{b}\right)\,2\mathrm{i}}{b}$$

```
[In] int((a*1i + b*x*1i + 1)^2/((a + b*x)^2 + 1),x)
[Out] (log(x + (a + 1i)/b)*2i)/b - x
```

3.176 $\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx=\frac{(i-a)\log(x)}{i+a}-\frac{2\log(i+a+bx)}{1-ia}$$

```
[Out] (I-a)*ln(x)/(I+a)-2*ln(I+a+b*x)/(1-I*a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx=\frac{(-a+i)\log(x)}{a+i}-\frac{2\log(a+bx+i)}{1-ia}$$

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] ((I - a)*Log[x])/(I + a) - (2*Log[I + a + b*x])/(1 - I*a)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+ia+ibx}{x(1-ia-ibx)}\,dx \\ &= \int \left(\frac{i-a}{(i+a)x}-\frac{2ib}{(i+a)(i+a+bx)}\right)dx \\ &= \frac{(i-a)\log(x)}{i+a}-\frac{2\log(i+a+bx)}{1-ia}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = -\frac{(-i+a)\log(x)+2i\log(i+a+bx)}{i+a}$$

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x,x]
[Out] -(((-I + a)*Log[x] + (2*I)*Log[I + a + b*x])/(I + a))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result	size
parallelrisch	$\frac{-2\ln(bx+a+i)+2ia\ln(x)-2i\ln(bx+a+i)a-a^2\ln(x)+\ln(x)}{a^2+1}$	47
risch	$\frac{i\ln(-x)}{i+a}-\frac{\ln(-x)a}{i+a}-\frac{i\ln(b^2x^2+2abx+a^2+1)}{i+a}-\frac{2\arctan(bx+a)}{i+a}$	69
default	$\frac{(-a^2+2ia+1)\ln(x)}{a^2+1}-\frac{2b\left(\frac{(iab+b)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{\left(ia^2-i+2a-\frac{(iab+b)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{a^2+1}$	110

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x,method=_RETURNVERBOSE)
[Out] (-2*ln(I+a+b*x)+2*I*a*ln(x)-2*I*ln(I+a+b*x)*a-a^2*ln(x)+ln(x))/(a^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = -\frac{(a-i)\log{(x)} + 2i\,\log\left(\frac{bx+a+i}{b}\right)}{a+i}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="fricas")
[Out] -((a - I)*log(x) + 2*I*log((b*x + a + I)/b))/(a + I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(24) = 48.

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.63

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = -\frac{(a-i)\log\left(-\frac{a^2(a-i)}{a+i} + a^2 - \frac{2ia(a-i)}{a+i} + x(ab-3ib) + \frac{a-i}{a+i} + 1\right)}{a+i} - \frac{2i\log\left(a^2 - \frac{2ia^2}{a+i} + \frac{4a}{a+i} + x(ab-3ib) + 1 + \frac{2i}{a+i}\right)}{a+i}$$

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x,x)
[Out] -(a - I)*log(-a**2*(a - I)/(a + I) + a**2 - 2*I*a*(a - I)/(a + I) + x*(a*b
- 3*I*b) + (a - I)/(a + I) + 1)/(a + I) - 2*I*log(a**2 - 2*I*a**2/(a + I) +
 4*a/(a + I) + x*(a*b - 3*I*b) + 1 + 2*I/(a + I))/(a + I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(29) = 58.

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = -\frac{2\,(a-i)\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1} - \frac{(i\,a+1)\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{a^2+1} - \frac{(a^2-2i\,a-1)\log{(x)}}{a^2+1}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="maxima")
[Out] -2*(a - I)*arctan((b^2*x + a*b)/b)/(a^2 + 1) - (I*a + 1)*log(b^2*x^2 + 2*a*
b*x + a^2 + 1)/(a^2 + 1) - (a^2 - 2*I*a - 1)*log(x)/(a^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = -\frac{2i\,b\log(bx+a+i)}{ab+i\,b} - \frac{(a-i)\log(|x|)}{a+i}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="giac")
[Out] -2*I*b*log(b*x + a + I)/(a*b + I*b) - (a - I)*log(abs(x))/(a + I)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = \ln(x)\left(-1+\frac{2\mathrm{i}}{a+1\mathrm{i}}\right) - \frac{\ln(a+b\,x+1\mathrm{i})\;2\mathrm{i}}{a+1\mathrm{i}}$$

```
[In] int((a*1i + b*x*1i + 1)^2/(x*((a + b*x)^2 + 1)),x)
[Out] log(x)*(2i/(a + 1i) - 1) - (log(a + b*x + 1i)*2i)/(a + 1i)
```

3.177 $\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = -\frac{i-a}{(i+a)x} - \frac{2ib\log(x)}{(i+a)^2} + \frac{2ib\log(i+a+bx)}{(i+a)^2}$$

```
[Out] (-I+a)/(I+a)/x-2*I*b*ln(x)/(I+a)^2+2*I*b*ln(I+a+b*x)/(I+a)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = -\frac{2ib\log(x)}{(a+i)^2} + \frac{2ib\log(a+bx+i)}{(a+i)^2} - \frac{-a+i}{(a+i)x}$$

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -((I - a)/((I + a)*x)) - ((2*I)*b*Log[x])/(I + a)^2 + ((2*I)*b*Log[I + a +
b*x])/(I + a)^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+ia+ibx}{x^2(1-ia-ibx)}\,dx \\
&= \int \left(\frac{i-a}{(i+a)x^2} - \frac{2ib}{(i+a)^2x} + \frac{2ib^2}{(i+a)^2(i+a+bx)}\right) dx \\
&= -\frac{i-a}{(i+a)x} - \frac{2ib\log(x)}{(i+a)^2} + \frac{2ib\log(i+a+bx)}{(i+a)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{1+a^2-2ibx\log(x)+2ibx\log(i+a+bx)}{(i+a)^2x}$$

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x^2,x]
[Out] (1 + a^2 - (2*I)*b*x*Log[x] + (2*I)*b*x*Log[I + a + b*x])/((I + a)^2*x)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(45) = 90.

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.75

method	result
parallelrisch	$-\frac{2i\ln(x)x\,a^2b-2i\ln(bx+a+i)x\,a^2b+1-2ib\ln(x)x+4\ln(x)xab+2ib\ln(bx+a+i)x-4\ln(bx+a+i)xab+2ia^3-a^4+2ia}{(a^2+1)^2x}$
default	$-\frac{-a^2+2ia+1}{(a^2+1)x}-\frac{2b(ia^2+2a-i)\ln(x)}{(a^2+1)^2}+\frac{2b^2\left(\frac{(ia^2b+2ab-ib)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{\left(ia^3-3ia+3a^2-1-\frac{(ia^2b+2ab-ib)a}{b}\right)\text{arcta}}{b}\right.}{(a^2+1)^2}$
risch	$-\frac{i}{(i+a)x}+\frac{a}{(i+a)x}-\frac{b\ln(4a^4b^2x^2+8a^5bx+4a^6+8a^2b^2x^2+16a^3bx+12a^4+4b^2x^2+8abx+12a^2+4)}{ia^2-2a-i}+\frac{2ib\arctan\left(\frac{(2a^2b+2b}{2a}\right.}{ia^2-2a-}$

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(2*I*ln(x)*x*a^2*b-2*I*ln(I+a+b*x)*x*a^2*b+1-2*I*b*ln(x)*x+4*ln(x)*x*a*b+2
*I*b*ln(I+a+b*x)*x-4*ln(I+a+b*x)*x*a*b+2*I*a^3-a^4+2*I*a)/(a^2+1)^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{-2i\,bx\log\left(x\right) + 2i\,bx\log\left(\frac{bx+a+i}{b}\right) + a^2 + 1}{(a^2 + 2i\,a - 1)x}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] (-2*I*b*x*log(x) + 2*I*b*x*log((b*x + a + I)/b) + a^2 + 1)/((a^2 + 2*I*a -
1)*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(39) = 78.

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.84

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = -\frac{2ib\log\left(-\frac{2a^3b}{(a+i)^2} - \frac{6ia^2b}{(a+i)^2} + 2ab + \frac{6ab}{(a+i)^2} + 4b^2x + 2ib + \frac{2ib}{(a+i)^2}\right)}{(a+i)^2} + \frac{2ib\log\left(\frac{2a^3b}{(a+i)^2} + \frac{6ia^2b}{(a+i)^2} + 2ab - \frac{6ab}{(a+i)^2} + 4b^2x + 2ib - \frac{2ib}{(a+i)^2}\right)}{(a+i)^2} - \frac{-a+i}{x\,(a+i)}$$

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**2,x)
```

```
[Out] -2*I*b*log(-2*a**3*b/(a + I)**2 - 6*I*a**2*b/(a + I)**2 + 2*a*b + 6*a*b/(a
+ I)**2 + 4*b**2*x + 2*I*b + 2*I*b/(a + I)**2)/(a + I)**2 + 2*I*b*log(2*a**
3*b/(a + I)**2 + 6*I*a**2*b/(a + I)**2 + 2*a*b - 6*a*b/(a + I)**2 + 4*b**2*
x + 2*I*b - 2*I*b/(a + I)**2)/(a + I)**2 - (-a + I)/(x*(a + I))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{2\,(a^2-2i\,a-1)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2\,a^2+1} + \frac{(i\,a^2+2\,a-i)b\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^4+2\,a^2+1} - \frac{2\,(i\,a^2+2\,a-i)b\log\left(x\right)}{a^4+2\,a^2+1} + \frac{a^2-2i\,a-1}{(a^2+1)x}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*(a^2 - 2*I*a - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + (I*a^2 +
2*a - I)*b*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*(I*a^2 +
2*a - I)*b*log(x)/(a^4 + 2*a^2 + 1) + (a^2 - 2*I*a - 1)/((a^2 + 1)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{2\,b^2\log\left(bx+a+i\right)}{-i\,a^2b+2\,ab+i\,b} + \frac{2\,b\log\left(|x|\right)}{i\,a^2-2\,a-i} + \frac{a^2+1}{(a+i)^2x}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] 2*b^2*log(b*x + a + I)/(-I*a^2*b + 2*a*b + I*b) + 2*b*log(abs(x))/(I*a^2 -
2*a - I) + (a^2 + 1)/((a + I)^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.78

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{a-\mathrm{i}}{x\,(a+1\mathrm{i})} + \frac{b\,\mathrm{atanh}\left(\frac{a^2+a\,2\mathrm{i}-1}{(a+1\mathrm{i})^2}-\frac{x\,(2\,a^4\,b^2+4\,a^2\,b^2+2\,b^2)}{(a+1\mathrm{i})^2\,(-b\,a^3+1\mathrm{i}\,b\,a^2-b\,a+b\,1\mathrm{i})}\right)\,4\mathrm{i}}{(a+1\mathrm{i})^2}$$

```
[In] int((a*1i + b*x*1i + 1)^2/(x^2*((a + b*x)^2 + 1)),x)
```

```
[Out] (a - 1i)/(x*(a + 1i)) + (b*atanh((a*2i + a^2 - 1)/(a + 1i)^2 - (x*(2*b^2 +
4*a^2*b^2 + 2*a^4*b^2))/((a + 1i)^2*(b*1i - a*b + a^2*b*1i - a^3*b)))*4i)/(
a + 1i)^2
```

3.178 $\int \frac{e^{2i \arctan(a+bx)}}{x^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{e^{2i \arctan(a+bx)}}{x^3} dx = -\frac{i-a}{2(i+a)x^2} + \frac{2ib}{(i+a)^2x} - \frac{2b^2 \log(x)}{(1-ia)^3} + \frac{2b^2 \log(i+a+bx)}{(1-ia)^3}$$

```
[Out] 1/2*(-I+a)/(I+a)/x^2+2*I*b/(I+a)^2/x-2*b^2*ln(x)/(1-I*a)^3+2*b^2*ln(I+a+b*x
)/(1-I*a)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int \frac{e^{2i \arctan(a+bx)}}{x^3} dx = -\frac{2b^2 \log(x)}{(1-ia)^3} + \frac{2b^2 \log(a+bx+i)}{(1-ia)^3} + \frac{2ib}{(a+i)^2x} - \frac{-a+i}{2(a+i)x^2}$$

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] -1/2*(I - a)/((I + a)*x^2) + ((2*I)*b)/((I + a)^2*x) - (2*b^2*Log[x])/(1 -
I*a)^3 + (2*b^2*Log[I + a + b*x])/(1 - I*a)^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+ia+ibx}{x^3(1-ia-ibx)}\,dx \\
&= \int \left(\frac{i-a}{(i+a)x^3} - \frac{2ib}{(i+a)^2x^2} + \frac{2ib^2}{(i+a)^3x} - \frac{2ib^3}{(i+a)^3(i+a+bx)}\right)dx \\
&= -\frac{i-a}{2(i+a)x^2} + \frac{2ib}{(i+a)^2x} - \frac{2b^2\log(x)}{(1-ia)^3} + \frac{2b^2\log(i+a+bx)}{(1-ia)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = \frac{(i+a)\left(1+a^2+4ibx\right)+4ib^2x^2\log(x)-4ib^2x^2\log(i+a+bx)}{2(i+a)^3x^2}$$

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x^3,x]
[Out] ((I + a)*(1 + a^2 + (4*I)*b*x) + (4*I)*b^2*x^2*Log[x] - (4*I)*b^2*x^2*Log[I
 + a + b*x])/(2*(I + a)^3*x^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(65) = 130.

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.64

method	result
parallelrisch	$\frac{4ia^4bx+12i\ln(bx+a+i)x^2ab^2-1+4i\ln(x)x^2a^3b^2+12\ln(x)x^2a^2b^2-4i\ln(bx+a+i)x^2a^3b^2-12\ln(bx+a+i)x^2a^2b^2-4ia^3-12i\ln(\cdots}{2(a^4+2a^2+1)(a^2+1)x^2}$
default	$-\frac{-a^2+2ia+1}{2(a^2+1)x^2}+\frac{2b(ia^2+2a-i)}{(a^2+1)^2x}+\frac{2b^2(ia^3+3a^2-3ia-1)\ln(x)}{(a^2+1)^3}-\frac{2b^3\left(\frac{(ia^3b+3a^2b-3iab-b)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{(ia^4-6\cdots}{\cdots}\right)}{(a^{\cdots}}$
risch	$\frac{\frac{2ibx}{a^2+2ia-1}+\frac{a-i}{2i+2a}}{x^2}+\frac{b^2\ln(4a^8b^2x^2+8a^9bx+4a^{10}+16a^6b^2x^2+32a^7bx+20a^8+24a^4b^2x^2+48a^5bx+40a^6+16a^2b^2x^2+32a^3bx+4\cdots}{ia^3-3a^2-3ia+1}$

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(4*I*x*a^4*b+12*I*ln(I+a+b*x)*x^2*a*b^2-1+4*I*ln(x)*x^2*a^3*b^2+12*ln(x
)*x^2*a^2*b^2-4*I*ln(I+a+b*x)*x^2*a^3*b^2-12*ln(I+a+b*x)*x^2*a^2*b^2-4*I*a^
3-12*I*ln(x)*x^2*a*b^2+a^6-4*ln(x)*x^2*b^2+4*ln(I+a+b*x)*x^2*b^2+8*a^3*b*x-
2*I*a^5+a^4-2*I*a+8*a*b*x-4*I*x*b-a^2)/(a^4+2*a^2+1)/(a^2+1)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = \frac{4i\,b^2x^2\log\left(x\right) - 4i\,b^2x^2\log\left(\frac{bx+a+i}{b}\right) + a^3 - 4\left(-i\,a+1\right)bx + i\,a^2 + a + i}{2\left(a^3 + 3i\,a^2 - 3\,a - i\right)x^2}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*I*b^2*x^2*log(x) - 4*I*b^2*x^2*log((b*x + a + I)/b) + a^3 - 4*(-I*a
+ 1)*b*x + I*a^2 + a + I)/((a^3 + 3*I*a^2 - 3*a - I)*x^2)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(60) = 120.

Time = 0.44 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.00

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = \frac{2ib^2\log\left(-\frac{2a^4b^2}{(a+i)^3} - \frac{8ia^3b^2}{(a+i)^3} + \frac{12a^2b^2}{(a+i)^3} + 2ab^2 + \frac{8iab^2}{(a+i)^3} + 4b^3x + 2ib^2 - \frac{2b^2}{(a+i)^3}\right)}{(a+i)^3} - \frac{2ib^2\log\left(\frac{2a^4b^2}{(a+i)^3} + \frac{8ia^3b^2}{(a+i)^3} - \frac{12a^2b^2}{(a+i)^3} + 2ab^2 - \frac{8iab^2}{(a+i)^3} + 4b^3x + 2ib^2 + \frac{2b^2}{(a+i)^3}\right)}{(a+i)^3} - \frac{-a^2 - 4ibx - 1}{x^2\cdot(2a^2 + 4ia - 2)}$$

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**3,x)
```

```
[Out] 2*I*b**2*log(-2*a**4*b**2/(a + I)**3 - 8*I*a**3*b**2/(a + I)**3 + 12*a**2*b
**2/(a + I)**3 + 2*a*b**2 + 8*I*a*b**2/(a + I)**3 + 4*b**3*x + 2*I*b**2 - 2
*b**2/(a + I)**3)/(a + I)**3 - 2*I*b**2*log(2*a**4*b**2/(a + I)**3 + 8*I*a*
*3*b**2/(a + I)**3 - 12*a**2*b**2/(a + I)**3 + 2*a*b**2 - 8*I*a*b**2/(a + I
)**3 + 4*b**3*x + 2*I*b**2 + 2*b**2/(a + I)**3)/(a + I)**3 - (-a**2 - 4*I*b
*x - 1)/(x**2*(2*a**2 + 4*I*a - 2))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.47

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = -\frac{2\,(a^3-3i\,a^2-3\,a+i)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1} - \frac{(i\,a^3+3\,a^2-3i\,a-1)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^6+3\,a^4+3\,a^2+1} - \frac{2\,(-i\,a^3-3\,a^2+3i\,a+1)b^2\log\left(x\right)}{a^6+3\,a^4+3\,a^2+1} + \frac{a^4-2i\,a^3-4\,(-i\,a^2-2\,a+i)bx-2i\,a-1}{2\,(a^4+2\,a^2+1)x^2}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*(a^3 - 3*I*a^2 - 3*a + I)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a
^2 + 1) - (I*a^3 + 3*a^2 - 3*I*a - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/
(a^6 + 3*a^4 + 3*a^2 + 1) - 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b^2*log(x)/(a^6
+ 3*a^4 + 3*a^2 + 1) + 1/2*(a^4 - 2*I*a^3 - 4*(-I*a^2 - 2*a + I)*b*x - 2*I*
a - 1)/((a^4 + 2*a^2 + 1)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = \frac{2\,b^3\log\left(bx+a+i\right)}{i\,a^3b-3\,a^2b-3i\,ab+b} + \frac{2\,b^2\log\left(|x|\right)}{-i\,a^3+3\,a^2+3i\,a-1} + \frac{a^3+i\,a^2+4i\,(ab+i\,b)x+a+i}{2\,(a+i)^3x^2}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(b*x + a + I)/(I*a^3*b - 3*a^2*b - 3*I*a*b + b) + 2*b^2*log(abs(x)
)/(-I*a^3 + 3*a^2 + 3*I*a - 1) + 1/2*(a^3 + I*a^2 + 4*I*(a*b + I*b)*x + a +
 I)/((a + I)^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.03

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx$$
$$= \frac{\frac{a-\mathrm{i}}{2\,(a+1\mathrm{i})}+\frac{b\,x\,2\mathrm{i}}{(a+1\mathrm{i})^2}}{x^2}+\frac{b^2\,\mathrm{atanh}\left(\frac{-a^3-a^2\,3\mathrm{i}+3\,a+1\mathrm{i}}{(a+1\mathrm{i})^3}+\frac{x\,\left(2\,a^8\,b^2+8\,a^6\,b^2+12\,a^4\,b^2+8\,a^2\,b^2+2\,b^2\right)}{(a+1\mathrm{i})^3\,\left(-b\,a^6+2\mathrm{i}\,b\,a^5-b\,a^4+4\mathrm{i}\,b\,a^3+b\,a^2+2\mathrm{i}\,b\,a+b\right)}\right)\,4\mathrm{i}}{(a+1\mathrm{i})^3}$$

```
[In] int((a*1i + b*x*1i + 1)^2/(x^3*((a + b*x)^2 + 1)),x)
```

```
[Out] ((a - 1i)/(2*(a + 1i)) + (b*x*2i)/(a + 1i)^2)/x^2 + (b^2*atanh((3*a - a^2*3
i - a^3 + 1i)/(a + 1i)^3 + (x*(2*b^2 + 8*a^2*b^2 + 12*a^4*b^2 + 8*a^6*b^2 +
 2*a^8*b^2))/((a + 1i)^3*(b + a*b*2i + a^2*b + a^3*b*4i - a^4*b + a^5*b*2i
- a^6*b)))*4i)/(a + 1i)^3
```

3.179 $\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = -\frac{i-a}{3(i+a)x^3}+\frac{ib}{(i+a)^2x^2}+\frac{2b^2}{(1-ia)^3x} - \frac{2ib^3\log(x)}{(i+a)^4}+\frac{2ib^3\log(i+a+bx)}{(i+a)^4}$$

```
[Out] 1/3*(-I+a)/(I+a)/x^3+I*b/(I+a)^2/x^2+2*b^2/(1-I*a)^3/x-2*I*b^3*ln(x)/(I+a)^
4+2*I*b^3*ln(I+a+b*x)/(I+a)^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = -\frac{2ib^3\log(x)}{(a+i)^4}+\frac{2ib^3\log(a+bx+i)}{(a+i)^4} + \frac{2b^2}{(1-ia)^3x}+\frac{ib}{(a+i)^2x^2}-\frac{-a+i}{3(a+i)x^3}$$

```
[In] Int[E^((2*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] -1/3*(I - a)/((I + a)*x^3) + (I*b)/((I + a)^2*x^2) + (2*b^2)/((1 - I*a)^3*x
) - ((2*I)*b^3*Log[x])/(I + a)^4 + ((2*I)*b^3*Log[I + a + b*x])/(I + a)^4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+ia+ibx}{x^4(1-ia-ibx)}\,dx\\ &= \int \left(\frac{i-a}{(i+a)x^4}-\frac{2ib}{(i+a)^2x^3}+\frac{2ib^2}{(i+a)^3x^2}-\frac{2ib^3}{(i+a)^4x}+\frac{2ib^4}{(i+a)^4(i+a+bx)}\right)dx\\ &= -\frac{i-a}{3(i+a)x^3}+\frac{ib}{(i+a)^2x^2}+\frac{2b^2}{(1-ia)^3x}-\frac{2ib^3\log(x)}{(i+a)^4}+\frac{2ib^3\log(i+a+bx)}{(i+a)^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = \frac{(i+a)\left(i+a+ia^2+a^3-3bx+3iabx-6ib^2x^2\right)-6ib^3x^3\log(x)+6ib^3x^3\log(i+a+bx)}{3(i+a)^4x^3}$$

```
[In] Integrate[E^((2*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] ((I + a)*(I + a + I*a^2 + a^3 - 3*b*x + (3*I)*a*b*x - (6*I)*b^2*x^2) - (6*I
)*b^3*x^3*Log[x] + (6*I)*b^3*x^3*Log[I + a + b*x])/(3*(I + a)^4*x^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(79) = 158.

Time = 0.35 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.87

method	result
default	$-\frac{-a^2+2ia+1}{3(a^2+1)x^3}+\frac{b(ia^2+2a-i)}{(a^2+1)^2x^2}-\frac{2b^2(ia^3+3a^2-3ia-1)}{(a^2+1)^3x}-\frac{2b^3(ia^4+4a^3-6ia^2-4a+i)\ln(x)}{(a^2+1)^4}+\frac{2b^4\left(\frac{(ia^4b+4a^3b-6ia^2b-4}{}\right.}{}$
parallelrisch	$-\frac{1+3bxi+12a^2b^2x^2+2a^2+6ix^2a^5b^2+6ib^3\ln(x)x^3+2ia-12a^3bx+2ia^7-6a^5bx+18a^4b^2x^2-6abx-a^8-2a^6-6b^2x^2+6ia^5+6i\ln(}{}$
risch	$\frac{-\frac{2ib^2x^2}{(a^2+2ia-1)(i+a)}+\frac{ibx}{a^2+2ia-1}+\frac{a-i}{3i+3a}}{x^3}+\frac{2b^3\ln((-2a^6b-6a^4b-6a^2b-2b)x)}{ia^4-4a^3-6ia^2+4a+i}-\frac{b^3\ln(4a^{12}b^2x^2+8a^{13}bx+4a^{14}+24a^{10}b^2x^2+}{}$

```
[In] int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2*I*a-a^2+1)/(a^2+1)/x^3+b*(I*a^2-I+2*a)/(a^2+1)^2/x^2-2*b^2*(I*a^3-3
*I*a+3*a^2-1)/(a^2+1)^3/x-2*b^3*(I*a^4-6*I*a^2+4*a^3+I-4*a)/(a^2+1)^4*ln(x)
+2*b^4/(a^2+1)^4*(1/2*(I*a^4*b-6*I*a^2*b+4*a^3*b+I*b-4*a*b)/b^2*ln(b^2*x^2+
2*a*b*x+a^2+1)+(I*a^5-10*I*a^3+5*a^4+5*I*a-10*a^2+1-(I*a^4*b-6*I*a^2*b+4*a^
3*b+I*b-4*a*b)*a/b)/b*arctan(1/2*(2*b^2*x+2*a*b)/b))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = \frac{-6i\,b^3x^3\log(x)+6i\,b^3x^3\log\left(\frac{bx+a+i}{b}\right)-6\,(i\,a-1)b^2x^2+a^4+2i\,a^3-3\,(-i\,a^2+2\,a+i)bx+2i\,a-1}{3\,(a^4+4i\,a^3-6\,a^2-4i\,a+1)x^3}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(-6*I*b^3*x^3*log(x) + 6*I*b^3*x^3*log((b*x + a + I)/b) - 6*(I*a - 1)*b
^2*x^2 + a^4 + 2*I*a^3 - 3*(-I*a^2 + 2*a + I)*b*x + 2*I*a - 1)/((a^4 + 4*I*
a^3 - 6*a^2 - 4*I*a + 1)*x^3)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(73) = 146.

Time = 0.56 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.08

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = -\frac{2ib^3\log\left(-\frac{2a^5b^3}{(a+i)^4}-\frac{10ia^4b^3}{(a+i)^4}+\frac{20a^3b^3}{(a+i)^4}+\frac{20ia^2b^3}{(a+i)^4}+2ab^3-\frac{10ab^3}{(a+i)^4}+4b^4x+2ib^3-\frac{2ib^3}{(a+i)^4}\right)}{(a+i)^4} + \frac{2ib^3\log\left(\frac{2a^5b^3}{(a+i)^4}+\frac{10ia^4b^3}{(a+i)^4}-\frac{20a^3b^3}{(a+i)^4}-\frac{20ia^2b^3}{(a+i)^4}+2ab^3+\frac{10ab^3}{(a+i)^4}+4b^4x+2ib^3+\frac{2ib^3}{(a+i)^4}\right)}{(a+i)^4} - \frac{-a^3-ia^2-a+6ib^2x^2+x(-3iab+3b)-i}{x^3\cdot(3a^3+9ia^2-9a-3i)}$$

```
[In] integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**4,x)
```

```
[Out] -2*I*b**3*log(-2*a**5*b**3/(a + I)**4 - 10*I*a**4*b**3/(a + I)**4 + 20*a**3
*b**3/(a + I)**4 + 20*I*a**2*b**3/(a + I)**4 + 2*a*b**3 - 10*a*b**3/(a + I)
**4 + 4*b**4*x + 2*I*b**3 - 2*I*b**3/(a + I)**4)/(a + I)**4 + 2*I*b**3*log(
2*a**5*b**3/(a + I)**4 + 10*I*a**4*b**3/(a + I)**4 - 20*a**3*b**3/(a + I)**
4 - 20*I*a**2*b**3/(a + I)**4 + 2*a*b**3 + 10*a*b**3/(a + I)**4 + 4*b**4*x
+ 2*I*b**3 + 2*I*b**3/(a + I)**4)/(a + I)**4 - (-a**3 - I*a**2 - a + 6*I*b*
*2*x**2 + x*(-3*I*a*b + 3*b) - I)/(x**3*(3*a**3 + 9*I*a**2 - 9*a - 3*I))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(69) = 138.

Time = 0.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.83

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = \frac{2\,(a^4-4i\,a^3-6\,a^2+4i\,a+1)b^3\arctan\left(\frac{b^2x+ab}{b}\right)}{a^8+4\,a^6+6\,a^4+4\,a^2+1} + \frac{(i\,a^4+4\,a^3-6i\,a^2-4\,a+i)b^3\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^8+4\,a^6+6\,a^4+4\,a^2+1} - \frac{2\,(i\,a^4+4\,a^3-6i\,a^2-4\,a+i)b^3\log\left(x\right)}{a^8+4\,a^6+6\,a^4+4\,a^2+1} + \frac{a^6-2i\,a^5+6\,(-i\,a^3-3\,a^2+3i\,a+1)b^2x^2+a^4-4i\,a^3+3\,(i\,a^4+2\,a^3+2\,a-i)bx-a^2-2i\,a-1}{3\,(a^6+3\,a^4+3\,a^2+1)x^3}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] 2*(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)*b^3*arctan((b^2*x + a*b)/b)/(a^8 + 4*
a^6 + 6*a^4 + 4*a^2 + 1) + (I*a^4 + 4*a^3 - 6*I*a^2 - 4*a + I)*b^3*log(b^2*
x^2 + 2*a*b*x + a^2 + 1)/(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1) - 2*(I*a^4 + 4*a
```

```
^3 - 6*I*a^2 - 4*a + I)*b^3*log(x)/(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1) + 1/3*
(a^6 - 2*I*a^5 + 6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b^2*x^2 + a^4 - 4*I*a^3 + 3
*(I*a^4 + 2*a^3 + 2*a - I)*b*x - a^2 - 2*I*a - 1)/((a^6 + 3*a^4 + 3*a^2 + 1
)*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.35

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = \frac{2\,b^4\log(bx+a+i)}{-i\,a^4b+4\,a^3b+6i\,a^2b-4\,ab-i\,b}+\frac{2\,b^3\log(|x|)}{i\,a^4-4\,a^3-6i\,a^2+4\,a+i}+\frac{a^4+2i\,a^3-6i\,(ab^2+i\,b^2)x^2+3i\,(a^2b+2i\,ab-b)x+2i\,a-1}{3\,(a+i)^4x^3}$$

```
[In] integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="giac")
```

```
[Out] 2*b^4*log(b*x + a + I)/(-I*a^4*b + 4*a^3*b + 6*I*a^2*b - 4*a*b - I*b) + 2*b
^3*log(abs(x))/(I*a^4 - 4*a^3 - 6*I*a^2 + 4*a + I) + 1/3*(a^4 + 2*I*a^3 - 6
*I*(a*b^2 + I*b^2)*x^2 + 3*I*(a^2*b + 2*I*a*b - b)*x + 2*I*a - 1)/((a + I)^
4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.14

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = \frac{\frac{a-\mathrm{i}}{3\,(a+1\mathrm{i})}-\frac{b^2\,x^2\,2\mathrm{i}}{(a+1\mathrm{i})^3}+\frac{b\,x\,1\mathrm{i}}{(a+1\mathrm{i})^2}}{x^3}+\frac{b^3\,\mathrm{atanh}\left(\frac{a^4+a^3\,4\mathrm{i}-6\,a^2-a\,4\mathrm{i}+1}{(a+1\mathrm{i})^4}-\frac{x\,(2\,a^{12}\,b^2+12\,a^{10}\,b^2+30\,a^8\,b^2+40\,a^6\,b^2+30\,a^4\,b^2+12\,a^2\,b^2+2\,b^2)}{(a+1\mathrm{i})^4\,(-b\,a^9+3\mathrm{i}\,b\,a^8+8\mathrm{i}\,b\,a^6+6\,b\,a^5+6\mathrm{i}\,b\,a^4+8\,b\,a^3+3\,b\,a-b\,1\mathrm{i})}\right)\,4\mathrm{i}}{(a+1\mathrm{i})^4}$$

```
[In] int((a*1i + b*x*1i + 1)^2/(x^4*((a + b*x)^2 + 1)),x)
```

```
[Out] ((a - 1i)/(3*(a + 1i)) - (b^2*x^2*2i)/(a + 1i)^3 + (b*x*1i)/(a + 1i)^2)/x^3
 + (b^3*atanh((a^3*4i - 6*a^2 - a*4i + a^4 + 1)/(a + 1i)^4 - (x*(2*b^2 + 12
*a^2*b^2 + 30*a^4*b^2 + 40*a^6*b^2 + 30*a^8*b^2 + 12*a^10*b^2 + 2*a^12*b^2)
)/((a + 1i)^4*(3*a*b - b*1i + 8*a^3*b + a^4*b*6i + 6*a^5*b + a^6*b*8i + a^8
*b*3i - a^9*b)))*4i)/(a + 1i)^4
```

3.180 $\int e^{3i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 324

$$\begin{aligned}
&\int e^{3i\arctan(a+bx)}x^4\,dx \\
&= -\frac{3(19i+68a-88ia^2-48a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5}-\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} \\
&\quad+\frac{3(17i+16a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3}-\frac{11x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} \\
&\quad-\frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(163-458ia-422a^2+112ia^3+2(61i+118a-52ia^2)\,bx\right)}{40b^5} \\
&\quad-\frac{3(19-68ia-88a^2+48ia^3+8a^4)\operatorname{arcsinh}(a+bx)}{8b^5}
\end{aligned}$$

```
[Out] -3/8*(19-68*I*a-88*a^2+48*I*a^3+8*a^4)*arcsinh(b*x+a)/b^5-2*I*x^4*(1+I*a+I*
b*x)^(3/2)/b/(1-I*a-I*b*x)^(1/2)+3/20*(17*I+16*a)*x^2*(1+I*a+I*b*x)^(3/2)*(
1-I*a-I*b*x)^(1/2)/b^3-11/5*x^3*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^2
-1/40*I*(1+I*a+I*b*x)^(3/2)*(163-458*I*a-422*a^2+112*I*a^3+2*(61*I+118*a-52
*I*a^2)*b*x)*(1-I*a-I*b*x)^(1/2)/b^5-3/8*(19*I+68*a-88*I*a^2-48*a^3+8*I*a^4
)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5203, 99, 158, 152, 52, 55, 633, 221}

$$\int e^{3i\arctan(a+bx)}x^4\,dx =$$
$$-\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(112ia^3+2(-52ia^2+118a+61i)\,bx-422a^2-458ia+163\right)}{40b^5}$$
$$-\frac{3(8a^4+48ia^3-88a^2-68ia+19)\operatorname{arcsinh}(a+bx)}{8b^5}$$
$$-\frac{3(8ia^4-48a^3-88ia^2+68a+19i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{8b^5}$$
$$+\frac{3(16a+17i)x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{20b^3}$$
$$-\frac{11x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}-\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] (-3*(19*I + 68*a - (88*I)*a^2 - 48*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*S
qrt[1 + I*a + I*b*x])/(8*b^5) - ((2*I)*x^4*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt
[1 - I*a - I*b*x]) + (3*(17*I + 16*a)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a +
I*b*x)^(3/2))/(20*b^3) - (11*x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3
/2))/(5*b^2) - ((I/40)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(163 -
 (458*I)*a - 422*a^2 + (112*I)*a^3 + 2*(61*I + 118*a - (52*I)*a^2)*b*x))/b^
5 - (3*(19 - (68*I)*a - 88*a^2 + (48*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b
^5)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
```

```
I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx\\
&= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}+\frac{(2i)\int \frac{x^3\sqrt{1+ia+ibx}\left(4(1+ia)+\frac{11ibx}{2}\right)}{\sqrt{1-ia-ibx}}\,dx}{b}\\
&= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}-\frac{11x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2}\\
&\quad+\frac{(2i)\int \frac{x^2\sqrt{1+ia+ibx}\left(-\frac{33}{2}(i-a)(1-ia)b+\frac{3}{2}(17-16ia)b^2x\right)}{\sqrt{1-ia-ibx}}\,dx}{5b^3}\\
&= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}+\frac{3(17i+16a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3}\\
&\quad-\frac{11x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2}\\
&\quad+\frac{i\int \frac{x\sqrt{1+ia+ibx}\left(3(1+ia)(i+a)(17i+16a)b^2-\frac{3}{2}\left(118a+i\left(61-52a^2\right)\right)b^3x\right)}{\sqrt{1-ia-ibx}}\,dx}{10b^5}\\
&= -\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}+\frac{3(17i+16a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3}\\
&\quad-\frac{11x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2}\\
&\quad-\frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(163-458ia-422a^2+112ia^3+2\left(61i+118a-52ia^2\right)bx\right)}{40b^5}\\
&\quad-\frac{\left(3\left(19-68ia-88a^2+48ia^3+8a^4\right)\right)\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{8b^4}\\
&= -\frac{3\left(19i+68a-88ia^2-48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5}\\
&\quad-\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}+\frac{3(17i+16a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3}\\
&\quad-\frac{11x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2}\\
&\quad-\frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(163-458ia-422a^2+112ia^3+2\left(61i+118a-52ia^2\right)bx\right)}{40b^5}\\
&\quad-\frac{\left(3\left(19-68ia-88a^2+48ia^3+8a^4\right)\right)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{8b^4}
\end{aligned}$$

$$= -\frac{3(19i + 68a - 88ia^2 - 48a^3 + 8ia^4)\sqrt{1 - ia - ibx}\sqrt{1 + ia + ibx}}{8b^5} - \frac{2ix^4(1 + ia + ibx)^{3/2}}{b\sqrt{1 - ia - ibx}} + \frac{3(17i + 16a)x^2\sqrt{1 - ia - ibx}(1 + ia + ibx)^{3/2}}{20b^3} - \frac{11x^3\sqrt{1 - ia - ibx}(1 + ia + ibx)^{3/2}}{5b^2} - \frac{i\sqrt{1 - ia - ibx}(1 + ia + ibx)^{3/2}(163 - 458ia - 422a^2 + 112ia^3 + 2(61i + 118a - 52ia^2)bx)}{40b^5} - \frac{(3(19 - 68ia - 88a^2 + 48ia^3 + 8a^4))\int\frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{8b^4}$$

$$= -\frac{3(19i + 68a - 88ia^2 - 48a^3 + 8ia^4)\sqrt{1 - ia - ibx}\sqrt{1 + ia + ibx}}{8b^5} - \frac{2ix^4(1 + ia + ibx)^{3/2}}{b\sqrt{1 - ia - ibx}} + \frac{3(17i + 16a)x^2\sqrt{1 - ia - ibx}(1 + ia + ibx)^{3/2}}{20b^3} - \frac{11x^3\sqrt{1 - ia - ibx}(1 + ia + ibx)^{3/2}}{5b^2} - \frac{i\sqrt{1 - ia - ibx}(1 + ia + ibx)^{3/2}(163 - 458ia - 422a^2 +}{40b^5} - \frac{(3(19 - 68ia - 88a^2 + 48ia^3 + 8a^4))\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab + 2b^2x\right)}{16b^6}$$

$$= -\frac{3(19i + 68a - 88ia^2 - 48a^3 + 8ia^4)\sqrt{1 - ia - ibx}\sqrt{1 + ia + ibx}}{8b^5} - \frac{2ix^4(1 + ia + ibx)^{3/2}}{b\sqrt{1 - ia - ibx}} + \frac{3(17i + 16a)x^2\sqrt{1 - ia - ibx}(1 + ia + ibx)^{3/2}}{20b^3} - \frac{11x^3\sqrt{1 - ia - ibx}(1 + ia + ibx)^{3/2}}{5b^2} - \frac{i\sqrt{1 - ia - ibx}(1 + ia + ibx)^{3/2}(163 - 458ia - 422a^2 + 112ia^3 + 2(61i + 118a - 52ia^2)bx)}{40b^5} - \frac{3(19 - 68ia - 88a^2 + 48ia^3 + 8a^4)\,\mathrm{arcsinh}(a + bx)}{8b^5}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.77

$$\int e^{3i\arctan(a+bx)}x^4\,dx = -\frac{\sqrt{1 + ia + ibx}(448i + 418ia^4 + 8a^5 + 163bx + 61ib^2x^2 - 34b^3x^3 - 22ib^4x^4 + 8b^5x^5 + 14ia^3(121i + 8bx}{40b^5\sqrt{-i(i + a + bx)}} + \frac{3(-1)^{3/4}(19 - 68ia - 88a^2 + 48ia^3 + 8a^4)\,\mathrm{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4\sqrt{-ib}\,b^{9/2}}$$

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x^4,x]
```

```
[Out] -1/40*(Sqrt[1 + I*a + I*b*x]*(448*I + (418*I)*a^4 + 8*a^5 + 163*b*x + (61*I
)*b^2*x^2 - 34*b^3*x^3 - (22*I)*b^4*x^4 + 8*b^5*x^5 + (14*I)*a^3*(121*I + 8
*b*x) - I*a^2*(2599 - (422*I)*b*x + 52*b^2*x^2) + a*(1763 - (458*I)*b*x + 1
18*b^2*x^2 + (32*I)*b^3*x^3)))/(b^5*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(3/
4)*(19 - (68*I)*a - 88*a^2 + (48*I)*a^3 + 8*a^4)*ArcSinh[((1/2 + I/2)*Sqrt[
b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*Sqrt[(-I)*b]*b^(9/2))
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{i\left(8x^4b^4-8a\,b^3x^3-30ib^3x^3+8a^2b^2x^2+70ia\,b^2x^2-8a^3bx-130ia^2bx+8a^4+250ia^3-64b^2x^2+252abx+125bxi-804a^2-835ia+288\right)\sqrt{b}}{40b^5}$
default	Expression too large to display

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*I*(8*x^4*b^4-30*I*b^3*x^3-8*a*b^3*x^3+70*I*a*b^2*x^2+8*a^2*b^2*x^2-13
0*I*a^2*b*x-8*a^3*b*x+250*I*a^3+8*a^4-64*b^2*x^2+125*I*b*x+252*a*b*x-835*I*
a-804*a^2+288)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^5-1/8/b^4*(57*ln((b^2*x+a*b)
/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+24*a^4*ln((b^2*x+a*
b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-264*a^2*ln((b^2*x
+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+144*I*a^3*ln((
b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-204*I*a*l
n((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*(128
*a^3-32*I*a^4-128*a+192*I*a^2-32*I)/b^2/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*
b*(x+(I+a)/b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.81

$$\int e^{3i\arctan(a+bx)}x^4\,dx$$
$$=\frac{-62i\,a^6+2687\,a^5+11575i\,a^4-20350\,a^3+\left(-62i\,a^5+2625\,a^4+8950i\,a^3-11400\,a^2-6340i\,a+1280\right)bx}{}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/320*(-62*I*a^6 + 2687*a^5 + 11575*I*a^4 - 20350*a^3 + (-62*I*a^5 + 2625*a
^4 + 8950*I*a^3 - 11400*a^2 - 6340*I*a + 1280)*b*x - 17740*I*a^2 + 120*(8*a
```

```
^5 + 56*I*a^4 - 136*a^3 + (8*a^4 + 48*I*a^3 - 88*a^2 - 68*I*a + 19)*b*x - 1
56*I*a^2 + 87*a + 19*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 8*(8*I*b^5*x^5 + 22*b^4*x^4 - 2*(16*a + 17*I)*b^3*x^3 + 8*I*a^5 + (52*a^2
+ 118*I*a - 61)*b^2*x^2 - 418*a^4 - 1694*I*a^3 - (112*a^3 + 422*I*a^2 - 458
*a - 163*I)*b*x + 2599*a^2 + 1763*I*a - 448)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1) + 7620*a + 1280*I)/(b^6*x + (a + I)*b^5)
```

Sympy [F]

$$\int e^{3i\arctan(a+bx)}x^4\,dx =$$

$$-i\left(\int \frac{ix^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2\right.$$

$$+\int\left(-\frac{3ax^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\right.$$

$$+\int \frac{a^3x^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}$$

$$+\int\left(-\frac{3bx^5}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\right.$$

$$+\int \frac{b^3x^7}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}$$

$$+\int\left(-\frac{3ia^2x^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\right.$$

$$+\int\left(-\frac{3ib^2x^6}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\right.$$

$$+\int \frac{3ab^2x^6}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}$$

$$+\int \frac{3a^2bx^5}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}$$

$$+\int\left(-\frac{6iabx^5}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\right.$$

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**4,x)
```

```
[Out] -I*(Integral(I*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x**4/(
a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x**4/(a**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2
```

```
 + 1)), x) + Integral(-3*b*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral
(b**3*x**7/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**4/(a**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**6/(a**2*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2
*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)), x) + Integral(3*a*b**2*x**6/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Int
egral(3*a**2*b*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x*
*5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3081 vs. 2(230) = 460.

Time = 0.22 (sec) , antiderivative size = 3081, normalized size of antiderivative = 9.51

$$\int e^{3i\arctan(a+bx)}x^4\,dx = \text{Too large to display}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] -1/5*I*b*x^6/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 11/20*I*a*x^5/sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1) - 693/4*I*a^7*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*b^2) - 33/20*I*a^2*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*b) + 2415/8*I*(a^2 + 1)*a^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b^2) + 231/40*I*a^3*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*b^2) - 2/5*(-I*a^2 - I)*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) -
3/4*(I*a*b^2 + b^2)*x^5/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 231/8*I*(
a^2 + 1)*a^6/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b
^3) + 945/4*(I*a*b^2 + b^2)*a^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*b^4) - 105*(I*a^2*b + 2*a*b - I*b)*a^5*x/((a^2*b^2 - (a
^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 2919/20*I*(a^2 + 1)^2
*a^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) -
15*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x
```

^2 + 2*a*b*x + a^2 + 1)*b^2) - 231/8*I*a^4*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*b^3) - 111/40*I*(a^2 + 1)*a*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b
^2) + 9/4*(I*a*b^2 + b^2)*a*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - (
I*a^2*b + 2*a*b - I*b)*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 189/5*
I*(a^2 + 1)^2*a^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*b^3) - 2835/8*(I*a*b^2 + b^2)*(a^2 + 1)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^2
)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 265/2*(I*a^2*b + 2*a*b - I*b)*(a
^2 + 1)*a^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
b^3) + 653/40*I*(a^2 + 1)^3*a*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)*b^2) + 31/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)*a^2*x/
((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 819/40*
I*(a^2 + 1)*a^2*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 63/8*(I*a*b^2
 + b^2)*a^2*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 7/2*(I*a^2*b + 2*
a*b - I*b)*a*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 1/2*(I*a^3 + 3*a
^2 - 3*I*a - 1)*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 693/8*I*a^5*a
rcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 315/8*(I*a
b^2 + b^2)(a^2 + 1)*a^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*b^5) - 35/2*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*a^4/((a^2*b^2 - (
a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 397/40*I*(a^2 + 1)^3
*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 5/
2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 507/4*(I*a*b^2 + b^2)*(a^2 + 1)^2*a^2
*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 61/2
*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)^2*a*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 3/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1
)^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 3
15/8*(I*a*b^2 + b^2)*a^3*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) - 35/2
*(I*a^2*b + 2*a*b - I*b)*a^2*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) -
8/5*I*(a^2 + 1)^2*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 5/2*(-I*a^3
 - 3*a^2 + 3*I*a + 1)*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 15/8*
(I*a*b^2 + b^2)*(a^2 + 1)*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 315
/4*I*(a^2 + 1)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2))/b^5 - 42*(I*a*b^2 + b^2)*(a^2 + 1)^2*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) - 231/4*I*(a^2 + 1)*a^4/(sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*b^5) + 29/2*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)^2*a^2/((a
^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 3/2*(-I*a^
3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)^2*a/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b^3) - 45/8*(I*a*b^2 + b^2)*(a^2 + 1)^3*x/((a^2*b^2
- (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 147/8*(I*a*b^2 +
b^2)*(a^2 + 1)*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) + 4*(I*a^2*b +
 2*a*b - I*b)*(a^2 + 1)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 945/8
*(I*a*b^2 + b^2)*a^4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2))/b^7 + 105/2*(I*a^2*b + 2*a*b - I*b)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(
-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 + 105/8*I*(a^2 + 1)^2*a*arcsinh(2*(b^2*x
 + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 15/2*(-I*a^3 - 3*a^2 + 3*

```
I*a + 1)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^
5 + 45/8*(I*a*b^2 + b^2)*(a^2 + 1)^3*a/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*b^5) + 819/20*I*(a^2 + 1)^2*a^2/(sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*b^5) + 315/4*(I*a*b^2 + b^2)*(a^2 + 1)*a^2*arcsinh(2*(b^2*
x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^7 - 45/2*(I*a^2*b + 2*a*b -
I*b)*(a^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
)/b^6 - 3/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 315/4*(I*a*b^2 + b^2)*(a^2 + 1)*a
^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^7) - 35*(I*a^2*b + 2*a*b - I*b)*(a^
2 + 1)*a^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^6) - 16/5*I*(a^2 + 1)^3/(sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) - 5*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2
+ 1)*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) - 45/8*(I*a*b^2 + b^2)*(a^2
+ 1)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^7 - 14
7/4*(I*a*b^2 + b^2)*(a^2 + 1)^2*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^7) +
 8*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b
^6)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.03

$$\int e^{3i\arctan(a+bx)}x^4\,dx = -\frac{1}{40}\sqrt{(bx+a)^2+1}\left(\left(2\left(x\left(\frac{4i\,x}{b}-\frac{4i\,ab^{17}-15\,b^{17}}{b^{19}}\right)-\frac{-4i\,a^2b^{16}+35\,ab^{16}+32i\,b^{16}}{b^{19}}\right)x-\frac{8i\,a^3b^{15}-13\cdots}{\cdots}\right.\right. + \frac{(8\,a^4+48i\,a^3-88\,a^2-68i\,a+19)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\cdots\right.\right.}{\cdots}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] -1/40*sqrt((b*x + a)^2 + 1)*((2*(x*(4*I*x/b - (4*I*a*b^17 - 15*b^17)/b^19)
- (-4*I*a^2*b^16 + 35*a*b^16 + 32*I*b^16)/b^19)*x - (8*I*a^3*b^15 - 130*a^2
*b^15 - 252*I*a*b^15 + 125*b^15)/b^19)*x - (-8*I*a^4*b^14 + 250*a^3*b^14 +
804*I*a^2*b^14 - 835*a*b^14 - 288*I*b^14)/b^19) + 1/8*(8*a^4 + 48*I*a^3 - 8
8*a^2 - 68*I*a + 19)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b
 + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x +
a)^2 + 1))*a^2*abs(b) + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^
2*b + 4*(I*x*abs(b) - I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) -
 sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}x^4\,dx=\int \frac{x^4\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

```
[In] int((x^4*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
[Out] int((x^4*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

3.181 $\int e^{3i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 249

$$\begin{aligned}\int e^{3i\arctan(a+bx)}x^3\,dx &= \frac{3(17-44ia-36a^2+8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4}\\ &\quad-\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}-\frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2}\\ &\quad-\frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(29i+54a-22ia^2-2(11-10ia)bx\right)}{8b^4}\\ &\quad-\frac{3(17i+44a-36ia^2-8a^3)\operatorname{arcsinh}(a+bx)}{8b^4}\end{aligned}$$

```
[Out] -3/8*(17*I+44*a-36*I*a^2-8*a^3)*arcsinh(b*x+a)/b^4-2*I*x^3*(1+I*a+I*b*x)^(3
/2)/b/(1-I*a-I*b*x)^(1/2)-9/4*x^2*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b
^2-1/8*I*(1+I*a+I*b*x)^(3/2)*(29*I+54*a-22*I*a^2-2*(11-10*I*a)*b*x)*(1-I*a-
I*b*x)^(1/2)/b^4+3/8*(17-44*I*a-36*a^2+8*I*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+
I*b*x)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{5203, 99, 158, 152, 52, 55, 633, 221\}$

$$\int e^{3i\arctan(a+bx)}x^3\,dx$$
$$= -\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(-22ia^2-2(11-10ia)bx+54a+29i\right)}{8b^4} - \frac{3(-8a^3-36ia^2+44a+17i)\operatorname{arcsinh}(a+bx)}{8b^4} + \frac{3(8ia^3-36a^2-44ia+17)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{8b^4} - \frac{9x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b^2} - \frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (3*(17 - (44*I)*a - 36*a^2 + (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a
+ I*b*x])/(8*b^4) - ((2*I)*x^3*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I
*b*x]) - (9*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b^2) - ((
I/8)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(29*I + 54*a - (22*I)*a^
2 - 2*(11 - (10*I)*a)*b*x))/b^4 - (3*(17*I + 44*a - (36*I)*a^2 - 8*a^3)*Arc
Sinh[a + b*x])/(8*b^4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^3(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} + \frac{(2i)\int \frac{x^2\sqrt{1+ia+ibx}\left(3(1+ia)+\frac{9ibx}{2}\right)}{\sqrt{1-ia-ibx}}\,dx}{b}$$

$$= -\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} + \frac{i\int \frac{x\sqrt{1+ia+ibx}\left(-9i(1+a^2)b+\frac{3}{2}(11-10ia)b^2x\right)}{\sqrt{1-ia-ibx}}\,dx}{2b^3}$$

$$= -\frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(29i+54a-22ia^2-2(11-10ia)bx\right)}{8b^4} - \frac{\left(3(17i+44a-36ia^2-8a^3)\right)\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{8b^3}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(29i+54a-22ia^2-2(11-10ia)bx\right)}{8b^4} - \frac{\left(3(17i+44a-36ia^2-8a^3)\right)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{8b^3}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(29i+54a-22ia^2-2(11-10ia)bx\right)}{8b^4} - \frac{\left(3(17i+44a-36ia^2-8a^3)\right)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{8b^3}$$

$$= \frac{3\left(17-44ia-36a^2+8ia^3\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}\left(29i+54a-22ia^2-2(11-10ia)bx\right)}{8b^4} - \frac{\left(3(17i+44a-36ia^2-8a^3)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{16b^5}$$

$$= \frac{3(17-44ia-36a^2+8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} - \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}(29i+54a-22ia^2-2(11-10ia)bx)}{8b^4} - \frac{3(17i+44a-36ia^2-8a^3)\operatorname{arcsinh}(a+bx)}{8b^4}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.81

$$\int e^{3i\arctan(a+bx)}x^3\,dx = \frac{\sqrt{1+ia+ibx}(80+78ia^3+2a^4-29ibx+11b^2x^2+6ib^3x^3-2b^4x^4+a^2(-233+22ibx)-ia(237-54ibx}{8b^4\sqrt{-i(i+a+bx)}} + \frac{3\sqrt[4]{-1}(-17i-44a+36ia^2+8a^3)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4b^{9/2}}$$

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(80 + (78*I)*a^3 + 2*a^4 - (29*I)*b*x + 11*b^2*x^2 +
 (6*I)*b^3*x^3 - 2*b^4*x^4 + a^2*(-233 + (22*I)*b*x) - I*a*(237 - (54*I)*b*
x + 10*b^2*x^2)))/(8*b^4*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(1/4)*(-17*I -
 44*a + (36*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.37

method	result
risch	$\frac{i(-2b^3x^3+2a\,b^2x^2+8ix^2b^2-2a^2bx-20iabx+2a^3+44ia^2+19bx-93a-48i)\sqrt{b^2x^2+2abx+a^2+1}}{8b^4} + \frac{-\frac{51i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+}\right.}{\sqrt{b^2}}}{}$
default	Expression too large to display

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*(-2*b^3*x^3+8*I*b^2*x^2+2*a*b^2*x^2-20*I*a*b*x-2*a^2*b*x+44*I*a^2+2*a
^3+19*b*x-48*I-93*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^4+1/8/b^3*(-51*I*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-132*a*ln((
b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+24*a^3*ln
```

```
((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+108*I*a
^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*
(96*a^2-32*I*a^3-32+96*I*a)/b^2/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(I+
a)/b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.87

$$\int e^{3i\arctan(a+bx)}x^3\,dx$$
$$= \frac{15i\,a^5 - 495\,a^4 - 1664i\,a^3 + (15i\,a^4 - 480\,a^3 - 1184i\,a^2 + 968\,a + 256i)bx + 2152\,a^2 - 24\,(8\,a^4 + 44i\,a^3 +}{}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/64*(15*I*a^5 - 495*a^4 - 1664*I*a^3 + (15*I*a^4 - 480*a^3 - 1184*I*a^2 +
968*a + 256*I)*b*x + 2152*a^2 - 24*(8*a^4 + 44*I*a^3 + (8*a^3 + 36*I*a^2 -
44*a - 17*I)*b*x - 80*a^2 - 61*I*a + 17)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)) - 8*(2*I*b^4*x^4 + 6*b^3*x^3 - (10*a + 11*I)*b^2*x^2 - 2*I*
a^4 + 78*a^3 + (22*a^2 + 54*I*a - 29)*b*x + 233*I*a^2 - 237*a - 80*I)*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1) + 1224*I*a - 256)/(b^5*x + (a + I)*b^4)
```

Sympy [F]

$$\int e^{3i\arctan(a+bx)}x^3\,dx =$$

$$-i\left(\int \frac{ix^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2a}\right.$$
$$+\int\left(-\frac{3ax^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2}\right.$$
$$+\int \frac{a^3x^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2abx}}$$
$$+\int\left(-\frac{3bx^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2}\right.$$
$$+\int \frac{b^3x^6}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2abx}}$$
$$+\int\left(-\frac{3ia^2x^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2}\right.$$
$$+\int\left(-\frac{3ib^2x^5}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2}\right.$$
$$+\int \frac{3ab^2x^5}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2abx}}$$
$$+\int \frac{3a^2bx^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2abx}}$$
$$+\int\left(-\frac{6iabx^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2}\right.$$

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**3,x)
```

```
[Out] -I*(Integral(I*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x**3/(
a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x**3/(a**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)), x) + Integral(-3*b*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral
(b**3*x**6/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**3/(a**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*
```

```
x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**5/(a**2*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2
*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)), x) + Integral(3*a*b**2*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Int
egral(3*a**2*b*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x*
*4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2295 vs. 2(175) = 350.

Time = 0.21 (sec) , antiderivative size = 2295, normalized size of antiderivative = 9.22

$$\int e^{3i\arctan(a+bx)}x^3\,dx = \text{Too large to display}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*b*x^5/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 315/4*I*a^6*x/((a^2*b^2 -
(a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 3/4*I*a*x^4/sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1) - 945/8*I*(a^2 + 1)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^
2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 21/8*I*a^2*x^3/(sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*b) + 105/8*I*(a^2 + 1)*a^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 105*(I*a*b^2 + b^2)*a^5*x/((a^2*b^2 -
(a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 45*(I*a^2*b + 2*a*b
 - I*b)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
b^2) + 169/4*I*(a^2 + 1)^2*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*b) + 6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*x/((a^2*b^2 - (a
^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 105/8*I*a^3*x^2/(sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 5/8*(-I*a^2 - I)*x^3/(sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)*b) - (I*a*b^2 + b^2)*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*b^2) - 14*I*(a^2 + 1)^2*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*b^2) + 265/2*(I*a*b^2 + b^2)*(a^2 + 1)*a^3*x/((a^2*b^2 - (a^
2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 93/2*(I*a^2*b + 2*a*b
- I*b)*(a^2 + 1)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*b^2) - 15/8*I*(a^2 + 1)^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b) - 5*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)*a*x/((
a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 49/8*I*(a^2
```

```
 + 1)*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 7/2*(I*a*b^2 + b^2)*a
*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 3/2*(I*a^2*b + 2*a*b - I*b)*
x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 315/8*I*a^4*arcsinh(2*(b^2*x
+ a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 - 35/2*(I*a*b^2 + b^2)*(a^2
+ 1)*a^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4)
+ 15/2*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 15/8*I*(a^2 + 1)^3*a/((a^2*b^2 - (a^2
 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + (-I*a^3 - 3*a^2 + 3*I*a
 + 1)*(a^2 + 1)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)*b^2) - 61/2*(I*a*b^2 + b^2)*(a^2 + 1)^2*a*x/((a^2*b^2 - (a^2 + 1)*b^2
)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 9/2*(I*a^2*b + 2*a*b - I*b)*(a^2
 + 1)^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2)
 - 35/2*(I*a*b^2 + b^2)*a^2*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 1
5/2*(I*a^2*b + 2*a*b - I*b)*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) +
 (-I*a^3 - 3*a^2 + 3*I*a + 1)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) +
 105/4*I*(a^2 + 1)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2))/b^4 + 29/2*(I*a*b^2 + b^2)*(a^2 + 1)^2*a^2/((a^2*b^2 - (a^2 + 1)*b^
2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 105/4*I*(a^2 + 1)*a^3/(sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 9/2*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)^2*a/
((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 4*(I*a*
b^2 + b^2)*(a^2 + 1)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 105/2*(I
*a*b^2 + b^2)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2
))/b^6 - 45/2*(I*a^2*b + 2*a*b - I*b)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 15/8*I*(a^2 + 1)^2*arcsinh(2*(b^2*x + a*b)
/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 - 3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a
*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 - 49/4*I*(
a^2 + 1)^2*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 45/2*(I*a*b^2 + b^2)
*(a^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^
6 + 9/2*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 35*(I*a*b^2 + b^2)*(a^2 + 1)*a^2/(sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*b^6) + 15*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*a/(sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^5) + 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2
+ 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 8*(I*a*b^2 + b^2)*(a^2 + 1)^
2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^6)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.14

$$\int e^{3i\arctan(a+bx)}x^3\,dx =$$
$$-\frac{1}{8}\sqrt{(bx+a)^2+1}\left(\left(2x\left(\frac{ix}{b}-\frac{i\,ab^{11}-4\,b^{11}}{b^{13}}\right)-\frac{-2i\,a^2b^{10}+20\,ab^{10}+19i\,b^{10}}{b^{13}}\right)x-\frac{2i\,a^3b^9-44\,a^2b^9-\cdots}{b^{13}}\right.$$
$$-\frac{(8\,a^3+36i\,a^2-44\,a-17i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|\cdots\right.}{\cdots}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt((b*x + a)^2 + 1)*((2*x*(I*x/b - (I*a*b^11 - 4*b^11)/b^13) - (-2*I
*a^2*b^10 + 20*a*b^10 + 19*I*b^10)/b^13)*x - (2*I*a^3*b^9 - 44*a^2*b^9 - 93
*I*a*b^9 + 48*b^9)/b^13) - 1/8*(8*a^3 + 36*I*a^2 - 44*a - 17*I)*log(3*(x*ab
s(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((b*x + a)^2
+ 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 2*I*(x*a
bs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b + 4*(I*x*abs(b) - I*sqrt((b*
x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/
(b^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}x^3\,dx = \int \frac{x^3\,{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}{{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

```
[Out] int((x^3*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

3.182 $\int e^{3i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 227

$$\int e^{3i\arctan(a+bx)}x^2\,dx=\frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3}+\frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3}-\frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}}+\frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{5/2}}{3b^3}+\frac{(11-18ia-6a^2)\operatorname{arcsinh}(a+bx)}{2b^3}$$

```
[Out] 1/2*(11-18*I*a-6*a^2)*arcsinh(b*x+a)/b^3-I*(I+a)^2*(1+I*a+I*b*x)^(5/2)/b^3/
(1-I*a-I*b*x)^(1/2)+1/6*(11*I+18*a-6*I*a^2)*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*
x)^(1/2)/b^3+1/3*I*(1+I*a+I*b*x)^(5/2)*(1-I*a-I*b*x)^(1/2)/b^3+1/2*(11*I+18
*a-6*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5203, 91, 81, 52, 55, 633, 221}

$$\int e^{3i\arctan(a+bx)}x^2\,dx=\frac{(-6a^2-18ia+11)\operatorname{arcsinh}(a+bx)}{2b^3}+\frac{(-6ia^2+18a+11i)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{6b^3}+\frac{(-6ia^2+18a+11i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^3}+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{5/2}}{3b^3}-\frac{i(a+i)^2(ia+ibx+1)^{5/2}}{b^3\sqrt{-ia-ibx+1}}$$

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] ((11*I + 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*
b^3) + ((11*I + 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(
3/2))/(6*b^3) - (I*(I + a)^2*(1 + I*a + I*b*x)^(5/2))/(b^3*Sqrt[1 - I*a - I
*b*x]) + ((I/3)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(5/2))/b^3 + ((11 -
 (18*I)*a - 6*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx \\
&= -\frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} - \frac{i\int \frac{(1+ia+ibx)^{3/2}((3-2ia)(i+a)b-b^2x)}{\sqrt{1-ia-ibx}}\,dx}{b^3} \\
&= -\frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} + \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{5/2}}{3b^3} \\
&\quad + \frac{(11-18ia-6a^2)\int \frac{(1+ia+ibx)^{3/2}}{\sqrt{1-ia-ibx}}\,dx}{3b^2} \\
&= \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} - \frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} \\
&\quad + \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{5/2}}{3b^3} + \frac{(11-18ia-6a^2)\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{2b^2} \\
&= \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} \\
&\quad + \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} - \frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} \\
&\quad + \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{5/2}}{3b^3} + \frac{(11-18ia-6a^2)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2b^2} \\
&= \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} \\
&\quad + \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} - \frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} \\
&\quad + \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{5/2}}{3b^3} + \frac{(11-18ia-6a^2)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{2b^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} \\
&\quad+ \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} \\
&\quad- \frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} + \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{5/2}}{3b^3} \\
&\quad+ \frac{(11-18ia-6a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{4b^4} \\
&= \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} \\
&\quad+ \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} - \frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} \\
&\quad+ \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{5/2}}{3b^3} + \frac{(11-18ia-6a^2)\operatorname{arcsinh}(a+bx)}{2b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.70

$$
\begin{aligned}
&\int e^{3i\arctan(a+bx)}x^2\,dx \\
&= \frac{\sqrt{1+ia+ibx}(52i-53ia^2-2a^3+19bx+7ib^2x^2-2b^3x^3+a(103-16ibx))}{6b^3\sqrt{-i(i+a+bx)}} \\
&\quad+ \frac{(-1)^{3/4}(-11+18ia+6a^2)\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{\sqrt{-ib}b^{5/2}}
\end{aligned}
$$

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(52*I - (53*I)*a^2 - 2*a^3 + 19*b*x + (7*I)*b^2*x^2
- 2*b^3*x^3 + a*(103 - (16*I)*b*x)))/(6*b^3*Sqrt[(-I)*(I + a + b*x)]) + ((-
1)^(3/4)*(-11 + (18*I)*a + 6*a^2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I
 + a + b*x)])/Sqrt[(-I)*b]])/(Sqrt[(-I)*b]*b^(5/2))
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{i\left(2b^2x^2-2abx-9bxi+2a^2+27ia-28\right)\sqrt{b^2x^2+2abx+a^2+1}}{6b^3}-\frac{-\frac{11\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}+\frac{6a^2\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}}{}$
default	Expression too large to display

[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x,method=_RETURNVERBOSE)

[Out] -1/6*I*(2*b^2*x^2-9*I*b*x-2*a*b*x+27*I*a+2*a^2-28)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^3-1/2/b^2*(-11*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+6*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+18*I*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*(16*a-8*I*a^2+8*I)/b^2/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(I+a)/b))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.77

$$\int e^{3i\arctan(a+bx)}x^2\,dx$$
$$=\frac{-7i\,a^4+166\,a^3+(-7i\,a^3+159\,a^2+249i\,a-96)bx+408i\,a^2+12\,(6\,a^3+(6\,a^2+18i\,a-11)bx+24i\,a^2}{}$$

[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="fricas")

[Out] 1/24*(-7*I*a^4 + 166*a^3 + (-7*I*a^3 + 159*a^2 + 249*I*a - 96)*b*x + 408*I*a^2 + 12*(6*a^3 + (6*a^2 + 18*I*a - 11)*b*x + 24*I*a^2 - 29*a - 11*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 4*(2*I*b^3*x^3 + 7*b^2*x^2 + 2*I*a^3 - (16*a + 19*I)*b*x - 53*a^2 - 103*I*a + 52)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 345*a - 96*I)/(b^4*x + (a + I)*b^3)

Sympy [F]

$$\int e^{3i\arctan(a+bx)}x^2\,dx =$$
$$-i\Bigg(\int \frac{ix^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2}}$$
$$+\int\Bigg(-\frac{3ax^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}$$
$$+\int\frac{a^3x^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}}$$
$$+\int\Bigg(-\frac{3bx^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}$$
$$+\int\frac{b^3x^5}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}}$$
$$+\int\Bigg(-\frac{3ia^2x^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}$$
$$+\int\Bigg(-\frac{3ib^2x^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}$$
$$+\int\frac{3ab^2x^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}}$$
$$+\int\frac{3a^2bx^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}}$$
$$+\int\Bigg(-\frac{6iabx^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}$$

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**2,x)
```

```
[Out] -I*(Integral(I*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x**2/(
a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x**2/(a**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)), x) + Integral(-3*b*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral
(b**3*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*
```

```
x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**4/(a**2*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2
*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)), x) + Integral(3*a*b**2*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Int
egral(3*a**2*b*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x*
*3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1608 vs. 2(155) = 310.

Time = 0.20 (sec) , antiderivative size = 1608, normalized size of antiderivative = 7.08

$$\int e^{3i\arctan(a+bx)}x^2\,dx = \text{Too large to display}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -35*I*a^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 1/3*I*b*x^4/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 265/6*I*(a^2 + 1)*a^3*x/((
a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 7/6*I*a*x^3/s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 35/6*I*(a^2 + 1)*a^4/((a^2*b^2 - (a^2 +
1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 61/6*I*(a^2 + 1)^2*a*x/((a^2
*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 2*(-I*a^3 - 3*a^
2 + 3*I*a + 1)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)) + 45*(I*a*b^2 + b^2)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*b^2) - 18*(I*a^2*b + 2*a*b - I*b)*a^3*x/((a^2*b^2 - (a
^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 35/6*I*a^2*x^2/(sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*b) + 29/6*I*(a^2 + 1)^2*a^2/((a^2*b^2 - (a^2 + 1
)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + (-I*a^3 - 3*a^2 + 3*I*a + 1)*
(a^2 + 1)*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 93/2*(I*a*b^2 + b^2)*(a^2 + 1)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*b^2) + 15*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*a*x/((a
^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 4/3*(-I*a^2
- I)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 3/2*(I*a*b^2 + b^2)*x^3/(s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 35/2*I*a^3*arcsinh(2*(b^2*x + a*b)/
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 15/2*(I*a*b^2 + b^2)*(a^2 + 1)*a^
3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 3*(I*
```

```
a^2*b + 2*a*b - I*b)*(a^2 + 1)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*b^2) - (-I*a^3 - 3*a^2 + 3*I*a + 1)*(a^2 + 1)*a/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 9/2*(I*a*b^2 +
b^2)*(a^2 + 1)^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*b^2) + 15/2*(I*a*b^2 + b^2)*a*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b
^3) - 3*(I*a^2*b + 2*a*b - I*b)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2)
 - 15/2*I*(a^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)
*b^2))/b^3 - 9/2*(I*a*b^2 + b^2)*(a^2 + 1)^2*a/((a^2*b^2 - (a^2 + 1)*b^2)*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 35/3*I*(a^2 + 1)*a^2/(sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*b^3) - 45/2*(I*a*b^2 + b^2)*a^2*arcsinh(2*(b^2*x + a*b
)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 9*(I*a^2*b + 2*a*b - I*b)*a*arc
sinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + (-I*a^3 - 3*
a^2 + 3*I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
)/b^3 + 8/3*I*(a^2 + 1)^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 9/2*(I*
a*b^2 + b^2)*(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2))/b^5 + 15*(I*a*b^2 + b^2)*(a^2 + 1)*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*b^5) - 6*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.07

$$\int e^{3i\arctan(a+bx)}x^2\,dx$$
$$= -\frac{1}{6}\sqrt{(bx+a)^2+1}\left(x\left(\frac{2i\,x}{b}-\frac{2i\,ab^6-9\,b^6}{b^8}\right)-\frac{-2i\,a^2b^5+27\,ab^5+28i\,b^5}{b^8}\right)$$
$$+\frac{(6\,a^2+18i\,a-11)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b\right.\right.}{}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt((b*x + a)^2 + 1)*(x*(2*I*x/b - (2*I*a*b^6 - 9*b^6)/b^8) - (-2*I*a
^2*b^5 + 27*a*b^5 + 28*I*b^5)/b^8) + 1/6*(6*a^2 + 18*I*a - 11)*log(3*(x*abs
(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((b*x + a)^2 +
 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 2*I*(x*ab
s(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b + 4*(I*x*abs(b) - I*sqrt((b*x
 + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(
b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}x^2\,dx = \int \frac{x^2\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

```
[In] int((x^2*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
[Out] int((x^2*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

3.183 $\int e^{3i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 163

$$\int e^{3i\arctan(a+bx)}x\,dx = -\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{3(3i+2a)\text{arcsinh}(a+bx)}{2b^2}$$

```
[Out] 3/2*(3*I+2*a)*arcsinh(b*x+a)/b^2-(1-I*a)*(1+I*a+I*b*x)^(5/2)/b^2/(1-I*a-I*b
*x)^(1/2)-1/2*(3-2*I*a)*(1+I*a+I*b*x)^(3/2)*(1-I*a-I*b*x)^(1/2)/b^2-3/2*(3-
2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5203, 79, 52, 55, 633, 221}

$$\int e^{3i\arctan(a+bx)}x\,dx = \frac{3(2a+3i)\text{arcsinh}(a+bx)}{2b^2} - \frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}} - \frac{(3-2ia)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2} - \frac{3(3-2ia)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2b^2}$$

```
[In] Int[E^((3*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] (-3*(3 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) - ((
3 - (2*I)*a)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b^2) - ((1 -
 I*a)*(1 + I*a + I*b*x)^(5/2))/(b^2*Sqrt[1 - I*a - I*b*x]) + (3*(3*I + 2*a)
*ArcSinh[a + b*x])/(2*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{(3i+2a)\int \frac{(1+ia+ibx)^{3/2}}{\sqrt{1-ia-ibx}}\,dx}{b}$$

$$= -\frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{(3(3i+2a))\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx}{2b}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{(3(3i+2a))\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2b}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{(3(3i+2a))\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{2b}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{(3(3i+2a))\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{4b^3}$$

$$= -\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2} - \frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}} + \frac{3(3i+2a)\text{arcsinh}(a+bx)}{2b^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int e^{3i\arctan(a+bx)}x\,dx = \frac{\sqrt{1+ia+ibx}(-14+15ia+a^2+5ibx-b^2x^2)}{2b^2\sqrt{-i(i+a+bx)}} + \frac{3\sqrt[4]{-1}(3i+2a)\sqrt{-ib}\,\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{5/2}}$$

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])*x,x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(-14 + (15*I)*a + a^2 + (5*I)*b*x - b^2*x^2))/(2*b^2
*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(1/4)*(3*I + 2*a)*Sqrt[(-I)*b]*ArcSinh
[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.14

method	result
risch	$\frac{i(-bx+a+6i)\sqrt{b^2x^2+2abx+a^2+1}}{2b^2} + \frac{\frac{9i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}} + \frac{6a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}} - \frac{i(-8ia+8)\sqrt{\left(x+\frac{i+a}{b}\right)^2}}{b^2\left(x+\frac{i+}{b}\right.}}{2b}$
default	Expression too large to display

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(-b*x+a+6*I)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^2+1/2/b*(9*I*ln((b^2*x+a
*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+6*a*ln((b^2*x+a*
b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*(8-8*I*a)/b^2/(
x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(I+a)/b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int e^{3i\arctan(a+bx)}x\,dx = \frac{3i\,a^3+(3i\,a^2-44\,a-32i)bx-47\,a^2-12\left((2\,a+3i)bx+2\,a^2+5i\,a-3\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+}\right.}{8\left(b^3x+(a+i)b^2\right)}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(3*I*a^3 + (3*I*a^2 - 44*a - 32*I)*b*x - 47*a^2 - 12*((2*a + 3*I)*b*x +
 2*a^2 + 5*I*a - 3)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 4*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*b^2*x^2 - I*a^2 + 5*b*x + 15*a + 14*I)
- 76*I*a + 32)/(b^3*x + (a + I)*b^2)
```

Sympy [F]

$$\int e^{3i\arctan(a+bx)}x\,dx =$$

$$-i\left(\int \frac{ix}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2\cdots}}\right.$$

$$+\int\left(-\frac{3ax}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\cdots}}\right.$$

$$+\int\frac{a^3x}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab\cdots}}$$

$$+\int\left(-\frac{3bx^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\cdots}}\right.$$

$$+\int\frac{b^3x^4}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab\cdots}}$$

$$+\int\left(-\frac{3ia^2x}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\cdots}}\right.$$

$$+\int\left(-\frac{3ib^2x^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\cdots}}\right.$$

$$+\int\frac{3ab^2x^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab\cdots}}$$

$$+\int\frac{3a^2bx^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab\cdots}}$$

$$+\int\left(-\frac{6iabx^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+\cdots}}\right.$$

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x,x)
```

```
[Out] -I*(Integral(I*x/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x/(a**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)), x) + Integral(a**3*x/(a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x
) + Integral(-3*b*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**
```

```
2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b**3*x**
4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2*x/(a**2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)), x) + Integral(-3*I*b**2*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
+ Integral(3*a*b**2*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b
*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a**2
*b*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x**2/(a**2*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1108 vs. 2(113) = 226.

Time = 0.22 (sec) , antiderivative size = 1108, normalized size of antiderivative = 6.80

$$\int e^{3i\arctan(a+bx)}x\,dx = \text{Too large to display}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 15*I*a^4*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
- 31/2*I*(a^2 + 1)*a^2*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)) - 1/2*I*b*x^3/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 5/2*I*(a^2
+ 1)*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 6*
(I*a^2*b + 2*a*b - I*b)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)) - 18*(I*a*b^2 + b^2)*a^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 3/2*I*(a^2 + 1)^2*b*x/((a^2*b^2 - (a^2 +
 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-I*a^3 - 3*a^2 + 3*I*a + 1)*
a*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 5/2*I
*a*x^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3/2*I*(a^2 + 1)^2*a/((a^2*b^2 -
(a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-I*a^3 - 3*a^2 + 3*I*a
 + 1)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3
*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)) + 15*(I*a*b^2 + b^2)*(a^2 + 1)*a*x/((a^2*b^2 - (a^2
 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 15/2*I*a^2*arcsinh(2*(b^2
```

```
*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 - 3*(I*a*b^2 + b^2)*(a^2
+ 1)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2)
+ 3*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*a/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*b) - 3*(I*a*b^2 + b^2)*x^2/(sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*b^2) - 3/2*(-I*a^2 - I)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^
2 + 4*(a^2 + 1)*b^2))/b^2 + 5*I*(a^2 + 1)*a/(sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*b^2) + 9*(I*a*b^2 + b^2)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*
(a^2 + 1)*b^2))/b^4 - 3*(I*a^2*b + 2*a*b - I*b)*arcsinh(2*(b^2*x + a*b)/sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + (I*a^3 + 3*a^2 - 3*I*a - 1)/(sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 6*(I*a*b^2 + b^2)*(a^2 + 1)/(sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.28

$$\int e^{3i\arctan(a+bx)}x\,dx = -\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{ix}{b}+\frac{-i\,ab^2+6\,b^2}{b^4}\right) - \frac{(2\,a+3i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b|-\sqrt{(bx\right.\right.}{}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(I*x/b + (-I*a*b^2 + 6*b^2)/b^4) - 1/2*(2*a + 3*
I)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqr
t((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs
(b) + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b + 4*(I*x*abs(b
) - I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2
+ 1))*abs(b))/(b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}x\,dx = \int \frac{x\,{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}{{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}}\,dx$$

```
[In] int((x*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

```
[Out] int((x*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

3.184 $\int e^{3i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 94

$$\int e^{3i\arctan(a+bx)}\,dx = -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{3\text{arcsinh}(a+bx)}{b}$$

```
[Out] -3*arcsinh(b*x+a)/b-2*I*(1+I*a+I*b*x)^(3/2)/b/(1-I*a-I*b*x)^(1/2)-3*I*(1-I*
a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5201, 49, 52, 55, 633, 221}

$$\int e^{3i\arctan(a+bx)}\,dx = -\frac{3\text{arcsinh}(a+bx)}{b} - \frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}} - \frac{3i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

```
[In] Int[E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-3*I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b - ((2*I)*(1 + I*a +
I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I*b*x]) - (3*ArcSinh[a + b*x])/b
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+ia+ibx)^{3/2}}{(1-ia-ibx)^{3/2}}\,dx \\
&= -\frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - 3\int \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\,dx \\
&= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - 3\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx
\end{aligned}
$$

$$= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - 3\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx$$

$$= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b^2}$$

$$= -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{3\text{arcsinh}(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int e^{3i\arctan(a+bx)}\,dx = \frac{\sqrt{1+(a+bx)^2}\left(-i+\frac{4}{i+a+bx}\right)}{b} - \frac{3\text{arcsinh}(a+bx)}{b}$$

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x]),x]
[Out] (Sqrt[1 + (a + b*x)^2]*(-I + 4/(I + a + b*x)))/b - (3*ArcSinh[a + b*x])/b
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{b} - \frac{3\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}} + \frac{4\sqrt{\left(x+\frac{i+a}{b}\right)^2 b^2-2ib\left(x+\frac{i+a}{b}\right)}}{b^2\left(x+\frac{i+a}{b}\right)}$
default	$-ib^3\left(\frac{x^2}{b^2\sqrt{b^2x^2+2abx+a^2+1}} - \frac{3a\left(-\frac{x}{b^2\sqrt{b^2x^2+2abx+a^2+1}} - \frac{a\left(-\frac{1}{b^2\sqrt{b^2x^2+2abx+a^2+1}} - \frac{2a\left(2b^2x+2ab\right)}{b\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\sqrt{b^2x^2+2abx+a^2+1}}\right)}{b}\right.}{b}\right.$

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+4/b^2/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*
(x+(I+a)/b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int e^{3i\arctan(a+bx)}\,dx = \frac{(-i\,a+8)bx - i\,a^2 + 6\,(bx+a+i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) - 2\,\sqrt{b^2x^2+2\,abx+a^2+1}(}{2\,(b^2x+(a+i)b)}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*a + 8)*b*x - I*a^2 + 6*(b*x + a + I)*log(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*b*x + I*a - 5
) + 9*a + 8*I)/(b^2*x + (a + I)*b)
```

Sympy [F]

$$\int e^{3i\arctan(a+bx)}\,dx =$$

$$-i\left(\int \frac{i}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2}}\right.$$
$$+\int\left(-\frac{3a}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}\right.$$
$$+\int\frac{a^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}}$$
$$+\int\left(-\frac{3ia^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}\right.$$
$$+\int\left(-\frac{3bx}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}\right.$$
$$+\int\frac{b^3x^3}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}}$$
$$+\int\left(-\frac{3ib^2x^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}\right.$$
$$+\int\frac{3ab^2x^2}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}}$$
$$+\int\frac{3a^2bx}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+2ab}}$$
$$+\int\left(-\frac{6iabx}{a^2\sqrt{a^2+2abx+b^2x^2+1}+2abx\sqrt{a^2+2abx+b^2x^2+1}+b^2x^2\sqrt{a^2+2abx+b^2x^2+1}+\sqrt{a^2+}}\right.$$

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] -I*(Integral(I/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a/(a**2*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1)), x) + Integral(a**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + In
tegral(-3*I*a**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))
, x) + Integral(-3*I*b**2*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(
3*a*b**2*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a**2*b*x/(a**2*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)), x) + Integral(-6*I*a*b*x/(a**2*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(66) = 132$.

Time = 0.21 (sec) , antiderivative size = 736, normalized size of antiderivative = 7.83

$$\int e^{3i\arctan(a+bx)}\,dx = -\frac{6i\,a^3b^2x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} + \frac{5i\,(a^2+1)ab^2x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} - \frac{i\,(a^2+1)a^2b}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} + \frac{6\,(i\,ab^2+b^2)a^2x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} - \frac{3\,(i\,a^2b+2\,ab-i\,b)abx}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} + \frac{(i\,a^3+3\,a^2-3i\,a-1)b^2x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} - \frac{i\,bx^2}{\sqrt{b^2x^2+2\,abx+a^2+1}} - \frac{3\,(i\,a^2b+2\,ab-i\,b)a^2}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} - \frac{(-i\,a^3-3\,a^2+3i\,a+1)ab}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} - \frac{3\,(i\,ab^2+b^2)(a^2+1)x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} + \frac{3i\,a\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b} + \frac{3\,(i\,ab^2+b^2)(a^2+1)a}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}b} - \frac{2\,(i\,a^2+i)}{\sqrt{b^2x^2+2\,abx+a^2+1}b} - \frac{3\,(i\,ab^2+b^2)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^3} + \frac{3\,(i\,a^2b+2\,ab-i\,b)}{\sqrt{b^2x^2+2\,abx+a^2+1}b^2}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -6*I*a^3*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) + 5*I*(a^2 + 1)*a*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)) - I*(a^2 + 1)*a^2*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)) + 6*(I*a*b^2 + b^2)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*(I*a^2*b + 2*a*b - I*b)*a*b*x/((a^2*b^
2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (I*a^3 + 3*a^2 - 3*
I*a - 1)*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
```

```
) - I*b*x^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3*(I*a^2*b + 2*a*b - I*b)*a
^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (-I*a^3
- 3*a^2 + 3*I*a + 1)*a*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)) - 3*(I*a*b^2 + b^2)*(a^2 + 1)*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 3*I*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2))/b + 3*(I*a*b^2 + b^2)*(a^2 + 1)*a/((a^2*b^2 - (a^2
+ 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 2*(I*a^2 + I)/(sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b) - 3*(I*a*b^2 + b^2)*arcsinh(2*(b^2*x + a*b)/sqrt(
-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 3*(I*a^2*b + 2*a*b - I*b)/(sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b^2)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(66) = 132.

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.91

$$\int e^{3i\arctan(a+bx)}\,dx$$

$$= \frac{\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b|-\sqrt{(bx+a)^2+1}\right)a^2 \cdots\right.}{\cdots} - \frac{i\sqrt{(bx+a)^2+1}}{b}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((
b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b)
 + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b + 4*(I*x*abs(b) -
 I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1
))*abs(b))/abs(b) - I*sqrt((b*x + a)^2 + 1)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}\,dx = \int \frac{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

```
[In] int((a*1i + b*x*1i + 1)^3/((a + b*x)^2 + 1)^(3/2),x)
```

```
[Out] int((a*1i + b*x*1i + 1)^3/((a + b*x)^2 + 1)^(3/2), x)
```

3.185 $\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx=\frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}}-i\operatorname{arcsinh}(a+bx)-\frac{2(i-a)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{3/2}}$$

```
[Out] -I*arcsinh(b*x+a)-2*(I-a)^(3/2)*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-
a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I+a)^(3/2)+4*(1+I*a+I*b*x)^(1/2)/(1-I*a)/(1-
I*a-I*b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5203, 100, 163, 55, 633, 221, 95, 214}

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx=-i\operatorname{arcsinh}(a+bx)-\frac{2(-a+i)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(a+i)^{3/2}}+\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}$$

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] (4*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*Sqrt[1 - I*a - I*b*x]) - I*ArcSinh[a +
 b*x] - (2*(I - a)^(3/2)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/(I + a)^(3/2)
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+ia+ibx)^{3/2}}{x(1-ia-ibx)^{3/2}}\,dx \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{2\int \frac{\frac{1}{2}i(i-a)^2b-\frac{1}{2}(1-ia)b^2x}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i+a)b} \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{(i-a)^2\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{1-ia} \\
&\quad -(ib)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{(2(i-a)^2)\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{1-ia} \\
&\quad -(ib)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - \frac{2(i-a)^{3/2}\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{3/2}} \\
&\quad - \frac{i\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b} \\
&= \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - i\text{arcsinh}(a+bx) - \frac{2(i-a)^{3/2}\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.46

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx = \frac{2\left(\frac{2i\sqrt{1+ia+ibx}}{\sqrt{-i(i+a+bx)}} + \frac{\sqrt[4]{-1}(i+a)(-ib)^{3/2}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{3/2}} + \frac{\sqrt{-1-ia}(-i+a)\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1+ia}}\right)}{i+a}$$

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*(((2*I)*Sqrt[1 + I*a + I*b*x])/Sqrt[(-I)*(I + a + b*x)] + ((-1)^(1/4)*(I
 + a)*((-I)*b)^(3/2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])
/Sqrt[(-I)*b]])/b^(3/2) + (Sqrt[-1 - I*a]*(-I + a)*ArcTanh[(Sqrt[-1 - I*a]*
Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1
+ I*a]))/(I + a)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 484 vs. 2(104) = 208.

Time = 0.50 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.62

method	result
default	$-ib^3\left(-\frac{x}{b^2\sqrt{b^2x^2+2abx+a^2+1}} - \frac{a\left(-\frac{1}{b^2\sqrt{b^2x^2+2abx+a^2+1}} - \frac{2a\left(2b^2x+2ab\right)}{b\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\sqrt{b^2x^2+2abx+a^2+1}}\right)}{b} + \frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+}\right.}{b^2\sqrt{b^2}}\right.$

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -I*b^3*(-x/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-a/b*(-1/b^2/(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)-2*a/b*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))+1/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/(b^2)^(1/2))+6*I*(1+I*a)^2*b*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*(1+I*a)*b^2*(-1/b^2/(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)-2*a/b*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^
2+1)^(1/2))+(-I*a^3-3*a^2+3*I*a+1)*(1/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
-2*a*b/(a^2+1)*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)-1/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(90) = 180.

Time = 0.28 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.66

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx =$$

$$-\frac{\left((a+i)bx+a^2+2i\,a-1\right)\sqrt{-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}}\log\left(-\frac{(a-i)bx-\sqrt{b^2x^2+2\,abx+a^2+1}(a-i)-(i\,a^2-2\,a-i)\sqrt{-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}}}{a-i}\right.}{}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -(((a + I)*b*x + a^2 + 2*I*a - 1)*sqrt(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*
I*a^2 - 3*a - I))*log(-((a - I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a
- I) - (I*a^2 - 2*a - I)*sqrt(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3
*a - I)))/(a - I)) - ((a + I)*b*x + a^2 + 2*I*a - 1)*sqrt(-(a^3 - 3*I*a^2 -
 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))*log(-((a - I)*b*x - sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)*(a - I) - (-I*a^2 + 2*a + I)*sqrt(-(a^3 - 3*I*a^2 - 3*a + I
)/(a^3 + 3*I*a^2 - 3*a - I)))/(a - I)) + 4*b*x + ((-I*a + 1)*b*x - I*a^2 +
2*a + I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 4*a + 4*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1) + 4*I)/((a + I)*b*x + a^2 + 2*I*a - 1)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx =$$

$$-i\left(\int \frac{i}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2+\cdots}}\right.$$
$$+\int\left(-\frac{3a}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2\cdots}}\right.$$
$$+\int \frac{a^3}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2+2\cdots}}$$
$$+\int\left(-\frac{3ia^2}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2\cdots}}\right.$$
$$+\int\left(-\frac{3bx}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2\cdots}}\right.$$
$$+\int \frac{b^3x^3}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2+2\cdots}}$$
$$+\int\left(-\frac{3ib^2x^2}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2\cdots}}\right.$$
$$+\int \frac{3ab^2x^2}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2+2\cdots}}$$
$$+\int \frac{3a^2bx}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2+2\cdots}}$$
$$+\int\left(-\frac{6iabx}{a^2x\sqrt{a^2+2abx+b^2x^2+1}+2abx^2\sqrt{a^2+2abx+b^2x^2+1}+b^2x^3\sqrt{a^2+2abx+b^2x^2+1}+x\sqrt{a^2\cdots}}\right.$$

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x,x)
```

```
[Out] -I*(Integral(I/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a/(a**
2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*x*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b
**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)),
x) + Integral(-3*b*x/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x
**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b**
3*x**3/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2
```

```
+ 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
 + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**2/(a
**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a*b**2*x**2/(a**2*x*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)), x) + Integral(3*a**2*b*x/(a**2*x*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**
2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)), x) + Integral(-6*I*a*b*x/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x
))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(90) = 180$.

Time = 0.19 (sec) , antiderivative size = 733, normalized size of antiderivative = 5.47

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx$$

$$= \frac{2i\,a^2b^3x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} + \frac{(-i\,a^2-i)b^3x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}}$$
$$+ \frac{(-i\,a^3-3\,a^2+3i\,a+1)ab^3x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}(a^2+1)}$$
$$+ \frac{i\,(a^2+1)ab^2}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}}$$
$$+ \frac{(-i\,a^3-3\,a^2+3i\,a+1)a^2b^2}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}(a^2+1)}$$
$$- \frac{3\,(i\,ab^2+b^2)abx}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} + \frac{3\,(i\,a^2b+2\,ab-i\,b)b^2x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}}$$
$$- \frac{3\,(i\,ab^2+b^2)a^2}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} + \frac{3\,(i\,a^2b+2\,ab-i\,b)ab}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}}$$
$$- \frac{(-i\,a^3-3\,a^2+3i\,a+1)\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{(a^2+1)^{\frac{3}{2}}}$$
$$+ \frac{-i\,a^3-3\,a^2+3i\,a+1}{\sqrt{b^2x^2+2\,abx+a^2+1}(a^2+1)} + \frac{3\,(i\,ab^2+b^2)}{\sqrt{b^2x^2+2\,abx+a^2+1}b^2}$$
$$- i\,\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2*I*a^2*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
 + (-I*a^2 - I)*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)) + (-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + I*(a^2 + 1)*a*b^2/((a^2*b^2
- (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-I*a^3 - 3*a^2 + 3*I
*a + 1)*a^2*b^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*(a^2 + 1)) - 3*(I*a*b^2 + b^2)*a*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)) + 3*(I*a^2*b + 2*a*b - I*b)*b^2*x/((a^2*b^2 - (a^
2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*(I*a*b^2 + b^2)*a^2/((a^
2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 3*(I*a^2*b + 2*
a*b - I*b)*a*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) - (-I*a^3 - 3*a^2 + 3*I*a + 1)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2
+ 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) + (-I*a^3 - 3*a
^2 + 3*I*a + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 3*(I*a*b^2
+ b^2)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - I*arcsinh(2*(b^2*x + a*b)/
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(90) = 180$.

Time = 0.40 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.88

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx$$
$$= \frac{i\,b\log\left(-3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab-a^3b-\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|-3\left(x|b|-\sqrt{(bx+a)^2+\cdots}\right.\right.}{\cdots}$$
$$-\frac{\left(i\,a^2+2\,a-i\right)\log\left(\left(\frac{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)\right)}{\sqrt{a^2+1}(a+i)}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*I*b*log(-3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b - a^3*b - (x*abs(b)
 - sqrt((b*x + a)^2 + 1))^3*abs(b) - 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a
^2*abs(b) - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b - 4*(I*x
*abs(b) - I*sqrt((b*x + a)^2 + 1))*a*abs(b) + a*b + (x*abs(b) - sqrt((b*x +
 a)^2 + 1))*abs(b))/abs(b) - (I*a^2 + 2*a - I)*log(abs(-2*x*abs(b) + 2*sqrt
((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 +
 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a + I))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx = \int \frac{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{x\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

```
[In] int((a*1i + b*x*1i + 1)^3/(x*((a + b*x)^2 + 1)^(3/2)),x)
[Out] int((a*1i + b*x*1i + 1)^3/(x*((a + b*x)^2 + 1)^(3/2)), x)
```

3.186 $\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 176

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} + \frac{6i\sqrt{i-a}b\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{5/2}}$$

```
[Out] 6*I*b*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/
2))*(I-a)^(1/2)/(I+a)^(5/2)-(1+I*a+I*b*x)^(3/2)/(1-I*a)/x/(1-I*a-I*b*x)^(1/
2)-6*I*b*(1+I*a+I*b*x)^(1/2)/(I+a)^2/(1-I*a-I*b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5203, 96, 95, 214}

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \frac{6i\sqrt{-a+i}b\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(a+i)^{5/2}} - \frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}} - \frac{6ib\sqrt{ia+ibx+1}}{(a+i)^2\sqrt{-ia-ibx+1}}$$

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] ((-6*I)*b*Sqrt[1 + I*a + I*b*x])/((I + a)^2*Sqrt[1 - I*a - I*b*x]) - (1 + I
*a + I*b*x)^(3/2)/((1 - I*a)*x*Sqrt[1 - I*a - I*b*x]) + ((6*I)*Sqrt[I - a]*
b*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I
*b*x])])/(I + a)^(5/2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1+ia+ibx)^{3/2}}{x^2(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} - \frac{(3b)\int \frac{\sqrt{1+ia+ibx}}{x(1-ia-ibx)^{3/2}}\,dx}{i+a}$$

$$= -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} - \frac{(3(i-a)b)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i+a)^2}$$

$$= -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} - \frac{(6(i-a)b)\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i+a)^2}$$

$$= -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} + \frac{6i\sqrt{i-a}b\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \frac{\frac{\sqrt{1+ia+ibx}(1+a^2-5ibx+abx)}{x\sqrt{-i(i+a+bx)}} + \frac{6i\sqrt{-1-ia}b\,\mathrm{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1+ia}}}{(i+a)^2}$$

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] ((Sqrt[1 + I*a + I*b*x]*(1 + a^2 - (5*I)*b*x + a*b*x))/(x*Sqrt[(-I)*(I + a
+ b*x)]) + ((6*I)*Sqrt[-1 - I*a]*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a
 + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 + I*a])/(I + a)^
2
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

method	result
risch	$\frac{i\sqrt{b^2x^2+2abx+a^2+1}\,(a-i)}{(i+a)^2x} + \frac{b\left(-\frac{(3a^2+3)\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(i+a)\sqrt{a^2+1}} - \frac{4i(ia-1)\sqrt{\left(x+\frac{i+a}{b}\right)^2b^2-2ib\left(x+\frac{i+a}{b}\right)}}{b(i+a)\left(x+\frac{i+a}{b}\right)}\right)}{a^2+2ia-1}$
default	$-\frac{6b^2(2b^2x+2ab)}{(4b^2(a^2+1)-4a^2b^2)\sqrt{b^2x^2+2abx+a^2+1}} - ib^3\left(-\frac{1}{b^2\sqrt{b^2x^2+2abx+a^2+1}} - \frac{2a(2b^2x+2ab)}{b(4b^2(a^2+1)-4a^2b^2)\sqrt{b^2x^2+2abx+a^2+1}}\right) - \frac{}{(4}$

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] I*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*(a-I)/(I+a)^2/x+1/(2*I*a+a^2-1)*b*(-(3*a^2+
3)/(I+a)/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/x)-4*I*(I*a-1)/b/(I+a)/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*
(x+(I+a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(116) = 232.

Time = 0.30 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.21

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx =$$

$$-\frac{(-i\,a-5)b^2x^2+(-i\,a^2-4\,a-5i)bx-3\left((a^2+2i\,a-1)bx^2+(a^3+3i\,a^2-3\,a-i)x\right)\sqrt{\frac{(a-i)}{a^5+5i\,a^4-10\,a^3-}}}{}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -((-I*a - 5)*b^2*x^2 + (-I*a^2 - 4*a - 5*I)*b*x - 3*((a^2 + 2*I*a - 1)*b*x^
2 + (a^3 + 3*I*a^2 - 3*a - I)*x)*sqrt((a - I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 -
 10*I*a^2 + 5*a + I))*log(-(b^2*x + (a^3 + 3*I*a^2 - 3*a - I)*sqrt((a - I)*
b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*I*a^2 + 5*a + I)) - sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*b)/b) + 3*((a^2 + 2*I*a - 1)*b*x^2 + (a^3 + 3*I*a^2 - 3*a - I)*
x)*sqrt((a - I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*I*a^2 + 5*a + I))*log(-(b^
2*x - (a^3 + 3*I*a^2 - 3*a - I)*sqrt((a - I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 -
10*I*a^2 + 5*a + I)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b)/b) + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*((-I*a - 5)*b*x - I*a^2 - I))/((a^2 + 2*I*a - 1)*b*
x^2 + (a^3 + 3*I*a^2 - 3*a - I)*x)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx =$$
$$-i\Bigg(\int \frac{i}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a^2}}$$
$$+\int\Bigg(-\frac{3a}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a}}$$
$$+\int \frac{a^3}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a^2+}}$$
$$+\int\Bigg(-\frac{3ia^2}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a}}$$
$$+\int\Bigg(-\frac{3bx}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a}}$$
$$+\int \frac{b^3x^3}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a^2+}}$$
$$+\int\Bigg(-\frac{3ib^2x^2}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a}}$$
$$+\int \frac{3ab^2x^2}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a^2+}}$$
$$+\int \frac{3a^2bx}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a^2+}}$$
$$+\int\Bigg(-\frac{6iabx}{a^2x^2\sqrt{a^2+2abx+b^2x^2+1}+2abx^3\sqrt{a^2+2abx+b^2x^2+1}+b^2x^4\sqrt{a^2+2abx+b^2x^2+1}+x^2\sqrt{a}}$$

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**2,x)
```

```
[Out] -I*(Integral(I/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*
a/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
 x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*x**2*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x**2*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*x**2*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*
```

```
x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)), x) + Integral(-3*I*b**2*x**2/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))
, x) + Integral(3*a*b**2*x**2/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
 + Integral(3*a**2*b*x/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*
a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Inte
gral(-6*I*a*b*x/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**
3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(116) = 232.

Time = 0.21 (sec) , antiderivative size = 992, normalized size of antiderivative = 5.64

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -I*a*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - I*a^2*b^3/((a^2*b^2 - (a^2 + 1)*b
^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3
*b^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)
^2) - 3*(I*a^2*b + 2*a*b - I*b)*a*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b^4*x
/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) -
3*(I*a^2*b + 2*a*b - I*b)*a^2*b^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^3/((a^2
*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 3*(I*a
*b^2 + b^2)*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)) + 3*(I*a*b^2 + b^2)*a*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)) + 3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b*arcsinh(2*a*b*x/(sqrt(
-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(
5/2) + I*b/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3*(-I*a^3 - 3*a^2 + 3*I*a +
1)*a*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + 3*(I*a^2*b + 2*a*b
 - I*b)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2
```

```
/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2
+ 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - 3*(I*a^2*b + 2*a*b - I*b)/(sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) - (-I*a^3 - 3*a^2 + 3*I*a + 1)/(sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x)
```

Giac [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{(i\,bx + i\,a + 1)^3}{\left((bx+a)^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{x^2\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

```
[In] int((a*1i + b*x*1i + 1)^3/(x^2*((a + b*x)^2 + 1)^(3/2)),x)
[Out] int((a*1i + b*x*1i + 1)^3/(x^2*((a + b*x)^2 + 1)^(3/2)), x)
```

3.187 $\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 264

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(1+ia)(i+a)^3\sqrt{1-ia-ibx}} + \frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1+ia)(i+a)^2x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/2}}{2(1+a^2)\,x^2\sqrt{1-ia-ibx}} + \frac{3(3+2ia)b^2\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}(i+a)^{7/2}}$$

```
[Out] 3*(3+2*I*a)*b^2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-
I*b*x)^(1/2))/(I+a)^(7/2)/(I-a)^(1/2)+1/2*(3*I-2*a)*b*(1+I*a+I*b*x)^(3/2)/(
1+I*a)/(I+a)^2/x/(1-I*a-I*b*x)^(1/2)-1/2*(1+I*a+I*b*x)^(5/2)/(a^2+1)/x^2/(1
-I*a-I*b*x)^(1/2)+3*(3*I-2*a)*b^2*(1+I*a+I*b*x)^(1/2)/(1+I*a)/(I+a)^3/(1-I*
a-I*b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5203, 98, 96, 95, 214}

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = -\frac{(ia+ibx+1)^{5/2}}{2(a^2+1)\,x^2\sqrt{-ia-ibx+1}} + \frac{3(3+2ia)b^2\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}(a+i)^{7/2}} + \frac{3(-2a+3i)b^2\sqrt{ia+ibx+1}}{(1+ia)(a+i)^3\sqrt{-ia-ibx+1}} + \frac{(-2a+3i)b(ia+ibx+1)^{3/2}}{2(1+ia)(a+i)^2x\sqrt{-ia-ibx+1}}$$

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] (3*(3*I - 2*a)*b^2*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*(I + a)^3*Sqrt[1 - I*a
 - I*b*x]) + ((3*I - 2*a)*b*(1 + I*a + I*b*x)^(3/2))/(2*(1 + I*a)*(I + a)^2
*x*Sqrt[1 - I*a - I*b*x]) - (1 + I*a + I*b*x)^(5/2)/(2*(1 + a^2)*x^2*Sqrt[1
 - I*a - I*b*x]) + (3*(3 + (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a +
 I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/(Sqrt[I - a]*(I + a)^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+ia+ibx)^{3/2}}{x^3(1-ia-ibx)^{3/2}}\,dx\\
&= -\frac{(1+ia+ibx)^{5/2}}{2\,(1+a^2)\,x^2\sqrt{1-ia-ibx}}+\frac{((3i-2a)b)\int\frac{(1+ia+ibx)^{3/2}}{x^2(1-ia-ibx)^{3/2}}\,dx}{2\,(1+a^2)}\\
&= -\frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\,(1+a^2)\,x\sqrt{1-ia-ibx}}\\
&\quad -\frac{(1+ia+ibx)^{5/2}}{2\,(1+a^2)\,x^2\sqrt{1-ia-ibx}}-\frac{(3(3i-2a)b^2)\int\frac{\sqrt{1+ia+ibx}}{x(1-ia-ibx)^{3/2}}\,dx}{2(i+a)\,(1+a^2)}\\
&= -\frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(i-a)(1-ia)^3\sqrt{1-ia-ibx}}-\frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\,(1+a^2)\,x\sqrt{1-ia-ibx}}\\
&\quad -\frac{(1+ia+ibx)^{5/2}}{2\,(1+a^2)\,x^2\sqrt{1-ia-ibx}}+\frac{(3(3i-2a)b^2)\int\frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2(i+a)^3}\\
&= -\frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(i-a)(1-ia)^3\sqrt{1-ia-ibx}}\\
&\quad -\frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\,(1+a^2)\,x\sqrt{1-ia-ibx}}-\frac{(1+ia+ibx)^{5/2}}{2\,(1+a^2)\,x^2\sqrt{1-ia-ibx}}\\
&\quad +\frac{(3(3i-2a)b^2)\,\text{Subst}\left(\int\frac{1}{-1-ia-(-1+ia)x^2}\,dx,x,\frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i+a)^3}\\
&= -\frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(i-a)(1-ia)^3\sqrt{1-ia-ibx}}-\frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1-ia)\,(1+a^2)\,x\sqrt{1-ia-ibx}}\\
&\quad -\frac{(1+ia+ibx)^{5/2}}{2\,(1+a^2)\,x^2\sqrt{1-ia-ibx}}+\frac{3(3+2ia)b^2\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}(i+a)^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.73

$$
\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \frac{\frac{\sqrt{1+ia+ibx}\,(i+a+ia^2+a^3-5bx+5iabx+14ib^2x^2-ab^2x^2)}{x^2\sqrt{-i(i+a+bx)}}-\frac{6i\sqrt{-1-ia}(-3i+2a)b^2\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1+ia}(-i+a)}}{2(i+a)^3}
$$

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x^3,x]
```

```
[Out] ((Sqrt[1 + I*a + I*b*x]*(I + a + I*a^2 + a^3 - 5*b*x + (5*I)*a*b*x + (14*I)
*b^2*x^2 - a*b^2*x^2))/(x^2*Sqrt[(-I)*(I + a + b*x)]) - ((6*I)*Sqrt[-1 - I*
a]*(-3*I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt
[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 + I*a]*(-I + a)))/(2*(I + a)^3
)
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.02

method	result
risch	$\frac{i\left(-a\,b^3x^3+6ib^3x^3-a^2b^2x^2+12ia\,b^2x^2+a^3bx+6ia^2bx+a^4+b^2x^2+abx+6bxi+2a^2+1\right)}{2x^2(i+a)^3\sqrt{b^2x^2+2abx+a^2+1}}+\frac{b^2\left(-\frac{\left(-6a^2+3ia-9\right)\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+}}{}\right.}{(i+a)\sqrt{a^2+1}}\right.}{}$
default	$-\frac{2ib^3\left(2b^2x+2ab\right)}{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\sqrt{b^2x^2+2abx+a^2+1}}-3(ia+1)\,b^2\left(\frac{1}{\left(a^2+1\right)\sqrt{b^2x^2+2abx+a^2+1}}-\frac{2ab\left(2b^2x+2ab\right)}{\left(a^2+1\right)\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\sqrt{b^2x^2+}}\right.$

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(-a*b^3*x^3-a^2*b^2*x^2+a^3*b*x+6*I*b^3*x^3+a^4+b^2*x^2+12*I*a*b^2*x^
2+a*b*x+6*I*a^2*b*x+2*a^2+6*I*b*x+1)/x^2/(I+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)+1/2/(3*I*a^2+a^3-I-3*a)*b^2*(-(3*I*a-6*a^2-9)/(I+a)/(a^2+1)^(1/2)*ln((2
*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+8*I*(I*a-1
)/b/(I+a)/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(I+a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(182) = 364.

Time = 0.28 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.17

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx$$

$$= \frac{(-i\,a-14)b^3x^3+(-i\,a^2-13\,a-14i)b^2x^2-3\left(\left(a^3+3i\,a^2-3\,a-i\right)bx^3+\left(a^4+4i\,a^3-6\,a^2-4i\,a+1\right)x^2\right.}{}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*((-I*a - 14)*b^3*x^3 + (-I*a^2 - 13*a - 14*I)*b^2*x^2 - 3*((a^3 + 3*I*a
^2 - 3*a - I)*b*x^3 + (a^4 + 4*I*a^3 - 6*a^2 - 4*I*a + 1)*x^2)*sqrt((4*a^2
- 12*I*a - 9)*b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 +
6*I*a - 1))*log(-((2*a - 3*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*
a - 3*I)*b^2 + (a^5 + 3*I*a^4 - 2*a^3 + 2*I*a^2 - 3*a - I)*sqrt((4*a^2 - 12
*I*a - 9)*b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*
a - 1)))/((2*a - 3*I)*b^2)) + 3*((a^3 + 3*I*a^2 - 3*a - I)*b*x^3 + (a^4 + 4
*I*a^3 - 6*a^2 - 4*I*a + 1)*x^2)*sqrt((4*a^2 - 12*I*a - 9)*b^4/(a^8 + 6*I*a
^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))*log(-((2*a - 3*I)*
b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - 3*I)*b^2 - (a^5 + 3*I*a^4
- 2*a^3 + 2*I*a^2 - 3*a - I)*sqrt((4*a^2 - 12*I*a - 9)*b^4/(a^8 + 6*I*a^7 -
 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1)))/((2*a - 3*I)*b^2)) +
((-I*a - 14)*b^2*x^2 + I*a^3 - 5*(a + I)*b*x - a^2 + I*a - 1)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))/((a^3 + 3*I*a^2 - 3*a - I)*b*x^3 + (a^4 + 4*I*a^3 - 6
*a^2 - 4*I*a + 1)*x^2)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx =$$

$$-i\Bigg(\int \frac{i}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{a^2\cdots}}$$

$$+\int\Bigg(-\frac{3a}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{\cdots}}$$

$$+\int \frac{a^3}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{a^2\cdots}}$$

$$+\int\Bigg(-\frac{3ia^2}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{\cdots}}$$

$$+\int\Bigg(-\frac{3bx}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{\cdots}}$$

$$+\int \frac{b^3x^3}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{a^2\cdots}}$$

$$+\int\Bigg(-\frac{3ib^2x^2}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{\cdots}}$$

$$+\int \frac{3ab^2x^2}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{a^2\cdots}}$$

$$+\int \frac{3a^2bx}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{a^2\cdots}}$$

$$+\int\Bigg(-\frac{6iabx}{a^2x^3\sqrt{a^2+2abx+b^2x^2+1}+2abx^4\sqrt{a^2+2abx+b^2x^2+1}+b^2x^5\sqrt{a^2+2abx+b^2x^2+1}+x^3\sqrt{\cdots}}$$

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**3,x)
```

```
[Out] -I*(Integral(I/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*
a/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
 x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*x**3*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x**3*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*x**3*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)), x) + Integral(-3*I*b**2*x**2/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))
, x) + Integral(3*a*b**2*x**2/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
 + Integral(3*a**2*b*x/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*
a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Inte
gral(-6*I*a*b*x/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**
4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1536 vs. $2(182) = 364$.

Time = 0.21 (sec) , antiderivative size = 1536, normalized size of antiderivative = 5.82

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 15/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*b^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) + 15/2*(-I*a^3 - 3*a^2 + 3*I*a +
```

```
 1)*a^4*b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a
^2 + 1)^3) + 9*(I*a^2*b + 2*a*b - I*b)*a^2*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + I*b^5*x/((a^2*b^2 - (a^2
+ 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 13/2*(-I*a^3 - 3*a^2 + 3*I*a
 + 1)*a*b^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
(a^2 + 1)^2) + 9*(I*a^2*b + 2*a*b - I*b)*a^3*b^3/((a^2*b^2 - (a^2 + 1)*b^2)
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + I*a*b^4/((a^2*b^2 - (a^2
+ 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 13/2*(-I*a^3 - 3*a^2 + 3*I*a
 + 1)*a^2*b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
(a^2 + 1)^2) - 3*(I*a*b^2 + b^2)*a*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) - 6*(I*a^2*b + 2*a*b - I*b)*b^4*x/((a
^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) - 3*(I
*a*b^2 + b^2)*a^2*b^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(a^2 + 1)) - 6*(I*a^2*b + 2*a*b - I*b)*a*b^3/((a^2*b^2 - (a^2 + 1)*
b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) - 15/2*(-I*a^3 - 3*a^2 +
3*I*a + 1)*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(
x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^
2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) + 15/2*(-I*a^3 - 3*a^2 + 3*I*
a + 1)*a^2*b^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - 9*(I*a^2*b
 + 2*a*b - I*b)*a*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs
(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + 3/2*(-I*a^3 - 3*a^2 + 3*I*
a + 1)*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*
a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a
^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + 9*(I*a^2*b + 2*a*b - I*b)*a*b/(sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - 3/2*(-I*a^3 - 3*a^2 + 3*I*a +
1)*b^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + 3*(I*a*b^2 + b^2)*
arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2
)*abs(x)))/(a^2 + 1)^(3/2) - 3*(I*a*b^2 + b^2)/(sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(a^2 + 1)) + 5/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b/(sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*(a^2 + 1)^2*x) + 3*(I*a^2*b + 2*a*b - I*b)/(sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*(a^2 + 1)*x) - 1/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)/(sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x^2)
```

Giac [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{(i\,bx + i\,a + 1)^3}{\left((bx+a)^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{x^3\,\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

```
[In] int((a*1i + b*x*1i + 1)^3/(x^3*((a + b*x)^2 + 1)^(3/2)),x)
[Out] int((a*1i + b*x*1i + 1)^3/(x^3*((a + b*x)^2 + 1)^(3/2)), x)
```

3.188 $\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 338

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \frac{(52+51ia-2a^2)\,b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19+16ia)b^2\sqrt{1+ia+ibx}}{6(i-a)(i+a)^3x\sqrt{1-ia-ibx}} - \frac{(11i-18a-6ia^2)\,b^3\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}(i+a)^{9/2}}$$

```
[Out] -(11*I-18*a-6*I*a^2)*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2
)/(1-I*a-I*b*x)^(1/2))/(I-a)^(3/2)/(I+a)^(9/2)+1/6*(52+51*I*a-2*a^2)*b^3*(1
+I*a+I*b*x)^(1/2)/(I-a)/(I+a)^4/(1-I*a-I*b*x)^(1/2)-1/3*(I-a)*(1+I*a+I*b*x)
^(1/2)/(I+a)/x^3/(1-I*a-I*b*x)^(1/2)+7/6*I*b*(1+I*a+I*b*x)^(1/2)/(I+a)^2/x^
2/(1-I*a-I*b*x)^(1/2)+1/6*(19+16*I*a)*b^2*(1+I*a+I*b*x)^(1/2)/(I-a)/(I+a)^3
/x/(1-I*a-I*b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used

$= \{5203, 100, 156, 157, 12, 95, 214\}$

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = -\frac{(-6ia^2-18a+11i)\,b^3\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}(a+i)^{9/2}} + \frac{(-2a^2+51ia+52)\,b^3\sqrt{ia+ibx+1}}{6(-a+i)(a+i)^4\sqrt{-ia-ibx+1}} + \frac{(19+16ia)b^2\sqrt{ia+ibx+1}}{6(-a+i)(a+i)^3x\sqrt{-ia-ibx+1}} - \frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}} + \frac{7ib\sqrt{ia+ibx+1}}{6(a+i)^2x^2\sqrt{-ia-ibx+1}}$$

```
[In] Int[E^((3*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] ((52 + (51*I)*a - 2*a^2)*b^3*Sqrt[1 + I*a + I*b*x])/(6*(I - a)*(I + a)^4*Sq
rt[1 - I*a - I*b*x]) - ((I - a)*Sqrt[1 + I*a + I*b*x])/(3*(I + a)*x^3*Sqrt[
1 - I*a - I*b*x]) + (((7*I)/6)*b*Sqrt[1 + I*a + I*b*x])/((I + a)^2*x^2*Sqrt
[1 - I*a - I*b*x]) + ((19 + (16*I)*a)*b^2*Sqrt[1 + I*a + I*b*x])/(6*(I - a)
*(I + a)^3*x*Sqrt[1 - I*a - I*b*x]) - ((11*I - 18*a - (6*I)*a^2)*b^3*ArcTan
h[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])
/((I - a)^(3/2)*(I + a)^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1+ia+ibx)^{3/2}}{x^4(1-ia-ibx)^{3/2}}\,dx$$

$$= -\frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} - \frac{\int \frac{-7(i-a)b+6b^2x}{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}\,dx}{3(1-ia)}$$

$$= -\frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{\int \frac{-((19+35ia-16a^2)b^2)-14(i-a)b^3x}{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}\,dx}{6(1-ia)\left(1+a^2\right)}$$

$$= -\frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-16a)b^2\sqrt{1+ia+ibx}}{6(i-a)(1-ia)^3x\sqrt{1-ia-ibx}} - \frac{\int \frac{3(i-a)(11+18ia-6a^2)b^3-(19+35ia-16a^2)b^4x}{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}dx}{6(1-ia)(1+a^2)^2}$$

$$= \frac{(52+51ia-2a^2)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-16a)b^2\sqrt{1+ia+ibx}}{6(i-a)(1-ia)^3x\sqrt{1-ia-ibx}} + \frac{i\int \frac{3(11+29ia-24a^2-6ia^3)b^4}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}dx}{6(i-a)^2(i+a)^4b}$$

$$= \frac{(52+51ia-2a^2)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-16a)b^2\sqrt{1+ia+ibx}}{6(i-a)(1-ia)^3x\sqrt{1-ia-ibx}} + \frac{((11+18ia-6a^2)b^3)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}dx}{2(i-a)(i+a)^4}$$

$$= \frac{(52+51ia-2a^2)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-16a)b^2\sqrt{1+ia+ibx}}{6(i-a)(1-ia)^3x\sqrt{1-ia-ibx}} + \frac{((11+18ia-6a^2)b^3)\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i-a)(i+a)^4}$$

$$= \frac{(52+51ia-2a^2)b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19i-16a)b^2\sqrt{1+ia+ibx}}{6(i-a)(1-ia)^3x\sqrt{1-ia-ibx}} + \frac{(18a-i(11-6a^2))b^3\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}(i+a)^{9/2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.83

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}dx =$$

$$-\frac{2(-1+ia)^{3/2}(1+ia)(i+a)^2(1+ia+ibx)^{5/2}+(3i-4a)(-1+ia)^{5/2}bx(1+ia+ibx)^{5/2}-i(-11-18}{6(-1+ia)}$$

```
[In] Integrate[E^((3*I)*ArcTan[a + b*x])/x^4,x]
```

```
[Out] -1/6*(2*(-1 + I*a)^(3/2)*(1 + I*a)*(I + a)^2*(1 + I*a + I*b*x)^(5/2) + (3*I
 - 4*a)*(-1 + I*a)^(5/2)*b*x*(1 + I*a + I*b*x)^(5/2) - I*(-11 - (18*I)*a +
6*a^2)*b^2*x^2*(I*Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x]*(1 + a^2 - (5*I)*b*x
 + a*b*x) - 6*Sqrt[-1 - I*a]*b*x*Sqrt[(-I)*(I + a + b*x)]*ArcTanh[(Sqrt[-1
- I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])]))/
((-1 + I*a)^(5/2)*(1 + a^2)^2*x^3*Sqrt[(-I)*(I + a + b*x)])
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.12

method	result
risch	$\frac{i(2a^2b^4x^4-27ia\,b^4x^4+2a^3b^3x^3-45ia^2b^3x^3-9ix^2a^3b^2-28x^4b^4+2a^5bx+9ia^4bx-58a\,b^3x^3+9ib^3x^3+2a^6-26a^2b^2x^2-9ia\,b^2x^2+4a^3b}{6x^3(a-i)(i+a)^4\sqrt{b^2x^2+2abx+a^2+1}}$
default	Expression too large to display

```
[In] int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*(-9*I*a*b^2*x^2+2*a^2*b^4*x^4-27*I*a*b^4*x^4+2*a^3*b^3*x^3-45*I*x^3*a
^2*b^3-28*x^4*b^4+9*I*x*a^4*b+18*I*a^2*b*x+2*a^5*b*x-58*a*b^3*x^3-9*I*b^2*x
^2*a^3+2*a^6-26*a^2*b^2*x^2+9*I*b^3*x^3+4*a^3*b*x+6*a^4-26*b^2*x^2+9*I*b*x+
2*a*b*x+6*a^2+2)/x^3/(a-I)/(I+a)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2/(a-I)/
(a^4-6*a^2+4*I*a^3+1-4*I*a)*b^3*(-(12*I*a^2-6*a^3+11*I-7*a)/(I+a)/(a^2+1)^(
1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-
8*(a^2+1)/b/(I+a)/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(I+a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(223) = 446.

Time = 0.31 (sec) , antiderivative size = 839, normalized size of antiderivative = 2.48

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx$$

$$= \frac{(2i\,a^2+51\,a-52i)b^4x^4+(2i\,a^3+49\,a^2-i\,a+52)b^3x^3+3\sqrt{\frac{(36\,a^4-216i\,a^3-456\,a^2+396i\,a+121)b^6}{a^{12}+6i\,a^{11}-12\,a^{10}-2i\,a^9-27\,a^8-36i\,a^7-36i\,a^5+27\,a^4-2}}}{}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((2*I*a^2 + 51*a - 52*I)*b^4*x^4 + (2*I*a^3 + 49*a^2 - I*a + 52)*b^3*x^
3 + 3*sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^12 + 6*I*a
```

```
^11 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^3 +
 12*a^2 + 6*I*a - 1))*((a^5 + 3*I*a^4 - 2*a^3 + 2*I*a^2 - 3*a - I)*b*x^4 +
(a^6 + 4*I*a^5 - 5*a^4 - 5*a^2 - 4*I*a + 1)*x^3)*log(-((6*a^2 - 18*I*a - 11
)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^2 - 18*I*a - 11)*b^3 + (a^
7 + 3*I*a^6 - a^5 + 5*I*a^4 - 5*a^3 + I*a^2 - 3*a - I)*sqrt((36*a^4 - 216*I
*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^12 + 6*I*a^11 - 12*a^10 - 2*I*a^9 -
27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^3 + 12*a^2 + 6*I*a - 1)))/((6
*a^2 - 18*I*a - 11)*b^3)) - 3*sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*a
+ 121)*b^6/(a^12 + 6*I*a^11 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*
a^5 + 27*a^4 - 2*I*a^3 + 12*a^2 + 6*I*a - 1))*((a^5 + 3*I*a^4 - 2*a^3 + 2*I
*a^2 - 3*a - I)*b*x^4 + (a^6 + 4*I*a^5 - 5*a^4 - 5*a^2 - 4*I*a + 1)*x^3)*lo
g(-((6*a^2 - 18*I*a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^2
- 18*I*a - 11)*b^3 - (a^7 + 3*I*a^6 - a^5 + 5*I*a^4 - 5*a^3 + I*a^2 - 3*a -
 I)*sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^12 + 6*I*a^1
1 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^3 + 1
2*a^2 + 6*I*a - 1)))/((6*a^2 - 18*I*a - 11)*b^3)) + ((2*I*a^2 + 51*a - 52*I
)*b^3*x^3 + 2*I*a^5 + (16*a^2 - 3*I*a + 19)*b^2*x^2 - 2*a^4 + 4*I*a^3 - 7*(
a^3 + I*a^2 + a + I)*b*x - 4*a^2 + 2*I*a - 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))/((a^5 + 3*I*a^4 - 2*a^3 + 2*I*a^2 - 3*a - I)*b*x^4 + (a^6 + 4*I*a^5 -
 5*a^4 - 5*a^2 - 4*I*a + 1)*x^3)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx =$$

$$-i\Bigg(\int \frac{i}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{}}$$

$$+\int\Bigg(-\frac{3a}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{}}$$

$$+\int \frac{a^3}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{a^2}}$$

$$+\int\Bigg(-\frac{3ia^2}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{}}$$

$$+\int\Bigg(-\frac{3bx}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{}}$$

$$+\int \frac{b^3x^3}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{a^2}}$$

$$+\int\Bigg(-\frac{3ib^2x^2}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{}}$$

$$+\int \frac{3ab^2x^2}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{a^2}}$$

$$+\int \frac{3a^2bx}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{a^2}}$$

$$+\int\Bigg(-\frac{6iabx}{a^2x^4\sqrt{a^2+2abx+b^2x^2+1}+2abx^5\sqrt{a^2+2abx+b^2x^2+1}+b^2x^6\sqrt{a^2+2abx+b^2x^2+1}+x^4\sqrt{}}$$

```
[In] integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**4,x)
```

```
[Out] -I*(Integral(I/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*
a/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) +
 x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*x**4*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x**4*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*x**4*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*
```

```
x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)), x) + Integral(-3*I*b**2*x**2/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))
, x) + Integral(3*a*b**2*x**2/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
 + Integral(3*a**2*b*x/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*
a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Inte
gral(-6*I*a*b*x/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**
5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2313 vs. 2(223) = 446.

Time = 0.21 (sec) , antiderivative size = 2313, normalized size of antiderivative = 6.84

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -35/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^4*b^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^4) - 35/2*(-I*a^3 - 3*a^2 + 3*I*a
+ 1)*a^5*b^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(
a^2 + 1)^4) - 45/2*(I*a^2*b + 2*a*b - I*b)*a^3*b^5*x/((a^2*b^2 - (a^2 + 1)*
b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - I*a*b^6*x/((a^2*b^2 -
 (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 115/6*(-I*a^
3 - 3*a^2 + 3*I*a + 1)*a^2*b^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - 45/2*(I*a^2*b + 2*a*b - I*b)*a^4*b^4/((a^
2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - I*a
^2*b^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 +
1)) + 115/6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*b^5/((a^2*b^2 - (a^2 + 1)*b^2)
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) + 9*(I*a*b^2 + b^2)*a^2*b^4
*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2
) + 39/2*(I*a^2*b + 2*a*b - I*b)*a*b^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - 8/3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*
b^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1
)^2) + 9*(I*a*b^2 + b^2)*a^3*b^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + 39/2*(I*a^2*b + 2*a*b - I*b)*a^2*b^4/((a^
2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - 8/3
```

```
*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - 6*(I*a*b^2 + b^2)*b^4*x/((a^2*b^2 - (a
^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 35/2*(-I*a^3 -
3*a^2 + 3*I*a + 1)*a^3*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b
^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(9/2) - 6*(I*a*b^2 + b^2)*a
*b^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)
) - 35/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*b^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)*(a^2 + 1)^4) + 45/2*(I*a^2*b + 2*a*b - I*b)*a^2*b^2*arcsinh(2*a*b*x/(
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^
2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 +
 1)^(7/2) + I*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x
)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2
 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - 15/2*(-I*a^3 - 3*a^2 + 3*I*a
 + 1)*a*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2
*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 45/2*(I*a^2*b + 2*a*b - I*b)*a^2*b
^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - I*b^3/(sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*(a^2 + 1)) + 15/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^3/(sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - 9*(I*a*b^2 + b^2)*a*b*arcsin
h(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(
x)))/(a^2 + 1)^(5/2) - 9/2*(I*a^2*b + 2*a*b - I*b)*b^2*arcsinh(2*a*b*x/(sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 +
 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)
^(5/2) + 9*(I*a*b^2 + b^2)*a*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)
^2) + 9/2*(I*a^2*b + 2*a*b - I*b)*b^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a
^2 + 1)^2) - 35/6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^2/(sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*(a^2 + 1)^3*x) - 15/2*(I*a^2*b + 2*a*b - I*b)*a*b/(sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2*x) + 4/3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*
b^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2*x) + 7/6*(-I*a^3 - 3*a^2
 + 3*I*a + 1)*a*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2*x^2) + 3*(
I*a*b^2 + b^2)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x) + 3/2*(I*a^2
*b + 2*a*b - I*b)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x^2) - 1/3*(
-I*a^3 - 3*a^2 + 3*I*a + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x^
3)
```

Giac [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \int \frac{(i\,bx + i\,a + 1)^3}{\left((bx+a)^2+1\right)^{\frac{3}{2}} x^4}\,dx$$

```
[In] integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \int \frac{(1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i})^3}{x^4\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

```
[In] int((a*1i + b*x*1i + 1)^3/(x^4*((a + b*x)^2 + 1)^(3/2)),x)
[Out] int((a*1i + b*x*1i + 1)^3/(x^4*((a + b*x)^2 + 1)^(3/2)), x)
```

3.189 $\int e^{-i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 276

$$\begin{aligned}\int e^{-i\arctan(a+bx)}x^4\,dx &= -\frac{(3i-12a-24ia^2+16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5}\\ &\quad+\frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3}+\frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2}\\ &\quad-\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(19i-114a-86ia^2+96a^3+2(13+14ia-36a^2)\,bx)}{120b^5}\\ &\quad+\frac{(3+12ia-24a^2-16ia^3+8a^4)\operatorname{arcsinh}(a+bx)}{8b^5}\end{aligned}$$

```
[Out] 1/8*(3+12*I*a-24*a^2-16*I*a^3+8*a^4)*arcsinh(b*x+a)/b^5+1/20*(I-8*a)*x^2*(1
-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^3+1/5*x^3*(1-I*a-I*b*x)^(3/2)*(1+I*
a+I*b*x)^(1/2)/b^2-1/120*(1-I*a-I*b*x)^(3/2)*(19*I-114*a-86*I*a^2+96*a^3+2*
(13+14*I*a-36*a^2)*b*x)*(1+I*a+I*b*x)^(1/2)/b^5-1/8*(3*I-12*a-24*I*a^2+16*a
^3+8*I*a^4)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{5203, 102, 158, 152, 52, 55, 633, 221\}$

$$\int e^{-i\arctan(a+bx)}x^4\,dx =$$
$$-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}(96a^3+2(-36a^2+14ia+13)\,bx-86ia^2-114a+19i)}{120b^5}$$
$$+\frac{(8a^4-16ia^3-24a^2+12ia+3)\operatorname{arcsinh}(a+bx)}{8b^5}$$
$$-\frac{(8ia^4+16a^3-24ia^2-12a+3i)\,\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{8b^5}$$
$$+\frac{(-8a+i)x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{20b^3}+\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2}$$

```
[In] Int[x^4/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] -1/8*((3*I - 12*a - (24*I)*a^2 + 16*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*
Sqrt[1 + I*a + I*b*x])/b^5 + ((I - 8*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1
+ I*a + I*b*x])/(20*b^3) + (x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*
x])/(5*b^2) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(19*I - 114*a
- (86*I)*a^2 + 96*a^3 + 2*(13 + (14*I)*a - 36*a^2)*b*x))/(120*b^5) + ((3 +
(12*I)*a - 24*a^2 - (16*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^5)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx$$

$$= \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{\int \frac{x^2\sqrt{1-ia-ibx}\left(-3(1+a^2)+(i-8a)bx\right)}{\sqrt{1+ia+ibx}}\,dx}{5b^2}$$

$$= \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} + \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{\int \frac{x\sqrt{1-ia-ibx}(2(i-8a)(i-a)(i+a)b-(13+14ia-36a^2)b^2x)}{\sqrt{1+ia+ibx}}\,dx}{20b^4}$$

$$= \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} + \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(19i-114a-86ia^2+96a^3+2(13+14ia-36a^2)\,bx)}{120b^5} + \frac{(3+12ia-24a^2-16ia^3+8a^4)\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{8b^4}$$

$$= -\frac{(3i-12a-24ia^2+16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} + \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(19i-114a-86ia^2+96a^3+2(13+14ia-36a^2)\,bx)}{120b^5} + \frac{(3+12ia-24a^2-16ia^3+8a^4)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{8b^4}$$

$$= -\frac{(3i-12a-24ia^2+16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} + \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(19i-114a-86ia^2+96a^3+2(13+14ia-36a^2)\,bx)}{120b^5} + \frac{(3+12ia-24a^2-16ia^3+8a^4)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{8b^4}$$

$$= -\frac{(3i-12a-24ia^2+16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} + \frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} + \frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(19i-114a-86ia^2+96a^3+2(13+14ia-36a^2)\,bx)}{120b^5} + \frac{(3+12ia-24a^2-16ia^3+8a^4)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{16b^6}$$

$$= -\frac{(3i - 12a - 24ia^2 + 16a^3 + 8ia^4)\sqrt{1 - ia - ibx}\sqrt{1 + ia + ibx}}{8b^5} + \frac{(i - 8a)x^2(1 - ia - ibx)^{3/2}\sqrt{1 + ia + ibx}}{20b^3} + \frac{x^3(1 - ia - ibx)^{3/2}\sqrt{1 + ia + ibx}}{5b^2} - \frac{(1 - ia - ibx)^{3/2}\sqrt{1 + ia + ibx}(19i - 114a - 86ia^2 + 96a^3 + 2(13 + 14ia - 36a^2)\,bx)}{120b^5} + \frac{(3 + 12ia - 24a^2 - 16ia^3 + 8a^4)\,\mathrm{arcsinh}(a + bx)}{8b^5}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.90

$$\int e^{-i\arctan(a+bx)}x^4\,dx = \frac{i\sqrt{1 + ia + ibx}(-64 + 226a^4 + 24ia^5 + 109ibx + 77b^2x^2 - 62ib^3x^3 - 54b^4x^4 + 24ib^5x^5 + 2a^3(-41i + 72b}{120b^5\sqrt{-i(i + a + bx)}} + \frac{\sqrt[4]{-1}(-3i + 12a + 24ia^2 - 16a^3 - 8ia^4)\,\mathrm{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4\sqrt{-ib}b^{9/2}}$$

```
[In] Integrate[x^4/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((I/120)*Sqrt[1 + I*a + I*b*x]*(-64 + 226*a^4 + (24*I)*a^5 + (109*I)*b*x +
77*b^2*x^2 - (62*I)*b^3*x^3 - 54*b^4*x^4 + (24*I)*b^5*x^5 + 2*a^3*(-41*I +
72*b*x) + a^2*(57 - (346*I)*b*x - 84*b^2*x^2) + a*(-211*I - 346*b*x + (154*
I)*b^2*x^2 + 64*b^3*x^3)))/(b^5*Sqrt[(-I)*(I + a + b*x)]) + ((-1)^(1/4)*(-3
*I + 12*a + (24*I)*a^2 - 16*a^3 - (8*I)*a^4)*ArcSinh[((1/2 + I/2)*Sqrt[b]*S
qrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*Sqrt[(-I)*b]*b^(9/2))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{i(24x^4b^4-24a\,b^3x^3+30ib^3x^3+24a^2b^2x^2-70ia\,b^2x^2-24a^3bx+130ia^2bx+24a^4-250ia^3-32b^2x^2+116abx-45bxi-332a^2+275ia+6}{120b^5}$
default	Expression too large to display

```
[In] int(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*I*(24*x^4*b^4+30*I*b^3*x^3-24*a*b^3*x^3-70*I*a*b^2*x^2+24*a^2*b^2*x^
2+130*I*a^2*b*x-24*a^3*b*x-250*I*a^3+24*a^4-32*b^2*x^2-45*I*b*x+116*a*b*x+2
75*I*a-332*a^2+64)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^5+1/8*(3+12*I*a-24*a^2-1
```

```
6*I*a^3+8*a^4)/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
)/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.64

$$\int e^{-i\arctan(a+bx)}x^4\,dx = \frac{-186i\,a^5 - 1345\,a^4 + 1730i\,a^3 + 1320\,a^2 - 120\,(8\,a^4 - 16i\,a^3 - 24\,a^2 + 12i\,a + 3)\log\left(-bx - a + \sqrt{b^2x^2 +$$

```
[In] integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/960*(-186*I*a^5 - 1345*a^4 + 1730*I*a^3 + 1320*a^2 - 120*(8*a^4 - 16*I*a^
3 - 24*a^2 + 12*I*a + 3)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
- 8*(24*I*b^4*x^4 + 6*(-4*I*a - 5)*b^3*x^3 + 2*(12*I*a^2 + 35*a - 16*I)*b^2
*x^2 + 24*I*a^4 + 250*a^3 + (-24*I*a^3 - 130*a^2 + 116*I*a + 45)*b*x - 332*
I*a^2 - 275*a + 64*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 300*I*a)/b^5
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}x^4\,dx = -i\int \frac{x^4\sqrt{a^2 + 2abx + b^2x^2 + 1}}{a + bx - i}\,dx$$

```
[In] integrate(x**4/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] -I*Integral(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(200) = 400.

Time = 0.28 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.65

$$\int e^{-i\arctan(a+bx)}x^4\,dx = \frac{2i\sqrt{b^2x^2+2\,abx+a^2+1}a^3x}{b^4} - \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}x^2}{5\,b^3} + \frac{a^4\,\mathrm{arsinh}\,(bx+a)}{b^5} + \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}a^4}{b^5} + \frac{3i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}ax}{5\,b^4} + \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}a^2x}{b^4} - \frac{2i\,a^3\,\mathrm{arsinh}\,(bx+a)}{b^5} - \frac{6i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a^2}{5\,b^5} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}a^3}{b^5} + \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}x}{4\,b^4} - \frac{5i\sqrt{b^2x^2+2\,abx+a^2+1}ax}{2\,b^4} - \frac{3\,a^2\,\mathrm{arsinh}\,(bx+a)}{b^5} - \frac{13\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{12\,b^5} + \frac{7i\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{2\,b^5} - \frac{5\sqrt{b^2x^2+2\,abx+a^2+1}x}{8\,b^4} + \frac{3i\,a\,\mathrm{arsinh}\,(bx+a)}{2\,b^5} + \frac{7i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{15\,b^5} + \frac{27\sqrt{b^2x^2+2\,abx+a^2+1}a}{8\,b^5} + \frac{3\,\mathrm{arsinh}\,(bx+a)}{8\,b^5} - \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{b^5}$$

```
[In] integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*x/b^4 - 1/5*I*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2)*x^2/b^3 + a^4*arcsinh(b*x + a)/b^5 + I*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*a^4/b^5 + 3/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^4
+ 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^4 - 2*I*a^3*arcsinh(b*x + a)/
b^5 - 6/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/b^5 - sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a^3/b^5 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^4 -
5/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^4 - 3*a^2*arcsinh(b*x + a)/b^
5 - 13/12*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^5 + 7/2*I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*a^2/b^5 - 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 +
3/2*I*a*arcsinh(b*x + a)/b^5 + 7/15*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b
^5 + 27/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^5 + 3/8*arcsinh(b*x + a)/b^
5 - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.74

$$\int e^{-i\arctan(a+bx)}x^4\,dx =$$
$$-\frac{1}{120}\sqrt{(bx+a)^2+1}\left(\left(2\left(3\,x\left(\frac{4i\,x}{b}-\frac{4i\,ab^7+5\,b^7}{b^9}\right)-\frac{-12i\,a^2b^6-35\,ab^6+16i\,b^6}{b^9}\right)x-\frac{24i\,a^3b^5+130\ldots}{}\right.\right.$$
$$-\frac{(8\,a^4-16i\,a^3-24\,a^2+12i\,a+3)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{8\,b^4|b|}$$

```
[In] integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/120*sqrt((b*x + a)^2 + 1)*((2*(3*x*(4*I*x/b - (4*I*a*b^7 + 5*b^7)/b^9) -
 (-12*I*a^2*b^6 - 35*a*b^6 + 16*I*b^6)/b^9)*x - (24*I*a^3*b^5 + 130*a^2*b^5
 - 116*I*a*b^5 - 45*b^5)/b^9)*x - (-24*I*a^4*b^4 - 250*a^3*b^4 + 332*I*a^2*
b^4 + 275*a*b^4 - 64*I*b^4)/b^9) - 1/8*(8*a^4 - 16*I*a^3 - 24*a^2 + 12*I*a
+ 3)*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}x^4\,dx = \int \frac{x^4\,\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

```
[In] int((x^4*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
```

```
[Out] int((x^4*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

3.190 $\int e^{-i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 201

$$\begin{aligned}\int e^{-i\arctan(a+bx)}x^3\,dx = &-\frac{(3+12ia-12a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4}\\ &+\frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2}\\ &-\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(7+10ia-18a^2-2(i-6a)bx)}{24b^4}\\ &-\frac{(3i-12a-12ia^2+8a^3)\operatorname{arcsinh}(a+bx)}{8b^4}\end{aligned}$$

```
[Out] -1/8*(3*I-12*a-12*I*a^2+8*a^3)*arcsinh(b*x+a)/b^4+1/4*x^2*(1-I*a-I*b*x)^(3/
2)*(1+I*a+I*b*x)^(1/2)/b^2-1/24*(1-I*a-I*b*x)^(3/2)*(7+10*I*a-18*a^2-2*(I-6
*a)*b*x)*(1+I*a+I*b*x)^(1/2)/b^4-1/8*(3+12*I*a-12*a^2-8*I*a^3)*(1-I*a-I*b*x
)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5203, 102, 152, 52, 55, 633, 221}

$$\begin{aligned}
&\int e^{-i\arctan(a+bx)}x^3\,dx \\
&= -\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}(-18a^2-2(-6a+i)bx+10ia+7)}{24b^4} \\
&\quad -\frac{(8a^3-12ia^2-12a+3i)\,\text{arcsinh}(a+bx)}{8b^4} \\
&\quad -\frac{(-8ia^3-12a^2+12ia+3)\,\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{8b^4} \\
&\quad +\frac{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b^2}
\end{aligned}$$

```
[In] Int[x^3/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] -1/8*((3 + (12*I)*a - 12*a^2 - (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*
a + I*b*x])/b^4 + (x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b^
2) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(7 + (10*I)*a - 18*a^2
- 2*(I - 6*a)*b*x))/(24*b^4) - ((3*I - 12*a - (12*I)*a^2 + 8*a^3)*ArcSinh[a
 + b*x])/(8*b^4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx \\
&= \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} + \frac{\int \frac{x\sqrt{1-ia-ibx}(-2(1+a^2)+(i-6a)bx)}{\sqrt{1+ia+ibx}}\,dx}{4b^2} \\
&= \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} \\
&\quad - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(7+10ia-18a^2-2(i-6a)bx)}{24b^4} \\
&\quad - \frac{(3i-12a-12ia^2+8a^3)\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{8b^3}
\end{aligned}$$

$$= -\frac{(3+12ia-12a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(7+10ia-18a^2-2(i-6a)bx)}{24b^4} - \frac{(3i-12a-12ia^2+8a^3)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{8b^3}$$

$$= -\frac{(3+12ia-12a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(7+10ia-18a^2-2(i-6a)bx)}{24b^4} - \frac{(3i-12a-12ia^2+8a^3)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{8b^3}$$

$$= -\frac{(3+12ia-12a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(7+10ia-18a^2-2(i-6a)bx)}{24b^4} - \frac{(3i-12a-12ia^2+8a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{16b^5}$$

$$= -\frac{(3+12ia-12a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(7+10ia-18a^2-2(i-6a)bx)}{24b^4} - \frac{(3i-12a-12ia^2+8a^3)\,\text{arcsinh}(a+bx)}{8b^4}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int e^{-i\arctan(a+bx)}x^3\,dx$$
$$=\frac{\sqrt{1+ia+ibx}(-16-38ia^3+6a^4+25ibx+17b^2x^2-14ib^3x^3-6b^4x^4+5a^2(1-6ibx)+ia(-23+50ibx\cdots}{24b^4\sqrt{-i(i+a+bx)}}$$
$$+\frac{(-1)^{3/4}\left(-3-12ia+12a^2+8ia^3\right)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4b^{9/2}}$$

```
[In] Integrate[x^3/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(-16 - (38*I)*a^3 + 6*a^4 + (25*I)*b*x + 17*b^2*x^2
- (14*I)*b^3*x^3 - 6*b^4*x^4 + 5*a^2*(1 - (6*I)*b*x) + I*a*(-23 + (50*I)*b*
x + 18*b^2*x^2)))/(24*b^4*Sqrt[(-I)*(I + a + b*x)]) + ((-1)^(3/4)*(-3 - (12
*I)*a + 12*a^2 + (8*I)*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.75

method	result
risch	$\frac{i\left(-6b^3x^3+6a\,b^2x^2-8ix^2b^2-6a^2bx+20iabx+6a^3-44ia^2+9bx-39a+16i\right)\sqrt{b^2x^2+2abx+a^2+1}}{24b^4}-\frac{\left(8a^3-12ia^2-12a+3i\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}\cdots\right)}{8b^3\sqrt{b^2}}$
default	$-\frac{i\left(\frac{x\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{4b^2}-\frac{5a\left(\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3b^2}-\frac{a\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+\cdots}\right)}{8b^2\sqrt{b^2}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{b}$

```
[In] int(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*I*(-6*b^3*x^3-8*I*b^2*x^2+6*a*b^2*x^2+20*I*a*b*x-6*a^2*b*x-44*I*a^2+6*
a^3+9*b*x+16*I-39*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^4-1/8*(3*I-12*a-12*I*a
^2+8*a^3)/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^
2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.69

$$\int e^{-i\arctan(a+bx)}x^3\,dx$$
$$=\frac{45i\,a^4+224\,a^3-192i\,a^2+24\left(8\,a^3-12i\,a^2-12\,a+3i\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-8\left(6i\,b\right.}{192}$$

```
[In] integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/192*(45*I*a^4 + 224*a^3 - 192*I*a^2 + 24*(8*a^3 - 12*I*a^2 - 12*a + 3*I)*
log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 8*(6*I*b^3*x^3 + 2*(-3*
I*a - 4)*b^2*x^2 - 6*I*a^3 + (6*I*a^2 + 20*a - 9*I)*b*x - 44*a^2 + 39*I*a +
 16)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 72*a)/b^4
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}x^3\,dx=-i\int\frac{x^3\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

```
[In] integrate(x**3/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] -I*Integral(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(145) = 290$.

Time = 0.30 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.53

$$\int e^{-i\arctan(a+bx)}x^3\,dx = -\frac{3i\,\sqrt{b^2x^2+2\,abx+a^2+1}a^2x}{2\,b^3} - \frac{a^3\,\mathrm{arsinh}\,(bx+a)}{b^4} - \frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}a^3}{2\,b^4} - \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}x}{4\,b^3} - \frac{3\,\sqrt{b^2x^2+2\,abx+a^2+1}ax}{2\,b^3} + \frac{3i\,a^2\,\mathrm{arsinh}\,(bx+a)}{2\,b^4} + \frac{3i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{4\,b^4} + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{2\,b^4} + \frac{5i\,\sqrt{b^2x^2+2\,abx+a^2+1}x}{8\,b^3} + \frac{3\,a\,\mathrm{arsinh}\,(bx+a)}{2\,b^4} + \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,b^4} - \frac{19i\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{8\,b^4} - \frac{3i\,\mathrm{arsinh}\,(bx+a)}{8\,b^4} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^4}$$

```
[In] integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^3 - a^3*arcsinh(b*x + a)/b
^4 - 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/b^4 - 1/4*I*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2)*x/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^3
 + 3/2*I*a^2*arcsinh(b*x + a)/b^4 + 3/4*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2)*a/b^4 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^4 + 5/8*I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*x/b^3 + 3/2*a*arcsinh(b*x + a)/b^4 + 1/3*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)/b^4 - 19/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/
b^4 - 3/8*I*arcsinh(b*x + a)/b^4 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.77

$$\int e^{-i\arctan(a+bx)}x^3\,dx = -\frac{1}{24}\sqrt{(bx+a)^2+1}\left(\left(2\,x\left(\frac{3i\,x}{b}-\frac{3i\,ab^5+4\,b^5}{b^7}\right)-\frac{-6i\,a^2b^4-20\,ab^4+9i\,b^4}{b^7}\right)x-\frac{6i\,a^3b^3+44\,a^2b^3-}{b^7} + \frac{(8\,a^3-12i\,a^2-12\,a+3i)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{8\,b^3|b|}$$

```
[In] integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt((b*x + a)^2 + 1)*((2*x*(3*I*x/b - (3*I*a*b^5 + 4*b^5)/b^7) - (-6
*I*a^2*b^4 - 20*a*b^4 + 9*I*b^4)/b^7)*x - (6*I*a^3*b^3 + 44*a^2*b^3 - 39*I*
a*b^3 - 16*b^3)/b^7) + 1/8*(8*a^3 - 12*I*a^2 - 12*a + 3*I)*log(-a*b - (x*ab
s(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}x^3\,dx=\int \frac{x^3\,\sqrt{{\left(a+b\,x\right)}^2+1}}{1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,dx$$

```
[In] int((x^3*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
```

```
[Out] int((x^3*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

3.191 $\int e^{-i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 171

$$\int e^{-i\arctan(a+bx)}x^2\,dx=\frac{(i-2a-2ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3}+\frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3}+\frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2}-\frac{(1+2ia-2a^2)\operatorname{arcsinh}(a+bx)}{2b^3}$$

```
[Out] -1/2*(1+2*I*a-2*a^2)*arcsinh(b*x+a)/b^3+1/6*(I-4*a)*(1-I*a-I*b*x)^(3/2)*(1+
I*a+I*b*x)^(1/2)/b^3+1/3*x*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2+1/2*
(I-2*a-2*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5203, 92, 81, 52, 55, 633, 221}

$$\int e^{-i\arctan(a+bx)}x^2\,dx=-\frac{(-2a^2+2ia+1)\operatorname{arcsinh}(a+bx)}{2b^3}+\frac{(-2ia^2-2a+i)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^3}+\frac{(-4a+i)\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{6b^3}+\frac{x\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{3b^2}$$

```
[In] Int[x^2/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((I - 2*a - (2*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3)
 + ((I - 4*a)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(6*b^3) + (x*(
1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(3*b^2) - ((1 + (2*I)*a - 2*a
^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx \\
&= \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} + \frac{\int \frac{\sqrt{1-ia-ibx}(-1-a^2+(i-4a)bx)}{\sqrt{1+ia+ibx}}\,dx}{3b^2} \\
&= \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad + \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} - \frac{(1+2ia-2a^2)\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{2b^2} \\
&= -\frac{(2a-i(1-2a^2))\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad + \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} - \frac{(1+2ia-2a^2)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2b^2} \\
&= -\frac{(2a-i(1-2a^2))\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad + \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} - \frac{(1+2ia-2a^2)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{2b^2} \\
&= -\frac{(2a-i(1-2a^2))\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad + \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} - \frac{(1+2ia-2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{4b^4} \\
&= -\frac{(2a-i(1-2a^2))\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} \\
&\quad + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad + \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} - \frac{(1+2ia-2a^2)\,\text{arcsinh}(a+bx)}{2b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

$$\int e^{-i\arctan(a+bx)}x^2\,dx = \frac{i\sqrt{1+ia+ibx}(4+7a^2+2ia^3-7ibx-5b^2x^2+2ib^3x^3+a(5i+8bx))}{6b^3\sqrt{-i(i+a+bx)}} + \frac{\sqrt[4]{-1}(-1-2ia+2a^2)\sqrt{-ib}\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{7/2}}$$

```
[In] Integrate[x^2/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((I/6)*Sqrt[1 + I*a + I*b*x]*(4 + 7*a^2 + (2*I)*a^3 - (7*I)*b*x - 5*b^2*x^2
 + (2*I)*b^3*x^3 + a*(5*I + 8*b*x)))/(b^3*Sqrt[(-I)*(I + a + b*x)]) + ((-1)
^(1/4)*(-1 - (2*I)*a + 2*a^2)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqr
t[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(7/2)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{i(2b^2x^2-2abx+3bxi+2a^2-9ia-4)\sqrt{b^2x^2+2abx+a^2+1}}{6b^3}+\frac{(2a^2-2ia-1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b^2\sqrt{b^2}}$
default	$-\frac{i\left(i\left(\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{(4b^2(a^2+1)-4a^2b^2)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b^2\sqrt{b^2}}\right)-a\left(\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\right.\right.}{}$

```
[In] int(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*(2*b^2*x^2+3*I*b*x-2*a*b*x-9*I*a+2*a^2-4)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)/b^3+1/2*(-2*I*a+2*a^2-1)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int e^{-i\arctan(a+bx)}x^2\,dx$$
$$= \frac{-7i\,a^3 - 21\,a^2 - 12\,(2\,a^2 - 2i\,a - 1)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) - 4\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{24\,b^3}$$

```
[In] integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/24*(-7*I*a^3 - 21*a^2 - 12*(2*a^2 - 2*I*a - 1)*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)) - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*I*b^2*x^2
+ (-2*I*a - 3)*b*x + 2*I*a^2 + 9*a - 4*I) + 9*I*a)/b^3
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}x^2\,dx = -i\int \frac{x^2\sqrt{a^2 + 2abx + b^2x^2 + 1}}{a + bx - i}\,dx$$

```
[In] integrate(x**2/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
[Out] -I*Integral(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\begin{aligned}\int e^{-i\arctan(a+bx)}x^2\,dx &= \frac{i\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}ax}{b^2} + \frac{a^2\operatorname{arsinh}(bx + a)}{b^3} \\ &+ \frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}x}{2\,b^2} - \frac{i\,a\operatorname{arsinh}(bx + a)}{b^3} \\ &- \frac{i\,(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}}}{3\,b^3} - \frac{3\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}a}{2\,b^3} \\ &- \frac{\operatorname{arsinh}(bx + a)}{2\,b^3} + \frac{i\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{b^3}\end{aligned}$$

```
[In] integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
[Out] I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^2 + a^2*arcsinh(b*x + a)/b^3 + 1/
2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - I*a*arcsinh(b*x + a)/b^3 - 1/3*
I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*a/b^3 - 1/2*arcsinh(b*x + a)/b^3 + I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int e^{-i\arctan(a+bx)}x^2\,dx = -\frac{1}{6}\sqrt{(bx+a)^2+1}\left(x\left(\frac{2i\,x}{b}-\frac{2i\,ab^3+3\,b^3}{b^5}\right)-\frac{-2i\,a^2b^2-9\,ab^2+4i\,b^2}{b^5}\right) - \frac{(2\,a^2-2i\,a-1)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2\,b^2|b|}$$

```
[In] integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] -1/6*sqrt((b*x + a)^2 + 1)*(x*(2*I*x/b - (2*I*a*b^3 + 3*b^3)/b^5) - (-2*I*a
^2*b^2 - 9*a*b^2 + 4*I*b^2)/b^5) - 1/2*(2*a^2 - 2*I*a - 1)*log(-a*b - (x*ab
s(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}x^2\,dx = \int \frac{x^2\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

```
[In] int((x^2*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
[Out] int((x^2*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

3.192 $\int e^{-i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int e^{-i\arctan(a+bx)}x\,dx=\frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}+\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}+\frac{(i-2a)\text{arcsinh}(a+bx)}{2b^2}$$

```
[Out] 1/2*(I-2*a)*arcsinh(b*x+a)/b^2+1/2*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/
b^2+1/2*(1+2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5203, 81, 52, 55, 633, 221}

$$\int e^{-i\arctan(a+bx)}x\,dx=\frac{(-2a+i)\text{arcsinh}(a+bx)}{2b^2}+\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}+\frac{(1+2ia)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^2}$$

```
[In] Int[x/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] ((1 + (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) + ((1 -
 I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*b^2) + ((I - 2*a)*ArcSinh[a +
 b*x])/(2*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx \\
&= \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}+\frac{(i-2a)\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{2b} \\
&= \frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} \\
&\quad +\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}+\frac{(i-2a)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} \\
&\quad + \frac{(i-2a)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{2b} \\
&= \frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} \\
&\quad + \frac{(i-2a)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{4b^3} \\
&= \frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} \\
&\quad + \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} + \frac{(i-2a)\text{arcsinh}(a+bx)}{2b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$
\begin{aligned}
\int e^{-i\arctan(a+bx)}x\,dx &= \frac{\sqrt{1+ia+ibx}(2-ia+a^2-3ibx-b^2x^2)}{2b^2\sqrt{-i(i+a+bx)}} \\
&\quad + \frac{(-1)^{3/4}(1+2ia)\sqrt{-ib}\,\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{5/2}}
\end{aligned}
$$

```
[In] Integrate[x/E^(I*ArcTan[a + b*x]),x]
```

```
[Out] (Sqrt[1 + I*a + I*b*x]*(2 - I*a + a^2 - (3*I)*b*x - b^2*x^2))/(2*b^2*Sqrt[(
-I)*(I + a + b*x)]) + ((-1)^(3/4)*(1 + (2*I)*a)*Sqrt[(-I)*b]*ArcSinh[((1/2
+ I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.79

method	result
risch	$\frac{i(-bx+a-2i)\sqrt{b^2x^2+2abx+a^2+1}}{2b^2} - \frac{(-i+2a)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b\sqrt{b^2}}$
default	$-\frac{i\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b^2\sqrt{b^2}}\right)}{b} + \frac{(ia+1)\Bigg(\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\Big(x-\frac{i}{}}}{}$

```
[In] int(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(-b*x+a-2*I)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^2-1/2*(-I+2*a)/b*ln((b^2
*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int e^{-i\arctan(a+bx)}x\,dx = \frac{3i\,a^2+4\,(2\,a-i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-4\,\sqrt{b^2x^2+2\,abx+a^2+1}(i\,bx-i\,a-2)+4\,a}{8\,b^2}$$

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*I*a^2 + 4*(2*a - I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*b*x - I*a - 2) + 4*a)/b^2
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}x\,dx = -i\int\frac{x\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] -I*Integral(x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\begin{aligned}\int e^{-i\arctan(a+bx)}x\,dx = &-\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}x}{2\,b}-\frac{a\,\mathrm{arsinh}\,(bx+a)}{b^2}\\&+\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{2\,b^2}\\&+\frac{i\,\mathrm{arsinh}\,(bx+a)}{2\,b^2}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^2}\end{aligned}$$

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b - a*arcsinh(b*x + a)/b^2 + 1/2
*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^2 + 1/2*I*arcsinh(b*x + a)/b^2 + s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int e^{-i\arctan(a+bx)}x\,dx = -\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{i\,x}{b}+\frac{-i\,ab-2\,b}{b^3}\right) + \frac{(2\,a-i)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2\,b|b|}$$

```
[In] integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(I*x/b + (-I*a*b - 2*b)/b^3) + 1/2*(2*a - I)*log
(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}x\,dx = \int \frac{x\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

```
[In] int((x*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
[Out] int((x*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

3.193 $\int e^{-i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int e^{-i\arctan(a+bx)}\,dx=-\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}+\frac{\operatorname{arcsinh}(a+bx)}{b}$$

```
[Out] arcsinh(b*x+a)/b-I*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5201, 52, 55, 633, 221}

$$\int e^{-i\arctan(a+bx)}\,dx=\frac{\operatorname{arcsinh}(a+bx)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

```
[In] Int[E^((-I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[a + b*x]/b
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx \\
&= -\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx \\
&= -\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx \\
&= -\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b^2} \\
&= -\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \frac{\operatorname{arcsinh}(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

$$\int e^{-i\arctan(a+bx)}\,dx = \frac{-i\sqrt{1+(a+bx)^2}+\operatorname{arcsinh}(a+bx)}{b}$$

```
[In] Integrate[E^((-I)*ArcTan[a + b*x]),x]
[Out] ((-I)*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x])/b
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result	size
risch	$-\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{b}+\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}$	69
default	$-\frac{i\left(\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}+\frac{ib\ln\left(\frac{ib+\left(x-\frac{i-a}{b}\right)b^2}{\sqrt{b^2}}+\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}\right)}{\sqrt{b^2}}\right)}{b}$	125

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int e^{-i\arctan(a+bx)}\,dx = \frac{-i\,a-2i\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,b}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(-I*a - 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*log(-b*x - a + sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int e^{-i\arctan(a+bx)}\,dx = \frac{\operatorname{arsinh}(bx+a)}{b} - \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*x + a)/b - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int e^{-i\arctan(a+bx)}\,dx = -\frac{\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|} - \frac{i\sqrt{(bx+a)^2+1}}{b}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] -log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/abs(b) - I*sqrt((b*x
 + a)^2 + 1)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}\,dx=\int\frac{\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(a*1i + b*x*1i + 1),x)
[Out] int(((a + b*x)^2 + 1)^(1/2)/(a*1i + b*x*1i + 1), x)
```

3.194 $\int \frac{e^{-i\arctan(a+bx)}}{x} dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{e^{-i\arctan(a+bx)}}{x} dx = -i\text{arcsinh}(a+bx) - \frac{2\sqrt{i+a}\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}}$$

```
[Out] -I*arcsinh(b*x+a)-2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-
I*a-I*b*x)^(1/2))*(I+a)^(1/2)/(I-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5203, 132, 55, 633, 221, 12, 95, 214}

$$\int \frac{e^{-i\arctan(a+bx)}}{x} dx = -i\text{arcsinh}(a+bx) - \frac{2\sqrt{a+i}\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}}$$

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x),x]
```

```
[Out] (-I)*ArcSinh[a + b*x] - (2*Sqrt[I + a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a +
I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I - a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1-ia-ibx}}{x\sqrt{1+ia+ibx}}\,dx\\
&= -\left((ib)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx\right)+\int \frac{1-ia}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx\\
&= (1-ia)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx-(ib)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx\\
&= (2(1-ia))\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx,x,\frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)\\
&\quad -\frac{i\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx,x,2ab+2b^2x\right)}{2b}\\
&= -i\text{arcsinh}(a+bx)-\frac{2\sqrt{i+a}\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$
\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = \frac{2\sqrt[4]{-1}(-ib)^{3/2}\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{3/2}} - \frac{2\sqrt{-1+ia}\,\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}}
$$

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*(-1)^(1/4)*((-I)*b)^(3/2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a
+ b*x)])/Sqrt[(-I)*b]])/b^(3/2) - (2*Sqrt[-1 + I*a]*ArcTanh[(Sqrt[-1 - I*a]
*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1
 - I*a]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(68) = 136.

Time = 0.46 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.92

method	result
default	$\frac{i\left(\sqrt{b^{2}x^{2}+2abx+a^{2}+1}+\frac{ab\ln\left(\frac{b^{2}x+ab}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+2abx+a^{2}+1}\right)}{\sqrt{b^{2}}}-\sqrt{a^{2}+1}\,\ln\left(\frac{2a^{2}+2+2abx+2\sqrt{a^{2}+1}\,\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{x}\right)\right)}{i-a}-\frac{i\left(\sqrt{\left(x-\right.}\right.}{}$

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] I/(I-a)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(
a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))-I/(I-a)*(((x-(I-a)/b)^2*b^2
+2*I*b*(x-(I-a)/b))^(1/2)+I*b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a
)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(59) = 118.

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.62

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = -\sqrt{-\frac{a+i}{a-i}}\log\left(-bx+(i\,a+1)\sqrt{-\frac{a+i}{a-i}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right) + \sqrt{-\frac{a+i}{a-i}}\log\left(-bx+(-i\,a-1)\sqrt{-\frac{a+i}{a-i}} + \sqrt{b^2x^2+2\,abx+a^2+1}\right) + i\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(-(a + I)/(a - I))*log(-b*x + (I*a + 1)*sqrt(-(a + I)/(a - I)) + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)) + sqrt(-(a + I)/(a - I))*log(-b*x + (-I*a - 1
)*sqrt(-(a + I)/(a - I)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + I*log(-b*x
- a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
```

Sympy [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax+bx^2-ix}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x + b*x**2 - I*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = -\frac{(-ia+1)\log\left(\frac{\left|2x|b|-2\sqrt{(bx+a)^2+1}-2\sqrt{a^2+1}\right|}{\left|2x|b|-2\sqrt{(bx+a)^2+1}+2\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}} + \frac{ib\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{|b|}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -(-I*a + 1)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))
/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/sqrt(a^2 + 1)
 + I*b*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx=\int \frac{\sqrt{{(a+b\,x)}^2+1}}{x\,\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(x*(a*1i + b*x*1i + 1)),x)
[Out] int(((a + b*x)^2 + 1)^(1/2)/(x*(a*1i + b*x*1i + 1)), x)
```

3.195 $\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1+ia)x} - \frac{2ib\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}\sqrt{i+a}}$$

```
[Out] -2*I*b*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1
/2))/(I-a)^(3/2)/(I+a)^(1/2)-(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1+I*a
)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5203, 96, 95, 214}

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = -\frac{2ib\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}\sqrt{a+i}} - \frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}$$

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*x)) - ((2*I)*b*A
rcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*
x])])/((I - a)^(3/2)*Sqrt[I + a])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1-ia-ibx}}{x^2\sqrt{1+ia+ibx}}\,dx\\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1+ia)x}+\frac{b\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{i-a}\\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1+ia)x}+\frac{(2b)\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx,x,\frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{i-a}\\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1+ia)x}-\frac{2ib\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}\sqrt{i+a}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = i\left(\frac{\sqrt{1+a^2+2abx+b^2x^2}}{(-i+a)x}+\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{(-1-ia)^{3/2}\sqrt{-1+ia}}\right)$$

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x^2),x]
```

```
[Out] I*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/((-I + a)*x) + (2*b*ArcTanh[(Sqrt[-1 -
 I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/((
-1 - I*a)^(3/2)*Sqrt[-1 + I*a]))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

method	result
risch	$\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{(a-i)x}+\frac{b\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(a-i)\sqrt{a^2+1}}$
default	$\frac{i\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x}+\frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\,\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx}}{x}\right.\right.}{a^2+1}\right.}{i-a}$

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] I*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(a-I)/x+1/(a-I)*b/(a^2+1)^(1/2)*ln((2*a^2+2
+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(86) = 172.

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.72

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx =$$

$$-\frac{(a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b^2x^2+2\,abx+a^2+1}\,b+(a^3-i\,a^2+a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}}{b}\right)-(a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i}}}{(a-i)x}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -((a - I)*sqrt(b^2/(a^4 - 2*I*a^3 - 2*I*a - 1))*x*log(-(b^2*x - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*b + (a^3 - I*a^2 + a - I)*sqrt(b^2/(a^4 - 2*I*a^3 -
2*I*a - 1)))/b) - (a - I)*sqrt(b^2/(a^4 - 2*I*a^3 - 2*I*a - 1))*x*log(-(b^2
*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b - (a^3 - I*a^2 + a - I)*sqrt(b^2/(
a^4 - 2*I*a^3 - 2*I*a - 1)))/b) - I*b*x - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1))/((a - I)*x)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax^2+bx^3-ix^2}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x**2 + b*x**3 - I*x**2)
, x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^2}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = \frac{b\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}(a-i)} - \frac{2\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2-a^2-1\right)(-i\,a-1)}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(2*x*a
bs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a - I))
 - 2*((x*abs(b) - sqrt((b*x + a)^2 + 1))*a*b + a^2*abs(b) + abs(b))/(((x*ab
s(b) - sqrt((b*x + a)^2 + 1))^2 - a^2 - 1)*(-I*a - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{{(a+b\,x)}^2+1}}{x^2\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(x^2*(a*1i + b*x*1i + 1)),x)
```

```
[Out] int(((a + b*x)^2 + 1)^(1/2)/(x^2*(a*1i + b*x*1i + 1)), x)
```

3.196 $\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 201

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = \frac{(1-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2(1+a^2)x^2} + \frac{(1-2ia)b^2\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}(i+a)^{3/2}}$$

```
[Out] (1-2*I*a)*b^2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*
b*x)^(1/2))/(I-a)^(5/2)/(I+a)^(3/2)-1/2*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(
1/2)/(a^2+1)/x^2+1/2*(1-2*I*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I
-a)^2/(I+a)/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5203, 98, 96, 95, 214}

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = -\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2(a^2+1)x^2} + \frac{(1-2ia)b^2\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}(a+i)^{3/2}} + \frac{(1-2ia)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(-a+i)^2(a+i)x}$$

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((1 - (2*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*(I - a)^2*
(I + a)*x) - ((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*(1 + a^2)*x
^2) + ((1 - (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(5/2)*(I + a)^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1-ia-ibx}}{x^3\sqrt{1+ia+ibx}}\,dx\\
&= -\frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} - \frac{((i+2a)b)\int \frac{\sqrt{1-ia-ibx}}{x^2\sqrt{1+ia+ibx}}\,dx}{2\left(1+a^2\right)}\\
&= \frac{(1-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2}\\
&\quad + \frac{\left((i+2a)b^2\right)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2(i-a)^2(i+a)}\\
&= \frac{(1-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2}\\
&\quad + \frac{\left((i+2a)b^2\right)\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i-a)^2(i+a)}\\
&= \frac{(1-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x}\\
&\quad - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2\left(1+a^2\right)x^2} + \frac{(1-2ia)b^2\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}(i+a)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.77

$$
\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = \frac{\frac{i\left(1+a^2-2ibx-abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{x^2} + \frac{2(i+2a)b^2\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}\sqrt{-1+ia}}}{2(-i+a)^2(i+a)}
$$

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x^3),x]
```

```
[Out] ((I*(1 + a^2 - (2*I)*b*x - a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2 +
(2*(I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1
 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*Sqrt[-1 + I*a]))/(2*(-I +
a)^2*(I + a))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.93

method	result
risch	$\frac{i(-a b^3x^3-2ib^3x^3-a^2b^2x^2-4ia b^2x^2+a^3bx-2ia^2bx+a^4+b^2x^2+abx-2bxi+2a^2+1)}{2x^2(i+a)(a-i)^2\sqrt{b^2x^2+2abx+a^2+1}}-\frac{b^2(i+2a)\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+}}{x}\right.}{2(a^2+1)^{\frac{3}{2}}(a-i)}$
default	$i\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)x^2}-\frac{ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x}+\frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\,\ln\left(\frac{2a^2}{}\right.\right.}{a^2+1}\right.}{}\right.$

[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)

[Out] 1/2*I*(-a*b^3*x^3-a^2*b^2*x^2+a^3*b*x-2*I*b^3*x^3+a^4+b^2*x^2-4*I*a*b^2*x^2+a*b*x-2*I*a^2*b*x+2*a^2-2*I*b*x+1)/x^2/(I+a)/(a-I)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*b^2*(I+2*a)/(a^2+1)^(3/2)/(a-I)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(135) = 270.

Time = 0.29 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.25

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx$$

$$=\frac{(-i\,a+2)b^2x^2+\sqrt{\frac{(4\,a^2+4i\,a-1)b^4}{a^8-2i\,a^7+2\,a^6-6i\,a^5-6i\,a^3-2\,a^2-2i\,a-1}}(a^3-i\,a^2+a-i)x^2\log\left(-\frac{(2\,a+i)b^3x-\sqrt{b^2x^2+2\,abx+a^2+1}(2}{}\right.}{}$$

[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")

[Out] 1/2*((-I*a + 2)*b^2*x^2 + sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))*(a^3 - I*a^2 + a - I)*x^2*log(-((2*a + I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + I)*b^2 + (a^5 - I*a^4 + 2*a^3 - 2*I*a^2 + a - I)*sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*

```
a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1)))/((2*a + I)*b^2)) - s
qrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*
a^2 - 2*I*a - 1))*(a^3 - I*a^2 + a - I)*x^2*log(-((2*a + I)*b^3*x - sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + I)*b^2 - (a^5 - I*a^4 + 2*a^3 - 2*I*a^2 +
 a - I)*sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I
*a^3 - 2*a^2 - 2*I*a - 1)))/((2*a + I)*b^2)) + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)*((-I*a + 2)*b*x + I*a^2 + I))/((a^3 - I*a^2 + a - I)*x^2)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax^3+bx^4-ix^3}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**3,x)
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x**3 + b*x**4 - I*x**3)
, x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^3}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^3), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(135) = 270.

Time = 0.34 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.34

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = -\frac{(2\,ab^2+i\,b^2)\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{2\,(a^3-i\,a^2+a-i)\sqrt{a^2+1}} - \frac{4\left(i\,x|b|-i\,\sqrt{(bx+a)^2+1}\right)a^4b^2+2i\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2a^3b|b|+2i\,a^5b|b|+2\left(x|b|-\sqrt{(bx+\right.}{}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b^2 + I*b^2)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqr
t(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/((
a^3 - I*a^2 + a - I)*sqrt(a^2 + 1)) - (4*(I*x*abs(b) - I*sqrt((b*x + a)^2 +
 1))*a^4*b^2 + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^3*b*abs(b) + 2*I*
a^5*b*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a*b^2 - 2*(x*abs(b) -
 sqrt((b*x + a)^2 + 1))*a^3*b^2 + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^
2*b*abs(b) - 2*a^4*b*abs(b) + I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*b^2 +
5*(I*x*abs(b) - I*sqrt((b*x + a)^2 + 1))*a^2*b^2 + 2*I*(x*abs(b) - sqrt((b*
x + a)^2 + 1))^2*a*b*abs(b) + 4*I*a^3*b*abs(b) - 2*(x*abs(b) - sqrt((b*x +
a)^2 + 1))*a*b^2 + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b*abs(b) - 4*a^2*
b*abs(b) - (-I*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*b^2 + 2*I*a*b*abs(b) - 2
*b*abs(b))/((a^3 - I*a^2 + a - I)*((x*abs(b) - sqrt((b*x + a)^2 + 1))^2 - a
^2 - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx=\int \frac{\sqrt{{(a+b\,x)}^2+1}}{x^3\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(x^3*(a*1i + b*x*1i + 1)),x)
```

```
[Out] int(((a + b*x)^2 + 1)^(1/2)/(x^3*(a*1i + b*x*1i + 1)), x)
```

3.197 $\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 283

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{(4-9ia-2a^2)\,b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1+ia)\,(1+a^2)^2\,x} + \frac{(2a+i(1-2a^2))\,b^3\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{7/2}(i+a)^{5/2}}$$

```
[Out] (2*a+I*(-2*a^2+1))*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/
(1-I*a-I*b*x)^(1/2))/(I-a)^(7/2)/(I+a)^(5/2)-1/3*(1-I*a-I*b*x)^(1/2)*(1+I*a
+I*b*x)^(1/2)/(1+I*a)/x^3+1/6*(3-2*I*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)
^(1/2)/(I-a)^2/(I+a)/x^2+1/6*(4-9*I*a-2*a^2)*b^2*(1-I*a-I*b*x)^(1/2)*(1+I*a
+I*b*x)^(1/2)/(1+I*a)/(a^2+1)^2/x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

= {5203, 101, 156, 12, 95, 214}

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = \frac{(-2ia^2+2a+i)\,b^3\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}(a+i)^{5/2}} + \frac{(-2a^2-9ia+4)\,b^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(1+ia)\,(a^2+1)^2\,x} - \frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{6(-a+i)^2(a+i)x^2}$$

```
[In] Int[1/(E^(I*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -1/3*(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*x^3) + ((3 -
(2*I)*a)*b*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(6*(I - a)^2*(I + a
)*x^2) + ((4 - (9*I)*a - 2*a^2)*b^2*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*
b*x])/(6*(1 + I*a)*(1 + a^2)^2*x) + ((I + 2*a - (2*I)*a^2)*b^3*ArcTanh[(Sqr
t[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I -
 a)^(7/2)*(I + a)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1-ia-ibx}}{x^4\sqrt{1+ia+ibx}}\,dx \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3}+\frac{\int \frac{-((3i+2a)b)-2b^2x}{x^3\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{3(1+ia)} \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} \\
&\quad+\frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2}-\frac{\int \frac{(4-9ia-2a^2)b^2-(3i+2a)b^3x}{x^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{6(1+ia)\left(1+a^2\right)} \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3}+\frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} \\
&\quad+\frac{\left(4-9ia-2a^2\right)b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1+ia)\left(1+a^2\right)^2x}+\frac{\int \frac{3\left(i+2a-2ia^2\right)b^3}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{6(1+ia)\left(1+a^2\right)^2} \\
&= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3}+\frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} \\
&\quad+\frac{\left(4-9ia-2a^2\right)b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1+ia)\left(1+a^2\right)^2x} \\
&\quad-\frac{\left(\left(1-2ia-2a^2\right)b^3\right)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2(i-a)^3(i+a)^2}
\end{aligned}
$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{(4-9ia-2a^2)\, b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1+ia)\left(1+a^2\right)^2 x} - \frac{\left((1-2ia-2a^2)\, b^3\right)\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\, dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i-a)^3(i+a)^2}$$

$$= -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3} + \frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2} + \frac{(4-9ia-2a^2)\, b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1+ia)\left(1+a^2\right)^2 x} + \frac{(i+2a-2ia^2)\, b^3\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{7/2}(i+a)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.82

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\, dx$$

$$= \frac{\frac{2(1+ia)(i+a)(i+a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{x^3} + \frac{(1-4ia)b(i+a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{x^2} - 3i(1-2ia-2a^2)\, b^2\left(\frac{\sqrt{1+a^2+2abx+b^2x^2}}{(-i+a)x}\right.}{6\left(1+a^2\right)^2}$$

```
[In] Integrate[1/(E^(I*ArcTan[a + b*x])*x^4),x]
```

```
[Out] ((2*(1 + I*a)*(I + a)*(I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^3
+ ((1 - (4*I)*a)*b*(I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2 - (
3*I)*(1 - (2*I)*a - 2*a^2)*b^2*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/((-I + a)
*x) + (2*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a
]*Sqrt[1 + I*a + I*b*x])])/((-1 - I*a)^(3/2)*Sqrt[-1 + I*a])))/(6*(1 + a^2)
^2)
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.99

method	result
risch	$\frac{i\left(2a^2b^4x^4+9ia\,b^4x^4+2a^3b^3x^3+15ia^2b^3x^3+3ix^2a^3b^2-4x^4b^4+2a^5bx-3ia^4bx-10a\,b^3x^3-3ib^3x^3+2a^6-2a^2b^2x^2+3ia\,b^2x^2+4a^3bx-6ia\right.}{6x^3(i+a)^2(a-i)^3\sqrt{b^2x^2+2abx+a^2+1}}$
default	Expression too large to display

```
[In] int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*(3*I*a*b^2*x^2+2*a^2*b^4*x^4+9*I*a*b^4*x^4+2*a^3*b^3*x^3+15*I*x^3*a^2
*b^3-4*x^4*b^4-3*I*x*a^4*b-6*I*a^2*b*x+2*a^5*b*x-10*a*b^3*x^3+3*I*b^2*x^2*a
^3+2*a^6-2*a^2*b^2*x^2-3*I*b^3*x^3+4*a^3*b*x+6*a^4-2*b^2*x^2-3*I*b*x+2*a*b*
x+6*a^2+2)/x^3/(I+a)^2/(a-I)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*b^3*(2*I*a
+2*a^2-1)/(a^2+1)^(5/2)/(a-I)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(194) = 388.

Time = 0.29 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.44

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx$$

$$= \frac{(2i\,a^2-9\,a-4i)b^3x^3-3\sqrt{\frac{(4\,a^4+8i\,a^3-8\,a^2-4i\,a+1)b^6}{a^{12}-2i\,a^{11}+4\,a^{10}-10i\,a^9+5\,a^8-20i\,a^7-20i\,a^5-5\,a^4-10i\,a^3-4\,a^2-2i\,a-1}}(a^5-i\,a^4+2\,a^3-2i\,a^2}{}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((2*I*a^2 - 9*a - 4*I)*b^3*x^3 - 3*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*
a + 1)*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a
^5 - 5*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1))*(a^5 - I*a^4 + 2*a^3 - 2*I*a^2
+ a - I)*x^3*log(-((2*a^2 + 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)*(2*a^2 + 2*I*a - 1)*b^3 + (a^7 - I*a^6 + 3*a^5 - 3*I*a^4 + 3*a^3 - 3*
I*a^2 + a - I)*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*a + 1)*b^6/(a^12 - 2*I*a
^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a^5 - 5*a^4 - 10*I*a^3 -
4*a^2 - 2*I*a - 1)))/((2*a^2 + 2*I*a - 1)*b^3)) + 3*sqrt((4*a^4 + 8*I*a^3 -
 8*a^2 - 4*I*a + 1)*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I
*a^7 - 20*I*a^5 - 5*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1))*(a^5 - I*a^4 + 2*a
^3 - 2*I*a^2 + a - I)*x^3*log(-((2*a^2 + 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 +
```

```
2*a*b*x + a^2 + 1)*(2*a^2 + 2*I*a - 1)*b^3 - (a^7 - I*a^6 + 3*a^5 - 3*I*a^4
 + 3*a^3 - 3*I*a^2 + a - I)*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*a + 1)*b^6/
(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a^5 - 5*a^4
- 10*I*a^3 - 4*a^2 - 2*I*a - 1)))/((2*a^2 + 2*I*a - 1)*b^3)) + ((2*I*a^2 -
9*a - 4*I)*b^2*x^2 + 2*I*a^4 + (-2*I*a^3 + 3*a^2 - 2*I*a + 3)*b*x + 4*I*a^2
 + 2*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((a^5 - I*a^4 + 2*a^3 - 2*I*a^2
+ a - I)*x^3)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax^4+bx^5-ix^4}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**4,x)
[Out] -I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x**4 + b*x**5 - I*x**4)
, x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^4}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^4), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(194) = 388.

Time = 0.34 (sec) , antiderivative size = 884, normalized size of antiderivative = 3.12

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
[Out] 1/2*(2*a^2*b^3 + 2*I*a*b^3 - b^3)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 +
 1) - 2*sqrt(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^
2 + 1)))/((a^5 - I*a^4 + 2*a^3 - 2*I*a^2 + a - I)*sqrt(a^2 + 1)) + 1/3*(-8*
```

```
I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^5*b^3 + 24*(-I*x*abs(b) + I*sqrt((
b*x + a)^2 + 1))*a^7*b^3 - 24*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^6*b^
2*abs(b) - 8*I*a^8*b^2*abs(b) + 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*a^2*
b^3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^4*b^3 + 18*(x*abs(b) - sqrt
((b*x + a)^2 + 1))*a^6*b^3 - 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^5*b^
2*abs(b) + 12*a^7*b^2*abs(b) + 6*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*a*b
^3 - 32*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^3*b^3 + 54*(-I*x*abs(b) +
I*sqrt((b*x + a)^2 + 1))*a^5*b^3 - 60*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^
2*a^4*b^2*abs(b) - 20*I*a^6*b^2*abs(b) - 3*(x*abs(b) - sqrt((b*x + a)^2 + 1
))^5*b^3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^2*b^3 + 39*(x*abs(b) -
 sqrt((b*x + a)^2 + 1))*a^4*b^3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a
^3*b^2*abs(b) + 36*a^5*b^2*abs(b) - 24*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))
^3*a*b^3 + 36*(-I*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*a^3*b^3 - 48*I*(x*abs
(b) - sqrt((b*x + a)^2 + 1))^2*a^2*b^2*abs(b) - 12*I*a^4*b^2*abs(b) + 24*(x
*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*b^3 - 12*(x*abs(b) - sqrt((b*x + a)^2
+ 1))^2*a*b^2*abs(b) + 36*a^3*b^2*abs(b) + 6*(-I*x*abs(b) + I*sqrt((b*x + a
)^2 + 1))*a*b^3 - 12*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b^2*abs(b) + 4*
I*a^2*b^2*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*b^3 + 12*a*b^2*abs(
b) + 4*I*b^2*abs(b))/((a^5 - I*a^4 + 2*a^3 - 2*I*a^2 + a - I)*((x*abs(b) -
sqrt((b*x + a)^2 + 1))^2 - a^2 - 1)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx=\int \frac{\sqrt{(a+b\,x)^2+1}}{x^4\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(1/2)/(x^4*(a*1i + b*x*1i + 1)),x)
[Out] int(((a + b*x)^2 + 1)^(1/2)/(x^4*(a*1i + b*x*1i + 1)), x)
```

3.198 $\int e^{-2i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int e^{-2i\arctan(a+bx)}x^4\,dx=-\frac{2(1+ia)^3x}{b^4}-\frac{i(i-a)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}-\frac{2i(i-a)^4\log(i-a-bx)}{b^5}$$

```
[Out] -2*(1+I*a)^3*x/b^4-I*(I-a)^2*x^2/b^3+2/3*(1+I*a)*x^3/b^2-1/2*I*x^4/b-1/5*x^
5-2*I*(I-a)^4*ln(I-a-b*x)/b^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int e^{-2i\arctan(a+bx)}x^4\,dx=-\frac{2i(-a+i)^4\log(-a-bx+i)}{b^5}-\frac{2(1+ia)^3x}{b^4}-\frac{i(-a+i)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}$$

```
[In] Int[x^4/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (-2*(1 + I*a)^3*x)/b^4 - (I*(I - a)^2*x^2)/b^3 + (2*(1 + I*a)*x^3)/(3*b^2)
- ((I/2)*x^4)/b - x^5/5 - ((2*I)*(I - a)^4*Log[I - a - b*x])/b^5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4(1-ia-ibx)}{1+ia+ibx}\,dx \\
&= \int \left(\frac{2(-1-ia)^3}{b^4}-\frac{2i(-i+a)^2x}{b^3}+\frac{2(1+ia)x^2}{b^2}-\frac{2ix^3}{b}-x^4-\frac{2i(-i+a)^4}{b^4(-i+a+bx)}\right)dx \\
&= -\frac{2(1+ia)^3x}{b^4}-\frac{i(i-a)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}-\frac{2i(i-a)^4\log(i-a-bx)}{b^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\begin{aligned}
\int e^{-2i\arctan(a+bx)}x^4\,dx = &-\frac{2(1+ia)^3x}{b^4}-\frac{i(-i+a)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2} \\
&-\frac{ix^4}{2b}-\frac{x^5}{5}-\frac{2i(-i+a)^4\log(i-a-bx)}{b^5}
\end{aligned}$$

```
[In] Integrate[x^4/E^((2*I)*ArcTan[a + b*x]),x]
[Out] (-2*(1 + I*a)^3*x)/b^4 - (I*(-I + a)^2*x^2)/b^3 + (2*(1 + I*a)*x^3)/(3*b^2)
 - ((I/2)*x^4)/b - x^5/5 - ((2*I)*(-I + a)^4*Log[I - a - b*x])/b^5
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.26

method	result
default	$-\frac{i\left(-\frac{1}{5}ib^4x^5+\frac{1}{2}b^3x^4+\frac{2}{3}ib^2x^3-\frac{2}{3}a\,b^2x^3-2iab\,x^2+a^2b\,x^2+6ia^2x-2a^3x-x^2b-2ix+6ax\right)}{b^4}+\frac{\left(-2ia^4-8a^3+12ia^2+8a-2i\right)\ln(-}{b^5}$
risch	$-\frac{x^5}{5}+\frac{2ia\,x^3}{3b^2}+\frac{2x^3}{3b^2}-\frac{ix^4}{2b}-\frac{2a\,x^2}{b^3}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^4}{b^5}+\frac{6a^2x}{b^4}-\frac{8i\arctan(bx+a)a^3}{b^5}+\frac{2ia^3x}{b^4}-\frac{2x}{b^4}+$
parallelrisch	$\underline{60-90a^2b^2x^2-600a^2+300a^4+300\ln(bx+a-i)a^4-600\ln(bx+a-i)a^2-600i\ln(bx+a-i)a^3+300i\ln(bx+a-i)a+9ix^5b^5+60i\ln(b}$

```
[In] int(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/b^4*(-1/5*I*b^4*x^5+1/2*b^3*x^4+2/3*I*b^2*x^3-2/3*a*b^2*x^3-2*I*a*b*x^2+
a^2*b*x^2+6*I*a^2*x-2*a^3*x-x^2*b-2*I*x+6*a*x)+(-2*I*a^4+12*I*a^2-8*a^3-2*I
+8*a)/b^5*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = -\frac{6\,b^5x^5+15i\,b^4x^4+20\,(-i\,a-1)b^3x^3+30\,(i\,a^2+2\,a-i)b^2x^2+60\,(-i\,a^3-3\,a^2+3i\,a+1)bx+60\,(i\,a}{30\,b^5}$$

```
[In] integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 + 15*I*b^4*x^4 + 20*(-I*a - 1)*b^3*x^3 + 30*(I*a^2 + 2*a -
 I)*b^2*x^2 + 60*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b*x + 60*(I*a^4 + 4*a^3 - 6*I
*a^2 - 4*a + I)*log((b*x + a - I)/b))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15

$$\begin{aligned}\int e^{-2i\arctan(a+bx)}x^4\,dx = &-\frac{x^5}{5} - x^3\left(-\frac{2ia}{3b^2} - \frac{2}{3b^2}\right) - x^2\left(\frac{ia^2}{b^3} + \frac{2a}{b^3} - \frac{i}{b^3}\right)\\ &- x\left(-\frac{2ia^3}{b^4} - \frac{6a^2}{b^4} + \frac{6ia}{b^4} + \frac{2}{b^4}\right) - \frac{ix^4}{2b} - \frac{2i(a-i)^4\log(a+bx-i)}{b^5}\end{aligned}$$

```
[In] integrate(x**4/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

```
[Out] -x**5/5 - x**3*(-2*I*a/(3*b**2) - 2/(3*b**2)) - x**2*(I*a**2/b**3 + 2*a/b**
3 - I/b**3) - x*(-2*I*a**3/b**4 - 6*a**2/b**4 + 6*I*a/b**4 + 2/b**4) - I*x*
*4/(2*b) - 2*I*(a - I)**4*log(a + b*x - I)/b**5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = -\frac{6\,b^4x^5+15i\,b^3x^4-20\,(i\,a+1)b^2x^3-30\,(-i\,a^2-2\,a+i)bx^2-60\,(i\,a^3+3\,a^2-3i\,a-1)x}{30\,b^4} - \frac{2\,(i\,a^4+4\,a^3-6i\,a^2-4\,a+i)\log{(i\,bx+i\,a+1)}}{b^5}$$

```
[In] integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 + 15*I*b^3*x^4 - 20*(I*a + 1)*b^2*x^3 - 30*(-I*a^2 - 2*a +
 I)*b*x^2 - 60*(I*a^3 + 3*a^2 - 3*I*a - 1)*x)/b^4 - 2*(I*a^4 + 4*a^3 - 6*I*
a^2 - 4*a + I)*log(I*b*x + I*a + 1)/b^5
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(73) = 146.

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.17

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = \frac{i\,(i\,bx+i\,a+1)^5\left(-\frac{15i\,(2\,ab-3i\,b)}{(i\,bx+i\,a+1)b}-\frac{20\,(3\,a^2b^2-10i\,ab^2-7\,b^2)}{(i\,bx+i\,a+1)^2b^2}+\frac{60i\,(a^3b^3-6i\,a^2b^3-9\,ab^3+4i\,b^3)}{(i\,bx+i\,a+1)^3b^3}+\frac{30\,(a^4b^4-12i\,a^3b^4-30\,a^2b^4+28i}{(i\,bx+i\,a+1)^4b^4}\right.}{30\,b^5} - \frac{2\,(-i\,a^4-4\,a^3+6i\,a^2+4\,a-i)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b^5}$$

```
[In] integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/30*I*(I*b*x + I*a + 1)^5*(-15*I*(2*a*b - 3*I*b)/((I*b*x + I*a + 1)*b) - 2
0*(3*a^2*b^2 - 10*I*a*b^2 - 7*b^2)/((I*b*x + I*a + 1)^2*b^2) + 60*I*(a^3*b^
3 - 6*I*a^2*b^3 - 9*a*b^3 + 4*I*b^3)/((I*b*x + I*a + 1)^3*b^3) + 30*(a^4*b^
4 - 12*I*a^3*b^4 - 30*a^2*b^4 + 28*I*a*b^4 + 9*b^4)/((I*b*x + I*a + 1)^4*b^
4) + 6)/b^5 - 2*(-I*a^4 - 4*a^3 + 6*I*a^2 + 4*a - I)*log(1/(sqrt((b*x + a)^
2 + 1)*abs(b)))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.67

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = \ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(\frac{8\,a-8\,a^3}{b^5}-\frac{\left(2\,a^4-12\,a^2+2\right)\,1\mathrm{i}}{b^5}\right)+x^4\left(\frac{a-\mathrm{i}}{4\,b}-\frac{a+1\mathrm{i}}{4\,b}\right)-\frac{x^5}{5}+\frac{x^2\,\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)\,{\left(a-\mathrm{i}\right)}^2}{2\,b^2}-\frac{x^3\,\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)\,\left(a-\mathrm{i}\right)}{3\,b}-\frac{x\,\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)\,{\left(a-\mathrm{i}\right)}^3}{b^3}$$

```
[In] int((x^4*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

```
[Out] log(x + (a - 1i)/b)*((8*a - 8*a^3)/b^5 - ((2*a^4 - 12*a^2 + 2)*1i)/b^5) + x
^4*((a - 1i)/(4*b) - (a + 1i)/(4*b)) - x^5/5 + (x^2*((a - 1i)/b - (a + 1i)/
b)*(a - 1i)^2)/(2*b^2) - (x^3*((a - 1i)/b - (a + 1i)/b)*(a - 1i))/(3*b) - (
x*((a - 1i)/b - (a + 1i)/b)*(a - 1i)^3)/b^3
```

3.199 $\int e^{-2i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int e^{-2i\arctan(a+bx)}x^3\,dx=-\frac{2i(i-a)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1+ia)^3\log(i-a-bx)}{b^4}$$

```
[Out] -2*I*(I-a)^2*x/b^3+(1+I*a)*x^2/b^2-2/3*I*x^3/b-1/4*x^4-2*(1+I*a)^3*ln(I-a-b
*x)/b^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int e^{-2i\arctan(a+bx)}x^3\,dx=-\frac{2(1+ia)^3\log(-a-bx+i)}{b^4}-\frac{2i(-a+i)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}$$

```
[In] Int[x^3/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*(I - a)^2*x)/b^3 + ((1 + I*a)*x^2)/b^2 - (((2*I)/3)*x^3)/b - x^4/4
- (2*(1 + I*a)^3*Log[I - a - b*x])/b^4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3(1-ia-ibx)}{1+ia+ibx}\,dx\\ &= \int \left(-\frac{2i(-i+a)^2}{b^3}+\frac{2(1+ia)x}{b^2}-\frac{2ix^2}{b}-x^3+\frac{2(-1-ia)^3}{b^3(-i+a+bx)}\right)dx\\ &= -\frac{2i(i-a)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1+ia)^3\log(i-a-bx)}{b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\frac{2i(i-a)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1+ia)^3\log(i-a-bx)}{b^4}$$

```
[In] Integrate[x^3/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*(I - a)^2*x)/b^3 + ((1 + I*a)*x^2)/b^2 - (((2*I)/3)*x^3)/b - x^4/4
- (2*(1 + I*a)^3*Log[I - a - b*x])/b^4
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

method	result
default	$\frac{i\left(\frac{1}{4}ib^3x^4-\frac{2}{3}b^2x^3-ib\,x^2+ab\,x^2+4iax-2a^2x+2x\right)}{b^3}+\frac{\left(2ia^3+6a^2-6ia-2\right)\ln(-bx-a+i)}{b^4}$
risch	$-\frac{x^4}{4}-\frac{2ix^3}{3b}+\frac{x^2}{b^2}+\frac{ia\,x^2}{b^2}-\frac{4ax}{b^3}-\frac{2ia^2x}{b^3}+\frac{2ix}{b^3}+\frac{3\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{b^4}+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^3}{b^4}-$
parallelrisch	$-\frac{-96a+4b^3x^3-24a\,b^2x^2-5ix^4b^4+4ix^3a\,b^3-144ia^2-24\ln(bx+a-i)xb+24i+24i\ln(bx+a-i)-12ix^2a^2b^2+24i\ln(bx+a-i)a^4}{}$

```
[In] int(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] I/b^3*(1/4*I*b^3*x^4-2/3*b^2*x^3-I*b*x^2+a*b*x^2+4*I*a*x-2*a^2*x+2*x)+(2*I*
a^3-6*I*a+6*a^2-2)/b^4*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\frac{3\,b^4x^4 + 8i\,b^3x^3 + 12\,(-i\,a-1)b^2x^2 + 24\,(i\,a^2+2\,a-i)bx + 24\,(-i\,a^3-3\,a^2+3i\,a+1)\log\left(\frac{bx+a-i}{b}\right)}{12\,b^4}$$

```
[In] integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
[Out] -1/12*(3*b^4*x^4 + 8*I*b^3*x^3 + 12*(-I*a - 1)*b^2*x^2 + 24*(I*a^2 + 2*a -
I)*b*x + 24*(-I*a^3 - 3*a^2 + 3*I*a + 1)*log((b*x + a - I)/b))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\frac{x^4}{4} - x^2\left(-\frac{ia}{b^2}-\frac{1}{b^2}\right) - x\left(\frac{2ia^2}{b^3}+\frac{4a}{b^3}-\frac{2i}{b^3}\right) - \frac{2ix^3}{3b} + \frac{2i(a-i)^3\log(a+bx-i)}{b^4}$$

```
[In] integrate(x**3/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
[Out] -x**4/4 - x**2*(-I*a/b**2 - 1/b**2) - x*(2*I*a**2/b**3 + 4*a/b**3 - 2*I/b**
3) - 2*I*x**3/(3*b) + 2*I*(a - I)**3*log(a + b*x - I)/b**4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\frac{i\,(-3i\,b^3x^4 + 8\,b^2x^3 - 12\,(a-i)bx^2 + 24\,(a^2-2i\,a-1)x)}{12\,b^3} - \frac{2\,(-i\,a^3-3\,a^2+3i\,a+1)\log(i\,bx+i\,a+1)}{b^4}$$

```
[In] integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
[Out] -1/12*I*(-3*I*b^3*x^4 + 8*b^2*x^3 - 12*(a - I)*b*x^2 + 24*(a^2 - 2*I*a - 1)
*x)/b^3 - 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*log(I*b*x + I*a + 1)/b^4
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(59) = 118$.

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.05

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\frac{(i\,bx+i\,a+1)^4\left(-\frac{4i\,(3\,ab-5i\,b)}{(i\,bx+i\,a+1)b}-\frac{18\,(a^2b^2-4i\,ab^2-3\,b^2)}{(i\,bx+i\,a+1)^2b^2}+\frac{12i\,(a^3b^3-9i\,a^2b^3-15\,ab^3+7i\,b^3)}{(i\,bx+i\,a+1)^3b^3}+3\right)}{12\,b^4} - \frac{2\,(i\,a^3+3\,a^2-3i\,a-1)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b^4}$$

```
[In] integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/12*(I*b*x + I*a + 1)^4*(-4*I*(3*a*b - 5*I*b)/((I*b*x + I*a + 1)*b) - 18*
(a^2*b^2 - 4*I*a*b^2 - 3*b^2)/((I*b*x + I*a + 1)^2*b^2) + 12*I*(a^3*b^3 - 9
*I*a^2*b^3 - 15*a*b^3 + 7*I*b^3)/((I*b*x + I*a + 1)^3*b^3) + 3)/b^4 - 2*(I*
a^3 + 3*a^2 - 3*I*a - 1)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b^4
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = x^3\left(\frac{a-\mathrm{i}}{3\,b}-\frac{a+1\mathrm{i}}{3\,b}\right)-\frac{x^4}{4} - \ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(-\frac{6\,a^2-2}{b^4}+\frac{(6\,a-2\,a^3)\ 1\mathrm{i}}{b^4}\right) - \frac{x^2\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)\,(a-\mathrm{i})}{2\,b}+\frac{x\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)\,(a-\mathrm{i})^2}{b^2}$$

```
[In] int((x^3*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

```
[Out] x^3*((a - 1i)/(3*b) - (a + 1i)/(3*b)) - x^4/4 - log(x + (a - 1i)/b)*(((6*a
- 2*a^3)*1i)/b^4 - (6*a^2 - 2)/b^4) - (x^2*((a - 1i)/b - (a + 1i)/b)*(a - 1
i))/(2*b) + (x*((a - 1i)/b - (a + 1i)/b)*(a - 1i)^2)/b^2
```

3.200 $\int e^{-2i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int e^{-2i\arctan(a+bx)}x^2\,dx=\frac{2(1+ia)x}{b^2}-\frac{ix^2}{b}-\frac{x^3}{3}-\frac{2i(i-a)^2\log(i-a-bx)}{b^3}$$

```
[Out] 2*(1+I*a)*x/b^2-I*x^2/b-1/3*x^3-2*I*(I-a)^2*ln(I-a-b*x)/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int e^{-2i\arctan(a+bx)}x^2\,dx=-\frac{2i(-a+i)^2\log(-a-bx+i)}{b^3}+\frac{2(1+ia)x}{b^2}-\frac{ix^2}{b}-\frac{x^3}{3}$$

```
[In] Int[x^2/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (2*(1 + I*a)*x)/b^2 - (I*x^2)/b - x^3/3 - ((2*I)*(I - a)^2*Log[I - a - b*x]
)/b^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2(1-ia-ibx)}{1+ia+ibx}\,dx\\ &= \int \left(\frac{2i(-i+a)}{b^2}-\frac{2ix}{b}-x^2-\frac{2i(-i+a)^2}{b^2(-i+a+bx)}\right)dx\\ &= \frac{2(1+ia)x}{b^2}-\frac{ix^2}{b}-\frac{x^3}{3}-\frac{2i(i-a)^2\log(i-a-bx)}{b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = \frac{bx(6+6ia-3ibx-b^2x^2)-6i(-i+a)^2\log(i-a-bx)}{3b^3}$$

```
[In] Integrate[x^2/E^((2*I)*ArcTan[a + b*x]),x]
[Out] (b*x*(6 + (6*I)*a - (3*I)*b*x - b^2*x^2) - (6*I)*(-I + a)^2*Log[I - a - b*x
])/(3*b^3)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result
default	$\frac{i\left(\frac{1}{3}ib^2x^3-x^2b-2ix+2ax\right)}{b^2}+\frac{(-2ia^2-4a+2i)\ln(-bx-a+i)}{b^3}$
risch	$-\frac{x^3}{3}-\frac{ix^2}{b}+\frac{2x}{b^2}+\frac{2iax}{b^2}-\frac{2\ln(b^2x^2+2abx+a^2+1)a}{b^3}-\frac{i\ln(b^2x^2+2abx+a^2+1)a^2}{b^3}+\frac{i\ln(b^2x^2+2abx+a^2+1)}{b^3}-\frac{4i}{}$
parallelrisch	$\frac{x^4b^4-3ia\,b^2x^2+a\,b^3x^3+6i\ln(bx+a-i)a^3+6i\ln(bx+a-i)x\,a^2b-18ia-6+6ia^3+2ib^3x^3+12\ln(bx+a-i)xab-3b^2x^2-18i\ln(bx+}{3b^3(-bx-a+i)}$

```
[In] int(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] I/b^2*(1/3*I*b^2*x^3-x^2*b-2*I*x+2*a*x)+(-2*I*a^2+2*I-4*a)/b^3*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\frac{b^3x^3 + 3i\,b^2x^2 + 6\,(-i\,a-1)bx + 6\,(i\,a^2+2\,a-i)\log\left(\frac{bx+a-i}{b}\right)}{3\,b^3}$$

```
[In] integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
[Out] -1/3*(b^3*x^3 + 3*I*b^2*x^2 + 6*(-I*a - 1)*b*x + 6*(I*a^2 + 2*a - I)*log((b
*x + a - I)/b))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\frac{x^3}{3} - x\left(-\frac{2ia}{b^2}-\frac{2}{b^2}\right) - \frac{ix^2}{b} - \frac{2i(a-i)^2\log(a+bx-i)}{b^3}$$

```
[In] integrate(x**2/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
[Out] -x**3/3 - x*(-2*I*a/b**2 - 2/b**2) - I*x**2/b - 2*I*(a - I)**2*log(a + b*x
- I)/b**3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\frac{b^2x^3 + 3i\,bx^2 + 6\,(-i\,a-1)x}{3\,b^2} - \frac{2\,(i\,a^2+2\,a-i)\log(i\,bx+i\,a+1)}{b^3}$$

```
[In] integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
[Out] -1/3*(b^2*x^3 + 3*I*b*x^2 + 6*(-I*a - 1)*x)/b^2 - 2*(I*a^2 + 2*a - I)*log(I
*b*x + I*a + 1)/b^3
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\frac{i\,(i\,bx+i\,a+1)^3\left(-\frac{3i\,(ab-2i\,b)}{(i\,bx+i\,a+1)b}-\frac{3\,(a^2b^2-6i\,ab^2-5\,b^2)}{(i\,bx+i\,a+1)^2b^2}+1\right)}{3\,b^3} - \frac{2\,(-i\,a^2-2\,a+i)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b^3}$$

```
[In] integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/3*I*(I*b*x + I*a + 1)^3*(-3*I*(a*b - 2*I*b)/((I*b*x + I*a + 1)*b) - 3*(a
^2*b^2 - 6*I*a*b^2 - 5*b^2)/((I*b*x + I*a + 1)^2*b^2) + 1)/b^3 - 2*(-I*a^2
- 2*a + I)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(\frac{4\,a}{b^3}+\frac{(2\,a^2-2)\,1\mathrm{i}}{b^3}\right) + x^2\left(\frac{a-\mathrm{i}}{2\,b}-\frac{a+1\mathrm{i}}{2\,b}\right)-\frac{x^3}{3}-\frac{x\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)(a-\mathrm{i})}{b}$$

```
[In] int((x^2*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

```
[Out] x^2*((a - 1i)/(2*b) - (a + 1i)/(2*b)) - log(x + (a - 1i)/b)*((4*a)/b^3 + ((
2*a^2 - 2)*1i)/b^3) - x^3/3 - (x*((a - 1i)/b - (a + 1i)/b)*(a - 1i))/b
```

3.201 $\int e^{-2i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int e^{-2i\arctan(a+bx)}x\,dx=-\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1+ia)\log(i-a-bx)}{b^2}$$

```
[Out] -2*I*x/b-1/2*x^2+2*(1+I*a)*ln(I-a-b*x)/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5203, 78}

$$\int e^{-2i\arctan(a+bx)}x\,dx=\frac{2(1+ia)\log(-a-bx+i)}{b^2}-\frac{2ix}{b}-\frac{x^2}{2}$$

```
[In] Int[x/E^((2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-2*I)*x)/b - x^2/2 + (2*(1 + I*a)*Log[I - a - b*x])/b^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(1-ia-ibx)}{1+ia+ibx}\,dx\\ &= \int \left(-\frac{2i}{b}-x+\frac{2(1+ia)}{b(-i+a+bx)}\right)dx\\ &= -\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1+ia)\log(i-a-bx)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(a+bx)}x\,dx = -\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1+ia)\log(i-a-bx)}{b^2}$$

```
[In] Integrate[x/E^((2*I)*ArcTan[a + b*x]),x]
[Out] ((-2*I)*x)/b - x^2/2 + (2*(1 + I*a)*Log[I - a - b*x])/b^2
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\frac{1}{2}x^2b+2ix}{b}+\frac{(2ia+2)\ln(-bx-a+i)}{b^2}$
risch	$-\frac{x^2}{2}-\frac{2ix}{b}+\frac{\ln(b^2x^2+2abx+a^2+1)}{b^2}+\frac{2i\arctan(bx+a)}{b^2}+\frac{ia\ln(b^2x^2+2abx+a^2+1)}{b^2}-\frac{2a\arctan(bx+a)}{b^2}$
parallelrisch	$-\frac{-b^3x^3+4i\ln(bx+a-i)xab-3ix^2b^2-a\,b^2x^2+4i\ln(bx+a-i)a^2-4i+4ia^2+4\ln(bx+a-i)xb-4i\ln(bx+a-i)+8\ln(bx+a-i)a+}{2b^2(-bx-a+i)}$

```
[In] int(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] -1/b*(1/2*x^2*b+2*I*x)+(2*I*a+2)/b^2*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{-2i\arctan(a+bx)}x\,dx = -\frac{b^2x^2 + 4i\,bx + 4\,(-i\,a - 1)\log\left(\frac{bx+a-i}{b}\right)}{2\,b^2}$$

```
[In] integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 + 4*I*b*x + 4*(-I*a - 1)*log((b*x + a - I)/b))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int e^{-2i\arctan(a+bx)}x\,dx = -\frac{x^2}{2} - \frac{2ix}{b} + \frac{2i(a-i)\log(a+bx-i)}{b^2}$$

```
[In] integrate(x/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
[Out] -x**2/2 - 2*I*x/b + 2*I*(a - I)*log(a + b*x - I)/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(a+bx)}x\,dx = \frac{i\,(i\,bx^2 - 4\,x)}{2\,b} - \frac{2\,(-i\,a - 1)\log(i\,bx + i\,a + 1)}{b^2}$$

```
[In] integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
[Out] 1/2*I*(I*b*x^2 - 4*x)/b - 2*(-I*a - 1)*log(I*b*x + I*a + 1)/b^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int e^{-2i\arctan(a+bx)}x\,dx = -\frac{i\left(\frac{(i\,bx+i\,a+1)^2\left(-\frac{2i\,(i\,ab+3\,b)}{(i\,bx+i\,a+1)b}+i\right)}{b} + \frac{4\,(a-i)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b}\right)}{2\,b}$$

```
[In] integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/2*I*((I*b*x + I*a + 1)^2*(-2*I*(I*a*b + 3*b)/((I*b*x + I*a + 1)*b) + I)/
b + 4*(a - I)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b))))/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int e^{-2i\arctan(a+bx)}x\,dx = \ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(\frac{2}{b^2}+\frac{a\,2\mathrm{i}}{b^2}\right)-\frac{x^2}{2}+x\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)$$

```
[In] int((x*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

```
[Out] log(x + (a - 1i)/b)*((a*2i)/b^2 + 2/b^2) - x^2/2 + x*((a - 1i)/b - (a + 1i)
/b)
```

3.202 $\int e^{-2i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int e^{-2i\arctan(a+bx)}\,dx = -x - \frac{2i\log(i-a-bx)}{b}$$

```
[Out] -x-2*I*ln(I-a-b*x)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5201, 45}

$$\int e^{-2i\arctan(a+bx)}\,dx = -x - \frac{2i\log(-a-bx+i)}{b}$$

```
[In] Int[E^((-2*I)*ArcTan[a + b*x]),x]
```

```
[Out] -x - ((2*I)*Log[I - a - b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
```

```
c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-ia-ibx}{1+ia+ibx}\,dx\\ &= \int \left(-1-\frac{2i}{-i+a+bx}\right)dx\\ &= -x-\frac{2i\log(i-a-bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int e^{-2i\arctan(a+bx)}\,dx = -x+\frac{2\arctan(a+bx)}{b}-\frac{i\log\left(1+(a+bx)^2\right)}{b}$$

```
[In] Integrate[E^((-2*I)*ArcTan[a + b*x]),x]
[Out] -x + (2*ArcTan[a + b*x])/b - (I*Log[1 + (a + b*x)^2])/b
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-x-\frac{2i\ln(-bx-a+i)}{b}$	22
risch	$-x-\frac{i\ln(b^2x^2+2abx+a^2+1)}{b}+\frac{2\arctan(bx+a)}{b}$	40
parallelrisch	$\frac{2i\ln(bx+a-i)xb+b^2x^2+2i\ln(bx+a-i)a+1+2ia-a^2+2\ln(bx+a-i)}{b(-bx-a+i)}$	70

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] -x-2*I*ln(I-a-b*x)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{-2i\arctan(a+bx)}\,dx = -\frac{bx + 2i\,\log\left(\frac{bx+a-i}{b}\right)}{b}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
[Out] -(b*x + 2*I*log((b*x + a - I)/b))/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int e^{-2i\arctan(a+bx)}\,dx = -x - \frac{2i\log\left(a + bx - i\right)}{b}$$

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
[Out] -x - 2*I*log(a + b*x - I)/b
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int e^{-2i\arctan(a+bx)}\,dx = -x - \frac{2i\,\log\left(i\,bx + i\,a + 1\right)}{b}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
[Out] -x - 2*I*log(I*b*x + I*a + 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int e^{-2i\arctan(a+bx)}\,dx = \frac{i\left(i\,bx + i\,a + 1\right)}{b} + \frac{2i\,\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
[Out] I*(I*b*x + I*a + 1)/b + 2*I*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int e^{-2i\arctan(a+bx)}\,dx = -x - \frac{\ln\left(x + \frac{a-\mathrm{i}}{b}\right)\,2\mathrm{i}}{b}$$

```
[In] int(((a + b*x)^2 + 1)/(a*1i + b*x*1i + 1)^2,x)
[Out] - x - (log(x + (a - 1i)/b)*2i)/b
```

3.203 $\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = \frac{(i+a)\log(x)}{i-a} - \frac{2\log(i-a-bx)}{1+ia}$$

```
[Out] (I+a)*ln(x)/(I-a)-2*ln(I-a-b*x)/(1+I*a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5203, 78}

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = \frac{(a+i)\log(x)}{-a+i} - \frac{2\log(-a-bx+i)}{1+ia}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x),x]
```

```
[Out] ((I + a)*Log[x])/(I - a) - (2*Log[I - a - b*x])/(1 + I*a)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1-ia-ibx}{x(1+ia+ibx)}\,dx \\
&= \int \left(\frac{-i-a}{(-i+a)x}+\frac{2ib}{(-i+a)(-i+a+bx)}\right)dx \\
&= \frac{(i+a)\log(x)}{i-a}-\frac{2\log(i-a-bx)}{1+ia}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx=\frac{-((i+a)\log(x))+2i\log(i-a-bx)}{-i+a}$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x),x]
[Out] (-((I + a)*Log[x]) + (2*I)*Log[I - a - b*x])/(-I + a)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

method	result
default	$\frac{(-a^2-1)\ln(x)}{(i-a)^2}-\frac{2i\ln(-bx-a+i)}{i-a}$
risch	$-\frac{i\ln(b^2x^2+2abx+a^2+1)}{i-a}+\frac{2\arctan(bx+a)}{i-a}+\frac{i\ln(x)}{i-a}+\frac{\ln(x)a}{i-a}$
parallelrisch	$-\frac{252\ln(bx+a-i)a^4-72\ln(bx+a-i)a^2-168i\ln(bx+a-i)a^3+18i\ln(bx+a-i)a+252i\ln(bx+a-i)a^5+112\ln(bx+a-i)x\,a^3b-1}{}$

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x,method=_RETURNVERBOSE)
[Out] (-a^2-1)/(I-a)^2*ln(x)-2*I/(I-a)*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = -\frac{(a+i)\log(x) - 2i\,\log\left(\frac{bx+a-i}{b}\right)}{a-i}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] -((a + I)*log(x) - 2*I*log((b*x + a - I)/b))/(a - I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(24) = 48.

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.41

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = -\frac{(a+i)\log\left(a^2 - \frac{a^2(a+i)}{a-i} + \frac{2ia(a+i)}{a-i} + x(ab+3ib) + 1 + \frac{a+i}{a-i}\right)}{a-i} + \frac{2i\log\left(a^2 + \frac{2ia^2}{a-i} + \frac{4a}{a-i} + x(ab+3ib) + 1 - \frac{2i}{a-i}\right)}{a-i}$$

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x,x)
```

```
[Out] -(a + I)*log(a**2 - a**2*(a + I)/(a - I) + 2*I*a*(a + I)/(a - I) + x*(a*b +
 3*I*b) + 1 + (a + I)/(a - I))/(a - I) + 2*I*log(a**2 + 2*I*a**2/(a - I) +
4*a/(a - I) + x*(a*b + 3*I*b) + 1 - 2*I/(a - I))/(a - I)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = -\frac{2\,(-i\,a-1)\log(i\,bx+i\,a+1)}{a^2-2i\,a-1} - \frac{(a^2+1)\log(x)}{a^2-2i\,a-1}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] -2*(-I*a - 1)*log(I*b*x + I*a + 1)/(a^2 - 2*I*a - 1) - (a^2 + 1)*log(x)/(a^
2 - 2*I*a - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = i\,b\left(\frac{(a+i)\log\left(\frac{i\,a}{i\,bx+i\,a+1}+\frac{1}{i\,bx+i\,a+1}-1\right)}{-i\,ab-b}-\frac{i\,\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b}\right)$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] I*b*((a + I)*log(I*a/(I*b*x + I*a + 1) + 1/(I*b*x + I*a + 1) - 1)/(-I*a*b -
 b) - I*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = -\frac{2\,\ln\left(a+b\,x-\mathrm{i}\right)}{1+a\,1\mathrm{i}}+\ln\left(x\right)\,\left(\frac{2}{1+a\,1\mathrm{i}}-1\right)$$

```
[In] int(((a + b*x)^2 + 1)/(x*(a*1i + b*x*1i + 1)^2),x)
```

```
[Out] log(x)*(2/(a*1i + 1) - 1) - (2*log(a + b*x - 1i))/(a*1i + 1)
```

3.204 $\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = -\frac{i+a}{(i-a)x} + \frac{2ib\log(x)}{(i-a)^2} - \frac{2ib\log(i-a-bx)}{(i-a)^2}$$

```
[Out] (-I-a)/(I-a)/x+2*I*b*ln(x)/(I-a)^2-2*I*b*ln(I-a-b*x)/(I-a)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{2ib\log(x)}{(-a+i)^2} - \frac{2ib\log(-a-bx+i)}{(-a+i)^2} - \frac{a+i}{(-a+i)x}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((I + a)/((I - a)*x)) + ((2*I)*b*Log[x])/(I - a)^2 - ((2*I)*b*Log[I - a -
b*x])/(I - a)^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1-ia-ibx}{x^2(1+ia+ibx)}\,dx \\
&= \int \left(\frac{-i-a}{(-i+a)x^2}+\frac{2ib}{(-i+a)^2x}-\frac{2ib^2}{(-i+a)^2(-i+a+bx)}\right)dx \\
&= -\frac{i+a}{(i-a)x}+\frac{2ib\log(x)}{(i-a)^2}-\frac{2ib\log(i-a-bx)}{(i-a)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

$$
\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{1+a^2+2ibx\log(x)-2ibx\log(i-a-bx)}{(-i+a)^2x}
$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^2),x]
[Out] (1 + a^2 + (2*I)*b*x*Log[x] - (2*I)*b*x*Log[I - a - b*x])/((-I + a)^2*x)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

method	result
default	$-\frac{-a^2-1}{x(i-a)^2}-\frac{2b(ia+1)\ln(x)}{(i-a)^3}+\frac{2b(ia+1)\ln(-bx-a+i)}{(i-a)^3}$
risch	$\frac{i}{(a-i)x}+\frac{a}{(a-i)x}+\frac{b\ln(4a^4b^2x^2+8a^5bx+4a^6+8a^2b^2x^2+16a^3bx+12a^4+4b^2x^2+8abx+12a^2+4)}{ia^2+2a-i}-\frac{2ib\arctan\left(\frac{(2a^2b+2b}{-2}\right.}{ia^2+2a-}$
parallelrisch	$\frac{-2\ln(x)xb^2-2ixa^3b^2-2ixab^2-2i\ln(x)x^2b^3+2i\ln(bx+a-i)x^2b^3-6\ln(bx+a-i)xa^2b^2+4\ln(x)x^2ab^3-4\ln(bx+a-i)x^2ab^3+}{}$

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(-a^2-1)/x/(I-a)^2-2*b*(1+I*a)/(I-a)^3*ln(x)+2*b*(1+I*a)/(I-a)^3*ln(I-a-b*
x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{2i\,bx\log(x) - 2i\,bx\log\left(\frac{bx+a-i}{b}\right) + a^2 + 1}{(a^2 - 2i\,a - 1)x}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*I*b*x*log(x) - 2*I*b*x*log((b*x + a - I)/b) + a^2 + 1)/((a^2 - 2*I*a - 1
)*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(41) = 82.

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.55

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{2ib\log\left(-\frac{2a^3b}{(a-i)^2} + \frac{6ia^2b}{(a-i)^2} + 2ab + \frac{6ab}{(a-i)^2} + 4b^2x - 2ib - \frac{2ib}{(a-i)^2}\right)}{(a-i)^2} - \frac{2ib\log\left(\frac{2a^3b}{(a-i)^2} - \frac{6ia^2b}{(a-i)^2} + 2ab - \frac{6ab}{(a-i)^2} + 4b^2x - 2ib + \frac{2ib}{(a-i)^2}\right)}{(a-i)^2} - \frac{-a-i}{x(a-i)}$$

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**2,x)
```

```
[Out] 2*I*b*log(-2*a**3*b/(a - I)**2 + 6*I*a**2*b/(a - I)**2 + 2*a*b + 6*a*b/(a -
 I)**2 + 4*b**2*x - 2*I*b - 2*I*b/(a - I)**2)/(a - I)**2 - 2*I*b*log(2*a**3
*b/(a - I)**2 - 6*I*a**2*b/(a - I)**2 + 2*a*b - 6*a*b/(a - I)**2 + 4*b**2*x
 - 2*I*b + 2*I*b/(a - I)**2)/(a - I)**2 - (-a - I)/(x*(a - I))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(41) = 82.

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = -\frac{2(a-i)b\log(i\,bx + i\,a + 1)}{-i\,a^3 - 3\,a^2 + 3i\,a + 1} + \frac{2(a-i)b\log(x)}{-i\,a^3 - 3\,a^2 + 3i\,a + 1} + \frac{a^3 + (a^2+1)bx - i\,a^2 + a - i}{(a^2 - 2i\,a - 1)bx^2 + (a^3 - 3i\,a^2 - 3\,a + i)x}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*(a - I)*b*log(I*b*x + I*a + 1)/(-I*a^3 - 3*a^2 + 3*I*a + 1) + 2*(a - I)*
b*log(x)/(-I*a^3 - 3*a^2 + 3*I*a + 1) + (a^3 + (a^2 + 1)*b*x - I*a^2 + a -
I)/((a^2 - 2*I*a - 1)*b*x^2 + (a^3 - 3*I*a^2 - 3*a + I)*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{2\,b^2\log\left(-\frac{i\,a}{i\,bx+i\,a+1}-\frac{1}{i\,bx+i\,a+1}+1\right)}{-i\,a^2b-2\,ab+i\,b}-\frac{ab+i\,b}{(a-i)^2\left(\frac{i\,a}{i\,bx+i\,a+1}+\frac{1}{i\,bx+i\,a+1}-1\right)}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] 2*b^2*log(-I*a/(I*b*x + I*a + 1) - 1/(I*b*x + I*a + 1) + 1)/(-I*a^2*b - 2*a
*b + I*b) - (a*b + I*b)/((a - I)^2*(I*a/(I*b*x + I*a + 1) + 1/(I*b*x + I*a
+ 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{-1+a\,1\mathrm{i}}{x\,(1+a\,1\mathrm{i})}-\frac{4\,b\,\mathrm{atan}\left(\frac{a^2\,1\mathrm{i}+2\,a-\mathrm{i}}{(a-\mathrm{i})^2}+\frac{x\,(2\,a^4\,b^2+4\,a^2\,b^2+2\,b^2)}{(a-\mathrm{i})^2\,(-1\mathrm{i}\,b\,a^3+b\,a^2-1\mathrm{i}\,b\,a+b)}\right)}{(a-\mathrm{i})^2}$$

```
[In] int(((a + b*x)^2 + 1)/(x^2*(a*1i + b*x*1i + 1)^2),x)
```

```
[Out] (a*1i - 1)/(x*(a*1i + 1)) - (4*b*atan((2*a + a^2*1i - 1i)/(a - 1i)^2 + (x*(
2*b^2 + 4*a^2*b^2 + 2*a^4*b^2))/((a - 1i)^2*(b - a*b*1i + a^2*b - a^3*b*1i)
)))/(a - 1i)^2
```

3.205 $\int \frac{e^{-2i \arctan(a+bx)}}{x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{e^{-2i \arctan(a+bx)}}{x^3} \, dx = \frac{-i-a}{2(i-a)x^2} - \frac{2ib}{(i-a)^2 x} - \frac{2b^2 \log(x)}{(1+ia)^3} + \frac{2b^2 \log(i-a-bx)}{(1+ia)^3}$$

```
[Out] 1/2*(-I-a)/(I-a)/x^2-2*I*b/(I-a)^2/x-2*b^2*ln(x)/(1+I*a)^3+2*b^2*ln(I-a-b*x
)/(1+I*a)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5203, 78}

$$\int \frac{e^{-2i \arctan(a+bx)}}{x^3} \, dx = -\frac{2b^2 \log(x)}{(1+ia)^3} + \frac{2b^2 \log(-a-bx+i)}{(1+ia)^3} - \frac{2ib}{(-a+i)^2 x} - \frac{a+i}{2(-a+i)x^2}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] -1/2*(I + a)/((I - a)*x^2) - ((2*I)*b)/((I - a)^2*x) - (2*b^2*Log[x])/(1 +
I*a)^3 + (2*b^2*Log[I - a - b*x])/(1 + I*a)^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-ia-ibx}{x^3(1+ia+ibx)}\,dx\\ &= \int \left(\frac{-i-a}{(-i+a)x^3}+\frac{2ib}{(-i+a)^2x^2}-\frac{2ib^2}{(-i+a)^3x}+\frac{2ib^3}{(-i+a)^3(-i+a+bx)}\right)dx\\ &= -\frac{i+a}{2(i-a)x^2}-\frac{2ib}{(i-a)^2x}-\frac{2b^2\log(x)}{(1+ia)^3}+\frac{2b^2\log(i-a-bx)}{(1+ia)^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = \frac{(-i+a)\left(1+a^2-4ibx\right)-4ib^2x^2\log(x)+4ib^2x^2\log(i-a-bx)}{2(-i+a)^3x^2}$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^3),x]
[Out] ((-I + a)*(1 + a^2 - (4*I)*b*x) - (4*I)*b^2*x^2*Log[x] + (4*I)*b^2*x^2*Log[
I - a - b*x])/(2*(-I + a)^3*x^2)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2b^2(ia+1)\ln(x)}{(i-a)^4}-\frac{2b\left(ia^2+2a-i\right)}{(i-a)^4x}-\frac{-a^4+2ia^3+2ia+1}{2(i-a)^4x^2}+\frac{2b^2(ia+1)\ln(-bx-a+i)}{(i-a)^4}$
risch	$\frac{-\frac{2ibx}{a^2-2ia-1}+\frac{i+a}{2a-2i}}{x^2}+\frac{2b^2\ln\left(\left(2a^4b+4a^2b+2b\right)x\right)}{ia^3+3a^2-3ia-1}-\frac{b^2\ln\left(4a^8b^2x^2+8a^9bx+4a^{10}+16a^6b^2x^2+32a^7bx+20a^8+24a^4b^2x^2+48\right.}{ia^3+3a^2-3}$
parallelrisch	$\frac{-b-42a^4b+8ia^9b-48ia^7b-8iab+4\ln(bx+a-i)x^2b^3+3ix\,b^2+24\ln(x)x^3a^5b^4+28\ln(x)x^2a^6b^3-80\ln(x)x^3a^3b^4-140\ln(x)x^2a^4}{}$

```
[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x,method=_RETURNVERBOSE)
[Out] -2*b^2*(1+I*a)/(I-a)^4*ln(x)-2*b*(I*a^2-I+2*a)/(I-a)^4/x-1/2/(I-a)^4*(-a^4+
2*I*a^3+2*I*a+1)/x^2+2*b^2*(1+I*a)/(I-a)^4*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = \frac{-4i\,b^2x^2\log(x) + 4i\,b^2x^2\log\left(\frac{bx+a-i}{b}\right) + a^3 - 4\,(i\,a+1)bx - i\,a^2 + a - i}{2\,(a^3 - 3i\,a^2 - 3\,a + i)x^2}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="fricas")
[Out] 1/2*(-4*I*b^2*x^2*log(x) + 4*I*b^2*x^2*log((b*x + a - I)/b) + a^3 - 4*(I*a
+ 1)*b*x - I*a^2 + a - I)/((a^3 - 3*I*a^2 - 3*a + I)*x^2)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(61) = 122.

Time = 0.44 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.72

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = -\frac{2ib^2\log\left(-\frac{2a^4b^2}{(a-i)^3} + \frac{8ia^3b^2}{(a-i)^3} + \frac{12a^2b^2}{(a-i)^3} + 2ab^2 - \frac{8iab^2}{(a-i)^3} + 4b^3x - 2ib^2 - \frac{2b^2}{(a-i)^3}\right)}{(a-i)^3} + \frac{2ib^2\log\left(\frac{2a^4b^2}{(a-i)^3} - \frac{8ia^3b^2}{(a-i)^3} - \frac{12a^2b^2}{(a-i)^3} + 2ab^2 + \frac{8iab^2}{(a-i)^3} + 4b^3x - 2ib^2 + \frac{2b^2}{(a-i)^3}\right)}{(a-i)^3} - \frac{-a^2 + 4ibx - 1}{x^2\cdot(2a^2 - 4ia - 2)}$$

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**3,x)
[Out] -2*I*b**2*log(-2*a**4*b**2/(a - I)**3 + 8*I*a**3*b**2/(a - I)**3 + 12*a**2*
b**2/(a - I)**3 + 2*a*b**2 - 8*I*a*b**2/(a - I)**3 + 4*b**3*x - 2*I*b**2 -
2*b**2/(a - I)**3)/(a - I)**3 + 2*I*b**2*log(2*a**4*b**2/(a - I)**3 - 8*I*a
**3*b**2/(a - I)**3 - 12*a**2*b**2/(a - I)**3 + 2*a*b**2 + 8*I*a*b**2/(a -
I)**3 + 4*b**3*x - 2*I*b**2 + 2*b**2/(a - I)**3)/(a - I)**3 - (-a**2 + 4*I*
b*x - 1)/(x**2*(2*a**2 - 4*I*a - 2))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(61) = 122$.

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.93

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = -\frac{2\,(-i\,a-1)b^2\log\left(i\,bx+i\,a+1\right)}{a^4-4i\,a^3-6\,a^2+4i\,a+1} - \frac{2\,(i\,a+1)b^2\log\left(x\right)}{a^4-4i\,a^3-6\,a^2+4i\,a+1} + \frac{4\,(-i\,a-1)b^2x^2+a^4-2i\,a^3+(a^3-5i\,a^2-7\,a+3i)bx-2i\,a-1}{2\,((a^3-3i\,a^2-3\,a+i)bx^3+(a^4-4i\,a^3-6\,a^2+4i\,a+1)x^2)}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*(-I*a - 1)*b^2*log(I*b*x + I*a + 1)/(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)
- 2*(I*a + 1)*b^2*log(x)/(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1) + 1/2*(4*(-I*a
 - 1)*b^2*x^2 + a^4 - 2*I*a^3 + (a^3 - 5*I*a^2 - 7*a + 3*I)*b*x - 2*I*a - 1
)/((a^3 - 3*I*a^2 - 3*a + I)*b*x^3 + (a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)*x^
2)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(61) = 122$.

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.71

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = \frac{2\,b^3\log\left(-\frac{i\,a}{i\,bx+i\,a+1}-\frac{1}{i\,bx+i\,a+1}+1\right)}{i\,a^3b+3\,a^2b-3i\,ab-b} + \frac{\frac{i\,ab^2-5\,b^2}{-i\,a-1}+\frac{2i\,(ab^3+3i\,b^3)}{(i\,bx+i\,a+1)b}}{2\,(a-i)^2\left(\frac{i\,a}{i\,bx+i\,a+1}+\frac{1}{i\,bx+i\,a+1}-1\right)^2}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(-I*a/(I*b*x + I*a + 1) - 1/(I*b*x + I*a + 1) + 1)/(I*a^3*b + 3*a^
2*b - 3*I*a*b - b) + 1/2*((I*a*b^2 - 5*b^2)/(-I*a - 1) + 2*I*(a*b^3 + 3*I*b
^3)/((I*b*x + I*a + 1)*b))/((a - I)^2*(I*a/(I*b*x + I*a + 1) + 1/(I*b*x + I
*a + 1) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.88

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = \frac{\frac{a+1\mathrm{i}}{2\,(a-\mathrm{i})}-\frac{b\,x\,2\mathrm{i}}{(a-\mathrm{i})^2}}{x^2} - \frac{b^2\,\mathrm{atanh}\left(\frac{-a^3+a^2\,3\mathrm{i}+3\,a-\mathrm{i}}{(a-\mathrm{i})^3}-\frac{x\,\left(2\,a^8\,b^2+8\,a^6\,b^2+12\,a^4\,b^2+8\,a^2\,b^2+2\,b^2\right)}{(a-\mathrm{i})^3\,\left(b\,a^6+2\mathrm{i}\,b\,a^5+b\,a^4+4\mathrm{i}\,b\,a^3-b\,a^2+2\mathrm{i}\,b\,a-b\right)}\right)\,4\mathrm{i}}{(a-\mathrm{i})^3}$$

```
[In] int(((a + b*x)^2 + 1)/(x^3*(a*1i + b*x*1i + 1)^2),x)
```

```
[Out] ((a + 1i)/(2*(a - 1i)) - (b*x*2i)/(a - 1i)^2)/x^2 - (b^2*atanh((3*a + a^2*3
i - a^3 - 1i)/(a - 1i)^3 - (x*(2*b^2 + 8*a^2*b^2 + 12*a^4*b^2 + 8*a^6*b^2 +
 2*a^8*b^2))/((a - 1i)^3*(a*b*2i - b - a^2*b + a^3*b*4i + a^4*b + a^5*b*2i
+ a^6*b)))*4i)/(a - 1i)^3
```

3.206 $\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{-i-a}{3(i-a)x^3} - \frac{ib}{(i-a)^2x^2} + \frac{2b^2}{(1+ia)^3x} + \frac{2ib^3\log(x)}{(i-a)^4} - \frac{2ib^3\log(i-a-bx)}{(i-a)^4}$$

```
[Out] 1/3*(-I-a)/(I-a)/x^3-I*b/(I-a)^2/x^2+2*b^2/(1+I*a)^3/x+2*I*b^3*ln(x)/(I-a)^
4-2*I*b^3*ln(I-a-b*x)/(I-a)^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5203, 78}

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{2ib^3\log(x)}{(-a+i)^4} - \frac{2ib^3\log(-a-bx+i)}{(-a+i)^4} + \frac{2b^2}{(1+ia)^3x} - \frac{ib}{(-a+i)^2x^2} - \frac{a+i}{3(-a+i)x^3}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -1/3*(I + a)/((I - a)*x^3) - (I*b)/((I - a)^2*x^2) + (2*b^2)/((1 + I*a)^3*x
) + ((2*I)*b^3*Log[x])/(I - a)^4 - ((2*I)*b^3*Log[I - a - b*x])/(I - a)^4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-ia-ibx}{x^4(1+ia+ibx)}\,dx\\ &= \int \left(\frac{-i-a}{(-i+a)x^4}+\frac{2ib}{(-i+a)^2x^3}-\frac{2ib^2}{(-i+a)^3x^2}+\frac{2ib^3}{(-i+a)^4x}-\frac{2ib^4}{(-i+a)^4(-i+a+bx)}\right)dx\\ &= -\frac{i+a}{3(i-a)x^3}-\frac{ib}{(i-a)^2x^2}+\frac{2b^2}{(1+ia)^3x}+\frac{2ib^3\log(x)}{(i-a)^4}-\frac{2ib^3\log(i-a-bx)}{(i-a)^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{(-i+a)\left(-i+a-ia^2+a^3-3bx-3iabx+6ib^2x^2\right)+6ib^3x^3\log(x)-6ib^3x^3\log(i-a-bx)}{3(-i+a)^4x^3}$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^4),x]
[Out] ((-I + a)*(-I + a - I*a^2 + a^3 - 3*b*x - (3*I)*a*b*x + (6*I)*b^2*x^2) + (6
*I)*b^3*x^3*Log[x] - (6*I)*b^3*x^3*Log[I - a - b*x])/(3*(-I + a)^4*x^3)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.44

method	result
default	$-\frac{2b^3(ia+1)\ln(x)}{(i-a)^5}-\frac{2b^2(ia^2+2a-i)}{(i-a)^5x}-\frac{a^5-3ia^4-2a^3-2ia^2-3a+i}{3(i-a)^5x^3}+\frac{b(ia^3+3a^2-3ia-1)}{(i-a)^5x^2}+\frac{2b^3(ia+1)\ln(-bx-a+i)}{(i-a)^5}$
risch	$\frac{\frac{2ib^2x^2}{(a^2-2ia-1)(a-i)}-\frac{ibx}{a^2-2ia-1}+\frac{i+a}{3a-3i}}{x^3}-\frac{2b^3\ln((-2a^6b-6a^4b-6a^2b-2b)x)}{ia^4+4a^3-6ia^2-4a+i}+\frac{b^3\ln(4a^{12}b^2x^2+8a^{13}bx+4a^{14}+24a^{10}b^2x^2+\cdots}{}$
parallelrisch	$-\frac{168a^3b^3x^2-9ia^{10}b+75ia^8b-42ia^6b+6\ln(bx+a-i)x^3b^4+3ix^2b^3+42a^5b-24ab^3x^2+81a^2b^2x-35a^9b+90a^7b-90ia^4b+35ia^2\cdots}{}$

[In] int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x,method=_RETURNVERBOSE)

[Out] -2*b^3*(1+I*a)/(I-a)^5*ln(x)-2*b^2*(I*a^2-I+2*a)/(I-a)^5/x-1/3/(I-a)^5*(-3*I*a^4+a^5-2*I*a^2-2*a^3+I-3*a)/x^3+b*(I*a^3-3*I*a+3*a^2-1)/(I-a)^5/x^2+2*b^3*(1+I*a)/(I-a)^5*ln(I-a-b*x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{6i\,b^3x^3\log(x)-6i\,b^3x^3\log\left(\frac{bx+a-i}{b}\right)-6(-i\,a-1)b^2x^2+a^4-2i\,a^3-3(i\,a^2+2\,a-i)bx-2i\,a-1}{3(a^4-4i\,a^3-6\,a^2+4i\,a+1)x^3}$$

[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="fricas")

[Out] 1/3*(6*I*b^3*x^3*log(x) - 6*I*b^3*x^3*log((b*x + a - I)/b) - 6*(-I*a - 1)*b^2*x^2 + a^4 - 2*I*a^3 - 3*(I*a^2 + 2*a - I)*b*x - 2*I*a - 1)/((a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)*x^3)

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(75) = 150.

Time = 0.57 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.75

$$\begin{aligned}\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx &= \frac{2ib^3\log\left(-\frac{2a^5b^3}{(a-i)^4}+\frac{10ia^4b^3}{(a-i)^4}+\frac{20a^3b^3}{(a-i)^4}-\frac{20ia^2b^3}{(a-i)^4}+2ab^3-\frac{10ab^3}{(a-i)^4}+4b^4x-2ib^3+\frac{2ib^3}{(a-i)^4}\right)}{(a-i)^4}\\ &\quad-\frac{2ib^3\log\left(\frac{2a^5b^3}{(a-i)^4}-\frac{10ia^4b^3}{(a-i)^4}-\frac{20a^3b^3}{(a-i)^4}+\frac{20ia^2b^3}{(a-i)^4}+2ab^3+\frac{10ab^3}{(a-i)^4}+4b^4x-2ib^3-\frac{2ib^3}{(a-i)^4}\right)}{(a-i)^4}\\ &\quad-\frac{-a^3+ia^2-a-6ib^2x^2+x(3iab+3b)+i}{x^3\cdot(3a^3-9ia^2-9a+3i)}\end{aligned}$$

```
[In] integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**4,x)
```

```
[Out] 2*I*b**3*log(-2*a**5*b**3/(a - I)**4 + 10*I*a**4*b**3/(a - I)**4 + 20*a**3*
b**3/(a - I)**4 - 20*I*a**2*b**3/(a - I)**4 + 2*a*b**3 - 10*a*b**3/(a - I)*
*4 + 4*b**4*x - 2*I*b**3 + 2*I*b**3/(a - I)**4)/(a - I)**4 - 2*I*b**3*log(2
*a**5*b**3/(a - I)**4 - 10*I*a**4*b**3/(a - I)**4 - 20*a**3*b**3/(a - I)**4
 + 20*I*a**2*b**3/(a - I)**4 + 2*a*b**3 + 10*a*b**3/(a - I)**4 + 4*b**4*x -
 2*I*b**3 - 2*I*b**3/(a - I)**4)/(a - I)**4 - (-a**3 + I*a**2 - a - 6*I*b**
2*x**2 + x*(3*I*a*b + 3*b) + I)/(x**3*(3*a**3 - 9*I*a**2 - 9*a + 3*I))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(72) = 144.

Time = 0.19 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.10

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{2\,(a-i)b^3\log\left(i\,bx+i\,a+1\right)}{i\,a^5+5\,a^4-10i\,a^3-10\,a^2+5i\,a+1} - \frac{2\,(a-i)b^3\log\left(x\right)}{i\,a^5+5\,a^4-10i\,a^3-10\,a^2+5i\,a+1} + \frac{6\,(a-i)b^3x^3-i\,a^5+3\,(a^2-2i\,a-1)b^2x^2-3\,a^4+2i\,a^3-(i\,a^4+5\,a^3-9i\,a^2-7\,a+2i)bx-2\,a^2+3i}{3\,((-i\,a^4-4\,a^3+6i\,a^2+4\,a-i)bx^4+(-i\,a^5-5\,a^4+10i\,a^3+10\,a^2-5i\,a-1)x^3)}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] 2*(a - I)*b^3*log(I*b*x + I*a + 1)/(I*a^5 + 5*a^4 - 10*I*a^3 - 10*a^2 + 5*I
*a + 1) - 2*(a - I)*b^3*log(x)/(I*a^5 + 5*a^4 - 10*I*a^3 - 10*a^2 + 5*I*a +
 1) + 1/3*(6*(a - I)*b^3*x^3 - I*a^5 + 3*(a^2 - 2*I*a - 1)*b^2*x^2 - 3*a^4
+ 2*I*a^3 - (I*a^4 + 5*a^3 - 9*I*a^2 - 7*a + 2*I)*b*x - 2*a^2 + 3*I*a + 1)/
((-I*a^4 - 4*a^3 + 6*I*a^2 + 4*a - I)*b*x^4 + (-I*a^5 - 5*a^4 + 10*I*a^3 +
10*a^2 - 5*I*a - 1)*x^3)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(72) = 144.

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{2\,b^4\log\left(-\frac{i\,a}{i\,bx+i\,a+1}-\frac{1}{i\,bx+i\,a+1}+1\right)}{-i\,a^4b-4\,a^3b+6i\,a^2b+4\,ab-i\,b} + \frac{\frac{-i\,ab^3+10\,b^3}{i\,a+1}+\frac{3i\,(ab^4+8i\,b^4)}{(i\,bx+i\,a+1)b}+\frac{3\,(a^2b^5+4i\,ab^5+5\,b^5)}{(i\,bx+i\,a+1)^2b^2}}{3\,(a-i)^3\left(\frac{i\,a}{i\,bx+i\,a+1}+\frac{1}{i\,bx+i\,a+1}-1\right)^3}$$

```
[In] integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="giac")
```

```
[Out] 2*b^4*log(-I*a/(I*b*x + I*a + 1) - 1/(I*b*x + I*a + 1) + 1)/(-I*a^4*b - 4*a
^3*b + 6*I*a^2*b + 4*a*b - I*b) + 1/3*((-I*a*b^3 + 10*b^3)/(I*a + 1) + 3*I*
(a*b^4 + 8*I*b^4)/((I*b*x + I*a + 1)*b) + 3*(a^2*b^5 + 4*I*a*b^5 + 5*b^5)/(
(I*b*x + I*a + 1)^2*b^2))/((a - I)^3*(I*a/(I*b*x + I*a + 1) + 1/(I*b*x + I*
a + 1) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.91

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{\frac{a+1\mathrm{i}}{3\,(a-\mathrm{i})}+\frac{b^2\,x^2\,2\mathrm{i}}{(a-\mathrm{i})^3}-\frac{b\,x\,1\mathrm{i}}{(a-\mathrm{i})^2}}{x^3} - \frac{4\,b^3\,\mathrm{atan}\left(\frac{\left(a^4-a^3\,4\mathrm{i}-6\,a^2+a\,4\mathrm{i}+1\right)\,1\mathrm{i}}{(a-\mathrm{i})^4}+\frac{x\,\left(2\,a^{12}\,b^2+12\,a^{10}\,b^2+30\,a^8\,b^2+40\,a^6\,b^2+30\,a^4\,b^2+12\,a^2\,b^2+2\,b^2\right)}{(a-\mathrm{i})^4\,\left(-1\mathrm{i}\,b\,a^9+3\,b\,a^8+8\,b\,a^6+6\mathrm{i}\,b\,a^5+6\,b\,a^4+8\mathrm{i}\,b\,a^3+3\mathrm{i}\,b\,a-b\right)}\right)}{(a-\mathrm{i})^4}$$

```
[In] int(((a + b*x)^2 + 1)/(x^4*(a*1i + b*x*1i + 1)^2),x)
```

```
[Out] ((a + 1i)/(3*(a - 1i)) + (b^2*x^2*2i)/(a - 1i)^3 - (b*x*1i)/(a - 1i)^2)/x^3
 - (4*b^3*atan(((a*4i - 6*a^2 - a^3*4i + a^4 + 1)*1i)/(a - 1i)^4 + (x*(2*b^
2 + 12*a^2*b^2 + 30*a^4*b^2 + 40*a^6*b^2 + 30*a^8*b^2 + 12*a^10*b^2 + 2*a^1
2*b^2))/((a - 1i)^4*(a*b*3i - b + a^3*b*8i + 6*a^4*b + a^5*b*6i + 8*a^6*b +
 3*a^8*b - a^9*b*1i))))/(a - 1i)^4
```

3.207 $\int e^{-3i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 324

$$\begin{aligned}
&\int e^{-3i\arctan(a+bx)}x^4\,dx \\
&= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}}+\frac{3(19i-68a-88ia^2+48a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} \\
&\quad-\frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3}-\frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} \\
&\quad+\frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(163+458ia-422a^2-112ia^3-2(61i-118a-52ia^2)\,bx)}{40b^5} \\
&\quad-\frac{3(19+68ia-88a^2-48ia^3+8a^4)\operatorname{arcsinh}(a+bx)}{8b^5}
\end{aligned}$$

```
[Out] -3/8*(19+68*I*a-88*a^2-48*I*a^3+8*a^4)*arcsinh(b*x+a)/b^5+2*I*x^4*(1-I*a-I*
b*x)^(3/2)/b/(1+I*a+I*b*x)^(1/2)-3/20*(17*I-16*a)*x^2*(1-I*a-I*b*x)^(3/2)*(
1+I*a+I*b*x)^(1/2)/b^3-11/5*x^3*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2
+1/40*I*(1-I*a-I*b*x)^(3/2)*(163+458*I*a-422*a^2-112*I*a^3-2*(61*I-118*a-52
*I*a^2)*b*x)*(1+I*a+I*b*x)^(1/2)/b^5+3/8*(19*I-68*a-88*I*a^2+48*a^3+8*I*a^4
)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5203, 99, 158, 152, 52, 55, 633, 221}

$$\int e^{-3i\arctan(a+bx)}x^4\,dx$$
$$=\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}(-112ia^3-2(-52ia^2-118a+61i)\,bx-422a^2+458ia+163)}{40b^5}$$
$$-\frac{3(8a^4-48ia^3-88a^2+68ia+19)\,\text{arcsinh}(a+bx)}{8b^5}$$
$$+\frac{3(8ia^4+48a^3-88ia^2-68a+19i)\,\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{8b^5}$$
$$-\frac{3(-16a+17i)x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{20b^3}$$
$$-\frac{11x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2}+\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}$$

```
[In] Int[x^4/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*x^4*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + (3*(19*I -
68*a - (88*I)*a^2 + 48*a^3 + (8*I)*a^4)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a
+ I*b*x])/(8*b^5) - (3*(17*I - 16*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I
*a + I*b*x])/(20*b^3) - (11*x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*
x])/(5*b^2) + ((I/40)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(163 +
(458*I)*a - 422*a^2 - (112*I)*a^3 - 2*(61*I - 118*a - (52*I)*a^2)*b*x))/b^5
 - (3*(19 + (68*I)*a - 88*a^2 - (48*I)*a^3 + 8*a^4)*ArcSinh[a + b*x])/(8*b^
5)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
```

```
I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{(2i)\int \frac{x^3\sqrt{1-ia-ibx}\left(4(1-ia)-\frac{11ibx}{2}\right)}{\sqrt{1+ia+ibx}}\,dx}{b}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{(2i)\int \frac{x^2\sqrt{1-ia-ibx}\left(\frac{33}{2}(1+ia)(i+a)b+\frac{3}{2}(17+16ia)b^2x\right)}{\sqrt{1+ia+ibx}}\,dx}{5b^3}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} - \frac{i\int \frac{x\sqrt{1-ia-ibx}\left(3(17i-16a)(i-a)(1-ia)b^2-\frac{3}{2}\left(118a-i\left(61-52a^2\right)\right)b^3x\right)}{\sqrt{1+ia+ibx}}\,dx}{10b^5}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}\left(163+458ia-422a^2-112ia^3-2\left(61i-118a-52ia^2\right)bx\right)}{40b^5} - \frac{\left(3\left(19+68ia-88a^2-48ia^3+8a^4\right)\right)\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{8b^4}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3\left(19i-68a-88ia^2+48a^3+8ia^4\right)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}\left(163+458ia-422a^2-112ia^3-2\left(61i-118a-52ia^2\right)bx\right)}{40b^5} - \frac{\left(3\left(19+68ia-88a^2-48ia^3+8a^4\right)\right)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{8b^4}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3(19i-68a-88ia^2+48a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(163+458ia-422a^2-112ia^3-2(61i-118a-52ia^2)bx)}{40b^5} - \frac{(3(19+68ia-88a^2-48ia^3+8a^4))\int\frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{8b^4}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3(19i-68a-88ia^2+48a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(163+458ia-422a^2-112ia^3-2(61i-118a-52ia^2)bx)}{40b^5} - \frac{(3(19+68ia-88a^2-48ia^3+8a^4))\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx,x,2ab+2b^2x\right)}{16b^6}$$

$$= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3(19i-68a-88ia^2+48a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} - \frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} - \frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} + \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(163+458ia-422a^2-112ia^3-2(61i-118a-52ia^2)bx)}{40b^5} - \frac{3(19+68ia-88a^2-48ia^3+8a^4)\,\mathrm{arcsinh}(a+bx)}{8b^5}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.92

$$\int e^{-3i\arctan(a+bx)}x^4\,dx$$

$$=\frac{\frac{448i+8ia^6+285bx+224ib^2x^2+95b^3x^3-56ib^4x^4-30b^5x^5+8ib^6x^6+a^5(410+8ibx)+2a^4(-638i+265bx)+a^3\left(-905-2004ibx+60b^2x^2\right)-a^2\left(836i+2\right.}{\sqrt{1+a^2+2abx+b^2x^2}}}{}$$

```
[In] Integrate[x^4/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((448*I + (8*I)*a^6 + 285*b*x + (224*I)*b^2*x^2 + 95*b^3*x^3 - (56*I)*b^4*x
^4 - 30*b^5*x^5 + (8*I)*b^6*x^6 + a^5*(410 + (8*I)*b*x) + 2*a^4*(-638*I + 2
65*b*x) + a^3*(-905 - (2004*I)*b*x + 60*b^2*x^2) - a^2*(836*I + 2635*b*x +
(356*I)*b^2*x^2 + 20*b^3*x^3) + a*(-1315 + (1468*I)*b*x - 515*b^2*x^2 + (11
6*I)*b^3*x^3 + 10*b^4*x^4 + (8*I)*b^5*x^5))/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^
2] + (30*(-1)^(1/4)*(19*I - 68*a - (88*I)*a^2 + 48*a^3 + (8*I)*a^4)*Sqrt[b]
*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/Sqrt
[(-I)*b])/(40*b^5)
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.37

method	result
risch	$\frac{i\left(8x^4b^4-8a\,b^3x^3+30ib^3x^3+8a^2b^2x^2-70ia\,b^2x^2-8a^3bx+130ia^2bx+8a^4-250ia^3-64b^2x^2+252abx-125bxi-804a^2+835ia+288\right)\sqrt{b^2}}{40b^5}$
default	Expression too large to display

```
[In] int(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*I*(8*x^4*b^4+30*I*b^3*x^3-8*a*b^3*x^3-70*I*a*b^2*x^2+8*a^2*b^2*x^2+130
*I*a^2*b*x-8*a^3*b*x-250*I*a^3+8*a^4-64*b^2*x^2-125*I*b*x+252*a*b*x+835*I*a
-804*a^2+288)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^5-1/8/b^4*(57*ln((b^2*x+a*b)/
(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+24*a^4*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-264*a^2*ln((b^2*x+
a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-144*I*a^3*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+204*I*a*ln
((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+I*(128*
a^3+32*I*a^4-128*a-192*I*a^2+32*I)/b^2/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b
*(x-(I-a)/b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.81

$$\int e^{-3i\arctan(a+bx)}x^4\,dx = \frac{62i\,a^6 + 2687\,a^5 - 11575i\,a^4 - 20350\,a^3 + (62i\,a^5 + 2625\,a^4 - 8950i\,a^3 - 11400\,a^2 + 6340i\,a + 1280)bx + 1}{}$$

```
[In] integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/320*(62*I*a^6 + 2687*a^5 - 11575*I*a^4 - 20350*a^3 + (62*I*a^5 + 2625*a^4
 - 8950*I*a^3 - 11400*a^2 + 6340*I*a + 1280)*b*x + 17740*I*a^2 + 120*(8*a^5
 - 56*I*a^4 - 136*a^3 + (8*a^4 - 48*I*a^3 - 88*a^2 + 68*I*a + 19)*b*x + 156
*I*a^2 + 87*a - 19*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 8
*(-8*I*b^5*x^5 + 22*b^4*x^4 - 2*(16*a - 17*I)*b^3*x^3 - 8*I*a^5 + (52*a^2 -
 118*I*a - 61)*b^2*x^2 - 418*a^4 + 1694*I*a^3 - (112*a^3 - 422*I*a^2 - 458*
a + 163*I)*b*x + 2599*a^2 - 1763*I*a - 448)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1) + 7620*a - 1280*I)/(b^6*x + (a - I)*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^4\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1368 vs. 2(230) = 460.

Time = 0.30 (sec) , antiderivative size = 1368, normalized size of antiderivative = 4.22

$$\int e^{-3i\arctan(a+bx)}x^4\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^4/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 -
2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^3/(b
```

```
^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*(b^2*x^2 +
2*a*b*x + a^2 + 1)^(3/2)*a^3/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) + 6*I*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*a^4/(I*b^6*x + I*a*b^5 + b^5) - 6*I*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^(3/2)*a^2/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I
*a*b^5 - b^5) - 12*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(2*I*b^6*x + 2
*I*a*b^5 + 2*b^5) + 24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/(I*b^6*x + I*a
*b^5 + b^5) - 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^7*x^2 + 2*a*b^6*x
+ a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) - 12*(b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2)*a/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) - 36*I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*a^2/(I*b^6*x + I*a*b^5 + b^5) + I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*I*(b^2*
x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) - 24*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a/(I*b^6*x + I*a*b^5 + b^5) - 3*a^4*arcsinh(b*x
+ a)/b^5 + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^6*x + I*a*b^5 + b^5)
- I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^4 - 3*sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 4*I*b*x + 4*I*a + 3)*a^2*x/b^4 + 18*I*a^3*arcsinh(b*x + a)/b^5 + I
*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/b^5 + 6*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*a^3/b^5 - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^
3/b^5 - 3/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^4 - 3/2*I*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*a*x/b^4 + 6*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x
+ 4*I*a + 3)*a*x/b^4 + 3*a^2*arcsin(I*b*x + I*a + 2)/b^5 + 36*a^2*arcsinh(b
*x + a)/b^5 + 1/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2)/b^5 + 13/4*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^5 - 39/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*a^2/b^5 + 12*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^2/b
^5 - 9/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 + 3*sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 4*I*b*x + 4*I*a + 3)*x/b^4 - 6*I*a*arcsin(I*b*x + I*a + 2)/b^5 -
63/2*I*a*arcsinh(b*x + a)/b^5 - 2*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^5
 - 153/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^5 + 15*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 4*I*b*x + 4*I*a + 3)*a/b^5 - 3*arcsin(I*b*x + I*a + 2)/b^5 - 81/
8*arcsinh(b*x + a)/b^5 + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5 - 6*I*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.03

$$\int e^{-3i\arctan(a+bx)}x^4\,dx =$$

$$-\frac{1}{40}\sqrt{(bx+a)^2+1}\left(\left(2\left(x\left(-\frac{4i\,x}{b}-\frac{-4i\,ab^{17}-15\,b^{17}}{b^{19}}\right)-\frac{4i\,a^2b^{16}+35\,ab^{16}-32i\,b^{16}}{b^{19}}\right)x-\frac{-8i\,a^3b^{15}}{\quad}\right.\right.$$

$$+\frac{(8\,a^4-48i\,a^3-88\,a^2+68i\,a+19)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+\right.\right.}{}$$

```
[In] integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/40*sqrt((b*x + a)^2 + 1)*((2*(x*(-4*I*x/b - (-4*I*a*b^17 - 15*b^17)/b^19
) - (4*I*a^2*b^16 + 35*a*b^16 - 32*I*b^16)/b^19)*x - (-8*I*a^3*b^15 - 130*a
^2*b^15 + 252*I*a*b^15 + 125*b^15)/b^19)*x - (8*I*a^4*b^14 + 250*a^3*b^14 -
 804*I*a^2*b^14 - 835*a*b^14 + 288*I*b^14)/b^19) + 1/8*(8*a^4 - 48*I*a^3 -
88*a^2 + 68*I*a + 19)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*
b + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x +
 a)^2 + 1))*a^2*abs(b) - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a
^2*b + 4*(-I*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b)
 - sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^4\,dx = \int \frac{x^4\,\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

```
[In] int((x^4*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
```

```
[Out] int((x^4*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

3.208 $\int e^{-3i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 249

$$\begin{aligned}\int e^{-3i\arctan(a+bx)}x^3\,dx &= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}}+\frac{3(17+44ia-36a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4}\\ &\quad-\frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2}\\ &\quad-\frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(29i-54a-22ia^2+2(11+10ia)bx)}{8b^4}\\ &\quad+\frac{3(17i-44a-36ia^2+8a^3)\operatorname{arcsinh}(a+bx)}{8b^4}\end{aligned}$$

```
[Out] 3/8*(17*I-44*a-36*I*a^2+8*a^3)*arcsinh(b*x+a)/b^4+2*I*x^3*(1-I*a-I*b*x)^(3/
2)/b/(1+I*a+I*b*x)^(1/2)-9/4*x^2*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^
2-1/8*I*(1-I*a-I*b*x)^(3/2)*(29*I-54*a-22*I*a^2+2*(11+10*I*a)*b*x)*(1+I*a+I
*b*x)^(1/2)/b^4+3/8*(17+44*I*a-36*a^2-8*I*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I
*b*x)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{5203, 99, 158, 152, 52, 55, 633, 221\}$

$$\int e^{-3i\arctan(a+bx)}x^3\,dx$$
$$= -\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}(-22ia^2+2(11+10ia)bx-54a+29i)}{8b^4}$$
$$+ \frac{3(8a^3-36ia^2-44a+17i)\operatorname{arcsinh}(a+bx)}{8b^4}$$
$$+ \frac{3(-8ia^3-36a^2+44ia+17)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{8b^4}$$
$$- \frac{9x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b^2} + \frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}$$

```
[In] Int[x^3/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*x^3*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + (3*(17 + (4
4*I)*a - 36*a^2 - (8*I)*a^3)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(
8*b^4) - (9*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b^2) - ((
I/8)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(29*I - 54*a - (22*I)*a^
2 + 2*(11 + (10*I)*a)*b*x))/b^4 + (3*(17*I - 44*a - (36*I)*a^2 + 8*a^3)*Arc
Sinh[a + b*x])/(8*b^4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^3(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{(2i)\int \frac{x^2\sqrt{1-ia-ibx}\left(3(1-ia)-\frac{9ibx}{2}\right)}{\sqrt{1+ia+ibx}}\,dx}{b}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{i\int \frac{x\sqrt{1-ia-ibx}\left(9i(1+a^2)b+\frac{3}{2}(11+10ia)b^2x\right)}{\sqrt{1+ia+ibx}}dx}{2b^3}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(29i-54a-22ia^2+2(11+10ia)bx)}{8b^4} + \frac{(3(17i-44a-36ia^2+8a^3))\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}dx}{8b^3}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3(17+44ia-36a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(29i-54a-22ia^2+2(11+10ia)bx)}{8b^4} + \frac{(3(17i-44a-36ia^2+8a^3))\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}dx}{8b^3}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3(17+44ia-36a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(29i-54a-22ia^2+2(11+10ia)bx)}{8b^4} + \frac{(3(17i-44a-36ia^2+8a^3))\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}dx}{8b^3}$$

$$= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3(17+44ia-36a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(29i-54a-22ia^2+2(11+10ia)bx)}{8b^4} + \frac{(3(17i-44a-36ia^2+8a^3))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}dx, x, 2ab+2b^2x\right)}{16b^5}$$

$$
\begin{aligned}
&= \frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3(17+44ia-36a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} \\
&\quad - \frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} \\
&\quad - \frac{i(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}(29i-54a-22ia^2+2(11+10ia)bx)}{8b^4} \\
&\quad + \frac{3(17i-44a-36ia^2+8a^3)\operatorname{arcsinh}(a+bx)}{8b^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$
\begin{aligned}
&\int e^{-3i\arctan(a+bx)}x^3\,dx \\
&= \frac{80-2ia^5-51ibx+40b^2x^2-17ib^3x^3-8b^4x^4+2ib^5x^5+a^4(-76-2ibx)-5a^3(-31i+20bx)+a^2(4+\cdots}{8b^4\sqrt{1+a^2+2abx+b^2x^2}} \\
&\quad + \frac{3\sqrt[4]{-1}(17i-44a-36ia^2+8a^3)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4b^{9/2}}
\end{aligned}
$$

```
[In] Integrate[x^3/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] (80 - (2*I)*a^5 - (51*I)*b*x + 40*b^2*x^2 - (17*I)*b^3*x^3 - 8*b^4*x^4 + (2
*I)*b^5*x^5 + a^4*(-76 - (2*I)*b*x) - 5*a^3*(-31*I + 20*b*x) + a^2*(4 + (26
5*I)*b*x - 12*b^2*x^2) + a*(157*I + 212*b*x + (53*I)*b^2*x^2 + 4*b^3*x^3 +
(2*I)*b^4*x^4))/(8*b^4*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + (3*(-1)^(1/4)*(
17*I - 44*a - (36*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]
*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{i\left(-2b^3x^3+2a\,b^2x^2-8ix^2b^2-2a^2bx+20iabx+2a^3-44ia^2+19bx-93a+48i\right)\sqrt{b^2x^2+2abx+a^2+1}}{8b^4}+\frac{\frac{51i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a\cdots}\right)}{\sqrt{b^2}}}{\cdots}$
default	$\frac{i\left(\frac{\left(2b^2x+2ab\right)\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{8b^2}+\frac{3\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+\cdots}\right)}{8b^2\sqrt{b^2}}\cdots\right)}{16b^2}\cdots\right)}{b^3}$

```
[In] int(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*(-2*b^3*x^3-8*I*b^2*x^2+2*a*b^2*x^2+20*I*a*b*x-2*a^2*b*x-44*I*a^2+2*
a^3+19*b*x+48*I-93*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^4+1/8/b^3*(51*I*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-132*a*ln((
b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+24*a^3*ln
((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-108*I*a
^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+I*
(96*a^2+32*I*a^3-32-96*I*a)/b^2/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-
a)/b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.87

$$\int e^{-3i\arctan(a+bx)}x^3\,dx$$
$$= \frac{-15i\,a^5 - 495\,a^4 + 1664i\,a^3 + (-15i\,a^4 - 480\,a^3 + 1184i\,a^2 + 968\,a - 256i)bx + 2152\,a^2 - 24\,(8\,a^4 - 44i\,a}{}$$

```
[In] integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(-15*I*a^5 - 495*a^4 + 1664*I*a^3 + (-15*I*a^4 - 480*a^3 + 1184*I*a^2
+ 968*a - 256*I)*b*x + 2152*a^2 - 24*(8*a^4 - 44*I*a^3 + (8*a^3 - 36*I*a^2
- 44*a + 17*I)*b*x - 80*a^2 + 61*I*a + 17)*log(-b*x - a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)) - 8*(-2*I*b^4*x^4 + 6*b^3*x^3 - (10*a - 11*I)*b^2*x^2 + 2
*I*a^4 + 78*a^3 + (22*a^2 - 54*I*a - 29)*b*x - 233*I*a^2 - 237*a + 80*I)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 1224*I*a - 256)/(b^5*x + (a - I)*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^3\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 979 vs. $2(175) = 350$.

Time = 0.28 (sec) , antiderivative size = 979, normalized size of antiderivative = 3.93

$$
\begin{aligned}
\int e^{-3i\arctan(a+bx)}x^3\,dx = &-\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a^3}{b^6x^2+2\,ab^5x+a^2b^4-2i\,b^5x-2i\,ab^4-b^4} \\
&-\frac{3\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a^2}{b^6x^2+2\,ab^5x+a^2b^4-2i\,b^5x-2i\,ab^4-b^4} \\
&-\frac{3\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a^2}{2i\,b^5x+2i\,ab^4+2\,b^4}-\frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}a^3}{i\,b^5x+i\,ab^4+b^4} \\
&+\frac{3i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a}{b^6x^2+2\,ab^5x+a^2b^4-2i\,b^5x-2i\,ab^4-b^4} \\
&+\frac{6i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a}{2i\,b^5x+2i\,ab^4+2\,b^4}-\frac{18\,\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{i\,b^5x+i\,ab^4+b^4} \\
&+\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{b^6x^2+2\,ab^5x+a^2b^4-2i\,b^5x-2i\,ab^4-b^4} \\
&+\frac{3\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{2i\,b^5x+2i\,ab^4+2\,b^4}+\frac{18i\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{i\,b^5x+i\,ab^4+b^4} \\
&+\frac{3\,a^3\operatorname{arsinh}(bx+a)}{b^4}+\frac{6\,\sqrt{b^2x^2+2\,abx+a^2+1}}{i\,b^5x+i\,ab^4+b^4} \\
&+\frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}x}{4\,b^3} \\
&+\frac{3\,\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}ax}{2\,b^3} \\
&-\frac{27i\,a^2\operatorname{arsinh}(bx+a)}{2\,b^4}-\frac{3i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a}{4\,b^4} \\
&-\frac{9\,\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{2\,b^4} \\
&+\frac{3\,\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}a^2}{2\,b^4} \\
&+\frac{3i\,\sqrt{b^2x^2+2\,abx+a^2+1}x}{8\,b^3} \\
&-\frac{3i\,\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}x}{2\,b^3} \\
&-\frac{3\,a\arcsin(i\,bx+i\,a+2)}{2\,b^4}-\frac{18\,a\operatorname{arsinh}(bx+a)}{b^4} \\
&-\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{b^4}+\frac{75i\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{8\,b^4} \\
&-\frac{9i\,\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}a}{2\,b^4} \\
&+\frac{3i\,\arcsin(i\,bx+i\,a+2)}{2\,b^4}+\frac{63i\,\operatorname{arsinh}(bx+a)}{8\,b^4} \\
&+\frac{9\,\sqrt{b^2x^2+2\,abx+a^2+1}}{2\,b^4} \\
&-\frac{3\,\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}}{b^4}
\end{aligned}
$$

```
[In] integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^3/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 -
 2*I*b^5*x - 2*I*a*b^4 - b^4) - 3*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(
b^6*x^2 + 2*a*b^5*x + a^2*b^4 - 2*I*b^5*x - 2*I*a*b^4 - b^4) - 3*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)*a^2/(2*I*b^5*x + 2*I*a*b^4 + 2*b^4) - 6*I*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a^3/(I*b^5*x + I*a*b^4 + b^4) + 3*I*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^(3/2)*a/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 - 2*I*b^5*x - 2*I*
a*b^4 - b^4) + 6*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(2*I*b^5*x + 2*I*a
*b^4 + 2*b^4) - 18*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/(I*b^5*x + I*a*b^4
 + b^4) + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^6*x^2 + 2*a*b^5*x + a^2*b^
4 - 2*I*b^5*x - 2*I*a*b^4 - b^4) + 3*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2
*I*b^5*x + 2*I*a*b^4 + 2*b^4) + 18*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/(I
*b^5*x + I*a*b^4 + b^4) + 3*a^3*arcsinh(b*x + a)/b^4 + 6*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)/(I*b^5*x + I*a*b^4 + b^4) + 1/4*I*(b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2)*x/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)
*a*x/b^3 - 27/2*I*a^2*arcsinh(b*x + a)/b^4 - 3/4*I*(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(3/2)*a/b^4 - 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^4 + 3/2*sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^2/b^4 + 3/8*I*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*x/b^3 - 3/2*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*
b*x + 4*I*a + 3)*x/b^3 - 3/2*a*arcsin(I*b*x + I*a + 2)/b^4 - 18*a*arcsinh(b
*x + a)/b^4 - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^4 + 75/8*I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a/b^4 - 9/2*I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x
 + 4*I*a + 3)*a/b^4 + 3/2*I*arcsin(I*b*x + I*a + 2)/b^4 + 63/8*I*arcsinh(b*
x + a)/b^4 + 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4 - 3*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.14

$$\int e^{-3i\arctan(a+bx)}x^3\,dx =$$

$$-\frac{1}{8}\sqrt{(bx+a)^2+1}\left(\left(2\,x\left(-\frac{i\,x}{b}-\frac{-i\,ab^{11}-4\,b^{11}}{b^{13}}\right)-\frac{2i\,a^2b^{10}+20\,ab^{10}-19i\,b^{10}}{b^{13}}\right)x-\frac{-2i\,a^3b^9-44\,a^2\cdots}{\cdots}\right.$$

$$-\frac{(8\,a^3-36i\,a^2-44\,a+17i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|\cdots\right.}{\cdots}$$

```
[In] integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt((b*x + a)^2 + 1)*((2*x*(-I*x/b - (-I*a*b^11 - 4*b^11)/b^13) - (2*
I*a^2*b^10 + 20*a*b^10 - 19*I*b^10)/b^13)*x - (-2*I*a^3*b^9 - 44*a^2*b^9 +
```

```
93*I*a*b^9 + 48*b^9)/b^13) - 1/8*(8*a^3 - 36*I*a^2 - 44*a + 17*I)*log(3*(x*
abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((b*x + a)^
2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) - 2*I*(x
*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b + 4*(-I*x*abs(b) + I*sqrt(
(b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b
))/(b^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^3\,dx = \int \frac{x^3\,\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

```
[In] int((x^3*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
[Out] int((x^3*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

3.209 $\int e^{-3i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\int e^{-3i\arctan(a+bx)}x^2\,dx = \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} - \frac{(11i-18a-6ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(11i-18a-6ia^2)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} + \frac{(11+18ia-6a^2)\,\mathrm{arcsinh}(a+bx)}{2b^3}$$

```
[Out] 1/2*(11+18*I*a-6*a^2)*arcsinh(b*x+a)/b^3+I*(I-a)^2*(1-I*a-I*b*x)^(5/2)/b^3/
(1+I*a+I*b*x)^(1/2)-1/6*(11*I-18*a-6*I*a^2)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*
x)^(1/2)/b^3-1/3*I*(1-I*a-I*b*x)^(5/2)*(1+I*a+I*b*x)^(1/2)/b^3-1/2*(11*I-18
*a-6*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used

$= \{5203, 91, 81, 52, 55, 633, 221\}$

$$\int e^{-3i\arctan(a+bx)}x^2\,dx = \frac{(-6a^2+18ia+11)\operatorname{arcsinh}(a+bx)}{2b^3} - \frac{(-6ia^2-18a+11i)\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{6b^3} - \frac{(-6ia^2-18a+11i)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^3} - \frac{i\sqrt{ia+ibx+1}(-ia-ibx+1)^{5/2}}{3b^3} + \frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}$$

```
[In] Int[x^2/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] (I*(I - a)^2*(1 - I*a - I*b*x)^(5/2))/(b^3*Sqrt[1 + I*a + I*b*x]) - ((11*I
- 18*a - (6*I)*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^3) -
((11*I - 18*a - (6*I)*a^2)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(
6*b^3) - ((I/3)*(1 - I*a - I*b*x)^(5/2)*Sqrt[1 + I*a + I*b*x])/b^3 + ((11 +
 (18*I)*a - 6*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx \\
&= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{i\int \frac{(1-ia-ibx)^{3/2}(-((i-a)(3+2ia)b)-b^2x)}{\sqrt{1+ia+ibx}}\,dx}{b^3} \\
&= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} \\
&\quad + \frac{(11+18ia-6a^2)\int \frac{(1-ia-ibx)^{3/2}}{\sqrt{1+ia+ibx}}\,dx}{3b^2} \\
&= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{(18a-i(11-6a^2))(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} + \frac{(11+18ia-6a^2)\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{2b^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{(18a-i(11-6a^2))\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} \\
&\quad + \frac{(18a-i(11-6a^2))(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} + \frac{(11+18ia-6a^2)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2b^2} \\
&= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{(18a-i(11-6a^2))\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} \\
&\quad + \frac{(18a-i(11-6a^2))(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} + \frac{(11+18ia-6a^2)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{2b^2} \\
&= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{(18a-i(11-6a^2))\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} \\
&\quad + \frac{(18a-i(11-6a^2))(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} \\
&\quad + \frac{(11+18ia-6a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{4b^4} \\
&= \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} + \frac{(18a-i(11-6a^2))\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} \\
&\quad + \frac{(18a-i(11-6a^2))(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} \\
&\quad - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} + \frac{(11+18ia-6a^2)\,\mathrm{arcsinh}(a+bx)}{2b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

$$
\begin{aligned}
&\int e^{-3i\arctan(a+bx)}x^2\,dx \\
&= \frac{2ia^4+a^3(51+2ibx)+a^2(-50i+69bx)+a(51-106ibx+9b^2x^2+2ib^3x^3)+i(-52+33ibx-26b^2x^2+9}{6b^3\sqrt{1+a^2+2abx+b^2x^2}} \\
&\quad + \frac{\sqrt[4]{-1}(11+18ia-6a^2)\sqrt{-ib}\,\mathrm{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{7/2}}
\end{aligned}
$$

```
[In] Integrate[x^2/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*a^4 + a^3*(51 + (2*I)*b*x) + a^2*(-50*I + 69*b*x) + a*(51 - (106*I)*
b*x + 9*b^2*x^2 + (2*I)*b^3*x^3) + I*(-52 + (33*I)*b*x - 26*b^2*x^2 + (9*I)
*b^3*x^3 + 2*b^4*x^4))/(6*b^3*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + ((-1)^(1
/4)*(11 + (18*I)*a - 6*a^2)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(7/2)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.17

method	result
risch	$\frac{i\left(2b^{2}x^{2}-2abx+9bxi+2a^{2}-27ia-28\right)\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{6b^{3}}-\frac{-\frac{11\ln\left(\frac{b^{2}x+ab}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+2abx+a^{2}+1}\right)}{\sqrt{b^{2}}}+\frac{6a^{2}\ln\left(\frac{b^{2}x+ab}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+2abx+a\cdots\right)}{\sqrt{b^{2}}}}{\cdots}$
default	$\frac{i\left(\frac{\left(\left(x-\frac{i-a}{b}\right)^{2}b^{2}+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}}}{3}+ib\left(\frac{\left(2\left(x-\frac{i-a}{b}\right)b^{2}+2ib\right)\sqrt{\left(x-\frac{i-a}{b}\right)^{2}b^{2}+2ib\left(x-\frac{i-a}{b}\right)}}{4b^{2}}+\frac{\ln\left(\frac{ib+\left(x-\frac{i-a}{b}\right)b^{2}}{\sqrt{b^{2}}}+\sqrt{\left(x-\frac{i-a}{b}\right)^{2}b^{2}+2ib\left(x-\cdots\right.}\right)}{2\sqrt{b^{2}}}\cdots\right.\right.}{b^{3}}$

```
[In] int(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*(2*b^2*x^2+9*I*b*x-2*a*b*x-27*I*a+2*a^2-28)*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)/b^3-1/2/b^2*(-11*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/(b^2)^(1/2)+6*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(
1/2))/(b^2)^(1/2)-18*I*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2)+I*(16*a+8*I*a^2-8*I)/b^2/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2
+2*I*b*(x-(I-a)/b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.76

$$\int e^{-3i\arctan(a+bx)}x^{2}\,dx$$
$$=\frac{7i\,a^{4}+166\,a^{3}+(7i\,a^{3}+159\,a^{2}-249i\,a-96)bx-408i\,a^{2}+12\,(6\,a^{3}+(6\,a^{2}-18i\,a-11)bx-24i\,a^{2}-2\cdots}{\cdots}$$

```
[In] integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(7*I*a^4 + 166*a^3 + (7*I*a^3 + 159*a^2 - 249*I*a - 96)*b*x - 408*I*a^
2 + 12*(6*a^3 + (6*a^2 - 18*I*a - 11)*b*x - 24*I*a^2 - 29*a + 11*I)*log(-b*
x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 4*(-2*I*b^3*x^3 + 7*b^2*x^2 -
2*I*a^3 - (16*a - 19*I)*b*x - 53*a^2 + 103*I*a + 52)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1) - 345*a + 96*I)/(b^4*x + (a - I)*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^2\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(155) = 310$.

Time = 0.28 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.72

$$\begin{aligned}\int e^{-3i\arctan(a+bx)}x^2\,dx &= \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a^2}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3} \\ &+ \frac{2\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3} \\ &+ \frac{2\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{2i\,b^4x+2i\,ab^3+2\,b^3} + \frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{i\,b^4x+i\,ab^3+b^3} \\ &- \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3} \\ &- \frac{2i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{2i\,b^4x+2i\,ab^3+2\,b^3} + \frac{12\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{i\,b^4x+i\,ab^3+b^3} \\ &- \frac{3\,a^2\operatorname{arsinh}(bx+a)}{b^3} - \frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{i\,b^4x+i\,ab^3+b^3} \\ &- \frac{\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}x}{2\,b^2} + \frac{9i\,a\operatorname{arsinh}(bx+a)}{b^3} \\ &+ \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,b^3} + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{b^3} \\ &- \frac{\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}a}{2\,b^3} + \frac{\arcsin(i\,bx+i\,a+2)}{2\,b^3} \\ &+ \frac{6\,\operatorname{arsinh}(bx+a)}{b^3} - \frac{3i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{b^3} \\ &+ \frac{i\,\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}}{b^3}\end{aligned}$$

```
[In] integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
[Out] I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 -
2*I*b^4*x - 2*I*a*b^3 - b^3) + 2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^5
```

```
*x^2 + 2*a*b^4*x + a^2*b^3 - 2*I*b^4*x - 2*I*a*b^3 - b^3) + 2*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^(3/2)*a/(2*I*b^4*x + 2*I*a*b^3 + 2*b^3) + 6*I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a^2/(I*b^4*x + I*a*b^3 + b^3) - I*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 - 2*I*b^4*x - 2*I*a*b^3 - b
^3) - 2*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2*I*b^4*x + 2*I*a*b^3 + 2*b^
3) + 12*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/(I*b^4*x + I*a*b^3 + b^3) - 3*a
^2*arcsinh(b*x + a)/b^3 - 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^4*x +
I*a*b^3 + b^3) - 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*x
/b^2 + 9*I*a*arcsinh(b*x + a)/b^3 + 1/3*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2)/b^3 + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3 - 1/2*sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a/b^3 + 1/2*arcsin(I*b*x + I*a + 2)/b^3
+ 6*arcsinh(b*x + a)/b^3 - 3*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3 + I*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.05

$$\int e^{-3i\arctan(a+bx)}x^2\,dx$$
$$= -\frac{1}{6}\sqrt{(bx+a)^2+1}\left(x\left(-\frac{2i\,x}{b}+\frac{2i\,ab^6+9\,b^6}{b^8}\right)+\frac{-2i\,a^2b^5-27\,ab^5+28i\,b^5}{b^8}\right)$$
$$+\frac{(6\,a^2-18i\,a-11)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b\right.\right.}{}$$

```
[In] integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt((b*x + a)^2 + 1)*(x*(-2*I*x/b + (2*I*a*b^6 + 9*b^6)/b^8) + (-2*I*
a^2*b^5 - 27*a*b^5 + 28*I*b^5)/b^8) + 1/6*(6*a^2 - 18*I*a - 11)*log(3*(x*ab
s(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((b*x + a)^2
+ 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) - 2*I*(x*a
bs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b + 4*(-I*x*abs(b) + I*sqrt((b
*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))
/(b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^2\,dx = \int \frac{x^2\,\left({\left(a+b\,x\right)}^2+1\right)^{3/2}}{{\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)}^3}\,dx$$

```
[In] int((x^2*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
[Out] int((x^2*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

3.210 $\int e^{-3i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 163

$$\int e^{-3i\arctan(a+bx)}x\,dx=-\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}}-\frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}-\frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2}-\frac{3(3i-2a)\text{arcsinh}(a+bx)}{2b^2}$$

```
[Out] -3/2*(3*I-2*a)*arcsinh(b*x+a)/b^2-(1+I*a)*(1-I*a-I*b*x)^(5/2)/b^2/(1+I*a+I*
b*x)^(1/2)-1/2*(3+2*I*a)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2-3/2*(3
+2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5203, 79, 52, 55, 633, 221}

$$\int e^{-3i\arctan(a+bx)}x\,dx=-\frac{3(-2a+3i)\text{arcsinh}(a+bx)}{2b^2}-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}-\frac{(3+2ia)\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}-\frac{3(3+2ia)\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{2b^2}$$

```
[In] Int[x/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] -(((1 + I*a)*(1 - I*a - I*b*x)^(5/2))/(b^2*Sqrt[1 + I*a + I*b*x])) - (3*(3
+ (2*I)*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/(2*b^2) - ((3 + (2*
```

```
I)*a)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*b^2) - (3*(3*I - 2*
a)*ArcSinh[a + b*x])/(2*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{(3i-2a)\int \frac{(1-ia-ibx)^{3/2}}{\sqrt{1+ia+ibx}}\,dx}{b}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3(3i-2a))\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx}{2b}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3(3i-2a))\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2b}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3(3i-2a))\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx}{2b}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3(3i-2a))\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{4b^3}$$

$$= -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} - \frac{3(3i-2a)\text{arcsinh}(a+bx)}{2b^2}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int e^{-3i\arctan(a+bx)}x\,dx$$
$$=\frac{i(14i-a^3+9bx+6ib^2x^2+b^3x^3+a^2(14i-bx)+a(-1+20ibx+b^2x^2))}{2b^2\sqrt{1+a^2+2abx+b^2x^2}}$$
$$+\frac{3\sqrt[4]{-1}(-3i+2a)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{5/2}}$$

```
[In] Integrate[x/E^((3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((I/2)*(14*I - a^3 + 9*b*x + (6*I)*b^2*x^2 + b^3*x^3 + a^2*(14*I - b*x) + a
*(-1 + (20*I)*b*x + b^2*x^2)))/(b^2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + (3
*(-1)^(1/4)*(-3*I + 2*a)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I
)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{i(-bx+a-6i)\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}+\frac{-\frac{9i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}+\frac{6a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}+\frac{i(8ia+8)\sqrt{\left(x-\frac{i-a}{b}\right.}}{b^2\left(x\right.}}{2b}$
default	$\frac{i\left(-\frac{i\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b\left(x-\frac{i-a}{b}\right)^2}+3ib\left(\frac{\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}}}{3}+ib\left(\frac{\left(2\left(x-\frac{i-a}{b}\right)b^2+2ib\right)\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}}{4b^2}+\frac{\ln\left(\right.}{}\right.\right.\right.}{b^3}$

```
[In] int(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(-b*x+a-6*I)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^2+1/2/b*(-9*I*ln((b^2*x
+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+6*a*ln((b^2*x+
a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+I*(8+8*I*a)/b^2
/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int e^{-3i\arctan(a+bx)}x\,dx$$
$$= \frac{-3i\,a^3 + (-3i\,a^2 - 44\,a + 32i)bx - 47\,a^2 - 12\,((2\,a - 3i)bx + 2\,a^2 - 5i\,a - 3)\log\left(-bx - a + \sqrt{b^2x^2 + 2}\right.}{8\,(b^3x + (a-i)b}$$

```
[In] integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(-3*I*a^3 + (-3*I*a^2 - 44*a + 32*I)*b*x - 47*a^2 - 12*((2*a - 3*I)*b*x
 + 2*a^2 - 5*I*a - 3)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 4
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*b^2*x^2 + I*a^2 + 5*b*x + 15*a - 14*
I) + 76*I*a + 32)/(b^3*x + (a - I)*b^2)
```

Sympy [F]

$$\int e^{-3i\arctan(a+bx)}x\,dx$$
$$= i\left(\int \frac{x\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx + \int \frac{a^2x\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx + \int \frac{b^2x^3\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx + \int \frac{2abx^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx\right)$$

```
[In] integrate(x/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] I*(Integral(x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I
*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x
 + I), x) + Integral(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*
a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**
2*x**2 - 3*b*x + I), x) + Integral(b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b*
*3*x**3 - 3*I*b**2*x**2 - 3*b*x + I), x) + Integral(2*a*b*x**2*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*
I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x + I), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(113) = 226.

Time = 0.26 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.80

$$\int e^{-3i\arctan(a+bx)}x\,dx = -\frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{b^4x^2+2\,ab^3x+a^2b^2-2i\,b^3x-2i\,ab^2-b^2} - \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{b^4x^2+2\,ab^3x+a^2b^2-2i\,b^3x-2i\,ab^2-b^2} - \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{2i\,b^3x+2i\,ab^2+2\,b^2} - \frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{i\,b^3x+i\,ab^2+b^2} + \frac{3\,a\,\mathrm{arsinh}\,(bx+a)}{b^2} - \frac{6\,\sqrt{b^2x^2+2\,abx+a^2+1}}{i\,b^3x+i\,ab^2+b^2} - \frac{9i\,\mathrm{arsinh}\,(bx+a)}{2\,b^2} - \frac{3\,\sqrt{b^2x^2+2\,abx+a^2+1}}{2\,b^2}$$

```
[In] integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^4*x^2 + 2*a*b^3*x + a^2*b^2 - 2
*I*b^3*x - 2*I*a*b^2 - b^2) - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^4*x^2
+ 2*a*b^3*x + a^2*b^2 - 2*I*b^3*x - 2*I*a*b^2 - b^2) - (b^2*x^2 + 2*a*b*x +
 a^2 + 1)^(3/2)/(2*I*b^3*x + 2*I*a*b^2 + 2*b^2) - 6*I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*a/(I*b^3*x + I*a*b^2 + b^2) + 3*a*arcsinh(b*x + a)/b^2 - 6*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^3*x + I*a*b^2 + b^2) - 9/2*I*arcsinh(b*
x + a)/b^2 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.29

$$\int e^{-3i\arctan(a+bx)}x\,dx = -\frac{1}{2}\sqrt{(bx+a)^2+1}\left(-\frac{i\,x}{b}-\frac{-i\,ab^2-6\,b^2}{b^4}\right) - \frac{(2\,a-3i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b|-\sqrt{(bx+\right.\right.}{}$$

```
[In] integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt((b*x + a)^2 + 1)*(-I*x/b - (-I*a*b^2 - 6*b^2)/b^4) - 1/2*(2*a - 3
*I)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sq
```

```
rt((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*ab
s(b) - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b + 4*(-I*x*abs
(b) + I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^
2 + 1))*abs(b))/(b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x\,dx = \int \frac{x\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

```
[In] int((x*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
[Out] int((x*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

3.211 $\int e^{-3i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 94

$$\int e^{-3i\arctan(a+bx)}\,dx = \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}}+\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}-\frac{3\text{arcsinh}(a+bx)}{b}$$

```
[Out] -3*arcsinh(b*x+a)/b+2*I*(1-I*a-I*b*x)^(3/2)/b/(1+I*a+I*b*x)^(1/2)+3*I*(1-I*
a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5201, 49, 52, 55, 633, 221}

$$\int e^{-3i\arctan(a+bx)}\,dx = -\frac{3\text{arcsinh}(a+bx)}{b}+\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}+\frac{3i\sqrt{ia+ibx+1}\sqrt{-ia-ibx+1}}{b}$$

```
[In] Int[E^((-3*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((2*I)*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) + ((3*I)*Sqrt[1 -
 I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b - (3*ArcSinh[a + b*x])/b
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-ia-ibx)^{3/2}}{(1+ia+ibx)^{3/2}}\,dx \\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} - 3\int \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\,dx \\
&= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - 3\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx
\end{aligned}$$

$$= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - 3\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx$$

$$= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b^2}$$

$$= \frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} + \frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{3\text{arcsinh}(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int e^{-3i\arctan(a+bx)}\,dx = \frac{\sqrt{1+(a+bx)^2}\left(i+\frac{4}{-i+a+bx}\right)}{b} - \frac{3\text{arcsinh}(a+bx)}{b}$$

```
[In] Integrate[E^((-3*I)*ArcTan[a + b*x]),x]
[Out] (Sqrt[1 + (a + b*x)^2]*(I + 4/(-I + a + b*x)))/b - (3*ArcSinh[a + b*x])/b
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

method	result
risch	$\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{b} - \frac{3\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}} + \frac{4\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}}{b^2\left(x-\frac{i-a}{b}\right)}$
default	$\frac{i\Big(\frac{i\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b\left(x-\frac{i-a}{b}\right)^3}-2ib\Big(-\frac{i\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b\left(x-\frac{i-a}{b}\right)^2}+3ib\Big(\frac{\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}}}{3}+ib\Big(\frac{\left(2\left(x-\frac{i-a}{b}\right)b^2+\cdots}{\cdots}}{b^3}$

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+4/b^2/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(
x-(I-a)/b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int e^{-3i\arctan(a+bx)}\,dx$$
$$=\frac{(i\,a+8)bx+i\,a^2+6\,(bx+a-i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-2\,\sqrt{b^2x^2+2\,abx+a^2+1}(-}{2\,(b^2x+(a-i)b)}$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((I*a + 8)*b*x + I*a^2 + 6*(b*x + a - I)*log(-b*x - a + sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*b*x - I*a - 5
) + 9*a - 8*I)/(b^2*x + (a - I)*b)
```

Sympy [F]

$$\int e^{-3i\arctan(a+bx)}\,dx$$
$$=i\left(\int\frac{\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx+\int\frac{a^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx+\int\frac{b^2x^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx+\int\frac{2abx\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx\right)$$

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

```
[Out] I*(Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a
**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x +
 I), x) + Integral(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2
*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x*
*2 - 3*b*x + I), x) + Integral(b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x
**3 - 3*I*b**2*x**2 - 3*b*x + I), x) + Integral(2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x
 - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x + I), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int e^{-3i\arctan(a+bx)}\,dx = \frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{b^3x^2+2\,ab^2x+a^2b-2i\,b^2x-2i\,ab-b} - \frac{3\,\operatorname{arsinh}\left(bx+a\right)}{b} + \frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{i\,b^2x+i\,ab+b}$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^3*x^2 + 2*a*b^2*x + a^2*b - 2*I*b^
2*x - 2*I*a*b - b) - 3*arcsinh(b*x + a)/b + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(I*b^2*x + I*a*b + b)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(66) = 132.

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.91

$$\int e^{-3i\arctan(a+bx)}\,dx = \frac{\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b|-\sqrt{(bx+a)^2+1}\right)a^2 \cdots\right.}{\cdots} + \frac{i\sqrt{(bx+a)^2+1}}{b}$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((
b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b)
 - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b + 4*(-I*x*abs(b)
+ I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 +
1))*abs(b))/abs(b) + I*sqrt((b*x + a)^2 + 1)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}\,dx = \int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(a*1i + b*x*1i + 1)^3,x)
[Out] int(((a + b*x)^2 + 1)^(3/2)/(a*1i + b*x*1i + 1)^3, x)
```

3.212 $\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} + i\text{arcsinh}(a+bx) - \frac{2(i+a)^{3/2}\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}}$$

```
[Out] I*arcsinh(b*x+a)-2*(I+a)^(3/2)*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a
)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)^(3/2)+4*(1-I*a-I*b*x)^(1/2)/(1+I*a)/(1+I
*a+I*b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5203, 100, 163, 55, 633, 221, 95, 214}

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = i\text{arcsinh}(a+bx) - \frac{2(a+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}} + \frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x),x]
```

```
[Out] (4*Sqrt[1 - I*a - I*b*x])/((1 + I*a)*Sqrt[1 + I*a + I*b*x]) + I*ArcSinh[a +
 b*x] - (2*(I + a)^(3/2)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[
I - a]*Sqrt[1 - I*a - I*b*x])])/(I - a)^(3/2)
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1-ia-ibx)^{3/2}}{x(1+ia+ibx)^{3/2}}\,dx$$

$$= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} + \frac{2\int \frac{-\frac{1}{2}i(i+a)^2b-\frac{1}{2}(1+ia)b^2x}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i-a)b}$$

$$= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} - \frac{(i+a)^2\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{1+ia} + (ib)\int \frac{1}{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx$$

$$= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} - \frac{(2(i+a)^2)\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{1+ia} + (ib)\int \frac{1}{\sqrt{(1-ia)(1+ia)+2abx+b^2x^2}}\,dx$$

$$= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} - \frac{2(i+a)^{3/2}\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}} + \frac{i\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)}{2b}$$

$$= \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} + i\text{arcsinh}(a+bx) - \frac{2(i+a)^{3/2}\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.41

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = \frac{2(-1)^{3/4}\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{\sqrt{b}} + \frac{2\left(-\frac{2\sqrt{1+a^2+2abx+b^2x^2}}{-i+a+bx}+\frac{\sqrt{-1+ia}(i+a)\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}}\right)}{-i+a}$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*(-1)^(3/4)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a +
b*x)])/Sqrt[(-I)*b]])/Sqrt[b] + (2*((-2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/
(-I + a + b*x) + (Sqrt[-1 + I*a]*(I + a)*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*
(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 - I*a]))/(
-I + a)
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1066 vs. 2(104) = 208.

Time = 0.81 (sec) , antiderivative size = 1067, normalized size of antiderivative = 7.96

method	result
default	$-\frac{i\left(\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3}+ab\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b^2\sqrt{b^2}}\right)+\left(a^2+1\right)\left(\right.\right.}{(i-a)^3}$

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -I/(I-a)^3*(1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b*(1/4*(2*b^2*x+2*a*b)/b^2*
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a
*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+(a^2+1)*((b^2*x
^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)))-I/(I-a)^2/b*(-I/b/(x-(I-a)/b)^2*((x-(I-a)
/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)+3*I*b*(1/3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(
I-a)/b))^(3/2)+I*b*(1/4*(2*(x-(I-a)/b)*b^2+2*I*b)/b^2*((x-(I-a)/b)^2*b^2+2*
I*b*(x-(I-a)/b))^(1/2)+1/2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b
)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2))))+I/(I-a)/b^2*(I/b/(x-(I-a)/
b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)-2*I*b*(-I/b/(x-(I-a)/b)^2*
```

```
((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)+3*I*b*(1/3*((x-(I-a)/b)^2*b^2+2
*I*b*(x-(I-a)/b))^(3/2)+I*b*(1/4*(2*(x-(I-a)/b)*b^2+2*I*b)/b^2*((x-(I-a)/b)
^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)+1/2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((
x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)))))+I/(I-a)^3*(1/3*(
(x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)+I*b*(1/4*(2*(x-(I-a)/b)*b^2+2*I*
b)/b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)+1/2*ln((I*b+(x-(I-a)/b)*
b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(90) = 180$.

Time = 0.27 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.66

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx$$
$$= \frac{((a-i)bx+a^2-2i\,a-1)\sqrt{-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}}\log\left(-\frac{(a+i)bx-\sqrt{b^2x^2+2\,abx+a^2+1}(a+i)-(i\,a^2+2\,a-i)\sqrt{-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}}}{a+i}\right) -}{}$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (((a - I)*b*x + a^2 - 2*I*a - 1)*sqrt(-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I
*a^2 - 3*a + I))*log(-((a + I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a +
 I) - (I*a^2 + 2*a - I)*sqrt(-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*
a + I)))/(a + I)) - ((a - I)*b*x + a^2 - 2*I*a - 1)*sqrt(-(a^3 + 3*I*a^2 -
3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))*log(-((a + I)*b*x - sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*(a + I) - (-I*a^2 - 2*a + I)*sqrt(-(a^3 + 3*I*a^2 - 3*a - I)
/(a^3 - 3*I*a^2 - 3*a + I)))/(a + I)) - 4*b*x - ((I*a + 1)*b*x + I*a^2 + 2*
a - I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 4*a - 4*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1) + 4*I)/((a - I)*b*x + a^2 - 2*I*a - 1)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx$$
$$= i\left(\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{a^3x+3a^2bx^2-3ia^2x+3ab^2x^3-6iabx^2-3ax+b^3x^4-3ib^2x^3-3bx^2+ix}\,dx\right.$$
$$+\int \frac{a^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3x+3a^2bx^2-3ia^2x+3ab^2x^3-6iabx^2-3ax+b^3x^4-3ib^2x^3-3bx^2+ix}\,dx$$
$$+\int \frac{b^2x^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3x+3a^2bx^2-3ia^2x+3ab^2x^3-6iabx^2-3ax+b^3x^4-3ib^2x^3-3bx^2+ix}\,dx$$
$$\left.+\int \frac{2abx\sqrt{a^2+2abx+b^2x^2+1}}{a^3x+3a^2bx^2-3ia^2x+3ab^2x^3-6iabx^2-3ax+b^3x^4-3ib^2x^3-3bx^2+ix}\,dx\right)$$

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x,x)
```

```
[Out] I*(Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*x + 3*a**2*b*x**2 -
3*I*a**2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*a*x + b**3*x**4 - 3*I*b**2*x*
*3 - 3*b*x**2 + I*x), x) + Integral(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)/(a**3*x + 3*a**2*b*x**2 - 3*I*a**2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*
a*x + b**3*x**4 - 3*I*b**2*x**3 - 3*b*x**2 + I*x), x) + Integral(b**2*x**2*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*x + 3*a**2*b*x**2 - 3*I*a**2*x +
 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*a*x + b**3*x**4 - 3*I*b**2*x**3 - 3*b*x**
2 + I*x), x) + Integral(2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*
x + 3*a**2*b*x**2 - 3*I*a**2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*a*x + b**
3*x**4 - 3*I*b**2*x**3 - 3*b*x**2 + I*x), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(90) = 180$.

Time = 0.41 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.88

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx =$$

$$-\frac{i\,b\log\left(-3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2 ab - a^3b - \left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b| - 3\left(x|b|-\sqrt{(bx+a)^2\right.\right.}{}$$

$$-\frac{\left(-i\,a^2+2\,a+i\right)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}(a-i)}$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/3*I*b*log(-3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b - a^3*b - (x*abs(b
) - sqrt((b*x + a)^2 + 1))^3*abs(b) - 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*
a^2*abs(b) + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b - 4*(-I
*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*a*abs(b) + a*b + (x*abs(b) - sqrt((b*x
 + a)^2 + 1))*abs(b))/abs(b) - (-I*a^2 + 2*a + I)*log(abs(-2*x*abs(b) + 2*s
qrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt((b*x + a)^
2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a - I))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = \int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(x*(a*1i + b*x*1i + 1)^3),x)
```

```
[Out] int(((a + b*x)^2 + 1)^(3/2)/(x*(a*1i + b*x*1i + 1)^3), x)
```

3.213 $\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 178

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} - \frac{6i\sqrt{i+a}\,b\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}}$$

```
[Out] -6*I*b*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1
/2))*(I+a)^(1/2)/(I-a)^(5/2)-(1-I*a-I*b*x)^(3/2)/(1+I*a)/x/(1+I*a+I*b*x)^(1
/2)+6*I*b*(1-I*a-I*b*x)^(1/2)/(I-a)^2/(1+I*a+I*b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5203, 96, 95, 214}

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = -\frac{6i\sqrt{a+i}\,b\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{5/2}} - \frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}} + \frac{6ib\sqrt{-ia-ibx+1}}{(-a+i)^2\sqrt{ia+ibx+1}}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] ((6*I)*b*Sqrt[1 - I*a - I*b*x])/((I - a)^2*Sqrt[1 + I*a + I*b*x]) - (1 - I*
a - I*b*x)^(3/2)/((1 + I*a)*x*Sqrt[1 + I*a + I*b*x]) - ((6*I)*Sqrt[I + a]*b
*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*
b*x])])/(I - a)^(5/2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1-ia-ibx)^{3/2}}{x^2(1+ia+ibx)^{3/2}}\,dx$$

$$= -\frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} + \frac{(3b)\int \frac{\sqrt{1-ia-ibx}}{x(1+ia+ibx)^{3/2}}\,dx}{i-a}$$

$$= \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} + \frac{(3(i+a)b)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{(i-a)^2}$$

$$= \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} + \frac{(6(i+a)b)\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i-a)^2}$$

$$= \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} - \frac{6i\sqrt{i+a}\,b\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \frac{\frac{\sqrt{-i(i+a+bx)}(1+a^2+5ibx+abx)}{x\sqrt{1+ia+ibx}} - \frac{6i\sqrt{-1+ia}b\,\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}}}{(-i+a)^2}$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] ((Sqrt[(-I)*(I + a + b*x)]*(1 + a^2 + (5*I)*b*x + a*b*x))/(x*Sqrt[1 + I*a +
 I*b*x]) - ((6*I)*Sqrt[-1 + I*a]*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a
 + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 - I*a])/(-I + a)
^2
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{i\sqrt{b^2x^2+2abx+a^2+1}\,(i+a)}{(a-i)^2 x} + \frac{b\left(-\frac{\left(-3a^2-3\right)\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(i-a)\sqrt{a^2+1}} + \frac{4i(ia+1)\sqrt{\left(x-\frac{i-a}{b}\right)^2 b^2+2ib\left(x-\frac{i-a}{b}\right.}}{b(i-a)\left(x-\frac{i-a}{b}\right)}\right.}{a^2-2ia-1}$
default	Expression too large to display

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -I*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*(I+a)/(a-I)^2/x+1/(-2*I*a+a^2-1)*b*(-(-3*a
^2-3)/(I-a)/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/x)+4*I*(1+I*a)/b/(I-a)/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I
*b*(x-(I-a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(116) = 232.

Time = 0.27 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.19

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx =$$

$$-\frac{(i\,a-5)b^2x^2 + (i\,a^2-4\,a+5i)bx - 3\left((a^2-2i\,a-1)bx^2 + (a^3-3i\,a^2-3\,a+i)x\right)\sqrt{\frac{(a+i)b^2}{a^5-5i\,a^4-10\,a^3+10i\,a}}}{}$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -((I*a - 5)*b^2*x^2 + (I*a^2 - 4*a + 5*I)*b*x - 3*((a^2 - 2*I*a - 1)*b*x^2
+ (a^3 - 3*I*a^2 - 3*a + I)*x)*sqrt((a + I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 1
0*I*a^2 + 5*a - I))*log(-(b^2*x + (a^3 - 3*I*a^2 - 3*a + I)*sqrt((a + I)*b^
2/(a^5 - 5*I*a^4 - 10*a^3 + 10*I*a^2 + 5*a - I)) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*b)/b) + 3*((a^2 - 2*I*a - 1)*b*x^2 + (a^3 - 3*I*a^2 - 3*a + I)*x)
*sqrt((a + I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10*I*a^2 + 5*a - I))*log(-(b^2*
x - (a^3 - 3*I*a^2 - 3*a + I)*sqrt((a + I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10
*I*a^2 + 5*a - I)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b)/b) + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*((I*a - 5)*b*x + I*a^2 + I))/((a^2 - 2*I*a - 1)*b*x^2
 + (a^3 - 3*I*a^2 - 3*a + I)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{\left(i\,bx+i\,a+1\right)^3x^2}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^2), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{\left(i\,bx+i\,a+1\right)^3x^2}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x^2\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(x^2*(a*1i + b*x*1i + 1)^3),x)
[Out] int(((a + b*x)^2 + 1)^(3/2)/(x^2*(a*1i + b*x*1i + 1)^3), x)
```

3.214 $\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 264

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx=-\frac{3(3i+2a)b^2\sqrt{1-ia-ibx}}{(1+ia)^3(i+a)\sqrt{1+ia+ibx}}+\frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}}-\frac{(1-ia-ibx)^{5/2}}{2\left(1+a^2\right)x^2\sqrt{1+ia+ibx}}+\frac{3(3-2ia)b^2\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{7/2}\sqrt{i+a}}$$

```
[Out] 3*(3-2*I*a)*b^2*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-
I*b*x)^(1/2))/(I-a)^(7/2)/(I+a)^(1/2)+1/2*(3-2*I*a)*b*(1-I*a-I*b*x)^(3/2)/(
I-a)^2/(I+a)/x/(1+I*a+I*b*x)^(1/2)-1/2*(1-I*a-I*b*x)^(5/2)/(a^2+1)/x^2/(1+I
*a+I*b*x)^(1/2)-3*(3*I+2*a)*b^2*(1-I*a-I*b*x)^(1/2)/(1+I*a)^3/(I+a)/(1+I*a+
I*b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5203, 98, 96, 95, 214}

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx=-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}+\frac{3(3-2ia)b^2\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{7/2}\sqrt{a+i}}-\frac{3(2a+3i)b^2\sqrt{-ia-ibx+1}}{(1+ia)^3(a+i)\sqrt{ia+ibx+1}}+\frac{(3-2ia)b(-ia-ibx+1)^{3/2}}{2(-a+i)^2(a+i)x\sqrt{ia+ibx+1}}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x^3),x]
```

```
[Out] (-3*(3*I + 2*a)*b^2*Sqrt[1 - I*a - I*b*x])/((1 + I*a)^3*(I + a)*Sqrt[1 + I*
a + I*b*x]) + ((3 - (2*I)*a)*b*(1 - I*a - I*b*x)^(3/2))/(2*(I - a)^2*(I + a
```

```
)*x*Sqrt[1 + I*a + I*b*x]) - (1 - I*a - I*b*x)^(5/2)/(2*(1 + a^2)*x^2*Sqrt[
1 + I*a + I*b*x]) + (3*(3 - (2*I)*a)*b^2*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a
+ I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/((I - a)^(7/2)*Sqrt[I + a])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1-ia-ibx)^{3/2}}{x^3(1+ia+ibx)^{3/2}}\,dx$$

$$= -\frac{(1-ia-ibx)^{5/2}}{2(1+a^2)x^2\sqrt{1+ia+ibx}} - \frac{((3i+2a)b)\int \frac{(1-ia-ibx)^{3/2}}{x^2(1+ia+ibx)^{3/2}}dx}{2(1+a^2)}$$

$$= \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2(1+a^2)x^2\sqrt{1+ia+ibx}} + \frac{(3(3i+2a)b^2)\int \frac{\sqrt{1-ia-ibx}}{x(1+ia+ibx)^{3/2}}dx}{2(i-a)^2(i+a)}$$

$$= -\frac{3(3-2ia)b^2\sqrt{1-ia-ibx}}{(i-a)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2(1+a^2)x^2\sqrt{1+ia+ibx}} + \frac{(3(3i+2a)b^2)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}dx}{2(i-a)^3}$$

$$= -\frac{3(3-2ia)b^2\sqrt{1-ia-ibx}}{(i-a)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2(1+a^2)x^2\sqrt{1+ia+ibx}} + \frac{(3(3i+2a)b^2)\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i-a)^3}$$

$$= -\frac{3(3-2ia)b^2\sqrt{1-ia-ibx}}{(i-a)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2(1+a^2)x^2\sqrt{1+ia+ibx}} + \frac{3(3-2ia)b^2\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{7/2}\sqrt{i+a}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.73

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}dx$$

$$= \frac{\frac{\sqrt{-i(i+a+bx)}(-i+a-ia^2+a^3-5bx-5iabx-14ib^2x^2-ab^2x^2)}{x^2\sqrt{1+ia+ibx}} + \frac{6i\sqrt{-1+ia}(3i+2a)b^2\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}(i+a)}}{2(-i+a)^3}$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^3),x]
[Out] ((Sqrt[(-I)*(I + a + b*x)]*(-I + a - I*a^2 + a^3 - 5*b*x - (5*I)*a*b*x - (1
4*I)*b^2*x^2 - a*b^2*x^2))/(x^2*Sqrt[1 + I*a + I*b*x]) + ((6*I)*Sqrt[-1 + I
*a]*(3*I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt
[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*(I + a)))/(2*(-I + a)^3
)
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{i\left(-a\,b^{3}x^{3}-6 i b^{3}x^{3}-a^{2}b^{2}x^{2}-12 i a\,b^{2}x^{2}+a^{3}bx-6 i a^{2}bx+a^{4}+b^{2}x^{2}+abx-6bxi+2a^{2}+1\right)}{2x^{2}\left(a-i\right)^{3}\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}+\frac{b^{2}\left(-\frac{\left(6a^{2}+3ia+9\right)\ln\left(\frac{2a^{2}+2+2abx+2\sqrt{}}{}\right)}{\left(i-a\right)\sqrt{a^{2}}}\right.}{}$
default	Expression too large to display

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(-a*b^3*x^3-a^2*b^2*x^2+a^3*b*x-6*I*b^3*x^3+a^4+b^2*x^2-12*I*a*b^2*x
^2+a*b*x-6*I*a^2*b*x+2*a^2-6*I*b*x+1)/x^2/(a-I)^3/(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)+1/2/(-3*I*a^2+a^3+I-3*a)*b^2*(-(3*I*a+6*a^2+9)/(I-a)/(a^2+1)^(1/2)*ln(
(2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-8*I*(1+I
*a)/b/(I-a)/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(180) = 360.

Time = 0.28 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.17

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx$$

$$= \frac{(i\,a-14)b^{3}x^{3}+(i\,a^{2}-13\,a+14i)b^{2}x^{2}-3\left((a^{3}-3i\,a^{2}-3\,a+i)bx^{3}+(a^{4}-4i\,a^{3}-6\,a^{2}+4i\,a+1)x^{2}\right)\sqrt{}}{}$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*((I*a - 14)*b^3*x^3 + (I*a^2 - 13*a + 14*I)*b^2*x^2 - 3*((a^3 - 3*I*a^2
 - 3*a + I)*b*x^3 + (a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)*x^2)*sqrt((4*a^2 +
12*I*a - 9)*b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*
I*a - 1))*log(-((2*a + 3*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a
+ 3*I)*b^2 + (a^5 - 3*I*a^4 - 2*a^3 - 2*I*a^2 - 3*a + I)*sqrt((4*a^2 + 12*I
*a - 9)*b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a
- 1)))/((2*a + 3*I)*b^2)) + 3*((a^3 - 3*I*a^2 - 3*a + I)*b*x^3 + (a^4 - 4*I
*a^3 - 6*a^2 + 4*I*a + 1)*x^2)*sqrt((4*a^2 + 12*I*a - 9)*b^4/(a^8 - 6*I*a^7
 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))*log(-((2*a + 3*I)*b^
3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + 3*I)*b^2 - (a^5 - 3*I*a^4 -
2*a^3 - 2*I*a^2 - 3*a + I)*sqrt((4*a^2 + 12*I*a - 9)*b^4/(a^8 - 6*I*a^7 - 1
4*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1)))/((2*a + 3*I)*b^2)) + ((
```

```
I*a - 14)*b^2*x^2 - I*a^3 - 5*(a - I)*b*x - a^2 - I*a - 1)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))/((a^3 - 3*I*a^2 - 3*a + I)*b*x^3 + (a^4 - 4*I*a^3 - 6*a^
2 + 4*I*a + 1)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^3}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^3), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^3}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x^3\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)^3}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(x^3*(a*1i + b*x*1i + 1)^3),x)
[Out] int(((a + b*x)^2 + 1)^(3/2)/(x^3*(a*1i + b*x*1i + 1)^3), x)
```

3.215 $\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 339

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = -\frac{(52-51ia-2a^2)\,b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19-16ia)b^2\sqrt{1-ia-ibx}}{6(i-a)^3(i+a)x\sqrt{1+ia+ibx}} + \frac{(11i+18a-6ia^2)\,b^3\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{9/2}(i+a)^{3/2}}$$

```
[Out] (11*I+18*a-6*I*a^2)*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)
/(1-I*a-I*b*x)^(1/2))/(I-a)^(9/2)/(I+a)^(3/2)-1/6*(52-51*I*a-2*a^2)*b^3*(1-
I*a-I*b*x)^(1/2)/(I-a)^4/(I+a)/(1+I*a+I*b*x)^(1/2)-1/3*(I+a)*(1-I*a-I*b*x)^
(1/2)/(I-a)/x^3/(1+I*a+I*b*x)^(1/2)-7/6*I*b*(1-I*a-I*b*x)^(1/2)/(I-a)^2/x^2
/(1+I*a+I*b*x)^(1/2)+1/6*(19-16*I*a)*b^2*(1-I*a-I*b*x)^(1/2)/(I-a)^3/(I+a)/
x/(1+I*a+I*b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used

$= \{5203, 100, 156, 157, 12, 95, 214\}$

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \frac{(-6ia^2+18a+11i)\,b^3\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{9/2}(a+i)^{3/2}} - \frac{(-2a^2-51ia+52)\,b^3\sqrt{-ia-ibx+1}}{6(-a+i)^4(a+i)\sqrt{ia+ibx+1}} + \frac{(19-16ia)b^2\sqrt{-ia-ibx+1}}{6(-a+i)^3(a+i)x\sqrt{ia+ibx+1}} - \frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}} - \frac{7ib\sqrt{-ia-ibx+1}}{6(-a+i)^2x^2\sqrt{ia+ibx+1}}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -1/6*((52 - (51*I)*a - 2*a^2)*b^3*Sqrt[1 - I*a - I*b*x])/((I - a)^4*(I + a)
*Sqrt[1 + I*a + I*b*x]) - ((I + a)*Sqrt[1 - I*a - I*b*x])/(3*(I - a)*x^3*Sq
rt[1 + I*a + I*b*x]) - (((7*I)/6)*b*Sqrt[1 - I*a - I*b*x])/((I - a)^2*x^2*S
qrt[1 + I*a + I*b*x]) + ((19 - (16*I)*a)*b^2*Sqrt[1 - I*a - I*b*x])/(6*(I -
 a)^3*(I + a)*x*Sqrt[1 + I*a + I*b*x]) + ((11*I + 18*a - (6*I)*a^2)*b^3*Arc
Tanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x]
)])/((I - a)^(9/2)*(I + a)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1-ia-ibx)^{3/2}}{x^4(1+ia+ibx)^{3/2}}\,dx$$

$$= -\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{\int \frac{7(i+a)b+6b^2x}{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}\,dx}{3(1+ia)}$$

$$= -\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{\int \frac{-((19-35ia-16a^2)b^2)+14(i+a)b^3x}{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}\,dx}{6(1+ia)\,(1+a^2)}$$

$$= -\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i+16a)b^2\sqrt{1-ia-ibx}}{6(1+ia)^3(i+a)x\sqrt{1+ia+ibx}} - \frac{\int \frac{-3(i+a)(11-18ia-6a^2)b^3-(19-35ia-16a^2)b^4x}{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}\,dx}{6(1+ia)\left(1+a^2\right)^2}$$

$$= -\frac{(52-51ia-2a^2)\,b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i+16a)b^2\sqrt{1-ia-ibx}}{6(1+ia)^3(i+a)x\sqrt{1+ia+ibx}} - \frac{i\int \frac{3(11-29ia-24a^2+6ia^3)b^4}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{6(i-a)^4(i+a)^2b}$$

$$= -\frac{(52-51ia-2a^2)\,b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i+16a)b^2\sqrt{1-ia-ibx}}{6(1+ia)^3(i+a)x\sqrt{1+ia+ibx}} - \frac{((11-18ia-6a^2)\,b^3)\int \frac{1}{x\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}\,dx}{2(i-a)^4(i+a)}$$

$$= -\frac{(52-51ia-2a^2)\,b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i+16a)b^2\sqrt{1-ia-ibx}}{6(1+ia)^3(i+a)x\sqrt{1+ia+ibx}} - \frac{((11-18ia-6a^2)\,b^3)\,\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^2}\,dx, x, \frac{\sqrt{1+ia+ibx}}{\sqrt{1-ia-ibx}}\right)}{(i-a)^4(i+a)}$$

$$= -\frac{(52-51ia-2a^2)\,b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}} - \frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}} - \frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}} + \frac{(19i+16a)b^2\sqrt{1-ia-ibx}}{6(1+ia)^3(i+a)x\sqrt{1+ia+ibx}} + \frac{(11i+18a-6ia^2)\,b^3\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{9/2}(i+a)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.81

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx =$$

$$-\frac{-2(-1-ia)^{7/2}(1-ia)(-i(i+a+bx))^{5/2}-(-1-ia)^{5/2}(3i+4a)bx(-i(i+a+bx))^{5/2}+i(-11+18}{6(-1-}$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^4),x]
```

```
[Out] -1/6*(-2*(-1 - I*a)^(7/2)*(1 - I*a)*((-I)*(I + a + b*x))^(5/2) - (-1 - I*a)
^(5/2)*(3*I + 4*a)*b*x*((-I)*(I + a + b*x))^(5/2) + I*(-11 + (18*I)*a + 6*a
^2)*b^2*x^2*((-I)*Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)]*(1 + a^2 + (5*I)*
b*x + a*b*x) - 6*Sqrt[-1 + I*a]*b*x*Sqrt[1 + I*a + I*b*x]*ArcTanh[(Sqrt[-1
- I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])]))/
((-1 - I*a)^(5/2)*(1 + a^2)^2*x^3*Sqrt[1 + I*a + I*b*x])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{i\left(2a^2b^4x^4+27ia\,b^4x^4+2a^3b^3x^3+45ia^2b^3x^3+9ib^2x^2a^3-28x^4b^4+2a^5bx-9ix\,a^4b-58a\,b^3x^3-9ib^3x^3+2a^6-26a^2b^2x^2+9ia\,b^2x^2+4a\right.}{6x^3(i+a)(a-i)^4\sqrt{b^2x^2+2abx+a^2+1}}$
default	Expression too large to display

```
[In] int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*(9*I*a*b^2*x^2+2*a^2*b^4*x^4+27*I*a*b^4*x^4+2*a^3*b^3*x^3+45*I*x^3*a
^2*b^3-28*x^4*b^4-9*I*x*a^4*b-18*I*a^2*b*x+2*a^5*b*x-58*a*b^3*x^3+9*I*b^2*x
^2*a^3+2*a^6-26*a^2*b^2*x^2-9*I*b^3*x^3+4*a^3*b*x+6*a^4-26*b^2*x^2-9*I*b*x+
2*a*b*x+6*a^2+2)/x^3/(I+a)/(a-I)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2/(I+a)/
(-4*I*a^3+a^4+4*I*a-6*a^2+1)*b^3*(-(12*I*a^2+6*a^3+11*I+7*a)/(I-a)/(a^2+1)^
(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
+8*(a^2+1)/b/(I-a)/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(223) = 446.

Time = 0.30 (sec) , antiderivative size = 839, normalized size of antiderivative = 2.47

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx$$

$$= \frac{(-2i\,a^2+51\,a+52i)b^4x^4+(-2i\,a^3+49\,a^2+i\,a+52)b^3x^3+3\sqrt{\frac{(36\,a^4+216i\,a^3-456\,a^2-396i\,a+121}{a^{12}-6i\,a^{11}-12\,a^{10}+2i\,a^9-27\,a^8+36i\,a^7+36i\,a^5+27\,a^4}}}{}$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((-2*I*a^2 + 51*a + 52*I)*b^4*x^4 + (-2*I*a^3 + 49*a^2 + I*a + 52)*b^3*
x^3 + 3*sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(a^12 - 6*I
*a^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*a^3
 + 12*a^2 - 6*I*a - 1))*((a^5 - 3*I*a^4 - 2*a^3 - 2*I*a^2 - 3*a + I)*b*x^4
+ (a^6 - 4*I*a^5 - 5*a^4 - 5*a^2 + 4*I*a + 1)*x^3)*log(-((6*a^2 + 18*I*a -
11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^2 + 18*I*a - 11)*b^3 + (
a^7 - 3*I*a^6 - a^5 - 5*I*a^4 - 5*a^3 - I*a^2 - 3*a + I)*sqrt((36*a^4 + 216
*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(a^12 - 6*I*a^11 - 12*a^10 + 2*I*a^9
- 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*a^3 + 12*a^2 - 6*I*a - 1)))/(
(6*a^2 + 18*I*a - 11)*b^3)) - 3*sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*I*
a + 121)*b^6/(a^12 - 6*I*a^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*
I*a^5 + 27*a^4 + 2*I*a^3 + 12*a^2 - 6*I*a - 1))*((a^5 - 3*I*a^4 - 2*a^3 - 2
*I*a^2 - 3*a + I)*b*x^4 + (a^6 - 4*I*a^5 - 5*a^4 - 5*a^2 + 4*I*a + 1)*x^3)*
log(-((6*a^2 + 18*I*a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^
2 + 18*I*a - 11)*b^3 - (a^7 - 3*I*a^6 - a^5 - 5*I*a^4 - 5*a^3 - I*a^2 - 3*a
 + I)*sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(a^12 - 6*I*a
^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*a^3 +
 12*a^2 - 6*I*a - 1)))/((6*a^2 + 18*I*a - 11)*b^3)) + ((-2*I*a^2 + 51*a + 5
2*I)*b^3*x^3 - 2*I*a^5 + (16*a^2 + 3*I*a + 19)*b^2*x^2 - 2*a^4 - 4*I*a^3 -
7*(a^3 - I*a^2 + a - I)*b*x - 4*a^2 - 2*I*a - 2)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1))/((a^5 - 3*I*a^4 - 2*a^3 - 2*I*a^2 - 3*a + I)*b*x^4 + (a^6 - 4*I*a^
5 - 5*a^4 - 5*a^2 + 4*I*a + 1)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**4,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^4}\,dx$$

```
[In] integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x^4\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

```
[In] int(((a + b*x)^2 + 1)^(3/2)/(x^4*(a*1i + b*x*1i + 1)^3),x)
[Out] int(((a + b*x)^2 + 1)^(3/2)/(x^4*(a*1i + b*x*1i + 1)^3), x)
```

3.216 $\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 494

$$\begin{aligned}\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx =& -\frac{(3i+4a-8ia^2)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} \\ &- \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} \\ &+ \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} \\ &+ \frac{(3i+4a-8ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &- \frac{(3i+4a-8ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &- \frac{(3i+4a-8ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} \\ &+ \frac{(3i+4a-8ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}\end{aligned}$$

```
[Out] -1/8*(3*I+4*a-8*I*a^2)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^3-1/12*(I+
8*a)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(5/4)/b^3+1/3*x*(1-I*a-I*b*x)^(3/4)*
(1+I*a+I*b*x)^(5/4)/b^2+1/16*(3*I+4*a-8*I*a^2)*arctan(1-(1-I*a-I*b*x)^(1/4)
*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/16*(3*I+4*a-8*I*a^2)*arctan(1+(
1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/32*(3*I+4*a-8
*I*a^2)*ln(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^
(1/2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)+1/32*(3*I+4*a-8*I*a^2)*ln(1+(1-I*a-I
```

```
*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(
1/2))/b^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5203, 92, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx = {}& \frac{(-8ia^2+4a+3i)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{8\sqrt{2}b^3} \\ & -\frac{(-8ia^2+4a+3i)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{8\sqrt{2}b^3} \\ & -\frac{(-8ia^2+4a+3i)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{8b^3} \\ & -\frac{(-8ia^2+4a+3i)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3} \\ & +\frac{(-8ia^2+4a+3i)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3} \\ & -\frac{(8a+i)(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{12b^3} \\ & +\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}\end{aligned}$$

```
[In] Int[E^((I/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] -1/8*((3*I + 4*a - (8*I)*a^2)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/
4))/b^3 - ((I + 8*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(12*b
^3) + (x*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(3*b^2) + ((3*I +
 4*a - (8*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I
*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*I + 4*a - (8*I)*a^2)*ArcTan[1 + (Sqrt[2
]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*
I + 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] -
(Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt[2]*b^3
) + ((3*I + 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I
*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqr
t[2]*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rule steps

$$\text{integral} = \int \frac{x^2 \sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx$$

$$= \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} + \frac{\int \frac{\sqrt[4]{1+ia+ibx}\left(-1-a^2-\frac{1}{2}(i+8a)bx\right)}{\sqrt[4]{1-ia-ibx}}\,dx}{3b^2}$$

$$= -\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} - \frac{(3-4ia-8a^2)\int \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx}{8b^2}$$

$$= -\frac{(3i+4a-8ia^2)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} - \frac{(3-4ia-8a^2)\int \frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx}{16b^2}$$

$$= -\frac{(3i+4a-8ia^2)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} - \frac{(3i+4a-8ia^2)\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{4b^3}$$

$$= -\frac{(3i+4a-8ia^2)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} - \frac{(3i+4a-8ia^2)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^3}$$

$$= -\frac{(3i+4a-8ia^2)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} + \frac{(3i+4a-8ia^2)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^3} - \frac{(3i+4a-8ia^2)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^3}$$

$$= -\frac{(3i+4a-8ia^2)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} - \frac{(3i+4a-8ia^2)\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16b^3} - \frac{(3i+4a-8ia^2)\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16b^3} - \frac{(3i+4a-8ia^2)\operatorname{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} - \frac{(3i+4a-8ia^2)\operatorname{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}$$

$$= -\frac{(3i+4a-8ia^2)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} - \frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} + \frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} - \frac{(3i+4a-8ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} + \frac{(3i+4a-8ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} - \frac{(3i+4a-8ia^2)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} + \frac{(3i+4a-8ia^2)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3}$$

$$
\begin{aligned}
&= -\frac{(3i+4a-8ia^2)\,(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} \\
&\quad -\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3}+\frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} \\
&\quad +\frac{(3i+4a-8ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\
&\quad -\frac{(3i+4a-8ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\
&\quad -\frac{(3i+4a-8ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} \\
&\quad +\frac{(3i+4a-8ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.24

$$
\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx
$$

$$
= \frac{(-i(i+a+bx))^{3/4}\left(-i\sqrt[4]{1+ia+ibx}(1+8a^2+5ibx-4b^2x^2+a(-7i+4bx))+2i\sqrt[4]{2}(-3+4ia+8a^2)\,\mathrm{H}\right.}{12b^3}
$$

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(3/4)*((-I)*(1 + I*a + I*b*x)^(1/4)*(1 + 8*a^2 + (5*I
)*b*x - 4*b^2*x^2 + a*(-7*I + 4*b*x)) + (2*I)*2^(1/4)*(-3 + (4*I)*a + 8*a^2
)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-1/2*I)*(I + a + b*x)]))/(12*b^3)
```

Maple [F]

$$
\int \sqrt{\frac{1+i\,(bx+a)}{\sqrt{1+(bx+a)^2}}}\,x^2dx
$$

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.12

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx$$

$$= \frac{3\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24\,a+9i}{b^6}}\log\left(\frac{i\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24\,a+9i}{b^6}}+(8\,a^2+4i\,a-3)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{8\,a^2+4i\,a-3}\right)-3\,b^3\sqrt{\frac{64i\,a^4-6}{}}}{}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="frica
s")
[Out] 1/48*(3*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6)*log((I*b^
3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6) + (8*a^2 + 4*I*a -
3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 + 4*I*a
- 3)) - 3*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6)*log((-I
*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6) + (8*a^2 + 4*I*a
 - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 + 4*I
*a - 3)) + 3*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6)*log
((I*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6) + (8*a^2 + 4
*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 +
 4*I*a - 3)) - 3*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6)
*log((-I*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6) + (8*a^
2 + 4*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*
a^2 + 4*I*a - 3)) + 2*(8*b^3*x^3 - 2*I*b^2*x^2 + 8*a^3 + (8*I*a - 1)*b*x +
34*I*a^2 - 37*a - 11*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I)))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)*x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int x^2\sqrt{\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2+1}}}\,dx$$

```
[In] integrate((((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="maxim
a")
[Out] integrate(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by -27, a substitution variable should perhaps be pur
ged.Wa
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int x^2\sqrt{\frac{1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}\,dx$$

```
[In] int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

3.217 $\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 16, antiderivative size = 410

$$\begin{aligned}\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx &= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} \\ &+ \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} \\ &- \frac{(1-4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} \\ &+ \frac{(1-4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} \\ &+ \frac{(1-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} \\ &- \frac{(1-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}\end{aligned}$$

```
[Out] 1/4*(1-4*I*a)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^2+1/2*(1-I*a-I*b*x)
^(3/4)*(1+I*a+I*b*x)^(5/4)/b^2-1/8*(1-4*I*a)*arctan(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)+1/8*(1-4*I*a)*arctan(1+(1-I*a-I*b*x
)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)+1/16*(1-4*I*a)*ln(1-(1-I*a
-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)
^(1/2))/b^2*2^(1/2)-1/16*(1-4*I*a)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+
I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5203, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = -\frac{(1-4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{4\sqrt{2}b^2} + \frac{(1-4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{4\sqrt{2}b^2} + \frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2} + \frac{(1-4ia)(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{4b^2} + \frac{(1-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{8\sqrt{2}b^2} - \frac{(1-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{8\sqrt{2}b^2}$$

```
[In] Int[E^((I/2)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((1 - (4*I)*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(2*b^2) - ((1 - (4*I)*a)*A
rcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*Sq
rt[2]*b^2) + ((1 - (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) + ((1 - (4*I)*a)*Log[1 + Sqrt[1 - I
*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) - ((1 - (4*I)*a)*Log[1 + Sqrt[1 - I*a
- I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a
 + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx$$

$$= \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} - \frac{(i+4a)\int \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx}{4b}$$

$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} - \frac{(i+4a)\int \frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx}{8b}$$

$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} + \frac{(1-4ia)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{2b^2}$$

$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} + \frac{(1-4ia)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b^2}$$

$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} - \frac{(1-4ia)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^2} + \frac{(1-4ia)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^2}$$

$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} + \frac{(1-4ia)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^2} + \frac{(1-4ia)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^2} + \frac{(1-4ia)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} + \frac{(1-4ia)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$

$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} + \frac{(1-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} - \frac{(1-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} + \frac{(1-4ia)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} - \frac{(1-4ia)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}$$

$$= \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} - \frac{(1-4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} + \frac{(1-4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} + \frac{(1-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} - \frac{(1-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.20

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = \frac{(-i(i+a+bx))^{3/4}\left(3(1+ia+ibx)^{5/4}+2\sqrt[4]{2}(1-4ia)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{1}{2}i(i+a+bx)\right)\right)}{6b^2}$$

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])*x,x]
[Out] (((-I)*(I + a + b*x))^(3/4)*(3*(1 + I*a + I*b*x)^(5/4) + 2*2^(1/4)*(1 - (4*
I)*a)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-1/2*I)*(I + a + b*x)]))/(6*b^2)
```

Maple [F]

$$\int \sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\,xdx$$

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.01

$$\int e^{\frac{1}{2} i \arctan(a+bx)} x \, dx =$$

$$-\frac{b^2\sqrt{\frac{16 i a^2-8a-i}{b^4}}\log\left(\frac{i b^2\sqrt{\frac{16 i a^2-8a-i}{b^4}}+(4a+i)\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}}{4a+i}\right)-b^2\sqrt{\frac{16 i a^2-8a-i}{b^4}}\log\left(\frac{-i b^2\sqrt{\frac{16 i a^2-8a-i}{b^4}}+(4a+\cdots}{4a-\cdots}\right.\cdots}{\cdots}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((16*I*a^2 - 8*a - I)/b^4)*log((I*b^2*sqrt((16*I*a^2 - 8*a -
I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
)/(4*a + I)) - b^2*sqrt((16*I*a^2 - 8*a - I)/b^4)*log((-I*b^2*sqrt((16*I*a^
2 - 8*a - I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
 + a + I)))/(4*a + I)) + b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4)*log((I*b^2*sqr
t((-16*I*a^2 + 8*a + I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(b*x + a + I)))/(4*a + I)) - b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4)*log
((-I*b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a + I)) - 2*(2*b^2*x^2 - 2*a^2 - I*b*
x - 5*I*a + 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{1}{2} i \arctan(a+bx)} x \, dx = \text{Timed out}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)*x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{1}{2} i \arctan(a+bx)} x \, dx = \int x\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}\,dx$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="maxima"
)
```

```
[Out] integrate(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by -27, a substitution variable should perhaps be pur
ged.Wa
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = \int x\,\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}\,dx$$

```
[In] int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

3.218 $\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 338

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$

```
[Out] I*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b-1/2*I*arctan(1-(1-I*a-I*b*x)^(1
/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+1/2*I*arctan(1+(1-I*a-I*b*x)^(1/
4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+1/4*I*ln(1-(1-I*a-I*b*x)^(1/4)*2^
(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2
)-1/4*I*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^
(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5201, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = -\frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}b} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}b} + \frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b} + \frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b} - \frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}$$

```
[In] Int[E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (I*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b - (I*ArcTan[1 - (Sqrt
[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + (I*Arc
Tan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2
]*b) + ((I/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2
]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) - ((I/2)*L
og[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*
b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\,dx \\
&= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{1}{2}\int \frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx \\
&= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(2i)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b} \\
&= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{(2i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} \\
&= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} \\
&\quad + \frac{i\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} \\
&= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} \\
&\quad + \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} \\
&\quad + \frac{i\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} \\
&\quad + \frac{i\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}
\end{aligned}$$

$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} + \frac{i\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{i\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}$$

$$= \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.13

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = -\frac{8ie^{\frac{5}{2}i\arctan(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},2,\frac{9}{4},-e^{2i\arctan(a+bx)}\right)}{5b}$$

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-8*I)/5)*E^(((5*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[5/4, 2, 9/4, -E
^((2*I)*ArcTan[a + b*x])])/b
```

Maple [F]

$$\int \sqrt{\frac{1+i\,(bx+a)}{\sqrt{1+(bx+a)^2}}}dx$$

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.75

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \frac{b\sqrt{\frac{i}{b^2}}\log\left(i\,b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{i}{b^2}}\log\left(-i\,b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)+b\sqrt{-\frac{i}{b^2}}\log\left(i\,b\cdots\right.}{\cdots}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*sqrt(I/b^2)*log(I*b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(b*x + a + I))) - b*sqrt(I/b^2)*log(-I*b*sqrt(I/b^2) + sqrt(I*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-I/b^2)*log(I*b*sqrt(-
I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(
-I/b^2)*log(-I*b*sqrt(-I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I))) + 2*(b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I)))/b
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \int \sqrt{\frac{i\,(a+bx)+1}{\sqrt{(a+bx)^2+1}}}\,dx$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt((I*(a + b*x) + 1)/sqrt((a + b*x)**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \int \sqrt{\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}}\,dx$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by -27, a substitution variable should perhaps be pur
ged.Wa
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \int \sqrt{\frac{1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2 + 1}}}\,dx$$

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

3.219 $\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 395

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = -\frac{2\sqrt[4]{i-a}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) - \frac{2\sqrt[4]{i-a}\operatorname{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{\sqrt{2}}$$

```
[Out] -2*(I-a)^(1/4)*arctan((I+a)^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)/(1-I*(b*x
+a))^(1/4))/(I+a)^(1/4)-2*(I-a)^(1/4)*arctanh((I+a)^(1/4)*(1+I*(b*x+a))^(1/
4)/(I-a)^(1/4)/(1-I*(b*x+a))^(1/4))/(I+a)^(1/4)-1/2*ln(1-(1+I*(b*x+a))^(1/4
)*2^(1/2)/(1-I*(b*x+a))^(1/4)+(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*2^(1
/2)+1/2*ln(1+(1+I*(b*x+a))^(1/4)*2^(1/2)/(1-I*(b*x+a))^(1/4)+(1+I*(b*x+a))^
(1/2)/(1-I*(b*x+a))^(1/2))*2^(1/2)-arctan(1-(1+I*(b*x+a))^(1/4)*2^(1/2)/(1-
```

```
I*(b*x+a))^(1/4))*2^(1/2)+arctan(1+(1+I*(b*x+a))^(1/4)*2^(1/2)/(1-I*(b*x+a)
)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5202, 456, 492, 217, 1179, 642, 1176, 631, 210, 218, 214, 211}

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = -\frac{2\sqrt[4]{-a+i}\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{a+i}} - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) - \frac{2\sqrt[4]{-a+i}\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{a+i}} - \frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}} + \frac{\log\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}$$

```
[In] Int[E^((I/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] (-2*(I - a)^(1/4)*ArcTan[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(
1/4)*(1 - I*(a + b*x))^(1/4))])/(I + a)^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)] - (2*(I - a)^(1/4)*
ArcTanh[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a +
b*x))^(1/4))])/(I + a)^(1/4) - Log[1 - (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1
 - I*(a + b*x))^(1/4) + Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]/Sqrt[2
] + Log[1 + (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4) + Sqr
t[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]/Sqrt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 456

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5202

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[x^(2/(I*n))*((1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m/(1 + x^(2/(I*n)))^(m + 2)), x], x, (1 - I*c*(a + b*x))^
(I*(n/2))/(1 + I*c*(a + b*x))^(I*(n/2))], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 8\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)\left(1-ia-\frac{1+ia}{x^4}\right)x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)\\
&= 8\text{Subst}\left(\int \frac{x^4}{(1+x^4)\,(-1-ia+(1-ia)x^4)}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)\\
&= 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)\\
&\quad + (4(1+ia))\text{Subst}\left(\int \frac{1}{-1-ia+(1-ia)x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)
\end{aligned}
$$

$$= 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) - \left(2\sqrt{i-a}\right)\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) - \left(2\sqrt{i-a}\right)\text{Subst}\left(\int \frac{1}{\sqrt{i-a}+\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)$$

$$= -\frac{2\sqrt[4]{i-a}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{2\sqrt[4]{i-a}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt{2}} + \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)$$

$$= -\frac{2\sqrt[4]{i-a}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{2\sqrt[4]{i-a}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{\sqrt{2}} + \sqrt{2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) - \sqrt{2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)$$

$$= -\frac{2\sqrt[4]{i-a}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right) - \frac{2\sqrt[4]{i-a}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}+\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.31

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \frac{2}{3}(-i(i+a+bx))^{3/4}\left(-\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{1}{2}i(i+a+bx)\right)+\frac{2(-i+a)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)}{(i+a)(1+ia+ibx)^{3/4}}\right)$$

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*((-I)*(I + a + b*x))^(3/4)*(-(2^(1/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (
-1/2*I)*(I + a + b*x)]) + (2*(-I + a)*Hypergeometric2F1[3/4, 1, 7/4, (1 + a
^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I + a)*(1 + I*a + I*b*x)^
(3/4))))/3
```

Maple [F]

$$\int \frac{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}{x}\,dx$$

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - \frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) + \frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - \frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - \left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\right) - i\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+i\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\right) + i\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}-i\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\right) + \left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}-\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\right)$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
(b*x + a + I))) - 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log
(-1/2*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))
 - (-(a - I)/(a + I))^(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I)) + (-(a - I)/(a + I))^(1/4)) - I*(-(a - I)/(a + I))^(1/4)*log(s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + I*(-(a - I)/(a + I
))^(1/4)) + I*(-(a - I)/(a + I))^(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)/(b*x + a + I)) - I*(-(a - I)/(a + I))^(1/4)) + (-(a - I)/(a + I))^
(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-(a -
I)/(a + I))^(1/4))
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}{x}\,dx$$

[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x,x)

[Out] Integral(sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x, x)

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}{x}\,dx$$

[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="maxima"
)

[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by -27, a substitution variable should perhaps be pur
ged.Wa

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}{x}\,dx$$

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x,x)
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x, x)
```

3.220 $\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 205

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{(i+a+bx)\sqrt[4]{1+i(a+bx)}}{(i+a)x\sqrt[4]{1-i(a+bx)}}+\frac{ib\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{(i-a)^{3/4}(i+a)^{5/4}}+\frac{ib\operatorname{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{(i-a)^{3/4}(i+a)^{5/4}}$$

```
[Out] -(I+a+b*x)*(1+I*(b*x+a))^(1/4)/(I+a)/x/(1-I*(b*x+a))^(1/4)+I*b*arctan((I+a)
^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)/(1-I*(b*x+a))^(1/4))/(I-a)^(3/4)/(I+
a)^(5/4)+I*b*arctanh((I+a)^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)/(1-I*(b*x+
a))^(1/4))/(I-a)^(3/4)/(I+a)^(5/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5202, 269, 294, 218, 214, 211}

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{ib\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}+\frac{ib\operatorname{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{(-a+i)^{3/4}(a+i)^{5/4}}-\frac{\sqrt[4]{1+i(a+bx)}(a+bx+i)}{(a+i)x\sqrt[4]{1-i(a+bx)}}$$

```
[In] Int[E^((I/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -(((I + a + b*x)*(1 + I*(a + b*x))^(1/4))/((I + a)*x*(1 - I*(a + b*x))^(1/4
))) + (I*b*ArcTan[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1
 - I*(a + b*x))^(1/4))]])/((I - a)^(3/4)*(I + a)^(5/4)) + (I*b*ArcTanh[((I +
 a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))]
)/((I - a)^(3/4)*(I + a)^(5/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5202

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[x^(2/(I*n))*((1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m/(1 + x^(2/(I*n)))^(m + 2)), x], x, (1 - I*c*(a + b*x))^
(I*(n/2))/(1 + I*c*(a + b*x))^(I*(n/2))], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rubi steps

$$\text{integral} = (8ib)\text{Subst}\left(\int \frac{1}{\left(1 - ia - \frac{1+ia}{x^4}\right)^2 x^4}\,dx, x, \frac{\sqrt[4]{1 + i(a + bx)}}{\sqrt[4]{1 - i(a + bx)}}\right)$$

$$= (8ib)\text{Subst}\left(\int \frac{x^4}{(-1 - ia + (1 - ia)x^4)^2}\,dx, x, \frac{\sqrt[4]{1 + i(a + bx)}}{\sqrt[4]{1 - i(a + bx)}}\right)$$

$$= -\frac{(i + a + bx)\sqrt[4]{1 + i(a + bx)}}{(i + a)x\sqrt[4]{1 - i(a + bx)}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{-1-ia+(1-ia)x^4}\,dx, x, \frac{\sqrt[4]{1 + i(a + bx)}}{\sqrt[4]{1 - i(a + bx)}}\right)}{i + a}$$

$$= -\frac{(i + a + bx)\sqrt[4]{1 + i(a + bx)}}{(i + a)x\sqrt[4]{1 - i(a + bx)}} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1 + i(a + bx)}}{\sqrt[4]{1 - i(a + bx)}}\right)}{\sqrt{i - a}(1 - ia)} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{i-a}+\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1 + i(a + bx)}}{\sqrt[4]{1 - i(a + bx)}}\right)}{\sqrt{i - a}(1 - ia)}$$

$$= -\frac{(i + a + bx)\sqrt[4]{1 + i(a + bx)}}{(i + a)x\sqrt[4]{1 - i(a + bx)}} + \frac{ib\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1 + i(a + bx)}}{\sqrt[4]{i-a}\sqrt[4]{1 - i(a + bx)}}\right)}{(i - a)^{3/4}(i + a)^{5/4}} + \frac{ib\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1 + i(a + bx)}}{\sqrt[4]{i-a}\sqrt[4]{1 - i(a + bx)}}\right)}{(i - a)^{3/4}(i + a)^{5/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.54

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{(-i(i + a + bx))^{3/4}\left(3(i + a)(-i + a + bx) + 2ibx\,\text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, \frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)\right)}{3(i + a)^2 x(1 + ia + ibx)^{3/4}}$$

```
[In] Integrate[E^((I/2)*ArcTan[a + b*x])/x^2,x]
[Out] (((-I)*(I + a + b*x))^(3/4)*(3*(I + a)*(-I + a + b*x) + (2*I)*b*x*Hypergeom
etric2F1[3/4, 1, 7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])
)/(3*(I + a)^2*x*(1 + I*a + I*b*x)^(3/4))
```

Maple [F]

$$\int \frac{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}{x^2}\,dx$$

[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)

[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(141) = 282$.

Time = 0.27 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.92

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{\left(-\frac{b^4}{a^8+2i\,a^7+2\,a^6+6i\,a^5+6i\,a^3-2\,a^2+2i\,a-1}\right)^{\frac{1}{4}}(-i\,a+1)x\log\left(\frac{b\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+\left(-\frac{b^4}{a^8+2i\,a^7+2\,a^6+6i\,a^5+6i\,a^3-2\,a^2+2i\,a-1}\right)}{b}\right.}{}$$

[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="frica s")

[Out] 1/2*((-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(-I*a + 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(a^2 + 1))/b) + (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(I*a - 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(a^2 + 1))/b) - (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(a + I)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(I*a^2 + I))/b) + (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(a + I)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(-I*a^2 - I))/b) - 2*(b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/((a + I)*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}{x^2}\,dx$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by -27, a substitution variable should perhaps be pur
ged.Wa
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}{x^2}\,dx$$

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x^2,x)
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x^2, x)
```

3.221 $\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 494

$$\begin{aligned}\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx =& -\frac{(17i+36a-24ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} \\ &- \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} \\ &+ \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} \\ &+ \frac{(17i+36a-24ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &- \frac{(17i+36a-24ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &+ \frac{(17i+36a-24ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} \\ &- \frac{(17i+36a-24ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}\end{aligned}$$

```
[Out] -1/24*(17*I+36*a-24*I*a^2)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^3-1/12
*(3*I+8*a)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(7/4)/b^3+1/3*x*(1-I*a-I*b*x)^
(1/4)*(1+I*a+I*b*x)^(7/4)/b^2+1/16*(17*I+36*a-24*I*a^2)*arctan(1-(1-I*a-I*b
*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/16*(17*I+36*a-24*I*a^2
)*arctan(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)+1/3
2*(17*I+36*a-24*I*a^2)*ln(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)
+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)-1/32*(17*I+36*a-24*I*
```

```
a^2)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/
2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5203, 92, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx =& \frac{(-24ia^2+36a+17i)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{8\sqrt{2}b^3}\\ &-\frac{(-24ia^2+36a+17i)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{8\sqrt{2}b^3}\\ &-\frac{(-24ia^2+36a+17i)\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{24b^3}\\ &+\frac{(-24ia^2+36a+17i)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}\\ &-\frac{(-24ia^2+36a+17i)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}\\ &-\frac{(8a+3i)\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{12b^3}\\ &+\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}\end{aligned}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] -1/24*((17*I + 36*a - (24*I)*a^2)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)
^(3/4))/b^3 - ((3*I + 8*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))
/(12*b^3) + (x*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(3*b^2) + (
(17*I + 36*a - (24*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
+ I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((17*I + 36*a - (24*I)*a^2)*ArcTan
[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]
*b^3) + ((17*I + 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 +
I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/
(16*Sqrt[2]*b^3) - ((17*I + 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x
]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*
x)^(1/4)])/(16*Sqrt[2]*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx$$

$$= \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} + \frac{\int \frac{(1+ia+ibx)^{3/4}\left(-1-a^2-\frac{1}{2}(3i+8a)bx\right)}{(1-ia-ibx)^{3/4}}\,dx}{3b^2}$$

$$= -\frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} - \frac{(17-36ia-24a^2)\int \frac{(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx}{24b^2}$$

$$= -\frac{(17i+36a-24ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} - \frac{(17-36ia-24a^2)\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx}{16b^2}$$

$$= -\frac{(17i+36a-24ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} - \frac{(17i+36a-24ia^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{4b^3}$$

$$= -\frac{(17i+36a-24ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} - \frac{(17i+36a-24ia^2)\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^3}$$

$$= -\frac{(17i+36a-24ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} - \frac{(17i+36a-24ia^2)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^3} - \frac{(17i+36a-24ia^2)\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^3}$$

$$= -\frac{(17i+36a-24ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} - \frac{(17i+36a-24ia^2)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16b^3} - \frac{(17i+36a-24ia^2)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16b^3} + \frac{(17i+36a-24ia^2)\,\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} + \frac{(17i+36a-24ia^2)\,\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}$$

$$= -\frac{(17i+36a-24ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} + \frac{(17i+36a-24ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} - \frac{(17i+36a-24ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} - \frac{(17i+36a-24ia^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} + \frac{(17i+36a-24ia^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3}$$

$$= -\frac{(17i+36a-24ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} - \frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} + \frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} + \frac{(17i+36a-24ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} - \frac{(17i+36a-24ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} + \frac{(17i+36a-24ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} - \frac{(17i+36a-24ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.24

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \frac{\sqrt[4]{-i(i+a+bx)}\left(-i(1+ia+ibx)^{3/4}\left(3+8a^2+7ibx-4b^2x^2+a(-5i+4bx)\right)+2i2^{3/4}(-17+36ia+24a\right.}{12b^3}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1/4)*((-I)*(1 + I*a + I*b*x)^(3/4)*(3 + 8*a^2 + (7*I
)*b*x - 4*b^2*x^2 + a*(-5*I + 4*b*x)) + (2*I)*2^(3/4)*(-17 + (36*I)*a + 24*
a^2)*Hypergeometric2F1[-3/4, 1/4, 5/4, (-1/2*I)*(I + a + b*x)]))/(12*b^3)
```

Maple [F]

$$\int \left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x^2dx$$

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.14

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx$$

$$= \frac{3\,b^3\sqrt{\frac{576i\,a^4-1728\,a^3-2112i\,a^2+1224\,a+289i}{b^6}}\log\left(\frac{b^3\sqrt{\frac{576i\,a^4-1728\,a^3-2112i\,a^2+1224\,a+289i}{b^6}}+(24\,a^2+36i\,a-17)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{24\,a^2+36i\,a-17}\right)}{}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 + 1224*a + 289*I)/b^6)*
log((b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 + 1224*a + 289*I)/b^6) + (
24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I
)))/(24*a^2 + 36*I*a - 17)) - 3*b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2
 + 1224*a + 289*I)/b^6)*log(-(b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 +
 1224*a + 289*I)/b^6) - (24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I)))/(24*a^2 + 36*I*a - 17)) - 3*b^3*sqrt((-576*I*a
^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)*log((b^3*sqrt((-576*I*a^4
 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6) + (24*a^2 + 36*I*a - 17)*sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 + 36*I*a - 1
7)) + 3*b^3*sqrt((-576*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)
*log(-(b^3*sqrt((-576*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)
- (24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
+ I)))/(24*a^2 + 36*I*a - 17)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(8*I*b
^2*x^2 - 2*(4*I*a - 7)*b*x + 8*I*a^2 - 46*a - 23*I)*sqrt(I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)/(b*x + a + I)))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)*x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \int x^2\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 14, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \int x^2\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}\,dx$$

```
[In] int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

3.222 $\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 16, antiderivative size = 410

$$\begin{aligned}\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx=&\frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2}\\&+\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2}\\&-\frac{3(3-4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}\\&+\frac{3(3-4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}\\&-\frac{3(3-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}\\&+\frac{3(3-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}\end{aligned}$$

```
[Out] 1/4*(3-4*I*a)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^2+1/2*(1-I*a-I*b*x)
^(1/4)*(1+I*a+I*b*x)^(7/4)/b^2-3/8*(3-4*I*a)*arctan(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)+3/8*(3-4*I*a)*arctan(1+(1-I*a-I*b*x
)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)-3/16*(3-4*I*a)*ln(1-(1-I*a
-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)
^(1/2))/b^2*2^(1/2)+3/16*(3-4*I*a)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+
I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5203, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = -\frac{3(3-4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{4\sqrt{2}b^2} + \frac{3(3-4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{4\sqrt{2}b^2} + \frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2} + \frac{(3-4ia)\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{4b^2} - \frac{3(3-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{8\sqrt{2}b^2} + \frac{3(3-4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{8\sqrt{2}b^2}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])*x,x]
```

```
[Out] ((3 - (4*I)*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(2*b^2) - (3*(3 - (4*I)*a)
*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*
Sqrt[2]*b^2) + (3*(3 - (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4)
)/(1 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) - (3*(3 - (4*I)*a)*Log[1 + Sqrt
[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))
/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) + (3*(3 - (4*I)*a)*Log[1 + Sqrt[
1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/
(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx \\
&= \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} - \frac{(3i+4a)\int \frac{(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx}{4b} \\
&= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} \\
&\quad - \frac{(3(3i+4a))\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx}{8b} \\
&= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} \\
&\quad + \frac{(3(3-4ia))\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{2b^2} \\
&= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} \\
&\quad + \frac{(3(3-4ia))\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b^2}
\end{aligned}
$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^2} + \frac{(3(3-4ia))\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^2}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} + \frac{(3(3-4ia))\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^2} + \frac{(3(3-4ia))\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^2} - \frac{(3(3-4ia))\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} - \frac{(3(3-4ia))\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} - \frac{3(3-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} + \frac{3(3-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} + \frac{(3(3-4ia))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} - \frac{(3(3-4ia))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}$$

$$= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2} - \frac{3(3-4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} + \frac{3(3-4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} - \frac{3(3-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} + \frac{3(3-4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.19

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \frac{\sqrt[4]{-i(i+a+bx)}\left((1+ia+ibx)^{7/4}+2\,2^{3/4}(3-4ia)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4},\frac{5}{4},-\frac{1}{2}i(i+a+bx)\right)\right)}{2b^2}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])*x,x]
[Out] (((-I)*(I + a + b*x))^(1/4)*((1 + I*a + I*b*x)^(7/4) + 2*2^(3/4)*(3 - (4*I)
*a)*Hypergeometric2F1[-3/4, 1/4, 5/4, (-1/2*I)*(I + a + b*x)]))/(2*b^2)
```

Maple [F]

$$\int \left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} xdx$$

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.05

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx$$
$$= \frac{3\,b^2\sqrt{-\frac{16i\,a^2-24\,a-9i}{b^4}}\log\left(\frac{b^2\sqrt{-\frac{16i\,a^2-24\,a-9i}{b^4}}+(4\,a+3i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a+3i}\right)-3\,b^2\sqrt{-\frac{16i\,a^2-24\,a-9i}{b^4}}\log\left(-\frac{b^2\sqrt{-\frac{16i}{}}}{}\right.}{}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="fricas"
)
```

```
[Out] 1/8*(3*b^2*sqrt(-(16*I*a^2 - 24*a - 9*I)/b^4)*log((b^2*sqrt(-(16*I*a^2 - 24
*a - 9*I)/b^4) + (4*a + 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
+ a + I)))/(4*a + 3*I)) - 3*b^2*sqrt(-(16*I*a^2 - 24*a - 9*I)/b^4)*log(-(b^
2*sqrt(-(16*I*a^2 - 24*a - 9*I)/b^4) - (4*a + 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a + 3*I)) - 3*b^2*sqrt(-(-16*I*a^2 + 24
*a + 9*I)/b^4)*log((b^2*sqrt(-(-16*I*a^2 + 24*a + 9*I)/b^4) + (4*a + 3*I)*s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a + 3*I)) + 3*b^
2*sqrt(-(-16*I*a^2 + 24*a + 9*I)/b^4)*log(-(b^2*sqrt(-(-16*I*a^2 + 24*a + 9
*I)/b^4) - (4*a + 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
I)))/(4*a + 3*I)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*I*b*x - 2*I*a +
5)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)*x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \int x\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="maxima"
)
```

```
[Out] integrate(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 14, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \int x\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}\,dx$$

```
[In] int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

3.223 $\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 338

$$\begin{aligned}\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx &= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b}-\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}\\&+\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}\\&-\frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}\\&+\frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}\end{aligned}$$

```
[Out] I*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b-3/2*I*arctan(1-(1-I*a-I*b*x)^(1
/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+3/2*I*arctan(1+(1-I*a-I*b*x)^(1/
4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)-3/4*I*ln(1-(1-I*a-I*b*x)^(1/4)*2^
(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2
)+3/4*I*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^
(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5201, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int e^{\frac{3}{2} i \arctan(a+bx)} \, dx = & -\frac{3 i \arctan\left(1-\frac{\sqrt{2} \sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2} b} \\ & +\frac{3 i \arctan\left(1+\frac{\sqrt{2} \sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2} b} \\ & +\frac{i \sqrt[4]{-ia-ibx+1} (ia+ibx+1)^{3/4}}{b} \\ & -\frac{3 i \log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2} \sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2 \sqrt{2} b} \\ & +\frac{3 i \log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2} \sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2 \sqrt{2} b}\end{aligned}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] (I*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b - ((3*I)*ArcTan[1 - (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + ((
3*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])
/(Sqrt[2]*b) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*
x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b
) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[
2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1+ia+ibx)^{3/4}}{(1-ia-ibx)^{3/4}}\,dx$$

$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{3}{2}\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx$$

$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(6i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b}$$

$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(6i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$

$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(3i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} + \frac{(3i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b}$$

$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(3i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} + \frac{(3i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} - \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} - \frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$

$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} - \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} + \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} + \frac{(3i)\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{(3i)\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}$$

$$= \frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} - \frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} + \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.13

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = -\frac{8ie^{\frac{7}{2}i\arctan(a+bx)}\,\mathrm{Hypergeometric2F1}\left(\frac{7}{4},2,\frac{11}{4},-e^{2i\arctan(a+bx)}\right)}{7b}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-8*I)/7)*E^(((7*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[7/4, 2, 11/4, -
E^((2*I)*ArcTan[a + b*x])])/b
```

Maple [F]

$$\int \left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} dx$$

[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)

[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.79

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx$$
$$= \frac{b\sqrt{\frac{9i}{b^2}}\log\left(\frac{1}{3}b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{9i}{b^2}}\log\left(-\frac{1}{3}b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{-\frac{9i}{b^2}}\log\left(\frac{1}{3}\right.}{}$$

[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas")

[Out] 1/2*(b*sqrt(9*I/b^2)*log(1/3*b*sqrt(9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(9*I/b^2)*log(-1/3*b*sqrt(9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(-9*I/b^2)*log(1/3*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-9*I/b^2)*log(-1/3*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \text{Timed out}$$

[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \int \left(\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 14, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \int \left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}\,dx$$

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

3.224 $\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 427

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \frac{2(i-a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) - \frac{2(i-a)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} + \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}}$$

```
[Out] 2*(I-a)^(3/4)*arctan((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b
*x)^(1/4))/(I+a)^(3/4)-2*(I-a)^(3/4)*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4
)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I+a)^(3/4)+1/2*ln(1-(1-I*a-I*b*x)^(1/4)
*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))*2^(1/
2)-1/2*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(
1/2)/(1+I*a+I*b*x)^(1/2))*2^(1/2)+arctan(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I
*a+I*b*x)^(1/4))*2^(1/2)-arctan(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)
^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5203, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 12, 95, 304, 211, 214}

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \frac{2(-a+i)^{3/4}\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(a+i)^{3/4}} + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right) - \frac{2(-a+i)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(a+i)^{3/4}} + \frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}} - \frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] (2*(I - a)^(3/4)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1
/4)*(1 - I*a - I*b*x)^(1/4))])/(I + a)^(3/4) + Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - (2*(I - a)^(3/4)*A
rcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*
b*x)^(1/4))])/(I + a)^(3/4) + Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a +
I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
 - Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a
- I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+ia+ibx)^{3/4}}{x(1-ia-ibx)^{3/4}}\,dx\\
&= (ib)\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx+\int \frac{1+ia}{x(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx\\
&= -\left(4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx,x,\sqrt[4]{1-ia-ibx}\right)\right)\\
&\quad+(1+ia)\int \frac{1}{x(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx\\
&= -\left(4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)\right)\\
&\quad+(4(1+ia))\text{Subst}\left(\int \frac{x^2}{-1-ia-(-1+ia)x^4}\,dx,x,\frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)\right)\\
&\quad-2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)\\
&\quad-\frac{(2(i-a))\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx,x,\frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{\sqrt{i+a}}\\
&\quad+\frac{(2(i-a))\text{Subst}\left(\int \frac{1}{\sqrt{i-a}+\sqrt{i+a}x^2}\,dx,x,\frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{\sqrt{i+a}}\\
&= \frac{2(i-a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}}-\frac{2(i-a)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}}\\
&\quad-\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)-\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)
\end{aligned}
$$

$$= \frac{2(i-a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} - \frac{2(i-a)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} + \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} - \sqrt{2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) + \sqrt{2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+i}}\right.$$

$$= \frac{2(i-a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) - \frac{2(i-a)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} + \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.29

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = 2\sqrt[4]{-i(i+a+bx)}\left(-2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{1}{2}i(i+a+bx)\right) + \frac{2(-i+a)\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)}{(i+a)\sqrt[4]{1+ia+ibx}}\right)$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])/x,x]
```

```
[Out] 2*((-I)*(I + a + b*x))^(1/4)*(-(2^(3/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (-
1/2*I)*(I + a + b*x)]) + (2*(-I + a)*Hypergeometric2F1[1/4, 1, 5/4, (1 + a^
2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I + a)*(1 + I*a + I*b*x)^(
1/4)))
```

Maple [F]

$$\int \frac{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}{x}dx$$

[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)

[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(284) = 568.

Time = 0.29 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.62

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)$$
$$-\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)$$
$$-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)$$
$$+\frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)$$
$$-\left(-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}\right)^{\frac{1}{4}}\log\left(\frac{(a^2+2i\,a-1)\left(-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}\right)^{\frac{3}{4}}+(a^2-2i\,a-1)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a^2-2i\,a-1}\right)$$
$$+\left(-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}\right)^{\frac{1}{4}}\log\left(-\frac{(a^2+2i\,a-1)\left(-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}\right)^{\frac{3}{4}}-(a^2-2i\,a-1)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a^2-2i\,a-1}\right)$$
$$+i\left(-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}\right)^{\frac{1}{4}}\log\left(\frac{(i\,a^2-2\,a-i)\left(-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}\right)^{\frac{3}{4}}+(a^2-2i\,a-1)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a^2-2i\,a-1}\right)$$
$$-i\left(-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}\right)^{\frac{1}{4}}\log\left(\frac{(-i\,a^2+2\,a+i)\left(-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}\right)^{\frac{3}{4}}+(a^2-2i\,a-1)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a^2-2i\,a-1}\right)$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)/(b*x + a + I))) - 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I
) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*
I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
a + I))) - (-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log
(((a^2 + 2*I*a - 1)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^
(3/4) + (a^2 - 2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
 + I)))/(a^2 - 2*I*a - 1)) + (-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3
*a - I))^(1/4)*log(-((a^2 + 2*I*a - 1)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3
*I*a^2 - 3*a - I))^(3/4) - (a^2 - 2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)/(b*x + a + I)))/(a^2 - 2*I*a - 1)) + I*(-(a^3 - 3*I*a^2 - 3*a +
I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log(((I*a^2 - 2*a - I)*(-(a^3 - 3*I*a^2
 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(3/4) + (a^2 - 2*I*a - 1)*sqrt(I*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a^2 - 2*I*a - 1)) - I*(-(a^
3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log(((-I*a^2 + 2*a
+ I)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(3/4) + (a^2 -
2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a^2 -
2*I*a - 1))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \text{Timed out}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 14, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}{x}\,dx$$

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x, x)
```

3.225 $\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 211

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{3ib\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{\sqrt[4]{i-a}(i+a)^{7/4}} + \frac{3ib\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{\sqrt[4]{i-a}(i+a)^{7/4}}$$

```
[Out] -(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/(1-I*a)/x-3*I*b*arctan((I+a)^(1/4)
*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I-a)^(1/4)/(I+a)^(7/
4)+3*I*b*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^
(1/4))/(I-a)^(1/4)/(I+a)^(7/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{5203, 96, 95, 304, 211, 214\}$

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{3ib\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}(a+i)^{7/4}} + \frac{3ib\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{\sqrt[4]{-a+i}(a+i)^{7/4}} - \frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{(1-ia)x}$$

```
[In] Int[E^(((3*I)/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] -(((1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/((1 - I*a)*x)) - ((3*I)
*b*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a -
 I*b*x)^(1/4))])/((I - a)^(1/4)*(I + a)^(7/4)) + ((3*I)*b*ArcTanh[((I + a)^
(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((
I - a)^(1/4)*(I + a)^(7/4))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+ia+ibx)^{3/4}}{x^2(1-ia-ibx)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{(3b)\int \frac{1}{x(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx}{2(i+a)} \\
&= -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{(6b)\text{Subst}\left(\int \frac{x^2}{-1-ia-(-1+ia)x^4}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{i+a} \\
&= -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} \\
&\quad + \frac{(3ib)\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/2}} \\
&\quad - \frac{(3ib)\text{Subst}\left(\int \frac{1}{\sqrt{i-a}+\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/2}} \\
&= -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{3ib\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{\sqrt[4]{i-a}(i+a)^{7/4}} \\
&\quad + \frac{3ib\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{\sqrt[4]{i-a}(i+a)^{7/4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.50

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{\sqrt[4]{-i(i+a+bx)}\left(1+a^2+ibx+abx+6ibx\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)\right)}{(i+a)^2x\sqrt[4]{1+ia+ibx}}$$

```
[In] Integrate[E^(((3*I)/2)*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1/4)*(1 + a^2 + I*b*x + a*b*x + (6*I)*b*x*Hypergeome
tric2F1[1/4, 1, 5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))
/((I + a)^2*x*(1 + I*a + I*b*x)^(1/4))
```

Maple [F]

$$\int \frac{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}{x^2}dx$$

```
[In] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 694 vs. 2(137) = 274.

Time = 0.27 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.29

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{3\left(-\frac{b^4}{a^8+6i\,a^7-14\,a^6-14i\,a^5-14i\,a^3+14\,a^2+6i\,a-1}\right)^{\frac{1}{4}}(-i\,a+1)x\log\left(\frac{b^3\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+(a^6+4i\,a^5-5\,a^4-5\,a^2-4i\,a+1)\left(-\frac{}{a^8}\right.}{b^3}\right.}{}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="frica
s")
```

```
[Out] 1/2*(3*(-b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a
 - 1))^(1/4)*(-I*a + 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
```

```
/(b*x + a + I)) + (a^6 + 4*I*a^5 - 5*a^4 - 5*a^2 - 4*I*a + 1)*(-b^4/(a^8 +
6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(3/4))/b^3) +
 3*(-b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1
))^(1/4)*(I*a - 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
 + a + I)) - (a^6 + 4*I*a^5 - 5*a^4 - 5*a^2 - 4*I*a + 1)*(-b^4/(a^8 + 6*I*a
^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(3/4))/b^3) + 3*(-
b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(1
/4)*(a + I)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
I)) - (I*a^6 - 4*a^5 - 5*I*a^4 - 5*I*a^2 + 4*a + I)*(-b^4/(a^8 + 6*I*a^7 -
14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(3/4))/b^3) - 3*(-b^4/(
a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(1/4)*(
a + I)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) -
 (-I*a^6 + 4*a^5 + 5*I*a^4 + 5*I*a^2 - 4*a - I)*(-b^4/(a^8 + 6*I*a^7 - 14*a
^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(3/4))/b^3) - 2*I*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
+ I)))/((a + I)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx=\int \frac{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 14, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}{x^2}\,dx$$

```
[In] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x^2, x)
```

3.226 $\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 494

$$\begin{aligned}\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx &= \frac{(3i-4a-8ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3}\\ &+\frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3}\\ &+\frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2}\\ &+\frac{(3i-4a-8ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3}\\ &-\frac{(3i-4a-8ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3}\\ &+\frac{(3i-4a-8ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}\\ &-\frac{(3i-4a-8ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}\end{aligned}$$

```
[Out] 1/8*(3*I-4*a-8*I*a^2)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^3+1/12*(I-8
*a)*(1-I*a-I*b*x)^(5/4)*(1+I*a+I*b*x)^(3/4)/b^3+1/3*x*(1-I*a-I*b*x)^(5/4)*(
1+I*a+I*b*x)^(3/4)/b^2+1/16*(3*I-4*a-8*I*a^2)*arctan(1-(1-I*a-I*b*x)^(1/4)*
2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/16*(3*I-4*a-8*I*a^2)*arctan(1+(1
-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)+1/32*(3*I-4*a-8*
I*a^2)*ln(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(
1/2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)-1/32*(3*I-4*a-8*I*a^2)*ln(1+(1-I*a-I*
```

```
b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1
/2))/b^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5203, 92, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx=&\frac{(-8ia^2-4a+3i)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{8\sqrt{2}b^3}\\&-\frac{(-8ia^2-4a+3i)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{8\sqrt{2}b^3}\\&+\frac{(-8ia^2-4a+3i)\,(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{8b^3}\\&+\frac{(-8ia^2-4a+3i)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}\\&-\frac{(-8ia^2-4a+3i)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3}\\&+\frac{(-8a+i)(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{12b^3}\\&+\frac{x(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{3b^2}\end{aligned}$$

```
[In] Int[x^2/E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((3*I - 4*a - (8*I)*a^2)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(
8*b^3) + ((I - 8*a)*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(12*b^
3) + (x*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(3*b^2) + ((3*I -
4*a - (8*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*
b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((3*I - 4*a - (8*I)*a^2)*ArcTan[1 + (Sqrt[2]
*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) + ((3*I
 - 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt[2]*b^3)
 - ((3*I - 4*a - (8*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*
b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(16*Sqrt
[2]*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx$$

$$= \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} + \frac{\int \frac{\sqrt[4]{1-ia-ibx}\left(-1-a^2+\frac{1}{2}(i-8a)bx\right)}{\sqrt[4]{1+ia+ibx}}\,dx}{3b^2}$$

$$= \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} - \frac{(3+4ia-8a^2)\int \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx}{8b^2}$$

$$= -\frac{(4a-i(3-8a^2))\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} - \frac{(3+4ia-8a^2)\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx}{16b^2}$$

$$= -\frac{(4a-i(3-8a^2))\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} - \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{4b^3}$$

$$= -\frac{(4a-i(3-8a^2))\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} - \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^3}$$

$$= -\frac{(4a-i(3-8a^2))\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} - \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^3} - \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^3}$$

$$= -\frac{(4a-i(3-8a^2))\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} - \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16b^3} - \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16b^3} + \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} + \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}$$

$$= -\frac{(4a-i(3-8a^2))\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} + \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} + \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} - \frac{(4a-i(3-8a^2))\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} + \frac{(4a-i(3-8a^2))\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} - \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} + \frac{(i(3+4ia-8a^2))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3}$$

$$= -\frac{(4a - i(3 - 8a^2))\sqrt[4]{1 - ia - ibx}(1 + ia + ibx)^{3/4}}{8b^3}$$
$$+ \frac{(i - 8a)(1 - ia - ibx)^{5/4}(1 + ia + ibx)^{3/4}}{12b^3} + \frac{x(1 - ia - ibx)^{5/4}(1 + ia + ibx)^{3/4}}{3b^2}$$
$$- \frac{(4a - i(3 - 8a^2))\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{1 - ia - ibx}}{\sqrt[4]{1 + ia + ibx}}\right)}{8\sqrt{2}b^3}$$
$$+ \frac{(4a - i(3 - 8a^2))\arctan\left(1 + \frac{\sqrt{2}\sqrt[4]{1 - ia - ibx}}{\sqrt[4]{1 + ia + ibx}}\right)}{8\sqrt{2}b^3}$$
$$- \frac{(4a - i(3 - 8a^2))\log\left(1 + \frac{\sqrt{1 - ia - ibx}}{\sqrt{1 + ia + ibx}} - \frac{\sqrt{2}\sqrt[4]{1 - ia - ibx}}{\sqrt[4]{1 + ia + ibx}}\right)}{16\sqrt{2}b^3}$$
$$+ \frac{(4a - i(3 - 8a^2))\log\left(1 + \frac{\sqrt{1 - ia - ibx}}{\sqrt{1 + ia + ibx}} + \frac{\sqrt{2}\sqrt[4]{1 - ia - ibx}}{\sqrt[4]{1 + ia + ibx}}\right)}{16\sqrt{2}b^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.20

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx$$
$$= \frac{(-i(i + a + bx))^{5/4}\left(5(1 + ia + ibx)^{3/4}(i - 8a + 4bx) + 3\,2^{3/4}(-3i + 4a + 8ia^2)\,\text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{5}{4}, \ldots\right.\right.}{60b^3}$$

```
[In] Integrate[x^2/E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-I)*(I + a + b*x))^(5/4)*(5*(1 + I*a + I*b*x)^(3/4)*(I - 8*a + 4*b*x) +
3*2^(3/4)*(-3*I + 4*a + (8*I)*a^2)*Hypergeometric2F1[1/4, 5/4, 9/4, (-1/2*I
)*(I + a + b*x)]))/(60*b^3)
```

Maple [F]

$$\int \frac{x^2}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}dx$$

```
[In] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.14

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx$$

$$= \frac{3\,b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2-24\,a+9i}{b^6}}\log\left(\frac{b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2-24\,a+9i}{b^6}}+(8\,a^2-4i\,a-3)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{8\,a^2-4i\,a-3}\right)-3\,b^3\sqrt{\frac{64i\,a^4+64\,a}{}}}{}$$

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6)*log((b^3*
sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6) + (8*a^2 - 4*I*a - 3)
*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a -
3)) - 3*b^3*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6)*log(-(b^3
*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6) - (8*a^2 - 4*I*a - 3
)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a -
 3)) - 3*b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6)*log((b^
3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6) + (8*a^2 - 4*I*a -
 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a
 - 3)) + 3*b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6)*log(-
(b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6) - (8*a^2 - 4*I*
a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*
I*a - 3)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-8*I*b^2*x^2 - 2*(-4*I*a -
 5)*b*x - 8*I*a^2 - 26*a + 11*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)))/b^3
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

```
[In] integrate(x**2/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x
)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxim
a")
[Out] integrate(x^2/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 14, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{1+a\,1{}i+b\,x\,1{}i}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

```
[In] int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
[Out] int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

3.227 $\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 16, antiderivative size = 410

$$\begin{aligned}\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx &= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2}\\ &\quad+\frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2}\\ &\quad+\frac{(1+4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}\\ &\quad-\frac{(1+4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}\\ &\quad+\frac{(1+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}\\ &\quad-\frac{(1+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}\end{aligned}$$

```
[Out] 1/4*(1+4*I*a)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^2+1/2*(1-I*a-I*b*x)
^(5/4)*(1+I*a+I*b*x)^(3/4)/b^2+1/8*(1+4*I*a)*arctan(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)-1/8*(1+4*I*a)*arctan(1+(1-I*a-I*b*x
)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)+1/16*(1+4*I*a)*ln(1-(1-I*a
-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)
^(1/2))/b^2*2^(1/2)-1/16*(1+4*I*a)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+
I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5203, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \frac{(1+4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{4\sqrt{2}b^2} - \frac{(1+4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{4\sqrt{2}b^2} + \frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2} + \frac{(1+4ia)(ia+ibx+1)^{3/4}\sqrt[4]{-ia-ibx+1}}{4b^2} + \frac{(1+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{8\sqrt{2}b^2} - \frac{(1+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{8\sqrt{2}b^2}$$

```
[In] Int[x/E^((I/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((1 + (4*I)*a)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(2*b^2) + ((1 + (4*I)*a)*A
rcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*Sq
rt[2]*b^2) - ((1 + (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1
 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) + ((1 + (4*I)*a)*Log[1 + Sqrt[1 - I
*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) - ((1 + (4*I)*a)*Log[1 + Sqrt[1 - I*a
- I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a
 + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx$$

$$= \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(i-4a)\int \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx}{4b}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} + \frac{(i-4a)\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx}{8b}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{2b^2}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} - \frac{(1+4ia)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b^2}$$

$$=\frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2}+\frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2}$$
$$-\frac{(1+4ia)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^2}$$
$$-\frac{(1+4ia)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^2}$$

$$=\frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2}+\frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2}$$
$$-\frac{(1+4ia)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^2}$$
$$-\frac{(1+4ia)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^2}$$
$$+\frac{(1+4ia)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$
$$+\frac{(1+4ia)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$

$$=\frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2}+\frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2}$$
$$+\frac{(1+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$
$$-\frac{(1+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$
$$-\frac{(1+4ia)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}$$
$$+\frac{(1+4ia)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}$$

$$= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} + \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2}$$
$$+ \frac{(1+4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}$$
$$- \frac{(1+4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}$$
$$+ \frac{(1+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$
$$- \frac{(1+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.20

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx =$$
$$-\frac{i(-i(i+a+bx))^{5/4}\left(5i(1+ia+ibx)^{3/4}+2^{3/4}(-i+4a)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},-\frac{1}{2}i(i+a+bx)\right)\right.}{10b^2}$$

```
[In] Integrate[x/E^((I/2)*ArcTan[a + b*x]),x]
[Out] ((-1/10*I)*((-I)*(I + a + b*x))^(5/4)*((5*I)*(1 + I*a + I*b*x)^(3/4) + 2^(3
/4)*(-I + 4*a)*Hypergeometric2F1[1/4, 5/4, 9/4, (-1/2*I)*(I + a + b*x)]))/b
^2
```

Maple [F]

$$\int \frac{x}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}dx$$

```
[In] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
[Out] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.03

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx =$$

$$-\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}\log\left(\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}+(4\,a-i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a-i}\right)-b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}\log\left(-\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}-(4\,a-i)\sqrt{}}{4\,a-i}\right.}{}$$

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/8*(b^2*sqrt((16*I*a^2 + 8*a - I)/b^4)*log((b^2*sqrt((16*I*a^2 + 8*a - I)
/b^4) + (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/
(4*a - I)) - b^2*sqrt((16*I*a^2 + 8*a - I)/b^4)*log(-(b^2*sqrt((16*I*a^2 +
8*a - I)/b^4) - (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
 + I)))/(4*a - I)) - b^2*sqrt((-16*I*a^2 - 8*a + I)/b^4)*log((b^2*sqrt((-16
*I*a^2 - 8*a + I)/b^4) + (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I)))/(4*a - I)) + b^2*sqrt((-16*I*a^2 - 8*a + I)/b^4)*log(-(b^2
*sqrt((-16*I*a^2 - 8*a + I)/b^4) - (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)/(b*x + a + I)))/(4*a - I)) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(-2*I*b*x + 2*I*a + 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I)))/b^2
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 14, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

```
[In] int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

3.228 $\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 338

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} - \frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} + \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$

```
[Out] -I*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b-1/2*I*arctan(1-(1-I*a-I*b*x)^(
1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+1/2*I*arctan(1+(1-I*a-I*b*x)^(1
/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)-1/4*I*ln(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/
2)+1/4*I*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)
^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5201, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = -\frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}b} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}b} - \frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b} - \frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b} + \frac{i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}$$

```
[In] Int[E^((-1/2*I)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b - (I*ArcTan[1 - (S
qrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + (I*
ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqr
t[2]*b) - ((I/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqr
t[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) + ((I/2
)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a -
 I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\,dx \\
&= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{1}{2}\int \frac{1}{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}\,dx \\
&= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(2i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right)}{b} \\
&= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{(2i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} \\
&= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} \\
&\quad + \frac{i\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} \\
&= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} + \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} \\
&\quad + \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} \\
&\quad - \frac{i\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} \\
&\quad - \frac{i\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}
\end{aligned}$$

$$= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$
$$+ \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$
$$+ \frac{i\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}$$
$$- \frac{i\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}$$

$$= -\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b} - \frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}$$
$$+ \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$
$$+ \frac{i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.13

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = -\frac{8ie^{\frac{3}{2}i\arctan(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2i\arctan(a+bx)}\right)}{3b}$$

```
[In] Integrate[E^((-1/2*I)*ArcTan[a + b*x]),x]
```

```
[Out] (((-8*I)/3)*E^(((3*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[3/4, 2, 7/4, -E
^((2*I)*ArcTan[a + b*x])])/b
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}dx$$

[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)

[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.79

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = \frac{b\sqrt{\frac{i}{b^2}}\log\left(b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{i}{b^2}}\log\left(-b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{-\frac{i}{b^2}}\log\left(b\sqrt{-\right.}$$

[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas"
)

[Out] 1/2*(b*sqrt(I/b^2)*log(b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)/(b*x + a + I))) - b*sqrt(I/b^2)*log(-b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(-I/b^2)*log(b*sqrt(-I/b^2)
 + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-I/b^2
)*log(-b*sqrt(-I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I))) - 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)/(b*x + a + I)))/b

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\sqrt{\frac{i(a+bx)+1}{\sqrt{(a+bx)^2+1}}}}\,dx$$

[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)

[Out] Integral(1/sqrt((I*(a + b*x) + 1)/sqrt((a + b*x)**2 + 1)), x)

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by -27, a substitution variable should perhaps be pur
ged.Wa
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

```
[In] int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

3.229 $\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 395

$$\begin{aligned}\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = &-\frac{2\sqrt[4]{i+a}\arctan\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} \\ &-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) \\ &+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) \\ &-\frac{2\sqrt[4]{i+a}\operatorname{arctanh}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} \\ &-\frac{\log\left(1+\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}} \\ &+\frac{\log\left(1+\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}}\end{aligned}$$

```
[Out] -2*(I+a)^(1/4)*arctan((I-a)^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)/(1+I*(b*x
+a))^(1/4))/(I-a)^(1/4)-2*(I+a)^(1/4)*arctanh((I-a)^(1/4)*(1-I*(b*x+a))^(1/
4)/(I+a)^(1/4)/(1+I*(b*x+a))^(1/4))/(I-a)^(1/4)-1/2*ln(1-(1-I*(b*x+a))^(1/4
)*2^(1/2)/(1+I*(b*x+a))^(1/4)+1/(1+I*(b*x+a))^(1/2)*(1-I*(b*x+a))^(1/2))*2^
(1/2)+1/2*ln(1+(1-I*(b*x+a))^(1/4)*2^(1/2)/(1+I*(b*x+a))^(1/4)+1/(1+I*(b*x+
a))^(1/2)*(1-I*(b*x+a))^(1/2))*2^(1/2)-arctan(1-(1-I*(b*x+a))^(1/4)*2^(1/2)
```

```
/(1+I*(b*x+a))^(1/4))*2^(1/2)+arctan(1+(1-I*(b*x+a))^(1/4)*2^(1/2)/(1+I*(b*
x+a))^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5202, 492, 217, 1179, 642, 1176, 631, 210, 218, 214, 211}

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = -\frac{2\sqrt[4]{a+i}\arctan\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{-a+i}} - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) - \frac{2\sqrt[4]{a+i}\,\text{arctanh}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{-a+i}} - \frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}} + \frac{\log\left(\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}+1\right)}{\sqrt{2}}$$

```
[In] Int[1/(E^((I/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (-2*(I + a)^(1/4)*ArcTan[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(
1/4)*(1 + I*(a + b*x))^(1/4))])/(I - a)^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)] - (2*(I + a)^(1/4)*
ArcTanh[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a +
b*x))^(1/4))])/(I - a)^(1/4) - Log[1 + Sqrt[1 - I*(a + b*x)]/Sqrt[1 + I*(a
+ b*x)] - (Sqrt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2
] + Log[1 + Sqrt[1 - I*(a + b*x)]/Sqrt[1 + I*(a + b*x)] + (Sqrt[2]*(1 - I*(
a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

& (LtQ[a, 0] || LtQ[b, 0])

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5202

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[x^(2/(I*n))*((1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m/(1 + x^(2/(I*n)))^(m + 2)), x], x, (1 - I*c*(a + b*x))^
(I*(n/2))/(1 + I*c*(a + b*x))^(I*(n/2))], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rubi steps

$$\text{integral} = -\left(8\text{Subst}\left(\int \frac{x^4}{(1+x^4)\left(1-ia-(1+ia)x^4\right)}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)\right)$$

$$= 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) - (4(1-ia))\text{Subst}\left(\int \frac{1}{1-ia+(-1-ia)x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)$$

$$= 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) - \left(2\sqrt{i+a}\right)\text{Subst}\left(\int \frac{1}{\sqrt{i+a}-\sqrt{i-a}x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) - \left(2\sqrt{i+a}\right)\text{Subst}\left(\int \frac{1}{\sqrt{i+a}+\sqrt{i-a}x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)$$

$$= -\frac{2\sqrt[4]{i+a}\arctan\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{2\sqrt[4]{i+a}\operatorname{arctanh}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}} + \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) + \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)$$

$$= -\frac{2\sqrt[4]{i+a}\arctan\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{2\sqrt[4]{i+a}\operatorname{arctanh}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{\log\left(1+\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}} + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)$$

$$= -\frac{2\sqrt[4]{i+a}\arctan\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) - \frac{2\sqrt[4]{i+a}\operatorname{arctanh}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} - \frac{\log\left(1+\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.32

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \frac{2\sqrt[4]{-i(i+a+bx)}\left(2^{3/4}\sqrt[4]{1+ia+ibx}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{1}{2}i(i+a+bx)\right)-2\,\text{Hypergeometric2}\right.}{\sqrt[4]{1+ia+ibx}}$$

```
[In] Integrate[1/(E^((I/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*((-I)*(I + a + b*x))^(1/4)*(2^(3/4)*(1 + I*a + I*b*x)^(1/4)*Hypergeometr
ic2F1[1/4, 1/4, 5/4, (-1/2*I)*(I + a + b*x)] - 2*Hypergeometric2F1[1/4, 1,
5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/(1 + I*a + I*b*
x)^(1/4)
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\,x}\,dx$$

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.19

$$\int \frac{e^{-\frac{1}{2} i \arctan(a+bx)}}{x} \, dx$$
$$= -\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) + \frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) + \frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - \frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) + \left(-\frac{a+i}{a-i}\right)^{\frac{1}{4}}\log\left(\frac{(a-i)\left(-\frac{a+i}{a-i}\right)^{\frac{3}{4}}+(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a+i}\right) - \left(-\frac{a+i}{a-i}\right)^{\frac{1}{4}}\log\left(-\frac{(a-i)\left(-\frac{a+i}{a-i}\right)^{\frac{3}{4}}-(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a+i}\right) - i\left(-\frac{a+i}{a-i}\right)^{\frac{1}{4}}\log\left(\frac{(i\,a+1)\left(-\frac{a+i}{a-i}\right)^{\frac{3}{4}}+(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a+i}\right) + i\left(-\frac{a+i}{a-i}\right)^{\frac{1}{4}}\log\left(\frac{(-i\,a-1)\left(-\frac{a+i}{a-i}\right)^{\frac{3}{4}}+(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a+i}\right)$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/(b*x + a + I))) + 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*
I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4
*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
 a + I))) + (-(a + I)/(a - I))^(1/4)*log(((a - I)*(-(a + I)/(a - I))^(3/4)
+ (a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I))
 - (-(a + I)/(a - I))^(1/4)*log(-((a - I)*(-(a + I)/(a - I))^(3/4) - (a + I
```

```
)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I)) - I*(-(
a + I)/(a - I))^(1/4)*log(((I*a + 1)*(-(a + I)/(a - I))^(3/4) + (a + I)*sqr
t(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I)) + I*(-(a + I
)/(a - I))^(1/4)*log(((-I*a - 1)*(-(a + I)/(a - I))^(3/4) + (a + I)*sqrt(I*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I))
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x,x)
[Out] Integral(1/(x*sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))),
x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="maxim
a")
[Out] integrate(1/(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 14, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

```
[In] int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)), x)
```

3.230 $\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 210

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{ib\arctan\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}} - \frac{ib\text{arctanh}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}}$$

```
[Out] -(I-a-b*x)*(1-I*(b*x+a))^(1/4)/(I-a)/x/(1+I*(b*x+a))^(1/4)-I*b*arctan((I-a)
^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)/(1+I*(b*x+a))^(1/4))/(I-a)^(5/4)/(I+
a)^(3/4)-I*b*arctanh((I-a)^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)/(1+I*(b*x+
a))^(1/4))/(I-a)^(5/4)/(I+a)^(3/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5202, 294, 218, 214, 211}

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{ib\arctan\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}} - \frac{ib\text{arctanh}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{(-a+i)^{5/4}(a+i)^{3/4}} - \frac{\sqrt[4]{1-i(a+bx)}(-a-bx+i)}{(-a+i)x\sqrt[4]{1+i(a+bx)}}$$

```
[In] Int[1/(E^((I/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -(((I - a - b*x)*(1 - I*(a + b*x))^(1/4))/((I - a)*x*(1 + I*(a + b*x))^(1/4
))) - (I*b*ArcTan[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1
 + I*(a + b*x))^(1/4))])/((I - a)^(5/4)*(I + a)^(3/4)) - (I*b*ArcTanh[((I -
 a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))]
)/((I - a)^(5/4)*(I + a)^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5202

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Dist
[4/(I^m*n*b^(m + 1)*c^(m + 1)), Subst[Int[x^(2/(I*n))*((1 - I*a*c - (1 + I*
a*c)*x^(2/(I*n)))^m/(1 + x^(2/(I*n)))^(m + 2)), x], x, (1 - I*c*(a + b*x))^
(I*(n/2))/(1 + I*c*(a + b*x))^(I*(n/2))], x] /; FreeQ[{a, b, c}, x] && ILtQ
[m, 0] && LtQ[-1, I*n, 1]
```

Rubi steps

$$\text{integral} = -\left((8ib)\text{Subst}\left(\int \frac{x^4}{(1-ia-(1+ia)x^4)^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)\right)$$

$$= -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{1-ia+(-1-ia)x^4}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{i-a}$$

$$= -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{i+a}-\sqrt{i-a}x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{(1+ia)\sqrt{i+a}} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{i+a}+\sqrt{i-a}x^2}\,dx, x, \frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{(1+ia)\sqrt{i+a}}$$

$$= -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{ib\arctan\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}} - \frac{ib\text{arctanh}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{\sqrt[4]{-i(i+a+bx)}\left(1+a^2+ibx+abx-2ibx\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)\right)}{(1+a^2)\,x\sqrt[4]{1+ia+ibx}}$$

```
[In] Integrate[1/(E^((I/2)*ArcTan[a + b*x])*x^2),x]
[Out] -((((-I)*(I + a + b*x))^(1/4)*(1 + a^2 + I*b*x + a*b*x - (2*I)*b*x*Hypergeo
metric2F1[1/4, 1, 5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]
))/((1 + a^2)*x*(1 + I*a + I*b*x)^(1/4)))
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\,x^2}dx$$

[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)

[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(141) = 282$.

Time = 0.28 (sec) , antiderivative size = 707, normalized size of antiderivative = 3.37

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{\left(-\frac{b^4}{a^8-2i\,a^7+2\,a^6-6i\,a^5-6i\,a^3-2\,a^2-2i\,a-1}\right)^{\frac{1}{4}}(-i\,a-1)x\log\left(\frac{b^3\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+(a^6-2i\,a^5+a^4-4i\,a^3-a^2-2i\,a-1)\left(-\frac{}{a^8-}\right.}{b^3}\right.}{}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2*((-b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))
^(1/4)*(-I*a - 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
+ a + I)) + (a^6 - 2*I*a^5 + a^4 - 4*I*a^3 - a^2 - 2*I*a - 1)*(-b^4/(a^8 -
2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(3/4))/b^3) + (-b
^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(1/4)*(
I*a + 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
 - (a^6 - 2*I*a^5 + a^4 - 4*I*a^3 - a^2 - 2*I*a - 1)*(-b^4/(a^8 - 2*I*a^7 +
 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(3/4))/b^3) + (-b^4/(a^8 -
 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(1/4)*(a - I)*x*
log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (I*a^6 +
 2*a^5 + I*a^4 + 4*a^3 - I*a^2 + 2*a - I)*(-b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*
I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(3/4))/b^3) - (-b^4/(a^8 - 2*I*a^7 +
2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(1/4)*(a - I)*x*log((b^3*sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-I*a^6 - 2*a^5 - I
*a^4 - 4*a^3 + I*a^2 - 2*a + I)*(-b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*
I*a^3 - 2*a^2 - 2*I*a - 1))^(3/4))/b^3) + 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/((a - I)*x)
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))
), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(1/(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 14, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

```
[In] int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)), x)
```

3.231 $\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 494

$$\begin{aligned}\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx &= \frac{(17i-36a-24ia^2)\,(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} \\ &+ \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} \\ &+ \frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2} \\ &+ \frac{(17i-36a-24ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &- \frac{(17i-36a-24ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &- \frac{(17i-36a-24ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} \\ &+ \frac{(17i-36a-24ia^2)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}\end{aligned}$$

```
[Out] 1/24*(17*I-36*a-24*I*a^2)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^3+1/12*
(3*I-8*a)*(1-I*a-I*b*x)^(7/4)*(1+I*a+I*b*x)^(1/4)/b^3+1/3*x*(1-I*a-I*b*x)^(
7/4)*(1+I*a+I*b*x)^(1/4)/b^2+1/16*(17*I-36*a-24*I*a^2)*arctan(1-(1-I*a-I*b*
x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/16*(17*I-36*a-24*I*a^2)
*arctan(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^3*2^(1/2)-1/32
*(17*I-36*a-24*I*a^2)*ln(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+
(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)+1/32*(17*I-36*a-24*I*a
```

```
^2)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2
)/(1+I*a+I*b*x)^(1/2))/b^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5203, 92, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx$$
$$= \frac{(-24ia^2-36a+17i)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{8\sqrt{2}b^3} - \frac{(-24ia^2-36a+17i)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{8\sqrt{2}b^3} + \frac{(-24ia^2-36a+17i)\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{3/4}}{24b^3} - \frac{(-24ia^2-36a+17i)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3} + \frac{(-24ia^2-36a+17i)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{16\sqrt{2}b^3} + \frac{(-8a+3i)\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{12b^3} + \frac{x\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{3b^2}$$

```
[In] Int[x^2/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((17*I - 36*a - (24*I)*a^2)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4)
)/(24*b^3) + ((3*I - 8*a)*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/
(12*b^3) + (x*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(3*b^2) + ((
17*I - 36*a - (24*I)*a^2)*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
 I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^3) - ((17*I - 36*a - (24*I)*a^2)*ArcTan[
1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*
b^3) - ((17*I - 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I
*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(
16*Sqrt[2]*b^3) + ((17*I - 36*a - (24*I)*a^2)*Log[1 + Sqrt[1 - I*a - I*b*x]
/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x
)^(1/4)])/(16*Sqrt[2]*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx$$

$$= \frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2} + \frac{\int \frac{(1-ia-ibx)^{3/4}\left(-1-a^2+\frac{1}{2}(3i-8a)bx\right)}{(1+ia+ibx)^{3/4}}\,dx}{3b^2}$$

$$
\begin{aligned}
&=\frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3}\\
&+\frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2}-\frac{(17+36ia-24a^2)\int\frac{(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx}{24b^2}\\
&=-\frac{(36a-i(17-24a^2))\,(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3}\\
&+\frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3}+\frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2}\\
&-\frac{(17+36ia-24a^2)\int\frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx}{16b^2}\\
&=-\frac{(36a-i(17-24a^2))\,(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3}\\
&+\frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3}+\frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2}\\
&-\frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int\frac{x^2}{(2-x^4)^{3/4}}\,dx,x,\sqrt[4]{1-ia-ibx}\right)}{4b^3}\\
&=-\frac{(36a-i(17-24a^2))\,(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3}\\
&+\frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3}+\frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2}\\
&-\frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^3}\\
&=-\frac{(36a-i(17-24a^2))\,(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3}\\
&+\frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3}+\frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2}\\
&+\frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^3}\\
&-\frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^3}
\end{aligned}
$$

$$= -\frac{(36a - i(17-24a^2))\,(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} + \frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2} - \frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16b^3} - \frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16b^3} - \frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} - \frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3}$$

$$= -\frac{(36a - i(17-24a^2))\,(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} + \frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2} + \frac{(36a-i(17-24a^2))\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} - \frac{(36a-i(17-24a^2))\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{16\sqrt{2}b^3} - \frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} + \frac{(i(17+36ia-24a^2))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3}$$

$$= -\frac{(36a - i(17 - 24a^2))(1 - ia - ibx)^{3/4}\sqrt[4]{1 + ia + ibx}}{24b^3} + \frac{(3i - 8a)(1 - ia - ibx)^{7/4}\sqrt[4]{1 + ia + ibx}}{12b^3} + \frac{x(1 - ia - ibx)^{7/4}\sqrt[4]{1 + ia + ibx}}{3b^2} - \frac{(36a - i(17 - 24a^2))\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{1 - ia - ibx}}{\sqrt[4]{1 + ia + ibx}}\right)}{8\sqrt{2}b^3} + \frac{(36a - i(17 - 24a^2))\arctan\left(1 + \frac{\sqrt{2}\sqrt[4]{1 - ia - ibx}}{\sqrt[4]{1 + ia + ibx}}\right)}{8\sqrt{2}b^3} + \frac{(36a - i(17 - 24a^2))\log\left(1 + \frac{\sqrt{1 - ia - ibx}}{\sqrt{1 + ia + ibx}} - \frac{\sqrt{2}\sqrt[4]{1 - ia - ibx}}{\sqrt[4]{1 + ia + ibx}}\right)}{16\sqrt{2}b^3} - \frac{(36a - i(17 - 24a^2))\log\left(1 + \frac{\sqrt{1 - ia - ibx}}{\sqrt{1 + ia + ibx}} + \frac{\sqrt{2}\sqrt[4]{1 - ia - ibx}}{\sqrt[4]{1 + ia + ibx}}\right)}{16\sqrt{2}b^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.20

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \frac{(-i(i + a + bx))^{7/4}\left(7\sqrt[4]{1 + ia + ibx}(3i - 8a + 4bx) + \sqrt[4]{2}(-17i + 36a + 24ia^2)\,\text{Hypergeometric2F1}\left(\frac{3}{4}, \frac{7}{4},\right.\right.}{84b^3}$$

```
[In] Integrate[x^2/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] (((-I)*(I + a + b*x))^(7/4)*(7*(1 + I*a + I*b*x)^(1/4)*(3*I - 8*a + 4*b*x)
+ 2^(1/4)*(-17*I + 36*a + (24*I)*a^2)*Hypergeometric2F1[3/4, 7/4, 11/4, (-1
/2*I)*(I + a + b*x)]))/(84*b^3)
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.12

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \frac{3\,b^3\sqrt{\frac{576i\,a^4+1728\,a^3-2112i\,a^2-1224\,a+289i}{b^6}}\log\left(\frac{i\,b^3\sqrt{\frac{576i\,a^4+1728\,a^3-2112i\,a^2-1224\,a+289i}{b^6}}+(24\,a^2-36i\,a-17)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{24\,a^2-36i\,a-17}\right)}{}$$

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(3*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a^2 - 1224*a + 289*I)/b^6)*
log((I*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a^2 - 1224*a + 289*I)/b^6) +
 (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I)))/(24*a^2 - 36*I*a - 17)) - 3*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a
^2 - 1224*a + 289*I)/b^6)*log((-I*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a
^2 - 1224*a + 289*I)/b^6) + (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 - 36*I*a - 17)) + 3*b^3*sqrt((-576
*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I)/b^6)*log((I*b^3*sqrt((-576
*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I)/b^6) + (24*a^2 - 36*I*a -
17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 - 36*I
*a - 17)) - 3*b^3*sqrt((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I
)/b^6)*log((-I*b^3*sqrt((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*
I)/b^6) + (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)))/(24*a^2 - 36*I*a - 17)) - 2*(8*b^3*x^3 + 22*I*b^2*x^2 + 8*a^
3 - (40*I*a + 37)*b*x - 38*I*a^2 + 23*a - 23*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)/(b*x + a + I)))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \text{Timed out}$$

```
[In] integrate(x**2/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx=\int\frac{x^2}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(x^2/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 71, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx=\int\frac{x^2}{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

```
[Out] int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

3.232 $\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 16, antiderivative size = 410

$$\begin{aligned}\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx &= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} \\ &+ \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} \\ &+ \frac{3(3+4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} \\ &- \frac{3(3+4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} \\ &- \frac{3(3+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} \\ &+ \frac{3(3+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}\end{aligned}$$

```
[Out] 1/4*(3+4*I*a)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^2+1/2*(1-I*a-I*b*x)
^(7/4)*(1+I*a+I*b*x)^(1/4)/b^2+3/8*(3+4*I*a)*arctan(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)-3/8*(3+4*I*a)*arctan(1+(1-I*a-I*b*x
)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b^2*2^(1/2)-3/16*(3+4*I*a)*ln(1-(1-I*a
-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)
^(1/2))/b^2*2^(1/2)+3/16*(3+4*I*a)*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+
I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5203, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx=\frac{3(3+4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{4\sqrt{2}b^2}-\frac{3(3+4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{4\sqrt{2}b^2}+\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}+\frac{(3+4ia)\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{3/4}}{4b^2}-\frac{3(3+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{8\sqrt{2}b^2}+\frac{3(3+4ia)\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{8\sqrt{2}b^2}$$

```
[In] Int[x/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((3 + (4*I)*a)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/(4*b^2) + (
(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(2*b^2) + (3*(3 + (4*I)*a)
*ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(4*
Sqrt[2]*b^2) - (3*(3 + (4*I)*a)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4)
)/(1 + I*a + I*b*x)^(1/4)])/(4*Sqrt[2]*b^2) - (3*(3 + (4*I)*a)*Log[1 + Sqrt
[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))
/(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2) + (3*(3 + (4*I)*a)*Log[1 + Sqrt[
1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/
(1 + I*a + I*b*x)^(1/4)])/(8*Sqrt[2]*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx\\
&= \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2}+\frac{(3i-4a)\int\frac{(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx}{4b}\\
&= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2}\\
&\quad+\frac{(3(3i-4a))\int\frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx}{8b}\\
&= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2}\\
&\quad-\frac{(3(3+4ia))\text{Subst}\left(\int\frac{x^2}{(2-x^4)^{3/4}}\,dx,x,\sqrt[4]{1-ia-ibx}\right)}{2b^2}\\
&= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2}\\
&\quad-\frac{(3(3+4ia))\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b^2}
\end{aligned}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{(3(3+4ia))\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^2} - \frac{(3(3+4ia))\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4b^2}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{(3(3+4ia))\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^2} - \frac{(3(3+4ia))\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8b^2} - \frac{(3(3+4ia))\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} - \frac{(3(3+4ia))\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} - \frac{3(3+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} + \frac{3(3+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} - \frac{(3(3+4ia))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} + \frac{(3(3+4ia))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}$$

$$= \frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2} + \frac{3(3+4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} - \frac{3(3+4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} - \frac{3(3+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2} + \frac{3(3+4ia)\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.20

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = -\frac{i(-i(i+a+bx))^{7/4}\left(7i\sqrt[4]{1+ia+ibx}+\sqrt[4]{2}(-3i+4a)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},-\frac{1}{2}i(i+a+bx)\right)\right)}{14b^2}$$

```
[In] Integrate[x/E^(((3*I)/2)*ArcTan[a + b*x]),x]
[Out] ((-1/14*I)*((-I)*(I + a + b*x))^(7/4)*((7*I)*(1 + I*a + I*b*x)^(1/4) + 2^(1
/4)*(-3*I + 4*a)*Hypergeometric2F1[3/4, 7/4, 11/4, (-1/2*I)*(I + a + b*x)])
)/b^2
```

Maple [F]

$$\int \frac{x}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}dx$$

```
[In] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
[Out] int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.06

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \frac{3\,b^2\sqrt{-\frac{16i\,a^2+24\,a-9i}{b^4}}\log\left(-\frac{i\,b^2\sqrt{-\frac{16i\,a^2+24\,a-9i}{b^4}}-(4\,a-3i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a-3i}\right)-3\,b^2\sqrt{-\frac{16i\,a^2+24\,a-9i}{b^4}}\log\left(-\frac{-i\,b^2\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/8*(3*b^2*sqrt(-(16*I*a^2 + 24*a - 9*I)/b^4)*log(-(I*b^2*sqrt(-(16*I*a^2 +
 24*a - 9*I)/b^4) - (4*a - 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I)))/(4*a - 3*I)) - 3*b^2*sqrt(-(16*I*a^2 + 24*a - 9*I)/b^4)*log(-
(-I*b^2*sqrt(-(16*I*a^2 + 24*a - 9*I)/b^4) - (4*a - 3*I)*sqrt(I*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a - 3*I)) + 3*b^2*sqrt(-(-16*I*a^
2 - 24*a + 9*I)/b^4)*log(-(I*b^2*sqrt(-(-16*I*a^2 - 24*a + 9*I)/b^4) - (4*a
 - 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a - 3*I
)) - 3*b^2*sqrt(-(-16*I*a^2 - 24*a + 9*I)/b^4)*log(-(-I*b^2*sqrt(-(-16*I*a^
2 - 24*a + 9*I)/b^4) - (4*a - 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I)))/(4*a - 3*I)) - 2*(2*b^2*x^2 - 2*a^2 + 7*I*b*x + 3*I*a - 5)
*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \text{Timed out}$$

```
[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima"
)

[Out] integrate(x/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 71, a substitution variable should perhaps be purg
ed.War

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{{\left(\frac{1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}{\sqrt{{\left(a+b\,x\right)}^2+1}}\right)}^{3/2}}\,dx$$

[In] int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)

[Out] int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)

3.233 $\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 338

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx=-\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b}-\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}+\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}+\frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}-\frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$

```
[Out] -I*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b-3/2*I*arctan(1-(1-I*a-I*b*x)^(
1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+3/2*I*arctan(1+(1-I*a-I*b*x)^(1
/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4))/b*2^(1/2)+3/4*I*ln(1-(1-I*a-I*b*x)^(1/4)*2
^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/
2)-3/4*I*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)
^(1/2)/(1+I*a+I*b*x)^(1/2))/b*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5201, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = -\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}b} + \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}b} - \frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b} + \frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b} - \frac{3i\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{2\sqrt{2}b}$$

```
[In] Int[E^(((-3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-I)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b - ((3*I)*ArcTan[1
- (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b) +
 ((3*I)*ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4
)])/(Sqrt[2]*b) + (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I
*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2
]*b) - (((3*I)/2)*Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sq
rt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)])/(Sqrt[2]*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-ia-ibx)^{3/4}}{(1+ia+ibx)^{3/4}}\,dx \\
&= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b}+\frac{3}{2}\int \frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx \\
&= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b}+\frac{(6i)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx,x,\sqrt[4]{1-ia-ibx}\right)}{b} \\
&= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b}+\frac{(6i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} \\
&= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b}-\frac{(3i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} \\
&\quad+\frac{(3i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{b} \\
&= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b}+\frac{(3i)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} \\
&\quad+\frac{(3i)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2b} \\
&\quad+\frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} \\
&\quad+\frac{(3i)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}
\end{aligned}$$

$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} + \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} - \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} + \frac{(3i)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{(3i)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}$$

$$= -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b} - \frac{3i\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{2\sqrt{2}b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.13

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = -\frac{8ie^{\frac{1}{2}i\arctan(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},2,\frac{5}{4},-e^{2i\arctan(a+bx)}\right)}{b}$$

```
[In] Integrate[E^(((-3*I)/2)*ArcTan[a + b*x]),x]
```

```
[Out] ((-8*I)*E^((I/2)*ArcTan[a + b*x])*Hypergeometric2F1[1/4, 2, 5/4, -E^((2*I)*
ArcTan[a + b*x])])/b
```

Maple [F]

$$\int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.75

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = \frac{b\sqrt{\frac{9i}{b^2}}\log\left(\frac{1}{3}i\,b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{9i}{b^2}}\log\left(-\frac{1}{3}i\,b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)+b\sqrt{-\frac{9i}{b^2}}\log\left(\right.}{}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(b*sqrt(9*I/b^2)*log(1/3*I*b*sqrt(9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(9*I/b^2)*log(-1/3*I*b*sqrt(9*I/b^2)
 + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-9*I/b
^2)*log(1/3*I*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I))) - b*sqrt(-9*I/b^2)*log(-1/3*I*b*sqrt(-9*I/b^2) + sqrt(I*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 2*(b*x + a + I)*sqrt(I*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\left(\frac{i(a+bx)+1}{\sqrt{(a+bx)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

```
[Out] Integral(((I*(a + b*x) + 1)/sqrt((a + b*x)**2 + 1))**(-3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima"
)
[Out] integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 71, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
[Out] int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

3.234 $\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 427

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx =& -\frac{2(i+a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} \\ &-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) \\ &+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) \\ &-\frac{2(i+a)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} \\ &+\frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} \\ &-\frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}}\end{aligned}$$

```
[Out] -2*(I+a)^(3/4)*arctan((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*
b*x)^(1/4))/(I-a)^(3/4)-2*(I+a)^(3/4)*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/
4)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I-a)^(3/4)+1/2*ln(1-(1-I*a-I*b*x)^(1/4
)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2))*2^(1
/2)-1/2*ln(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x)^(1/4)+(1-I*a-I*b*x)^
(1/2)/(1+I*a+I*b*x)^(1/2))*2^(1/2)-arctan(1-(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+
I*a+I*b*x)^(1/4))*2^(1/2)+arctan(1+(1-I*a-I*b*x)^(1/4)*2^(1/2)/(1+I*a+I*b*x
)^(1/4))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5203, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 12, 95, 218, 214, 211}

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = -\frac{2(a+i)^{3/4}\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{3/4}} - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right) - \frac{2(a+i)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{3/4}} + \frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}} - \frac{\log\left(\frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}$$

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (-2*(I + a)^(3/4)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(
1/4)*(1 - I*a - I*b*x)^(1/4))])/(I - a)^(3/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]
*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sq
rt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)] - (2*(I + a)^(3/4)*
ArcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I
*b*x)^(1/4))])/(I - a)^(3/4) + Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a +
 I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2
] - Log[1 + Sqrt[1 - I*a - I*b*x]/Sqrt[1 + I*a + I*b*x] + (Sqrt[2]*(1 - I*a
 - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1-ia-ibx)^{3/4}}{x(1+ia+ibx)^{3/4}}\,dx$$

$$= -\left((ib)\int \frac{1}{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx\right) + \int \frac{1-ia}{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx$$

$$= 4\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ia-ibx}\right) + (1-ia)\int \frac{1}{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx$$

$$= 4\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) + (4(1-ia))\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^4}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)$$

$$= -\left(2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)\right) + 2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) - \frac{(2(i+a))\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{\sqrt{i-a}} - \frac{(2(i+a))\text{Subst}\left(\int \frac{1}{\sqrt{i-a}+\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{\sqrt{i-a}}$$

$$= -\frac{2(i+a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} - \frac{2(i+a)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} + \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) + \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right.$$

$$= -\frac{2(i+a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} - \frac{2(i+a)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} + \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} + \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) - \sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+i}}\right.$$

$$= -\frac{2(i+a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) - \frac{2(i+a)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} + \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.30

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx$$
$$= \frac{2(-i(i+a+bx))^{3/4}\left(\sqrt[4]{2}(1+ia+ibx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{1}{2}i(i+a+bx)\right)-2\,\text{Hypergeomet}\right.}{3(1+ia+ibx)^{3/4}}$$

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x),x]
```

```
[Out] (2*((-I)*(I + a + b*x))^(3/4)*(2^(1/4)*(1 + I*a + I*b*x)^(3/4)*Hypergeometr
ic2F1[3/4, 3/4, 7/4, (-1/2*I)*(I + a + b*x)] - 2*Hypergeometric2F1[3/4, 1,
7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/(3*(1 + I*a + I
*b*x)^(3/4))
```

Maple [F]

$$\int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x}\,dx$$

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(284) = 568.

Time = 0.28 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.47

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = -\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)$$
$$+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)$$
$$-\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)$$
$$+\frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)$$
$$+\left(-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}\right)^{\frac{1}{4}}\log\left(\frac{(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+(a-i)\left(-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}\right)^{\frac{1}{4}}}{a+i}\right)$$
$$-\left(-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}\right)^{\frac{1}{4}}\log\left(\frac{(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}-(a-i)\left(-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}\right)^{\frac{1}{4}}}{a+i}\right)$$
$$+i\left(-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}\right)^{\frac{1}{4}}\log\left(\frac{(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+(i\,a+1)\left(-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}\right)^{\frac{1}{4}}}{a+i}\right)$$
$$-i\left(-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}\right)^{\frac{1}{4}}\log\left(\frac{(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+(-i\,a-1)\left(-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}\right)^{\frac{1}{4}}}{a+i}\right)$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I))) + 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*lo
g(-1/2*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
) + (-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a +
I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (a - I)*(-(a^3
```

```
 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) - (-(a^3 +
 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (a - I)*(-(a^3 + 3*I*a^2 -
 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) + I*(-(a^3 + 3*I*a^2 -
 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (I*a + 1)*(-(a^3 + 3*I*a^2 - 3*a - I
)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) - I*(-(a^3 + 3*I*a^2 - 3*a - I
)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)/(b*x + a + I)) + (-I*a - 1)*(-(a^3 + 3*I*a^2 - 3*a - I)/(a^3
- 3*I*a^2 - 3*a + I))^(1/4))/(a + I))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx=\text{Timed out}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx=\int \frac{1}{x\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="maxim
a")
[Out] integrate(1/(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 71, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{x\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)),x)
[Out] int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)), x)
```

3.235 $\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 211

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} - \frac{3ib\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}} - \frac{3ib\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}}$$

```
[Out] -(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/(1+I*a)/x-3*I*b*arctan((I+a)^(1/4)
*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I-a)^(7/4)/(I+a)^(1/
4)-3*I*b*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^
(1/4))/(I-a)^(7/4)/(I+a)^(1/4)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{5203, 96, 95, 218, 214, 211\}$

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{3ib\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}} - \frac{3ib\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{(-a+i)^{7/4}\sqrt[4]{a+i}} - \frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}$$

```
[In] Int[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -(((1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/((1 + I*a)*x)) - ((3*I)
*b*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a -
 I*b*x)^(1/4))])/((I - a)^(7/4)*(I + a)^(1/4)) - ((3*I)*b*ArcTanh[((I + a)^
(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((
I - a)^(7/4)*(I + a)^(1/4))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-ia-ibx)^{3/4}}{x^2(1+ia+ibx)^{3/4}}\,dx \\
&= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} + \frac{(3b)\int \frac{1}{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}\,dx}{2(i-a)} \\
&= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} + \frac{(6b)\text{Subst}\left(\int \frac{1}{-1-ia-(-1+ia)x^4}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{i-a} \\
&= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} \\
&\quad - \frac{(3ib)\text{Subst}\left(\int \frac{1}{\sqrt{i-a}-\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/2}} \\
&\quad - \frac{(3ib)\text{Subst}\left(\int \frac{1}{\sqrt{i-a}+\sqrt{i+a}x^2}\,dx, x, \frac{\sqrt[4]{1+ia+ibx}}{\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/2}} \\
&= -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} - \frac{3ib\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}} \\
&\quad - \frac{3ib\,\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{(-i(i+a+bx))^{3/4}\left(1+a^2+ibx+abx-2ibx\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)\right)}{(1+a^2)\,x(1+ia+ibx)^{3/4}}$$

```
[In] Integrate[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x^2),x]
```

```
[Out] -((((-I)*(I + a + b*x))^(3/4)*(1 + a^2 + I*b*x + a*b*x - (2*I)*b*x*Hypergeo
metric2F1[3/4, 1, 7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]
))/((1 + a^2)*x*(1 + I*a + I*b*x)^(3/4)))
```

Maple [F]

$$\int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x^2}\,dx$$

```
[In] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(137) = 274.

Time = 0.28 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.91

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{3\left(-\frac{b^4}{a^8-6i\,a^7-14\,a^6+14i\,a^5+14i\,a^3+14\,a^2-6i\,a-1}\right)^{\frac{1}{4}}(-i\,a-1)x\log\left(\frac{b\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+\left(-\frac{b^4}{a^8-6i\,a^7-14\,a^6+14i\,a^5+14i\,a^3+14}\right.}{b}\right.}{}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2*(3*(-b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a
 - 1))^(1/4)*(-I*a - 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
```

```
b*x + a + I)) + (-b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^
2 - 6*I*a - 1))^(1/4)*(a^2 - 2*I*a - 1))/b) + 3*(-b^4/(a^8 - 6*I*a^7 - 14*a
^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))^(1/4)*(I*a + 1)*x*log((b*sq
rt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-b^4/(a^8 - 6*I*a^
7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))^(1/4)*(a^2 - 2*I*a
- 1))/b) - 3*(-b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 -
 6*I*a - 1))^(1/4)*(a - I)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)/(b*x + a + I)) - (-b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14
*a^2 - 6*I*a - 1))^(1/4)*(I*a^2 + 2*a - I))/b) + 3*(-b^4/(a^8 - 6*I*a^7 - 1
4*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))^(1/4)*(a - I)*x*log((b*s
qrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-b^4/(a^8 - 6*I*a
^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))^(1/4)*(-I*a^2 - 2*
a + I))/b) + 2*(b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
+ a + I)))/((a - I)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(1/(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]Warning, replacing 0 by 71, a substitution variable should perhaps be purg
ed.War
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)), x)
```

3.236 $\int e^{n\arctan(a+bx)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 140

$$\int e^{n\arctan(a+bx)}x^m\,dx = \frac{x^{1+m}(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(1-\frac{bx}{i-a}\right)^{\frac{in}{2}}\left(1+\frac{bx}{i+a}\right)^{-\frac{in}{2}}\text{AppellF1}\left(1+m,-\frac{in}{2},\frac{in}{2},2+m,-\frac{bx}{i+a},\frac{bx}{i-a}\right)}{1+m}$$

```
[Out] x^(1+m)*(1-I*a-I*b*x)^(1/2*I*n)*(1-b*x/(I-a))^(1/2*I*n)*AppellF1(1+m,1/2*I*
n,-1/2*I*n,2+m,b*x/(I-a),-b*x/(I+a))/(1+m)/((1+I*a+I*b*x)^(1/2*I*n))/((1+b*
x/(I+a))^(1/2*I*n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5203, 140, 138}

$$\int e^{n\arctan(a+bx)}x^m\,dx = \frac{x^{m+1}(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(1-\frac{bx}{-a+i}\right)^{\frac{in}{2}}\left(1+\frac{bx}{a+i}\right)^{-\frac{in}{2}}\text{AppellF1}\left(m+1,-\frac{in}{2},\frac{in}{2},m+2,-\frac{bx}{a+i},\frac{bx}{i-a}\right)}{m+1}$$

```
[In] Int[E^(n*ArcTan[a + b*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(1 - I*a - I*b*x)^((I/2)*n)*(1 - (b*x)/(I - a))^((I/2)*n)*Appell
F1[1 + m, (-1/2*I)*n, (I/2)*n, 2 + m, -((b*x)/(I + a)), (b*x)/(I - a)])/((1
 + m)*(1 + I*a + I*b*x)^((I/2)*n)*(1 + (b*x)/(I + a))^((I/2)*n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^m (1-ia-ibx)^{\frac{in}{2}} (1+ia+ibx)^{-\frac{in}{2}}\, dx \\
&= \left((1-ia-ibx)^{\frac{in}{2}} \left(1-\frac{ibx}{1-ia}\right)^{-\frac{in}{2}}\right) \int x^m (1+ia+ibx)^{-\frac{in}{2}} \left(1-\frac{ibx}{1-ia}\right)^{\frac{in}{2}} dx \\
&= \left((1-ia-ibx)^{\frac{in}{2}} (1+ia+ibx)^{-\frac{in}{2}} \left(1-\frac{ibx}{1-ia}\right)^{-\frac{in}{2}} \left(1+\frac{ibx}{1+ia}\right)^{\frac{in}{2}}\right) \int x^m \left(1 - \frac{ibx}{1-ia}\right)^{\frac{in}{2}} \left(1+\frac{ibx}{1+ia}\right)^{-\frac{in}{2}} dx \\
&= \frac{x^{1+m} (1-ia-ibx)^{\frac{in}{2}} (1+ia+ibx)^{-\frac{in}{2}} \left(1-\frac{bx}{i-a}\right)^{\frac{in}{2}} \left(1+\frac{bx}{i+a}\right)^{-\frac{in}{2}} \text{AppellF1}\left(1+m,-\frac{in}{2},\frac{in}{2},2+m\right.}{1+m}
\end{aligned}$$

Mathematica [F]

$$\int e^{n\arctan(a+bx)} x^m\, dx = \int e^{n\arctan(a+bx)} x^m\, dx$$

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x^m,x]
[Out] Integrate[E^(n*ArcTan[a + b*x])*x^m, x]
```

Maple [F]

$$\int e^{n\arctan(bx+a)}x^m dx$$

[In] int(exp(n*arctan(b*x+a))*x^m,x)

[Out] int(exp(n*arctan(b*x+a))*x^m,x)

Fricas [F]

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int x^m e^{(n\arctan(bx+a))}\,dx$$

[In] integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="fricas")

[Out] integral(x^m*e^(n*arctan(b*x + a)), x)

Sympy [F]

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int x^m e^{n\operatorname{atan}(a+bx)}\,dx$$

[In] integrate(exp(n*atan(b*x+a))*x**m,x)

[Out] Integral(x**m*exp(n*atan(a + b*x)), x)

Maxima [F]

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int x^m e^{(n\arctan(bx+a))}\,dx$$

[In] integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="maxima")

[Out] integrate(x^m*e^(n*arctan(b*x + a)), x)

Giac [F]

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int x^m e^{(n\arctan(bx+a))}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int x^m\,\mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

```
[In] int(x^m*exp(n*atan(a + b*x)),x)
[Out] int(x^m*exp(n*atan(a + b*x)), x)
```

3.237 $\int e^{n\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 260

$$\int e^{n\arctan(a+bx)}x^3\,dx = \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} - \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}\left(6-18a^2-10an-n^2+2b(6a+n)x\right)}{24b^4} + \frac{2^{-2-\frac{in}{2}}\left(24a^3+36a^2n-12a(2-n^2)-n(8-n^2)\right)(1-ia-ibx)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\right.}{3b^4(2i-n)}$$

```
[Out] 1/4*x^2*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^2-1/24*(1-I*a
-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)*(6-18*a^2-10*a*n-n^2+2*b*(6*a
+n)*x)/b^4+1/3*2^(-2-1/2*I*n)*(24*a^3+36*a^2*n-12*a*(-n^2+2)-n*(-n^2+8))*(1
-I*a-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*I*n],1/2-1/2*
I*a-1/2*I*b*x)/b^4/(2*I-n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5203, 102, 152, 71}

$$\int e^{n\arctan(a+bx)}x^3\,dx = -\frac{(-ia-ibx+1)^{1+\frac{in}{2}}\left(-18a^2+2bx(6a+n)-10an-n^2+6\right)(ia+ibx+1)^{1-\frac{in}{2}}}{24b^4} + \frac{2^{-2-\frac{in}{2}}\left(24a^3+36a^2n-12a(2-n^2)-n(8-n^2)\right)(-ia-ibx+1)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2},\frac{in}{2}\right.}{3b^4(-n+2i)} + \frac{x^2(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{4b^2}$$

```
[In] Int[E^(n*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (x^2*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(4*b^
2) - ((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n)*(6 -
18*a^2 - 10*a*n - n^2 + 2*b*(6*a + n)*x))/(24*b^4) + (2^(-2 - (I/2)*n)*(24*
a^3 + 36*a^2*n - 12*a*(2 - n^2) - n*(8 - n^2))*(1 - I*a - I*b*x)^(1 + (I/2)
*n)*Hypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/
2])/(3*b^4*(2*I - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int x^3(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx$$
$$= \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} + \frac{\int x(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(-2(1+a^2)-b(6a+n)x\right)\,dx}{4b^2}$$
$$= \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} - \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}\left(6-18a^2-10an-n^2+2b(6a+n)x\right)}{24b^4} - \frac{\left(24a^3+36a^2n-12a(2-n^2)-n(8-n^2)\right)\int(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx}{24b^3}$$
$$= \frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2} - \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}\left(6-18a^2-10an-n^2+2b(6a+n)x\right)}{24b^4} + \frac{2^{-2-\frac{in}{2}}\left(24a^3+36a^2n-12a(2-n^2)-n(8-n^2)\right)(1-ia-ibx)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2}\right.}{3b^4(2i-n)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.05

$$\int e^{n\arctan(a+bx)}x^3\,dx$$
$$= \frac{(-i(i+a+bx))^{1+\frac{in}{2}}\left(b^2(2i-n)x^2(1+ia+ibx)^{1-\frac{in}{2}} - 2^{3-\frac{in}{2}}(6a+n)\,\text{Hypergeometric2F1}\left(-2+\frac{in}{2},1+\right.\right.}{}$$

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x^3,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1 + (I/2)*n)*(b^2*(2*I - n)*x^2*(1 + I*a + I*b*x)^(1
 - (I/2)*n) - 2^(3 - (I/2)*n)*(6*a + n)*Hypergeometric2F1[-2 + (I/2)*n, 1 +
 (I/2)*n, 2 + (I/2)*n, (-1/2*I)*(I + a + b*x)] + 2^(3 - (I/2)*n)*(1 + I*a)*
(-I + 5*a + n)*Hypergeometric2F1[-1 + (I/2)*n, 1 + (I/2)*n, 2 + (I/2)*n, (-
1/2*I)*(I + a + b*x)] + 2^(1 - (I/2)*n)*(-I + a)^2*(-2*I + 4*a + n)*Hyperge
ometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (-1/2*I)*(I + a + b*x)]))/(4*
b^4*(2*I - n))
```

Maple [F]

$$\int \mathrm{e}^{n\arctan(bx+a)}x^3dx$$

```
[In] int(exp(n*arctan(b*x+a))*x^3,x)
[Out] int(exp(n*arctan(b*x+a))*x^3,x)
```

Fricas [F]

$$\int e^{n\arctan(a+bx)}x^3\,dx = \int x^3e^{(n\arctan(bx+a))}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="fricas")
[Out] integral(x^3*e^(n*arctan(b*x + a)), x)
```

Sympy [F]

$$\int e^{n\arctan(a+bx)}x^3\,dx = \int x^3e^{n\,\mathrm{atan}\,(a+bx)}\,dx$$

```
[In] integrate(exp(n*atan(b*x+a))*x**3,x)
[Out] Integral(x**3*exp(n*atan(a + b*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(a+bx)}x^3\,dx = \int x^3e^{(n\arctan(bx+a))}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="maxima")
[Out] integrate(x^3*e^(n*arctan(b*x + a)), x)
```

Giac [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x^3\,dx = \text{Timed out}$$

```
[In] integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x^3\,dx = \int x^3\,\mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

```
[In] int(x^3*exp(n*atan(a + b*x)),x)
[Out] int(x^3*exp(n*atan(a + b*x)), x)
```

3.238 $\int e^{n\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 220

$$\int e^{n\arctan(a+bx)}x^2\,dx = -\frac{(4a+n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{6b^3}+\frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2}+\frac{2^{-\frac{in}{2}}\left(2-6a^2-6an-n^2\right)(1-ia-ibx)^{1+\frac{in}{2}}\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{3b^3(2i-n)}$$

```
[Out] -1/6*(4*a+n)*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^3+1/3*x*
(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^2+1/3*(-6*a^2-6*a*n-n
^2+2)*(1-I*a-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*I*n],
1/2-1/2*I*a-1/2*I*b*x)/(2^(1/2*I*n))/b^3/(2*I-n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5203, 92, 81, 71}

$$\int e^{n\arctan(a+bx)}x^2\,dx = \frac{2^{-\frac{in}{2}}\left(-6a^2-6an-n^2+2\right)(-ia-ibx+1)^{1+\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2},\frac{in}{2}+2,\frac{1}{2}(-ia-ibx+1\right.}{3b^3(-n+2i)}-\frac{(4a+n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{6b^3}+\frac{x(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{3b^2}$$

```
[In] Int[E^(n*ArcTan[a + b*x])*x^2,x]
```

```
[Out] -1/6*((4*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2
)*n))/b^3 + (x*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)
*n))/(3*b^2) + ((2 - 6*a^2 - 6*a*n - n^2)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*H
ypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(
3*2^((I/2)*n)*b^3*(2*I - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx\\ &= \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2}\\ &\quad+\frac{\int(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\left(-1-a^2-b(4a+n)x\right)\,dx}{3b^2}\end{aligned}$$

$$= -\frac{(4a+n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{6b^3} + \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2} - \frac{(2-6a^2-6an-n^2)\int(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx}{6b^2}$$

$$= -\frac{(4a+n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{6b^3} + \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2} + \frac{2^{-\frac{in}{2}}(2-6a^2-6an-n^2)(1-ia-ibx)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-ia-\right.}{3b^3(2i-n)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int e^{n\arctan(a+bx)}x^2\,dx = \frac{(-i(i+a+bx))^{1+\frac{in}{2}}\left(-\left((4a+n)(1+ia+ibx)^{1-\frac{in}{2}}\right)+2bx(1+ia+ibx)^{1-\frac{in}{2}}+\frac{2^{1-\frac{in}{2}}(-2+6a^2+6an+n^2)\,\text{Hyp}}{}\right.}{6b^3}$$

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x^2,x]
```

```
[Out] (((-I)*(I + a + b*x))^(1 + (I/2)*n)*(-((4*a + n)*(1 + I*a + I*b*x)^(1 - (I/
2)*n)) + 2*b*x*(1 + I*a + I*b*x)^(1 - (I/2)*n) + (2^(1 - (I/2)*n)*(-2 + 6*a
^2 + 6*a*n + n^2)*Hypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (-1/
2*I)*(I + a + b*x)])/(-2*I + n)))/(6*b^3)
```

Maple [F]

$$\int e^{n\arctan(bx+a)}x^2dx$$

```
[In] int(exp(n*arctan(b*x+a))*x^2,x)
```

```
[Out] int(exp(n*arctan(b*x+a))*x^2,x)
```

Fricas [F]

$$\int e^{n \arctan(a+bx)} x^2 \, dx = \int x^2 e^{(n \arctan(bx+a))} \, dx$$

[In] integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="fricas")

[Out] integral(x^2*e^(n*arctan(b*x + a)), x)

Sympy [F]

$$\int e^{n \arctan(a+bx)} x^2 \, dx = \int x^2 e^{n \operatorname{atan}(a+bx)} \, dx$$

[In] integrate(exp(n*atan(b*x+a))*x**2,x)

[Out] Integral(x**2*exp(n*atan(a + b*x)), x)

Maxima [F]

$$\int e^{n \arctan(a+bx)} x^2 \, dx = \int x^2 e^{(n \arctan(bx+a))} \, dx$$

[In] integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="maxima")

[Out] integrate(x^2*e^(n*arctan(b*x + a)), x)

Giac [F(-1)]

Timed out.

$$\int e^{n \arctan(a+bx)} x^2 \, dx = \text{Timed out}$$

[In] integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int e^{n \arctan(a+bx)} x^2 \, dx = \int x^2 \, e^{n \operatorname{atan}(a+b\,x)} \, dx$$

```
[In] int(x^2*exp(n*atan(a + b*x)),x)
[Out] int(x^2*exp(n*atan(a + b*x)), x)
```

3.239 $\int e^{n\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 147

$$\int e^{n\arctan(a+bx)}x\,dx = \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2b^2} + \frac{2^{-\frac{in}{2}}(2a+n)(1-ia-ibx)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{b^2(2i-n)}$$

```
[Out] 1/2*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^2+(2*a+n)*(1-I*a-
I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*I*n],1/2-1/2*I*a-1
/2*I*b*x)/(2^(1/2*I*n))/b^2/(2*I-n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5203, 81, 71}

$$\int e^{n\arctan(a+bx)}x\,dx = \frac{2^{-\frac{in}{2}}(2a+n)(-ia-ibx+1)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2},\frac{in}{2}+2,\frac{1}{2}(-ia-ibx+1)\right)}{b^2(-n+2i)} + \frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2b^2}$$

```
[In] Int[E^(n*ArcTan[a + b*x])*x,x]
```

```
[Out] ((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(2*b^2) +
 ((2*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n,
(I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(2^((I/2)*n)*b^2*(2*I - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx\\
&= \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2b^2} - \frac{(2a+n)\int(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx}{2b}\\
&= \frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2b^2}\\
&\quad + \frac{2^{-\frac{in}{2}}(2a+n)(1-ia-ibx)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{b^2(2i-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

$$\int e^{n\arctan(a+bx)}x\,dx$$

$$= \frac{i(-i(i+a+bx))^{1+\frac{in}{2}}\left((1+ia+ibx)^{-\frac{in}{2}}(-i+a+bx)+\frac{2^{1-\frac{in}{2}}(2a+n)\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},-\frac{1}{2}i(i+a+bx\right.}{-2-in}\right.}{2b^2}$$

```
[In] Integrate[E^(n*ArcTan[a + b*x])*x,x]
```

[Out] ((I/2)*((-I)*(I + a + b*x))^(1 + (I/2)*n)*((-I + a + b*x)/(1 + I*a + I*b*x)
^((I/2)*n) + (2^(1 - (I/2)*n)*(2*a + n)*Hypergeometric2F1[1 + (I/2)*n, (I/2
)*n, 2 + (I/2)*n, (-1/2*I)*(I + a + b*x)])/(-2 - I*n)))/b^2

Maple [F]

$$\int \mathrm{e}^{n \arctan(bx+a)} x dx$$

[In] int(exp(n*arctan(b*x+a))*x,x)

[Out] int(exp(n*arctan(b*x+a))*x,x)

Fricas [F]

$$\int e^{n \arctan(a+bx)} x \, dx = \int x e^{(n \arctan(bx+a))} \, dx$$

[In] integrate(exp(n*arctan(b*x+a))*x,x, algorithm="fricas")

[Out] integral(x*e^(n*arctan(b*x + a)), x)

Sympy [F]

$$\int e^{n \arctan(a+bx)} x \, dx = \int x e^{n \operatorname{atan}(a+bx)} \, dx$$

[In] integrate(exp(n*atan(b*x+a))*x,x)

[Out] Integral(x*exp(n*atan(a + b*x)), x)

Maxima [F]

$$\int e^{n \arctan(a+bx)} x \, dx = \int x e^{(n \arctan(bx+a))} \, dx$$

[In] integrate(exp(n*arctan(b*x+a))*x,x, algorithm="maxima")

[Out] integrate(x*e^(n*arctan(b*x + a)), x)

Giac [F]

$$\int e^{n\arctan(a+bx)}x\,dx = \int xe^{(n\arctan(bx+a))}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a))*x,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x\,dx = \int x\,\mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

```
[In] int(x*exp(n*atan(a + b*x)),x)
[Out] int(x*exp(n*atan(a + b*x)), x)
```

3.240 $\int e^{n\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 91

$$\int e^{n\arctan(a+bx)}\,dx = -\frac{2^{1-\frac{in}{2}}(1-ia-ibx)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{b(2i-n)}$$

```
[Out] -2^(1-1/2*I*n)*(1-I*a-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+
1/2*I*n],1/2-1/2*I*a-1/2*I*b*x)/b/(2*I-n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5201, 71}

$$\int e^{n\arctan(a+bx)}\,dx = -\frac{2^{1-\frac{in}{2}}(-ia-ibx+1)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2},\frac{in}{2}+2,\frac{1}{2}(-ia-ibx+1)\right)}{b(-n+2i)}$$

```
[In] Int[E^(n*ArcTan[a + b*x]),x]
```

```
[Out] -((2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I
/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(b*(2*I - n)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5201

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Rubi steps

$$\text{integral} = \int (1 - ia - ibx)^{\frac{in}{2}} (1 + ia + ibx)^{-\frac{in}{2}} \, dx$$
$$= -\frac{2^{1-\frac{in}{2}} (1 - ia - ibx)^{1+\frac{in}{2}} \text{Hypergeometric2F1}\left(1 + \frac{in}{2}, \frac{in}{2}, 2 + \frac{in}{2}, \frac{1}{2}(1 - ia - ibx)\right)}{b(2i - n)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int e^{n \arctan(a+bx)} \, dx$$
$$= \frac{4e^{(2i+n) \arctan(a+bx)} \text{Hypergeometric2F1}\left(2, 1 - \frac{in}{2}, 2 - \frac{in}{2}, -e^{2i \arctan(a+bx)}\right)}{b(2i + n)}$$

```
[In] Integrate[E^(n*ArcTan[a + b*x]),x]
[Out] (4*E^((2*I + n)*ArcTan[a + b*x])*Hypergeometric2F1[2, 1 - (I/2)*n, 2 - (I/2
)*n, -E^((2*I)*ArcTan[a + b*x])])/(b*(2*I + n))
```

Maple [F]

$$\int \mathrm{e}^{n \arctan(bx+a)} dx$$

```
[In] int(exp(n*arctan(b*x+a)),x)
[Out] int(exp(n*arctan(b*x+a)),x)
```

Fricas [F]

$$\int e^{n\arctan(a+bx)}\,dx = \int e^{(n\arctan(bx+a))}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a)),x, algorithm="fricas")
[Out] integral(e^(n*arctan(b*x + a)), x)
```

Sympy [F]

$$\int e^{n\arctan(a+bx)}\,dx = \int e^{n\operatorname{atan}(a+bx)}\,dx$$

```
[In] integrate(exp(n*atan(b*x+a)),x)
[Out] Integral(exp(n*atan(a + b*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(a+bx)}\,dx = \int e^{(n\arctan(bx+a))}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a)),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(b*x + a)), x)
```

Giac [F]

$$\int e^{n\arctan(a+bx)}\,dx = \int e^{(n\arctan(bx+a))}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}\,dx = \int \mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

```
[In] int(exp(n*atan(a + b*x)),x)
[Out] int(exp(n*atan(a + b*x)), x)
```

3.241 $\int \frac{e^{n\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 191

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \frac{2i(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{in}{2},1+\frac{in}{2},\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{n} - \frac{i2^{1-\frac{in}{2}}(1-ia-ibx)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{n}$$

```
[Out] 2*I*(1-I*a-I*b*x)^(1/2*I*n)*hypergeom([1, 1/2*I*n],[1+1/2*I*n],(I-a)*(1-I*a
-I*b*x)/(I+a)/(1+I*a+I*b*x))/n/((1+I*a+I*b*x)^(1/2*I*n))-I*2^(1-1/2*I*n)*(1
-I*a-I*b*x)^(1/2*I*n)*hypergeom([1/2*I*n, 1/2*I*n],[1+1/2*I*n],1/2-1/2*I*a-
1/2*I*b*x)/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5203, 132, 71, 12, 133}

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \frac{2i(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{in}{2},\frac{in}{2}+1,\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{n} - \frac{i2^{1-\frac{in}{2}}(-ia-ibx+1)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},\frac{in}{2}+1,\frac{1}{2}(-ia-ibx+1)\right)}{n}$$

```
[In] Int[E^(n*ArcTan[a + b*x])/x,x]
```

```
[Out] ((2*I)*(1 - I*a - I*b*x)^((I/2)*n)*Hypergeometric2F1[1, (I/2)*n, 1 + (I/2)*
n, ((I - a)*(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/(n*(1 + I*a +
I*b*x)^((I/2)*n)) - (I*2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^((I/2)*n)*Hypergeo
metric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a - I*b*x)/2])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x}\,dx$$

$$= -\left((ib)\int (1-ia-ibx)^{-1+\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,dx\right) + \int \frac{(1-ia)(1-ia-ibx)^{-1+\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x}\,dx$$

$$= -\frac{i2^{1-\frac{in}{2}}(1-ia-ibx)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{n} + (1-ia)\int \frac{(1-ia-ibx)^{-1+\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x}\,dx$$

$$= \frac{2i(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{in}{2},1+\frac{in}{2},\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{n} - \frac{i2^{1-\frac{in}{2}}(1-ia-ibx)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{n}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.89

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \frac{2i(1+ia+ibx)^{-\frac{in}{2}}(-i(i+a+bx))^{\frac{in}{2}}\left(\text{Hypergeometric2F1}\left(1,\frac{in}{2},1+\frac{in}{2},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)-2^{-\frac{in}{2}}(1+ia+i$$

```
[In] Integrate[E^(n*ArcTan[a + b*x])/x,x]
```

```
[Out] ((2*I)*((-I)*(I + a + b*x))^((I/2)*n)*(Hypergeometric2F1[1, (I/2)*n, 1 + (I
/2)*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)] - ((1 + I*a + I
*b*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (-1/2*I)*(
I + a + b*x)])/2^((I/2)*n)))/(n*(1 + I*a + I*b*x)^((I/2)*n))
```

Maple [F]

$$\int \frac{e^{n\arctan(bx+a)}}{x}dx$$

```
[In] int(exp(n*arctan(b*x+a))/x,x)
```

```
[Out] int(exp(n*arctan(b*x+a))/x,x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \int \frac{e^{(n\arctan(bx+a))}}{x}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(e^(n*arctan(b*x + a))/x, x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \int \frac{e^{n\operatorname{atan}(a+bx)}}{x}\,dx$$

```
[In] integrate(exp(n*atan(b*x+a))/x,x)
[Out] Integral(exp(n*atan(a + b*x))/x, x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \int \frac{e^{(n\arctan(bx+a))}}{x}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(e^(n*arctan(b*x + a))/x, x)
```

Giac [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \int \frac{e^{(n\arctan(bx+a))}}{x}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a))/x,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(a+bx)}}{x} \, dx = \int \frac{\mathrm{e}^{n\, \mathrm{atan}(a+b\,x)}}{x} \, dx$$

```
[In] int(exp(n*atan(a + b*x))/x,x)
[Out] int(exp(n*atan(a + b*x))/x, x)
```

3.242 $\int \frac{e^{n \arctan(a+bx)}}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 128

$$\int \frac{e^{n \arctan(a+bx)}}{x^2} dx = -\frac{4b(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}} \text{Hypergeometric2F1}\left(2,1+\frac{in}{2},2+\frac{in}{2},\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{(i+a)^2(2i-n)}$$

```
[Out] -4*b*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(-1-1/2*I*n)*hypergeom([2, 1+1
/2*I*n],[2+1/2*I*n],(I-a)*(1-I*a-I*b*x)/(I+a)/(1+I*a+I*b*x))/(I+a)^2/(2*I-n
)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5203, 133}

$$\int \frac{e^{n \arctan(a+bx)}}{x^2} dx = -\frac{4b(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}} \text{Hypergeometric2F1}\left(2,\frac{in}{2}+1,\frac{in}{2}+2,\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{(a+i)^2(-n+2i)}$$

```
[In] Int[E^(n*ArcTan[a + b*x])/x^2,x]
```

```
[Out] (-4*b*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(-1 - (I/2)*n)*Hype
rgeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I - a)*(1 - I*a - I*b*x))/((I
+ a)*(1 + I*a + I*b*x))])/((I + a)^2*(2*I - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x^2}\,dx$$

$$= -\frac{4b(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(2,1+\frac{in}{2},2+\frac{in}{2},\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{(i+a)^2(2i-n)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int \frac{e^{n\arctan(a+bx)}}{x^2}\,dx = -\frac{4ib(1+ia+ibx)^{-\frac{in}{2}}(-i(i+a+bx))^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(2,1+\frac{in}{2},2+\frac{in}{2},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)}{(i+a)^2(-2i+n)(-i+a+bx)}$$

```
[In] Integrate[E^(n*ArcTan[a + b*x])/x^2,x]
```

```
[Out] ((-4*I)*b*((-I)*(I + a + b*x))^(1 + (I/2)*n)*Hypergeometric2F1[2, 1 + (I/2)
*n, 2 + (I/2)*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I
+ a)^2*(-2*I + n)*(1 + I*a + I*b*x)^((I/2)*n)*(-I + a + b*x))
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(bx+a)}}{x^2} dx$$

```
[In] int(exp(n*arctan(b*x+a))/x^2,x)
[Out] int(exp(n*arctan(b*x+a))/x^2,x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(a+bx)}}{x^2} \, dx = \int \frac{e^{(n \arctan(bx+a))}}{x^2} \, dx$$

```
[In] integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="fricas")
[Out] integral(e^(n*arctan(b*x + a))/x^2, x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(a+bx)}}{x^2} \, dx = \int \frac{e^{n \operatorname{atan}(a+bx)}}{x^2} \, dx$$

```
[In] integrate(exp(n*atan(b*x+a))/x**2,x)
[Out] Integral(exp(n*atan(a + b*x))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(a+bx)}}{x^2} \, dx = \int \frac{e^{(n \arctan(bx+a))}}{x^2} \, dx$$

```
[In] integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="maxima")
[Out] integrate(e^(n*arctan(b*x + a))/x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(a+bx)}}{x^2} \, dx = \text{Timed out}$$

```
[In] integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(a+bx)}}{x^2} \, dx = \int \frac{e^{n \operatorname{atan}(a+b\,x)}}{x^2} \, dx$$

```
[In] int(exp(n*atan(a + b*x))/x^2,x)
[Out] int(exp(n*atan(a + b*x))/x^2, x)
```

3.243 $\int \frac{e^{n \arctan(a+bx)}}{x^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 207

$$\int \frac{e^{n \arctan(a+bx)}}{x^3} \, dx = -\frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2(1+a^2)x^2} - \frac{2b^2(2a-n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}} \text{Hypergeometric2F1}\left(2,1+\frac{in}{2},2+\frac{in}{2},\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{(i-a)(i+a)^3(2i-n)}$$

```
[Out] -1/2*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/(a^2+1)/x^2-2*b^2*
(2*a-n)*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(-1-1/2*I*n)*hypergeom([2,
1+1/2*I*n],[2+1/2*I*n],(I-a)*(1-I*a-I*b*x)/(I+a)/(1+I*a+I*b*x))/(I-a)/(I+a)
^3/(2*I-n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5203, 98, 133}

$$\int \frac{e^{n \arctan(a+bx)}}{x^3} \, dx = -\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2(a^2+1)x^2} - \frac{2b^2(2a-n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}} \text{Hypergeometric2F1}\left(2,\frac{in}{2}+1,\frac{in}{2}+2,\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{(-a+i)(a+i)^3(-n+2i)}$$

```
[In] Int[E^(n*ArcTan[a + b*x])/x^3,x]
```

```
[Out] -1/2*((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/((1
+ a^2)*x^2) - (2*b^2*(2*a - n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I
```

```
*b*x)^(-1 - (I/2)*n)*Hypergeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I - a
)*(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/((I - a)*(I + a)^3*(2*I
- n))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 5203

```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x^3}\,dx$$

$$= -\frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2(1+a^2)x^2} - \frac{(b(2a-n))\int \frac{(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}}{x^2}\,dx}{2(1+a^2)}$$

$$= -\frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2(1+a^2)x^2} + \frac{2b^2(2a-n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(2,1+\frac{in}{2},2+\frac{in}{2},\frac{(i-a)(1-}{(i+a)(1+}\right.}{(i+a)^2(1+a^2)(2i-n)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx = -\frac{i(1+ia+ibx)^{-\frac{in}{2}}(-i(i+a+bx))^{1+\frac{in}{2}}\left((i+a)^2(-2i+n)(-i+a+bx)^2+4b^2(-2a+n)x^2\,\text{Hypergeo}\right.}{2(-i+a)(i+a)^3(-2i+n)x^2(-i+a+bx)}$$

```
[In] Integrate[E^(n*ArcTan[a + b*x])/x^3,x]
[Out] ((-1/2*I)*((-I)*(I + a + b*x))^(1 + (I/2)*n)*((I + a)^2*(-2*I + n)*(-I + a
+ b*x)^2 + 4*b^2*(-2*a + n)*x^2*Hypergeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)
*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/((-I + a)*(I + a
)^3*(-2*I + n)*x^2*(1 + I*a + I*b*x)^((I/2)*n)*(-I + a + b*x))
```

Maple [F]

$$\int \frac{e^{n\arctan(bx+a)}}{x^3}dx$$

```
[In] int(exp(n*arctan(b*x+a))/x^3,x)
[Out] int(exp(n*arctan(b*x+a))/x^3,x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx = \int \frac{e^{(n\arctan(bx+a))}}{x^3}\,dx$$

```
[In] integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="fricas")
[Out] integral(e^(n*arctan(b*x + a))/x^3, x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx = \int \frac{e^{n\,\mathrm{atan}\,(a+bx)}}{x^3}\,dx$$

```
[In] integrate(exp(n*atan(b*x+a))/x**3,x)
[Out] Integral(exp(n*atan(a + b*x))/x**3, x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(a+bx)}}{x^3} \, dx = \int \frac{e^{(n \arctan(bx+a))}}{x^3} \, dx$$

```
[In] integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="maxima")
[Out] integrate(e^(n*arctan(b*x + a))/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(a+bx)}}{x^3} \, dx = \text{Timed out}$$

```
[In] integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(a+bx)}}{x^3} \, dx = \int \frac{e^{n \operatorname{atan}(a+b\,x)}}{x^3} \, dx$$

```
[In] int(exp(n*atan(a + b*x))/x^3,x)
[Out] int(exp(n*atan(a + b*x))/x^3, x)
```

3.244 $\int e^{\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{\left(1-\frac{i}{2}\right)+p}(1-iax)^{\left(1+\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(\frac{i}{2}-p,\left(1+\frac{i}{2}\right)+p,\left(2+\frac{i}{2}\right)+p,\right.}{a((2+i)+2p)}$$

```
[Out] I*2^(1-1/2*I+p)*(1-I*a*x)^(1+1/2*I+p)*(a^2*c*x^2+c)^p*hypergeom([1/2*I-p, 1
+1/2*I+p],[2+1/2*I+p],1/2-1/2*I*a*x)/a/(2+I+2*p)/((a^2*x^2+1)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5184, 5181, 71}

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{p+\left(1-\frac{i}{2}\right)}(1-iax)^{p+\left(1+\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\,\text{Hypergeometric2F1}\left(\frac{i}{2}-p,p+\left(1+\frac{i}{2}\right),p+\left(2+\frac{i}{2}\right)\right.}{a(2p+(2+i))}$$

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^((1 - I/2) + p)*(1 - I*a*x)^((1 + I/2) + p)*(c + a^2*c*x^2)^p*Hypergeo
metric2F1[I/2 - p, (1 + I/2) + p, (2 + I/2) + p, (1 - I*a*x)/2])/(a*((2 + I
) + 2*p)*(1 + a^2*x^2)^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{\arctan(ax)}\left(1+a^2x^2\right)^p\,dx$$

$$= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{\frac{i}{2}+p}(1+iax)^{-\frac{i}{2}+p}\,dx$$

$$= \frac{i2^{\left(1-\frac{i}{2}\right)+p}(1-iax)^{\left(1+\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(\frac{i}{2}-p,\left(1+\frac{i}{2}\right)+p,\left(2+\frac{i}{2}\right.\right.}{a((2+i)+2p)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$

$$= \frac{i2^{-\frac{i}{2}+p}(1-iax)^{\left(1+\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(\frac{i}{2}-p,\left(1+\frac{i}{2}\right)+p,\left(2+\frac{i}{2}\right)+p,\frac{1}{2}(1\right.}{a\left(\left(1+\frac{i}{2}\right)+p\right)}$$

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^(-1/2*I + p)*(1 - I*a*x)^((1 + I/2) + p)*(c + a^2*c*x^2)^p*Hypergeomet
ric2F1[I/2 - p, (1 + I/2) + p, (2 + I/2) + p, (1 - I*a*x)/2])/(a*((1 + I/2)
 + p)*(1 + a^2*x^2)^p)
```

Maple [F]

$$\int \mathrm{e}^{\arctan(ax)} \left(a^2 c\, x^2 + c\right)^p dx$$

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x)
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{\arctan(ax)} \left(c + a^2 c x^2\right)^p \, dx = \int \left(a^2 c x^2 + c\right)^p e^{(\arctan(ax))} \, dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)} \left(c + a^2 c x^2\right)^p \, dx = \int \left(c \left(a^2 x^2 + 1\right)\right)^p e^{\operatorname{atan}(ax)} \, dx$$

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**p,x)
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(atan(a*x)), x)
```

Maxima [F]

$$\int e^{\arctan(ax)} \left(c + a^2 c x^2\right)^p \, dx = \int \left(a^2 c x^2 + c\right)^p e^{(\arctan(ax))} \, dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

Giac [F]

$$\int e^{\arctan(ax)} (c + a^2 c x^2)^p \, dx = \int (a^2 c x^2 + c)^p e^{(\arctan(ax))} \, dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)} (c + a^2 c x^2)^p \, dx = \int e^{\operatorname{atan}(a\,x)} \left(c\,a^2\,x^2 + c\right)^p \, dx$$

```
[In] int(exp(atan(a*x))*(c + a^2*c*x^2)^p,x)
[Out] int(exp(atan(a*x))*(c + a^2*c*x^2)^p, x)
```

3.245 $\int e^{\arctan(ax)}(c+a^2cx^2)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = \frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\text{Hypergeometric2F1}\left(-2+\frac{i}{2},3+\frac{i}{2},4+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/37+6/37*I)*2^(3-1/2*I)*c^2*(1-I*a*x)^(3+1/2*I)*hypergeom([3+1/2*I, -2+1/
2*I],[4+1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5181, 71}

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = \frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\text{Hypergeometric2F1}\left(-2+\frac{i}{2},3+\frac{i}{2},4+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/37 + (6*I)/37)*2^(3 - I/2)*c^2*(1 - I*a*x)^(3 + I/2)*Hypergeometric2F1[
-2 + I/2, 3 + I/2, 4 + I/2, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = c^2\int(1-iax)^{2+\frac{i}{2}}(1+iax)^{2-\frac{i}{2}}\,dx$$
$$= \frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2+\frac{i}{2},3+\frac{i}{2},4+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx$$
$$= \frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2+\frac{i}{2},3+\frac{i}{2},4+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^2,x]
[Out] ((1/37 + (6*I)/37)*2^(3 - I/2)*c^2*(1 - I*a*x)^(3 + I/2)*Hypergeometric2F1[
-2 + I/2, 3 + I/2, 4 + I/2, (1 - I*a*x)/2])/a
```

Maple [F]

$$\int e^{\arctan(ax)}\left(a^2cx^2+c\right)^2dx$$

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x)
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(\arctan(ax))}\,dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=c^2\left(\int 2a^2x^2e^{\operatorname{atan}(ax)}\,dx+\int a^4x^4e^{\operatorname{atan}(ax)}\,dx+\int e^{\operatorname{atan}(ax)}\,dx\right)$$

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**2,x)
[Out] c**2*(Integral(2*a**2*x**2*exp(atan(a*x)), x) + Integral(a**4*x**4*exp(atan
(a*x)), x) + Integral(exp(atan(a*x)), x))
```

Maxima [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(\arctan(ax))}\,dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^2*e^(arctan(a*x)), x)
```

Giac [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(\arctan(ax))}\,dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \mathrm{e}^{\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

```
[In] int(exp(atan(a*x))*(c + a^2*c*x^2)^2,x)
[Out] int(exp(atan(a*x))*(c + a^2*c*x^2)^2, x)
```

3.246 $\int e^{\arctan(ax)}(c+a^2cx^2)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx = \frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1+\frac{i}{2},2+\frac{i}{2},3+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/17+4/17*I)*2^(2-1/2*I)*c*(1-I*a*x)^(2+1/2*I)*hypergeom([2+1/2*I, -1+1/2*
I],[3+1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5181, 71}

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx = \frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1+\frac{i}{2},2+\frac{i}{2},3+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2),x]
```

```
[Out] ((1/17 + (4*I)/17)*2^(2 - I/2)*c*(1 - I*a*x)^(2 + I/2)*Hypergeometric2F1[-1
 + I/2, 2 + I/2, 3 + I/2, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = c\int (1-iax)^{1+\frac{i}{2}}(1+iax)^{1-\frac{i}{2}}\,dx$$
$$= \frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1+\frac{i}{2},2+\frac{i}{2},3+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx$$
$$= \frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1+\frac{i}{2},2+\frac{i}{2},3+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2),x]
[Out] ((1/17 + (4*I)/17)*2^(2 - I/2)*c*(1 - I*a*x)^(2 + I/2)*Hypergeometric2F1[-1
 + I/2, 2 + I/2, 3 + I/2, (1 - I*a*x)/2])/a
```

Maple [F]

$$\int \mathrm{e}^{\arctan(ax)}\left(a^2cx^2+c\right)dx$$

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c),x)
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(\arctan(ax))}\,dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=c\left(\int a^2x^2e^{\operatorname{atan}(ax)}\,dx+\int e^{\operatorname{atan}(ax)}\,dx\right)$$

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c),x)
[Out] c*(Integral(a**2*x**2*exp(atan(a*x)), x) + Integral(exp(atan(a*x)), x))
```

Maxima [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(\arctan(ax))}\,dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Giac [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(\arctan(ax))}\,dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}(c+a^2cx^2)\ dx=\int \mathrm{e}^{\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

```
[In] int(exp(atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(atan(a*x))*(c + a^2*c*x^2), x)
```

3.247 $\int e^{\arctan(ax)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 60

$$\int e^{\arctan(ax)}\,dx = \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(\frac{i}{2},1+\frac{i}{2},2+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/5+2/5*I)*2^(1-1/2*I)*(1-I*a*x)^(1+1/2*I)*hypergeom([1/2*I, 1+1/2*I],[2+1
/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5169, 71}

$$\int e^{\arctan(ax)}\,dx = \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(\frac{i}{2},1+\frac{i}{2},2+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[E^ArcTan[a*x],x]
```

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I/2)*(1 - I*a*x)^(1 + I/2)*Hypergeometric2F1[I/2, 1
 + I/2, 2 + I/2, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int (1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}\,dx$$
$$= \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(\frac{i}{2},1+\frac{i}{2},2+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int e^{\arctan(ax)}\,dx = \frac{\left(\frac{4}{5}-\frac{8i}{5}\right)e^{(1+2i)\arctan(ax)}\,\text{Hypergeometric2F1}\left(1-\frac{i}{2},2,2-\frac{i}{2},-e^{2i\arctan(ax)}\right)}{a}$$

```
[In] Integrate[E^ArcTan[a*x],x]
[Out] ((4/5 - (8*I)/5)*E^((1 + 2*I)*ArcTan[a*x])*Hypergeometric2F1[1 - I/2, 2, 2
- I/2, -E^((2*I)*ArcTan[a*x])])/a
```

Maple [F]

$$\int \mathrm{e}^{\arctan(ax)}dx$$

```
[In] int(exp(arctan(a*x)),x)
[Out] int(exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{\arctan(ax)}\,dx = \int e^{(\arctan(ax))}\,dx$$

```
[In] integrate(exp(arctan(a*x)),x, algorithm="fricas")
[Out] integral(e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)}\,dx = \int e^{\operatorname{atan}{(ax)}}\,dx$$

```
[In] integrate(exp(atan(a*x)),x)
[Out] Integral(exp(atan(a*x)), x)
```

Maxima [F]

$$\int e^{\arctan(ax)}\,dx = \int e^{\left(\arctan(ax)\right)}\,dx$$

```
[In] integrate(exp(arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(arctan(a*x)), x)
```

Giac [F]

$$\int e^{\arctan(ax)}\,dx = \int e^{\left(\arctan(ax)\right)}\,dx$$

```
[In] integrate(exp(arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}\,dx = \int \mathrm{e}^{\mathrm{atan}(a\,x)}\,dx$$

```
[In] int(exp(atan(a*x)),x)
[Out] int(exp(atan(a*x)), x)
```

3.248 $\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{\arctan(ax)}}{ac}$$

```
[Out] exp(arctan(a*x))/a/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{5179\}$

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{\arctan(ax)}}{ac}$$

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2),x]
```

```
[Out] E^ArcTan[a*x]/(a*c)
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\text{integral} = \frac{e^{\arctan(ax)}}{ac}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}}{ac}$$

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2),x]
[Out] (1 - I*a*x)^(I/2)/(a*c*(1 + I*a*x)^(I/2))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{e^{\arctan(ax)}}{ac}$	13
parallelrisch	$\frac{e^{\arctan(ax)}}{ac}$	13
risch	$\frac{(-iax+1)^{\frac{i}{2}}(iax+1)^{-\frac{i}{2}}}{ac}$	28

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] exp(arctan(a*x))/a/c
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(\arctan(ax))}}{ac}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] e^(arctan(a*x))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx = \begin{cases} \frac{e^{\operatorname{atan}(ax)}}{ac} & \text{for } a \neq 0 \\ \frac{x}{c} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c),x)
[Out] Piecewise((exp(atan(a*x))/(a*c), Ne(a, 0)), (x/c, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(\arctan(ax))}}{ac}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] e^(arctan(a*x))/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(\arctan(ax))}}{ac}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
[Out] e^(arctan(a*x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{\operatorname{atan}(a\,x)}}{a\,c}$$

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2),x)
[Out] exp(atan(a*x))/(a*c)
```

3.249 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{2e^{\arctan(ax)}}{5ac^2} + \frac{e^{\arctan(ax)}(1+2ax)}{5ac^2(1+a^2x^2)}$$

```
[Out] 2/5*exp(arctan(a*x))/a/c^2+1/5*exp(arctan(a*x))*(2*a*x+1)/a/c^2/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5178, 5179}

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{(2ax+1)e^{\arctan(ax)}}{5ac^2(a^2x^2+1)} + \frac{2e^{\arctan(ax)}}{5ac^2}$$

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

```
[Out] (2*E^ArcTan[a*x])/(5*a*c^2) + (E^ArcTan[a*x]*(1 + 2*a*x))/(5*a*c^2*(1 + a^2
*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{\arctan(ax)}(1+2ax)}{5ac^2\left(1+a^2x^2\right)}+\frac{2\int\frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx}{5c}\\&=\frac{2e^{\arctan(ax)}}{5ac^2}+\frac{e^{\arctan(ax)}(1+2ax)}{5ac^2\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int\frac{e^{\arctan(ax)}}{\left(c+a^2cx^2\right)^2}\,dx=\frac{(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}\left(3+2ax+2a^2x^2\right)}{5c^2\left(a+a^3x^2\right)}$$

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
[Out] ((1 - I*a*x)^(I/2)*(3 + 2*a*x + 2*a^2*x^2))/(5*c^2*(1 + I*a*x)^(I/2)*(a + a
^3*x^2))
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{e^{\arctan(ax)}\left(2a^2x^2+2ax+3\right)}{5\left(a^2x^2+1\right)a\,c^2}$	39
parallelrisch	$\frac{2x^2e^{\arctan(ax)}a^2+2\,e^{\arctan(ax)}ax+3\,e^{\arctan(ax)}}{5c^2\left(a^2x^2+1\right)a}$	50

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/5*exp(arctan(a*x))*(2*a^2*x^2+2*a*x+3)/(a^2*x^2+1)/a/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{(2\,a^2x^2+2\,ax+3)e^{(\arctan(ax))}}{5\,(a^3c^2x^2+ac^2)}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/5*(2*a^2*x^2 + 2*a*x + 3)*e^(arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(42) = 84.

Time = 1.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \begin{cases} \frac{2a^2x^2e^{\operatorname{atan}(ax)}}{5a^3c^2x^2+5ac^2} + \frac{2axe^{\operatorname{atan}(ax)}}{5a^3c^2x^2+5ac^2} + \frac{3e^{\operatorname{atan}(ax)}}{5a^3c^2x^2+5ac^2} & \text{for } a \neq 0 \\ \frac{x}{c^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**2,x)
[Out] Piecewise((2*a**2*x**2*exp(atan(a*x))/(5*a**3*c**2*x**2 + 5*a*c**2) + 2*a*x
*exp(atan(a*x))/(5*a**3*c**2*x**2 + 5*a*c**2) + 3*exp(atan(a*x))/(5*a**3*c*
*2*x**2 + 5*a*c**2), Ne(a, 0)), (x/c**2, True))
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{e^{\operatorname{atan}(a\,x)}\,\left(\frac{3}{5\,a^3\,c^2}+\frac{2\,x}{5\,a^2\,c^2}+\frac{2\,x^2}{5\,a\,c^2}\right)}{\frac{1}{a^2}+x^2}$$

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^2,x)
```

```
[Out] (exp(atan(a*x))*(3/(5*a^3*c^2) + (2*x)/(5*a^2*c^2) + (2*x^2)/(5*a*c^2)))/(1
/a^2 + x^2)
```

3.250 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{24e^{\arctan(ax)}}{85ac^3} + \frac{e^{\arctan(ax)}(1+4ax)}{17ac^3(1+a^2x^2)^2} + \frac{12e^{\arctan(ax)}(1+2ax)}{85ac^3(1+a^2x^2)}$$

```
[Out] 24/85*exp(arctan(a*x))/a/c^3+1/17*exp(arctan(a*x))*(4*a*x+1)/a/c^3/(a^2*x^2
+1)^2+12/85*exp(arctan(a*x))*(2*a*x+1)/a/c^3/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5178, 5179}

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{12(2ax+1)e^{\arctan(ax)}}{85ac^3(a^2x^2+1)} + \frac{(4ax+1)e^{\arctan(ax)}}{17ac^3(a^2x^2+1)^2} + \frac{24e^{\arctan(ax)}}{85ac^3}$$

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] (24*E^ArcTan[a*x])/(85*a*c^3) + (E^ArcTan[a*x]*(1 + 4*a*x))/(17*a*c^3*(1 +
a^2*x^2)^2) + (12*E^ArcTan[a*x]*(1 + 2*a*x))/(85*a*c^3*(1 + a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
```

```
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{\arctan(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12\int\frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx}{17c}\\
&= \frac{e^{\arctan(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12e^{\arctan(ax)}(1+2ax)}{85ac^3\left(1+a^2x^2\right)}+\frac{24\int\frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx}{85c^2}\\
&= \frac{24e^{\arctan(ax)}}{85ac^3}+\frac{e^{\arctan(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12e^{\arctan(ax)}(1+2ax)}{85ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int\frac{e^{\arctan(ax)}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{5e^{\arctan(ax)}(1+4ax)+12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}\left(1+a^2x^2\right)\left(3+2ax+2a^2x^2\right)}{85ac^3\left(1+a^2x^2\right)^2}$$

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

```
[Out] (5*E^ArcTan[a*x]*(1 + 4*a*x) + (12*(1 - I*a*x)^(I/2)*(1 + a^2*x^2)*(3 + 2*a
*x + 2*a^2*x^2))/(1 + I*a*x)^(I/2))/(85*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A] (verified)

Time = 10.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{e^{\arctan(ax)}\left(24a^4x^4+24a^3x^3+60a^2x^2+44ax+41\right)}{85\left(a^2x^2+1\right)^2c^3a}$	55
parallelrisch	$\frac{24a^4e^{\arctan(ax)}x^4+24a^3x^3e^{\arctan(ax)}+60x^2e^{\arctan(ax)}a^2+44\,e^{\arctan(ax)}ax+41\,e^{\arctan(ax)}}{85c^3\left(a^2x^2+1\right)^2a}$	76

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/85*exp(arctan(a*x))*(24*a^4*x^4+24*a^3*x^3+60*a^2*x^2+44*a*x+41)/(a^2*x^2
+1)^2/c^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{(24\,a^4x^4+24\,a^3x^3+60\,a^2x^2+44\,ax+41)e^{(\arctan(ax))}}{85\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/85*(24*a^4*x^4 + 24*a^3*x^3 + 60*a^2*x^2 + 44*a*x + 41)*e^(arctan(a*x))/(
a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(75) = 150$.

Time = 2.87 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.69

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \begin{cases} \frac{24a^4x^4e^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{24a^3x^3e^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{60a^2x^2e^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{44axe^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{}{85a^5c} \\ \frac{x}{c^3} \end{cases}$$

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((24*a**4*x**4*exp(atan(a*x))/(85*a**5*c**3*x**4 + 170*a**3*c**3*x
**2 + 85*a*c**3) + 24*a**3*x**3*exp(atan(a*x))/(85*a**5*c**3*x**4 + 170*a**
3*c**3*x**2 + 85*a*c**3) + 60*a**2*x**2*exp(atan(a*x))/(85*a**5*c**3*x**4 +
 170*a**3*c**3*x**2 + 85*a*c**3) + 44*a*x*exp(atan(a*x))/(85*a**5*c**3*x**4
 + 170*a**3*c**3*x**2 + 85*a*c**3) + 41*exp(atan(a*x))/(85*a**5*c**3*x**4 +
 170*a**3*c**3*x**2 + 85*a*c**3), Ne(a, 0)), (x/c**3, True))
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{24\,\mathrm{e}^{\mathrm{atan}(a\,x)}}{85\,a\,c^3} + \frac{12\,\mathrm{e}^{\mathrm{atan}(a\,x)}\,(2\,a\,x+1)}{85\,a\,c^3\,\left(a^2\,x^2+1\right)} + \frac{\mathrm{e}^{\mathrm{atan}(a\,x)}\,(4\,a\,x+1)}{17\,a\,c^3\,{\left(a^2\,x^2+1\right)}^2}$$

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^3,x)
[Out] (24*exp(atan(a*x)))/(85*a*c^3) + (12*exp(atan(a*x))*(2*a*x + 1))/(85*a*c^3*
(a^2*x^2 + 1)) + (exp(atan(a*x))*(4*a*x + 1))/(17*a*c^3*(a^2*x^2 + 1)^2)
```

3.251 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{144e^{\arctan(ax)}}{629ac^4} + \frac{e^{\arctan(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3} + \frac{30e^{\arctan(ax)}(1+4ax)}{629ac^4(1+a^2x^2)^2} + \frac{72e^{\arctan(ax)}(1+2ax)}{629ac^4(1+a^2x^2)}$$

```
[Out] 144/629*exp(arctan(a*x))/a/c^4+1/37*exp(arctan(a*x))*(6*a*x+1)/a/c^4/(a^2*x
^2+1)^3+30/629*exp(arctan(a*x))*(4*a*x+1)/a/c^4/(a^2*x^2+1)^2+72/629*exp(ar
ctan(a*x))*(2*a*x+1)/a/c^4/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5178, 5179}

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{72(2ax+1)e^{\arctan(ax)}}{629ac^4(a^2x^2+1)} + \frac{30(4ax+1)e^{\arctan(ax)}}{629ac^4(a^2x^2+1)^2} + \frac{(6ax+1)e^{\arctan(ax)}}{37ac^4(a^2x^2+1)^3} + \frac{144e^{\arctan(ax)}}{629ac^4}$$

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^4,x]
```

```
[Out] (144*E^ArcTan[a*x])/(629*a*c^4) + (E^ArcTan[a*x]*(1 + 6*a*x))/(37*a*c^4*(1
+ a^2*x^2)^3) + (30*E^ArcTan[a*x]*(1 + 4*a*x))/(629*a*c^4*(1 + a^2*x^2)^2)
+ (72*E^ArcTan[a*x]*(1 + 2*a*x))/(629*a*c^4*(1 + a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{\arctan(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3}+\frac{30\int\frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx}{37c}\\
&= \frac{e^{\arctan(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3}+\frac{30e^{\arctan(ax)}(1+4ax)}{629ac^4(1+a^2x^2)^2}+\frac{360\int\frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx}{629c^2}\\
&= \frac{e^{\arctan(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3}+\frac{30e^{\arctan(ax)}(1+4ax)}{629ac^4(1+a^2x^2)^2}+\frac{72e^{\arctan(ax)}(1+2ax)}{629ac^4(1+a^2x^2)}+\frac{144\int\frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx}{629c^3}\\
&= \frac{144e^{\arctan(ax)}}{629ac^4}+\frac{e^{\arctan(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3}+\frac{30e^{\arctan(ax)}(1+4ax)}{629ac^4(1+a^2x^2)^2}+\frac{72e^{\arctan(ax)}(1+2ax)}{629ac^4(1+a^2x^2)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int\frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{17ce^{\arctan(ax)}(1+6ax)+6(c+a^2cx^2)\left(5e^{\arctan(ax)}(1+4ax)+12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(-i+ax)(i+ax)(\right.}{629ac^2(c+a^2cx^2)^3}$$

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^4,x]
```

```
[Out] (17*c*E^ArcTan[a*x]*(1 + 6*a*x) + 6*(c + a^2*c*x^2)*(5*E^ArcTan[a*x]*(1 + 4
*a*x) + (12*(1 - I*a*x)^(I/2)*(-I + a*x)*(I + a*x)*(3 + 2*a*x + 2*a^2*x^2))
/(1 + I*a*x)^(I/2)))/(629*a*c^2*(c + a^2*c*x^2)^3)
```

Maple [A] (verified)

Time = 32.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

method	result
gosper	$\frac{e^{\arctan(ax)}(144a^6x^6+144a^5x^5+504a^4x^4+408a^3x^3+606a^2x^2+366ax+263)}{629(a^2x^2+1)^3c^4a}$
parallelrisch	$\frac{144a^6e^{\arctan(ax)}x^6+144a^5e^{\arctan(ax)}x^5+504a^4e^{\arctan(ax)}x^4+408a^3x^3e^{\arctan(ax)}+606x^2e^{\arctan(ax)}a^2+366\,e^{\arctan(ax)}ax+26}{629c^4(a^2x^2+1)^3a}$

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/629*exp(arctan(a*x))*(144*a^6*x^6+144*a^5*x^5+504*a^4*x^4+408*a^3*x^3+606
*a^2*x^2+366*a*x+263)/(a^2*x^2+1)^3/c^4/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{(144\,a^6x^6+144\,a^5x^5+504\,a^4x^4+408\,a^3x^3+606\,a^2x^2+366\,ax+263)e^{(\arctan(ax))}}{629\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/629*(144*a^6*x^6 + 144*a^5*x^5 + 504*a^4*x^4 + 408*a^3*x^3 + 606*a^2*x^2
+ 366*a*x + 263)*e^(arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x
^2 + a*c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(107) = 214.

Time = 7.51 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.43

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \begin{cases} \frac{144a^6x^6e^{\operatorname{atan}(ax)}}{629a^7c^4x^6+1887a^5c^4x^4+1887a^3c^4x^2+629ac^4} + \frac{144a^5x^5e^{\operatorname{atan}(ax)}}{629a^7c^4x^6+1887a^5c^4x^4+1887a^3c^4x^2+629ac^4} + \frac{504a^4x^4e^{\operatorname{atan}(ax)}}{629a^7c^4x^6+1887a^5c^4x^4+1887a^3c^4x^2+} \\ \frac{x}{c^4} \end{cases}$$

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Piecewise((144*a**6*x**6*exp(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c**
4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4) + 144*a**5*x**5*exp(atan(a*x))/(
629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4
) + 504*a**4*x**4*exp(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4
+ 1887*a**3*c**4*x**2 + 629*a*c**4) + 408*a**3*x**3*exp(atan(a*x))/(629*a**
7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4) + 606
*a**2*x**2*exp(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*
a**3*c**4*x**2 + 629*a*c**4) + 366*a*x*exp(atan(a*x))/(629*a**7*c**4*x**6 +
 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4) + 263*exp(atan(a*x
))/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*
c**4), Ne(a, 0)), (x/c**4, True))
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Giac [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{144\,e^{\mathrm{atan}(a\,x)}}{629\,a\,c^4} + \frac{72\,e^{\mathrm{atan}(a\,x)}\,(2\,a\,x+1)}{629\,a\,c^4\,(a^2\,x^2+1)} + \frac{30\,e^{\mathrm{atan}(a\,x)}\,(4\,a\,x+1)}{629\,a\,c^4\,(a^2\,x^2+1)^2} + \frac{e^{\mathrm{atan}(a\,x)}\,(6\,a\,x+1)}{37\,a\,c^4\,(a^2\,x^2+1)^3}$$

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^4,x)
```

```
[Out] (144*exp(atan(a*x)))/(629*a*c^4) + (72*exp(atan(a*x))*(2*a*x + 1))/(629*a*c
^4*(a^2*x^2 + 1)) + (30*exp(atan(a*x))*(4*a*x + 1))/(629*a*c^4*(a^2*x^2 + 1
)^2) + (exp(atan(a*x))*(6*a*x + 1))/(37*a*c^4*(a^2*x^2 + 1)^3)
```

3.252 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5} dx = \frac{8064e^{\arctan(ax)}}{40885ac^5} + \frac{e^{\arctan(ax)}(1+8ax)}{65ac^5(1+a^2x^2)^4} + \frac{56e^{\arctan(ax)}(1+6ax)}{2405ac^5(1+a^2x^2)^3} + \frac{336e^{\arctan(ax)}(1+4ax)}{8177ac^5(1+a^2x^2)^2} + \frac{4032e^{\arctan(ax)}(1+2ax)}{40885ac^5(1+a^2x^2)}$$

```
[Out] 8064/40885*exp(arctan(a*x))/a/c^5+1/65*exp(arctan(a*x))*(8*a*x+1)/a/c^5/(a^
2*x^2+1)^4+56/2405*exp(arctan(a*x))*(6*a*x+1)/a/c^5/(a^2*x^2+1)^3+336/8177*
exp(arctan(a*x))*(4*a*x+1)/a/c^5/(a^2*x^2+1)^2+4032/40885*exp(arctan(a*x))*
(2*a*x+1)/a/c^5/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5178, 5179}

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5} dx = \frac{4032(2ax+1)e^{\arctan(ax)}}{40885ac^5(a^2x^2+1)} + \frac{336(4ax+1)e^{\arctan(ax)}}{8177ac^5(a^2x^2+1)^2} + \frac{56(6ax+1)e^{\arctan(ax)}}{2405ac^5(a^2x^2+1)^3} + \frac{(8ax+1)e^{\arctan(ax)}}{65ac^5(a^2x^2+1)^4} + \frac{8064e^{\arctan(ax)}}{40885ac^5}$$

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^5,x]
```

```
[Out] (8064*E^ArcTan[a*x])/(40885*a*c^5) + (E^ArcTan[a*x]*(1 + 8*a*x))/(65*a*c^5*
(1 + a^2*x^2)^4) + (56*E^ArcTan[a*x]*(1 + 6*a*x))/(2405*a*c^5*(1 + a^2*x^2)
```

```
^3) + (336*E^ArcTan[a*x]*(1 + 4*a*x))/(8177*a*c^5*(1 + a^2*x^2)^2) + (4032*
E^ArcTan[a*x]*(1 + 2*a*x))/(40885*a*c^5*(1 + a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{\arctan(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4}+\frac{56\int\frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx}{65c}\\
&= \frac{e^{\arctan(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4}+\frac{56e^{\arctan(ax)}(1+6ax)}{2405ac^5\left(1+a^2x^2\right)^3}+\frac{336\int\frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx}{481c^2}\\
&= \frac{e^{\arctan(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4}+\frac{56e^{\arctan(ax)}(1+6ax)}{2405ac^5\left(1+a^2x^2\right)^3}+\frac{336e^{\arctan(ax)}(1+4ax)}{8177ac^5\left(1+a^2x^2\right)^2}+\frac{4032\int\frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx}{8177c^3}\\
&= \frac{e^{\arctan(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4}+\frac{56e^{\arctan(ax)}(1+6ax)}{2405ac^5\left(1+a^2x^2\right)^3}+\frac{336e^{\arctan(ax)}(1+4ax)}{8177ac^5\left(1+a^2x^2\right)^2}\\
&\quad+\frac{4032e^{\arctan(ax)}(1+2ax)}{40885ac^5\left(1+a^2x^2\right)}+\frac{8064\int\frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx}{40885c^4}\\
&= \frac{8064e^{\arctan(ax)}}{40885ac^5}+\frac{e^{\arctan(ax)}(1+8ax)}{65ac^5\left(1+a^2x^2\right)^4}+\frac{56e^{\arctan(ax)}(1+6ax)}{2405ac^5\left(1+a^2x^2\right)^3}\\
&\quad+\frac{336e^{\arctan(ax)}(1+4ax)}{8177ac^5\left(1+a^2x^2\right)^2}+\frac{4032e^{\arctan(ax)}(1+2ax)}{40885ac^5\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx = \frac{629e^{\arctan(ax)}(1+8ax)+\frac{56(c+a^2cx^2)\left(17ce^{\arctan(ax)}(1+6ax)+6(c+a^2cx^2)\left(5e^{\arctan(ax)}(1+4ax)+12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(-i+ax)(i}{c^2}}{40885ac\,(c+a^2cx^2)^4}$$

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^5,x]
```

```
[Out] (629*E^ArcTan[a*x]*(1 + 8*a*x) + (56*(c + a^2*c*x^2)*(17*c*E^ArcTan[a*x]*(1
 + 6*a*x) + 6*(c + a^2*c*x^2)*(5*E^ArcTan[a*x]*(1 + 4*a*x) + (12*(1 - I*a*x
)^(I/2)*(-I + a*x)*(I + a*x)*(3 + 2*a*x + 2*a^2*x^2))/(1 + I*a*x)^(I/2))))/
c^2)/(40885*a*c*(c + a^2*c*x^2)^4)
```

Maple [A] (verified)

Time = 85.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

method	result
gosper	$\frac{e^{\arctan(ax)}\left(8064a^8x^8+8064a^7x^7+36288a^6x^6+30912a^5x^5+62160a^4x^4+43344a^3x^3+48664a^2x^2+25528ax+15357\right)}{40885\left(a^2x^2+1\right)^4c^5a}$
parallelrisch	$\frac{8064a^8e^{\arctan(ax)}x^8+8064a^7e^{\arctan(ax)}x^7+36288a^6e^{\arctan(ax)}x^6+30912a^5e^{\arctan(ax)}x^5+62160a^4e^{\arctan(ax)}x^4+43344a^3x^3e}{40885c^5\left(a^2x^2+1\right)^4a}$

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40885*exp(arctan(a*x))*(8064*a^8*x^8+8064*a^7*x^7+36288*a^6*x^6+30912*a^5
*x^5+62160*a^4*x^4+43344*a^3*x^3+48664*a^2*x^2+25528*a*x+15357)/(a^2*x^2+1)
^4/c^5/a
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx = \frac{(8064\,a^8x^8+8064\,a^7x^7+36288\,a^6x^6+30912\,a^5x^5+62160\,a^4x^4+43344\,a^3x^3+48664\,a^2x^2+25528\,ax+}{40885\,(a^9c^5x^8+4\,a^7c^5x^6+6\,a^5c^5x^4+4\,a^3c^5x^2+ac^5)}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] 1/40885*(8064*a^8*x^8 + 8064*a^7*x^7 + 36288*a^6*x^6 + 30912*a^5*x^5 + 6216
0*a^4*x^4 + 43344*a^3*x^3 + 48664*a^2*x^2 + 25528*a*x + 15357)*e^(arctan(a*
x))/(a^9*c^5*x^8 + 4*a^7*c^5*x^6 + 6*a^5*c^5*x^4 + 4*a^3*c^5*x^2 + a*c^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(139) = 278$.

Time = 21.90 (sec) , antiderivative size = 620, normalized size of antiderivative = 4.16

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx = \begin{cases} \frac{8064a^8x^8e^{\operatorname{atan}(ax)}}{40885a^9c^5x^8+163540a^7c^5x^6+245310a^5c^5x^4+163540a^3c^5x^2+40885ac^5} + \frac{8064a^7x^7e^{\operatorname{atan}(ax)}}{40885a^9c^5x^8+163540a^7c^5x^6+245310a^5c^5x^4+163540a^3c^5x^2+4088} \\ \frac{x}{c^5} \end{cases}$$

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**5,x)
```

```
[Out] Piecewise((8064*a**8*x**8*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**
7*c**5*x**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5)
 + 8064*a**7*x**7*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x
**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 36288
*a**6*x**6*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 2
45310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 30912*a**5*x
**5*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a
**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 62160*a**4*x**4*exp
(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**
5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 43344*a**3*x**3*exp(atan(a
*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**5*x**4
+ 163540*a**3*c**5*x**2 + 40885*a*c**5) + 48664*a**2*x**2*exp(atan(a*x))/(4
0885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**5*x**4 + 16354
0*a**3*c**5*x**2 + 40885*a*c**5) + 25528*a*x*exp(atan(a*x))/(40885*a**9*c**
5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x
**2 + 40885*a*c**5) + 15357*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a
**7*c**5*x**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**
5), Ne(a, 0)), (x/c**5, True))
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^5}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^5, x)
```

Giac [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^5}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.90

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx = \frac{e^{\mathrm{atan}(a\,x)}\left(\frac{15357}{40885\,a^9\,c^5}+\frac{25528\,x}{40885\,a^8\,c^5}+\frac{8064\,x^8}{40885\,a\,c^5}+\frac{8064\,x^7}{40885\,a^2\,c^5}+\frac{36288\,x^6}{40885\,a^3\,c^5}+\frac{30912\,x^5}{40885\,a^4\,c^5}+\frac{336\,x^4}{221\,a^5\,c^5}+\frac{43344\,x^3}{40885\,a^6\,c^5}+\frac{48664}{40885\,a}\right.}{\frac{1}{a^8}+x^8+\frac{4\,x^6}{a^2}+\frac{6\,x^4}{a^4}+\frac{4\,x^2}{a^6}}$$

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^5,x)
```

```
[Out] (exp(atan(a*x))*(15357/(40885*a^9*c^5) + (25528*x)/(40885*a^8*c^5) + (8064*
x^8)/(40885*a*c^5) + (8064*x^7)/(40885*a^2*c^5) + (36288*x^6)/(40885*a^3*c^
5) + (30912*x^5)/(40885*a^4*c^5) + (336*x^4)/(221*a^5*c^5) + (43344*x^3)/(4
0885*a^6*c^5) + (48664*x^2)/(40885*a^7*c^5)))/(1/a^8 + x^8 + (4*x^6)/a^2 +
(6*x^4)/a^4 + (4*x^2)/a^6)
```

3.253 $\int e^{\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int e^{\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx = \frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{7}{2}+\frac{i}{2},\frac{1}{2}(1-i$$

$$a\sqrt{1+a^2x^2}$$

```
[Out] (1/13+5/13*I)*2^(3/2-1/2*I)*c*(1-I*a*x)^(5/2+1/2*I)*hypergeom([5/2+1/2*I, -
3/2+1/2*I],[7/2+1/2*I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/
2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5184, 5181, 71}

$$\int e^{\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx = \frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{7}{2}+\frac{i}{2},\frac{1}{2}(1-i$$

$$a\sqrt{a^2x^2+1}$$

```
[In] Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1/13 + (5*I)/13)*2^(3/2 - I/2)*c*(1 - I*a*x)^(5/2 + I/2)*Sqrt[c + a^2*c*x
^2]*Hypergeometric2F1[-3/2 + I/2, 5/2 + I/2, 7/2 + I/2, (1 - I*a*x)/2])/(a*
Sqrt[1 + a^2*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{\arctan(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}} \\
&= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int (1-iax)^{\frac{3}{2}+\frac{i}{2}}(1+iax)^{\frac{3}{2}-\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}} \\
&= \frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{7}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = \frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{7}{2}+\frac{i}{2},\frac{1}{2}(1-\right.}{a\sqrt{1+a^2x^2}}$$

```
[In] Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1/13 + (5*I)/13)*2^(3/2 - I/2)*c*(1 - I*a*x)^(5/2 + I/2)*Sqrt[c + a^2*c*x
^2]*Hypergeometric2F1[-3/2 + I/2, 5/2 + I/2, 7/2 + I/2, (1 - I*a*x)/2])/(a*
Sqrt[1 + a^2*x^2])
```

Maple [F]

$$\int \mathrm{e}^{\arctan(ax)} \left(a^2 c\, x^2 + c\right)^{\frac{3}{2}} dx$$

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int e^{\arctan(ax)} \left(c + a^2 c x^2\right)^{3/2} dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} e^{(\arctan(ax))} dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)} \left(c + a^2 c x^2\right)^{3/2} dx = \int \left(c\left(a^2 x^2 + 1\right)\right)^{\frac{3}{2}} e^{\operatorname{atan}(ax)} dx$$

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*exp(atan(a*x)), x)
```

Maxima [F]

$$\int e^{\arctan(ax)} \left(c + a^2 c x^2\right)^{3/2} dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} e^{(\arctan(ax))} dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx = \int e^{\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

[In] int(exp(atan(a*x))*(c + a^2*c*x^2)^(3/2),x)

[Out] int(exp(atan(a*x))*(c + a^2*c*x^2)^(3/2), x)

3.254 $\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] (1/5+3/5*I)*2^(1/2-1/2*I)*(1-I*a*x)^(3/2+1/2*I)*hypergeom([3/2+1/2*I, -1/2+
1/2*I],[5/2+1/2*I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5184, 5181, 71}

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[In] Int[E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1/2 - I/2)*(1 - I*a*x)^(3/2 + I/2)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-1/2 + I/2, 3/2 + I/2, 5/2 + I/2, (1 - I*a*x)/2])/(a*Sqrt
[1 + a^2*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c+a^2cx^2}\int e^{\arctan(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}\\
&= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}+\frac{i}{2}}(1+iax)^{\frac{1}{2}-\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}}\\
&= \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$$
$$=\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[In] Integrate[E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1/2 - I/2)*(1 - I*a*x)^(3/2 + I/2)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-1/2 + I/2, 3/2 + I/2, 5/2 + I/2, (1 - I*a*x)/2])/(a*Sqrt
[1 + a^2*x^2])
```

Maple [F]

$$\int e^{\arctan(ax)}\sqrt{a^2c\,x^2+c}dx$$

```
[In] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}e^{(\arctan(ax))}\,dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{c\,(a^2x^2+1)}e^{\operatorname{atan}(ax)}\,dx$$

```
[In] integrate(exp(atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(atan(a*x)), x)
```

Maxima [F]

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}e^{(\arctan(ax))}\,dx$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int e^{\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

```
[In] int(exp(atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

3.255 $\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}} dx = \frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

```
[Out] (1+I)*2^(-1/2-1/2*I)*(1-I*a*x)^(1/2+1/2*I)*hypergeom([1/2+1/2*I, 1/2+1/2*I]
,[3/2+1/2*I],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5184, 5181, 71}

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}} dx = \frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{a^2x^2+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 + I)*(1 - I*a*x)^(1/2 + I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 +
I/2, 1/2 + I/2, 3/2 + I/2, (1 - I*a*x)/2])/(2^(1/2 + I/2)*a*Sqrt[c + a^2*c*
x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\int \frac{e^{\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{i}{2}}(1+iax)^{-\frac{1}{2}-\frac{i}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 + I)*(1 - I*a*x)^(1/2 + I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 +
I/2, 1/2 + I/2, 3/2 + I/2, (1 - I*a*x)/2])/(2^(1/2 + I/2)*a*Sqrt[c + a^2*c*
x^2])
```

Maple [F]

$$\int \frac{\mathrm{e}^{\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}dx$$

[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)

[Out] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)

Fricas [F]

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] integral(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)

Sympy [F]

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{\operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(1/2),x)

[Out] Integral(exp(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)

Giac [F]

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{\operatorname{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.256 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{\arctan(ax)}(1+ax)}{2ac\sqrt{c+a^2cx^2}}$$

```
[Out] 1/2*exp(arctan(a*x))*(a*x+1)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{5177\}$

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(ax+1)e^{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^ArcTan[a*x]*(1 + a*x))/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\text{integral} = \frac{e^{\arctan(ax)}(1+ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{e^{\arctan(ax)}(1+ax)}{2ac\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
[Out] (E^ArcTan[a*x]*(1 + a*x))/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{(a^2x^2+1)(ax+1)\mathrm{e}^{\arctan(ax)}}{2a(a^2c\,x^2+c)^{\frac{3}{2}}}$	37

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a^2*x^2+1)*(a*x+1)*exp(arctan(a*x))/a/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{a^2cx^2+c}(ax+1)e^{(\arctan(ax))}}{2\,(a^3c^2x^2+ac^2)}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] 1/2*sqrt(a^2*c*x^2 + c)*(a*x + 1)*e^(arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]

$$\int \frac{e^{\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{e^{\operatorname{atan}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{e^{(\arctan(ax))}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{e^{\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{e^{\operatorname{atan}(a\,x)}\left(\frac{x}{2\,c}+\frac{1}{2\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] (exp(atan(a*x))*(x/(2*c) + 1/(2*a*c)))/(c + a^2*c*x^2)^(1/2)
```

3.257 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{\arctan(ax)}(1+3ax)}{10ac\,(c+a^2cx^2)^{3/2}} + \frac{3e^{\arctan(ax)}(1+ax)}{10ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/10*exp(arctan(a*x))*(3*a*x+1)/a/c/(a^2*c*x^2+c)^(3/2)+3/10*exp(arctan(a*x
))*(a*x+1)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5177}

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{3(ax+1)e^{\arctan(ax)}}{10ac^2\sqrt{a^2cx^2+c}} + \frac{(3ax+1)e^{\arctan(ax)}}{10ac\,(a^2cx^2+c)^{3/2}}$$

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^ArcTan[a*x]*(1 + 3*a*x))/(10*a*c*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a*
x]*(1 + a*x))/(10*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] && !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{\arctan(ax)}(1+3ax)}{10ac\,(c+a^2cx^2)^{3/2}}+\frac{3\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{5c}\\ &= \frac{e^{\arctan(ax)}(1+3ax)}{10ac\,(c+a^2cx^2)^{3/2}}+\frac{3e^{\arctan(ax)}(1+ax)}{10ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\frac{e^{\arctan(ax)}(4+6ax+3a^2x^2+3a^3x^3)}{10c^2\,(a+a^3x^2)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
[Out] (E^ArcTan[a*x]*(4 + 6*a*x + 3*a^2*x^2 + 3*a^3*x^3))/(10*c^2*(a + a^3*x^2)*S
qrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{(a^2x^2+1)(3a^3x^3+3a^2x^2+6ax+4)\,e^{\arctan(ax)}}{10a(a^2cx^2+c)^{\frac{5}{2}}}$	54

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/10*(a^2*x^2+1)*(3*a^3*x^3+3*a^2*x^2+6*a*x+4)*exp(arctan(a*x))/a/(a^2*c*x^
2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(3\,a^3x^3+3\,a^2x^2+6\,ax+4)\sqrt{a^2cx^2+c}e^{(\arctan(ax))}}{10\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/10*(3*a^3*x^3 + 3*a^2*x^2 + 6*a*x + 4)*sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)
)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
[Out] Integral(exp(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{\operatorname{atan}(a\,x)}\,\left(\frac{2}{5\,a^3\,c^2}+\frac{3\,x^3}{10\,c^2}+\frac{3\,x}{5\,a^2\,c^2}+\frac{3\,x^2}{10\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] (exp(atan(a*x))*(2/(5*a^3*c^2) + (3*x^3)/(10*c^2) + (3*x)/(5*a^2*c^2) + (3*
x^2)/(10*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)^(1/2))
```

3.258 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{\arctan(ax)}(1+5ax)}{26ac\,(c+a^2cx^2)^{5/2}} + \frac{e^{\arctan(ax)}(1+3ax)}{13ac^2\,(c+a^2cx^2)^{3/2}} + \frac{3e^{\arctan(ax)}(1+ax)}{13ac^3\sqrt{c+a^2cx^2}}$$

```
[Out] 1/26*exp(arctan(a*x))*(5*a*x+1)/a/c/(a^2*c*x^2+c)^(5/2)+1/13*exp(arctan(a*x
))*(3*a*x+1)/a/c^2/(a^2*c*x^2+c)^(3/2)+3/13*exp(arctan(a*x))*(a*x+1)/a/c^3/
(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5177}

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{3(ax+1)e^{\arctan(ax)}}{13ac^3\sqrt{a^2cx^2+c}} + \frac{(3ax+1)e^{\arctan(ax)}}{13ac^2\,(a^2cx^2+c)^{3/2}} + \frac{(5ax+1)e^{\arctan(ax)}}{26ac\,(a^2cx^2+c)^{5/2}}$$

```
[In] Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^ArcTan[a*x]*(1 + 5*a*x))/(26*a*c*(c + a^2*c*x^2)^(5/2)) + (E^ArcTan[a*x]
*(1 + 3*a*x))/(13*a*c^2*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a*x]*(1 + a*x)
)/(13*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{\arctan(ax)}(1+5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}+\frac{10\int\frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx}{13c}\\ &= \frac{e^{\arctan(ax)}(1+5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}+\frac{e^{\arctan(ax)}(1+3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{6\int\frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{13c^2}\\ &= \frac{e^{\arctan(ax)}(1+5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}+\frac{e^{\arctan(ax)}(1+3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{3e^{\arctan(ax)}(1+ax)}{13ac^3\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.73

$$\int\frac{e^{\arctan(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx=\frac{e^{\arctan(ax)}\left(9+17ax+14a^2x^2+18a^3x^3+6a^4x^4+6a^5x^5\right)}{26ac^3\left(1+a^2x^2\right)^2\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(7/2),x]
[Out] (E^ArcTan[a*x]*(9 + 17*a*x + 14*a^2*x^2 + 18*a^3*x^3 + 6*a^4*x^4 + 6*a^5*x^
5))/(26*a*c^3*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{(a^2x^2+1)(6a^5x^5+6a^4x^4+18a^3x^3+14a^2x^2+17ax+9)e^{\arctan(ax)}}{26a(a^2c\,x^2+c)^{\frac{7}{2}}}$	70

```
[In] int(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/26*(a^2*x^2+1)*(6*a^5*x^5+6*a^4*x^4+18*a^3*x^3+14*a^2*x^2+17*a*x+9)*exp(a
rctan(a*x))/a/(a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{(6\,a^5x^5+6\,a^4x^4+18\,a^3x^3+14\,a^2x^2+17\,ax+9)\sqrt{a^2cx^2+c}e^{(\arctan(ax))}}{26\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
[Out] 1/26*(6*a^5*x^5 + 6*a^4*x^4 + 18*a^3*x^3 + 14*a^2*x^2 + 17*a*x + 9)*sqrt(a^
2*c*x^2 + c)*e^(arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 +
 a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{\mathrm{atan}(a\,x)}\,\left(\frac{9}{26\,a^5\,c^3}+\frac{3\,x^5}{13\,c^3}+\frac{17\,x}{26\,a^4\,c^3}+\frac{3\,x^4}{13\,a\,c^3}+\frac{9\,x^3}{13\,a^2\,c^3}+\frac{7\,x^2}{13\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

```
[In] int(exp(atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] (exp(atan(a*x))*(9/(26*a^5*c^3) + (3*x^5)/(13*c^3) + (17*x)/(26*a^4*c^3) +
(3*x^4)/(13*a*c^3) + (9*x^3)/(13*a^2*c^3) + (7*x^2)/(13*a^3*c^3)))/((c + a^
2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*(c + a^2*c*x^2)^(1/
2))/a^2)
```

3.259 $\int e^{2\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{-i+p}(1-iax)^{(1+i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(i-p,(1+i)+p,(2+i)+p,\tfrac{1}{2}(1-\right.}{a((1+i)+p)}$$

```
[Out] I*2^(-I+p)*(1-I*a*x)^(1+I+p)*(a^2*c*x^2+c)^p*hypergeom([I-p, 1+I+p],[2+I+p]
,1/2-1/2*I*a*x)/a/(1+I+p)/((a^2*x^2+1)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5184, 5181, 71}

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{p-i}(1-iax)^{p+(1+i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\text{Hypergeometric2F1}\left(i-p,p+(1+i),p+(2+i),\tfrac{1}{2}(1-\right.}{a(p+(1+i))}$$

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^(-I + p)*(1 - I*a*x)^((1 + I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1
[I - p, (1 + I) + p, (2 + I) + p, (1 - I*a*x)/2])/(a*((1 + I) + p)*(1 + a^2
*x^2)^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{2\arctan(ax)}\left(1+a^2x^2\right)^p\,dx$$

$$= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{i+p}(1+iax)^{-i+p}\,dx$$

$$= \frac{i2^{-i+p}(1-iax)^{(1+i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(i-p,(1+i)+p,(2+i)+p\right.}{a((1+i)+p)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$= \frac{i2^{-i+p}(1-iax)^{(1+i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(i-p,(1+i)+p,(2+i)+p,\frac{1}{2}(1-i\right.}{a((1+i)+p)}$$

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (I*2^(-I + p)*(1 - I*a*x)^((1 + I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1
[I - p, (1 + I) + p, (2 + I) + p, (1 - I*a*x)/2])/(a*((1 + I) + p)*(1 + a^2
*x^2)^p)
```

Maple [F]

$$\int e^{2\arctan(ax)}\left(a^2 c\, x^2+c\right)^p dx$$

[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x)

[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x)

Fricas [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\, dx=\int \left(a^2cx^2+c\right)^p e^{(2\,\arctan(ax))}\, dx$$

[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")

[Out] integral((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)

Sympy [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\, dx=\int \left(c\left(a^2x^2+1\right)\right)^p e^{2\,\mathrm{atan}\,(ax)}\, dx$$

[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**p,x)

[Out] Integral((c*(a**2*x**2 + 1))**p*exp(2*atan(a*x)), x)

Maxima [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\, dx=\int \left(a^2cx^2+c\right)^p e^{(2\,\arctan(ax))}\, dx$$

[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)

Giac [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(2\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int e^{2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^p\,dx$$

```
[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^p,x)
[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^p, x)
```

3.260 $\int e^{2\arctan(ax)}(c+a^2cx^2)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,\text{Hypergeometric2F1}\left(-2+i,3+i,4+i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/5+3/5*I)*2^(1-I)*c^2*(1-I*a*x)^(3+I)*hypergeom([3+I, -2+I],[4+I],1/2-1/2
*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5181, 71}

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,\text{Hypergeometric2F1}\left(-2+i,3+i,4+i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] ((1/5 + (3*I)/5)*2^(1 - I)*c^2*(1 - I*a*x)^(3 + I)*Hypergeometric2F1[-2 + I
, 3 + I, 4 + I, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= c^2 \int (1-iax)^{2+i}(1+iax)^{2-i}\,dx\\ &= \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,\text{Hypergeometric2F1}\left(-2+i,3+i,4+i,\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,\text{Hypergeometric2F1}\left(-2+i,3+i,4+i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
[Out] ((1/5 + (3*I)/5)*2^(1 - I)*c^2*(1 - I*a*x)^(3 + I)*Hypergeometric2F1[-2 + I
, 3 + I, 4 + I, (1 - I*a*x)/2])/a
```

Maple [F]

$$\int \mathrm{e}^{2\arctan(ax)}\left(a^2c\,x^2+c\right)^2dx$$

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x)
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(2\arctan(ax))}\,dx$$

[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(2*arctan(a*x)), x)

Sympy [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=c^2\left(\int 2a^2x^2e^{2\operatorname{atan}(ax)}\,dx+\int a^4x^4e^{2\operatorname{atan}(ax)}\,dx+\int e^{2\operatorname{atan}(ax)}\,dx\right)$$

[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**2,x)

[Out] c**2*(Integral(2*a**2*x**2*exp(2*atan(a*x)), x) + Integral(a**4*x**4*exp(2*atan(a*x)), x) + Integral(exp(2*atan(a*x)), x))

Maxima [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(2\arctan(ax))}\,dx$$

[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^2*e^(2*arctan(a*x)), x)

Giac [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(2\arctan(ax))}\,dx$$

[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

```
[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^2,x)
[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^2, x)
```

3.261 $\int e^{2\arctan(ax)}(c+a^2cx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int e^{2\arctan(ax)}(c+a^2cx^2)\,dx = \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,\text{Hypergeometric2F1}\left(-1+i,2+i,3+i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1/5+2/5*I)*2^(1-I)*c*(1-I*a*x)^(2+I)*hypergeom([-1+I, 2+I],[3+I],1/2-1/2*I
*a*x)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5181, 71}

$$\int e^{2\arctan(ax)}(c+a^2cx^2)\,dx = \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,\text{Hypergeometric2F1}\left(-1+i,2+i,3+i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] ((1/5 + (2*I)/5)*2^(1 - I)*c*(1 - I*a*x)^(2 + I)*Hypergeometric2F1[-1 + I,
2 + I, 3 + I, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = c\int (1-iax)^{1+i}(1+iax)^{1-i}\,dx$$
$$= \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,\text{Hypergeometric2F1}\left(-1+i,2+i,3+i,\frac{1}{2}(1-iax)\right)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx$$
$$= \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,\text{Hypergeometric2F1}\left(-1+i,2+i,3+i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2),x]
[Out] ((1/5 + (2*I)/5)*2^(1 - I)*c*(1 - I*a*x)^(2 + I)*Hypergeometric2F1[-1 + I,
2 + I, 3 + I, (1 - I*a*x)/2])/a
```

Maple [F]

$$\int e^{2\arctan(ax)}\left(a^2c\,x^2+c\right)dx$$

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c),x)
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(2\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=c\left(\int a^2x^2e^{2\operatorname{atan}(ax)}\,dx+\int e^{2\operatorname{atan}(ax)}\,dx\right)$$

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c),x)
[Out] c*(Integral(a**2*x**2*exp(2*atan(a*x)), x) + Integral(exp(2*atan(a*x)), x))
```

Maxima [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(2\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(2\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int e^{2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

```
[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2), x)
```

3.262 $\int e^{2\arctan(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int e^{2\arctan(ax)}\,dx=\frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,\text{Hypergeometric2F1}\left(i,1+i,2+i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (1+I)*2^(-1-I)*(1-I*a*x)^(1+I)*hypergeom([I, 1+I],[2+I],1/2-1/2*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5169, 71}

$$\int e^{2\arctan(ax)}\,dx=\frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,\text{Hypergeometric2F1}\left(i,1+i,2+i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[E^(2*ArcTan[a*x]),x]
```

```
[Out] ((1 + I)*(1 - I*a*x)^(1 + I)*Hypergeometric2F1[I, 1 + I, 2 + I, (1 - I*a*x)
/2])/(2^(1 + I)*a)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int (1-iax)^i(1+iax)^{-i}\,dx$$
$$= \frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,\text{Hypergeometric2F1}\left(i,1+i,2+i,\frac{1}{2}(1-iax)\right)}{a}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int e^{2\arctan(ax)}\,dx = \frac{(1-i)e^{(2+2i)\arctan(ax)}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{2i\arctan(ax)}\right)}{a}$$

```
[In] Integrate[E^(2*ArcTan[a*x]),x]
[Out] ((1 - I)*E^((2 + 2*I)*ArcTan[a*x])*Hypergeometric2F1[1 - I, 2, 2 - I, -E^((
2*I)*ArcTan[a*x])])/a
```

Maple [F]

$$\int \mathrm{e}^{2\arctan(ax)}dx$$

```
[In] int(exp(2*arctan(a*x)),x)
[Out] int(exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{2\arctan(ax)}\,dx = \int e^{(2\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral(e^(2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{2\arctan(ax)}\,dx = \int e^{2\operatorname{atan}(ax)}\,dx$$

```
[In] integrate(exp(2*atan(a*x)),x)
[Out] Integral(exp(2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{2\arctan(ax)}\,dx = \int e^{(2\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{2\arctan(ax)}\,dx = \int e^{(2\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\,dx = \int \mathrm{e}^{2\operatorname{atan}(a\,x)}\,dx$$

```
[In] int(exp(2*atan(a*x)),x)
[Out] int(exp(2*atan(a*x)), x)
```

3.263 $\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{e^{2\arctan(ax)}}{2ac}$$

```
[Out] 1/2*exp(2*arctan(a*x))/a/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5179}

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{e^{2\arctan(ax)}}{2ac}$$

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] E^(2*ArcTan[a*x])/(2*a*c)
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\text{integral}=\frac{e^{2\arctan(ax)}}{2ac}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{(1-iax)^i(1+iax)^{-i}}{2ac}$$

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2),x]
[Out] (1 - I*a*x)^I/(2*a*c*(1 + I*a*x)^I)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$\frac{e^{2\arctan(ax)}}{2ac}$	16
parallelrisch	$\frac{e^{2\arctan(ax)}}{2ac}$	16
risch	$\frac{(-iax+1)^i(iax+1)^{-i}}{2ac}$	29

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(2*arctan(a*x))/a/c
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(2\arctan(ax))}}{2\,ac}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/2*e^(2*arctan(a*x))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \begin{cases} \frac{e^{2\operatorname{atan}(ax)}}{2ac} & \text{for } a \neq 0 \\ \frac{x}{c} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c),x)
[Out] Piecewise((exp(2*atan(a*x))/(2*a*c), Ne(a, 0)), (x/c, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(2\arctan(ax))}}{2\,ac}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] 1/2*e^(2*arctan(a*x))/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(2\arctan(ax))}}{2\,ac}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/2*e^(2*arctan(a*x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{2\operatorname{atan}(a\,x)}}{2\,a\,c}$$

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2),x)
[Out] exp(2*atan(a*x))/(2*a*c)
```

3.264 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2} dx = \frac{e^{2\arctan(ax)}}{8ac^2} + \frac{e^{2\arctan(ax)}(1+ax)}{4ac^2(1+a^2x^2)}$$

```
[Out] 1/8*exp(2*arctan(a*x))/a/c^2+1/4*exp(2*arctan(a*x))*(a*x+1)/a/c^2/(a^2*x^2+
1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5179}

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2} dx = \frac{(ax+1)e^{2\arctan(ax)}}{4ac^2(a^2x^2+1)} + \frac{e^{2\arctan(ax)}}{8ac^2}$$

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

```
[Out] E^(2*ArcTan[a*x])/(8*a*c^2) + (E^(2*ArcTan[a*x])*(1 + a*x))/(4*a*c^2*(1 + a
^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\text{integral} = \frac{e^{2\arctan(ax)}(1+ax)}{4ac^2(1+a^2x^2)} + \frac{\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx}{4c}$$
$$= \frac{e^{2\arctan(ax)}}{8ac^2} + \frac{e^{2\arctan(ax)}(1+ax)}{4ac^2(1+a^2x^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{(1-iax)^i(1+iax)^{-i}(3+2ax+a^2x^2)}{8c^2(a+a^3x^2)}$$

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
[Out] ((1 - I*a*x)^I*(3 + 2*a*x + a^2*x^2))/(8*c^2*(1 + I*a*x)^I*(a + a^3*x^2))
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{e^{2\arctan(ax)}(a^2x^2+2ax+3)}{8(a^2x^2+1)a\,c^2}$	40
parallelrisch	$\frac{x^2e^{2\arctan(ax)}a^2+2\,e^{2\arctan(ax)}ax+3\,e^{2\arctan(ax)}}{8c^2(a^2x^2+1)a}$	55

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*exp(2*arctan(a*x))*(a^2*x^2+2*a*x+3)/(a^2*x^2+1)/a/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{(a^2x^2+2\,ax+3)e^{(2\arctan(ax))}}{8\,(a^3c^2x^2+ac^2)}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/8*(a^2*x^2 + 2*a*x + 3)*e^(2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(42) = 84$.

Time = 1.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.87

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \begin{cases} \frac{a^2x^2e^{2\operatorname{atan}(ax)}}{8a^3c^2x^2+8ac^2} + \frac{2axe^{2\operatorname{atan}(ax)}}{8a^3c^2x^2+8ac^2} + \frac{3e^{2\operatorname{atan}(ax)}}{8a^3c^2x^2+8ac^2} & \text{for}\ a \neq 0 \\ \frac{x}{c^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**2,x)
[Out] Piecewise((a**2*x**2*exp(2*atan(a*x))/(8*a**3*c**2*x**2 + 8*a*c**2) + 2*a*x
*exp(2*atan(a*x))/(8*a**3*c**2*x**2 + 8*a*c**2) + 3*exp(2*atan(a*x))/(8*a**
3*c**2*x**2 + 8*a*c**2), Ne(a, 0)), (x/c**2, True))
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,\left(\frac{3}{8\,a^3\,c^2}+\frac{x}{4\,a^2\,c^2}+\frac{x^2}{8\,a\,c^2}\right)}{\frac{1}{a^2}+x^2}$$

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

```
[Out] (exp(2*atan(a*x))*(3/(8*a^3*c^2) + x/(4*a^2*c^2) + x^2/(8*a*c^2)))/(1/a^2 +
 x^2)
```

3.265 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3} dx = \frac{3e^{2\arctan(ax)}}{40ac^3} + \frac{e^{2\arctan(ax)}(1+2ax)}{10ac^3(1+a^2x^2)^2} + \frac{3e^{2\arctan(ax)}(1+ax)}{20ac^3(1+a^2x^2)}$$

```
[Out] 3/40*exp(2*arctan(a*x))/a/c^3+1/10*exp(2*arctan(a*x))*(2*a*x+1)/a/c^3/(a^2*
x^2+1)^2+3/20*exp(2*arctan(a*x))*(a*x+1)/a/c^3/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5179}

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3} dx = \frac{3(ax+1)e^{2\arctan(ax)}}{20ac^3(a^2x^2+1)} + \frac{(2ax+1)e^{2\arctan(ax)}}{10ac^3(a^2x^2+1)^2} + \frac{3e^{2\arctan(ax)}}{40ac^3}$$

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (3*E^(2*ArcTan[a*x]))/(40*a*c^3) + (E^(2*ArcTan[a*x])*(1 + 2*a*x))/(10*a*c^
3*(1 + a^2*x^2)^2) + (3*E^(2*ArcTan[a*x])*(1 + a*x))/(20*a*c^3*(1 + a^2*x^2
))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
```

```
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{2\arctan(ax)}(1+2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3\int\frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx}{5c}\\
&= \frac{e^{2\arctan(ax)}(1+2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3e^{2\arctan(ax)}(1+ax)}{20ac^3\left(1+a^2x^2\right)}+\frac{3\int\frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx}{20c^2}\\
&= \frac{3e^{2\arctan(ax)}}{40ac^3}+\frac{e^{2\arctan(ax)}(1+2ax)}{10ac^3\left(1+a^2x^2\right)^2}+\frac{3e^{2\arctan(ax)}(1+ax)}{20ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int\frac{e^{2\arctan(ax)}}{\left(c+a^2cx^2\right)^3}\,dx
=\frac{4e^{2\arctan(ax)}(1+2ax)+3(1-iax)^i(1+iax)^{-i}\left(1+a^2x^2\right)\left(3+2ax+a^2x^2\right)}{40ac^3\left(1+a^2x^2\right)^2}$$

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

```
[Out] (4*E^(2*ArcTan[a*x])*(1 + 2*a*x) + (3*(1 - I*a*x)^I*(1 + a^2*x^2)*(3 + 2*a*
x + a^2*x^2))/(1 + I*a*x)^I)/(40*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A] (verified)

Time = 10.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{e^{2\arctan(ax)}\left(3a^4x^4+6a^3x^3+12a^2x^2+14ax+13\right)}{40\left(a^2x^2+1\right)^2c^3a}$	57
parallelrisch	$\frac{3a^4e^{2\arctan(ax)}x^4+6a^3x^3e^{2\arctan(ax)}+12x^2e^{2\arctan(ax)}a^2+14\,e^{2\arctan(ax)}ax+13\,e^{2\arctan(ax)}}{40c^3\left(a^2x^2+1\right)^2a}$	86

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*exp(2*arctan(a*x))*(3*a^4*x^4+6*a^3*x^3+12*a^2*x^2+14*a*x+13)/(a^2*x^2
+1)^2/c^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{(3\,a^4x^4+6\,a^3x^3+12\,a^2x^2+14\,ax+13)e^{(2\,\arctan(ax))}}{40\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(3*a^4*x^4 + 6*a^3*x^3 + 12*a^2*x^2 + 14*a*x + 13)*e^(2*arctan(a*x))/(
a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(78) = 156$.

Time = 2.87 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.62

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \begin{cases} \frac{3a^4x^4e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{6a^3x^3e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{12a^2x^2e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{14axe^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{13}{40a^5c^3x^4} \\ \frac{x}{c^3} \end{cases}$$

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((3*a**4*x**4*exp(2*atan(a*x))/(40*a**5*c**3*x**4 + 80*a**3*c**3*x
**2 + 40*a*c**3) + 6*a**3*x**3*exp(2*atan(a*x))/(40*a**5*c**3*x**4 + 80*a**
3*c**3*x**2 + 40*a*c**3) + 12*a**2*x**2*exp(2*atan(a*x))/(40*a**5*c**3*x**4
 + 80*a**3*c**3*x**2 + 40*a*c**3) + 14*a*x*exp(2*atan(a*x))/(40*a**5*c**3*x
**4 + 80*a**3*c**3*x**2 + 40*a*c**3) + 13*exp(2*atan(a*x))/(40*a**5*c**3*x*
*4 + 80*a**3*c**3*x**2 + 40*a*c**3), Ne(a, 0)), (x/c**3, True))
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")

[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)

Giac [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{3\,e^{2\,\mathrm{atan}(a\,x)}}{40\,a\,c^3} + \frac{3\,e^{2\,\mathrm{atan}(a\,x)}\,(a\,x+1)}{20\,a\,c^3\,(a^2\,x^2+1)} + \frac{e^{2\,\mathrm{atan}(a\,x)}\,(2\,a\,x+1)}{10\,a\,c^3\,(a^2\,x^2+1)^2}$$

[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^3,x)

[Out] (3*exp(2*atan(a*x)))/(40*a*c^3) + (3*exp(2*atan(a*x))*(a*x + 1))/(20*a*c^3*(a^2*x^2 + 1)) + (exp(2*atan(a*x))*(2*a*x + 1))/(10*a*c^3*(a^2*x^2 + 1)^2)

3.266 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{9e^{2\arctan(ax)}}{160ac^4}+\frac{e^{2\arctan(ax)}(1+3ax)}{20ac^4(1+a^2x^2)^3}+\frac{3e^{2\arctan(ax)}(1+2ax)}{40ac^4(1+a^2x^2)^2}+\frac{9e^{2\arctan(ax)}(1+ax)}{80ac^4(1+a^2x^2)}$$

```
[Out] 9/160*exp(2*arctan(a*x))/a/c^4+1/20*exp(2*arctan(a*x))*(3*a*x+1)/a/c^4/(a^2
*x^2+1)^3+3/40*exp(2*arctan(a*x))*(2*a*x+1)/a/c^4/(a^2*x^2+1)^2+9/80*exp(2*
arctan(a*x))*(a*x+1)/a/c^4/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5179}

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{9(ax+1)e^{2\arctan(ax)}}{80ac^4(a^2x^2+1)}+\frac{3(2ax+1)e^{2\arctan(ax)}}{40ac^4(a^2x^2+1)^2}+\frac{(3ax+1)e^{2\arctan(ax)}}{20ac^4(a^2x^2+1)^3}+\frac{9e^{2\arctan(ax)}}{160ac^4}$$

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (9*E^(2*ArcTan[a*x]))/(160*a*c^4) + (E^(2*ArcTan[a*x])*(1 + 3*a*x))/(20*a*c
^4*(1 + a^2*x^2)^3) + (3*E^(2*ArcTan[a*x])*(1 + 2*a*x))/(40*a*c^4*(1 + a^2*
x^2)^2) + (9*E^(2*ArcTan[a*x])*(1 + a*x))/(80*a*c^4*(1 + a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{2\arctan(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3\int\frac{e^{2\arctan(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{4c}\\
&= \frac{e^{2\arctan(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3e^{2\arctan(ax)}(1+2ax)}{40ac^4\left(1+a^2x^2\right)^2}+\frac{9\int\frac{e^{2\arctan(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{20c^2}\\
&= \frac{e^{2\arctan(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3e^{2\arctan(ax)}(1+2ax)}{40ac^4\left(1+a^2x^2\right)^2}+\frac{9e^{2\arctan(ax)}(1+ax)}{80ac^4\left(1+a^2x^2\right)}+\frac{9\int\frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx}{80c^3}\\
&= \frac{9e^{2\arctan(ax)}}{160ac^4}+\frac{e^{2\arctan(ax)}(1+3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3e^{2\arctan(ax)}(1+2ax)}{40ac^4\left(1+a^2x^2\right)^2}+\frac{9e^{2\arctan(ax)}(1+ax)}{80ac^4\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int\frac{e^{2\arctan(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = \frac{8ce^{2\arctan(ax)}(1+3ax)+3(c+a^2cx^2)\left(4e^{2\arctan(ax)}(1+2ax)+3(1-iax)^i(1+iax)^{-i}(-i+ax)(i+ax)(3\right.}{160ac^2\left(c+a^2cx^2\right)^3}$$

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (8*c*E^(2*ArcTan[a*x])*(1 + 3*a*x) + 3*(c + a^2*c*x^2)*(4*E^(2*ArcTan[a*x])
*(1 + 2*a*x) + (3*(1 - I*a*x)^I*(-I + a*x)*(I + a*x)*(3 + 2*a*x + a^2*x^2))
/(1 + I*a*x)^I))/(160*a*c^2*(c + a^2*c*x^2)^3)
```

Maple [A] (verified)

Time = 31.91 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

method	result
gosper	$\frac{e^{2\arctan(ax)}(9a^6x^6+18a^5x^5+45a^4x^4+60a^3x^3+75a^2x^2+66ax+47)}{160(a^2x^2+1)^3c^4a}$
parallelrisch	$\frac{9a^6e^{2\arctan(ax)}x^6+18a^5e^{2\arctan(ax)}x^5+45a^4e^{2\arctan(ax)}x^4+60a^3x^3e^{2\arctan(ax)}+75x^2e^{2\arctan(ax)}a^2+66\,e^{2\arctan(ax)}ax+4}{160c^4(a^2x^2+1)^3a}$

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/160*exp(2*arctan(a*x))*(9*a^6*x^6+18*a^5*x^5+45*a^4*x^4+60*a^3*x^3+75*a^2
*x^2+66*a*x+47)/(a^2*x^2+1)^3/c^4/a
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{(9\,a^6x^6+18\,a^5x^5+45\,a^4x^4+60\,a^3x^3+75\,a^2x^2+66\,ax+47)e^{(2\arctan(ax))}}{160\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/160*(9*a^6*x^6 + 18*a^5*x^5 + 45*a^4*x^4 + 60*a^3*x^3 + 75*a^2*x^2 + 66*a
*x + 47)*e^(2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 + a
*c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(112) = 224.

Time = 7.64 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.33

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \begin{cases} \frac{9a^6x^6e^{2\operatorname{atan}(ax)}}{160a^7c^4x^6+480a^5c^4x^4+480a^3c^4x^2+160ac^4} + \frac{18a^5x^5e^{2\operatorname{atan}(ax)}}{160a^7c^4x^6+480a^5c^4x^4+480a^3c^4x^2+160ac^4} + \frac{45a^4x^4e^{2\operatorname{atan}(ax)}}{160a^7c^4x^6+480a^5c^4x^4+480a^3c^4x^2+160ac^4} \\ \frac{x}{c^4} \end{cases}$$

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Piecewise((9*a**6*x**6*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**4
*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) + 18*a**5*x**5*exp(2*atan(a*x))/(1
60*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) +
 45*a**4*x**4*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 4
80*a**3*c**4*x**2 + 160*a*c**4) + 60*a**3*x**3*exp(2*atan(a*x))/(160*a**7*c
**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) + 75*a**2*
x**2*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c
**4*x**2 + 160*a*c**4) + 66*a*x*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*
a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) + 47*exp(2*atan(a*x))/(16
0*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4), N
e(a, 0)), (x/c**4, True))
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Giac [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{9\,e^{2\,\mathrm{atan}(a\,x)}}{160\,a\,c^4} + \frac{9\,e^{2\,\mathrm{atan}(a\,x)}\,(a\,x+1)}{80\,a\,c^4\,(a^2\,x^2+1)} + \frac{3\,e^{2\,\mathrm{atan}(a\,x)}\,(2\,a\,x+1)}{40\,a\,c^4\,(a^2\,x^2+1)^2} + \frac{e^{2\,\mathrm{atan}(a\,x)}\,(3\,a\,x+1)}{20\,a\,c^4\,(a^2\,x^2+1)^3}$$

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^4,x)
[Out] (9*exp(2*atan(a*x)))/(160*a*c^4) + (9*exp(2*atan(a*x))*(a*x + 1))/(80*a*c^4
*(a^2*x^2 + 1)) + (3*exp(2*atan(a*x))*(2*a*x + 1))/(40*a*c^4*(a^2*x^2 + 1)^
2) + (exp(2*atan(a*x))*(3*a*x + 1))/(20*a*c^4*(a^2*x^2 + 1)^3)
```

3.267 $\int e^{2\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int e^{2\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx=\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+i,\frac{5}{2}+i,\frac{7}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] (2/29+5/29*I)*2^(5/2-I)*c*(1-I*a*x)^(5/2+I)*hypergeom([5/2+I, -3/2+I],[7/2+
I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int e^{2\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx=\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+i,\frac{5}{2}+i,\frac{7}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[In] Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((2/29 + (5*I)/29)*2^(5/2 - I)*c*(1 - I*a*x)^(5/2 + I)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-3/2 + I, 5/2 + I, 7/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a
^2*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{2\arctan(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int (1-iax)^{\frac{3}{2}+i}(1+iax)^{\frac{3}{2}-i}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+i,\frac{5}{2}+i,\frac{7}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = \frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+i,\frac{5}{2}+i,\frac{7}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[In] Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((2/29 + (5*I)/29)*2^(5/2 - I)*c*(1 - I*a*x)^(5/2 + I)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-3/2 + I, 5/2 + I, 7/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a
^2*x^2])
```

Maple [F]

$$\int \mathrm{e}^{2\arctan(ax)}\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}d x$$

[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)

[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)

Fricas [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} e^{\left(2 \, \arctan\left(a x\right)\right)}\,{d x}$$

[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(2*arctan(a*x)), x)

Sympy [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} e^{2 \operatorname{atan}{\left(a x \right)}}\, dx$$

[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**(3/2),x)

[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*exp(2*atan(a*x)), x)

Maxima [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} e^{\left(2 \, \arctan\left(a x\right)\right)}\,{d x}$$

[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(2*arctan(a*x)), x)

Giac [F(-2)]

Exception generated.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

```
[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

3.268 $\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+i,\frac{3}{2}+i,\frac{5}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] (2/13+3/13*I)*2^(3/2-I)*(1-I*a*x)^(3/2+I)*hypergeom([-1/2+I, 3/2+I],[5/2+I]
,1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+i,\frac{3}{2}+i,\frac{5}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[In] Int[E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((2/13 + (3*I)/13)*2^(3/2 - I)*(1 - I*a*x)^(3/2 + I)*Sqrt[c + a^2*c*x^2]*Hy
pergeometric2F1[-1/2 + I, 3/2 + I, 5/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^2
*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+a^2cx^2}\int e^{2\arctan(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}+i}(1+iax)^{\frac{1}{2}-i}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+i,\frac{3}{2}+i,\frac{5}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$$
$$= \frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+i,\frac{3}{2}+i,\frac{5}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[In] Integrate[E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
[Out] ((2/13 + (3*I)/13)*2^(3/2 - I)*(1 - I*a*x)^(3/2 + I)*Sqrt[c + a^2*c*x^2]*Hy
pergeometric2F1[-1/2 + I, 3/2 + I, 5/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^2
*x^2])
```

Maple [F]

$$\int e^{2\arctan(ax)}\sqrt{a^2c\,x^2+c}dx$$

```
[In] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}e^{(2\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{c\,(a^2x^2+1)}e^{2\,\mathrm{atan}\,(ax)}\,dx$$

```
[In] integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}e^{(2\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int e^{2\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

[In] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)

[Out] int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)

3.269 $\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+i,\frac{1}{2}+i,\frac{3}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

```
[Out] (2/5+1/5*I)*2^(1/2-I)*(1-I*a*x)^(1/2+I)*hypergeom([1/2+I, 1/2+I],[3/2+I],1/
2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{a^2x^2+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+i,\frac{1}{2}+i,\frac{3}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^(2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((2/5 + I/5)*2^(1/2 - I)*(1 - I*a*x)^(1/2 + I)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + I, 1/2 + I, 3/2 + I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{2\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+i}(1+iax)^{-\frac{1}{2}-i}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+i,\frac{1}{2}+i,\frac{3}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+i,\frac{1}{2}+i,\frac{3}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] ((2/5 + I/5)*2^(1/2 - I)*(1 - I*a*x)^(1/2 + I)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + I, 1/2 + I, 3/2 + I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [F]

$$\int \frac{e^{2 \arctan(ax)}}{\sqrt{a^2 c\,x^2+c}}dx$$

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{2 \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(2 \arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{2 \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{2 \operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(2*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{2 \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(2 \arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{2\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.270 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 37

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{e^{2\arctan(ax)}(2+ax)}{5ac\sqrt{c+a^2cx^2}}$$

```
[Out] 1/5*exp(2*arctan(a*x))*(a*x+2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5177}

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{(ax+2)e^{2\arctan(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
[Out] (E^(2*ArcTan[a*x])*(2 + a*x))/(5*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\text{integral}=\frac{e^{2\arctan(ax)}(2+ax)}{5ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{2\arctan(ax)}(2+ax)}{5ac\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
[Out] (E^(2*ArcTan[a*x])*(2 + a*x))/(5*a*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{(a^2x^2+1)(ax+2)\mathrm{e}^{2\arctan(ax)}}{5a(a^2cx^2+c)^{\frac{3}{2}}}$	39

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/5*(a^2*x^2+1)*(a*x+2)*exp(2*arctan(a*x))/a/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax+2)e^{(2\arctan(ax))}}{5\,(a^3c^2x^2+ac^2)}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] 1/5*sqrt(a^2*c*x^2 + c)*(a*x + 2)*e^(2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{2\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(2*atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{2\operatorname{atan}(a\,x)}\left(\frac{x}{5\,c}+\frac{2}{5\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] (exp(2*atan(a*x))*(x/(5*c) + 2/(5*a*c)))/(c + a^2*c*x^2)^(1/2)
```

3.271 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{2\arctan(ax)}(2+3ax)}{13ac\,(c+a^2cx^2)^{3/2}} + \frac{6e^{2\arctan(ax)}(2+ax)}{65ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/13*exp(2*arctan(a*x))*(3*a*x+2)/a/c/(a^2*c*x^2+c)^(3/2)+6/65*exp(2*arctan
(a*x))*(a*x+2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5178, 5177}

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{6(ax+2)e^{2\arctan(ax)}}{65ac^2\sqrt{a^2cx^2+c}} + \frac{(3ax+2)e^{2\arctan(ax)}}{13ac\,(a^2cx^2+c)^{3/2}}$$

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(2 + 3*a*x))/(13*a*c*(c + a^2*c*x^2)^(3/2)) + (6*E^(2*Ar
cTan[a*x])*(2 + a*x))/(65*a*c^2*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{2\arctan(ax)}(2+3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{2\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{13c}\\ &= \frac{e^{2\arctan(ax)}(2+3ax)}{13ac\left(c+a^2cx^2\right)^{3/2}}+\frac{6e^{2\arctan(ax)}(2+ax)}{65ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx=\frac{e^{2\arctan(ax)}\left(22+21ax+12a^2x^2+6a^3x^3\right)}{65c^2\left(a+a^3x^2\right)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
[Out] (E^(2*ArcTan[a*x])*(22 + 21*a*x + 12*a^2*x^2 + 6*a^3*x^3))/(65*c^2*(a + a^3
*x^2)*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{\left(a^2x^2+1\right)\left(6a^3x^3+12a^2x^2+21ax+22\right)\mathrm{e}^{2\arctan(ax)}}{65a\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}$	56

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/65*(a^2*x^2+1)*(6*a^3*x^3+12*a^2*x^2+21*a*x+22)*exp(2*arctan(a*x))/a/(a^2
*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(6\,a^3x^3+12\,a^2x^2+21\,ax+22)\sqrt{a^2cx^2+c}e^{(2\,\arctan(ax))}}{65\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/65*(6*a^3*x^3 + 12*a^2*x^2 + 21*a*x + 22)*sqrt(a^2*c*x^2 + c)*e^(2*arctan
(a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{2\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
[Out] Integral(exp(2*atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(2\,\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(2\,\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{2\,\mathrm{atan}(a\,x)}\left(\frac{22}{65\,a^3\,c^2}+\frac{6\,x^3}{65\,c^2}+\frac{21\,x}{65\,a^2\,c^2}+\frac{12\,x^2}{65\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] (exp(2*atan(a*x))*(22/(65*a^3*c^2) + (6*x^3)/(65*c^2) + (21*x)/(65*a^2*c^2)
 + (12*x^2)/(65*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)^(
1/2))
```

3.272 $\int \frac{e^{2\arctan(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{e^{2\arctan(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx=\frac{e^{2\arctan(ax)}(2+5ax)}{29ac\left(c+a^2cx^2\right)^{5/2}}+\frac{20e^{2\arctan(ax)}(2+3ax)}{377ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{24e^{2\arctan(ax)}(2+ax)}{377ac^3\sqrt{c+a^2cx^2}}$$

```
[Out] 1/29*exp(2*arctan(a*x))*(5*a*x+2)/a/c/(a^2*c*x^2+c)^(5/2)+20/377*exp(2*arct
an(a*x))*(3*a*x+2)/a/c^2/(a^2*c*x^2+c)^(3/2)+24/377*exp(2*arctan(a*x))*(a*x
+2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5178, 5177}

$$\int \frac{e^{2\arctan(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx=\frac{24(ax+2)e^{2\arctan(ax)}}{377ac^3\sqrt{a^2cx^2+c}}+\frac{20(3ax+2)e^{2\arctan(ax)}}{377ac^2\left(a^2cx^2+c\right)^{3/2}}+\frac{(5ax+2)e^{2\arctan(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

```
[In] Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (E^(2*ArcTan[a*x])*(2 + 5*a*x))/(29*a*c*(c + a^2*c*x^2)^(5/2)) + (20*E^(2*A
rcTan[a*x])*(2 + 3*a*x))/(377*a*c^2*(c + a^2*c*x^2)^(3/2)) + (24*E^(2*ArcTa
n[a*x])*(2 + a*x))/(377*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^{2\arctan(ax)}(2+5ax)}{29ac\,(c+a^2cx^2)^{5/2}}+\frac{20\int\frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx}{29c}\\ &= \frac{e^{2\arctan(ax)}(2+5ax)}{29ac\,(c+a^2cx^2)^{5/2}}+\frac{20e^{2\arctan(ax)}(2+3ax)}{377ac^2\,(c+a^2cx^2)^{3/2}}+\frac{120\int\frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{377c^2}\\ &= \frac{e^{2\arctan(ax)}(2+5ax)}{29ac\,(c+a^2cx^2)^{5/2}}+\frac{20e^{2\arctan(ax)}(2+3ax)}{377ac^2\,(c+a^2cx^2)^{3/2}}+\frac{24e^{2\arctan(ax)}(2+ax)}{377ac^3\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.71

$$\int\frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx=\frac{e^{2\arctan(ax)}(114+149ax+136a^2x^2+108a^3x^3+48a^4x^4+24a^5x^5)}{377ac^3\,(1+a^2x^2)^2\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(7/2),x]
[Out] (E^(2*ArcTan[a*x])*(114 + 149*a*x + 136*a^2*x^2 + 108*a^3*x^3 + 48*a^4*x^4
+ 24*a^5*x^5))/(377*a*c^3*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{(a^2x^2+1)(24a^5x^5+48a^4x^4+108a^3x^3+136a^2x^2+149ax+114)e^{2\arctan(ax)}}{377a(a^2cx^2+c)^{\frac{7}{2}}}$	72

```
[In] int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/377*(a^2*x^2+1)*(24*a^5*x^5+48*a^4*x^4+108*a^3*x^3+136*a^2*x^2+149*a*x+11
4)*exp(2*arctan(a*x))/a/(a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{(24\,a^5x^5+48\,a^4x^4+108\,a^3x^3+136\,a^2x^2+149\,ax+114)\sqrt{a^2cx^2+c}\,e^{(2\arctan(ax))}}{377\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
[Out] 1/377*(24*a^5*x^5 + 48*a^4*x^4 + 108*a^3*x^3 + 136*a^2*x^2 + 149*a*x + 114)
*sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3
*c^4*x^2 + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{\mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,\left(\frac{114}{377\,a^5\,c^3}+\frac{24\,x^5}{377\,c^3}+\frac{149\,x}{377\,a^4\,c^3}+\frac{48\,x^4}{377\,a\,c^3}+\frac{108\,x^3}{377\,a^2\,c^3}+\frac{136\,x^2}{377\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

```
[In] int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] (exp(2*atan(a*x))*(114/(377*a^5*c^3) + (24*x^5)/(377*c^3) + (149*x)/(377*a^
4*c^3) + (48*x^4)/(377*a*c^3) + (108*x^3)/(377*a^2*c^3) + (136*x^2)/(377*a^
3*c^3)))/((c + a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*(c
 + a^2*c*x^2)^(1/2))/a^2)
```

3.273 $\int e^{-\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int e^{-\arctan(ax)}(c+a^2cx^2)^p\,dx = \frac{2^{(1+\frac{i}{2})+p}(1-iax)^{(1-\frac{i}{2})+p}\,(1+a^2x^2)^{-p}\,(c+a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-\frac{i}{2}-p,\left(1-\frac{i}{2}\right)+p,\left(2-\frac{i}{2}\right)+p,\right.}{a((-1-2i)-2ip)}$$

```
[Out] 2^(1+1/2*I+p)*(1-I*a*x)^(1-1/2*I+p)*(a^2*c*x^2+c)^p*hypergeom([-1/2*I-p, 1-
1/2*I+p],[2-1/2*I+p],1/2-1/2*I*a*x)/a/(-1-2*I-2*I*p)/((a^2*x^2+1)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5184, 5181, 71}

$$\int e^{-\arctan(ax)}(c+a^2cx^2)^p\,dx = \frac{2^{p+(1+\frac{i}{2})}(1-iax)^{p+(1-\frac{i}{2})}\,(a^2x^2+1)^{-p}\,(a^2cx^2+c)^p\,\text{Hypergeometric2F1}\left(-p-\frac{i}{2},p+\left(1-\frac{i}{2}\right),p+\left(2-\frac{i}{2}\right)\right.}{a(-2ip-(1+2i))}$$

```
[In] Int[(c + a^2*c*x^2)^p/E^ArcTan[a*x],x]
```

```
[Out] (2^((1 + I/2) + p)*(1 - I*a*x)^((1 - I/2) + p)*(c + a^2*c*x^2)^p*Hypergeome
tric2F1[-1/2*I - p, (1 - I/2) + p, (2 - I/2) + p, (1 - I*a*x)/2])/(a*((-1 -
 2*I) - (2*I)*p)*(1 + a^2*x^2)^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{-\arctan(ax)}\left(1+a^2x^2\right)^p\,dx$$

$$= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{-\frac{i}{2}+p}(1+iax)^{\frac{i}{2}+p}\,dx$$

$$= \frac{2^{\left(1+\frac{i}{2}\right)+p}(1-iax)^{\left(1-\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-\frac{i}{2}-p,\left(1-\frac{i}{2}\right)+p,(2\right.}{a((-1-2i)-2ip)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$= \frac{i2^{\frac{i}{2}+p}(1-iax)^{\left(1-\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-\frac{i}{2}-p,\left(1-\frac{i}{2}\right)+p,\left(2-\frac{i}{2}\right)+p,\frac{1}{2}\right.}{a\left(\left(1-\frac{i}{2}\right)+p\right)}$$

```
[In] Integrate[(c + a^2*c*x^2)^p/E^ArcTan[a*x],x]
```

```
[Out] (I*2^(I/2 + p)*(1 - I*a*x)^((1 - I/2) + p)*(c + a^2*c*x^2)^p*Hypergeometric
2F1[-1/2*I - p, (1 - I/2) + p, (2 - I/2) + p, (1 - I*a*x)/2])/(a*((1 - I/2)
 + p)*(1 + a^2*x^2)^p)
```

Maple [F]

$$\int \left(a^2 c\,x^2+c\right)^p \mathrm{e}^{-\arctan(ax)}dx$$

```
[In] int((a^2*c*x^2+c)^p/exp(arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^p/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \left(a^2cx^2+c\right)^p e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \left(c\left(a^2x^2+1\right)\right)^p e^{-\operatorname{atan}(ax)}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**p/exp(atan(a*x)),x)
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(-atan(a*x)), x)
```

Maxima [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \left(a^2cx^2+c\right)^p e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

Giac [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \left(a^2cx^2+c\right)^p e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int e^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^p\,dx$$

```
[In] int(exp(-atan(a*x))*(c + a^2*c*x^2)^p,x)
[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2)^p, x)
```

3.274 $\int e^{-\arctan(ax)}(c+a^2cx^2)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2-\frac{i}{2},3-\frac{i}{2},4-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/37+6/37*I)*2^(3+1/2*I)*c^2*(1-I*a*x)^(3-1/2*I)*hypergeom([-2-1/2*I, 3-1
/2*I],[4-1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5181, 71}

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2-\frac{i}{2},3-\frac{i}{2},4-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[(c + a^2*c*x^2)^2/E^ArcTan[a*x],x]
```

```
[Out] ((-1/37 + (6*I)/37)*2^(3 + I/2)*c^2*(1 - I*a*x)^(3 - I/2)*Hypergeometric2F1
[-2 - I/2, 3 - I/2, 4 - I/2, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = c^2 \int (1-iax)^{2-\frac{i}{2}}(1+iax)^{2+\frac{i}{2}}\,dx$$

$$= -\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2-\frac{i}{2},3-\frac{i}{2},4-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx$$
$$= -\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2-\frac{i}{2},3-\frac{i}{2},4-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Integrate[(c + a^2*c*x^2)^2/E^ArcTan[a*x],x]
[Out] ((-1/37 + (6*I)/37)*2^(3 + I/2)*c^2*(1 - I*a*x)^(3 - I/2)*Hypergeometric2F1
[-2 - I/2, 3 - I/2, 4 - I/2, (1 - I*a*x)/2])/a
```

Maple [F]

$$\int \left(a^2c\,x^2+c\right)^2 \mathrm{e}^{-\arctan(ax)}dx$$

```
[In] int((a^2*c*x^2+c)^2/exp(arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^2/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=c^2\left(\int 2a^2x^2e^{-\operatorname{atan}(ax)}\,dx+\int a^4x^4e^{-\operatorname{atan}(ax)}\,dx+\int e^{-\operatorname{atan}(ax)}\,dx\right)$$

```
[In] integrate((a**2*c*x**2+c)**2/exp(atan(a*x)),x)
[Out] c**2*(Integral(2*a**2*x**2*exp(-atan(a*x)), x) + Integral(a**4*x**4*exp(-at
an(a*x)), x) + Integral(exp(-atan(a*x)), x))
```

Maxima [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^2*e^(-arctan(a*x)), x)
```

Giac [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int e^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

```
[In] int(exp(-atan(a*x))*(c + a^2*c*x^2)^2,x)
[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2)^2, x)
```

3.275 $\int e^{-\arctan(ax)}(c+a^2cx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx = -\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1-\frac{i}{2},2-\frac{i}{2},3-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/17+4/17*I)*2^(2+1/2*I)*c*(1-I*a*x)^(2-1/2*I)*hypergeom([-1-1/2*I, 2-1/2
*I],[3-1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5181, 71}

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx = -\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1-\frac{i}{2},2-\frac{i}{2},3-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[(c + a^2*c*x^2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/17 + (4*I)/17)*2^(2 + I/2)*c*(1 - I*a*x)^(2 - I/2)*Hypergeometric2F1[-
1 - I/2, 2 - I/2, 3 - I/2, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = c\int (1-iax)^{1-\frac{i}{2}}(1+iax)^{1+\frac{i}{2}}\,dx$$
$$= -\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1-\frac{i}{2},2-\frac{i}{2},3-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx$$
$$= -\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1-\frac{i}{2},2-\frac{i}{2},3-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Integrate[(c + a^2*c*x^2)/E^ArcTan[a*x],x]
[Out] ((-1/17 + (4*I)/17)*2^(2 + I/2)*c*(1 - I*a*x)^(2 - I/2)*Hypergeometric2F1[-
1 - I/2, 2 - I/2, 3 - I/2, (1 - I*a*x)/2])/a
```

Maple [F]

$$\int \left(a^2c\,x^2+c\right)\mathrm{e}^{-\arctan(ax)}dx$$

```
[In] int((a^2*c*x^2+c)/exp(arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx=c\left(\int a^2x^2e^{-\operatorname{atan}(ax)}\,dx+\int e^{-\operatorname{atan}(ax)}\,dx\right)$$

```
[In] integrate((a**2*c*x**2+c)/exp(atan(a*x)),x)
[Out] c*(Integral(a**2*x**2*exp(-atan(a*x)), x) + Integral(exp(-atan(a*x)), x))
```

Maxima [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Giac [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

```
[In] int(exp(-atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2), x)
```

3.276 $\int e^{-\arctan(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int e^{-\arctan(ax)}\,dx = -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-\frac{i}{2},1-\frac{i}{2},2-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/5+2/5*I)*2^(1+1/2*I)*(1-I*a*x)^(1-1/2*I)*hypergeom([-1/2*I, 1-1/2*I],[2
-1/2*I],1/2-1/2*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5169, 71}

$$\int e^{-\arctan(ax)}\,dx = -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-\frac{i}{2},1-\frac{i}{2},2-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[E^(-ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I/2)*(1 - I*a*x)^(1 - I/2)*Hypergeometric2F1[-1/2*
I, 1 - I/2, 2 - I/2, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] && !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int (1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}\,dx$$
$$= -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-\frac{i}{2},1-\frac{i}{2},2-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int e^{-\arctan(ax)}\,dx$$
$$= -\frac{\left(\frac{4}{5}+\frac{8i}{5}\right)e^{(-1+2i)\arctan(ax)}\,\text{Hypergeometric2F1}\left(1+\frac{i}{2},2,2+\frac{i}{2},-e^{2i\arctan(ax)}\right)}{a}$$

```
[In] Integrate[E^(-ArcTan[a*x]),x]
[Out] ((-4/5 - (8*I)/5)*Hypergeometric2F1[1 + I/2, 2, 2 + I/2, -E^((2*I)*ArcTan[a
*x])])/(a*E^((1 - 2*I)*ArcTan[a*x]))
```

Maple [F]

$$\int \mathrm{e}^{-\arctan(ax)}dx$$

```
[In] int(exp(-arctan(a*x)),x)
[Out] int(exp(-arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)}\,dx = \int e^{(-\arctan(ax))}\,dx$$

```
[In] integrate(exp(-arctan(a*x)),x, algorithm="fricas")
[Out] integral(e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan(ax)}\,dx = \int e^{-\operatorname{atan}(ax)}\,dx$$

```
[In] integrate(exp(-atan(a*x)),x)
[Out] Integral(exp(-atan(a*x)), x)
```

Maxima [F]

$$\int e^{-\arctan(ax)}\,dx = \int e^{(-\arctan(ax))}\,dx$$

```
[In] integrate(exp(-arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x)), x)
```

Giac [F]

$$\int e^{-\arctan(ax)}\,dx = \int e^{(-\arctan(ax))}\,dx$$

```
[In] integrate(exp(-arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\,dx = \int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,dx$$

```
[In] int(exp(-atan(a*x)),x)
[Out] int(exp(-atan(a*x)), x)
```

3.277 $\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx=-\frac{e^{-\arctan(ax)}}{ac}$$

```
[Out] -1/a/c/exp(arctan(a*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5179}

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx=-\frac{e^{-\arctan(ax)}}{ac}$$

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)),x]
```

```
[Out] -(1/(a*c*E^ArcTan[a*x]))
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\text{integral}=-\frac{e^{-\arctan(ax)}}{ac}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}}{ac}$$

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)),x]
[Out] -((1 + I*a*x)^(I/2)/(a*c*(1 - I*a*x)^(I/2)))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{e^{-\arctan(ax)}}{ac}$	16
parallelrisch	$-\frac{e^{-\arctan(ax)}}{ac}$	16
risch	$-\frac{(-iax+1)^{-\frac{i}{2}}(iax+1)^{\frac{i}{2}}}{ac}$	33

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] -1/a/c/exp(arctan(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-\arctan(ax))}}{ac}$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] -e^(-arctan(a*x))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = \begin{cases} -\frac{e^{-\operatorname{atan}(ax)}}{ac} & \text{for } a \neq 0 \\ \frac{x}{c} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c),x)
[Out] Piecewise((-exp(-atan(a*x))/(a*c), Ne(a, 0)), (x/c, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{2\,e^{(-\arctan(ax))}}{a^3cx^2+ac}$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] -2*e^(-arctan(a*x))/(a^3*c*x^2 + a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-\arctan(ax))}}{ac}$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
[Out] -e^(-arctan(a*x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}}{a\,c}$$

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2),x)
[Out] -exp(-atan(a*x))/(a*c)
```

3.278 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2} dx = -\frac{2e^{-\arctan(ax)}}{5ac^2} - \frac{e^{-\arctan(ax)}(1-2ax)}{5ac^2(1+a^2x^2)}$$

```
[Out] -2/5/a/c^2/exp(arctan(a*x))+1/5*(2*a*x-1)/a/c^2/exp(arctan(a*x))/(a^2*x^2+1
)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5179}

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2} dx = -\frac{(1-2ax)e^{-\arctan(ax)}}{5ac^2(a^2x^2+1)} - \frac{2e^{-\arctan(ax)}}{5ac^2}$$

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^2),x]
```

```
[Out] -2/(5*a*c^2*E^ArcTan[a*x]) - (1 - 2*a*x)/(5*a*c^2*E^ArcTan[a*x]*(1 + a^2*x^
2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{-\arctan(ax)}(1-2ax)}{5ac^2\left(1+a^2x^2\right)} + \frac{2\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx}{5c} \\ &= -\frac{2e^{-\arctan(ax)}}{5ac^2} - \frac{e^{-\arctan(ax)}(1-2ax)}{5ac^2\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}\left(3-2ax+2a^2x^2\right)}{5c^2\left(a+a^3x^2\right)}$$

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^2),x]
[Out] -1/5*((1 + I*a*x)^(I/2)*(3 - 2*a*x + 2*a^2*x^2))/(c^2*(1 - I*a*x)^(I/2)*(a
+ a^3*x^2))
```

Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{\left(2a^2x^2-2ax+3\right)\mathrm{e}^{-\arctan(ax)}}{5\left(a^2x^2+1\right)c^2a}$	41
parallelrisch	$\frac{\left(-2a^2x^2+2ax-3\right)\mathrm{e}^{-\arctan(ax)}}{5c^2\left(a^2x^2+1\right)a}$	41

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/5*(2*a^2*x^2-2*a*x+3)/(a^2*x^2+1)/c^2/exp(arctan(a*x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{(2\,a^2x^2-2\,ax+3)e^{(-\arctan(ax))}}{5\,(a^3c^2x^2+ac^2)}$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] -1/5*(2*a^2*x^2 - 2*a*x + 3)*e^(-arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(44) = 88.

Time = 19.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.15

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \begin{cases} -\frac{2a^2x^2}{5a^3c^2x^2e^{\operatorname{atan}(ax)}+5ac^2e^{\operatorname{atan}(ax)}} + \frac{2ax}{5a^3c^2x^2e^{\operatorname{atan}(ax)}+5ac^2e^{\operatorname{atan}(ax)}} - \frac{3}{5a^3c^2x^2e^{\operatorname{atan}(ax)}+5ac^2e^{\operatorname{atan}(ax)}} & \text{for } a \neq 0 \\ \frac{x}{c^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**2,x)
[Out] Piecewise((-2*a**2*x**2/(5*a**3*c**2*x**2*exp(atan(a*x)) + 5*a*c**2*exp(ata
n(a*x))) + 2*a*x/(5*a**3*c**2*x**2*exp(atan(a*x)) + 5*a*c**2*exp(atan(a*x))
) - 3/(5*a**3*c**2*x**2*exp(atan(a*x)) + 5*a*c**2*exp(atan(a*x))), Ne(a, 0)
), (x/c**2, True))
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{e^{-\mathrm{atan}(a\,x)}\,\left(\frac{3}{5\,a^3\,c^2}-\frac{2\,x}{5\,a^2\,c^2}+\frac{2\,x^2}{5\,a\,c^2}\right)}{\frac{1}{a^2}+x^2}$$

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^2,x)
[Out] -(exp(-atan(a*x))*(3/(5*a^3*c^2) - (2*x)/(5*a^2*c^2) + (2*x^2)/(5*a*c^2)))/
(1/a^2 + x^2)
```

3.279 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3} dx = -\frac{24e^{-\arctan(ax)}}{85ac^3} - \frac{e^{-\arctan(ax)}(1-4ax)}{17ac^3(1+a^2x^2)^2} - \frac{12e^{-\arctan(ax)}(1-2ax)}{85ac^3(1+a^2x^2)}$$

```
[Out] -24/85/a/c^3/exp(arctan(a*x))+1/17*(4*a*x-1)/a/c^3/exp(arctan(a*x))/(a^2*x^
2+1)^2-12/85*(-2*a*x+1)/a/c^3/exp(arctan(a*x))/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5179}

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3} dx = -\frac{(1-4ax)e^{-\arctan(ax)}}{17ac^3(a^2x^2+1)^2} - \frac{12(1-2ax)e^{-\arctan(ax)}}{85ac^3(a^2x^2+1)} - \frac{24e^{-\arctan(ax)}}{85ac^3}$$

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^3),x]
```

```
[Out] -24/(85*a*c^3*E^ArcTan[a*x]) - (1 - 4*a*x)/(17*a*c^3*E^ArcTan[a*x]*(1 + a^2
*x^2)^2) - (12*(1 - 2*a*x))/(85*a*c^3*E^ArcTan[a*x]*(1 + a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
```

```
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^{-\arctan(ax)}(1-4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{17c}\\
&= -\frac{e^{-\arctan(ax)}(1-4ax)}{17ac^3\left(1+a^2x^2\right)^2}-\frac{12e^{-\arctan(ax)}(1-2ax)}{85ac^3\left(1+a^2x^2\right)}+\frac{24\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx}{85c^2}\\
&= -\frac{24e^{-\arctan(ax)}}{85ac^3}-\frac{e^{-\arctan(ax)}(1-4ax)}{17ac^3\left(1+a^2x^2\right)^2}-\frac{12e^{-\arctan(ax)}(1-2ax)}{85ac^3\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{5e^{-\arctan(ax)}(-1+4ax)-12(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}\left(1+a^2x^2\right)\left(3-2ax+2a^2x^2\right)}{85ac^3\left(1+a^2x^2\right)^2}$$

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^3),x]
[Out] ((5*(-1 + 4*a*x))/E^ArcTan[a*x] - (12*(1 + I*a*x)^(I/2)*(1 + a^2*x^2)*(3 -
2*a*x + 2*a^2*x^2))/(1 - I*a*x)^(I/2))/(85*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A] (verified)

Time = 12.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{\left(24a^4x^4-24a^3x^3+60a^2x^2-44ax+41\right)\mathrm{e}^{-\arctan(ax)}}{85\left(a^2x^2+1\right)^2c^3a}$	57
parallelrisch	$\frac{\left(-24a^4x^4+24a^3x^3-60a^2x^2+44ax-41\right)\mathrm{e}^{-\arctan(ax)}}{85c^3\left(a^2x^2+1\right)^2a}$	57

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/85*(24*a^4*x^4-24*a^3*x^3+60*a^2*x^2-44*a*x+41)/(a^2*x^2+1)^2/c^3/exp(ar
ctan(a*x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{(24\,a^4x^4-24\,a^3x^3+60\,a^2x^2-44\,ax+41)e^{(-\arctan(ax))}}{85\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/85*(24*a^4*x^4 - 24*a^3*x^3 + 60*a^2*x^2 - 44*a*x + 41)*e^(-arctan(a*x))
/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(76) = 152$.

Time = 87.60 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.27

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \begin{cases} -\frac{24a^4x^4}{85a^5c^3x^4e^{\operatorname{atan}(ax)}+170a^3c^3x^2e^{\operatorname{atan}(ax)}+85ac^3e^{\operatorname{atan}(ax)}} + \frac{24a^3x^3}{85a^5c^3x^4e^{\operatorname{atan}(ax)}+170a^3c^3x^2e^{\operatorname{atan}(ax)}+85ac^3e^{\operatorname{atan}(ax)}} - \frac{}{85a^5c^3x^4e^{\operatorname{atan}} \cdots} \\ \frac{x}{c^3} \end{cases}$$

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((-24*a**4*x**4/(85*a**5*c**3*x**4*exp(atan(a*x)) + 170*a**3*c**3*
x**2*exp(atan(a*x)) + 85*a*c**3*exp(atan(a*x))) + 24*a**3*x**3/(85*a**5*c**
3*x**4*exp(atan(a*x)) + 170*a**3*c**3*x**2*exp(atan(a*x)) + 85*a*c**3*exp(a
tan(a*x))) - 60*a**2*x**2/(85*a**5*c**3*x**4*exp(atan(a*x)) + 170*a**3*c**3
*x**2*exp(atan(a*x)) + 85*a*c**3*exp(atan(a*x))) + 44*a*x/(85*a**5*c**3*x**
4*exp(atan(a*x)) + 170*a**3*c**3*x**2*exp(atan(a*x)) + 85*a*c**3*exp(atan(a
*x))) - 41/(85*a**5*c**3*x**4*exp(atan(a*x)) + 170*a**3*c**3*x**2*exp(atan(
a*x)) + 85*a*c**3*exp(atan(a*x))), Ne(a, 0)), (x/c**3, True))
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{12\,\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{85\,a\,c^3\,\left(a^2\,x^2+1\right)} - \frac{24\,\mathrm{e}^{-\mathrm{atan}(a\,x)}}{85\,a\,c^3} + \frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(4\,a\,x-1)}{17\,a\,c^3\,\left(a^2\,x^2+1\right)^2}$$

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^3,x)
[Out] (12*exp(-atan(a*x))*(2*a*x - 1))/(85*a*c^3*(a^2*x^2 + 1)) - (24*exp(-atan(a
*x)))/(85*a*c^3) + (exp(-atan(a*x))*(4*a*x - 1))/(17*a*c^3*(a^2*x^2 + 1)^2)
```

3.280 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{144e^{-\arctan(ax)}}{629ac^4} - \frac{e^{-\arctan(ax)}(1-6ax)}{37ac^4(1+a^2x^2)^3} - \frac{30e^{-\arctan(ax)}(1-4ax)}{629ac^4(1+a^2x^2)^2} - \frac{72e^{-\arctan(ax)}(1-2ax)}{629ac^4(1+a^2x^2)}$$

```
[Out] -144/629/a/c^4/exp(arctan(a*x))+1/37*(6*a*x-1)/a/c^4/exp(arctan(a*x))/(a^2*
x^2+1)^3-30/629*(-4*a*x+1)/a/c^4/exp(arctan(a*x))/(a^2*x^2+1)^2-72/629*(-2*
a*x+1)/a/c^4/exp(arctan(a*x))/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5179}

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{(1-6ax)e^{-\arctan(ax)}}{37ac^4(a^2x^2+1)^3} - \frac{72(1-2ax)e^{-\arctan(ax)}}{629ac^4(a^2x^2+1)} - \frac{30(1-4ax)e^{-\arctan(ax)}}{629ac^4(a^2x^2+1)^2} - \frac{144e^{-\arctan(ax)}}{629ac^4}$$

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^4),x]
```

```
[Out] -144/(629*a*c^4*E^ArcTan[a*x]) - (1 - 6*a*x)/(37*a*c^4*E^ArcTan[a*x]*(1 + a
^2*x^2)^3) - (30*(1 - 4*a*x))/(629*a*c^4*E^ArcTan[a*x]*(1 + a^2*x^2)^2) - (
72*(1 - 2*a*x))/(629*a*c^4*E^ArcTan[a*x]*(1 + a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^{-\arctan(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3}+\frac{30\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{37c}\\
&= -\frac{e^{-\arctan(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3}-\frac{30e^{-\arctan(ax)}(1-4ax)}{629ac^4\left(1+a^2x^2\right)^2}+\frac{360\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{629c^2}\\
&= -\frac{e^{-\arctan(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3}-\frac{30e^{-\arctan(ax)}(1-4ax)}{629ac^4\left(1+a^2x^2\right)^2}\\
&\quad -\frac{72e^{-\arctan(ax)}(1-2ax)}{629ac^4\left(1+a^2x^2\right)}+\frac{144\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx}{629c^3}\\
&= -\frac{144e^{-\arctan(ax)}}{629ac^4}-\frac{e^{-\arctan(ax)}(1-6ax)}{37ac^4\left(1+a^2x^2\right)^3}-\frac{30e^{-\arctan(ax)}(1-4ax)}{629ac^4\left(1+a^2x^2\right)^2}-\frac{72e^{-\arctan(ax)}(1-2ax)}{629ac^4\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$$
$$=\frac{17ce^{-\arctan(ax)}(-1+6ax)-6(c+a^2cx^2)\left(5e^{-\arctan(ax)}(1-4ax)+12(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}(-i+ax)(i+\right.}{629ac^2\left(c+a^2cx^2\right)^3}$$

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^4),x]
```

```
[Out] ((17*c*(-1 + 6*a*x))/E^ArcTan[a*x] - 6*(c + a^2*c*x^2)*((5*(1 - 4*a*x))/E^A
rcTan[a*x] + (12*(1 + I*a*x)^(I/2)*(-I + a*x)*(I + a*x)*(3 - 2*a*x + 2*a^2*
x^2))/(1 - I*a*x)^(I/2)))/(629*a*c^2*(c + a^2*c*x^2)^3)
```

Maple [A] (verified)

Time = 40.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(144 a^{6} x^{6}-144 a^{5} x^{5}+504 a^{4} x^{4}-408 a^{3} x^{3}+606 a^{2} x^{2}-366 a x +263\right) {\mathrm e}^{-\arctan \left(a x \right)}}{629 \left(a^{2} x^{2}+1\right)^{3} c^{4} a}$	73
parallelrisch	$\frac{\left(-144 a^{6} x^{6}+144 a^{5} x^{5}-504 a^{4} x^{4}+408 a^{3} x^{3}-606 a^{2} x^{2}+366 a x -263\right) {\mathrm e}^{-\arctan \left(a x \right)}}{629 c^{4} \left(a^{2} x^{2}+1\right)^{3} a}$	73

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/629*(144*a^6*x^6-144*a^5*x^5+504*a^4*x^4-408*a^3*x^3+606*a^2*x^2-366*a*x
+263)/(a^2*x^2+1)^3/c^4/exp(arctan(a*x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{(144\,a^6x^6-144\,a^5x^5+504\,a^4x^4-408\,a^3x^3+606\,a^2x^2-366\,ax+263)e^{(-\arctan(ax))}}{629\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/629*(144*a^6*x^6 - 144*a^5*x^5 + 504*a^4*x^4 - 408*a^3*x^3 + 606*a^2*x^2
 - 366*a*x + 263)*e^(-arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4
*x^2 + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Giac [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{72\,\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{629\,a\,c^4\,\left(a^2\,x^2+1\right)} - \frac{144\,\mathrm{e}^{-\mathrm{atan}(a\,x)}}{629\,a\,c^4} + \frac{30\,\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(4\,a\,x-1)}{629\,a\,c^4\,{\left(a^2\,x^2+1\right)}^2} + \frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(6\,a\,x-1)}{37\,a\,c^4\,{\left(a^2\,x^2+1\right)}^3}$$

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^4,x)
[Out] (72*exp(-atan(a*x))*(2*a*x - 1))/(629*a*c^4*(a^2*x^2 + 1)) - (144*exp(-atan
(a*x)))/(629*a*c^4) + (30*exp(-atan(a*x))*(4*a*x - 1))/(629*a*c^4*(a^2*x^2
+ 1)^2) + (exp(-atan(a*x))*(6*a*x - 1))/(37*a*c^4*(a^2*x^2 + 1)^3)
```

3.281 $\int e^{-\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{7}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] (-1/13+5/13*I)*2^(3/2+1/2*I)*c*(1-I*a*x)^(5/2-1/2*I)*hypergeom([5/2-1/2*I,
-3/2-1/2*I],[7/2-1/2*I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1
/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{7}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[In] Int[(c + a^2*c*x^2)^(3/2)/E^ArcTan[a*x],x]
```

```
[Out] ((-1/13 + (5*I)/13)*2^(3/2 + I/2)*c*(1 - I*a*x)^(5/2 - I/2)*Sqrt[c + a^2*c*
x^2]*Hypergeometric2F1[-3/2 - I/2, 5/2 - I/2, 7/2 - I/2, (1 - I*a*x)/2])/(a
*Sqrt[1 + a^2*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{-\arctan(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}\\
&= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int (1-iax)^{\frac{3}{2}-\frac{i}{2}}(1+iax)^{\frac{3}{2}+\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}}\\
&= -\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{7}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$
\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{7}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}
$$

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/E^ArcTan[a*x],x]
[Out] ((-1/13 + (5*I)/13)*2^(3/2 + I/2)*c*(1 - I*a*x)^(5/2 - I/2)*Sqrt[c + a^2*c*
x^2]*Hypergeometric2F1[-3/2 - I/2, 5/2 - I/2, 7/2 - I/2, (1 - I*a*x)/2])/(a
*Sqrt[1 + a^2*x^2])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} {\mathrm e}^{-\arctan \left(a x \right)}d x$$

```
[In] int((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{\left(-\arctan(ax)\right)}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(-arctan(a*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((a**2*c*x**2+c)**(3/2)/exp(atan(a*x)),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{\left(-\arctan(ax)\right)}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(-arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

[In] integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

[In] int(exp(-atan(a*x))*(c + a^2*c*x^2)^(3/2),x)

[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2)^(3/2), x)

3.282 $\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] (-1/5+3/5*I)*2^(1/2+1/2*I)*(1-I*a*x)^(3/2-1/2*I)*hypergeom([3/2-1/2*I, -1/2
-1/2*I],[5/2-1/2*I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5184, 5181, 71}

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[In] Int[Sqrt[c + a^2*c*x^2]/E^ArcTan[a*x],x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1/2 + I/2)*(1 - I*a*x)^(3/2 - I/2)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-1/2 - I/2, 3/2 - I/2, 5/2 - I/2, (1 - I*a*x)/2])/(a*Sqr
t[1 + a^2*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c+a^2cx^2}\int e^{-\arctan(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\sqrt{c+a^2cx^2}\int(1-iax)^{\frac{1}{2}-\frac{i}{2}}(1+iax)^{\frac{1}{2}+\frac{i}{2}}\,dx}{\sqrt{1+a^2x^2}}\\ &= -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/E^ArcTan[a*x],x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1/2 + I/2)*(1 - I*a*x)^(3/2 - I/2)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-1/2 - I/2, 3/2 - I/2, 5/2 - I/2, (1 - I*a*x)/2])/(a*Sqr
t[1 + a^2*x^2])
```

Maple [F]

$$\int \sqrt{a^2c\,x^2+c}\,\mathrm{e}^{-\arctan(ax)}dx$$

```
[In] int((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int \sqrt{a^2cx^2+c}e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int \sqrt{c\,(a^2x^2+1)}e^{-\operatorname{atan}(ax)}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**(1/2)/exp(atan(a*x)),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(-atan(a*x)), x)
```

Maxima [F]

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int \sqrt{a^2cx^2+c}e^{(-\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int e^{-\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

```
[In] int(exp(-atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(-atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

3.283 $\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

```
[Out] (-1+I)*2^(-1/2+1/2*I)*(1-I*a*x)^(1/2-1/2*I)*hypergeom([1/2-1/2*I, 1/2-1/2*I
],[3/2-1/2*I],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{a^2x^2+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-1 + I)*(1 - I*a*x)^(1/2 - I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 -
 I/2, 1/2 - I/2, 3/2 - I/2, (1 - I*a*x)/2])/(2^(1/2 - I/2)*a*Sqrt[c + a^2*c
*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}-\frac{i}{2}}(1+iax)^{-\frac{1}{2}+\frac{i}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2]),x]
[Out] ((-1 + I)*(1 - I*a*x)^(1/2 - I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 -
 I/2, 1/2 - I/2, 3/2 - I/2, (1 - I*a*x)/2])/(2^(1/2 - I/2)*a*Sqrt[c + a^2*c
*x^2])
```

Maple [F]

$$\int \frac{\mathrm{e}^{-\arctan(ax)}}{\sqrt{a^{2} c\,x^{2}+c}}dx$$

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
[Out] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{\left(-\arctan(ax)\right)}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{-\operatorname{atan}(ax)}}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(-atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{\left(-\arctan(ax)\right)}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{\sqrt{a^2cx^2 + c}}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{e^{-\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2 + c}}\,dx$$

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.284 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{e^{-\arctan(ax)}(1-ax)}{2ac\sqrt{c+a^2cx^2}}$$

```
[Out] 1/2*(a*x-1)/a/c/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5177}

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{(1-ax)e^{-\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/2*(1 - a*x)/(a*c*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\text{integral} = -\frac{e^{-\arctan(ax)}(1-ax)}{2ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{-\arctan(ax)}(-1+ax)}{2ac\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)),x]
[Out] (-1 + a*x)/(2*a*c*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result	size
gosper	$\frac{(a^2x^2+1)(ax-1)\mathrm{e}^{-\arctan(ax)}}{2a(a^2c\,x^2+c)^{\frac{3}{2}}}$	39

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a^2*x^2+1)*(a*x-1)/a/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax-1)e^{(-\arctan(ax))}}{2\,(a^3c^2x^2+ac^2)}$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] 1/2*sqrt(a^2*c*x^2 + c)*(a*x - 1)*e^(-arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{-\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(-atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{-\operatorname{atan}(a\,x)}\left(\frac{x}{2\,c}-\frac{1}{2\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] (exp(-atan(a*x))*(x/(2*c) - 1/(2*a*c)))/(c + a^2*c*x^2)^(1/2)
```

3.285 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{e^{-\arctan(ax)}(1-3ax)}{10ac\,(c+a^2cx^2)^{3/2}} - \frac{3e^{-\arctan(ax)}(1-ax)}{10ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/10*(3*a*x-1)/a/c/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)-3/10*(-a*x+1)/a/c^2
/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5178, 5177}

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{3(1-ax)e^{-\arctan(ax)}}{10ac^2\sqrt{a^2cx^2+c}} - \frac{(1-3ax)e^{-\arctan(ax)}}{10ac\,(a^2cx^2+c)^{3/2}}$$

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/10*(1 - 3*a*x)/(a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x))
/(10*a*c^2*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{-\arctan(ax)}(1-3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}+\frac{3\int\frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{5c}\\ &= -\frac{e^{-\arctan(ax)}(1-3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}-\frac{3e^{-\arctan(ax)}(1-ax)}{10ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int\frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx=\frac{e^{-\arctan(ax)}\left(-4+6ax-3a^2x^2+3a^3x^3\right)}{10c^2\left(a+a^3x^2\right)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)),x]
[Out] (-4 + 6*a*x - 3*a^2*x^2 + 3*a^3*x^3)/(10*c^2*E^ArcTan[a*x]*(a + a^3*x^2)*Sq
rt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{\left(a^2x^2+1\right)\left(3a^3x^3-3a^2x^2+6ax-4\right)\mathrm{e}^{-\arctan(ax)}}{10a\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}$	56

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/10*(a^2*x^2+1)*(3*a^3*x^3-3*a^2*x^2+6*a*x-4)/a/exp(arctan(a*x))/(a^2*c*x^
2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(3\,a^3x^3 - 3\,a^2x^2 + 6\,ax - 4)\sqrt{a^2cx^2 + c}e^{(-\arctan(ax))}}{10\,(a^5c^3x^4 + 2\,a^3c^3x^2 + ac^3)}$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/10*(3*a^3*x^3 - 3*a^2*x^2 + 6*a*x - 4)*sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x
))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{e^{-\mathrm{atan}(a\,x)}\,\left(\frac{2}{5\,a^3\,c^2}-\frac{3\,x^3}{10\,c^2}-\frac{3\,x}{5\,a^2\,c^2}+\frac{3\,x^2}{10\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] -(exp(-atan(a*x))*(2/(5*a^3*c^2) - (3*x^3)/(10*c^2) - (3*x)/(5*a^2*c^2) + (
3*x^2)/(10*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)^(1/2))
```

3.286 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{e^{-\arctan(ax)}(1-5ax)}{26ac\,(c+a^2cx^2)^{5/2}}-\frac{e^{-\arctan(ax)}(1-3ax)}{13ac^2\,(c+a^2cx^2)^{3/2}}-\frac{3e^{-\arctan(ax)}(1-ax)}{13ac^3\sqrt{c+a^2cx^2}}$$

```
[Out] 1/26*(5*a*x-1)/a/c/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2)+1/13*(3*a*x-1)/a/c^
2/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)-3/13*(-a*x+1)/a/c^3/exp(arctan(a*x))
/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5178, 5177}

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{3(1-ax)e^{-\arctan(ax)}}{13ac^3\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\arctan(ax)}}{13ac^2\,(a^2cx^2+c)^{3/2}}-\frac{(1-5ax)e^{-\arctan(ax)}}{26ac\,(a^2cx^2+c)^{5/2}}$$

```
[In] Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] -1/26*(1 - 5*a*x)/(a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)) - (1 - 3*a*x)/(
13*a*c^2*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x))/(13*a*c^3*E^A
rcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^{-\arctan(ax)}(1-5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}+\frac{10\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx}{13c}\\
&= -\frac{e^{-\arctan(ax)}(1-5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}-\frac{e^{-\arctan(ax)}(1-3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{13c^2}\\
&= -\frac{e^{-\arctan(ax)}(1-5ax)}{26ac\left(c+a^2cx^2\right)^{5/2}}-\frac{e^{-\arctan(ax)}(1-3ax)}{13ac^2\left(c+a^2cx^2\right)^{3/2}}-\frac{3e^{-\arctan(ax)}(1-ax)}{13ac^3\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^{7/2}}\,dx=\frac{e^{-\arctan(ax)}\left(-9+17ax-14a^2x^2+18a^3x^3-6a^4x^4+6a^5x^5\right)}{26ac^3\left(1+a^2x^2\right)^2\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(7/2)),x]
[Out] (-9 + 17*a*x - 14*a^2*x^2 + 18*a^3*x^3 - 6*a^4*x^4 + 6*a^5*x^5)/(26*a*c^3*E
^ArcTan[a*x]*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{(a^2x^2+1)\left(6a^5x^5-6a^4x^4+18a^3x^3-14a^2x^2+17ax-9\right)\mathrm{e}^{-\arctan(ax)}}{26a\left(a^2c\,x^2+c\right)^{\frac{7}{2}}}$	72

```
[In] int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/26*(a^2*x^2+1)*(6*a^5*x^5-6*a^4*x^4+18*a^3*x^3-14*a^2*x^2+17*a*x-9)/a/exp
(arctan(a*x))/(a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{(6\,a^5x^5-6\,a^4x^4+18\,a^3x^3-14\,a^2x^2+17\,ax-9)\sqrt{a^2cx^2+c}e^{(-\arctan(ax))}}{26\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
[Out] 1/26*(6*a^5*x^5 - 6*a^4*x^4 + 18*a^3*x^3 - 14*a^2*x^2 + 17*a*x - 9)*sqrt(a^
2*c*x^2 + c)*e^(-arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2
+ a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{e^{-\mathrm{atan}(a\,x)}\left(\frac{9}{26\,a^5\,c^3}-\frac{3\,x^5}{13\,c^3}-\frac{17\,x}{26\,a^4\,c^3}+\frac{3\,x^4}{13\,a\,c^3}-\frac{9\,x^3}{13\,a^2\,c^3}+\frac{7\,x^2}{13\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

```
[In] int(exp(-atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

```
[Out] -(exp(-atan(a*x))*(9/(26*a^5*c^3) - (3*x^5)/(13*c^3) - (17*x)/(26*a^4*c^3)
+ (3*x^4)/(13*a*c^3) - (9*x^3)/(13*a^2*c^3) + (7*x^2)/(13*a^3*c^3)))/((c +
a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*(c + a^2*c*x^2)^(
1/2))/a^2)
```

3.287 $\int e^{-2\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{i+p}(1-iax)^{(1-i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-i-p,(1-i)+p,(2-i)+p,\frac{1}{2}(1-\right.}{a((1-i)+p)}$$

```
[Out] I*2^(I+p)*(1-I*a*x)^(1-I+p)*(a^2*c*x^2+c)^p*hypergeom([-I-p, 1-I+p],[2-I+p]
,1/2-1/2*I*a*x)/a/(1-I+p)/((a^2*x^2+1)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5184, 5181, 71}

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{p+i}(1-iax)^{p+(1-i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\text{Hypergeometric2F1}\left(-p-i,p+(1-i),p+(2-i),\frac{1}{2}(1-\right.}{a(p+(1-i))}$$

```
[In] Int[(c + a^2*c*x^2)^p/E^(2*ArcTan[a*x]),x]
```

```
[Out] (I*2^(I + p)*(1 - I*a*x)^((1 - I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1[
-I - p, (1 - I) + p, (2 - I) + p, (1 - I*a*x)/2])/(a*((1 - I) + p)*(1 + a^2
*x^2)^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{-2\arctan(ax)}\left(1+a^2x^2\right)^p\,dx$$

$$= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int(1-iax)^{-i+p}(1+iax)^{i+p}\,dx$$

$$= \frac{i2^{i+p}(1-iax)^{(1-i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-i-p,(1-i)+p,(2-i)+\right.}{a((1-i)+p)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$= \frac{i2^{i+p}(1-iax)^{(1-i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-i-p,(1-i)+p,(2-i)+p,\frac{1}{2}(1-\right.}{a((1-i)+p)}$$

```
[In] Integrate[(c + a^2*c*x^2)^p/E^(2*ArcTan[a*x]),x]
```

```
[Out] (I*2^(I + p)*(1 - I*a*x)^((1 - I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1[
-I - p, (1 - I) + p, (2 - I) + p, (1 - I*a*x)/2]])/(a*((1 - I) + p)*(1 + a^2
*x^2)^p)
```

Maple [F]

$$\int \left(a^{2} c\,x^{2}+c\right)^{p} \mathrm{e}^{-2 \arctan \left(a x \right)}d x$$

```
[In] int((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \int \left(a^2cx^2+c\right)^pe^{(-2\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \int \left(c\left(a^2x^2+1\right)\right)^pe^{-2\operatorname{atan}(ax)}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**p/exp(2*atan(a*x)),x)
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(-2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \int \left(a^2cx^2+c\right)^pe^{(-2\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \left(a^2cx^2+c\right)^p e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^p\,dx$$

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^p,x)
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^p, x)
```

3.288 $\int e^{-2\arctan(ax)}(c+a^2cx^2)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\,\text{Hypergeometric2F1}\left(-2-i,3-i,4-i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/5+3/5*I)*2^(1+I)*c^2*(1-I*a*x)^(3-I)*hypergeom([3-I, -2-I],[4-I],1/2-1/
2*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5181, 71}

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\,\text{Hypergeometric2F1}\left(-2-i,3-i,4-i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[(c + a^2*c*x^2)^2/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (3*I)/5)*2^(1 + I)*c^2*(1 - I*a*x)^(3 - I)*Hypergeometric2F1[-2 -
I, 3 - I, 4 - I, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = c^2 \int (1-iax)^{2-i}(1+iax)^{2+i}\,dx$$
$$= -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\,\text{Hypergeometric2F1}\left(-2-i,3-i,4-i,\frac{1}{2}(1-iax)\right)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx$$
$$= -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\,\text{Hypergeometric2F1}\left(-2-i,3-i,4-i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Integrate[(c + a^2*c*x^2)^2/E^(2*ArcTan[a*x]),x]
[Out] ((-1/5 + (3*I)/5)*2^(1 + I)*c^2*(1 - I*a*x)^(3 - I)*Hypergeometric2F1[-2 -
I, 3 - I, 4 - I, (1 - I*a*x)/2])/a
```

Maple [F]

$$\int \left(a^2c\,x^2+c\right)^2 \mathrm{e}^{-2\arctan(ax)}dx$$

```
[In] int((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=c^2\left(\int 2a^2x^2e^{-2\operatorname{atan}(ax)}\,dx+\int a^4x^4e^{-2\operatorname{atan}(ax)}\,dx+\int e^{-2\operatorname{atan}(ax)}\,dx\right)$$

```
[In] integrate((a**2*c*x**2+c)**2/exp(2*atan(a*x)),x)
[Out] c**2*(Integral(2*a**2*x**2*exp(-2*atan(a*x)), x) + Integral(a**4*x**4*exp(-
2*atan(a*x)), x) + Integral(exp(-2*atan(a*x)), x))
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^2*e^(-2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^2,x)
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^2, x)
```

3.289 $\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx = -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,\text{Hypergeometric2F1}\left(-1-i,2-i,3-i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1/5+2/5*I)*2^(1+I)*c*(1-I*a*x)^(2-I)*hypergeom([-1-I, 2-I],[3-I],1/2-1/2*
I*a*x)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5181, 71}

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx = -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,\text{Hypergeometric2F1}\left(-1-i,2-i,3-i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[(c + a^2*c*x^2)/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-1/5 + (2*I)/5)*2^(1 + I)*c*(1 - I*a*x)^(2 - I)*Hypergeometric2F1[-1 - I,
 2 - I, 3 - I, (1 - I*a*x)/2])/a
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\int (1-iax)^{1-i}(1+iax)^{1+i}\,dx\\ &= -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,\text{Hypergeometric2F1}\left(-1-i,2-i,3-i,\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx = -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,\text{Hypergeometric2F1}\left(-1-i,2-i,3-i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Integrate[(c + a^2*c*x^2)/E^(2*ArcTan[a*x]),x]
[Out] ((-1/5 + (2*I)/5)*2^(1 + I)*c*(1 - I*a*x)^(2 - I)*Hypergeometric2F1[-1 - I,
 2 - I, 3 - I, (1 - I*a*x)/2])/a
```

Maple [F]

$$\int \left(a^2c\,x^2+c\right)\mathrm{e}^{-2\arctan(ax)}dx$$

```
[In] int((a^2*c*x^2+c)/exp(2*arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=c\left(\int a^2x^2e^{-2\operatorname{atan}(ax)}\,dx+\int e^{-2\operatorname{atan}(ax)}\,dx\right)$$

```
[In] integrate((a**2*c*x**2+c)/exp(2*atan(a*x)),x)
[Out] c*(Integral(a**2*x**2*exp(-2*atan(a*x)), x) + Integral(exp(-2*atan(a*x)), x
))
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int e^{-2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2), x)
```

3.290 $\int e^{-2\arctan(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int e^{-2\arctan(ax)}\,dx = -\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,\text{Hypergeometric2F1}\left(-i,1-i,2-i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[Out] (-1+I)*2^(-1+I)*(1-I*a*x)^(1-I)*hypergeom([-I, 1-I],[2-I],1/2-1/2*I*a*x)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5169, 71}

$$\int e^{-2\arctan(ax)}\,dx = -\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,\text{Hypergeometric2F1}\left(-i,1-i,2-i,\frac{1}{2}(1-iax)\right)}{a}$$

```
[In] Int[E^(-2*ArcTan[a*x]),x]
```

```
[Out] ((-1 + I)*(1 - I*a*x)^(1 - I)*Hypergeometric2F1[-I, 1 - I, 2 - I, (1 - I*a*
x)/2])/(2^(1 - I)*a)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (1-iax)^{-i}(1+iax)^i\,dx\\ &= -\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,\text{Hypergeometric2F1}\left(-i,1-i,2-i,\frac{1}{2}(1-iax)\right)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int e^{-2\arctan(ax)}\,dx = -\frac{(1+i)e^{(-2+2i)\arctan(ax)}\,\text{Hypergeometric2F1}\left(1+i,2,2+i,-e^{2i\arctan(ax)}\right)}{a}$$

```
[In] Integrate[E^(-2*ArcTan[a*x]),x]
[Out] ((-1 - I)*Hypergeometric2F1[1 + I, 2, 2 + I, -E^((2*I)*ArcTan[a*x])])/(a*E^
((2 - 2*I)*ArcTan[a*x]))
```

Maple [F]

$$\int \mathrm{e}^{-2\arctan(ax)}dx$$

```
[In] int(exp(-2*arctan(a*x)),x)
[Out] int(exp(-2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2\arctan(ax)}\,dx = \int e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate(exp(-2*arctan(a*x)),x, algorithm="fricas")
[Out] integral(e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2\arctan(ax)}\,dx = \int e^{-2\operatorname{atan}(ax)}\,dx$$

```
[In] integrate(exp(-2*atan(a*x)),x)
[Out] Integral(exp(-2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\,dx = \int e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate(exp(-2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(-2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{-2\arctan(ax)}\,dx = \int e^{(-2\arctan(ax))}\,dx$$

```
[In] integrate(exp(-2*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\,dx=\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,dx$$

```
[In] int(exp(-2*atan(a*x)),x)
[Out] int(exp(-2*atan(a*x)), x)
```

3.291 $\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{-2\arctan(ax)}}{2ac}$$

```
[Out] -1/2/a/c/exp(2*arctan(a*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{5179\}$

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{-2\arctan(ax)}}{2ac}$$

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)),x]
```

```
[Out] -1/2*1/(a*c*E^(2*ArcTan[a*x]))
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\text{integral} = -\frac{e^{-2\arctan(ax)}}{2ac}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{(1-iax)^{-i}(1+iax)^i}{2ac}$$

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)),x]
[Out] -1/2*(1 + I*a*x)^I/(a*c*(1 - I*a*x)^I)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{e^{-2\arctan(ax)}}{2ac}$	18
parallelrisch	$-\frac{e^{-2\arctan(ax)}}{2ac}$	18
risch	$-\frac{(-iax+1)^{-i}(iax+1)^i}{2ac}$	33

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] -1/2/a/c/exp(2*arctan(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-2\arctan(ax))}}{2ac}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] -1/2*e^(-2*arctan(a*x))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = \begin{cases} -\frac{e^{-2\operatorname{atan}(ax)}}{2ac} & \text{for } a \neq 0 \\ \frac{x}{c} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c),x)
[Out] Piecewise((-exp(-2*atan(a*x))/(2*a*c), Ne(a, 0)), (x/c, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-2\arctan(ax))}}{a^3cx^2+ac}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] -e^(-2*arctan(a*x))/(a^3*c*x^2 + a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-2\arctan(ax))}}{2\,ac}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
[Out] -1/2*e^(-2*arctan(a*x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{-2\operatorname{atan}(a\,x)}}{2\,a\,c}$$

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2),x)
[Out] -exp(-2*atan(a*x))/(2*a*c)
```

3.292 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{e^{-2\arctan(ax)}}{8ac^2} - \frac{e^{-2\arctan(ax)}(1-ax)}{4ac^2(1+a^2x^2)}$$

```
[Out] -1/8/a/c^2/exp(2*arctan(a*x))+1/4*(a*x-1)/a/c^2/exp(2*arctan(a*x))/(a^2*x^2
+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5179}

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{(1-ax)e^{-2\arctan(ax)}}{4ac^2(a^2x^2+1)} - \frac{e^{-2\arctan(ax)}}{8ac^2}$$

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2),x]
```

```
[Out] -1/8*1/(a*c^2*E^(2*ArcTan[a*x])) - (1 - a*x)/(4*a*c^2*E^(2*ArcTan[a*x])*(1
+ a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{-2\arctan(ax)}(1-ax)}{4ac^2\left(1+a^2x^2\right)}+\frac{\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx}{4c}\\ &= -\frac{e^{-2\arctan(ax)}}{8ac^2}-\frac{e^{-2\arctan(ax)}(1-ax)}{4ac^2\left(1+a^2x^2\right)}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{e^{-2\arctan(ax)}}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{(1-iax)^{-i}(1+iax)^{i}\left(3-2ax+a^2x^2\right)}{8c^2\left(a+a^3x^2\right)}$$

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2),x]
[Out] -1/8*((1 + I*a*x)^I*(3 - 2*a*x + a^2*x^2))/(c^2*(1 - I*a*x)^I*(a + a^3*x^2)
)
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{\left(a^2x^2-2ax+3\right)\mathrm{e}^{-2\arctan(ax)}}{8\left(a^2x^2+1\right)c^2a}$	42
parallelrisch	$\frac{\left(-a^2x^2+2ax-3\right)\mathrm{e}^{-2\arctan(ax)}}{8c^2\left(a^2x^2+1\right)a}$	43

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/8*(a^2*x^2-2*a*x+3)/(a^2*x^2+1)/c^2/exp(2*arctan(a*x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{(a^2x^2-2\,ax+3)e^{(-2\ \arctan(ax))}}{8\,(a^3c^2x^2+ac^2)}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] -1/8*(a^2*x^2 - 2*a*x + 3)*e^(-2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(46) = 92.

Time = 39.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.30

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \begin{cases} -\frac{a^2x^2}{8a^3c^2x^2e^{2\operatorname{atan}(ax)}+8ac^2e^{2\operatorname{atan}(ax)}} + \frac{2ax}{8a^3c^2x^2e^{2\operatorname{atan}(ax)}+8ac^2e^{2\operatorname{atan}(ax)}} - \frac{3}{8a^3c^2x^2e^{2\operatorname{atan}(ax)}+8ac^2e^{2\operatorname{atan}(ax)}} & \text{for } a \neq 0 \\ \frac{x}{c^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**2,x)
[Out] Piecewise((-a**2*x**2/(8*a**3*c**2*x**2*exp(2*atan(a*x)) + 8*a*c**2*exp(2*a
tan(a*x))) + 2*a*x/(8*a**3*c**2*x**2*exp(2*atan(a*x)) + 8*a*c**2*exp(2*atan
(a*x))) - 3/(8*a**3*c**2*x**2*exp(2*atan(a*x)) + 8*a*c**2*exp(2*atan(a*x)))
, Ne(a, 0)), (x/c**2, True))
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(-2\ \arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(-2\,\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{e^{-2\,\mathrm{atan}(a\,x)}\,\left(\frac{3}{8\,a^3\,c^2}-\frac{x}{4\,a^2\,c^2}+\frac{x^2}{8\,a\,c^2}\right)}{\frac{1}{a^2}+x^2}$$

[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^2,x)

[Out] -(exp(-2*atan(a*x))*(3/(8*a^3*c^2) - x/(4*a^2*c^2) + x^2/(8*a*c^2)))/(1/a^2 + x^2)

3.293 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{3e^{-2\arctan(ax)}}{40ac^3} - \frac{e^{-2\arctan(ax)}(1-2ax)}{10ac^3(1+a^2x^2)^2} - \frac{3e^{-2\arctan(ax)}(1-ax)}{20ac^3(1+a^2x^2)}$$

```
[Out] -3/40/a/c^3/exp(2*arctan(a*x))+1/10*(2*a*x-1)/a/c^3/exp(2*arctan(a*x))/(a^2
*x^2+1)^2-3/20*(-a*x+1)/a/c^3/exp(2*arctan(a*x))/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5178, 5179}

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{(1-2ax)e^{-2\arctan(ax)}}{10ac^3(a^2x^2+1)^2} - \frac{3(1-ax)e^{-2\arctan(ax)}}{20ac^3(a^2x^2+1)} - \frac{3e^{-2\arctan(ax)}}{40ac^3}$$

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

```
[Out] -3/(40*a*c^3*E^(2*ArcTan[a*x])) - (1 - 2*a*x)/(10*a*c^3*E^(2*ArcTan[a*x])*(
1 + a^2*x^2)^2) - (3*(1 - a*x))/(20*a*c^3*E^(2*ArcTan[a*x])*(1 + a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
```

```
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{-2\arctan(ax)}(1-2ax)}{10ac^3(1+a^2x^2)^2}+\frac{3\int\frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx}{5c}\\ &= -\frac{e^{-2\arctan(ax)}(1-2ax)}{10ac^3(1+a^2x^2)^2}-\frac{3e^{-2\arctan(ax)}(1-ax)}{20ac^3(1+a^2x^2)}+\frac{3\int\frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx}{20c^2}\\ &= -\frac{3e^{-2\arctan(ax)}}{40ac^3}-\frac{e^{-2\arctan(ax)}(1-2ax)}{10ac^3(1+a^2x^2)^2}-\frac{3e^{-2\arctan(ax)}(1-ax)}{20ac^3(1+a^2x^2)}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int\frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{e^{-2\arctan(ax)}(-4+8ax)-3(1-iax)^{-i}(1+iax)^{i}(1+a^2x^2)(3-2ax+a^2x^2)}{40ac^3(1+a^2x^2)^2}$$

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
[Out] ((-4 + 8*a*x)/E^(2*ArcTan[a*x]) - (3*(1 + I*a*x)^I*(1 + a^2*x^2)*(3 - 2*a*x
 + a^2*x^2))/(1 - I*a*x)^I)/(40*a*c^3*(1 + a^2*x^2)^2)
```

Maple [A] (verified)

Time = 13.84 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(3a^4x^4-6a^3x^3+12a^2x^2-14ax+13)\mathrm{e}^{-2\arctan(ax)}}{40(a^2x^2+1)^2c^3a}$	59
parallelrisch	$\frac{(-3a^4x^4+6a^3x^3-12a^2x^2+14ax-13)\mathrm{e}^{-2\arctan(ax)}}{40c^3(a^2x^2+1)^2a}$	59

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(3*a^4*x^4-6*a^3*x^3+12*a^2*x^2-14*a*x+13)/(a^2*x^2+1)^2/c^3/exp(2*ar
ctan(a*x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{(3\,a^4x^4-6\,a^3x^3+12\,a^2x^2-14\,ax+13)e^{(-2\,\arctan(ax))}}{40\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(3*a^4*x^4 - 6*a^3*x^3 + 12*a^2*x^2 - 14*a*x + 13)*e^(-2*arctan(a*x))
/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(80) = 160.

Time = 163.45 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.55

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \begin{cases} -\frac{3a^4x^4}{40a^5c^3x^4e^{2\operatorname{atan}(ax)}+80a^3c^3x^2e^{2\operatorname{atan}(ax)}+40ac^3e^{2\operatorname{atan}(ax)}} + \frac{6a^3x^3}{40a^5c^3x^4e^{2\operatorname{atan}(ax)}+80a^3c^3x^2e^{2\operatorname{atan}(ax)}+40ac^3e^{2\operatorname{atan}(ax)}} - \frac{}{40a^5c^3x} \\ \frac{x}{c^3} \end{cases}$$

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((-3*a**4*x**4/(40*a**5*c**3*x**4*exp(2*atan(a*x)) + 80*a**3*c**3*
x**2*exp(2*atan(a*x)) + 40*a*c**3*exp(2*atan(a*x))) + 6*a**3*x**3/(40*a**5*
c**3*x**4*exp(2*atan(a*x)) + 80*a**3*c**3*x**2*exp(2*atan(a*x)) + 40*a*c**3
*exp(2*atan(a*x))) - 12*a**2*x**2/(40*a**5*c**3*x**4*exp(2*atan(a*x)) + 80*
a**3*c**3*x**2*exp(2*atan(a*x)) + 40*a*c**3*exp(2*atan(a*x))) + 14*a*x/(40*
a**5*c**3*x**4*exp(2*atan(a*x)) + 80*a**3*c**3*x**2*exp(2*atan(a*x)) + 40*a
*c**3*exp(2*atan(a*x))) - 13/(40*a**5*c**3*x**4*exp(2*atan(a*x)) + 80*a**3*
c**3*x**2*exp(2*atan(a*x)) + 40*a*c**3*exp(2*atan(a*x))), Ne(a, 0)), (x/c**
3, True))
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{3\,\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,(a\,x-1)}{20\,a\,c^3\,(a^2\,x^2+1)} - \frac{3\,\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}}{40\,a\,c^3} + \frac{\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{10\,a\,c^3\,(a^2\,x^2+1)^2}$$

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^3,x)
[Out] (3*exp(-2*atan(a*x))*(a*x - 1))/(20*a*c^3*(a^2*x^2 + 1)) - (3*exp(-2*atan(a
*x)))/(40*a*c^3) + (exp(-2*atan(a*x))*(2*a*x - 1))/(10*a*c^3*(a^2*x^2 + 1)^
2)
```

3.294 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{9e^{-2\arctan(ax)}}{160ac^4} - \frac{e^{-2\arctan(ax)}(1-3ax)}{20ac^4(1+a^2x^2)^3} - \frac{3e^{-2\arctan(ax)}(1-2ax)}{40ac^4(1+a^2x^2)^2} - \frac{9e^{-2\arctan(ax)}(1-ax)}{80ac^4(1+a^2x^2)}$$

```
[Out] -9/160/a/c^4/exp(2*arctan(a*x))+1/20*(3*a*x-1)/a/c^4/exp(2*arctan(a*x))/(a^
2*x^2+1)^3-3/40*(-2*a*x+1)/a/c^4/exp(2*arctan(a*x))/(a^2*x^2+1)^2-9/80*(-a*
x+1)/a/c^4/exp(2*arctan(a*x))/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5179}

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{(1-3ax)e^{-2\arctan(ax)}}{20ac^4(a^2x^2+1)^3} - \frac{9(1-ax)e^{-2\arctan(ax)}}{80ac^4(a^2x^2+1)} - \frac{3(1-2ax)e^{-2\arctan(ax)}}{40ac^4(a^2x^2+1)^2} - \frac{9e^{-2\arctan(ax)}}{160ac^4}$$

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^4),x]
```

```
[Out] -9/(160*a*c^4*E^(2*ArcTan[a*x])) - (1 - 3*a*x)/(20*a*c^4*E^(2*ArcTan[a*x])*
(1 + a^2*x^2)^3) - (3*(1 - 2*a*x))/(40*a*c^4*E^(2*ArcTan[a*x])*(1 + a^2*x^2
)^2) - (9*(1 - a*x))/(80*a*c^4*E^(2*ArcTan[a*x])*(1 + a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^{-2\arctan(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3}+\frac{3\int\frac{e^{-2\arctan(ax)}}{\left(c+a^2cx^2\right)^3}\,dx}{4c}\\
&= -\frac{e^{-2\arctan(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3}-\frac{3e^{-2\arctan(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2}+\frac{9\int\frac{e^{-2\arctan(ax)}}{\left(c+a^2cx^2\right)^2}\,dx}{20c^2}\\
&= -\frac{e^{-2\arctan(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3}-\frac{3e^{-2\arctan(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2}-\frac{9e^{-2\arctan(ax)}(1-ax)}{80ac^4\left(1+a^2x^2\right)}+\frac{9\int\frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx}{80c^3}\\
&= -\frac{9e^{-2\arctan(ax)}}{160ac^4}-\frac{e^{-2\arctan(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3}-\frac{3e^{-2\arctan(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2}-\frac{9e^{-2\arctan(ax)}(1-ax)}{80ac^4\left(1+a^2x^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int\frac{e^{-2\arctan(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = \frac{8ce^{-2\arctan(ax)}(-1+3ax)-3\left(c+a^2cx^2\right)\left(e^{-2\arctan(ax)}(4-8ax)+3(1-iax)^{-i}(1+iax)^{i}(-i+ax)(i+ax)\right.}{160ac^2\left(c+a^2cx^2\right)^3}$$

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^4),x]
```

```
[Out] ((8*c*(-1 + 3*a*x))/E^(2*ArcTan[a*x]) - 3*(c + a^2*c*x^2)*((4 - 8*a*x)/E^(2
*ArcTan[a*x]) + (3*(1 + I*a*x)^I*(-I + a*x)*(I + a*x)*(3 - 2*a*x + a^2*x^2)
)/(1 - I*a*x)^I))/(160*a*c^2*(c + a^2*c*x^2)^3)
```

Maple [A] (verified)

Time = 42.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{\left(9a^{6}x^{6}-18a^{5}x^{5}+45a^{4}x^{4}-60a^{3}x^{3}+75a^{2}x^{2}-66ax+47\right){\mathrm e}^{-2\arctan\left(ax\right)}}{160\left(a^{2}x^{2}+1\right)^{3}c^{4}a}$	75
parallelrisch	$\frac{\left(-9a^{6}x^{6}+18a^{5}x^{5}-45a^{4}x^{4}+60a^{3}x^{3}-75a^{2}x^{2}+66ax-47\right){\mathrm e}^{-2\arctan\left(ax\right)}}{160c^{4}\left(a^{2}x^{2}+1\right)^{3}a}$	75

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
[Out] -1/160*(9*a^6*x^6-18*a^5*x^5+45*a^4*x^4-60*a^3*x^3+75*a^2*x^2-66*a*x+47)/(a
^2*x^2+1)^3/c^4/exp(2*arctan(a*x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{\left(9\,a^{6}x^{6}-18\,a^{5}x^{5}+45\,a^{4}x^{4}-60\,a^{3}x^{3}+75\,a^{2}x^{2}-66\,ax+47\right)e^{\left(-2\,\arctan\left(ax\right)\right)}}{160\,\left(a^{7}c^{4}x^{6}+3\,a^{5}c^{4}x^{4}+3\,a^{3}c^{4}x^{2}+ac^{4}\right)}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
[Out] -1/160*(9*a^6*x^6 - 18*a^5*x^5 + 45*a^4*x^4 - 60*a^3*x^3 + 75*a^2*x^2 - 66*
a*x + 47)*e^(-2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 +
 a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")

[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)

Giac [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{9\,e^{-2\,\mathrm{atan}(a\,x)}\,(a\,x-1)}{80\,a\,c^4\,(a^2\,x^2+1)} - \frac{9\,e^{-2\,\mathrm{atan}(a\,x)}}{160\,a\,c^4} + \frac{3\,e^{-2\,\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{40\,a\,c^4\,(a^2\,x^2+1)^2} + \frac{e^{-2\,\mathrm{atan}(a\,x)}\,(3\,a\,x-1)}{20\,a\,c^4\,(a^2\,x^2+1)^3}$$

[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^4,x)

[Out] (9*exp(-2*atan(a*x))*(a*x - 1))/(80*a*c^4*(a^2*x^2 + 1)) - (9*exp(-2*atan(a*x)))/(160*a*c^4) + (3*exp(-2*atan(a*x))*(2*a*x - 1))/(40*a*c^4*(a^2*x^2 + 1)^2) + (exp(-2*atan(a*x))*(3*a*x - 1))/(20*a*c^4*(a^2*x^2 + 1)^3)

3.295 $\int e^{-2\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int e^{-2\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx = -\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-i,\frac{5}{2}-i,\frac{7}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] (-2/29+5/29*I)*2^(5/2+I)*c*(1-I*a*x)^(5/2-I)*hypergeom([5/2-I, -3/2-I],[7/2
-I],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int e^{-2\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx = -\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-i,\frac{5}{2}-i,\frac{7}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[In] Int[(c + a^2*c*x^2)^(3/2)/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-2/29 + (5*I)/29)*2^(5/2 + I)*c*(1 - I*a*x)^(5/2 - I)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-3/2 - I, 5/2 - I, 7/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 +
a^2*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{-2\arctan(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int (1-iax)^{\frac{3}{2}-i}(1+iax)^{\frac{3}{2}+i}\,dx}{\sqrt{1+a^2x^2}}\\ &= -\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-i,\frac{5}{2}-i,\frac{7}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-i,\frac{5}{2}-i,\frac{7}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/E^(2*ArcTan[a*x]),x]
[Out] ((-2/29 + (5*I)/29)*2^(5/2 + I)*c*(1 - I*a*x)^(5/2 - I)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-3/2 - I, 5/2 - I, 7/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 +
a^2*x^2])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} {\mathrm e}^{-2 \arctan \left(a x \right)}d x$$

```
[In] int((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(a^{2} c x^{2} + c\right)^{\frac{3}{2}} e^{\left(-2 \, \arctan\left(a x\right)\right)}\,{d x}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(-2*arctan(a*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((a**2*c*x**2+c)**(3/2)/exp(2*atan(a*x)),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(a^{2} c x^{2} + c\right)^{\frac{3}{2}} e^{\left(-2 \, \arctan\left(a x\right)\right)}\,{d x}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(-2*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \mathrm{e}^{-2\,\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

3.296 $\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-i,\frac{3}{2}-i,\frac{5}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[Out] (-2/13+3/13*I)*2^(3/2+I)*(1-I*a*x)^(3/2-I)*hypergeom([-1/2-I, 3/2-I],[5/2-I
],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5184, 5181, 71}

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-i,\frac{3}{2}-i,\frac{5}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

```
[In] Int[Sqrt[c + a^2*c*x^2]/E^(2*ArcTan[a*x]),x]
```

```
[Out] ((-2/13 + (3*I)/13)*2^(3/2 + I)*(1 - I*a*x)^(3/2 - I)*Sqrt[c + a^2*c*x^2]*H
ypergeometric2F1[-1/2 - I, 3/2 - I, 5/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^
2*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+a^2cx^2}\int e^{-2\arctan(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}-i}(1+iax)^{\frac{1}{2}+i}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-i,\frac{3}{2}-i,\frac{5}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-i,\frac{3}{2}-i,\frac{5}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/E^(2*ArcTan[a*x]),x]
[Out] ((-2/13 + (3*I)/13)*2^(3/2 + I)*(1 - I*a*x)^(3/2 - I)*Sqrt[c + a^2*c*x^2]*H
ypergeometric2F1[-1/2 - I, 3/2 - I, 5/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 + a^
2*x^2])
```

Maple [F]

$$\int \sqrt{a^2c\,x^2+c}\,\mathrm{e}^{-2\arctan(ax)}dx$$

```
[In] int((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x)
[Out] int((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int \sqrt{a^2cx^2+c}e^{\left(-2\arctan(ax)\right)}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int \sqrt{c\,(a^2x^2+1)}e^{-2\,\mathrm{atan}\,(ax)}\,dx$$

```
[In] integrate((a**2*c*x**2+c)**(1/2)/exp(2*atan(a*x)),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(-2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int \sqrt{a^2cx^2+c}e^{\left(-2\arctan(ax)\right)}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int e^{-2\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

```
[In] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

3.297 $\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-i,\frac{1}{2}-i,\frac{3}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

```
[Out] (-2/5+1/5*I)*2^(1/2+I)*(1-I*a*x)^(1/2-I)*hypergeom([1/2-I, 1/2-I],[3/2-I],1
/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{a^2x^2+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-i,\frac{1}{2}-i,\frac{3}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-2/5 + I/5)*2^(1/2 + I)*(1 - I*a*x)^(1/2 - I)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 - I, 1/2 - I, 3/2 - I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\int \frac{e^{-2\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}-i}(1+iax)^{-\frac{1}{2}+i}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-i,\frac{1}{2}-i,\frac{3}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-i,\frac{1}{2}-i,\frac{3}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
[Out] ((-2/5 + I/5)*2^(1/2 + I)*(1 - I*a*x)^(1/2 - I)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 - I, 1/2 - I, 3/2 - I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [F]

$$\int \frac{\mathrm{e}^{-2\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}dx$$

[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)

[Out] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)

Fricas [F]

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(-2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] integral(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)

Sympy [F]

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{-2\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)

[Out] Integral(exp(-2*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(-2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)

Giac [F(-1)]

Timed out.

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{-2\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.298 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{e^{-2\arctan(ax)}(2-ax)}{5ac\sqrt{c+a^2cx^2}}$$

```
[Out] 1/5*(a*x-2)/a/c/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5177}

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{(2-ax)e^{-2\arctan(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/5*(2 - a*x)/(a*c*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\text{integral} = -\frac{e^{-2\arctan(ax)}(2-ax)}{5ac\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{-2\arctan(ax)}(-2+ax)}{5ac\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
[Out] (-2 + a*x)/(5*a*c*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{(a^2x^2+1)(ax-2)\mathrm{e}^{-2\arctan(ax)}}{5a(a^2cx^2+c)^{\frac{3}{2}}}$	41

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/5*(a^2*x^2+1)*(a*x-2)/a/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax-2)e^{(-2\arctan(ax))}}{5(a^3c^2x^2+ac^2)}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] 1/5*sqrt(a^2*c*x^2 + c)*(a*x - 2)*e^(-2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{-2\,\mathrm{atan}\,(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(-2*atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(-2\,\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(-2\,\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(\frac{x}{5\,c}-\frac{2}{5\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
[Out] (exp(-2*atan(a*x))*(x/(5*c) - 2/(5*a*c)))/(c + a^2*c*x^2)^(1/2)
```

3.299 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{e^{-2\arctan(ax)}(2-3ax)}{13ac\,(c+a^2cx^2)^{3/2}} - \frac{6e^{-2\arctan(ax)}(2-ax)}{65ac^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/13*(3*a*x-2)/a/c/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)-6/65*(-a*x+2)/a/c
^2/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5178, 5177}

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{6(2-ax)e^{-2\arctan(ax)}}{65ac^2\sqrt{a^2cx^2+c}} - \frac{(2-3ax)e^{-2\arctan(ax)}}{13ac\,(a^2cx^2+c)^{3/2}}$$

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/13*(2 - 3*a*x)/(a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (6*(2 - a
*x))/(65*a*c^2*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{-2\arctan(ax)}(2-3ax)}{13ac\,(c+a^2cx^2)^{3/2}}+\frac{6\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{13c}\\ &= -\frac{e^{-2\arctan(ax)}(2-3ax)}{13ac\,(c+a^2cx^2)^{3/2}}-\frac{6e^{-2\arctan(ax)}(2-ax)}{65ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\frac{e^{-2\arctan(ax)}(-22+21ax-12a^2x^2+6a^3x^3)}{65c^2\,(a+a^3x^2)\,\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)),x]
[Out] (-22 + 21*a*x - 12*a^2*x^2 + 6*a^3*x^3)/(65*c^2*E^(2*ArcTan[a*x])*(a + a^3*
x^2)*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{(a^2x^2+1)\,(6a^3x^3-12a^2x^2+21ax-22)\,\mathrm{e}^{-2\arctan(ax)}}{65a(a^2c\,x^2+c)^{\frac{5}{2}}}$	58

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/65*(a^2*x^2+1)*(6*a^3*x^3-12*a^2*x^2+21*a*x-22)/a/exp(2*arctan(a*x))/(a^2
*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(6\,a^3x^3 - 12\,a^2x^2 + 21\,ax - 22)\sqrt{a^2cx^2+c}e^{(-2\,\arctan(ax))}}{65\,(a^5c^3x^4 + 2\,a^3c^3x^2 + ac^3)}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/65*(6*a^3*x^3 - 12*a^2*x^2 + 21*a*x - 22)*sqrt(a^2*c*x^2 + c)*e^(-2*arcta
n(a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(-2\,\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(-2\,\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(\frac{22}{65\,a^3\,c^2}-\frac{6\,x^3}{65\,c^2}-\frac{21\,x}{65\,a^2\,c^2}+\frac{12\,x^2}{65\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

```
[Out] -(exp(-2*atan(a*x))*(22/(65*a^3*c^2) - (6*x^3)/(65*c^2) - (21*x)/(65*a^2*c^
2) + (12*x^2)/(65*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)
^(1/2))
```

3.300 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{e^{-2\arctan(ax)}(2-5ax)}{29ac\,(c+a^2cx^2)^{5/2}} - \frac{20e^{-2\arctan(ax)}(2-3ax)}{377ac^2\,(c+a^2cx^2)^{3/2}} - \frac{24e^{-2\arctan(ax)}(2-ax)}{377ac^3\sqrt{c+a^2cx^2}}$$

```
[Out] 1/29*(5*a*x-2)/a/c/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2)-20/377*(-3*a*x+2)
/a/c^2/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)-24/377*(-a*x+2)/a/c^3/exp(2*a
rctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5178, 5177}

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{24(2-ax)e^{-2\arctan(ax)}}{377ac^3\sqrt{a^2cx^2+c}} - \frac{20(2-3ax)e^{-2\arctan(ax)}}{377ac^2\,(a^2cx^2+c)^{3/2}} - \frac{(2-5ax)e^{-2\arctan(ax)}}{29ac\,(a^2cx^2+c)^{5/2}}$$

```
[In] Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(7/2)),x]
```

```
[Out] -1/29*(2 - 5*a*x)/(a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)) - (20*(2 -
3*a*x))/(377*a*c^2*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (24*(2 - a*x)
)/(377*a*c^3*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{-2\arctan(ax)}(2-5ax)}{29ac\,(c+a^2cx^2)^{5/2}}+\frac{20\int\frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx}{29c}\\&= -\frac{e^{-2\arctan(ax)}(2-5ax)}{29ac\,(c+a^2cx^2)^{5/2}}-\frac{20e^{-2\arctan(ax)}(2-3ax)}{377ac^2\,(c+a^2cx^2)^{3/2}}+\frac{120\int\frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx}{377c^2}\\&= -\frac{e^{-2\arctan(ax)}(2-5ax)}{29ac\,(c+a^2cx^2)^{5/2}}-\frac{20e^{-2\arctan(ax)}(2-3ax)}{377ac^2\,(c+a^2cx^2)^{3/2}}-\frac{24e^{-2\arctan(ax)}(2-ax)}{377ac^3\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int\frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx=\frac{e^{-2\arctan(ax)}(-114+149ax-136a^2x^2+108a^3x^3-48a^4x^4+24a^5x^5)}{377ac^3\,(1+a^2x^2)^2\,\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(7/2)),x]
[Out] (-114 + 149*a*x - 136*a^2*x^2 + 108*a^3*x^3 - 48*a^4*x^4 + 24*a^5*x^5)/(377
*a*c^3*E^(2*ArcTan[a*x])*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{(a^2x^2+1)(24a^5x^5-48a^4x^4+108a^3x^3-136a^2x^2+149ax-114)e^{-2\arctan(ax)}}{377a(a^2cx^2+c)^{\frac{7}{2}}}$	74

```
[In] int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/377*(a^2*x^2+1)*(24*a^5*x^5-48*a^4*x^4+108*a^3*x^3-136*a^2*x^2+149*a*x-11
4)/a/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{(24\,a^5x^5-48\,a^4x^4+108\,a^3x^3-136\,a^2x^2+149\,ax-114)\sqrt{a^2cx^2+c}e^{(-2\arctan(ax))}}{377\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
[Out] 1/377*(24*a^5*x^5 - 48*a^4*x^4 + 108*a^3*x^3 - 136*a^2*x^2 + 149*a*x - 114)
*sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^
3*c^4*x^2 + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{e^{-2\,\mathrm{atan}(a\,x)}\left(\frac{114}{377\,a^5\,c^3}-\frac{24\,x^5}{377\,c^3}-\frac{149\,x}{377\,a^4\,c^3}+\frac{48\,x^4}{377\,a\,c^3}-\frac{108\,x^3}{377\,a^2\,c^3}+\frac{136\,x^2}{377\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

```
[In] int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
[Out] -(exp(-2*atan(a*x))*(114/(377*a^5*c^3) - (24*x^5)/(377*c^3) - (149*x)/(377*
a^4*c^3) + (48*x^4)/(377*a*c^3) - (108*x^3)/(377*a^2*c^3) + (136*x^2)/(377*
a^3*c^3)))/((c + a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*
(c + a^2*c*x^2)^(1/2))/a^2)
```

3.301 $\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2i}{a(1-iax)^2}+\frac{4i}{a(1-iax)}+\frac{i\log(i+ax)}{a}$$

```
[Out] -2*I/a/(1-I*a*x)^2+4*I/a/(1-I*a*x)+I*ln(I+a*x)/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5181, 45}

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{4i}{a(1-iax)}-\frac{2i}{a(1-iax)^2}+\frac{i\log(ax+i)}{a}$$

```
[In] Int[E^((5*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-2*I)/(a*(1 - I*a*x)^2) + (4*I)/(a*(1 - I*a*x)) + (I*Log[I + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
```

```
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^2}{(1-iax)^3}\,dx \\
&= \int \left(\frac{4}{(1-iax)^3} - \frac{4}{(1-iax)^2} + \frac{1}{1-iax}\right) dx \\
&= -\frac{2i}{a(1-iax)^2} + \frac{4i}{a(1-iax)} + \frac{i\log(i+ax)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i(-2+4iax+(i+ax)^2\log(i+ax))}{a(i+ax)^2}$$

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (I*(-2 + (4*I)*a*x + (I + a*x)^2*Log[I + a*x]))/(a*(I + a*x)^2)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

method	result
default	$\frac{-4x-\frac{2i}{a}}{(ax+i)^2}+\frac{i\ln(ax+i)}{a}$
risch	$\frac{-4x-\frac{2i}{a}}{(ax+i)^2}+\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$
parallelrisch	$\frac{i\ln(ax+i)x^4a^4-2ix^4a^4+2i\ln(ax+i)x^2a^2-4a^3x^3+2ix^2a^2+i\ln(ax+i)}{(a^2x^2+1)^2a}$
meijerg	$\frac{\frac{x\sqrt{a^2}\left(3a^2x^2+5\right)}{2\left(a^2x^2+1\right)^2}+\frac{3\sqrt{a^2}\arctan(ax)}{2a}}{4\sqrt{a^2}}+\frac{5iax^2(a^2x^2+2)}{4(a^2x^2+1)^2}-\frac{5\left(-\frac{x\left(a^2\right)^{\frac{3}{2}}\left(-3a^2x^2+3\right)}{6a^2\left(a^2x^2+1\right)^2}+\frac{\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{2a^3}\right)}{2\sqrt{a^2}}-\frac{5ia^3x^4}{2(a^2x^2+1)^2}-$

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] (-4*x-2*I/a)/(I+a*x)^2+I*ln(I+a*x)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=-\frac{4\,ax-\left(i\,a^2x^2-2\,ax-i\right)\log\left(\frac{ax+i}{a}\right)+2i}{a^3x^2+2i\,a^2x-a}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -(4*a*x - (I*a^2*x^2 - 2*a*x - I)*log((a*x + I)/a) + 2*I)/(a^3*x^2 + 2*I*a^
2*x - a)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=\frac{-4ax-2i}{a^3x^2+2ia^2x-a}+\frac{i\log\left(ax+i\right)}{a}$$

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**3,x)
```

```
[Out] (-4*a*x - 2*I)/(a**3*x**2 + 2*I*a**2*x - a) + I*log(a*x + I)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=-\frac{2\left(2\,a^3x^3-3i\,a^2x^2-i\right)}{a^5x^4+2\,a^3x^2+a}+\frac{\arctan\left(ax\right)}{a}+\frac{i\,\log\left(a^2x^2+1\right)}{2\,a}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -2*(2*a^3*x^3 - 3*I*a^2*x^2 - I)/(a^5*x^4 + 2*a^3*x^2 + a) + arctan(a*x)/a
+ 1/2*I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\,\log\left(ax+i\right)}{a} - \frac{2\left(2\,ax+i\right)}{\left(ax+i\right)^2a}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="giac")
[Out] I*log(a*x + I)/a - 2*(2*a*x + I)/((a*x + I)^2*a)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,1\mathrm{i}}{a} - \frac{\frac{4\,x}{a^2}+\frac{2\mathrm{i}}{a^3}}{x^2-\frac{1}{a^2}+\frac{x\,2\mathrm{i}}{a}}$$

```
[In] int((a*x*1i + 1)^5/(a^2*x^2 + 1)^3,x)
[Out] (log(x + 1i/a)*1i)/a - ((4*x)/a^2 + 2i/a^3)/((x*2i)/a - 1/a^2 + x^2)
```

3.302 $\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \frac{\operatorname{arcsinh}(ax)}{a}$$

```
[Out] -2/3*I*(1+I*a*x)^(3/2)/a/(1-I*a*x)^(3/2)+arcsinh(a*x)/a+2*I*(1+I*a*x)^(1/2)
/a/(1-I*a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5181, 49, 41, 221}

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arcsinh}(ax)}{a} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - (((2*I)/3)*(1 + I*a*x)^(3/2))
/(a*(1 - I*a*x)^(3/2)) + ArcSinh[a*x]/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+iax)^{3/2}}{(1-iax)^{5/2}}\,dx \\
&= -\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} - \int \frac{\sqrt{1+iax}}{(1-iax)^{3/2}}\,dx \\
&= \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx \\
&= \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \int \frac{1}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}} + \frac{\operatorname{arcsinh}(ax)}{a}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{4i\sqrt{2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1-iax)\right)}{3a(1-iax)^{3/2}}$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (((-4*I)/3)*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - I*a*x)/2])/(a*
(1 - I*a*x)^(3/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(57) = 114$.

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.44

method	result
meijerg	$\frac{x(2a^2x^2+3)}{3(a^2x^2+1)^{\frac{3}{2}}} + \frac{8i\left(\frac{\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{2\left(a^2x^2+1\right)^{\frac{3}{2}}}\right)}{3a\sqrt{\pi}} - \frac{2a^2x^3}{(a^2x^2+1)^{\frac{3}{2}}} - \frac{8i\left(\sqrt{\pi} - \frac{\sqrt{\pi}\left(12a^2x^2+8\right)}{8\left(a^2x^2+1\right)^{\frac{3}{2}}}\right)}{3a\sqrt{\pi}} + \frac{-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{5}{2}}\left(20a^2x^2+15\right)}{15a^4\left(a^2x^2+1\right)^{\frac{3}{2}}} + \frac{\sqrt{\pi}\left(a^2\right)^{\frac{5}{2}}\,\mathrm{ar}}{a^5}}{\sqrt{\pi}\,\sqrt{a^2}}$
default	$\frac{x}{3(a^2x^2+1)^{\frac{3}{2}}} + \frac{2x}{3\sqrt{a^2x^2+1}} + a^4\left(-\frac{x^3}{3a^2(a^2x^2+1)^{\frac{3}{2}}} + \frac{-\frac{x}{a^2\sqrt{a^2x^2+1}} + \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}} + \sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}}{a^2}\right) - 6a^2\left(-\frac{x}{2a^2(a^2x^2+1)^{\frac{3}{2}}} +\right.$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(2*a^2*x^2+3)/(a^2*x^2+1)^(3/2)+8/3*I/a/Pi^(1/2)*(1/2*Pi^(1/2)-1/2*Pi
^(1/2)/(a^2*x^2+1)^(3/2))-2*a^2*x^3/(a^2*x^2+1)^(3/2)-8/3*I/a/Pi^(1/2)*(Pi^
(1/2)-1/8*Pi^(1/2)*(12*a^2*x^2+8)/(a^2*x^2+1)^(3/2))+2/3/Pi^(1/2)/(a^2)^(1/
2)*(-1/10*Pi^(1/2)*x*(a^2)^(5/2)*(20*a^2*x^2+15)/a^4/(a^2*x^2+1)^(3/2)+3/2*
Pi^(1/2)*(a^2)^(5/2)/a^5*arcsinh(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{8\,a^2x^2 + 16i\,ax + 3\,(a^2x^2 + 2i\,ax - 1)\log\left(-ax + \sqrt{a^2x^2+1}\right) + 4\,\sqrt{a^2x^2+1}(2\,ax + i) - 8}{3\,(a^3x^2 + 2i\,a^2x - a)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*a^2*x^2 + 16*I*a*x + 3*(a^2*x^2 + 2*I*a*x - 1)*log(-a*x + sqrt(a^2*
x^2 + 1)) + 4*sqrt(a^2*x^2 + 1)*(2*a*x + I) - 8)/(a^3*x^2 + 2*I*a^2*x - a)
```

Sympy [F]

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{(ax-i)^4}{(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**(5/2),x)
[Out] Integral((a*x - I)**4/(a**2*x**2 + 1)**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{3}a^4x\left(\frac{3x^2}{(a^2x^2+1)^{\frac{3}{2}}a^2} + \frac{2}{(a^2x^2+1)^{\frac{3}{2}}a^4}\right) + \frac{4i\,ax^2}{(a^2x^2+1)^{\frac{3}{2}}} - \frac{5x}{3\sqrt{a^2x^2+1}} + \frac{\operatorname{arsinh}(ax)}{a} + \frac{7x}{3(a^2x^2+1)^{\frac{3}{2}}} + \frac{4i}{3(a^2x^2+1)^{\frac{3}{2}}a}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="maxima")
[Out] -1/3*a^4*x*(3*x^2/((a^2*x^2 + 1)^(3/2)*a^2) + 2/((a^2*x^2 + 1)^(3/2)*a^4))
+ 4*I*a*x^2/(a^2*x^2 + 1)^(3/2) - 5/3*x/sqrt(a^2*x^2 + 1) + arcsinh(a*x)/a
+ 7/3*x/(a^2*x^2 + 1)^(3/2) + 4/3*I/((a^2*x^2 + 1)^(3/2)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.33

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{|a|}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="giac")
[Out] -log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} - \frac{8\sqrt{a^2x^2+1}}{3\left(x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}} + \frac{a\sqrt{a^2x^2+1}\,4\mathrm{i}}{3\left(a^4x^2+a^3x\,2\mathrm{i}-a^2\right)}$$

```
[In] int((a*x*1i + 1)^4/(a^2*x^2 + 1)^(5/2),x)
```

```
[Out] asinh(x*(a^2)^(1/2))/(a^2)^(1/2) - (8*(a^2*x^2 + 1)^(1/2))/(3*(((a^2)^(1/2)
*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2)) + (a*(a^2*x^2 + 1)^(1/2)*4i)/(3*(a^3*x
*2i - a^2 + a^4*x^2))
```

3.303 $\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 30

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{a(i+ax)} - \frac{i\log(i+ax)}{a}$$

```
[Out] 2/a/(I+a*x)-I*ln(I+a*x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5181, 45}

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{a(ax+i)} - \frac{i\log(ax+i)}{a}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] 2/(a*(I + a*x)) - (I*Log[I + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
```

```
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+iax}{(1-iax)^2}\,dx\\ &= \int \left(-\frac{2}{(i+ax)^2}-\frac{i}{i+ax}\right)dx\\ &= \frac{2}{a(i+ax)}-\frac{i\log(i+ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{a(i+ax)}-\frac{i\log(i+ax)}{a}$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] 2/(a*(I + a*x)) - (I*Log[I + a*x])/a
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{2}{a(ax+i)}-\frac{i\ln(ax+i)}{a}$	28
risch	$\frac{2}{a(ax+i)}-\frac{i\ln(a^2x^2+1)}{2a}-\frac{\arctan(ax)}{a}$	40
parallelrisch	$-\frac{i\ln(ax+i)x^2a^2-2ix^2a^2+i\ln(ax+i)-2ax}{(a^2x^2+1)a}$	57
meijerg	$\frac{\frac{2x\sqrt{a^2}}{2a^2x^2+2}+\frac{\sqrt{a^2}\,\arctan(ax)}{a}}{2\sqrt{a^2}}+\frac{3iax^2}{2(a^2x^2+1)}-\frac{3\left(-\frac{x\left(a^2\right)^{\frac{3}{2}}}{a^2\left(a^2x^2+1\right)}+\frac{\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{a^3}\right)}{2\sqrt{a^2}}-\frac{i\left(-\frac{a^2x^2}{a^2x^2+1}+\ln\left(a^2x^2+1\right)\right)}{2a}$	140

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 2/a/(I+a*x)-I*ln(I+a*x)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{(-i\,ax+1)\log\left(\frac{ax+i}{a}\right)+2}{a^2x+i\,a}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="fricas")
[Out] ((-I*a*x + 1)*log((a*x + I)/a) + 2)/(a^2*x + I*a)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{a^2x+ia} - \frac{i\log(ax+i)}{a}$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**2,x)
[Out] 2/(a**2*x + I*a) - I*log(a*x + I)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2(ax-i)}{a^3x^2+a} - \frac{\arctan(ax)}{a} - \frac{i\,\log(a^2x^2+1)}{2\,a}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="maxima")
[Out] 2*(a*x - I)/(a^3*x^2 + a) - arctan(a*x)/a - 1/2*I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\,\log(ax+i)}{a} + \frac{2}{(ax+i)a}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="giac")
[Out] -I*log(a*x + I)/a + 2/((a*x + I)*a)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{x\,a^2 + a\,1\mathrm{i}} - \frac{\ln\left(a\,x + 1\mathrm{i}\right)\,1\mathrm{i}}{a}$$

```
[In] int((a*x*1i + 1)^3/(a^2*x^2 + 1)^2,x)
[Out] 2/(a*1i + a^2*x) - (log(a*x + 1i)*1i)/a
```

3.304 $\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{\operatorname{arcsinh}(ax)}{a}$$

```
[Out] -arcsinh(a*x)/a-2*I*(1+I*a*x)^(1/2)/a/(1-I*a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5181, 49, 41, 221}

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\operatorname{arcsinh}(ax)}{a} - \frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - ArcSinh[a*x]/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+iax}}{(1-iax)^{3/2}}\,dx \\
&= -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx \\
&= -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \int \frac{1}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}} - \frac{\operatorname{arcsinh}(ax)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2i\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}} + \arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a}$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] ((-2*I)*(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x] + ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]])
)/a
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.12

method	result	size
default	$\frac{x}{\sqrt{a^2x^2+1}} - a^2\left(-\frac{x}{a^2\sqrt{a^2x^2+1}} + \frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}\right) - \frac{2i}{a\sqrt{a^2x^2+1}}$	87
meijerg	$\frac{x}{\sqrt{a^2x^2+1}} + \frac{2i\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}\right)}{a\sqrt{\pi}} - \frac{-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{3}{2}}}{a^2\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\left(a^2\right)^{\frac{3}{2}}\operatorname{arcsinh}(ax)}{a^3}}{\sqrt{\pi}\,\sqrt{a^2}}$	96

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/(a^2*x^2+1)^(1/2)-a^2*(-x/a^2/(a^2*x^2+1)^(1/2)+1/a^2*ln(a^2*x/(a^2)^(1/2
)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2))-2*I/a/(a^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2\,ax + (ax+i)\log\left(-ax+\sqrt{a^2x^2+1}\right) + 2\sqrt{a^2x^2+1} + 2i}{a^2x + i\,a}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*a*x + (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1) + 2*
I)/(a^2*x + I*a)
```

Sympy [F]

$$\begin{aligned}\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = &-\int \frac{a^2x^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx \\ &-\int \left(-\frac{2iax}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx \\ &-\int \left(-\frac{1}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\end{aligned}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)**(3/2),x)
```

```
[Out] -Integral(a**2*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x) - Integral(-2*I*a*x/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)
), x) - Integral(-1/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2x}{\sqrt{a^2x^2+1}} - \frac{\operatorname{arsinh}(ax)}{a} - \frac{2i}{\sqrt{a^2x^2+1}a}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] 2*x/sqrt(a^2*x^2 + 1) - arcsinh(a*x)/a - 2*I/(sqrt(a^2*x^2 + 1)*a)
```

Giac [F]

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{(i\,ax+1)^2}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{2\sqrt{a^2x^2+1}}{\left(x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}}$$

```
[In] int((a*x*1i + 1)^2/(a^2*x^2 + 1)^(3/2),x)
[Out] (2*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
- asinh(x*(a^2)^(1/2))/(a^2)^(1/2)
```

3.305 $\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 15

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\log(i+ax)}{a}$$

```
[Out] I*ln(I+a*x)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5181, 31}

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\log(ax+i)}{a}$$

```
[In] Int[E^(I*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (I*Log[I + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \int \frac{1}{1-iax}\,dx$$
$$= \frac{i\log(i+ax)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\log(i+ax)}{a}$$

```
[In] Integrate[E^(I*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
[Out] (I*Log[I + a*x])/a
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\frac{i\ln(ax+i)}{a}$	14
default	$\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$	26
meijerg	$\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$	26
risch	$\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$	26

```
[In] int((1+I*a*x)/(a^2*x^2+1),x,method=_RETURNVERBOSE)
[Out] I*ln(I+a*x)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\,\log\left(\frac{ax+i}{a}\right)}{a}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="fricas")
[Out] I*log((a*x + I)/a)/a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\log\left(ax+i\right)}{a}$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1),x)
[Out] I*log(a*x + I)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\arctan\left(ax\right)}{a} + \frac{i\,\log\left(a^2x^2+1\right)}{2\,a}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="maxima")
[Out] arctan(a*x)/a + 1/2*I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^{i \arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i \log(ax+i)}{a}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="giac")
[Out] I*log(a*x + I)/a
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^{i \arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,1\mathrm{i}}{a}$$

```
[In] int((a*x*1i + 1)/(a^2*x^2 + 1),x)
[Out] (log(x + 1i/a)*1i)/a
```

3.306 $\int \frac{e^{-i \arctan(ax)}}{\sqrt{1+a^2x^2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 16

$$\int \frac{e^{-i \arctan(ax)}}{\sqrt{1+a^2x^2}} dx = -\frac{i \log(i-ax)}{a}$$

```
[Out] -I*ln(I-a*x)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5181, 31}

$$\int \frac{e^{-i \arctan(ax)}}{\sqrt{1+a^2x^2}} dx = -\frac{i \log(-ax+i)}{a}$$

```
[In] Int[1/(E^(I*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ((-I)*Log[I - a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{1+iax}\,dx\\ &= -\frac{i\log(i-ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\log(i-ax)}{a}$$

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
[Out] ((-I)*Log[I - a*x])/a
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$-\frac{i\ln(ax-i)}{a}$	14
default	$-\frac{i\ln(iax+1)}{a}$	15
meijerg	$-\frac{i\ln(iax+1)}{a}$	15
risch	$-\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$	26

```
[In] int(1/(1+I*a*x),x,method=_RETURNVERBOSE)
[Out] -I*ln(a*x-I)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\log\left(\frac{ax-i}{a}\right)}{a}$$

```
[In] integrate(1/(1+I*a*x),x, algorithm="fricas")
[Out] -I*log((a*x - I)/a)/a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\log(ax-i)}{a}$$

```
[In] integrate(1/(1+I*a*x),x)
[Out] -I*log(a*x - I)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\,\log(i\,ax+1)}{a}$$

```
[In] integrate(1/(1+I*a*x),x, algorithm="maxima")
[Out] -I*log(I*a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\,\log(i\,ax+1)}{a}$$

```
[In] integrate(1/(1+I*a*x),x, algorithm="giac")
[Out] -I*log(I*a*x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\ln\left(x-\frac{1\mathrm{i}}{a}\right)\,1\mathrm{i}}{a}$$

```
[In] int(1/(a*x*1i + 1),x)
[Out] -(log(x - 1i/a)*1i)/a
```

3.307 $\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \frac{\operatorname{arcsinh}(ax)}{a}$$

```
[Out] -arcsinh(a*x)/a+2*I*(1-I*a*x)^(1/2)/a/(1+I*a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5181, 49, 41, 221}

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\operatorname{arcsinh}(ax)}{a} + \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[1 - I*a*x])/(a*Sqrt[1 + I*a*x]) - ArcSinh[a*x]/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{1-iax}}{(1+iax)^{3/2}}\,dx\\ &= \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx\\ &= \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \int \frac{1}{\sqrt{1+a^2x^2}}\,dx\\ &= \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} - \frac{\operatorname{arcsinh}(ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2\left(\sqrt{1+a^2x^2}+(-1-iax)\arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a(-i+ax)}$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
[Out] (2*(Sqrt[1 + a^2*x^2] + (-1 - I*a*x)*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(a*(
-I + a*x))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.63

method	result	size
default	$-\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{a\left(x-\frac{i}{a}\right)^2}-ia\left(\sqrt{\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)}+\frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)}\right)}{\sqrt{a^2}}\right)}{a^2}$	149

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(I/a/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)-I*a*(((x-I/a)^2*a
^2+2*I*a*(x-I/a))^(1/2)+I*a*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2
+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2\,ax+(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}\right)+2\sqrt{a^2x^2+1}-2i}{a^2x-i\,a}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*x + (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1) - 2*
I)/(a^2*x - I*a)
```

Sympy [F]

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\int \frac{\sqrt{a^2x^2+1}}{a^2x^2-2iax-1}\,dx$$

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(sqrt(a**2*x**2 + 1)/(a**2*x**2 - 2*I*a*x - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\operatorname{arsinh}(ax)}{a} + \frac{2i\sqrt{a^2x^2+1}}{i\,a^2x+a}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -arcsinh(a*x)/a + 2*I*sqrt(a^2*x^2 + 1)/(I*a^2*x + a)
```

Giac [F]

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)^2}\,dx$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] undef
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} - \frac{2\sqrt{a^2\,x^2+1}}{\left(-x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}}$$

```
[In] int((a^2*x^2 + 1)^(1/2)/(a*x*1i + 1)^2,x)
[Out] - asinh(x*(a^2)^(1/2))/(a^2)^(1/2) - (2*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)
*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

3.308 $\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2}{a(i-ax)}+\frac{i\log(i-ax)}{a}$$

```
[Out] -2/a/(I-a*x)+I*ln(I-a*x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5181, 45}

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
[Out] -2/(a*(I - a*x)) + (I*Log[I - a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
```

```
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-iax}{(1+iax)^2}\,dx \\ &= \int \left(-\frac{2}{(-i+ax)^2}+\frac{i}{-i+ax}\right)dx \\ &= -\frac{2}{a(i-ax)}+\frac{i\log(i-ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2}{a(i-ax)}+\frac{i\log(i-ax)}{a}$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
[Out] -2/(a*(I - a*x)) + (I*Log[I - a*x])/a
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2}{a(-ax+i)}+\frac{i\ln(-ax+i)}{a}$	30
risch	$\frac{2}{a(ax-i)}+\frac{i\ln(a^2x^2+1)}{2a}-\frac{\arctan(ax)}{a}$	40
meijerg	$\frac{i\left(-\frac{ixa(9iax+6)}{3(iax+1)^2}+2\ln(iax+1)\right)}{2a}+\frac{x(iax+2)}{2(iax+1)^2}$	59
parallelrisch	$\frac{i\ln(ax-i)x^2a^2+2\ln(ax-i)xa-2ix^2a^2-i\ln(ax-i)-2ax}{(-ax+i)^2a}$	65

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -2/a/(I-a*x)+I*ln(I-a*x)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{(i\,ax+1)\log\left(\frac{ax-i}{a}\right)+2}{a^2x-i\,a}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="fricas")
[Out] ((I*a*x + 1)*log((a*x - I)/a) + 2)/(a^2*x - I*a)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{a^2x-ia}+\frac{i\log(ax-i)}{a}$$

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1),x)
[Out] 2/(a**2*x - I*a) + I*log(a*x - I)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{4\,(-i\,ax-1)}{2i\,a^3x^2+4\,a^2x-2i\,a}+\frac{i\,\log(i\,ax+1)}{a}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="maxima")
[Out] -4*(-I*a*x - 1)/(2*I*a^3*x^2 + 4*a^2*x - 2*I*a) + I*log(I*a*x + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\,\log(ax-i)}{a}+\frac{2}{(ax-i)a}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="giac")
[Out] I*log(a*x - I)/a + 2/((a*x - I)*a)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2}{-a^2\,x + a\,1\mathrm{i}} + \frac{\ln\left(a\,x - \mathrm{i}\right)\,1\mathrm{i}}{a}$$

```
[In] int((a^2*x^2 + 1)/(a*x*1i + 1)^3,x)
[Out] (log(a*x - 1i)*1i)/a - 2/(a*1i - a^2*x)
```

3.309 $\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \frac{\operatorname{arcsinh}(ax)}{a}$$

```
[Out] 2/3*I*(1-I*a*x)^(3/2)/a/(1+I*a*x)^(3/2)+arcsinh(a*x)/a-2*I*(1-I*a*x)^(1/2)/
a/(1+I*a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5181, 49, 41, 221}

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{arcsinh}(ax)}{a} + \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (((2*I)/3)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(3/2)) - ((2*I)*Sqrt[1 - I*a*x
])/(a*Sqrt[1 + I*a*x]) + ArcSinh[a*x]/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1-iax)^{3/2}}{(1+iax)^{5/2}}\,dx \\
&= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \int \frac{\sqrt{1-iax}}{(1+iax)^{3/2}}\,dx \\
&= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \int \frac{1}{\sqrt{1-iax}\sqrt{1+iax}}\,dx \\
&= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \int \frac{1}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}} - \frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}} + \frac{\operatorname{arcsinh}(ax)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2i\left(\frac{2\sqrt{1+iax}(1+iax+2a^2x^2)}{\sqrt{1-iax}(-i+ax)^2} + 3\arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{3a}$$

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
[Out] (((2*I)/3)*((2*Sqrt[1 + I*a*x]*(1 + I*a*x + 2*a^2*x^2))/(Sqrt[1 - I*a*x]*(-
I + a*x)^2) + 3*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/a
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(57) = 114.

Time = 0.35 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.18

method	result
default	$\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{3a\left(x-\frac{i}{a}\right)^{4}}-\frac{ia\left(\frac{i\left(\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^{3}}-2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^{2}}+3ia\left(\frac{\left(\left(x-\frac{i}{a}\right)^{2}a^{2}+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{3}+ia\left(\right.\right.\right.\right.}{3}}{a^{4}}$

[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/a^4*(1/3*I/a/(x-I/a)^4*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-1/3*I*a*(I/a/(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-2*I*a*(-I/a/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)+3*I*a*(1/3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)+I*a*(1/4*(2*(x-I/a)*a^2+2*I*a)/a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)+1/2*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2))))))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{8\,a^2x^2-16i\,ax+3\,(a^2x^2-2i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+4\,\sqrt{a^2x^2+1}(2\,ax-i)-8}{3\,(a^3x^2-2i\,a^2x-a)}$$

[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="fricas")

[Out] -1/3*(8*a^2*x^2 - 16*I*a*x + 3*(a^2*x^2 - 2*I*a*x - 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + 4*sqrt(a^2*x^2 + 1)*(2*a*x - I) - 8)/(a^3*x^2 - 2*I*a^2*x - a)

Sympy [F]

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(ax-i)^4}\,dx$$

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**(3/2),x)
[Out] Integral((a**2*x**2 + 1)**(3/2)/(a*x - I)**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(51) = 102$.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.47

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\,(a^2x^2+1)^{\frac{3}{2}}}{-3i\,a^4x^3-9\,a^3x^2+9i\,a^2x+3\,a} + \frac{\operatorname{arsinh}(ax)}{a} - \frac{2i\,\sqrt{a^2x^2+1}}{3\,(a^3x^2-2i\,a^2x-a)} - \frac{7i\,\sqrt{a^2x^2+1}}{3i\,a^2x+3\,a}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] I*(a^2*x^2 + 1)^(3/2)/(-3*I*a^4*x^3 - 9*a^3*x^2 + 9*I*a^2*x + 3*a) + arcsin
h(a*x)/a - 2/3*I*sqrt(a^2*x^2 + 1)/(a^3*x^2 - 2*I*a^2*x - a) - 7*I*sqrt(a^2
*x^2 + 1)/(3*I*a^2*x + 3*a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.33

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] -log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{8\sqrt{a^2\,x^2+1}}{3\left(-x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}} + \frac{a\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{3\left(-a^4\,x^2+a^3\,x\,2\mathrm{i}+a^2\right)}$$

```
[In] int((a^2*x^2 + 1)^(3/2)/(a*x*1i + 1)^4,x)
```

```
[Out] asinh(x*(a^2)^(1/2))/(a^2)^(1/2) + (8*(a^2*x^2 + 1)^(1/2))/(3*(((a^2)^(1/2)
*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2)) + (a*(a^2*x^2 + 1)^(1/2)*4i)/(3*(a^3*x
*2i + a^2 - a^4*x^2))
```

3.310 $\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2i\sqrt{1+a^2x^2}}{a(1-iax)^2\sqrt{c+a^2cx^2}} + \frac{4i\sqrt{1+a^2x^2}}{a(1-iax)\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

```
[Out] -2*I*(a^2*x^2+1)^(1/2)/a/(1-I*a*x)^2/(a^2*c*x^2+c)^(1/2)+4*I*(a^2*x^2+1)^(1
/2)/a/(1-I*a*x)/(a^2*c*x^2+c)^(1/2)+I*ln(I+a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*
x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5184, 5181, 45}

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{4i\sqrt{a^2x^2+1}}{a(1-iax)\sqrt{a^2cx^2+c}} - \frac{2i\sqrt{a^2x^2+1}}{a(1-iax)^2\sqrt{a^2cx^2+c}} + \frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^((5*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + a^2*x^2])/(a*(1 - I*a*x)^2*Sqrt[c + a^2*c*x^2]) + ((4*I)*S
qrt[1 + a^2*x^2])/(a*(1 - I*a*x)*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*x^2
]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \frac{(1+iax)^2}{(1-iax)^3}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \left(\frac{4}{(1-iax)^3}-\frac{4}{(1-iax)^2}+\frac{1}{1-iax}\right)dx}{\sqrt{c+a^2cx^2}}\\
&= -\frac{2i\sqrt{1+a^2x^2}}{a(1-iax)^2\sqrt{c+a^2cx^2}}+\frac{4i\sqrt{1+a^2x^2}}{a(1-iax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$
\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{i\sqrt{1+a^2x^2}(-2+4iax+(i+ax)^2\log(i+ax))}{a(i+ax)^2\sqrt{c+a^2cx^2}}
$$

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] (I*Sqrt[1 + a^2*x^2]*(-2 + (4*I)*a*x + (I + a*x)^2*Log[I + a*x]))/(a*(I + a
*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\sqrt{a^{2}x^{2}+1}\,\left(-4x-\frac{2i}{a}\right)}{\sqrt{c(a^{2}x^{2}+1)}\,(ax+i)^{2}}+\frac{i\sqrt{a^{2}x^{2}+1}\,\ln(ax+i)}{\sqrt{c(a^{2}x^{2}+1)}\,a}$	82
default	$\frac{\sqrt{c(a^{2}x^{2}+1)}\,\left(i\ln(ax+i)x^{2}a^{2}-2\ln(ax+i)ax-i\ln(ax+i)-4ax-2i\right)}{\sqrt{a^{2}x^{2}+1}\,ca(ax+i)^{2}}$	84

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] (a^2*x^2+1)^(1/2)/(c*(a^2*x^2+1))^(1/2)*(-4*x-2*I/a)/(I+a*x)^2+I*(a^2*x^2+1
)^(1/2)/(c*(a^2*x^2+1))^(1/2)/a*ln(I+a*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(107) = 214.

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.78

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^{2}cx^{2}}}\,dx = \frac{-4i\sqrt{a^{2}cx^{2}+c}\sqrt{a^{2}x^{2}+1}ax^{2}+\left(i\,a^{4}cx^{4}-2\,a^{3}cx^{3}-2\,acx-i\,c\right)\sqrt{\frac{1}{a^{2}c}}\log\left(\frac{\left(i\,a^{6}x^{2}-2\,a^{5}x-2i\,a^{4}\right)\sqrt{a^{2}cx^{2}+c}\sqrt{a^{2}x^{2}-}}{8\,(a^{3}x^{3}+}\right.}{}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*(-4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a*x^2 + (I*a^4*c*x^4 - 2*a^
3*c*x^3 - 2*a*c*x - I*c)*sqrt(1/(a^2*c))*log(1/8*((I*a^6*x^2 - 2*a^5*x - 2*
I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 - 2*a^8*c*x^3 +
 I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3 + I*a^2*x^2 + a*x + I))
 + (-I*a^4*c*x^4 + 2*a^3*c*x^3 + 2*a*c*x + I*c)*sqrt(1/(a^2*c))*log(1/8*((I
*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (-I*a
^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3
 + I*a^2*x^2 + a*x + I)))/(a^4*c*x^4 + 2*I*a^3*c*x^3 + 2*I*a*c*x - c)
```

Sympy [F]

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= i\left(\int \left(-\frac{i}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx\right.$$
$$+\int \frac{5ax}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx$$
$$+\int \left(-\frac{10a^3x^3}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx$$
$$+\int \frac{a^5x^5}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx$$
$$+\int \frac{10ia^2x^2}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx$$
$$\left.+\int \left(-\frac{5ia^4x^4}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] I*(Integral(-I/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**
2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt
(a**2*c*x**2 + c)), x) + Integral(5*a*x/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt
(a**2*c*x**2 + c) + 2*a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) +
 sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-10*a**3*x**3/(a
**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*x**2*sqrt(a**2*
x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)
), x) + Integral(a**5*x**5/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2
+ c) + 2*a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x*
*2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(10*I*a**2*x**2/(a**4*x**4*sqr
t(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*x**2*sqrt(a**2*x**2 + 1)*sq
rt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Inte
gral(-5*I*a**4*x**4/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) +
2*a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)
*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F]

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^5}{\sqrt{a^2cx^2+c}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
[Out] integrate((I*a*x + 1)^5/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^5}{\sqrt{a^2cx^2+c}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
[Out] integrate((I*a*x + 1)^5/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(1+a\,x\,1\mathrm{i})^5}{\sqrt{c\,a^2\,x^2+c}\,(a^2\,x^2+1)^{5/2}}\,dx$$

```
[In] int((a*x*1i + 1)^5/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(5/2)),x)
[Out] int((a*x*1i + 1)^5/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(5/2)), x)
```

3.311 $\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}}+\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] -2/3*I*c*(1+I*a*x)^3/a/(a^2*c*x^2+c)^(3/2)+arctanh(a*x*c^(1/2)/(a^2*c*x^2+c
)^(1/2))/a/c^(1/2)+2*I*(1+I*a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5183, 683, 667, 223, 212}

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}-\frac{2ic(1+iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}+\frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (((-2*I)/3)*c*(1 + I*a*x)^3)/(a*(c + a^2*c*x^2)^(3/2)) + ((2*I)*(1 + I*a*x)
)/(a*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a*S
qrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5183

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^(I*(n/2)), Int[(c + d*x^2)^(p + I*(n/2))/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[I*(n/2), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^2 \int \frac{(1+iax)^4}{(c+a^2cx^2)^{5/2}}\,dx \\
&= -\frac{2ic(1+iax)^3}{3a\,(c+a^2cx^2)^{3/2}} - c\int \frac{(1+iax)^2}{(c+a^2cx^2)^{3/2}}\,dx \\
&= -\frac{2ic(1+iax)^3}{3a\,(c+a^2cx^2)^{3/2}} + \frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} + \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx \\
&= -\frac{2ic(1+iax)^3}{3a\,(c+a^2cx^2)^{3/2}} + \frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} + \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right) \\
&= -\frac{2ic(1+iax)^3}{3a\,(c+a^2cx^2)^{3/2}} + \frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} + \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.74

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{4i\sqrt{2+2a^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1-iax)\right)}{3a(1-iax)^{3/2}\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] (((-4*I)/3)*Sqrt[2 + 2*a^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - I*
a*x)/2])/(a*(1 - I*a*x)^(3/2)*Sqrt[c + a^2*c*x^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(80) = 160.

Time = 0.45 (sec) , antiderivative size = 526, normalized size of antiderivative = 5.48

method	result
default	$\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2+c}\right)}{\sqrt{a^2c}}+\frac{2\left(i\sqrt{-a^2}-a\right)\left(\frac{\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c-2c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}}{3c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2}-\frac{\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c-2c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}}{3c\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}\right)}{a^3}-$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)+2/a^3*(I*(-a^2)
^(1/2)-a)*(1/3/c/(-a^2)^(1/2)/(x+(-a^2)^(1/2)/a^2)^2*((x+(-a^2)^(1/2)/a^2)^
2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)-1/3/c/(x+(-a^2)^(1/2)/
a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(
1/2))-2/a^3*(I*(-a^2)^(1/2)+a)*(-1/3/c/(-a^2)^(1/2)/(x-(-a^2)^(1/2)/a^2)^2*
((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)-
1/3/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(
x-(-a^2)^(1/2)/a^2))^(1/2))-2/a^3*(I*(-a^2)^(1/2)+a)/c/(x-(-a^2)^(1/2)/a^2)
*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)
+2/a^3*(I*(-a^2)^(1/2)-a)/c/(x+(-a^2)^(1/2)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^
2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(75) = 150.

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.94

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{3\left(a^3cx^2+2i\,a^2cx-ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-3\left(a^3cx^2+2i\,a^2cx-ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\right.}{}\right.}{6\left(a^3cx^2+2i\,a^2cx-ac\right)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(3*(a^3*c*x^2 + 2*I*a^2*c*x - a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqr
t(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 3*(a^3*c*x^2 + 2*I*a^2*c*x - a
*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*
c)))/x) - 8*sqrt(a^2*c*x^2 + c)*(2*a*x + I))/(a^3*c*x^2 + 2*I*a^2*c*x - a*c
)
```

Sympy [F]

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(ax-i)^4}{\sqrt{c\left(a^2x^2+1\right)}\left(a^2x^2+1\right)^2}\,dx$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x - I)**4/(sqrt(c*(a**2*x**2 + 1))*(a**2*x**2 + 1)**2), x)
```

Maxima [F]

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^4}{\sqrt{a^2cx^2+c}\left(a^2x^2+1\right)^2}\,dx$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((I*a*x + 1)^4/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.38

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} - \frac{8\left(3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2+3i\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)\sqrt{c}-2c\right)}{3\left(i\sqrt{a^2c}x-i\sqrt{a^2cx^2+c}-\sqrt{c}\right)^3a}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 8/3*(3*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 + c))^2 + 3*I*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c
))*sqrt(c) - 2*c)/((I*sqrt(a^2*c)*x - I*sqrt(a^2*c*x^2 + c) - sqrt(c))^3*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(1+a\,x\,1\mathrm{i})^4}{\sqrt{c\,a^2\,x^2+c}\,(a^2\,x^2+1)^2}\,dx$$

```
[In] int((a*x*1i + 1)^4/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^2),x)
```

```
[Out] int((a*x*1i + 1)^4/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^2), x)
```

3.312 $\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{1+a^2x^2}}{a(i+ax)\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

```
[Out] 2*(a^2*x^2+1)^(1/2)/a/(I+a*x)/(a^2*c*x^2+c)^(1/2)-I*ln(I+a*x)*(a^2*x^2+1)^(
1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5184, 5181, 45}

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{a^2x^2+1}}{a(ax+i)\sqrt{a^2cx^2+c}} - \frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2])/(a*(I + a*x)*Sqrt[c + a^2*c*x^2]) - (I*Sqrt[1 + a^2*x
^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \frac{1+iax}{(1-iax)^2}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{2}{(i+ax)^2}-\frac{i}{i+ax}\right)dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{2\sqrt{1+a^2x^2}}{a(i+ax)\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{1+a^2x^2}\left(\frac{2}{i+ax}-i\log(i+ax)\right)}{a\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] (Sqrt[1 + a^2*x^2]*(2/(I + a*x) - I*Log[I + a*x]))/(a*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{(-i\ln(ax+i)ax+\ln(ax+i)+2)\sqrt{c(a^2x^2+1)}}{\sqrt{a^2x^2+1}\,ca(ax+i)}$	61
risch	$\frac{2\sqrt{a^2x^2+1}}{\sqrt{c(a^2x^2+1)}\,a(ax+i)} - \frac{i\sqrt{a^2x^2+1}\,\ln(ax+i)}{\sqrt{c(a^2x^2+1)}\,a}$	76

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] (-I*ln(I+a*x)*a*x+ln(I+a*x)+2)/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c/a/
(I+a*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(70) = 140.

Time = 0.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.25

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{(-i\,a^3cx^3 + a^2cx^2 - i\,acx + c)\sqrt{\frac{1}{a^2c}}\log\left(\frac{(i\,a^6x^2-2\,a^5x-2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(i\,a^9cx^4-2\,a^8cx^3+i\,a^7cx^2-2\,a^6cx)\sqrt{\frac{1}{a^2c}}}{8\,(a^3x^3+i\,a^2x^2+ax+i)}\right.}{}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*((-I*a^3*c*x^3 + a^2*c*x^2 - I*a*c*x + c)*sqrt(1/(a^2*c))*log(1/8*((I*a
^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*
c*x^4 - 2*a^8*c*x^3 + I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3 +
I*a^2*x^2 + a*x + I)) + (I*a^3*c*x^3 - a^2*c*x^2 + I*a*c*x - c)*sqrt(1/(a^2
*c))*log(1/8*((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*
x^2 + 1) + (-I*a^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a
^2*c)))/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2
*x^2 + 1)*x)/(a^3*c*x^3 + I*a^2*c*x^2 + a*c*x + I*c)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -i\left(\int \frac{i}{a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx + \int \left(-\frac{3ax}{a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int \frac{a^3x^3}{a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -I*(Integral(I/(a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(
a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*a*x/(a**2*x**2*sqrt
(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**
2 + c)), x) + Integral(a**3*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c
*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*I
*a**2*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2
*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F]

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^3}{\sqrt{a^2cx^2+c}\,(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((I*a*x + 1)^3/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^3}{\sqrt{a^2cx^2+c}\,(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^3/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(1+a\,x\,1\mathrm{i})^3}{\sqrt{c\,a^2\,x^2+c}\,(a^2\,x^2+1)^{3/2}}\,dx$$

```
[In] int((a*x*1i + 1)^3/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(3/2)),x)
[Out] int((a*x*1i + 1)^3/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(3/2)), x)
```

3.313 $\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] -arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a/c^(1/2)-2*I*(1+I*a*x)/a/(a^2*c*
x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5183, 667, 223, 212}

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}} - \frac{2i(1+iax)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((-2*I)*(1 + I*a*x))/(a*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c
 + a^2*c*x^2]]/(a*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5183

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^(I*(n/2)), Int[(c + d*x^2)^(p + I*(n/2))/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[I*(n/2), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\int \frac{(1+iax)^2}{(c+a^2cx^2)^{3/2}}\,dx \\
&= -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx \\
&= -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right) \\
&= -\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2i\sqrt{1+a^2x^2}\left(\sqrt{1+iax}+\sqrt{1-iax}\arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-iax}\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] ((-2*I)*Sqrt[1 + a^2*x^2]*(Sqrt[1 + I*a*x] + Sqrt[1 - I*a*x]*ArcSin[Sqrt[1
- I*a*x]/Sqrt[2]]))/(a*Sqrt[1 - I*a*x]*Sqrt[c + a^2*c*x^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(53) = 106.

Time = 0.36 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.24

method	result
default	$-\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+c}\right)}{\sqrt{a^2c}}+\frac{\left(i\sqrt{-a^2}+a\right)\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c+2c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}}{a^3c\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}-\frac{\left(i\sqrt{-a^2}-a\right)\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c-2c\sqrt{-c}}}{a^3c\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}$

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)+1/a^3*(I*(-a^2
)^(1/2)+a)/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^
(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)-1/a^3*(I*(-a^2)^(1/2)-a)/c/(x+(-a^2)^(1/2
)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))
^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(50) = 100.

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.41

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(a^2cx+i\,ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-(a^2cx+i\,ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-4}{2\,(a^2cx+i\,ac)}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/2*((a^2*c*x + I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(a^2*c*x^2 + c
)*a^2*c*sqrt(1/(a^2*c)))/x) - (a^2*c*x + I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*
c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 4*sqrt(a^2*c*x^2 + c)
)/(a^2*c*x + I*a*c)
```

Sympy [F]

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\int \frac{a^2x^2}{a^2x^2\sqrt{a^2cx^2+c}+\sqrt{a^2cx^2+c}}\,dx - \int \left(-\frac{2iax}{a^2x^2\sqrt{a^2cx^2+c}+\sqrt{a^2cx^2+c}}\right)dx - \int \left(-\frac{1}{a^2x^2\sqrt{a^2cx^2+c}+\sqrt{a^2cx^2+c}}\right)dx$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a**2*x**2/(a**2*x**2*sqrt(a**2*c*x**2 + c) + sqrt(a**2*c*x**2 + c
)), x) - Integral(-2*I*a*x/(a**2*x**2*sqrt(a**2*c*x**2 + c) + sqrt(a**2*c*x
**2 + c)), x) - Integral(-1/(a**2*x**2*sqrt(a**2*c*x**2 + c) + sqrt(a**2*c*
x**2 + c)), x)
```

Maxima [F]

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^2}{\sqrt{a^2cx^2+c}(a^2x^2+1)}\,dx$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*x + 1)^2/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} - \frac{4}{\left(i\sqrt{a^2c}x - i\sqrt{a^2cx^2+c}-\sqrt{c}\right)a}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 4/((I*sqrt(a^2
*c)*x - I*sqrt(a^2*c*x^2 + c) - sqrt(c))*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(1+a\,x\,1\mathrm{i})^2}{\sqrt{c\,a^2\,x^2+c}\,\left(a^2\,x^2+1\right)}\,dx$$

```
[In] int((a*x*1i + 1)^2/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)),x)
[Out] int((a*x*1i + 1)^2/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)), x)
```

3.314 $\int \frac{e^{i \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 42

$$\int \frac{e^{i \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

```
[Out] I*ln(I+a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5184, 5181, 31}

$$\int \frac{e^{i \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^(I*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (I*Sqrt[1 + a^2*x^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{1-iax}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(I*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
[Out] (I*Sqrt[1 + a^2*x^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{i\sqrt{a^2x^2+1}\,\ln(ax+i)}{\sqrt{c(a^2x^2+1)}\,a}$	38
default	$\frac{\sqrt{c(a^2x^2+1)}\,(i\ln(a^2x^2+1)+2\arctan(ax))}{2\sqrt{a^2x^2+1}\,ca}$	53

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
[Out] I*(a^2*x^2+1)^(1/2)/(c*(a^2*x^2+1))^(1/2)/a*ln(I+a*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 6.02

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{(i\,a^6x^2-2\,a^5x-2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(i\,a^9cx^4-2\,a^8cx^3+i\,a^7cx^2-2\,a^6cx)\sqrt{\frac{1}{a^2c}}}{8\,(a^3x^3+i\,a^2x^2+ax+i)}\right)$$
$$-\frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{(i\,a^6x^2-2\,a^5x-2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(-i\,a^9cx^4+2\,a^8cx^3-i\,a^7cx^2+2\,a^6cx)\sqrt{\frac{1}{a^2c}}}{8\,(a^3x^3+i\,a^2x^2+ax+i)}\right)$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*I*sqrt(1/(a^2*c))*log(1/8*((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x
^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 - 2*a^8*c*x^3 + I*a^7*c*x^2 - 2*a^
6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3 + I*a^2*x^2 + a*x + I)) - 1/2*I*sqrt(1/(a^
2*c))*log(1/8*((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2
*x^2 + 1) + (-I*a^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(
a^2*c)))/(a^3*x^3 + I*a^2*x^2 + a*x + I))
```

Sympy [F]

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = i\left(\int \left(-\frac{i}{\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx\right)$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] I*(Integral(-I/(sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a
*x/(sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{i\,ax+1}{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((I*a*x + 1)/(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{1+a\,x\,1\mathrm{i}}{\sqrt{c\,a^2\,x^2+c}\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((a*x*1i + 1)/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int((a*x*1i + 1)/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(1/2)), x)
```

3.315 $\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 43

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=-\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}$$

```
[Out] -I*ln(I-a*x)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5184, 5181, 31}

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=-\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^(I*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + a^2*x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{1+iax}\,dx}{\sqrt{c+a^2cx^2}}\\ &= -\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
[Out] ((-I)*Sqrt[1 + a^2*x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{i\sqrt{a^2x^2+1}\,\ln(-ax+i)}{\sqrt{c(a^2x^2+1)}\,a}$	39
default	$-\frac{i\sqrt{c(a^2x^2+1)}\,\ln(iax+1)}{\sqrt{a^2x^2+1}\,ca}$	42

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
[Out] -I*(a^2*x^2+1)^(1/2)/(c*(a^2*x^2+1))^(1/2)/a*ln(I-a*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(35) = 70.

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 5.88

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{(-i\,a^6x^2-2\,a^5x+2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(i\,a^9cx^4+2\,a^8cx^3+i\,a^7cx^2+2\,a^6cx)\sqrt{\cdots}}{8\,(a^3x^3-i\,a^2x^2+ax-i)}\right.$$
$$-\frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{(-i\,a^6x^2-2\,a^5x+2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(-i\,a^9cx^4-2\,a^8cx^3-i\,a^7cx^2-2\,a^6c\cdots}{8\,(a^3x^3-i\,a^2x^2+ax-i)}\right.$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*I*sqrt(1/(a^2*c))*log(1/8*((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*
x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 + 2*a^8*c*x^3 + I*a^7*c*x^2 + 2*a
^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3 - I*a^2*x^2 + a*x - I)) - 1/2*I*sqrt(1/(a
^2*c))*log(1/8*((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a
^2*x^2 + 1) + (-I*a^9*c*x^4 - 2*a^8*c*x^3 - I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1
/(a^2*c)))/(a^3*x^3 - I*a^2*x^2 + a*x - I))
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax\sqrt{a^2cx^2+c}-i\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a*x*sqrt(a**2*c*x**2 + c) - I*sqrt(a**2*c*
x**2 + c)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.35

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{i\,\log\left(i\,ax+1\right)}{a\sqrt{c}}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] -I*log(I*a*x + 1)/(a*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{a^2\,x^2+1}}{\sqrt{c\,a^2\,x^2+c}\,\left(1+a\,x\,1\mathrm{i}\right)}\,dx$$

```
[In] int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)),x)
```

```
[Out] int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)), x)
```

3.316 $\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] -arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a/c^(1/2)+2*I*(1-I*a*x)/a/(a^2*c*
x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5182, 667, 223, 212}

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}} + \frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((2*I)*(1 - I*a*x))/(a*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c
+ a^2*c*x^2]]/(a*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5182

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^(I*(n/2)), Int[(c + d*x^2)^(p - I*(n/2))*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[I*(n/2), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\int \frac{(1-iax)^2}{(c+a^2cx^2)^{3/2}}\,dx \\
&= \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx \\
&= \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right) \\
&= \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{1+a^2x^2}\left((1-iax)\sqrt{1+iax}-i\sqrt{1-iax}(-i+ax)\arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-iax}(-i+ax)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
[Out] (2*Sqrt[1 + a^2*x^2]*((1 - I*a*x)*Sqrt[1 + I*a*x] - I*Sqrt[1 - I*a*x]*(-I +
 a*x)*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(a*Sqrt[1 - I*a*x]*(-I + a*x)*Sqrt[
c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\ln\left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\,x^{2}+c}\right)}{\sqrt{a^{2} c}}+\frac{2 \sqrt{\left(x-\frac{i}{a}\right)^{2} a^{2} c+2 i a c \left(x-\frac{i}{a}\right)}}{a^{2} c \left(x-\frac{i}{a}\right)}$	87

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)+2/a^2/c/(x-I/a
)*((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.41

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(a^2cx-i\,ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-(a^2cx-i\,ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-}{2\,(a^2cx-i\,ac)}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
[Out] -1/2*((a^2*c*x - I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(a^2*c*x^2 + c
)*a^2*c*sqrt(1/(a^2*c)))/x) - (a^2*c*x - I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*
c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 4*sqrt(a^2*c*x^2 + c)
)/(a^2*c*x - I*a*c)
```

Sympy [F]

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\int \frac{a^2x^2}{a^2x^2\sqrt{a^2cx^2+c}-2iax\sqrt{a^2cx^2+c}-\sqrt{a^2cx^2+c}}\,dx - \int \frac{1}{a^2x^2\sqrt{a^2cx^2+c}-2iax\sqrt{a^2cx^2+c}-\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**(1/2),x)
[Out] -Integral(a**2*x**2/(a**2*x**2*sqrt(a**2*c*x**2 + c) - 2*I*a*x*sqrt(a**2*c*
x**2 + c) - sqrt(a**2*c*x**2 + c)), x) - Integral(1/(a**2*x**2*sqrt(a**2*c*
x**2 + c) - 2*I*a*x*sqrt(a**2*c*x**2 + c) - sqrt(a**2*c*x**2 + c)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2i\sqrt{a^2cx^2+c}}{i\,a^2cx+ac} - \frac{\operatorname{arsinh}(ax)}{a\sqrt{c}}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2*I*sqrt(a^2*c*x^2 + c)/(I*a^2*c*x + a*c) - arcsinh(a*x)/(a*sqrt(c))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} - \frac{4}{\left(-i\sqrt{a^2c}x+i\sqrt{a^2cx^2+c}-\sqrt{c}\right)a}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 4/((-I*sqrt(a^
2*c)*x + I*sqrt(a^2*c*x^2 + c) - sqrt(c))*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{a^2\,x^2+1}{\sqrt{c\,a^2\,x^2+c}\,{\left(1+a\,x\,1{}\mathrm{i}\right)}^2}\,dx$$

```
[In] int((a^2*x^2 + 1)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^2),x)
```

```
[Out] int((a^2*x^2 + 1)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^2), x)
```

3.317 $\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{a(i-ax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}$$

```
[Out] -2*(a^2*x^2+1)^(1/2)/a/(I-a*x)/(a^2*c*x^2+c)^(1/2)+I*ln(I-a*x)*(a^2*x^2+1)^
(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5184, 5181, 45}

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a(-ax+i)\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(I - a*x)*Sqrt[c + a^2*c*x^2]) + (I*Sqrt[1 + a^2*
x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \frac{1-iax}{(1+iax)^2}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{2}{(-i+ax)^2}+\frac{i}{-i+ax}\right)\,dx}{\sqrt{c+a^2cx^2}} \\
&= -\frac{2\sqrt{1+a^2x^2}}{a(i-ax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

$$
\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(-\frac{2}{a(i-ax)}+\frac{i\log(i-ax)}{a}\right)}{\sqrt{c+a^2cx^2}}
$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
[Out] (Sqrt[1 + a^2*x^2]*(-2/(a*(I - a*x)) + (I*Log[I - a*x])/a))/Sqrt[c + a^2*c*
x^2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\left(-i \ln \left(-a x+i\right) a x-\ln \left(-a x+i\right)-2\right) \sqrt{c \left(a^{2} x^{2}+1\right)}}{\sqrt{a^{2} x^{2}+1}\, c \left(-a x+i\right) a}$	66
risch	$\frac{2 \sqrt{a^{2} x^{2}+1}}{\sqrt{c \left(a^{2} x^{2}+1\right)}\, a \left(a x-i\right)}+\frac{i \sqrt{a^{2} x^{2}+1}\, \ln \left(a x-i\right)}{\sqrt{c \left(a^{2} x^{2}+1\right)}\, a}$	76

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] (-I*ln(I-a*x)*a*x-ln(I-a*x)-2)/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c/(I
-a*x)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(71) = 142.

Time = 0.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.15

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\left(-i\,a^3cx^3-a^2cx^2-i\,acx-c\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{\left(-i\,a^6x^2-2\,a^5x+2i\,a^4\right)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+\left(i\,a^9cx^4+2\,a^8cx^3+i\,a^7cx^2+2\,a^6cx\right)\sqrt{\frac{1}{a^2c}}}{8\left(a^3x^3-i\,a^2x^2+ax-i\right)}\right.}{}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 1/2*((-I*a^3*c*x^3 - a^2*c*x^2 - I*a*c*x - c)*sqrt(1/(a^2*c))*log(1/8*((-I*
a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9
*c*x^4 + 2*a^8*c*x^3 + I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3 -
 I*a^2*x^2 + a*x - I)) + (I*a^3*c*x^3 + a^2*c*x^2 + I*a*c*x + c)*sqrt(1/(a^
2*c))*log(1/8*((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^
2*x^2 + 1) + (-I*a^9*c*x^4 - 2*a^8*c*x^3 - I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1/
(a^2*c)))/(a^3*x^3 - I*a^2*x^2 + a*x - I)) - 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a
^2*x^2 + 1)*x)/(a^3*c*x^3 - I*a^2*c*x^2 + a*c*x - I*c)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^3\sqrt{a^2cx^2+c}-3ia^2x^2\sqrt{a^2cx^2+c}-3ax\sqrt{a^2cx^2+c}+i\sqrt{a^2cx^2+c}}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^3\sqrt{a^2cx^2+c}-3ia^2x^2\sqrt{a^2cx^2+c}-3ax\sqrt{a^2cx^2+c}+i\sqrt{a^2cx^2+c}}\,dx\right)$$

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**3*sqrt(a**2*c*x**2 + c) - 3*I*a**2
*x**2*sqrt(a**2*c*x**2 + c) - 3*a*x*sqrt(a**2*c*x**2 + c) + I*sqrt(a**2*c*x
**2 + c)), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**3*sqrt(a**2
*c*x**2 + c) - 3*I*a**2*x**2*sqrt(a**2*c*x**2 + c) - 3*a*x*sqrt(a**2*c*x**2
 + c) + I*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{i\log(i\,ax+1)}{a\sqrt{c}} + \frac{2}{a^2\sqrt{c}x - i\,a\sqrt{c}}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] I*log(I*a*x + 1)/(a*sqrt(c)) + 2/(a^2*sqrt(c)*x - I*a*sqrt(c))
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}(i\,ax+1)^3}\,dx$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a^2*x^2 + 1)^(3/2)/(sqrt(a^2*c*x^2 + c)*(I*a*x + 1)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{{\left(a^2\,x^2+1\right)}^{3/2}}{\sqrt{c\,a^2\,x^2+c}\,{\left(1+a\,x\,1\mathrm{i}\right)}^3}\,dx$$

```
[In] int((a^2*x^2 + 1)^(3/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^3),x)
[Out] int((a^2*x^2 + 1)^(3/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^3), x)
```

3.318 $\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2ic(1-iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}}-\frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}}+\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] 2/3*I*c*(1-I*a*x)^3/a/(a^2*c*x^2+c)^(3/2)+arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)
^(1/2))/a/c^(1/2)-2*I*(1-I*a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5182, 683, 667, 223, 212}

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a\sqrt{c}}+\frac{2ic(1-iax)^3}{3a\left(a^2cx^2+c\right)^{3/2}}-\frac{2i(1-iax)}{a\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (((2*I)/3)*c*(1 - I*a*x)^3)/(a*(c + a^2*c*x^2)^(3/2)) - ((2*I)*(1 - I*a*x))
/(a*Sqrt[c + a^2*c*x^2]) + ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a*Sq
rt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 5182

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^(I*(n/2)), Int[(c + d*x^2)^(p - I*(n/2))*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[I*(n/2), 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c^2 \int \frac{(1-iax)^4}{(c+a^2cx^2)^{5/2}}\,dx \\
&= \frac{2ic(1-iax)^3}{3a\,(c+a^2cx^2)^{3/2}} - c\int \frac{(1-iax)^2}{(c+a^2cx^2)^{3/2}}\,dx \\
&= \frac{2ic(1-iax)^3}{3a\,(c+a^2cx^2)^{3/2}} - \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} + \int \frac{1}{\sqrt{c+a^2cx^2}}\,dx \\
&= \frac{2ic(1-iax)^3}{3a\,(c+a^2cx^2)^{3/2}} - \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} + \text{Subst}\left(\int \frac{1}{1-a^2cx^2}\,dx, x, \frac{x}{\sqrt{c+a^2cx^2}}\right) \\
&= \frac{2ic(1-iax)^3}{3a\,(c+a^2cx^2)^{3/2}} - \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} + \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.38

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{1+a^2x^2}\left(2i\sqrt{1+iax}(1+iax+2a^2x^2)+3i\sqrt{1-iax}(-i+ax)^2\arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{3a\sqrt{1-iax}(-i+ax)^2\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*((2*I)*Sqrt[1 + I*a*x]*(1 + I*a*x + 2*a^2*x^2) + (3*I)
*Sqrt[1 - I*a*x]*(-I + a*x)^2*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(3*a*Sqrt[1
 - I*a*x]*(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(80) = 160.

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.96

method	result	size
default	$\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+c}\right)}{\sqrt{a^2c}}-\frac{4\left(\frac{i\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}{3ac\left(x-\frac{i}{a}\right)^2}-\frac{\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}{3c\left(x-\frac{i}{a}\right)}\right)}{a^2}-\frac{4\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}{a^2c\left(x-\frac{i}{a}\right)}$	188

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)-4/a^2*(1/3*I/a/
c/(x-I/a)^2*((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/2)-1/3/c/(x-I/a)*((x-I/a)^
2*a^2*c+2*I*a*c*(x-I/a))^(1/2))-4/a^2/c/(x-I/a)*((x-I/a)^2*a^2*c+2*I*a*c*(x
-I/a))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(75) = 150.

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.94

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{3\,(a^3cx^2-2i\,a^2cx-ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-3\,(a^3cx^2-2i\,a^2cx-ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\right.}{}\right.}{6\,(a^3cx^2-2i\,a^2cx-ac)}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/6*(3*(a^3*c*x^2 - 2*I*a^2*c*x - a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqr
t(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 3*(a^3*c*x^2 - 2*I*a^2*c*x - a
*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*
c)))/x) - 8*sqrt(a^2*c*x^2 + c)*(2*a*x - I))/(a^3*c*x^2 - 2*I*a^2*c*x - a*c
)
```

Sympy [F]

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(a^2x^2+1)^2}{\sqrt{c\,(a^2x^2+1)}\,(ax-i)^4}\,dx$$

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a**2*x**2 + 1)**2/(sqrt(c*(a**2*x**2 + 1))*(a*x - I)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{4i\sqrt{a^2cx^2+c}}{3\,(a^3cx^2-2i\,a^2cx-ac)}-\frac{8i\sqrt{a^2cx^2+c}}{3i\,a^2cx+3\,ac}+\frac{\operatorname{arsinh}(ax)}{a\sqrt{c}}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -4/3*I*sqrt(a^2*c*x^2 + c)/(a^3*c*x^2 - 2*I*a^2*c*x - a*c) - 8*I*sqrt(a^2*c
*x^2 + c)/(3*I*a^2*c*x + 3*a*c) + arcsinh(a*x)/(a*sqrt(c))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.38

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} - \frac{8\left(3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2-3i\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)\sqrt{c}-2c\right)}{3\left(-i\sqrt{a^2c}x+i\sqrt{a^2cx^2+c}-\sqrt{c}\right)^3a}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] -log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 8/3*(3*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 + c))^2 - 3*I*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c
))*sqrt(c) - 2*c)/((-I*sqrt(a^2*c)*x + I*sqrt(a^2*c*x^2 + c) - sqrt(c))^3*a
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\left(a^2\,x^2+1\right)^2}{\sqrt{c\,a^2\,x^2+c}\,\left(1+a\,x\,1\mathrm{i}\right)^4}\,dx$$

```
[In] int((a^2*x^2 + 1)^2/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^4),x)
```

```
[Out] int((a^2*x^2 + 1)^2/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^4), x)
```

3.319 $\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}} dx = -\frac{2}{3a(i+ax)^3} - \frac{i}{2a(i+ax)^2}$$

```
[Out] -2/3/a/(I+a*x)^3-1/2*I/a/(I+a*x)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5181, 45}

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}} dx = -\frac{i}{2a(ax+i)^2} - \frac{2}{3a(ax+i)^3}$$

```
[In] Int[E^((5*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
[Out] -2/(3*a*(I + a*x)^3) - (I/2)/(a*(I + a*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
```

```
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+iax}{(1-iax)^4}\,dx\\ &= \int \left(\frac{2}{(i+ax)^4}+\frac{i}{(i+ax)^3}\right)dx\\ &= -\frac{2}{3a(i+ax)^3}-\frac{i}{2a(i+ax)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{1+3iax}{6a(i+ax)^3}$$

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
[Out] -1/6*(1 + (3*I)*a*x)/(a*(I + a*x)^3)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

method	result
default	$\frac{-\frac{ix}{2}-\frac{1}{6a}}{(ax+i)^3}$
risch	$\frac{-\frac{ix}{2}-\frac{1}{6a}}{(ax+i)^3}$
norman	$\frac{x+\frac{5}{2}ia\,x^2+\frac{1}{6}ia^5x^6-\frac{5}{3}a^2x^3}{(a^2x^2+1)^3}$
parallelrisch	$\frac{ia^5x^6-10a^2x^3+15ia\,x^2+6x}{6(a^2x^2+1)^3}$
gosper	$-\frac{(ax+i)(-3ax+i)(iax+1)^5}{6a(-ax+i)(a^2x^2+1)^4}$
meijerg	$\frac{\frac{x\sqrt{a^2}\left(15a^4x^4+40a^2x^2+33\right)}{4\left(a^2x^2+1\right)^3}+\frac{15\sqrt{a^2}\,\arctan(ax)}{4a}}{12\sqrt{a^2}}+\frac{5ia\,x^2(a^4x^4+3a^2x^2+3)}{6(a^2x^2+1)^3}-\frac{5\left(-\frac{x\left(a^2\right)^{\frac{3}{2}}\left(-3a^4x^4-8a^2x^2+3\right)}{4a^2\left(a^2x^2+1\right)^3}+\frac{3\left(a^2\right)^{\frac{3}{2}}\arctan}{4a^3}\right.}{6\sqrt{a^2}}$

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^4,x,method=_RETURNVERBOSE)
[Out] (-1/2*I*x-1/6/a)/(I+a*x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{-3i\,ax-1}{6\,(a^4x^3+3i\,a^3x^2-3\,a^2x-i\,a)}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="fricas")
[Out] 1/6*(-3*I*a*x - 1)/(a^4*x^3 + 3*I*a^3*x^2 - 3*a^2*x - I*a)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{-3iax-1}{6a^4x^3+18ia^3x^2-18a^2x-6ia}$$

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**4,x)
[Out] (-3*I*a*x - 1)/(6*a**4*x**3 + 18*I*a**3*x**2 - 18*a**2*x - 6*I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(25) = 50.

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{3i\,a^4x^4+10\,a^3x^3-12i\,a^2x^2-6\,ax+i}{6\,(a^7x^6+3\,a^5x^4+3\,a^3x^2+a)}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="maxima")
[Out] -1/6*(3*I*a^4*x^4 + 10*a^3*x^3 - 12*I*a^2*x^2 - 6*a*x + I)/(a^7*x^6 + 3*a^5
*x^4 + 3*a^3*x^2 + a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{3i\,ax+1}{6\,(ax+i)^3a}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="giac")
[Out] -1/6*(3*I*a*x + 1)/((a*x + I)^3*a)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{3\,a\,x-\mathrm{i}}{6\,a\,(-1+a\,x\,1\mathrm{i})^3}$$

```
[In] int((a*x*1i + 1)^5/(a^2*x^2 + 1)^4,x)
[Out] -(3*a*x - 1i)/(6*a*(a*x*1i - 1)^3)
```

3.320 $\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}} dx = -\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}} - \frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}}$$

```
[Out] -1/5*I*(1+I*a*x)^(3/2)/a/(1-I*a*x)^(5/2)-1/15*I*(1+I*a*x)^(3/2)/a/(1-I*a*x)
^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5181, 47, 37}

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}} dx = -\frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}} - \frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((-1/5*I)*(1 + I*a*x)^(3/2))/(a*(1 - I*a*x)^(5/2)) - ((I/15)*(1 + I*a*x)^(3
/2))/(a*(1 - I*a*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+iax}}{(1-iax)^{7/2}}\,dx \\
&= -\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}} + \frac{1}{5}\int \frac{\sqrt{1+iax}}{(1-iax)^{5/2}}\,dx \\
&= -\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}} - \frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{(1+iax)^{3/2}(4i+ax)}{15a\sqrt{1-iax}(i+ax)^2}$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
[Out] ((1 + I*a*x)^(3/2)*(4*I + a*x))/(15*a*Sqrt[1 - I*a*x]*(I + a*x)^2)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

method	result
gosper	$\frac{(-ax+i)(ax+i)(ax+4i)(iax+1)^4}{15a(a^2x^2+1)^{\frac{7}{2}}}$
trager	$\frac{-a^5x^5-10a^3x^3+20ix^2a^2+15ax-4i}{15(a^2x^2+1)^{\frac{5}{2}}a}$
meijerg	$\frac{x(8a^4x^4+20a^2x^2+15)}{15(a^2x^2+1)^{\frac{5}{2}}}+\frac{16i\left(\frac{3\sqrt{\pi}}{4}-\frac{3\sqrt{\pi}}{4\left(a^2x^2+1\right)^{\frac{5}{2}}}\right)}{15a\sqrt{\pi}}-\frac{2a^2x^3(2a^2x^2+5)}{5(a^2x^2+1)^{\frac{5}{2}}}-\frac{16i\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}\left(20a^2x^2+8\right)}{16\left(a^2x^2+1\right)^{\frac{5}{2}}}\right)}{15a\sqrt{\pi}}+\frac{a^4x^5}{5(a^2x^2+1)^{\frac{5}{2}}}$
default	$\frac{x}{5(a^2x^2+1)^{\frac{5}{2}}}+\frac{4x}{15(a^2x^2+1)^{\frac{3}{2}}}+\frac{8x}{15\sqrt{a^2x^2+1}}+a^4\left(-\frac{x^3}{2a^2(a^2x^2+1)^{\frac{5}{2}}}+\frac{-\frac{3x}{8a^2\left(a^2x^2+1\right)^{\frac{5}{2}}}+\frac{3\left(\frac{x}{5\left(a^2x^2+1\right)^{\frac{5}{2}}}+\frac{4x}{15\left(a^2x^2+1\right)^{\frac{3}{2}}}\right.}{8a^2}}{a^2}\right.$

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(I-a*x)*(I+a*x)*(a*x+4*I)*(1+I*a*x)^4/a/(a^2*x^2+1)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{a^3x^3+3i\,a^2x^2-3\,ax+(a^2x^2+3i\,ax+4)\sqrt{a^2x^2+1}-i}{15\,(a^4x^3+3i\,a^3x^2-3\,a^2x-i\,a)}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(a^3*x^3 + 3*I*a^2*x^2 - 3*a*x + (a^2*x^2 + 3*I*a*x + 4)*sqrt(a^2*x^2
 + 1) - I)/(a^4*x^3 + 3*I*a^3*x^2 - 3*a^2*x - I*a)
```

Sympy [F]

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \int \frac{(ax-i)^4}{(a^2x^2+1)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**(7/2),x)
```

```
[Out] Integral((a*x - I)**4/(a**2*x**2 + 1)**(7/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(43) = 86$.

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{a^2x^3}{2\,(a^2x^2+1)^{\frac{5}{2}}} - \frac{x}{15\,\sqrt{a^2x^2+1}} + \frac{4i\,ax^2}{3\,(a^2x^2+1)^{\frac{5}{2}}} - \frac{x}{30\,(a^2x^2+1)^{\frac{3}{2}}} + \frac{11\,x}{10\,(a^2x^2+1)^{\frac{5}{2}}} - \frac{4i}{15\,(a^2x^2+1)^{\frac{5}{2}}a}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x^3/(a^2*x^2 + 1)^(5/2) - 1/15*x/sqrt(a^2*x^2 + 1) + 4/3*I*a*x^2/(
a^2*x^2 + 1)^(5/2) - 1/30*x/(a^2*x^2 + 1)^(3/2) + 11/10*x/(a^2*x^2 + 1)^(5/
2) - 4/15*I/((a^2*x^2 + 1)^(5/2)*a)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.66

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{2\left(4\,a^4 - 25\,a^2\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2 + 15i\,a\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3 + 15\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^4 - 5\right.}{15\left(i\,a+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^5}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(4*a^4 - 25*a^2*(sqrt(a^2 + 1/x^2) - 1/x)^2 + 15*I*a*(sqrt(a^2 + 1/x^2
) - 1/x)^3 + 15*(sqrt(a^2 + 1/x^2) - 1/x)^4 - 5*a^3*(I*sqrt(a^2 + 1/x^2) -
I/x))/(I*a + sqrt(a^2 + 1/x^2) - 1/x)^5
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(a^2\,x^2\,1\mathrm{i} - 3\,a\,x + 4\mathrm{i}\right)}{15\,a\,\left(-1+a\,x\,1\mathrm{i}\right)^3}$$

```
[In] int((a*x*1i + 1)^4/(a^2*x^2 + 1)^(7/2),x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*(a^2*x^2*1i - 3*a*x + 4i))/(15*a*(a*x*1i - 1)^3)
```

3.321 $\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i}{2a(1-iax)^2}$$

```
[Out] -1/2*I/a/(1-I*a*x)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5181, 32}

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i}{2a(1-iax)^2}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] (-1/2*I)/(a*(1 - I*a*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> 
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \int \frac{1}{(1-iax)^3}\,dx$$
$$= -\frac{i}{2a(1-iax)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2a(i+ax)^2}$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
[Out] (I/2)/(a*(I + a*x)^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	s
default	$\frac{i}{2a(ax+i)^2}$	1
risch	$\frac{i}{2a(ax+i)^2}$	1
norman	$\frac{x+\frac{3}{2}iax^2+\frac{1}{2}ia^3x^4}{(a^2x^2+1)^2}$	3
gosper	$-\frac{(ax+i)(iax+1)^3}{2a(a^2x^2+1)^3}$	3
parallelrisch	$\frac{ia^3x^4+3iax^2+2x}{2(a^2x^2+1)^2}$	3
meijerg	$\frac{\frac{x\sqrt{a^2}\left(3a^2x^2+5\right)}{2\left(a^2x^2+1\right)^2}+\frac{3\sqrt{a^2}\arctan(ax)}{2a}}{4\sqrt{a^2}}+\frac{3iax^2(a^2x^2+2)}{4(a^2x^2+1)^2}-\frac{3\left(-\frac{x\left(a^2\right)^{\frac{3}{2}}\left(-3a^2x^2+3\right)}{6a^2\left(a^2x^2+1\right)^2}+\frac{\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{2a^3}\right)}{4\sqrt{a^2}}-\frac{ia^3x^4}{4(a^2x^2+1)^2}$	1

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/2*I/a/(I+a*x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2\left(a^3x^2+2i\,a^2x-a\right)}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="fricas")
[Out] 1/2*I/(a^3*x^2 + 2*I*a^2*x - a)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2a^3x^2+4ia^2x-2a}$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**3,x)
[Out] I/(2*a**3*x**2 + 4*I*a**2*x - 2*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{-i\,a^2x^2-2\,ax+i}{2\left(a^5x^4+2\,a^3x^2+a\right)}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="maxima")
[Out] -1/2*(-I*a^2*x^2 - 2*a*x + I)/(a^5*x^4 + 2*a^3*x^2 + a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2\,(ax+i)^2a}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="giac")
[Out] 1/2*I/((a*x + I)^2*a)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{1\mathrm{i}}{2\,\left(a^3\,x^2+a^2\,x\,2\mathrm{i}-a\right)}$$

```
[In] int((a*x*1i + 1)^3/(a^2*x^2 + 1)^3,x)
[Out] 1i/(2*(a^2*x*2i - a + a^3*x^2))
```

3.322 $\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}} - \frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}}$$

```
[Out] -1/3*I*(1+I*a*x)^(1/2)/a/(1-I*a*x)^(3/2)-1/3*I*(1+I*a*x)^(1/2)/a/(1-I*a*x)^
(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5181, 47, 37}

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}} - \frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] ((-1/3*I)*Sqrt[1 + I*a*x])/(a*(1 - I*a*x)^(3/2)) - ((I/3)*Sqrt[1 + I*a*x])/
(a*Sqrt[1 - I*a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(1-iax)^{5/2}\sqrt{1+iax}}\,dx\\ &= -\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}+\frac{1}{3}\int \frac{1}{(1-iax)^{3/2}\sqrt{1+iax}}\,dx\\ &= -\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}-\frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx=\frac{(2-iax)\sqrt{1+iax}}{3a\sqrt{1-iax}(i+ax)}$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
[Out] ((2 - I*a*x)*Sqrt[1 + I*a*x])/(3*a*Sqrt[1 - I*a*x]*(I + a*x))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

method	result	size
trager	$\frac{a^{3} x^{3}+3 a x -2 i}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}} a}$	31
gosper	$\frac{\left(-a x +i \right) \left(a x +i \right) \left(a x +2 i \right) \left(i a x +1\right)^{2}}{3 a \left(a^{2} x^{2}+1\right)^{\frac{5}{2}}}$	45
meijerg	$\frac{x \left(2 a^{2} x^{2}+3\right)}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}+\frac{4 i \left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}\right)}{3 a \sqrt{\pi}}-\frac{a^{2} x^{3}}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}$	76
default	$\frac{x}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}+\frac{2 x}{3 \sqrt{a^{2} x^{2}+1}}-a^{2} \left(-\frac{x}{2 a^{2} \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}+\frac{\frac{x}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}+\frac{2 x}{3 \sqrt{a^{2} x^{2}+1}}}{2 a^{2}}\right)-\frac{2 i}{3 a \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}$	104

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(a^3*x^3+3*a*x-2*I)/(a^2*x^2+1)^(3/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{a^2x^2 + 2i\,ax + \sqrt{a^2x^2+1}(ax+2i) - 1}{3\left(a^3x^2 + 2i\,a^2x - a\right)}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="fricas")
[Out] 1/3*(a^2*x^2 + 2*I*a*x + sqrt(a^2*x^2 + 1)*(a*x + 2*I) - 1)/(a^3*x^2 + 2*I*
a^2*x - a)
```

Sympy [F]

$$\begin{aligned}\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx &= -\int \frac{a^2x^2}{a^4x^4\sqrt{a^2x^2+1} + 2a^2x^2\sqrt{a^2x^2+1} + \sqrt{a^2x^2+1}}\,dx \\ &\quad - \int \left(-\frac{2iax}{a^4x^4\sqrt{a^2x^2+1} + 2a^2x^2\sqrt{a^2x^2+1} + \sqrt{a^2x^2+1}}\right)dx \\ &\quad - \int \left(-\frac{1}{a^4x^4\sqrt{a^2x^2+1} + 2a^2x^2\sqrt{a^2x^2+1} + \sqrt{a^2x^2+1}}\right)dx\end{aligned}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)**(5/2),x)
[Out] -Integral(a**2*x**2/(a**4*x**4*sqrt(a**2*x**2 + 1) + 2*a**2*x**2*sqrt(a**2*
x**2 + 1) + sqrt(a**2*x**2 + 1)), x) - Integral(-2*I*a*x/(a**4*x**4*sqrt(a*
```

```
*2*x**2 + 1) + 2*a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x) -
 Integral(-1/(a**4*x**4*sqrt(a**2*x**2 + 1) + 2*a**2*x**2*sqrt(a**2*x**2 +
1) + sqrt(a**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{x}{3\sqrt{a^2x^2+1}} + \frac{2x}{3\,(a^2x^2+1)^{\frac{3}{2}}} - \frac{2i}{3\,(a^2x^2+1)^{\frac{3}{2}}a}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*x/sqrt(a^2*x^2 + 1) + 2/3*x/(a^2*x^2 + 1)^(3/2) - 2/3*I/((a^2*x^2 + 1)^
(3/2)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{2\left(2\,a^2 - 3\left(\sqrt{a^2+\frac{1}{x^2}} - \frac{1}{x}\right)^2 + 3\,a\left(-i\sqrt{a^2+\frac{1}{x^2}} + \frac{i}{x}\right)\right)}{3\left(i\,a + \sqrt{a^2+\frac{1}{x^2}} - \frac{1}{x}\right)^3}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="giac")
[Out] -2/3*(2*a^2 - 3*(sqrt(a^2 + 1/x^2) - 1/x)^2 + 3*a*(-I*sqrt(a^2 + 1/x^2) + I
/x))/(I*a + sqrt(a^2 + 1/x^2) - 1/x)^3
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\sqrt{a^2\,x^2+1}\,(-2+a\,x\,1\mathrm{i})\,1\mathrm{i}}{3\,a\,(-1+a\,x\,1\mathrm{i})^2}$$

```
[In] int((a*x*1i + 1)^2/(a^2*x^2 + 1)^(5/2),x)
[Out] ((a^2*x^2 + 1)^(1/2)*(a*x*1i - 2)*1i)/(3*a*(a*x*1i - 1)^2)
```

3.323 $\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 28

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{1}{2a(i+ax)} + \frac{\arctan(ax)}{2a}$$

```
[Out] 1/2/a/(I+a*x)+1/2*arctan(a*x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5181, 46, 209}

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\arctan(ax)}{2a} + \frac{1}{2a(ax+i)}$$

```
[In] Int[E^(I*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

```
[Out] 1/(2*a*(I + a*x)) + ArcTan[a*x]/(2*a)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(1-iax)^2(1+iax)}\,dx\\ &= \int \left(-\frac{1}{2(i+ax)^2}+\frac{1}{2\,(1+a^2x^2)}\right)dx\\ &= \frac{1}{2a(i+ax)}+\frac{1}{2}\int \frac{1}{1+a^2x^2}\,dx\\ &= \frac{1}{2a(i+ax)}+\frac{\arctan(ax)}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\frac{1}{i+ax}+\arctan(ax)}{2a}$$

```
[In] Integrate[E^(I*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
[Out] ((I + a*x)^(-1) + ArcTan[a*x])/(2*a)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{1}{2a(ax+i)}+\frac{\arctan(ax)}{2a}$	24
default	$\frac{2a^2x-2ia}{4a^2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}$	38
meijerg	$\frac{\frac{2x\sqrt{a^2}}{2a^2x^2+2}+\frac{\sqrt{a^2}\,\arctan(ax)}{a}}{2\sqrt{a^2}}+\frac{ia\,x^2}{2a^2x^2+2}$	61
parallelrisch	$-\frac{i\ln(ax-i)x^2a^2-i\ln(ax+i)x^2a^2-2ix^2a^2+i\ln(ax-i)-i\ln(ax+i)-2ax}{4(a^2x^2+1)a}$	83

```
[In] int((1+I*a*x)/(a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2/a/(I+a*x)+1/2*arctan(a*x)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{(i\,ax-1)\log\left(\frac{ax+i}{a}\right)+(-i\,ax+1)\log\left(\frac{ax-i}{a}\right)+2}{4\,(a^2x+i\,a)}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="fricas")
[Out] 1/4*((I*a*x - 1)*log((a*x + I)/a) + (-I*a*x + 1)*log((a*x - I)/a) + 2)/(a^2
*x + I*a)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = i\left(-\frac{i}{2a^2x+2ia}+\frac{-\frac{\log\left(x-\frac{i}{a}\right)}{4}+\frac{\log\left(x+\frac{i}{a}\right)}{4}}{a}\right)$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**2,x)
[Out] I*(-I/(2*a**2*x + 2*I*a) + (-log(x - I/a)/4 + log(x + I/a)/4)/a)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{ax-i}{2\,(a^3x^2+a)}+\frac{\arctan(ax)}{2\,a}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="maxima")
[Out] 1/2*(a*x - I)/(a^3*x^2 + a) + 1/2*arctan(a*x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i\,\log\left(ax+i\right)}{4\,a} - \frac{i\,\log\left(ax-i\right)}{4\,a} + \frac{1}{2\,(ax+i)a}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="giac")
[Out] 1/4*I*log(a*x + I)/a - 1/4*I*log(a*x - I)/a + 1/2/((a*x + I)*a)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{1}{2\,\left(x\,a^2+a\,1\mathrm{i}\right)} + \frac{\mathrm{atan}(a\,x)}{2\,a}$$

```
[In] int((a*x*1i + 1)/(a^2*x^2 + 1)^2,x)
[Out] 1/(2*(a*1i + a^2*x)) + atan(a*x)/(2*a)
```

3.324 $\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{1}{2a(i-ax)}+\frac{\arctan(ax)}{2a}$$

```
[Out] -1/2/a/(I-a*x)+1/2*arctan(a*x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5181, 46, 209}

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\arctan(ax)}{2a}-\frac{1}{2a(-ax+i)}$$

```
[In] Int[1/(E^(I*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
[Out] -1/2*1/(a*(I - a*x)) + ArcTan[a*x]/(2*a)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(1-iax)(1+iax)^2}\,dx \\ &= \int \left(-\frac{1}{2(-i+ax)^2}+\frac{1}{2\left(1+a^2x^2\right)}\right)dx \\ &= -\frac{1}{2a(i-ax)}+\frac{1}{2}\int \frac{1}{1+a^2x^2}\,dx \\ &= -\frac{1}{2a(i-ax)}+\frac{\arctan(ax)}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{e^{-i\arctan(ax)}}{\left(1+a^2x^2\right)^{3/2}}\,dx=\frac{\frac{1}{-i+ax}+\arctan(ax)}{2a}$$

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
[Out] ((-I + a*x)^(-1) + ArcTan[a*x])/(2*a)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{1}{2a(ax-i)}+\frac{\arctan(ax)}{2a}$	24
default	$-\frac{i\ln(-ax+i)}{4a}-\frac{1}{2a(-ax+i)}+\frac{i\ln(ax+i)}{4a}$	43
parallelrisch	$\frac{i\ln(ax-i)xa-i\ln(ax+i)ax+2iax+\ln(ax-i)-\ln(ax+i)}{4(-ax+i)a}$	61

```
[In] int(1/(1+I*a*x)/(a^2*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2/a/(a*x-I)+1/2*arctan(a*x)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(22) = 44$.

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{(i\,ax+1)\log\left(\frac{ax+i}{a}\right) + (-i\,ax-1)\log\left(\frac{ax-i}{a}\right) + 2}{4\,(a^2x - i\,a)}$$

```
[In] integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*((I*a*x + 1)*log((a*x + I)/a) + (-I*a*x - 1)*log((a*x - I)/a) + 2)/(a^2
*x - I*a)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -i\left(\frac{i}{2a^2x-2ia} + \frac{\frac{\log\left(x-\frac{i}{a}\right)}{4} - \frac{\log\left(x+\frac{i}{a}\right)}{4}}{a}\right)$$

```
[In] integrate(1/(1+I*a*x)/(a**2*x**2+1),x)
```

```
[Out] -I*(I/(2*a**2*x - 2*I*a) + (log(x - I/a)/4 - log(x + I/a)/4)/a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i\,\log\left(ax+i\right)}{4\,a} - \frac{i\,\log\left(ax-i\right)}{4\,a} + \frac{1}{2\,(ax-i)a}$$

```
[In] integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="giac")
[Out] 1/4*I*log(a*x + I)/a - 1/4*I*log(a*x - I)/a + 1/2/((a*x - I)*a)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\mathrm{atan}(a\,x)}{2\,a} - \frac{1}{2\,\left(-a^2\,x+a\,1\mathrm{i}\right)}$$

```
[In] int(1/((a^2*x^2 + 1)*(a*x*1i + 1)),x)
[Out] atan(a*x)/(2*a) - 1/(2*(a*1i - a^2*x))
```

3.325 $\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}} + \frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}}$$

```
[Out] 1/3*I*(1-I*a*x)^(1/2)/a/(1+I*a*x)^(3/2)+1/3*I*(1-I*a*x)^(1/2)/a/(1+I*a*x)^(
1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5181, 47, 37}

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}} + \frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] ((I/3)*Sqrt[1 - I*a*x])/(a*(1 + I*a*x)^(3/2)) + ((I/3)*Sqrt[1 - I*a*x])/(a*
Sqrt[1 + I*a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{1-iax}(1+iax)^{5/2}}\,dx\\ &= \frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{1-iax}(1+iax)^{3/2}}\,dx\\ &= \frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}+\frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\sqrt{1-iax}(2+iax)}{3a\sqrt{1+iax}(-i+ax)}$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
[Out] (Sqrt[1 - I*a*x]*(2 + I*a*x))/(3*a*Sqrt[1 + I*a*x]*(-I + a*x))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{(-ax+i)(ax+i)(-ax+2i)}{3a(iax+1)^2\sqrt{a^2x^2+1}}$	46
default	$-\frac{\frac{i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{3a\left(x-\frac{i}{a}\right)^2}-\frac{\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{3\left(x-\frac{i}{a}\right)}}{a^2}$	93

```
[In] int(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(I-a*x)*(I+a*x)*(-a*x+2*I)/a/(1+I*a*x)^2/(a^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{a^2x^2-2i\,ax+\sqrt{a^2x^2+1}(ax-2i)-1}{3\,(a^3x^2-2i\,a^2x-a)}$$

```
[In] integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(a^2*x^2 - 2*I*a*x + sqrt(a^2*x^2 + 1)*(a*x - 2*I) - 1)/(a^3*x^2 - 2*I*
a^2*x - a)
```

Sympy [F]

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\int \frac{1}{a^2x^2\sqrt{a^2x^2+1}-2iax\sqrt{a^2x^2+1}-\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate(1/(1+I*a*x)**2/(a**2*x**2+1)**(1/2),x)
[Out] -Integral(1/(a**2*x**2*sqrt(a**2*x**2 + 1) - 2*I*a*x*sqrt(a**2*x**2 + 1) -
sqrt(a**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i\sqrt{a^2x^2+1}}{3\,(a^3x^2-2i\,a^2x-a)}+\frac{i\sqrt{a^2x^2+1}}{3i\,a^2x+3\,a}$$

```
[In] integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*I*sqrt(a^2*x^2 + 1)/(a^3*x^2 - 2*I*a^2*x - a) + I*sqrt(a^2*x^2 + 1)/(3
*I*a^2*x + 3*a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{2\left(2\,a^2-3\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2+3\,a\left(i\,\sqrt{a^2+\frac{1}{x^2}}-\frac{i}{x}\right)\right)}{3\left(-i\,a+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3}$$

```
[In] integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -2/3*(2*a^2 - 3*(sqrt(a^2 + 1/x^2) - 1/x)^2 + 3*a*(I*sqrt(a^2 + 1/x^2) - I/
x))/(-I*a + sqrt(a^2 + 1/x^2) - 1/x)^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{\sqrt{a^2\,x^2+1}\,(a\,x-2\mathrm{i})}{3\,a\,(1+a\,x\,1\mathrm{i})^2}$$

```
[In] int(1/((a^2*x^2 + 1)^(1/2)*(a*x*1i + 1)^2),x)
[Out] -((a^2*x^2 + 1)^(1/2)*(a*x - 2i))/(3*a*(a*x*1i + 1)^2)
```

3.326 $\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2a(1+iax)^2}$$

```
[Out] 1/2*I/a/(1+I*a*x)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5181, 32}

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2a(1+iax)^2}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] (I/2)/(a*(1 + I*a*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \int \frac{1}{(1+iax)^3}\,dx$$
$$= \frac{i}{2a(1+iax)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i}{2a(-i+ax)^2}$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
[Out] (-1/2*I)/(a*(-I + a*x)^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{i}{2a(ax-i)^2}$	15
default	$\frac{i}{2a(iax+1)^2}$	16
meijerg	$\frac{x(iax+2)}{2(iax+1)^2}$	20
gosper	$\frac{-ax+i}{2a(iax+1)^3}$	22
parallelrisch	$-\frac{ia\,x^2+2x}{2(-ax+i)^2}$	23
norman	$\frac{x-\frac{3}{2}ia\,x^2-\frac{1}{2}ia^3x^4}{(a^2x^2+1)^2}$	31

```
[In] int(1/(1+I*a*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*I/a/(a*x-I)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i}{2\,(a^3x^2-2i\,a^2x-a)}$$

```
[In] integrate(1/(1+I*a*x)^3,x, algorithm="fricas")
[Out] -1/2*I/(a^3*x^2 - 2*I*a^2*x - a)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i}{2a^3x^2-4ia^2x-2a}$$

```
[In] integrate(1/(1+I*a*x)**3,x)
[Out] -I/(2*a**3*x**2 - 4*I*a**2*x - 2*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2\,(i\,ax+1)^2a}$$

```
[In] integrate(1/(1+I*a*x)^3,x, algorithm="maxima")
[Out] 1/2*I/((I*a*x + 1)^2*a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2\,(i\,ax+1)^2a}$$

```
[In] integrate(1/(1+I*a*x)^3,x, algorithm="giac")
[Out] 1/2*I/((I*a*x + 1)^2*a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{1\mathrm{i}}{2\,\left(-a^3\,x^2+a^2\,x\,2\mathrm{i}+a\right)}$$

```
[In] int(1/(a*x*1i + 1)^3,x)
[Out] 1i/(2*(a + a^2*x*2i - a^3*x^2))
```

3.327 $\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}} + \frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}}$$

```
[Out] 1/5*I*(1-I*a*x)^(3/2)/a/(1+I*a*x)^(5/2)+1/15*I*(1-I*a*x)^(3/2)/a/(1+I*a*x)^
(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5181, 47, 37}

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}} + \frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}$$

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

```
[Out] ((I/5)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(5/2)) + ((I/15)*(1 - I*a*x)^(3/2)
)/(a*(1 + I*a*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{1-iax}}{(1+iax)^{7/2}}\,dx\\ &= \frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}+\frac{1}{5}\int \frac{\sqrt{1-iax}}{(1+iax)^{5/2}}\,dx\\ &= \frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}+\frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{(1-iax)^{3/2}(-4i+ax)}{15a\sqrt{1+iax}(-i+ax)^2}$$

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
[Out] ((1 - I*a*x)^(3/2)*(-4*I + a*x))/(15*a*Sqrt[1 + I*a*x]*(-I + a*x)^2)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{(-ax+i)(ax+i)(-ax+4i)\sqrt{a^2x^2+1}}{15a(iax+1)^4}$	46
default	$\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{5a\left(x-\frac{i}{a}\right)^4}-\frac{\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{15\left(x-\frac{i}{a}\right)^3}}{a^4}$	92

```
[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(I-a*x)*(I+a*x)*(-a*x+4*I)*(a^2*x^2+1)^(1/2)/a/(1+I*a*x)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{a^3x^3-3i\,a^2x^2-3\,ax+(a^2x^2-3i\,ax+4)\sqrt{a^2x^2+1}+i}{15\,(a^4x^3-3i\,a^3x^2-3\,a^2x+i\,a)}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/15*(a^3*x^3 - 3*I*a^2*x^2 - 3*a*x + (a^2*x^2 - 3*I*a*x + 4)*sqrt(a^2*x^2
 + 1) + I)/(a^4*x^3 - 3*I*a^3*x^2 - 3*a^2*x + I*a)
```

Sympy [F]

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(ax-i)^4}\,dx$$

```
[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**(1/2),x)
[Out] Integral(sqrt(a**2*x**2 + 1)/(a*x - I)**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(43) = 86$.

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.48

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{2i\sqrt{a^2x^2+1}}{-5i\,a^4x^3-15\,a^3x^2+15i\,a^2x+5\,a} + \frac{i\sqrt{a^2x^2+1}}{15\,(a^3x^2-2i\,a^2x-a)} - \frac{i\sqrt{a^2x^2+1}}{15i\,a^2x+15\,a}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*sqrt(a^2*x^2 + 1)/(-5*I*a^4*x^3 - 15*a^3*x^2 + 15*I*a^2*x + 5*a) + 1/15
*I*sqrt(a^2*x^2 + 1)/(a^3*x^2 - 2*I*a^2*x - a) - I*sqrt(a^2*x^2 + 1)/(15*I*
a^2*x + 15*a)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(43) = 86.

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.66

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{2\left(4\,a^4-25\,a^2\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2-15i\,a\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3+15\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^4-5\right.}{15\left(-i\,a+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^5}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(4*a^4 - 25*a^2*(sqrt(a^2 + 1/x^2) - 1/x)^2 - 15*I*a*(sqrt(a^2 + 1/x^2
) - 1/x)^3 + 15*(sqrt(a^2 + 1/x^2) - 1/x)^4 - 5*a^3*(-I*sqrt(a^2 + 1/x^2) +
 I/x))/(-I*a + sqrt(a^2 + 1/x^2) - 1/x)^5
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(a^2\,x^2-a\,x\,3\mathrm{i}+4\right)\,1\mathrm{i}}{15\,a\,{\left(1+a\,x\,1\mathrm{i}\right)}^3}$$

```
[In] int((a^2*x^2 + 1)^(1/2)/(a*x*1i + 1)^4,x)
```

```
[Out] ((a^2*x^2 + 1)^(1/2)*(a^2*x^2 - a*x*3i + 4)*1i)/(15*a*(a*x*1i + 1)^3)
```

3.328 $\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{3ac(i+ax)^3\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}}{2ac(i+ax)^2\sqrt{c+a^2cx^2}}$$

```
[Out] -2/3*(a^2*x^2+1)^(1/2)/a/c/(I+a*x)^3/(a^2*c*x^2+c)^(1/2)-1/2*I*(a^2*x^2+1)^
(1/2)/a/c/(I+a*x)^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5184, 5181, 45}

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{i\sqrt{a^2x^2+1}}{2ac(ax+i)^2\sqrt{a^2cx^2+c}} - \frac{2\sqrt{a^2x^2+1}}{3ac(ax+i)^3\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^((5*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*c*(I + a*x)^3*Sqrt[c + a^2*c*x^2]) - ((I/2)*Sqr
t[1 + a^2*x^2])/(a*c*(I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int\frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int\frac{1+iax}{(1-iax)^4}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int\left(\frac{2}{(i+ax)^4}+\frac{i}{(i+ax)^3}\right)dx}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{2\sqrt{1+a^2x^2}}{3ac(i+ax)^3\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}}{2ac(i+ax)^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int\frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{i(-i+3ax)\sqrt{1+a^2x^2}}{6ac(i+ax)^3\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^((5*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
[Out] ((-1/6*I)*(-I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a*c*(I + a*x)^3*Sqrt[c + a^2*c*x
^2])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{\sqrt{a^{2}x^{2}+1}\,\left(-\frac{ix}{2}-\frac{1}{6a}\right)}{c\sqrt{c\left(a^{2}x^{2}+1\right)}\,\left(ax+i\right)^{3}}$	47
default	$-\frac{\sqrt{c\left(a^{2}x^{2}+1\right)}\,\left(3iax+1\right)}{6\sqrt{a^{2}x^{2}+1}\,c^{2}a\left(ax+i\right)^{3}}$	48
gosper	$-\frac{\left(ax+i\right)\left(-3ax+i\right)\left(iax+1\right)^{5}}{6a\left(-ax+i\right)\left(a^{2}x^{2}+1\right)^{\frac{5}{2}}\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$	60

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/c*(a^2*x^2+1)^(1/2)/(c*(a^2*x^2+1))^(1/2)*(-1/2*I*x-1/6/a)/(I+a*x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(i\,a^2x^3-3\,ax^2-6i\,x)\sqrt{a^2x^2+1}}{6\,(a^5c^2x^5+3i\,a^4c^2x^4-2\,a^3c^2x^3+2i\,a^2c^2x^2-3\,ac^2x-i\,c^2)}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*sqrt(a^2*c*x^2 + c)*(I*a^2*x^3 - 3*a*x^2 - 6*I*x)*sqrt(a^2*x^2 + 1)/(a^
5*c^2*x^5 + 3*I*a^4*c^2*x^4 - 2*a^3*c^2*x^3 + 2*I*a^2*c^2*x^2 - 3*a*c^2*x -
 I*c^2)
```

Sympy [F]

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = i\Bigg(\int \Bigg(-\frac{i}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+}}$$
$$+\int \frac{5ax}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+}}$$
$$+\int \Bigg(-\frac{10a^3x^3}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2}}$$
$$+\int \frac{a^5x^5}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+}}$$
$$+\int \frac{10ia^2x^2}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+}}$$
$$+\int \Bigg(-\frac{5ia^4x^4}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2}}$$

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] I*(Integral(-I/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a
**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt(a
**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**
2 + c)), x) + Integral(5*a*x/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x
**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2
*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*s
qrt(a**2*c*x**2 + c)), x) + Integral(-10*a**3*x**3/(a**6*c*x**6*sqrt(a**2*x
**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**
2*c*x**2 + c) + 3*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c
*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a**5*x**5/(a**6*
c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*
x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a*
*2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integra
l(10*I*a**2*x**2/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3
*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt
(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x
**2 + c)), x) + Integral(-5*I*a**4*x**4/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sq
rt(a**2*c*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 +
c) + 3*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*
x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F]

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)^5}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((I*a*x + 1)^5/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)^5}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^5/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.51

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{c\,(a^2\,x^2+1)}\,(3\,a\,x-\mathrm{i})}{6\,a\,c^2\,\sqrt{a^2\,x^2+1}\,(-1+a\,x\,1\mathrm{i})^3}$$

```
[In] int((a*x*1i + 1)^5/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(5/2)),x)
```

```
[Out] -((c*(a^2*x^2 + 1))^(1/2)*(3*a*x - 1i))/(6*a*c^2*(a^2*x^2 + 1)^(1/2)*(a*x*1
i - 1)^3)
```

3.329 $\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{ic(1+iax)^4}{3a\,(c+a^2cx^2)^{5/2}} + \frac{ic(1+iax)^5}{15a\,(c+a^2cx^2)^{5/2}}$$

```
[Out] -1/3*I*c*(1+I*a*x)^4/a/(a^2*c*x^2+c)^(5/2)+1/15*I*c*(1+I*a*x)^5/a/(a^2*c*x^
2+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5183, 673, 665}

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{ic(1+iax)^5}{15a\,(a^2cx^2+c)^{5/2}} - \frac{ic(1+iax)^4}{3a\,(a^2cx^2+c)^{5/2}}$$

```
[In] Int[E^((4*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((-1/3*I)*c*(1 + I*a*x)^4)/(a*(c + a^2*c*x^2)^(5/2)) + ((I/15)*c*(1 + I*a*x
)^5)/(a*(c + a^2*c*x^2)^(5/2))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 5183

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^(I*(n/2)), Int[(c + d*x^2)^(p + I*(n/2))/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[I*(n/2), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c^2\int \frac{(1+iax)^4}{(c+a^2cx^2)^{7/2}}\,dx\\ &= -\frac{ic(1+iax)^4}{3a\,(c+a^2cx^2)^{5/2}}-\frac{1}{3}c^2\int \frac{(1+iax)^5}{(c+a^2cx^2)^{7/2}}\,dx\\ &= -\frac{ic(1+iax)^4}{3a\,(c+a^2cx^2)^{5/2}}+\frac{ic(1+iax)^5}{15a\,(c+a^2cx^2)^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{(1+iax)^{3/2}(4i+ax)\sqrt{1+a^2x^2}}{15ac\sqrt{1-iax}(i+ax)^2\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^((4*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 + I*a*x)^(3/2)*(4*I + a*x)*Sqrt[1 + a^2*x^2])/(15*a*c*Sqrt[1 - I*a*x]*(
I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

method	result
gosper	$\frac{(-ax+i)(ax+i)(ax+4i)(iax+1)^4}{15a(a^2x^2+1)^2(a^2c\,x^2+c)^{\frac{3}{2}}}$
trager	$\frac{(-a^5x^5-10a^3x^3+20ix^2a^2+15ax-4i)\sqrt{a^2c\,x^2+c}}{15c^2(a^2x^2+1)^3a}$
default	$\frac{x}{c\sqrt{a^2c\,x^2+c}}+\frac{2\left(i\sqrt{-a^2}-a\right)\left(\frac{1}{5c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c-2c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}}+\frac{3a^2\left(\frac{1}{3c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right.}}\right.}{}\right.}{a^3}$

[In] int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/15*(I-a*x)*(I+a*x)*(a*x+4*I)*(1+I*a*x)^4/a/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{a^2cx^2+c}(a^2x^2+3i\,ax+4)}{15\,(a^4c^2x^3+3i\,a^3c^2x^2-3\,a^2c^2x-i\,ac^2)}$$

[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] -1/15*sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 3*I*a*x + 4)/(a^4*c^2*x^3 + 3*I*a^3*c^2*x^2 - 3*a^2*c^2*x - I*a*c^2)

Sympy [F]

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(ax-i)^4}{(c\,(a^2x^2+1))^{\frac{3}{2}}\,(a^2x^2+1)^2}\,dx$$

[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2/(a**2*c*x**2+c)**(3/2),x)

[Out] Integral((a*x - I)**4/((c*(a**2*x**2 + 1))**(3/2)*(a**2*x**2 + 1)**2), x)

Maxima [F]

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)^4}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)^2}\,dx$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((I*a*x + 1)^4/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^2), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(53) = 106.

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.94

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2\left(15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^3\sqrt{c}-5i\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2c-5\left(\sqrt{a^2c}x-\sqrt{a^2 \cdots\right.\right.}{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}+i\sqrt{c}\right)^5ac}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] 2/15*(15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^3*sqrt(c) - 5*I*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 + c))^2*c - 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))*c^(
3/2) - I*c^2)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c) + I*sqrt(c))^5*a*c)
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c\,(a^2\,x^2+1)}\,(a^2\,x^2\,1\mathrm{i}-3\,a\,x+4\mathrm{i})}{15\,a\,c^2\,(-1+a\,x\,1\mathrm{i})^3}$$

```
[In] int((a*x*1i + 1)^4/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^2),x)
```

```
[Out] ((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2*1i - 3*a*x + 4i))/(15*a*c^2*(a*x*1i - 1)^
3)
```

3.330 $\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 49

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{i\sqrt{1+a^2x^2}}{2ac(1-iax)^2\sqrt{c+a^2cx^2}}$$

```
[Out] -1/2*I*(a^2*x^2+1)^(1/2)/a/c/(1-I*a*x)^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5184, 5181, 32}

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{i\sqrt{a^2x^2+1}}{2ac(1-iax)^2\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^((3*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + a^2*x^2])/(a*c*(1 - I*a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
```

```
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1-iax)^3}\,dx}{c\sqrt{c+a^2cx^2}}$$
$$= -\frac{i\sqrt{1+a^2x^2}}{2ac(1-iax)^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{i\sqrt{1+a^2x^2}}{2ac(i+ax)^2\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^((3*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
[Out] ((I/2)*Sqrt[1 + a^2*x^2])/(a*c*(I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{i\sqrt{c(a^2x^2+1)}}{2\sqrt{a^2x^2+1}\,c^2a(ax+i)^2}$	42
risch	$\frac{i\sqrt{a^2x^2+1}}{2c\sqrt{c(a^2x^2+1)}\,a(ax+i)^2}$	42
gosper	$-\frac{(ax+i)(iax+1)^3}{2a(a^2x^2+1)^{\frac{3}{2}}(a^2c\,x^2+c)^{\frac{3}{2}}}$	44

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/2*I/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c^2/a/(I+a*x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}(i\,ax^2-2\,x)}{2\,(a^4c^2x^4+2i\,a^3c^2x^3+2i\,ac^2x-c^2)}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(I*a*x^2 - 2*x)/(a^4*c^2*x^4 + 2*
I*a^3*c^2*x^3 + 2*I*a*c^2*x - c^2)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -i\left(\int \frac{i}{a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx + \int \left(-\frac{3ax}{a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int \frac{a^3x^3}{a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx + \int \left(-\frac{3ia^2x^2}{a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx\right)$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -I*(Integral(I/(a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a
**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1
)*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*a*x/(a**4*c*x**4*sqrt(a**2*x**2
+ 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*
x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a**
3*x**3/(a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x*
*2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a
**2*c*x**2 + c)), x) + Integral(-3*I*a**2*x**2/(a**4*c*x**4*sqrt(a**2*x**2
+ 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*
x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)^3}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((I*a*x + 1)^3/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c\,(a^2\,x^2+1)}\,1\mathrm{i}}{2\,a\,c^2\,\sqrt{a^2\,x^2+1}\,(a\,x+1\mathrm{i})^2}$$

```
[In] int((a*x*1i + 1)^3/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] ((c*(a^2*x^2 + 1))^(1/2)*1i)/(2*a*c^2*(a^2*x^2 + 1)^(1/2)*(a*x + 1i)^2)
```

3.331 $\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2i(1+iax)}{3a\,(c+a^2cx^2)^{3/2}} + \frac{x}{3c\sqrt{c+a^2cx^2}}$$

```
[Out] -2/3*I*(1+I*a*x)/a/(a^2*c*x^2+c)^(3/2)+1/3*x/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5183, 667, 197}

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x}{3c\sqrt{a^2cx^2+c}} - \frac{2i(1+iax)}{3a\,(a^2cx^2+c)^{3/2}}$$

```
[In] Int[E^((2*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (((-2*I)/3)*(1 + I*a*x))/(a*(c + a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c + a^2*c*
x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
```

```
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5183

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[1/c^(I*(n/2)), Int[(c + d*x^2)^(p + I*(n/2))/(1 + I*a*x)^(I*n), x], x] /
; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &
& ILtQ[I*(n/2), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\int \frac{(1+iax)^2}{(c+a^2cx^2)^{5/2}}\,dx \\
&= -\frac{2i(1+iax)}{3a\,(c+a^2cx^2)^{3/2}}+\frac{1}{3}\int \frac{1}{(c+a^2cx^2)^{3/2}}\,dx \\
&= -\frac{2i(1+iax)}{3a\,(c+a^2cx^2)^{3/2}}+\frac{x}{3c\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{(2-iax)\sqrt{1+iax}\sqrt{1+a^2x^2}}{3ac\sqrt{1-iax}(i+ax)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^((2*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
[Out] ((2 - I*a*x)*Sqrt[1 + I*a*x]*Sqrt[1 + a^2*x^2])/(3*a*c*Sqrt[1 - I*a*x]*(I +
 a*x)*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result
trager	$\frac{(a^3x^3+3ax-2i)\sqrt{a^2c\,x^2+c}}{3c^2(a^2x^2+1)^2a}$
gosper	$\frac{(-ax+i)(ax+i)(ax+2i)(iax+1)^2}{3a(a^2x^2+1)(a^2c\,x^2+c)^{\frac{3}{2}}}$
default	$-\frac{x}{c\sqrt{a^2c\,x^2+c}}+\frac{\left(i\sqrt{-a^2}+a\right)\left(-\frac{1}{3c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c+2c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}}-\frac{2\left(x-\frac{\sqrt{-a^2}}{a^2}\right)a^2c+2c\sqrt{-}}{3c^2\sqrt{-a^2}\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c+2c\sqrt{-a}}}\right.}{a\sqrt{-a^2}}$

[In] int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3/c^2*(a^3*x^3+3*a*x-2*I)/(a^2*x^2+1)^2/a*(a^2*c*x^2+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax+2i)}{3\left(a^3c^2x^2+2i\,a^2c^2x-ac^2\right)}$$

[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)

[Out] 1/3*sqrt(a^2*c*x^2 + c)*(a*x + 2*I)/(a^3*c^2*x^2 + 2*I*a^2*c^2*x - a*c^2)

Sympy [F]

$$\begin{aligned}\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = &-\int \frac{a^2x^2}{a^4cx^4\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2cx^2+c}+c\sqrt{a^2cx^2+c}}\,dx\\ &-\int \left(-\frac{2iax}{a^4cx^4\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2cx^2+c}+c\sqrt{a^2cx^2+c}}\right)dx\\ &-\int \left(-\frac{1}{a^4cx^4\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2cx^2+c}+c\sqrt{a^2cx^2+c}}\right)dx\end{aligned}$$

[In] integrate((1+I*a*x)**2/(a**2*x**2+1)/(a**2*c*x**2+c)**(3/2),x)

[Out] -Integral(a**2*x**2/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*c*x**2 + c)), x) - Integral(-2*I*a*x/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*c*x**2 + c)), x) - Integral(-1/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*c*x**2 + c)), x)

Maxima [F]

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)^2}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)}\,dx$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
[Out] integrate((I*a*x + 1)^2/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2\sqrt{a^2c}\left(3\sqrt{a^2c}x-3\sqrt{a^2cx^2+c}+i\sqrt{c}\right)}{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}+i\sqrt{c}\right)^3a^2c}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] -2/3*sqrt(a^2*c)*(3*sqrt(a^2*c)*x - 3*sqrt(a^2*c*x^2 + c) + I*sqrt(c))/((sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 + c) + I*sqrt(c))^3*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{a^3\,x^3+3\,a\,x-2\mathrm{i}}{3\,a\,\left(c\,\left(a^2\,x^2+1\right)\right)^{3/2}}$$

```
[In] int((a*x*1i + 1)^2/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)),x)
[Out] (3*a*x + a^3*x^3 - 2i)/(3*a*(c*(a^2*x^2 + 1))^(3/2))
```

3.332 $\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}}{2ac(i+ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\arctan(ax)}{2ac\sqrt{c+a^2cx^2}}$$

```
[Out] 1/2*(a^2*x^2+1)^(1/2)/a/c/(I+a*x)/(a^2*c*x^2+c)^(1/2)+1/2*arctan(a*x)*(a^2*
x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5184, 5181, 46, 209}

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2x^2+1}\arctan(ax)}{2ac\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}}{2ac(ax+i)\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^(I*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 + a^2*x^2]/(2*a*c*(I + a*x)*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2
]*ArcTan[a*x])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1-iax)^2(1+iax)}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{1}{2(i+ax)^2}+\frac{1}{2(1+a^2x^2)}\right)dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}}{2ac(i+ax)\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\int \frac{1}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}}{2ac(i+ax)\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\arctan(ax)}{2ac\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(\frac{1}{i+ax}+\arctan(ax)\right)}{2ac\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(I*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
[Out] (Sqrt[1 + a^2*x^2]*((I + a*x)^(-1) + ArcTan[a*x]))/(2*a*c*Sqrt[c + a^2*c*x^
2])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(-\arctan(ax)a^2x^2-ax+i-\arctan(ax))}{2(a^2x^2+1)^{\frac{3}{2}}\,a\,c^2}$	58
risch	$\frac{\sqrt{a^2x^2+1}}{2c\sqrt{c(a^2x^2+1)}\,a(ax+i)}-\frac{i\sqrt{a^2x^2+1}\,\ln(-iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a}+\frac{i\sqrt{a^2x^2+1}\,\ln(iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a}$	124

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*(c*(a^2*x^2+1))^(1/2)*(-arctan(a*x)*a^2*x^2-a*x+I-arctan(a*x))/(a^2*x^
2+1)^(3/2)/a/c^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.60

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(i\,a^3c^2x^3-a^2c^2x^2+i\,ac^2x-c^2)\sqrt{\frac{1}{a^2c^3}}\log\left(\frac{2\left(2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}a^6x-(i\,a^{10}c^2x^4-i\,a^6c^2)\sqrt{\frac{1}{a^2c^3}}\cdots\right.}{a^4x^4+2\,a^2x^2+1}\cdots\right.}{\cdots}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/8*((I*a^3*c^2*x^3 - a^2*c^2*x^2 + I*a*c^2*x - c^2)*sqrt(1/(a^2*c^3))*log(
2*(2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^6*x - (I*a^10*c^2*x^4 - I*a^6*
c^2)*sqrt(1/(a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + (-I*a^3*c^2*x^3 + a^2*
c^2*x^2 - I*a*c^2*x + c^2)*sqrt(1/(a^2*c^3))*log(2*(2*sqrt(a^2*c*x^2 + c)*s
qrt(a^2*x^2 + 1)*a^6*x - (-I*a^10*c^2*x^4 + I*a^6*c^2)*sqrt(1/(a^2*c^3)))/(
a^4*x^4 + 2*a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*x)/(a
^3*c^2*x^3 + I*a^2*c^2*x^2 + a*c^2*x + I*c^2)
```

Sympy [F]

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = i\left(\int \left(-\frac{i}{a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int \frac{ax}{a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx\right)$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] I*(Integral(-I/(a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*s
qrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a*x/(a**2*c*x**2*s
qrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*
c*x**2 + c)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i \arctan(ax)}}{(c + a^2cx^2)^{3/2}}\, dx = \int \frac{1 + a\,x\,1\mathrm{i}}{(c\,a^2\,x^2 + c)^{3/2}\,\sqrt{a^2\,x^2 + 1}}\, dx$$

```
[In] int((a*x*1i + 1)/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int((a*x*1i + 1)/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)), x)
```

3.333 $\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{1+a^2x^2}}{2ac(i-ax)\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\arctan(ax)}{2ac\sqrt{c+a^2cx^2}}$$

```
[Out] -1/2*(a^2*x^2+1)^(1/2)/a/c/(I-a*x)/(a^2*c*x^2+c)^(1/2)+1/2*arctan(a*x)*(a^2
*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5184, 5181, 46, 209}

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2x^2+1}\arctan(ax)}{2ac\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2x^2+1}}{2ac(-ax+i)\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^2]/(a*c*(I - a*x)*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*
x^2]*ArcTan[a*x])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1-iax)(1+iax)^2}\,dx}{c\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \left(-\frac{1}{2(-i+ax)^2}+\frac{1}{2(1+a^2x^2)}\right)dx}{c\sqrt{c+a^2cx^2}} \\
&= -\frac{\sqrt{1+a^2x^2}}{2ac(i-ax)\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\int \frac{1}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}} \\
&= -\frac{\sqrt{1+a^2x^2}}{2ac(i-ax)\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\arctan(ax)}{2ac\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(-\frac{1}{2a(i-ax)}+\frac{\arctan(ax)}{2a}\right)}{c\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
[Out] (Sqrt[1 + a^2*x^2]*(-1/2*1/(a*(I - a*x)) + ArcTan[a*x]/(2*a)))/(c*Sqrt[c +
a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{c(a^2x^2+1)}\,(i\ln(-ax+i)ax-i\ln(ax+i)ax+\ln(-ax+i)-\ln(ax+i)-2)}{4\sqrt{a^2x^2+1}\,c^2(-ax+i)a}$	86
risch	$\frac{\sqrt{a^2x^2+1}}{2c\sqrt{c(a^2x^2+1)}\,a(ax-i)}+\frac{i\sqrt{a^2x^2+1}\,\ln(iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a}-\frac{i\sqrt{a^2x^2+1}\,\ln(-iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a}$	124

```
[In] int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(I-a*x)*a*x-I*ln(I+a*x)*a*
x+ln(I-a*x)-ln(I+a*x)-2)/c^2/(I-a*x)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.56

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(i\,a^3c^2x^3+a^2c^2x^2+i\,ac^2x+c^2)\sqrt{\frac{1}{a^2c^3}}\log\left(\frac{2\left(2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}a^6x-(i\,a^{10}c^2x^4-i\,a^6c^2)\sqrt{\frac{1}{a^2c^3}}\cdots\right)}{a^4x^4+2\,a^2x^2+1}\cdots\right.}{\cdots}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/8*((I*a^3*c^2*x^3 + a^2*c^2*x^2 + I*a*c^2*x + c^2)*sqrt(1/(a^2*c^3))*log(
2*(2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^6*x - (I*a^10*c^2*x^4 - I*a^6*
c^2)*sqrt(1/(a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + (-I*a^3*c^2*x^3 - a^2*
c^2*x^2 - I*a*c^2*x - c^2)*sqrt(1/(a^2*c^3))*log(2*(2*sqrt(a^2*c*x^2 + c)*s
qrt(a^2*x^2 + 1)*a^6*x - (-I*a^10*c^2*x^4 + I*a^6*c^2)*sqrt(1/(a^2*c^3)))/(
a^4*x^4 + 2*a^2*x^2 + 1)) - 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*x)/(a
^3*c^2*x^3 - I*a^2*c^2*x^2 + a*c^2*x - I*c^2)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{a^3cx^3\sqrt{a^2cx^2+c}-ia^2cx^2\sqrt{a^2cx^2+c}+acx\sqrt{a^2cx^2+c}-ic\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -I*Integral(sqrt(a**2*x**2 + 1)/(a**3*c*x**3*sqrt(a**2*c*x**2 + c) - I*a**2
*c*x**2*sqrt(a**2*c*x**2 + c) + a*c*x*sqrt(a**2*c*x**2 + c) - I*c*sqrt(a**2
*c*x**2 + c)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}}{2\,(a^2c^2x-i\,ac^2)} - \frac{i\,\log{(ax-i)}}{4\,ac^{\frac{3}{2}}} + \frac{i\,\log{(i\,ax-1)}}{4\,ac^{\frac{3}{2}}}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] 1/2*sqrt(c)/(a^2*c^2*x - I*a*c^2) - 1/4*I*log(a*x - I)/(a*c^(3/2)) + 1/4*I*
log(I*a*x - 1)/(a*c^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{a^2\,x^2+1}}{\left(c\,a^2\,x^2+c\right)^{3/2}\,\left(1+a\,x\,1\mathrm{i}\right)}\,dx$$

```
[In] int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)),x)
[Out] int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)), x)
```

3.334 $\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2i(1-iax)}{3a\,(c+a^2cx^2)^{3/2}} + \frac{x}{3c\sqrt{c+a^2cx^2}}$$

```
[Out] 2/3*I*(1-I*a*x)/a/(a^2*c*x^2+c)^(3/2)+1/3*x/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5182, 667, 197}

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x}{3c\sqrt{a^2cx^2+c}} + \frac{2i(1-iax)}{3a\,(a^2cx^2+c)^{3/2}}$$

```
[In] Int[1/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] (((2*I)/3)*(1 - I*a*x))/(a*(c + a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c + a^2*c*x
^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
```

```
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 5182

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^(I*(n/2)), Int[(c + d*x^2)^(p - I*(n/2))*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[I*(n/2), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\int \frac{(1-iax)^2}{(c+a^2cx^2)^{5/2}}\,dx\\ &= \frac{2i(1-iax)}{3a\,(c+a^2cx^2)^{3/2}}+\frac{1}{3}\int \frac{1}{(c+a^2cx^2)^{3/2}}\,dx\\ &= \frac{2i(1-iax)}{3a\,(c+a^2cx^2)^{3/2}}+\frac{x}{3c\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{1-iax}(2+iax)\sqrt{1+a^2x^2}}{3ac\sqrt{1+iax}(-i+ax)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
[Out] (Sqrt[1 - I*a*x]*(2 + I*a*x)*Sqrt[1 + a^2*x^2])/(3*a*c*Sqrt[1 + I*a*x]*(-I
+ a*x)*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{(-ax+i)(ax+i)(-ax+2i)\left(a^{2}x^{2}+1\right)}{3a(iax+1)^{2}\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$	56
default	$-\frac{x}{c\sqrt{a^{2}c\,x^{2}+c}}-\frac{2i\left(\frac{i}{3ac\left(x-\frac{i}{a}\right)\sqrt{\left(x-\frac{i}{a}\right)^{2}a^{2}c+2iac\left(x-\frac{i}{a}\right)}}+\frac{i\left(2\left(x-\frac{i}{a}\right)a^{2}c+2iac\right)}{3a\,c^{2}\sqrt{\left(x-\frac{i}{a}\right)^{2}a^{2}c+2iac\left(x-\frac{i}{a}\right)}}\right)}{a}$	137

```
[In] int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(I-a*x)*(I+a*x)*(-a*x+2*I)*(a^2*x^2+1)/a/(1+I*a*x)^2/(a^2*c*x^2+c)^(3/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax-2i)}{3\left(a^3c^2x^2-2i\,a^2c^2x-ac^2\right)}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
[Out] 1/3*sqrt(a^2*c*x^2 + c)*(a*x - 2*I)/(a^3*c^2*x^2 - 2*I*a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\int \frac{a^2x^2}{a^4cx^4\sqrt{a^2cx^2+c}-2ia^3cx^3\sqrt{a^2cx^2+c}-2iacx\sqrt{a^2cx^2+c}-c\sqrt{a^2cx^2+c}}\,dx - \int \frac{1}{a^4cx^4\sqrt{a^2cx^2+c}-2ia^3cx^3\sqrt{a^2cx^2+c}-2iacx\sqrt{a^2cx^2+c}-c\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**(3/2),x)
[Out] -Integral(a**2*x**2/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*I*a**3*c*x**3*sq
rt(a**2*c*x**2 + c) - 2*I*a*c*x*sqrt(a**2*c*x**2 + c) - c*sqrt(a**2*c*x**2
+ c)), x) - Integral(1/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*I*a**3*c*x**3
*sqrt(a**2*c*x**2 + c) - 2*I*a*c*x*sqrt(a**2*c*x**2 + c) - c*sqrt(a**2*c*x*
*2 + c)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x}{3\sqrt{a^2cx^2+c}c} + \frac{2i}{3i\sqrt{a^2cx^2+c}a^2cx+3\sqrt{a^2cx^2+c}ac}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
[Out] 1/3*x/(sqrt(a^2*c*x^2 + c)*c) + 2*I/(3*I*sqrt(a^2*c*x^2 + c)*a^2*c*x + 3*sq
rt(a^2*c*x^2 + c)*a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2\sqrt{a^2c}\left(3\sqrt{a^2c}x-3\sqrt{a^2cx^2+c}-i\sqrt{c}\right)}{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}-i\sqrt{c}\right)^3a^2c}$$

```
[In] integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -2/3*sqrt(a^2*c)*(3*sqrt(a^2*c)*x - 3*sqrt(a^2*c*x^2 + c) - I*sqrt(c))/((sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 + c) - I*sqrt(c))^3*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{a^3\,x^3+3\,a\,x+2\mathrm{i}}{3\,a\,\left(c\,\left(a^2\,x^2+1\right)\right)^{3/2}}$$

```
[In] int((a^2*x^2 + 1)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)^2),x)
```

```
[Out] (3*a*x + a^3*x^3 + 2i)/(3*a*(c*(a^2*x^2 + 1))^(3/2))
```

3.335 $\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 49

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{i\sqrt{1+a^2x^2}}{2ac(1+iax)^2\sqrt{c+a^2cx^2}}$$

```
[Out] 1/2*I*(a^2*x^2+1)^(1/2)/a/c/(1+I*a*x)^2/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5184, 5181, 32}

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{i\sqrt{a^2x^2+1}}{2ac(1+iax)^2\sqrt{a^2cx^2+c}}$$

```
[In] Int[1/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
[Out] ((I/2)*Sqrt[1 + a^2*x^2])/(a*c*(1 + I*a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
```

```
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{1}{(1+iax)^3}\,dx}{c\sqrt{c+a^2cx^2}}\\ &= \frac{i\sqrt{1+a^2x^2}}{2ac(1+iax)^2\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{i\sqrt{1+a^2x^2}}{2ac(-i+ax)^2\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
[Out] ((-1/2*I)*Sqrt[1 + a^2*x^2])/(a*c*(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{i\sqrt{a^2x^2+1}}{2c\sqrt{c(a^2x^2+1)}\,a(ax-i)^2}$	42
default	$\frac{i\sqrt{c(a^2x^2+1)}}{2\sqrt{a^2x^2+1}\,c^2a(iax+1)^2}$	43
gosper	$\frac{(-ax+i)(a^2x^2+1)^{\frac{3}{2}}}{2a(iax+1)^3(a^2c\,x^2+c)^{\frac{3}{2}}}$	45

```
[In] int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/2*I/c*(a^2*x^2+1)^(1/2)/(c*(a^2*x^2+1))^(1/2)/a/(a*x-I)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}(-i\,ax^2-2\,x)}{2\,(a^4c^2x^4-2i\,a^3c^2x^3-2i\,ac^2x-c^2)}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(-I*a*x^2 - 2*x)/(a^4*c^2*x^4 - 2
*I*a^3*c^2*x^3 - 2*I*a*c^2*x - c^2)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^5cx^5\sqrt{a^2cx^2+c}-3ia^4cx^4\sqrt{a^2cx^2+c}-2a^3cx^3\sqrt{a^2cx^2+c}-2ia^2cx^2\sqrt{a^2cx^2+ \cdots}}\right.$$

$$+\int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^5cx^5\sqrt{a^2cx^2+c}-3ia^4cx^4\sqrt{a^2cx^2+c}-2a^3cx^3\sqrt{a^2cx^2+c}-2ia^2cx^2\sqrt{a^2cx^2+c}-3acx\sqrt{a^2cx^2+c}+i \cdots}$$

```
[In] integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] I*(Integral(sqrt(a**2*x**2 + 1)/(a**5*c*x**5*sqrt(a**2*c*x**2 + c) - 3*I*a*
*4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*a**3*c*x**3*sqrt(a**2*c*x**2 + c) - 2*I
*a**2*c*x**2*sqrt(a**2*c*x**2 + c) - 3*a*c*x*sqrt(a**2*c*x**2 + c) + I*c*sq
rt(a**2*c*x**2 + c)), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**5*c*x
**5*sqrt(a**2*c*x**2 + c) - 3*I*a**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*a**3*
c*x**3*sqrt(a**2*c*x**2 + c) - 2*I*a**2*c*x**2*sqrt(a**2*c*x**2 + c) - 3*a*
c*x*sqrt(a**2*c*x**2 + c) + I*c*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{1}{2i\,a^3c^{\frac{3}{2}}x^2 + 4\,a^2c^{\frac{3}{2}}x - 2i\,ac^{\frac{3}{2}}}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] 1/(2*I*a^3*c^(3/2)*x^2 + 4*a^2*c^(3/2)*x - 2*I*a*c^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\sqrt{a^2\,x^2+1}}{2\,a\,c^2\,\left(a\,x+1{}\mathrm{i}\right)\,{\left(1+a\,x\,1{}\mathrm{i}\right)}^3}$$

```
[In] int((a^2*x^2 + 1)^(3/2)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)^3),x)
```

```
[Out] -((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2 + 1)^(1/2))/(2*a*c^2*(a*x + 1i)*(a*x*1i
+ 1)^3)
```

3.336 $\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{ic(1-iax)^4}{3a(c+a^2cx^2)^{5/2}} - \frac{ic(1-iax)^5}{15a(c+a^2cx^2)^{5/2}}$$

```
[Out] 1/3*I*c*(1-I*a*x)^4/a/(a^2*c*x^2+c)^(5/2)-1/15*I*c*(1-I*a*x)^5/a/(a^2*c*x^2
+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5182, 673, 665}

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{ic(1-iax)^4}{3a(a^2cx^2+c)^{5/2}} - \frac{ic(1-iax)^5}{15a(a^2cx^2+c)^{5/2}}$$

```
[In] Int[1/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((I/3)*c*(1 - I*a*x)^4)/(a*(c + a^2*c*x^2)^(5/2)) - ((I/15)*c*(1 - I*a*x)^5
)/(a*(c + a^2*c*x^2)^(5/2))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 5182

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Di
st[c^(I*(n/2)), Int[(c + d*x^2)^(p - I*(n/2))*(1 - I*a*x)^(I*n), x], x] /;
FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0]) &&
IGtQ[I*(n/2), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c^2 \int \frac{(1-iax)^4}{(c+a^2cx^2)^{7/2}}\,dx\\ &= \frac{ic(1-iax)^4}{3a\,(c+a^2cx^2)^{5/2}} - \frac{1}{3}c^2 \int \frac{(1-iax)^5}{(c+a^2cx^2)^{7/2}}\,dx\\ &= \frac{ic(1-iax)^4}{3a\,(c+a^2cx^2)^{5/2}} - \frac{ic(1-iax)^5}{15a\,(c+a^2cx^2)^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(1-iax)^{3/2}(-4i+ax)\sqrt{1+a^2x^2}}{15ac\sqrt{1+iax}(-i+ax)^2\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[1/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
[Out] ((1 - I*a*x)^(3/2)*(-4*I + a*x)*Sqrt[1 + a^2*x^2])/(15*a*c*Sqrt[1 + I*a*x]*
(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result
gosper	$-\frac{(-ax+i)(ax+i)(-ax+4i)(a^2x^2+1)^2}{15a(iax+1)^4(a^2cx^2+c)^{\frac{3}{2}}}$
default	$\frac{x}{c\sqrt{a^2cx^2+c}}-\frac{4\left(\frac{i}{5ac\left(x-\frac{i}{a}\right)^2\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}+\frac{3ia\left(\frac{i}{3ac\left(x-\frac{i}{a}\right)\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}+\frac{i\left(2\left(x-\frac{i}{a}\right)a^2c+2iac\right)}{3a\,c^2\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}\right)}{5}\right.}{a^2}$

[In] int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)

[Out] -1/15*(I-a*x)*(I+a*x)*(-a*x+4*I)*(a^2*x^2+1)^2/a/(1+I*a*x)^4/(a^2*c*x^2+c)^
(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{a^2cx^2+c}(a^2x^2-3i\,ax+4)}{15\,(a^4c^2x^3-3i\,a^3c^2x^2-3\,a^2c^2x+i\,ac^2)}$$

[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")

[Out] -1/15*sqrt(a^2*c*x^2 + c)*(a^2*x^2 - 3*I*a*x + 4)/(a^4*c^2*x^3 - 3*I*a^3*c^
2*x^2 - 3*a^2*c^2*x + I*a*c^2)

Sympy [F]

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(a^2x^2+1)^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\,(ax-i)^4}\,dx$$

[In] integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**(3/2),x)

[Out] Integral((a**2*x**2 + 1)**2/((c*(a**2*x**2 + 1))**(3/2)*(a*x - I)**4), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(53) = 106.

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{x}{15\sqrt{a^2cx^2+c}c} - \frac{4i}{5\left(\sqrt{a^2cx^2+c}a^3cx^2 - 2i\sqrt{a^2cx^2+c}a^2cx - \sqrt{a^2cx^2+c}ac\right)} - \frac{8i}{15i\sqrt{a^2cx^2+c}a^2cx + 15\sqrt{a^2cx^2+c}ac}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/15*x/(sqrt(a^2*c*x^2 + c)*c) - 4/5*I/(sqrt(a^2*c*x^2 + c)*a^3*c*x^2 - 2*
I*sqrt(a^2*c*x^2 + c)*a^2*c*x - sqrt(a^2*c*x^2 + c)*a*c) - 8*I/(15*I*sqrt(a
^2*c*x^2 + c)*a^2*c*x + 15*sqrt(a^2*c*x^2 + c)*a*c)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(53) = 106.

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.94

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2\left(15\left(\sqrt{a^2c}x - \sqrt{a^2cx^2+c}\right)^3\sqrt{c} + 5i\left(\sqrt{a^2c}x - \sqrt{a^2cx^2+c}\right)^2c - 5\left(\sqrt{a^2c}x - \sqrt{a^2\right.}{15\left(\sqrt{a^2c}x - \sqrt{a^2cx^2+c} - i\sqrt{c}\right)^5ac}$$

```
[In] integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] 2/15*(15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^3*sqrt(c) + 5*I*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 + c))^2*c - 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))*c^(
3/2) + I*c^2)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c) - I*sqrt(c))^5*a*c)
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\left(a^2\,x^2-a\,x\,3\mathrm{i}+4\right)\,1\mathrm{i}}{15\,a\,c^2\,{\left(1+a\,x\,1\mathrm{i}\right)}^3}$$

```
[In] int((a^2*x^2 + 1)^2/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)^4),x)
```

```
[Out] ((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2 - a*x*3i + 4)*1i)/(15*a*c^2*(a*x*1i + 1)^
3)
```

3.337 $\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{2^{3-\frac{in}{2}}c^2(1-iax)^{3+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(-2+\frac{in}{2},3+\frac{in}{2},4+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a(6i-n)}$$

```
[Out] -2^(3-1/2*I*n)*c^2*(1-I*a*x)^(3+1/2*I*n)*hypergeom([-2+1/2*I*n, 3+1/2*I*n],
[4+1/2*I*n],1/2-1/2*I*a*x)/a/(6*I-n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5181, 71}

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{c^2 2^{3-\frac{in}{2}}(1-iax)^{3+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2}-2,\frac{in}{2}+3,\frac{in}{2}+4,\frac{1}{2}(1-iax)\right)}{a(-n+6i)}$$

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

```
[Out] -((2^(3 - (I/2)*n)*c^2*(1 - I*a*x)^(3 + (I/2)*n)*Hypergeometric2F1[-2 + (I/
2)*n, 3 + (I/2)*n, 4 + (I/2)*n, (1 - I*a*x)/2])/(a*(6*I - n)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = c^2 \int (1 - iax)^{2+\frac{in}{2}} (1 + iax)^{2-\frac{in}{2}} \, dx$$
$$= -\frac{2^{3-\frac{in}{2}} c^2 (1 - iax)^{3+\frac{in}{2}} \text{Hypergeometric2F1}\left(-2 + \frac{in}{2}, 3 + \frac{in}{2}, 4 + \frac{in}{2}, \frac{1}{2}(1 - iax)\right)}{a(6i - n)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^2 \, dx = \frac{i 2^{2-\frac{in}{2}} c^2 (1 - iax)^{3+\frac{in}{2}} \text{Hypergeometric2F1}\left(-2 + \frac{in}{2}, 3 + \frac{in}{2}, 4 + \frac{in}{2}, \frac{1}{2}(1 - iax)\right)}{a \left(3 + \frac{in}{2}\right)}$$

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
[Out] (I*2^(2 - (I/2)*n)*c^2*(1 - I*a*x)^(3 + (I/2)*n)*Hypergeometric2F1[-2 + (I/
2)*n, 3 + (I/2)*n, 4 + (I/2)*n, (1 - I*a*x)/2])/(a*(3 + (I/2)*n))
```

Maple [F]

$$\int e^{n \arctan(ax)} \left(a^2 c x^2 + c\right)^2 dx$$

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x)
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^2cx^2+c\right)^2 e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=c^2\left(\int 2a^2x^2e^{n\operatorname{atan}(ax)}\,dx+\int a^4x^4e^{n\operatorname{atan}(ax)}\,dx+\int e^{n\operatorname{atan}(ax)}\,dx\right)$$

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**2,x)
[Out] c**2*(Integral(2*a**2*x**2*exp(n*atan(a*x)), x) + Integral(a**4*x**4*exp(n*
atan(a*x)), x) + Integral(exp(n*atan(a*x)), x))
```

Maxima [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^2cx^2+c\right)^2 e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^2*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^2cx^2+c\right)^2 e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^2,x)
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^2, x)
```

3.338 $\int e^{n\arctan(ax)}(c+a^2cx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int e^{n\arctan(ax)}(c+a^2cx^2)\,dx = -\frac{2^{2-\frac{in}{2}}c(1-iax)^{2+\frac{in}{2}}\operatorname{Hypergeometric2F1}\left(-1+\frac{in}{2},2+\frac{in}{2},3+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a(4i-n)}$$

```
[Out] -2^(2-1/2*I*n)*c*(1-I*a*x)^(2+1/2*I*n)*hypergeom([2+1/2*I*n, -1+1/2*I*n],[3
+1/2*I*n],1/2-1/2*I*a*x)/a/(4*I-n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5181, 71}

$$\int e^{n\arctan(ax)}(c+a^2cx^2)\,dx = -\frac{c2^{2-\frac{in}{2}}(1-iax)^{2+\frac{in}{2}}\operatorname{Hypergeometric2F1}\left(\frac{in}{2}-1,\frac{in}{2}+2,\frac{in}{2}+3,\frac{1}{2}(1-iax)\right)}{a(-n+4i)}$$

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

```
[Out] -((2^(2 - (I/2)*n)*c*(1 - I*a*x)^(2 + (I/2)*n)*Hypergeometric2F1[-1 + (I/2)
*n, 2 + (I/2)*n, 3 + (I/2)*n, (1 - I*a*x)/2])/(a*(4*I - n)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= c\int (1-iax)^{1+\frac{in}{2}}(1+iax)^{1-\frac{in}{2}}\,dx \\ &= -\frac{2^{2-\frac{in}{2}}c(1-iax)^{2+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(-1+\frac{in}{2},2+\frac{in}{2},3+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a(4i-n)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)\,dx = \frac{i2^{1-\frac{in}{2}}c(1-iax)^{2+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(-1+\frac{in}{2},2+\frac{in}{2},3+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a\left(2+\frac{in}{2}\right)}$$

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2),x]
[Out] (I*2^(1 - (I/2)*n)*c*(1 - I*a*x)^(2 + (I/2)*n)*Hypergeometric2F1[-1 + (I/2)
*n, 2 + (I/2)*n, 3 + (I/2)*n, (1 - I*a*x)/2])/(a*(2 + (I/2)*n))
```

Maple [F]

$$\int e^{n\arctan(ax)}\left(a^2c\,x^2+c\right)dx$$

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}(c + a^2cx^2)\ dx = \int (a^2cx^2 + c)e^{(n\arctan(ax))}\, dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}(c + a^2cx^2)\ dx = c\left(\int a^2x^2e^{n\operatorname{atan}(ax)}\, dx + \int e^{n\operatorname{atan}(ax)}\, dx\right)$$

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c),x)
[Out] c*(Integral(a**2*x**2*exp(n*atan(a*x)), x) + Integral(exp(n*atan(a*x)), x))
```

Maxima [F]

$$\int e^{n\arctan(ax)}(c + a^2cx^2)\ dx = \int (a^2cx^2 + c)e^{(n\arctan(ax))}\, dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}(c + a^2cx^2)\ dx = \int (a^2cx^2 + c)e^{(n\arctan(ax))}\, dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2), x)
```

3.339 $\int e^{n\arctan(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 81

$$\int e^{n\arctan(ax)}\,dx=-\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a(2i-n)}$$

```
[Out] -2^(1-1/2*I*n)*(1-I*a*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*
I*n],1/2-1/2*I*a*x)/a/(2*I-n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5169, 71}

$$\int e^{n\arctan(ax)}\,dx=-\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2},\frac{in}{2}+2,\frac{1}{2}(1-iax)\right)}{a(-n+2i)}$$

```
[In] Int[E^(n*ArcTan[a*x]),x]
```

```
[Out] -((2^(1 - (I/2)*n)*(1 - I*a*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n,
 (I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(a*(2*I - n)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5169

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Rubi steps

$$\text{integral} = \int (1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,dx$$
$$= -\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a(2i-n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int e^{n\arctan(ax)}\,dx = \frac{4e^{(2i+n)\arctan(ax)}\text{Hypergeometric2F1}\left(2,1-\frac{in}{2},2-\frac{in}{2},-e^{2i\arctan(ax)}\right)}{a(2i+n)}$$

```
[In] Integrate[E^(n*ArcTan[a*x]),x]
[Out] (4*E^((2*I + n)*ArcTan[a*x])*Hypergeometric2F1[2, 1 - (I/2)*n, 2 - (I/2)*n,
 -E^((2*I)*ArcTan[a*x])])/(a*(2*I + n))
```

Maple [F]

$$\int \mathrm{e}^{n\arctan(ax)}dx$$

```
[In] int(exp(n*arctan(a*x)),x)
[Out] int(exp(n*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}\,dx = \int e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x)),x, algorithm="fricas")
[Out] integral(e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}\,dx = \int e^{n\operatorname{atan}(ax)}\,dx$$

```
[In] integrate(exp(n*atan(a*x)),x)
[Out] Integral(exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}\,dx = \int e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x)),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}\,dx = \int e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\,dx = \int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,dx$$

```
[In] int(exp(n*atan(a*x)),x)
[Out] int(exp(n*atan(a*x)), x)
```

3.340 $\int \frac{e^{n \arctan(ax)}x^3}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \frac{e^{n\arctan(ax)}(2i+n-in^2)}{2a^4cn} - \frac{e^{n\arctan(ax)}nx}{2a^3c} + \frac{e^{n\arctan(ax)}x^2}{2a^2c} + \frac{ie^{n\arctan(ax)}(-2+n^2)\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},-e^{2i\arctan(ax)}\right)}{a^4cn}$$

```
[Out] 1/2*exp(n*arctan(a*x))*(2*I+n-I*n^2)/a^4/c/n-1/2*exp(n*arctan(a*x))*n*x/a^3
/c+1/2*exp(n*arctan(a*x))*x^2/a^2/c+I*exp(n*arctan(a*x))*(n^2-2)*hypergeom(
[1, -1/2*I*n],[1-1/2*I*n],-(1+I*a*x)^2/(a^2*x^2+1))/a^4/c/n
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5190, 102, 148, 71}

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \frac{2^{-1-\frac{in}{2}}(2-n^2)(1-iax)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2}+1,\frac{in}{2}+2,\frac{1}{2}(1-iax)\right)}{a^4c(2+in)} + \frac{i(1+iax)^{-\frac{in}{2}}(ian^2x-n^2-in+2)(1-iax)^{\frac{in}{2}}}{2a^4cn} + \frac{x^2(1+iax)^{-\frac{in}{2}}(1-iax)^{\frac{in}{2}}}{2a^2c}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^3)/(c + a^2*c*x^2),x]
```

```
[Out] (x^2*(1 - I*a*x)^((I/2)*n))/(2*a^2*c*(1 + I*a*x)^((I/2)*n)) + ((I/2)*(1 - I
*a*x)^((I/2)*n)*(2 - I*n - n^2 + I*a*n^2*x))/(a^4*c*n*(1 + I*a*x)^((I/2)*n)
) + (2^(-1 - (I/2)*n)*(2 - n^2)*(1 - I*a*x)^(1 + (I/2)*n)*Hypergeometric2F1
[1 + (I/2)*n, 1 + (I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(a^4*c*(2 + I*n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int x^3(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}\\ &= \frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2c}+\frac{\int x(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}(-2-anx)\,dx}{2a^2c}\end{aligned}$$

$$= \frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2c} + \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}(2-in-n^2+ian^2x)}{2a^4cn} - \frac{(i(2-n^2))\int(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{2a^3c}$$

$$= \frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2c} + \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}(2-in-n^2+ian^2x)}{2a^4cn} + \frac{2^{-1-\frac{in}{2}}(2-n^2)(1-iax)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},1+\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a^4c(2+in)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \frac{(1-iax)^{\frac{in}{2}}\left(\frac{(1+iax)^{-\frac{in}{2}}(2i+n+a^2nx^2-n^2(i+ax))}{n} + \frac{2^{-\frac{in}{2}}(-2+n^2)(i+ax)\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},1+\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{-2i+n}\right)}{2a^4c}$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^3)/(c + a^2*c*x^2),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*((2*I + n + a^2*n*x^2 - n^2*(I + a*x))/(n*(1 + I*a*x
)^((I/2)*n)) + ((-2 + n^2)*(I + a*x)*Hypergeometric2F1[1 + (I/2)*n, 1 + (I/
2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(2^((I/2)*n)*(-2*I + n))))/(2*a^4*c)
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{a^2c\,x^2+c}dx$$

```
[In] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \int \frac{x^3e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \frac{\int \frac{x^3e^{n\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

```
[In] integrate(exp(n*atan(a*x))*x**3/(a**2*c*x**2+c),x)
[Out] Integral(x**3*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \int \frac{x^3e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \int \frac{x^3e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \int \frac{x^3\,\mathrm{e}^{n\operatorname{atan}(a\,x)}}{c\,a^2\,x^2+c}\,dx$$

```
[In] int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
[Out] int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

3.341 $\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = -\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn}+\frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}+\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a^3c}$$

```
[Out] -(1+I*n)*(1-I*a*x)^(1/2*I*n)/a^3/c/n/((1+I*a*x)^(1/2*I*n))+x*(1-I*a*x)^(1/2
*I*n)/a^2/c/((1+I*a*x)^(1/2*I*n))+I*2^(1-1/2*I*n)*(1-I*a*x)^(1/2*I*n)*hyper
geom([1/2*I*n, 1/2*I*n],[1+1/2*I*n],1/2-1/2*I*a*x)/a^3/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5190, 92, 80, 71}

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},\frac{in}{2}+1,\frac{1}{2}(1-iax)\right)}{a^3c}-\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn}+\frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^2)/(c + a^2*c*x^2),x]
```

```
[Out] -(((1 + I*n)*(1 - I*a*x)^((I/2)*n))/(a^3*c*n*(1 + I*a*x)^((I/2)*n))) + (x*(
1 - I*a*x)^((I/2)*n))/(a^2*c*(1 + I*a*x)^((I/2)*n)) + (I*2^(1 - (I/2)*n)*(1
 - I*a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I
*a*x)/2])/(a^3*c)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int x^2(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}\\
&= \frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c} + \frac{\int (1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}(-1-anx)\,dx}{a^2c}\\
&= -\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn} + \frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c}\\
&\quad + \frac{(in)\int (1-iax)^{-1+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,dx}{a^2c}
\end{aligned}$$

$$= -\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn} + \frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c} + \frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\, \text{Hypergeometric2F1}\left(\frac{in}{2}, \frac{in}{2}, 1+\frac{in}{2}, \frac{1}{2}(1-iax)\right)}{a^3c}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \frac{(1-iax)^{\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}(-1+n(-i+ax)) + 2in(1+iax)^{\frac{in}{2}}\, \text{Hypergeometric2F1}\left(\frac{in}{2}, \frac{in}{2}, 1+\frac{in}{2}, \frac{1}{2}(1\right.\right.}{a^3cn}$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^2)/(c + a^2*c*x^2),x]
[Out] ((1 - I*a*x)^((I/2)*n)*(2^((I/2)*n)*(-1 + n*(-I + a*x)) + (2*I)*n*(1 + I*a*
x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a*x)/2
]))/(a^3*c*n*(2 + (2*I)*a*x)^((I/2)*n))
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{a^2c\,x^2+c}dx$$

```
[In] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \int \frac{x^2e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \frac{\int \frac{x^2e^{n\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

```
[In] integrate(exp(n*atan(a*x))*x**2/(a**2*c*x**2+c),x)
[Out] Integral(x**2*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \int \frac{x^2e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \int \frac{x^2e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \int \frac{x^2\,\mathrm{e}^{n\operatorname{atan}(a\,x)}}{c\,a^2\,x^2+c}\,dx$$

```
[In] int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
[Out] int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

3.342 $\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx = \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn} - \frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a^2cn}$$

```
[Out] I*(1-I*a*x)^(1/2*I*n)/a^2/c/n/((1+I*a*x)^(1/2*I*n))-I*2^(1-1/2*I*n)*(1-I*a*
x)^(1/2*I*n)*hypergeom([1/2*I*n, 1/2*I*n],[1+1/2*I*n],1/2-1/2*I*a*x)/a^2/c/
n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5190, 80, 71}

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx = \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn} - \frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},\frac{in}{2}+1,\frac{1}{2}(1-iax)\right)}{a^2cn}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x)/(c + a^2*c*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^((I/2)*n))/(a^2*c*n*(1 + I*a*x)^((I/2)*n)) - (I*2^(1 - (I/2)
*n)*(1 - I*a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n,
(1 - I*a*x)/2])/(a^2*c*n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int x(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}\\ &= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn} - \frac{i\int(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,dx}{ac}\\ &= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn} - \frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a^2cn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx = \frac{i(1-iax)^{\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}-2(1+iax)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-iax)\right)\right)}{a^2cn}$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x)/(c + a^2*c*x^2),x]
```

```
[Out] (I*(1 - I*a*x)^((I/2)*n)*(2^((I/2)*n) - 2*(1 + I*a*x)^((I/2)*n)*Hypergeomet
ric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a*x)/2]))/(a^2*c*n*(2 + (2*I)*
a*x)^((I/2)*n))
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(ax)} x}{a^2 c\, x^2 + c} dx$$

```
[In] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x}{c + a^2 c x^2} \, dx = \int \frac{x e^{(n \arctan(ax))}}{a^2 c x^2 + c} \, dx$$

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x}{c + a^2 c x^2} \, dx = \frac{\int \frac{x e^{n \operatorname{atan}(ax)}}{a^2 x^2 + 1} \, dx}{c}$$

```
[In] integrate(exp(n*atan(a*x))*x/(a**2*c*x**2+c),x)
[Out] Integral(x*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)} x}{c + a^2 c x^2} \, dx = \int \frac{x e^{(n \arctan(ax))}}{a^2 c x^2 + c} \, dx$$

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx = \int \frac{xe^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{c\,a^2\,x^2+c}\,dx$$

```
[In] int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
[Out] int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

3.343 $\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{n\arctan(ax)}}{acn}$$

```
[Out] exp(n*arctan(a*x))/a/c/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5179}

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{n\arctan(ax)}}{acn}$$

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

```
[Out] E^(n*ArcTan[a*x])/(a*c*n)
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\text{integral} = \frac{e^{n\arctan(ax)}}{acn}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{acn}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2),x]
[Out] (1 - I*a*x)^((I/2)*n)/(a*c*n*(1 + I*a*x)^((I/2)*n))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{e^{n\arctan(ax)}}{acn}$	18
parallelrisch	$\frac{e^{n\arctan(ax)}}{acn}$	18
risch	$\frac{(-iax+1)^{\frac{in}{2}}(iax+1)^{-\frac{in}{2}}}{can}$	35

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] exp(n*arctan(a*x))/a/c/n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(n\arctan(ax))}}{acn}$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] e^(n*arctan(a*x))/(a*c*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx = \begin{cases} \frac{x}{c} & \text{for } a=0 \wedge (a=0 \vee n=0) \\ \frac{\operatorname{atan}(ax)}{ac} & \text{for } n=0 \\ \frac{e^{n\operatorname{atan}(ax)}}{acn} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c),x)
[Out] Piecewise((x/c, Eq(a, 0) & (Eq(a, 0) | Eq(n, 0))), (atan(a*x)/(a*c), Eq(n,
0)), (exp(n*atan(a*x))/(a*c*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(n\arctan(ax))}}{acn}$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] e^(n*arctan(a*x))/(a*c*n)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(n\arctan(ax))}}{acn}$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
[Out] e^(n*arctan(a*x))/(a*c*n)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{n \arctan(ax)}}{c + a^2 c x^2} \, dx = \frac{e^{n \operatorname{atan}(a\,x)}}{a\,c\,n}$$

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2),x)
[Out] exp(n*atan(a*x))/(a*c*n)
```

3.344 $\int \frac{e^{n\arctan(ax)}}{x(c+a^2cx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{e^{n\arctan(ax)}}{x(c+a^2cx^2)}\,dx = \frac{ie^{n\arctan(ax)}}{cn} - \frac{2ie^{n\arctan(ax)}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},e^{2i\arctan(ax)}\right)}{cn}$$

```
[Out] I*exp(n*arctan(a*x))/c/n-2*I*exp(n*arctan(a*x))*hypergeom([1, -1/2*I*n],[1-
1/2*I*n],(1+I*a*x)^2/(a^2*x^2+1))/c/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5190, 98, 133}

$$\int \frac{e^{n\arctan(ax)}}{x(c+a^2cx^2)}\,dx = \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{2i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},\frac{iax+1}{1-iax}\right)}{cn}$$

```
[In] Int[E^(n*ArcTan[a*x])/(x*(c + a^2*c*x^2)),x]
```

```
[Out] (I*(1 - I*a*x)^((I/2)*n))/(c*n*(1 + I*a*x)^((I/2)*n)) - ((2*I)*(1 - I*a*x)^
((I/2)*n)*Hypergeometric2F1[1, (-1/2*I)*n, 1 - (I/2)*n, (1 + I*a*x)/(1 - I*
a*x)])/(c*n*(1 + I*a*x)^((I/2)*n))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{c}\\
&= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} + \frac{\int \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{c}\\
&= \frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn}\\
&\quad - \frac{2i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},\frac{1+iax}{1-iax}\right)}{cn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85

$$\int \frac{e^{n\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left((2+in)(-i+ax)+2(n-ianx)\,\text{Hypergeometric2F1}\left(1,1+\frac{in}{2},2+\frac{in}{2},\frac{i+ax}{i-ax}\right)\right)}{cn(-2i+n)(-i+ax)}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(x*(c + a^2*c*x^2)),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*((2 + I*n)*(-I + a*x) + 2*(n - I*a*n*x)*Hypergeometr
ic2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/(I - a*x)]))/(c*n*(-2*I + n)*(
1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}}{x\,(a^2c\,x^2+c)}dx$$

```
[In] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^3 + c*x), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{e^{n\operatorname{atan}(ax)}}{a^2x^3+x}\,dx}{c}$$

```
[In] integrate(exp(n*atan(a*x))/x/(a**2*c*x**2+c),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(a**2*x**3 + x), x)/c
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{x\,(c + a^2cx^2)}\,dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2 + c)x}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x), x)
```

Giac [F]

$$\int \frac{e^{n \arctan(ax)}}{x\,(c + a^2cx^2)}\,dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2 + c)x}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)}}{x\,(c + a^2cx^2)}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x\,\left(c\,a^2\,x^2 + c\right)}\,dx$$

```
[In] int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)),x)
[Out] int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)), x)
```

3.345 $\int \frac{e^{n\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{e^{n\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \frac{iae^{n\arctan(ax)}(i+n)}{cn} - \frac{e^{n\arctan(ax)}}{cx} - \frac{2iae^{n\arctan(ax)}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},-1+\frac{2i}{i+ax}\right)}{c}$$

```
[Out] I*a*exp(n*arctan(a*x))*(I+n)/c/n-exp(n*arctan(a*x))/c/x-2*I*a*exp(n*arctan(
a*x))*hypergeom([1, -1/2*I*n],[1-1/2*I*n],-1+2*I/(I+a*x))/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5190, 105, 160, 12, 133}

$$\int \frac{e^{n\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = -\frac{2ia(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},\frac{iax+1}{1-iax}\right)}{c} - \frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx}$$

```
[In] Int[E^(n*ArcTan[a*x])/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] -((a*(1 - I*n)*(1 - I*a*x)^((I/2)*n))/(c*n*(1 + I*a*x)^((I/2)*n))) - (1 - I
*a*x)^((I/2)*n)/(c*x*(1 + I*a*x)^((I/2)*n)) - ((2*I)*a*(1 - I*a*x)^((I/2)*n
```

```
)*Hypergeometric2F1[1, (-1/2*I)*n, 1 - (I/2)*n, (1 + I*a*x)/(1 - I*a*x)])/(
c*(1 + I*a*x)^((I/2)*n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x^2}\,dx}{c}$$
$$= -\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx} - \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}(-an+a^2x)}{x}\,dx}{c}$$
$$= -\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx} + \frac{\int \frac{a^2n^2(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{acn}$$
$$= -\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx} + \frac{(an)\int \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{c}$$
$$= -\frac{a(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{cx} - \frac{2ia(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},\frac{1+iax}{1-iax}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{e^{n\arctan(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx$$
$$= \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left((-2i+n)(1+iax)(iax+n(i+ax)) + 2an^2x(1-iax)\,\text{Hypergeometric2F1}\left(1,1\right.\right.}{cn(-2i+n)x(-i+ax)}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^2*(c + a^2*c*x^2)),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*((-2*I + n)*(1 + I*a*x)*(I*a*x + n*(I + a*x)) + 2*a*
n^2*x*(1 - I*a*x)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/
(I - a*x)]))/(c*n*(-2*I + n)*x*(1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{x^2 (a^2 c x^2 + c)} dx$$

```
[In] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{x^2 (c + a^2 c x^2)} dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c) x^2} dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{x^2 (c + a^2 c x^2)} dx = \frac{\int \frac{e^{n \operatorname{atan}(ax)}}{a^2 x^4 + x^2} dx}{c}$$

```
[In] integrate(exp(n*atan(a*x))/x**2/(a**2*c*x**2+c),x)
[Out] Integral(exp(n*atan(a*x))/(a**2*x**4 + x**2), x)/c
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{x^2 (c + a^2 c x^2)} dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c) x^2} dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x^2}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

```
[In] int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)),x)
[Out] int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)), x)
```

3.346 $\int \frac{e^{n \arctan(ax)}}{x^3(c+a^2cx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{e^{n \arctan(ax)}}{x^3(c+a^2cx^2)}\,dx = \frac{ia^2e^{n\arctan(ax)}(-2+in+n^2)}{2cn}-\frac{e^{n\arctan(ax)}}{2cx^2}-\frac{ae^{n\arctan(ax)}n}{2cx}-\frac{ia^2e^{n\arctan(ax)}(-2+n^2)\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},e^{2i\arctan(ax)}\right)}{cn}$$

```
[Out] 1/2*I*a^2*exp(n*arctan(a*x))*(-2+I*n+n^2)/c/n-1/2*exp(n*arctan(a*x))/c/x^2-
1/2*a*exp(n*arctan(a*x))*n/c/x-I*a^2*exp(n*arctan(a*x))*(n^2-2)*hypergeom([
1, -1/2*I*n],[1-1/2*I*n],(1+I*a*x)^2/(a^2*x^2+1))/c/n
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5190, 105, 156, 160, 12, 133}

$$\int \frac{e^{n \arctan(ax)}}{x^3(c+a^2cx^2)}\,dx = \frac{ia^2(2-n^2)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},\frac{iax+1}{1-iax}\right)}{cn}-\frac{a^2(-in^2+n+2i)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn}-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2}-\frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx}$$

```
[In] Int[E^(n*ArcTan[a*x])/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] -1/2*(a^2*(2*I + n - I*n^2)*(1 - I*a*x)^((I/2)*n))/(c*n*(1 + I*a*x)^((I/2)*
n)) - (1 - I*a*x)^((I/2)*n)/(2*c*x^2*(1 + I*a*x)^((I/2)*n)) - (a*n*(1 - I*a
*x)^((I/2)*n))/(2*c*x*(1 + I*a*x)^((I/2)*n)) + (I*a^2*(2 - n^2)*(1 - I*a*x)
^((I/2)*n)*Hypergeometric2F1[1, (-1/2*I)*n, 1 - (I/2)*n, (1 + I*a*x)/(1 - I
*a*x)])/(c*n*(1 + I*a*x)^((I/2)*n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x^3}\,dx}{c} \\
&= -\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}(-an+2a^2x)}{x^2}\,dx}{2c} \\
&= -\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx} \\
&\quad + \frac{\int \frac{(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}(-a^2(2-n^2)-a^3nx)}{x}\,dx}{2c} \\
&= -\frac{a^2(2i+n-in^2)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} \\
&\quad - \frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx} - \frac{\int \frac{a^3n(2-n^2)(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{2acn} \\
&= -\frac{a^2(2i+n-in^2)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn} - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} \\
&\quad - \frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx} - \frac{(a^2(2-n^2))\int \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}}{x}\,dx}{2c} \\
&= -\frac{a^2(2i+n-in^2)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cn} \\
&\quad - \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx^2} - \frac{an(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2cx} \\
&\quad + \frac{ia^2(2-n^2)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},\frac{1+iax}{1-iax}\right)}{cn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.38

$$\int \frac{e^{n \arctan(ax)}}{x^3 (c + a^2 c x^2)} \, dx$$
$$= \frac{(1 - iax)^{\frac{in}{2}} (1 + iax)^{-\frac{in}{2}} \left(i(-2i + n)(-i + ax) \left(-2a^2x^2 + an^2x(i + ax) + in(1 + a^2x^2) \right) + 2a^2n(-2 + n^2) x}{2cn(-2i + n)x^2(-i + ax)}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^3*(c + a^2*c*x^2)),x]
```

```
[Out] ((1 - I*a*x)^((I/2)*n)*(I*(-2*I + n)*(-I + a*x)*(-2*a^2*x^2 + a*n^2*x*(I +
a*x) + I*n*(1 + a^2*x^2)) + 2*a^2*n*(-2 + n^2)*x^2*(1 - I*a*x)*Hypergeometr
ic2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/(I - a*x)]))/(2*c*n*(-2*I + n)
*x^2*(1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 (a^2 c x^2 + c)} dx$$

```
[In] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 (c + a^2 c x^2)} \, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c) x^3} \, dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 (c + a^2 c x^2)} \, dx = \frac{\int \frac{e^{n \operatorname{atan}(ax)}}{a^2 x^5 + x^3} \, dx}{c}$$

```
[In] integrate(exp(n*atan(a*x))/x**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(exp(n*atan(a*x))/(a**2*x**5 + x**3), x)/c
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 (c + a^2cx^2)} \, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2 + c)x^3} \, dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^3), x)
```

Giac [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 (c + a^2cx^2)} \, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2 + c)x^3} \, dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)}}{x^3 (c + a^2cx^2)} \, dx = \int \frac{e^{n \operatorname{atan}(a\,x)}}{x^3 \,(c\,a^2\,x^2 + c)} \, dx$$

```
[In] int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)),x)
[Out] int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)), x)
```

3.347 $\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 181

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4} dx = \frac{720e^{n \arctan(ax)}}{ac^4n(4+n^2)(16+n^2)(36+n^2)} + \frac{e^{n \arctan(ax)}(n+6ax)}{ac^4(36+n^2)(1+a^2x^2)^3} + \frac{30e^{n \arctan(ax)}(n+4ax)}{ac^4(16+n^2)(36+n^2)(1+a^2x^2)^2} + \frac{360e^{n \arctan(ax)}(n+2ax)}{ac^4(4+n^2)(16+n^2)(36+n^2)(1+a^2x^2)}$$

```
[Out] 720*exp(n*arctan(a*x))/a/c^4/n/(n^2+36)/(n^4+20*n^2+64)+exp(n*arctan(a*x))*
(6*a*x+n)/a/c^4/(n^2+36)/(a^2*x^2+1)^3+30*exp(n*arctan(a*x))*(4*a*x+n)/a/c^
4/(n^2+16)/(n^2+36)/(a^2*x^2+1)^2+360*exp(n*arctan(a*x))*(2*a*x+n)/a/c^4/(n
^2+36)/(n^4+20*n^2+64)/(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5178, 5179}

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4} dx = \frac{360(2ax+n)e^{n \arctan(ax)}}{ac^4(n^2+4)(n^2+16)(n^2+36)(a^2x^2+1)} + \frac{30(4ax+n)e^{n \arctan(ax)}}{ac^4(n^2+16)(n^2+36)(a^2x^2+1)^2} + \frac{(6ax+n)e^{n \arctan(ax)}}{ac^4(n^2+36)(a^2x^2+1)^3} + \frac{720e^{n \arctan(ax)}}{ac^4n(n^2+4)(n^2+16)(n^2+36)}$$

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (720*E^(n*ArcTan[a*x]))/(a*c^4*n*(4 + n^2)*(16 + n^2)*(36 + n^2)) + (E^(n*A
rcTan[a*x])*(n + 6*a*x))/(a*c^4*(36 + n^2)*(1 + a^2*x^2)^3) + (30*E^(n*ArcT
an[a*x])*(n + 4*a*x))/(a*c^4*(16 + n^2)*(36 + n^2)*(1 + a^2*x^2)^2) + (360*
E^(n*ArcTan[a*x])*(n + 2*a*x))/(a*c^4*(4 + n^2)*(16 + n^2)*(36 + n^2)*(1 +
a^2*x^2))
```

Rule 5178

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2))), In
t[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x] &
& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
0] && IntegerQ[2*p]
```

Rule 5179

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^{n\arctan(ax)}(n+6ax)}{ac^4(36+n^2)(1+a^2x^2)^3}+\frac{30\int\frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^3}\,dx}{c(36+n^2)}\\
&= \frac{e^{n\arctan(ax)}(n+6ax)}{ac^4(36+n^2)(1+a^2x^2)^3}+\frac{30e^{n\arctan(ax)}(n+4ax)}{ac^4(16+n^2)(36+n^2)(1+a^2x^2)^2}+\frac{360\int\frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^2}\,dx}{c^2(16+n^2)(36+n^2)}\\
&= \frac{e^{n\arctan(ax)}(n+6ax)}{ac^4(36+n^2)(1+a^2x^2)^3}+\frac{30e^{n\arctan(ax)}(n+4ax)}{ac^4(16+n^2)(36+n^2)(1+a^2x^2)^2}\\
&\quad+\frac{360e^{n\arctan(ax)}(n+2ax)}{ac^4(4+n^2)(16+n^2)(36+n^2)(1+a^2x^2)}+\frac{720\int\frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx}{c^3(4+n^2)(16+n^2)(36+n^2)}\\
&= \frac{720e^{n\arctan(ax)}}{ac^4n(4+n^2)(16+n^2)(36+n^2)}+\frac{e^{n\arctan(ax)}(n+6ax)}{ac^4(36+n^2)(1+a^2x^2)^3}\\
&\quad+\frac{30e^{n\arctan(ax)}(n+4ax)}{ac^4(16+n^2)(36+n^2)(1+a^2x^2)^2}+\frac{360e^{n\arctan(ax)}(n+2ax)}{ac^4(4+n^2)(16+n^2)(36+n^2)(1+a^2x^2)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.91

$$\int \frac{e^{n\arctan(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$$
$$= \frac{e^{n\arctan(ax)}(n+6ax)+\frac{30(c+a^2cx^2)\left(e^{n\arctan(ax)}n(-2i+n)(2i+n)(n+4ax)+12(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}(-i+ax)(i+ax)(2+n^2+2anx+2a\ldots\right.}{cn(64+20n^2+n^4)}}{ac\left(36+n^2\right)\left(c+a^2cx^2\right)^3}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

```
[Out] (E^(n*ArcTan[a*x])*(n + 6*a*x) + (30*(c + a^2*c*x^2)*(E^(n*ArcTan[a*x])*n*(
-2*I + n)*(2*I + n)*(n + 4*a*x) + (12*(1 - I*a*x)^((I/2)*n)*(-I + a*x)*(I +
 a*x)*(2 + n^2 + 2*a*n*x + 2*a^2*x^2))/(1 + I*a*x)^((I/2)*n)))/(c*n*(64 + 2
0*n^2 + n^4)))/(a*c*(36 + n^2)*(c + a^2*c*x^2)^3)
```

Maple [A] (verified)

Time = 35.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

method	result
gosper	$\frac{(720a^6x^6+720a^5n\,x^5+360a^4n^2x^4+120a^3n^3x^3+2160a^4x^4+30a^2n^4x^2+1920a^3n\,x^3+6a\,n^5x+840a^2n^2x^2+n^6+240a\,n^3x+2160a\ldots}{(a^2x^2+1)^3c^4an(n^6+56n^4+784n^2+2304)}$
parallelrisch	$720a^6\mathrm{e}^{n\arctan(ax)}x^6+720\,\mathrm{e}^{n\arctan(ax)}+2160a^2\mathrm{e}^{n\arctan(ax)}x^2+360x^4\mathrm{e}^{n\arctan(ax)}a^4n^2+120x^3\mathrm{e}^{n\arctan(ax)}a^3n^3+30x^2\mathrm{e}^{n\,\mathrm{arctan}}\ldots$

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (720*a^6*x^6+720*a^5*n*x^5+360*a^4*n^2*x^4+120*a^3*n^3*x^3+2160*a^4*x^4+30*
a^2*n^4*x^2+1920*a^3*n*x^3+6*a*n^5*x+840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160*a
^2*x^2+50*n^4+1584*a*n*x+544*n^2+720)*exp(n*arctan(a*x))/(a^2*x^2+1)^3/c^4/
a/n/(n^6+56*n^4+784*n^2+2304)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.65

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{(720\,a^6x^6 + 720\,a^5nx^5 + n^6 + 360\,(a^4n^2 + 6\,a^4)x^4 + 50\,n^4 + 120\,(a^3n^3 + 16\,a^3n)x^3 + 30\,(a}{ac^4n^7 + 56\,ac^4n^5 + 784\,ac^4n^3 + (a^7c^4n^7 + 56\,a^7c^4n^5 + 784\,a^7c^4n^3 + 2304\,a^7c^4n)x^6 + 2304\,ac^4n + 3\,(a^5c^4n}$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] (720*a^6*x^6 + 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 + 6*a^4)*x^4 + 50*n^4 + 1
20*(a^3*n^3 + 16*a^3*n)*x^3 + 30*(a^2*n^4 + 28*a^2*n^2 + 72*a^2)*x^2 + 544*
n^2 + 6*(a*n^5 + 40*a*n^3 + 264*a*n)*x + 720)*e^(n*arctan(a*x))/(a*c^4*n^7
+ 56*a*c^4*n^5 + 784*a*c^4*n^3 + (a^7*c^4*n^7 + 56*a^7*c^4*n^5 + 784*a^7*c^
4*n^3 + 2304*a^7*c^4*n)*x^6 + 2304*a*c^4*n + 3*(a^5*c^4*n^7 + 56*a^5*c^4*n^
5 + 784*a^5*c^4*n^3 + 2304*a^5*c^4*n)*x^4 + 3*(a^3*c^4*n^7 + 56*a^3*c^4*n^5
 + 784*a^3*c^4*n^3 + 2304*a^3*c^4*n)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Giac [F]

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.55

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{e^{n\,\mathrm{atan}(a\,x)}\left(\frac{720\,x^5}{a^2\,c^4\,(n^6+56\,n^4+784\,n^2+2304)}+\frac{n^6+50\,n^4+544\,n^2+720}{a^7\,c^4\,n\,(n^6+56\,n^4+784\,n^2+2304)}+\frac{720\,x^6}{a\,c^4\,n\,(n^6+56\,n^4+784\,n^2+2304)}+\frac{6\,x\,(n^4+40\,n^2\cdots}{a^6\,c^4\,(n^6+56\,n^4+7\cdots}\right.}{\frac{1}{a^6}+x^6+\frac{3\,x^4}{a^2}+\cdots}$$

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^4,x)
```

```
[Out] (exp(n*atan(a*x))*((720*x^5)/(a^2*c^4*(784*n^2 + 56*n^4 + n^6 + 2304)) + (5
44*n^2 + 50*n^4 + n^6 + 720)/(a^7*c^4*n*(784*n^2 + 56*n^4 + n^6 + 2304)) +
(720*x^6)/(a*c^4*n*(784*n^2 + 56*n^4 + n^6 + 2304)) + (6*x*(40*n^2 + n^4 +
264))/(a^6*c^4*(784*n^2 + 56*n^4 + n^6 + 2304)) + (120*x^3*(n^2 + 16))/(a^4
*c^4*(784*n^2 + 56*n^4 + n^6 + 2304)) + (360*x^4*(n^2 + 6))/(a^3*c^4*n*(784
*n^2 + 56*n^4 + n^6 + 2304)) + (30*x^2*(28*n^2 + n^4 + 72))/(a^5*c^4*n*(784
*n^2 + 56*n^4 + n^6 + 2304))))/(1/a^6 + x^6 + (3*x^4)/a^2 + (3*x^2)/a^4)
```

3.348 $\int e^{n\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{2^{\frac{5}{2}-\frac{in}{2}}c(1-iax)^{\frac{1}{2}(5+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+in),\frac{1}{2}(5+in),\frac{1}{2}(7+in),\frac{1}{2}(1-iax)\right)}{a(5i-n)\sqrt{1+a^2x^2}}$$

```
[Out] -2^(5/2-1/2*I*n)*c*(1-I*a*x)^(5/2+1/2*I*n)*hypergeom([5/2+1/2*I*n, -3/2+1/2
*I*n],[7/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(5*I-n)/(a^2*x^2+1
)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{c2^{\frac{5}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5),\frac{1}{2}(in+7),\frac{1}{2}(1-iax)\right)}{a(-n+5i)\sqrt{a^2x^2+1}}$$

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -((2^(5/2 - (I/2)*n)*c*(1 - I*a*x)^((5 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hyperg
eometric2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(a*(5*I
 - n)*Sqrt[1 + a^2*x^2]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{n\arctan(ax)}\left(1+a^2x^2\right)^{3/2}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}+\frac{in}{2}}(1+iax)^{\frac{3}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2^{\frac{5}{2}-\frac{in}{2}}c(1-iax)^{\frac{1}{2}(5+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+in),\frac{1}{2}(5+in),\frac{1}{2}(7+in),\frac{1}{2}(1-\right.}{a(5i-n)\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = \frac{2^{\frac{5}{2}-\frac{in}{2}}c(1-iax)^{\frac{5}{2}+\frac{in}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(5+in),\frac{1}{2}i(3i+n),\frac{1}{2}(7+in),\frac{1}{2}(1-\right.}{a(-5i+n)\sqrt{1+a^2x^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2^(5/2 - (I/2)*n)*c*(1 - I*a*x)^(5/2 + (I/2)*n)*Sqrt[c + a^2*c*x^2]*Hyperg
eometric2F1[(5 + I*n)/2, (I/2)*(3*I + n), (7 + I*n)/2, (1 - I*a*x)/2])/(a*(
-5*I + n)*Sqrt[1 + a^2*x^2])
```

Maple [F]

$$\int e^{n\arctan(ax)}\left(a^2c\,x^2+c\right)^{\frac{3}{2}}dx$$

[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)

[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)

Fricas [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(n\arctan(ax))}\,dx$$

[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] integral((a^2*c*x^2 + c)^(3/2)*e^(n*arctan(a*x)), x)

Sympy [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(3/2),x)

[Out] Integral((c*(a**2*x**2 + 1))**(3/2)*exp(n*atan(a*x)), x)

Maxima [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(n\arctan(ax))}\,dx$$

[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 + c)^(3/2)*e^(n*arctan(a*x)), x)

Giac [F(-2)]

Exception generated.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
```

```
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

3.349 $\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{2^{\frac{3}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(3+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(3+in),\frac{1}{2}(5+in),\frac{1}{2}(1-iax)\right)}{a(3i-n)\sqrt{1+a^2x^2}}$$

```
[Out] -2^(3/2-1/2*I*n)*(1-I*a*x)^(3/2+1/2*I*n)*hypergeom([-1/2+1/2*I*n, 3/2+1/2*I
*n],[5/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a/(3*I-n)/(a^2*x^2+1)^
(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{2^{\frac{3}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+in)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3),\frac{1}{2}(in+5),\frac{1}{2}(1-iax)\right)}{a(-n+3i)\sqrt{a^2x^2+1}}$$

```
[In] Int[E^(n*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((2^(3/2 - (I/2)*n)*(1 - I*a*x)^((3 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeo
metric2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(a*(3*I -
 n)*Sqrt[1 + a^2*x^2]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c+a^2cx^2}\int e^{n\arctan(ax)}\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}\\ &= \frac{\sqrt{c+a^2cx^2}\int (1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}\\ &= -\frac{2^{\frac{3}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(3+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(3+in),\frac{1}{2}(5+in),\frac{1}{2}(1-i\right.}{a(3i-n)\sqrt{1+a^2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{2^{\frac{3}{2}-\frac{in}{2}}(1-iax)^{\frac{3}{2}+\frac{in}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(3+in),\frac{1}{2}i(i+n),\frac{1}{2}(5+in),\frac{1}{2}(1-iax)\right)}{a(-3i+n)\sqrt{1+a^2x^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(3/2 - (I/2)*n)*(1 - I*a*x)^(3/2 + (I/2)*n)*Sqrt[c + a^2*c*x^2]*Hypergeo
metric2F1[(3 + I*n)/2, (I/2)*(I + n), (5 + I*n)/2, (1 - I*a*x)/2])/(a*(-3*I
 + n)*Sqrt[1 + a^2*x^2])
```

Maple [F]

$$\int \mathrm{e}^{n\arctan(ax)}\sqrt{a^2c\,x^2+c}dx$$

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{c\,(a^2x^2+1)}e^{n\operatorname{atan}(ax)}\,dx$$

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

3.350 $\int \frac{e^{n \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-iax)\right)}{a(i-n)\sqrt{c+a^2cx^2}}$$

```
[Out] -2^(1/2-1/2*I*n)*(1-I*a*x)^(1/2+1/2*I*n)*hypergeom([1/2+1/2*I*n, 1/2+1/2*I*
n],[3/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(I-n)/(a^2*c*x^2+c)^(1/
2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)*S
qrt[c + a^2*c*x^2]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-iax\right.}{a(i-n)\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}+\frac{in}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{in}{2},\frac{1}{2}+\frac{in}{2},\frac{3}{2}+\frac{in}{2},\frac{1}{2}-\frac{iax}{2}\right)}{a(-i+n)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + (I/2)*n, 1/2 + (I/2)*n, 3/2 + (I/2)*n, 1/2 - (I/2)*a*x])/(a*(
-I + n)*Sqrt[c + a^2*c*x^2])
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}dx$$

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.351 $\int e^{n\arctan(ax)}x^2(c+a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 283

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=-\frac{cn(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{30a^3\sqrt{1+a^2x^2}}+\frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{6a^2\sqrt{1+a^2x^2}}+\frac{2^{\frac{3}{2}-\frac{in}{2}}c(5-n^2)\,(1-iax)^{\frac{1}{2}(5+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+in),\frac{1}{2}(5+in),\frac{1}{2}(7+in),\frac{1}{2}(1-i\right.}{15a^3(5i-n)\sqrt{1+a^2x^2}}$$

```
[Out] -1/30*c*n*(1-I*a*x)^(5/2+1/2*I*n)*(1+I*a*x)^(5/2-1/2*I*n)*(a^2*c*x^2+c)^(1/
2)/a^3/(a^2*x^2+1)^(1/2)+1/6*c*x*(1-I*a*x)^(5/2+1/2*I*n)*(1+I*a*x)^(5/2-1/2
*I*n)*(a^2*c*x^2+c)^(1/2)/a^2/(a^2*x^2+1)^(1/2)+1/15*2^(3/2-1/2*I*n)*c*(-n^
2+5)*(1-I*a*x)^(5/2+1/2*I*n)*hypergeom([5/2+1/2*I*n, -3/2+1/2*I*n],[7/2+1/2
*I*n],1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a^3/(5*I-n)/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5193, 5190, 92, 81, 71}

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\frac{cx\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}}{6a^2\sqrt{a^2x^2+1}}+\frac{c2^{\frac{3}{2}-\frac{in}{2}}(5-n^2)\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5),\frac{1}{2}(in+7),\frac{1}{2}(1-iax\right.}{15a^3(-n+5i)\sqrt{a^2x^2+1}}-\frac{cn\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}}{30a^3\sqrt{a^2x^2+1}}$$

```
[In] Int[E^(n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/30*(c*n*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*x)^((5 - I*n)/2)*Sqrt[c + a^2
*c*x^2])/(a^3*Sqrt[1 + a^2*x^2]) + (c*x*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*
x)^((5 - I*n)/2)*Sqrt[c + a^2*c*x^2])/(6*a^2*Sqrt[1 + a^2*x^2]) + (2^(3/2 -
 (I/2)*n)*c*(5 - n^2)*(1 - I*a*x)^((5 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hyperge
ometric2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(15*a^3*
(5*I - n)*Sqrt[1 + a^2*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+a^2cx^2}\right)\int e^{n\arctan(ax)}x^2(1+a^2x^2)^{3/2}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\int x^2(1-iax)^{\frac{3}{2}+\frac{in}{2}}(1+iax)^{\frac{3}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}$$
$$= \frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{6a^2\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}+\frac{in}{2}}(1+iax)^{\frac{3}{2}-\frac{in}{2}}(-1-anx)\,dx}{6a^2\sqrt{1+a^2x^2}}$$
$$= -\frac{cn(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{30a^3\sqrt{1+a^2x^2}} + \frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{6a^2\sqrt{1+a^2x^2}} + \frac{\left(c(-5+n^2)\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{3}{2}+\frac{in}{2}}(1+iax)^{\frac{3}{2}-\frac{in}{2}}\,dx}{30a^2\sqrt{1+a^2x^2}}$$
$$= -\frac{cn(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{30a^3\sqrt{1+a^2x^2}} + \frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{6a^2\sqrt{1+a^2x^2}} + \frac{2^{\frac{3}{2}-\frac{in}{2}}c(5-n^2)(1-iax)^{\frac{1}{2}(5+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+in),\frac{1}{2}(5+in),\frac{1}{2}(7+i\right.}{15a^3(5i-n)\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.77

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx = \frac{2^{-1-\frac{in}{2}}c(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}(i+ax)^2\sqrt{c+a^2cx^2}\left(2^{\frac{in}{2}}(-5i+n)\sqrt{1+iax}(-i+ax)^2\right.}{15a}$$

```
[In] Integrate[E^(n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2^(-1 - (I/2)*n)*c*(1 - I*a*x)^(1/2 + (I/2)*n)*(I + a*x)^2*Sqrt[c + a^2*c*
x^2]*(2^((I/2)*n)*(-5*I + n)*Sqrt[1 + I*a*x]*(-I + a*x)^2*(-n + 5*a*x) - 4*
Sqrt[2]*(-5 + n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(5 + I*n)/2, (I/
2)*(3*I + n), (7 + I*n)/2, (1 - I*a*x)/2]))/(15*a^3*(-5*I + n)*(1 + I*a*x)^
((I/2)*n)*Sqrt[1 + a^2*x^2])
```

Maple [F]

$$\int e^{n\arctan(ax)}x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}dx$$

```
[In] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(exp(n*atan(a*x))*x**2*(a**2*c*x**2+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(3/2)*x^2*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\int x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

```
[In] int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
[Out] int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

3.352 $\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 280

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=-\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{12a^3\sqrt{1+a^2x^2}}+\frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{4a^2\sqrt{1+a^2x^2}}+\frac{2^{-\frac{1}{2}-\frac{in}{2}}(3-n^2)(1-iax)^{\frac{1}{2}(3+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(3+in),\frac{1}{2}(5+in),\frac{1}{2}(1-\right.}{3a^3(3i-n)\sqrt{1+a^2x^2}}$$

```
[Out] -1/12*n*(1-I*a*x)^(3/2+1/2*I*n)*(1+I*a*x)^(3/2-1/2*I*n)*(a^2*c*x^2+c)^(1/2)
/a^3/(a^2*x^2+1)^(1/2)+1/4*x*(1-I*a*x)^(3/2+1/2*I*n)*(1+I*a*x)^(3/2-1/2*I*n
)*(a^2*c*x^2+c)^(1/2)/a^2/(a^2*x^2+1)^(1/2)+1/3*2^(-1/2-1/2*I*n)*(-n^2+3)*(
1-I*a*x)^(3/2+1/2*I*n)*hypergeom([-1/2+1/2*I*n, 3/2+1/2*I*n],[5/2+1/2*I*n],
1/2-1/2*I*a*x)*(a^2*c*x^2+c)^(1/2)/a^3/(3*I-n)/(a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5193, 5190, 92, 81, 71}

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\frac{x\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}}{4a^2\sqrt{a^2x^2+1}}+\frac{2^{-\frac{1}{2}-\frac{in}{2}}(3-n^2)\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+in)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3),\frac{1}{2}(in+5),\frac{1}{2}(1-i\right.}{3a^3(-n+3i)\sqrt{a^2x^2+1}}-\frac{n\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}}{12a^3\sqrt{a^2x^2+1}}$$

```
[In] Int[E^(n*ArcTan[a*x])*x^2*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -1/12*(n*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^((3 - I*n)/2)*Sqrt[c + a^2*c
*x^2])/(a^3*Sqrt[1 + a^2*x^2]) + (x*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^(
(3 - I*n)/2)*Sqrt[c + a^2*c*x^2])/(4*a^2*Sqrt[1 + a^2*x^2]) + (2^(-1/2 - (I
/2)*n)*(3 - n^2)*(1 - I*a*x)^((3 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hypergeometr
ic2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(3*a^3*(3*I -
 n)*Sqrt[1 + a^2*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+a^2cx^2}\int e^{n\arctan(ax)}x^2\sqrt{1+a^2x^2}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\int x^2(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{4a^2\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\int(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{\frac{1}{2}-\frac{in}{2}}(-1-anx)\,dx}{4a^2\sqrt{1+a^2x^2}}$$

$$= -\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{12a^3\sqrt{1+a^2x^2}} + \frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{4a^2\sqrt{1+a^2x^2}} + \frac{\left((-3+n^2)\sqrt{c+a^2cx^2}\right)\int(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{\frac{1}{2}-\frac{in}{2}}\,dx}{12a^2\sqrt{1+a^2x^2}}$$

$$= -\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{12a^3\sqrt{1+a^2x^2}} + \frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{4a^2\sqrt{1+a^2x^2}} + \frac{2^{-\frac{1}{2}-\frac{in}{2}}(3-n^2)(1-iax)^{\frac{1}{2}(3+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(3+in),\frac{1}{2}(5+in\right.}{3a^3(3i-n)\sqrt{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.76

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx = \frac{2^{-2-\frac{in}{2}}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}(i+ax)\sqrt{c+a^2cx^2}\left(2^{\frac{in}{2}}(-3i+n)\sqrt{1+iax}(-i+ax)(-n+3ax)-2i\sqrt{2}\right.}{3a^3(-3i+n)\sqrt{1+a}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])*x^2*Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(-2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*(I + a*x)*Sqrt[c + a^2*c*x^2]
*(2^((I/2)*n)*(-3*I + n)*Sqrt[1 + I*a*x]*(-I + a*x)*(-n + 3*a*x) - (2*I)*Sq
rt[2]*(-3 + n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(3 + I*n)/2, (I/2)
*(I + n), (5 + I*n)/2, (1 - I*a*x)/2]))/(3*a^3*(-3*I + n)*(1 + I*a*x)^((I/2
)*n)*Sqrt[1 + a^2*x^2])
```

Maple [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{a^2c\,x^2+c}dx$$

```
[In] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}x^2e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^2*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\int x^2\sqrt{c\,(a^2x^2+1)}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

```
[In] integrate(exp(n*atan(a*x))*x**2*(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*sqrt(c*(a**2*x**2 + 1))*exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}x^2e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2*c*x^2 + c)*x^2*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx = \int \sqrt{a^2cx^2+c}x^2e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx = \int x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

```
[In] int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
[Out] int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

3.353 $\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 322

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} - \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\left(4-in-n^2+a(1+in)nx\right)\sqrt{1+a^2x^2}}{6a^4(1+in)\sqrt{c+a^2cx^2}} + \frac{2^{-\frac{1}{2}-\frac{in}{2}}n(5-n^2)\left(1-iax\right)^{\frac{1}{2}(3+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(5+in),\frac{1}{2}(1-\right.}{3a^4\left(4n-i\left(3-n^2\right)\right)\sqrt{c+a^2cx^2}}$$

```
[Out] 1/3*x^2*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a
^2/(a^2*c*x^2+c)^(1/2)-1/6*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*
(4-I*n-n^2+a*(1+I*n)*n*x)*(a^2*x^2+1)^(1/2)/a^4/(1+I*n)/(a^2*c*x^2+c)^(1/2)
+1/3*2^(-1/2-1/2*I*n)*n*(-n^2+5)*(1-I*a*x)^(3/2+1/2*I*n)*hypergeom([1/2+1/2
*I*n, 3/2+1/2*I*n],[5/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a^4/(4*n-
I*(-n^2+3))/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5193, 5190, 102, 151, 71}

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{x^2\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{3a^2\sqrt{a^2cx^2+c}} + \frac{2^{-\frac{1}{2}-\frac{in}{2}}n(5-n^2)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(3+in)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1}{2}(in+5),\frac{1}{2}(1-\right.}{3a^4\left(4n-i\left(3-n^2\right)\right)\sqrt{a^2cx^2+c}} - \frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\left(a(1+in)nx-n^2-in+4\right)(1+iax)^{\frac{1}{2}(1-in)}}{6a^4(1+in)\sqrt{a^2cx^2+c}}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (x^2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])
/(3*a^2*Sqrt[c + a^2*c*x^2]) - ((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 -
 I*n)/2)*(4 - I*n - n^2 + a*(1 + I*n)*n*x)*Sqrt[1 + a^2*x^2])/(6*a^4*(1 + I
*n)*Sqrt[c + a^2*c*x^2]) + (2^(-1/2 - (I/2)*n)*n*(5 - n^2)*(1 - I*a*x)^((3
+ I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[(1 + I*n)/2, (3 + I*n)/2, (5
+ I*n)/2, (1 - I*a*x)/2])/(3*a^4*(4*n - I*(3 - n^2))*Sqrt[c + a^2*c*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int x^3(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\int x(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}(-2-anx)\,dx}{3a^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} - \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\left(4-in-n^2+a(1+in)nx\right)\sqrt{1+a^2x^2}}{6a^4(1+in)\sqrt{c+a^2cx^2}} + \frac{\left(n(5-n^2)\sqrt{1+a^2x^2}\right)\int (1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{6a^3(1+in)\sqrt{c+a^2cx^2}}$$

$$= \frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}} - \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\left(4-in-n^2+a(1+in)nx\right)\sqrt{1+a^2x^2}}{6a^4(1+in)\sqrt{c+a^2cx^2}} + \frac{2^{-\frac{1}{2}-\frac{in}{2}}n(5-n^2)(1-iax)^{\frac{1}{2}(3+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(5+in\right.}{3a^4\left(4n-i\left(3-n^2\right)\right)\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.77

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{2^{-\frac{3}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\sqrt{1+a^2x^2}\left(2^{\frac{1}{2}+\frac{in}{2}}(-3i+n)\sqrt{1+iax}(-n^2(i+ax)-2i(-2+a^2x^2)+n(\right.}{3a^4\left(-3-4in\right.}$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^3)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(-3/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^(1/2 +
 (I/2)*n)*(-3*I + n)*Sqrt[1 + I*a*x]*(-(n^2*(I + a*x)) - (2*I)*(-2 + a^2*x^
2) + n*(1 + I*a*x + 2*a^2*x^2)) + 2*n*(-5 + n^2)*(1 + I*a*x)^((I/2)*n)*(I +
 a*x)*Hypergeometric2F1[1/2 + (I/2)*n, 3/2 + (I/2)*n, 5/2 + (I/2)*n, 1/2 -
(I/2)*a*x]))/(3*a^4*(-3 - (4*I)*n + n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt[c + a^2
*c*x^2])
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{a^2c\,x^2+c}}dx$$

```
[In] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3e^{n\,\mathrm{atan}\,(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

```
[In] integrate(exp(n*atan(a*x))*x**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**3*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

3.354 $\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 291

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^3(i+n)\sqrt{c+a^2cx^2}} + \frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^2\sqrt{c+a^2cx^2}} - \frac{i2^{\frac{1}{2}-\frac{in}{2}}(1-n^2)(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-i\right.}{a^3(1+n^2)\sqrt{c+a^2cx^2}}$$

```
[Out] -1/2*(1+I*n)*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1
/2)/a^3/(I+n)/(a^2*c*x^2+c)^(1/2)+1/2*x*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(
1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-I*2^(1/2-1/2*I*n)*(-
n^2+1)*(1-I*a*x)^(1/2+1/2*I*n)*hypergeom([-1/2+1/2*I*n, 1/2+1/2*I*n],[3/2+1
/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a^3/(n^2+1)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5193, 5190, 92, 80, 71}

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{x\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2a^2\sqrt{a^2cx^2+c}} - \frac{i2^{\frac{1}{2}-\frac{in}{2}}(1-n^2)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1}{2}(1-ia\right.}{a^3(n^2+1)\sqrt{a^2cx^2+c}} - \frac{(1+in)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2a^3(n+i)\sqrt{a^2cx^2+c}}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -1/2*((1 + I*n)*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1
+ a^2*x^2])/(a^3*(I + n)*Sqrt[c + a^2*c*x^2]) + (x*(1 - I*a*x)^((1 + I*n)/2
)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(2*a^2*Sqrt[c + a^2*c*x^2])
- (I*2^(1/2 - (I/2)*n)*(1 - n^2)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2
]*Hypergeometric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])
/(a^3*(1 + n^2)*Sqrt[c + a^2*c*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
```

```
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int x^2(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^2\sqrt{c+a^2cx^2}}\\
&\quad+\frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}(-1-anx)\,dx}{2a^2\sqrt{c+a^2cx^2}}\\
&= -\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^3(i+n)\sqrt{c+a^2cx^2}}\\
&\quad+\frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^2\sqrt{c+a^2cx^2}}\\
&\quad-\frac{\left((1-n^2)\sqrt{1+a^2x^2}\right)\int (1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}i(i+n)}\,dx}{2a^2(1-in)\sqrt{c+a^2cx^2}}\\
&= -\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^3(i+n)\sqrt{c+a^2cx^2}}\\
&\quad+\frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^2\sqrt{c+a^2cx^2}}\\
&\quad-\frac{i2^{\frac{1}{2}-\frac{in}{2}}(1-n^2)(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in)\right.}{a^3(1+n^2)\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.71

$$
\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx
= \frac{2^{-1-\frac{in}{2}}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\sqrt{1+a^2x^2}\Big(2^{\frac{in}{2}}(-i+n)\sqrt{1+iax}(-1+iax+n(-i+ax))+2i\sqrt{2}(-1+}{a^3(1+n^2)\sqrt{c+a^2cx^2}}
$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(-1 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^((I/2)*n
)*(-I + n)*Sqrt[1 + I*a*x]*(-1 + I*a*x + n*(-I + a*x)) + (2*I)*Sqrt[2]*(-1
```

```
+ n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(1 + I*n)/2, (I/2)*(I + n),
(3 + I*n)/2, (1 - I*a*x)/2]))/(a^3*(1 + n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt[c +
 a^2*c*x^2])
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(ax)} x^2}{\sqrt{a^2 c\, x^2 + c}} dx$$

```
[In] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x^2}{\sqrt{c + a^2 c x^2}} \, dx = \int \frac{x^2 e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}} \, dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x^2}{\sqrt{c + a^2 c x^2}} \, dx = \int \frac{x^2 e^{n \operatorname{atan}(ax)}}{\sqrt{c (a^2 x^2 + 1)}} \, dx$$

```
[In] integrate(exp(n*atan(a*x))*x**2/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)} x^2}{\sqrt{c + a^2 c x^2}} \, dx = \int \frac{x^2 e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}} \, dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

3.355 $\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{a^2(1-in)\sqrt{c+a^2cx^2}} - \frac{i2^{\frac{3}{2}-\frac{in}{2}}n(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-iax)\right)}{a^2(1+n^2)\sqrt{c+a^2cx^2}}$$

```
[Out] (1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a^2/(1-I*
n)/(a^2*c*x^2+c)^(1/2)-I*2^(3/2-1/2*I*n)*n*(1-I*a*x)^(1/2+1/2*I*n)*hypergeo
m([-1/2+1/2*I*n, 1/2+1/2*I*n],[3/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2
)/a^2/(n^2+1)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5193, 5190, 80, 71}

$$\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{a^2(1-in)\sqrt{a^2cx^2+c}} - \frac{i2^{\frac{3}{2}-\frac{in}{2}}n\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1}{2}(1-iax)\right)}{a^2(n^2+1)\sqrt{a^2cx^2+c}}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] ((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(a^
2*(1 - I*n)*Sqrt[c + a^2*c*x^2]) - (I*2^(3/2 - (I/2)*n)*n*(1 - I*a*x)^((1 +
```

```
I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3
+ I*n)/2, (1 - I*a*x)/2])/(a^2*(1 + n^2)*Sqrt[c + a^2*c*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}x}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int x(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{a^2(1-in)\sqrt{c+a^2cx^2}}\\
&\quad-\frac{\left(n\sqrt{1+a^2x^2}\right)\int(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}i(i+n)}\,dx}{a(1-in)\sqrt{c+a^2cx^2}}
\end{aligned}$$

$$= \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{a^2(1-in)\sqrt{c+a^2cx^2}} - \frac{i2^{\frac{3}{2}-\frac{in}{2}}n(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-\right.}{a^2(1+n^2)\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.87

$$\int \frac{e^{n\arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \frac{(1-iax)^{\frac{1}{2}+\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\sqrt{1+a^2x^2}\left(2^{\frac{in}{2}}(1+in)\sqrt{1+iax}-2i\sqrt{2}n(1+iax)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{a^2(1+n^2)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x)/Sqrt[c + a^2*c*x^2],x]
[Out] ((1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^((I/2)*n)*(1 + I*n)*Sqrt[
1 + I*a*x] - (2*I)*Sqrt[2]*n*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(1 + I
*n)/2, (I/2)*(I + n), (3 + I*n)/2, (1 - I*a*x)/2]))/(a^2*(1 + n^2)*(2 + (2*
I)*a*x)^((I/2)*n)*Sqrt[c + a^2*c*x^2])
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x}{\sqrt{a^2c\,x^2+c}}dx$$

```
[In] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{xe^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{xe^{n\,\mathrm{atan}\,(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

```
[In] integrate(exp(n*atan(a*x))*x/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{xe^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{xe^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

3.356 $\int \frac{e^{n \arctan(ax)}}{\sqrt{c+a^2cx^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c+a^2cx^2}} \, dx = -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-iax)\right)}{a(i-n)\sqrt{c+a^2cx^2}}$$

```
[Out] -2^(1/2-1/2*I*n)*(1-I*a*x)^(1/2+1/2*I*n)*hypergeom([1/2+1/2*I*n, 1/2+1/2*I*
n],[3/2+1/2*I*n],1/2-1/2*I*a*x)*(a^2*x^2+1)^(1/2)/a/(I-n)/(a^2*c*x^2+c)^(1/
2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c+a^2cx^2}} \, dx = -\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] -((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)*S
qrt[c + a^2*c*x^2]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int (1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}} \\
&= -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-iax)\right)}{a(i-n)\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}+\frac{in}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{in}{2},\frac{1}{2}+\frac{in}{2},\frac{3}{2}+\frac{in}{2},\frac{1}{2}-\frac{iax}{2}\right)}{a(-i+n)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2^(1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*Hypergeome
tric2F1[1/2 + (I/2)*n, 1/2 + (I/2)*n, 3/2 + (I/2)*n, 1/2 - (I/2)*a*x])/(a*(
-I + n)*Sqrt[c + a^2*c*x^2])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{a^2 c\, x^2 + c}} dx$$

[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)

[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}}\, dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] integral(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}}\, dx = \int \frac{e^{n \operatorname{atan}(ax)}}{\sqrt{c (a^2 x^2 + 1)}}\, dx$$

[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)

[Out] Integral(exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}}\, dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.357 $\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = -\frac{2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1-iax}{1+iax}\right)}{(1+in)\sqrt{c+a^2cx^2}}$$

```
[Out] -2*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(-1/2-1/2*I*n)*hypergeom([1, 1/2+1/2*I
*n],[3/2+1/2*I*n],(1-I*a*x)/(1+I*a*x))*(a^2*x^2+1)^(1/2)/(1+I*n)/(a^2*c*x^2
+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5193, 5190, 133}

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = -\frac{2\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^(n*ArcTan[a*x])/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (-2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Sqrt[1 + a^2*x^2]*
Hypergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/((
1 + I*n)*Sqrt[c + a^2*c*x^2])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}}{x\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x}\,dx}{\sqrt{c+a^2cx^2}}\\ &= -\frac{2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1-iax}{1+iax}\right)}{(1+in)\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \frac{2(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{in}{2},\frac{3}{2}+\frac{in}{2},\frac{i+ax}{i-ax}\right)}{(-1-in)\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(x*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] (2*(1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^2*x^
2]*Hypergeometric2F1[1, 1/2 + (I/2)*n, 3/2 + (I/2)*n, (I + a*x)/(I - a*x)])
/((-1 - I*n)*Sqrt[c + a^2*c*x^2])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{x\sqrt{a^2c\,x^2+c}}dx$$

```
[In] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}x}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^3 + c*x), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n \operatorname{atan}(ax)}}{x\sqrt{c(a^2x^2+1)}}\,dx$$

```
[In] integrate(exp(n*atan(a*x))/x/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*atan(a*x))/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}x}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.358 $\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 196

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{x\sqrt{c+a^2cx^2}} - \frac{2an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1-iax}{1+iax}\right)}{(1+in)\sqrt{c+a^2cx^2}}$$

```
[Out] -(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/x/(a^2*c
*x^2+c)^(1/2)-2*a*n*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(-1/2-1/2*I*n)*hyperg
eom([1, 1/2+1/2*I*n],[3/2+1/2*I*n],(1-I*a*x)/(1+I*a*x))*(a^2*x^2+1)^(1/2)/(
1+I*n)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5193, 5190, 98, 133}

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{2an\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}} - \frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{x\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^(n*ArcTan[a*x])/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -((((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(
x*Sqrt[c + a^2*c*x^2])) - (2*a*n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1
 - I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2,
 (1 - I*a*x)/(1 + I*a*x)])/((1 + I*n)*Sqrt[c + a^2*c*x^2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x^2}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{x\sqrt{c+a^2cx^2}} + \frac{\left(an\sqrt{1+a^2x^2}\right)\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x}\,dx}{\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{x\sqrt{c+a^2cx^2}} - \frac{2an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1-}{1+}\right.}{(1+in)\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.72

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\sqrt{1+a^2x^2}\left(-((-i+n)(-i+ax))+2anx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{in}{2},\frac{3}{2}\right.\right.}{(-1-in)x\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^2*Sqrt[c + a^2*c*x^2]),x]
[Out] ((1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^2*x^2]
*(-((-I + n)*(-I + a*x)) + 2*a*n*x*Hypergeometric2F1[1, 1/2 + (I/2)*n, 3/2
+ (I/2)*n, (I + a*x)/(I - a*x)]))/((-1 - I*n)*x*Sqrt[c + a^2*c*x^2])
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{a^2c\,x^2+c}}dx$$

```
[In] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx^2}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}\,(ax)}}{x^2\sqrt{c\,(a^2x^2+1)}}\,dx$$

```
[In] integrate(exp(n*atan(a*x))/x**2/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*atan(a*x))/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx^2}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx^2}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.359 $\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 281

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} + \frac{a^2(1-n^2)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1-i}{1+i}\right.}{(1+in)\sqrt{c+a^2cx^2}}$$

```
[Out] -1/2*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/x^2/
(a^2*c*x^2+c)^(1/2)-1/2*a*n*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)
*(a^2*x^2+1)^(1/2)/x/(a^2*c*x^2+c)^(1/2)+a^2*(-n^2+1)*(1-I*a*x)^(1/2+1/2*I*
n)*(1+I*a*x)^(-1/2-1/2*I*n)*hypergeom([1, 1/2+1/2*I*n],[3/2+1/2*I*n],(1-I*a
*x)/(1+I*a*x))*(a^2*x^2+1)^(1/2)/(1+I*n)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5193, 5190, 105, 156, 12, 133}

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{a^2(1-n^2)\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}$$
$$- \frac{an\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2x\sqrt{a^2cx^2+c}}$$
$$- \frac{\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2x^2\sqrt{a^2cx^2+c}}$$

```
[In] Int[E^(n*ArcTan[a*x])/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] -1/2*((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*Sqrt[1 + a^2*x^2]
)/(x^2*Sqrt[c + a^2*c*x^2]) - (a*n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((
1 - I*n)/2)*Sqrt[1 + a^2*x^2])/(2*x*Sqrt[c + a^2*c*x^2]) + (a^2*(1 - n^2)*(
1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Sqrt[1 + a^2*x^2]*Hyper
geometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/((1 + I
*n)*Sqrt[c + a^2*c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
```

```
|| !SumSimplerQ[p, 1]) && !ILtQ[m, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x^3}\,dx}{\sqrt{c+a^2cx^2}} \\
&= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} \\
&\quad - \frac{\sqrt{1+a^2x^2}\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}(-an+a^2x)}{x^2}\,dx}{2\sqrt{c+a^2cx^2}} \\
&= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} \\
&\quad - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} \\
&\quad - \frac{\sqrt{1+a^2x^2}\int \frac{a^2(1-n^2)(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x}\,dx}{2\sqrt{c+a^2cx^2}}
\end{aligned}
$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} - \frac{\left(a^2(1-n^2)\sqrt{1+a^2x^2}\right)\int \frac{(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}}{x}\,dx}{2\sqrt{c+a^2cx^2}}$$

$$= -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} + \frac{a^2(1-n^2)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+in),\frac{1}{2}(3+i\right.}{(1+in)\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.57

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{i(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\sqrt{1+a^2x^2}\left(-((-i+n)(-i+ax)(1+anx))+2a^2(-1+n^2)x^2\,\text{Hypergeome}\right.}{2(-i+n)x^2\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

```
[Out] ((I/2)*(1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^
2*x^2]*(-((-I + n)*(-I + a*x)*(1 + a*n*x)) + 2*a^2*(-1 + n^2)*x^2*Hypergeom
etric2F1[1, 1/2 + (I/2)*n, 3/2 + (I/2)*n, (I + a*x)/(I - a*x)]))/((-I + n)*
x^2*Sqrt[c + a^2*c*x^2])
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{a^2c\,x^2+c}}dx$$

```
[In] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx^3}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}\,(ax)}}{x^3\sqrt{c\,(a^2x^2+1)}}\,dx$$

```
[In] integrate(exp(n*atan(a*x))/x**3/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*atan(a*x))/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx^3}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx^3}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

```
[In] int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)^(1/2)),x)
[Out] int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.360 $\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = -\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(8+3in)}\sqrt[3]{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in),\frac{1}{6}(14+3in),\frac{1}{2}(1-\right.}{a(8i-3n)\sqrt[3]{1+a^2x^2}}$$

```
[Out] -3*2^(4/3-1/2*I*n)*(1-I*a*x)^(4/3+1/2*I*n)*(a^2*c*x^2+c)^(1/3)*hypergeom([4
/3+1/2*I*n, -1/3+1/2*I*n],[7/3+1/2*I*n],1/2-1/2*I*a*x)/a/(8*I-3*n)/(a^2*x^2
+1)^(1/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = -\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}\sqrt[3]{a^2cx^2+c}(1-iax)^{\frac{1}{6}(8+3in)}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8),\frac{1}{6}(3in+14),\frac{1}{2}(1-ia\right.}{a(-3n+8i)\sqrt[3]{a^2x^2+1}}$$

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (-3*2^(4/3 - (I/2)*n)*(1 - I*a*x)^((8 + (3*I)*n)/6)*(c + a^2*c*x^2)^(1/3)*H
ypergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (14 + (3*I)*n)/6, (1 -
I*a*x)/2])/(a*(8*I - 3*n)*(1 + a^2*x^2)^(1/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{c+a^2cx^2}\int e^{n\arctan(ax)}\sqrt[3]{1+a^2x^2}\,dx}{\sqrt[3]{1+a^2x^2}}$$

$$= \frac{\sqrt[3]{c+a^2cx^2}\int (1-iax)^{\frac{1}{3}+\frac{in}{2}}(1+iax)^{\frac{1}{3}-\frac{in}{2}}\,dx}{\sqrt[3]{1+a^2x^2}}$$

$$= -\frac{3\ 2^{\frac{4}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(8+3in)}\sqrt[3]{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in),\frac{1}{6}(14+3in),\right.}{a(8i-3n)\sqrt[3]{1+a^2x^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = \frac{3\ 2^{\frac{4}{3}-\frac{in}{2}}(1-iax)^{\frac{4}{3}+\frac{in}{2}}\sqrt[3]{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3}+\frac{in}{2},\frac{4}{3}+\frac{in}{2},\frac{7}{3}+\frac{in}{2},\frac{1}{2}-\frac{iax}{2}\right)}{a(-8i+3n)\sqrt[3]{1+a^2x^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (3*2^(4/3 - (I/2)*n)*(1 - I*a*x)^(4/3 + (I/2)*n)*(c + a^2*c*x^2)^(1/3)*Hype
rgeometric2F1[-1/3 + (I/2)*n, 4/3 + (I/2)*n, 7/3 + (I/2)*n, 1/2 - (I/2)*a*x
])/(a*(-8*I + 3*n)*(1 + a^2*x^2)^(1/3))
```

Maple [F]

$$\int e^{n\arctan(ax)}\left(a^2c\,x^2+c\right)^{\frac{1}{3}}dx$$

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x)
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = \int\left(a^2cx^2+c\right)^{\frac{1}{3}}e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(1/3)*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = \int\sqrt[3]{c\left(a^2x^2+1\right)}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(1/3),x)
[Out] Integral((c*(a**2*x**2 + 1))**(1/3)*exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = \int\left(a^2cx^2+c\right)^{\frac{1}{3}}e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^(1/3)*e^(n*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = \int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{1/3}\,dx$$

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/3),x)
```

```
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/3), x)
```

3.361 $\int \frac{e^{n \arctan(ax)}}{\sqrt[3]{c+a^2cx^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{e^{n \arctan(ax)}}{\sqrt[3]{c+a^2cx^2}} \, dx = -\frac{3\, 2^{\frac{2}{3}-\frac{in}{2}} (1-iax)^{\frac{1}{6}(4+3in)} \sqrt[3]{1+a^2x^2} \, \text{Hypergeometric2F1}\left(\frac{1}{6}(2+3in), \frac{1}{6}(4+3in), \frac{1}{6}(10+3in), \frac{1}{2}(1-iax\right.}{a(4i-3n)\sqrt[3]{c+a^2cx^2}}$$

```
[Out] -3*2^(2/3-1/2*I*n)*(1-I*a*x)^(2/3+1/2*I*n)*(a^2*x^2+1)^(1/3)*hypergeom([2/3
+1/2*I*n, 1/3+1/2*I*n],[5/3+1/2*I*n],1/2-1/2*I*a*x)/a/(4*I-3*n)/(a^2*c*x^2+
c)^(1/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int \frac{e^{n \arctan(ax)}}{\sqrt[3]{c+a^2cx^2}} \, dx = -\frac{3\, 2^{\frac{2}{3}-\frac{in}{2}} \sqrt[3]{a^2x^2+1}(1-iax)^{\frac{1}{6}(4+3in)} \, \text{Hypergeometric2F1}\left(\frac{1}{6}(3in+2), \frac{1}{6}(3in+4), \frac{1}{6}(3in+10), \frac{1}{2}(1-iax\right.}{a(-3n+4i)\sqrt[3]{a^2cx^2+c}}$$

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (-3*2^(2/3 - (I/2)*n)*(1 - I*a*x)^((4 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*Hyp
ergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (10 + (3*I)*n)/6, (1 - I*a
*x)/2])/(a*(4*I - 3*n)*(c + a^2*c*x^2)^(1/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+a^2x^2}\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{1+a^2x^2}}\,dx}{\sqrt[3]{c+a^2cx^2}}$$

$$= \frac{\sqrt[3]{1+a^2x^2}\int (1-iax)^{-\frac{1}{3}+\frac{in}{2}}(1+iax)^{-\frac{1}{3}-\frac{in}{2}}\,dx}{\sqrt[3]{c+a^2cx^2}}$$

$$= -\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(4+3in)}\sqrt[3]{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(2+3in),\frac{1}{6}(4+3in),\frac{1}{6}(10+3in),\frac{1}{2}(\right.}{a(4i-3n)\sqrt[3]{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx = \frac{3\,2^{\frac{2}{3}-\frac{in}{2}}(1-iax)^{\frac{2}{3}+\frac{in}{2}}\sqrt[3]{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{3}+\frac{in}{2},\frac{2}{3}+\frac{in}{2},\frac{5}{3}+\frac{in}{2},\frac{1}{2}-\frac{iax}{2}\right)}{a(-4i+3n)\sqrt[3]{c+a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(1/3),x]
```

```
[Out] (3*2^(2/3 - (I/2)*n)*(1 - I*a*x)^(2/3 + (I/2)*n)*(1 + a^2*x^2)^(1/3)*Hyperg
eometric2F1[1/3 + (I/2)*n, 2/3 + (I/2)*n, 5/3 + (I/2)*n, 1/2 - (I/2)*a*x])/
(a*(-4*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{(a^2 c\,x^2 + c)^{\frac{1}{3}}} dx$$

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x)
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt[3]{c + a^2 c x^2}} dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c)^{\frac{1}{3}}} dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="fricas")
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt[3]{c + a^2 c x^2}} dx = \int \frac{e^{n \operatorname{atan}(ax)}}{\sqrt[3]{c(a^2 x^2 + 1)}} dx$$

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/3),x)
[Out] Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(1/3), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt[3]{c + a^2 c x^2}} dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c)^{\frac{1}{3}}} dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx=\int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx=\int \frac{e^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{1/3}}\,dx$$

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/3),x)
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/3), x)
```

3.362 $\int \frac{e^{n \arctan(ax)}}{\left(c+a^2cx^2\right)^{2/3}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{e^{n \arctan(ax)}}{\left(c+a^2cx^2\right)^{2/3}} \, dx = -\frac{3\, 2^{\frac{1}{3}-\frac{in}{2}} (1-iax)^{\frac{1}{6}(2+3in)} \left(1+a^2x^2\right)^{2/3} \text{Hypergeometric2F1} \left(\frac{1}{6}(2+3in), \frac{1}{6}(4+3in), \frac{1}{6}(8+3in), \frac{1}{2}(1-iax\right.}{a(2i-3n) \left(c+a^2cx^2\right)^{2/3}}$$

```
[Out] -3*2^(1/3-1/2*I*n)*(1-I*a*x)^(1/3+1/2*I*n)*(a^2*x^2+1)^(2/3)*hypergeom([2/3
+1/2*I*n, 1/3+1/2*I*n],[4/3+1/2*I*n],1/2-1/2*I*a*x)/a/(2*I-3*n)/(a^2*c*x^2+
c)^(2/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int \frac{e^{n \arctan(ax)}}{\left(c+a^2cx^2\right)^{2/3}} \, dx = -\frac{3\, 2^{\frac{1}{3}-\frac{in}{2}} \left(a^2x^2+1\right)^{2/3} (1-iax)^{\frac{1}{6}(2+3in)} \text{Hypergeometric2F1} \left(\frac{1}{6}(3in+2), \frac{1}{6}(3in+4), \frac{1}{6}(3in+8), \frac{1}{2}(1-iax\right.}{a(-3n+2i) \left(a^2cx^2+c\right)^{2/3}}$$

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(2/3),x]
```

```
[Out] (-3*2^(1/3 - (I/2)*n)*(1 - I*a*x)^((2 + (3*I)*n)/6)*(1 + a^2*x^2)^(2/3)*Hyp
ergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (8 + (3*I)*n)/6, (1 - I*a*
x)/2])/(a*(2*I - 3*n)*(c + a^2*c*x^2)^(2/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{(1+a^2x^2)^{2/3}\int \frac{e^{n\arctan(ax)}}{(1+a^2x^2)^{2/3}}\,dx}{(c+a^2cx^2)^{2/3}}$$

$$= \frac{(1+a^2x^2)^{2/3}\int (1-iax)^{-\frac{2}{3}+\frac{in}{2}}(1+iax)^{-\frac{2}{3}-\frac{in}{2}}\,dx}{(c+a^2cx^2)^{2/3}}$$

$$= -\frac{3\,2^{\frac{1}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(2+3in)}\,(1+a^2x^2)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(2+3in),\frac{1}{6}(4+3in),\frac{1}{6}(8+3in),\frac{1}{2}\right.}{a(2i-3n)\,(c+a^2cx^2)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \frac{3\,2^{\frac{1}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{3}+\frac{in}{2}}\,(1+a^2x^2)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3}+\frac{in}{2},\frac{2}{3}+\frac{in}{2},\frac{4}{3}+\frac{in}{2},\frac{1}{2}-\frac{ia}{2}\right.}{a(-2i+3n)\,(c+a^2cx^2)^{2/3}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(2/3),x]
```

```
[Out] (3*2^(1/3 - (I/2)*n)*(1 - I*a*x)^(1/3 + (I/2)*n)*(1 + a^2*x^2)^(2/3)*Hyperg
eometric2F1[1/3 + (I/2)*n, 2/3 + (I/2)*n, 4/3 + (I/2)*n, 1/2 - (I/2)*a*x])/
(a*(-2*I + 3*n)*(c + a^2*c*x^2)^(2/3))
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(ax)}}{(a^{2} c\,x^{2}+c)^{\frac{2}{3}}}dx$$

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x)
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \int \frac{e^{(n \arctan(ax))}}{(a^{2} c x^{2} + c)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="fricas")
[Out] integral(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \int \frac{e^{n \operatorname{atan}{(ax)}}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(2/3),x)
[Out] Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(2/3), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \int \frac{e^{(n \arctan(ax))}}{(a^{2} c x^{2} + c)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx=\int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx=\int \frac{e^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{2/3}}\,dx$$

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(2/3),x)
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(2/3), x)
```

3.363 $\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(-2+3in)}\sqrt[3]{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in),\right.}{ac(2i+3n)\sqrt[3]{c+a^2cx^2}}$$

```
[Out] 3*2^(-1/3-1/2*I*n)*(1-I*a*x)^(-1/3+1/2*I*n)*(a^2*x^2+1)^(1/3)*hypergeom([4/
3+1/2*I*n, -1/3+1/2*I*n],[2/3+1/2*I*n],1/2-1/2*I*a*x)/a/c/(2*I+3*n)/(a^2*c*
x^2+c)^(1/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5184, 5181, 71}

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}(1-iax)^{\frac{1}{6}(-2+3in)}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8),\frac{1}{6}\right.}{ac(3n+2i)\sqrt[3]{a^2cx^2+c}}$$

```
[In] Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(4/3),x]
```

```
[Out] (3*2^(-1/3 - (I/2)*n)*(1 - I*a*x)^((-2 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*Hy
pergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (4 + (3*I)*n)/6, (1 - I*
a*x)/2])/(a*c*(2*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

<u>Rule 71</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+a^2x^2}\int \frac{e^{n\arctan(ax)}}{(1+a^2x^2)^{4/3}}\,dx}{c\sqrt[3]{c+a^2cx^2}}$$

$$= \frac{\sqrt[3]{1+a^2x^2}\int (1-iax)^{-\frac{4}{3}+\frac{in}{2}}(1+iax)^{-\frac{4}{3}-\frac{in}{2}}\,dx}{c\sqrt[3]{c+a^2cx^2}}$$

$$= \frac{3\ 2^{-\frac{1}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(-2+3in)}\sqrt[3]{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in),\frac{1}{6}(4+3in),\frac{1}{2}(\ldots\right.}{ac(2i+3n)\sqrt[3]{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \frac{3\ 2^{-\frac{1}{3}-\frac{in}{2}}(1-iax)^{-\frac{1}{3}+\frac{in}{2}}\sqrt[3]{1+a^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3}+\frac{in}{2},\frac{4}{3}+\frac{in}{2},\frac{2}{3}+\frac{in}{2},\frac{1}{2}-\ldots\right.}{ac(2i+3n)\sqrt[3]{c+a^2cx^2}}$$

```
[In] Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(4/3),x]
```

```
[Out] (3*2^(-1/3 - (I/2)*n)*(1 - I*a*x)^(-1/3 + (I/2)*n)*(1 + a^2*x^2)^(1/3)*Hype
rgeometric2F1[-1/3 + (I/2)*n, 4/3 + (I/2)*n, 2/3 + (I/2)*n, 1/2 - (I/2)*a*x
])/(a*c*(2*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{(a^2 c\, x^2 + c)^{\frac{4}{3}}} dx$$

```
[In] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x)
[Out] int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{(c + a^2 c x^2)^{4/3}}\, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c)^{\frac{4}{3}}}\, dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^(2/3)*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x
^2 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{(c + a^2 c x^2)^{4/3}}\, dx = \int \frac{e^{n \operatorname{atan}(ax)}}{(c\, (a^2 x^2 + 1))^{\frac{4}{3}}}\, dx$$

```
[In] integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(4/3),x)
[Out] Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(4/3), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{(c + a^2 c x^2)^{4/3}}\, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c)^{\frac{4}{3}}}\, dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="maxima")
[Out] integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(4/3), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{4/3}}\,dx$$

```
[In] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(4/3),x)
[Out] int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(4/3), x)
```

3.364 $\int e^{n\arctan(ax)}x^m(c+a^2cx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 49

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx = \frac{cx^{1+m}\,\text{AppellF1}\left(1+m,-1-\frac{in}{2},-1+\frac{in}{2},2+m,iax,-iax\right)}{1+m}$$

```
[Out] c*x^(1+m)*AppellF1(1+m,-1+1/2*I*n,-1-1/2*I*n,2+m,-I*a*x,I*a*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5190, 138}

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx = \frac{cx^{m+1}\,\text{AppellF1}\left(m+1,-\frac{in}{2}-1,\frac{in}{2}-1,m+2,iax,-iax\right)}{m+1}$$

```
[In] Int[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2),x]
```

```
[Out] (c*x^(1 + m)*AppellF1[1 + m, -1 - (I/2)*n, -1 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\int x^m(1-iax)^{1+\frac{in}{2}}(1+iax)^{1-\frac{in}{2}}\,dx\\ &= \frac{cx^{1+m}\,\text{AppellF1}\left(1+m,-1-\frac{in}{2},-1+\frac{in}{2},2+m,iax,-iax\right)}{1+m}\end{aligned}$$

Mathematica [F]

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx=\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx$$

```
[In] Integrate[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2),x]
[Out] Integrate[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2), x]
```

Maple [F]

$$\int \mathrm{e}^{n\arctan(ax)}x^m\left(a^2c\,x^2+c\right)dx$$

```
[In] int(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx=\int \left(a^2cx^2+c\right)x^me^{\left(n\arctan(ax)\right)}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx=c\left(\int x^me^{n\,\mathrm{atan}\,(ax)}\,dx+\int a^2x^2x^me^{n\,\mathrm{atan}\,(ax)}\,dx\right)$$

```
[In] integrate(exp(n*atan(a*x))*x**m*(a**2*c*x**2+c),x)
[Out] c*(Integral(x**m*exp(n*atan(a*x)), x) + Integral(a**2*x**2*x**m*exp(n*atan(
a*x)), x))
```

Maxima [F]

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)x^me^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)x^me^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx=\int x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

```
[In] int(x^m*exp(n*atan(a*x))*(c + a^2*c*x^2),x)
[Out] int(x^m*exp(n*atan(a*x))*(c + a^2*c*x^2), x)
```

3.365 $\int \frac{e^{n\arctan(ax)}x^m}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{e^{n\arctan(ax)}x^m}{c+a^2cx^2}\,dx = \frac{x^{1+m}\,\text{AppellF1}\left(1+m,1-\frac{in}{2},1+\frac{in}{2},2+m,iax,-iax\right)}{c(1+m)}$$

```
[Out] x^(1+m)*AppellF1(1+m,1+1/2*I*n,1-1/2*I*n,2+m,-I*a*x,I*a*x)/c/(1+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5190, 138}

$$\int \frac{e^{n\arctan(ax)}x^m}{c+a^2cx^2}\,dx = \frac{x^{m+1}\,\text{AppellF1}\left(m+1,1-\frac{in}{2},\frac{in}{2}+1,m+2,iax,-iax\right)}{c(m+1)}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1 - (I/2)*n, 1 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c*(1 + m))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\int x^m(1-iax)^{-1+\frac{in}{2}}(1+iax)^{-1-\frac{in}{2}}\,dx}{c}$$
$$= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,1-\frac{in}{2},1+\frac{in}{2},2+m,iax,-iax\right)}{c(1+m)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

$$\int \frac{e^{n\arctan(ax)}x^m}{c+a^2cx^2}\,dx$$
$$= \frac{e^{n\arctan(ax)}\left(1-e^{2i\arctan(ax)}\right)^{-m}\left(1+e^{2i\arctan(ax)}\right)^{m}x^m\,\text{AppellF1}\left(-\frac{in}{2},m,-m,1-\frac{in}{2},-e^{2i\arctan(ax)},e^{2i\arcta\ldots}\right.}{acn}$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2),x]
[Out] (E^(n*ArcTan[a*x])*(1 + E^((2*I)*ArcTan[a*x]))^m*x^m*AppellF1[(-1/2*I)*n, m
, -m, 1 - (I/2)*n, -E^((2*I)*ArcTan[a*x]), E^((2*I)*ArcTan[a*x])])/(a*c*(1
- E^((2*I)*ArcTan[a*x]))^m*n)
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{a^2c\,x^2+c}dx$$

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{c+a^2cx^2}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="fricas")

[Out] integral(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m e^{n\,\mathrm{atan}\,(ax)}}{a^2x^2+1}\,dx}{c}$$

[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c),x)

[Out] Integral(x**m*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{c+a^2cx^2}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="maxima")

[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{c+a^2cx^2}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^m}{c+a^2cx^2}\,dx = \int \frac{x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{c\,a^2\,x^2+c}\,dx$$

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

3.366 $\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx = \frac{x^{1+m}\,\text{AppellF1}\left(1+m,2-\frac{in}{2},2+\frac{in}{2},2+m,iax,-iax\right)}{c^2(1+m)}$$

```
[Out] x^(1+m)*AppellF1(1+m,2+1/2*I*n,2-1/2*I*n,2+m,-I*a*x,I*a*x)/c^2/(1+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5190, 138}

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx = \frac{x^{m+1}\,\text{AppellF1}\left(m+1,2-\frac{in}{2},\frac{in}{2}+2,m+2,iax,-iax\right)}{c^2(m+1)}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 2 - (I/2)*n, 2 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c^2*(1 + m))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int x^m(1-iax)^{-2+\frac{in}{2}}(1+iax)^{-2-\frac{in}{2}}\,dx}{c^2}\\ &= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,2-\frac{in}{2},2+\frac{in}{2},2+m,iax,-iax\right)}{c^2(1+m)}\end{aligned}$$

Mathematica [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx=\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2,x]
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2, x]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\arctan(ax)}x^m}{(a^2c\,x^2+c)^2}dx$$

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx=\int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(x^m*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m e^{n\,\mathrm{atan}\,(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**2,x)
[Out] Integral(x**m*exp(n*atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m\,e^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^2}\,dx$$

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^2,x)
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^2, x)
```

3.367 $\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^3} \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(1+m, 3-\frac{in}{2}, 3+\frac{in}{2}, 2+m, iax, -iax\right)}{c^3(1+m)}$$

```
[Out] x^(1+m)*AppellF1(1+m,3+1/2*I*n,3-1/2*I*n,2+m,-I*a*x,I*a*x)/c^3/(1+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5190, 138}

$$\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^3} \, dx = \frac{x^{m+1} \operatorname{AppellF1}\left(m+1, 3-\frac{in}{2}, \frac{in}{2}+3, m+2, iax, -iax\right)}{c^3(m+1)}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3 - (I/2)*n, 3 + (I/2)*n, 2 + m, I*a*x, (-I)*a*x
])/(c^3*(1 + m))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int x^m(1-iax)^{-3+\frac{in}{2}}(1+iax)^{-3-\frac{in}{2}}\,dx}{c^3}\\ &= \frac{x^{1+m}\,\text{AppellF1}\left(1+m,3-\frac{in}{2},3+\frac{in}{2},2+m,iax,-iax\right)}{c^3(1+m)}\end{aligned}$$

Mathematica [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^3}\,dx$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3,x]
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3, x]
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(a^2c\,x^2+c)^3}dx$$

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] integral(x^m*e^(n*arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^m e^{n\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**3,x)
[Out] Integral(x**m*exp(n*atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{\left(c+a^2cx^2\right)^3}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{\left(c+a^2cx^2\right)^3}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{\left(a^2cx^2+c\right)^3}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^m}{\left(c+a^2cx^2\right)^3}\,dx = \int \frac{x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^3,x)
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^3, x)
```

3.368 $\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c+a^2cx^2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c+a^2cx^2}} dx = \frac{x^{1+m}\sqrt{1+a^2x^2} \operatorname{AppellF1}\left(1+m, \frac{1}{2}(1-in), \frac{1}{2}(1+in), 2+m, iax, -iax\right)}{(1+m)\sqrt{c+a^2cx^2}}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/2+1/2*I*n,1/2-1/2*I*n,2+m,-I*a*x,I*a*x)*(a^2*x^2+1)^
(1/2)/(1+m)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5193, 5190, 138}

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c+a^2cx^2}} dx = \frac{\sqrt{a^2x^2+1} x^{m+1} \operatorname{AppellF1}\left(m+1, \frac{1}{2}(1-in), \frac{1}{2}(in+1), m+2, iax, -iax\right)}{(m+1)\sqrt{a^2cx^2+c}}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (1 - I*n)/2, (1 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/((1 + m)*Sqrt[c + a^2*c*x^2])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
```

```
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}x^m}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int x^m(1-iax)^{-\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\,dx}{\sqrt{c+a^2cx^2}}\\
&= \frac{x^{1+m}\sqrt{1+a^2x^2}\,\text{AppellF1}\left(1+m,\frac{1}{2}(1-in),\frac{1}{2}(1+in),2+m,iax,-iax\right)}{(1+m)\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{e^{n\arctan(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\arctan(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx
$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2],x]
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2], x]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(ax)} x^m}{\sqrt{a^2 c\, x^2 + c}} dx$$

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c + a^2 c x^2}}\, dx = \int \frac{x^m e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c + a^2 c x^2}}\, dx = \int \frac{x^m e^{n \operatorname{atan}(ax)}}{\sqrt{c (a^2 x^2 + 1)}}\, dx$$

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**m*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c + a^2 c x^2}}\, dx = \int \frac{x^m e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c + a^2 c x^2}}\, dx = \int \frac{x^m e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c + a^2 c x^2}}\, dx = \int \frac{x^m\, \mathrm{e}^{n\, \mathrm{atan}(a\, x)}}{\sqrt{c\, a^2\, x^2 + c}}\, dx$$

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

3.369 $\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x^{1+m}\sqrt{1+a^2x^2}\,\mathrm{AppellF1}\left(1+m,\frac{1}{2}(3-in),\frac{1}{2}(3+in),2+m,iax,-iax\right)}{c(1+m)\sqrt{c+a^2cx^2}}$$

```
[Out] x^(1+m)*AppellF1(1+m,3/2+1/2*I*n,3/2-1/2*I*n,2+m,-I*a*x,I*a*x)*(a^2*x^2+1)^
(1/2)/c/(1+m)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5193, 5190, 138}

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2x^2+1}x^{m+1}\,\mathrm{AppellF1}\left(m+1,\frac{1}{2}(3-in),\frac{1}{2}(in+3),m+2,iax,-iax\right)}{c(m+1)\sqrt{a^2cx^2+c}}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (3 - I*n)/2, (3 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/(c*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}x^m}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int x^m(1-iax)^{-\frac{3}{2}+\frac{in}{2}}(1+iax)^{-\frac{3}{2}-\frac{in}{2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{x^{1+m}\sqrt{1+a^2x^2}\,\text{AppellF1}\left(1+m,\frac{1}{2}(3-in),\frac{1}{2}(3+in),2+m,iax,-iax\right)}{c(1+m)\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2),x]
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2
*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m e^{n\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**m*exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(3/2),x)
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(3/2), x)
```

3.370 $\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^{1+m}\sqrt{1+a^2x^2}\,\text{AppellF1}\left(1+m,\frac{1}{2}(5-in),\frac{1}{2}(5+in),2+m,iax,-iax\right)}{c^2(1+m)\sqrt{c+a^2cx^2}}$$

```
[Out] x^(1+m)*AppellF1(1+m,5/2+1/2*I*n,5/2-1/2*I*n,2+m,-I*a*x,I*a*x)*(a^2*x^2+1)^
(1/2)/c^2/(1+m)/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5193, 5190, 138}

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{a^2x^2+1}x^{m+1}\,\text{AppellF1}\left(m+1,\frac{1}{2}(5-in),\frac{1}{2}(in+5),m+2,iax,-iax\right)}{c^2(m+1)\sqrt{a^2cx^2+c}}$$

```
[In] Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (5 - I*n)/2, (5 + I*n)/2, 2 +
m, I*a*x, (-I)*a*x])/(c^2*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\int \frac{e^{n\arctan(ax)}x^m}{(1+a^2x^2)^{5/2}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int x^m(1-iax)^{-\frac{5}{2}+\frac{in}{2}}(1+iax)^{-\frac{5}{2}-\frac{in}{2}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$
$$= \frac{x^{1+m}\sqrt{1+a^2x^2}\,\text{AppellF1}\left(1+m,\frac{1}{2}(5-in),\frac{1}{2}(5+in),2+m,iax,-iax\right)}{c^2(1+m)\sqrt{c+a^2cx^2}}$$

Mathematica [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx$$

```
[In] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2),x]
[Out] Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2), x]
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

```
[In] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x)
[Out] int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3
*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

```
[In] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(5/2),x)
[Out] int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(5/2), x)
```

3.371 $\int e^{n \arctan(ax)}(c+a^2cx^2)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int e^{n \arctan(ax)}(c+a^2cx^2)^p \, dx = \frac{2^{1-\frac{in}{2}+p}(1-iax)^{1+\frac{in}{2}+p}(1+a^2x^2)^{-p}(c+a^2cx^2)^p \text{Hypergeometric2F1}\left(\frac{in}{2}-p, 1+\frac{in}{2}+p, 2+\frac{in}{2}+p, \frac{1}{2}(1-\right.}{a(n-2i(1+p))}$$

```
[Out] 2^(1-1/2*I*n+p)*(1-I*a*x)^(1+1/2*I*n+p)*(a^2*c*x^2+c)^p*hypergeom([1/2*I*n-
p, 1+1/2*I*n+p],[2+1/2*I*n+p],1/2-1/2*I*a*x)/a/(n-2*I*(p+1))/((a^2*x^2+1)^p
)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5184, 5181, 71}

$$\int e^{n \arctan(ax)}(c+a^2cx^2)^p \, dx = \frac{2^{-\frac{in}{2}+p+1}(a^2x^2+1)^{-p}(a^2cx^2+c)^p(1-iax)^{\frac{in}{2}+p+1} \text{Hypergeometric2F1}\left(\frac{in}{2}-p, \frac{in}{2}+p+1, \frac{in}{2}+p+2, \frac{1}{2}(1\right.}{a(n-2i(p+1))}$$

```
[In] Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 - (I/2)*n + p)*(1 - I*a*x)^(1 + (I/2)*n + p)*(c + a^2*c*x^2)^p*Hyperg
eometric2F1[(I/2)*n - p, 1 + (I/2)*n + p, 2 + (I/2)*n + p, (1 - I*a*x)/2])/
(a*(n - (2*I)*(1 + p))*(1 + a^2*x^2)^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{n\arctan(ax)}\left(1+a^2x^2\right)^p\,dx$$

$$= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1-iax)^{\frac{in}{2}+p}(1+iax)^{-\frac{in}{2}+p}\,dx$$

$$= \frac{2^{1-\frac{in}{2}+p}(1-iax)^{1+\frac{in}{2}+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{in}{2}-p,1+\frac{in}{2}+p,2+\frac{in}{2}\right.}{a(n-2i(1+p))}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$= \frac{2^{1-\frac{in}{2}+p}(1-iax)^{1+\frac{in}{2}+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p \text{Hypergeometric2F1}\left(\frac{in}{2}-p,1+\frac{in}{2}+p,2+\frac{in}{2}+p,\frac{1}{2}(1\right.}{a(n-2i(1+p))}$$

```
[In] Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 - (I/2)*n + p)*(1 - I*a*x)^(1 + (I/2)*n + p)*(c + a^2*c*x^2)^p*Hyperg
eometric2F1[(I/2)*n - p, 1 + (I/2)*n + p, 2 + (I/2)*n + p, (1 - I*a*x)/2])/
(a*(n - (2*I)*(1 + p))*(1 + a^2*x^2)^p)
```

Maple [F]

$$\int e^{n \arctan(ax)} \left(a^2 c\, x^2 + c\right)^p dx$$

```
[In] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x)
[Out] int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^p \, dx = \int \left(a^2 c x^2 + c\right)^p e^{(n \arctan(ax))} \, dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^p \, dx = \int \left(c\left(a^2 x^2 + 1\right)\right)^p e^{n \operatorname{atan}(ax)} \, dx$$

```
[In] integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**p,x)
[Out] Integral((c*(a**2*x**2 + 1))**p*exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^p \, dx = \int \left(a^2 c x^2 + c\right)^p e^{(n \arctan(ax))} \, dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^p e^{(n\arctan(ax))}\,dx$$

```
[In] integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^p\,dx$$

```
[In] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^p,x)
[Out] int(exp(n*atan(a*x))*(c + a^2*c*x^2)^p, x)
```

3.372 $\int e^{-2ip\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{i(1-iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}$$

```
[Out] I*(1-I*a*x)^(1+2*p)*(a^2*c*x^2+c)^p/a/(1+2*p)/((a^2*x^2+1)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5184, 5181, 32}

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{i(1-iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

```
[In] Int[(c + a^2*c*x^2)^p/E^((2*I)*p*ArcTan[a*x]),x]
[Out] (I*(1 - I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^p)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{-2ip\arctan(ax)}\left(1+a^2x^2\right)^p\,dx\\ &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int(1-iax)^{2p}\,dx\\ &= \frac{i(1-iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{e^{-2ip\arctan(ax)}(i+ax)\left(c+a^2cx^2\right)^p}{a+2ap}$$

```
[In] Integrate[(c + a^2*c*x^2)^p/E^((2*I)*p*ArcTan[a*x]),x]
[Out] ((I + a*x)*(c + a^2*c*x^2)^p)/(E^((2*I)*p*ArcTan[a*x])*(a + 2*a*p))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{(ax+i)\left(a^2c\,x^2+c\right)^p\mathrm{e}^{-2ip\arctan(ax)}}{a(1+2p)}$
parallelrisch	$-\frac{\left(-x\left(a^2c\,x^2+c\right)^pa-i\left(a^2c\,x^2+c\right)^p\right)\mathrm{e}^{-2ip\arctan(ax)}}{a(1+2p)}$
risch	$\frac{((ax+i)^p)^2c^p(ax+i)\mathrm{e}^{-\frac{ip\pi\left(\mathrm{csgn}(ax+i)^3-\mathrm{csgn}(ax+i)^2\,\mathrm{csgn}(i(ax+i))+\mathrm{csgn}(i(ax+i))\,\mathrm{csgn}(i(ax-i))\,\mathrm{csgn}(i(ax-i)(ax+i))-\mathrm{csgn}(i(ax+i))\,\mathrm{c}}{}}}{}$

```
[In] int((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x,method=_RETURNVERBOSE)
[Out] (I+a*x)/a/(1+2*p)*(a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{(ax+i)(a^2cx^2+c)^p\left(-\frac{ax+i}{ax-i}\right)^p}{2\,ap+a}$$

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="fricas")
[Out] (a*x + I)*(a^2*c*x^2 + c)^p*(-(a*x + I)/(a*x - I))^p/(2*a*p + a)
```

Sympy [F]

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$=\begin{cases}\frac{x}{\sqrt{c}} & \text{for } a=0\wedge p=-\frac{1}{2}\\ c^px & \text{for } a=0\\ \int\frac{e^{i\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx & \text{for } p=-\frac{1}{2}\\ \frac{ax(a^2cx^2+c)^p}{2ape^{2ip\operatorname{atan}(ax)}+ae^{2ip\operatorname{atan}(ax)}}+\frac{i(a^2cx^2+c)^p}{2ape^{2ip\operatorname{atan}(ax)}+ae^{2ip\operatorname{atan}(ax)}} & \text{otherwise}\end{cases}$$

```
[In] integrate((a**2*c*x**2+c)**p/exp(2*I*p*atan(a*x)),x)
[Out] Piecewise((x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x, Eq(a, 0)), (Integra
l(exp(I*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x), Eq(p, -1/2)), (a*x*(a**2*c*
x**2 + c)**p/(2*a*p*exp(2*I*p*atan(a*x)) + a*exp(2*I*p*atan(a*x))) + I*(a**
2*c*x**2 + c)**p/(2*a*p*exp(2*I*p*atan(a*x)) + a*exp(2*I*p*atan(a*x))), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$=\frac{(ac^px+i\,c^p)(a^2x^2+1)^p\cos\left(2\,p\arctan\left(ax\right)\right)-(i\,ac^px-c^p)(a^2x^2+1)^p\sin\left(2\,p\arctan\left(ax\right)\right)}{2\,ap+a}$$

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="maxima")
[Out] ((a*c^p*x + I*c^p)*(a^2*x^2 + 1)^p*cos(2*p*arctan(a*x)) - (I*a*c^p*x - c^p)
*(a^2*x^2 + 1)^p*sin(2*p*arctan(a*x)))/(2*a*p + a)
```

Giac [F]

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(-2i\,p\arctan(ax))}\,dx$$

```
[In] integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\left(\frac{x\,\mathrm{e}^{-p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}}{2\,p+1}+\frac{\mathrm{e}^{-p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}\,1\mathrm{i}}{a\,\left(2\,p+1\right)}\right)\,\left(c\,a^2\,x^2+c\right)^p$$

```
[In] int(exp(-p*atan(a*x)*2i)*(c + a^2*c*x^2)^p,x)
[Out] ((x*exp(-p*atan(a*x)*2i))/(2*p + 1) + (exp(-p*atan(a*x)*2i)*1i)/(a*(2*p + 1
)))*(c + a^2*c*x^2)^p
```

3.373 $\int e^{2ip\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=-\frac{i(1+iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}$$

```
[Out] -I*(1+I*a*x)^(1+2*p)*(a^2*c*x^2+c)^p/a/(1+2*p)/((a^2*x^2+1)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5184, 5181, 32}

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=-\frac{i(1+iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

```
[In] Int[E^((2*I)*p*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

```
[Out] ((-I)*(1 + I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^p
)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5181

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x] /
```

```
; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 5184

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> D
ist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p]), Int[(
1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && Eq
Q[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int e^{2ip\arctan(ax)}\left(1+a^2x^2\right)^p\,dx \\
&= \left(\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\right)\int (1+iax)^{2p}\,dx \\
&= -\frac{i(1+iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$
\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{e^{2ip\arctan(ax)}(-i+ax)\left(c+a^2cx^2\right)^p}{a+2ap}
$$

```
[In] Integrate[E^((2*I)*p*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
[Out] (E^((2*I)*p*ArcTan[a*x])*(-I + a*x)*(c + a^2*c*x^2)^p)/(a + 2*a*p)
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

method	result
gosper	$-\frac{(-ax+i)\mathrm{e}^{2ip\arctan(ax)}\left(a^2c\,x^2+c\right)^p}{a(1+2p)}$
parallelrisch	$-\frac{-\mathrm{e}^{2ip\arctan(ax)}x\left(a^2c\,x^2+c\right)^p a+i\mathrm{e}^{2ip\arctan(ax)}\left(a^2c\,x^2+c\right)^p}{a(1+2p)}$
risch	$(ax+i)^p c^p (ax-i)^{2p}(ax+i)^{-p}(ax-i)\mathrm{e}^{-\frac{ip\pi\left(-\operatorname{csgn}(ax+i)^3+\operatorname{csgn}(ax+i)^2\operatorname{csgn}(i(ax+i))+\operatorname{csgn}(i(ax+i))\operatorname{csgn}(i(ax-i))\operatorname{csgn}(i(ax-i)(ax+\right.}{}}$

```
[In] int(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x,method=_RETURNVERBOSE)
[Out] -(I-a*x)/a/(1+2*p)*exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{(ax-i)(a^2cx^2+c)^p}{(2\,ap+a)\left(-\frac{ax+i}{ax-i}\right)^p}$$

```
[In] integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] (a*x - I)*(a^2*c*x^2 + c)^p/((2*a*p + a)*(-(a*x + I)/(a*x - I))^p)
```

Sympy [F]

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$=\begin{cases}\frac{x}{\sqrt{c}} & \text{for } a=0\wedge p=-\frac{1}{2}\\ c^px & \text{for } a=0\\ \int\frac{e^{-i\operatorname{atan}{(ax)}}}{\sqrt{c(a^2x^2+1)}}\,dx & \text{for } p=-\frac{1}{2}\\ \frac{ax(a^2cx^2+c)^pe^{2ip\operatorname{atan}{(ax)}}}{2ap+a}-\frac{i(a^2cx^2+c)^pe^{2ip\operatorname{atan}{(ax)}}}{2ap+a} & \text{otherwise}\end{cases}$$

```
[In] integrate(exp(2*I*p*atan(a*x))*(a**2*c*x**2+c)**p,x)
[Out] Piecewise((x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x, Eq(a, 0)), (Integra
l(exp(-I*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x), Eq(p, -1/2)), (a*x*(a**2*c
*x**2 + c)**p*exp(2*I*p*atan(a*x))/(2*a*p + a) - I*(a**2*c*x**2 + c)**p*exp
(2*I*p*atan(a*x))/(2*a*p + a), True))
```

Maxima [F]

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(2i\,p\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 + c)^p*e^(2*I*p*arctan(a*x)), x)
```

Giac [F]

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(2i\,p\arctan(ax))}\,dx$$

```
[In] integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\left(\frac{x\,\mathrm{e}^{p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}}{2\,p+1}-\frac{\mathrm{e}^{p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}\,1\mathrm{i}}{a\,\left(2\,p+1\right)}\right)\,\left(c\,a^2\,x^2+c\right)^p$$

```
[In] int(exp(p*atan(a*x)*2i)*(c + a^2*c*x^2)^p,x)
[Out] ((x*exp(p*atan(a*x)*2i))/(2*p + 1) - (exp(p*atan(a*x)*2i)*1i)/(a*(2*p + 1))
)*(c + a^2*c*x^2)^p
```

3.374 $\int e^{in\arctan(ax)}x^2(c+a^2cx^2)^{-1-\frac{n^2}{2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 60

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\frac{ie^{in\arctan(ax)}(1-ianx)\left(c+a^2cx^2\right)^{-\frac{n^2}{2}}}{a^3cn\,(1-n^2)}$$

```
[Out] I*exp(I*n*arctan(a*x))*(1-I*a*n*x)/a^3/c/n/(-n^2+1)/((a^2*c*x^2+c)^(1/2*n^2
))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}=0.029$, Rules used = {5187}

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\frac{i(1-ianx)e^{in\arctan(ax)}\left(a^2cx^2+c\right)^{-\frac{n^2}{2}}}{a^3cn\,(1-n^2)}$$

```
[In] Int[E^(I*n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(-1 - n^2/2),x]
```

```
[Out] (I*E^(I*n*ArcTan[a*x])*(1 - I*a*n*x))/(a^3*c*n*(1 - n^2)*(c + a^2*c*x^2)^(n
^2/2))
```

Rule 5187

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[(-(1 - a*n*x))*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*d*n*(n
^2 + 1))), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && EqQ[n^2 - 2*(p
+ 1), 0] &&  !IntegerQ[I*n]
```

Rubi steps

$$\text{integral} = \frac{ie^{in\arctan(ax)}(1-ianx)\left(c+a^2cx^2\right)^{-\frac{n^2}{2}}}{a^3cn\left(1-n^2\right)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx = -\frac{e^{in\arctan(ax)}(i+anx)\left(c+a^2cx^2\right)^{-\frac{n^2}{2}}}{a^3cn\left(-1+n^2\right)}$$

```
[In] Integrate[E^(I*n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(-1 - n^2/2),x]
[Out] -((E^(I*n*ArcTan[a*x])*(I + a*n*x))/(a^3*c*n*(-1 + n^2)*(c + a^2*c*x^2)^(n^
2/2)))
```

Maple [A] (verified)

Time = 5.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{(-ax+i)(ax+i)(nax+i)\mathrm{e}^{in\arctan(ax)}\left(a^2c\,x^2+c\right)^{-1-\frac{n^2}{2}}}{a^3n(n^2-1)}$
parallelrisch	$-\frac{\mathrm{e}^{in\arctan(ax)}x^3\left(a^2c\,x^2+c\right)^{-1-\frac{n^2}{2}}a^3n+i\mathrm{e}^{in\arctan(ax)}x^2\left(a^2c\,x^2+c\right)^{-1-\frac{n^2}{2}}a^2+\mathrm{e}^{in\arctan(ax)}\left(a^2c\,x^2+c\right)^{-1-\frac{n^2}{2}}xan+i\mathrm{e}^{in\,\mathrm{ar}}\cdots}{a^3n(n^2-1)}$

```
[In] int(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x,method=_RETURNVER
BOSE)
[Out] (I-a*x)*(I+a*x)*(n*a*x+I)*exp(I*n*arctan(a*x))*(a^2*c*x^2+c)^(-1-1/2*n^2)/a
^3/n/(n^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx = -\frac{(a^3nx^3+i\,a^2x^2+anx+i)(a^2cx^2+c)^{-\frac{1}{2}\,n^2-1}}{(a^3n^3-a^3n)\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}\,n}}$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"fricas")
```

```
[Out] -(a^3*n*x^3 + I*a^2*x^2 + a*n*x + I)*(a^2*c*x^2 + c)^(-1/2*n^2 - 1)/((a^3*n
^3 - a^3*n)*(-(a*x + I)/(a*x - I))^(1/2*n))
```

Sympy [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\text{Timed out}$$

```
[In] integrate(exp(I*n*atan(a*x))*x**2*(a**2*c*x**2+c)**(-1-1/2*n**2),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\int\left(a^2cx^2+c\right)^{-\frac{1}{2}n^2-1}x^2e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"maxima")
[Out] integrate((a^2*c*x^2 + c)^(-1/2*n^2 - 1)*x^2*e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\int\left(a^2cx^2+c\right)^{-\frac{1}{2}n^2-1}x^2e^{(i\,n\arctan(ax))}\,dx$$

```
[In] integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\int\frac{x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{{\left(c\,a^2\,x^2+c\right)}^{\frac{n^2}{2}+1}}\,dx$$

```
[In] int((x^2*exp(n*atan(a*x)*1i))/(c + a^2*c*x^2)^(n^2/2 + 1),x)
[Out] int((x^2*exp(n*atan(a*x)*1i))/(c + a^2*c*x^2)^(n^2/2 + 1), x)
```

3.375 $\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx = -\frac{i+6ax}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}$$

```
[Out] 1/210*(-I-6*a*x)/a^3/c^19/(1-I*a*x)^21/(1+I*a*x)^15
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5190, 82}

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx = -\frac{6ax+i}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}$$

```
[In] Int[(E^((6*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^19,x]
```

```
[Out] -1/210*(I + 6*a*x)/(a^3*c^19*(1 - I*a*x)^21*(1 + I*a*x)^15)
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{x^2}{(1-iax)^{22}(1+iax)^{16}}\,dx}{c^{19}}$$

$$= -\frac{i+6ax}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx = \frac{i+6ax}{210a^3c^{19}(-i+ax)^{15}(i+ax)^{21}}$$

```
[In] Integrate[(E^((6*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^19,x]
[Out] (I + 6*a*x)/(210*a^3*c^19*(-I + a*x)^15*(I + a*x)^21)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{x}{35a^2}+\frac{i}{210a^3}}{c^{19}(ax+i)^{21}(ax-i)^{15}}$
risch	$\frac{\frac{x}{35a^2}+\frac{i}{210a^3}}{c^{19}(ax+i)^{21}(ax-i)^{15}}$
gosper	$\frac{(-ax+i)(ax+i)(6ax+i)(iax+1)^6}{210a^3(a^2x^2+1)^{22}c^{19}}$
parallelrisch	$\frac{ix^{42}a^{39}+21ix^{40}a^{37}+210ix^{38}a^{35}+1330ix^{36}a^{33}+5985ix^{34}a^{31}+20349ix^{32}a^{29}+54264ix^{30}a^{27}+116280ix^{28}a^{25}+203490ix^{26}a^{23}+}{}$

```
[In] int((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x,method=_RETURNVERBOSE)
[Out] 1/c^19*(1/35*x/a^2+1/210*I/a^3)/(I+a*x)^21/(a*x-I)^15
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(30) = 60.

Time = 0.62 (sec) , antiderivative size = 379, normalized size of antiderivative = 9.97

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx$$

$$= \frac{}{210\,(a^{39}c^{19}x^{36} + 6i\,a^{38}c^{19}x^{35} + 70i\,a^{36}c^{19}x^{33} - 105\,a^{35}c^{19}x^{32} + 336i\,a^{34}c^{19}x^{31} - 896\,a^{33}c^{19}x^{30} + 720i\,a^{32}c^{19}}$$

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="fric
as")
```

```
[Out] 1/210*(6*a*x + I)/(a^39*c^19*x^36 + 6*I*a^38*c^19*x^35 + 70*I*a^36*c^19*x^3
3 - 105*a^35*c^19*x^32 + 336*I*a^34*c^19*x^31 - 896*a^33*c^19*x^30 + 720*I*
a^32*c^19*x^29 - 3900*a^31*c^19*x^28 - 280*I*a^30*c^19*x^27 - 10752*a^29*c^
19*x^26 - 6552*I*a^28*c^19*x^25 - 20020*a^27*c^19*x^24 - 21840*I*a^26*c^19*
x^23 - 24960*a^25*c^19*x^22 - 43472*I*a^24*c^19*x^21 - 18018*a^23*c^19*x^20
 - 60060*I*a^22*c^19*x^19 - 60060*I*a^20*c^19*x^17 + 18018*a^19*c^19*x^16 -
 43472*I*a^18*c^19*x^15 + 24960*a^17*c^19*x^14 - 21840*I*a^16*c^19*x^13 + 2
0020*a^15*c^19*x^12 - 6552*I*a^14*c^19*x^11 + 10752*a^13*c^19*x^10 - 280*I*
a^12*c^19*x^9 + 3900*a^11*c^19*x^8 + 720*I*a^10*c^19*x^7 + 896*a^9*c^19*x^6
 + 336*I*a^8*c^19*x^5 + 105*a^7*c^19*x^4 + 70*I*a^6*c^19*x^3 + 6*I*a^4*c^19
*x - a^3*c^19)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(32) = 64.

Time = 2.71 (sec) , antiderivative size = 439, normalized size of antiderivative = 11.55

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx =$$

$$-\frac{}{210a^{39}c^{19}x^{36} + 1260ia^{38}c^{19}x^{35} + 14700ia^{36}c^{19}x^{33} - 22050a^{35}c^{19}x^{32} + 70560ia^{34}c^{19}x^{31} - 188160a^{33}c^{19}x}$$

```
[In] integrate((1+I*a*x)**6/(a**2*x**2+1)**3*x**2/(a**2*c*x**2+c)**19,x)
```

```
[Out] -(-6*a*x - I)/(210*a**39*c**19*x**36 + 1260*I*a**38*c**19*x**35 + 14700*I*a
**36*c**19*x**33 - 22050*a**35*c**19*x**32 + 70560*I*a**34*c**19*x**31 - 18
8160*a**33*c**19*x**30 + 151200*I*a**32*c**19*x**29 - 819000*a**31*c**19*x*
*28 - 58800*I*a**30*c**19*x**27 - 2257920*a**29*c**19*x**26 - 1375920*I*a**
28*c**19*x**25 - 4204200*a**27*c**19*x**24 - 4586400*I*a**26*c**19*x**23 -
```

```
5241600*a**25*c**19*x**22 - 9129120*I*a**24*c**19*x**21 - 3783780*a**23*c**
19*x**20 - 12612600*I*a**22*c**19*x**19 - 12612600*I*a**20*c**19*x**17 + 37
83780*a**19*c**19*x**16 - 9129120*I*a**18*c**19*x**15 + 5241600*a**17*c**19
*x**14 - 4586400*I*a**16*c**19*x**13 + 4204200*a**15*c**19*x**12 - 1375920*
I*a**14*c**19*x**11 + 2257920*a**13*c**19*x**10 - 58800*I*a**12*c**19*x**9
+ 819000*a**11*c**19*x**8 + 151200*I*a**10*c**19*x**7 + 188160*a**9*c**19*x
**6 + 70560*I*a**8*c**19*x**5 + 22050*a**7*c**19*x**4 + 14700*I*a**6*c**19*
x**3 + 1260*I*a**4*c**19*x - 210*a**3*c**19)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(30) = 60$.

Time = 0.35 (sec) , antiderivative size = 292, normalized size of antiderivative = 7.68

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx$$

$$= \frac{}{210\,(a^{45}c^{19}x^{42} + 21\,a^{43}c^{19}x^{40} + 210\,a^{41}c^{19}x^{38} + 1330\,a^{39}c^{19}x^{36} + 5985\,a^{37}c^{19}x^{34} + 20349\,a^{35}c^{19}x^{32} + 54264\,a}$$

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="maxi
ma")
```

```
[Out] 1/210*(6*a^7*x^7 - 35*I*a^6*x^6 - 84*a^5*x^5 + 105*I*a^4*x^4 + 70*a^3*x^3 -
 21*I*a^2*x^2 - I)/(a^45*c^19*x^42 + 21*a^43*c^19*x^40 + 210*a^41*c^19*x^38
 + 1330*a^39*c^19*x^36 + 5985*a^37*c^19*x^34 + 20349*a^35*c^19*x^32 + 54264
*a^33*c^19*x^30 + 116280*a^31*c^19*x^28 + 203490*a^29*c^19*x^26 + 293930*a^
27*c^19*x^24 + 352716*a^25*c^19*x^22 + 352716*a^23*c^19*x^20 + 293930*a^21*
c^19*x^18 + 203490*a^19*c^19*x^16 + 116280*a^17*c^19*x^14 + 54264*a^15*c^19
*x^12 + 20349*a^13*c^19*x^10 + 5985*a^11*c^19*x^8 + 1330*a^9*c^19*x^6 + 210
*a^7*c^19*x^4 + 21*a^5*c^19*x^2 + a^3*c^19)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 7.87

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx =$$

$$-\frac{358229025\,a^{14}x^{14} - 5340869100i\,a^{13}x^{13} - 37114698075\,a^{12}x^{12} + 159416118225i\,a^{11}x^{11} + 473088806190\,a}{}$$

$$+\frac{358229025\,a^{20}x^{20} + 7555375800i\,a^{19}x^{19} - 75901131600\,a^{18}x^{18} - 483051354975i\,a^{17}x^{17} + 2184946607340}{}$$

```
[In] integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="giac
")
```

```
[Out] -1/901943132160*(358229025*a^14*x^14 - 5340869100*I*a^13*x^13 - 37114698075
*a^12*x^12 + 159416118225*I*a^11*x^11 + 473088806190*a^10*x^10 - 1026819468
675*I*a^9*x^9 - 1682288472150*a^8*x^8 + 2115551402250*I*a^7*x^7 + 205443504
6125*a^6*x^6 - 1535397250002*I*a^5*x^5 - 870854759775*a^4*x^4 + 36430753320
5*I*a^3*x^3 + 106553746740*a^2*x^2 - 19571887695*I*a*x - 1710785408)/((a*x
- I)^15*a^3*c^19) + 1/901943132160*(358229025*a^20*x^20 + 7555375800*I*a^19
*x^19 - 75901131600*a^18*x^18 - 483051354975*I*a^17*x^17 + 2184946607340*a^
16*x^16 + 7469205450840*I*a^15*x^15 - 20031221295000*a^14*x^14 - 4317700403
7300*I*a^13*x^13 + 76013078916950*a^12*x^12 + 110448380006328*I*a^11*x^11 -
 133277726128008*a^10*x^10 - 133908931763530*I*a^9*x^9 + 111933156213900*a^
8*x^8 + 77492989590120*I*a^7*x^7 - 44041557267624*a^6*x^6 - 20244576347604*
I*a^5*x^5 + 7349182966545*a^4*x^4 + 2026362494800*I*a^3*x^3 - 396520754280*
a^2*x^2 - 48177926223*I*a*x + 2584181888)/((a*x + I)^21*a^3*c^19)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx = \text{Hanged}$$

```
[In] int((x^2*(a*x*1i + 1)^6)/((c + a^2*c*x^2)^19*(a^2*x^2 + 1)^3),x)
```

```
[Out] \text{Hanged}
```

3.376 $\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{i+4ax}{60a^3c^9(1-iax)^{10}(1+iax)^6}$$

```
[Out] 1/60*(-I-4*a*x)/a^3/c^9/(1-I*a*x)^10/(1+I*a*x)^6
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5190, 82}

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{4ax+i}{60a^3c^9(1-iax)^{10}(1+iax)^6}$$

```
[In] Int[(E^((4*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^9,x]
```

```
[Out] -1/60*(I + 4*a*x)/(a^3*c^9*(1 - I*a*x)^10*(1 + I*a*x)^6)
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{x^2}{(1-iax)^{11}(1+iax)^7}\,dx}{c^9}\\ &= -\frac{i+4ax}{60a^3c^9(1-iax)^{10}(1+iax)^6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{i+4ax}{60a^3c^9(-i+ax)^6(i+ax)^{10}}$$

```
[In] Integrate[(E^((4*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^9,x]
[Out] -1/60*(I + 4*a*x)/(a^3*c^9*(-I + a*x)^6*(I + a*x)^10)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{-\frac{i}{60a^3}-\frac{x}{15a^2}}{(ax+i)^{10}c^9(ax-i)^6}$	34
default	$-\frac{\frac{i}{60a^3}+\frac{x}{15a^2}}{c^9(ax+i)^{10}(ax-i)^6}$	35
gosper	$\frac{(-ax+i)(ax+i)(4ax+i)(iax+1)^4}{60a^3(a^2x^2+1)^{11}c^9}$	49
parallelrisch	$\frac{ix^{20}a^{17}+10ix^{18}a^{15}+45ix^{16}a^{13}+120ix^{14}a^{11}+210ix^{12}a^9+252ix^{10}a^7+210ix^8a^5+120ix^6a^3-4a^2x^5+60ix^4a+20x^3}{60c^9(a^2x^2+1)^{10}}$	110
norman	$\frac{\frac{iax^4}{c}+\frac{x^3}{3c}-\frac{a^2x^5}{15c}+\frac{2ia^3x^6}{c}+\frac{7ia^5x^8}{2c}+\frac{21ia^7x^{10}}{5c}+\frac{7ia^9x^{12}}{2c}+\frac{2ia^{11}x^{14}}{c}+\frac{3ia^{13}x^{16}}{4c}+\frac{ia^{15}x^{18}}{6c}+\frac{ia^{17}x^{20}}{60c}}{(a^2x^2+1)^{10}c^8}$	142

```
[In] int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x,method=_RETURNVERBOSE)
[Out] (-1/60*I/a^3-1/15*x/a^2)/(I+a*x)^10/c^9/(a*x-I)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.45

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx =$$

$$-\frac{4\,ax+i}{60\,(a^{19}c^9x^{16}+4i\,a^{18}c^9x^{15}+20i\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}+36i\,a^{14}c^9x^{11}-64\,a^{13}c^9x^{10}+20i\,a^{12}c^9x^9-90\,a^{1}}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] -1/60*(4*a*x + I)/(a^19*c^9*x^16 + 4*I*a^18*c^9*x^15 + 20*I*a^16*c^9*x^13 -
 20*a^15*c^9*x^12 + 36*I*a^14*c^9*x^11 - 64*a^13*c^9*x^10 + 20*I*a^12*c^9*x
^9 - 90*a^11*c^9*x^8 - 20*I*a^10*c^9*x^7 - 64*a^9*c^9*x^6 - 36*I*a^8*c^9*x^
5 - 20*a^7*c^9*x^4 - 20*I*a^6*c^9*x^3 - 4*I*a^4*c^9*x + a^3*c^9)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(32) = 64$.

Time = 0.81 (sec) , antiderivative size = 194, normalized size of antiderivative = 5.11

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$$

$$= \frac{-}{60a^{19}c^9x^{16}+240ia^{18}c^9x^{15}+1200ia^{16}c^9x^{13}-1200a^{15}c^9x^{12}+2160ia^{14}c^9x^{11}-3840a^{13}c^9x^{10}+1200ia^{12}c^9x^9}$$

```
[In] integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**2/(a**2*c*x**2+c)**9,x)
```

```
[Out] (-4*a*x - I)/(60*a**19*c**9*x**16 + 240*I*a**18*c**9*x**15 + 1200*I*a**16*c
**9*x**13 - 1200*a**15*c**9*x**12 + 2160*I*a**14*c**9*x**11 - 3840*a**13*c*
*9*x**10 + 1200*I*a**12*c**9*x**9 - 5400*a**11*c**9*x**8 - 1200*I*a**10*c**
9*x**7 - 3840*a**9*c**9*x**6 - 2160*I*a**8*c**9*x**5 - 1200*a**7*c**9*x**4
- 1200*I*a**6*c**9*x**3 - 240*I*a**4*c**9*x + 60*a**3*c**9)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.08

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{4\,a^5x^5 - 15i\,a^4x^4 - 20\,a^3x^3 + 10i\,a^2x^2 + i}{60\,(a^{23}c^9x^{20} + 10\,a^{21}c^9x^{18} + 45\,a^{19}c^9x^{16} + 120\,a^{17}c^9x^{14} + 210\,a^{15}c^9x^{12} + 252\,a^{13}c^9x^{10} + 210\,a^{11}c^9x^8 + 1}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] -1/60*(4*a^5*x^5 - 15*I*a^4*x^4 - 20*a^3*x^3 + 10*I*a^2*x^2 + I)/(a^23*c^9*
x^20 + 10*a^21*c^9*x^18 + 45*a^19*c^9*x^16 + 120*a^17*c^9*x^14 + 210*a^15*c
^9*x^12 + 252*a^13*c^9*x^10 + 210*a^11*c^9*x^8 + 120*a^9*c^9*x^6 + 45*a^7*c
^9*x^4 + 10*a^5*c^9*x^2 + a^3*c^9)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$$

$$= -\frac{2145\,a^5x^5 - 12540i\,a^4x^4 - 30030\,a^3x^3 + 37080i\,a^2x^2 + 23841\,ax - 6476i}{983040\,(ax-i)^6a^3c^9} + \frac{2145\,a^9x^9 + 21780i\,a^8x^8 - 99660\,a^7x^7 - 270480i\,a^6x^6 + 481446\,a^5x^5 + 584920i\,a^4x^4 - 486220\,a^3x^3 - 2}{983040\,(ax+i)^{10}a^3c^9}$$

```
[In] integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 - 12540*I*a^4*x^4 - 30030*a^3*x^3 + 37080*I*a^2*x^2
 + 23841*a*x - 6476*I)/((a*x - I)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 + 217
80*I*a^8*x^8 - 99660*a^7*x^7 - 270480*I*a^6*x^6 + 481446*a^5*x^5 + 584920*I
*a^4*x^4 - 486220*a^3*x^3 - 265680*I*a^2*x^2 + 84065*a*x + 9908*I)/((a*x +
I)^10*a^3*c^9)
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.21

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx =$$

$$-\frac{4\,a^5\,x^5 - a^4\,x^4\,15\mathrm{i} - 20\,a^3\,x^3 + a^2\,x^2\,10\mathrm{i} +}{60\,a^{23}\,c^9\,x^{20} + 600\,a^{21}\,c^9\,x^{18} + 2700\,a^{19}\,c^9\,x^{16} + 7200\,a^{17}\,c^9\,x^{14} + 12600\,a^{15}\,c^9\,x^{12} + 15120\,a^{13}\,c^9\,x^{10} + 126}$$

```
[In] int((x^2*(a*x*1i + 1)^4)/((c + a^2*c*x^2)^9*(a^2*x^2 + 1)^2),x)
```

```
[Out] -(a^2*x^2*10i - 20*a^3*x^3 - a^4*x^4*15i + 4*a^5*x^5 + 1i)/(60*a^3*c^9 + 60
0*a^5*c^9*x^2 + 2700*a^7*c^9*x^4 + 7200*a^9*c^9*x^6 + 12600*a^11*c^9*x^8 +
15120*a^13*c^9*x^10 + 12600*a^15*c^9*x^12 + 7200*a^17*c^9*x^14 + 2700*a^19*
c^9*x^16 + 600*a^21*c^9*x^18 + 60*a^23*c^9*x^20)
```

3.377 $\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{i+2ax}{6a^3c^3(1-iax)^3(1+iax)}$$

```
[Out] 1/6*(-I-2*a*x)/a^3/c^3/(1-I*a*x)^3/(1+I*a*x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5190, 82}

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{2ax+i}{6a^3c^3(1-iax)^3(1+iax)}$$

```
[In] Int[(E^((2*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^3,x]
```

```
[Out] -1/6*(I + 2*a*x)/(a^3*c^3*(1 - I*a*x)^3*(1 + I*a*x))
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{x^2}{(1-iax)^4(1+iax)^2}\,dx}{c^3}$$
$$= -\frac{i+2ax}{6a^3c^3(1-iax)^3(1+iax)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{i+2ax}{6a^3c^3(-i+ax)(i+ax)^3}$$

```
[In] Integrate[(E^((2*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^3,x]
[Out] (I + 2*a*x)/(6*a^3*c^3*(-I + a*x)*(I + a*x)^3)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{x}{3a^2}+\frac{i}{6a^3}}{c^3(ax+i)^3(ax-i)}$	34
risch	$\frac{\frac{x}{3a^2}+\frac{i}{6a^3}}{c^3(ax+i)^3(ax-i)}$	34
parallelrisch	$\frac{ix^6a^3+3ix^4a+2x^3}{6c^3(a^2x^2+1)^3}$	39
norman	$\frac{\frac{x^3}{3c}+\frac{ia\,x^4}{2c}+\frac{ia^3x^6}{6c}}{(a^2x^2+1)^3c^2}$	47
gosper	$\frac{(-ax+i)(ax+i)(2ax+i)(iax+1)^2}{6a^3(a^2x^2+1)^4c^3}$	49

```
[In] int((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/c^3*(1/3*x/a^2+1/6*I/a^3)/(I+a*x)^3/(a*x-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{2\,ax+i}{6\,(a^7c^3x^4+2i\,a^6c^3x^3+2i\,a^4c^3x-a^3c^3)}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas"
)
[Out] 1/6*(2*a*x + I)/(a^7*c^3*x^4 + 2*I*a^6*c^3*x^3 + 2*I*a^4*c^3*x - a^3*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{-2ax-i}{6a^7c^3x^4+12ia^6c^3x^3+12ia^4c^3x-6a^3c^3}$$

```
[In] integrate((1+I*a*x)**2/(a**2*x**2+1)*x**2/(a**2*c*x**2+c)**3,x)
[Out] -(-2*a*x - I)/(6*a**7*c**3*x**4 + 12*I*a**6*c**3*x**3 + 12*I*a**4*c**3*x -
6*a**3*c**3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(30) = 60.

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{2\,a^3x^3-3i\,a^2x^2-i}{6\,(a^9c^3x^6+3\,a^7c^3x^4+3\,a^5c^3x^2+a^3c^3)}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima"
)
[Out] 1/6*(2*a^3*x^3 - 3*I*a^2*x^2 - I)/(a^9*c^3*x^6 + 3*a^7*c^3*x^4 + 3*a^5*c^3*
x^2 + a^3*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{1}{16\,(ax-i)a^3c^3} + \frac{3\,a^2x^2 + 12i\,ax - 5}{48\,(ax+i)^3a^3c^3}$$

```
[In] integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] -1/16/((a*x - I)*a^3*c^3) + 1/48*(3*a^2*x^2 + 12*I*a*x - 5)/((a*x + I)^3*a^
3*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{\frac{x}{3\,a^6\,c^3} + \frac{1\mathrm{i}}{6\,a^7\,c^3}}{\frac{x\,2\mathrm{i}}{a^3} - \frac{1}{a^4} + x^4 + \frac{x^3\,2\mathrm{i}}{a}}$$

```
[In] int((x^2*(a*x*1i + 1)^2)/((c + a^2*c*x^2)^3*(a^2*x^2 + 1)),x)
[Out] (1i/(6*a^7*c^3) + x/(3*a^6*c^3))/((x*2i)/a^3 - 1/a^4 + x^4 + (x^3*2i)/a)
```

3.378 $\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{i-2ax}{6a^3c^3(1-iax)(1+iax)^3}$$

```
[Out] 1/6*(I-2*a*x)/a^3/c^3/(1-I*a*x)/(1+I*a*x)^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5190, 82}

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{-2ax+i}{6a^3c^3(1-iax)(1+iax)^3}$$

```
[In] Int[x^2/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

```
[Out] (I - 2*a*x)/(6*a^3*c^3*(1 - I*a*x)*(1 + I*a*x)^3)
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{x^2}{(1-iax)^2(1+iax)^4}\,dx}{c^3}$$
$$= \frac{i-2ax}{6a^3c^3(1-iax)(1+iax)^3}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{-i+2ax}{6a^3c^3(-i+ax)^3(i+ax)}$$

```
[In] Integrate[x^2/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
[Out] (-I + 2*a*x)/(6*a^3*c^3*(-I + a*x)^3*(I + a*x))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\frac{x}{3a^2}-\frac{i}{6a^3}}{c^3(ax-i)^3(ax+i)}$	34
parallelrisch	$-\frac{ix^4a+2x^3}{6c^3(-ax+i)^2(a^2x^2+1)}$	39
norman	$\frac{\frac{x^3}{3c}-\frac{ia\,x^4}{2c}-\frac{ia^3x^6}{6c}}{(a^2x^2+1)^3c^2}$	47
gosper	$-\frac{(-2ax+i)(ax+i)(-ax+i)}{6(a^2x^2+1)^2c^3(iax+1)^2a^3}$	49
default	$\frac{-\frac{i}{8a^3(-ax+i)^2}-\frac{1}{12a^3(-ax+i)^3}-\frac{1}{16a^3(-ax+i)}-\frac{1}{16a^3(ax+i)}}{c^3}$	62

```
[In] int(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
[Out] (1/3*x/a^2-1/6*I/a^3)/c^3/(a*x-I)^3/(I+a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{2\,ax-i}{6\,(a^7c^3x^4-2i\,a^6c^3x^3-2i\,a^4c^3x-a^3c^3)}$$

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="fricas"
)
[Out] 1/6*(2*a*x - I)/(a^7*c^3*x^4 - 2*I*a^6*c^3*x^3 - 2*I*a^4*c^3*x - a^3*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{-2ax+i}{6a^7c^3x^4-12ia^6c^3x^3-12ia^4c^3x-6a^3c^3}$$

```
[In] integrate(x**2/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**3,x)
[Out] -(-2*a*x + I)/(6*a**7*c**3*x**4 - 12*I*a**6*c**3*x**3 - 12*I*a**4*c**3*x -
6*a**3*c**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="maxima"
)
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{1}{32\,a^3c^3\left(\frac{2i}{i\,ax+1}-i\right)} - \frac{-\frac{3i\,a^3c^6}{i\,ax+1} - \frac{6i\,a^3c^6}{(i\,ax+1)^2} + \frac{4i\,a^3c^6}{(i\,ax+1)^3}}{48\,a^6c^9}$$

```
[In] integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/32/(a^3*c^3*(2*I/(I*a*x + 1) - I)) - 1/48*(-3*I*a^3*c^6/(I*a*x + 1) - 6*
I*a^3*c^6/(I*a*x + 1)^2 + 4*I*a^3*c^6/(I*a*x + 1)^3)/(a^6*c^9)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{2\,a^3\,x^3 + a^2\,x^2\,3\mathrm{i} + 1\mathrm{i}}{6\,a^9\,c^3\,x^6 + 18\,a^7\,c^3\,x^4 + 18\,a^5\,c^3\,x^2 + 6\,a^3\,c^3}$$

```
[In] int((x^2*(a^2*x^2 + 1))/((c + a^2*c*x^2)^3*(a*x*1i + 1)^2),x)
```

```
[Out] (a^2*x^2*3i + 2*a^3*x^3 + 1i)/(6*a^3*c^3 + 18*a^5*c^3*x^2 + 18*a^7*c^3*x^4
+ 6*a^9*c^3*x^6)
```

3.379 $\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = \frac{i-4ax}{60a^3c^9(1-iax)^6(1+iax)^{10}}$$

```
[Out] 1/60*(I-4*a*x)/a^3/c^9/(1-I*a*x)^6/(1+I*a*x)^10
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5190, 82}

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = \frac{-4ax+i}{60a^3c^9(1-iax)^6(1+iax)^{10}}$$

```
[In] Int[x^2/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^9),x]
```

```
[Out] (I - 4*a*x)/(60*a^3*c^9*(1 - I*a*x)^6*(1 + I*a*x)^10)
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{x^2}{(1-iax)^7(1+iax)^{11}}\,dx}{c^9}\\ &= \frac{i-4ax}{60a^3c^9(1-iax)^6(1+iax)^{10}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = \frac{i-4ax}{60a^3c^9(-i+ax)^{10}(i+ax)^6}$$

```
[In] Integrate[x^2/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^9),x]
```

```
[Out] (I - 4*a*x)/(60*a^3*c^9*(-I + a*x)^10*(I + a*x)^6)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result
risch	$\frac{\frac{i}{60a^3}-\frac{x}{15a^2}}{c^9(ax-i)^{10}(ax+i)^6}$
gosper	$-\frac{(-4ax+i)(ax+i)(-ax+i)}{60(a^2x^2+1)^7c^9(iax+1)^4a^3}$
parallelrisch	$-\frac{ix^{16}a^{13}+4x^{15}a^{12}+20x^{13}a^{10}-20ix^{12}a^9+36x^{11}a^8-64ix^{10}a^7+20x^9a^6-90ix^8a^5-20a^4x^7-64ix^6a^3-36a^2x^5-20ix^4a-20x^3}{60c^9(-ax+i)^4(a^2x^2+1)^6}$
norman	$\frac{-\frac{ia\,x^4}{c}+\frac{x^3}{3c}-\frac{a^2x^5}{15c}-\frac{2ia^3x^6}{c}-\frac{7ia^5x^8}{2c}-\frac{21ia^7x^{10}}{5c}-\frac{7ia^9x^{12}}{2c}-\frac{2ia^{11}x^{14}}{c}-\frac{3ia^{13}x^{16}}{4c}-\frac{ia^{15}x^{18}}{6c}-\frac{ia^{17}x^{20}}{60c}}{(a^2x^2+1)^{10}c^8}$
default	$\frac{21i}{8192a^3(-ax+i)^4}+\frac{i}{1280a^3(-ax+i)^{10}}-\frac{i}{1024a^3(-ax+i)^8}-\frac{7i}{6144a^3(-ax+i)^6}-\frac{165i}{65536a^3(-ax+i)^2}+\frac{1}{768a^3(-ax+i)^9}-\frac{21}{10240a^3(-ax+i)^5}+$

```
[In] int(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] (1/60*I/a^3-1/15*x/a^2)/c^9/(a*x-I)^10/(I+a*x)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.45

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx =$$

$$-\frac{4\,ax-i}{60\,(a^{19}c^9x^{16}-4i\,a^{18}c^9x^{15}-20i\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}-36i\,a^{14}c^9x^{11}-64\,a^{13}c^9x^{10}-20i\,a^{12}c^9x^9-90\,a$$

```
[In] integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] -1/60*(4*a*x - I)/(a^19*c^9*x^16 - 4*I*a^18*c^9*x^15 - 20*I*a^16*c^9*x^13 -
 20*a^15*c^9*x^12 - 36*I*a^14*c^9*x^11 - 64*a^13*c^9*x^10 - 20*I*a^12*c^9*x
^9 - 90*a^11*c^9*x^8 + 20*I*a^10*c^9*x^7 - 64*a^9*c^9*x^6 + 36*I*a^8*c^9*x^
5 - 20*a^7*c^9*x^4 + 20*I*a^6*c^9*x^3 + 4*I*a^4*c^9*x + a^3*c^9)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(31) = 62$.

Time = 0.71 (sec) , antiderivative size = 192, normalized size of antiderivative = 5.05

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$$

$$= \frac{-}{60a^{19}c^9x^{16}-240ia^{18}c^9x^{15}-1200ia^{16}c^9x^{13}-1200a^{15}c^9x^{12}-2160ia^{14}c^9x^{11}-3840a^{13}c^9x^{10}-1200ia^{12}c^9x$$

```
[In] integrate(x**2/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**9,x)
```

```
[Out] (-4*a*x + I)/(60*a**19*c**9*x**16 - 240*I*a**18*c**9*x**15 - 1200*I*a**16*c
**9*x**13 - 1200*a**15*c**9*x**12 - 2160*I*a**14*c**9*x**11 - 3840*a**13*c*
*9*x**10 - 1200*I*a**12*c**9*x**9 - 5400*a**11*c**9*x**8 + 1200*I*a**10*c**
9*x**7 - 3840*a**9*c**9*x**6 + 2160*I*a**8*c**9*x**5 - 1200*a**7*c**9*x**4
+ 1200*I*a**6*c**9*x**3 + 240*I*a**4*c**9*x + 60*a**3*c**9)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(30) = 60.

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{2145\,a^5x^5 + 12540i\,a^4x^4 - 30030\,a^3x^3 - 37080i\,a^2x^2 + 23841\,ax + 6476i}{983040\,(ax+i)^6a^3c^9} + \frac{2145\,a^9x^9 - 21780i\,a^8x^8 - 99660\,a^7x^7 + 270480i\,a^6x^6 + 481446\,a^5x^5 - 584920i\,a^4x^4 - 486220\,a^3x^3 + 26}{983040\,(ax-i)^{10}a^3c^9}$$

```
[In] integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 + 12540*I*a^4*x^4 - 30030*a^3*x^3 - 37080*I*a^2*x^2
 + 23841*a*x + 6476*I)/((a*x + I)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 - 217
80*I*a^8*x^8 - 99660*a^7*x^7 + 270480*I*a^6*x^6 + 481446*a^5*x^5 - 584920*I
*a^4*x^4 - 486220*a^3*x^3 + 265680*I*a^2*x^2 + 84065*a*x - 9908*I)/((a*x -
I)^10*a^3*c^9)
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 159, normalized size of antiderivative = 4.18

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = \frac{-4\,a^5\,x^5 - a^4\,x^4\,15\mathrm{i} + 20\,a^3\,x^3 + a^2\,x^2\,10\mathrm{i} +}{60\,a^{23}\,c^9\,x^{20} + 600\,a^{21}\,c^9\,x^{18} + 2700\,a^{19}\,c^9\,x^{16} + 7200\,a^{17}\,c^9\,x^{14} + 12600\,a^{15}\,c^9\,x^{12} + 15120\,a^{13}\,c^9\,x^{10} + 12600}$$

```
[In] int((x^2*(a^2*x^2 + 1)^2)/((c + a^2*c*x^2)^9*(a*x*1i + 1)^4),x)
```

```
[Out] (a^2*x^2*10i + 20*a^3*x^3 - a^4*x^4*15i - 4*a^5*x^5 + 1i)/(60*a^3*c^9 + 600
*a^5*c^9*x^2 + 2700*a^7*c^9*x^4 + 7200*a^9*c^9*x^6 + 12600*a^11*c^9*x^8 + 1
5120*a^13*c^9*x^10 + 12600*a^15*c^9*x^12 + 7200*a^17*c^9*x^14 + 2700*a^19*c
^9*x^16 + 600*a^21*c^9*x^18 + 60*a^23*c^9*x^20)
```

3.380 $\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = -\frac{(i+5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{c+a^2cx^2}}$$

```
[Out] -1/120*(I+5*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^13/(1-I*a*x)^15/(1+I*a*x)^10/(a^2*
c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5193, 5190, 82}

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = -\frac{(5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{a^2cx^2+c}}$$

```
[In] Int[(E^((5*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(27/2),x]
```

```
[Out] -1/120*((I + 5*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^13*(1 - I*a*x)^15*(1 + I*a*x)
^10*Sqrt[c + a^2*c*x^2])
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
```

```
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+a^2x^2}\int \frac{e^{5i\arctan(ax)}x^2}{(1+a^2x^2)^{27/2}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^{16}(1+iax)^{11}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}$$
$$= -\frac{(i+5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{c+a^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{(1-5iax)\sqrt{1+a^2x^2}}{120a^3c^{13}(-i+ax)^{10}(i+ax)^{15}\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[(E^((5*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(27/2),x]
[Out] ((1 - (5*I)*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(-I + a*x)^10*(I + a*x)^1
5*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(5iax-1)}{120\sqrt{a^2x^2+1}\,c^{14}a^3(ax+i)^{15}(-ax+i)^{10}}$	57
gosper	$\frac{(-ax+i)(ax+i)(5ax+i)(iax+1)^5}{120a^3(a^2x^2+1)^{\frac{5}{2}}(a^2c\,x^2+c)^{\frac{27}{2}}}$	58
risch	$\frac{\sqrt{a^2x^2+1}\left(-\frac{ix}{24a^2}+\frac{1}{120a^3}\right)}{c^{13}\sqrt{c(a^2x^2+1)}\,(ax+i)^{15}(ax-i)^{10}}$	58

```
[In] int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/120/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(5*I*a*x-1)/c^14/a^3/(I+a*x)
^15/(I-a*x)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(53) = 106.

Time = 0.36 (sec) , antiderivative size = 496, normalized size of antiderivative = 7.63

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{(i\,a^{22}x^{25}-5\,a^{21}x^{24}-4}{120\,(a^{27}c^{14}x^{27}+5i\,a^{26}c^{14}x^{26}+a^{25}c^{14}x^{25}+45i\,a^{24}c^{14}x^{24}-50\,a^{23}c^{14}x^{23}+166i\,a^{22}c^{14}x}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] 1/120*(I*a^22*x^25 - 5*a^21*x^24 - 40*a^19*x^22 - 50*I*a^18*x^21 - 126*a^17
*x^20 - 280*I*a^16*x^19 - 160*a^15*x^18 - 765*I*a^14*x^17 + 105*a^13*x^16 -
 1248*I*a^12*x^15 + 720*a^11*x^14 - 1260*I*a^10*x^13 + 1260*a^9*x^12 - 720*
I*a^8*x^11 + 1248*a^7*x^10 - 105*I*a^6*x^9 + 765*a^5*x^8 + 160*I*a^4*x^7 +
280*a^3*x^6 + 126*I*a^2*x^5 + 50*a*x^4 + 40*I*x^3)*sqrt(a^2*c*x^2 + c)*sqrt
(a^2*x^2 + 1)/(a^27*c^14*x^27 + 5*I*a^26*c^14*x^26 + a^25*c^14*x^25 + 45*I*
a^24*c^14*x^24 - 50*a^23*c^14*x^23 + 166*I*a^22*c^14*x^22 - 330*a^21*c^14*x
^21 + 286*I*a^20*c^14*x^20 - 1045*a^19*c^14*x^19 + 55*I*a^18*c^14*x^18 - 20
13*a^17*c^14*x^17 - 825*I*a^16*c^14*x^16 - 2508*a^15*c^14*x^15 - 1980*I*a^1
4*c^14*x^14 - 1980*a^13*c^14*x^13 - 2508*I*a^12*c^14*x^12 - 825*a^11*c^14*x
^11 - 2013*I*a^10*c^14*x^10 + 55*a^9*c^14*x^9 - 1045*I*a^8*c^14*x^8 + 286*a
^7*c^14*x^7 - 330*I*a^6*c^14*x^6 + 166*a^5*c^14*x^5 - 50*I*a^4*c^14*x^4 + 4
5*a^3*c^14*x^3 + I*a^2*c^14*x^2 + 5*a*c^14*x + I*c^14)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \text{Timed out}$$

```
[In] integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)*x**2/(a**2*c*x**2+c)**(27/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \int \frac{(i\,ax+1)^5x^2}{(a^2cx^2+c)^{\frac{27}{2}}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
[Out] integrate((I*a*x + 1)^5*x^2/((a^2*c*x^2 + c)^(27/2)*(a^2*x^2 + 1)^(5/2)), x
)
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = -\frac{c\,(a\,x-\mathrm{i})^5\,(5\,a\,x+1\mathrm{i})\,1\mathrm{i}}{120\,a^3\,\left(c\,\left(a^2\,x^2+1\right)\right)^{29/2}\,\sqrt{a^2\,x^2+1}}$$

```
[In] int((x^2*(a*x*1i + 1)^5)/((c + a^2*c*x^2)^(27/2)*(a^2*x^2 + 1)^(5/2)),x)
[Out] -(c*(a*x - 1i)^5*(5*a*x + 1i)*1i)/(120*a^3*(c*(a^2*x^2 + 1))^(29/2)*(a^2*x^
2 + 1)^(1/2))
```

3.381 $\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = -\frac{(i+3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{c+a^2cx^2}}$$

```
[Out] -1/24*(I+3*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^5/(1-I*a*x)^6/(1+I*a*x)^3/(a^2*c*x^
2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5193, 5190, 82}

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = -\frac{(3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{a^2cx^2+c}}$$

```
[In] Int[(E^((3*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(11/2),x]
```

```
[Out] -1/24*((I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(1 - I*a*x)^6*(1 + I*a*x)^3*
Sqrt[c + a^2*c*x^2])
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
```

```
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{3i\arctan(ax)}x^2}{(1+a^2x^2)^{11/2}}\,dx}{c^5\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^7(1+iax)^4}\,dx}{c^5\sqrt{c+a^2cx^2}}\\ &= -\frac{(i+3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{i(i+3ax)\sqrt{1+a^2x^2}}{24a^3c^5(-i+ax)^3(i+ax)^6\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[(E^((3*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(11/2),x]
[Out] ((I/24)*(I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(-I + a*x)^3*(I + a*x)^6*Sq
rt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(3iax-1)}{24\sqrt{a^2x^2+1}\,c^6a^3(ax+i)^6(-ax+i)^3}$	57
gosper	$\frac{(-ax+i)(ax+i)(3ax+i)(iax+1)^3}{24a^3(a^2x^2+1)^{\frac{3}{2}}(a^2c\,x^2+c)^{\frac{11}{2}}}$	58
risch	$\frac{\sqrt{a^2x^2+1}\left(\frac{ix}{8a^2}-\frac{1}{24a^3}\right)}{c^5\sqrt{c(a^2x^2+1)}\,(ax+i)^6(ax-i)^3}$	58

```
[In] int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/24/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(3*I*a*x-1)/c^6/a^3/(I+a*x)^6
/(I-a*x)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(53) = 106.

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.95

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{(i\,a^6x^9-3\,a^5x^8-8\,a^3x^6-6i\,a^2x^5-6\,ax^4-8i\,x^3)\sqrt{a}}{24\,(a^{11}c^6x^{11}+3i\,a^{10}c^6x^{10}+a^9c^6x^9+11i\,a^8c^6x^8-6\,a^7c^6x^7+14i\,a^6c^6x^6-14\,a^5c^6x^5}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] 1/24*(I*a^6*x^9 - 3*a^5*x^8 - 8*a^3*x^6 - 6*I*a^2*x^5 - 6*a*x^4 - 8*I*x^3)*
sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/(a^11*c^6*x^11 + 3*I*a^10*c^6*x^10 +
a^9*c^6*x^9 + 11*I*a^8*c^6*x^8 - 6*a^7*c^6*x^7 + 14*I*a^6*c^6*x^6 - 14*a^5*
c^6*x^5 + 6*I*a^4*c^6*x^4 - 11*a^3*c^6*x^3 - I*a^2*c^6*x^2 - 3*a*c^6*x - I*
c^6)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx =$$

$$-i\left(\int \frac{}{a^{12}c^5x^{12}\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+6a^{10}c^5x^{10}\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+15a^8c^5x^8\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\cdots}\right.$$

$$+\int\left(-\frac{}{a^{12}c^5x^{12}\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+6a^{10}c^5x^{10}\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+15a^8c^5x^8\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}\cdots}\right.$$

$$+\int \frac{}{a^{12}c^5x^{12}\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+6a^{10}c^5x^{10}\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+15a^8c^5x^8\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2\cdots}$$

$$+\int\left(-\frac{}{a^{12}c^5x^{12}\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+6a^{10}c^5x^{10}\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+15a^8c^5x^8\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}\cdots}\right.$$

```
[In] integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**2/(a**2*c*x**2+c)**(11/2),x)
```

```
[Out] -I*(Integral(I*x**2/(a**12*c**5*x**12*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2
+ c) + 6*a**10*c**5*x**10*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a*
*8*c**5*x**8*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 20*a**6*c**5*x**6*
sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**4*c**5*x**4*sqrt(a**2*x**
2 + 1)*sqrt(a**2*c*x**2 + c) + 6*a**2*c**5*x**2*sqrt(a**2*x**2 + 1)*sqrt(a*
*2*c*x**2 + c) + c**5*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Inte
gral(-3*a*x**3/(a**12*c**5*x**12*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)
+ 6*a**10*c**5*x**10*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**8*c*
*5*x**8*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 20*a**6*c**5*x**6*sqrt(
a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**4*c**5*x**4*sqrt(a**2*x**2 + 1
)*sqrt(a**2*c*x**2 + c) + 6*a**2*c**5*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*
x**2 + c) + c**5*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(
a**3*x**5/(a**12*c**5*x**12*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 6*a
**10*c**5*x**10*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**8*c**5*x*
*8*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 20*a**6*c**5*x**6*sqrt(a**2*
x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**4*c**5*x**4*sqrt(a**2*x**2 + 1)*sqr
t(a**2*c*x**2 + c) + 6*a**2*c**5*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2
+ c) + c**5*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*I*
a**2*x**4/(a**12*c**5*x**12*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 6*a
**10*c**5*x**10*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**8*c**5*x*
*8*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 20*a**6*c**5*x**6*sqrt(a**2*
x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**4*c**5*x**4*sqrt(a**2*x**2 + 1)*sqr
t(a**2*c*x**2 + c) + 6*a**2*c**5*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2
+ c) + c**5*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \int \frac{(i\,ax+1)^3x^2}{(a^2cx^2+c)^{\frac{11}{2}}(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
```

```
[Out] integrate((I*a*x + 1)^3*x^2/((a^2*c*x^2 + c)^(11/2)*(a^2*x^2 + 1)^(3/2)), x
)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{\sqrt{c\,(a^2\,x^2+1)}\,(a\,x-\mathrm{i})^3\,(3\,a\,x+1\mathrm{i})\,1\mathrm{i}}{24\,a^3\,c^6\,(a^2\,x^2+1)^{13/2}}$$

```
[In] int((x^2*(a*x*1i + 1)^3)/((c + a^2*c*x^2)^(11/2)*(a^2*x^2 + 1)^(3/2)),x)
```

```
[Out] ((c*(a^2*x^2 + 1))^(1/2)*(a*x - 1i)^3*(3*a*x + 1i)*1i)/(24*a^3*c^6*(a^2*x^2
 + 1)^(13/2))
```

3.382 $\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 142

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{1+a^2x^2}}{2a^3c(i+ax)\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}$$

```
[Out] -1/2*(a^2*x^2+1)^(1/2)/a^3/c/(I+a*x)/(a^2*c*x^2+c)^(1/2)+1/4*I*ln(I-a*x)*(a
^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+3/4*I*ln(I+a*x)*(a^2*x^2+1)^(1/2)
/a^3/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5193, 5190, 90}

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{a^2x^2+1}}{2a^3c(ax+i)\sqrt{a^2cx^2+c}} + \frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}} + \frac{3i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

```
[In] Int[(E^(I*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^2]/(a^3*c*(I + a*x)*Sqrt[c + a^2*c*x^2]) + ((I/4)*Sqrt[
1 + a^2*x^2]*Log[I - a*x])/(a^3*c*Sqrt[c + a^2*c*x^2]) + (((3*I)/4)*Sqrt[1
+ a^2*x^2]*Log[I + a*x])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{i\arctan(ax)}x^2}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^2(1+iax)}\,dx}{c\sqrt{c+a^2cx^2}}\\
&= \frac{\sqrt{1+a^2x^2}\int \left(\frac{i}{4a^2(-i+ax)}+\frac{1}{2a^2(i+ax)^2}+\frac{3i}{4a^2(i+ax)}\right)dx}{c\sqrt{c+a^2cx^2}}\\
&= -\frac{\sqrt{1+a^2x^2}}{2a^3c(i+ax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}}+\frac{3i\sqrt{1+a^2x^2}\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(-\frac{2}{i+ax}+i\log(i-ax)+3i\log(i+ax)\right)}{4a^3c\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[(E^(I*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(3/2),x]
[Out] (Sqrt[1 + a^2*x^2]*(-2/(I + a*x) + I*Log[I - a*x] + (3*I)*Log[I + a*x]))/(4
*a^3*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\sqrt{c(a^2x^2+1)}\,(i\ln(-ax+i)ax+3i\ln(ax+i)ax-\ln(-ax+i)-3\ln(ax+i)-2)}{4\sqrt{a^2x^2+1}\,c^2a^3(ax+i)}$	87
risch	$-\frac{\sqrt{a^2x^2+1}}{2c\sqrt{c(a^2x^2+1)}\,a^3(ax+i)}+\frac{3i\sqrt{a^2x^2+1}\,\ln(iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a^3}+\frac{i\sqrt{a^2x^2+1}\,\ln(-iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a^3}$	124

```
[In] int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(I-a*x)*a*x+3*I*ln(I+a*x)*
a*x-ln(I-a*x)-3*ln(I+a*x)-2)/c^2/a^3/(I+a*x)
```

Fricas [F]

$$\int \frac{e^{i\arctan(ax)}x^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{(i\,ax+1)x^2}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -1/8*(3*(I*a^5*c^2*x^3 - a^4*c^2*x^2 + I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c
^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3))
 + I*a^2*x^3 + I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + 3*(-I*a^5*c^2*x^3 +
a^4*c^2*x^2 - I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3))*log((-I*sqrt(a^2*c*x
^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + I*a^2*x^3 + I*x)/(a^3
*x^3 + I*a^2*x^2 + a*x + I)) - (I*a^5*c^2*x^3 - a^4*c^2*x^2 + I*a^3*c^2*x -
 a^2*c^2)*sqrt(1/(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^
3*c*x*sqrt(1/(a^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I))
 - (-I*a^5*c^2*x^3 + a^4*c^2*x^2 - I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3))
*log((-I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) -
I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I)) + 4*(-I*a^5*c^2*x^3 + a^4
*c^2*x^2 - I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3))*log((sqrt(a^2*c*x^2 + c
)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + a^2*x^3 + x)/(a^2*x^2 + 1))
 + 4*(I*a^5*c^2*x^3 - a^4*c^2*x^2 + I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c^3)
)*log(-(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) - a
^2*x^3 - x)/(a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*x -
8*(a^5*c^2*x^3 + I*a^4*c^2*x^2 + a^3*c^2*x + I*a^2*c^2)*integral(1/2*sqrt(a
^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(2*I*a*x + 1)/(a^6*c^2*x^4 + 2*a^4*c^2*x^2
+ a^2*c^2), x))/(a^5*c^2*x^3 + I*a^4*c^2*x^2 + a^3*c^2*x + I*a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = i\left(\int \left(-\frac{ix^2}{a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int \frac{ax^3}{a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx\right)$$

```
[In] integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**2/(a**2*c*x**2+c)**(3/2),x)
[Out] I*(Integral(-I*x**2/(a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)
+ c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a*x**3/(a**2*
c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sq
rt(a**2*c*x**2 + c)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)x^2}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{a^2x^2+1}}\,dx$$

```
[In] integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
[Out] integrate((I*a*x + 1)*x^2/((a^2*c*x^2 + c)^(3/2)*sqrt(a^2*x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,(1+a\,x\,1\mathrm{i})}{(c\,a^2\,x^2+c)^{3/2}\,\sqrt{a^2\,x^2+1}}\,dx$$

```
[In] int((x^2*(a*x*1i + 1))/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)),x)
[Out] int((x^2*(a*x*1i + 1))/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)), x)
```

3.383 $\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 143

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}}{2a^3c(i-ax)\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a^2x^2}\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}$$

```
[Out] 1/2*(a^2*x^2+1)^(1/2)/a^3/c/(I-a*x)/(a^2*c*x^2+c)^(1/2)-3/4*I*ln(I-a*x)*(a^
2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-1/4*I*ln(I+a*x)*(a^2*x^2+1)^(1/2)/
a^3/c/(a^2*c*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5193, 5190, 90}

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2x^2+1}}{2a^3c(-ax+i)\sqrt{a^2cx^2+c}} - \frac{3i\sqrt{a^2x^2+1}\log(-ax+i)}{4a^3c\sqrt{a^2cx^2+c}} - \frac{i\sqrt{a^2x^2+1}\log(ax+i)}{4a^3c\sqrt{a^2cx^2+c}}$$

```
[In] Int[x^2/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

```
[Out] Sqrt[1 + a^2*x^2]/(2*a^3*c*(I - a*x)*Sqrt[c + a^2*c*x^2]) - (((3*I)/4)*Sqrt
[1 + a^2*x^2]*Log[I - a*x])/(a^3*c*Sqrt[c + a^2*c*x^2]) - ((I/4)*Sqrt[1 + a
^2*x^2]*Log[I + a*x])/(a^3*c*Sqrt[c + a^2*c*x^2])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-i\arctan(ax)}x^2}{(1+a^2x^2)^{3/2}}\,dx}{c\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)(1+iax)^2}\,dx}{c\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \left(\frac{1}{2a^2(-i+ax)^2}-\frac{3i}{4a^2(-i+ax)}-\frac{i}{4a^2(i+ax)}\right)dx}{c\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}}{2a^3c(i-ax)\sqrt{c+a^2cx^2}}-\frac{3i\sqrt{1+a^2x^2}\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(\frac{2}{i-ax}-3i\log(i-ax)-i\log(i+ax)\right)}{4a^3c\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[x^2/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
[Out] (Sqrt[1 + a^2*x^2]*(2/(I - a*x) - (3*I)*Log[I - a*x] - I*Log[I + a*x]))/(4*
a^3*c*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\sqrt{c(a^2x^2+1)}\,(3i\ln(-ax+i)ax+i\ln(ax+i)ax+3\ln(-ax+i)+\ln(ax+i)+2)}{4\sqrt{a^2x^2+1}\,c^2a^3(-ax+i)}$	86
risch	$-\frac{\sqrt{a^2x^2+1}}{2c\sqrt{c(a^2x^2+1)}\,a^3(ax-i)}-\frac{i\sqrt{a^2x^2+1}\,\ln(iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a^3}-\frac{3i\sqrt{a^2x^2+1}\,\ln(-iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a^3}$	124

```
[In] int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(3*I*ln(I-a*x)*a*x+I*ln(I+a*x)*
a*x+3*ln(I-a*x)+ln(I+a*x)+2)/c^2/a^3/(I-a*x)
```

Fricas [F]

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\sqrt{a^2x^2+1}x^2}{(a^2cx^2+c)^{\frac{3}{2}}(i\,ax+1)}\,dx$$

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/8*((I*a^5*c^2*x^3 + a^4*c^2*x^2 + I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3)
)*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) +
I*a^2*x^3 + I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + (-I*a^5*c^2*x^3 - a^4*c
^2*x^2 - I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c^3))*log((-I*sqrt(a^2*c*x^2 +
c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + I*a^2*x^3 + I*x)/(a^3*x^3
+ I*a^2*x^2 + a*x + I)) - 3*(I*a^5*c^2*x^3 + a^4*c^2*x^2 + I*a^3*c^2*x + a^
2*c^2)*sqrt(1/(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c
*x*sqrt(1/(a^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I)) -
3*(-I*a^5*c^2*x^3 - a^4*c^2*x^2 - I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c^3))*
log((-I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) - I
*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I)) - 4*(I*a^5*c^2*x^3 + a^4*c
^2*x^2 + I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3))*log((sqrt(a^2*c*x^2 + c)*
sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + a^2*x^3 + x)/(a^2*x^2 + 1)) -
 4*(-I*a^5*c^2*x^3 - a^4*c^2*x^2 - I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c^3))
*log(-(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) - a^
2*x^3 - x)/(a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*x + 8
*(a^5*c^2*x^3 - I*a^4*c^2*x^2 + a^3*c^2*x - I*a^2*c^2)*integral(1/2*sqrt(a^
2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(-2*I*a*x + 1)/(a^6*c^2*x^4 + 2*a^4*c^2*x^2
+ a^2*c^2), x))/(a^5*c^2*x^3 - I*a^4*c^2*x^2 + a^3*c^2*x - I*a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = -i\int \frac{x^2\sqrt{a^2x^2+1}}{a^3cx^3\sqrt{a^2cx^2+c}-ia^2cx^2\sqrt{a^2cx^2+c}+acx\sqrt{a^2cx^2+c}-ic\sqrt{a^2cx^2+c}}\,dx$$

```
[In] integrate(x**2/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
[Out] -I*Integral(x**2*sqrt(a**2*x**2 + 1)/(a**3*c*x**3*sqrt(a**2*c*x**2 + c) - I
*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + a*c*x*sqrt(a**2*c*x**2 + c) - I*c*sqrt
(a**2*c*x**2 + c)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.38

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{c}}{2\,(a^4c^2x-i\,a^3c^2)}-\frac{3i\,\log(ax-i)}{4\,a^3c^{\frac{3}{2}}}-\frac{i\,\log(i\,ax-1)}{4\,a^3c^{\frac{3}{2}}}$$

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
[Out] -1/2*sqrt(c)/(a^4*c^2*x - I*a^3*c^2) - 3/4*I*log(a*x - I)/(a^3*c^(3/2)) - 1
/4*I*log(I*a*x - 1)/(a^3*c^(3/2))
```

Giac [F]

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{a^2x^2+1}x^2}{(a^2cx^2+c)^{\frac{3}{2}}(i\,ax+1)}\,dx$$

```
[In] integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
[Out] integrate(sqrt(a^2*x^2 + 1)*x^2/((a^2*c*x^2 + c)^(3/2)*(I*a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(ax)}x^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{x^2\,\sqrt{a^2\,x^2+1}}{\left(c\,a^2\,x^2+c\right)^{3/2}\,\left(1+a\,x\,1\mathrm{i}\right)}\,dx$$

```
[In] int((x^2*(a^2*x^2 + 1)^(1/2))/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)),x)
[Out] int((x^2*(a^2*x^2 + 1)^(1/2))/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)), x)
```

3.384 $\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{(i-3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{c+a^2cx^2}}$$

```
[Out] 1/24*(I-3*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^5/(1-I*a*x)^3/(1+I*a*x)^6/(a^2*c*x^2
+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5193, 5190, 82}

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{(-3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{a^2cx^2+c}}$$

```
[In] Int[x^2/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(11/2)),x]
```

```
[Out] ((I - 3*a*x)*Sqrt[1 + a^2*x^2])/(24*a^3*c^5*(1 - I*a*x)^3*(1 + I*a*x)^6*Sqr
t[c + a^2*c*x^2])
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
```

```
1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-3i\arctan(ax)}x^2}{(1+a^2x^2)^{11/2}}\,dx}{c^5\sqrt{c+a^2cx^2}} \\
&= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^4(1+iax)^7}\,dx}{c^5\sqrt{c+a^2cx^2}} \\
&= \frac{(i-3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{c+a^2cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = -\frac{i(-i+3ax)\sqrt{1+a^2x^2}}{24a^3c^5(-i+ax)^6(i+ax)^3\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[x^2/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(11/2)),x]
[Out] ((-1/24*I)*(-I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(-I + a*x)^6*(I + a*x)^
3*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{-\frac{ix}{8a^2}-\frac{1}{24a^3}}{c^5(a^2x^2+1)^{\frac{5}{2}}\sqrt{c(a^2x^2+1)}\,(ax-i)^3}$	50
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(3iax+1)}{24\sqrt{a^2x^2+1}\,c^6a^3(-ax+i)^6(ax+i)^3}$	57
gosper	$-\frac{(-ax+i)(ax+i)(-3ax+i)(a^2x^2+1)^{\frac{3}{2}}}{24a^3(iax+1)^3(a^2c\,x^2+c)^{\frac{11}{2}}}$	58

```
[In] int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/c^5/(a^2*x^2+1)^(5/2)/(c*(a^2*x^2+1))^(1/2)*(-1/8*I/a^2*x-1/24/a^3)/(a*x-
I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(53) = 106.

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.95

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{(-i\,a^6x^9-3\,a^5x^8-8\,a^3x^6+6i\,a^2x^5-6\,ax^4+8i\,x^3)\sqrt{}}{24\,(a^{11}c^6x^{11}-3i\,a^{10}c^6x^{10}+a^9c^6x^9-11i\,a^8c^6x^8-6\,a^7c^6x^7-14i\,a^6c^6x^6-14\,a^5c^6x^5}$$

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] 1/24*(-I*a^6*x^9 - 3*a^5*x^8 - 8*a^3*x^6 + 6*I*a^2*x^5 - 6*a*x^4 + 8*I*x^3)
*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/(a^11*c^6*x^11 - 3*I*a^10*c^6*x^10 +
 a^9*c^6*x^9 - 11*I*a^8*c^6*x^8 - 6*a^7*c^6*x^7 - 14*I*a^6*c^6*x^6 - 14*a^5
*c^6*x^5 - 6*I*a^4*c^6*x^4 - 11*a^3*c^6*x^3 + I*a^2*c^6*x^2 - 3*a*c^6*x + I
*c^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(11/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{3\,ax-i}{24i\,a^{12}c^{\frac{11}{2}}x^9+72\,a^{11}c^{\frac{11}{2}}x^8+192\,a^9c^{\frac{11}{2}}x^6-144i\,a^8c^{\frac{11}{2}}x^5+144\,a^7c^{\frac{11}{2}}x^4-192i\,a^6c^{\frac{11}{2}}x^3}$$

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
[Out] (3*a*x - I)/(24*I*a^12*c^(11/2)*x^9 + 72*a^11*c^(11/2)*x^8 + 192*a^9*c^(11/
2)*x^6 - 144*I*a^8*c^(11/2)*x^5 + 144*a^7*c^(11/2)*x^4 - 192*I*a^6*c^(11/2)
*x^3 - 72*I*a^4*c^(11/2)*x - 24*a^3*c^(11/2))
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}x^2}{(a^2cx^2+c)^{\frac{11}{2}}(i\,ax+1)^3}\,dx$$

```
[In] integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
[Out] integrate((a^2*x^2 + 1)^(3/2)*x^2/((a^2*c*x^2 + c)^(11/2)*(I*a*x + 1)^3), x
)
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\sqrt{a^2\,x^2+1}\,\left(1+a\,x\,3\mathrm{i}\right)\,1\mathrm{i}}{24\,a^3\,c^6\,\left(a\,x+1\mathrm{i}\right)^4\,\left(1+a\,x\,1\mathrm{i}\right)^7}$$

```
[In] int((x^2*(a^2*x^2 + 1)^(3/2))/((c + a^2*c*x^2)^(11/2)*(a*x*1i + 1)^3),x)
[Out] ((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2 + 1)^(1/2)*(a*x*3i + 1)*1i)/(24*a^3*c^6*(
a*x + 1i)^4*(a*x*1i + 1)^7)
```

3.385 $\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{(i-5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{c+a^2cx^2}}$$

```
[Out] 1/120*(I-5*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^13/(1-I*a*x)^10/(1+I*a*x)^15/(a^2*c
*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5193, 5190, 82}

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{(-5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{a^2cx^2+c}}$$

```
[In] Int[x^2/(E^((5*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((I - 5*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(1 - I*a*x)^10*(1 + I*a*x)^15
*Sqrt[c + a^2*c*x^2])
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
```

```
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 5190

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(
n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ
[p] || GtQ[c, 0])
```

Rule 5193

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Dist[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p]), Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+a^2x^2}\int \frac{e^{-5i\arctan(ax)}x^2}{(1+a^2x^2)^{27/2}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}\\ &= \frac{\sqrt{1+a^2x^2}\int \frac{x^2}{(1-iax)^{11}(1+iax)^{16}}\,dx}{c^{13}\sqrt{c+a^2cx^2}}\\ &= \frac{(i-5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{c+a^2cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{(1+5iax)\sqrt{1+a^2x^2}}{120a^3c^{13}(-i+ax)^{15}(i+ax)^{10}\sqrt{c+a^2cx^2}}$$

```
[In] Integrate[x^2/(E^((5*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((1 + (5*I)*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(-I + a*x)^15*(I + a*x)^1
0*Sqrt[c + a^2*c*x^2])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{\frac{ix}{24a^2}+\frac{1}{120a^3}}{c^{13}(a^2x^2+1)^{\frac{19}{2}}\sqrt{c(a^2x^2+1)}\,(ax-i)^5}$	50
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(5iax+1)}{120\sqrt{a^2x^2+1}\,c^{14}a^3(-ax+i)^{15}(ax+i)^{10}}$	57
gosper	$-\frac{(-ax+i)(ax+i)(-5ax+i)(a^2x^2+1)^{\frac{5}{2}}}{120a^3(iax+1)^5(a^2c\,x^2+c)^{\frac{27}{2}}}$	58

```
[In] int(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/c^13/(a^2*x^2+1)^(19/2)/(c*(a^2*x^2+1))^(1/2)*(1/24*I/a^2*x+1/120/a^3)/(a
*x-I)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(53) = 106.

Time = 0.36 (sec) , antiderivative size = 496, normalized size of antiderivative = 7.63

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{(-i\,a^{22}x^{25}-5\,a^{21}x^{24}-}{120\,(a^{27}c^{14}x^{27}-5i\,a^{26}c^{14}x^{26}+a^{25}c^{14}x^{25}-45i\,a^{24}c^{14}x^{24}-50\,a^{23}c^{14}x^{23}-166i\,a^{22}c^{14}x}$$

```
[In] integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] 1/120*(-I*a^22*x^25 - 5*a^21*x^24 - 40*a^19*x^22 + 50*I*a^18*x^21 - 126*a^1
7*x^20 + 280*I*a^16*x^19 - 160*a^15*x^18 + 765*I*a^14*x^17 + 105*a^13*x^16
+ 1248*I*a^12*x^15 + 720*a^11*x^14 + 1260*I*a^10*x^13 + 1260*a^9*x^12 + 720
*I*a^8*x^11 + 1248*a^7*x^10 + 105*I*a^6*x^9 + 765*a^5*x^8 - 160*I*a^4*x^7 +
 280*a^3*x^6 - 126*I*a^2*x^5 + 50*a*x^4 - 40*I*x^3)*sqrt(a^2*c*x^2 + c)*sqr
t(a^2*x^2 + 1)/(a^27*c^14*x^27 - 5*I*a^26*c^14*x^26 + a^25*c^14*x^25 - 45*I
*a^24*c^14*x^24 - 50*a^23*c^14*x^23 - 166*I*a^22*c^14*x^22 - 330*a^21*c^14*
x^21 - 286*I*a^20*c^14*x^20 - 1045*a^19*c^14*x^19 - 55*I*a^18*c^14*x^18 - 2
013*a^17*c^14*x^17 + 825*I*a^16*c^14*x^16 - 2508*a^15*c^14*x^15 + 1980*I*a^
14*c^14*x^14 - 1980*a^13*c^14*x^13 + 2508*I*a^12*c^14*x^12 - 825*a^11*c^14*
x^11 + 2013*I*a^10*c^14*x^10 + 55*a^9*c^14*x^9 + 1045*I*a^8*c^14*x^8 + 286*
a^7*c^14*x^7 + 330*I*a^6*c^14*x^6 + 166*a^5*c^14*x^5 + 50*I*a^4*c^14*x^4 +
45*a^3*c^14*x^3 - I*a^2*c^14*x^2 + 5*a*c^14*x - I*c^14)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(1+I*a*x)**5*(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(27/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.22

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{}{120\,(a^{28}c^{14}x^{25} - 5i\,a^{27}c^{14}x^{24} - 40i\,a^{25}c^{14}x^{22} - 50\,a^{24}c^{14}x^{21} - 126i\,a^{23}c^{14}x^{20} - 280\,a}$$

```
[In] integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
[Out] 1/120*(5*I*a*sqrt(c)*x + sqrt(c))/(a^28*c^14*x^25 - 5*I*a^27*c^14*x^24 - 40
*I*a^25*c^14*x^22 - 50*a^24*c^14*x^21 - 126*I*a^23*c^14*x^20 - 280*a^22*c^1
4*x^19 - 160*I*a^21*c^14*x^18 - 765*a^20*c^14*x^17 + 105*I*a^19*c^14*x^16 -
 1248*a^18*c^14*x^15 + 720*I*a^17*c^14*x^14 - 1260*a^16*c^14*x^13 + 1260*I*
a^15*c^14*x^12 - 720*a^14*c^14*x^11 + 1248*I*a^13*c^14*x^10 - 105*a^12*c^14
*x^9 + 765*I*a^11*c^14*x^8 + 160*a^10*c^14*x^7 + 280*I*a^9*c^14*x^6 + 126*a
^8*c^14*x^5 + 50*I*a^7*c^14*x^4 + 40*a^6*c^14*x^3 + 5*a^4*c^14*x - I*a^3*c^
14)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0,0
]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0,0]e
xt_red
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{c^2\,\sqrt{a^2\,x^2+1}\,(a\,x+1\mathrm{i})^5\,(1+a\,x\,5\mathrm{i})}{120\,a^3\,\left(c\,\left(a^2\,x^2+1\right)\right)^{31/2}}$$

```
[In] int((x^2*(a^2*x^2 + 1)^(5/2))/((c + a^2*c*x^2)^(27/2)*(a*x*1i + 1)^5),x)
[Out] (c^2*(a^2*x^2 + 1)^(1/2)*(a*x + 1i)^5*(a*x*5i + 1))/(120*a^3*(c*(a^2*x^2 +
1))^(31/2))
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```