Computer Algebra Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.3-Inverse-tangent/153-5.3.7-Inverse-tangent-functions

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [153]. This is test number [153].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (153)	0.00 (0)
Mathematica	98.69 (151)	1.31 (2)
Fricas	93.46 (143)	6.54 (10)
Maple	87.58 (134)	12.42 (19)
Maxima	56.86 (87)	43.14 (66)
Giac	38.56 (59)	61.44 (94)
Mupad	35.95 (55)	64.05 (98)
Sympy	33.33 (51)	66.67 (102)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

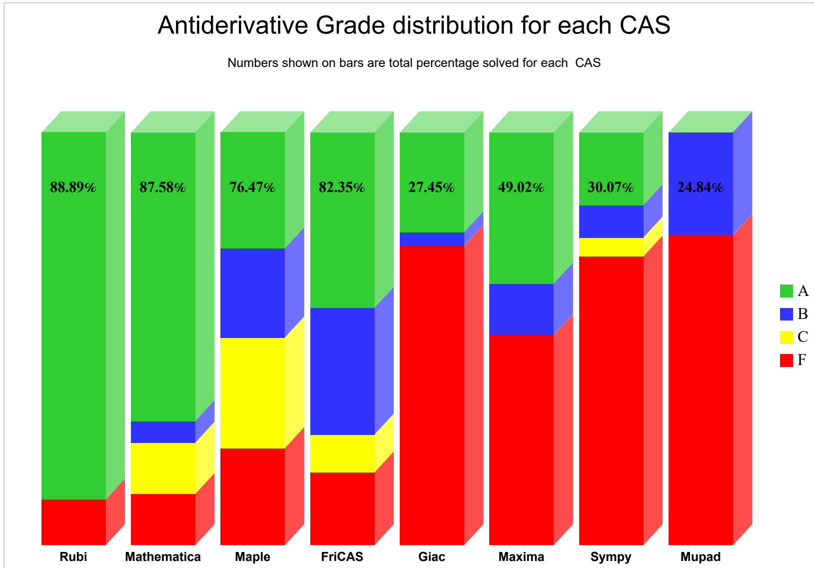
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

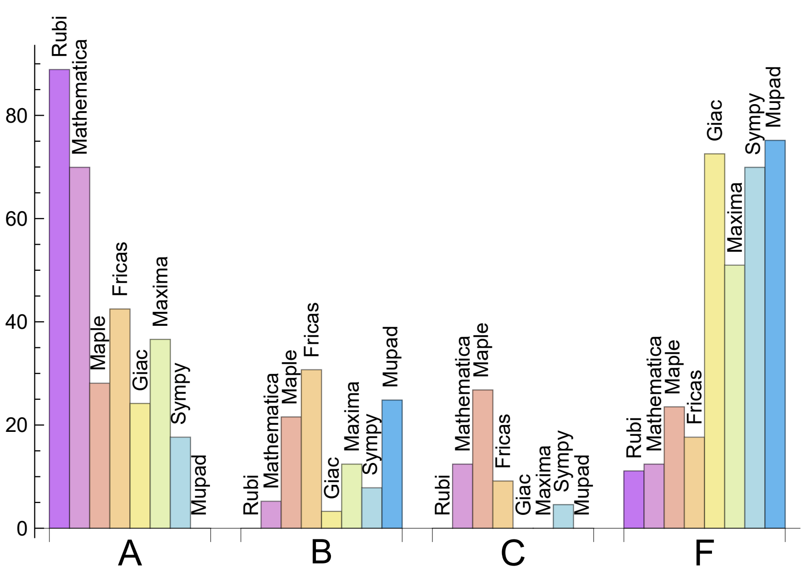
System	% A grade	% B grade	% C grade	% F grade
Rubi	88.889	0.000	0.000	11.111
Mathematica	69.935	5.229	12.418	12.418
Fricas	42.484	30.719	9.150	17.647
Maxima	36.601	12.418	0.000	50.980
Maple	28.105	21.569	26.797	23.529
Giac	24.183	3.268	0.000	72.549
Sympy	17.647	7.843	4.575	69.935
Mupad	0.000	24.837	0.000	75.163

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Fricas	10	100.00	0.00	0.00
Maple	19	100.00	0.00	0.00
Maxima	66	80.30	1.52	18.18
Sympy	102	46.08	30.39	23.53
Giac	94	98.94	1.06	0.00
Mupad	98	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.10
Fricas	0.27
Mupad	0.78
Mathematica	1.15
Maxima	3.14
Maple	3.58
Giac	3.78
Sympy	12.80

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	44.58	1.10	25.00	1.00
Giac	56.32	1.12	34.00	0.95
Sympy	93.04	1.73	66.00	1.01
Maxima	110.82	1.49	68.00	1.00
Rubi	113.85	1.00	85.00	1.00
Mathematica	141.72	1.32	79.00	0.95
Fricas	241.43	1.71	83.00	1.29
Maple	927.45	5.37	163.00	1.86

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

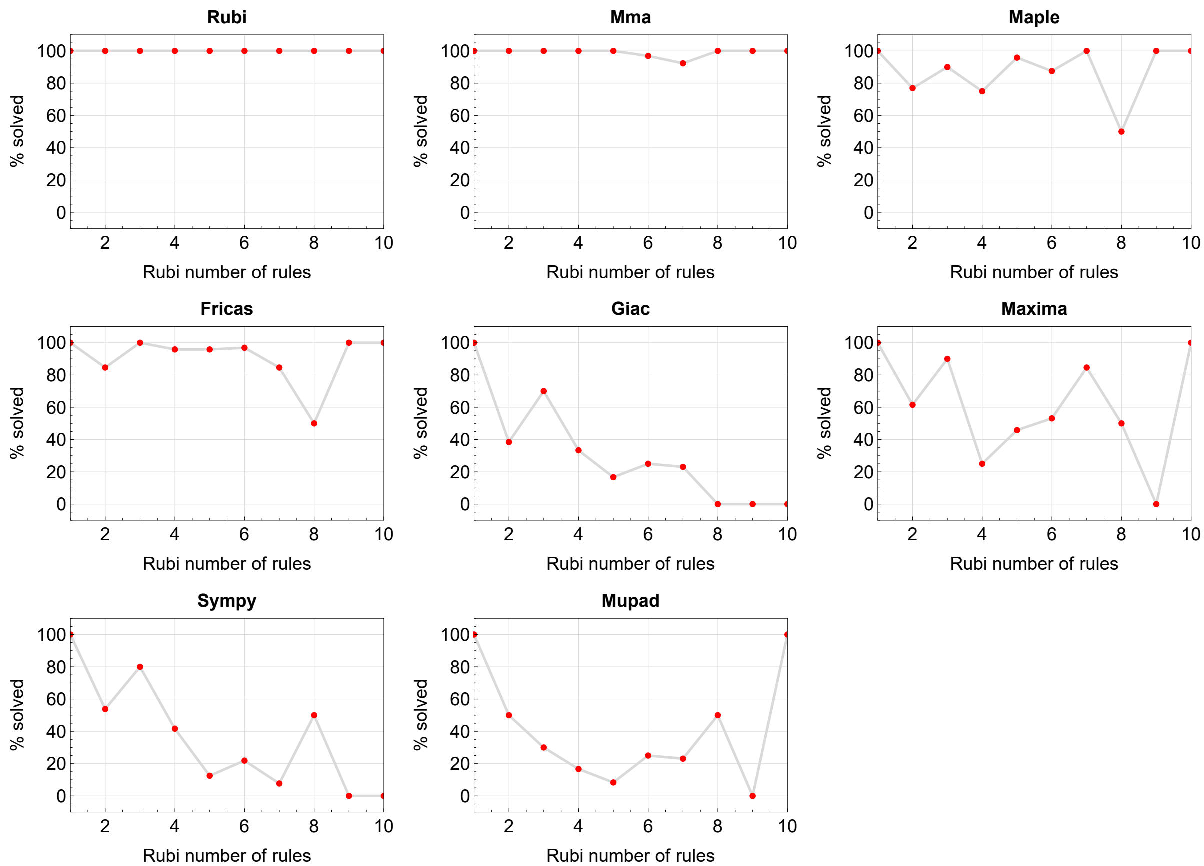


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

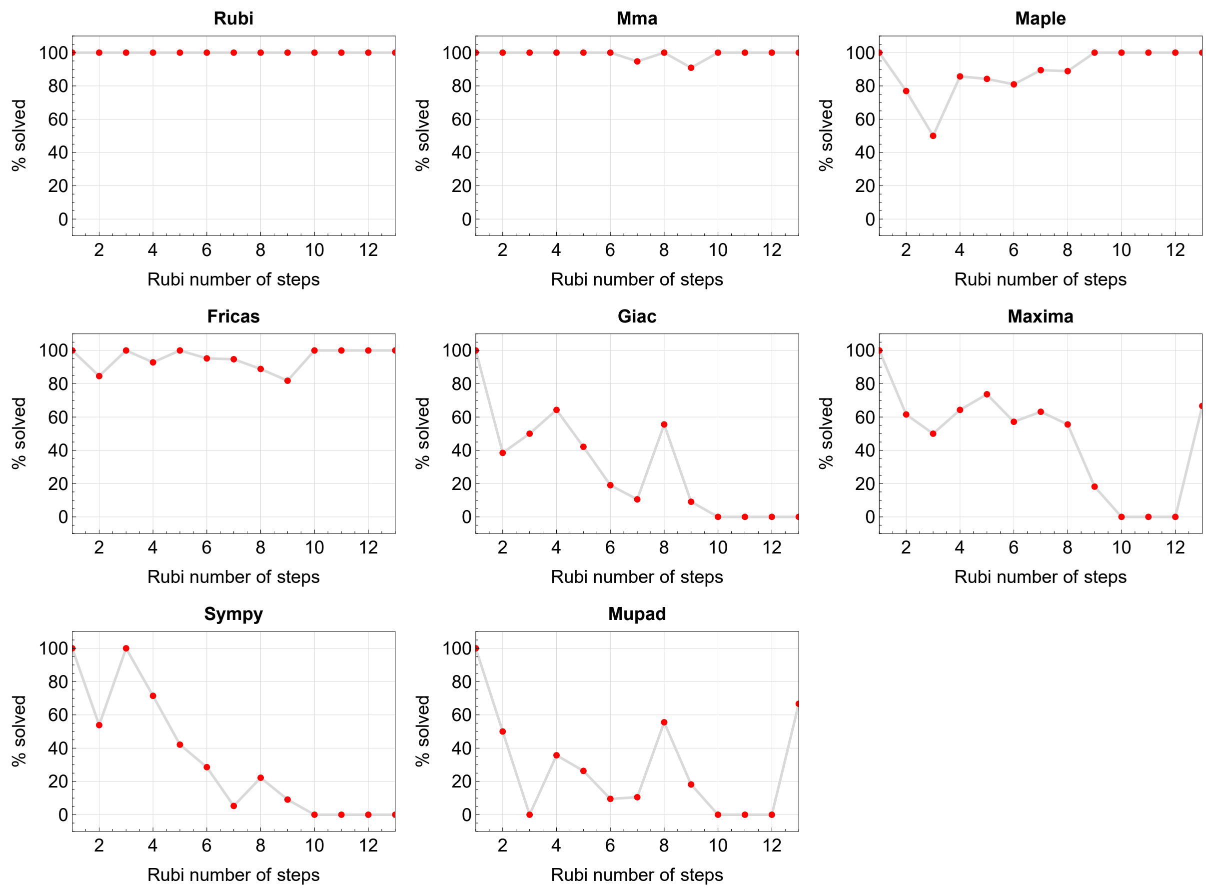


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

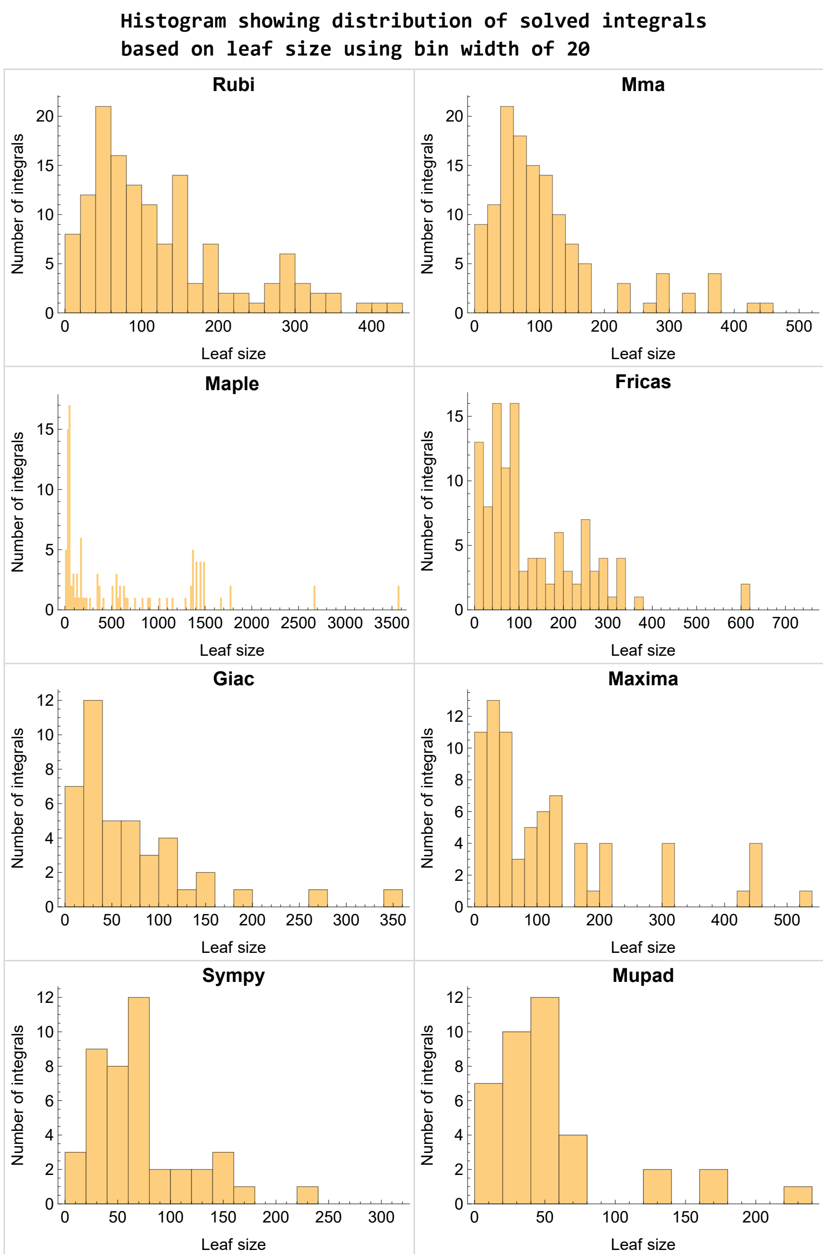


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

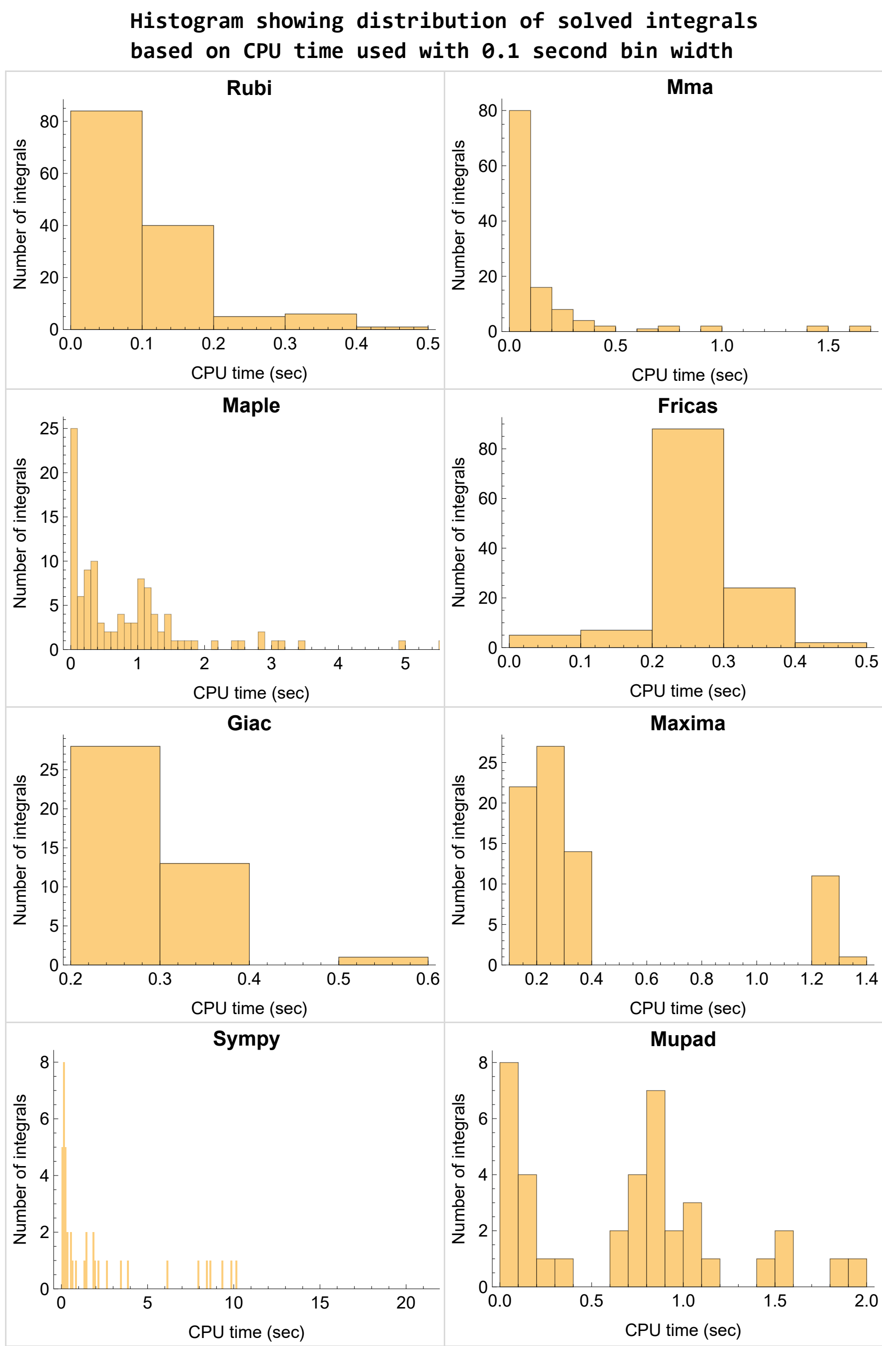


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

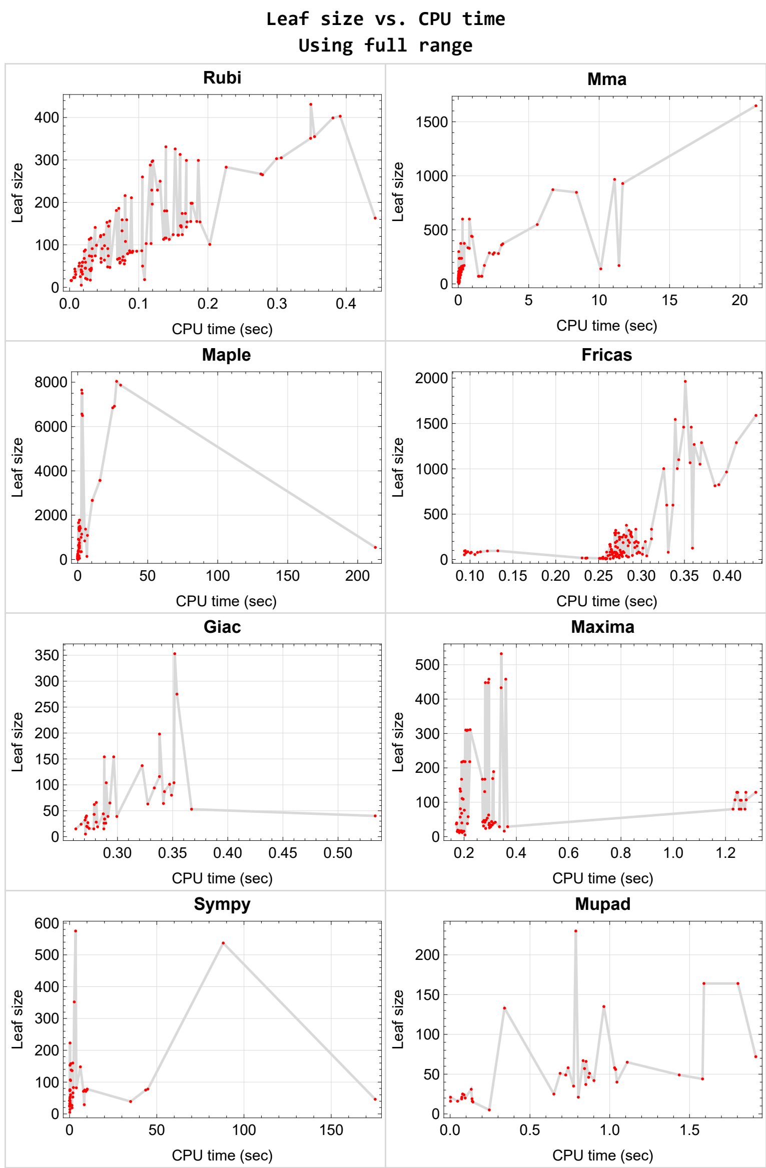


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{31, 35, 36, 51, 55, 59, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{50, 54, 58, 63, 67, 71\}$

Maple $\{48, 49, 52, 53, 56, 57, 61, 62, 65, 66, 69, 70, 74, 75, 76, 77, 78, 81, 82, 85, 86, 89, 90, 93, 94, 95, 98, 99, 102, 103, 106, 107, 146, 147, 148, 149, 150, 151, 152, 153\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

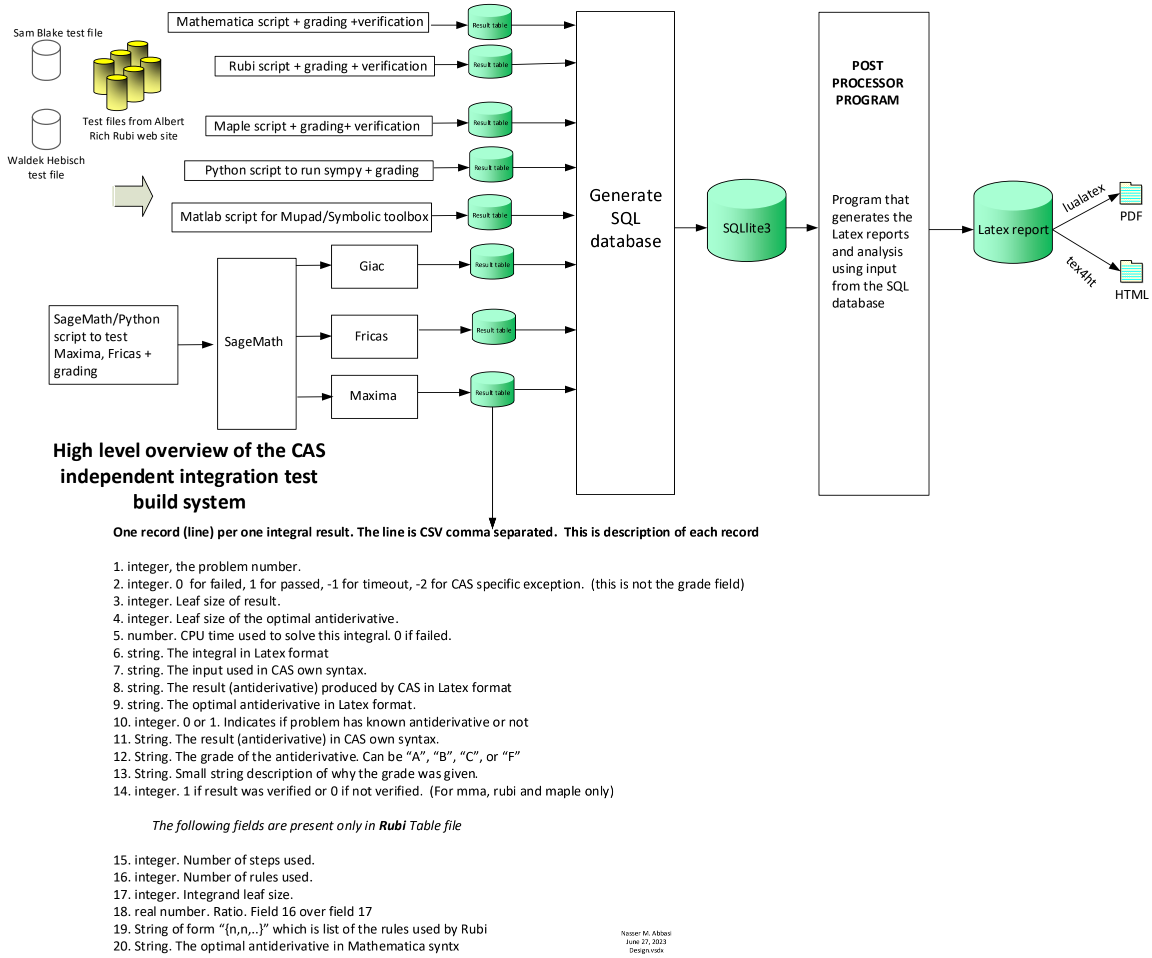
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 56, 57, 58, 60, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 30, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 56, 57, 60, 61, 62, 65, 66, 69, 70, 73, 74, 75, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152 }

B grade { 50, 54, 58, 63, 67, 71, 76, 93 }

C grade { 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 121, 148, 149, 150, 151, 153 }

F normal fail { 32, 33 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 8, 9, 10, 15, 16, 17, 30, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 60, 73, 111, 112, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { 3, 4, 5, 7, 11, 12, 13, 14, 32, 33, 34, 50, 54, 58, 63, 67, 71, 79, 83, 87, 91, 96, 100, 104, 108, 110, 113, 114, 115, 117, 118, 130, 131 }

C grade { 2, 48, 49, 52, 53, 56, 57, 61, 62, 65, 66, 69, 70, 74, 75, 76, 77, 78, 81, 82, 85, 86, 89, 90, 93, 94, 95, 98, 99, 102, 103, 106, 107, 146, 147, 148, 149, 150, 151, 152, 153 }

F normal fail { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 140, 141, 142, 143, 144, 145 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 30, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 60, 65, 66, 67, 69, 70, 71, 74, 75, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 137, 138, 139, 140, 143, 144, 145, 147 }

B grade { 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 73, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 113, 114, 134, 148, 151, 152, 153 }

C grade { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 149, 150 }

F normal fail { 6, 32, 33, 34, 130, 135, 136, 141, 142, 146 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 7, 8, 9, 10, 11, 12, 13, 30, 37, 40, 41, 42, 43, 44, 45, 46, 47, 60, 85, 86, 87, 89, 90, 91, 102, 103, 104, 106, 107, 108, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 138, 139, 147, 148, 149, 150, 151, 152 }

B grade { 14, 38, 39, 50, 52, 53, 54, 56, 57, 58, 63, 65, 66, 67, 69, 70, 71, 110, 113 }

C grade { }

F normal fail { 3, 4, 5, 6, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 48, 49, 61, 62, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 93, 94, 95, 96, 98, 99, 100, 111, 112, 114, 115, 117, 118, 135, 136, 137, 153 }

F(-1) timedout fail { 64 }

F(-2) exception fail { 55, 59, 68, 72, 134, 140, 141, 142, 143, 144, 145, 146 }

Giac

A grade { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 30, 40, 42, 43, 44, 45, 46, 47, 60, 119, 120, 122, 123, 124, 126, 127, 128, 129, 131, 132, 133, 147, 148, 151, 152 }

B grade { 7, 8, 9, 10, 125 }

C grade { }

F normal fail { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 37, 38, 39, 41, 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 130, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 153 }

F(-1) timedout fail { 76 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 14, 30, 38, 39, 40, 43, 44, 45, 47, 60, 110, 113, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 137, 138, 139, 147, 148, 149, 150, 151, 152 }

C grade { }

F normal fail { }

F(-1) timedout fail { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 37, 41, 42, 46, 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 111, 112, 114, 115, 116, 117, 118, 130, 135, 136, 140, 141, 142, 143, 144, 145, 146, 153 }

F(-2) exception fail { }

Sympy

A grade { 1, 3, 4, 5, 7, 8, 11, 12, 13, 14, 15, 16, 17, 30, 38, 41, 43, 45, 46, 60, 119, 122, 123, 124, 127, 128, 129 }

B grade { 9, 10, 37, 39, 40, 42, 44, 47, 120, 121, 125, 131 }

C grade { 19, 20, 21, 22, 26, 27, 28 }

F normal fail { 6, 32, 33, 34, 50, 63, 73, 74, 75, 76, 77, 78, 79, 82, 83, 93, 94, 95, 96, 110, 111, 112, 113, 114, 115, 116, 117, 118, 130, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152 }

F(-1) timedout fail { 2, 18, 23, 24, 25, 29, 48, 49, 51, 55, 59, 61, 62, 64, 68, 72, 81, 84, 88, 92, 97, 98, 99, 100, 105, 109, 126, 132, 133, 136, 153 }

F(-2) exception fail { 52, 53, 54, 56, 57, 58, 65, 66, 67, 69, 70, 71, 85, 86, 87, 89, 90, 91, 102, 103, 104, 106, 107, 108 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	37	37	45	60	37	230
N.S.	1	1.00	0.88	0.88	0.88	1.07	1.43	0.88	5.48
time (sec)	N/A	0.029	0.015	0.286	0.171	0.289	0.562	0.271	0.787

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	40	140	40	58	0	40	58
N.S.	1	1.00	0.89	3.11	0.89	1.29	0.00	0.89	1.29
time (sec)	N/A	0.031	0.027	6.573	0.172	0.276	0.000	0.272	0.739

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	86	260	0	76	138	94	0
N.S.	1	1.00	0.60	1.81	0.00	0.53	0.96	0.65	0.00
time (sec)	N/A	0.056	0.055	0.031	0.000	0.280	0.895	0.333	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	74	212	0	65	107	80	0
N.S.	1	1.00	0.64	1.83	0.00	0.56	0.92	0.69	0.00
time (sec)	N/A	0.031	0.038	0.025	0.000	0.277	0.389	0.349	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	59	164	0	49	73	63	0
N.S.	1	1.00	0.67	1.86	0.00	0.56	0.83	0.72	0.00
time (sec)	N/A	0.023	0.028	0.023	0.000	0.275	0.126	0.327	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	171	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	1.852	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	122	58	44	53	104	0
N.S.	1	1.00	0.95	2.14	1.02	0.77	0.93	1.82	0.00
time (sec)	N/A	0.014	0.028	0.023	0.216	0.274	1.872	0.351	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	67	69	68	58	83	198	0
N.S.	1	1.00	0.79	0.81	0.80	0.68	0.98	2.33	0.00
time (sec)	N/A	0.021	0.036	0.026	0.191	0.301	2.186	0.338	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	78	117	109	68	352	275	0
N.S.	1	1.00	0.69	1.04	0.96	0.60	3.12	2.43	0.00
time (sec)	N/A	0.028	0.043	0.024	0.197	0.295	2.676	0.354	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	89	165	132	80	575	353	0
N.S.	1	1.00	0.63	1.17	0.94	0.57	4.08	2.50	0.00
time (sec)	N/A	0.037	0.053	0.027	0.186	0.331	3.466	0.352	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	83	231	167	79	136	137	0
N.S.	1	1.00	0.67	1.86	1.35	0.64	1.10	1.10	0.00
time (sec)	N/A	0.049	0.076	0.038	0.190	0.270	1.434	0.322	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	72	183	139	68	105	101	0
N.S.	1	1.00	0.73	1.85	1.40	0.69	1.06	1.02	0.00
time (sec)	N/A	0.039	0.067	0.026	0.185	0.266	0.589	0.347	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	60	135	111	56	75	64	0
N.S.	1	1.00	0.81	1.82	1.50	0.76	1.01	0.86	0.00
time (sec)	N/A	0.029	0.070	0.023	0.192	0.266	0.276	0.342	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	86	77	41	41	40	35
N.S.	1	1.00	1.00	2.00	1.79	0.95	0.95	0.93	0.81
time (sec)	N/A	0.008	0.013	0.021	0.201	0.269	0.129	0.534	0.773

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	86	90	0	148	66	53	0
N.S.	1	1.00	1.46	1.53	0.00	2.51	1.12	0.90	0.00
time (sec)	N/A	0.023	0.042	0.023	0.000	0.295	1.930	0.367	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	101	130	0	198	82	87	0
N.S.	1	1.00	1.11	1.43	0.00	2.18	0.90	0.96	0.00
time (sec)	N/A	0.033	0.063	0.022	0.000	0.304	3.887	0.343	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	114	178	0	228	148	116	0
N.S.	1	1.00	0.96	1.50	0.00	1.92	1.24	0.97	0.00
time (sec)	N/A	0.045	0.083	0.027	0.000	0.312	6.188	0.338	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	170	0	0	96	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.089	11.410	0.000	0.000	0.132	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	158	0	0	85	75	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.47	0.41	0.00	0.00
time (sec)	N/A	0.068	0.312	0.000	0.000	0.112	43.609	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	147	0	0	71	75	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.46	0.49	0.00	0.00
time (sec)	N/A	0.054	0.213	0.000	0.000	0.100	8.640	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	115	0	0	52	71	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.43	0.58	0.00	0.00
time (sec)	N/A	0.045	0.093	0.000	0.000	0.093	7.972	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	150	0	0	73	78	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.47	0.50	0.00	0.00
time (sec)	N/A	0.058	0.196	0.000	0.000	0.095	44.905	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	186	186	162	0	0	86	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.071	0.223	0.000	0.000	0.097	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	171	0	0	97	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.080	0.419	0.000	0.000	0.095	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	326	139	0	0	94	0	0	0
N.S.	1	1.00	0.43	0.00	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.153	10.120	0.000	0.000	0.120	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	296	119	0	0	79	75	0	0
N.S.	1	1.00	0.40	0.00	0.00	0.27	0.25	0.00	0.00
time (sec)	N/A	0.119	0.083	0.000	0.000	0.109	9.881	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	260	89	0	0	55	71	0	0
N.S.	1	1.00	0.34	0.00	0.00	0.21	0.27	0.00	0.00
time (sec)	N/A	0.105	0.081	0.000	0.000	0.106	9.308	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	121	0	0	80	78	0	0
N.S.	1	1.00	0.41	0.00	0.00	0.27	0.26	0.00	0.00
time (sec)	N/A	0.120	0.098	0.000	0.000	0.101	10.110	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	331	331	137	0	0	94	0	0	0
N.S.	1	1.00	0.41	0.00	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.139	0.086	0.000	0.000	0.094	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	43	42	47	41	43	42
N.S.	1	1.00	1.00	0.86	0.84	0.94	0.82	0.86	0.84
time (sec)	N/A	0.106	0.014	0.361	0.319	0.274	0.286	0.279	0.901

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	34	38	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.85	0.95	0.98
time (sec)	N/A	0.036	0.135	0.415	0.963	0.292	3.748	0.387	1.017

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	0	1664	0	0	0	0	0
N.S.	1	1.00	0.00	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.000	0.580	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	0	916	0	0	0	0	0
N.S.	1	1.00	0.00	3.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.000	0.476	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	93	368	0	0	0	0	0
N.S.	1	1.00	0.95	3.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.029	0.324	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	38	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.95	0.98
time (sec)	N/A	0.034	0.224	0.279	0.426	0.243	2.756	0.371	0.804

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	163	91	126	38	39
N.S.	1	1.00	1.05	0.90	4.08	2.28	3.15	0.95	0.98
time (sec)	N/A	0.033	1.019	0.296	0.536	0.245	6.120	0.517	1.553

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	41	38	33	158	0	0
N.S.	1	1.00	0.92	1.11	1.03	0.89	4.27	0.00	0.00
time (sec)	N/A	0.018	0.047	0.270	0.213	0.278	0.665	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	81	13	32	0	19
N.S.	1	1.00	0.87	0.87	3.52	0.57	1.39	0.00	0.83
time (sec)	N/A	0.007	0.014	0.114	0.186	0.253	0.102	0.000	0.137

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	57	13	76	0	19
N.S.	1	1.00	0.87	0.87	2.48	0.57	3.30	0.00	0.83
time (sec)	N/A	0.006	0.012	0.110	0.185	0.251	0.167	0.000	0.072

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	10	42	26	16
N.S.	1	1.00	1.12	0.94	0.75	0.62	2.62	1.62	1.00
time (sec)	N/A	0.002	0.006	0.101	0.182	0.260	0.066	0.289	0.047

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	23	42	8	34	0	0
N.S.	1	1.00	0.90	1.10	2.00	0.38	1.62	0.00	0.00
time (sec)	N/A	0.023	0.014	0.127	0.280	0.263	0.335	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	56	40	42	160	62	0
N.S.	1	1.00	0.86	1.56	1.11	1.17	4.44	1.72	0.00
time (sec)	N/A	0.017	0.035	0.317	0.191	0.261	1.857	0.279	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	26	17	17	26	19	25
N.S.	1	1.00	0.87	1.13	0.74	0.74	1.13	0.83	1.09
time (sec)	N/A	0.008	0.014	0.247	0.191	0.231	0.104	0.282	0.650

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	26	17	17	49	19	25
N.S.	1	1.00	0.87	1.13	0.74	0.74	2.13	0.83	1.09
time (sec)	N/A	0.006	0.014	0.221	0.184	0.236	0.162	0.273	0.078

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	22	15	15	24	15	21
N.S.	1	1.00	1.12	1.38	0.94	0.94	1.50	0.94	1.31
time (sec)	N/A	0.003	0.007	0.118	0.188	0.236	0.059	0.279	0.073

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	27	14	14	27	15	0
N.S.	1	1.00	1.00	1.42	0.74	0.74	1.42	0.79	0.00
time (sec)	N/A	0.025	0.014	0.168	0.191	0.258	1.351	0.262	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	10	42	26	16
N.S.	1	1.00	1.12	0.94	0.75	0.62	2.62	1.62	1.00
time (sec)	N/A	0.002	0.000	0.040	0.193	0.255	0.066	0.288	0.002

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	403	403	371	8039	0	1965	0	0	0
N.S.	1	1.00	0.92	19.95	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	0.391	3.162	27.782	0.000	0.351	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	305	305	281	7647	0	1545	0	0	0
N.S.	1	1.00	0.92	25.07	0.00	5.07	0.00	0.00	0.00
time (sec)	N/A	0.306	2.847	2.852	0.000	0.339	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	549	1001	433	1101	0	0	0
N.S.	1	1.00	2.77	5.06	2.19	5.56	0.00	0.00	0.00
time (sec)	N/A	0.177	5.607	2.408	0.342	0.343	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	1.13	1.13
time (sec)	N/A	0.105	2.147	0.082	232.907	0.264	0.000	1.319	0.733

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	154	140	1487	310	322	0	0	0
N.S.	1	1.00	0.91	9.66	2.01	2.09	0.00	0.00	0.00
time (sec)	N/A	0.170	0.310	1.445	0.206	0.270	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1452	218	271	0	0	0
N.S.	1	1.00	0.89	11.80	1.77	2.20	0.00	0.00	0.00
time (sec)	N/A	0.157	0.181	1.071	0.204	0.279	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	967	563	448	202	0	0	0
N.S.	1	1.00	11.38	6.62	5.27	2.38	0.00	0.00	0.00
time (sec)	N/A	0.091	11.097	1.112	0.281	0.271	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	37	0	21	22
N.S.	1	1.00	1.10	0.90	0.00	1.76	0.00	1.00	1.05
time (sec)	N/A	0.104	0.575	0.080	0.000	0.256	0.000	1.005	0.868

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	155	137	1488	310	322	0	0	0
N.S.	1	1.00	0.88	9.60	2.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.175	0.315	1.480	0.214	0.285	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	124	111	1453	218	271	0	0	0
N.S.	1	1.00	0.90	11.72	1.76	2.19	0.00	0.00	0.00
time (sec)	N/A	0.150	0.211	0.981	0.222	0.269	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	847	594	448	200	0	0	0
N.S.	1	1.00	9.85	6.91	5.21	2.33	0.00	0.00	0.00
time (sec)	N/A	0.087	8.388	1.169	0.293	0.294	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	37	0	21	22
N.S.	1	1.00	1.10	0.90	0.00	1.76	0.00	1.00	1.05
time (sec)	N/A	0.114	0.876	0.082	0.000	0.260	0.000	1.170	1.074

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	22	15	15	24	15	21
N.S.	1	1.00	1.12	1.38	0.94	0.94	1.50	0.94	1.31
time (sec)	N/A	0.002	0.001	0.054	0.174	0.235	0.059	0.288	0.002

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	399	399	360	7869	0	1589	0	0	0
N.S.	1	1.00	0.90	19.72	0.00	3.98	0.00	0.00	0.00
time (sec)	N/A	0.381	3.069	30.662	0.000	0.433	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	303	303	275	7501	0	1289	0	0	0
N.S.	1	1.00	0.91	24.76	0.00	4.25	0.00	0.00	0.00
time (sec)	N/A	0.299	2.469	3.151	0.000	0.411	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	1648	1145	532	965	0	0	0
N.S.	1	1.00	8.32	5.78	2.69	4.87	0.00	0.00	0.00
time (sec)	N/A	0.176	21.134	2.872	0.342	0.399	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	0	17	0	17	17
N.S.	1	1.00	1.13	1.00	0.00	1.13	0.00	1.13	1.13
time (sec)	N/A	0.106	1.983	0.138	0.000	0.257	0.000	3.784	0.785

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	154	136	1487	309	166	0	0	0
N.S.	1	1.00	0.88	9.66	2.01	1.08	0.00	0.00	0.00
time (sec)	N/A	0.189	0.282	1.420	0.210	0.263	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1452	217	144	0	0	0
N.S.	1	1.00	0.89	11.80	1.76	1.17	0.00	0.00	0.00
time (sec)	N/A	0.158	0.195	1.101	0.191	0.264	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	929	584	458	112	0	0	0
N.S.	1	1.00	10.93	6.87	5.39	1.32	0.00	0.00	0.00
time (sec)	N/A	0.097	11.672	1.070	0.360	0.265	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	23	0	37	0	25	23
N.S.	1	1.00	1.10	1.10	0.00	1.76	0.00	1.19	1.10
time (sec)	N/A	0.102	0.539	0.125	0.000	0.260	0.000	1.798	1.062

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	155	140	1488	311	166	0	0	0
N.S.	1	1.00	0.90	9.60	2.01	1.07	0.00	0.00	0.00
time (sec)	N/A	0.184	0.291	1.497	0.223	0.263	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	124	110	1453	219	144	0	0	0
N.S.	1	1.00	0.89	11.72	1.77	1.16	0.00	0.00	0.00
time (sec)	N/A	0.163	0.191	1.098	0.197	0.270	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	872	625	458	112	0	0	0
N.S.	1	1.00	10.14	7.27	5.33	1.30	0.00	0.00	0.00
time (sec)	N/A	0.104	6.715	1.186	0.295	0.265	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	23	0	37	0	25	23
N.S.	1	1.00	1.10	1.10	0.00	1.76	0.00	1.19	1.10
time (sec)	N/A	0.102	0.540	0.135	0.000	0.259	0.000	2.043	1.121

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	56	52	0	58	0	0	0
N.S.	1	1.00	1.44	1.33	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.031	0.046	0.652	0.000	0.270	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	74	93	632	0	93	0	0	0
N.S.	1	1.00	1.26	8.54	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.056	0.042	0.561	0.000	0.265	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	108	108	121	658	0	125	0	0	0
N.S.	1	1.00	1.12	6.09	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.081	0.072	0.744	0.000	0.359	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	299	600	3570	0	1460	0	0	0
N.S.	1	1.00	2.01	11.94	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	0.186	0.785	15.921	0.000	0.349	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	375	2668	0	1002	0	0	0
N.S.	1	1.00	1.64	11.65	0.00	4.38	0.00	0.00	0.00
time (sec)	N/A	0.127	0.427	10.447	0.000	0.326	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	159	159	237	1776	0	600	0	0	0
N.S.	1	1.00	1.49	11.17	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.083	0.235	1.237	0.000	0.329	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	91	160	0	334	0	0	0
N.S.	1	1.00	1.23	2.16	0.00	4.51	0.00	0.00	0.00
time (sec)	N/A	0.037	0.024	0.825	0.000	0.312	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	3	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	0.20	1.13
time (sec)	N/A	0.029	11.143	0.129	1.555	0.271	2.953	105.119	0.696

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	355	355	438	6917	0	1289	0	0	0
N.S.	1	1.00	1.23	19.48	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	0.354	0.990	26.221	0.000	0.370	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	267	267	330	6567	0	1067	0	0	0
N.S.	1	1.00	1.24	24.60	0.00	4.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.783	3.002	0.000	0.357	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	288	352	0	825	0	0	0
N.S.	1	1.00	1.66	2.02	0.00	4.74	0.00	0.00	0.00
time (sec)	N/A	0.167	2.551	2.177	0.000	0.390	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	1.13	1.13
time (sec)	N/A	0.097	6.491	0.108	1.060	0.264	0.000	0.495	0.777

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	142	134	1406	129	293	0	0	0
N.S.	1	1.00	0.94	9.90	0.91	2.06	0.00	0.00	0.00
time (sec)	N/A	0.161	0.168	1.599	1.316	0.273	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	113	103	1370	106	247	0	0	0
N.S.	1	1.00	0.91	12.12	0.94	2.19	0.00	0.00	0.00
time (sec)	N/A	0.136	0.073	0.908	1.257	0.274	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	545	80	187	0	0	0
N.S.	1	1.00	0.90	6.90	1.01	2.37	0.00	0.00	0.00
time (sec)	N/A	0.084	1.656	1.027	1.229	0.296	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	72	37	0	19	20
N.S.	1	1.00	1.11	0.89	3.79	1.95	0.00	1.00	1.05
time (sec)	N/A	0.092	3.373	0.146	0.590	0.264	0.000	0.337	1.215

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	145	134	1409	129	293	0	0	0
N.S.	1	1.00	0.92	9.72	0.89	2.02	0.00	0.00	0.00
time (sec)	N/A	0.161	0.163	1.619	1.278	0.270	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	116	103	1373	107	247	0	0	0
N.S.	1	1.00	0.89	11.84	0.92	2.13	0.00	0.00	0.00
time (sec)	N/A	0.138	0.089	0.968	1.279	0.282	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	516	80	187	0	0	0
N.S.	1	1.00	0.87	6.29	0.98	2.28	0.00	0.00	0.00
time (sec)	N/A	0.090	1.679	1.015	1.274	0.269	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	77	37	0	19	20
N.S.	1	1.00	1.09	0.91	3.50	1.68	0.00	0.86	0.91
time (sec)	N/A	0.094	3.306	0.141	0.614	0.254	0.000	0.341	1.093

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	299	600	3570	0	1460	0	0	0
N.S.	1	1.00	2.01	11.94	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	0.169	0.299	15.857	0.000	0.358	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	375	2668	0	1002	0	0	0
N.S.	1	1.00	1.64	11.65	0.00	4.38	0.00	0.00	0.00
time (sec)	N/A	0.119	0.176	10.369	0.000	0.342	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	159	159	237	1777	0	600	0	0	0
N.S.	1	1.00	1.49	11.18	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.076	0.116	1.340	0.000	0.337	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	91	160	0	334	0	0	0
N.S.	1	1.00	1.25	2.19	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.032	0.021	0.837	0.000	0.293	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	3	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	0.20	1.13
time (sec)	N/A	0.029	3.179	0.135	1.554	0.251	0.000	90.476	0.713

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	351	351	442	6845	0	1269	0	0	0
N.S.	1	1.00	1.26	19.50	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	0.348	0.940	24.977	0.000	0.361	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	265	265	334	6495	0	1051	0	0	0
N.S.	1	1.00	1.26	24.51	0.00	3.97	0.00	0.00	0.00
time (sec)	N/A	0.279	0.682	3.421	0.000	0.368	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	287	352	0	813	0	0	0
N.S.	1	1.00	1.65	2.02	0.00	4.67	0.00	0.00	0.00
time (sec)	N/A	0.163	2.203	2.573	0.000	0.386	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.092	6.785	0.117	1.032	0.266	161.550	0.546	0.788

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	142	134	1405	129	293	0	0	0
N.S.	1	1.00	0.94	9.89	0.91	2.06	0.00	0.00	0.00
time (sec)	N/A	0.169	0.163	1.720	1.243	0.269	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	113	103	1369	106	247	0	0	0
N.S.	1	1.00	0.91	12.12	0.94	2.19	0.00	0.00	0.00
time (sec)	N/A	0.144	0.075	1.013	1.260	0.278	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	545	80	187	0	0	0
N.S.	1	1.00	0.90	6.90	1.01	2.37	0.00	0.00	0.00
time (sec)	N/A	0.084	1.476	1.153	1.252	0.286	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	77	37	0	19	20
N.S.	1	1.00	1.11	0.89	4.05	1.95	0.00	1.00	1.05
time (sec)	N/A	0.093	3.430	0.190	0.597	0.267	0.000	0.366	1.082

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	145	134	1410	129	293	0	0	0
N.S.	1	1.00	0.92	9.72	0.89	2.02	0.00	0.00	0.00
time (sec)	N/A	0.160	0.163	1.826	1.246	0.288	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	116	103	1374	107	247	0	0	0
N.S.	1	1.00	0.89	11.84	0.92	2.13	0.00	0.00	0.00
time (sec)	N/A	0.139	0.100	1.035	1.236	0.275	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	516	80	187	0	0	0
N.S.	1	1.00	0.87	6.29	0.98	2.28	0.00	0.00	0.00
time (sec)	N/A	0.087	1.430	1.144	1.259	0.292	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	72	37	0	19	20
N.S.	1	1.00	1.09	0.91	3.27	1.68	0.00	0.86	0.91
time (sec)	N/A	0.093	3.445	0.167	0.641	0.257	0.000	0.358	1.119

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	59	53	34	40	0	0	21
N.S.	1	1.00	1.90	1.71	1.10	1.29	0.00	0.00	0.68
time (sec)	N/A	0.019	0.018	0.307	0.303	0.277	0.000	0.000	0.803

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	44	0	65	0	0	0
N.S.	1	1.00	0.79	0.70	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.033	0.012	0.431	0.000	0.272	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	80	70	0	87	0	0	0
N.S.	1	1.00	0.88	0.77	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.055	0.013	0.644	0.000	0.279	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	83	95	63	103	0	0	37
N.S.	1	1.00	1.84	2.11	1.40	2.29	0.00	0.00	0.82
time (sec)	N/A	0.023	0.167	0.245	0.294	0.275	0.000	0.000	0.850

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	71	349	0	151	0	0	0
N.S.	1	1.00	0.78	3.84	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.047	0.013	0.378	0.000	0.284	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	115	407	0	187	0	0	0
N.S.	1	1.00	0.86	3.06	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.076	0.020	0.417	0.000	0.279	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	167	162	189	212	0	0	0
N.S.	1	1.00	0.85	0.83	0.96	1.08	0.00	0.00	0.00
time (sec)	N/A	0.120	0.310	0.740	0.313	0.284	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	250	236	672	0	304	0	0	0
N.S.	1	1.08	1.02	2.90	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.131	0.063	0.880	0.000	0.288	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	313	299	758	0	378	0	0	0
N.S.	1	1.04	0.99	2.51	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.160	0.033	1.247	0.000	0.282	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	19	28	19	20	20
N.S.	1	1.00	1.00	0.92	0.76	1.12	0.76	0.80	0.80
time (sec)	N/A	0.016	0.013	0.085	0.176	0.271	1.417	0.272	0.094

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	35	32	31	50	153	32	31
N.S.	1	1.00	0.78	0.71	0.69	1.11	3.40	0.71	0.69
time (sec)	N/A	0.023	0.025	0.267	0.274	0.263	0.220	0.271	0.132

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	81	54	54	77	223	0	46
N.S.	1	1.00	1.27	0.84	0.84	1.20	3.48	0.00	0.72
time (sec)	N/A	0.050	0.036	0.296	0.289	0.269	0.254	0.000	0.868

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	22	25	24	18	26	24	24
N.S.	1	1.00	0.73	0.83	0.80	0.60	0.87	0.80	0.80
time (sec)	N/A	0.009	0.019	0.023	0.282	0.274	0.089	0.267	0.087

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.017	0.046	0.246	0.203	0.258	0.110	0.271	0.245

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	16	15	14	16	15
N.S.	1	1.00	1.00	0.82	0.94	0.88	0.82	0.94	0.88
time (sec)	N/A	0.030	0.073	0.247	0.199	0.267	0.194	0.274	0.143

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	26	53	104	16
N.S.	1	1.00	1.00	0.94	0.89	1.44	2.94	5.78	0.89
time (sec)	N/A	0.108	0.024	0.322	0.354	0.255	0.257	0.290	0.139

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	58	45	44	40	0	44	72
N.S.	1	1.00	0.85	0.66	0.65	0.59	0.00	0.65	1.06
time (sec)	N/A	0.021	0.042	0.032	0.305	0.306	0.000	0.287	1.915

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	40	39	35	46	39	65
N.S.	1	1.00	0.90	0.68	0.66	0.59	0.78	0.66	1.10
time (sec)	N/A	0.019	0.023	0.034	0.313	0.268	175.227	0.291	1.110

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	35	34	28	39	34	58
N.S.	1	1.00	0.96	0.70	0.68	0.56	0.78	0.68	1.16
time (sec)	N/A	0.013	0.022	0.034	0.308	0.285	34.992	0.288	1.030

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	28	26	22	29	27	40
N.S.	1	1.00	0.84	0.76	0.70	0.59	0.78	0.73	1.08
time (sec)	N/A	0.008	0.073	0.030	0.297	0.267	8.412	0.273	1.045

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	84	374	43	0	0	0	0
N.S.	1	1.00	2.00	8.90	1.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.140	0.765	0.271	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	57	29	28	537	28	44
N.S.	1	1.00	0.98	1.39	0.71	0.68	13.10	0.68	1.07
time (sec)	N/A	0.017	0.022	0.033	0.302	0.284	88.100	0.281	1.582

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	35	34	35	0	34	49
N.S.	1	1.00	0.96	0.70	0.68	0.70	0.00	0.68	0.98
time (sec)	N/A	0.018	0.023	0.045	0.295	0.273	0.000	0.288	1.435

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	51	40	39	40	0	39	56
N.S.	1	1.00	0.86	0.68	0.66	0.68	0.00	0.66	0.95
time (sec)	N/A	0.019	0.029	0.052	0.296	0.282	0.000	0.299	1.037

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	59	0	126	0	0	57
N.S.	1	1.00	1.00	0.94	0.00	2.00	0.00	0.00	0.90
time (sec)	N/A	0.080	0.044	0.758	0.000	0.302	0.000	0.000	0.842

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	0	0	0	0	0
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.022	0.372	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	0	0	0	0	0
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	0.016	0.382	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	55	0	83	0	0	49
N.S.	1	1.00	1.00	1.00	0.00	1.51	0.00	0.00	0.89
time (sec)	N/A	0.079	0.121	1.294	0.000	0.296	0.000	0.000	0.724

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	57	29	82	0	0	51
N.S.	1	1.00	1.00	1.00	0.51	1.44	0.00	0.00	0.89
time (sec)	N/A	0.073	0.019	0.356	0.335	0.259	0.000	0.000	0.689

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	29	82	0	0	51
N.S.	1	1.00	1.00	0.97	0.49	1.39	0.00	0.00	0.86
time (sec)	N/A	0.073	0.018	0.352	0.366	0.263	0.000	0.000	0.873

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	66	0	0	132	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	0.077	0.106	0.000	0.000	0.292	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.058	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	58	0	0	83	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.077	0.051	0.000	0.000	0.276	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	60	0	0	82	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.069	0.050	0.000	0.000	0.269	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	83	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.072	0.044	0.000	0.000	0.266	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	101	101	79	1087	0	0	0	0	0
N.S.	1	1.00	0.78	10.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.105	6.954	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	48	48	61	1299	48	75	0	65	66
N.S.	1	1.00	1.27	27.06	1.00	1.56	0.00	1.35	1.38
time (sec)	N/A	0.055	0.057	1.111	0.284	0.298	0.000	0.293	0.852

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	103	146	1371	131	221	0	154	133
N.S.	1	1.00	1.42	13.31	1.27	2.15	0.00	1.50	1.29
time (sec)	N/A	0.118	0.083	5.592	0.279	0.274	0.000	0.297	0.341

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1355	167	258	0	0	164
N.S.	1	1.00	0.49	7.53	0.93	1.43	0.00	0.00	0.91
time (sec)	N/A	0.141	0.066	1.384	0.270	0.282	0.000	0.000	1.591

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1355	167	258	0	0	164
N.S.	1	1.00	0.49	7.53	0.93	1.43	0.00	0.00	0.91
time (sec)	N/A	0.137	0.064	1.226	0.280	0.288	0.000	0.000	1.803

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	103	145	838	169	276	0	154	135
N.S.	1	1.00	1.41	8.14	1.64	2.68	0.00	1.50	1.31
time (sec)	N/A	0.111	0.079	4.969	0.310	0.287	0.000	0.288	0.963

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	47	47	57	885	47	131	0	66	67
N.S.	1	1.00	1.21	18.83	1.00	2.79	0.00	1.40	1.43
time (sec)	N/A	0.058	0.051	1.017	0.276	0.272	0.000	0.281	0.833

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	163	163	116	547	0	250	0	0	0
N.S.	1	1.00	0.71	3.36	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.442	0.202	212.623	0.000	0.289	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [73] had the largest ratio of [1.3329999999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	12	0.333
2	A	4	4	1.00	14	0.286
3	A	6	4	1.00	25	0.160
4	A	5	4	1.00	25	0.160
5	A	4	4	1.00	23	0.174
6	A	8	8	1.00	25	0.320
7	A	2	2	1.00	25	0.080
8	A	3	3	1.00	25	0.120
9	A	4	3	1.00	25	0.120
10	A	5	3	1.00	25	0.120
11	A	4	3	1.00	25	0.120
12	A	4	3	1.00	25	0.120
13	A	4	3	1.00	25	0.120
14	A	2	2	1.00	21	0.095
15	A	4	4	1.00	25	0.160
16	A	5	5	1.00	25	0.200
17	A	6	5	1.00	25	0.200
18	A	6	4	1.00	27	0.148
19	A	5	4	1.00	27	0.148
20	A	4	4	1.00	27	0.148
21	A	3	3	1.00	27	0.111
22	A	4	4	1.00	27	0.148
23	A	5	4	1.00	27	0.148
24	A	6	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	7	6	1.00	27	0.222
26	A	6	6	1.00	27	0.222
27	A	5	5	1.00	27	0.185
28	A	6	6	1.00	27	0.222
29	A	7	6	1.00	27	0.222
30	A	8	7	1.00	11	0.636
31	N/A	0	0	1.00	40	0.000
32	A	9	7	1.00	40	0.175
33	A	7	6	1.00	40	0.150
34	A	4	4	1.00	38	0.105
35	N/A	0	0	1.00	40	0.000
36	N/A	0	0	1.00	40	0.000
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	11	0.182
39	A	2	2	1.00	9	0.222
40	A	2	2	1.00	7	0.286
41	A	2	2	1.00	11	0.182
42	A	2	2	1.00	11	0.182
43	A	2	2	1.00	11	0.182
44	A	2	2	1.00	9	0.222
45	A	2	2	1.00	7	0.286
46	A	2	2	1.00	11	0.182
47	A	2	2	1.00	7	0.286
48	A	11	6	1.00	15	0.400
49	A	9	5	1.00	13	0.385
50	A	7	4	1.00	11	0.364
51	N/A	0	0	1.00	15	0.000
52	A	7	7	1.00	21	0.333
53	A	6	6	1.00	19	0.316
54	A	5	5	1.00	17	0.294
55	N/A	0	0	1.00	21	0.000
56	A	7	7	1.00	21	0.333
57	A	6	6	1.00	19	0.316
58	A	5	5	1.00	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	N/A	0	0	1.00	21	0.000
60	A	2	2	1.00	7	0.286
61	A	11	6	1.00	15	0.400
62	A	9	5	1.00	13	0.385
63	A	7	4	1.00	11	0.364
64	N/A	0	0	1.00	15	0.000
65	A	7	7	1.00	21	0.333
66	A	6	6	1.00	19	0.316
67	A	5	5	1.00	17	0.294
68	N/A	0	0	1.00	21	0.000
69	A	7	7	1.00	21	0.333
70	A	6	6	1.00	19	0.316
71	A	5	5	1.00	17	0.294
72	N/A	0	0	1.00	21	0.000
73	A	6	4	1.00	3	1.333
74	A	8	5	1.00	5	1.000
75	A	10	6	1.00	7	0.857
76	A	12	6	1.00	15	0.400
77	A	10	6	1.00	15	0.400
78	A	8	5	1.00	13	0.385
79	A	6	4	1.00	7	0.571
80	N/A	0	0	1.00	15	0.000
81	A	11	6	1.00	15	0.400
82	A	9	5	1.00	13	0.385
83	A	7	4	1.00	11	0.364
84	N/A	0	0	1.00	15	0.000
85	A	7	7	1.00	19	0.368
86	A	6	6	1.00	17	0.353
87	A	5	5	1.00	15	0.333
88	N/A	0	0	1.00	19	0.000
89	A	7	7	1.00	22	0.318
90	A	6	6	1.00	20	0.300
91	A	5	5	1.00	18	0.278
92	N/A	0	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	12	6	1.00	15	0.400
94	A	10	6	1.00	15	0.400
95	A	8	5	1.00	13	0.385
96	A	6	4	1.00	7	0.571
97	N/A	0	0	1.00	15	0.000
98	A	11	6	1.00	15	0.400
99	A	9	5	1.00	13	0.385
100	A	7	4	1.00	11	0.364
101	N/A	0	0	1.00	15	0.000
102	A	7	7	1.00	19	0.368
103	A	6	6	1.00	17	0.353
104	A	5	5	1.00	15	0.333
105	N/A	0	0	1.00	19	0.000
106	A	7	7	1.00	22	0.318
107	A	6	6	1.00	20	0.300
108	A	5	5	1.00	18	0.278
109	N/A	0	0	1.00	22	0.000
110	A	4	3	1.00	4	0.750
111	A	7	4	1.00	6	0.667
112	A	9	5	1.00	8	0.625
113	A	4	3	1.00	8	0.375
114	A	7	4	1.00	10	0.400
115	A	9	5	1.00	12	0.417
116	A	6	6	1.00	12	0.500
117	A	9	5	1.08	14	0.357
118	A	11	6	1.04	16	0.375
119	A	5	6	1.00	10	0.600
120	A	5	4	1.00	8	0.500
121	A	9	8	1.00	14	0.571
122	A	4	4	1.00	8	0.500
123	A	1	1	1.00	14	0.071
124	A	1	1	1.00	19	0.053
125	A	5	3	1.00	26	0.115
126	A	9	6	1.00	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
127	A	8	6	1.00	21	0.286
128	A	7	6	1.00	19	0.316
129	A	6	6	1.00	18	0.333
130	A	6	5	1.00	21	0.238
131	A	6	6	1.00	21	0.286
132	A	7	6	1.00	21	0.286
133	A	8	6	1.00	21	0.286
134	A	2	2	1.00	39	0.051
135	A	2	2	1.00	39	0.051
136	A	2	2	1.00	37	0.054
137	A	2	2	1.00	39	0.051
138	A	2	2	1.00	39	0.051
139	A	2	2	1.00	39	0.051
140	A	2	2	1.00	40	0.050
141	A	2	2	1.00	40	0.050
142	A	2	2	1.00	38	0.053
143	A	2	2	1.00	40	0.050
144	A	2	2	1.00	40	0.050
145	A	2	2	1.00	40	0.050
146	A	9	7	1.00	24	0.292
147	A	5	5	1.00	20	0.250
148	A	8	7	1.00	20	0.350
149	A	13	10	1.00	20	0.500
150	A	13	10	1.00	20	0.500
151	A	8	7	1.00	20	0.350
152	A	5	5	1.00	20	0.250
153	A	13	9	1.00	24	0.375

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3 \arctan(a+bx^4)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int x^3 \arctan(a+bx^4)\,dx = \frac{(a+bx^4)\arctan(a+bx^4)}{4b} - \frac{\log\left(1+(a+bx^4)^2\right)}{8b}$$

```
[Out] 1/4*(b*x^4+a)*arctan(b*x^4+a)/b-1/8*ln(1+(b*x^4+a)^2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6847, 5147, 4930, 266}

$$\int x^3 \arctan(a+bx^4)\,dx = \frac{(a+bx^4)\arctan(a+bx^4)}{4b} - \frac{\log\left((a+bx^4)^2+1\right)}{8b}$$

```
[In] Int[x^3*ArcTan[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcTan[a + b*x^4])/(4*b) - Log[1 + (a + b*x^4)^2]/(8*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \arctan(a+bx)\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \arctan(x)\,dx, x, a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\arctan\left(a+bx^4\right)}{4b} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\arctan\left(a+bx^4\right)}{4b} - \frac{\log\left(1+(a+bx^4)^2\right)}{8b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$
\int x^3 \arctan\left(a+bx^4\right)\,dx = -\frac{-2(a+bx^4)\arctan\left(a+bx^4\right)+\log\left(1+(a+bx^4)^2\right)}{8b}
$$

```
[In] Integrate[x^3*ArcTan[a + b*x^4],x]
[Out] -1/8*(-2*(a + b*x^4)*ArcTan[a + b*x^4] + Log[1 + (a + b*x^4)^2])/b
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{(b\,x^4+a)\arctan(b\,x^4+a)-\frac{\ln\left(1+\left(b\,x^4+a\right)^2\right)}{2}}{4b}$
default	$\frac{(b\,x^4+a)\arctan(b\,x^4+a)-\frac{\ln\left(1+\left(b\,x^4+a\right)^2\right)}{2}}{4b}$
parallelrisch	$-\frac{-2\arctan(b\,x^4+a)x^4b^2-2a\arctan(b\,x^4+a)b+\ln(b^2x^8+2ab\,x^4+a^2+1)b}{8b^2}$
parts	$\frac{x^4\arctan(b\,x^4+a)}{4}-b\left(\frac{\ln(b^2x^8+2ab\,x^4+a^2+1)}{8b^2}-\frac{a\arctan\left(\frac{2b^2x^4+2ab}{2b}\right)}{4b^2}\right)$
risch	$-\frac{ix^4\ln(1+i(b\,x^4+a))}{8}+\frac{ix^4\ln(1-i(b\,x^4+a))}{8}+\frac{a\arctan\left(\frac{x^4b}{a^2+1}+\frac{a^2b\,x^4}{a^2+1}+\frac{a^3}{a^2+1}+\frac{a}{a^2+1}\right)}{4b}-\frac{a\arctan(a)}{4b}-\frac{\ln(a^6b^2}{}$

[In] int(x^3*arctan(b*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/4/b*((b*x^4+a)*arctan(b*x^4+a)-1/2*ln(1+(b*x^4+a)^2))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int x^3\arctan\left(a+bx^4\right)\,dx=\frac{2\,(bx^4+a)\arctan\left(bx^4+a\right)-\log\left(b^2x^8+2\,abx^4+a^2+1\right)}{8\,b}$$

[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="fricas")

[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))/
b

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int x^3\arctan\left(a+bx^4\right)\,dx=\begin{cases}\frac{a\,\mathrm{atan}\,(a+bx^4)}{4b}+\frac{x^4\,\mathrm{atan}\,(a+bx^4)}{4}-\frac{\log\left(a^2+2abx^4+b^2x^8+1\right)}{8b} & \text{for } b\neq 0\\ \frac{x^4\,\mathrm{atan}\,(a)}{4} & \text{otherwise}\end{cases}$$

[In] integrate(x**3*atan(b*x**4+a),x)

[Out] Piecewise((a*atan(a + b*x**4)/(4*b) + x**4*atan(a + b*x**4)/4 - log(a**2 +
2*a*b*x**4 + b**2*x**8 + 1)/(8*b), Ne(b, 0)), (x**4*atan(a)/4, True))

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \arctan \left(a+bx^4\right) \, dx = \frac{2 \left(bx^4+a\right) \arctan \left(bx^4+a\right) - \log \left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \arctan \left(a+bx^4\right) \, dx = \frac{2 \left(bx^4+a\right) \arctan \left(bx^4+a\right) - \log \left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 230, normalized size of antiderivative = 5.48

$$\int x^3 \arctan \left(a+bx^4\right) \, dx = \frac{x^4\,\mathrm{atan}(b\,x^4+a)}{4} - \frac{\ln\left(a^2+2\,a\,b\,x^4+b^2\,x^8+1\right)}{8\,b} + \frac{a\,\mathrm{atan}\left(\frac{a}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^3}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^5}{a^6+3\,a^4+3\,a^2+1}+\frac{a^7}{a^6+3\,a^4+3\,a^2+1}+\frac{b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^2\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\dots\right.}{4\,b}$$

```
[In] int(x^3*atan(a + b*x^4),x)
```

```
[Out] (x^4*atan(a + b*x^4))/4 - log(a^2 + b^2*x^8 + 2*a*b*x^4 + 1)/(8*b) + (a*ata
n(a/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^3)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^5)
/(3*a^2 + 3*a^4 + a^6 + 1) + a^7/(3*a^2 + 3*a^4 + a^6 + 1) + (b*x^4)/(3*a^2
 + 3*a^4 + a^6 + 1) + (3*a^2*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^4*b*x^
4)/(3*a^2 + 3*a^4 + a^6 + 1) + (a^6*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1)))/(4*b
)
```

3.2 $\int x^{-1+n} \arctan(a + bx^n)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int x^{-1+n} \arctan(a + bx^n)\, dx = \frac{(a + bx^n) \arctan(a + bx^n)}{bn} - \frac{\log\left(1 + (a + bx^n)^2\right)}{2bn}$$

```
[Out] (a+b*x^n)*arctan(a+b*x^n)/b/n-1/2*ln(1+(a+b*x^n)^2)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6847, 5147, 4930, 266}

$$\int x^{-1+n} \arctan(a + bx^n)\, dx = \frac{(a + bx^n) \arctan(a + bx^n)}{bn} - \frac{\log\left((a + bx^n)^2 + 1\right)}{2bn}$$

```
[In] Int[x^(-1 + n)*ArcTan[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcTan[a + b*x^n])/(b*n) - Log[1 + (a + b*x^n)^2]/(2*b*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \arctan(a+bx)\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \arctan(x)\,dx, x, a+bx^n\right)}{bn}\\ &= \frac{(a+bx^n)\arctan\left(a+bx^n\right)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx^n\right)}{bn}\\ &= \frac{(a+bx^n)\arctan\left(a+bx^n\right)}{bn} - \frac{\log\left(1+(a+bx^n)^2\right)}{2bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x^{-1+n}\arctan\left(a+bx^n\right)\,dx = -\frac{-2(a+bx^n)\arctan\left(a+bx^n\right)+\log\left(1+(a+bx^n)^2\right)}{2bn}$$

```
[In] Integrate[x^(-1 + n)*ArcTan[a + b*x^n],x]
[Out] -1/2*(-2*(a + b*x^n)*ArcTan[a + b*x^n] + Log[1 + (a + b*x^n)^2])/(b*n)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.57 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.11

method	result	size
risch	$-\frac{i x^{n} \ln\left(1+i\left(a+b\,x^{n}\right)\right)}{2n}+\frac{i x^{n} \ln\left(1-i\left(a+b\,x^{n}\right)\right)}{2n}-\frac{\ln\left(x^{n}-\frac{i-a}{b}\right)}{2bn}-\frac{\ln\left(\frac{i+a}{b}+x^{n}\right)}{2bn}-\frac{i\ln\left(x^{n}-\frac{i-a}{b}\right)a}{2bn}+\frac{i\ln\left(\frac{i+a}{b}+x^{n}\right)a}{2bn}$	140

```
[In] int(x^(-1+n)*arctan(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/n*x^n*ln(1+I*(a+b*x^n))+1/2*I/n*x^n*ln(1-I*(a+b*x^n))-1/2/b/n*ln(x^n
-(I-a)/b)-1/2/b/n*ln((I+a)/b+x^n)-1/2*I/b/n*ln(x^n-(I-a)/b)*a+1/2*I/b/n*ln(
(I+a)/b+x^n)*a
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int x^{-1+n}\arctan\left(a+bx^{n}\right)\,dx = \frac{2\,bx^{n}\arctan\left(bx^{n}+a\right)+2\,a\arctan\left(bx^{n}+a\right)-\log\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}+1\right)}{2\,bn}$$

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^n*arctan(b*x^n + a) + 2*a*arctan(b*x^n + a) - log(b^2*x^(2*n) +
2*a*b*x^n + a^2 + 1))/(b*n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}\arctan\left(a+bx^{n}\right)\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+n)*atan(a+b*x**n),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x^{-1+n} \arctan(a+bx^n)\,dx = \frac{2\,(bx^n+a)\arctan(bx^n+a) - \log\left((bx^n+a)^2+1\right)}{2\,bn}$$

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="maxima")
[Out] 1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x^{-1+n} \arctan(a+bx^n)\,dx = \frac{2\,(bx^n+a)\arctan(bx^n+a) - \log\left((bx^n+a)^2+1\right)}{2\,bn}$$

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="giac")
[Out] 1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int x^{-1+n} \arctan(a+bx^n)\,dx = \frac{x^n\,\mathrm{atan}(a+b\,x^n)}{n} - \frac{\ln\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1\right)-2\,a\,\mathrm{atan}(a+b\,x^n)}{2\,b\,n}$$

```
[In] int(x^(n - 1)*atan(a + b*x^n),x)
[Out] (x^n*atan(a + b*x^n))/n - (log(a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1) - 2*a*ata
n(a + b*x^n))/(2*b*n)
```

3.3 $\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{5d^3\sqrt{-e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^{7/2}}$$

```
[Out] 1/6*x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+5/96*d^3*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))*(-e)^(1/2)/e^(7/2)+5/96*d^2*x*(e*x^2+d)^(1/2)/(-e)^(5/2)+5/14
4*d*x^3*(e*x^2+d)^(1/2)/(-e)^(3/2)+1/36*x^5*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5259, 327, 223, 212}

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{6}x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{5d^3\sqrt{-e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^{7/2}} + \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}}$$

```
[In] Int[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (5*d^2*x*Sqrt[d + e*x^2])/(96*(-e)^(5/2)) + (5*d*x^3*Sqrt[d + e*x^2])/(144*
(-e)^(3/2)) + (x^5*Sqrt[d + e*x^2])/(36*Sqrt[-e]) + (x^6*ArcTan[(Sqrt[-e]*x
```

```
)/Sqrt[d + e*x^2]])/6 + (5*d^3*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/(96*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{-e}\int\frac{x^6}{\sqrt{d+ex^2}}\,dx\\
&= \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}}+\frac{1}{6}x^6\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(5d)\int\frac{x^4}{\sqrt{d+ex^2}}\,dx}{36\sqrt{-e}}\\
&= \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}}+\frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}}+\frac{1}{6}x^6\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(5d^2)\int\frac{x^2}{\sqrt{d+ex^2}}\,dx}{48(-e)^{3/2}}\\
&= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}}+\frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}}+\frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}}\\
&\quad+\frac{1}{6}x^6\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(5d^3)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{96(-e)^{5/2}}
\end{aligned}$$

$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(5d^3)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{96(-e)^{5/2}}$$

$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{5d^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96(-e)^{5/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{\sqrt{-e}x\sqrt{d+ex^2}(-15d^2+10dex^2-8e^2x^4)+3(5d^3+16e^3x^6)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{288e^3}$$

```
[In] Integrate[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-15*d^2 + 10*d*e*x^2 - 8*e^2*x^4) + 3*(5*d^3 +
 16*e^3*x^6)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(288*e^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(110) = 220$.

Time = 0.03 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.81

method	result
default	$\frac{x^6 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{6} + \frac{\sqrt{-e}\,e\left(\frac{x^7\sqrt{e\,x^2+d}}{8e} - \frac{7d\left(\frac{x^5\sqrt{e\,x^2+d}}{6e} - \frac{5d\left(\frac{x^3\sqrt{e\,x^2+d}}{4e} - \frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e} - \frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{8e}\right)}{6d} - \cdots$
parts	$\frac{x^6 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{6} + \frac{\sqrt{-e}\,e\left(\frac{x^7\sqrt{e\,x^2+d}}{8e} - \frac{7d\left(\frac{x^5\sqrt{e\,x^2+d}}{6e} - \frac{5d\left(\frac{x^3\sqrt{e\,x^2+d}}{4e} - \frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e} - \frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{8e}\right)}{6d} - \cdots$

```
[In] int(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/6*(-e)^(1/2)*e/d*(1/8*x^7/e*
(e*x^2+d)^(1/2)-7/8*d/e*(1/6*x^5/e*(e*x^2+d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*x^
2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x
^2+d)^(1/2))))))-1/6*(-e)^(1/2)/d*(1/8*x^5*(e*x^2+d)^(3/2)/e-5/8*d/e*(1/6*x
^3*(e*x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e*x^2
+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.53

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = -\frac{(8\,e^2x^5-10\,dex^3+15\,d^2x)\sqrt{ex^2+d}\sqrt{-e}-3\,(16\,e^3x^6+5\,d^3)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{288\,e^3}$$

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
[Out] -1/288*((8*e^2*x^5 - 10*d*e*x^3 + 15*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e) - 3*(1
6*e^3*x^6 + 5*d^3)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \begin{cases} \frac{5d^3\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{96e^3} - \frac{5d^2x\sqrt{-e}\sqrt{d+ex^2}}{96e^3} + \frac{5dx^3\sqrt{-e}\sqrt{d+ex^2}}{144e^2} + \frac{x^6\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{6} - \frac{x^5\sqrt{-e}\sqrt{d+ex^2}}{36e} & \text{for}\ e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Piecewise((5*d**3*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(96*e**3) - 5*d**2*x*sq
rt(-e)*sqrt(d + e*x**2)/(96*e**3) + 5*d*x**3*sqrt(-e)*sqrt(d + e*x**2)/(144
*e**2) + x**6*atan(x*sqrt(-e)/sqrt(d + e*x**2))/6 - x**5*sqrt(-e)*sqrt(d +
e*x**2)/(36*e), Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
[Out] 1/6*x^6*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - d*sqrt(-e)*integrate(-1/6*sq
rt(e*x^2 + d)*x^6/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{6} x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{1}{288} \sqrt{-e^2x^2-de}\left(2x^2\left(\frac{4x^2}{e} - \frac{5d}{e^2}\right) + \frac{15d^2}{e^3}\right)x - \frac{5d^3 \arcsin\left(\frac{ex}{\sqrt{-de}}\right) \operatorname{sgn}(e)}{96e^2|e|}$$

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/6*x^6*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/288*sqrt(-e^2*x^2 - d*e)*(2*
x^2*(4*x^2/e - 5*d/e^2) + 15*d^2/e^3)*x - 5/96*d^3*arcsin(e*x/sqrt(-d*e))*s
gn(e)/(e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^5 \operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

```
[In] int(x^5*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^5*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.4 $\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{3d^2\sqrt{-e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}$$

```
[Out] 1/4*x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-3/32*d^2*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))*(-e)^(1/2)/e^(5/2)+3/32*d*x*(e*x^2+d)^(1/2)/(-e)^(3/2)+1/16*x
^3*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5259, 327, 223, 212}

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{4}x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{3d^2\sqrt{-e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}} + \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}$$

```
[In] Int[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (3*d*x*Sqrt[d + e*x^2])/(32*(-e)^(3/2)) + (x^3*Sqrt[d + e*x^2])/(16*Sqrt[-e
]) + (x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/4 - (3*d^2*Sqrt[-e]*ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{4}\sqrt{-e}\int\frac{x^4}{\sqrt{d+ex^2}}\,dx\\
&= \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}+\frac{1}{4}x^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(3d)\int\frac{x^2}{\sqrt{d+ex^2}}\,dx}{16\sqrt{-e}}\\
&= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}}+\frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}+\frac{1}{4}x^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(3d^2)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{32(-e)^{3/2}}\\
&= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}}+\frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}+\frac{1}{4}x^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(3d^2)\,\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{32(-e)^{3/2}}\\
&= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}}+\frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}+\frac{1}{4}x^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{3d^2\sqrt{-e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$
$$= \frac{\sqrt{-e}x(3d-2ex^2)\sqrt{d+ex^2}+(-3d^2+8e^2x^4)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{32e^2}$$

```
[In] Integrate[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[-e]*x*(3*d - 2*e*x^2)*Sqrt[d + e*x^2] + (-3*d^2 + 8*e^2*x^4)*ArcTan[(
Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(32*e^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(88) = 176.

Time = 0.02 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.83

method	result
default	$\frac{x^4 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{4}+\frac{\sqrt{-e}\,e\left(\frac{x^5\sqrt{e\,x^2+d}}{6e}-\frac{5d\left(\frac{x^3\sqrt{e\,x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{4d}-\frac{\sqrt{-e}\left(\frac{x^3\left(e\,x^2+d\right)^{\frac{3}{2}}}{6e}-\cdots\right.}{\cdots}$
parts	$\frac{x^4 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{4}+\frac{\sqrt{-e}\,e\left(\frac{x^5\sqrt{e\,x^2+d}}{6e}-\frac{5d\left(\frac{x^3\sqrt{e\,x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{4d}-\frac{\sqrt{-e}\left(\frac{x^3\left(e\,x^2+d\right)^{\frac{3}{2}}}{6e}-\cdots\right.}{\cdots}$

```
[In] int(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/4*(-e)^(1/2)*e/d*(1/6*x^5/e*
(e*x^2+d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+
d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))-1/4*(-e)^(1/2)/d*(1
/6*x^3*(e*x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e
*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{(2\,ex^3 - 3\,dx)\sqrt{ex^2+d}\sqrt{-e} - (8\,e^2x^4 - 3\,d^2)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{32\,e^2}$$

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/32*((2*e*x^3 - 3*d*x)*sqrt(e*x^2 + d)*sqrt(-e) - (8*e^2*x^4 - 3*d^2)*arc
tan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e^2
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} -\frac{3d^2 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{32e^2} + \frac{3dx\sqrt{-e}\sqrt{d+ex^2}}{32e^2} + \frac{x^4 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4} - \frac{x^3\sqrt{-e}\sqrt{d+ex^2}}{16e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-3*d**2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(32*e**2) + 3*d*x*sqrt
(-e)*sqrt(d + e*x**2)/(32*e**2) + x**4*atan(x*sqrt(-e)/sqrt(d + e*x**2))/4
- x**3*sqrt(-e)*sqrt(d + e*x**2)/(16*e), Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) dx$$

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - d*sqrt(-e)*integrate(-1/4*sq
rt(e*x^2 + d)*x^4/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{1}{4}\,x^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{1}{32}\,\sqrt{-e^2x^2-dex}\left(\frac{2\,x^2}{e} - \frac{3\,d}{e^2}\right) + \frac{3\,d^2\arcsin\left(\frac{ex}{\sqrt{-de}}\right)\operatorname{sgn}(e)}{32\,e|e|}$$

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] 1/4*x^4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/32*sqrt(-e^2*x^2 - d*e)*x*(2
*x^2/e - 3*d/e^2) + 3/32*d^2*arcsin(e*x/sqrt(-d*e))*sgn(e)/(e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^3\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

```
[In] int(x^3*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^3*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.5 $\int x \arctan \left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int x \arctan \left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}} + \frac{1}{2}x^2 \arctan \left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{d\sqrt{-e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e^{3/2}}$$

```
[Out] 1/2*x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/4*d*arctanh(x*e^(1/2)/(e*x^2
+d)^(1/2))*(-e)^(1/2)/e^(3/2)+1/4*x*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5259, 327, 223, 212}

$$\int x \arctan \left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{2}x^2 \arctan \left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{d\sqrt{-e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e^{3/2}} + \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}$$

```
[In] Int[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(4*Sqrt[-e]) + (x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
])/2 + (d*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(4*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{2}\sqrt{-e}\int\frac{x^2}{\sqrt{d+ex^2}}\,dx\\
&= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{d\int\frac{1}{\sqrt{d+ex^2}}\,dx}{4\sqrt{-e}}\\
&= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{d\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-e}}\\
&= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-e^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{-\sqrt{-e}x\sqrt{d+ex^2}+(d+2ex^2)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4e}$$

```
[In] Integrate[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-(Sqrt[-e]*x*Sqrt[d + e*x^2]) + (d + 2*e*x^2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d +
 e*x^2]])/(4*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(66) = 132.

Time = 0.02 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.86

method	result
default	$\frac{x^2\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{2}+\frac{\sqrt{-e}\,e\left(\frac{x^3\sqrt{e\,x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{2d}-\frac{\sqrt{-e}\left(\frac{x\left(e\,x^2+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(\right.}{}\right.}{4e}\right.}{2d}$
parts	$\frac{x^2\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{2}+\frac{\sqrt{-e}\,e\left(\frac{x^3\sqrt{e\,x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{2d}-\frac{\sqrt{-e}\left(\frac{x\left(e\,x^2+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(\right.}{}\right.}{4e}\right.}{2d}$

```
[In] int(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/2*(-e)^(1/2)*e/d*(1/4*x^3/e*
(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)
+(e*x^2+d)^(1/2))))-1/2*(-e)^(1/2)/d*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*
x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=-\frac{\sqrt{ex^2+d}\sqrt{-e}x-(2\,ex^2+d)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{4\,e}$$

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(e*x^2 + d)*sqrt(-e)*x - (2*e*x^2 + d)*arctan(sqrt(-e)*x/sqrt(e*x
^2 + d)))/e
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} \frac{d \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4e} + \frac{x^2 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{2} - \frac{x\sqrt{-e}\sqrt{d+ex^2}}{4e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((d*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(4*e) + x**2*atan(x*sqrt(-e)
/sqrt(d + e*x**2))/2 - x*sqrt(-e)*sqrt(d + e*x**2)/(4*e), Ne(e, 0)), (0, Tr
ue))
```

Maxima [F]

$$\int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) dx$$

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - d*sqrt(-e)*integrate(-1/2*sq
rt(e*x^2 + d)*x^2/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{2}x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{d \arcsin\left(\frac{ex}{\sqrt{-de}}\right) \operatorname{sgn}(e)}{4|e|} - \frac{\sqrt{-e^2x^2-dex}}{4e}$$

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/4*d*arcsin(e*x/sqrt(-d*e))*s
gn(e)/abs(e) - 1/4*sqrt(-e^2*x^2 - d*e)*x/e
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

```
[In] int(x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.6 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 288

$$\begin{aligned}\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = &-\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}}\\ &+\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}}\\ &-\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}}\\ &+\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x)\\ &+\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))*ln(x)-1/2*arcsinh(x*e^(1/2)/d^(1/2))^2
*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+arcsinh(x*e^(
1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(-e)^(1
/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)-arcsinh(x*e^(1/2)/d^(1/2))*ln
(x)*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+1/2*polylo
g(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)
^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5257, 2364, 2362, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \frac{\sqrt{d}\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{-e}\log(x)\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\sqrt{d+ex^2}} + \log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] -1/2*(Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/
(Sqrt[e]*Sqrt[d + e*x^2]) + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(
Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(Sqrt[e]*S
qrt[d + e*x^2]) - (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)
/Sqrt[d]]*Log[x])/(Sqrt[e]*Sqrt[d + e*x^2]) + ArcTan[(Sqrt[-e]*x)/Sqrt[d +
e*x^2]]*Log[x] + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*PolyLog[2, E^(2*ArcS
inh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[e]*Sqrt[d + e*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2364

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e/d)*x^2]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e/d)*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5257

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]/(x_), x_Symbol] :> Simp
[ArcTan[c*(x/Sqrt[a + b*x^2])]*Log[x], x] - Dist[c, Int[Log[x]/Sqrt[a + b*x
^2], x], x] /; FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) - \sqrt{-e}\int\frac{\log(x)}{\sqrt{d+ex^2}}\,dx \\
&= \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{\log(x)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}
\end{aligned}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,dx}{\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(2\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} + \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\,\text{PolyLog}\left(2, e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.59

$$\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\left(\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)^2 + 2\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)\log\left(1-e^{-2\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)}\right) - 2\log(x)\log\left(\sqrt{\frac{e}{d}}x+\sqrt{1\right.\right.}{2\sqrt{\frac{e}{d}}\sqrt{d+ex^2}}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[-e]*Sqrt[1 + (e*x^2)/d]
*(ArcSinh[Sqrt[e/d]*x]^2 + 2*ArcSinh[Sqrt[e/d]*x]*Log[1 - E^(-2*ArcSinh[Sqr
t[e/d]*x])] - 2*Log[x]*Log[Sqrt[e/d]*x + Sqrt[1 + (e*x^2)/d]] - PolyLog[2,
E^(-2*ArcSinh[Sqrt[e/d]*x])]))/(2*Sqrt[e/d]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x}dx$$

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

Fricas [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="fricas")
[Out] integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

Sympy [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x,x)
[Out] Integral(atan(x*sqrt(-e)/sqrt(d + e*x**2))/x, x)
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="maxima")
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="giac")
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x,x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x, x)
```

3.7 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 57

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx=-\frac{\sqrt{-e}\sqrt{d+ex^2}}{2dx}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

```
[Out] -1/2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*(-e)^(1/2)*(e*x^2+d)^(1/2
)/d/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5259, 270}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx=-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{2dx}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]

[Out] -1/2*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
]/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}+\frac{1}{2}\sqrt{-e}\int\frac{1}{x^2\sqrt{d+ex^2}}\,dx\\ &= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{2dx}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx=-\frac{\sqrt{-e}x\sqrt{d+ex^2}+d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2dx^2}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]
[Out] -1/2*(Sqrt[-e]*x*Sqrt[d + e*x^2] + d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/
(d*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(45) = 90.

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{2x^2}-\frac{\sqrt{-e}\,\sqrt{e}\,\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{2d}$	122
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{2x^2}-\frac{\sqrt{-e}\,\sqrt{e}\,\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{2d}$	122

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*(-e)^(1/2)*e^(1/2)/d*ln(x
*e^(1/2)+(e*x^2+d)^(1/2))+1/2*(-e)^(1/2)/d*(-1/d/x*(e*x^2+d)^(3/2)+2*e/d*(1
/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\sqrt{ex^2+d}\sqrt{-e}x + d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,dx^2}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(e*x^2 + d)*sqrt(-e)*x + d*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(d
*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{2x^2} - \frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{2d}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**3,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(2*x**2) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(2*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,x^2} - \frac{\sqrt{-e}ex^2 + d\sqrt{-e}}{2\,\sqrt{ex^2+d}dx}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^2 - 1/2*(sqrt(-e)*e*x^2 + d*sqrt(
-e))/(sqrt(e*x^2 + d)*d*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(45) = 90$.

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.82

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = \frac{e^4x}{4\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)d|e|} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,x^2} - \frac{\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|}{4\,dx|e|}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*e^4*x/((sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))*d*abs(e)) - 1/2*arc
tan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^2 - 1/4*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d
*e)*abs(e))/(d*x*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^3}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^3,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^3, x)
```

3.8 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{12dx^3} - \frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

```
[Out] -1/4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4-1/6*(-e)^(3/2)*(e*x^2+d)^(1/2
)/d^2/x-1/12*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5259, 277, 270}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4} - \frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\sqrt{-e}\sqrt{d+ex^2}}{12dx^3}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] -1/12*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^3) - ((-e)^(3/2)*Sqrt[d + e*x^2])/(6*
d^2*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}+\frac{1}{4}\sqrt{-e}\int\frac{1}{x^4\sqrt{d+ex^2}}\,dx\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{12dx^3}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}+\frac{(-e)^{3/2}\int\frac{1}{x^2\sqrt{d+ex^2}}\,dx}{6d}\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{12dx^3}-\frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx=\frac{\sqrt{-e}x\sqrt{d+ex^2}(-d+2ex^2)-3d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{12d^2x^4}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-d + 2*e*x^2) - 3*d^2*ArcTan[(Sqrt[-e]*x)/Sqrt
[d + e*x^2]])/(12*d^2*x^4)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{4x^{4}}+\frac{\sqrt{-e}\,e\sqrt{e\,x^{2}+d}}{4d^{2}x}-\frac{\sqrt{-e}\,\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{12d^{2}x^{3}}$	69
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{4x^{4}}+\frac{\sqrt{-e}\,e\sqrt{e\,x^{2}+d}}{4d^{2}x}-\frac{\sqrt{-e}\,\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{12d^{2}x^{3}}$	69

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4+1/4*(-e)^(1/2)*e/d^2/x*(e*x^2
+d)^(1/2)-1/12*(-e)^(1/2)/d^2/x^3*(e*x^2+d)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{3\,d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(2\,ex^3-dx)\sqrt{ex^2+d}\sqrt{-e}}{12\,d^2x^4}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(3*d^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*e*x^3 - d*x)*sqrt(e*x^
2 + d)*sqrt(-e))/(d^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4x^4}-\frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{12dx^2}+\frac{e^{\frac{3}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{6d^2}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**5,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(4*x**4) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(12*d*x**2) + e**(3/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(6*d**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = \frac{\sqrt{ex^2+d}\sqrt{-e}e}{4\,d^2x} - \frac{(ex^2+d)^{\frac{3}{2}}\sqrt{-e}}{12\,d^2x^3} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{4\,x^4}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(e*x^2 + d)*sqrt(-e)*e/(d^2*x) - 1/12*(e*x^2 + d)^(3/2)*sqrt(-e)/(d
^2*x^3) - 1/4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(67) = 134.

Time = 0.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.33

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{\left(e^3 + \frac{9\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^2}{ex^2}\right)e^6x^3}{96\left(\sqrt{-de}e + \sqrt{-e^2x^2-de}|e|\right)^3 d^2|e|} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{4\,x^4} + \frac{\frac{9\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)d^4e^6}{x} + \frac{\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^3 d^4e^2}{x^3}}{96\,d^6e^5|e|}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] -1/96*(e^3 + 9*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^2/(e*x^2))*e^6*
x^3/((sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^2*abs(e)) - 1/4*arcta
n(sqrt(-e)*x/sqrt(e*x^2 + d))/x^4 + 1/96*(9*(sqrt(-d*e)*e + sqrt(-e^2*x^2 -
 d*e)*abs(e))*d^4*e^6/x + (sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^
4*e^2/x^3)/(d^6*e^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^5}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^5,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^5, x)
```

3.9 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5} - \frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

```
[Out] -1/6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6-2/45*(-e)^(3/2)*(e*x^2+d)^(1/
2)/d^2/x^3-4/45*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x-1/30*(-e)^(1/2)*(e*x^2+d)^
(1/2)/d/x^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5259, 277, 270}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6} - \frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x} - \frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] -1/30*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^5) - (2*(-e)^(3/2)*Sqrt[d + e*x^2])/(
45*d^2*x^3) - (4*(-e)^(5/2)*Sqrt[d + e*x^2])/(45*d^3*x) - ArcTan[(Sqrt[-e]*
x)/Sqrt[d + e*x^2]]/(6*x^6)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}+\frac{1}{6}\sqrt{-e}\int\frac{1}{x^6\sqrt{d+ex^2}}\,dx\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}+\frac{\left(2(-e)^{3/2}\right)\int\frac{1}{x^4\sqrt{d+ex^2}}\,dx}{15d}\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}+\frac{\left(4(-e)^{5/2}\right)\int\frac{1}{x^2\sqrt{d+ex^2}}\,dx}{45d^2}\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.69

$$\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx=\frac{\sqrt{-e}x\sqrt{d+ex^2}(-3d^2+4dex^2-8e^2x^4)-15d^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{90d^3x^6}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 4*d*e*x^2 - 8*e^2*x^4) - 15*d^3*ArcTa
n[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(90*d^3*x^6)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{6x^{6}}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^{2}+d}}{3d\,x^{3}}+\frac{2e\sqrt{e\,x^{2}+d}}{3d^{2}x}\right)}{6d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5d\,x^{5}}+\frac{2e\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15d^{2}x^{3}}\right)}{6d}$	117
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{6x^{6}}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^{2}+d}}{3d\,x^{3}}+\frac{2e\sqrt{e\,x^{2}+d}}{3d^{2}x}\right)}{6d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5d\,x^{5}}+\frac{2e\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15d^{2}x^{3}}\right)}{6d}$	117

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6-1/6*(-e)^(1/2)*e/d*(-1/3/d/x^
3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))+1/6*(-e)^(1/2)/d*(-1/5/d/x^5
*(e*x^2+d)^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx=-\frac{15\,d^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+(8\,e^2x^5-4\,dex^3+3\,d^2x)\sqrt{ex^2+d}\sqrt{-e}}{90\,d^3x^6}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(15*d^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (8*e^2*x^5 - 4*d*e*x^3 +
 3*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^3*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(102) = 204.

Time = 2.68 (sec) , antiderivative size = 352, normalized size of antiderivative = 3.12

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{d^4e^{\frac{9}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{30d^5e^4x^4+60d^4e^5x^6+30d^3e^6x^8} - \frac{d^3e^{\frac{11}{2}}x^2\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{45d^5e^4x^4+90d^4e^5x^6+45d^3e^6x^8} - \frac{d^2e^{\frac{13}{2}}x^4\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{30d^5e^4x^4+60d^4e^5x^6+30d^3e^6x^8} - \frac{2de^{\frac{15}{2}}x^6\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{15d^5e^4x^4+30d^4e^5x^6+15d^3e^6x^8} - \frac{4e^{\frac{17}{2}}x^8\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{45d^5e^4x^4+90d^4e^5x^6+45d^3e^6x^8} - \frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{6x^6}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**7,x)
```

```
[Out] -d**4*e**(9/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(30*d**5*e**4*x**4 + 60*d**4*e
**5*x**6 + 30*d**3*e**6*x**8) - d**3*e**(11/2)*x**2*sqrt(-e)*sqrt(d/(e*x**2
) + 1)/(45*d**5*e**4*x**4 + 90*d**4*e**5*x**6 + 45*d**3*e**6*x**8) - d**2*e
**(13/2)*x**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(30*d**5*e**4*x**4 + 60*d**4*e*
*5*x**6 + 30*d**3*e**6*x**8) - 2*d*e**(15/2)*x**6*sqrt(-e)*sqrt(d/(e*x**2)
+ 1)/(15*d**5*e**4*x**4 + 30*d**4*e**5*x**6 + 15*d**3*e**6*x**8) - 4*e**(17
/2)*x**8*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(45*d**5*e**4*x**4 + 90*d**4*e**5*x*
*6 + 45*d**3*e**6*x**8) - atan(x*sqrt(-e)/sqrt(d + e*x**2))/(6*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{(2\,e^2x^4+dex^2-d^2)\sqrt{-e}e}{18\,\sqrt{ex^2+d}d^3x^3} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{6\,x^6} + \frac{(2\,e^2x^4-dex^2-3\,d^2)\sqrt{ex^2+d}\sqrt{-e}}{90\,d^3x^5}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/18*(2*e^2*x^4 + d*e*x^2 - d^2)*sqrt(-e)*e/(sqrt(e*x^2 + d)*d^3*x^3) - 1/
6*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^6 + 1/90*(2*e^2*x^4 - d*e*x^2 - 3*d^
2)*sqrt(e*x^2 + d)*sqrt(-e)/(d^3*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(89) = 178.

Time = 0.35 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.43

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = \frac{\left(3e^4 + \frac{25\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^2}{x^2} + \frac{150\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^4}{e^4x^4}\right)e^{10}x^5}{2880\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^5 d^3|e|} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{6x^6} - \frac{\frac{150\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)d^{12}e^{16}}{x} + \frac{25\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^3 d^{12}e^{12}}{x^3} + \frac{3\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^5 d^{12}e^8}{x^5}}{2880\,d^{15}e^{14}|e|}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")
```

```
[Out] 1/2880*(3*e^4 + 25*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^2/x^2 + 150
*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^4/(e^4*x^4))*e^10*x^5/((sqrt(
-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^5*d^3*abs(e)) - 1/6*arctan(sqrt(-e)*
x/sqrt(e*x^2 + d))/x^6 - 1/2880*(150*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*a
bs(e))*d^12*e^16/x + 25*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^12
*e^12/x^3 + 3*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^5*d^12*e^8/x^5)/
(d^15*e^14*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^7}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^7,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^7, x)
```

3.10 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

```
[Out] -1/8*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^8-3/140*(-e)^(3/2)*(e*x^2+d)^(1
/2)/d^2/x^5-1/35*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x^3-2/35*(-e)^(7/2)*(e*x^2+
d)^(1/2)/d^4/x-1/56*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^7
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5259, 277, 270}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} - \frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] -1/56*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^7) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(
140*d^2*x^5) - ((-e)^(5/2)*Sqrt[d + e*x^2])/(35*d^3*x^3) - (2*(-e)^(7/2)*Sq
rt[d + e*x^2])/(35*d^4*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{1}{8}\sqrt{-e}\int \frac{1}{x^8\sqrt{d+ex^2}}\,dx \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(3(-e)^{3/2}\right)\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{28d} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(3(-e)^{5/2}\right)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{35d^2} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} \\
&\quad - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(2(-e)^{7/2}\right)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{35d^3} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} \\
&\quad - \frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = \frac{\sqrt{-e}x\sqrt{d+ex^2}(-5d^3+6d^2ex^2-8de^2x^4+16e^3x^6)-35d^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{280d^4x^8}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-5*d^3 + 6*d^2*e*x^2 - 8*d*e^2*x^4 + 16*e^3*x^
6) - 35*d^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(280*d^4*x^8)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{8x^8}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^2+d}}{5d\,x^5}-\frac{4e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{5d}\right)}{8d}+\frac{\sqrt{-e}\left(-\frac{(e\,x^2+d)^{\frac{3}{2}}}{7d\,x^7}-\frac{4e\left(-\frac{(e\,x^2+d)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e(e\,x^2+d)^{\frac{3}{2}}}{15d^2x^3}\right)}{7d}\right)}{8d}$
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{8x^8}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^2+d}}{5d\,x^5}-\frac{4e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{5d}\right)}{8d}+\frac{\sqrt{-e}\left(-\frac{(e\,x^2+d)^{\frac{3}{2}}}{7d\,x^7}-\frac{4e\left(-\frac{(e\,x^2+d)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e(e\,x^2+d)^{\frac{3}{2}}}{15d^2x^3}\right)}{7d}\right)}{8d}$

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^8-1/8*(-e)^(1/2)*e/d*(-1/5/d/x^
5*(e*x^2+d)^(1/2)-4/5*e/d*(-1/3/d/x^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)
^(1/2)))+1/8*(-e)^(1/2)/d*(-1/7/d/x^7*(e*x^2+d)^(3/2)-4/7*e/d*(-1/5/d/x^5*(
e*x^2+d)^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{35\,d^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - (16\,e^3x^7 - 8\,de^2x^5 + 6\,d^2ex^3 - 5\,d^3x)\sqrt{ex^2+d}\sqrt{-e}}{280\,d^4x^8}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="fricas")
[Out] -1/280*(35*d^4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (16*e^3*x^7 - 8*d*e^2*x
^5 + 6*d^2*e*x^3 - 5*d^3*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^4*x^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(128) = 256.

Time = 3.47 (sec) , antiderivative size = 575, normalized size of antiderivative = 4.08

$$\begin{aligned}\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = &-\frac{5d^6e^{\frac{19}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}} \\ &- \frac{9d^5e^{\frac{21}{2}}x^2\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}} \\ &- \frac{5d^4e^{\frac{23}{2}}x^4\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}} \\ &+ \frac{5d^3e^{\frac{25}{2}}x^6\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}} \\ &+ \frac{15d^2e^{\frac{27}{2}}x^8\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{140d^7e^9x^6+420d^6e^{10}x^8+420d^5e^{11}x^{10}+140d^4e^{12}x^{12}} \\ &+ \frac{5de^{\frac{29}{2}}x^{10}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{35d^7e^9x^6+105d^6e^{10}x^8+105d^5e^{11}x^{10}+35d^4e^{12}x^{12}} \\ &+ \frac{2e^{\frac{31}{2}}x^{12}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{35d^7e^9x^6+105d^6e^{10}x^8+105d^5e^{11}x^{10}+35d^4e^{12}x^{12}} \\ &- \frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{8x^8}\end{aligned}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**9,x)
```

```
[Out] -5*d**6*e**(19/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d
**6*e**10*x**8 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) - 9*d**5*e**(
21/2)*x**2*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**
10*x**8 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) - 5*d**4*e**(23/2)*x
**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**10*x**8
 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) + 5*d**3*e**(25/2)*x**6*sqr
t(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**10*x**8 + 840*
d**5*e**11*x**10 + 280*d**4*e**12*x**12) + 15*d**2*e**(27/2)*x**8*sqrt(-e)*
sqrt(d/(e*x**2) + 1)/(140*d**7*e**9*x**6 + 420*d**6*e**10*x**8 + 420*d**5*e
**11*x**10 + 140*d**4*e**12*x**12) + 5*d*e**(29/2)*x**10*sqrt(-e)*sqrt(d/(e
*x**2) + 1)/(35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*e**11*x**10
 + 35*d**4*e**12*x**12) + 2*e**(31/2)*x**12*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(
35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*e**11*x**10 + 35*d**4*e*
*12*x**12) - atan(x*sqrt(-e)/sqrt(d + e*x**2))/(8*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = \frac{(8\,e^3x^6 + 4\,de^2x^4 - d^2ex^2 + 3\,d^3)\sqrt{-e}e}{120\,\sqrt{ex^2+d}d^4x^5} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{8\,x^8} - \frac{(8\,e^3x^6 - 4\,de^2x^4 + 3\,d^2ex^2 + 15\,d^3)\sqrt{ex^2+d}\sqrt{-e}}{840\,d^4x^7}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="maxima")
```

```
[Out] 1/120*(8*e^3*x^6 + 4*d*e^2*x^4 - d^2*e*x^2 + 3*d^3)*sqrt(-e)*e/(sqrt(e*x^2
+ d)*d^4*x^5) - 1/8*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^8 - 1/840*(8*e^3*x
^6 - 4*d*e^2*x^4 + 3*d^2*e*x^2 + 15*d^3)*sqrt(e*x^2 + d)*sqrt(-e)/(d^4*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(111) = 222$.

Time = 0.35 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.50

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{\left(5\,e^5 + \frac{49\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^2 e}{x^2} + \frac{245\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^4}{e^3x^4} + \frac{1225\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^6}{e^7x^6}\right)e^{14}x^7}{35840\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^7 d^4|e|} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{8\,x^8} + \frac{\frac{1225\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)d^{24}e^{30}}{x} + \frac{245\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^3 d^{24}e^{26}}{x^3} + \frac{49\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^5 d^{24}e^{22}}{x^5} + \frac{5\left(\sqrt{-de}e+\cdots\right.}{}}{35840\,d^{28}e^{27}|e|}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

```
[Out] -1/35840*(5*e^5 + 49*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^2*e/x^2 +
 245*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^4/(e^3*x^4) + 1225*(sqrt(
-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^6/(e^7*x^6))*e^14*x^7/((sqrt(-d*e)*e
 + sqrt(-e^2*x^2 - d*e)*abs(e))^7*d^4*abs(e)) - 1/8*arctan(sqrt(-e)*x/sqrt(
e*x^2 + d))/x^8 + 1/35840*(1225*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e)
)*d^24*e^30/x + 245*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^24*e^2
6/x^3 + 49*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^5*d^24*e^22/x^5 + 5
*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^7*d^24*e^18/x^7)/(d^28*e^27*a
bs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^9}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^9,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^9, x)
```

3.11 $\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}} - \frac{d^2(d+ex^2)^{3/2}}{7(-e)^{7/2}} + \frac{3d(d+ex^2)^{5/2}}{35(-e)^{7/2}} - \frac{(d+ex^2)^{7/2}}{49(-e)^{7/2}} + \frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/7*d^2*(e*x^2+d)^(3/2)/(-e)^(7/2)+3/35*d*(e*x^2+d)^(5/2)/(-e)^(7/2)-1/49*
(e*x^2+d)^(7/2)/(-e)^(7/2)+1/7*x^7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/7
*d^3*(e*x^2+d)^(1/2)/(-e)^(7/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5259, 272, 45}

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}} - \frac{d^2(d+ex^2)^{3/2}}{7(-e)^{7/2}} - \frac{(d+ex^2)^{7/2}}{49(-e)^{7/2}} + \frac{3d(d+ex^2)^{5/2}}{35(-e)^{7/2}}$$

```
[In] Int[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^3*Sqrt[d + e*x^2])/(7*(-e)^(7/2)) - (d^2*(d + e*x^2)^(3/2))/(7*(-e)^(7/2
)) + (3*d*(d + e*x^2)^(5/2))/(35*(-e)^(7/2)) - (d + e*x^2)^(7/2)/(49*(-e)^(
7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7}x^7\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{7}\sqrt{-e}\int\frac{x^7}{\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{7}x^7\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{14}\sqrt{-e}\text{Subst}\left(\int\frac{x^3}{\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= \frac{1}{7}x^7\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{14}\sqrt{-e}\text{Subst}\left(\int\left(-\frac{d^3}{e^3\sqrt{d+ex}}+\frac{3d^2\sqrt{d+ex}}{e^3}-\frac{3d(d+ex)^{3/2}}{e^3}+\frac{(d+ex)^{5/2}}{e^3}\right)dx,x,x^2\right)\\
&= \frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}}-\frac{d^2(d+ex^2)^{3/2}}{7(-e)^{7/2}}+\frac{3d(d+ex^2)^{5/2}}{35(-e)^{7/2}}-\frac{(d+ex^2)^{7/2}}{49(-e)^{7/2}}+\frac{1}{7}x^7\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

$$\int x^6\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\frac{\sqrt{d+ex^2}(16d^3-8d^2ex^2+6de^2x^4-5e^3x^6)}{245(-e)^{7/2}}+\frac{1}{7}x^7\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[In] Integrate[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
[Out] (Sqrt[d + e*x^2]*(16*d^3 - 8*d^2*e*x^2 + 6*d*e^2*x^4 - 5*e^3*x^6))/(245*(-e
)^(7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(94) = 188.

Time = 0.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.86

method	result
default	$\frac{x^7 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{7} - \frac{\sqrt{-e}\left(\frac{x^6 \left(e x^2+d\right)^{\frac{3}{2}}}{9e} - \frac{2d\left(\frac{x^4 \left(e x^2+d\right)^{\frac{3}{2}}}{7e} - \frac{4d\left(\frac{x^2\left(e x^2+d\right)^{\frac{3}{2}}}{5e} - \frac{2d\left(e x^2+d\right)^{\frac{3}{2}}}{15e^2}\right)}{7e}\right)}{3e}\right)}{7d} + \frac{\sqrt{-e}\, e\left(\frac{x^8 \sqrt{e x^2+d}}{9e} - \cdots\right)}{\cdots}$
parts	$\frac{x^7 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{7} - \frac{\sqrt{-e}\left(\frac{x^6 \left(e x^2+d\right)^{\frac{3}{2}}}{9e} - \frac{2d\left(\frac{x^4 \left(e x^2+d\right)^{\frac{3}{2}}}{7e} - \frac{4d\left(\frac{x^2\left(e x^2+d\right)^{\frac{3}{2}}}{5e} - \frac{2d\left(e x^2+d\right)^{\frac{3}{2}}}{15e^2}\right)}{7e}\right)}{3e}\right)}{7d} + \frac{\sqrt{-e}\, e\left(\frac{x^8 \sqrt{e x^2+d}}{9e} - \cdots\right)}{\cdots}$

```
[In] int(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-1/7*(-e)^(1/2)/d*(1/9*x^6*(e*x
^2+d)^(3/2)/e-2/3*d/e*(1/7*x^4*(e*x^2+d)^(3/2)/e-4/7*d/e*(1/5*x^2*(e*x^2+d)
^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))))+1/7*(-e)^(1/2)*e/d*(1/9*x^8/e*(e*x^2
+d)^(1/2)-8/9*d/e*(1/7*x^6/e*(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(
1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{35\, e^4 x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - (5\, e^3 x^6 - 6\, de^2 x^4 + 8\, d^2 e x^2 - 16\, d^3)\sqrt{ex^2+d}\sqrt{-e}}{245\, e^4}$$

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*e^4*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (5*e^3*x^6 - 6*d*e^2
*x^4 + 8*d^2*e*x^2 - 16*d^3)*sqrt(e*x^2 + d)*sqrt(-e))/e^4
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$
$$= \begin{cases} \frac{16d^3\sqrt{-e}\sqrt{d+ex^2}}{245e^4} - \frac{8d^2x^2\sqrt{-e}\sqrt{d+ex^2}}{245e^3} + \frac{6dx^4\sqrt{-e}\sqrt{d+ex^2}}{245e^2} + \frac{x^7\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^6\sqrt{-e}\sqrt{d+ex^2}}{49e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**6*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((16*d**3*sqrt(-e)*sqrt(d + e*x**2)/(245*e**4) - 8*d**2*x**2*sqrt(
-e)*sqrt(d + e*x**2)/(245*e**3) + 6*d*x**4*sqrt(-e)*sqrt(d + e*x**2)/(245*e
**2) + x**7*atan(x*sqrt(-e)/sqrt(d + e*x**2))/7 - x**6*sqrt(-e)*sqrt(d + e*
x**2)/(49*e), Ne(e, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.35

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)$$
$$- \frac{\left(35\,(ex^2+d)^{\frac{9}{2}} - 135\,(ex^2+d)^{\frac{7}{2}}d + 189\,(ex^2+d)^{\frac{5}{2}}d^2 - 105\,(ex^2+d)^{\frac{3}{2}}d^3\right)\sqrt{-e}}{2205\,de^4}$$
$$+ \frac{\left(35\,(ex^2+d)^{\frac{9}{2}} - 180\,(ex^2+d)^{\frac{7}{2}}d + 378\,(ex^2+d)^{\frac{5}{2}}d^2 - 420\,(ex^2+d)^{\frac{3}{2}}d^3 + 315\,\sqrt{ex^2+d}\,d^4\right)\sqrt{-e}}{2205\,de^4}$$

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/2205*(35*(e*x^2 + d)^(9/2) -
 135*(e*x^2 + d)^(7/2)*d + 189*(e*x^2 + d)^(5/2)*d^2 - 105*(e*x^2 + d)^(3/2
)*d^3)*sqrt(-e)/(d*e^4) + 1/2205*(35*(e*x^2 + d)^(9/2) - 180*(e*x^2 + d)^(7
/2)*d + 378*(e*x^2 + d)^(5/2)*d^2 - 420*(e*x^2 + d)^(3/2)*d^3 + 315*sqrt(e*
x^2 + d)*d^4)*sqrt(-e)/(d*e^4)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) + \frac{\sqrt{-e^2x^2-de}d^3}{7e^4} + \frac{35(-e^2x^2-de)^{\frac{3}{2}}d^2e^2 + 21(e^2x^2+de)^2\sqrt{-e^2x^2-de}de - 5(e^2x^2+de)^3\sqrt{-e^2x^2-de}}{245e^7}$$

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] 1/7*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 1/7*sqrt(-e^2*x^2 - d*e)*d^3/e
^4 + 1/245*(35*(-e^2*x^2 - d*e)^(3/2)*d^2*e^2 + 21*(e^2*x^2 + d*e)^2*sqrt(-
e^2*x^2 - d*e)*d*e - 5*(e^2*x^2 + d*e)^3*sqrt(-e^2*x^2 - d*e))/e^7
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^6 \operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

```
[In] int(x^6*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^6*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.12 $\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}} - \frac{2d(d+ex^2)^{3/2}}{15(-e)^{5/2}} + \frac{(d+ex^2)^{5/2}}{25(-e)^{5/2}} + \frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -2/15*d*(e*x^2+d)^(3/2)/(-e)^(5/2)+1/25*(e*x^2+d)^(5/2)/(-e)^(5/2)+1/5*x^5*
arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/5*d^2*(e*x^2+d)^(1/2)/(-e)^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5259, 272, 45}

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}} + \frac{(d+ex^2)^{5/2}}{25(-e)^{5/2}} - \frac{2d(d+ex^2)^{3/2}}{15(-e)^{5/2}}$$

```
[In] Int[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^2*Sqrt[d + e*x^2])/(5*(-e)^(5/2)) - (2*d*(d + e*x^2)^(3/2))/(15*(-e)^(5/
2)) + (d + e*x^2)^(5/2)/(25*(-e)^(5/2)) + (x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d +
 e*x^2]])/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\sqrt{-e}\int\frac{x^5}{\sqrt{d+ex^2}}\,dx\\ &= \frac{1}{5}x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{-e}\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex}}\,dx,x,x^2\right)\\ &= \frac{1}{5}x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\\ &\quad -\frac{1}{10}\sqrt{-e}\text{Subst}\left(\int\left(\frac{d^2}{e^2\sqrt{d+ex}}-\frac{2d\sqrt{d+ex}}{e^2}+\frac{(d+ex)^{3/2}}{e^2}\right)\,dx,x,x^2\right)\\ &= \frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}}-\frac{2d(d+ex^2)^{3/2}}{15(-e)^{5/2}}+\frac{(d+ex^2)^{5/2}}{25(-e)^{5/2}}+\frac{1}{5}x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int x^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{75(-e)^{5/2}}+\frac{1}{5}x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[In] Integrate[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
[Out] (Sqrt[d + e*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4))/(75*(-e)^(5/2)) + (x^5*Ar
cTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/5
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(75) = 150.

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.85

method	result
default	$\frac{x^{5} \arctan\left(\frac{x \sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{5}+\frac{\sqrt{-e}\,e\left(\frac{x^{6} \sqrt{e\,x^{2}+d}}{7 e}-\frac{6 d\left(\frac{x^{4} \sqrt{e\,x^{2}+d}}{5 e}-\frac{4 d\left(\frac{x^{2} \sqrt{e\,x^{2}+d}}{3 e}-\frac{2 d \sqrt{e\,x^{2}+d}}{3 e^{2}}\right)}{5 e}\right)}{7 e}\right)}{5 d}-\frac{\sqrt{-e}\left(\frac{x^{4}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{7 e}-\frac{4 d\left(x^{2} \cdots\right.}{\cdots}\right.}{5 \cdots}$
parts	$\frac{x^{5} \arctan\left(\frac{x \sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{5}+\frac{\sqrt{-e}\,e\left(\frac{x^{6} \sqrt{e\,x^{2}+d}}{7 e}-\frac{6 d\left(\frac{x^{4} \sqrt{e\,x^{2}+d}}{5 e}-\frac{4 d\left(\frac{x^{2} \sqrt{e\,x^{2}+d}}{3 e}-\frac{2 d \sqrt{e\,x^{2}+d}}{3 e^{2}}\right)}{5 e}\right)}{7 e}\right)}{5 d}-\frac{\sqrt{-e}\left(\frac{x^{4}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{7 e}-\frac{4 d\left(x^{2} \cdots\right.}{\cdots}\right.}{5 \cdots}$

```
[In] int(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/5*(-e)^(1/2)*e/d*(1/7*x^6/e*
(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^
2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2))))-1/5*(-e)^(1/2)/d*(1/7*x^4*(e*x^2+d)
^(3/2)/e-4/7*d/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.69

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{15\,e^3x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - (3\,e^2x^4 - 4\,dex^2 + 8\,d^2)\sqrt{ex^2+d}\sqrt{-e}}{75\,e^3}$$

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*e^3*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (3*e^2*x^4 - 4*d*e*x^
2 + 8*d^2)*sqrt(e*x^2 + d)*sqrt(-e))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} -\frac{8d^2\sqrt{-e}\sqrt{d+ex^2}}{75e^3} + \frac{4dx^2\sqrt{-e}\sqrt{d+ex^2}}{75e^2} + \frac{x^5 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5} - \frac{x^4\sqrt{-e}\sqrt{d+ex^2}}{25e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-8*d**2*sqrt(-e)*sqrt(d + e*x**2)/(75*e**3) + 4*d*x**2*sqrt(-e)*
sqrt(d + e*x**2)/(75*e**2) + x**5*atan(x*sqrt(-e)/sqrt(d + e*x**2))/5 - x**
4*sqrt(-e)*sqrt(d + e*x**2)/(25*e), Ne(e, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.40

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{\left(15\,(ex^2+d)^{\frac{7}{2}} - 42\,(ex^2+d)^{\frac{5}{2}}d + 35\,(ex^2+d)^{\frac{3}{2}}d^2\right)\sqrt{-e}}{525\,de^3} + \frac{\left(5\,(ex^2+d)^{\frac{7}{2}} - 21\,(ex^2+d)^{\frac{5}{2}}d + 35\,(ex^2+d)^{\frac{3}{2}}d^2 - 35\,\sqrt{ex^2+d}\,d^3\right)\sqrt{-e}}{175\,de^3}$$

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/525*(15*(e*x^2 + d)^(7/2) -
42*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2)*sqrt(-e)/(d*e^3) + 1/175
*(5*(e*x^2 + d)^(7/2) - 21*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2 -
 35*sqrt(e*x^2 + d)*d^3)*sqrt(-e)/(d*e^3)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{\sqrt{-e^2x^2-de}d^2}{5e^3} - \frac{10(-e^2x^2-de)^{\frac{3}{2}}de + 3(e^2x^2+de)^2\sqrt{-e^2x^2-de}}{75e^5}$$

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] 1/5*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/5*sqrt(-e^2*x^2 - d*e)*d^2/e
^3 - 1/75*(10*(-e^2*x^2 - d*e)^(3/2)*d*e + 3*(e^2*x^2 + d*e)^2*sqrt(-e^2*x^
2 - d*e))/e^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^4 \operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

```
[In] int(x^4*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^4*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.13 $\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}} - \frac{(d+ex^2)^{3/2}}{9(-e)^{3/2}} + \frac{1}{3}x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/9*(e*x^2+d)^(3/2)/(-e)^(3/2)+1/3*x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)
)+1/3*d*(e*x^2+d)^(1/2)/(-e)^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5259, 272, 45}

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{3}x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(d+ex^2)^{3/2}}{9(-e)^{3/2}} + \frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}$$

```
[In] Int[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d*Sqrt[d + e*x^2])/(3*(-e)^(3/2)) - (d + e*x^2)^(3/2)/(9*(-e)^(3/2)) + (x^
3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\sqrt{-e}\int\frac{x^3}{\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{3}x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{-e}\text{Subst}\left(\int\frac{x}{\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{-e}\text{Subst}\left(\int\left(-\frac{d}{e\sqrt{d+ex}}+\frac{\sqrt{d+ex}}{e}\right)\,dx,x,x^2\right)\\
&= \frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}-\frac{\left(d+ex^2\right)^{3/2}}{9(-e)^{3/2}}+\frac{1}{3}x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{1}{9}\left(\frac{\left(2d-ex^2\right)\sqrt{d+ex^2}}{(-e)^{3/2}}+3x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)$$

```
[In] Integrate[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
[Out] (((2*d - e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 3*x^3*ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]])/9
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(56) = 112$.

Time = 0.02 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.82

method	result	size
default	$\frac{x^{3} \arctan\left(\frac{x \sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{3}+\frac{\sqrt{-e}\, e \left(\frac{x^{4} \sqrt{e\,x^{2}+d}}{5 e}-\frac{4 d \left(\frac{x^{2} \sqrt{e\,x^{2}+d}}{3 e}-\frac{2 d \sqrt{e\,x^{2}+d}}{3 e^{2}}\right)}{5 e}\right)}{3 d}-\frac{\sqrt{-e}\, \left(\frac{x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5 e}-\frac{2 d \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15 e^{2}}\right)}{3 d}$	135
parts	$\frac{x^{3} \arctan\left(\frac{x \sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{3}+\frac{\sqrt{-e}\, e \left(\frac{x^{4} \sqrt{e\,x^{2}+d}}{5 e}-\frac{4 d \left(\frac{x^{2} \sqrt{e\,x^{2}+d}}{3 e}-\frac{2 d \sqrt{e\,x^{2}+d}}{3 e^{2}}\right)}{5 e}\right)}{3 d}-\frac{\sqrt{-e}\, \left(\frac{x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5 e}-\frac{2 d \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15 e^{2}}\right)}{3 d}$	135

```
[In] int(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/3*(-e)^(1/2)*e/d*(1/5*x^4/e*
(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2
)))-1/3*(-e)^(1/2)/d*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{3\,e^2x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \sqrt{ex^2+d}(ex^2-2\,d)\sqrt{-e}}{9\,e^2}$$

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
[Out] 1/9*(3*e^2*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(e*x^2 + d)*(e*x^2
- 2*d)*sqrt(-e))/e^2
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \begin{cases} \frac{2d\sqrt{-e}\sqrt{d+ex^2}}{9e^2} + \frac{x^3\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3} - \frac{x^2\sqrt{-e}\sqrt{d+ex^2}}{9e} & \text{for}\: e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Piecewise((2*d*sqrt(-e)*sqrt(d + e*x**2)/(9*e**2) + x**3*atan(x*sqrt(-e)/sq
rt(d + e*x**2))/3 - x**2*sqrt(-e)*sqrt(d + e*x**2)/(9*e), Ne(e, 0)), (0, Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int x^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{1}{3}\,x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\frac{\left(3\,(ex^2+d)^{\frac{5}{2}}-5\,(ex^2+d)^{\frac{3}{2}}d\right)\sqrt{-e}}{45\,de^2}+\frac{\left(3\,(ex^2+d)^{\frac{5}{2}}-10\,(ex^2+d)^{\frac{3}{2}}d+15\,\sqrt{ex^2+d}d^2\right)\sqrt{-e}}{45\,de^2}$$

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
[Out] 1/3*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/45*(3*(e*x^2 + d)^(5/2) - 5*
(e*x^2 + d)^(3/2)*d)*sqrt(-e)/(d*e^2) + 1/45*(3*(e*x^2 + d)^(5/2) - 10*(e*x
^2 + d)^(3/2)*d + 15*sqrt(e*x^2 + d)*d^2)*sqrt(-e)/(d*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{1}{3}\,x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+\frac{\sqrt{-e^2x^2-de}d}{3\,e^2}+\frac{(-e^2x^2-de)^{\frac{3}{2}}}{9\,e^3}$$

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] 1/3*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 1/3*sqrt(-e^2*x^2 - d*e)*d/e^2
 + 1/9*(-e^2*x^2 - d*e)^(3/2)/e^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^2\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

```
[In] int(x^2*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^2*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.14 $\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5255, 267}

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{\sqrt{d+ex^2}}{\sqrt{-e}}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5255

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]], x_Symbol] :> Simp[x*Ar
cTan[(c*x)/Sqrt[a + b*x^2]], x] - Dist[c, Int[x/Sqrt[a + b*x^2], x], x] /;
```

```
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rubi steps

$$\text{integral} = x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \sqrt{-e}\int\frac{x}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
[Out] Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(35) = 70.

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

method	result	size
default	$x\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)+\frac{\sqrt{-e}\,e\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{d}-\frac{\sqrt{-e}\,(e\,x^2+d)^{\frac{3}{2}}}{3de}$	86
parts	$x\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)+\frac{\sqrt{-e}\,e\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{d}-\frac{\sqrt{-e}\,(e\,x^2+d)^{\frac{3}{2}}}{3de}$	86

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+(-e)^(1/2)*e/d*(1/3*x^2/e*(e*x^2+d)^
(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2))-1/3*(-e)^(1/2)/d*(e*x^2+d)^(3/2)/e
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{ex\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \sqrt{ex^2+d}\sqrt{-e}}{e}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
[Out] (e*x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(e*x^2 + d)*sqrt(-e))/e
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} x\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right) - \frac{\sqrt{-e}\sqrt{d+ex^2}}{e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Piecewise((x*atan(x*sqrt(-e)/sqrt(d + e*x**2)) - sqrt(-e)*sqrt(d + e*x**2)/
e, Ne(e, 0)), (0, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(35) = 70$.

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.79

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{(ex^2+d)^{\frac{3}{2}}\sqrt{-e}}{3\,de} + \frac{\left((ex^2+d)^{\frac{3}{2}} - 3\sqrt{ex^2+d}d\right)\sqrt{-e}}{3\,de}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/3*(e*x^2 + d)^(3/2)*sqrt(-e)/(d*e)
 + 1/3*((e*x^2 + d)^(3/2) - 3*sqrt(e*x^2 + d)*d)*sqrt(-e)/(d*e)
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{\sqrt{-e^2x^2-de}}{e}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(-e^2*x^2 - d*e)/e
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{\sqrt{e\,x^2+d}}{\sqrt{-e}} + x\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] (d + e*x^2)^(1/2)/(-e)^(1/2) + x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))
```

3.15 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 59

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2} dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\sqrt{-e}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x-arctanh((e*x^2+d)^(1/2)/d^(1/2))*(-
e)^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5259, 272, 65, 214}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2} dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\sqrt{-e}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[-e]*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/Sqrt[d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} + \sqrt{-e}\int \frac{1}{x\sqrt{d+ex^2}}\,dx \\
&= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} + \frac{1}{2}\sqrt{-e}\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right) \\
&= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{\sqrt{-e}} \\
&= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\sqrt{-e}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$
\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} + \frac{i\sqrt{e}\log\left(\frac{2i\sqrt{d}}{\sqrt{e}x} - \frac{2\sqrt{-e}\sqrt{d+ex^2}}{ex}\right)}{\sqrt{d}}
$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
[Out] -(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) + (I*Sqrt[e]*Log[((2*I)*Sqrt[d])/
(Sqrt[e]*x) - (2*Sqrt[-e]*Sqrt[d + e*x^2])/(e*x)])/Sqrt[d]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x}-\frac{\sqrt{-e}\,\sqrt{e\,x^{2}+d}}{d}+\frac{\sqrt{-e}\,\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{d}$	90
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x}-\frac{\sqrt{-e}\,\sqrt{e\,x^{2}+d}}{d}+\frac{\sqrt{-e}\,\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{d}$	90

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x-(-e)^(1/2)/d*(e*x^2+d)^(1/2)+(-e)^(
1/2)/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.51

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \left[\frac{x\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2+2\sqrt{ex^2+d}d\sqrt{-e}\sqrt{-\frac{e}{d}}+2\,de}{x^2}\right)-2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,x},\ -\frac{x\sqrt{\frac{e}{d}}\arctan\left(\frac{\sqrt{ex^2+d}d\sqrt{-e}\sqrt{\frac{e}{d}}}{e^2x^2+de}\right)+\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\right]$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(-e/d)
+ 2*d*e)/x^2) - 2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/x, -(x*sqrt(e/d)*arct
an(sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) + arctan(sqrt(-e)*
x/sqrt(e*x^2 + d)))/x]
```

Sympy [A] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \frac{\sqrt{-e}\left(\begin{cases}\frac{2\,\mathrm{atan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{-d}}\right)}{\sqrt{-d}} & \text{for } e\neq 0\\ \frac{\log\left(x^2\right)}{\sqrt{d}} & \text{otherwise}\end{cases}\right)}{2} - \frac{\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{x}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**2,x)
```

```
[Out] sqrt(-e)*Piecewise((2*atan(sqrt(d + e*x**2)/sqrt(-d))/sqrt(-d), Ne(e, 0)),
(log(x**2)/sqrt(d), True))/2 - atan(x*sqrt(-e)/sqrt(d + e*x**2))/x
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^2}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] (d*sqrt(-e)*x*integrate(-sqrt(e*x^2 + d)/(e^2*x^5 + d*e*x^3 - (e*x^3 + d*x)
*(e*x^2 + d)), x) - arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = -\frac{e\arctan\left(\frac{\sqrt{-e^2x^2-de}}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -e*arctan(sqrt(-e^2*x^2 - d*e)/sqrt(d*e))/sqrt(d*e) - arctan(sqrt(-e)*x/sqr
t(e*x^2 + d))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^2}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^2,x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^2, x)
```

3.16 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3} - \frac{(-e)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}$$

```
[Out] -1/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3-1/6*(-e)^(3/2)*arctanh((e*x^2
+d)^(1/2)/d^(1/2))/d^(3/2)-1/6*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5259, 272, 44, 65, 214}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3} - \frac{(-e)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}} - \frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -1/6*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^
2]]/(3*x^3) - ((-e)^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3} + \frac{1}{3}\sqrt{-e}\int \frac{1}{x^3\sqrt{d+ex^2}}\,dx \\
&= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3} + \frac{1}{6}\sqrt{-e}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3} + \frac{(-e)^{3/2}\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{12d} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3} - \frac{\sqrt{-e}\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{6d} \\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3} - \frac{(-e)^{3/2}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{6d^{3/2}} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -1/6*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^2) + (e^(3/2)*ArcTan[(Sqrt[d]*Sqrt[-e]
)/(Sqrt[e]*Sqrt[d + e*x^2])])/(6*d^(3/2)) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*
x^2]]/(3*x^3)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{3x^3}+\frac{\sqrt{-e}\,e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{3d^{\frac{3}{2}}}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}+\frac{e\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)}{2d}\right)}{3d}$	130
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{3x^3}+\frac{\sqrt{-e}\,e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{3d^{\frac{3}{2}}}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}+\frac{e\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)}{2d}\right)}{3d}$	130

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3+1/3*(-e)^(1/2)*e/d^(3/2)*ln((
2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)+1/3*(-e)^(1/2)/d*(-1/2/d/x^2*(e*x^2+d)^(3
/2)+1/2*e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.18

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$$

$$= \left[\frac{ex^3\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2-2\sqrt{ex^2+d}d\sqrt{-e}\sqrt{-\frac{e}{d}}+2\,de}{x^2}\right)-2\sqrt{ex^2+d}\sqrt{-e}x-4\,d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{12\,dx^3}, \frac{ex^3\sqrt{\frac{e}{d}}\arctan\right.$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(e*x^3*sqrt(-e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(-
e/d) + 2*d*e)/x^2) - 2*sqrt(e*x^2 + d)*sqrt(-e)*x - 4*d*arctan(sqrt(-e)*x/s
qrt(e*x^2 + d)))/(d*x^3), 1/6*(e*x^3*sqrt(e/d)*arctan(sqrt(e*x^2 + d)*d*sqr
t(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) - sqrt(e*x^2 + d)*sqrt(-e)*x - 2*d*arctan(
sqrt(-e)*x/sqrt(e*x^2 + d)))/(d*x^3)]
```

Sympy [A] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^3} - \frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{6dx} + \frac{e\sqrt{-e}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{6d^{\frac{3}{2}}}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**4,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**3) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(6*d*x) + e*sqrt(-e)*asinh(sqrt(d)/(sqrt(e)*x))/(6*d**(3/2))
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^4}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*d*sqrt(-e)*x^3*integrate(-1/3*sqrt(e*x^2 + d)/(e^2*x^7 + d*e*x^5 - (
e*x^5 + d*x^3)*(e*x^2 + d)), x) - arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \frac{\frac{e^3\arctan\left(\frac{\sqrt{-e^2x^2-de}}{\sqrt{de}}\right)}{\sqrt{de}d} - \frac{\sqrt{-e^2x^2-de}e}{dx^2}}{6\,e} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{3\,x^3}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(e^3*arctan(sqrt(-e^2*x^2 - d*e)/sqrt(d*e))/(sqrt(d*e)*d) - sqrt(-e^2*x
^2 - d*e)*e/(d*x^2))/e - 1/3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^4}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^4,x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^4, x)
```

3.17 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{3(-e)^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}$$

```
[Out] -1/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5-3/40*(-e)^(5/2)*arctanh((e*x^
2+d)^(1/2)/d^(1/2))/d^(5/2)-3/40*(-e)^(3/2)*(e*x^2+d)^(1/2)/d^2/x^2-1/20*(-
e)^(1/2)*(e*x^2+d)^(1/2)/d/x^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5259, 272, 44, 65, 214}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{3(-e)^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] -1/20*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^4) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(
40*d^2*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5) - (3*(-e)^(5/2)*
ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(40*d^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{5}\sqrt{-e}\int\frac{1}{x^5\sqrt{d+ex^2}}\,dx\\
&= -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{10}\sqrt{-e}\text{Subst}\left(\int\frac{1}{x^3\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{\left(3(-e)^{3/2}\right)\text{Subst}\left(\int\frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)}{40d}
\end{aligned}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5} + \frac{\left(3(-e)^{5/2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right)}{80d^2}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{\left(3(-e)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{40d^2}$$

$$= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{3(-e)^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \sqrt{-e}\left(-\frac{1}{20dx^4} + \frac{3e}{40d^2x^2}\right)\sqrt{d+ex^2} - \frac{3e^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{40d^{5/2}} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] Sqrt[-e]*(-1/20*1/(d*x^4) + (3*e)/(40*d^2*x^2))*Sqrt[d + e*x^2] - (3*e^(5/2
)*ArcTan[(Sqrt[d]*Sqrt[-e])/(Sqrt[e]*Sqrt[d + e*x^2])])/(40*d^(5/2)) - ArcT
an[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

method	result
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{5x^{5}}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^{2}+d}}{2d\,x^{2}}+\frac{e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)}{2d^{\frac{3}{2}}}\right)}{5d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4d\,x^{4}}-\frac{e\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}+\frac{e\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\right.}{}\right.}{4d}\right.}{5d}$
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{5x^{5}}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^{2}+d}}{2d\,x^{2}}+\frac{e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)}{2d^{\frac{3}{2}}}\right)}{5d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4d\,x^{4}}-\frac{e\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}+\frac{e\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\right.}{}\right.}{4d}\right.}{5d}$

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5-1/5*(-e)^(1/2)*e/d*(-1/2/d/x^
2*(e*x^2+d)^(1/2)+1/2*e/d^(3/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))+1/5*
(-e)^(1/2)/d*(-1/4/d/x^4*(e*x^2+d)^(3/2)-1/4*e/d*(-1/2/d/x^2*(e*x^2+d)^(3/2
)+1/2*e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.92

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \left[\frac{3\,e^2x^5\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2+2\,\sqrt{ex^2+d}d\sqrt{-e}\sqrt{-\frac{e}{d}}+2\,de}{x^2}\right)-16\,d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+2\,(3\,ex^3-2\,dx)\sqrt{ex^2+d}\sqrt{-e}}{80\,d^2x^5}, -\frac{3\,e^2x^5\sqrt{\frac{e}{d}}\arctan\left(\frac{\sqrt{ex^2+d}d\sqrt{-e}\sqrt{\frac{e}{d}}}{e^2x^2+de}\right)+8\,d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(3\,ex^3-2\,dx)\sqrt{ex^2+d}\sqrt{-e}}{40\,d^2x^5}\right]$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(3*e^2*x^5*sqrt(-e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sq
rt(-e/d) + 2*d*e)/x^2) - 16*d^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(3*e
*x^3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^2*x^5), -1/40*(3*e^2*x^5*sqrt(e/
d)*arctan(sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) + 8*d^2*arc
tan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (3*e*x^3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(-e
))/(d^2*x^5)]
```

Sympy [A] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.24

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{\sqrt{-e}}{20\sqrt{e}x^5\sqrt{\frac{d}{ex^2}+1}} + \frac{\sqrt{e}\sqrt{-e}}{40dx^3\sqrt{\frac{d}{ex^2}+1}} + \frac{3e^{\frac{3}{2}}\sqrt{-e}}{40d^2x\sqrt{\frac{d}{ex^2}+1}} - \frac{3e^2\sqrt{-e}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{40d^{\frac{5}{2}}}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**6,x)
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(5*x**5) - sqrt(-e)/(20*sqrt(e)*x**5*sqr
t(d/(e*x**2) + 1)) + sqrt(e)*sqrt(-e)/(40*d*x**3*sqrt(d/(e*x**2) + 1)) + 3*
e**(3/2)*sqrt(-e)/(40*d**2*x*sqrt(d/(e*x**2) + 1)) - 3*e**2*sqrt(-e)*asinh(
sqrt(d)/(sqrt(e)*x))/(40*d**(5/2))
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^6}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="maxima")
[Out] 1/5*(5*d*sqrt(-e)*x^5*integrate(-1/5*sqrt(e*x^2 + d)/(e^2*x^9 + d*e*x^7 - (
e*x^7 + d*x^5)*(e*x^2 + d)), x) - arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = -\frac{\frac{3\,e^4\arctan\left(\frac{\sqrt{-e^2x^2-de}}{\sqrt{de}}\right)}{\sqrt{de}d^2} + \frac{5\,\sqrt{-e^2x^2-de}de^5+3\,(-e^2x^2-de)^{\frac{3}{2}}e^4}{d^2e^4x^4}}{40\,e} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{5\,x^5}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="giac")
[Out] -1/40*(3*e^4*arctan(sqrt(-e^2*x^2 - d*e)/sqrt(d*e))/(sqrt(d*e)*d^2) + (5*sq
rt(-e^2*x^2 - d*e)*d*e^5 + 3*(-e^2*x^2 - d*e)^(3/2)*e^4)/(d^2*e^4*x^4))/e -
 1/5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^6}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^6,x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^6, x)
```

3.18 $\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 211

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{30d^{11/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{847e^{13/4}\sqrt{d+ex^2}}$$

```
[Out] 2/11*x^(11/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+36/847*d*x^(5/2)*(e*x^2+
d)^(1/2)/(-e)^(3/2)+4/121*x^(9/2)*(e*x^2+d)^(1/2)/(-e)^(1/2)+60/847*d^2*x^(
1/2)*(e*x^2+d)^(1/2)/(-e)^(5/2)+30/847*d^(11/4)*(cos(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(
2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/
2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(13/4)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5259, 327, 335, 226}

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{30d^{11/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{847e^{13/4}\sqrt{d+ex^2}}+\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}$$

```
[In] Int[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (60*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(847*(-e)^(5/2)) + (36*d*x^(5/2)*Sqrt[d +
e*x^2])/(847*(-e)^(3/2)) + (4*x^(9/2)*Sqrt[d + e*x^2])/(121*Sqrt[-e]) + (2*
x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 + (30*d^(11/4)*Sqrt[-e]*(
Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*
ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(847*e^(13/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{11}(2\sqrt{-e})\int\frac{x^{11/2}}{\sqrt{d+ex^2}}\,dx \\
&= \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(18d)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx}{121\sqrt{-e}} \\
&= \frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}} + \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(90d^2)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx}{847(-e)^{3/2}}
\end{aligned}$$

$$= \frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}} + \frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}} + \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(30d^3)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{847(-e)^{5/2}}$$

$$= \frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}} + \frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}} + \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(60d^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{847(-e)^{5/2}}$$

$$= \frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}} + \frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}} + \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}} + \frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{30d^{11/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right), \frac{1}{2}\right)}{847(-e)^{5/2}\sqrt[4]{e}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.41 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{4\sqrt{x}\sqrt{d+ex^2}(15d^2-9dex^2+7e^2x^4)}{847(-e)^{5/2}} + \frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{60id^3\sqrt{1+\frac{d}{ex^2}}\,x\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right), -1\right)}{847\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}(-e)^{5/2}\sqrt{d+ex^2}}$$

```
[In] Integrate[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2]*(15*d^2 - 9*d*e*x^2 + 7*e^2*x^4))/(847*(-e)^(5/2
)) + (2*x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 - (((60*I)/847)*d
^3*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt
[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*(-e)^(5/2)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^{\frac{9}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right) dx$$

[In] int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)

[Out] int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.45

$$\int x^{9/2} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{2\left(77\,e^4x^{\frac{11}{2}} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) + 30\,d^3\sqrt{-e}\sqrt{e}\text{weierstrassPInverse}\left(-\frac{4\,d}{e}, 0, x\right.}{847\,e^4}$$

[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")

[Out] 2/847*(77*e^4*x^(11/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 30*d^3*sqrt(-e)
*sqrt(e)*weierstrassPInverse(-4*d/e, 0, x) - 2*(7*e^3*x^4 - 9*d*e^2*x^2 + 1
5*d^2*e)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(x))/e^4

Sympy [F(-1)]

Timed out.

$$\int x^{9/2} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \text{Timed out}$$

[In] integrate(x**(9/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)

[Out] Timed out

Maxima [F]

$$\int x^{9/2} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^{\frac{9}{2}} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) dx$$

[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")

[Out] 2/11*x^(11/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate
(-1/11*x*e^(1/2*log(e*x^2 + d) + 9/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 +
d)^2), x)

Giac [F]

$$\int x^{9/2} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^{\frac{9}{2}} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) dx$$

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(x^(9/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{9/2} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^{9/2} \operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

```
[In] int(x^(9/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(9/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.19 $\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 181

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{147e^{9/4}\sqrt{d+ex^2}}$$

```
[Out] 2/7*x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/49*x^(5/2)*(e*x^2+d)^(1/
2)/(-e)^(1/2)+20/147*d*x^(1/2)*(e*x^2+d)^(1/2)/(-e)^(3/2)-10/147*d^(7/4)*(c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-
e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(9/4
)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5259, 327, 335, 226}

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{147e^{9/4}\sqrt{d+ex^2}}+\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}$$

```
[In] Int[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (20*d*Sqrt[x]*Sqrt[d + e*x^2])/(147*(-e)^(3/2)) + (4*x^(5/2)*Sqrt[d + e*x^2
])/(49*Sqrt[-e]) + (2*x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7 - (10
*d^(7/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]
*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(147*e^(9/4)*Sq
rt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{7}\left(2\sqrt{-e}\right)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx\\
&= \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(10d)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx}{49\sqrt{-e}}\\
&= \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(10d^2)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{147(-e)^{3/2}}
\end{aligned}$$

$$= \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}} + \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}} + \frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(20d^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{147(-e)^{3/2}}$$

$$= \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}} + \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}} + \frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right.\right.}{147e^{9/4}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.87

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{147}\sqrt{x}\left(\frac{2(5d-3ex^2)\sqrt{d+ex^2}}{(-e)^{3/2}} + 21x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right) - \frac{20id^2\sqrt{1+\frac{d}{ex^2}}\,x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right), -1\right)}{147\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}(-e)^{3/2}\sqrt{d+ex^2}}$$

```
[In] Integrate[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((2*(5*d - 3*e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 21*x^3*ArcTan[
(Sqrt[-e]*x)/Sqrt[d + e*x^2]]))/147 - (((20*I)/147)*d^2*Sqrt[1 + d/(e*x^2)]
*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I*Sq
rt[d])/Sqrt[e]]*(-e)^(3/2)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^{\frac{5}{2}}\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)dx$$

```
[In] int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.47

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2\left(21\,e^3x^{\frac{7}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - 10\,d^2\sqrt{-e}\sqrt{e}\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)\right.}{147\,e^3}$$

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] 2/147*(21*e^3*x^(7/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 10*d^2*sqrt(-e)*
sqrt(e)*weierstrassPInverse(-4*d/e, 0, x) - 2*(3*e^2*x^2 - 5*d*e)*sqrt(e*x^
2 + d)*sqrt(-e)*sqrt(x))/e^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.61 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2x^{\frac{7}{2}}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^{\frac{9}{2}}\sqrt{-e}\,\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{ex^2e^{i\pi}}{d}\right)}{7\sqrt{d}\,\Gamma\left(\frac{13}{4}\right)}$$

```
[In] integrate(x**(5/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(7/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/7 - x**(9/2)*sqrt(-e)*gamma(9/
4)*hyper((1/2, 9/4), (13/4,), e*x**2*exp_polar(I*pi)/d)/(7*sqrt(d)*gamma(13
/4))
```

Maxima [F]

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int x^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] 2/7*x^(7/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(-
1/7*x*e^(1/2*log(e*x^2 + d) + 5/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^
2), x)
```

Giac [F]

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(x^(5/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

```
[In] int(x^(5/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(5/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.20 $\int \sqrt{x} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 153

$$\int \sqrt{x} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}} + \frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{2d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{9e^{5/4}\sqrt{d+ex^2}}$$

```
[Out] 2/3*x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/9*x^(1/2)*(e*x^2+d)^(1/2
)/(-e)^(1/2)+2/9*d^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)
+x*e^(1/2))^2)^(1/2)/e^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

= {5259, 327, 335, 226}

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx$$
$$=\frac{2d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{9e^{5/4}\sqrt{d+ex^2}}$$
$$+\frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}$$

```
[In] Int[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/S
qrt[d + e*x^2]])/3 + (2*d^(3/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*
x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)]
, 1/2])/(9*e^(5/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\left(2\sqrt{-e}\right)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx\\
&= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(2d)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{9\sqrt{-e}}\\
&= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(4d)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{9\sqrt{-e}}\\
&= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\\
&\quad-\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{9\sqrt{-e}\sqrt[4]{e}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\\
&\quad-\frac{4id\sqrt{1+\frac{d}{ex^2}}\,x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{9\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{-e}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[In] Integrate[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/S
qrt[d + e*x^2]])/3 - (((4*I)/9)*d*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[-
e]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \sqrt{x}\,\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)dx$$

[In] int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)

[Out] int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2\left(3\,e^2x^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+2\,d\sqrt{-e}\sqrt{e}\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)-2\,\sqrt{ex^2+d}\sqrt{-e}e\sqrt{x}\right)}{9\,e^2}$$

[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")

[Out] 2/9*(3*e^2*x^(3/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*d*sqrt(-e)*sqrt(e
)*weierstrassPInverse(-4*d/e, 0, x) - 2*sqrt(e*x^2 + d)*sqrt(-e)*e*sqrt(x))
/e^2

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.64 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.49

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3} - \frac{x^{\frac{5}{2}}\sqrt{-e}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{9}{4}\right)}$$

[In] integrate(x**(1/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)

[Out] 2*x**(3/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/3 - x**(5/2)*sqrt(-e)*gamma(5/
4)*hyper((1/2, 5/4), (9/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(9/4
))

Maxima [F]

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
[Out] 2/3*x^(3/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(-
1/3*x*e^(1/2*log(e*x^2 + d) + 1/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^
2), x)
```

Giac [F]

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \int \sqrt{x}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

```
[In] int(x^(1/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(1/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.21 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt[4]{e}\sqrt{d+ex^2}}$$

```
[Out] -2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2)+2*(cos(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(si
n(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(
1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(1/4)/e^(1/4)/(e*x^2+d)^(1/
2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5259, 335, 226}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt[4]{e}\sqrt{d+ex^2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*Sqrt[-e]*(Sqrt[d] +
Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^
(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*e^(1/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(2\sqrt{-e}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx \\
&= -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(4\sqrt{-e}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right), \frac{1}{2}\right)}{\sqrt[4]{d}\sqrt[4]{e}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{4i\sqrt{-e}\sqrt{1+\frac{d}{ex^2}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
[Out] (-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + ((4*I)*Sqrt[-e]*Sqrt[1
+ d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])
/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{3}{2}}}dx$$

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \frac{2\left(2\sqrt{-e}\sqrt{e}x\text{weierstrassPInverse}\left(-\frac{4d}{e},0,x\right) - e\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\right)}{ex}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas
")
[Out] 2*(2*sqrt(-e)*sqrt(e)*x*weierstrassPInverse(-4*d/e, 0, x) - e*sqrt(x)*arcta
n(sqrt(-e)*x/sqrt(e*x^2 + d)))/(e*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.58

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{\sqrt{x}\sqrt{-e}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{\sqrt{d}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(3/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/sqrt(x) + sqrt(x)*sqrt(-e)*gamma(1/4)*
hyper((1/4, 1/2), (5/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(5/4))
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="maxima
")
```

```
[Out] 2*(d*sqrt(-e)*sqrt(x)*integrate(-sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(
3/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 3/2*log(x))), x) - arctan2(sqrt(-e)*
x, sqrt(e*x^2 + d)))/sqrt(x)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{3/2}}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2),x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2), x)
```

3.22 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}} dx = -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{2\sqrt{-e}e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}$$

```
[Out] -2/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2)-4/15*(-e)^(1/2)*(e*x^2+d)
^(1/2)/d/x^(3/2)-2/15*e^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1
/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x
^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^
(1/2)+x*e^(1/2))^2)^(1/2)/d^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5259, 331, 335, 226}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\sqrt{-e}e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(15*d*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[
d + e*x^2]])/(5*x^(5/2)) - (2*Sqrt[-e]*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(
d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^
(1/4)], 1/2])/(15*d^(5/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
```

```
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} + \frac{1}{5}\left(2\sqrt{-e}\right)\int\frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx \\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} + \frac{\left(2(-e)^{3/2}\right)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{15d} \\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} + \frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{15d} \\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} \\
&\quad + \frac{2(-e)^{3/2}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right), \frac{1}{2}\right)}{15d^{5/4}\sqrt[4]{e}\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\left(2\sqrt{-e}x\sqrt{d+ex^2} + 3d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)}{15dx^{5/2}} + \frac{4i(-e)^{3/2}\sqrt{1+\frac{d}{ex^2}}\,x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right), -1\right)}{15d\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-2*(2*Sqrt[-e]*x*Sqrt[d + e*x^2] + 3*d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
]))/(15*d*x^(5/2)) + (((4*I)/15)*(-e)^(3/2)*Sqrt[1 + d/(e*x^2)]*x*EllipticF
[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(d*Sqrt[(I*Sqrt[d])/Sqr
t[e]]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{7}{2}}}dx$$

[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)

[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\left(2\sqrt{-e}\sqrt{e}x^3\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)+2\sqrt{ex^2+d}\sqrt{-e}x^{\frac{3}{2}}+3\,d\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\right)}{15\,dx^3}$$

[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas")

[Out] -2/15*(2*sqrt(-e)*sqrt(e)*x^3*weierstrassPInverse(-4*d/e, 0, x) + 2*sqrt(e*x^2 + d)*sqrt(-e)*x^(3/2) + 3*d*sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(d*x^3)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 44.90 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.50

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\,\text{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5x^{\frac{5}{2}}}+\frac{\sqrt{-e}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{5\sqrt{d}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(7/2),x)

[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(5*x**(5/2)) + sqrt(-e)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), e*x**2*exp_polar(I*pi)/d)/(5*sqrt(d)*x**(3/2)*gamma(1/4))

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="maxima
")
```

```
[Out] 2/5*(5*d*sqrt(-e)*x^(5/2)*integrate(-1/5*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*
x^2)*x^(7/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 7/2*log(x))), x) - arctan2(s
qrt(-e)*x, sqrt(e*x^2 + d)))/x^(5/2)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{7/2}}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2), x)
```

3.23 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{10\sqrt{-e}e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}$$

```
[Out] -2/9*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2)-20/189*(-e)^(3/2)*(e*x^2+
d)^(1/2)/d^2/x^(3/2)-4/63*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^(7/2)+10/189*e^(7/
4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2
))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d
^(9/4)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5259, 331, 335, 226}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \frac{10\sqrt{-e}e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(63*d*x^(7/2)) - (20*(-e)^(3/2)*Sqrt[d + e*x^
2])/(189*d^2*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)
) + (10*Sqrt[-e]*e^(7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] +
Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(189*d^(
9/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{1}{9}\left(2\sqrt{-e}\right)\int\frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(10(-e)^{3/2}\right)\int\frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{63d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(10(-e)^{5/2}\right)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{189d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{(20(-e)^{5/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{189d^2}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{10\sqrt{-e}e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right), \frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \frac{4\sqrt{-e}x\sqrt{d+ex^2}(-3d+5ex^2) - 42d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{189d^2x^{9/2}} + \frac{20i(-e)^{5/2}\sqrt{1+\frac{d}{ex^2}}\,x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right), -1\right)}{189d^2\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (4*Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d + 5*e*x^2) - 42*d^2*ArcTan[(Sqrt[-e]*x)
/Sqrt[d + e*x^2]])/(189*d^2*x^(9/2)) + (((20*I)/189)*(-e)^(5/2)*Sqrt[1 + d/
(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(d^
2*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{11}{2}}}dx$$

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \frac{2\left(10\sqrt{-e}e^{\frac{3}{2}}x^5\text{weierstrassPInverse}\left(-\frac{4d}{e},0,x\right) - 21\,d^2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) + 2\,(5\,e\right.}{189\,d^2x^5}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="frica
s")
```

```
[Out] 2/189*(10*sqrt(-e)*e^(3/2)*x^5*weierstrassPInverse(-4*d/e, 0, x) - 21*d^2*s
qrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(5*e*x^3 - 3*d*x)*sqrt(e*x^2
+ d)*sqrt(-e)*sqrt(x))/(d^2*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="maxim
a")
```

```
[Out] 2/9*(9*d*sqrt(-e)*x^(9/2)*integrate(-1/9*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*
x^2)*x^(11/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 11/2*log(x))), x) - arctan2
(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^(9/2)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="giac"
)
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{11/2}}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2), x)
```

3.24 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 216

$$\begin{aligned}\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx &= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}\\ &\quad -\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}\\ &\quad -\frac{30\sqrt{-e}e^{11/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] -2/13*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(13/2)-36/1001*(-e)^(3/2)*(e*x
^2+d)^(1/2)/d^2/x^(7/2)-60/1001*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x^(3/2)-4/14
3*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^(11/2)-30/1001*e^(11/4)*(cos(2*arctan(e^(1
/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellip
ticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2
)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(13/4)/(e*x^2+d)^(1/
2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5259, 331, 335, 226}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{30\sqrt{-e}e^{11/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(143*d*x^(11/2)) - (36*(-e)^(3/2)*Sqrt[d + e*
x^2])/(1001*d^2*x^(7/2)) - (60*(-e)^(5/2)*Sqrt[d + e*x^2])/(1001*d^3*x^(3/2
)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) - (30*Sqrt[-e]*
e^(11/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*El
lipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(1001*d^(13/4)*Sqrt[d +
e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}+\frac{1}{13}\left(2\sqrt{-e}\right)\int\frac{1}{x^{13/2}\sqrt{d+ex^2}}\,dx\\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}}-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}+\frac{\left(18(-e)^{3/2}\right)\int\frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx}{143d}\\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}+\frac{\left(90(-e)^{5/2}\right)\int\frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{1001d^2}\\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}\\
&\quad-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}+\frac{\left(30(-e)^{7/2}\right)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{1001d^3}\\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}\\
&\quad-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}+\frac{\left(60(-e)^{7/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{1001d^3}\\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}\\
&\quad+\frac{30(-e)^{7/2}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{1001d^{13/4}\sqrt[4]{e}\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.79

$$\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx=\frac{2\left(-\frac{2\sqrt{-e}\sqrt{d+ex^2}\left(7d^2x-9dex^3+15e^2x^5\right)}{d^3}-77\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{30i(-e)^{7/2}\sqrt{1+\frac{d}{ex^2}}x^{15/2}\,\text{Elliptic}}{d^3\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{}}\right.}{1001x^{13/2}}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (2*((-2*Sqrt[-e]*Sqrt[d + e*x^2]*(7*d^2*x - 9*d*e*x^3 + 15*e^2*x^5))/d^3 -
77*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + ((30*I)*(-e)^(7/2)*Sqrt[1 + d/(e*
x^2)]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])
/(d^3*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])))/(1001*x^(13/2))
```

Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{15}{2}}}dx$$

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.45

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{2\left(30\sqrt{-e}e^{\frac{5}{2}}x^7\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)+77\,d^3\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+2\left(15\,e^2x^5-9\,dex^3+7\,d^2x\right)\right.}{1001\,d^3x^7}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="frica
s")
```

```
[Out] -2/1001*(30*sqrt(-e)*e^(5/2)*x^7*weierstrassPInverse(-4*d/e, 0, x) + 77*d^3
*sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(15*e^2*x^5 - 9*d*e*x^3 + 7
*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(x))/(d^3*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \text{Timed out}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(15/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="maxim
a")
```

```
[Out] 2/13*(13*d*sqrt(-e)*x^(13/2)*integrate(-1/13*sqrt(e*x^2 + d)*x/((e^2*x^4 +
d*e*x^2)*x^(15/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 15/2*log(x))), x) - arc
tan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^(13/2)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="giac"
)
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{15/2}}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2), x)
```

3.25 $\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 326

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}-\frac{28d^2\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}x\right)}$$
$$+\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{28d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}}-\frac{14d^{9/4}\sqrt{-e}}{}$$

```
[Out] 2/9*x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+28/405*d*x^(3/2)*(e*x^2+d)
^(1/2)/(-e)^(3/2)+4/81*x^(7/2)*(e*x^2+d)^(1/2)/(-e)^(1/2)-28/135*d^2*(-e)^(
1/2)*x^(1/2)*(e*x^2+d)^(1/2)/e^(5/2)/(d^(1/2)+x*e^(1/2))+28/135*d^(9/4)*(co
s(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/
d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e
)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(11/4
)/(e*x^2+d)^(1/2)-14/135*d^(9/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)
^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4
)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/
(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(11/4)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5259, 327, 335, 311, 226, 1210}

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = -\frac{14d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}} + \frac{28d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}} + \frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{28d^2\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}x\right)} + \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}} + \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}$$

```
[In] Int[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (28*d*x^(3/2)*Sqrt[d + e*x^2])/(405*(-e)^(3/2)) + (4*x^(7/2)*Sqrt[d + e*x^2
])/(81*Sqrt[-e]) - (28*d^2*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(135*e^(5/2)*(
Sqrt[d] + Sqrt[e]*x)) + (2*x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/9
+ (28*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sq
rt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(11
/4)*Sqrt[d + e*x^2]) - (14*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(135*e^(11/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{9}\left(2\sqrt{-e}\right)\int\frac{x^{9/2}}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}} + \frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(14d)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}\,dx}{81\sqrt{-e}}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}} + \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}} + \frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(14d^2)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{135(-e)^{3/2}}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}} + \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}} + \frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{(28d^2)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{135(-e)^{3/2}}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}} + \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}} + \frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{\left(28d^{5/2}\sqrt{-e}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{135e^{5/2}} + \frac{\left(28d^{5/2}\sqrt{-e}\right)\text{Subst}\left(\int\right.}{135e^5}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}} + \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}} - \frac{28d^2\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}x\right)}$$
$$+ \frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{28d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}} - 1$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.43

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{2x^{3/2}\left(2\sqrt{-e}(7d^2+2dex^2-5e^2x^4)+45e^2x^3\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-1\right.}{405e^2\sqrt{d+ex^2}}$$

```
[In] Integrate[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
[Out] (2*x^(3/2)*(2*Sqrt[-e]*(7*d^2 + 2*d*e*x^2 - 5*e^2*x^4) + 45*e^2*x^3*Sqrt[d
+ e*x^2]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 14*d^2*Sqrt[-e]*Sqrt[1 + (e
*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(405*e^2*Sqrt[d +
 e*x^2])
```

Maple [F]

$$\int x^{\frac{7}{2}}\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)dx$$

```
[In] int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
[Out] int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{2\left(45\,e^3x^{\frac{9}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+42\,d^2\sqrt{-e}\sqrt{e}\mathrm{weierstrassZeta}\left(-\frac{4\,d}{e},0,\mathrm{weierst}\right.\right.}{405\,e^3}$$

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
[Out] 2/405*(45*e^3*x^(9/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 42*d^2*sqrt(-e)*
sqrt(e)*weierstrassZeta(-4*d/e, 0, weierstrassPInverse(-4*d/e, 0, x)) - 2*(
5*e^2*x^3 - 7*d*e*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(x))/e^3
```

Sympy [F(-1)]

Timed out.

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \text{Timed out}$$

```
[In] integrate(x**(7/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Timed out
```

Maxima [F]

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int x^{\frac{7}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
[Out] 2/9*x^(9/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(-
1/9*x*e^(1/2*log(e*x^2 + d) + 7/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^
2), x)
```

Giac [F]

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int x^{\frac{7}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(x^(7/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int x^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)dx$$

```
[In] int(x^(7/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(7/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.26 $\int x^{3/2} \arctan \left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 296

$$\int x^{3/2} \arctan \left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}} + \frac{12d\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{25e^{3/2}\left(\sqrt{d}+\sqrt{e}x\right)} + \frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{12d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}} + \frac{6d^{5/4}\sqrt{-e}\left(\cdots\right.}{\cdots}$$

```
[Out] 2/5*x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/25*x^(3/2)*(e*x^2+d)^(1/
2)/(-e)^(1/2)+12/25*d*(-e)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/e^(3/2)/(d^(1/2)+x
*e^(1/2))-12/25*d^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/co
s(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+
x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1/2)+6/25*d^(5/4)*(cos(2*arctan(e^(1
/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellip
ticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2
)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1/2
)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5259, 327, 335, 311, 226, 1210}

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{6d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}} - \frac{12d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}} + \frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{12d\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{25e^{3/2}\left(\sqrt{d}+\sqrt{e}x\right)} + \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}$$

```
[In] Int[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*x^(3/2)*Sqrt[d + e*x^2])/(25*Sqrt[-e]) + (12*d*Sqrt[-e]*Sqrt[x]*Sqrt[d +
 e*x^2])/(25*e^(3/2)*(Sqrt[d] + Sqrt[e]*x)) + (2*x^(5/2)*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/5 - (12*d^(5/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
+ e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1
/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2]) + (6*d^(5/4)*Sqrt[-e]*(Sqrt[d] + S
qrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(
1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\left(2\sqrt{-e}\right)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}\,dx\\
&= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(6d)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{25\sqrt{-e}}\\
&= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{(12d)\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{25\sqrt{-e}}\\
&= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\\
&\quad-\frac{\left(12d^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{25\sqrt{-e^2}}+\frac{\left(12d^{3/2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{e}x^2}{\sqrt{d}}}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{25\sqrt{-e^2}}
\end{aligned}
$$

$$= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}} - \frac{12d\sqrt{x}\sqrt{d+ex^2}}{25\sqrt{-e^2}\left(\sqrt{d}+\sqrt{e}x\right)} + \frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$
$$+ \frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25\sqrt[4]{e}\sqrt{-e^2}\sqrt{d+ex^2}}$$
$$- \frac{6d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{25\sqrt[4]{e}\sqrt{-e^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.40

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2x^{3/2}\left(-2\sqrt{-e}(d+ex^2)+5ex\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+2d\sqrt{-e}\sqrt{1+\right.}{25e\sqrt{d+ex^2}}$$

```
[In] Integrate[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(-2*Sqrt[-e]*(d + e*x^2) + 5*e*x*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-e
]*x)/Sqrt[d + e*x^2]] + 2*d*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[
1/2, 3/4, 7/4, -((e*x^2)/d)]))/(25*e*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^{\frac{3}{2}}\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)dx$$

```
[In] int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.27

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2\left(5\,e^2x^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-2\sqrt{ex^2+d}\sqrt{-e}ex^{\frac{3}{2}}-6\,d\sqrt{-e}\sqrt{e}\text{weierstrass}\right.}{25\,e^2}$$

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] 2/25*(5*e^2*x^(5/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 2*sqrt(e*x^2 + d)*
sqrt(-e)*e*x^(3/2) - 6*d*sqrt(-e)*sqrt(e)*weierstrassZeta(-4*d/e, 0, weiers
trassPInverse(-4*d/e, 0, x)))/e^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.25

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\frac{2x^{\frac{5}{2}}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5}-\frac{x^{\frac{7}{2}}\sqrt{-e}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{ex^2e^{i\pi}}{d}\right)}{5\sqrt{d}\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate(x**(3/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(5/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/5 - x**(7/2)*sqrt(-e)*gamma(7/
4)*hyper((1/2, 7/4), (11/4,), e*x**2*exp_polar(I*pi)/d)/(5*sqrt(d)*gamma(11
/4))
```

Maxima [F]

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\int x^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] 2/5*x^(5/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(-
1/5*x*e^(1/2*log(e*x^2 + d) + 3/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^
2), x)
```

Giac [F]

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\int x^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

```
[In] int(x^(3/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(3/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.27 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 260

$$\begin{aligned}
&\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx \\
&= -\frac{4\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}x\right)}+2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) \\
&+\frac{4\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}} \\
&-\frac{2\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}
\end{aligned}$$

```
[Out] 2*x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-4*(-e)^(1/2)*x^(1/2)*(e*x^2+
d)^(1/2)/e^(1/2)/(d^(1/2)+x*e^(1/2))+4*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2
*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2
))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(3/4)/(e*x^2+d)^(1/2)-2*d^(1/4
)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2)
)*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^
(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5259, 335, 311, 226, 1210}

$$\begin{aligned}
&\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx \\
&= -\frac{2\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}} \\
&\quad+\frac{4\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}} \\
&\quad+2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{4\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}x\right)}
\end{aligned}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[e]*(Sqrt[d] + Sqrt[e]*x)) + 2*S
qrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + (4*d^(1/4)*Sqrt[-e]*(Sqrt[d]
+ Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(
e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2]) - (2*d^(1/4)*Sqr
t[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellip
ticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \left(2\sqrt{-e}\right)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx\\
&= 2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \left(4\sqrt{-e}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{\left(4\sqrt{d}\sqrt{-e}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{e}}\\
&\quad + \frac{\left(4\sqrt{d}\sqrt{-e}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{e}x^2}{\sqrt{d}}}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{e}}\\
&= -\frac{4\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}x\right)} + 2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\\
&\quad + \frac{4\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}\\
&\quad - \frac{2\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.34

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{4\sqrt{-e}x^{3/2}\sqrt{1+\frac{ex^2}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ex^2}{d}\right)}{3\sqrt{d+ex^2}}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - (4*Sqrt[-e]*x^(3/2)*Sqrt[1
 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(3*Sqrt[d + e
*x^2])
```

Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}dx$$

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.21

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \frac{2\left(e\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) + 2\sqrt{-e}\sqrt{e}\text{weierstrassZeta}\left(-\frac{4d}{e},0,\text{weierstrassPInverse}\left(-\frac{4d}{e},0,x\right)\right)\right)}{e}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas
")
```

```
[Out] 2*(e*sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*sqrt(-e)*sqrt(e)*weiers
trassZeta(-4*d/e, 0, weierstrassPInverse(-4*d/e, 0, x)))/e
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.27

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right) - \frac{x^{\frac{3}{2}}\sqrt{-e}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{\sqrt{d}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*atan(x*sqrt(-e)/sqrt(d + e*x**2)) - x**(3/2)*sqrt(-e)*gamma(3/4)*
hyper((1/2, 3/4), (7/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="maxima
")
```

```
[Out] -2*d*sqrt(-e)*integrate(sqrt(e*x^2 + d)*x/((e*x^2 + d)*e^(log(e*x^2 + d) +
1/2*log(x)) - (e^2*x^4 + d*e*x^2)*sqrt(x)), x) + 2*sqrt(x)*arctan2(sqrt(-e)
*x, sqrt(e*x^2 + d))
```

Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2),x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2), x)
```

3.28 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 298

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}} dx = -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} + \frac{4\sqrt{-e^2}\sqrt{x}\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{4\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}} + \frac{2\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}$$

```
[Out] -2/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2)-4/3*(-e)^(1/2)*(e*x^2+d)^
(1/2)/d/x^(1/2)+4/3*(-e^2)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/d/(d^(1/2)+x*e^(1/
2))-4/3*e^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arct
an(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)
)),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2
))^2)^(1/2)/d^(3/4)/(e*x^2+d)^(1/2)+2/3*e^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(
2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/
2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5259, 331, 335, 311, 226, 1210}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \frac{2\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}} - \frac{4\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{4\sqrt{-e^2}\sqrt{x}\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) + (4*Sqrt[-e^2]*Sqrt[x]*Sqrt[d
+ e*x^2])/(3*d*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x
^2]])/(3*x^(3/2)) - (4*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x
^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)],
 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2]) + (2*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]
*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*S
qrt[x])/d^(1/4)], 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}+\frac{1}{3}\left(2\sqrt{-e}\right)\int\frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx\\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}}-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}-\frac{\left(2(-e)^{3/2}\right)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{3d}\\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}}-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}-\frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{3d}\\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}}-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}-\frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{d}\sqrt{e}}\\
&\quad+\frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{e}x^2}{\sqrt{d}}}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{d}\sqrt{e}}
\end{aligned}
$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{4(-e)^{3/2}\sqrt{x}\sqrt{d+ex^2}}{3d\sqrt{e}\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{4(-e)^{3/2}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}e^{3/4}\sqrt{d+ex^2}} - \frac{2(-e)^{3/2}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{3/4}e^{3/4}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.41

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{2\left(6\sqrt{-e}x(d+ex^2) + 3d\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + 2(-e)^{3/2}x^3\sqrt{1+\frac{ex^2}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\right.\right.}{9dx^{3/2}\sqrt{d+ex^2}}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-2*(6*Sqrt[-e]*x*(d + e*x^2) + 3*d*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]] + 2*(-e)^(3/2)*x^3*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2,
 3/4, 7/4, -((e*x^2)/d)]))/(9*d*x^(3/2)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{5}{2}}}dx$$

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx =$$
$$-\frac{2\left(2\sqrt{-e}\sqrt{e}x^2\text{weierstrassZeta}\left(-\frac{4d}{e},0,\text{weierstrassPInverse}\left(-\frac{4d}{e},0,x\right)\right)+2\sqrt{ex^2+d}\sqrt{-e}x^{\frac{3}{2}}+d\sqrt{x}\,\text{arcta}\right.}{3\,dx^2}$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas
")
```

```
[Out] -2/3*(2*sqrt(-e)*sqrt(e)*x^2*weierstrassZeta(-4*d/e, 0, weierstrassPInverse
(-4*d/e, 0, x)) + 2*sqrt(e*x^2 + d)*sqrt(-e)*x^(3/2) + d*sqrt(x)*arctan(sqr
t(-e)*x/sqrt(e*x^2 + d)))/(d*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.26

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{2\,\text{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^{\frac{3}{2}}}+\frac{\sqrt{-e}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3\sqrt{d}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(5/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**(3/2)) + sqrt(-e)*gamma(-1/4)*hy
per((-1/4, 1/2), (3/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*sqrt(x)*gamma
(3/4))
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="maxima
")
```

```
[Out] 2/3*(3*d*sqrt(-e)*x^(3/2)*integrate(-1/3*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*
x^2)*x^(5/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 5/2*log(x))), x) - arctan2(s
qrt(-e)*x, sqrt(e*x^2 + d)))/x^(3/2)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="giac")
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{5/2}}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2),x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2), x)
```

3.29 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 331

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}} dx = -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{12\sqrt{-e}e^{3/2}\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{12\sqrt{-e}e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}} - \frac{6\sqrt{-e}e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}$$

```
[Out] -2/7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2)-4/35*(-e)^(1/2)*(e*x^2+d)
^(1/2)/d/x^(5/2)-12/35*(-e)^(3/2)*(e*x^2+d)^(1/2)/d^2/x^(1/2)-12/35*e^(3/2)
*(-e)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/d^2/(d^(1/2)+x*e^(1/2))+12/35*e^(5/4)*(
cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(
-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/
4)/(e*x^2+d)^(1/2)-6/35*e^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^
(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)
*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(
d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5259, 331, 335, 311, 226, 1210}

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = -\frac{6e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}} + \frac{12e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{12e^{3/2}\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}}$$

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(35*d*x^(5/2)) - (12*(-e)^(3/2)*Sqrt[d + e*x^
2])/(35*d^2*Sqrt[x]) - (12*Sqrt[-e]*e^(3/2)*Sqrt[x]*Sqrt[d + e*x^2])/(35*d^
2*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(7*x^(7
/2)) + (12*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d]
 + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d
^(7/4)*Sqrt[d + e*x^2]) - (6*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
 + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(
1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{1}{7}\left(2\sqrt{-e}\right)\int \frac{1}{x^{7/2}\sqrt{d+ex^2}}\,dx \\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{\left(6(-e)^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx}{35d} \\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(6(-e)^{5/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{35d^2} \\
&= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} \\
&\quad - \frac{\left(12(-e)^{5/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{35d^2}
\end{aligned}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12\sqrt{-e}e^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{35d^{3/2}} + \frac{\left(12\sqrt{-e}e^{3/2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{e}x^2}{\sqrt{d}}}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{35d^{3/2}}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{12\sqrt{-e}e^{3/2}\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{12\sqrt{-e}e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}} - \frac{6\sqrt{-e}e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.41

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \frac{4\sqrt{-e}x(-d^2+2dex^2+3e^2x^4) - 10d^2\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - 4(-e)^{5/2}x^5\sqrt{1}}{35d^2x^{7/2}\sqrt{d+ex^2}}$$

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (4*Sqrt[-e]*x*(-d^2 + 2*d*e*x^2 + 3*e^2*x^4) - 10*d^2*Sqrt[d + e*x^2]*ArcTa
n[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 4*(-e)^(5/2)*x^5*Sqrt[1 + (e*x^2)/d]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(35*d^2*x^(7/2)*Sqrt[d + e*x^2]
)
```

Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{9}{2}}}dx$$

[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)

[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.28

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \frac{2\left(6\sqrt{-e}e^{\frac{3}{2}}x^4\text{weierstrassZeta}\left(-\frac{4\,d}{e},0,\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)\right)-5\,d^2\sqrt{x}\,\text{ar}\right.}{35\,d^2x^4}$$

[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas
")

[Out] 2/35*(6*sqrt(-e)*e^(3/2)*x^4*weierstrassZeta(-4*d/e, 0, weierstrassPInverse
(-4*d/e, 0, x)) - 5*d^2*sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(3*e
*x^3 - d*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(x))/(d^2*x^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \text{Timed out}$$

[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(9/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="maxima
")

[Out] 2/7*(7*d*sqrt(-e)*x^(7/2)*integrate(-1/7*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*
x^2)*x^(9/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 9/2*log(x))), x) - arctan2(s
qrt(-e)*x, sqrt(e*x^2 + d)))/x^(7/2)

Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="giac")
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{9/2}}\,dx$$

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2),x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2), x)
```

3.30 $\int \frac{\arctan(1+x+x^2)}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{\arctan(1+x+x^2)}{x^2} dx = \frac{1}{2}\arctan(1+x) - \frac{\arctan(1+x+x^2)}{x} + \frac{\log(x)}{2} - \frac{1}{2}\log(1+x^2) + \frac{1}{4}\log(2+2x+x^2)$$

```
[Out] 1/2*arctan(1+x)-arctan(x^2+x+1)/x+1/2*ln(x)-1/2*ln(x^2+1)+1/4*ln(x^2+2*x+2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5313, 6874, 266, 648, 631, 210, 642}

$$\int \frac{\arctan(1+x+x^2)}{x^2} dx = -\frac{\arctan(x^2+x+1)}{x} + \frac{1}{2}\arctan(x+1) - \frac{1}{2}\log(x^2+1) + \frac{1}{4}\log(x^2+2x+2) + \frac{\log(x)}{2}$$

```
[In] Int[ArcTan[1 + x + x^2]/x^2,x]
```

```
[Out] ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[2
 + 2*x + x^2]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\arctan(1+x+x^2)}{x} + \int \frac{1+2x}{x(2+2x+3x^2+2x^3+x^4)}\,dx \\ &= -\frac{\arctan(1+x+x^2)}{x} + \int \left(\frac{1}{2x} - \frac{x}{1+x^2} + \frac{2+x}{2(2+2x+x^2)}\right)dx\end{aligned}$$

$$= -\frac{\arctan(1+x+x^2)}{x} + \frac{\log(x)}{2} + \frac{1}{2}\int \frac{2+x}{2+2x+x^2}\,dx - \int \frac{x}{1+x^2}\,dx$$

$$= -\frac{\arctan(1+x+x^2)}{x} + \frac{\log(x)}{2} - \frac{1}{2}\log(1+x^2) + \frac{1}{4}\int \frac{2+2x}{2+2x+x^2}\,dx + \frac{1}{2}\int \frac{1}{2+2x+x^2}\,dx$$

$$= -\frac{\arctan(1+x+x^2)}{x} + \frac{\log(x)}{2} - \frac{1}{2}\log(1+x^2) + \frac{1}{4}\log(2+2x+x^2) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+x\right)$$

$$= \frac{1}{2}\arctan(1+x) - \frac{\arctan(1+x+x^2)}{x} + \frac{\log(x)}{2} - \frac{1}{2}\log(1+x^2) + \frac{1}{4}\log(2+2x+x^2)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = \frac{1}{2}\arctan(1+x) - \frac{\arctan(1+x+x^2)}{x} + \frac{\log(x)}{2} - \frac{1}{2}\log(1+x^2) + \frac{1}{4}\log(2+2x+x^2)$$

```
[In] Integrate[ArcTan[1 + x + x^2]/x^2,x]
[Out] ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[2
 + 2*x + x^2]/4
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result
default	$\frac{\arctan(1+x)}{2} - \frac{\arctan(x^2+x+1)}{x} + \frac{\ln(x)}{2} - \frac{\ln(x^2+1)}{2} + \frac{\ln(x^2+2x+2)}{4}$
parts	$\frac{\arctan(1+x)}{2} - \frac{\arctan(x^2+x+1)}{x} + \frac{\ln(x)}{2} - \frac{\ln(x^2+1)}{2} + \frac{\ln(x^2+2x+2)}{4}$
risch	$\frac{i\ln(1+i(x^2+x+1))}{2x} - \frac{i(i\ln(x+1-i)x+i\ln(x+1+i)x+2i\ln(x)x-2i\ln(x^2+1)x+\ln(x+1-i)x-\ln(x+1+i)x+2\ln(1-i(x^2+x+}{4x}$
parallelrisch	$\frac{-4i\ln(i+x)x+7i\ln(x+1+i)x-7i\ln(x+1-i)x+4i\ln(x-i)x+6\ln(x)x-6\ln(x-i)x-6\ln(i+x)x+3\ln(x+1-i)x+3\ln(x+1+i)x+8}{12x}$

```
[In] int(arctan(x^2+x+1)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(1+x)-arctan(x^2+x+1)/x+1/2*ln(x)-1/2*ln(x^2+1)+1/4*ln(x^2+2*x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = \frac{2\,x\arctan(x+1) + x\log(x^2+2\,x+2) - 2\,x\log(x^2+1) + 2\,x\log(x) - 4\,\arctan(x^2+x+1)}{4\,x}$$

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*x*arctan(x + 1) + x*log(x^2 + 2*x + 2) - 2*x*log(x^2 + 1) + 2*x*log(
x) - 4*arctan(x^2 + x + 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = \frac{\log(x)}{2} - \frac{\log(x^2+1)}{2} + \frac{\log(x^2+2x+2)}{4} + \frac{\operatorname{atan}(x+1)}{2} - \frac{\operatorname{atan}(x^2+x+1)}{x}$$

```
[In] integrate(atan(x**2+x+1)/x**2,x)
```

```
[Out] log(x)/2 - log(x**2 + 1)/2 + log(x**2 + 2*x + 2)/4 + atan(x + 1)/2 - atan(x
**2 + x + 1)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = -\frac{\arctan(x^2+x+1)}{x} + \frac{1}{2}\arctan(x+1) + \frac{1}{4}\log\left(x^2+2\,x+2\right) - \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log(x)$$

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*l
og(x^2 + 1) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = -\frac{\arctan(x^2+x+1)}{x} + \frac{1}{2}\arctan(x+1) + \frac{1}{4}\log\left(x^2+2x+2\right) - \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log(|x|)$$

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="giac")
[Out] -arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*l
og(x^2 + 1) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = \frac{\mathrm{atan}(x+1)}{2} + \frac{\ln(x^2+2x+2)}{4} - \frac{\ln(x^2+1)}{2} + \frac{\ln(x)}{2} - \frac{\mathrm{atan}(x^2+x+1)}{x}$$

```
[In] int(atan(x + x^2 + 1)/x^2,x)
[Out] atan(x + 1)/2 + log(2*x + x^2 + 2)/4 - log(x^2 + 1)/2 + log(x)/2 - atan(x +
 x^2 + 1)/x
```

3.31 $\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\text{Int}\left(\frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
[Out] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="fricas")
[Out] integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 3.75 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
[Out] -Integral((a + b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="maxima")
[Out] -integrate((b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="giac")
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
[Out] -int((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.32 $\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 431

$$\begin{aligned}
&\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx \\
&= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&\quad+\frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad-\frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad+\frac{3b^2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad-\frac{3b^2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}+\frac{3ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}
\end{aligned}$$

```
[Out] 2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3*arctanh(-1+2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+3/2*I*b*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*I*b*(a+b*arctan((-
c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(
```

```
1/2)))/c+3/2*b^2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2/(
1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b^2*(a+b*arctan((-c*x+1)^(1/2)/(c*
x+1)^(1/2)))*polylog(3,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/4*I*b^3
*polylog(4,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*I*b^3*polylog(4,-1
+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6813, 4942, 5108, 5004, 5114, 5118, 6745}

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c} + \frac{3b^2\,\text{PolyLog}\left(3,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c} - \frac{3b^2\,\text{PolyLog}\left(3,\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c} + \frac{3ib\,\text{PolyLog}\left(2,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c} - \frac{3ib\,\text{PolyLog}\left(2,\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c} - \frac{3ib^3\,\text{PolyLog}\left(4,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{4c} + \frac{3ib^3\,\text{PolyLog}\left(4,\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)}{4c}$$

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/S
qrt[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
 - (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, -1
 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (3*b^2*(a + b*ArcTan[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c
*x])])/(2*c) - (3*b^2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3
, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/(2*c) - (((3*I)/4)*b^3*Pol
```

```
yLog[4, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/4)*b^3*Po
lyLog[4, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}\\
&\quad+\frac{(6b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2\operatorname{arctanh}\left(1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}\\
&\quad+\frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2\log\left(2-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&\quad-\frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(x))^2\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}\\
&\quad+\frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}\\
&\quad-\frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}\\
&\quad-\frac{(3ib^2)\text{Subst}\left(\int \frac{(a+b\arctan(x))\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&\quad+\frac{(3ib^2)\text{Subst}\left(\int \frac{(a+b\arctan(x))\text{PolyLog}\left(2,-1+\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}
\end{aligned}$$

$$= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} + \frac{3b^2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{3b^2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{(3b^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} + \frac{(3b^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

$$= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} + \frac{3b^2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{3b^2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c} + \frac{3ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}$$

Mathematica [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1663 vs. 2(360) = 720.

Time = 0.58 (sec) , antiderivative size = 1664, normalized size of antiderivative = 3.86

method	result	size
default	Expression too large to display	1664
parts	Expression too large to display	1664

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
[Out] -1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-b^3*(1/c*arctan((-c*x+1)^(1/2)/(c*
x+1)^(1/2))^3*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^
(1/2))-3*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(1+I*(-c*x+1)
^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+6/c*arctan((-c*x+1)^(1/2)
/(c*x+1)^(1/2))*polylog(3,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x
+1)+1)^(1/2))+6*I/c*polylog(4,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/
(c*x+1)+1)^(1/2))-1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(1+I*(-c*
x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+3/2*I/c*arctan((-c*x+1)^(
1/2)/(c*x+1)^(1/2))^2*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*
x+1)/(c*x+1)+1))-3/2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(1+I
*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-3/4*I/c*polylog(4,-(
1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*arctan((-c*x+
1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)
/(c*x+1)+1)^(1/2))-3*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-
(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+6/c*arctan((
-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/
((-c*x+1)/(c*x+1)+1)^(1/2))+6*I/c*polylog(4,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1
/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))-3*a*b^2*(1/c*arctan((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/
2))-2*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(1+I*(-c*x+1)^(1/2
)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2/c*polylog(3,(1+I*(-c*x+1)^(1
```

```
/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-1/c*arctan((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+
1))+I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)
/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/2/c*polylog(3,-(1+I*(-c*x+1)^(1/2
)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^
(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)
)-2*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)
/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2/c*polylog(3,-(1+I*(-c*x+1)^(1
/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))-3*a^2*b*(1/c*arctan((-c*x+1
)^(1/2)/(c*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c
*x+1)+1)^(1/2))-I/c*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/
(c*x+1)+1)^(1/2))-1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(1+I*(-c*x+
1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2*I/c*polylog(2,-(1+I*(-c
*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*arctan((-c*x+1)^(1/2
)/(c*x+1)^(1/2))*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+
1)^(1/2))-I/c*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+
1)+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^3*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arctan(sqrt
(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))
+ a^3)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{atan}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*atan(sqrt(-c*x + 1)/sqrt
(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*atan(sqrt(-c*x + 1)/s
qrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*atan(sqrt(-c*x + 1
)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/64*(4*(b^3*log(c*x + 1) - b^3
*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^3 - 3*(b^3*log(2)^2*
log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x
 + 1)) - 64*c*integrate(1/128*(112*b^3*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1
))^3 + 384*a*b^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - 3*(b^3*log(2)^2
*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1) - 4*(b^3*log(c*x + 1) - b^3*log(
-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2)*sqrt(c*x + 1)*sqrt(-c*
x + 1) + 12*(b^3*log(2)^2 + 32*a^2*b)*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)
))/(c^2*x^2 - 1), x))/c
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.33 $\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 283

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}$$

```
[Out] 2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*arctanh(-1+2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+I*b*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-I*b*(a+b*arctan((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))*polylog(2,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+1/2
*b^2*polylog(3,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-1/2*b^2*polylog(3,
-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6813, 4942, 5108, 5004, 5114, 6745}

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} - \frac{ib\,\text{PolyLog}\left(2,\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} + \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{2c} - \frac{b^2\,\text{PolyLog}\left(3,\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)}{2c}$$

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])]])/c + (I*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])]])/c - (I*b*(a
 + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 + (I*Sqrt[1
- c*x])/Sqrt[1 + c*x])]])/c + (b^2*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/S
qrt[1 + c*x])]])/(2*c) - (b^2*PolyLog[3, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[
1 + c*x])]])/(2*c)
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(a+b\arctan(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4b)\text{Subst}\left(\int \frac{(a+b\arctan(x))\operatorname{arctanh}\left(1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(2b)\text{Subst}\left(\int \frac{(a+b\arctan(x))\log\left(2-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{(2b)\text{Subst}\left(\int \frac{(a+b\arctan(x))\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2, 1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2, -1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} + \frac{(ib^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-1+\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2, 1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2, -1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{b^2\,\text{PolyLog}\left(3, 1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{b^2\,\text{PolyLog}\left(3, -1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}$$

Mathematica [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]

[Out] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(240) = 480.

Time = 0.48 (sec) , antiderivative size = 916, normalized size of antiderivative = 3.24

method	result
default	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{2i\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\mathrm{polylog}\left(2,\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right.}{c}\right.$
parts	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{2i\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\mathrm{polylog}\left(2,\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right.}{c}\right.$

[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RETURNVERBOSE)

[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2*(1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2/c*polylog(3,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/2/c*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2/c*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))-2*a*b*(1/c*arctan((-c*x+

```
1)^(1/2)/(c*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(
c*x+1)+1)^(1/2))-I/c*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)
/(c*x+1)+1)^(1/2))-1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(1+I*(-c*x
+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2*I/c*polylog(2,-(1+I*(-
c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*arctan((-c*x+1)^(1/
2)/(c*x+1)^(1/2))*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)
+1)^(1/2))-I/c*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x
+1)+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^2*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arctan(sqrt(-
c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=-\int \frac{a^2}{c^2x^2-1}\,dx-\int \frac{b^2\operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{2ab\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*atan(sqrt(-c*x + 1)/sqrt
(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*atan(sqrt(-c*x + 1)/sqrt
(c*x + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/32*(b^2*log(2)^2*log(c*x + 1)
 - b^2*log(2)^2*log(-c*x + 1) - 4*(b^2*log(c*x + 1) - b^2*log(-c*x + 1))*ar
ctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - (b^2*(log(c*x + 1)/c - log(c*x - 1
)/c)*log(2)^2 - 64*b^2*integrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(s
qrt(-c*x + 1)/sqrt(c*x + 1))*log(c*x + 1)/(c^2*x^2 - 1), x) + 64*b^2*integr
ate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))*
log(-c*x + 1)/(c^2*x^2 - 1), x) - 384*b^2*integrate(1/16*arctan(sqrt(-c*x +
 1)/sqrt(c*x + 1))^2/(c^2*x^2 - 1), x) - 1024*a*b*integrate(1/16*arctan(sqr
t(-c*x + 1)/sqrt(c*x + 1))/(c^2*x^2 - 1), x))*c)/c
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.34 $\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 98

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

```
[Out] -a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c-1/2*I*b*polylog(2,-I*(-c*x+1)^(1/2)/(
c*x+1)^(1/2))/c+1/2*I*b*polylog(2,I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {212, 6813, 4940, 2438}

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}$$

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) - ((I/2)*b*PolyLog[2, ((-I)*Sqrt[
1 - c*x])/Sqrt[1 + c*x]])/c + ((I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1
+ c*x]])/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{a+b\arctan(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} + \frac{(ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}\\
&= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{ib\,\text{PolyLog}\left(2, -\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} + \frac{ib\,\text{PolyLog}\left(2, \frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right) + \frac{1}{2}ib\,\text{PolyLog}\left(2, -\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right) - \frac{1}{2}ib\,\text{PolyLog}\left(2, \frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] + (I/2)*b*PolyLog[2, ((-I)*Sqrt[1 - c
*x])/Sqrt[1 + c*x]] - (I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/
c)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(78) = 156$.

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.76

method	result
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{i\,\mathrm{polylog}\left(2,\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln}{}\right.$
parts	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{i\,\mathrm{polylog}\left(2,\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln}{}\right.$

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)-b*(1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(
1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-I
/c*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))
-1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^
(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2*I/c*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*
ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-I/c*pol
ylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\int \frac{a}{c^2x^2-1}\,dx-\int \frac{b\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*atan(sqrt(-c*x + 1)/sqrt(c*x +
 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*((log(c*x + 1) - log(-c*x + 1
))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) - 2*c*integrate(1/2*(e^(1/2*log(c
*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) - e^(1/2*log(c*x + 1) + 1/2*log(-
c*x + 1))*log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1) - (c^2*x^2 - 1)*(c*x - 1)
), x))*b/c
```

Giac [F]

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.35 $\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
[Out] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="fricas")
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - a), x)
```

Sympy [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$= -\int \frac{1}{ac^2x^2-a+bc^2x^2\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*atan(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="maxima")
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="giac")
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

```
[In] int(-1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.36 $\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x
]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}dx$$

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arctan(sqrt(-c*x + 1)/sqrt(c
*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1
))), x)
```

Sympy [N/A]

Not integrable

Time = 6.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{a^2c^2x^2-a^2+2abc^2x^2\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^2c^2x^2\operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^2\operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}$$

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*atan(sqrt(-c*x + 1)/sq
rt(c*x + 1)) - 2*a*b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*at
an(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*atan(sqrt(-c*x + 1)/sqrt(c*x + 1
))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.08

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="maxima")
```

```
[Out] 2*(2*(b^2*c^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c^2)*sqrt(c*x +
1)*sqrt(-c*x + 1)*integrate(1/2*x/((a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2)
*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
+ 1)/((b^2*c*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c)*sqrt(c*x + 1)*
sqrt(-c*x + 1))
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2)
, x)
```

Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=-\int \frac{1}{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

```
[In] int(-1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.37 $\int x^m \arctan(\tan(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int x^m \arctan(\tan(a+bx))\,dx=-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\arctan(\tan(a+bx))}{1+m}$$

```
[Out] -b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arctan(tan(b*x+a))/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2199, 30}

$$\int x^m \arctan(\tan(a+bx))\,dx=\frac{x^{m+1}\arctan(\tan(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

```
[In] Int[x^m*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcTan[Tan[a + b*x]])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{1+m}\arctan(\tan(a+bx))}{1+m} - \frac{b\int x^{1+m}\,dx}{1+m}\\ &= -\frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m}\arctan(\tan(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m \arctan(\tan(a+bx))\,dx = x^m\left(\frac{bx^2}{2+m} + \frac{x(-bx+\arctan(\tan(a+bx)))}{1+m}\right)$$

```
[In] Integrate[x^m*ArcTan[Tan[a + b*x]],x]
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcTan[Tan[a + b*x]]))/(1 + m))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result
default	$\frac{b\,x^2 e^{m\ln(x)}}{2+m} + \frac{(\arctan(\tan(bx+a))-bx)x\,e^{m\ln(x)}}{1+m}$
parallelrisch	$-\frac{-2\arctan(\tan(bx+a))x^m x+b\,x^m x^2-x\,x^m\arctan(\tan(bx+a))m}{(1+m)(2+m)}$
risch	$-\frac{ix\,x^m\ln\left(e^{i(bx+a)}\right)}{1+m} - \frac{x\left(2\pi\,\mathrm{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right)^3+2\pi\,\mathrm{csgn}\left(ie^{2i(bx+a)}\right)^3+4bx+\pi m\ \mathrm{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(ie^{2i(bx+a)}\right)\mathrm{cs}\right.}{}$

```
[In] int(x^m*arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] b/(2+m)*x^2*exp(m*ln(x))+(arctan(tan(b*x+a))-b*x)/(1+m)*x*exp(m*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int x^m \arctan(\tan(a+bx))\,dx = \frac{((bm+b)x^2+(am+2\,a)x)x^m}{m^2+3\,m+2}$$

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(31) = 62.

Time = 0.67 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.27

$$\int x^m \arctan(\tan(a+bx))\,dx$$
$$= \begin{cases} b\log(x) - \frac{\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor}{x} & \text{for}\ m=-2 \\ -bx\log(x)+bx+\left(\operatorname{atan}(\tan(a+bx))+2\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)\log(x) & \text{for}\ m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2}+\frac{mxx^m\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{m^2+3m+2}+\frac{2xx^m\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{m^2+3m+2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**m*atan(tan(b*x+a)),x)
[Out] Piecewise((b*log(x) - (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/
x, Eq(m, -2)), (-b*x*log(x) + b*x + (atan(tan(a + b*x)) + 2*pi*floor((a + b
*x - pi/2)/pi))*log(x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x*
*m*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/(m**2 + 3*m + 2) +
2*x*x**m*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/(m**2 + 3*m +
 2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^m \arctan(\tan(a+bx))\,dx = -\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1}\arctan(\tan(bx+a))}{m+1}$$

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] -b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arctan(tan(b*x + a))/(m + 1)
```

Giac [F]

$$\int x^m \arctan(\tan(a+bx))\,dx = \int x^m \arctan\left(\tan\left(bx+a\right)\right)\,dx$$

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^m \arctan(\tan(a+bx))\,dx = \int x^m \operatorname{atan}(\tan(a+b\,x))\,dx$$

```
[In] int(x^m*atan(tan(a + b*x)),x)
[Out] int(x^m*atan(tan(a + b*x)), x)
```

3.38 $\int x^2 \arctan(\tan(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2 \arctan(\tan(a+bx))\,dx = -\frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(\tan(a+bx))$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctan(tan(b*x+a))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 30}

$$\int x^2 \arctan(\tan(a+bx))\,dx = \frac{1}{3}x^3 \arctan(\tan(a+bx)) - \frac{bx^4}{12}$$

```
[In] Int[x^2*ArcTan[Tan[a + b*x]],x]
[Out] -1/12*(b*x^4) + (x^3*ArcTan[Tan[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}x^3\arctan(\tan(a+bx)) - \frac{1}{3}b\int x^3\,dx \\ &= -\frac{bx^4}{12} + \frac{1}{3}x^3\arctan(\tan(a+bx)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2\arctan(\tan(a+bx))\,dx = -\frac{1}{12}x^3(bx - 4\arctan(\tan(a+bx)))$$

```
[In] Integrate[x^2*ArcTan[Tan[a + b*x]],x]
[Out] -1/12*(x^3*(b*x - 4*ArcTan[Tan[a + b*x]]))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^4}{12}+\frac{x^3\arctan(\tan(bx+a))}{3}$
parallelrisch	$-\frac{b\,x^4}{12}+\frac{x^3\arctan(\tan(bx+a))}{3}$
parts	$-\frac{b\,x^4}{12}+\frac{x^3\arctan(\tan(bx+a))}{3}$
risch	$-\frac{ix^3\ln\left(e^{i(bx+a)}\right)}{3}-\frac{\pi\,x^3\operatorname{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\operatorname{csgn}\left(ie^{2i(bx+a)}\right)\operatorname{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right)}{12}+\frac{\pi\,x^3\operatorname{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\operatorname{csgn}\left(\frac{ie^{2i(}}{e^{2i(b}}\right.}{12}$

```
[In] int(x^2*arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/12*b*x^4+1/3*x^3*arctan(tan(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2 \arctan(\tan(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int x^2 \arctan(\tan(a+bx))\,dx = -\frac{bx^4}{12} + \frac{x^3\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right) + \pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

```
[In] integrate(x**2*atan(tan(b*x+a)),x)
[Out] -b*x**4/12 + x**3*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(19) = 38.

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.52

$$\int x^2 \arctan(\tan(a+bx))\,dx = \frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan(\tan(bx+a))}{b^2} - \frac{(bx+a)^4-4\,(bx+a)^3a+6\,(bx+a)^2a^2}{b^2}}{12\,b}$$

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan(tan(b*x +
a))/b^2 - ((b*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2)/b^2)/b
```

Giac [F]

$$\int x^2 \arctan(\tan(a+bx))\,dx = \int x^2 \arctan\left(\tan\left(bx+a\right)\right)\,dx$$

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \arctan(\tan(a+bx))\,dx = \frac{x^3\,\mathrm{atan}(\tan(a+b\,x))}{3} - \frac{b\,x^4}{12}$$

```
[In] int(x^2*atan(tan(a + b*x)),x)
```

```
[Out] (x^3*atan(tan(a + b*x)))/3 - (b*x^4)/12
```

3.39 $\int x \arctan(\tan(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \arctan(\tan(a+bx))\,dx = -\frac{bx^3}{6} + \frac{1}{2}x^2 \arctan(\tan(a+bx))$$

```
[Out] -1/6*b*x^3+1/2*x^2*arctan(tan(b*x+a))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5279, 30}

$$\int x \arctan(\tan(a+bx))\,dx = \frac{1}{2}x^2 \arctan(\tan(a+bx)) - \frac{bx^3}{6}$$

```
[In] Int[x*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcTan[Tan[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
```

```
Q[(c + I*d)^2, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}x^2 \arctan(\tan(a+bx)) - \frac{1}{2}b\int x^2\,dx \\ &= -\frac{bx^3}{6} + \frac{1}{2}x^2 \arctan(\tan(a+bx)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x \arctan(\tan(a+bx))\,dx = -\frac{1}{6}x^2(bx - 3\arctan(\tan(a+bx)))$$

```
[In] Integrate[x*ArcTan[Tan[a + b*x]],x]
[Out] -1/6*(x^2*(b*x - 3*ArcTan[Tan[a + b*x]]))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^3}{6}+\frac{x^2\arctan(\tan(bx+a))}{2}$
parallelrisch	$-\frac{b\,x^3}{6}+\frac{x^2\arctan(\tan(bx+a))}{2}$
parts	$-\frac{b\,x^3}{6}+\frac{x^2\arctan(\tan(bx+a))}{2}$
risch	$-\frac{ix^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{2}-\frac{\pi\,x^2\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)}{8}+\frac{\pi\,x^2\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(}}{\mathrm{e}^{2i(b}}\right.}{8}$

```
[In] int(x*arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/6*b*x^3+1/2*x^2*arctan(tan(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x\arctan(\tan(a+bx))\,dx = \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(19) = 38.

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int x\arctan(\tan(a+bx))\,dx = \begin{cases} \frac{x\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} - \frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^3}{6b^2} & \text{for}\ b\neq 0 \\ \frac{x^2\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*atan(tan(b*x+a)),x)
[Out] Piecewise((x*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b)
- (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**3/(6*b**2), Ne(b, 0
)), (x**2*(atan(tan(a)) + pi*floor((a - pi/2)/pi))/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(19) = 38.

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.48

$$\int x\arctan(\tan(a+bx))\,dx = \frac{\frac{3\left((bx+a)^2-2\,(bx+a)a\right)\arctan(\tan(bx+a))}{b} - \frac{(bx+a)^3-3\,(bx+a)^2a}{b}}{6\,b}$$

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/6*(3*((b*x + a)^2 - 2*(b*x + a)*a)*arctan(tan(b*x + a))/b - ((b*x + a)^3
- 3*(b*x + a)^2*a)/b)/b
```

Giac [F]

$$\int x \arctan(\tan(a+bx))\,dx = \int x \arctan\left(\tan\left(bx+a\right)\right)\,dx$$

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \arctan(\tan(a+bx))\,dx = \frac{x^2\,\mathrm{atan}(\tan(a+b\,x))}{2} - \frac{b\,x^3}{6}$$

```
[In] int(x*atan(tan(a + b*x)),x)
[Out] (x^2*atan(tan(a + b*x)))/2 - (b*x^3)/6
```

3.40 $\int \arctan(\tan(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \arctan(\tan(a+bx))\,dx = \frac{\arctan(\tan(a+bx))^2}{2b}$$

```
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2188, 30}

$$\int \arctan(\tan(a+bx))\,dx = \frac{\arctan(\tan(a+bx))^2}{2b}$$

```
[In] Int[ArcTan[Tan[a + b*x]],x]
```

```
[Out] ArcTan[Tan[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int x\,dx, x, \arctan(\tan(a+bx)))}{b}$$
$$= \frac{\arctan(\tan(a+bx))^2}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \arctan(\tan(a+bx))\,dx = -\frac{bx^2}{2} + x\arctan(\tan(a+bx))$$

```
[In] Integrate[ArcTan[Tan[a + b*x]],x]
[Out] -1/2*(b*x^2) + x*ArcTan[Tan[a + b*x]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))^2}{2b}$
default	$\frac{\arctan(\tan(bx+a))^2}{2b}$
parallelrisch	$-\frac{x^2 b}{2} + x\arctan\left(\tan\left(bx+a\right)\right)$
parts	$-\frac{x^2 b}{2} + x\arctan\left(\tan\left(bx+a\right)\right)$
risch	$-ix\ln\left(\mathrm{e}^{i(bx+a)}\right) - \frac{\pi x\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)}{4} + \frac{\pi x\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}}{4}$

```
[In] int(arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \arctan(\tan(a+bx))\,dx = \frac{1}{2}bx^2 + ax$$

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \arctan(\tan(a+bx))\,dx = \begin{cases} \frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for } b \neq 0 \\ x\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(atan(tan(b*x+a)),x)
[Out] Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b), N
e(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \arctan(\tan(a+bx))\,dx = \frac{(bx+a)^2}{2\,b}$$

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/2*(b*x + a)^2/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \arctan(\tan(a+bx))\,dx = \frac{1}{2}\,bx^2 - \pi x \left\lfloor \frac{bx+a}{\pi} + \frac{1}{2} \right\rfloor + ax$$

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="giac")
[Out] 1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \arctan(\tan(a+bx))\,dx = x\,\mathrm{atan}(\tan(a+b\,x)) - \frac{b\,x^2}{2}$$

```
[In] int(atan(tan(a + b*x)),x)
[Out] x*atan(tan(a + b*x)) - (b*x^2)/2
```

3.41 $\int \frac{\arctan(\tan(a+bx))}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\arctan(\tan(a+bx))}{x} dx = bx - (bx - \arctan(\tan(a+bx))) \log(x)$$

```
[Out] b*x-(b*x-arctan(tan(b*x+a)))*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2189, 29}

$$\int \frac{\arctan(\tan(a+bx))}{x} dx = bx - \log(x)(bx - \arctan(\tan(a+bx)))$$

```
[In] Int[ArcTan[Tan[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcTan[Tan[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= bx - (bx - \arctan(\tan(a+bx))) \int \frac{1}{x}\,dx \\ &= bx - (bx - \arctan(\tan(a+bx)))\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx = bx + (-bx + \arctan(\tan(a+bx)))\log(x)$$

```
[In] Integrate[ArcTan[Tan[a + b*x]]/x,x]
[Out] b*x + (-(b*x) + ArcTan[Tan[a + b*x]])*Log[x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result
default	$\ln(x)\arctan(\tan(bx+a)) - b(\ln(x)x - x)$
parts	$\ln(x)\arctan(\tan(bx+a)) - b(\ln(x)x - x)$
risch	$-i\ln(x)\ln\left(e^{i(bx+a)}\right) - \ln(x)bx + bx - \frac{\pi\left(\operatorname{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\operatorname{csgn}\left(ie^{2i(bx+a)}\right)\operatorname{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right)-\operatorname{csgn}\left(\frac{i}{e^{2i(bx+a)}\right.\right.}{}$

```
[In] int(arctan(tan(b*x+a))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*arctan(tan(b*x+a))-b*(ln(x)*x-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx = bx + a\log(x)$$

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="fricas")
[Out] b*x + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx$$
$$= -bx\log(x) + bx + \left(\operatorname{atan}(\tan(a+bx)) + \pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)\log(x)$$

```
[In] integrate(atan(tan(b*x+a))/x,x)
[Out] -b*x*log(x) + b*x + (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))*lo
g(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx$$
$$= \frac{b\arctan(\tan(bx+a))\log(bx) + (bx-(bx+a)\log(bx) + a\log(bx) + a)b}{b}$$

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="maxima")
[Out] (b*arctan(tan(b*x + a))*log(b*x) + (b*x - (b*x + a)*log(b*x) + a*log(b*x) +
 a)*b)/b
```

Giac [F]

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx = \int \frac{\arctan(\tan(bx+a))}{x}\,dx$$

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(\tan(a+b\,x))}{x}\,dx$$

```
[In] int(atan(tan(a + b*x))/x,x)
[Out] int(atan(tan(a + b*x))/x, x)
```

3.42 $\int x^m \arctan(\cot(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int x^m \arctan(\cot(a+bx))\,dx = \frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m}\arctan(\cot(a+bx))}{1+m}$$

```
[Out] b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*(1/2*Pi-arccot(cot(b*x+a)))/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2199, 30}

$$\int x^m \arctan(\cot(a+bx))\,dx = \frac{x^{m+1}\arctan(\cot(a+bx))}{m+1} + \frac{bx^{m+2}}{m^2+3m+2}$$

```
[In] Int[x^m*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^(2 + m))/(2 + 3*m + m^2) + (x^(1 + m)*ArcTan[Cot[a + b*x]])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{1+m}\arctan(\cot(a+bx))}{1+m}+\frac{b\int x^{1+m}\,dx}{1+m}\\ &= \frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\arctan(\cot(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int x^m\arctan(\cot(a+bx))\,dx=\frac{x^{1+m}(bx+(2+m)\arctan(\cot(a+bx)))}{(1+m)(2+m)}$$

```
[In] Integrate[x^m*ArcTan[Cot[a + b*x]],x]
[Out] (x^(1 + m)*(b*x + (2 + m)*ArcTan[Cot[a + b*x]]))/((1 + m)*(2 + m))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

method	result
default	$\frac{\pi x^{1+m}}{2+2m}-\frac{b\,x^2 \mathrm{e}^{m\ln(x)}}{2+m}-\frac{(\operatorname{arccot}(\cot(bx+a))-bx)x\,\mathrm{e}^{m\ln(x)}}{1+m}$
parts	$\frac{\pi x^{1+m}}{2+2m}-\frac{b\,x^2 \mathrm{e}^{m\ln(x)}}{2+m}-\frac{(\operatorname{arccot}(\cot(bx+a))-bx)x\,\mathrm{e}^{m\ln(x)}}{1+m}$
parallelrisch	$\frac{2\pi x\,x^m-4\operatorname{arccot}(\cot(bx+a))x^m x+2b\,x^m x^2+\pi x\,x^m m-2\operatorname{arccot}(\cot(bx+a))x^m x m}{2(1+m)(2+m)}$
risch	$\frac{ix\,x^m\ln\left(\mathrm{e}^{i(bx+a)}\right)}{1+m}+\frac{x\left(4\pi+2\pi m\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}-1}\right)^3+\pi m\operatorname{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}-1}\right)^3+2m\pi-2\pi\operatorname{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2i(bx}}{\mathrm{e}^{2i(bx+}}\right.\right.}{}$

```
[In] int(x^m*(1/2*Pi-arccot(cot(b*x+a))),x,method=_RETURNVERBOSE)
[Out] 1/2*Pi*x^(1+m)/(1+m)-b/(2+m)*x^2*exp(m*ln(x))-(arccot(cot(b*x+a))-b*x)/(1+m
)*x*exp(m*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int x^m \arctan(\cot(a+bx))\,dx = -\frac{(2\,(bm+b)x^2-(\pi(m+2)-2\,am-4\,a)x)x^m}{2\,(m^2+3\,m+2)}$$

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
[Out] -1/2*(2*(b*m + b)*x^2 - (pi*(m + 2) - 2*a*m - 4*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(34) = 68.

Time = 1.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.44

$$\int x^m \arctan(\cot(a+bx))\,dx = \begin{cases} -b\log(x)+\frac{\operatorname{acot}(\cot(a+bx))}{x}-\frac{\pi}{2x} & \text{for}\ m=-2 \\ bx\log(x)-bx-\log(x)\operatorname{acot}(\cot(a+bx))+\frac{\pi\log(x)}{2} & \text{for}\ m=-1 \\ \frac{2bx^2x^m}{2m^2+6m+4}-\frac{2mxx^m\operatorname{acot}(\cot(a+bx))}{2m^2+6m+4}+\frac{\pi mxx^m}{2m^2+6m+4}-\frac{4xx^m\operatorname{acot}(\cot(a+bx))}{2m^2+6m+4}+\frac{2\pi xx^m}{2m^2+6m+4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**m*(1/2*pi-acot(cot(b*x+a))),x)
[Out] Piecewise((-b*log(x) + acot(cot(a + b*x))/x - pi/(2*x), Eq(m, -2)), (b*x*lo
g(x) - b*x - log(x)*acot(cot(a + b*x)) + pi*log(x)/2, Eq(m, -1)), (2*b*x**2
*x**m/(2*m**2 + 6*m + 4) - 2*m*x*x**m*acot(cot(a + b*x))/(2*m**2 + 6*m + 4)
 + pi*m*x*x**m/(2*m**2 + 6*m + 4) - 4*x*x**m*acot(cot(a + b*x))/(2*m**2 + 6
*m + 4) + 2*pi*x*x**m/(2*m**2 + 6*m + 4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x^m \arctan(\cot(a+bx))\,dx = -\frac{bx^{m+2}}{m+2}+\frac{\pi x^{m+1}}{2\,(m+1)}-\frac{ax^{m+1}}{m+1}$$

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
[Out] -b*x^(m + 2)/(m + 2) + 1/2*pi*x^(m + 1)/(m + 1) - a*x^(m + 1)/(m + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.72

$$\int x^m \arctan(\cot(a+bx))\,dx = -\frac{2\,bmx^2x^m - \pi mxx^m + 2\,amxx^m + 2\,bx^2x^m - 2\,\pi xx^m + 4\,axx^m}{2\,(m^2+3\,m+2)}$$

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
[Out] -1/2*(2*b*m*x^2*x^m - pi*m*x*x^m + 2*a*m*x*x^m + 2*b*x^2*x^m - 2*pi*x*x^m +
 4*a*x*x^m)/(m^2 + 3*m + 2)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \arctan(\cot(a+bx))\,dx = \int x^m \left(\frac{\Pi}{2} - \mathrm{acot}(\cot(a+b\,x))\right)\,dx$$

```
[In] int(x^m*(Pi/2 - acot(cot(a + b*x))),x)
[Out] int(x^m*(Pi/2 - acot(cot(a + b*x))), x)
```

3.43 $\int x^2 \arctan(\cot(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2 \arctan(\cot(a+bx))\,dx = \frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(\cot(a+bx))$$

```
[Out] 1/12*b*x^4+1/3*x^3*(1/2*Pi-arccot(cot(b*x+a)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2199, 30}

$$\int x^2 \arctan(\cot(a+bx))\,dx = \frac{1}{3}x^3 \arctan(\cot(a+bx)) + \frac{bx^4}{12}$$

```
[In] Int[x^2*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTan[Cot[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^3\arctan(\cot(a+bx))+\frac{1}{3}b\int x^3\,dx\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\arctan(\cot(a+bx))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2\arctan(\cot(a+bx))\,dx=\frac{1}{12}x^3(bx+4\arctan(\cot(a+bx)))$$

```
[In] Integrate[x^2*ArcTan[Cot[a + b*x]],x]
[Out] (x^3*(b*x + 4*ArcTan[Cot[a + b*x]]))/12
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result
parallelrisch	$-\frac{x^3\operatorname{arccot}(\cot(bx+a))}{3}+\frac{\pi x^3}{6}+\frac{b x^4}{12}$
default	$\frac{\pi x^3}{6}-\frac{x^3\operatorname{arccot}(\cot(bx+a))}{3}-\frac{-\frac{(bx+a)^4}{4}+a(bx+a)^3-\frac{3a^2(bx+a)^2}{2}+a^3(bx+a)}{3b^3}$
parts	$\frac{\pi x^3}{6}-\frac{x^3\operatorname{arccot}(\cot(bx+a))}{3}-\frac{-\frac{(bx+a)^4}{4}+a(bx+a)^3-\frac{3a^2(bx+a)^2}{2}+a^3(bx+a)}{3b^3}$
risch	$\frac{ix^3\ln\left(e^{i(bx+a)}\right)}{3}+\frac{\pi x^3\operatorname{csgn}\left(ie^{i(bx+a)}\right)^2\operatorname{csgn}\left(ie^{2i(bx+a)}\right)}{12}-\frac{\pi x^3\operatorname{csgn}\left(ie^{i(bx+a)}\right)\operatorname{csgn}\left(ie^{2i(bx+a)}\right)^2}{6}+\frac{\pi x^3\operatorname{csgn}\left(ie^{2i(bx+a)}\right.}{12}$

```
[In] int(x^2*(1/2*Pi-arccot(cot(b*x+a))),x,method=_RETURNVERBOSE)
[Out] -1/3*x^3*arccot(cot(b*x+a))+1/6*Pi*x^3+1/12*b*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x^2 \arctan(\cot(a+bx))\,dx = -\frac{1}{4}\,bx^4 + \frac{1}{6}\,(\pi - 2\,a)x^3$$

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
[Out] -1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x^2 \arctan(\cot(a+bx))\,dx = \frac{bx^4}{12} - \frac{x^3 \operatorname{acot}\left(\cot\left(a+bx\right)\right)}{3} + \frac{\pi x^3}{6}$$

```
[In] integrate(x**2*(1/2*pi-acot(cot(b*x+a))),x)
[Out] b*x**4/12 - x**3*acot(cot(a + b*x))/3 + pi*x**3/6
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x^2 \arctan(\cot(a+bx))\,dx = -\frac{1}{4}\,bx^4 + \frac{1}{6}\,(\pi - 2\,a)x^3$$

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
[Out] -1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \arctan(\cot(a+bx))\,dx = -\frac{1}{4}\,bx^4 + \frac{1}{6}\,\pi x^3 - \frac{1}{3}\,ax^3$$

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
[Out] -1/4*b*x^4 + 1/6*pi*x^3 - 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2 \arctan(\cot(a+bx))\,dx = \frac{\Pi\,x^3}{6} + \frac{b\,x^4}{12} - \frac{x^3\,\mathrm{acot}(\cot(a+b\,x))}{3}$$

```
[In] int(x^2*(Pi/2 - acot(cot(a + b*x))),x)
[Out] (Pi*x^3)/6 + (b*x^4)/12 - (x^3*acot(cot(a + b*x)))/3
```

3.44 $\int x \arctan(\cot(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \arctan(\cot(a+bx))\,dx = \frac{bx^3}{6} + \frac{1}{2}x^2 \arctan(\cot(a+bx))$$

```
[Out] 1/6*b*x^3+1/2*x^2*(1/2*Pi-arccot(cot(b*x+a)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5281, 30}

$$\int x \arctan(\cot(a+bx))\,dx = \frac{1}{2}x^2 \arctan(\cot(a+bx)) + \frac{bx^3}{6}$$

```
[In] Int[x*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[Cot[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
```

```
Q[(c - I*d)^2, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2 \arctan(\cot(a+bx)) + \frac{1}{2}b\int x^2\,dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2 \arctan(\cot(a+bx))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x \arctan(\cot(a+bx))\,dx = \frac{1}{6}x^2(bx + 3\arctan(\cot(a+bx)))$$

```
[In] Integrate[x*ArcTan[Cot[a + b*x]],x]
[Out] (x^2*(b*x + 3*ArcTan[Cot[a + b*x]]))/6
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result
parallelrisch	$\frac{b\,x^3}{6} - \frac{x^2 \operatorname{arccot}(\cot(bx+a))}{2} + \frac{\pi\,x^2}{4}$
default	$\frac{\pi\,x^2}{4} - \frac{x^2 \operatorname{arccot}(\cot(bx+a))}{2} - \frac{-\frac{(bx+a)^3}{3} + (bx+a)^2 a - a^2(bx+a)}{2b^2}$
parts	$\frac{\pi\,x^2}{4} - \frac{x^2 \operatorname{arccot}(\cot(bx+a))}{2} - \frac{-\frac{(bx+a)^3}{3} + (bx+a)^2 a - a^2(bx+a)}{2b^2}$
risch	$\frac{ix^2 \ln\left(e^{i(bx+a)}\right)}{2} + \frac{\pi\,x^2 \operatorname{csgn}\left(ie^{i(bx+a)}\right)^2 \operatorname{csgn}\left(ie^{2i(bx+a)}\right)}{8} - \frac{\pi\,x^2 \operatorname{csgn}\left(ie^{i(bx+a)}\right) \operatorname{csgn}\left(ie^{2i(bx+a)}\right)^2}{4} + \frac{\pi\,x^2 \operatorname{csgn}\left(ie^{2i(bx+a)}\right.}{8}$

```
[In] int(x*(1/2*Pi-arccot(cot(b*x+a))),x,method=_RETURNVERBOSE)
[Out] 1/6*b*x^3-1/2*x^2*arccot(cot(b*x+a))+1/4*Pi*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\arctan(\cot(a+bx))\,dx = -\frac{1}{3}\,bx^3 + \frac{1}{4}\,(\pi - 2\,a)x^2$$

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
[Out] -1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(22) = 44$.

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int x\arctan(\cot(a+bx))\,dx = \begin{cases} \frac{\pi x^2}{4} - \frac{x\operatorname{acot}^2(\cot(a+bx))}{2b} + \frac{\operatorname{acot}^3(\cot(a+bx))}{6b^2} & \text{for } b \neq 0 \\ \frac{x^2\left(-\operatorname{acot}(\cot(a)) + \frac{\pi}{2}\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(1/2*pi-acot(cot(b*x+a))),x)
[Out] Piecewise((pi*x**2/4 - x*acot(cot(a + b*x))**2/(2*b) + acot(cot(a + b*x))**
3/(6*b**2), Ne(b, 0)), (x**2*(-acot(cot(a)) + pi/2)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\arctan(\cot(a+bx))\,dx = -\frac{1}{3}\,bx^3 + \frac{1}{4}\,(\pi - 2\,a)x^2$$

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
[Out] -1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\arctan(\cot(a+bx))\,dx=-\frac{1}{3}\,bx^3+\frac{1}{4}\,\pi x^2-\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
[Out] -1/3*b*x^3 + 1/4*pi*x^2 - 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\arctan(\cot(a+bx))\,dx=\frac{\Pi\,x^2}{4}+\frac{b\,x^3}{6}-\frac{x^2\,\mathrm{acot}(\cot(a+b\,x))}{2}$$

```
[In] int(x*(Pi/2 - acot(cot(a + b*x))),x)
[Out] (Pi*x^2)/4 + (b*x^3)/6 - (x^2*acot(cot(a + b*x)))/2
```

3.45 $\int \arctan(\cot(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \arctan(\cot(a+bx))\,dx = -\frac{\arctan(\cot(a+bx))^2}{2b}$$

```
[Out] -1/2*(1/2*Pi-arccot(cot(b*x+a)))^2/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2188, 30}

$$\int \arctan(\cot(a+bx))\,dx = -\frac{\arctan(\cot(a+bx))^2}{2b}$$

```
[In] Int[ArcTan[Cot[a + b*x]],x]
```

```
[Out] -1/2*ArcTan[Cot[a + b*x]]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}(\int x\,dx, x, \arctan(\cot(a+bx)))}{b}$$
$$= -\frac{\arctan(\cot(a+bx))^2}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \arctan(\cot(a+bx))\,dx = \frac{bx^2}{2} + x\arctan(\cot(a+bx))$$

```
[In] Integrate[ArcTan[Cot[a + b*x]],x]
[Out] (b*x^2)/2 + x*ArcTan[Cot[a + b*x]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result
parallelrisch	$\frac{x^2 b}{2} - x\ \text{arccot}\left(\cot\left(bx+a\right)\right) + \frac{\pi x}{2}$
derivativedivides	$\frac{-\pi\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)-\text{arccot}(\cot(bx+a))^2}{2b}$
default	$\frac{\pi x}{2} - \frac{-\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\text{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
parts	$\frac{\pi x}{2} - \frac{-\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\text{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
risch	$\frac{\pi x}{2} + ix\ln\left(e^{i(bx+a)}\right) + \frac{\pi x\,\text{csgn}\left(ie^{i(bx+a)}\right)^2\text{csgn}\left(ie^{2i(bx+a)}\right)}{4} - \frac{\pi x\ \text{csgn}\left(ie^{i(bx+a)}\right)\text{csgn}\left(ie^{2i(bx+a)}\right)^2}{2} + \frac{\pi x\,\text{csg}}{}$

```
[In] int(1/2*Pi-arccot(cot(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*b-x*arccot(cot(b*x+a))+1/2*Pi*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,(\pi - 2\,a)x$$

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")
[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \arctan(\cot(a+bx))\,dx = \frac{\pi x}{2} - \begin{cases} \frac{\operatorname{acot}^2(\cot(a+bx))}{2b} & \text{for } b \neq 0 \\ x \operatorname{acot}(\cot(a)) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/2*pi-acot(cot(b*x+a)),x)
[Out] pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a))
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \arctan(\cot(a+bx))\,dx = \frac{\Pi\,x}{2} - x\,\mathrm{acot}(\cot(a+b\,x)) + \frac{b\,x^2}{2}$$

```
[In] int(Pi/2 - acot(cot(a + b*x)),x)
[Out] (Pi*x)/2 - x*acot(cot(a + b*x)) + (b*x^2)/2
```

3.46 $\int \frac{\arctan(\cot(a+bx))}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\arctan(\cot(a+bx))}{x} dx = -bx + (bx + \arctan(\cot(a+bx))) \log(x)$$

```
[Out] -b*x+(b*x+1/2*Pi-arccot(cot(b*x+a)))*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2189, 29}

$$\int \frac{\arctan(\cot(a+bx))}{x} dx = \log(x)(\arctan(\cot(a+bx)) + bx) - bx$$

```
[In] Int[ArcTan[Cot[a + b*x]]/x,x]
```

```
[Out] -(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, 
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -bx - (-bx - \arctan(\cot(a+bx)))\int \frac{1}{x}\,dx \\ &= -bx + (bx + \arctan(\cot(a+bx)))\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = -bx + (bx + \arctan(\cot(a+bx)))\log(x)$$

```
[In] Integrate[ArcTan[Cot[a + b*x]]/x,x]
[Out] -(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

method	result
default	$\frac{\pi\ln(x)}{2} - bx - (\operatorname{arccot}(\cot(bx+a)) - bx)\ln(x)$
parts	$\frac{\pi\ln(x)}{2} - bx - (\operatorname{arccot}(\cot(bx+a)) - bx)\ln(x)$
risch	$i\ln(x)\ln\left(e^{i(bx+a)}\right) + \ln(x)\,bx - bx + \frac{\pi\left(\operatorname{csgn}\left(ie^{i(bx+a)}\right)^2 \operatorname{csgn}\left(ie^{2i(bx+a)}\right) - 2\operatorname{csgn}\left(ie^{i(bx+a)}\right)\operatorname{csgn}\left(ie^{2i(bx+a)}\right)^2 + \operatorname{csgn}\right.}{}$

```
[In] int((1/2*Pi-arccot(cot(b*x+a)))/x,x,method=_RETURNVERBOSE)
[Out] 1/2*Pi*ln(x)-b*x-(arccot(cot(b*x+a))-b*x)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = -bx + \frac{1}{2}(\pi - 2a)\log(x)$$

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="fricas")
[Out] -b*x + 1/2*(pi - 2*a)*log(x)
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = bx\log(x) - bx - \log(x)\operatorname{acot}(\cot(a+bx)) + \frac{\pi\log(x)}{2}$$

```
[In] integrate((1/2*pi-acot(cot(b*x+a)))/x,x)
[Out] b*x*log(x) - b*x - log(x)*acot(cot(a + b*x)) + pi*log(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = -bx + \frac{1}{2}(\pi - 2a)\log(x)$$

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="maxima")
[Out] -b*x + 1/2*(pi - 2*a)*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = -bx + \frac{1}{2}(\pi - 2a)\log(|x|)$$

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="giac")
[Out] -b*x + 1/2*(pi - 2*a)*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = \int \frac{\frac{\Pi}{2} - \operatorname{acot}(\cot(a+b\,x))}{x}\,dx$$

```
[In] int((Pi/2 - acot(cot(a + b*x)))/x,x)
[Out] int((Pi/2 - acot(cot(a + b*x)))/x, x)
```

3.47 $\int \arctan(\tan(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \arctan(\tan(a+bx))\,dx = \frac{\arctan(\tan(a+bx))^2}{2b}$$

```
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2188, 30}

$$\int \arctan(\tan(a+bx))\,dx = \frac{\arctan(\tan(a+bx))^2}{2b}$$

```
[In] Int[ArcTan[Tan[a + b*x]],x]
```

```
[Out] ArcTan[Tan[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int x\,dx, x, \arctan(\tan(a+bx)))}{b}$$
$$= \frac{\arctan(\tan(a+bx))^2}{2b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \arctan(\tan(a+bx))\,dx = -\frac{bx^2}{2} + x\arctan(\tan(a+bx))$$

```
[In] Integrate[ArcTan[Tan[a + b*x]],x]
[Out] -1/2*(b*x^2) + x*ArcTan[Tan[a + b*x]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))^2}{2b}$
default	$\frac{\arctan(\tan(bx+a))^2}{2b}$
parallelrisch	$-\frac{x^2 b}{2} + x\arctan\left(\tan\left(bx+a\right)\right)$
parts	$-\frac{x^2 b}{2} + x\arctan\left(\tan\left(bx+a\right)\right)$
risch	$-ix\ln\left(\mathrm{e}^{i(bx+a)}\right) - \frac{\pi x\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)}{4} + \frac{\pi x\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{cs}}{4}$

```
[In] int(arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \arctan(\tan(a+bx))\,dx = \frac{1}{2}\,bx^2 + ax$$

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \arctan(\tan(a+bx))\,dx = \begin{cases} \frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for } b\neq 0 \\ x\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(atan(tan(b*x+a)),x)
[Out] Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b), N
e(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \arctan(\tan(a+bx))\,dx = \frac{(bx+a)^2}{2\,b}$$

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/2*(b*x + a)^2/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \arctan(\tan(a+bx))\,dx = \frac{1}{2}\,bx^2 - \pi x\left\lfloor \frac{bx+a}{\pi} + \frac{1}{2}\right\rfloor + ax$$

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="giac")
[Out] 1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \arctan(\tan(a+bx))\,dx = x\,\mathrm{atan}(\tan(a+b\,x)) - \frac{b\,x^2}{2}$$

```
[In] int(atan(tan(a + b*x)),x)
[Out] x*atan(tan(a + b*x)) - (b*x^2)/2
```

3.48 $\int x^2 \arctan(c+d\tan(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 403

$$\begin{aligned}\int x^2\arctan(c+d\tan(a+bx))\,dx &= \frac{1}{3}x^3\arctan(c+d\tan(a+bx))\\ &+\frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)\\ &-\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)\\ &+\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b}\\ &-\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}\\ &+\frac{ix\operatorname{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2}\\ &-\frac{ix\operatorname{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2}\\ &-\frac{\operatorname{PolyLog}\left(4,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^3}\\ &+\frac{\operatorname{PolyLog}\left(4,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^3}\end{aligned}$$

```
[Out] 1/3*x^3*arctan(c+d*tan(b*x+a))+1/6*I*x^3*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))-1/6*I*x^3*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*x
^2*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*x^2*polylog(2,-
(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b+1/4*I*x*polylog(3,-(1+I*c+d)*
exp(2*I*a+2*I*b*x)/(1+I*c-d))/b^2-1/4*I*x*polylog(3,-(c+I*(1-d))*exp(2*I*a+
```

```
2*I*b*x)/(c+I*(1+d)))/b^2-1/8*polylog(4,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*
c-d))/b^3+1/8*polylog(4,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5283, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(c+d\tan(a+bx))\,dx = \frac{1}{3}x^3\arctan(d\tan(a+bx)+c) - \frac{\text{PolyLog}\left(4,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^3} + \frac{\text{PolyLog}\left(4,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^3} + \frac{ix\,\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2} - \frac{ix\,\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2} + \frac{x^2\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b} - \frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b} + \frac{1}{6}ix^3\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right) - \frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)$$

```
[In] Int[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/6)*x^3*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x^2*PolyLog[2, -(((1 + I*c + d)*E
^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x^2*PolyLog[2, -(((c + I*
(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/4)*x*PolyL
og[3, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/
4)*x*PolyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d))
)])/b^2 - PolyLog[4, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d
))]/(8*b^3) + PolyLog[4, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I
*(1 + d)))]/(8*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5283

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (Dist[b*((1 - I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2
*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
- Dist[b*((1 + I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*I
*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c+d\tan(a+bx)) \\
&\quad+\frac{1}{3}(b(1-ic-d))\int\frac{e^{2ia+2ibx}x^3}{1-ic+d+(1-ic-d)e^{2ia+2ibx}}\,dx \\
&\quad-\frac{1}{3}(b(1+ic+d))\int\frac{e^{2ia+2ibx}x^3}{1+ic-d+(1+ic+d)e^{2ia+2ibx}}\,dx \\
&= \frac{1}{3}x^3\arctan(c+d\tan(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad-\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad+\frac{1}{2}i\int x^2\log\left(1+\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx \\
&\quad-\frac{1}{2}i\int x^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx \\
&= \frac{1}{3}x^3\arctan(c+d\tan(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad-\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad+\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} \\
&\quad+\frac{\int x\,\mathrm{PolyLog}\left(2,-\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx}{2b}-\frac{\int x\,\mathrm{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx}{2b} \\
&= \frac{1}{3}x^3\arctan(c+d\tan(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad-\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad+\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} \\
&\quad+\frac{ix\,\mathrm{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2}-\frac{ix\,\mathrm{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2} \\
&\quad+\frac{i\int\mathrm{PolyLog}\left(3,-\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx}{4b^2}-\frac{i\int\mathrm{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx}{4b^2}
\end{aligned}
$$

$$= \frac{1}{3}x^3 \arctan(c + d\tan(a + bx)) + \frac{1}{6}ix^3 \log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) - \frac{1}{6}ix^3 \log\left(1 + \frac{(c + i(1 - d))e^{2ia+2ibx}}{c + i(1 + d)}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} - \frac{x^2 \operatorname{PolyLog}\left(2, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2} - \frac{ix \operatorname{PolyLog}\left(3, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2} - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, -\frac{(1+ic+d)x}{1+ic-d}\right)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^3} + \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, -\frac{(c-i(-1+d))x}{c+i(1+d)}\right)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^3}$$

$$= \frac{1}{3}x^3 \arctan(c + d\tan(a + bx)) + \frac{1}{6}ix^3 \log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) - \frac{1}{6}ix^3 \log\left(1 + \frac{(c + i(1 - d))e^{2ia+2ibx}}{c + i(1 + d)}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} - \frac{x^2 \operatorname{PolyLog}\left(2, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2} - \frac{ix \operatorname{PolyLog}\left(3, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^3} + \frac{\operatorname{PolyLog}\left(4, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.92

$$\int x^2 \arctan(c + d\tan(a + bx))\, dx$$

$$= \frac{8b^3x^3 \arctan(c + d\tan(a + bx)) + 4ib^3x^3 \log\left(1 + \frac{(c+i(-1+d))e^{-2i(a+bx)}}{c-i(1+d)}\right) - 4ib^3x^3 \log\left(1 + \frac{(c+i(1+d))e^{-2i(a+bx)}}{i+c-id}\right)}{}$$

```
[In] Integrate[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcTan[c + d*Tan[a + b*x]] + (4*I)*b^3*x^3*Log[1 + (c + I*(-1 +
d))/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - (4*I)*b^3*x^3*Log[1 + (c + I*(
1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, (-c - I
*(1 + d))/((c - I*(-1 + d))*E^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (I
```

```
 - c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - (6*I)*b*x*PolyLog[3, (
-c - I*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] + (6*I)*b*x*PolyLog[3,
 (I - c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - 3*PolyLog[4, (-c -
I*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (I - c - I*d
)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.78 (sec) , antiderivative size = 8039, normalized size of antiderivative = 19.95

method	result	size
risch	Expression too large to display	8039

```
[In] int(x^2*arctan(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1965 vs. 2(290) = 580.

Time = 0.35 (sec) , antiderivative size = 1965, normalized size of antiderivative = 4.88

$$\int x^2 \arctan(c + d\tan(a + bx))\, dx = \text{Too large to display}$$

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arctan(d*tan(b*x + a) + c) + 6*b^2*x^2*dilog(2*((I*c*d - d
^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x
+ a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)
+ 1) - 6*b^2*x^2*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
(I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*ta
n(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^2*dilog(2*((-I*c*d - d^2
 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x +
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
 1) - 6*b^2*x^2*dilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d +
(-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 4*I*a^3*log(((I*c*d + d^2 + d)*
tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d
 - 1)/(tan(b*x + a)^2 + 1)) - 4*I*a^3*log(((I*c*d + d^2 - d)*tan(b*x + a)^2
 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x
 + a)^2 + 1)) + 4*I*a^3*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d
```

```
 + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1))
- 4*I*a^3*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*
d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) + 6*I*b*x*poly
log(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*
(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b
*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^
2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)
*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d
+ 1)) - 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2
- 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 +
d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + 6*I*b*x*polylog(3,
((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2
+ 2*c*d - I*d^2 + I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 - 2*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log(-2*((I*c*d - d^2 + d)
*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) +
d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) - 4*(-
I*b^3*x^3 - I*a^3)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
 (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log(-2*((-I*
c*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*
tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2
*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2
- c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 +
d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2
+ 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c
*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)
^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*tan(b*x + a) - 1)/
((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 3*polylog(4
, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*(-I*c
^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x +
a)^2 + c^2 + d^2 - 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(
b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*tan(b*x +
a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\tan(a + bx))\, dx = \text{Timed out}$$

```
[In] integrate(x**2*atan(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2 \arctan(c + d\tan(a + bx)) \, dx = \int x^2 \arctan\left(d \tan\left(bx + a\right) + c\right) \, dx$$

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*x^3*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)*
cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/6*x^3*arctan2(c*cos(2*b*
x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*sin(
2*b*x + 2*a) + d + 1) + 4*b*d*integrate(-1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2*b
*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*x
+ 2*a)^2 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*
a) - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*co
s(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c
^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(
4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*
a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^
4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4
+ d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2
*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c
^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*
d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2
*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x^2 \arctan(c + d\tan(a + bx)) \, dx = \int x^2 \arctan\left(d \tan\left(bx + a\right) + c\right) \, dx$$

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\tan(a + bx)) \, dx = \int x^2 \operatorname{atan}(c + d\tan(a + b\,x)) \, dx$$

```
[In] int(x^2*atan(c + d*tan(a + b*x)),x)
```

```
[Out] int(x^2*atan(c + d*tan(a + b*x)), x)
```

3.49 $\int x \arctan(c + d\tan(a + bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 305

$$\begin{aligned}\int x\arctan(c+d\tan(a+bx))\,dx =& \frac{1}{2}x^2\arctan(c+d\tan(a+bx)) \\ &+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\ &-\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\ &+\frac{x\,\mathrm{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} \\ &-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} \\ &+\frac{i\,\mathrm{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^2} \\ &-\frac{i\,\mathrm{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^2}\end{aligned}$$

```
[Out] 1/2*x^2*arctan(c+d*tan(b*x+a))+1/4*I*x^2*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))-1/4*I*x^2*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*x
*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*x*polylog(2,-(c+I
*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b+1/8*I*polylog(3,-(1+I*c+d)*exp(2*
I*a+2*I*b*x)/(1+I*c-d))/b^2-1/8*I*polylog(3,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)
/(c+I*(1+d)))/b^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5283, 2221, 2611, 2320, 6724}

$$\int x\arctan(c+d\tan(a+bx))\,dx=\frac{1}{2}x^2\arctan(d\tan(a+bx)+c)+\frac{i\operatorname{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^2}-\frac{i\operatorname{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^2}+\frac{x\operatorname{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)-\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)$$

```
[In] Int[x*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/4)*x^2*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x*PolyLog[2, -(((1 + I*c + d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x*PolyLog[2, -(((c + I*(1 -
 d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/8)*PolyLog[3,
-(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/8)*Pol
yLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5283

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (Dist[b*((1 - I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2
*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
- Dist[b*((1 + I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*I
*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c+d\tan(a+bx)) \\
&\quad+\frac{1}{2}(b(1-ic-d))\int\frac{e^{2ia+2ibx}x^2}{1-ic+d+(1-ic-d)e^{2ia+2ibx}}\,dx \\
&\quad-\frac{1}{2}(b(1+ic+d))\int\frac{e^{2ia+2ibx}x^2}{1+ic-d+(1+ic+d)e^{2ia+2ibx}}\,dx \\
&= \frac{1}{2}x^2\arctan(c+d\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad-\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad+\frac{1}{2}i\int x\log\left(1+\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx \\
&\quad-\frac{1}{2}i\int x\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx
\end{aligned}$$

$$= \frac{1}{2}x^2 \arctan(c + d\tan(a + bx)) + \frac{1}{4}ix^2 \log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) - \frac{1}{4}ix^2 \log\left(1 + \frac{(c + i(1 - d))e^{2ia+2ibx}}{c + i(1 + d)}\right) + \frac{x \operatorname{PolyLog}\left(2, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} - \frac{x \operatorname{PolyLog}\left(2, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} + \frac{\int \operatorname{PolyLog}\left(2, -\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right) dx}{4b} - \frac{\int \operatorname{PolyLog}\left(2, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) dx}{4b}$$

$$= \frac{1}{2}x^2 \arctan(c + d\tan(a + bx)) + \frac{1}{4}ix^2 \log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) - \frac{1}{4}ix^2 \log\left(1 + \frac{(c + i(1 - d))e^{2ia+2ibx}}{c + i(1 + d)}\right) + \frac{x \operatorname{PolyLog}\left(2, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} - \frac{x \operatorname{PolyLog}\left(2, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} + \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, -\frac{(1+ic+d)x}{1+ic-d}\right)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^2} - \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, -\frac{(c-i(-1+d))x}{c+i(1+d)}\right)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^2}$$

$$= \frac{1}{2}x^2 \arctan(c + d\tan(a + bx)) + \frac{1}{4}ix^2 \log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) - \frac{1}{4}ix^2 \log\left(1 + \frac{(c + i(1 - d))e^{2ia+2ibx}}{c + i(1 + d)}\right) + \frac{x \operatorname{PolyLog}\left(2, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} - \frac{x \operatorname{PolyLog}\left(2, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} + \frac{i \operatorname{PolyLog}\left(3, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^2} - \frac{i \operatorname{PolyLog}\left(3, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.92

$$\int x \arctan(c + d\tan(a + bx))\, dx$$

$$= \frac{4b^2x^2 \arctan(c + d\tan(a + bx)) + 2ib^2x^2 \log\left(1 + \frac{(c+i(-1+d))e^{-2i(a+bx)}}{c-i(1+d)}\right) - 2ib^2x^2 \log\left(1 + \frac{(c+i(1+d))e^{-2i(a+bx)}}{i+c-id}\right.}{}$$

```
[In] Integrate[x*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (4*b^2*x^2*ArcTan[c + d*Tan[a + b*x]] + (2*I)*b^2*x^2*Log[1 + (c + I*(-1 +
d))/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - (2*I)*b^2*x^2*Log[1 + (c + I*(
1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] + 2*b*x*PolyLog[2, (-c - I*(1
+ d))/((c - I*(-1 + d))*E^((2*I)*(a + b*x)))] - 2*b*x*PolyLog[2, (I - c - I
*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - I*PolyLog[3, (-c - I*(1 + d))/
((I + c - I*d)*E^((2*I)*(a + b*x)))] + I*PolyLog[3, (I - c - I*d)/((c - I*(
1 + d))*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.85 (sec) , antiderivative size = 7647, normalized size of antiderivative = 25.07

method	result	size
risch	Expression too large to display	7647

```
[In] int(x*arctan(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(217) = 434.

Time = 0.34 (sec) , antiderivative size = 1545, normalized size of antiderivative = 5.07

$$\int x\arctan(c+d\tan(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arctan(d*tan(b*x + a) + c) + 2*b*x*dilog(2*((I*c*d - d^2 +
d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a)
+ d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1)
- 2*b*x*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 -
2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a
)^2 + c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(2*((-I*c*d - d^2 + d)*tan(b*x
 + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/
((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) - 2*b*x*d
ilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d +
 I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*d + 1) + 1) - 2*I*a^2*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 -
c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x +
a)^2 + 1)) + 2*I*a^2*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d +
```

```
(I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) - 2
*I*a^2*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2
 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) + 2*I*a^2*log(((I
*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*
tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - 2*(I*b^2*x^2 - I*a^2)*log(-2*
((I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 +
I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
- 2*d + 1)) - 2*(-I*b^2*x^2 + I*a^2)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)
^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2
+ d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(-I*b^2*x^2 + I
*a^2)*log(-2*((-I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2
*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)
^2 + c^2 + d^2 - 2*d + 1)) - 2*(I*b^2*x^2 - I*a^2)*log(-2*((-I*c*d - d^2 -
d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a)
 - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + I
*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2
 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*
tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - I*polylog(3, ((c^2 - 2*I*c*d - d^2
 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*
tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d +
 1)) - I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c
*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2
*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + I*polylog(3, ((c^2 - 2*I*c
*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d
^2 + I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
 - 2*d + 1)))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int x\arctan(c+d\tan(a+bx))\,dx = \text{Timed out}$$

```
[In] integrate(x*atan(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\arctan(c+d\tan(a+bx))\,dx=\int x\arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)*
cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/4*x^2*arctan2(c*cos(2*b*
x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*sin(
2*b*x + 2*a) + d + 1) + 2*b*d*integrate(-(2*(c^2 + d^2 + 1)*x^2*cos(2*b*x +
 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x + 2*
a)^2 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) -
 (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*
b*x + 2*a) + (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 +
 d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*b*
x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)^2
 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 +
d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 + d^
4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a)
- 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 -
 d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*d^3
+ (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a))
*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x\arctan(c+d\tan(a+bx))\,dx=\int x\arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+d\tan(a+bx))\,dx=\int x\,\mathrm{atan}(c+d\tan(a+b\,x))\,dx$$

```
[In] int(x*atan(c + d*tan(a + b*x)),x)
```

```
[Out] int(x*atan(c + d*tan(a + b*x)), x)
```

3.50 $\int \arctan(c+d\tan(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 198

$$\int \arctan(c+d\tan(a+bx))\,dx = x\arctan(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) + \frac{\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} - \frac{\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}$$

```
[Out] x*arctan(c+d*tan(b*x+a))+1/2*I*x*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d
))-1/2*I*x*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*polylog(2,-
(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*polylog(2,-(c+I*(1-d))*exp(2*
I*a+2*I*b*x)/(c+I*(1+d)))/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{5275, 2221, 2317, 2438\}$

$$\int \arctan(c+d\tan(a+bx))\,dx = x\arctan(d\tan(a+bx)+c)+\frac{\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b} - \frac{\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b} + \frac{1}{2}ix\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)$$

```
[In] Int[ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tan[a + b*x]] + (I/2)*x*Log[1 + ((1 + I*c + d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c - d)] - (I/2)*x*Log[1 + ((c + I*(1 - d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 + d))] + PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*
I)*b*x))/(1 + I*c - d))]/(4*b) - PolyLog[2, -(((c + I*(1 - d))*E^((2*I)*a +
 (2*I)*b*x))/(c + I*(1 + d)))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5275

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] + (Dist[b*(1 - I*c - d), Int[x*(E^(2*I*a + 2*I*b
*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] - Dist[b*(1
+ I*c + d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*
a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rubi steps

$$\text{integral} = x\arctan(c+d\tan(a+bx)) + (b(1-ic-d))\int \frac{e^{2ia+2ibx}x}{1-ic+d+(1-ic-d)e^{2ia+2ibx}}\,dx - (b(1+ic+d))\int \frac{e^{2ia+2ibx}x}{1+ic-d+(1+ic+d)e^{2ia+2ibx}}\,dx$$

$$= x\arctan(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) + \frac{1}{2}i\int \log\left(1+\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx - \frac{1}{2}i\int \log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx$$

$$= x\arctan(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{(1-ic-d)x}{1-ic+d}\right)}{x}\,dx, x, e^{2ia+2ibx}\right)}{4b} - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{(1+ic+d)x}{1+ic-d}\right)}{x}\,dx, x, e^{2ia+2ibx}\right)}{4b}$$

$$= x\arctan(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) + \frac{\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} - \frac{\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 549 vs. 2(198) = 396.

Time = 5.61 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.77

$$\int \arctan(c+d\tan(a+bx))\,dx = x\arctan(c+d\tan(a+bx)) - \frac{x\left(4a\sqrt{-d^2}\arctan(c+d\tan(a+bx)) - id\log(1-i\tan(a+bx))\log\left(\frac{-cd+\sqrt{-d^2}-d^2\tan(a+bx)}{-cd+id^2+\sqrt{-d^2}}\right) + id\log(1\right.}{}$$

```
[In] Integrate[ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tan[a + b*x]] - (x*(4*a*Sqrt[-d^2]*ArcTan[c + d*Tan[a + b*x]
] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - d^2*Tan[a + b*x]
)/(-(c*d) + I*d^2 + Sqrt[-d^2])] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - S
qrt[-d^2] + d^2*Tan[a + b*x])/(c*d + I*d^2 - Sqrt[-d^2])] + I*d*Log[1 - I*T
an[a + b*x]]*Log[(c*d + Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d - I*d^2 + Sqrt[
-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + d^2*Tan[a + b
*x])/(c*d + I*d^2 + Sqrt[-d^2])] - I*d*PolyLog[2, (d^2*(1 - I*Tan[a + b*x])
)/(I*c*d + d^2 - I*Sqrt[-d^2])] + I*d*PolyLog[2, (d^2*(1 - I*Tan[a + b*x]))
/(I*c*d + d^2 + I*Sqrt[-d^2])] + I*d*PolyLog[2, (d^2*(1 + I*Tan[a + b*x]))/
((-I)*c*d + d^2 + I*Sqrt[-d^2])] - I*d*PolyLog[2, (d^2*(1 + I*Tan[a + b*x])
)/(d^2 - I*(c*d + Sqrt[-d^2]))]))/(2*Sqrt[-d^2]*(2*a - I*Log[1 - I*Tan[a +
b*x]] + I*Log[1 + I*Tan[a + b*x]]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(168) = 336.

Time = 2.41 (sec) , antiderivative size = 1001, normalized size of antiderivative = 5.06

method	result	size
derivativedivides	Expression too large to display	1001
default	Expression too large to display	1001
risch	Expression too large to display	4973

```
[In] int(arctan(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(d*arctan(tan(b*x+a))*arctan(c+d*tan(b*x+a))-d^2*(1/2*I/d*arctan(-(c+
d*tan(b*x+a))/d+c/d)*ln(1-(c-I*d-I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d
*tan(b*x+a))/d-c/d)^2+1)/(-I*d+I-c))-1/2/d*arctan(-(c+d*tan(b*x+a))/d+c/d)^
2-1/4/d*polylog(2,(c-I*d-I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x
+a))/d-c/d)^2+1)/(-I*d+I-c))+1/2/(I+c+I*d)*ln(1-(c-I*d+I)*(1+I*((c+d*tan(b*
x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan(-(c+d*tan
(b*x+a))/d+c/d)+1/2/d/(I+c+I*d)*ln(1-(c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d
))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan(-(c+d*tan(b*x+a))/d+
c/d)-1/2*I/d/(I+c+I*d)*ln(1-(c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c
+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*c*arctan(-(c+d*tan(b*x+a))/d+c/d)+1/
2*I/(I+c+I*d)*arctan(-(c+d*tan(b*x+a))/d+c/d)^2+1/4*I/(I+c+I*d)*polylog(2,(
c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-
I*d-I-c))+1/2*I/d/(I+c+I*d)*arctan(-(c+d*tan(b*x+a))/d+c/d)^2+1/2/d/(I+c+I*
d)*c*arctan(-(c+d*tan(b*x+a))/d+c/d)^2+1/4*I/d/(I+c+I*d)*polylog(2,(c-I*d+I
)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c
))+1/4/d/(I+c+I*d)*polylog(2,(c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((
c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1101 vs. 2(141) = 282.

Time = 0.34 (sec) , antiderivative size = 1101, normalized size of antiderivative = 5.56

$$\int \arctan(c + d\tan(a + bx))\,dx = \text{Too large to display}$$

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arctan(d*tan(b*x + a) + c) - 2*(I*b*x + I*a)*log(-2*((I*c*d - d^
2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x +
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1))
- 2*(-I*b*x - I*a)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
 (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(-I*b*x - I*a)*log(-2*((-I*c*d -
d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*
x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1
)) - 2*(I*b*x + I*a)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*
d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 2*I*a*log(((I*c*d + d^2 + d)*ta
n(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d -
 1)/(tan(b*x + a)^2 + 1)) - 2*I*a*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c
^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a
)^2 + 1)) + 2*I*a*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*
c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) - 2*I*
a*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*
I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) + dilog(2*((I*c*d - d^
2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x +
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
 1) - dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*
c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 + 2*d + 1) + 1) + dilog(2*((-I*c*d - d^2 + d)*tan(b*x + a)^2
- c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/((c^2 +
d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) - dilog(2*((-I*c*
d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*ta
n(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d
 + 1) + 1))/b
```

Sympy [F]

$$\int \arctan(c+d\tan(a+bx))\,dx = \int \operatorname{atan}\left(c+d\tan\left(a+bx\right)\right)dx$$

```
[In] integrate(atan(c+d*tan(b*x+a)),x)
[Out] Integral(atan(c + d*tan(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(141) = 282.

Time = 0.34 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.19

$$\int \arctan(c+d\tan(a+bx))\,dx = \frac{d\left(\frac{8\,(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{d} - \frac{4\,(bx+a)\arctan\left(\frac{cd+\left(d^2+d\right)\tan(bx+a)}{c^2+d^2+2\,d+1}, \frac{cd\tan(bx+a)+c^2+d+1}{c^2+d^2+2\,d+1}\right) - 4\,(bx+a)\arctan\left(\frac{cd+\left(d^2-d\right)\tan(bx+a}{c^2+d^2-2\,d+1}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$$

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
[Out] 1/8*(d*(8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d)/d - (4*(b*x + a)*arc
tan2((c*d + (d^2 + d)*tan(b*x + a))/(c^2 + d^2 + 2*d + 1), (c*d*tan(b*x + a
) + c^2 + d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(b*x + a)*arctan2((c*d + (d^2 -
 d)*tan(b*x + a))/(c^2 + d^2 - 2*d + 1), (c*d*tan(b*x + a) + c^2 - d + 1)/(
c^2 + d^2 - 2*d + 1)) + log(tan(b*x + a)^2 + 1)*log((d^2*tan(b*x + a)^2 + 2
*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) - log(tan(b*x + a)^2 +
1)*log((d^2*tan(b*x + a)^2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 - 2*d
 + 1)) + 2*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + 1)) - 2*dilog(-(I*d*tan
(b*x + a) - d)/(I*c + d - 1)) + 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
1)) - 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - 1)))/d) + 8*(b*x + a)*arct
an(d*tan(b*x + a) + c) - 8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d))/b
```

Giac [F]

$$\int \arctan(c+d\tan(a+bx))\,dx = \int \arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan(d*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c+d\tan(a+bx))\,dx = \int \mathrm{atan}(c+d\tan(a+b\,x))\,dx$$

```
[In] int(atan(c + d*tan(a + b*x)),x)
[Out] int(atan(c + d*tan(a + b*x)), x)
```

3.51 $\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c+d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*tan(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx = \int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\tan(a + bx))}{x} \, dx = \int \frac{\arctan(c + d\tan(a + bx))}{x} \, dx$$

```
[In] Integrate[ArcTan[c + d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + d*Tan[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(c + d\tan\left(bx + a\right)\right)}{x} dx$$

```
[In] int(arctan(c+d*tan(b*x+a))/x,x)
[Out] int(arctan(c+d*tan(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\tan(a + bx))}{x} \, dx = \int \frac{\arctan\left(d\tan\left(bx + a\right) + c\right)}{x} \, dx$$

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*tan(b*x + a) + c)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c + d\tan(a + bx))}{x} \, dx = \text{Timed out}$$

```
[In] integrate(atan(c+d*tan(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 232.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arctan(d*tan(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*tan(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c+d\tan(a+b\,x))}{x}\,dx$$

```
[In] int(atan(c + d*tan(a + b*x))/x,x)
[Out] int(atan(c + d*tan(a + b*x))/x, x)
```

3.52 $\int x^2 \arctan(c+(1+ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int x^2 \arctan(c+(1+ic)\tan(a+bx))\,dx = -\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) - \frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctan(c+(1+I*c)*tan(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*I*a+2*I*b*x))-1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog(3,I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5279, 2215, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx)) + \frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3} - \frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2} - \frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} - \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{bx^4}{12}$$

[In] Int[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]

[Out] -1/12*(b*x^4) + (x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - (I/6)*x^3*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/(4*b) - ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + PolyLog[4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$

$$= -\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx)) + \frac{1}{3}(bc)\int \frac{e^{2ia+2ibx}x^3}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$

$$= -\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) + \frac{1}{2}i\int x^2\log\left(1+\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx$$

$$= -\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) - \frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{\int x\,\text{PolyLog}\left(2,-\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx}{2b}$$

$$= -\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) - \frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} - \frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2} + \frac{i\int \text{PolyLog}\left(3,-\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx}{4b^2}$$

$$= -\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) - \frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} - \frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2} + \frac{\text{Subst}\left(\int \frac{\text{PolyLog}(3,icx)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^3}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c + (1+ic)\tan(a+bx)) - \frac{1}{6}ix^3 \log\left(1 - ice^{2ia+2ibx}\right) - \frac{x^2 \operatorname{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} - \frac{ix \operatorname{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4, ice^{2ia+2ibx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int x^2 \arctan(c + (1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3 \arctan(c + (1+ic)\tan(a+bx)) - \frac{4ib^3x^3 \log\left(1 + \frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2 \operatorname{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx \operatorname{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{c}\right) + 3 \operatorname{PolyLo}}{24b^3}$$

```
[In] Integrate[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 + I/(c*E^
((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))] +
(6*I)*b*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (-I)/(c*E
^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.44 (sec) , antiderivative size = 1487, normalized size of antiderivative = 9.66

method	result	size
risch	Expression too large to display	1487

```
[In] int(x^2*arctan(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*x^3*ln(exp(I*(b*x+a)))-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1
/2))*x-1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln(1
-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/6*I*x^3*ln(1-I*exp(2*I*(b*x+a))*c)-1/4*
I*x*polylog(3,I*exp(2*I*(b*x+a))*c)/b^2-1/4*x^2*polylog(2,I*exp(2*I*(b*x+a)
)*c)/b+1/4/b^3*polylog(2,I*exp(2*I*(b*x+a))*c)*a^2+1/2*I/b^2*ln(1-I*exp(2*I
*(b*x+a))*c)*x*a^2+1/8*polylog(4,I*exp(2*I*(b*x+a))*c)/b^3+1/6*I*x^3*ln(exp
(2*I*(b*x+a))*c+I)+1/12*I*(-2*I*Pi-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I
*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1)
)-2*ln(c-I)-I*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(e
xp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))
*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1
))+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b
*x+a))+1))+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))
```

```
*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))+I*Pi*csgn(exp(2*I*(b*x
+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^
3+I*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*ex
p(2*I*(b*x+a)))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^
3-I*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn((exp(2
*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(
exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))
+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(c-I)/
(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2
-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+I*P
i*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-I*Pi*csgn(I*exp(2*I*(b*
x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(c
-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2-2*I*Pi*csgn(I*exp(I*(b*x+a)))*csg
n(I*exp(2*I*(b*x+a)))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(
b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*cs
gn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*exp(2*I*(b*
x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x
+a))+1)))*x^3+1/6*I/b^3*a^3*ln(exp(2*I*(b*x+a))*c+I)-1/2*I/b^3*a^3*ln(1-I*e
xp(I*(b*x+a))*(-I*c)^(1/2))+1/3*I/b^3*ln(1-I*exp(2*I*(b*x+a))*c)*a^3-1/2/b^
3*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2/b^3*a^2*dilog(1+I*exp(I*(b
*x+a))*(-I*c)^(1/2))-1/12*b*x^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(107) = 214$.

Time = 0.27 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.09

$$\int x^2 \arctan(c + (1 + ic)\tan(a + bx))\,dx =$$

$$-\frac{b^4x^4 - 2i\,b^3x^3\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx-}\right.}{}$$

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(b^4*x^4 - 2*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x -
 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*b^2
*x^2*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a) -
 I*sqrt(4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) +
 12*I*b*x*polylog(3, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 2*(I*b^3*x^3 + I*a
^3)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) + 2*(I*b^3*x^3 + I*a^3)*log(-1
/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(4*I*c)*e^(I*b*
x + I*a)) - 12*polylog(4, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c + (1 + ic) \tan(a + bx))\, dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(107) = 214$.

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.01

$$\int x^2 \arctan(c + (1 + ic) \tan(a + bx))\, dx$$
$$= \frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((i\,c+1)\tan(bx+a)+c)}{b^2} - \frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx+a)^2\right.\right.}{}}{}$$

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c + 1)*
tan(b*x + a) + c)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*ar
ctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*
I*(b*x + a)*a + 3*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 -
9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2*b
*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^(2
*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b^2*
(c - I)))/b
```

Giac [F]

$$\int x^2 \arctan(c + (1 + ic) \tan(a + bx))\, dx = \int x^2 \arctan\left((i\,c + 1) \tan\left(bx + a\right) + c\right)\, dx$$

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + (1 + ic)\tan(a + bx))\,dx = \int x^2\,\mathrm{atan}(c + \tan(a + b\,x)\ (1 + c\,1\mathrm{i}))\ dx$$

```
[In] int(x^2*atan(c + tan(a + b*x)*(c*1i + 1)),x)
[Out] int(x^2*atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.53 $\int x \arctan(c+(1+ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\begin{aligned}\int x\arctan(c+(1+ic)\tan(a+bx))\,dx = &-\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx))\\ &-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)\\ &-\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}\\ &-\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}\end{aligned}$$

```
[Out] -1/6*b*x^3+1/2*x^2*arctan(c+(1+I*c)*tan(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*I*
a+2*I*b*x))-1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,I*c*e
xp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5279, 2215, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int x\arctan(c+(1+ic)\tan(a+bx))\,dx = &\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx))\\ &-\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}\\ &-\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}\\ &-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-\frac{bx^3}{6}\end{aligned}$$

```
[In] Int[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - (I/4)*x^2*Log[1
 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) - ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{2}(ib)\int \frac{x^2}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) + \frac{1}{2}(bc)\int \frac{e^{2ia+2ibx}x^2}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) \\
&\quad -\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}i\int x\log\left(1+\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) \\
&\quad -\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{\int \text{PolyLog}\left(2,-\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx}{4b} \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) \\
&\quad -\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{i\text{Subst}\left(\int \frac{\text{PolyLog}(2,icx)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^2} \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) \\
&\quad -\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x\arctan(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) - \frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

```
[In] Integrate[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.07 (sec) , antiderivative size = 1452, normalized size of antiderivative = 11.80

method	result	size
risch	Expression too large to display	1452

```
[In] int(x*arctan(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*x^2*ln(exp(2*I*(b*x+a))*c+I)-1/6*b*x^3-1/4*I*ln(1-I*exp(2*I*(b*x+a))*
c)*x^2+1/8*I*(-2*I*Pi-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*
x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))-2*ln(c-I)-I
*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a
))*(c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*
I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))+I*Pi*csgn(
I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))+I*
Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2
*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))+I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(e
xp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I
*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*exp(2*I*(b*x+a)
))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3-I*Pi*csgn((
exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn((exp(2*I*(b*x+a))*c
+I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+
a))+1))^3+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((ex
p(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(c-I)/(exp(2*I*(b*x
+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I/
(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*exp(
I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I
*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(c-I))*csgn(I*(
c-I)/(exp(2*I*(b*x+a))+1))^2-2*I*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(
b*x+a)))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/
(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I
*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(
exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1)))*x^2
-1/4*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*a^2+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a
))*(-I*c)^(1/2))-1/4/b*polylog(2,I*exp(2*I*(b*x+a))*c)*x-1/4/b^2*polylog(2,
I*exp(2*I*(b*x+a))*c)*a+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1
/2*I*x^2*ln(exp(I*(b*x+a)))-1/8*I/b^2*polylog(3,I*exp(2*I*(b*x+a))*c)-1/4*I
/b^2*a^2*ln(exp(2*I*(b*x+a))*c+I)-1/2*I/b*ln(1-I*exp(2*I*(b*x+a))*c)*a*x+1/
2*I/b*a*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/2/b^2*a*dilog(1-I*exp(I*(b*
x+a))*(-I*c)^(1/2))+1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2*I/
b*a*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(85) = 170.

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.20

$$\int x\arctan(c+(1+ic)\tan(a+bx))\,dx = -\frac{2\,b^3x^3-3i\,b^2x^2\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right)+2\,a^3+6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right)+6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(}\right.}{}$$

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*x^3 - 3*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) +
 6*b*x*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*s
qrt(4*I*c))/c) + 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)
+ 1) + 3*(I*b^2*x^2 - I*a^2)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) + 6*
I*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*sqrt(4*
I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c+(1+ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(85) = 170.

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.77

$$\int x\arctan(c+(1+ic)\tan(a+bx))\,dx = \frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan((i\,c+1)\tan(bx+a)+c)}{b}-\frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right)\mathrm{arc}\right.}{}}{}$$

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c + 1)*tan(b*x + a) + c)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x + 2
*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), c*
sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x +
 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(3,
 I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(c - I)))/b
```

Giac [F]

$$\int x\arctan(c+(1+ic)\tan(a+bx))\,dx=\int x\arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(1+ic)\tan(a+bx))\,dx=\int x\,\mathrm{atan}(c+\tan(a+b\,x)\;(1+c\,1\mathrm{i}))\,dx$$

```
[In] int(x*atan(c + tan(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(x*atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.54 $\int \arctan(c+(1+ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = -\frac{bx^2}{2}+x\arctan(c+(1+ic)\tan(a+bx)) -\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] -1/2*b*x^2+x*arctan(c+(1+I*c)*tan(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b*
x))-1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5271, 2215, 2221, 2317, 2438}

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = x\arctan(c+(1+ic)\tan(a+bx)) -\frac{\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} -\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{bx^2}{2}$$

```
[In] Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] - (I/2)*x*Log[1 - I*c*E
^((2*I)*a + (2*I)*b*x)] - PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5271

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan(c+(1+ic)\tan(a+bx))-(ib)\int\frac{x}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^2}{2}+x\arctan(c+(1+ic)\tan(a+bx))+(bc)\int\frac{e^{2ia+2ibx}x}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^2}{2}+x\arctan(c+(1+ic)\tan(a+bx))\\
&\quad-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}i\int\log\left(1+\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)\,dx\\
&= -\frac{bx^2}{2}+x\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{cx}{i(1+ic)+c}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{4b}
\end{aligned}$$

$$= -\frac{bx^2}{2} + x\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) - \frac{\text{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 967 vs. 2(85) = 170.

Time = 11.10 (sec) , antiderivative size = 967, normalized size of antiderivative = 11.38

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = x\arctan(c+(1+ic)\tan(a+bx))$$

$$+ \frac{}{((i+c)\cos(a+bx)+(1+ic)\sin(a+bx))\left(2bx - i\log\left(1-\frac{\sec(bx)((-i+c)\cos(a)+i(i+c)\sin(a))(\cos(a+bx)-i\sin(a+}{2c}\right.\right.}$$

```
[In] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] + (I*x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[
b*x] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a + b*
x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*((
1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*
Log[1 + I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (
Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x
]))/(2*c)] + PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] - (I + c)*Sin[a])*(Cos[
a + b*x] + I*Sin[a + b*x]))/2])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*C
os[b*x] + Sin[b*x])*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(((I
+ c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x])*(2*b*x - I*Log[1 - (Sec[b*x]*((
-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]
- I*Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] +
I*Sin[a + b*x]))/2] - (I*(-I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1
 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]) + ((I + c)
*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((I + c)*Cos[a +
 b*x] + (1 + I*c)*Sin[a + b*x]) - (2*I)*b*x*Tan[b*x] + Log[1 - (Sec[b*x]*((
-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*
Tan[b*x] - Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a +
b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - Log[1 - I*Tan[b*x]]*Tan[b*x] + Cos[a]
^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (
Log[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] -
 (2*I)*Sin[a])]*Sec[b*x]^2)/(-I + Tan[b*x]) - (Log[(Sec[b*x]*(Cos[a] - I*Si
n[a])*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(
I + Tan[b*x]))*(-I + Tan[a + b*x])*(1 - I*c + (-I + c)*Tan[a + b*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 562 vs. 2(69) = 138.

Time = 1.11 (sec) , antiderivative size = 563, normalized size of antiderivative = 6.62

method	result
derivativedivides	$\frac{\frac{\arctan(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c^2}{2i-2c}-\frac{2i\arctan(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c}{2i-2c}-\frac{\arctan(c}{}}{}$
default	$\frac{\frac{\arctan(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c^2}{2i-2c}-\frac{2i\arctan(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c}{2i-2c}-\frac{\arctan(c}{}}{}$
risch	Expression too large to display

```
[In] int(arctan(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I*c+1)*(arctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+a
)+I)*c^2-2*I*arctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+a
)+I)*c-arctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+a)+I)-a
rctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))*c^2+2*I*a
rctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))*c+arctan(
c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))-(I*c+1)^2*(1/2/
(I-c)*(-1/4*I*ln(-I+c+(I*c+1)*tan(b*x+a))^2+1/2*I*(dilog(-1/2*I*(c+(I*c+1)*
tan(b*x+a)+I))+ln(-I+c+(I*c+1)*tan(b*x+a))*ln(-1/2*I*(c+(I*c+1)*tan(b*x+a)+
I))))-1/2/(I-c)*(1/2*I*(dilog(1/2*(c+(I*c+1)*tan(b*x+a)+I)/c)+ln(-c+(I*c+1)
*tan(b*x+a)+I)*ln(1/2*(c+(I*c+1)*tan(b*x+a)+I)/c))-1/2*I*(dilog((-I+c+(I*c+
1)*tan(b*x+a))/(-2*I+2*c))+ln(-c+(I*c+1)*tan(b*x+a)+I)*ln((-I+c+(I*c+1)*tan
(b*x+a))/(-2*I+2*c))))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(60) = 120.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.38

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = -\frac{b^2x^2-i\,bx\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right)-a^2-(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}+1\right)-(-i\,bx-i}{}$$

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)
/(c - I)) - a^2 - (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) -
 (-I*b*x - I*a)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*
c*e^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) -
I*sqrt(4*I*c))/c) + dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sqr
t(4*I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 448, normalized size of antiderivative = 5.27

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = -\frac{(ic+1)\left(\frac{4i(bx+a)\log\left(-\frac{2\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)}{2ic^2-2\left(c^2-2ic-1\right)\tan(bx+a)+2i}\right)}{ic+1} - \frac{i\left(4(bx+a)\left(\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)-\log\left(-ic^2+\right.\right.\right.}{}\right.}{}$$

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c + 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x +
 a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/(I*c + 1
) - I*(4*(b*x + a)*(log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c + I)
- log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2 -
 2*I*c - 1)*tan(b*x + a) - 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*
tan(b*x + a) - I)*log(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) + 2*I*log(
-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c + 1)*tan(b*x +
a) + c + I)/c + 1) - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c
+ I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(1/2*(c - I)*tan(b*x + a) -
1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c + 1)*tan(b*x + a) + c + I)/c) - 2*I*di
log(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)) - 8*(b*x + a)*arctan((I*c + 1)*ta
n(b*x + a) + c) + 4*(-I*b*x - I*a)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b
*x + a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I)))/b
```

Giac [F]

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = \int \arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = \int \mathrm{atan}(c+\tan(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

```
[In] int(atan(c + tan(a + b*x)*(c*1i + 1)),x)
[Out] int(atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.55 $\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\arctan(c+(1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\arctan\left(c+(ic+1)\tan\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arctan(c+(I*c+1)*tan(b*x+a))/x,x)
[Out] int(arctan(c+(I*c+1)*tan(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(atan(c+(1+I*c)*tan(b*x+a))/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan((I*c + 1)*tan(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}\left(c+\tan\left(a+b\,x\right)\,\left(1+c\,1\mathrm{i}\right)\right)}{x}\,dx$$

```
[In] int(atan(c + tan(a + b*x)*(c*1i + 1))/x,x)
[Out] int(atan(c + tan(a + b*x)*(c*1i + 1))/x, x)
```

3.56 $\int x^2 \arctan(c + (-1 + ic) \tan(a + bx)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\begin{aligned}\int x^2 \arctan(c + (-1 + ic) \tan(a + bx)) \, dx = &\frac{bx^4}{12} + \frac{1}{3} x^3 \arctan(c - (1 - ic) \tan(a + bx)) \\ &+ \frac{1}{6} i x^3 \log \left(1 + ice^{2ia+2ibx}\right) \\ &+ \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} \\ &+ \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} \\ &- \frac{\operatorname{PolyLog}\left(4, -ice^{2ia+2ibx}\right)}{8b^3}\end{aligned}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arctan(c-(1-I*c)*tan(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2*I*
a+2*I*b*x))+1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(3,
-I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{5279, 2215, 2221, 2611, 6744, 2320, 6724\}$

$$\int x^2 \arctan(c+(-1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx)) - \frac{\text{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3} + \frac{ix\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} + \frac{x^2\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) + \frac{bx^4}{12}$$

```
[In] Int[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) + ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - Po
lyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx)) - \frac{1}{3}(bc)\int \frac{e^{2ia+2ibx}x^3}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx)) \\
&\quad + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) - \frac{1}{2}i\int x^2\log\left(1+\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) \\
&\quad + \frac{x^2\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} - \frac{\int x\,\text{PolyLog}\left(2,-\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)dx}{2b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c - (1 - ic)\tan(a + bx)) \\
&\quad + \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} \\
&\quad + \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} - \frac{i \int \operatorname{PolyLog}\left(3, -\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right) dx}{4b^2} \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c - (1 - ic)\tan(a + bx)) + \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) \\
&\quad + \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-icx)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^3} \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c - (1 - ic)\tan(a + bx)) \\
&\quad + \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} \\
&\quad + \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, -ice^{2ia+2ibx}\right)}{8b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int x^2 \arctan(c + (-1 + ic)\tan(a + bx))\, dx = \frac{1}{24}\Bigg(& 8x^3 \arctan(c + i(i + c)\tan(a + bx)) \\
& + 4ix^3 \log\left(1 - \frac{ie^{-2i(a+bx)}}{c}\right) \\
& - \frac{6x^2 \operatorname{PolyLog}\left(2, \frac{ie^{-2i(a+bx)}}{c}\right)}{b} \\
& + \frac{6ix \operatorname{PolyLog}\left(3, \frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} \\
& + \frac{3 \operatorname{PolyLog}\left(4, \frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\Bigg)
\end{aligned}
$$

```
[In] Integrate[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (4*I)*x^3*Log[1 - I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)*x*
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, I/(c*E^((2*I)*(a
 + b*x)))])/b^3)/24
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.48 (sec) , antiderivative size = 1488, normalized size of antiderivative = 9.60

method	result	size
risch	Expression too large to display	1488

```
[In] int(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*(-I*Pi*csgn(I*exp(2*I*(b*x+a)))^3-2*I*Pi+I*Pi*csgn(I/(exp(2*I*(b*x+a
))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+2*ln(I+c
)-I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*csgn(I*(exp(2
*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3-I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)
/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a
))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-I*Pi*csgn(exp(2*I*
(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+
c))^3-I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I
*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))+I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp
(2*I*(b*x+a))+1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c))*csgn(I/
(exp(2*I*(b*x+a))+1)*(I+c))-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(
b*x+a))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))+I*Pi*
csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I
*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn
(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(exp(2*I*(b*x
+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(
I/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2+I*Pi*csgn(I*(I
+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csg
n(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(exp(2*I*(b*x+a
))*(I+c)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*
(b*x+a))+1))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csg
n((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+2*I*Pi*csgn(I*exp(I*(b*x+a
)))*csgn(I*exp(2*I*(b*x+a)))^2+I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*
(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2)*x^3-1/6*I
/b^3*a^3*ln(-exp(2*I*(b*x+a))*c+I)+1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(I*c
)^(1/2))+1/6*I*x^3*ln(I*exp(2*I*(b*x+a))*c+1)+1/2*I/b^3*a^3*ln(1-I*exp(I*(b
*x+a))*(I*c)^(1/2))+1/4*I*x*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2+1/4*x^2*po
lylog(2,-I*exp(2*I*(b*x+a))*c)/b-1/4/b^3*polylog(2,-I*exp(2*I*(b*x+a))*c)*a
^2-1/3*I/b^3*ln(I*exp(2*I*(b*x+a))*c+1)*a^3-1/8*polylog(4,-I*exp(2*I*(b*x+a
))*c)/b^3+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/2*I/b^2*ln(I
*exp(2*I*(b*x+a))*c+1)*x*a^2-1/6*I*x^3*ln(exp(2*I*(b*x+a))*c-I)+1/3*I*x^3*l
n(exp(I*(b*x+a)))+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/2/b^
3*a^2*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2/b^3*a^2*dilog(1-I*exp(I*(b*
x+a))*(I*c)^(1/2))+1/12*b*x^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(108) = 216.

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.08

$$\int x^2 \arctan(c + (-1 + ic)\tan(a + bx))\,dx$$
$$= \frac{b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) - a^4 -}{}$$

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x
+ 2*I*a) - I)) + 6*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*b^2*
x^2*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a)
- I*sqrt(-4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)
) + 12*I*b*x*polylog(3, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - 2*(-I*b^3*x^3
- I*a^3)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 2*(-I*b^3*x^3 - I*a^3)
*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(-4*I*c
)*e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c + (-1 + ic)\tan(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(108) = 216.

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.00

$$\int x^2 \arctan(c + (-1 + ic)\tan(a + bx))\,dx$$
$$= \frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((i\,c-1)\tan(bx+a)+c)}{b^2} + \frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(bx+a\right.\right.}{}}{}$$

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c - 1)*
tan(b*x + a) + c)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)*a
rctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 -
6*I*(b*x + a)*a + 3*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3
 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(
2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*c*
e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/
(b^2*(c + I)))/b
```

Giac [F]

$$\int x^2 \arctan(c + (-1 + ic)\tan(a + bx))\,dx = \int x^2 \arctan\left((i\,c - 1)\tan\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + (-1 + ic)\tan(a + bx))\,dx = \int x^2\,\mathrm{atan}(c + \tan(a + b\,x)\,\left(-1 + c\,1\mathrm{i}\right))\,dx$$

```
[In] int(x^2*atan(c + tan(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x^2*atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.57 $\int x \arctan(c+(-1+ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx=\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{x\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{i\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}$$

```
[Out] 1/6*b*x^3+1/2*x^2*arctan(c-(1-I*c)*tan(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*I*a
+2*I*b*x))+1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,-I*c*
exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5279, 2215, 2221, 2611, 2320, 6724}

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx=\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))+\frac{i\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{bx^3}{6}$$

```
[In] Int[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 +
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx)) - \frac{1}{2}(ib)\int \frac{x^2}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx)) - \frac{1}{2}(bc)\int \frac{e^{2ia+2ibx}x^2}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx)) \\
&\quad + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) - \frac{1}{2}i\int x\log\left(1+\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) \\
&\quad + \frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} - \frac{\int \text{PolyLog}\left(2,-\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)dx}{4b} \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) \\
&\quad + \frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} + \frac{i\text{Subst}\left(\int \frac{\text{PolyLog}(2,-icx)}{x}\,dx, x, e^{2ia+2ibx}\right)}{8b^2} \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) \\
&\quad + \frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx \\
&= \frac{1}{2}x^2\arctan(c+i(i+c)\tan(a+bx)) \\
&\quad + \frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) + 2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right) + \text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}
\end{aligned}$$

```
[In] Integrate[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
[Out] (x^2*ArcTan[c + I*(I + c)*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 1453, normalized size of antiderivative = 11.72

method	result	size
risch	Expression too large to display	1453

```
[In] int(x*arctan(c+(-1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b*ln(I*exp(2*I*(b*x+a))*c+1)*a*x+1/8*I/b^2*polylog(3,-I*exp(2*I*(b*x+
a))*c)-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/8*I*(-I*Pi*csgn(I*e
xp(2*I*(b*x+a)))^3-2*I*Pi+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))*csgn(I*ex
p(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+2*ln(I+c)-I*Pi*csgn(I*exp(I*(b
*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(
2*I*(b*x+a))+1))^3-I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))
^3+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(
b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-I*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*
I*(b*x+a))+1))^3-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^3-I*Pi*csgn(I*exp(
2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2
*I*(b*x+a))+1))+I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2-I*
Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(
I+c))-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))*csgn
(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))+I*Pi*csgn(I/(exp(2*I*(b*x+a
))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*
I*(b*x+a))+1))-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c
-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp
(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1)
)*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2+I*Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(
b*x+a))+1)*(I+c))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(
I+c)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x
+a))+1))^2-I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3-I*Pi*cs
gn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-
I)/(exp(2*I*(b*x+a))+1))^2+2*I*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*
x+a)))^2+I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(
2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2)*x^2-1/2*I/b^2*a^2*ln(1-I*exp(I*
(b*x+a))*(I*c)^(1/2))+1/4/b*polylog(2,-I*exp(2*I*(b*x+a))*c)*x+1/4/b^2*poly
log(2,-I*exp(2*I*(b*x+a))*c)*a+1/4*I/b^2*ln(I*exp(2*I*(b*x+a))*c+1)*a^2-1/2
*I/b*a*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/2*I*x^2*ln(exp(I*(b*x+a)))-1/
2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/4*I*x^2*ln(exp(2*I*(b*x+a)
)*c-I)+1/4*I*ln(I*exp(2*I*(b*x+a))*c+1)*x^2-1/2/b^2*a*dilog(1+I*exp(I*(b*x+
a))*(I*c)^(1/2))-1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/6*b*x^3+
1/4*I/b^2*a^2*ln(-exp(2*I*(b*x+a))*c+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(86) = 172.

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.19

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx$$
$$=\frac{2\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)+2\,a^3+6\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right)+6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right)+$$

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) +
6*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*
sqrt(-4*I*c))/c) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I
*a) + 1) - 3*(-I*b^2*x^2 + I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1
) + 6*I*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*
sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx=\text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(86) = 172.

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.76

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx$$
$$=\frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan((i\,c-1)\tan(bx+a)+c)}{b}+\frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx+a)a\right)\right.}{}}{}$$

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c - 1)*tan(b*x + a) + c)/b
+ (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x +
2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
-c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*
x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog
(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(c + I)))/b
```

Giac [F]

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx=\int x\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx=\int x\,\mathrm{atan}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

```
[In] int(x*atan(c + tan(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x*atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.58 $\int \arctan(c+(-1+ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx=\frac{bx^2}{2}+x\arctan(c-(1-ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arctan(c-(1-I*c)*tan(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b*x
))+1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5271, 2215, 2221, 2317, 2438}

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx=x\arctan(c-(1-ic)\tan(a+bx))+\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{bx^2}{2}$$

```
[In] Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[c - (1 - I*c)*Tan[a + b*x]] + (I/2)*x*Log[1 + I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5271

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan(c-(1-ic)\tan(a+bx))-(ib)\int\frac{x}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^2}{2}+x\arctan(c-(1-ic)\tan(a+bx))-(bc)\int\frac{e^{2ia+2ibx}x}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^2}{2}+x\arctan(c-(1-ic)\tan(a+bx))\\
&\quad+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{1}{2}i\int\log\left(1+\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)\,dx
\end{aligned}$$

$$= \frac{bx^2}{2} + x\arctan(c-(1-ic)\tan(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{cx}{i(-1+ic)+c}\right)}{x}\,dx, x, e^{2ia+2ibx}\right)}{4b}$$

$$= \frac{bx^2}{2} + x\arctan(c-(1-ic)\tan(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) + \frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 847 vs. $2(86) = 172$.

Time = 8.39 (sec) , antiderivative size = 847, normalized size of antiderivative = 9.85

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx = x\arctan(c+i(i+c)\tan(a+bx)) + \frac{ix\left(-2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+\log\left(\frac{\sec(bx)(\cos(a)-i}{}\right.\right.}{((-i+c)\cos(a+bx)+i(i+c)\sin(a+bx))\left(-2bx+i\log\left(1-\frac{\sec(bx)((i+c)\cos(a)+(1+ic)\sin(a))(\cos(a+bx)-i\sin(}{2c}\right.\right.}$$

```
[In] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (I*x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos
[b*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a +
b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*x]*
(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]*Log
[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (Sec
[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/
(2*c)] - PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a]
)*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + S
in[b*x]))/(((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x])*(-2*b*x + I*Log
[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a
+ b*x]))/(2*c)] + (I*(I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*
Tan[b*x]]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) + ((1 + I*c)*(
Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-1 - I*c)*Cos[a
+ b*x] + (I + c)*Sin[a + b*x]) + (2*I)*b*x*Tan[b*x] - Log[1 - (Sec[b*x]*((I
 + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Ta
n[b*x] + Log[1 - I*Tan[b*x]]*Tan[b*x] - Log[1 + I*Tan[b*x]]*Tan[b*x] - (Log
[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*
x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*((I + c
)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]
) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a + b*x] + I*(I + c)*S
in[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I + Tan[a + b*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(70) = 140$.

Time = 1.17 (sec) , antiderivative size = 594, normalized size of antiderivative = 6.91

method	result
derivativedivides	$\frac{\frac{\arctan(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\arctan(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\arctan(c+(\cdots}{\cdots}}{\cdots}$
default	$\frac{\frac{\arctan(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\arctan(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\arctan(c+(\cdots}{\cdots}}{\cdots}$
risch	Expression too large to display

```
[In] int(arctan(c+(-1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(-1+I*c)*(arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b
*x+a))*c^2+2*I*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(
b*x+a))*c-arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b*x+a
))-arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*c^2-
2*I*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*c+a
rctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)+(-1+I*c)
^2*(1/2/(I+c)*(1/4*I*ln(I+c+(-1+I*c)*tan(b*x+a))^2-1/2*I*((ln(I+c+(-1+I*c)*
tan(b*x+a))-ln(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a))))*ln(-1/2*I*(I-c-(-1+I*c)*t
an(b*x+a)))-dilog(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))))-1/2/(I+c)*(1/2*I*(dil
og((-I-c-(-1+I*c)*tan(b*x+a))/(-2*I-2*c))+ln(c-(-1+I*c)*tan(b*x+a)+I)*ln((-
I-c-(-1+I*c)*tan(b*x+a))/(-2*I-2*c)))-1/2*I*(dilog(-1/2*(I-c-(-1+I*c)*tan(b
*x+a))/c)+ln(c-(-1+I*c)*tan(b*x+a)+I)*ln(-1/2*(I-c-(-1+I*c)*tan(b*x+a))/c))
)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(61) = 122$.

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.33

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx = \frac{b^2x^2+i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)-a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{-\cdots}\right.}{\cdots}$$

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a
) - I)) - a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) + (
I*b*x + I*a)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*c*
e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) - I
*sqrt(-4*I*c))/c) + dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sq
rt(-4*I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \arctan(c + (-1 + ic)\tan(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(61) = 122.

Time = 0.29 (sec) , antiderivative size = 448, normalized size of antiderivative = 5.21

$$\int \arctan(c + (-1 + ic)\tan(a + bx))\,dx = -\frac{(i\,c-1)\left(\frac{4i\,(bx+a)\log\left(-\frac{2\left(-i\,c^2+\left(c^2+2i\,c-1\right)\tan(bx+a)-i\right)}{2i\,c^2-2\left(c^2+2i\,c-1\right)\tan(bx+a)-4\,c-2i}\right)}{i\,c-1}+\frac{i\left(4\,(bx+a)\left(\log\left(-i\,c^2+\left(c^2+2i\,c-1\right)\tan(bx+a)+2\,c+i\right)-\log\left(-i\,c^2+\left(\right.\right.\right.\right.}{}\right.}{}$$

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c - 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x +
 a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I))/(I*c - 1
) + I*(4*(b*x + a)*(log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c + I)
- log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2 +
 2*I*c - 1)*tan(b*x + a) + 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*
tan(b*x + a) - I)*log(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) + 2*I*log(-
I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c - 1)*tan(b*x + a
) + c - I)/c + 1) - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c +
 I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(-1/2*(c + I)*tan(b*x + a) +
1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c - 1)*tan(b*x + a) + c - I)/c) - 2*I*di
log(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)) - 8*(b*x + a)*arctan((I*c - 1)*ta
n(b*x + a) + c) + 4*(-I*b*x - I*a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b
*x + a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I)))/b
```

Giac [F]

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx=\int \arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan((I*c - 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx=\int \mathrm{atan}\left(c+\tan\left(a+b\,x\right)\,\left(-1+c\,1{}\mathrm{i}\right)\right)\,dx$$

```
[In] int(atan(c + tan(a + b*x)*(c*1i - 1)),x)
[Out] int(atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.59 $\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\arctan(c+(-1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx=\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\arctan\left(c+(ic-1)\tan\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
[Out] int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx=\int \frac{\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) - I)
)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(atan(c+(-1+I*c)*tan(b*x+a))/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan((I*c - 1)*tan(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))}{x}\,dx$$

```
[In] int(atan(c + tan(a + b*x)*(c*1i - 1))/x,x)
[Out] int(atan(c + tan(a + b*x)*(c*1i - 1))/x, x)
```

3.60 $\int \arctan(\cot(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \arctan(\cot(a+bx))\,dx=-\frac{\arctan(\cot(a+bx))^2}{2b}$$

```
[Out] -1/2*(1/2*Pi-arccot(cot(b*x+a)))^2/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2188, 30}

$$\int \arctan(\cot(a+bx))\,dx=-\frac{\arctan(\cot(a+bx))^2}{2b}$$

```
[In] Int[ArcTan[Cot[a + b*x]],x]
```

```
[Out] -1/2*ArcTan[Cot[a + b*x]]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}(\int x\,dx, x, \arctan(\cot(a+bx)))}{b}\\ &= -\frac{\arctan(\cot(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \arctan(\cot(a+bx))\,dx = \frac{bx^2}{2} + x\arctan(\cot(a+bx))$$

```
[In] Integrate[ArcTan[Cot[a + b*x]],x]
[Out] (b*x^2)/2 + x*ArcTan[Cot[a + b*x]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result
parallelrisch	$\frac{x^2 b}{2} - x\,\text{arccot}\left(\cot\left(bx+a\right)\right) + \frac{\pi x}{2}$
derivativedivides	$\frac{-\pi\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)-\text{arccot}(\cot(bx+a))^2}{2b}$
default	$\frac{\pi x}{2} - \frac{-\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\text{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
parts	$\frac{\pi x}{2} - \frac{-\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\text{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
risch	$\frac{\pi x}{2} + ix\ln\left(e^{i(bx+a)}\right) + \frac{\pi x\,\text{csgn}\left(ie^{i(bx+a)}\right)^2\text{csgn}\left(ie^{2i(bx+a)}\right)}{4} - \frac{\pi x\,\text{csgn}\left(ie^{i(bx+a)}\right)\text{csgn}\left(ie^{2i(bx+a)}\right)^2}{2} + \frac{\pi x\,\text{csg}}{}$

```
[In] int(1/2*Pi-arccot(cot(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*b-x*arccot(cot(b*x+a))+1/2*Pi*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,(\pi - 2\,a)x$$

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")
[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \arctan(\cot(a+bx))\,dx = \frac{\pi x}{2} - \begin{cases} \frac{\operatorname{acot}^2(\cot(a+bx))}{2b} & \text{for } b \neq 0 \\ x \operatorname{acot}(\cot(a)) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/2*pi-acot(cot(b*x+a)),x)
[Out] pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a))
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \arctan(\cot(a+bx))\,dx = \frac{\Pi\,x}{2} - x\,\mathrm{acot}(\cot(a+b\,x)) + \frac{b\,x^2}{2}$$

```
[In] int(Pi/2 - acot(cot(a + b*x)),x)
[Out] (Pi*x)/2 - x*acot(cot(a + b*x)) + (b*x^2)/2
```

3.61 $\int x^2 \arctan(c+d\cot(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 399

$$\begin{aligned}\int x^2\arctan(c+d\cot(a+bx))\,dx &= \frac{1}{3}x^3\arctan(c+d\cot(a+bx)) \\ &+ \frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\ &- \frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\ &+ \frac{x^2\operatorname{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\ &- \frac{x^2\operatorname{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} \\ &+ \frac{ix\operatorname{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2} \\ &- \frac{ix\operatorname{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2} \\ &- \frac{\operatorname{PolyLog}\left(4,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^3} \\ &+ \frac{\operatorname{PolyLog}\left(4,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3}\end{aligned}$$

```
[Out] 1/3*x^3*arctan(c+d*cot(b*x+a))+1/6*I*x^3*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))-1/6*I*x^3*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*x
^2*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*x^2*polylog(2,(c
+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b+1/4*I*x*polylog(3,(1+I*c-d)*exp
(2*I*a+2*I*b*x)/(1+I*c+d))/b^2-1/4*I*x*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*
```

```
b*x)/(c+I*(1-d)))/b^2-1/8*polylog(4,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))
/b^3+1/8*polylog(4,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5285, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(c+d\cot(a+bx))\,dx = \frac{1}{3}x^3\arctan(d\cot(a+bx)+c) - \frac{\text{PolyLog}\left(4,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^3} + \frac{\text{PolyLog}\left(4,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3} + \frac{ix\,\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2} - \frac{ix\,\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2} + \frac{x^2\,\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b} - \frac{x^2\,\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} + \frac{1}{6}ix^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right) - \frac{1}{6}ix^3\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)$$

```
[In] Int[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/6)*x^3*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x^2*PolyLog[2, ((1 + I*c - d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x^2*PolyLog[2, ((c + I*(1 +
d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/4)*x*PolyLog[3,
((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/4)*x*Poly
Log[3, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2 - Po
lyLog[4, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)]/(8*b^3) + P
olyLog[4, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]/(8*b^3
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5285

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (Dist[b*((1 + I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2
*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
- Dist[b*((1 - I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*I
*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c+d\cot(a+bx)) \\
&\quad +\frac{1}{3}(b(1+ic-d))\int\frac{e^{2ia+2ibx}x^3}{1+ic+d+(-1-ic+d)e^{2ia+2ibx}}\,dx \\
&\quad -\frac{1}{3}(b(1-ic+d))\int\frac{e^{2ia+2ibx}x^3}{1-ic-d+(-1+ic-d)e^{2ia+2ibx}}\,dx \\
&= \frac{1}{3}x^3\arctan(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad -\frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\
&\quad +\frac{1}{2}i\int x^2\log\left(1+\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)\,dx \\
&\quad -\frac{1}{2}i\int x^2\log\left(1+\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)\,dx \\
&= \frac{1}{3}x^3\arctan(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad -\frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)+\frac{x^2\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\
&\quad -\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{\int x\,\text{PolyLog}\left(2,-\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)\,dx}{2b} \\
&\quad -\frac{\int x\,\text{PolyLog}\left(2,-\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)\,dx}{2b} \\
&= \frac{1}{3}x^3\arctan(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad -\frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)+\frac{x^2\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\
&\quad -\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{ix\,\text{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2} \\
&\quad -\frac{ix\,\text{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}+\frac{i\int\text{PolyLog}\left(3,-\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)\,dx}{4b^2} \\
&\quad -\frac{i\int\text{PolyLog}\left(3,-\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)\,dx}{4b^2}
\end{aligned}
$$

$$= \frac{1}{3}x^3 \arctan(c + d\cot(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right) - \frac{1}{6}ix^3 \log\left(1 - \frac{(c + i(1 + d))e^{2ia+2ibx}}{c + i(1 - d)}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} - \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2} - \frac{ix \operatorname{PolyLog}\left(3, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2} - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, -\frac{(-1-ic+d)x}{1+ic+d}\right)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^3} + \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, \frac{(c+i(1+d))x}{c-i(-1+d)}\right)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^3}$$

$$= \frac{1}{3}x^3 \arctan(c + d\cot(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right) - \frac{1}{6}ix^3 \log\left(1 - \frac{(c + i(1 + d))e^{2ia+2ibx}}{c + i(1 - d)}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} - \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2} - \frac{ix \operatorname{PolyLog}\left(3, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^3} + \frac{\operatorname{PolyLog}\left(4, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 3.07 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.90

$$\int x^2 \arctan(c + d\cot(a + bx)) \, dx = \frac{8b^3x^3 \arctan(c + d\cot(a + bx)) + 4ib^3x^3 \log\left(1 + \frac{(-c+i(1+d))e^{-2i(a+bx)}}{c+i(-1+d)}\right) - 4ib^3x^3 \log\left(1 + \frac{(-c+i(-1+d))e^{-2i(a+}}{c+i(1+d)}\right.}{}$$

```
[In] Integrate[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcTan[c + d*Cot[a + b*x]] + (4*I)*b^3*x^3*Log[1 + (-c + I*(1 +
d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - (4*I)*b^3*x^3*Log[1 + (-c + I
*(-1 + d))/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (c
 - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + 6*b^2*x^2*PolyLog[2
```

```
, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + (6*I)*b*x*PolyLog[
3, (c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - (6*I)*b*x*Poly
Log[3, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + 3*PolyLog[4,
(c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - 3*PolyLog[4, (I +
 c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 30.66 (sec) , antiderivative size = 7869, normalized size of antiderivative = 19.72

method	result	size
risch	Expression too large to display	7869

```
[In] int(x^2*arctan(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1589 vs. 2(283) = 566.

Time = 0.43 (sec) , antiderivative size = 1589, normalized size of antiderivative = 3.98

$$\int x^2 \arctan(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arctan(d*cot(b*x + a) + c) + 6*b^2*x^2*dilog(-(c^2 + d^2 -
 (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*
sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I
*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^
2*dilog(-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1)
 - 6*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a)
 + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d
 + 1) + 1) - 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1
/2) + 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos
(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) + 4
*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x
 + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 4*I*a^3
```

```
*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a
) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 6*I*b*x*polyl
og(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2
+ I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 +
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a))/(c^2 + d^2 - 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2
 + d^2 + 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d
+ 1)) - 4*(-I*b^3*x^3 - I*a^3)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*c
os(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/
(c^2 + d^2 + 2*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log((c^2 + d^2 - (c^2 - 2*I*
c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2
*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log((c^2 + d^
2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 -
I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 4*(-I*b^3*x^3 - I*a
^3)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 +
2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 3*p
olylog(4, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*
d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x +
 2*a))/(c^2 + d^2 - 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos
(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 +
 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I
*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\cot(a + bx))\,dx = \text{Timed out}$$

```
[In] integrate(x**2*atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2 \arctan(c + d\cot(a + bx))\, dx = \int x^2 \arctan\left(d\cot\left(bx + a\right) + c\right)\, dx$$

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/6*x^3*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1
)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/6*x^3*arctan2(-c*cos(2
*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) - c*s
in(2*b*x + 2*a) - d - 1) + 4*b*d*integrate(1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2
*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*
x + 2*a)^2 - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x +
2*a) + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*
cos(2*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/
(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*co
s(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x +
2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(
c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^
4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x +
 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*
(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(
c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x +
 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x^2 \arctan(c + d\cot(a + bx))\, dx = \int x^2 \arctan\left(d\cot\left(bx + a\right) + c\right)\, dx$$

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*cot(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\cot(a + bx))\, dx = \int x^2\, \mathrm{atan}(c + d\cot(a + b\,x))\, dx$$

```
[In] int(x^2*atan(c + d*cot(a + b*x)),x)
```

```
[Out] int(x^2*atan(c + d*cot(a + b*x)), x)
```

3.62 $\int x \arctan(c + d\cot(a + bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 303

$$\int x\arctan(c+d\cot(a+bx))\,dx=\frac{1}{2}x^2\arctan(c+d\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)+\frac{x\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^2}-\frac{i\,\text{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}$$

```
[Out] 1/2*x^2*arctan(c+d*cot(b*x+a))+1/4*I*x^2*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))-1/4*I*x^2*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*x
*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*x*polylog(2,(c+I*(
1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b+1/8*I*polylog(3,(1+I*c-d)*exp(2*I*a
+2*I*b*x)/(1+I*c+d))/b^2-1/8*I*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+
I*(1-d)))/b^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5285, 2221, 2611, 2320, 6724}

$$\int x\arctan(c+d\cot(a+bx))\,dx=\frac{1}{2}x^2\arctan(d\cot(a+bx)+c)+\frac{i\,\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^2}-\frac{i\,\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}+\frac{x\,\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)$$

```
[In] Int[x*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/4)*x^2*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x*PolyLog[2, ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x*PolyLog[2, ((c + I*(1 + d))*
E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/8)*PolyLog[3, ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/8)*PolyLog[3, (
(c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5285

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (Dist[b*((1 + I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2
*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
- Dist[b*((1 - I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*I
*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c+d\cot(a+bx)) \\
&\quad+\frac{1}{2}(b(1+ic-d))\int\frac{e^{2ia+2ibx}x^2}{1+ic+d+(-1-ic+d)e^{2ia+2ibx}}\,dx \\
&\quad-\frac{1}{2}(b(1-ic+d))\int\frac{e^{2ia+2ibx}x^2}{1-ic-d+(-1+ic-d)e^{2ia+2ibx}}\,dx \\
&= \frac{1}{2}x^2\arctan(c+d\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad-\frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\
&\quad+\frac{1}{2}i\int x\log\left(1+\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)\,dx \\
&\quad-\frac{1}{2}i\int x\log\left(1+\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)\,dx
\end{aligned}
$$

$$= \frac{1}{2}x^2\arctan(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) + \frac{x\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} - \frac{x\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} + \frac{\int \text{PolyLog}\left(2,-\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)dx}{4b} - \frac{\int \text{PolyLog}\left(2,-\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)dx}{4b}$$

$$= \frac{1}{2}x^2\arctan(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) + \frac{x\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} - \frac{x\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} + \frac{i\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-\frac{(-1-ic+d)x}{1+ic+d}\right)}{x}dx,x,e^{2ia+2ibx}\right)}{8b^2} - \frac{i\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{(c+i(1+d))x}{c-i(-1+d)}\right)}{x}dx,x,e^{2ia+2ibx}\right)}{8b^2}$$

$$= \frac{1}{2}x^2\arctan(c+d\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) + \frac{x\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} - \frac{x\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} + \frac{i\,\text{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^2} - \frac{i\,\text{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.91

$$\int x\arctan(c+d\cot(a+bx))\,dx = \frac{4b^2x^2\arctan(c+d\cot(a+bx)) + 2ib^2x^2\log\left(1+\frac{(-c+i(1+d))e^{-2i(a+bx)}}{c+i(-1+d)}\right) - 2ib^2x^2\log\left(1+\frac{(-c+i(-1+d))e^{-2i(a+bx}}{c+i(1+d)}\right.}{}$$

```
[In] Integrate[x*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (4*b^2*x^2*ArcTan[c + d*Cot[a + b*x]] + (2*I)*b^2*x^2*Log[1 + (-c + I*(1 +
d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - (2*I)*b^2*x^2*Log[1 + (-c + I
*(-1 + d))/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - 2*b*x*PolyLog[2, (c - I
*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + 2*b*x*PolyLog[2, (I + c
 - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + I*PolyLog[3, (c - I*(1 + d
))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - I*PolyLog[3, (I + c - I*d)/((c
 + I*(1 + d))*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.15 (sec) , antiderivative size = 7501, normalized size of antiderivative = 24.76

method	result	size
risch	Expression too large to display	7501

```
[In] int(x*arctan(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(213) = 426.

Time = 0.41 (sec) , antiderivative size = 1289, normalized size of antiderivative = 4.25

$$\int x \arctan(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arctan(d*cot(b*x + a) + c) + 2*b*x*dilog(-(c^2 + d^2 - (c^2
 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2
*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(-(c^2 + d^2
 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)
*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(-(c^2
 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - 2*b*x*dilo
g(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d
 - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) + 2*I*
a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2
*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a^2*log
(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a^2*log(-1/2*c
```

```
^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*
c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 2*I*a^2*log(-1/2*c^2 + I
*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 2*(-I*b^2*x^2 + I*a^2)*log((c^
2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*
d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 2*(I*b^2*x^2
- I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c
^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1))
- 2*(I*b^2*x^2 - I*a^2)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 +
d^2 - 2*d + 1)) - 2*(-I*b^2*x^2 + I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d -
d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) -
2*d + 1)/(c^2 + d^2 - 2*d + 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*c
os(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2
+ 2*d + 1)) - I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I
*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)) - I*poly
log(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^
2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + I*polylog(3, ((c^2 - 2*I*
c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x +
2*a))/(c^2 + d^2 - 2*d + 1)))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int x\arctan(c+d\cot(a+bx))\,dx=\text{Timed out}$$

```
[In] integrate(x*atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\arctan(c+d\cot(a+bx))\,dx=\int x\arctan\left(d\cot\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^2*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1
)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/4*x^2*arctan2(-c*cos(2
*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) - c*s
in(2*b*x + 2*a) - d - 1) + 2*b*d*integrate((2*(c^2 + d^2 + 1)*x^2*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x +
2*a)^2 - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a)
```

```
 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(
2*b*x + 2*a) - (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4
 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*
b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)
^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4
+ d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 +
d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a
) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4
 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(c*d^
3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a
))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x\arctan(c+d\cot(a+bx))\,dx=\int x\arctan\left(d\cot\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctan(d*cot(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+d\cot(a+bx))\,dx=\int x\,\mathrm{atan}(c+d\cot(a+b\,x))\,dx$$

```
[In] int(x*atan(c + d*cot(a + b*x)),x)
[Out] int(x*atan(c + d*cot(a + b*x)), x)
```

3.63 $\int \arctan(c+d\cot(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 198

$$\int \arctan(c+d\cot(a+bx))\,dx = x\arctan(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) + \frac{\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} - \frac{\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}$$

```
[Out] x*arctan(c+d*cot(b*x+a))+1/2*I*x*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d
))-1/2*I*x*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*polylog(2,(
1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*polylog(2,(c+I*(1+d))*exp(2*I*
a+2*I*b*x)/(c+I*(1-d)))/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{5277, 2221, 2317, 2438\}$

$$\int \arctan(c+d\cot(a+bx))\,dx = x\arctan(d\cot(a+bx)+c) + \frac{\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b} - \frac{\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} + \frac{1}{2}ix\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right) - \frac{1}{2}ix\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)$$

```
[In] Int[ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Cot[a + b*x]] + (I/2)*x*Log[1 - ((1 + I*c - d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c + d)] - (I/2)*x*Log[1 - ((c + I*(1 + d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 - d))] + PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)
*b*x))/(1 + I*c + d)]/(4*b) - PolyLog[2, ((c + I*(1 + d))*E^((2*I)*a + (2*I
)*b*x))/(c + I*(1 - d))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5277

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] + (Dist[b*(1 + I*c - d), Int[x*(E^(2*I*a + 2*I*b
*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] - Dist[b*(1
- I*c + d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*
a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan(c+d\cot(a+bx))+(b(1+ic-d))\int\frac{e^{2ia+2ibx}x}{1+ic+d+(-1-ic+d)e^{2ia+2ibx}}\,dx \\
&\quad-(b(1-ic+d))\int\frac{e^{2ia+2ibx}x}{1-ic-d+(-1+ic-d)e^{2ia+2ibx}}\,dx \\
&= x\arctan(c+d\cot(a+bx))+\frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad-\frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)+\frac{1}{2}i\int\log\left(1+\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)dx \\
&\quad-\frac{1}{2}i\int\log\left(1+\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)dx \\
&= x\arctan(c+d\cot(a+bx))+\frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad-\frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{(-1+ic-d)x}{1-ic-d}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{4b} \\
&\quad-\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{(-1-ic+d)x}{1+ic+d}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{4b} \\
&= x\arctan(c+d\cot(a+bx))+\frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad-\frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\
&\quad+\frac{\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b}-\frac{\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1648 vs. 2(198) = 396.

Time = 21.13 (sec) , antiderivative size = 1648, normalized size of antiderivative = 8.32

$$\int\arctan(c+d\cot(a+bx))\,dx = x\arctan(c+d\cot(a+bx)) + \frac{d\left(4a\sqrt{-d^2}\arctan\left(\frac{cd+\tan(a+bx)+c^2\tan(a+bx)}{d}\right)+id\log(1+i\tan(a+bx))\log\left(\frac{cd-\sqrt{-d^2}+\tan(a+bx)+c^2\tan(a+bx)}{i+ic^2+cd-\sqrt{-d^2}}\right.\right.}{\frac{d\log\left(1-\frac{\left(1+c^2\right)(1-i\tan(a+bx))}{1+c^2+icd-i\sqrt{-d^2}}\right)\sec^2(a+bx)}{1-i\tan(a+bx)}-\frac{d\log\left(1-\frac{\left(1+c^2\right)(1-i\tan(a+bx))}{1+c^2+icd+i\sqrt{-d^2}}\right)\sec^2(a+bx}{1-i\tan(a+bx)}}$$

```
[In] Integrate[ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Cot[a + b*x]] + (d*(4*a*Sqrt[-d^2]*ArcTan[(c*d + Tan[a + b*x
] + c^2*Tan[a + b*x])/d] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2
] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d - Sqrt[-d^2])] + I*d*
Log[1 - I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a +
b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c
*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[
-d^2])] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*
Tan[a + b*x])/(I + I*c^2 - c*d + Sqrt[-d^2])] - I*d*PolyLog[2, ((1 + c^2)*(
1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])] + I*d*PolyLog[2, ((1
 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])] - I*d*PolyL
og[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])] +
I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-
d^2])])*((2*a)/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)]
 - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])) - (2*(a + b*x))/(b*(-1 -
 c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)]
 - 2*c*d*Sin[2*(a + b*x)]))))/((d*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/
(1 + c^2 + I*c*d - I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d
*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])]
*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) + (d*Log[(c*d + Sqrt[-d^2] + Tan[a +
b*x] + c^2*Tan[a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(
1 - I*Tan[a + b*x]) - (d*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x])
/(I + I*c^2 - c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d*
Log[1 - ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])]*
Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[1 - ((1 + c^2)*(1 + I*Tan[a +
 b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*
x]) - (d*Log[(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^
2 + c*d - Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[(c*d +
 Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[-d^2
])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (I*d*Log[1 + I*Tan[a + b*x]]*(Se
c[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*
Tan[a + b*x]) + (I*d*Log[1 - I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a +
b*x]^2))/(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x]) - (I*d*Log[1
+ I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d + Sqrt[-d^2]
+ Tan[a + b*x] + c^2*Tan[a + b*x]) + (I*(1 + c^2)*d*Log[1 - I*Tan[a + b*x]]
*Sec[a + b*x]^2)/(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x]) + (4*a*Sqrt
[-d^2]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(d*(1 + (c*d + Tan[a + b*x] +
 c^2*Tan[a + b*x])^2/d^2)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1144 vs. 2(168) = 336.

Time = 2.87 (sec) , antiderivative size = 1145, normalized size of antiderivative = 5.78

method	result	size
derivativedivides	Expression too large to display	1145
default	Expression too large to display	1145
risch	Expression too large to display	4986

```
[In] int(arctan(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arctan(c+d*cot(b*x+a))+d^2*(-1/d*arct
an(d*((c+d*cot(b*x+a))/d-c/d)+c)*arctan(-(c+d*cot(b*x+a))/d+c/d)-1/d^2*(-1/
2*I*d*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)*ln(1-(I+c+I*d)*(1+I*(d*((c+d*cot
(b*x+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))-1/2*
d*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^2-1/4*d*polylog(2,(I+c+I*d)*(1+I*(d*
((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I
-c))+1/2*I*d^2*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((
c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)
+c)/(1+I*c+d)+1/2*I*d*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2
/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))
/d-c/d)+c)/(1+I*c+d)+1/2*I*d/(c-I*d-I)*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x
+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c*arctan(
d*((c+d*cot(b*x+a))/d-c/d)+c)+1/2*d^2*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^
2/(1+I*c+d)+1/4*d^2*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c)
)^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/2*d*arctan(
d*((c+d*cot(b*x+a))/d-c/d)+c)^2/(1+I*c+d)+1/2*d/(c-I*d-I)*c*arctan(d*((c+d*
cot(b*x+a))/d-c/d)+c)^2+1/4*d*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))
/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/4
*d/(c-I*d-I)*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d
*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 965 vs. 2(140) = 280.

Time = 0.40 (sec) , antiderivative size = 965, normalized size of antiderivative = 4.87

$$\int \arctan(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arctan(d*cot(b*x + a) + c) - 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2
 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d
+ I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c
^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(
2*b*x + 2*a) + 1/2) + 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2
 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x +
2*a) - 1/2) - 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1
/2) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x
 + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d
^2 + 2*d + 1)) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)
*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)
/(c^2 + d^2 + 2*d + 1)) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d -
 d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a)
- 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2
- 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b
*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + dilog(-(c^2 + d^2 - (c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
+ 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + dilog(-(c^2 + d^2 - (c^2 - 2
*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
+ 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - dilog(-(c^2 + d^2 - (c^2 + 2
*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
 + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - dilog(-(c^2 + d^2 - (c^2 -
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1))/b
```

Sympy [F]

$$\int \arctan(c + d\cot(a + bx))\,dx = \int \operatorname{atan}\left(c + d\cot\left(a + bx\right)\right)dx$$

```
[In] integrate(atan(c+d*cot(b*x+a)),x)
```

```
[Out] Integral(atan(c + d*cot(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(140) = 280$.

Time = 0.34 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.69

$$\int \arctan(c+d\cot(a+bx))\,dx =$$

$$-\frac{d\left(\frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)}{d}-\frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)-4\,\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)\arctan\left(\frac{cd+\left(c^2+a\right.}{c^2+}\right.}{}\right.}{}$$

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(d*(8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)/d - (8*(b*x +
 a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d) - 4*arctan((c*d + (c^2 + 1)*ta
n(b*x + a))/d)*arctan2((c*d + (c^2 + d + 1)*tan(b*x + a))/(c^2 + d^2 + 2*d
+ 1), -(c*d*tan(b*x + a) - c^2 - d - 1)/(c^2 + d^2 + 2*d + 1)) + 4*arctan((
c*d + (c^2 + 1)*tan(b*x + a))/d)*arctan2(-(c*d + (c^2 - d + 1)*tan(b*x + a)
)/(c^2 + d^2 - 2*d + 1), -(c*d*tan(b*x + a) - c^2 + d - 1)/(c^2 + d^2 - 2*d
 + 1)) - (log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) -
 log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 - 2*d + 1)))*log((c^2
+ 1)*d^2 + 2*(c^3 + c)*d*tan(b*x + a) + (c^4 + 2*c^2 + 1)*tan(b*x + a)^2) -
 2*dilog(((I*c - 1)*tan(b*x + a) + I*d)/(c + I*d + I)) + 2*dilog(((I*c + 1)
*tan(b*x + a) + I*d)/(c + I*d - I)) + 2*dilog(-((I*c - 1)*tan(b*x + a) + I*
d)/(c - I*d + I)) - 2*dilog(-((I*c + 1)*tan(b*x + a) + I*d)/(c - I*d - I)))
/d) - 8*(b*x + a)*arctan(c + d/tan(b*x + a)) - 8*(b*x + a)*arctan((c*d + (c
^2 + 1)*tan(b*x + a))/d))/b
```

Giac [F]

$$\int \arctan(c+d\cot(a+bx))\,dx = \int \arctan\left(d\cot\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*cot(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c + d\cot(a + bx))\,dx = \int \operatorname{atan}(c + d\cot(a + b\,x))\,dx$$

```
[In] int(atan(c + d*cot(a + b*x)),x)
[Out] int(atan(c + d*cot(a + b*x)), x)
```

3.64 $\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c+d\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*cot(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx = \int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c + d*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\cot(a + bx))}{x}\,dx = \int \frac{\arctan(c + d\cot(a + bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c + d*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + d*Cot[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(c + d\cot\left(bx + a\right)\right)}{x}dx$$

```
[In] int(arctan(c+d*cot(b*x+a))/x,x)
[Out] int(arctan(c+d*cot(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\cot(a + bx))}{x}\,dx = \int \frac{\arctan\left(d\cot\left(bx + a\right) + c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*cot(b*x + a) + c)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c + d\cot(a + bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(atan(c+d*cot(b*x+a))/x,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(c + d\cot(a + bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\cot(a + bx))}{x}\,dx = \int \frac{\arctan\left(d\cot\left(bx + a\right) + c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*cot(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\cot(a + bx))}{x}\,dx = \int \frac{\mathrm{atan}(c + d\cot(a + b\,x))}{x}\,dx$$

```
[In] int(atan(c + d*cot(a + b*x))/x,x)
[Out] int(atan(c + d*cot(a + b*x))/x, x)
```

3.65 $\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) + \frac{x^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}$$

```
[Out] 1/12*b*x^4-1/3*x^3*arctan(-c-(1-I*c)*cot(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*I
*a+2*I*b*x))+1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(3,
I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {5281, 2215, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3 \arctan(c+(1-ic)\cot(a+bx)) - \frac{\text{PolyLog}\left(4, ice^{2ia+2ibx}\right)}{8b^3} + \frac{ix\,\text{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{4b^2} + \frac{x^2\,\text{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} + \frac{1}{6}ix^3 \log\left(1-ice^{2ia+2ibx}\right) + \frac{bx^4}{12}$$

```
[In] Int[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - PolyLog[
4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))-\frac{1}{3}(ib)\int\frac{x^3}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))+\frac{1}{3}(bc)\int\frac{e^{2ia+2ibx}x^3}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))\\
&\quad+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{1}{2}i\int x^2\log\left(1-\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)\,dx\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad+\frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{\int x\,\text{PolyLog}\left(2,\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)\,dx}{2b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c + (1 - ic)\cot(a + bx)) \\
&\quad + \frac{1}{6}ix^3 \log\left(1 - ice^{2ia+2ibx}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} \\
&\quad + \frac{ix \operatorname{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{4b^2} - \frac{i \int \operatorname{PolyLog}\left(3, \frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right) dx}{4b^2} \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c + (1 - ic)\cot(a + bx)) \\
&\quad + \frac{1}{6}ix^3 \log\left(1 - ice^{2ia+2ibx}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} \\
&\quad + \frac{ix \operatorname{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, icx)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^3} \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c + (1 - ic)\cot(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - ice^{2ia+2ibx}\right) \\
&\quad + \frac{x^2 \operatorname{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, ice^{2ia+2ibx}\right)}{8b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$
\int x^2 \arctan(c + (1 - ic)\cot(a + bx))\, dx = \frac{1}{24}\left(8x^3 \arctan(c + (1 - ic)\cot(a + bx)) + 4ix^3 \log\left(1 + \frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2 \operatorname{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix \operatorname{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3 \operatorname{PolyLog}\left(4, -\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)
$$

```
[In] Integrate[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (4*I)*x^3*Log[1 + I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)
*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, (-I)/(c*E^
((2*I)*(a + b*x)))])/b^3)/24
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.42 (sec) , antiderivative size = 1487, normalized size of antiderivative = 9.66

method	result	size
risch	Expression too large to display	1487

```
[In] int(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I*(2*I*Pi-2*ln(I+c)+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c))*c
sgn(I*(I+c)/(exp(2*I*(b*x+a))-1))+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c
)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))
+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I
*(b*x+a))-1))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a
))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))
-1))-I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*
(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*cs
gn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn((exp(2*I*(b*x
+a))*c+I)/(exp(2*I*(b*x+a))-1))^3-I*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I
*(b*x+a))-1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+
I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+
a))*(I+c)/(exp(2*I*(b*x+a))-1))-I*Pi*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b
*x+a))-1))^2-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(e
xp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2
*I*(b*x+a))-1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a)
)*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))-I*Pi*csgn(I*(I+
c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1)
)^2+I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*csgn(I*(I+c
)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+
a))-1))^3+I*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3-I*Pi*csg
n(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+
a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a)
)-1))^2-2*I*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2)*x^3+1/2*I
/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*
x+a))*(-I*c)^(1/2))*x+1/4*I*x*polylog(3,I*exp(2*I*(b*x+a))*c)/b^2+1/2*I/b^2
*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/6*I/b^3*a^3*ln(exp(2*I*(b*x+a)
)*c+I)+1/4*x^2*polylog(2,I*exp(2*I*(b*x+a))*c)/b-1/4/b^3*polylog(2,I*exp(2*
I*(b*x+a))*c)*a^2-1/3*I/b^3*ln(1-I*exp(2*I*(b*x+a))*c)*a^3-1/8*polylog(4,I*
exp(2*I*(b*x+a))*c)/b^3+1/6*I*x^3*ln(1-I*exp(2*I*(b*x+a))*c)-1/2*I/b^2*ln(1
-I*exp(2*I*(b*x+a))*c)*x*a^2+1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/
2))-1/6*I*x^3*ln(exp(2*I*(b*x+a))*c+I)+1/3*I*x^3*ln(exp(I*(b*x+a)))+1/2/b^3
*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2/b^3*a^2*dilog(1+I*exp(I*(b*
x+a))*(-I*c)^(1/2))+1/12*b*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx$$
$$= \frac{2\,b^4x^4 + 4i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right) - 2\,a^4 - 4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right) + 6i\,bx\mathrm{pol}}{24\,b^3}$$

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*b^4*x^4 + 4*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) + I)) + 6*b^2*x^2*dilog(I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 - 4*I*a
^3*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*I*b*x*polylog(3, I*c*e^(2*I*b*x +
 2*I*a)) - 4*(-I*b^3*x^3 - I*a^3)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) - 3*pol
ylog(4, I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(-x**2*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(110) = 220.

Time = 0.21 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.01

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx$$
$$= \frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((-i\,c+1)\cot(bx+a)+c)}{b^2} + \frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx+a\right.\right.}{}}{}$$

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c + 1)
*cot(b*x + a) + c)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*a
rctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6
*I*(b*x + a)*a + 3*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 -
 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2*
b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^(
2*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b^2
*(c + I)))/b
```

Giac [F]

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx=\int -x^2 \arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx=\int x^2\,\mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(-1+c\,1{}\mathrm{i}\right)\right)\,dx$$

```
[In] int(x^2*atan(c - cot(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x^2*atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.66 $\int x\arctan(c+(1-ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int x\arctan(c+(1-ic)\cot(a+bx))\,dx=\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}$$

```
[Out] 1/6*b*x^3-1/2*x^2*arctan(-c-(1-I*c)*cot(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*I*
a+2*I*b*x))+1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,I*c*e
xp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5281, 2215, 2221, 2611, 2320, 6724}

$$\int x\arctan(c+(1-ic)\cot(a+bx))\,dx=\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx))+\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{bx^3}{6}$$

```
[In] Int[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 -
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/
(4*b) + ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx)) - \frac{1}{2}(ib)\int \frac{x^2}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx)) + \frac{1}{2}(bc)\int \frac{e^{2ia+2ibx}x^2}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) - \frac{1}{2}i\int x\log\left(1-\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) + \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} - \frac{\int \text{PolyLog}\left(2,\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)dx}{4b}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) + \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{i\text{Subst}\left(\int \frac{\text{PolyLog}(2,icx)}{x}\,dx, x, e^{2ia+2ibx}\right)}{8b^2}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) + \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x\arctan(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx)) + \frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

```
[In] Integrate[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 1452, normalized size of antiderivative = 11.80

method	result	size
risch	Expression too large to display	1452

```
[In] int(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/6*b*x^3+1/2*I/b*ln(1-I*e
xp(2*I*(b*x+a))*c)*a*x+1/2*I*x^2*ln(exp(I*(b*x+a)))-1/4*I*ln(exp(2*I*(b*x+a
))*c+I)*x^2+1/8*I/b^2*polylog(3,I*exp(2*I*(b*x+a))*c)-1/2*I/b*a*ln(1-I*exp(
I*(b*x+a))*(-I*c)^(1/2))*x+1/4/b*polylog(2,I*exp(2*I*(b*x+a))*c)*x+1/4/b^2*
polylog(2,I*exp(2*I*(b*x+a))*c)*a-1/8*I*(2*I*Pi-2*ln(I+c)+I*Pi*csgn(I/(exp(
2*I*(b*x+a))-1))*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))+I*Pi*csgn
(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x
+a))*(I+c)/(exp(2*I*(b*x+a))-1))+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I
*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)
))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I
*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))-I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(ex
p(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+I*Pi
*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a
))-1))^2+I*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3-I*Pi*csgn
((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a
))*c+I)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I
*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))-I*Pi*csgn(I
*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*exp(2*I*(b*x+a)))*
csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I/(exp(2*I*
(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I/(exp(2*I*(b*x
+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(
2*I*(b*x+a))-1))-I*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*
x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*ex
p(2*I*(b*x+a)))+I*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(I*exp(2
*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/
(exp(2*I*(b*x+a))-1))^3-I*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-
1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*
I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-2*I*Pi*csgn(I*exp(I*(b*x+a)))*csgn(
I*exp(2*I*(b*x+a)))^2)*x^2+1/4*I*ln(1-I*exp(2*I*(b*x+a))*c)*x^2+1/4*I/b^2*a
^2*ln(exp(2*I*(b*x+a))*c+I)+1/4*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*a^2-1/2*I/
b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x
+a))*(-I*c)^(1/2))-1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2/b^2
*a*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int x \arctan(c + (1 - ic)\cot(a + bx))\,dx = \frac{4\,b^3x^3 + 6i\,b^2x^2\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right) + 4\,a^3 + 6\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right) + 6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right) - 6\,(-i\,b^2x^2}{24\,b^2}$$

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*b^3*x^3 + 6*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) + I)) + 4*a^3 + 6*b*x*dilog(I*c*e^(2*I*b*x + 2*I*a)) + 6*I*a^2*l
og((c*e^(2*I*b*x + 2*I*a) + I)/c) - 6*(-I*b^2*x^2 + I*a^2)*log(-I*c*e^(2*I*
b*x + 2*I*a) + 1) + 3*I*polylog(3, I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x \arctan(c + (1 - ic)\cot(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(-x*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(88) = 176.

Time = 0.19 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.76

$$\int x \arctan(c + (1 - ic)\cot(a + bx))\,dx = \frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan((-i\,c+1)\cot(bx+a)+c)}{b} + \frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right)\mathrm{ar}}{}}{}$$

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c + 1)*cot(b*x + a) + c)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x +
```

```
2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), c
*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x
+ 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(3
, I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(c + I)))/b
```

Giac [F]

$$\int x\arctan(c+(1-ic)\cot(a+bx))\,dx = \int -x\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(-x*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(1-ic)\cot(a+bx))\,dx = \int x\,\mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(-1+c\,1\mathrm{i}\right)\right)\,dx$$

```
[In] int(x*atan(c - cot(a + b*x)*(c*1i - 1)),x)
[Out] int(x*atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.67 $\int \arctan(c+(1-ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx=\frac{bx^2}{2}+x\arctan(c+(1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*b*x^2-x*arctan(-c-(1-I*c)*cot(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b*
x))+1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5273, 2215, 2221, 2317, 2438}

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx=x\arctan(c+(1-ic)\cot(a+bx))+\frac{\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{bx^2}{2}$$

```
[In] Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (I/2)*x*Log[1 - I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5273

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan(c+(1-ic)\cot(a+bx))-(ib)\int\frac{x}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&=\frac{bx^2}{2}+x\arctan(c+(1-ic)\cot(a+bx))+(bc)\int\frac{e^{2ia+2ibx}x}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&=\frac{bx^2}{2}+x\arctan(c+(1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad-\frac{1}{2}i\int\log\left(1-\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)dx\\
&=\frac{bx^2}{2}+x\arctan(c+(1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad-\frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{cx}{-i(1-ic)+c}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{4b}
\end{aligned}$$

$$= \frac{bx^2}{2} + x\arctan(c+(1-ic)\cot(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) + \frac{\text{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 929 vs. 2(85) = 170.

Time = 11.67 (sec) , antiderivative size = 929, normalized size of antiderivative = 10.93

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx = x\arctan(c+(1-ic)\cot(a+bx))$$

$$-\frac{}{(i+\cot(a+bx))(1+ic+(i+c)\cot(a+bx))\left(2ibx+\log\left(1-\frac{\sec(bx)((-i+c)\cos(a)+i(i+c)\sin(a))(\cos(a+bx)-i\sin}{2c}\right.\right.}$$

```
[In] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] - (I*x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos
[b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I +
c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - I*L
og[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] -
(2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x
]] + I*PolyLog[2, (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b
*x] - I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] -
(I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2])*(Cos[b*x] - I*Sin[b*x]
)*(Cos[b*x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(1 + I*c + (I + c)*Cot[a + b
*x])*((2*I)*b*x + Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(C
os[a + b*x] - I*Sin[a + b*x]))/(2*c)] + Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a
] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2] + ((-I + c)*Cos[a +
 b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] +
(1 + I*c)*Sin[a + b*x]) + ((I + c)*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x
]])*Sin[a + b*x])/((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]) + 2*b*x*
Tan[b*x] + I*Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a
+ b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-1 - I*c)
*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - I*
Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Cos[a]^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + I*
Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (I*Log[(Sec[b*x]*((1 - I*c)*Cos[a +
 b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I
+ Tan[b*x]) - (I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a + b*x] +
(1 + I*c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + Tan[b*x])))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(73) = 146.

Time = 1.07 (sec) , antiderivative size = 584, normalized size of antiderivative = 6.87

method	result
derivativedivides	$-\frac{\frac{\arctan(-c+\cot(bx+a)(ic-1))\ln(-i+\cot(bx+a)(ic-1)-c)c^2}{2i+2c}+\frac{2i\arctan(-c+\cot(bx+a)(ic-1))\ln(-i+\cot(bx+a)(ic-1)-c)c}{2i+2c}-\text{arct}}{}$
default	$-\frac{\frac{\arctan(-c+\cot(bx+a)(ic-1))\ln(-i+\cot(bx+a)(ic-1)-c)c^2}{2i+2c}+\frac{2i\arctan(-c+\cot(bx+a)(ic-1))\ln(-i+\cot(bx+a)(ic-1)-c)c}{2i+2c}-\text{arct}}{}$
risch	Expression too large to display

```
[In] int(-arctan(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(-1+I*c)*(arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)*(-
1+I*c)-c)*c^2+2*I*arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)
*(-1+I*c)-c)*c-arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)*(-
1+I*c)-c)-arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)+c
+I)*c^2-2*I*arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)
+c+I)*c+arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)+c+I
)-(-1+I*c)^2*(-1/2/(I+c)*(1/2*I*(dilog(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))+ln
(-I+cot(b*x+a)*(-1+I*c)-c)*ln(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I)))-1/4*I*ln(-
I+cot(b*x+a)*(-1+I*c)-c)^2)+1/2/(I+c)*(1/2*I*(dilog(-1/2*(cot(b*x+a)*(-1+I*
c)-c+I)/c)+ln(cot(b*x+a)*(-1+I*c)+c+I)*ln(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)
)-1/2*I*(dilog((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))+ln(cot(b*x+a)*(-1+I*c
)+c+I)*ln((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx = \frac{2\,b^2x^2+2i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)-2\,a^2-2\left(-i\,bx-i\,a\right)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right)-2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)}-}{c}\right.}{4\,b}$$

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2 + 2*I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2
*I*a) + I)) - 2*a^2 - 2*(-I*b*x - I*a)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) -
2*I*a*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + dilog(I*c*e^(2*I*b*x + 2*I*a)))/
b
```

Sympy [F(-2)]

Exception generated.

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(-atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(64) = 128$.

Time = 0.36 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.39

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx =$$

$$-\frac{(i\,c-1)\left(\frac{4i\,(bx+a)\log\left(-\frac{2\left(i\,c^2-\left(c^2+1\right)\tan(bx+a)-2\,c-i\right)}{-2i\,c^2+2\left(c^2+1\right)\tan(bx+a)-2i}\right)}{i\,c-1}-\frac{i\left(4\,(bx+a)\left(\log\left(-i\,c^2+\left(c^2+1\right)\tan(bx+a)+2\,c+i\right)-\log\left(-i\,c^2+\left(c^2+1\right)\tan\right.\right.\right.}{}\right.}{}$$

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c - 1)*(4*I*(b*x + a)*log(-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) - 2*c
 - I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 2*I))/(I*c - 1) - I*(4*(b*x +
a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I) - log(-I*c^2 + (c^2 + 1)
*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I)*lo
g(-1/2*((I*c + 1)*tan(b*x + a) + c + I)/c + 1) + 2*I*log(-I*c^2 + (c^2 + 1)
*tan(b*x + a) + 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(1/2*(c - I)*tan(b*
x + a) - 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a) - I)^2 -
 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) - I)*log(-
1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a) + 1) - 2*I
*dilog(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) - 2*I*dilog(1/2*((I*c + 1
)*tan(b*x + a) + c + I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)
) - 8*(b*x + a)*arctan(c + (-I*c + 1)/tan(b*x + a)) + 4*(-I*b*x - I*a)*log(
-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) - 2*c - I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b
*x + a) - 2*I)))/b
```

Giac [F]

$$\int \arctan(c + (1 - ic)\cot(a + bx))\,dx = \int -\arctan\left(-(-i\,c + 1)\cot\left(bx + a\right) - c\right)\,dx$$

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c + (1 - ic)\cot(a + bx))\,dx = \int \mathrm{atan}(c - \cot(a + b\,x)\,\left(-1 + c\,1\mathrm{i}\right))\,dx$$

```
[In] int(atan(c - cot(a + b*x)*(c*1i - 1)),x)
[Out] int(atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.68 $\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c+(1-ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int -\frac{\arctan\left(-c-(-ic+1)\cot\left(bx+a\right)\right)}{x}dx$$

```
[In] int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
[Out] int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \int -\frac{\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) + I)
)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(-atan(-c-(1-I*c)*cot(b*x+a))/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \int -\frac{\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="giac")
[Out] integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}\left(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right)\right)}{x}\,dx$$

```
[In] int(atan(c - cot(a + b*x)*(c*1i - 1))/x,x)
[Out] int(atan(c - cot(a + b*x)*(c*1i - 1))/x, x)
```

3.69 $\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = -\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx)) -\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) -\frac{x^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} -\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} +\frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}$$

```
[Out] -1/12*b*x^4-1/3*x^3*arctan(-c+(1+I*c)*cot(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*
I*a+2*I*b*x))-1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog(
3,-I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{5281, 2215, 2221, 2611, 6744, 2320, 6724\}$

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx)) + \frac{\text{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3} - \frac{ix\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} - \frac{x^2\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) - \frac{bx^4}{12}$$

```
[In] Int[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/3 - (I/6)*x^3*Log[
1 + I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I
)*b*x)])/(4*b) - ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 +
 PolyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{3}(ib)\int\frac{x^3}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&=-\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{3}(bc)\int\frac{e^{2ia+2ibx}x^3}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&=-\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx))\\
&\quad-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}i\int x^2\log\left(1-\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)dx\\
&=-\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad-\frac{x^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{\int x\operatorname{PolyLog}\left(2,\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)dx}{2b}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c - (1+ic)\cot(a+bx)) \\
&\quad - \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) - \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} \\
&\quad - \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} + \frac{i \int \operatorname{PolyLog}\left(3, \frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right) dx}{4b^2} \\
&= -\frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c - (1+ic)\cot(a+bx)) \\
&\quad - \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) - \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} \\
&\quad - \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} + \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-icx)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^3} \\
&= -\frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(c - (1+ic)\cot(a+bx)) - \frac{1}{6}ix^3 \log\Big(1 \\
&\qquad + ice^{2ia+2ibx}\Big) - \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} \\
&\quad - \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4, -ice^{2ia+2ibx}\right)}{8b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$
\int x^2 \arctan(c + (-1-ic)\cot(a+bx))\, dx = \frac{1}{3}x^3 \arctan(c + (-1-ic)\cot(a+bx)) \\
- \frac{4ib^3x^3 \log\left(1 - \frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2 \operatorname{PolyLog}\left(2, \frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx \operatorname{PolyLog}\left(3, \frac{ie^{-2i(a+bx)}}{c}\right) + 3 \operatorname{PolyLog}\left(4\right.}{24b^3}
$$

```
[In] Integrate[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 - I/(c*E
^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] + (6
*I)*b*x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, I/(c*E^((2*I)*
(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.50 (sec) , antiderivative size = 1488, normalized size of antiderivative = 9.60

method	result	size
risch	Expression too large to display	1488

```
[In] int(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/12*I*(-I*Pi*csgn(I*ex
p(2*I*(b*x+a)))^3+2*I*Pi-I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))
-1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^3-I*Pi*csgn(I*exp(2*I*(b*x+a
))*(c-I)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*
(b*x+a))-1))^3-I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*
csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(
b*x+a))-1))^2+2*ln(c-I)-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I))*csg
n(I/(exp(2*I*(b*x+a))-1)*(c-I))-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2
*I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))-I
*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+
a))*c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I/(
exp(2*I*(b*x+a))-1)*(c-I))^2+I*Pi*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+a))-1)
*(c-I))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(
2*I*(b*x+a))-1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))
^3+I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3-I*Pi*csgn(I*exp
(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(
2*I*(b*x+a))-1))-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))
*c-I)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(e
xp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*
c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1)
)+I*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*
x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(
I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1
))-I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+2*I*Pi*csgn(I*e
xp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2)*x^3-1/4*I*x*polylog(3,-I*exp(2*I
*(b*x+a))*c)/b^2+1/6*I*x^3*ln(exp(2*I*(b*x+a))*c-I)+1/2*I/b^2*ln(I*exp(2*I*
(b*x+a))*c+1)*x*a^2-1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/4*x^
2*polylog(2,-I*exp(2*I*(b*x+a))*c)/b+1/4/b^3*polylog(2,-I*exp(2*I*(b*x+a))*
c)*a^2+1/6*I/b^3*a^3*ln(-exp(2*I*(b*x+a))*c+I)+1/8*polylog(4,-I*exp(2*I*(b*
x+a))*c)/b^3-1/3*I*x^3*ln(exp(I*(b*x+a)))-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a
))*(I*c)^(1/2))*x-1/6*I*x^3*ln(I*exp(2*I*(b*x+a))*c+1)-1/2*I/b^3*a^3*ln(1-I
*exp(I*(b*x+a))*(I*c)^(1/2))+1/3*I/b^3*ln(I*exp(2*I*(b*x+a))*c+1)*a^3-1/2/b
^3*a^2*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/2/b^3*a^2*dilog(1-I*exp(I*(b
*x+a))*(I*c)^(1/2))-1/12*b*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = -\frac{2\,b^4x^4 - 4i\,b^3x^3\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}-i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right) - 2\,a^4 - 4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}}{c}\right.}{2}$$

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/24*(2*b^4*x^4 - 4*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(-I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 - 4*
I*a^3*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*I*b*x*polylog(3, -I*c*e^(2*I*b
*x + 2*I*a)) + 4*(I*b^3*x^3 + I*a^3)*log(I*c*e^(2*I*b*x + 2*I*a) + 1) - 3*p
olylog(4, -I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(-x**2*atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(109) = 218.

Time = 0.22 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.01

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = \frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((-i\,c-1)\cot(bx+a)+c)}{b^2} - \frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(bx+\right.\right.}{}}{}$$

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c - 1)
*cot(b*x + a) + c)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)*
arctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 -
 6*I*(b*x + a)*a + 3*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^
3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin
(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*c
*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)
/(b^2*(c - I)))/b
```

Giac [F]

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = \int -x^2 \arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)\,dx$$

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(-x^2*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = \int x^2\,\mathrm{atan}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

```
[In] int(x^2*atan(c - cot(a + b*x)*(c*1i + 1)),x)
[Out] int(x^2*atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.70 $\int x \arctan(c+(-1-ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx=-\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)-\frac{x\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{i\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}$$

```
[Out] -1/6*b*x^3-1/2*x^2*arctan(-c+(1+I*c)*cot(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*I
*a+2*I*b*x))-1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,-I*
c*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5281, 2215, 2221, 2611, 2320, 6724}

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx=\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{i\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)-\frac{bx^3}{6}$$

```
[In] Int[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/2 - (I/4)*x^2*Log[1
 + I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) - ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{2}(ib)\int\frac{x^2}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{2}(bc)\int\frac{e^{2ia+2ibx}x^2}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))\\
&\quad -\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}i\int x\log\left(1-\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)dx\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad -\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{\int\text{PolyLog}\left(2,\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)dx}{4b}\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad -\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-icx)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^2}\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad -\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{i\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\begin{aligned}
&\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx\\
&= \frac{1}{2}x^2\arctan(c+(-1-ic)\cot(a+bx))\\
&\quad -\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}
\end{aligned}$$

```
[In] Integrate[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
[Out] (x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
 PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 1453, normalized size of antiderivative = 11.72

method	result	size
risch	Expression too large to display	1453

```
[In] int(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/4*I*ln(I*exp(2*I*(b*x+a)
)*c+1)*x^2-1/4*I/b^2*ln(I*exp(2*I*(b*x+a))*c+1)*a^2+1/2*I/b^2*a^2*ln(1-I*ex
p(I*(b*x+a))*(I*c)^(1/2))+1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/
4/b*polylog(2,-I*exp(2*I*(b*x+a))*c)*x-1/4/b^2*polylog(2,-I*exp(2*I*(b*x+a)
)*c)*a-1/8*I/b^2*polylog(3,-I*exp(2*I*(b*x+a))*c)-1/4*I/b^2*a^2*ln(-exp(2*I
*(b*x+a))*c+I)+1/4*I*x^2*ln(exp(2*I*(b*x+a))*c-I)-1/8*I*(-I*Pi*csgn(I*exp(2
*I*(b*x+a)))^3+2*I*Pi-I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1)
)^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^3-I*Pi*csgn(I*exp(2*I*(b*x+a))*
(c-I)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*
x+a))-1))^3-I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*csg
n(I/(exp(2*I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x
+a))-1))^2+2*ln(c-I)-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I))*csgn(I
/(exp(2*I*(b*x+a))-1)*(c-I))-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*
(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))-I*Pi
*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))
*c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I/(exp
(2*I*(b*x+a))-1)*(c-I))^2+I*Pi*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+a))-1)*(c
-I))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I
*(b*x+a))-1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+
I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3-I*Pi*csgn(I*exp(2*
I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I
*(b*x+a))-1))-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-
I)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(
2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I
)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))+I
*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a
))*(c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(
exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))-
I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+2*I*Pi*csgn(I*exp(
I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2)*x^2-1/2*I*x^2*ln(exp(I*(b*x+a)))-1/
2*I/b*ln(I*exp(2*I*(b*x+a))*c+1)*a*x+1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I*
c)^(1/2))+1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/6*b*x^3+1/2*I/b
*a*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx = -\frac{4\,b^3x^3 - 6i\,b^2x^2\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}-i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 4\,a^3 + 6\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right) + 6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right.}{24\,b^2}$$

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b^3*x^3 - 6*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 4*a^3 + 6*b*x*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + 6*I*a^
2*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*(I*b^2*x^2 - I*a^2)*log(I*c*e^(2*I
*b*x + 2*I*a) + 1) + 3*I*polylog(3, -I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(-x*atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(87) = 174.

Time = 0.20 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.77

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx = \frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan((-i\,c-1)\cot(bx+a)+c)}{b} - \frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx+a)a\right.\right.}{}}{}$$

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c - 1)*cot(b*x + a) + c)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x +
```

```
 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
 -c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b
*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polylo
g(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(c - I)))/b
```

Giac [F]

$$\int x \arctan(c + (-1 - ic)\cot(a + bx))\,dx = \int -x \arctan\left(-(-i\,c - 1)\cot\left(bx + a\right) - c\right)\,dx$$

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(-x*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \arctan(c + (-1 - ic)\cot(a + bx))\,dx = \int x\,\mathrm{atan}\left(c - \cot\left(a + b\,x\right)\,\left(1 + c\,1\mathrm{i}\right)\right)\,dx$$

```
[In] int(x*atan(c - cot(a + b*x)*(c*1i + 1)),x)
[Out] int(x*atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.71 $\int \arctan(c+(-1-ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \arctan(c+(-1-ic)\cot(a+bx))\,dx = -\frac{bx^2}{2} + x\arctan(c-(1+ic)\cot(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) - \frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] -1/2*b*x^2-x*arctan(-c+(1+I*c)*cot(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b
*x))-1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5273, 2215, 2221, 2317, 2438}

$$\int \arctan(c+(-1-ic)\cot(a+bx))\,dx = x\arctan(c-(1+ic)\cot(a+bx)) - \frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} - \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) - \frac{bx^2}{2}$$

```
[In] Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcTan[c - (1 + I*c)*Cot[a + b*x]] - (I/2)*x*Log[1 + I*c*E
^((2*I)*a + (2*I)*b*x)] - PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5273

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\arctan(c-(1+ic)\cot(a+bx))-(ib)\int\frac{x}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^2}{2}+x\arctan(c-(1+ic)\cot(a+bx))-(bc)\int\frac{e^{2ia+2ibx}x}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^2}{2}+x\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad+\frac{1}{2}i\int\log\left(1-\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)dx
\end{aligned}
$$

$$= -\frac{bx^2}{2} + x\arctan(c-(1+ic)\cot(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) + \frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{cx}{-i(-1-ic)+c}\right)}{x}\,dx, x, e^{2ia+2ibx}\right)}{4b}$$

$$= -\frac{bx^2}{2} + x\arctan(c-(1+ic)\cot(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) - \frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 872 vs. $2(86) = 172$.

Time = 6.71 (sec) , antiderivative size = 872, normalized size of antiderivative = 10.14

$$\int \arctan(c+(-1-ic)\cot(a+bx))\,dx = x\arctan(c+(-1-ic)\cot(a+bx)) + \frac{ix\csc(a+bx)\left(2bx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+i\log\left(\frac{\sec(}{}\right.\right.}{(i+\cot(a+bx))((-i+c)\cos(a+bx)+i(i+c)\sin(a+bx))\left(-2ibx-\log\left(1-\frac{\sec(bx)((i+c)\cos(a)+(1+ic)\sin}{2}\right.\right.}$$

```
[In] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]] + (I*x*Csc[a + b*x]*(2*b*x*Log[2*Cos[
b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I +
c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - I*L
og[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a +
b*x]))/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] +
I*PolyLog[2, (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] -
I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a]
 + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x]
 + I*Sin[b*x]))/((I + Cot[a + b*x])*((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[
a + b*x])*((-2*I)*b*x - Log[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a
])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)] - (Log[1 - I*Tan[b*x]]*((I + c)*
Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*
Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[
a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) + (Log[(Sec[b*x
]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]*S
ec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*Tan[b*x] - I*Log[1 - (Sec[b*x]*((I + c)
*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x
] + I*Log[1 - I*Tan[b*x]]*Tan[b*x] - I*Log[1 + I*Tan[b*x]]*Tan[b*x] + (I*Lo
g[1 - ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*
```

```
x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*
((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + T
an[b*x])))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(72) = 144$.

Time = 1.19 (sec) , antiderivative size = 625, normalized size of antiderivative = 7.27

method	result
derivativedivides	$-\frac{-\frac{\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c^2}{2i-2c}+\frac{2i\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c}{2i-2c}+\mathrm{arct}}{}$
default	$-\frac{-\frac{\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c^2}{2i-2c}+\frac{2i\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c}{2i-2c}+\mathrm{arct}}{}$
risch	Expression too large to display

```
[In] int(-arctan(-c-(-1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(I*c+1)*(-arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x
+a)-c)*c^2+2*I*arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x
+a)-c)*c+arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x+a)-c)
+arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(b*x+a)-c+I)*c^2-2*
I*arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(b*x+a)-c+I)*c-arc
tan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(b*x+a)-c+I)+(I*c+1)^2*
(-1/2/(I-c)*(-1/2*I*((ln(I+(I*c+1)*cot(b*x+a)-c)-ln(-1/2*I*(I+(I*c+1)*cot(b
*x+a)-c)))*ln(-1/2*I*(I-(I*c+1)*cot(b*x+a)+c))-dilog(-1/2*I*(I+(I*c+1)*cot(
b*x+a)-c)))+1/4*I*ln(I+(I*c+1)*cot(b*x+a)-c)^2)+1/2/(I-c)*(1/2*I*(dilog((-I
-(I*c+1)*cot(b*x+a)+c)/(-2*I+2*c))+ln(-(I*c+1)*cot(b*x+a)-c+I)*ln((-I-(I*c+
1)*cot(b*x+a)+c)/(-2*I+2*c)))-1/2*I*(dilog(1/2*(I-(I*c+1)*cot(b*x+a)+c)/c)+
ln(-(I*c+1)*cot(b*x+a)-c+I)*ln(1/2*(I-(I*c+1)*cot(b*x+a)+c)/c)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \arctan(c + (-1 - ic)\cot(a + bx))\,dx = -\frac{2\,b^2x^2 - 2i\,bx\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}-i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) - 2\,a^2 + 2\,(i\,bx + i\,a)\log\left(i\,ce^{(2i\,bx+2i\,a)} + 1\right) - 2i\,a\log\left(\frac{ce^{(2}}{}\right.}{4\,b}$$

[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")

[Out] -1/4*(2*b^2*x^2 - 2*I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2*I*a)/(c - I)) - 2*a^2 + 2*(I*b*x + I*a)*log(I*c*e^(2*I*b*x + 2*I*a) + 1) - 2*I*a*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + dilog(-I*c*e^(2*I*b*x + 2*I*a)))/b

Sympy [F(-2)]

Exception generated.

$$\int \arctan(c + (-1 - ic)\cot(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

[In] integrate(-atan(-c-(-1-I*c)*cot(b*x+a)),x)

[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(63) = 126.

Time = 0.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.33

$$\int \arctan(c + (-1 - ic)\cot(a + bx))\,dx = -\frac{(i\,c+1)\left(\frac{4i\,(bx+a)\log\left(-\frac{2\left(i\,c^2-\left(c^2+1\right)\tan(bx+a)+i\right)}{-2i\,c^2+2\left(c^2+1\right)\tan(bx+a)-4\,c+2i}\right)}{i\,c+1} + \frac{i\left(4\,(bx+a)(\log(-i\,c^2+(c^2+1)\tan(bx+a)-2\,c+i)-\log(-i\,c^2+(c^2+1)\,\mathrm{ta}\right.}{}\right.}{}$$

[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")

```
[Out] -1/8*((I*c + 1)*(4*I*(b*x + a)*log(-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) + I)/
(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 4*c + 2*I))/(I*c + 1) + I*(4*(b*x +
a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I) - log(-I*c^2 + (c^2 + 1)
*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I)*lo
g(-1/2*((I*c - 1)*tan(b*x + a) + c - I)/c + 1) + 2*I*log(-I*c^2 + (c^2 + 1)
*tan(b*x + a) - 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(-1/2*(c + I)*tan(b
*x + a) + 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a) - I)^2
- 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) - I)*log(
-1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a) + 1) - 2*
I*dilog(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) - 2*I*dilog(1/2*((I*c - 1
)*tan(b*x + a) + c - I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)
) - 8*(b*x + a)*arctan(c + (-I*c - 1)/tan(b*x + a)) + 4*(-I*b*x - I*a)*log(
-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) + I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a
) - 4*c + 2*I)))/b
```

Giac [F]

$$\int \arctan(c+(-1-ic)\cot(a+bx))\,dx=\int -\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)\,dx$$

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c+(-1-ic)\cot(a+bx))\,dx=\int \mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(1+c\,1{}\mathrm{i}\right)\right)\,dx$$

```
[In] int(atan(c - cot(a + b*x)*(c*1i + 1)),x)
[Out] int(atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.72 $\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx=\text{Int}\left(\frac{\arctan(c+(-1-ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx=\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int -\frac{\arctan\left(-c-(-ic-1)\cot\left(bx+a\right)\right)}{x}dx$$

```
[In] int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
[Out] int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \int -\frac{\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(-atan(-c-(-1-I*c)*cot(b*x+a))/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \int -\frac{\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="giac")
[Out] integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(1+c\,1\mathrm{i}\right)\right)}{x}\,dx$$

```
[In] int(atan(c - cot(a + b*x)*(c*1i + 1))/x,x)
[Out] int(atan(c - cot(a + b*x)*(c*1i + 1))/x, x)
```

3.73 $\int \arctan(\sinh(x))\,dx$

Optimal result

Integrand size = 3, antiderivative size = 39

$$\int \arctan(\sinh(x))\,dx = -2x\arctan\left(e^x\right) + x\arctan(\sinh(x)) + i\operatorname{PolyLog}\left(2,-ie^x\right) - i\operatorname{PolyLog}\left(2,ie^x\right)$$

```
[Out] -2*x*arctan(exp(x))+x*arctan(sinh(x))+I*polylog(2,-I*exp(x))-I*polylog(2,I*
exp(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {5311, 4265, 2317, 2438}

$$\int \arctan(\sinh(x))\,dx = -2x\arctan\left(e^x\right) + x\arctan(\sinh(x)) + i\operatorname{PolyLog}\left(2,-ie^x\right) - i\operatorname{PolyLog}\left(2,ie^x\right)$$

```
[In] Int[ArcTan[Sinh[x]],x]
```

```
[Out] -2*x*ArcTan[E^x] + x*ArcTan[Sinh[x]] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2
, I*E^x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan(\sinh(x)) - \int x\text{sech}(x)\,dx \\
&= -2x\arctan\left(e^x\right) + x\arctan(\sinh(x)) + i\int \log\left(1-ie^x\right)\,dx - i\int \log\left(1+ie^x\right)\,dx \\
&= -2x\arctan\left(e^x\right) + x\arctan(\sinh(x)) + i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^x\right) \\
&\quad - i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^x\right) \\
&= -2x\arctan\left(e^x\right) + x\arctan(\sinh(x)) + i\,\text{PolyLog}\left(2,-ie^x\right) - i\,\text{PolyLog}\left(2,ie^x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \arctan(\sinh(x))\,dx = x\arctan(\sinh(x)) - i(x(\log\left(1-ie^x\right) - \log\left(1+ie^x\right)) - \text{PolyLog}\left(2,-ie^x\right) + \text{PolyLog}\left(2,ie^x\right))$$

```
[In] Integrate[ArcTan[Sinh[x]],x]
```

```
[Out] x*ArcTan[Sinh[x]] - I*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I
)*E^x] + PolyLog[2, I*E^x])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

method	result	size
default	$x \arctan(\sinh(x)) - ix(\ln(1 - ie^{x}) - \ln(1 + ie^{x})) + i \,\mathrm{dilog}(1 + ie^{x}) - i \,\mathrm{dilog}(1 - ie^{x})$	52
parts	$x \arctan(\sinh(x)) - ix(\ln(1 - ie^{x}) - \ln(1 + ie^{x})) + i \,\mathrm{dilog}(1 + ie^{x}) - i \,\mathrm{dilog}(1 - ie^{x})$	52
risch	Expression too large to display	651

```
[In] int(arctan(sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arctan(sinh(x))-I*x*(ln(1-I*exp(x))-ln(1+I*exp(x)))+I*dilog(1+I*exp(x))-I
*dilog(1-I*exp(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \arctan(\sinh(x))\,dx = {} & x\arctan(\sinh(x)) + i\,x\log(i\cosh(x) + i\sinh(x) + 1) \\ & - i\,x\log(-i\cosh(x) - i\sinh(x) + 1) \\ & - i\,\mathrm{Li}_2(i\cosh(x) + i\sinh(x)) + i\,\mathrm{Li}_2(-i\cosh(x) - i\sinh(x))\end{aligned}$$

```
[In] integrate(arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] x*arctan(sinh(x)) + I*x*log(I*cosh(x) + I*sinh(x) + 1) - I*x*log(-I*cosh(x)
 - I*sinh(x) + 1) - I*dilog(I*cosh(x) + I*sinh(x)) + I*dilog(-I*cosh(x) - I
*sinh(x))
```

Sympy [F]

$$\int \arctan(\sinh(x))\,dx = \int \operatorname{atan}(\sinh(x))\,dx$$

```
[In] integrate(atan(sinh(x)),x)
```

```
[Out] Integral(atan(sinh(x)), x)
```

Maxima [F]

$$\int \arctan(\sinh(x))\,dx = \int \arctan\left(\sinh\left(x\right)\right)\,dx$$

```
[In] integrate(arctan(sinh(x)),x, algorithm="maxima")
[Out] x*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(x*e^x/(e^(2*x) + 1), x)
```

Giac [F]

$$\int \arctan(\sinh(x))\,dx = \int \arctan\left(\sinh\left(x\right)\right)\,dx$$

```
[In] integrate(arctan(sinh(x)),x, algorithm="giac")
[Out] integrate(arctan(sinh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(\sinh(x))\,dx = \int \mathrm{atan}(\sinh(x))\,dx$$

```
[In] int(atan(sinh(x)),x)
[Out] int(atan(sinh(x)), x)
```

3.74 $\int x \arctan(\sinh(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 74

$$\int x \arctan(\sinh(x))\,dx = -x^2 \arctan\left(e^x\right) + \frac{1}{2}x^2 \arctan(\sinh(x)) + ix \operatorname{PolyLog}\left(2, -ie^x\right) - ix \operatorname{PolyLog}\left(2, ie^x\right) - i \operatorname{PolyLog}\left(3, -ie^x\right) + i \operatorname{PolyLog}\left(3, ie^x\right)$$

```
[Out] -x^2*arctan(exp(x))+1/2*x^2*arctan(sinh(x))+I*x*polylog(2,-I*exp(x))-I*x*po
lylog(2,I*exp(x))-I*polylog(3,-I*exp(x))+I*polylog(3,I*exp(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5313, 4265, 2611, 2320, 6724}

$$\int x \arctan(\sinh(x))\,dx = x^2\left(-\arctan\left(e^x\right)\right) + \frac{1}{2}x^2 \arctan(\sinh(x)) + ix \operatorname{PolyLog}\left(2, -ie^x\right) - ix \operatorname{PolyLog}\left(2, ie^x\right) - i \operatorname{PolyLog}\left(3, -ie^x\right) + i \operatorname{PolyLog}\left(3, ie^x\right)$$

```
[In] Int[x*ArcTan[Sinh[x]],x]
```

```
[Out] -(x^2*ArcTan[E^x]) + (x^2*ArcTan[Sinh[x]])/2 + I*x*PolyLog[2, (-I)*E^x] - I
*x*PolyLog[2, I*E^x] - I*PolyLog[3, (-I)*E^x] + I*PolyLog[3, I*E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(\sinh(x)) - \frac{1}{2}\int x^2\operatorname{sech}(x)\,dx \\
&= -x^2\arctan\left(e^x\right)+\frac{1}{2}x^2\arctan(\sinh(x))+i\int x\log\left(1-ie^x\right)\,dx-i\int x\log\left(1+ie^x\right)\,dx \\
&= -x^2\arctan\left(e^x\right)+\frac{1}{2}x^2\arctan(\sinh(x))+ix\operatorname{PolyLog}\left(2,-ie^x\right) \\
&\quad -ix\operatorname{PolyLog}\left(2,ie^x\right)-i\int\operatorname{PolyLog}\left(2,-ie^x\right)\,dx+i\int\operatorname{PolyLog}\left(2,ie^x\right)\,dx
\end{aligned}$$

$$= -x^2 \arctan\left(e^x\right) + \frac{1}{2}x^2 \arctan(\sinh(x)) + ix\,\text{PolyLog}\left(2, -ie^x\right) - ix\,\text{PolyLog}\left(2, ie^x\right) - i\text{Subst}\left(\int \frac{\text{PolyLog}(2, -ix)}{x}\,dx, x, e^x\right) + i\text{Subst}\left(\int \frac{\text{PolyLog}(2, ix)}{x}\,dx, x, e^x\right)$$

$$= -x^2 \arctan\left(e^x\right) + \frac{1}{2}x^2 \arctan(\sinh(x)) + ix\,\text{PolyLog}\left(2, -ie^x\right) - ix\,\text{PolyLog}\left(2, ie^x\right) - i\,\text{PolyLog}\left(3, -ie^x\right) + i\,\text{PolyLog}\left(3, ie^x\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int x \arctan(\sinh(x))\,dx = \frac{1}{2}x^2 \arctan(\sinh(x)) - \frac{1}{2}i\left(x^2 \log\left(1 - ie^x\right) - x^2 \log\left(1 + ie^x\right) - 2x\,\text{PolyLog}\left(2, -ie^x\right) + 2x\,\text{PolyLog}\left(2, ie^x\right) + 2\,\text{PolyLog}\left(3, -ie^x\right) - 2\,\text{PolyLog}\left(3, ie^x\right)\right)$$

```
[In] Integrate[x*ArcTan[Sinh[x]],x]
```

```
[Out] (x^2*ArcTan[Sinh[x]])/2 - (I/2)*(x^2*Log[1 - I*E^x] - x^2*Log[1 + I*E^x] -
2*x*PolyLog[2, (-I)*E^x] + 2*x*PolyLog[2, I*E^x] + 2*PolyLog[3, (-I)*E^x] -
 2*PolyLog[3, I*E^x])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 632, normalized size of antiderivative = 8.54

method	result	size
risch	Expression too large to display	632

```
[In] int(x*arctan(sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*x^2*ln(exp(x)+I)-1/2*I*x^2*ln(exp(x)-I)+1/2*I*x^2*ln(1+I*exp(x))+I*x*
polylog(2,-I*exp(x))-I*polylog(3,-I*exp(x))-1/8*Pi*(csgn(I*(exp(x)-I))^2*cs
gn(I*(exp(x)-I)^2)-2*csgn(I*(exp(x)-I))*csgn(I*(exp(x)-I)^2)^2+csgn(I*(exp(
x)-I)^2)^3+csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2
)-csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x)*(exp(x)-I)^2)^2-csgn(I*(exp(x)+I))^2*
csgn(I*(exp(x)+I)^2)+2*csgn(I*(exp(x)+I))*csgn(I*(exp(x)+I)^2)^2-csgn(I*(ex
p(x)+I)^2)^3-csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)
^2)+csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x)*(exp(x)+I)^2)^2-csgn(I*exp(-x))*csg
n(I*exp(-x)*(exp(x)-I)^2)^2+csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)^2-
csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)+csgn(exp(-x)*(exp(x
```

```
)+I)^2)^2+csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)+csgn(exp(
-x)*(exp(x)-I)^2)^2+csgn(I*exp(-x)*(exp(x)-I)^2)^3-csgn(I*exp(-x)*(exp(x)-I
)^2)*csgn(exp(-x)*(exp(x)-I)^2)^2-csgn(I*exp(-x)*(exp(x)+I)^2)^3+csgn(I*exp
(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)^2-csgn(exp(-x)*(exp(x)+I)^2)^
3-csgn(exp(-x)*(exp(x)-I)^2)^3-2)*x^2-1/2*I*x^2*ln(1-I*exp(x))-I*x*polylog(
2,I*exp(x))+I*polylog(3,I*exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int x\arctan(\sinh(x))\,dx = \frac{1}{2}\,x^2\arctan\left(\sinh\left(x\right)\right) + \frac{1}{2}i\,x^2\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) - \frac{1}{2}i\,x^2\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) - i\,x\mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) + i\,x\mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right)) + i\,\mathrm{polylog}(3, i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) - i\,\mathrm{polylog}(3, -i\,\cosh\left(x\right) - i\,\sinh\left(x\right))$$

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arctan(sinh(x)) + 1/2*I*x^2*log(I*cosh(x) + I*sinh(x) + 1) - 1/2*I*
x^2*log(-I*cosh(x) - I*sinh(x) + 1) - I*x*dilog(I*cosh(x) + I*sinh(x)) + I*
x*dilog(-I*cosh(x) - I*sinh(x)) + I*polylog(3, I*cosh(x) + I*sinh(x)) - I*p
olylog(3, -I*cosh(x) - I*sinh(x))
```

Sympy [F]

$$\int x\arctan(\sinh(x))\,dx = \int x\,\mathrm{atan}\left(\sinh\left(x\right)\right)\,dx$$

```
[In] integrate(x*atan(sinh(x)),x)
```

```
[Out] Integral(x*atan(sinh(x)), x)
```

Maxima [F]

$$\int x\arctan(\sinh(x))\,dx = \int x\arctan\left(\sinh\left(x\right)\right)\,dx$$

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - integrate(x^2*e^x/(e^(2*x) + 1),
 x)
```

Giac [F]

$$\int x\arctan(\sinh(x))\,dx = \int x\arctan\left(\sinh\left(x\right)\right)\,dx$$

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(sinh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan(\sinh(x))\,dx = \int x\,\mathrm{atan}(\sinh(x))\,dx$$

```
[In] int(x*atan(sinh(x)),x)
```

```
[Out] int(x*atan(sinh(x)), x)
```

3.75 $\int x^2 \arctan(\sinh(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 108

$$\int x^2 \arctan(\sinh(x))\,dx = -\frac{2}{3}x^3 \arctan\left(e^x\right) + \frac{1}{3}x^3 \arctan(\sinh(x)) + ix^2 \operatorname{PolyLog}\left(2,-ie^x\right) - ix^2 \operatorname{PolyLog}\left(2,ie^x\right) - 2ix \operatorname{PolyLog}\left(3,-ie^x\right) + 2ix \operatorname{PolyLog}\left(3,ie^x\right) + 2i \operatorname{PolyLog}\left(4,-ie^x\right) - 2i \operatorname{PolyLog}\left(4,ie^x\right)$$

```
[Out] -2/3*x^3*arctan(exp(x))+1/3*x^3*arctan(sinh(x))+I*x^2*polylog(2,-I*exp(x))-
I*x^2*polylog(2,I*exp(x))-2*I*x*polylog(3,-I*exp(x))+2*I*x*polylog(3,I*exp(
x))+2*I*polylog(4,-I*exp(x))-2*I*polylog(4,I*exp(x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5313, 4265, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(\sinh(x))\,dx = -\frac{2}{3}x^3 \arctan\left(e^x\right) + \frac{1}{3}x^3 \arctan(\sinh(x)) + ix^2 \operatorname{PolyLog}\left(2,-ie^x\right) - ix^2 \operatorname{PolyLog}\left(2,ie^x\right) - 2ix \operatorname{PolyLog}\left(3,-ie^x\right) + 2ix \operatorname{PolyLog}\left(3,ie^x\right) + 2i \operatorname{PolyLog}\left(4,-ie^x\right) - 2i \operatorname{PolyLog}\left(4,ie^x\right)$$

```
[In] Int[x^2*ArcTan[Sinh[x]],x]
```

```
[Out] (-2*x^3*ArcTan[E^x])/3 + (x^3*ArcTan[Sinh[x]])/3 + I*x^2*PolyLog[2, (-I)*E^
x] - I*x^2*PolyLog[2, I*E^x] - (2*I)*x*PolyLog[3, (-I)*E^x] + (2*I)*x*PolyL
og[3, I*E^x] + (2*I)*PolyLog[4, (-I)*E^x] - (2*I)*PolyLog[4, I*E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(\sinh(x)) - \frac{1}{3}\int x^3\operatorname{sech}(x)\,dx \\
&= -\frac{2}{3}x^3\arctan(e^x)+\frac{1}{3}x^3\arctan(\sinh(x))+i\int x^2\log(1-ie^x)\,dx-i\int x^2\log(1+ie^x)\,dx \\
&= -\frac{2}{3}x^3\arctan(e^x)+\frac{1}{3}x^3\arctan(\sinh(x))+ix^2\operatorname{PolyLog}(2,-ie^x) \\
&\quad - ix^2\operatorname{PolyLog}(2,ie^x) - 2i\int x\operatorname{PolyLog}(2,-ie^x)\,dx + 2i\int x\operatorname{PolyLog}(2,ie^x)\,dx \\
&= -\frac{2}{3}x^3\arctan(e^x)+\frac{1}{3}x^3\arctan(\sinh(x))+ix^2\operatorname{PolyLog}(2,-ie^x) \\
&\quad - ix^2\operatorname{PolyLog}(2,ie^x) - 2ix\operatorname{PolyLog}(3,-ie^x) + 2ix\operatorname{PolyLog}(3,ie^x) \\
&\quad + 2i\int\operatorname{PolyLog}(3,-ie^x)\,dx - 2i\int\operatorname{PolyLog}(3,ie^x)\,dx \\
&= -\frac{2}{3}x^3\arctan(e^x)+\frac{1}{3}x^3\arctan(\sinh(x))+ix^2\operatorname{PolyLog}(2,-ie^x) \\
&\quad - ix^2\operatorname{PolyLog}(2,ie^x) - 2ix\operatorname{PolyLog}(3,-ie^x) + 2ix\operatorname{PolyLog}(3,ie^x) \\
&\quad + 2i\text{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-ix)}{x}\,dx,x,e^x\right) - 2i\text{Subst}\left(\int\frac{\operatorname{PolyLog}(3,ix)}{x}\,dx,x,e^x\right) \\
&= -\frac{2}{3}x^3\arctan(e^x)+\frac{1}{3}x^3\arctan(\sinh(x))+ix^2\operatorname{PolyLog}(2,-ie^x)-ix^2\operatorname{PolyLog}(2,ie^x) \\
&\quad -2ix\operatorname{PolyLog}(3,-ie^x)+2ix\operatorname{PolyLog}(3,ie^x)+2i\operatorname{PolyLog}(4,-ie^x)-2i\operatorname{PolyLog}(4,ie^x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\begin{aligned}
\int x^2\arctan(\sinh(x))\,dx &= \frac{1}{3}x^3\arctan(\sinh(x)) \\
&\quad - \frac{1}{3}i\big(x^3\log(1-ie^x) - x^3\log(1+ie^x) - 3x^2\operatorname{PolyLog}(2,-ie^x) \\
&\quad + 3x^2\operatorname{PolyLog}(2,ie^x) + 6x\operatorname{PolyLog}(3,-ie^x) \\
&\quad - 6x\operatorname{PolyLog}(3,ie^x) - 6\operatorname{PolyLog}(4,-ie^x) + 6\operatorname{PolyLog}(4,ie^x)\big)
\end{aligned}$$

```
[In] Integrate[x^2*ArcTan[Sinh[x]],x]
```

```
[Out] (x^3*ArcTan[Sinh[x]])/3 - (I/3)*(x^3*Log[1 - I*E^x] - x^3*Log[1 + I*E^x] -
3*x^2*PolyLog[2, (-I)*E^x] + 3*x^2*PolyLog[2, I*E^x] + 6*x*PolyLog[3, (-I)*
E^x] - 6*x*PolyLog[3, I*E^x] - 6*PolyLog[4, (-I)*E^x] + 6*PolyLog[4, I*E^x]
)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 658, normalized size of antiderivative = 6.09

method	result	size
risch	Expression too large to display	658

```
[In] int(x^2*arctan(sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*x^3*ln(exp(x)+I)-1/3*I*x^3*ln(exp(x)-I)+1/3*I*x^3*ln(1+I*exp(x))+I*x^
2*polylog(2,-I*exp(x))-2*I*x*polylog(3,-I*exp(x))+2*I*polylog(4,-I*exp(x))-
1/12*Pi*(csgn(I*(exp(x)-I))^2*csgn(I*(exp(x)-I)^2)-2*csgn(I*(exp(x)-I))*csg
n(I*(exp(x)-I)^2)^2+csgn(I*(exp(x)-I)^2)^3+csgn(I*(exp(x)-I)^2)*csgn(I*exp(
-x))*csgn(I*exp(-x)*(exp(x)-I)^2)-csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x)*(exp(
x)-I)^2)^2-csgn(I*(exp(x)+I))^2*csgn(I*(exp(x)+I)^2)+2*csgn(I*(exp(x)+I))*c
sgn(I*(exp(x)+I)^2)^2-csgn(I*(exp(x)+I)^2)^3-csgn(I*(exp(x)+I)^2)*csgn(I*ex
p(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)+csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x)*(ex
p(x)+I)^2)^2-csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)^2+csgn(I*exp(-x))
*csgn(I*exp(-x)*(exp(x)+I)^2)^2-csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(
exp(x)+I)^2)+csgn(exp(-x)*(exp(x)+I)^2)^2+csgn(I*exp(-x)*(exp(x)-I)^2)*csgn
(exp(-x)*(exp(x)-I)^2)+csgn(exp(-x)*(exp(x)-I)^2)^2+csgn(I*exp(-x)*(exp(x)-
I)^2)^3-csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)^2-csgn(I*ex
p(-x)*(exp(x)+I)^2)^3+csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^
2)^2-csgn(exp(-x)*(exp(x)+I)^2)^3-csgn(exp(-x)*(exp(x)-I)^2)^3-2)*x^3-1/3*I
*x^3*ln(1-I*exp(x))-I*x^2*polylog(2,I*exp(x))+2*I*x*polylog(3,I*exp(x))-2*I
*polylog(4,I*exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16

$$\begin{aligned}\int x^2 \arctan(\sinh(x))\,dx = {} & \frac{1}{3}x^3\arctan(\sinh(x)) + \frac{1}{3}i\,x^3\log(i\cosh(x) + i\sinh(x) + 1) \\ & - \frac{1}{3}i\,x^3\log(-i\cosh(x) - i\sinh(x) + 1) \\ & - i\,x^2\mathrm{Li}_2(i\cosh(x) + i\sinh(x)) + i\,x^2\mathrm{Li}_2(-i\cosh(x) - i\sinh(x)) \\ & + 2i\,x\mathrm{polylog}(3, i\cosh(x) + i\sinh(x)) \\ & - 2i\,x\mathrm{polylog}(3, -i\cosh(x) - i\sinh(x)) \\ & - 2i\,\mathrm{polylog}(4, i\cosh(x) + i\sinh(x)) \\ & + 2i\,\mathrm{polylog}(4, -i\cosh(x) - i\sinh(x))\end{aligned}$$

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(sinh(x)) + 1/3*I*x^3*log(I*cosh(x) + I*sinh(x) + 1) - 1/3*I*
x^3*log(-I*cosh(x) - I*sinh(x) + 1) - I*x^2*dilog(I*cosh(x) + I*sinh(x)) +
I*x^2*dilog(-I*cosh(x) - I*sinh(x)) + 2*I*x*polylog(3, I*cosh(x) + I*sinh(x
)) - 2*I*x*polylog(3, -I*cosh(x) - I*sinh(x)) - 2*I*polylog(4, I*cosh(x) +
I*sinh(x)) + 2*I*polylog(4, -I*cosh(x) - I*sinh(x))
```

Sympy [F]

$$\int x^2 \arctan(\sinh(x))\,dx = \int x^2 \operatorname{atan}\left(\sinh\left(x\right)\right)dx$$

```
[In] integrate(x**2*atan(sinh(x)),x)
```

```
[Out] Integral(x**2*atan(sinh(x)), x)
```

Maxima [F]

$$\int x^2 \arctan(\sinh(x))\,dx = \int x^2 \arctan\left(\sinh\left(x\right)\right)\,dx$$

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(1/3*x^3*e^x/(e^(2*x)
 + 1), x)
```

Giac [F]

$$\int x^2 \arctan(\sinh(x))\,dx = \int x^2 \arctan\left(\sinh\left(x\right)\right)\,dx$$

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(sinh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(\sinh(x))\,dx = \int x^2\,\mathrm{atan}(\sinh(x))\;dx$$

```
[In] int(x^2*atan(sinh(x)),x)
[Out] int(x^2*atan(sinh(x)), x)
```

3.76 $\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 299

$$\begin{aligned}\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx =& -\frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} \\ &+ \frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f} \\ &+ \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ &- \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ &- \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} \\ &+ \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\ &+ \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ &- \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\ &- \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} \\ &+ \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}\end{aligned}$$

```
[Out] -1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+1/4*(f*x+e)^4*arctan(tanh(b*x+a))/f
+1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^3*polylog(2,I
*exp(2*b*x+2*a))/b-3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2+3/8*I
*f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2+3/8*I*f^2*(f*x+e)*polylog(4,-I
*exp(2*b*x+2*a))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3-3/16
```

```
*I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+3/16*I*f^3*polylog(5,I*exp(2*b*x+2*
a))/b^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5291, 4265, 2611, 6744, 2320, 6724}

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = -\frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f} - \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} + \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[In] Int[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -1/4*((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/f + ((e + f*x)^4*ArcTan[Tanh[a +
 b*x]])/(4*f) + ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b - (((3*I)/8)*f*(e + f*x)^
2*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f*(e + f*x)^2*PolyLog[
3, I*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*
a + 2*b*x)])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/
b^3 - (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 + (((3*I)/16)*f
^3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5291

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Tanh[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f} - \frac{b\int (e+fx)^4 \text{sech}(2a+2bx)\,dx}{4f}$$

$$
\begin{aligned}
&= -\frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f} \\
&\quad + \frac{1}{2} i \int (e+fx)^3 \log\left(1-ie^{2a+2bx}\right) \, dx - \frac{1}{2} i \int (e+fx)^3 \log\left(1+ie^{2a+2bx}\right) \, dx \\
&= -\frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f} \\
&\quad + \frac{i(e+fx)^3 \text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3 \text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad - \frac{(3if) \int (e+fx)^2 \text{PolyLog}\left(2,-ie^{2a+2bx}\right) \, dx}{4b} \\
&\quad + \frac{(3if) \int (e+fx)^2 \text{PolyLog}\left(2,ie^{2a+2bx}\right) \, dx}{4b} \\
&= -\frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f} \\
&\quad + \frac{i(e+fx)^3 \text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3 \text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad - \frac{3if(e+fx)^2 \text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2 \text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\
&\quad + \frac{(3if^2) \int (e+fx) \text{PolyLog}\left(3,-ie^{2a+2bx}\right) \, dx}{4b^2} \\
&\quad - \frac{(3if^2) \int (e+fx) \text{PolyLog}\left(3,ie^{2a+2bx}\right) \, dx}{4b^2} \\
&= -\frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f} \\
&\quad + \frac{i(e+fx)^3 \text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3 \text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad - \frac{3if(e+fx)^2 \text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2 \text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\
&\quad + \frac{3if^2(e+fx) \text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{3if^2(e+fx) \text{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\
&\quad - \frac{(3if^3) \int \text{PolyLog}\left(4,-ie^{2a+2bx}\right) \, dx}{8b^3} + \frac{(3if^3) \int \text{PolyLog}\left(4,ie^{2a+2bx}\right) \, dx}{8b^3}
\end{aligned}
$$

$$= -\frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\arctan(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} - \frac{(3if^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(4,-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{16b^4} + \frac{(3if^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(4,ix)}{x}\,dx,x,e^{2a+2bx}\right)}{16b^4}$$

$$= -\frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4\arctan(\tanh(a+bx))}{4f} + \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} - \frac{3if^3\operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} + \frac{3if^3\operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. 2(299) = 598.

Time = 0.79 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.01

$$\int(e+fx)^3\arctan(\tanh(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\arctan(\tanh(a+bx)) - \frac{i\left(8b^4e^3x\log\left(1-ie^{2(a+bx)}\right)+12b^4e^2fx^2\log\left(1-ie^{2(a+bx)}\right)+8b^4ef^2x^3\log\left(1-ie^{2(a+bx)}\right)+2b^4f^3x^4\log\left(\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Tanh[a + b*x]])/4 - (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
```

```
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.92 (sec) , antiderivative size = 3570, normalized size of antiderivative = 11.94

method	result	size
risch	Expression too large to display	3570

```
[In] int((f*x+e)^3*arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I/f*ln(exp(2*b*x+2*a)-I)*e^4+1/8*I*(f*x+e)^4/f*ln(exp(2*b*x+2*a)+I)-1/
8*I*f^3*ln(exp(2*b*x+2*a)-I)*x^4-1/8*I*f^3*ln(1-I*exp(2*b*x+2*a))*x^4-1/8*I
/f*e^4*ln(exp(2*b*x+2*a)+I)-1/2*I/b*e^3*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^
(1/2))-1/2*I/b*e^3*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*e^3*ln((
(-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(
-I)^(1/2))*x+1/8*I*f^3*ln(1+I*exp(2*b*x+2*a))*x^4+1/8*I/f*e^4*ln(-exp(2*b*x
+2*a)+I)+1/2*I/b*e^3*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I/b*e^3*dilog(1-exp
(b*x+a)*(-1)^(3/4))+1/2*I*e^3*ln(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*e^3*ln(1-
exp(b*x+a)*(-1)^(3/4))*x-1/16*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*
b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+
2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*
a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*
a)+1))^2-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*
a)+1))^2+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*
a)+1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*
x+2*a)+I)/(exp(2*b*x+2*a)+1))+csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)
+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+
2*a)-I)/(exp(2*b*x+2*a)+1))+csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1
))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(exp(2*b*x+2*a)
-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2
-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(exp(2*b*x+2*a)+I)/
(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csg
n((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-csgn((1-I)*(exp(2*b*x+2*a)
-I)/(exp(2*b*x+2*a)+1))^3-1)*(f*x+e)^4/f-1/2*I*f^2*ln(exp(2*b*x+2*a)-I)*x^3
```

*e-3/4*I*f*ln(exp(2*b*x+2*a)-I)*x^2*e^2-1/2*I*ln(exp(2*b*x+2*a)-I)*x*e^3+3/
16*I*f^3*polylog(5,I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,-I*exp(2*b*x+
2*a))/b^4+1/4*I*f^3/b*polylog(2,-I*exp(2*b*x+2*a))*x^3+1/4*I*f^3/b^4*polylo
g(2,-I*exp(2*b*x+2*a))*a^3-3/8*I*f^3/b^2*polylog(3,-I*exp(2*b*x+2*a))*x^2+3
/8*I*f^3/b^3*polylog(4,-I*exp(2*b*x+2*a))*x+3/8*I*f^2/b^3*e*polylog(4,-I*ex
p(2*b*x+2*a))-3/8*I*f/b^2*e^2*polylog(3,-I*exp(2*b*x+2*a))-1/2*I*f^3/b^4*a^
4*ln(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f^3/b^4*a^4*ln(1-exp(b*x+a)*(-1)^(3/4))
-1/2*I*f^3/b^4*a^3*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f^3/b^4*a^3*dilog(1
-exp(b*x+a)*(-1)^(3/4))+1/8*I*f^3/b^4*a^4*ln(-exp(2*b*x+2*a)+I)+1/2*I*f^2*e
*ln(1+I*exp(2*b*x+2*a))*x^3+3/4*I*f*e^2*ln(1+I*exp(2*b*x+2*a))*x^2+1/2*I/b*
e^3*ln(1+exp(b*x+a)*(-1)^(3/4))*a+1/2*I/b*e^3*ln(1-exp(b*x+a)*(-1)^(3/4))*a
-1/2*I/b*a*e^3*ln(-exp(2*b*x+2*a)+I)+3/8*I*f^3/b^4*ln(1+I*exp(2*b*x+2*a))*a
^4+1/2*I*f^3/b^3*ln(1+I*exp(2*b*x+2*a))*a^3*x-1/2*I*f^3/b^3*a^3*ln(1+exp(b*
x+a)*(-1)^(3/4))*x-1/2*I*f^3/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))*x+3/2*I*f^
2/b^3*a^3*e*ln(1+exp(b*x+a)*(-1)^(3/4))+3/2*I*f^2/b^3*a^3*e*ln(1-exp(b*x+a)
*(-1)^(3/4))+3/2*I*f^2/b^3*a^2*e*dilog(1+exp(b*x+a)*(-1)^(3/4))+3/2*I*f^2/b
^3*a^2*e*dilog(1-exp(b*x+a)*(-1)^(3/4))-3/2*I*f/b^2*a^2*e^2*ln(1+exp(b*x+a)
*(-1)^(3/4))-3/2*I*f/b^2*a^2*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))-3/2*I*f/b^2*a*
e^2*dilog(1+exp(b*x+a)*(-1)^(3/4))-3/2*I*f/b^2*a*e^2*dilog(1-exp(b*x+a)*(-1
)^(3/4))-1/2*I*f^2/b^3*a^3*e*ln(-exp(2*b*x+2*a)+I)+3/4*I*f/b^2*a^2*e^2*ln(-
exp(2*b*x+2*a)+I)-I*f^2/b^3*e*ln(1+I*exp(2*b*x+2*a))*a^3+3/4*I*f^2/b*e*poly
log(2,-I*exp(2*b*x+2*a))*x^2-3/4*I*f^2/b^3*e*polylog(2,-I*exp(2*b*x+2*a))*a
^2-3/4*I*f^2/b^2*e*polylog(3,-I*exp(2*b*x+2*a))*x+3/4*I*f/b^2*e^2*ln(1+I*ex
p(2*b*x+2*a))*a^2+3/4*I*f/b*e^2*polylog(2,-I*exp(2*b*x+2*a))*x+3/4*I*f/b^2*
e^2*polylog(2,-I*exp(2*b*x+2*a))*a+I*f^2/b^3*e*ln(1-I*exp(2*b*x+2*a))*a^3+1
/2*I*f^2/b^3*a^3*e*ln(exp(2*b*x+2*a)+I)-3/4*I*f/b^2*a^2*e^2*ln(exp(2*b*x+2*
a)+I)-3/4*I*f/b^2*e^2*ln(1-I*exp(2*b*x+2*a))*a^2-3/4*I*f/b*e^2*polylog(2,I*
exp(2*b*x+2*a))*x-3/4*I*f/b^2*e^2*polylog(2,I*exp(2*b*x+2*a))*a-3/2*I*f^2/b
^3*a^3*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-3/2*I*f^2/b^3*a^3*e*ln(((-I
)^(1/2)+exp(b*x+a))/(-I)^(1/2))-3/2*I*f^2/b^3*a^2*e*dilog(((-I)^(1/2)-exp(b
*x+a))/(-I)^(1/2))-3/2*I*f^2/b^3*a^2*e*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(
1/2))+3/2*I*f/b^2*a^2*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+3/2*I*f/b^
2*a^2*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+3/2*I*f/b^2*a*e^2*dilog(((
-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+3/2*I*f/b^2*a*e^2*dilog(((-I)^(1/2)+exp(b
*x+a))/(-I)^(1/2))-1/2*I*f^3/b^3*ln(1-I*exp(2*b*x+2*a))*a^3*x+1/2*I*f^3/b^3
*a^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f^3/b^3*a^3*ln(((-I)^(1
/2)+exp(b*x+a))/(-I)^(1/2))*x-3/4*I*f^2/b*e*polylog(2,I*exp(2*b*x+2*a))*x^2
+3/4*I*f^2/b^3*e*polylog(2,I*exp(2*b*x+2*a))*a^2+3/4*I*f^2/b^2*e*polylog(3,
I*exp(2*b*x+2*a))*x+1/2*I/b*a*e^3*ln(exp(2*b*x+2*a)+I)-1/2*I/b*e^3*ln(((-I)
^(1/2)-exp(b*x+a))/(-I)^(1/2))*a-1/2*I/b*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I
)^(1/2))*a-3/8*I*f^3/b^4*ln(1-I*exp(2*b*x+2*a))*a^4-1/4*I*f^3/b*polylog(2,I
*exp(2*b*x+2*a))*x^3-1/4*I*f^3/b^4*polylog(2,I*exp(2*b*x+2*a))*a^3+3/8*I*f^
3/b^2*polylog(3,I*exp(2*b*x+2*a))*x^2-3/8*I*f^3/b^3*polylog(4,I*exp(2*b*x+2
*a))*x+1/2*I*f^3/b^4*a^4*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f^3/b
^4*a^4*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-3/2*I*f/b*a*e^2*ln(1+exp(b*x+

```
a)*(-1)^(3/4))*x-3/2*I*f/b*a*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))*x+3/2*I*f^2/b^
2*a^2*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x+3/2*I*f^2/b^2*a^2*e*ln(1-exp(b*x+a)*(
-1)^(3/4))*x-3/2*I*f^2/b^2*e*ln(1+I*exp(2*b*x+2*a))*a^2*x+3/2*I*f/b*e^2*ln(
1+I*exp(2*b*x+2*a))*a*x+3/2*I*f/b*a*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/
2))*x-3/2*I*f^2/b^2*a^2*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-3/2*I*f^
2/b^2*a^2*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+3/2*I*f/b*a*e^2*ln(((-
I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+3/2*I*f^2/b^2*e*ln(1-I*exp(2*b*x+2*a))*a
^2*x-3/2*I*f/b*e^2*ln(1-I*exp(2*b*x+2*a))*a*x+1/2*I*f^3/b^4*a^3*dilog(((-I)
^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)+exp(b*x+
a))/(-I)^(1/2))-1/8*I*f^3/b^4*a^4*ln(exp(2*b*x+2*a)+I)-3/8*I*f^2/b^3*e*poly
log(4,I*exp(2*b*x+2*a))+3/8*I*f/b^2*e^2*polylog(3,I*exp(2*b*x+2*a))-3/4*I*f
*e^2*ln(1-I*exp(2*b*x+2*a))*x^2-1/2*I*f^2*e*ln(1-I*exp(2*b*x+2*a))*x^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(236) = 472$.

Time = 0.35 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2
4*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f
^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f^3*po
lylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
+ 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(sinh(b*x + a)/cos
h(b*x + a)) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^
3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*
x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 -
 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*
x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*
e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-I*b^4*f^3*x
^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3
+ 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(1/2*sqrt(4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*
b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3
*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) +
1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x
 + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(1/2
*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4
*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^
```

```
2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2
 - I*a^4*f^3)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (4*I*a
*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-I*sqrt(4*I
) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*
f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sin
h(b*x + a)) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4
*f^3)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 24*(I*b*f^3*
x + I*b*e*f^2)*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
24*(I*b*f^3*x + I*b*e*f^2)*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(
b*x + a))) - 24*(-I*b*f^3*x - I*b*e*f^2)*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*
x + a) + sinh(b*x + a))) - 24*(-I*b*f^3*x - I*b*e*f^2)*polylog(4, -1/2*sqrt
(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*e*f^
2*x - I*b^2*e^2*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))
) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*e*f^2*x - I*b^2*e^2*f)*polylog(3, -1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*e*f^2
*x + I*b^2*e^2*f)*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*e*f^2*x + I*b^2*e^2*f)*polylog(3, -1/2*sqrt
(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = \int (e+fx)^3 \operatorname{atan}(\tanh(a+bx))\,dx$$

```
[In] integrate((f*x+e)**3*atan(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*atan(tanh(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = \int (fx+e)^3 \arctan(\tanh(bx+a))\,dx$$

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan((e^(2*b*x + 2*a)
 - 1)/(e^(2*b*x + 2*a) + 1)) - integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2
*x^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*b
*x + 4*a) + 1), x)
```

Giac [F(-1)]

Timed out.

$$\int (e+fx)^3 \arctan(\tanh(a+bx)) \, dx = \text{Timed out}$$

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \arctan(\tanh(a+bx)) \, dx = \int \operatorname{atan}(\tanh(a+b\,x)) \, (e+f\,x)^3 \, dx$$

```
[In] int(atan(tanh(a + b*x))*(e + f*x)^3,x)
[Out] int(atan(tanh(a + b*x))*(e + f*x)^3, x)
```

3.77 $\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 229

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = -\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} + \frac{if(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} + \frac{if^2\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{if^2\operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}$$

```
[Out] -1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+1/3*(f*x+e)^3*arctan(tanh(b*x+a))/f
+1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^2*polylog(2,I
*exp(2*b*x+2*a))/b-1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/4*I*f
*(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2+1/8*I*f^2*polylog(4,-I*exp(2*b*x+2
*a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5291, 4265, 2611, 6744, 2320, 6724}

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = -\frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \arctan(\tanh(a+bx))}{3f} + \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} + \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[In] Int[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -1/3*((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/f + ((e + f*x)^3*ArcTan[Tanh[a +
 b*x]])/(3*f) + ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/4)*f*(e + f*x)*Poly
Log[3, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a
+ 2*b*x)])/b^2 + ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 - ((I/8)*
f^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5291

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Tanh[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f}-\frac{b\int(e+fx)^3\operatorname{sech}(2a+2bx)\,dx}{3f}\\ &= -\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f}\\ &\quad+\frac{1}{2}i\int(e+fx)^2\log\left(1-ie^{2a+2bx}\right)\,dx-\frac{1}{2}i\int(e+fx)^2\log\left(1+ie^{2a+2bx}\right)\,dx\end{aligned}$$

$$= -\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{(if)\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)\,dx}{2b} + \frac{(if)\int(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)\,dx}{2b}$$

$$= -\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} + \frac{if(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} + \frac{(if^2)\int\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)\,dx}{4b^2} - \frac{(if^2)\int\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)\,dx}{4b^2}$$

$$= -\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} + \frac{if(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} + \frac{(if^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3} - \frac{(if^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3}$$

$$= -\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f} + \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} + \frac{if(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} + \frac{if^2\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{if^2\operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.64

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\arctan(\tanh(a+bx)) - \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(1\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Tanh[a + b*x]])/3 - ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.45 (sec) , antiderivative size = 2668, normalized size of antiderivative = 11.65

method	result	size
risch	Expression too large to display	2668

```
[In] int((f*x+e)^2*arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+
2*a))/b^3-1/12*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I))*cs
gn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn(I
*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I
*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn(I
*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(
2*b*x+2*a)+1))+csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(
exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*
b*x+2*a)+1))+csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(ex
p(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+
```

2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*
b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)
+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn((1+I)*(exp(2*
b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+
2*a)+1))^3-1)*(f*x+e)^3/f+I*f/b^2*a^2*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/
2))+I*f/b^2*a^2*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+I*f/b^2*a*e*dilog(
((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+I*f/b^2*a*e*dilog(((-I)^(1/2)+exp(b*x+a
))/(-I)^(1/2))-1/2*I*f/b^2*e*ln(1-I*exp(2*b*x+2*a))*a^2-1/2*I*f/b*e*polylog
(2,I*exp(2*b*x+2*a))*x-1/2*I*f/b^2*e*polylog(2,I*exp(2*b*x+2*a))*a-1/2*I*f/
b^2*a^2*e*ln(exp(2*b*x+2*a)+I)+1/2*I*f^2/b^2*ln(1-I*exp(2*b*x+2*a))*a^2*x-1
/2*I*f^2/b^2*a^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*f^2/b^2*a^2
*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+I*f/b*e*ln(1+I*exp(2*b*x+2*a))*a*
x-I*f/b*a*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x-I*f/b*a*e*ln(1-exp(b*x+a)*(-1)^(3
/4))*x+I*f/b*a*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+I*f/b*a*e*ln(((-I
)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x-I*f/b*e*ln(1-I*exp(2*b*x+2*a))*a*x-1/6*I/
f*ln(exp(2*b*x+2*a)-I)*e^3-1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I
)^(1/2))-1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*
f^2/b^3*a^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/4*I*f/b^2*e*polylog
(3,I*exp(2*b*x+2*a))-1/2*I*f*e*ln(1-I*exp(2*b*x+2*a))*x^2+1/6*I*f^2/b^3*a^3
*ln(exp(2*b*x+2*a)+I)+1/3*I*f^2/b^3*ln(1-I*exp(2*b*x+2*a))*a^3-1/4*I*f^2/b*
polylog(2,I*exp(2*b*x+2*a))*x^2+1/4*I*f^2/b^3*polylog(2,I*exp(2*b*x+2*a))*a
^2+1/4*I*f^2/b^2*polylog(3,I*exp(2*b*x+2*a))*x-1/6*I/f*e^3*ln(exp(2*b*x+2*a
)+I)-1/6*I*f^2*ln(1-I*exp(2*b*x+2*a))*x^3-1/2*I/b*e^2*dilog(((-I)^(1/2)-exp
(b*x+a))/(-I)^(1/2))-1/2*I/b*e^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-
1/2*I*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*e^2*ln(((-I)^(1/2)
+exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f^2/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))+1/
2*I*f^2/b^3*a^2*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f^2/b^3*a^2*dilog(1-ex
p(b*x+a)*(-1)^(3/4))-1/4*I*f/b^2*e*polylog(3,-I*exp(2*b*x+2*a))-1/6*I*f^2/b
^3*a^3*ln(-exp(2*b*x+2*a)+I)+1/2*I*f*e*ln(1+I*exp(2*b*x+2*a))*x^2+1/2*I*f/b
^2*e*ln(1+I*exp(2*b*x+2*a))*a^2+1/2*I*f/b*e*polylog(2,-I*exp(2*b*x+2*a))*x+
1/2*I*f/b^2*e*polylog(2,-I*exp(2*b*x+2*a))*a-I*f/b^2*a^2*e*ln(1+exp(b*x+a)*
(-1)^(3/4))-I*f/b^2*a^2*e*ln(1-exp(b*x+a)*(-1)^(3/4))-I*f/b^2*a*e*dilog(1+e
xp(b*x+a)*(-1)^(3/4))-I*f/b^2*a*e*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f^2/
b^2*ln(1+I*exp(2*b*x+2*a))*a^2*x+1/2*I*f^2/b^2*a^2*ln(1+exp(b*x+a)*(-1)^(3/
4))*x+1/2*I*f^2/b^2*a^2*ln(1-exp(b*x+a)*(-1)^(3/4))*x+1/2*I*f/b^2*a^2*e*ln(
-exp(2*b*x+2*a)+I)+1/6*I*f^2*ln(1+I*exp(2*b*x+2*a))*x^3+1/6*I/f*e^3*ln(-exp
(2*b*x+2*a)+I)+1/2*I/b*e^2*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I/b*e^2*dilog
(1-exp(b*x+a)*(-1)^(3/4))+1/2*I*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*e^2
*ln(1-exp(b*x+a)*(-1)^(3/4))*x+1/6*I*(f*x+e)^3/f*ln(exp(2*b*x+2*a)+I)-1/6*I
*f^2*ln(exp(2*b*x+2*a)-I)*x^3-1/2*I*ln(exp(2*b*x+2*a)-I)*x*e^2-1/2*I/b*e^2*
ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*a-1/2*I/b*e^2*ln(((-I)^(1/2)+exp(b*x
+a))/(-I)^(1/2))*a+1/2*I/b*a*e^2*ln(exp(2*b*x+2*a)+I)-1/2*I*f^2/b^3*a^3*ln(
((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f*ln(exp(2*b*x+2*a)-I)*x^2*e+1/2*
I/b*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*a+1/2*I/b*e^2*ln(1-exp(b*x+a)*(-1)^(3/4
))*a-1/2*I/b*a*e^2*ln(-exp(2*b*x+2*a)+I)-1/3*I*f^2/b^3*ln(1+I*exp(2*b*x+2*a

```
))*a^3+1/4*I*f^2/b*polylog(2,-I*exp(2*b*x+2*a))*x^2-1/4*I*f^2/b^3*polylog(2
,-I*exp(2*b*x+2*a))*a^2-1/4*I*f^2/b^2*polylog(3,-I*exp(2*b*x+2*a))*x+1/2*I*
f^2/b^3*a^3*ln(1+exp(b*x+a)*(-1)^(3/4))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(180) = 360.

Time = 0.33 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.38

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(-6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6
*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2
*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2*polyl
og(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3
*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) - 3*(I*b^2*
f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + s
inh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f
*x - I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(
-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x
 + a) + sinh(b*x + a))) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b^3*e^2*x
 - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(4*I)*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b^3*e^2
*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt(4*I)*(cosh(b*
x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3*e
^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3
*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(-4*I)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*
f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (3*I*a*b^2*e^2
- 3*I*a^2*b*e*f + I*a^3*f^2)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*
x + a)) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(I*sqrt(-4*I) + 2
*cosh(b*x + a) + 2*sinh(b*x + a)) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3
*f^2)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 6*(-I*b*f^2*
x - I*b*e*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*
(-I*b*f^2*x - I*b*e*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 6*(I*b*f^2*x + I*b*e*f)*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) - 6*(I*b*f^2*x + I*b*e*f)*polylog(3, -1/2*sqrt(-4*I)*(cosh
(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \int (e+fx)^2 \operatorname{atan}(\tanh(a+bx))\,dx$$

```
[In] integrate((f*x+e)**2*atan(tanh(b*x+a)),x)
[Out] Integral((e + f*x)**2*atan(tanh(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \int (fx+e)^2 \arctan(\tanh(bx+a))\,dx$$

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="maxima")
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x
+ 2*a) + 1)) - integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b
*e^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \int (fx+e)^2 \arctan(\tanh(bx+a))\,dx$$

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \int \operatorname{atan}(\tanh(a+b\,x))\,(e+f\,x)^2\,dx$$

```
[In] int(atan(tanh(a + b*x))*(e + f*x)^2,x)
[Out] int(atan(tanh(a + b*x))*(e + f*x)^2, x)
```

3.78 $\int (e+fx)\arctan(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 159

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = -\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f} + \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{if\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

```
[Out] -1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+1/2*(f*x+e)^2*arctan(tanh(b*x+a))/f
+1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)*polylog(2,I*exp
(2*b*x+2*a))/b-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/8*I*f*polylog(3,I
*exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5291, 4265, 2611, 2320, 6724}

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = -\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f} - \frac{if\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{if\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[In] Int[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -1/2*((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/f + ((e + f*x)^2*ArcTan[Tanh[a +
 b*x]])/(2*f) + ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I/
4)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/8)*f*PolyLog[3, (-I)*E^
(2*a + 2*b*x)])/b^2 + ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5291

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Tanh[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}-\frac{b\int(e+fx)^2\text{sech}(2a+2bx)\,dx}{2f}\\
&= -\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}\\
&\quad+\frac{1}{2}i\int(e+fx)\log\left(1-ie^{2a+2bx}\right)\,dx-\frac{1}{2}i\int(e+fx)\log\left(1+ie^{2a+2bx}\right)\,dx\\
&= -\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}\\
&\quad+\frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}\\
&\quad-\frac{(if)\int\text{PolyLog}\left(2,-ie^{2a+2bx}\right)\,dx}{4b}+\frac{(if)\int\text{PolyLog}\left(2,ie^{2a+2bx}\right)\,dx}{4b}\\
&= -\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}\\
&\quad+\frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}\\
&\quad-\frac{(if)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&\quad+\frac{(if)\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

$$= -\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f} + \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{if\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.49

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = ex\arctan(\tanh(a+bx)) + \frac{1}{2}fx^2\arctan(\tanh(a+bx)) - \frac{ie\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b} - \frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\,\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\,\text{PolyLog}\left(2,\right.\right.}{8b^2}$$

```
[In] Integrate[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] e*x*ArcTan[Tanh[a + b*x]] + (f*x^2*ArcTan[Tanh[a + b*x]])/2 - ((I/4)*e*(2*b
*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x))]) - PolyLog[2, (
-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^(2*(a + b*x))]))/b - ((I/8)*f*(2*b^2*
x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a + b*x))] - 2*b
*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*(a + b*x))] +
 PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 1776, normalized size of antiderivative = 11.17

method	result	size
risch	Expression too large to display	1776

```
[In] int((f*x+e)*arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(
2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*
b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*
x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*
x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*
```

```
x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)
+1))+csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+
2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1
))+csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*
a)-I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*c
sgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)+I
)/(exp(2*b*x+2*a)+1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn(
(1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn((1+I)*(exp(2*b*x+2*a)+I
)/(exp(2*b*x+2*a)+1))^3-csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3
-1)*(1/2*f*x^2+e*x)-1/4*I/b^2*f*a^2*ln(exp(2*b*x+2*a)+I)+1/2*I/b*e*a*ln(exp
(2*b*x+2*a)+I)+1/2*I*(1/2*f*x^2+e*x)*ln(exp(2*b*x+2*a)+I)-1/2*I*e*ln(((-I)^
(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/
2))*x-1/2*I*e/b*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*e/b*dilog((
(-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*e/b*ln(((-I)^(1/2)-exp(b*x+a))/(-I
)^(1/2))*a-1/2*I*e/b*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a+1/2*I*f/b^2*a
^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*e/b*a*ln(-exp(2*b*x+2*a)+I)
+1/4*I*f/b^2*a^2*ln(-exp(2*b*x+2*a)+I)-1/4*I*ln(exp(2*b*x+2*a)-I)*x^2*f-1/2
*I*ln(exp(2*b*x+2*a)-I)*e*x+1/4*I*f*ln(1+I*exp(2*b*x+2*a))*x^2+1/8*I*f*poly
log(3,I*exp(2*b*x+2*a))/b^2+1/2*I*e/b*ln(1+exp(b*x+a)*(-1)^(3/4))*a+1/2*I*e
/b*ln(1-exp(b*x+a)*(-1)^(3/4))*a-1/2*I*f/b^2*a^2*ln(1-exp(b*x+a)*(-1)^(3/4)
)-1/2*I*f/b^2*a*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a*dilog(1-exp(b*
x+a)*(-1)^(3/4))-1/4*I*f/b^2*ln(1-I*exp(2*b*x+2*a))*a^2-1/4*I*f/b*polylog(2
,I*exp(2*b*x+2*a))*x-1/4*I*f/b^2*polylog(2,I*exp(2*b*x+2*a))*a+1/4*I*f/b^2*
ln(1+I*exp(2*b*x+2*a))*a^2+1/4*I*f/b*polylog(2,-I*exp(2*b*x+2*a))*x+1/4*I*f
/b^2*polylog(2,-I*exp(2*b*x+2*a))*a-1/2*I*f/b^2*a^2*ln(1+exp(b*x+a)*(-1)^(3
/4))+1/2*I*f/b^2*a^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f/b^2*a*d
ilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f/b^2*a*dilog(((-I)^(1/2)+ex
p(b*x+a))/(-I)^(1/2))+1/2*I*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*e*ln(1-ex
p(b*x+a)*(-1)^(3/4))*x+1/2*I*e/b*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*e/b*d
ilog(1-exp(b*x+a)*(-1)^(3/4))-1/4*I*f*ln(1-I*exp(2*b*x+2*a))*x^2+1/2*I*f/b*
ln(1+I*exp(2*b*x+2*a))*a*x-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/2*I*f
/b*a*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*f/b*a*ln(1-exp(b*x+a)*(-1)^(3/4))*
x-1/2*I*f/b*ln(1-I*exp(2*b*x+2*a))*a*x+1/2*I*f/b*a*ln(((-I)^(1/2)-exp(b*x+a
))/(-I)^(1/2))*x+1/2*I*f/b*a*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(130) = 260$.

Time = 0.33 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.77

$$\int (e + fx)\arctan(\tanh(a + bx))\,dx$$

$$= \frac{2\,(b^2fx^2 + 2\,b^2ex)\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right) - 2\,(i\,bfx + i\,be)\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i}(\cosh\left(bx + a\right) + \sinh\left(bx + a\right))\right) - 2\,(i\,bf}{}$$

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) - 2*(I*b
*f*x + I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(I*b
*f*x + I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(-I
*b*f*x - I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(
-I*b*f*x - I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
(-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e +
I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f
*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(b*x + a)
 + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*
log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (2*I*a*b*e - I*a
^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2*I*a*b*e - I
*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2*I*a*b*e
 + I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2*I*a
*b*e + I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + 2*
I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polyl
og(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polylog(3, 1/
2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polylog(3, -1/2*sqrt(
-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = \int (e+fx)\,\mathrm{atan}\,(\tanh{(a+bx)})\,dx$$

```
[In] integrate((f*x+e)*atan(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atan(tanh(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = \int (fx+e)\arctan(\tanh(bx+a))\,dx$$

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - i
ntegrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1
), x)
```

Giac [F]

$$\int (e + fx) \arctan(\tanh(a + bx)) \, dx = \int (fx + e) \arctan(\tanh(bx + a)) \, dx$$

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e + fx) \arctan(\tanh(a + bx)) \, dx = \int \operatorname{atan}(\tanh(a + b\,x)) \, (e + f\,x) \, dx$$

```
[In] int(atan(tanh(a + b*x))*(e + f*x),x)
[Out] int(atan(tanh(a + b*x))*(e + f*x), x)
```

3.79 $\int \arctan(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 74

$$\int \arctan(\tanh(a+bx))\,dx = -x\arctan\left(e^{2a+2bx}\right)+x\arctan(\tanh(a+bx))+\frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] -x*arctan(exp(2*b*x+2*a))+x*arctan(tanh(b*x+a))+1/4*I*polylog(2,-I*exp(2*b*
x+2*a))/b-1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5287, 4265, 2317, 2438}

$$\int \arctan(\tanh(a+bx))\,dx = -x\arctan\left(e^{2a+2bx}\right)+x\arctan(\tanh(a+bx))+\frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[In] Int[ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -(x*ArcTan[E^(2*a + 2*b*x)]) + x*ArcTan[Tanh[a + b*x]] + ((I/4)*PolyLog[2,
(-I)*E^(2*a + 2*b*x)])/b - ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5287

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Tanh[a + b
*x]], x] - Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\arctan(\tanh(a+bx)) - b\int x\operatorname{sech}(2a+2bx)\,dx \\
&= -x\arctan\left(e^{2a+2bx}\right) + x\arctan(\tanh(a+bx)) \\
&\quad + \frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)\,dx - \frac{1}{2}i\int \log\left(1+ie^{2a+2bx}\right)\,dx \\
&= -x\arctan\left(e^{2a+2bx}\right) + x\arctan(\tanh(a+bx)) \\
&\quad + \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} - \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&= -x\arctan\left(e^{2a+2bx}\right) + x\arctan(\tanh(a+bx)) \\
&\quad + \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \arctan(\tanh(a+bx))\,dx = x\arctan(\tanh(a+bx)) - \frac{i\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\operatorname{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\operatorname{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b}$$

```
[In] Integrate[ArcTan[Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[Tanh[a + b*x]] - ((I/4)*(2*b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1
 + I*E^(2*(a + b*x))]) - PolyLog[2, (-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^
(2*(a + b*x))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(63) = 126.

Time = 0.82 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.16

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))\arctan(\tanh(bx+a))-\frac{i\operatorname{arctanh}(\tanh(bx+a))\left(\ln\left(1-\frac{i(\tanh(bx+a)+1)^2}{1-\tanh(bx+a)^2}\right)-\ln\left(1+\frac{i(\tanh(bx+a)+1)^2}{1-\tanh(bx+a)^2}\right)\right)}{2} \ldots}{b}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))\arctan(\tanh(bx+a))-\frac{i\operatorname{arctanh}(\tanh(bx+a))\left(\ln\left(1-\frac{i(\tanh(bx+a)+1)^2}{1-\tanh(bx+a)^2}\right)-\ln\left(1+\frac{i(\tanh(bx+a)+1)^2}{1-\tanh(bx+a)^2}\right)\right)}{2} \ldots}{b}$
risch	Expression too large to display

```
[In] int(arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(tanh(b*x+a))*arctan(tanh(b*x+a))-1/2*I*arctanh(tanh(b*x+a))*(l
n(1-I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2))-ln(1+I*(tanh(b*x+a)+1)^2/(1-tanh
(b*x+a)^2)))+1/4*I*dilog(1+I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2))-1/4*I*dil
og(1-I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(57) = 114.

Time = 0.31 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.51

$$\int \arctan(\tanh(a+bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right) + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a) + \sinh(bx+a)) + 1\right) + (-i\,bx - i\,a)\log\left(\right.}{}$$

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-I*b*x - I*a)*log(1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(1/2*sqrt(-4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) + I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) - I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) + I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
+ I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \arctan(\tanh(a+bx))\,dx = \int \operatorname{atan}(\tanh(a+bx))\,dx$$

```
[In] integrate(atan(tanh(b*x+a)),x)
```

```
[Out] Integral(atan(tanh(a + b*x)), x)
```

Maxima [F]

$$\int \arctan(\tanh(a+bx))\,dx = \int \arctan(\tanh(bx+a))\,dx$$

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - 2*b*integrate(x*e^(
2*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int \arctan(\tanh(a+bx))\,dx = \int \arctan\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan(tanh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(\tanh(a+bx))\,dx = \int \mathrm{atan}(\tanh(a+b\,x))\,dx$$

```
[In] int(atan(tanh(a + b*x)),x)
[Out] int(atan(tanh(a + b*x)), x)
```

3.80 $\int \frac{\arctan(\tanh(a+bx))}{e+fx} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx} \, dx = \text{Int}\left(\frac{\arctan(\tanh(a+bx))}{e+fx}, x\right)$$

```
[Out] CannotIntegrate(arctan(tanh(b*x+a))/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx} \, dx = \int \frac{\arctan(\tanh(a+bx))}{e+fx} \, dx$$

```
[In] Int[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
[Out] Defer[Int][ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(\tanh(a+bx))}{e+fx} \, dx$$

Mathematica [N/A]

Not integrable

Time = 11.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx=\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx$$

```
[In] Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e}dx$$

```
[In] int(arctan(tanh(b*x+a))/(f*x+e),x)
[Out] int(arctan(tanh(b*x+a))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx=\int \frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e}\,dx$$

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arctan(tanh(b*x + a))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{atan}\left(\tanh\left(a+bx\right)\right)}{e+fx}\,dx$$

```
[In] integrate(atan(tanh(b*x+a))/(f*x+e),x)
[Out] Integral(atan(tanh(a + b*x))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx=\int \frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e}\,dx$$

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arctan(tanh(b*x + a))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 105.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.20

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx=\int \frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e}\,dx$$

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx=\int \frac{\mathrm{atan}(\tanh(a+b\,x))}{e+f\,x}\,dx$$

```
[In] int(atan(tanh(a + b*x))/(e + f*x),x)
[Out] int(atan(tanh(a + b*x))/(e + f*x), x)
```

3.81 $\int x^2 \arctan(c + d\tanh(a + bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 355

$$
\begin{aligned}
\int x^2 \arctan(c+d\tanh(a+bx))\,dx &= \frac{1}{3}x^3\arctan(c+d\tanh(a+bx)) \\
&+ \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&- \frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\
&+ \frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&- \frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\
&- \frac{ix\operatorname{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\
&+ \frac{ix\operatorname{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} \\
&+ \frac{i\operatorname{PolyLog}\left(4,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} \\
&- \frac{i\operatorname{PolyLog}\left(4,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}
\end{aligned}
$$

```
[Out] 1/3*x^3*arctan(c+d*tanh(b*x+a))+1/6*I*x^3*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/6*I*x^3*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x^2*polylog(2,-(I-
c-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x^2*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/
(I+c-d))/b-1/4*I*x*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/4*I*x*p
olylog(3,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2+1/8*I*polylog(4,-(I-c-d)*exp(
```

```
2*b*x+2*a)/(I-c+d))/b^3-1/8*I*polylog(4,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^
3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5307, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^2 \arctan(c+d\tanh(a+bx))\,dx = {} & \frac{1}{3}x^3\arctan(d\tanh(a+bx)+c) \\ & + \frac{i\,\text{PolyLog}\left(4,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3} \\ & - \frac{i\,\text{PolyLog}\left(4,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3} \\ & - \frac{ix\,\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2} \\ & + \frac{ix\,\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2} \\ & + \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} \\ & - \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} \\ & + \frac{1}{6}ix^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) \\ & - \frac{1}{6}ix^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)\end{aligned}$$

```
[In] Int[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tanh[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c
+ d))])/b - ((I/4)*x^2*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c -
d))])/b - ((I/4)*x*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))]
)/b^2 + ((I/4)*x*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/
b^2 + ((I/8)*PolyLog[4, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^3
- ((I/8)*PolyLog[4, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5307

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b
*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c -
 d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c+d\tanh(a+bx)) \\
&\quad+\frac{1}{3}(b(1-i(c+d)))\int\frac{e^{2a+2bx}x^3}{i+c-d+(i+c+d)e^{2a+2bx}}\,dx \\
&\quad-\frac{1}{3}(b(1+i(c+d)))\int\frac{e^{2a+2bx}x^3}{i-c+d+(i-c-d)e^{2a+2bx}}\,dx \\
&= \frac{1}{3}x^3\arctan(c+d\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad-\frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{1}{2}i\int x^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx \\
&\quad+\frac{1}{2}i\int x^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx \\
&= \frac{1}{3}x^3\arctan(c+d\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad-\frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&\quad-\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}-\frac{i\int x\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx}{2b} \\
&\quad+\frac{i\int x\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx}{2b} \\
&= \frac{1}{3}x^3\arctan(c+d\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad-\frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&\quad-\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}-\frac{ix\,\text{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\
&\quad+\frac{ix\,\text{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2}+\frac{i\int\text{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx}{4b^2} \\
&\quad-\frac{i\int\text{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx}{4b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{3}x^3\arctan(c+d\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&- \frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&- \frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\
&+ \frac{ix\operatorname{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} + \frac{i\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,-\frac{(-i+c+d)x}{-i+c-d}\right)}{x}dx,x,e^{2a+2bx}\right)}{8b^3} \\
&- \frac{i\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,-\frac{(i+c+d)x}{i+c-d}\right)}{x}dx,x,e^{2a+2bx}\right)}{8b^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{3}x^3\arctan(c+d\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&- \frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\
&+ \frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\
&- \frac{ix\operatorname{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} + \frac{ix\operatorname{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} \\
&+ \frac{i\operatorname{PolyLog}\left(4,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} - \frac{i\operatorname{PolyLog}\left(4,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.23

$$
\begin{aligned}
&\int x^2\arctan(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\arctan(c+d\tanh(a+bx)) \\
&- \frac{d\left(4b^3x^3\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right) - 4b^3x^3\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) + 6b^2x^2\operatorname{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2}{-1-c^2+d^2+}\right.\right.}{}
\end{aligned}
$$

```
[In] Integrate[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tanh[a + b*x]])/3 - (d*(4*b^3*x^3*Log[1 + (2*(1 + (c + d)
^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] - 4*b^3*x^3*Log[1
+ ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 6*b^2
*x^2*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 +
 2*Sqrt[-d^2])] - 6*b^2*x^2*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/
(1 + c^2 - d^2 + 2*Sqrt[-d^2]))] - 6*b*x*PolyLog[3, ((1 + c^2 + 2*c*d + d^2
```

```
)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 6*b*x*PolyLog[3, -(((
1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))] + 3*PolyLo
g[4, (-2*(1 + (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2]
)] - 3*PolyLog[4, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sq
rt[-d^2]))]))/(24*b^3*Sqrt[-d^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.22 (sec) , antiderivative size = 6917, normalized size of antiderivative = 19.48

method	result	size
risch	Expression too large to display	6917

```
[In] int(x^2*arctan(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(263) = 526.

Time = 0.37 (sec) , antiderivative size = 1289, normalized size of antiderivative = 3.63

$$\int x^2 \arctan(c + d\tanh(a + bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
3*I*b^2*x^2*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(c
osh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 + 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*
x^2*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x
+ a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^3*log(2*(c^2
 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a)
 + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d
^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*
c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 +
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)
*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2
*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*l
og(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh
```

```
(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) - 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -
sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a))) + 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -sqrt(-
(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
+ a))) + (I*b^3*x^3 + I*a^3)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log
(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^
3 - I*a^3)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) + 6*I*polylog(4, sqrt(-(c^2 - d^2 + 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*polylo
g(4, -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a)
 + sinh(b*x + a))) - 6*I*polylog(4, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*
c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, -sqrt(-(c
^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
a))))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\tanh(a + bx))\,dx = \text{Timed out}$$

```
[In] integrate(x**2*atan(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2 \arctan(c + d\tanh(a + bx))\,dx = \int x^2 \arctan\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x + 2*a)
 + 1)) - 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*
e^(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a)
 - d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Giac [F]

$$\int x^2 \arctan(c + d\tanh(a + bx))\,dx = \int x^2 \arctan\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arctan(d*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\tanh(a + bx))\,dx = \int x^2\,\mathrm{atan}(c + d\tanh(a + b\,x))\,dx$$

```
[In] int(x^2*atan(c + d*tanh(a + b*x)),x)
[Out] int(x^2*atan(c + d*tanh(a + b*x)), x)
```

3.82 $\int x \arctan(c + d \tanh(a + bx)) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 267

$$\begin{aligned}\int x \arctan(c + d\tanh(a + bx)) \, dx = {} & \frac{1}{2}x^2 \arctan(c + d\tanh(a + bx)) \\ & + \frac{1}{4} i x^2 \log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) \\ & - \frac{1}{4} i x^2 \log\left(1 + \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) \\ & + \frac{ix \operatorname{PolyLog}\left(2, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ & - \frac{ix \operatorname{PolyLog}\left(2, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ & - \frac{i \operatorname{PolyLog}\left(3, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} \\ & + \frac{i \operatorname{PolyLog}\left(3, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}\end{aligned}$$

```
[Out] 1/2*x^2*arctan(c+d*tanh(b*x+a))+1/4*I*x^2*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/4*I*x^2*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x*polylog(2,-(I-c-
d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c
-d))/b-1/8*I*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/8*I*polylog(3
,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5307, 2221, 2611, 2320, 6724}

$$\int x\arctan(c+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\arctan(d\tanh(a+bx)+c) - \frac{i\,\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2} + \frac{i\,\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2} + \frac{ix\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} - \frac{ix\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} + \frac{1}{4}ix^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) - \frac{1}{4}ix^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)$$

```
[In] Int[x*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d))])/b - ((I/4)*x*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]
)/b - ((I/8)*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^2
+ ((I/8)*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5307

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b
*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c -
 d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c+d\tanh(a+bx)) \\
&\quad +\frac{1}{2}(b(1-i(c+d)))\int\frac{e^{2a+2bx}x^2}{i+c-d+(i+c+d)e^{2a+2bx}}\,dx \\
&\quad -\frac{1}{2}(b(1+i(c+d)))\int\frac{e^{2a+2bx}x^2}{i-c+d+(i-c-d)e^{2a+2bx}}\,dx \\
&= \frac{1}{2}x^2\arctan(c+d\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad -\frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{1}{2}i\int x\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx \\
&\quad +\frac{1}{2}i\int x\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx
\end{aligned}$$

$$= \frac{1}{2}x^2\arctan(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{i\int\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx}{4b} + \frac{i\int\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx}{4b}$$

$$= \frac{1}{2}x^2\arctan(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{i\text{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{(-i+c+d)x}{-i+c-d}\right)}{x}dx,x,e^{2a+2bx}\right)}{8b^2} + \frac{i\text{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{(i+c+d)x}{i+c-d}\right)}{x}dx,x,e^{2a+2bx}\right)}{8b^2}$$

$$= \frac{1}{2}x^2\arctan(c+d\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{i\operatorname{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} + \frac{i\operatorname{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.24

$$\int x\arctan(c+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\arctan(c+d\tanh(a+bx)) - \frac{d\left(2b^2x^2\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right) - 2b^2x^2\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) + 2bx\operatorname{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2)e^{2(}}{-1-c^2+d^2+2\sqrt{-}}\right.\right.}{}$$

```
[In] Integrate[x*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tanh[a + b*x]])/2 - (d*(2*b^2*x^2*Log[1 + (2*(1 + (c + d)
^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] - 2*b^2*x^2*Log[1
+ ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 2*b*x
*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*S
qrt[-d^2])] - 2*b*x*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2
 - d^2 + 2*Sqrt[-d^2]))] - PolyLog[3, (-2*(1 + (c + d)^2)*E^(2*(a + b*x)))/
(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + PolyLog[3, -(((1 + (c + d)^2)*E^(2*(a
 + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))]))/(8*b^2*Sqrt[-d^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.00 (sec) , antiderivative size = 6567, normalized size of antiderivative = 24.60

method	result	size
risch	Expression too large to display	6567

```
[In] int(x*arctan(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. 2(197) = 394.

Time = 0.36 (sec) , antiderivative size = 1067, normalized size of antiderivative = 4.00

$$\int x\arctan(c+d\tanh(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
2*I*b*x*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(
b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(sq
rt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(
b*x + a))) - 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^
2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 +
 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2
- 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^
2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2
 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
```

```
2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-
(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*
d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c
^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1)
)) + (I*b^2*x^2 - I*a^2)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d
^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(-sq
rt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(
b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 +
I*a^2)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a)) + 1) - 2*I*polylog(3, sqrt(-(c^2 - d^2 + 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3,
 -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a))) + 2*I*polylog(3, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d
+ d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, -sqrt(-(c^2 -
 d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
)/b^2
```

Sympy [F]

$$\int x\arctan(c+d\tanh(a+bx))\,dx=\int x\operatorname{atan}\left(c+d\tanh\left(a+bx\right)\right)dx$$

```
[In] integrate(x*atan(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x*atan(c + d*tanh(a + b*x)), x)
```

Maxima [F]

$$\int x\arctan(c+d\tanh(a+bx))\,dx=\int x\arctan\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x + 2*a)
 + 1)) - 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4
*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d
^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Giac [F]

$$\int x \arctan(c + d \tanh(a + bx)) \, dx = \int x \arctan\left(d \tanh\left(bx + a\right) + c\right) \, dx$$

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctan(d*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \arctan(c + d \tanh(a + bx)) \, dx = \int x \operatorname{atan}(c + d \tanh(a + b\,x)) \, dx$$

```
[In] int(x*atan(c + d*tanh(a + b*x)),x)
[Out] int(x*atan(c + d*tanh(a + b*x)), x)
```

3.83 $\int \arctan(c+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 174

$$\int \arctan(c+d\tanh(a+bx))\,dx = x\arctan(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{i\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}$$

```
[Out] x*arctan(c+d*tanh(b*x+a))+1/2*I*x*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+d))-1/2*
I*x*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*polylog(2,-(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b-1/4*I*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{5299, 2221, 2317, 2438\}$

$$\int \arctan(c + d\tanh(a + bx))\,dx = x\arctan(d\tanh(a + bx) + c) + \frac{i\,\text{PolyLog}\left(2, -\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2, -\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} + \frac{1}{2}ix\log\left(1 + \frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) - \frac{1}{2}ix\log\left(1 + \frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)$$

```
[In] Int[ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tanh[a + b*x]] + (I/2)*x*Log[1 + ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b - ((
I/4)*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5299

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] + (Dist[I*b*(I - c - d), Int[x*(E^(2*a + 2*b*x
)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b*(I + c + d)
, Int[x*(E^(2*a + 2*b*x)/(I + c - d + (I + c + d)*E^(2*a + 2*b*x))), x], x]
```

```
) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\arctan(c+d\tanh(a+bx)) + (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(i+c+d)e^{2a+2bx}}\,dx \\
&\quad - (b(1+i(c+d)))\int \frac{e^{2a+2bx}x}{i-c+d+(i-c-d)e^{2a+2bx}}\,dx \\
&= x\arctan(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad - \frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{1}{2}i\int \log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx \\
&\quad + \frac{1}{2}i\int \log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx \\
&= x\arctan(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad - \frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{i\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(i-c-d)x}{i-c+d}\right)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&\quad + \frac{i\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(i+c+d)x}{i+c-d}\right)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&= x\arctan(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad - \frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\
&\quad + \frac{i\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.66

$$
\int \arctan(c+d\tanh(a+bx))\,dx = x\arctan(c+d\tanh(a+bx)) + \frac{4a\sqrt{-d^2}\arctan\left(\frac{1+c^2-d^2+(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right) - 2d(a+bx)\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right) + 2d(a+bx)\log}{4b\sqrt{-d}}
$$

```
[In] Integrate[ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tanh[a + b*x]] + (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 + (1
+ c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] - 2*d*(a + b*x)*Log[1 + (2*(1
+ (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + 2*d*(a
+ b*x)*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d
^2])] + d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 -
 d^2 + 2*Sqrt[-d^2]))] - d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*
x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])])/(4*b*Sqrt[-d^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(150) = 300$.

Time = 2.18 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\right.}{2}\right.}{}}{}$
default	$\frac{\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\right.}{2}\right.}{}}{}$
risch	Expression too large to display

```
[In] int(arctan(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(1/2*arctan(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)-1/2*arctan(c+d*ta
nh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*d^2*(1/d*(1/2*I*ln(-d*tanh(b*x+a)+d)*
ln((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I*ln(-d*tanh(b*x+a)+d)*ln((I-d*tanh(b*x
+a)-c)/(I-c-d))+1/2*I*dilog((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I*dilog((I-d*t
anh(b*x+a)-c)/(I-c-d)))-1/d*(1/2*I*ln(-d*tanh(b*x+a)-d)*ln((I+d*tanh(b*x+a)
+c)/(I+c-d))-1/2*I*ln(-d*tanh(b*x+a)-d)*ln((I-d*tanh(b*x+a)-c)/(I-c+d))+1/2
*I*dilog((I+d*tanh(b*x+a)+c)/(I+c-d))-1/2*I*dilog((I-d*tanh(b*x+a)-c)/(I-c+
d)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(128) = 256.

Time = 0.39 (sec) , antiderivative size = 825, normalized size of antiderivative = 4.74

$$\int \arctan(c+d\tanh(a+bx))\,dx$$

$$= \frac{2\,bx\arctan\left(\frac{c\cosh(bx+a)+d\sinh(bx+a)}{\cosh(bx+a)}\right)-i\,a\log\left(2\left(c^2+2\,cd+d^2+1\right)\cosh\left(bx+a\right)+2\left(c^2+2\,cd+d^2+1\right)\mathrm{si}\right.}{}$$

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 -
 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(
c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d
^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I
*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*
sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c
^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)
*log(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a)
 + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*
a)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
a) + sinh(b*x + a)) + 1) + I*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c
*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-sqrt(-(c^2 - d^2
 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I
*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a
) + sinh(b*x + a))) - I*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \arctan(c + d\tanh(a + bx))\,dx = \int \operatorname{atan}\left(c + d\tanh\left(a + bx\right)\right)\,dx$$

```
[In] integrate(atan(c+d*tanh(b*x+a)),x)
[Out] Integral(atan(c + d*tanh(a + b*x)), x)
```

Maxima [F]

$$\int \arctan(c + d\tanh(a + bx))\,dx = \int \arctan\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
[Out] -4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*
d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2*e^(2*a)
 + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x
) + c - d)/(e^(2*b*x + 2*a) + 1))
```

Giac [F]

$$\int \arctan(c + d\tanh(a + bx))\,dx = \int \arctan\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan(d*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c + d\tanh(a + bx))\,dx = \int \operatorname{atan}(c + d\tanh(a + b\,x))\,dx$$

```
[In] int(atan(c + d*tanh(a + b*x)),x)
[Out] int(atan(c + d*tanh(a + b*x)), x)
```

3.84 $\int \frac{\arctan(c+d\tanh(a+bx))}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x} dx = \text{Int}\left(\frac{\arctan(c+d\tanh(a+bx))}{x}, x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*tanh(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x} dx = \int \frac{\arctan(c+d\tanh(a+bx))}{x} dx$$

```
[In] Int[ArcTan[c + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c+d\tanh(a+bx))}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 6.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c + d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(c+d\tanh\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arctan(c+d*tanh(b*x+a))/x,x)
[Out] int(arctan(c+d*tanh(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\arctan\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*tanh(b*x + a) + c)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(atan(c+d*tanh(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\tanh(a + bx))}{x}\, dx = \int \frac{\arctan\left(d\tanh\left(bx + a\right) + c\right)}{x}\, dx$$

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\tanh(a + bx))}{x}\, dx = \int \frac{\arctan\left(d\tanh\left(bx + a\right) + c\right)}{x}\, dx$$

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\tanh(a + bx))}{x}\, dx = \int \frac{\mathrm{atan}(c + d\tanh(a + b\,x))}{x}\, dx$$

```
[In] int(atan(c + d*tanh(a + b*x))/x,x)
[Out] int(atan(c + d*tanh(a + b*x))/x, x)
```

3.85 $\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = -\frac{1}{12}ibx^4+\frac{1}{3}x^3\arctan(c+(i+c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{ix^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{ix\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{i\,\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arctan(c+(I+c)*tanh(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,-I*c
*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5303, 2215, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\arctan(c+(c+i)\tanh(a+bx))+\frac{i\,\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}-\frac{ix\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{ix^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{1}{12}ibx^4$$

```
[In] Int[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/12*I)*b*x^4 + (x^3*ArcTan[c + (I + c)*Tanh[a + b*x]])/3 + (I/6)*x^3*Log
[1 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/
b - ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, (-
I)*c*E^(2*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5303

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c+(i+c)\tanh(a+bx)) - \frac{1}{3}b\int\frac{x^3}{-i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\tanh(a+bx)) + \frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{-i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\tanh(a+bx))\\
&\quad + \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) - \frac{1}{2}i\int x^2\log\left(1+ice^{2a+2bx}\right)\,dx\\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)\\
&\quad + \frac{ix^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} - \frac{i\int x\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)\,dx}{2b}\\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\tanh(a+bx))\\
&\quad + \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) + \frac{ix^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}\\
&\quad - \frac{ix\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2} + \frac{i\int\text{PolyLog}\left(3,-ice^{2a+2bx}\right)\,dx}{4b^2}\\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\tanh(a+bx))\\
&\quad + \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) + \frac{ix^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}\\
&\quad - \frac{ix\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2} + \frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(3,-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3}
\end{aligned}
$$

$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\tanh(a+bx)) + \frac{1}{6}ix^3\log\left(1 + ice^{2a+2bx}\right) + \frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2} + \frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int x^2\arctan(c+(i+c)\tanh(a+bx))\,dx = \frac{8b^3x^3\arctan(c+(i+c)\tanh(a+bx)) + 4ib^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - 6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right) - 6ibx}{24b^3}$$

```
[In] Integrate[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcTan[c + (I + c)*Tanh[a + b*x]] + (4*I)*b^3*x^3*Log[1 - I/(c*E
^(2*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - (6*I)*
b*x*PolyLog[3, I/(c*E^(2*(a + b*x)))] - (3*I)*PolyLog[4, I/(c*E^(2*(a + b*x
)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.60 (sec) , antiderivative size = 1406, normalized size of antiderivative = 9.90

method	result	size
risch	Expression too large to display	1406

```
[In] int(x^2*arctan(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))-1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^2*x
-1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c-2*I)+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I*c)
^(1/2))*x+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/3*I/b^3*ln(1+I*c
*exp(2*b*x+2*a))*a^3-1/12*I*b*c/(I+c)*x^4+1/12*I/b^3*c/(I+c)*a^4-1/4*I*x*po
lylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3+1
/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1+I*exp(b*x
+a)*(I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/4*I*x^2*
polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a
^2-1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)-1/12*Pi*(csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b
*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b
```

```
*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+
1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2
*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2
*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp
(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn(
(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-csgn(I*(2*I*exp(2*b*x+2*a)+2*e
xp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(
2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
+1))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+csg
n((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn((2*exp
(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp
(2*b*x+2*a)+1))^2-2)*x^3-1/12/b^3/(I+c)*a^4+1/12*b/(I+c)*x^4+1/6*I*x^3*ln(2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(I*c)
^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.06

$$\int x^2 \arctan(c + (i + c)\tanh(a + bx))\,dx$$

$$= \frac{-i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + i\,a}{}$$

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*d
ilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) - 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*b*x*polylo
g(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(-
4*I*c)*e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(-4*I*c)*e^(b
*x + a) + 1) + 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*polylog
(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\arctan\left((c+i)\tanh\left(bx+a\right)+c\right) + \frac{4}{9}\left(\frac{3x^4}{4ic-4} - \frac{4b^3x^3\log\left(ice^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-ice^{(2bx+2a)}\right)-6bx\mathrm{Li}_3(-ice^{(2bx+2a)})+3\mathrm{Li}_4(-i}{-2b^4(-ic+1)}\right.$$

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c + I)*tanh(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c - 2)))*b*(c + I)
```

Giac [F]

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = \int x^2\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c + I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = \int x^2\,\mathrm{atan}(c+\tanh(a+b\,x)\,\,(c+1\mathrm{i}))\,\,dx$$

```
[In] int(x^2*atan(c + tanh(a + b*x)*(c + 1i)),x)
[Out] int(x^2*atan(c + tanh(a + b*x)*(c + 1i)), x)
```

3.86 $\int x \arctan(c+(i+c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int x \arctan(c+(i+c)\tanh(a+bx))\,dx = -\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c+(i+c)\tanh(a+bx)) +\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right) +\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} -\frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arctan(c+(I+c)*tanh(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,-I*c*exp(
2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5303, 2215, 2221, 2611, 2320, 6724}

$$\int x \arctan(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{2}x^2\arctan(c+(c+i)\tanh(a+bx)) -\frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2} +\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} +\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{1}{6}ibx^3$$

```
[In] Int[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/6*I)*b*x^3 + (x^2*ArcTan[c + (I + c)*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[
1 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b -
 ((I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5303

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c+(i+c)\tanh(a+bx))-\frac{1}{2}b\int\frac{x^2}{-i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c+(i+c)\tanh(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{-i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c+(i+c)\tanh(a+bx))\\
&\quad+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{1}{2}i\int x\log\left(1+ice^{2a+2bx}\right)\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad+\frac{ix\,\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\int\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)\,dx}{4b}\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad+\frac{ix\,\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}(2,-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad+\frac{ix\,\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\,\mathrm{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x\arctan(c+(i+c)\tanh(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\arctan(c+(i+c)\tanh(a+bx))+i\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)-2ibx\,\mathrm{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-i\,\mathrm{PolyLo}}{8b^2}$$

```
[In] Integrate[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTan[c + (I + c)*Tanh[a + b*x]] + I*Log[1 - I/(c*E^(2*(a +
b*x)))]) - (2*I)*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - I*PolyLog[3, I/(c*
E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 1370, normalized size of antiderivative = 12.12

method	result	size
risch	Expression too large to display	1370

```
[In] int(x*arctan(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a*dilog(1-I*exp(b*
x+a)*(I*c)^(1/2))-1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c-2*I)+1/4*I/b^2*polylog(2,
-I*c*exp(2*b*x+2*a))*a+1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)-1/6*I*b*c/(I+c
)*x^3-1/2*I/b^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*a^2+1/4*I*x^2*ln(1+I*c*exp(2
*b*x+2*a))-1/2*I/b^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*a^2+1/4*I/b^2*ln(1+I*c*
exp(2*b*x+2*a))*a^2-1/8*I*polylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/2*I/b*ln(1+I
*c*exp(2*b*x+2*a))*a*x-1/6*I/b^2*c/(I+c)*a^3+1/4*I*x*polylog(2,-I*c*exp(2*b
*x+2*a))/b+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)-1/8*Pi*(csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x
+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp
(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a
)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2
*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a
)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*I
*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2
*a)+1))^3-csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2
*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp
(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-csgn(I*(2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+csgn(I*(2*I*exp
(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*ex
p(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c)/(exp(2*b*x+2*a)+1))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp
(2*b*x+2*a)+1))^3+csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2
*a)+1))^2-csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+csgn((2*exp(2
*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-2)*x^2+1/6*b/(I+c)*x^3+1/6/b^2/(I+c)
*a^3-1/2*I/b*a*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/2*I/b*a*ln(1+I*exp(b*x+a)
*(I*c)^(1/2))*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.19

$$\int x\arctan(c+(i+c)\tanh(a+bx))\,dx$$
$$= \frac{-2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)-2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right)+}{}$$

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) - I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b
*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c
) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^
2*x^2 + I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*
sqrt(-4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/
b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int x\arctan(c+(i+c)\tanh(a+bx))\,dx$$
$$= \left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i)$$
$$+\frac{1}{2}\,x^2\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)$$

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(
-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2))
)*b*(c + I) + 1/2*x^2*arctan((c + I)*tanh(b*x + a) + c)
```

Giac [F]

$$\int x\arctan(c+(i+c)\tanh(a+bx))\,dx=\int x\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c + I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(i+c)\tanh(a+bx))\,dx=\int x\,\mathrm{atan}(c+\tanh(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

```
[In] int(x*atan(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x*atan(c + tanh(a + b*x)*(c + 1i)), x)
```

3.87 $\int \arctan(c+(i+c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

```
[Out] -1/2*I*b*x^2+x*arctan(c+(I+c)*tanh(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5295, 2215, 2221, 2317, 2438}

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = x\arctan(c+(c+i)\tanh(a+bx)) + \frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{1}{2}ibx^2$$

```
[In] Int[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/2*I)*b*x^2 + x*ArcTan[c + (I + c)*Tanh[a + b*x]] + (I/2)*x*Log[1 + I*c*
E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

```
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5295

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan(c+(i+c)\tanh(a+bx)) - b\int \frac{x}{-i+ce^{2a+2bx}}\,dx \\
&= -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\tanh(a+bx)) + (ibc)\int \frac{e^{2a+2bx}x}{-i+ce^{2a+2bx}}\,dx \\
&= -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\tanh(a+bx)) \\
&\quad + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{1}{2}i\int \log\left(1+ice^{2a+2bx}\right)\,dx \\
&= -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\tanh(a+bx)) \\
&\quad + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{i\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx, x, e^{2a+2bx}\right)}{4b}
\end{aligned}$$

$$= -\frac{1}{2}ibx^2 + x\arctan(c + (i + c)\tanh(a + bx)) + \frac{1}{2}ix\log\left(1 + ice^{2a+2bx}\right) + \frac{i\operatorname{PolyLog}\left(2, -ice^{2a+2bx}\right)}{4b}$$

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \arctan(c + (i + c)\tanh(a + bx))\,dx = x\arctan(c + (i + c)\tanh(a + bx)) + \frac{i\left(2bx\log\left(1 - \frac{ie^{-2(a+bx)}}{c}\right) - \operatorname{PolyLog}\left(2, \frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

```
[In] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (I + c)*Tanh[a + b*x]] + ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(65) = 130.

Time = 1.03 (sec) , antiderivative size = 545, normalized size of antiderivative = 6.90

method	result
derivativedivides	$-\frac{\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\arctan(c+(i+$
default	$-\frac{\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\arctan(c+(i+$
risch	Expression too large to display

```
[In] int(arctan(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I+c)*(-arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))
+2*I*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c+arct
an(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c^2+arctan(c+(I
+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)-2*I*arctan(c+(I+c)*tan
h(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c-arctan(c+(I+c)*tanh(b*x+a))
/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c^2-(I+c)^2*(1/2/(I+c)*(-1/2*I*((ln(I+
c+(I+c)*tanh(b*x+a))-ln(-1/2*I*(I+c+(I+c)*tanh(b*x+a))))*ln(-1/2*I*(I-c-(I+
```

```
c)*tanh(b*x+a)))-dilog(-1/2*I*(I+c+(I+c)*tanh(b*x+a))))+1/4*I*ln(I+c+(I+c)*
tanh(b*x+a))^2)-1/2/(I+c)*(-1/2*I*(dilog(-1/2*(I-c-(I+c)*tanh(b*x+a))/c)+ln
(c-(I+c)*tanh(b*x+a)+I)*ln(-1/2*(I-c-(I+c)*tanh(b*x+a))/c))+1/2*I*(dilog((-
I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c))+ln(c-(I+c)*tanh(b*x+a)+I)*ln((-I-c-(I+c)
*tanh(b*x+a))/(-2*I-2*c)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.37

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = \frac{-i\,b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) + i\,a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{-\cdots}\right.}{\cdots}$$

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (I*b*x +
 I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c) +
 I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x +
 a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx$$
$$= 2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2} - \frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)$$
$$+ x\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)$$

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dilo
g(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*tanh(b*x + a
) + c)
```

Giac [F]

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = \int \arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = \int \mathrm{atan}\left(c+\tanh\left(a+b\,x\right)\,\left(c+1\mathrm{i}\right)\right)\,dx$$

```
[In] int(atan(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(atan(c + tanh(a + b*x)*(c + 1i)), x)
```

3.88 $\int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx = \mathrm{Int}\left(\frac{\arctan(c+(i+c)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(I+c)*tanh(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x} \, dx = \int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x} \, dx$$

```
[In] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\arctan\left(c + (i + c)\tanh\left(bx + a\right)\right)}{x} dx$$

```
[In] int(arctan(c+(I+c)*tanh(b*x+a))/x,x)
[Out] int(arctan(c+(I+c)*tanh(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x} \, dx = \int \frac{\arctan\left((c + i)\tanh\left(bx + a\right) + c\right)}{x} \, dx$$

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x} \, dx = \text{Timed out}$$

```
[In] integrate(atan(c+(I+c)*tanh(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.79

$$\int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x}\,dx = \int \frac{\arctan\left((c + i)\tanh\left(bx + a\right) + c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x - 1/4*(4*pi - 4*I*a - 2*arctan(c) - I*log(c^2 + 1))*log(x) + 1/2*inte
grate(arctan(c*e^(2*b*x + 2*a))/x, x) - 1/4*I*integrate(log(c^2*e^(4*b*x +
4*a) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x}\,dx = \int \frac{\arctan\left((c + i)\tanh\left(bx + a\right) + c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*tanh(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x}\,dx = \int \frac{\mathrm{atan}(c + \tanh(a + b\,x)\ (c + 1\mathrm{i}))}{x}\,dx$$

```
[In] int(atan(c + tanh(a + b*x)*(c + 1i))/x,x)
```

```
[Out] int(atan(c + tanh(a + b*x)*(c + 1i))/x, x)
```

3.89 $\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{12}ibx^4+\frac{1}{3}x^3\arctan(c-(i-c)\tanh(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arctan(c-(I-c)*tanh(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*b
*x+2*a))-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,I*c*ex
p(2*b*x+2*a))/b^2-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5303, 2215, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\arctan(c-(-c+i)\tanh(a+bx)) - \frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3} + \frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2} - \frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{12}ibx^4$$

```
[In] Int[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Tanh[a + b*x]])/3 - (I/6)*x^3*Log[1
- I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((
I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, I*c*E^(2*a
+ 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5303

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c-(i-c)\tanh(a+bx)) - \frac{1}{3}b\int\frac{x^3}{i+ce^{2a+2bx}}\,dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\tanh(a+bx)) - \frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{i+ce^{2a+2bx}}\,dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\tanh(a+bx)) \\
&\quad - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{1}{2}i\int x^2\log\left(1-ice^{2a+2bx}\right)\,dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) \\
&\quad - \frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} + \frac{i\int x\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)\,dx}{2b} \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1\right. \\
&\qquad \left.-ice^{2a+2bx}\right) - \frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} \\
&\quad + \frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2} - \frac{i\int\text{PolyLog}\left(3,ice^{2a+2bx}\right)\,dx}{4b^2} \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\tanh(a+bx)) \\
&\quad - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} \\
&\quad + \frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2} - \frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(3,icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3} \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) \\
&\quad - \frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} + \frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2} - \frac{i\,\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx$$
$$=\frac{8b^3x^3\arctan(c+(-i+c)\tanh(a+bx))-4ib^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)+6ib^2x^2\,\mathrm{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)+}{24b^3}$$

```
[In] Integrate[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcTan[c + (-I + c)*Tanh[a + b*x]] - (4*I)*b^3*x^3*Log[1 + I/(c*
E^(2*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] + (6
*I)*b*x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] + (3*I)*PolyLog[4, (-I)/(c*E^(
2*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.62 (sec) , antiderivative size = 1409, normalized size of antiderivative = 9.72

method	result	size
risch	Expression too large to display	1409

```
[In] int(x^2*arctan(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/4*I*x*polylog(3,I*c*exp(2*
b*x+2*a))/b^2-1/12*I*b*c/(I-c)*x^4-1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(-I*c
)^(1/2))+1/12*I/b^3*c*a^4/(I-c)-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1
/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c
)^(1/2))*x+1/6*I*x^3*ln(-2*exp(2*b*x+2*a)*c-2*I)+1/6*I/b^3*a^3*ln(exp(2*b*x
+2*a)*c+I)-1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/4*I/b^3*po
lylog(2,I*c*exp(2*b*x+2*a))*a^2-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3-1/2
*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^3*ln(1+I*exp(b*x+
a)*(-I*c)^(1/2))+1/2*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2*x+1/12*Pi*(csgn(I/(
exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a
)*c+2*I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*
c)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*
c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)
*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I
*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+
```

```
2*a)+1))^3+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(
2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(ex
p(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))+csgn(I*(
-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-csgn(I*(-2*I*
exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x
+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*
c)/(exp(2*b*x+2*a)+1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp
(2*b*x+2*a)+1))^2+csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3+csgn(
(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-2)*x^3-1/2*I/b^2*a^2*ln(1+I*
exp(b*x+a)*(-I*c)^(1/2))*x-1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))-1/12*b/(I-c)*
x^4+1/12/b^3/(I-c)*a^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.02

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c-i}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) -$$

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)
/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*di
log(-1/2*sqrt(4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*b*x*polylog(3,
-1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(4*I*c)*e
^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) +
1) - 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*polylog(4, -1/2*sq
rt(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{3}x^3 \arctan\left((c-i)\tanh\left(bx+a\right)+c\right) - \frac{4}{9}\left(\frac{3\,x^4}{4i\,c+4} - \frac{4\,b^3x^3\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_4\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^4(-i\,c-1)}\right)$$

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c - I)*tanh(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c + 2)))*b*(c - I)
```

Giac [F]

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \int x^2 \arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \int x^2\,\mathrm{atan}\left(c+\tanh(a+b\,x)\,\left(c-\mathrm{i}\right)\right)\,dx$$

```
[In] int(x^2*atan(c + tanh(a + b*x)*(c - 1i)),x)
[Out] int(x^2*atan(c + tanh(a + b*x)*(c - 1i)), x)
```

3.90 $\int x \arctan(c-(i-c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int x \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) - \frac{ix\,\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} + \frac{i\,\mathrm{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arctan(c-(I-c)*tanh(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,I*c*exp(2*b
*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5303, 2215, 2221, 2611, 2320, 6724}

$$\int x \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{2}x^2\arctan(c-(-c+i)\tanh(a+bx)) + \frac{i\,\mathrm{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2} - \frac{ix\,\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} - \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{6}ibx^3$$

```
[In] Int[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Tanh[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((I/8
)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5303

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c-(i-c)\tanh(a+bx))-\frac{1}{2}b\int\frac{x^2}{i+ce^{2a+2bx}}\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\tanh(a+bx))-\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{i+ce^{2a+2bx}}\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\tanh(a+bx))\\
&\quad -\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}i\int x\log\left(1-ice^{2a+2bx}\right)\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\int\text{PolyLog}\left(2,ice^{2a+2bx}\right)\,dx}{4b}\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int x\arctan(c-(i-c)\tanh(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\arctan(c+(-i+c)\tanh(a+bx))-i\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)+i\,\text{P}}{8b^2}$$

```
[In] Integrate[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTan[c + (-I + c)*Tanh[a + b*x]] - I*Log[1 + I/(c*E^(2*(a +
 b*x)))]) + (2*I)*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] + I*PolyLog[3, (
-I)/(c*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 1373, normalized size of antiderivative = 11.84

method	result	size
risch	Expression too large to display	1373

```
[In] int(x*arctan(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b*a*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x+1/4*I*x^2*ln(-2*exp(2*b*x+2*a)*
c-2*I)+1/2*I/b^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*a^2-1/4*I/b^2*a^2*ln(exp(2
*b*x+2*a)*c+I)+1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x+1/2*I/b^2*a*dilo
g(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/6*I*b*c/(I-c)*x^3+1/2*I/b^2*ln(1+I*exp(b*x
+a)*(-I*c)^(1/2))*a^2-1/6*I/b^2*c/(I-c)*a^3-1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*
a))*a^2-1/4*I/b^2*polylog(2,I*c*exp(2*b*x+2*a))*a-1/4*I*x^2*ln(2*I*exp(2*b*
x+2*a)-2*exp(2*b*x+2*a)*c)-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*I*po
lylog(3,I*c*exp(2*b*x+2*a))/b^2+1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/
2))+1/8*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csg
n(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn
(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a
)+1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2
*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a
)*c+2*I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2
*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2
*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b
*x+2*a)+1))+csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
+1))^3-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*
csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*
(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp
(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+csgn((-2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*ex
p(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*
b*x+2*a)+1))^3+csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-2)*x^2-1
/6/b^2/(I-c)*a^3-1/6*b/(I-c)*x^3-1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))*a*x-1/4*I
*x^2*ln(1-I*c*exp(2*b*x+2*a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.13

$$\int x\arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c-i}\right)+2i\,a^3-6i\,bx\text{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right)-6i\,bx\text{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx}\right.}{}$$

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*
a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b*
x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) -
3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*x^2 -
 I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sqrt(4*I
*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int x\arctan(c-(i-c)\tanh(a+bx))\,dx = -\left(\frac{2\,x^3}{3i\,c+3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\text{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\text{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i) + \frac{1}{2}\,x^2\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)$$

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2))
)*b*(c - I) + 1/2*x^2*arctan((c - I)*tanh(b*x + a) + c)
```

Giac [F]

$$\int x \arctan(c-(i-c)\tanh(a+bx))\,dx = \int x \arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \arctan(c-(i-c)\tanh(a+bx))\,dx = \int x\,\mathrm{atan}\left(c+\tanh(a+b\,x)\,\left(c-\mathrm{i}\right)\right)\,dx$$

```
[In] int(x*atan(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x*atan(c + tanh(a + b*x)*(c - 1i)), x)
```

3.91 $\int \arctan(c-(i-c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{i\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arctan(c-(I-c)*tanh(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5295, 2215, 2221, 2317, 2438}

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx = x\arctan(c-(-c+i)\tanh(a+bx)) - \frac{i\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{1}{2}ibx^2$$

```
[In] Int[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Tanh[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

```
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5295

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan(c-(i-c)\tanh(a+bx)) - b\int \frac{x}{i+ce^{2a+2bx}}\,dx \\
&= \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\tanh(a+bx)) - (ibc)\int \frac{e^{2a+2bx}x}{i+ce^{2a+2bx}}\,dx \\
&= \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\tanh(a+bx)) \\
&\quad - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{1}{2}i\int \log\left(1-ice^{2a+2bx}\right)\,dx \\
&= \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\tanh(a+bx)) \\
&\quad - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{i\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&= \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{i\,\text{PolyLog}\left(2, ice^{2a+2bx}\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx$$
$$= x\arctan(c+(-i+c)\tanh(a+bx)) - \frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

```
[In] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-I + c)*Tanh[a + b*x]] - ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(68) = 136.

Time = 1.02 (sec) , antiderivative size = 516, normalized size of antiderivative = 6.29

method	result
derivativedivides	$\frac{\frac{\arctan(c+\tanh(bx+a)(c-i))\ln(-i+\tanh(bx+a)(c-i)+c)}{2i-2c}+\frac{2i\arctan(c+\tanh(bx+a)(c-i))\ln(-i+\tanh(bx+a)(c-i)+c)c}{2i-2c}-\frac{\arctan(c+t}{}}{}$
default	$\frac{\frac{\arctan(c+\tanh(bx+a)(c-i))\ln(-i+\tanh(bx+a)(c-i)+c)}{2i-2c}+\frac{2i\arctan(c+\tanh(bx+a)(c-i))\ln(-i+\tanh(bx+a)(c-i)+c)c}{2i-2c}-\frac{\arctan(c+t}{}}{}$
risch	Expression too large to display

```
[In] int(arctan(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(c-I)*(arctan(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)
+2*I*arctan(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)*c-arc
tan(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)*c^2-arctan(c+
tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)-2*I*arctan(c+tanh(b*
x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)*c+arctan(c+tanh(b*x+a)*(c-I
))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)*c^2+(I-c)^2*(1/2/(I-c)*(-1/4*I*ln(-I
+tanh(b*x+a)*(c-I)+c)^2+1/2*I*(dilog(-1/2*I*(tanh(b*x+a)*(c-I)+c+I))+ln(-I+
tanh(b*x+a)*(c-I)+c)*ln(-1/2*I*(tanh(b*x+a)*(c-I)+c+I))))-1/2/(I-c)*(-1/2*I
*(dilog((-I+tanh(b*x+a)*(c-I)+c)/(-2*I+2*c))+ln(tanh(b*x+a)*(c-I)-c+I)*ln((
-I+tanh(b*x+a)*(c-I)+c)/(-2*I+2*c)))+1/2*I*(dilog(1/2*(tanh(b*x+a)*(c-I)+c+
I)/c)+ln(tanh(b*x+a)*(c-I)-c+I)*ln(1/2*(tanh(b*x+a)*(c-I)+c+I)/c))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.28

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx$$
$$= \frac{i\,b^2x^2 + i\,bx\log\left(-\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c-i}\right) - i\,a^2 + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)} + 1\right) + (-i\,bx - i\,a)\log$$

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c - I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-I*b*x
- I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - I
*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a))
)/b
```

Sympy [F(-2)]

Exception generated.

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx$$
$$= -2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2} - \frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right) + \text{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)$$
$$+ x\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)$$

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + di
log(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*tanh(b*x +
a) + c)
```

Giac [F]

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx = \int \arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx = \int \mathrm{atan}(c+\tanh(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

```
[In] int(atan(c + tanh(a + b*x)*(c - 1i)),x)
[Out] int(atan(c + tanh(a + b*x)*(c - 1i)), x)
```

3.92 $\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\arctan(c-(i-c)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx=\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan\left(c-(i-c)\tanh\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
[Out] int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx=\int \frac{\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c - I))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx=\text{Timed out}$$

```
[In] integrate(atan(c-(I-c)*tanh(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.50

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/2*pi*log(x) - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))*
log(x) + 1/2*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) + 1/4*I*integrate(lo
g(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*tanh(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c+\tanh(a+b\,x)\ (c-\mathrm{i}))}{x}\,dx$$

```
[In] int(atan(c + tanh(a + b*x)*(c - 1i))/x,x)
```

```
[Out] int(atan(c + tanh(a + b*x)*(c - 1i))/x, x)
```

3.93 $\int (e+fx)^3 \arctan(\coth(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 299

$$\begin{aligned}\int (e+fx)^3 \arctan(\coth(a+bx))\,dx =& \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} \\ &+ \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} \\ &- \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ &+ \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ &+ \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} \\ &- \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\ &- \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ &+ \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\ &+ \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} \\ &- \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}\end{aligned}$$

```
[Out] 1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+1/4*(f*x+e)^4*arctan(coth(b*x+a))/f-
1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^3*polylog(2,I*
exp(2*b*x+2*a))/b+3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2-3/8*I*
f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2-3/8*I*f^2*(f*x+e)*polylog(4,-I*
exp(2*b*x+2*a))/b^3+3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3+3/16*
```

```
I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a
))/b^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5293, 4265, 2611, 6744, 2320, 6724}

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} + \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} - \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4} - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[In] Int[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/(4*f) + ((e + f*x)^4*ArcTan[Coth[a +
b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (((3*I)/8)*f*(e + f*x)^2
*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f*(e + f*x)^2*PolyLog[3
, I*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*a
 + 2*b*x)])/b^3 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/b
^3 + (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 - (((3*I)/16)*f^
3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5293

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Coth[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} + \frac{b\int (e+fx)^4 \text{sech}(2a+2bx)\,dx}{4f}$$

$$
\begin{aligned}
&= \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} \\
&\quad - \frac{1}{2} i \int (e+fx)^3 \log\left(1-ie^{2a+2bx}\right) \, dx + \frac{1}{2} i \int (e+fx)^3 \log\left(1+ie^{2a+2bx}\right) \, dx \\
&= \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} \\
&\quad - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad + \frac{(3if) \int (e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right) \, dx}{4b} \\
&\quad - \frac{(3if) \int (e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right) \, dx}{4b} \\
&= \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} \\
&\quad - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\
&\quad - \frac{(3if^2) \int (e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right) \, dx}{4b^2} \\
&\quad + \frac{(3if^2) \int (e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right) \, dx}{4b^2} \\
&= \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} \\
&\quad - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\
&\quad - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\
&\quad + \frac{(3if^3) \int \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right) \, dx}{8b^3} - \frac{(3if^3) \int \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right) \, dx}{8b^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} \\
&\quad - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\
&\quad - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\
&\quad + \frac{(3if^3) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,-ix)}{x} \, dx, x, e^{2a+2bx}\right)}{16b^4} \\
&\quad - \frac{(3if^3) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,ix)}{x} \, dx, x, e^{2a+2bx}\right)}{16b^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} \\
&\quad - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\
&\quad - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\
&\quad + \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} - \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. 2(299) = 598.

Time = 0.30 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.01

$$
\int (e+fx)^3 \arctan(\coth(a+bx)) \, dx = \frac{1}{4} x \left(4e^3 + 6e^2 fx + 4ef^2x^2 + f^3x^3\right) \arctan(\coth(a+bx))
$$
$$
+ \frac{i\left(8b^4e^3x \log\left(1-ie^{2(a+bx)}\right) + 12b^4e^2fx^2 \log\left(1-ie^{2(a+bx)}\right) + 8b^4ef^2x^3 \log\left(1-ie^{2(a+bx)}\right) + 2b^4f^3x^4 \log\right.}{}
$$

```
[In] Integrate[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Coth[a + b*x]])/4 + (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
```

```
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.86 (sec) , antiderivative size = 3570, normalized size of antiderivative = 11.94

method	result	size
risch	Expression too large to display	3570

```
[In] int((f*x+e)^3*arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/f*e^4*ln(exp(2*b*x+2*a)+I)+1/8*I*f^3*ln(1-I*exp(2*b*x+2*a))*x^4+1/2*I
/b*e^3*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I/b*e^3*dilog(((-I)^(1
/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*e^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2)
)*x+1/2*I*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x-1/8*I*f^3*ln(1+I*exp
(2*b*x+2*a))*x^4-1/8*I/f*e^4*ln(-exp(2*b*x+2*a)+I)-1/2*I*e^3*ln(1+exp(b*x+a
)*(-1)^(3/4))*x-1/2*I*e^3*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/2*I/b*e^3*dilog(1
+exp(b*x+a)*(-1)^(3/4))-1/2*I/b*e^3*dilog(1-exp(b*x+a)*(-1)^(3/4))+1/8*I*f^
3*ln(exp(2*b*x+2*a)-I)*x^4+1/2*I*ln(exp(2*b*x+2*a)-I)*x*e^3+3/16*I*f^3*poly
log(5,-I*exp(2*b*x+2*a))/b^4+1/8*I/f*ln(exp(2*b*x+2*a)-I)*e^4-3/16*I*f^3*po
lylog(5,I*exp(2*b*x+2*a))/b^4+1/16*Pi*(csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(e
xp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn(I*(exp
(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp
(2*b*x+2*a)+I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2
*b*x+2*a)-1))+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b
*x+2*a)-1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp
(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x
+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2
*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))-csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2
*a)-1))^2-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2
*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2
*a)-1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(exp(2*b*x
+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-
1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(exp(2*b*x+2*a
)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^
2+csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+csgn((1-I)*(exp(2*b*x
```

```
+2*a)+I)/(exp(2*b*x+2*a)-1))^3+1)*(f*x+e)^4/f-1/8*I*(f*x+e)^4/f*ln(exp(2*b*
x+2*a)+I)-1/2*I*f^3/b^4*a^4*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f^
3/b^4*a^4*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*f^3/b^4*a^3*dilog(((
-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)+exp(b
*x+a))/(-I)^(1/2))+3/8*I*f^2/b^3*e*polylog(4,I*exp(2*b*x+2*a))-3/8*I*f/b^2*
e^2*polylog(3,I*exp(2*b*x+2*a))+1/8*I*f^3/b^4*a^4*ln(exp(2*b*x+2*a)+I)+3/8*
I*f^3/b^4*ln(1-I*exp(2*b*x+2*a))*a^4+1/4*I*f^3/b*polylog(2,I*exp(2*b*x+2*a)
)*x^3+1/4*I*f^3/b^4*polylog(2,I*exp(2*b*x+2*a))*a^3-3/8*I*f^3/b^2*polylog(3
,I*exp(2*b*x+2*a))*x^2+3/8*I*f^3/b^3*polylog(4,I*exp(2*b*x+2*a))*x+1/2*I*f^
2*e*ln(1-I*exp(2*b*x+2*a))*x^3+3/4*I*f*e^2*ln(1-I*exp(2*b*x+2*a))*x^2-1/2*I
/b*a*e^3*ln(exp(2*b*x+2*a)+I)+1/2*I/b*e^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(
1/2))*a+1/2*I/b*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a-3/2*I*f/b*e^2*
ln(1+I*exp(2*b*x+2*a))*a*x-3/2*I*f^2/b^2*a^2*e*ln(1+exp(b*x+a)*(-1)^(3/4))*
x-3/2*I*f^2/b^2*a^2*e*ln(1-exp(b*x+a)*(-1)^(3/4))*x+3/2*I*f/b*a*e^2*ln(1+ex
p(b*x+a)*(-1)^(3/4))*x+3/2*I*f/b*a*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))*x+3/2*I*
f^2/b^2*e*ln(1+I*exp(2*b*x+2*a))*a^2*x+3/2*I*f^2/b^2*a^2*e*ln(((-I)^(1/2)-e
xp(b*x+a))/(-I)^(1/2))*x+3/2*I*f^2/b^2*a^2*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I
)^(1/2))*x+1/2*I*f^2*ln(exp(2*b*x+2*a)-I)*x^3*e+3/4*I*f*ln(exp(2*b*x+2*a)-I
)*x^2*e^2-1/2*I*f^2/b^3*a^3*e*ln(exp(2*b*x+2*a)+I)+3/4*I*f/b^2*a^2*e^2*ln(e
xp(2*b*x+2*a)+I)+1/2*I*f^3/b^3*ln(1-I*exp(2*b*x+2*a))*a^3*x-1/2*I*f^3/b^3*a
^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*f^3/b^3*a^3*ln(((-I)^(1/2
)+exp(b*x+a))/(-I)^(1/2))*x-3/2*I*f/b^2*a*e^2*dilog(((-I)^(1/2)+exp(b*x+a))
/(-I)^(1/2))-3/2*I*f/b^2*a^2*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-3/2
*I*f/b^2*a^2*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-3/2*I*f/b^2*a*e^2*d
ilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-I*f^2/b^3*e*ln(1-I*exp(2*b*x+2*a))
*a^3+3/4*I*f^2/b*e*polylog(2,I*exp(2*b*x+2*a))*x^2-3/4*I*f^2/b^3*e*polylog(
2,I*exp(2*b*x+2*a))*a^2-3/4*I*f^2/b^2*e*polylog(3,I*exp(2*b*x+2*a))*x+I*f^2
/b^3*e*ln(1+I*exp(2*b*x+2*a))*a^3+3/4*I*f/b^2*e^2*ln(1-I*exp(2*b*x+2*a))*a^
2+3/4*I*f/b*e^2*polylog(2,I*exp(2*b*x+2*a))*x+3/4*I*f/b^2*e^2*polylog(2,I*e
xp(2*b*x+2*a))*a+3/2*I*f^2/b^3*a^3*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))
+3/2*I*f^2/b^3*a^3*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+3/2*I*f^2/b^3*a
^2*e*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+3/2*I*f^2/b^3*a^2*e*dilog(((
-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f^3/b^4*a^4*ln(1-exp(b*x+a)*(-1)^(3
/4))+1/2*I*f^3/b^4*a^3*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f^3/b^4*a^3*dil
og(1-exp(b*x+a)*(-1)^(3/4))-1/8*I*f^3/b^4*a^4*ln(-exp(2*b*x+2*a)+I)-3/4*I*f
*e^2*ln(1+I*exp(2*b*x+2*a))*x^2-1/2*I*f^2*e*ln(1+I*exp(2*b*x+2*a))*x^3+1/2*
I/b*a*e^3*ln(-exp(2*b*x+2*a)+I)-1/2*I/b*e^3*ln(1+exp(b*x+a)*(-1)^(3/4))*a-1
/2*I/b*e^3*ln(1-exp(b*x+a)*(-1)^(3/4))*a-3/8*I*f^3/b^4*ln(1+I*exp(2*b*x+2*a
))*a^4-1/4*I*f^3/b*polylog(2,-I*exp(2*b*x+2*a))*x^3-1/4*I*f^3/b^4*polylog(2
,-I*exp(2*b*x+2*a))*a^3+3/8*I*f^3/b^2*polylog(3,-I*exp(2*b*x+2*a))*x^2-3/8*
I*f^3/b^3*polylog(4,-I*exp(2*b*x+2*a))*x-3/8*I*f^2/b^3*e*polylog(4,-I*exp(2
*b*x+2*a))+3/8*I*f/b^2*e^2*polylog(3,-I*exp(2*b*x+2*a))+1/2*I*f^3/b^4*a^4*l
n(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f^2/b^3*a^3*e*ln(-exp(2*b*x+2*a)+I)-3/4*I*
f/b^2*a^2*e^2*ln(-exp(2*b*x+2*a)+I)+3/4*I*f^2/b^3*e*polylog(2,-I*exp(2*b*x+
2*a))*a^2+3/4*I*f^2/b^2*e*polylog(3,-I*exp(2*b*x+2*a))*x-3/4*I*f/b^2*e^2*ln
```

(1+I*exp(2*b*x+2*a))*a^2-3/4*I*f/b*e^2*polylog(2,-I*exp(2*b*x+2*a))*x-3/4*I*f/b^2*e^2*polylog(2,-I*exp(2*b*x+2*a))*a-1/2*I*f^3/b^3*ln(1+I*exp(2*b*x+2*a))*a^3*x-3/4*I*f^2/b*e*polylog(2,-I*exp(2*b*x+2*a))*x^2+1/2*I*f^3/b^3*a^3*ln(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*f^3/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))*x-3/2*I*f^2/b^3*a^3*e*ln(1+exp(b*x+a)*(-1)^(3/4))-3/2*I*f^2/b^3*a^3*e*ln(1-exp(b*x+a)*(-1)^(3/4))-3/2*I*f^2/b^3*a^2*e*dilog(1+exp(b*x+a)*(-1)^(3/4))-3/2*I*f^2/b^3*a^2*e*dilog(1-exp(b*x+a)*(-1)^(3/4))+3/2*I*f/b^2*a^2*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))+3/2*I*f/b^2*a^2*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))+3/2*I*f/b^2*a*e^2*dilog(1+exp(b*x+a)*(-1)^(3/4))+3/2*I*f/b^2*a*e^2*dilog(1-exp(b*x+a)*(-1)^(3/4))-3/2*I*f^2/b^2*e*ln(1-I*exp(2*b*x+2*a))*a^2*x+3/2*I*f/b*e^2*ln(1-I*exp(2*b*x+2*a))*a*x-3/2*I*f/b*a*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-3/2*I*f/b*a*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(236) = 472$.

Time = 0.36 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int (e + fx)^3 \arctan(\coth(a + bx))\, dx = \text{Too large to display}$$

[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="fricas")

[Out] 1/8*(-24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*f^3*polylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b

```
^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a)
+ sinh(b*x + a)) + 1) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f
^2 + I*a^4*f^3)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-4*
I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-I*sqrt(
4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^
2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*s
inh(b*x + a)) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^
4*f^3)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 24*(-I*b*f^
3*x - I*b*e*f^2)*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- 24*(-I*b*f^3*x - I*b*e*f^2)*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + si
nh(b*x + a))) - 24*(I*b*f^3*x + I*b*e*f^2)*polylog(4, 1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a))) - 24*(I*b*f^3*x + I*b*e*f^2)*polylog(4, -1/2*sqr
t(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*e*f^
2*x + I*b^2*e^2*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))
) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*e*f^2*x + I*b^2*e^2*f)*polylog(3, -1/2*sqrt
(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*e*f^2
*x - I*b^2*e^2*f)*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*e*f^2*x - I*b^2*e^2*f)*polylog(3, -1/2*sqr
t(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \int (e+fx)^3 \operatorname{atan}(\coth(a+bx))\,dx$$

```
[In] integrate((f*x+e)**3*atan(coth(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*atan(coth(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \int (fx+e)^3 \arctan(\coth(bx+a))\,dx$$

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan2(e^(2*b*x + 2*a)
 + 1, e^(2*b*x + 2*a) - 1) + integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2*x
^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x
 + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \int (fx+e)^3 \arctan\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \int \operatorname{atan}(\coth(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

```
[In] int(atan(coth(a + b*x))*(e + f*x)^3,x)
[Out] int(atan(coth(a + b*x))*(e + f*x)^3, x)
```

3.94 $\int (e+fx)^2 \arctan(\coth(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 229

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \arctan(\coth(a+bx))}{3f} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} - \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}$$

```
[Out] 1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+1/3*(f*x+e)^3*arctan(coth(b*x+a))/f-
1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^2*polylog(2,I*
exp(2*b*x+2*a))/b+1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*I*f*
(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2-1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*
a))/b^3+1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5293, 4265, 2611, 6744, 2320, 6724}

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \arctan(\coth(a+bx))}{3f} - \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} + \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[In] Int[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/(3*f) + ((e + f*x)^3*ArcTan[Coth[a +
b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/4)*f*(e + f*x)*PolyL
og[3, (-I)*E^(2*a + 2*b*x)])/b^2 - ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a +
 2*b*x)])/b^2 - ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 + ((I/8)*f
^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5293

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Coth[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(e+fx)^3\arctan(\coth(a+bx))}{3f}+\frac{b\int(e+fx)^3\operatorname{sech}(2a+2bx)\,dx}{3f}\\
&= \frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\arctan(\coth(a+bx))}{3f}\\
&\quad -\frac{1}{2}i\int(e+fx)^2\log\left(1-ie^{2a+2bx}\right)\,dx+\frac{1}{2}i\int(e+fx)^2\log\left(1+ie^{2a+2bx}\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \arctan(\coth(a+bx))}{3f} \\
&\quad - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad + \frac{(if)\int (e+fx) \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)\,dx}{2b} \\
&\quad - \frac{(if)\int (e+fx) \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)\,dx}{2b} \\
&= \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \arctan(\coth(a+bx))}{3f} \\
&\quad - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad + \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} \\
&\quad - \frac{(if^2)\int \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)\,dx}{4b^2} + \frac{(if^2)\int \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)\,dx}{4b^2} \\
&= \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \arctan(\coth(a+bx))}{3f} \\
&\quad - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad + \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} \\
&\quad - \frac{(if^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{8b^3} \\
&\quad + \frac{(if^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,ix)}{x}\,dx, x, e^{2a+2bx}\right)}{8b^3} \\
&= \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \arctan(\coth(a+bx))}{3f} \\
&\quad - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad + \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} \\
&\quad - \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.64

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \frac{1}{3}x(3e^2+3efx+f^2x^2)\arctan(\coth(a+bx)) + \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Coth[a + b*x]])/3 + ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.37 (sec) , antiderivative size = 2668, normalized size of antiderivative = 11.65

method	result	size
risch	Expression too large to display	2668

```
[In] int((f*x+e)^2*arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3+1/2*I*e^2*ln(((-I)^(1/2)+exp(b*x+
a))/(-I)^(1/2))*x+1/6*I/f*e^3*ln(exp(2*b*x+2*a)+I)+1/6*I*f^2*ln(1-I*exp(2*b
*x+2*a))*x^3+1/2*I/b*e^2*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I/b*
e^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*e^2*ln(((-I)^(1/2)-exp(
b*x+a))/(-I)^(1/2))*x+I*f/b*a*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x+I*f/b*a*e*ln(
1-exp(b*x+a)*(-1)^(3/4))*x+1/3*I*f^2/b^3*ln(1+I*exp(2*b*x+2*a))*a^3-1/4*I*f
^2/b*polylog(2,-I*exp(2*b*x+2*a))*x^2+1/4*I*f^2/b^3*polylog(2,-I*exp(2*b*x+
2*a))*a^2+1/4*I*f^2/b^2*polylog(3,-I*exp(2*b*x+2*a))*x-1/2*I*f^2/b^3*a^3*ln
(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f^2/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))-1/2
*I/b*a*e^2*ln(exp(2*b*x+2*a)+I)+1/2*I/b*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)
^(1/2))*a+1/2*I/b*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a-1/6*I*f^2/b^
3*a^3*ln(exp(2*b*x+2*a)+I)-1/4*I*f/b^2*e*polylog(3,I*exp(2*b*x+2*a))+1/2*I*
f*e*ln(1-I*exp(2*b*x+2*a))*x^2-1/3*I*f^2/b^3*ln(1-I*exp(2*b*x+2*a))*a^3+1/4
```

*I*f^2/b*polylog(2,I*exp(2*b*x+2*a))*x^2-1/4*I*f^2/b^3*polylog(2,I*exp(2*b*x+2*a))*a^2-1/4*I*f^2/b^2*polylog(3,I*exp(2*b*x+2*a))*x+1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/6*I*(f*x+e)^3/f*ln(exp(2*b*x+2*a)+I)+1/6*I*f^2*ln(exp(2*b*x+2*a)-I)*x^3+1/2*I*ln(exp(2*b*x+2*a)-I)*x*e^2+1/6*I/f*ln(exp(2*b*x+2*a)-I)*e^3+1/2*I*f*ln(exp(2*b*x+2*a)-I)*x^2*e+1/2*I*f/b^2*a^2*e*ln(exp(2*b*x+2*a)+I)+1/2*I*f^2/b^2*a^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f^2/b^2*a^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f/b^2*e*ln(1-I*exp(2*b*x+2*a))*a^2+1/2*I*f/b*e*polylog(2,I*exp(2*b*x+2*a))*x+1/2*I*f/b^2*e*polylog(2,I*exp(2*b*x+2*a))*a-I*f/b^2*a^2*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-I*f/b^2*a^2*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-I*f/b^2*a*e*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-I*f/b^2*a*e*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*f^2/b^2*ln(1-I*exp(2*b*x+2*a))*a^2*x-1/2*I*f^2/b^3*a^2*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f^2/b^3*a^2*dilog(1-exp(b*x+a)*(-1)^(3/4))+1/4*I*f/b^2*e*polylog(3,-I*exp(2*b*x+2*a))+1/6*I*f^2/b^3*a^3*ln(-exp(2*b*x+2*a)+I)-1/2*I*f*e*ln(1+I*exp(2*b*x+2*a))*x^2+1/2*I/b*a*e^2*ln(-exp(2*b*x+2*a)+I)-1/2*I/b*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*a-1/2*I/b*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))*a-1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*a))/b^3-I*f/b*e*ln(1+I*exp(2*b*x+2*a))*a*x+I*f/b*e*ln(1-I*exp(2*b*x+2*a))*a*x-I*f/b*a*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-I*f/b*a*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x-1/2*I/b*e^2*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I/b*e^2*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/6*I*f^2*ln(1+I*exp(2*b*x+2*a))*x^3-1/6*I/f*e^3*ln(-exp(2*b*x+2*a)+I)+I*f/b^2*a^2*e*ln(1+exp(b*x+a)*(-1)^(3/4))+I*f/b^2*a^2*e*ln(1-exp(b*x+a)*(-1)^(3/4))+I*f/b^2*a*e*dilog(1+exp(b*x+a)*(-1)^(3/4))+I*f/b^2*a*e*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a^2*e*ln(-exp(2*b*x+2*a)+I)+1/2*I*f^2/b^2*ln(1+I*exp(2*b*x+2*a))*a^2*x-1/2*I*f^2/b^2*a^2*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*f^2/b^2*a^2*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/2*I*f/b^2*e*ln(1+I*exp(2*b*x+2*a))*a^2-1/2*I*f/b*e*polylog(2,-I*exp(2*b*x+2*a))*x-1/2*I*f/b^2*e*polylog(2,-I*exp(2*b*x+2*a))*a+1/12*Pi*(csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))-csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(

```
exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn
((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+csgn((1-I)*(exp(2*b*x+2*a)+
I)/(exp(2*b*x+2*a)-1))^3+1)*(f*x+e)^3/f
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(180) = 360.

Time = 0.34 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.38

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*
polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*polylo
g(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3*
b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) - 3*(-I*b^2*
f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + s
inh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(-1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*e*f
*x + I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(
I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x
+ a) + sinh(b*x + a))) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3*e^2*x +
 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(4*I)*(cosh(b*x + a
) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3*e^2*x
+ 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(4*I)*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b^3*e^2
*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(-4*I)*(cosh(b*
x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b^3*
e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt(-4*I)*(cos
h(b*x + a) + sinh(b*x + a)) + 1) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*
f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-3*I*a*b^2*e^2
 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b
*x + a)) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(I*sqrt(-4*I) + 2
*cosh(b*x + a) + 2*sinh(b*x + a)) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*
f^2)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 6*(I*b*f^2*x
+ I*b*e*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*(I
*b*f^2*x + I*b*e*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a
))) - 6*(-I*b*f^2*x - I*b*e*f)*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + s
inh(b*x + a))) - 6*(-I*b*f^2*x - I*b*e*f)*polylog(3, -1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]

$$\int (e+fx)^2 \arctan(\coth(a+bx))\, dx = \int (e+fx)^2 \operatorname{atan}(\coth(a+bx))\, dx$$

```
[In] integrate((f*x+e)**2*atan(coth(b*x+a)),x)
[Out] Integral((e + f*x)**2*atan(coth(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^2 \arctan(\coth(a+bx))\, dx = \int (fx+e)^2 \arctan(\coth(bx+a))\, dx$$

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="maxima")
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x +
 2*a) - 1) + integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b*e
^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)^2 \arctan(\coth(a+bx))\, dx = \int (fx+e)^2 \arctan(\coth(bx+a))\, dx$$

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \arctan(\coth(a+bx))\, dx = \int \operatorname{atan}(\coth(a+b\,x))\,(e+f\,x)^2\, dx$$

```
[In] int(atan(coth(a + b*x))*(e + f*x)^2,x)
[Out] int(atan(coth(a + b*x))*(e + f*x)^2, x)
```

3.95 $\int (e+fx)\arctan(\coth(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 159

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\arctan(\coth(a+bx))}{2f} - \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{if\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

```
[Out] 1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+1/2*(f*x+e)^2*arctan(coth(b*x+a))/f-
1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)*polylog(2,I*exp(
2*b*x+2*a))/b+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/8*I*f*polylog(3,I*
exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5293, 4265, 2611, 2320, 6724}

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\arctan(\coth(a+bx))}{2f} + \frac{if\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{if\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} - \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[In] Int[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/(2*f) + ((e + f*x)^2*ArcTan[Coth[a +
b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I/4
)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/8)*f*PolyLog[3, (-I)*E^(
2*a + 2*b*x)])/b^2 - ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5293

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Coth[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}+\frac{b\int(e+fx)^2\operatorname{sech}(2a+2bx)\,dx}{2f}\\
&= \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}\\
&\quad -\frac{1}{2}i\int(e+fx)\log\left(1-ie^{2a+2bx}\right)\,dx+\frac{1}{2}i\int(e+fx)\log\left(1+ie^{2a+2bx}\right)\,dx\\
&= \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}\\
&\quad -\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}\\
&\quad +\frac{(if)\int\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)\,dx}{4b}-\frac{(if)\int\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)\,dx}{4b}\\
&= \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}\\
&\quad -\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}\\
&\quad +\frac{(if)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&\quad -\frac{(if)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

$$= \frac{(e+fx)^2 \arctan\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2 \arctan(\coth(a+bx))}{2f} - \frac{i(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)\,\mathrm{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if\,\mathrm{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{if\,\mathrm{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.49

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = ex\arctan(\coth(a+bx)) + \frac{1}{2}fx^2\arctan(\coth(a+bx)) + \frac{ie\left(2bx\left(\log\left(1-ie^{2}\right.\right.\right.}{} + \frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right) - 2b^2x^2\log\left(1+ie^{2(a+bx)}\right) - 2bx\,\mathrm{PolyLog}\left(2,-ie^{2(a+bx)}\right) + 2bx\,\mathrm{PolyLog}\left(2,i\right.\right.}{8b^2}$$

```
[In] Integrate[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

```
[Out] e*x*ArcTan[Coth[a + b*x]] + (f*x^2*ArcTan[Coth[a + b*x]])/2 + ((I/4)*e*(2*b
*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x))]) - PolyLog[2, (
-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^(2*(a + b*x))]))/b + ((I/8)*f*(2*b^2*
x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a + b*x))] - 2*b
*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*(a + b*x))] +
 PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 1777, normalized size of antiderivative = 11.18

method	result	size
risch	Expression too large to display	1777

```
[In] int((f*x+e)*arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*f*ln(1+I*exp(2*b*x+2*a))*x^2+1/4*I*ln(exp(2*b*x+2*a)-I)*x^2*f+1/2*I*
ln(exp(2*b*x+2*a)-I)*e*x+1/2*I*(-1/2*f*x^2-e*x)*ln(exp(2*b*x+2*a)+I)+1/4*Pi
*(csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2
*a)-I)/(exp(2*b*x+2*a)-1))-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a
)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp(2*b*x+2*a
)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))+csgn(I*(exp(2*b*x+2*a)+
I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(exp(2*b*x+2*a)-
I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-cs
gn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I)
```

```
/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))-csgn
((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(
exp(2*b*x+2*a)-1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+
I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(
2*b*x+2*a)-1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(
exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(
2*b*x+2*a)-1))^3+csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+1)*(1/
2*f*x^2+e*x)-1/4*I*f/b^2*a^2*ln(-exp(2*b*x+2*a)+I)+1/2*I*e/b*a*ln(-exp(2*b*
x+2*a)+I)+1/4*I*f*ln(1-I*exp(2*b*x+2*a))*x^2+1/2*I*e*ln(((-I)^(1/2)-exp(b*x
+a))/(-I)^(1/2))*x+1/2*I*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+1/2*I*e
/b*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*e/b*dilog(((-I)^(1/2)+ex
p(b*x+a))/(-I)^(1/2))+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/2*I*e*ln(1
+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e/b*d
ilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*e/b*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*
I*e/b*ln(1+exp(b*x+a)*(-1)^(3/4))*a-1/2*I*e/b*ln(1-exp(b*x+a)*(-1)^(3/4))*a
+1/2*I*f/b^2*a^2*ln(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a^2*ln(1-exp(b*x+a
)*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*di
log(1-exp(b*x+a)*(-1)^(3/4))+1/2*I*e/b*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2
))*a+1/2*I*e/b*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a-1/4*I*f/b^2*ln(1+I*
exp(2*b*x+2*a))*a^2-1/4*I*f/b*polylog(2,-I*exp(2*b*x+2*a))*x-1/4*I*f/b^2*po
lylog(2,-I*exp(2*b*x+2*a))*a+1/2*I*f/b*a*ln(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*
I*f/b*a*ln(1-exp(b*x+a)*(-1)^(3/4))*x+1/2*I*f/b*ln(1-I*exp(2*b*x+2*a))*a*x-
1/2*I*f/b*a*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*f/b*a*ln(((-I)^(
1/2)+exp(b*x+a))/(-I)^(1/2))*x-1/2*I*f/b^2*a^2*ln(((-I)^(1/2)-exp(b*x+a))/(
-I)^(1/2))-1/2*I*f/b^2*a^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*f/b
^2*a*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f/b^2*a*dilog(((-I)^(1
/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I/b*e*a*ln(exp(2*b*x+2*a)+I)+1/4*I/b^2*f*a^
2*ln(exp(2*b*x+2*a)+I)+1/4*I*f/b^2*ln(1-I*exp(2*b*x+2*a))*a^2+1/4*I*f/b*pol
ylog(2,I*exp(2*b*x+2*a))*x+1/4*I*f/b^2*polylog(2,I*exp(2*b*x+2*a))*a-1/2*I*
f/b*ln(1+I*exp(2*b*x+2*a))*a*x-1/8*I*f*polylog(3,I*exp(2*b*x+2*a))/b^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(130) = 260$.

Time = 0.34 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.77

$$\int (e+fx)\arctan(\coth(a+bx))\,dx$$

$$= \frac{2\left(b^2fx^2+2\,b^2ex\right)\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)-2\left(-i\,bfx-i\,be\right)\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-2\,(\cdots}{}$$

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) - 2*(-I*
b*f*x - I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(-I
*b*f*x - I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(
I*b*f*x + I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*
(I*b*f*x + I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
(I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(4*I)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*
a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*
x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(b*x + a)
+ sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*
log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-2*I*a*b*e + I*
a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2*I*a*b*e +
 I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2*I*a*b*
e - I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2*I*a
*b*e - I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 2*
I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polyl
og(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polylog(3, 1/
2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polylog(3, -1/2*sqrt(
-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \int (e+fx)\,\mathrm{atan}\,(\coth\,(a+bx))\,dx$$

```
[In] integrate((f*x+e)*atan(coth(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atan(coth(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \int (fx+e)\arctan\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + int
egrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1),
 x)
```

Giac [F]

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \int (fx+e)\arctan\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \int \mathrm{atan}(\coth(a+b\,x))\,\left(e+f\,x\right)\,dx$$

```
[In] int(atan(coth(a + b*x))*(e + f*x),x)
[Out] int(atan(coth(a + b*x))*(e + f*x), x)
```

3.96 $\int \arctan(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 73

$$\int \arctan(\coth(a+bx))\,dx = x\arctan\left(e^{2a+2bx}\right) + x\arctan(\coth(a+bx)) - \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] x*arctan(exp(2*b*x+2*a))+x*arctan(coth(b*x+a))-1/4*I*polylog(2,-I*exp(2*b*x
+2*a))/b+1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5289, 4265, 2317, 2438}

$$\int \arctan(\coth(a+bx))\,dx = x\arctan\left(e^{2a+2bx}\right) + x\arctan(\coth(a+bx)) - \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[In] Int[ArcTan[Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[E^(2*a + 2*b*x)] + x*ArcTan[Coth[a + b*x]] - ((I/4)*PolyLog[2, (-I
)*E^(2*a + 2*b*x)])/b + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5289

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Coth[a + b
*x]], x] + Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\arctan(\coth(a+bx)) + b\int x\operatorname{sech}(2a+2bx)\,dx \\
&= x\arctan\left(e^{2a+2bx}\right) + x\arctan(\coth(a+bx)) \\
&\quad -\frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)\,dx + \frac{1}{2}i\int \log\left(1+ie^{2a+2bx}\right)\,dx \\
&= x\arctan\left(e^{2a+2bx}\right) + x\arctan(\coth(a+bx)) \\
&\quad -\frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} + \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&= x\arctan\left(e^{2a+2bx}\right) + x\arctan(\coth(a+bx)) \\
&\quad -\frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \arctan(\coth(a+bx))\,dx = x\arctan(\coth(a+bx)) + \frac{i\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\operatorname{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\operatorname{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b}$$

```
[In] Integrate[ArcTan[Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[Coth[a + b*x]] + ((I/4)*(2*b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1
 + I*E^(2*(a + b*x))]) - PolyLog[2, (-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^
(2*(a + b*x))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(62) = 124.

Time = 0.84 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.19

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\coth(bx+a))\arctan(\coth(bx+a))-\frac{i\operatorname{arctanh}(\coth(bx+a))\left(\ln\left(1-\frac{i(\coth(bx+a)+1)^2}{1-\coth(bx+a)^2}\right)-\ln\left(1+\frac{i(\coth(bx+a)+1)^2}{1-\coth(bx+a)^2}\right)\right)}{2}+}{b}$
default	$\frac{\operatorname{arctanh}(\coth(bx+a))\arctan(\coth(bx+a))-\frac{i\operatorname{arctanh}(\coth(bx+a))\left(\ln\left(1-\frac{i(\coth(bx+a)+1)^2}{1-\coth(bx+a)^2}\right)-\ln\left(1+\frac{i(\coth(bx+a)+1)^2}{1-\coth(bx+a)^2}\right)\right)}{2}+}{b}$
risch	Expression too large to display

```
[In] int(arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(coth(b*x+a))*arctan(coth(b*x+a))-1/2*I*arctanh(coth(b*x+a))*(l
n(1-I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2))-ln(1+I*(coth(b*x+a)+1)^2/(1-coth
(b*x+a)^2)))+1/4*I*dilog(1+I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2))-1/4*I*dil
og(1-I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(56) = 112$.

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.58

$$\int \arctan(\coth(a+bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right) + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a)+\sinh(bx+a))+1\right) + (i\,bx+i\,a)\log\left(-\right.}{}$$

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(cosh(b*x + a)/sinh(b*x + a)) + (I*b*x + I*a)*log(1/2*sqrt
(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(4*
I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt(-4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) - I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) - I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) + I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) - I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \arctan(\coth(a+bx))\,dx = \int \operatorname{atan}(\coth(a+bx))\,dx$$

```
[In] integrate(atan(coth(b*x+a)),x)
```

```
[Out] Integral(atan(coth(a + b*x)), x)
```

Maxima [F]

$$\int \arctan(\coth(a+bx))\,dx = \int \arctan(\coth(bx+a))\,dx$$

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + 2*b*integrate(x*e^(2*
b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int \arctan(\coth(a+bx))\,dx = \int \arctan\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan(coth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(\coth(a+bx))\,dx = \int \mathrm{atan}(\mathrm{coth}(a+b\,x))\,dx$$

```
[In] int(atan(coth(a + b*x)),x)
[Out] int(atan(coth(a + b*x)), x)
```

3.97 $\int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx = \text{Int}\left(\frac{\arctan(\coth(a+bx))}{e+fx}, x\right)$$

```
[Out] CannotIntegrate(arctan(coth(b*x+a))/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx = \int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx$$

```
[In] Int[ArcTan[Coth[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx$$

Mathematica [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx$$

```
[In] Integrate[ArcTan[Coth[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}dx$$

```
[In] int(arctan(coth(b*x+a))/(f*x+e),x)
[Out] int(arctan(coth(b*x+a))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arctan(coth(b*x + a))/(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \text{Timed out}$$

```
[In] integrate(atan(coth(b*x+a))/(f*x+e),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arctan(coth(b*x + a))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 90.48 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.20

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \int \frac{\mathrm{atan}(\coth(a+b\,x))}{e+f\,x}\,dx$$

```
[In] int(atan(coth(a + b*x))/(e + f*x),x)
[Out] int(atan(coth(a + b*x))/(e + f*x), x)
```

3.98 $\int x^2 \arctan(c + d\coth(a + bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 351

$$\begin{aligned}\int x^2 \arctan(c+d\coth(a+bx))\,dx &= \frac{1}{3}x^3\arctan(c+d\coth(a+bx)) \\ &+ \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\ &- \frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\ &+ \frac{ix^2\operatorname{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ &- \frac{ix^2\operatorname{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ &- \frac{ix\operatorname{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\ &+ \frac{ix\operatorname{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} \\ &+ \frac{i\operatorname{PolyLog}\left(4,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} \\ &- \frac{i\operatorname{PolyLog}\left(4,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}\end{aligned}$$

```
[Out] 1/3*x^3*arctan(c+d*coth(b*x+a))+1/6*I*x^3*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/6*I*x^3*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x^2*polylog(2,(I-c
-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x^2*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I
+c-d))/b-1/4*I*x*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/4*I*x*poly
```

```
log(3,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2+1/8*I*polylog(4,(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b^3-1/8*I*polylog(4,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5309, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^2\arctan(c+d\coth(a+bx))\,dx=&\ \frac{1}{3}x^3\arctan(d\coth(a+bx)+c)\\&+\frac{i\,\text{PolyLog}\left(4,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}\\&-\frac{i\,\text{PolyLog}\left(4,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3}\\&-\frac{ix\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}\\&+\frac{ix\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}\\&+\frac{ix^2\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}\\&-\frac{ix^2\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}\\&+\frac{1}{6}ix^3\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)\\&-\frac{1}{6}ix^3\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)\end{aligned}$$

```
[In] Int[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Coth[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d)])/b - ((I/4)*x^2*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/
b - ((I/4)*x*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + (
(I/4)*x*PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2 + ((I/8)
*PolyLog[4, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^3 - ((I/8)*PolyLo
g[4, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5309

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*
b*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
- d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3 \arctan(c + d\coth(a + bx)) \\
&\quad - \frac{1}{3}(b(1 - i(c + d))) \int \frac{e^{2a+2bx}x^3}{i + c - d + (-i - c - d)e^{2a+2bx}}\,dx \\
&\quad + \frac{1}{3}(b(1 + i(c + d))) \int \frac{e^{2a+2bx}x^3}{i - c + d + (-i + c + d)e^{2a+2bx}}\,dx \\
&= \frac{1}{3}x^3 \arctan(c + d\coth(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) \\
&\quad - \frac{1}{6}ix^3 \log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) + \frac{1}{2}i \int x^2 \log\left(1 + \frac{(-i - c - d)e^{2a+2bx}}{i + c - d}\right) dx \\
&\quad - \frac{1}{2}i \int x^2 \log\left(1 + \frac{(-i + c + d)e^{2a+2bx}}{i - c + d}\right) dx \\
&= \frac{1}{3}x^3 \arctan(c + d\coth(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) \\
&\quad - \frac{1}{6}ix^3 \log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) + \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&\quad - \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{i \int x \operatorname{PolyLog}\left(2, -\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right) dx}{2b} \\
&\quad - \frac{i \int x \operatorname{PolyLog}\left(2, -\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right) dx}{2b} \\
&= \frac{1}{3}x^3 \arctan(c + d\coth(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) \\
&\quad - \frac{1}{6}ix^3 \log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) + \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&\quad - \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{ix \operatorname{PolyLog}\left(3, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\
&\quad + \frac{ix \operatorname{PolyLog}\left(3, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} - \frac{i \int \operatorname{PolyLog}\left(3, -\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right) dx}{4b^2} \\
&\quad + \frac{i \int \operatorname{PolyLog}\left(3, -\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right) dx}{4b^2}
\end{aligned}
$$

$$= \frac{1}{3}x^3 \arctan(c + d\coth(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3 \log\left(1 - \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{ix \operatorname{PolyLog}\left(3, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} + \frac{ix \operatorname{PolyLog}\left(3, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} + \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, \frac{(-i+c+d)x}{-i+c-d}\right)}{x} dx, x, e^{2a+2bx}\right)}{8b^3} - \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, \frac{(i+c+d)x}{i+c-d}\right)}{x} dx, x, e^{2a+2bx}\right)}{8b^3}$$

$$= \frac{1}{3}x^3 \arctan(c + d\coth(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{6}ix^3 \log\left(1 - \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{ix \operatorname{PolyLog}\left(3, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} + \frac{ix \operatorname{PolyLog}\left(3, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} + \frac{i \operatorname{PolyLog}\left(4, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} - \frac{i \operatorname{PolyLog}\left(4, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.26

$$\int x^2 \arctan(c + d\coth(a + bx))\, dx = \frac{1}{3}x^3 \arctan(c + d\coth(a + bx)) + \frac{d\left(4b^3x^3 \log\left(1 - \frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) - 4b^3x^3 \log\left(1 + \frac{(1+(c+d)^2)e^{2(a+bx)}}{-1-c^2+d^2+2\sqrt{-d^2}}\right) + 6b^2x^2 \operatorname{PolyLog}\left(2, \frac{(1+c^2+2cd+d^2)e^{2}}{1+c^2-d^2+2\sqrt{-}}\right.}{}$$

```
[In] Integrate[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Coth[a + b*x]])/3 + (d*(4*b^3*x^3*Log[1 - ((1 + (c + d)^2
)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 4*b^3*x^3*Log[1 + ((1
+ (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 6*b^2*x^2*
PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqr
t[-d^2])] - 6*b^2*x^2*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 -
c^2 + d^2 + 2*Sqrt[-d^2]))] - 6*b*x*PolyLog[3, ((1 + c^2 + 2*c*d + d^2)*E^(
```

```
2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 6*b*x*PolyLog[3, -(((1 + (c
 + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2]))] - 3*PolyLog[4,
((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 - 2*Sqrt[-d^2])] +
 3*PolyLog[4, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*
Sqrt[-d^2])]))/(24*b^3*Sqrt[-d^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.98 (sec) , antiderivative size = 6845, normalized size of antiderivative = 19.50

method	result	size
risch	Expression too large to display	6845

```
[In] int(x^2*arctan(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1269 vs. 2(259) = 518.

Time = 0.36 (sec) , antiderivative size = 1269, normalized size of antiderivative = 3.62

$$\int x^2 \arctan(c + d\coth(a + bx))\, dx = \text{Too large to display}$$

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
3*I*b^2*x^2*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 + 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^
2*dilog(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a
) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^3*log(2*(c^2 + 2
*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2
*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d +
 d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d
 + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(
b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d +
 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c
^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x +
```

```
a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))) - 6*I*b*x*polylog(3, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -sqrt((c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)
)) + 6*I*b*x*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1
))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -sqrt((c^2 - d^2 -
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + (I*
b^3*x^3 + I*a^3)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log(-sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))
+ 1) + (-I*b^3*x^3 - I*a^3)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + 6*I*polylog(4, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*polylog(4, -sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 - 6*I*polylog(4, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, -sqrt((c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d \coth(a + bx))\, dx = \text{Timed out}$$

```
[In] integrate(x**2*atan(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2 \arctan(c + d \coth(a + bx))\, dx = \int x^2 \arctan\left(d \coth\left(bx + a\right) + c\right)\, dx$$

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
- 1) + 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^
(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) -
 d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Giac [F]

$$\int x^2 \arctan(c + d \coth(a + bx)) \, dx = \int x^2 \arctan\left(d \coth\left(bx + a\right) + c\right) \, dx$$

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arctan(d*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d \coth(a + bx)) \, dx = \int x^2 \operatorname{atan}(c + d \coth(a + b\,x)) \, dx$$

```
[In] int(x^2*atan(c + d*coth(a + b*x)),x)
[Out] int(x^2*atan(c + d*coth(a + b*x)), x)
```

3.99 $\int x \arctan(c + d \coth(a + bx)) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 265

$$\begin{aligned}\int x \arctan(c+d\coth(a+bx))\,dx &= \frac{1}{2}x^2\arctan(c+d\coth(a+bx)) \\ &\quad+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\ &\quad-\frac{1}{4}ix^2\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\ &\quad+\frac{ix\,\text{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ &\quad-\frac{ix\,\text{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ &\quad-\frac{i\,\text{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} \\ &\quad+\frac{i\,\text{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}\end{aligned}$$

```
[Out] 1/2*x^2*arctan(c+d*coth(b*x+a))+1/4*I*x^2*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/4*I*x^2*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x*polylog(2,(I-c-d
)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d
))/b-1/8*I*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/8*I*polylog(3,(I
+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5309, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int x \arctan(c+d\coth(a+bx))\,dx = &\frac{1}{2}x^2\arctan(d\coth(a+bx)+c)\\ &-\frac{i\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}\\ &+\frac{i\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}\\ &+\frac{ix\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}\\ &-\frac{ix\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}\\ &+\frac{1}{4}ix^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)\\ &-\frac{1}{4}ix^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)\end{aligned}$$

```
[In] Int[x*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Coth[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)
])/b - ((I/4)*x*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b -
((I/8)*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + ((I/8)*
PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5309

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*
b*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
- d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c+d\coth(a+bx)) \\
&\quad -\frac{1}{2}(b(1-i(c+d)))\int\frac{e^{2a+2bx}x^2}{i+c-d+(-i-c-d)e^{2a+2bx}}\,dx \\
&\quad +\frac{1}{2}(b(1+i(c+d)))\int\frac{e^{2a+2bx}x^2}{i-c+d+(-i+c+d)e^{2a+2bx}}\,dx \\
&= \frac{1}{2}x^2\arctan(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad -\frac{1}{4}ix^2\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{1}{2}i\int x\log\left(1+\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right)\,dx \\
&\quad -\frac{1}{2}i\int x\log\left(1+\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right)\,dx
\end{aligned}$$

$$= \frac{1}{2}x^2 \arctan(c + d \coth(a + bx)) + \frac{1}{4}ix^2 \log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) - \frac{1}{4}ix^2 \log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) + \frac{ix \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{ix \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{i \int \operatorname{PolyLog}\left(2, -\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right) dx}{4b} - \frac{i \int \operatorname{PolyLog}\left(2, -\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right) dx}{4b}$$

$$= \frac{1}{2}x^2 \arctan(c + d \coth(a + bx)) + \frac{1}{4}ix^2 \log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) - \frac{1}{4}ix^2 \log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) + \frac{ix \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{ix \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, \frac{(-i+c+d)x}{-i+c-d}\right)}{x} dx, x, e^{2a+2bx}\right)}{8b^2} + \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, \frac{(i+c+d)x}{i+c-d}\right)}{x} dx, x, e^{2a+2bx}\right)}{8b^2}$$

$$= \frac{1}{2}x^2 \arctan(c + d \coth(a + bx)) + \frac{1}{4}ix^2 \log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) - \frac{1}{4}ix^2 \log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) + \frac{ix \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{ix \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{i \operatorname{PolyLog}\left(3, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} + \frac{i \operatorname{PolyLog}\left(3, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.26

$$\int x \arctan(c + d \coth(a + bx)) \, dx = \frac{1}{2}x^2 \arctan(c + d \coth(a + bx)) + \frac{d\left(2b^2x^2 \log\left(1 - \frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) - 2b^2x^2 \log\left(1 + \frac{(1+(c+d)^2)e^{2(a+bx)}}{-1-c^2+d^2+2\sqrt{-d^2}}\right) + 2bx \operatorname{PolyLog}\left(2, \frac{(1+c^2+2cd+d^2)e^{2}}{1+c^2-d^2+2\sqrt{-}}\right.\right.}{}$$

```
[In] Integrate[x*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Coth[a + b*x]])/2 + (d*(2*b^2*x^2*Log[1 - ((1 + (c + d)^2
)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*b^2*x^2*Log[1 + ((1
+ (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 2*b*x*Poly
Log[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d
^2])] - 2*b*x*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^
2 + 2*Sqrt[-d^2]))] + PolyLog[3, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/
(1 + c^2 - d^2 - 2*Sqrt[-d^2])] - PolyLog[3, ((1 + c^2 + 2*c*d + d^2)*E^(2*
(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])]))/(8*b^2*Sqrt[-d^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.42 (sec) , antiderivative size = 6495, normalized size of antiderivative = 24.51

method	result	size
risch	Expression too large to display	6495

```
[In] int(x*arctan(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1051 vs. 2(195) = 390.

Time = 0.37 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.97

$$\int x\arctan(c+d\coth(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
2*I*b*x*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b
*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(sqrt
((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) - 2*I*b*x*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*
cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 - 2*
I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log
(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b
*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c
*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^
```

```
2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 -
d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2
 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^
2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + (I*
b^2*x^2 - I*a^2)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(-sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))
+ 1) + (-I*b^2*x^2 + I*a^2)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 2*I*polylog(3, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3, -sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 + 2*I*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, -sqrt((c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int x \arctan(c + d \coth(a + bx))\, dx = \text{Timed out}$$

```
[In] integrate(x*atan(c+d*coth(b*x+a)),x)
[Out] Timed out
```

Maxima [F]

$$\int x \arctan(c + d \coth(a + bx))\, dx = \int x \arctan\left(d \coth\left(bx + a\right) + c\right)\, dx$$

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
[Out] 1/2*x^2*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
- 1) + 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a
) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2
*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Giac [F]

$$\int x \arctan(c + d \coth(a + bx)) \, dx = \int x \arctan\left(d \coth\left(bx + a\right) + c\right) \, dx$$

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctan(d*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \arctan(c + d \coth(a + bx)) \, dx = \int x \operatorname{atan}(c + d \coth(a + b x)) \, dx$$

```
[In] int(x*atan(c + d*coth(a + b*x)),x)
[Out] int(x*atan(c + d*coth(a + b*x)), x)
```

3.100 $\int \arctan(c+d\coth(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 174

$$\begin{aligned}\int \arctan(c+d\coth(a+bx))\,dx = {} & x\arctan(c+d\coth(a+bx)) \\ & + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\ & - \frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\ & + \frac{i\operatorname{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ & - \frac{i\operatorname{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}\end{aligned}$$

```
[Out] x*arctan(c+d*coth(b*x+a))+1/2*I*x*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))-1/2*
I*x*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*polylog(2,(I-c-d)*exp(2*b*x+
2*a)/(I-c+d))/b-1/4*I*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{5301, 2221, 2317, 2438\}$

$$\int \arctan(c+d\coth(a+bx))\,dx = x\arctan(d\coth(a+bx)+c) + \frac{i\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} + \frac{1}{2}ix\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) - \frac{1}{2}ix\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)$$

```
[In] Int[ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Coth[a + b*x]] + (I/2)*x*Log[1 - ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b - ((I/4
)*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5301

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] + (-Dist[I*b*(I - c - d), Int[x*(E^(2*a + 2*b*
x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*b*(I + c + d
), Int[x*(E^(2*a + 2*b*x)/(I + c - d - (I + c + d)*E^(2*a + 2*b*x))), x], x
```

```
]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\arctan(c+d\coth(a+bx))-(b(1-i(c+d)))\int\frac{e^{2a+2bx}x}{i+c-d+(-i-c-d)e^{2a+2bx}}\,dx \\
&\quad+(b(1+i(c+d)))\int\frac{e^{2a+2bx}x}{i-c+d+(-i+c+d)e^{2a+2bx}}\,dx \\
&= x\arctan(c+d\coth(a+bx))+\frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad-\frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{1}{2}i\int\log\left(1+\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right)dx \\
&\quad-\frac{1}{2}i\int\log\left(1+\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right)dx \\
&= x\arctan(c+d\coth(a+bx))+\frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad-\frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{i\text{Subst}\left(\int\frac{\log\left(1+\frac{(-i-c-d)x}{i+c-d}\right)}{x}\,dx,x,e^{2a+2bx}\right)}{4b} \\
&\quad-\frac{i\text{Subst}\left(\int\frac{\log\left(1+\frac{(-i+c+d)x}{i-c+d}\right)}{x}\,dx,x,e^{2a+2bx}\right)}{4b} \\
&= x\arctan(c+d\coth(a+bx))+\frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad-\frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\
&\quad+\frac{i\,\text{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b}-\frac{i\,\text{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.65

$$
\int\arctan(c+d\coth(a+bx))\,dx = x\arctan(c+d\coth(a+bx)) + \frac{4a\sqrt{-d^2}\arctan\left(\frac{1+c^2-d^2-(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right)+2d(a+bx)\log\left(1-\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)-2d(a+bx)\log}{4b\sqrt{-}}
$$

```
[In] Integrate[ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Coth[a + b*x]] + (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 - (1
+ c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + 2*d*(a + b*x)*Log[1 - ((1 +
(c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*d*(a + b*x)
*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])]
 + d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 +
2*Sqrt[-d^2])] - d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(
-1 - c^2 + d^2 + 2*Sqrt[-d^2]))])/(4*b*Sqrt[-d^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(150) = 300$.

Time = 2.57 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln\left(\frac{i+d}{}\right.}{2}\right.}{}}{}$
default	$\frac{\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln\left(\frac{i+d}{}\right.}{2}\right.}{}}{}$
risch	Expression too large to display

```
[In] int(arctan(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(1/2*arctan(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*arctan(c+d*co
th(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*d^2*(1/d*(1/2*I*ln(-d*coth(b*x+a)+d)*
ln((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I*ln(-d*coth(b*x+a)+d)*ln((I-d*coth(b*x
+a)-c)/(I-c-d))+1/2*I*dilog((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I*dilog((I-d*c
oth(b*x+a)-c)/(I-c-d)))-1/d*(1/2*I*ln(-d*coth(b*x+a)-d)*ln((I+d*coth(b*x+a)
+c)/(I+c-d))-1/2*I*ln(-d*coth(b*x+a)-d)*ln((I-d*coth(b*x+a)-c)/(I-c+d))+1/2
*I*dilog((I+d*coth(b*x+a)+c)/(I+c-d))-1/2*I*dilog((I-d*coth(b*x+a)-c)/(I-c+
d)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(128) = 256.

Time = 0.39 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.67

$$\int \arctan(c + d\coth(a + bx))\,dx$$

$$= \frac{2\,bx\arctan\left(\frac{d\cosh(bx+a)+c\sinh(bx+a)}{\sinh(bx+a)}\right) - i\,a\log\left(2\left(c^2+2\,cd+d^2+1\right)\cosh\left(bx+a\right)+2\left(c^2+2\,cd+d^2+1\right)\right.}{}$$

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c
^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^2
+ 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2
 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(
-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + (-I*b*x - I*a)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2
*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + I*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2
+ 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-sqrt((c^2 - d^2 + 2*I*d +
 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(sqr
t((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*
x + a))) - I*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F(-1)]

Timed out.

$$\int \arctan(c+d\coth(a+bx))\,dx = \text{Timed out}$$

```
[In] integrate(atan(c+d*coth(b*x+a)),x)
[Out] Timed out
```

Maxima [F]

$$\int \arctan(c+d\coth(a+bx))\,dx = \int \arctan\left(d\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
[Out] 4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*d
*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2*e^(2*a)
+ e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x)
 - c + d, e^(2*b*x + 2*a) - 1)
```

Giac [F]

$$\int \arctan(c+d\coth(a+bx))\,dx = \int \arctan\left(d\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan(d*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c+d\coth(a+bx))\,dx = \int \mathrm{atan}(c+d\coth(a+b\,x))\,dx$$

```
[In] int(atan(c + d*coth(a + b*x)),x)
[Out] int(atan(c + d*coth(a + b*x)), x)
```

3.101 $\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\arctan(c+d\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*coth(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx=\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx=\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c + d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + d*Coth[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(c+d\coth\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arctan(c+d*coth(b*x+a))/x,x)
[Out] int(arctan(c+d*coth(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx=\int \frac{\arctan\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*coth(b*x + a) + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 161.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{atan}\left(c+d\coth\left(a+bx\right)\right)}{x}\,dx$$

```
[In] integrate(atan(c+d*coth(b*x+a))/x,x)
[Out] Integral(atan(c + d*coth(a + b*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\coth(a + bx))}{x} dx = \int \frac{\arctan(d\coth(bx + a) + c)}{x} dx$$

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\coth(a + bx))}{x} dx = \int \frac{\arctan(d\coth(bx + a) + c)}{x} dx$$

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\coth(a + bx))}{x} dx = \int \frac{\mathrm{atan}(c + d\coth(a + b\,x))}{x} dx$$

```
[In] int(atan(c + d*coth(a + b*x))/x,x)
[Out] int(atan(c + d*coth(a + b*x))/x, x)
```

3.102 $\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx = -\frac{1}{12}ibx^4+\frac{1}{3}x^3\arctan(c+(i+c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}+\frac{i\,\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arctan(c+(I+c)*coth(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,I*c*e
xp(2*b*x+2*a))/b^2+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5305, 2215, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx = \frac{1}{3}x^3\arctan(c+(c+i)\coth(a+bx)) + \frac{i\,\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3} - \frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2} + \frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)-\frac{1}{12}ibx^4$$

```
[In] Int[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-1/12*I)*b*x^4 + (x^3*ArcTan[c + (I + c)*Coth[a + b*x]])/3 + (I/6)*x^3*Log
[1 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b -
 ((I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, I*c*E^(2
*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5305

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c+(i+c)\coth(a+bx)) - \frac{1}{3}b\int\frac{x^3}{-i-ce^{2a+2bx}}\,dx \\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\coth(a+bx)) - \frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{-i-ce^{2a+2bx}}\,dx \\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\coth(a+bx)) \\
&\quad + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{1}{2}i\int x^2\log\left(1-ice^{2a+2bx}\right)\,dx \\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) \\
&\quad + \frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} - \frac{i\int x\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)\,dx}{2b} \\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1\right. \\
&\qquad \left.-ice^{2a+2bx}\right) + \frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} \\
&\quad - \frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2} + \frac{i\int\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)\,dx}{4b^2} \\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\coth(a+bx)) \\
&\quad + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} \\
&\quad - \frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2} + \frac{i\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3} \\
&= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) \\
&\quad + \frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2} + \frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx$$
$$= \frac{8b^3x^3\arctan(c+(i+c)\coth(a+bx))+4ib^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-6ib^2x^2\,\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-6i}{24b^3}$$

```
[In] Integrate[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcTan[c + (I + c)*Coth[a + b*x]] + (4*I)*b^3*x^3*Log[1 + I/(c*E
^(2*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - (6*
I)*b*x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - (3*I)*PolyLog[4, (-I)/(c*E^(2
*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 1405, normalized size of antiderivative = 9.89

method	result	size
risch	Expression too large to display	1405

```
[In] int(x^2*arctan(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-I
*c)^(1/2))+1/12*I/b^3*c/(I+c)*a^4-1/4*I*x*polylog(3,I*c*exp(2*b*x+2*a))/b^2
+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3+1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)
*(-I*c)^(1/2))-1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c+2*I)+1/2*I/b^3*a^2*dilog(1+I
*exp(b*x+a)*(-I*c)^(1/2))-1/12*I*b*c/(I+c)*x^4+1/2*I/b^2*a^2*ln(1-I*exp(b*x
+a)*(-I*c)^(1/2))*x+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/4*I/b^3
*polylog(2,I*c*exp(2*b*x+2*a))*a^2+1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))+1/4*I
*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)-1/6*I/b^3*a^3*ln(exp(2*b*x+2*a)*c+I)-1/12*Pi*(csgn(I/(exp(2*b
*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I
)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2
*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(e
xp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*c
sgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*I*exp(2*b*x+
2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-c
```

```
sgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c
+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)
-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-csgn(I*(2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(
2*b*x+2*a)-1))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
-1))^3+csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-c
sgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+csgn((2*exp(2*b*x+2*a)*c
+2*I)/(exp(2*b*x+2*a)-1))^2-2)*x^3+1/12*b/(I+c)*x^4-1/2*I/b^2*ln(1-I*c*exp(
2*b*x+2*a))*a^2*x+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x-1/12/b^3/
(I+c)*a^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.06

$$\int x^2 \arctan(c + (i + c)\coth(a + bx))\,dx$$

$$= \frac{-i\,b^4x^4 + 2i\,b^3x^3 \log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right) + i\,a^4 -}{}$$

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) + I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*di
log(-1/2*sqrt(4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) - 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*b*x*polylog(3,
-1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(4*I*c)*
e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a)
+ 1) + 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*polylog(4, -1/2*
sqrt(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c + (i + c) \coth(a + bx))\, dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int x^2 \arctan(c + (i + c) \coth(a + bx))\, dx = \frac{1}{3} x^3 \arctan\left((c + i) \coth\left(bx + a\right) + c\right) + \frac{4}{9} \left(\frac{3\, x^4}{4i\, c - 4} - \frac{4\, b^3 x^3 \log\left(-i\, ce^{(2\, bx + 2\, a)} + 1\right) + 6\, b^2 x^2 \mathrm{Li}_2\left(i\, ce^{(2\, bx + 2\, a)}\right) - 6\, bx \mathrm{Li}_3(i\, ce^{(2\, bx + 2\, a)}) + 3\, \mathrm{Li}_4(i\, ce$$

$$-2\, b^4(-i\, c + 1)$$

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c + I)*coth(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c - 2)))*b*(c + I)
```

Giac [F]

$$\int x^2 \arctan(c + (i + c) \coth(a + bx))\, dx = \int x^2 \arctan\left((c + i) \coth\left(bx + a\right) + c\right)\, dx$$

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c + I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + (i + c)\coth(a + bx))\,dx = \int x^2\,\mathrm{atan}(c + \coth(a + b\,x)\,\left(c + 1\mathrm{i}\right))\,dx$$

```
[In] int(x^2*atan(c + coth(a + b*x)*(c + 1i)),x)
[Out] int(x^2*atan(c + coth(a + b*x)*(c + 1i)), x)
```

3.103 $\int x \arctan(c+(i+c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx = -\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c+(i+c)\coth(a+bx)) +\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) +\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} -\frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arctan(c+(I+c)*coth(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,I*c*exp(2*
b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5305, 2215, 2221, 2611, 2320, 6724}

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx = \frac{1}{2}x^2\arctan(c+(c+i)\coth(a+bx)) -\frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2} +\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} +\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{1}{6}ibx^3$$

```
[In] Int[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-1/6*I)*b*x^3 + (x^2*ArcTan[c + (I + c)*Coth[a + b*x]])/2 + (I/4)*x^2*Log[
1 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - ((
I/8)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5305

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c+(i+c)\coth(a+bx)) - \frac{1}{2}b\int \frac{x^2}{-i-ce^{2a+2bx}}\,dx \\
&= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\arctan(c+(i+c)\coth(a+bx)) - \frac{1}{2}(ibc)\int \frac{e^{2a+2bx}x^2}{-i-ce^{2a+2bx}}\,dx \\
&= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\arctan(c+(i+c)\coth(a+bx)) \\
&\quad + \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) - \frac{1}{2}i\int x\log\left(1-ice^{2a+2bx}\right)\,dx \\
&= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\arctan(c+(i+c)\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) \\
&\quad + \frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} - \frac{i\int \text{PolyLog}\left(2,ice^{2a+2bx}\right)\,dx}{4b} \\
&= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\arctan(c+(i+c)\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) \\
&\quad + \frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} - \frac{i\text{Subst}\left(\int \frac{\text{PolyLog}(2,icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2} \\
&= -\frac{1}{6}ibx^3 + \frac{1}{2}x^2\arctan(c+(i+c)\coth(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) \\
&\quad + \frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx = \frac{2b^2x^2\left(2\arctan(c+(i+c)\coth(a+bx)) + i\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right) - 2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right) - i\,\text{Poly}}{8b^2}$$

```
[In] Integrate[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTan[c + (I + c)*Coth[a + b*x]] + I*Log[1 + I/(c*E^(2*(a +
b*x)))]) - (2*I)*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - I*PolyLog[3, (-
I)/(c*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.01 (sec) , antiderivative size = 1369, normalized size of antiderivative = 12.12

method	result	size
risch	Expression too large to display	1369

```
[In] int(x*arctan(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))-1/6*I/b^2*c/(I+c)*a^3+1/2*I/b*ln(1-I*c*e
xp(2*b*x+2*a))*a*x-1/6*I*b*c/(I+c)*x^3-1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*c)^(
1/2))*x+1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2+1/4*I/b^2*polylog(2,I*c*exp(
2*b*x+2*a))*a+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)-1/2*I/b^2
*a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c+2*I)+
1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+I)-1/2*I/b^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/
2))*a^2-1/8*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/2*I/b^2*ln(1+I*exp(b*x+a)
*(-I*c)^(1/2))*a^2-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/8*Pi*(c
sgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*
b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*
exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+
2*a)*c)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+
2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+
2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(
2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*
x+2*a)-1))^3-csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*ex
p(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(
exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-csgn(I
*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*I*
exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+
2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(
exp(2*b*x+2*a)-1))^3+csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*
x+2*a)-1))^2-csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+csgn((2*ex
p(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-2)*x^2+1/6/b^2/(I+c)*a^3+1/6*b/(I
+c)*x^3-1/2*I/b*a*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x+1/4*I*x*polylog(2,I*c*e
xp(2*b*x+2*a))/b
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.19

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx$$
$$=\frac{-2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right)-2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right)+\ldots}{}$$

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) + I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b*
x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) -
3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^2*x^2
 + I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*sqrt(4
*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx=\text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx$$
$$=\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i)$$
$$+\frac{1}{2}\,x^2\arctan\left((c+i)\coth\left(bx+a\right)+c\right)$$

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2)))
*b*(c + I) + 1/2*x^2*arctan((c + I)*coth(b*x + a) + c)
```

Giac [F]

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx = \int x\arctan\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c + I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx = \int x\,\mathrm{atan}(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

```
[In] int(x*atan(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x*atan(c + coth(a + b*x)*(c + 1i)), x)
```

3.104 $\int \arctan(c+(i+c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx = -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{i\,\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

```
[Out] -1/2*I*b*x^2+x*arctan(c+(I+c)*coth(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5297, 2215, 2221, 2317, 2438}

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx = x\arctan(c+(c+i)\coth(a+bx)) + \frac{i\,\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{1}{2}ibx^2$$

```
[In] Int[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-1/2*I)*b*x^2 + x*ArcTan[c + (I + c)*Coth[a + b*x]] + (I/2)*x*Log[1 - I*c*
E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

```
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5297

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan(c+(i+c)\coth(a+bx)) - b\int \frac{x}{-i-ce^{2a+2bx}}\,dx \\
&= -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\coth(a+bx)) - (ibc)\int \frac{e^{2a+2bx}x}{-i-ce^{2a+2bx}}\,dx \\
&= -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\coth(a+bx)) \\
&\quad + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{1}{2}i\int \log\left(1-ice^{2a+2bx}\right)\,dx \\
&= -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\coth(a+bx)) \\
&\quad + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) - \frac{i\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&= -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{i\,\text{PolyLog}\left(2, ice^{2a+2bx}\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx$$
$$= x\arctan(c+(i+c)\coth(a+bx)) + \frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

```
[In] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (I + c)*Coth[a + b*x]] + ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(65) = 130.

Time = 1.15 (sec) , antiderivative size = 545, normalized size of antiderivative = 6.90

method	result
derivativedivides	$\frac{\frac{\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\arctan(c+(i+c}{}}{}$
default	$\frac{\frac{\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\arctan(c+(i+c}{}}{}$
risch	Expression too large to display

```
[In] int(arctan(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I+c)*(arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)-
2*I*arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c-arcta
n(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c^2-arctan(c+(I+
c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))+2*I*arctan(c+(I+c)*coth
(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c+arctan(c+(I+c)*coth(b*x+a))/
(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c^2-(I+c)^2*(1/2/(I+c)*(-1/2*I*((ln(I+c
+(I+c)*coth(b*x+a))-ln(-1/2*I*(I+c+(I+c)*coth(b*x+a))))*ln(-1/2*I*(I-c-(I+c
)*coth(b*x+a)))-dilog(-1/2*I*(I+c+(I+c)*coth(b*x+a))))+1/4*I*ln(I+c+(I+c)*c
oth(b*x+a))^2)-1/2/(I+c)*(-1/2*I*(dilog(-1/2*(I-c-(I+c)*coth(b*x+a))/c)+ln(
c-(I+c)*coth(b*x+a)+I)*ln(-1/2*(I-c-(I+c)*coth(b*x+a))/c))+1/2*I*(dilog((-I
-c-(I+c)*coth(b*x+a))/(-2*I-2*c))+ln(c-(I+c)*coth(b*x+a)+I)*ln((-I-c-(I+c)*
coth(b*x+a))/(-2*I-2*c))))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.37

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx$$
$$= \frac{-i\,b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) + i\,a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{4i\,c}\right.}{}$$

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (I*b*x +
I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a) +
 I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) + I*d
ilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)))/
b
```

Sympy [F(-2)]

Exception generated.

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx$$
$$= 2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2} - \frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right) + \mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right) + x\arctan\left((c+i)\coth\left(bx+a\right)+c\right)$$

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + dil
og(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*coth(b*x + a
) + c)
```

Giac [F]

$$\int \arctan(c + (i + c)\coth(a + bx))\,dx = \int \arctan\left((c + i)\coth\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan((c + I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c + (i + c)\coth(a + bx))\,dx = \int \operatorname{atan}(c + \coth(a + b\,x)\ (c + 1\mathrm{i}))\,dx$$

```
[In] int(atan(c + coth(a + b*x)*(c + 1i)),x)
[Out] int(atan(c + coth(a + b*x)*(c + 1i)), x)
```

3.105 $\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx = \mathrm{Int}\left(\frac{\arctan(c+(i+c)\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\arctan \left(c+(i+c)\coth \left(bx+a\right)\right)}{x}dx$$

```
[In] int(arctan(c+(I+c)*coth(b*x+a))/x,x)
[Out] int(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\arctan\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx=\text{Timed out}$$

```
[In] integrate(atan(c+(I+c)*coth(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.05

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x + 1/2*pi*log(x) - 1/4*(4*pi - 4*I*a - 2*arctan(c) - I*log(c^2 + 1))*l
og(x) - 1/2*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) - 1/4*I*integrate(log
(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*coth(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c+\mathrm{coth}(a+b\,x)\ (c+1\mathrm{i}))}{x}\,dx$$

```
[In] int(atan(c + coth(a + b*x)*(c + 1i))/x,x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c + 1i))/x, x)
```

3.106 $\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{1}{12}ibx^4+\frac{1}{3}x^3\arctan(c-(i-c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) -\frac{ix^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{ix\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{i\,\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arctan(c-(I-c)*coth(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*b
*x+2*a))-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,-I*c*
exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5305, 2215, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{1}{3}x^3\arctan(c-(-c+i)\coth(a+bx)) -\frac{i\,\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3} +\frac{ix\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2} -\frac{ix^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} -\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{12}ibx^4$$

```
[In] Int[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Coth[a + b*x]])/3 - (I/6)*x^3*Log[1
+ I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b +
 ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, (-I)*
c*E^(2*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5305

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan(c-(i-c)\coth(a+bx)) - \frac{1}{3}b\int\frac{x^3}{i-ce^{2a+2bx}}\,dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\coth(a+bx)) + \frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{i-ce^{2a+2bx}}\,dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\coth(a+bx)) \\
&\quad - \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) + \frac{1}{2}i\int x^2\log\left(1+ice^{2a+2bx}\right)\,dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) \\
&\quad - \frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} + \frac{i\int x\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)\,dx}{2b} \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\coth(a+bx)) \\
&\quad - \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) - \frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} \\
&\quad + \frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2} - \frac{i\int\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)\,dx}{4b^2} \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\arctan(c-(i-c)\coth(a+bx)) \\
&\quad - \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) - \frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} \\
&\quad + \frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2} - \frac{i\text{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3}
\end{aligned}$$

$$= \frac{1}{12} i b x^4 + \frac{1}{3} x^3 \arctan(c - (i - c)\coth(a + bx)) - \frac{1}{6} i x^3 \log\left(1 + ice^{2a+2bx}\right) - \frac{ix^2 \operatorname{PolyLog}\left(2, -ice^{2a+2bx}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, -ice^{2a+2bx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, -ice^{2a+2bx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int x^2 \arctan(c - (i - c)\coth(a + bx))\, dx = \frac{8b^3x^3 \arctan(c + (-i + c)\coth(a + bx)) - 4ib^3x^3 \log\left(1 - \frac{ie^{-2(a+bx)}}{c}\right) + 6ib^2x^2 \operatorname{PolyLog}\left(2, \frac{ie^{-2(a+bx)}}{c}\right) + 6ib}{24b^3}$$

```
[In] Integrate[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcTan[c + (-I + c)*Coth[a + b*x]] - (4*I)*b^3*x^3*Log[1 - I/(c*
E^(2*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] + (6*I)
*b*x*PolyLog[3, I/(c*E^(2*(a + b*x)))] + (3*I)*PolyLog[4, I/(c*E^(2*(a + b*
x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.83 (sec) , antiderivative size = 1410, normalized size of antiderivative = 9.72

method	result	size
risch	Expression too large to display	1410

```
[In] int(x^2*arctan(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^2*x-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I
*c)^(1/2))*x-1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a^2*ln(
1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(1/2
))+1/3*I/b^3*ln(1+I*c*exp(2*b*x+2*a))*a^3-1/8*I*polylog(4,-I*c*exp(2*b*x+2*
a))/b^3+1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/12*I/b^3*c*a^4/(I-c)-1
/6*I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/6*I*x^3*ln(1+I*c*exp(2
*b*x+2*a))+1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)+1/6*I*x^3*ln(-2*exp(2*b*x+
2*a)*c+2*I)-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(I*c)^(1/2))-1/4*I*x^2*polylog(
2,-I*c*exp(2*b*x+2*a))/b+1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2+1/12*
Pi*(csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
```

```
(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp
(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*ex
p(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-
csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I)
/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*
csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a
)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-
1))+csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-c
sgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2
*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+
2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)
-1))^3+csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-2)*x^3-1/12*I*b*
c/(I-c)*x^4-1/12*b/(I-c)*x^4-1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2)
)+1/12/b^3/(I-c)*a^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(105) = 210.

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.02

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx$$
$$= \frac{i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c-i}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a}\right.}{}$$

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)
/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*d
ilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(-4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*b*x*polylo
g(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(-4
*I*c)*e^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(-4*I*c)*e^(b*x
 + a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*polylog(4
, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{1}{3}x^3 \arctan\left((c-i)\coth(bx+a)+c\right) - \frac{4}{9}\left(\frac{3x^4}{4ic+4} - \frac{4b^3x^3 \log\left(ice^{(2bx+2a)}+1\right) + 6b^2x^2\text{Li}_2\left(-ice^{(2bx+2a)}\right) - 6bx\text{Li}_3\left(-ice^{(2bx+2a)}\right) + 3\text{Li}_4\left(-i\right.}{-2b^4(-ic-1)}\right.$$

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c - I)*coth(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c + 2)))*b*(c - I)
```

Giac [F]

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \int x^2 \arctan\left((c-i)\coth(bx+a)+c\right)\,dx$$

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c - I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c - (i - c)\coth(a + bx))\,dx = \int x^2\,\mathrm{atan}(c + \coth(a + b\,x)\,\,(c - \mathrm{i}))\,\,dx$$

```
[In] int(x^2*atan(c + coth(a + b*x)*(c - 1i)),x)
[Out] int(x^2*atan(c + coth(a + b*x)*(c - 1i)), x)
```

3.107 $\int x \arctan(c-(i-c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int x \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\coth(a+bx)) - \frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right) - \frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arctan(c-(I-c)*coth(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,-I*c*exp(2
*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5305, 2215, 2221, 2611, 2320, 6724}

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx = \frac{1}{2}x^2\arctan(c-(-c+i)\coth(a+bx)) + \frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2} - \frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} - \frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{6}ibx^3$$

```
[In] Int[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Coth[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b + ((
I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5305

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan(c-(i-c)\coth(a+bx))-\frac{1}{2}b\int\frac{x^2}{i-ce^{2a+2bx}}\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\coth(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{i-ce^{2a+2bx}}\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\coth(a+bx))\\
&\quad-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}i\int x\log\left(1+ice^{2a+2bx}\right)\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad-\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\int\text{PolyLog}\left(2,-ice^{2a+2bx}\right)\,dx}{4b}\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad-\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad-\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx$$

$$=\frac{2b^2x^2\left(2\arctan(c+(-i+c)\coth(a+bx))-i\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)+i\,\text{PolyI}}{8b^2}$$

```
[In] Integrate[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTan[c + (-I + c)*Coth[a + b*x]] - I*Log[1 - I/(c*E^(2*(a +
 b*x)))]) + (2*I)*b*x*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] + I*PolyLog[3, I/(c
*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.04 (sec) , antiderivative size = 1374, normalized size of antiderivative = 11.84

method	result	size
risch	Expression too large to display	1374

```
[In] int(x*arctan(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*b*c/(I-c)*x^3-1/4*I*x^2*ln(1+I*c*exp(2*b*x+2*a))+1/2*I/b*a*ln(1+I*ex
p(b*x+a)*(I*c)^(1/2))*x-1/6*I/b^2*c/(I-c)*a^3-1/4*I/b^2*ln(1+I*c*exp(2*b*x+
2*a))*a^2-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I/b^2*polylog(2,-I*c
*exp(2*b*x+2*a))*a-1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)+1/2*I/b^2*ln(1+I*e
xp(b*x+a)*(I*c)^(1/2))*a^2+1/2*I/b^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*a^2+1/2
*I/b^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/8*I*polylog(3,-I*c*exp(2*b*x+2
*a))/b^2-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/4*I*x^2*ln(-
2*exp(2*b*x+2*a)*c+2*I)-1/2*I/b*ln(1+I*c*exp(2*b*x+2*a))*a*x+1/8*Pi*(csgn(I
/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2
*a)*c-2*I)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a
)*c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*
a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*
x+2*a)-1))^3+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*ex
p(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))+csgn(I
*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-csgn(I*(-2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)-1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)-1))^2+csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+csg
n((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-2)*x^2-1/6/b^2/(I-c)*a^3-1
/6*b/(I-c)*x^3+1/2*I/b*a*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x+1/2*I/b^2*a*dilog
(1+I*exp(b*x+a)*(I*c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.13

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx$$
$$= \frac{2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c-i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(}\right.}{}$$

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*
a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b
*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c
) - 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*
x^2 - I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sq
rt(-4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^
2
```

Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx =$$
$$-\left(\frac{2\,x^3}{3i\,c+3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i)$$
$$+\frac{1}{2}\,x^2\arctan\left((c-i)\coth\left(bx+a\right)+c\right)$$

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2)
))*b*(c - I) + 1/2*x^2*arctan((c - I)*coth(b*x + a) + c)
```

Giac [F]

$$\int x \arctan(c-(i-c)\coth(a+bx))\,dx = \int x \arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c - I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \arctan(c-(i-c)\coth(a+bx))\,dx = \int x \operatorname{atan}(c+\coth(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

```
[In] int(x*atan(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x*atan(c + coth(a + b*x)*(c - 1i)), x)
```

3.108 $\int \arctan(c-(i-c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arctan(c-(I-c)*coth(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5297, 2215, 2221, 2317, 2438}

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx = x\arctan(c-(-c+i)\coth(a+bx)) - \frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{1}{2}ibx^2$$

```
[In] Int[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Coth[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

```
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5297

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan(c-(i-c)\coth(a+bx)) - b\int \frac{x}{i-ce^{2a+2bx}}\,dx \\
&= \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\coth(a+bx)) + (ibc)\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}\,dx \\
&= \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\coth(a+bx)) \\
&\quad -\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{1}{2}i\int \log\left(1+ice^{2a+2bx}\right)\,dx \\
&= \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\coth(a+bx)) \\
&\quad -\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{i\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&= \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx$$
$$= x\arctan(c+(-i+c)\coth(a+bx)) - \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\operatorname{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

```
[In] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-I + c)*Coth[a + b*x]] - ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(68) = 136.

Time = 1.14 (sec) , antiderivative size = 516, normalized size of antiderivative = 6.29

method	result
derivativedivides	$\frac{\frac{\arctan(c+\coth(bx+a)(c-i))\ln(-i+\coth(bx+a)(c-i)+c)}{2i-2c}+\frac{2i\arctan(c+\coth(bx+a)(c-i))\ln(-i+\coth(bx+a)(c-i)+c)c}{2i-2c}-\frac{\arctan(c+\coth}{}}{}$
default	$\frac{\frac{\arctan(c+\coth(bx+a)(c-i))\ln(-i+\coth(bx+a)(c-i)+c)}{2i-2c}+\frac{2i\arctan(c+\coth(bx+a)(c-i))\ln(-i+\coth(bx+a)(c-i)+c)c}{2i-2c}-\frac{\arctan(c+\coth}{}}{}$
risch	Expression too large to display

```
[In] int(arctan(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(c-I)*(arctan(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)
+2*I*arctan(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)*c-arc
tan(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)*c^2-arctan(c+
coth(b*x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)-2*I*arctan(c+coth(b*
x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)*c+arctan(c+coth(b*x+a)*(c-I
))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)*c^2+(I-c)^2*(1/2/(I-c)*(1/2*I*(dilog
(-1/2*I*(coth(b*x+a)*(c-I)+c+I))+ln(-I+coth(b*x+a)*(c-I)+c)*ln(-1/2*I*(coth
(b*x+a)*(c-I)+c+I)))-1/4*I*ln(-I+coth(b*x+a)*(c-I)+c)^2)-1/2/(I-c)*(-1/2*I*
(dilog((-I+coth(b*x+a)*(c-I)+c)/(-2*I+2*c))+ln(coth(b*x+a)*(c-I)-c+I)*ln((-
I+coth(b*x+a)*(c-I)+c)/(-2*I+2*c)))+1/2*I*(dilog(1/2*(coth(b*x+a)*(c-I)+c+I
)/c)+ln(coth(b*x+a)*(c-I)-c+I)*ln(1/2*(coth(b*x+a)*(c-I)+c+I)/c)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.28

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx$$
$$= \frac{i\,b^2x^2 + i\,bx\log\left(-\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c-i}\right) - i\,a^2 + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}+1\right) + (-i\,bx - i\,a)\ldots}{}$$

[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")

[Out] 1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c - I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-I*b*x
 - I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
 - I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x
 + a)))/b

Sympy [F(-2)]

Exception generated.

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

[In] integrate(atan(c-(I-c)*coth(b*x+a)),x)

[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]

Maxima [A] (verification not implemented)

none

Time = 1.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx$$
$$= -2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2} - \frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right) + x\arctan\left((c-i)\coth\left(bx+a\right)+c\right)$$

[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")

```
[Out] -2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dil
og(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*coth(b*x +
a) + c)
```

Giac [F]

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx=\int \arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx=\int \mathrm{atan}\left(c+\mathrm{coth}\left(a+b\,x\right)\,\left(c-\mathrm{i}\right)\right)\,dx$$

```
[In] int(atan(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c - 1i)), x)
```

3.109 $\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\arctan(c-(i-c)\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx$$

```
[In] Int[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan\left(c-(i-c)\coth\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arctan(c-(I-c)*coth(b*x+a))/x,x)
[Out] int(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c - I))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(atan(c-(I-c)*coth(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))*log(x) - 1/2*int
egrate(arctan(c*e^(2*b*x + 2*a))/x, x) + 1/4*I*integrate(log(c^2*e^(4*b*x +
 4*a) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*coth(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c+\coth(a+b\,x)\ (c-\mathrm{i}))}{x}\,dx$$

```
[In] int(atan(c + coth(a + b*x)*(c - 1i))/x,x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c - 1i))/x, x)
```

3.110 $\int \arctan(e^x)\, dx$

Optimal result

Integrand size = 4, antiderivative size = 31

$$\int \arctan(e^x)\, dx = \frac{1}{2} i \operatorname{PolyLog}(2, -ie^x) - \frac{1}{2} i \operatorname{PolyLog}(2, ie^x)$$

```
[Out] 1/2*I*polylog(2,-I*exp(x))-1/2*I*polylog(2,I*exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2320, 4940, 2438}

$$\int \arctan(e^x)\, dx = \frac{1}{2} i \operatorname{PolyLog}(2, -ie^x) - \frac{1}{2} i \operatorname{PolyLog}(2, ie^x)$$

```
[In] Int[ArcTan[E^x],x]
```

```
[Out] (I/2)*PolyLog[2, (-I)*E^x] - (I/2)*PolyLog[2, I*E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{\arctan(x)}{x}\,dx, x, e^x\right)\\ &= \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^x\right) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^x\right)\\ &= \frac{1}{2}i\,\text{PolyLog}\left(2,-ie^x\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,ie^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \arctan\left(e^x\right)\,dx = x\arctan\left(e^x\right) - \frac{1}{2}i(x(\log\left(1-ie^x\right) - \log\left(1+ie^x\right)) - \text{PolyLog}\left(2,-ie^x\right) + \text{PolyLog}\left(2,ie^x\right))$$

```
[In] Integrate[ArcTan[E^x],x]
[Out] x*ArcTan[E^x] - (I/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I
)*E^x] + PolyLog[2, I*E^x])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

method	result	size
parts	$x\arctan\left(\mathrm{e}^{x}\right)+\frac{ix\ln(1+i\mathrm{e}^{x})}{2}-\frac{ix\ln(1-i\mathrm{e}^{x})}{2}+\frac{i\,\mathrm{dilog}(1+i\mathrm{e}^{x})}{2}-\frac{i\,\mathrm{dilog}(1-i\mathrm{e}^{x})}{2}$	53
derivativedivides	$\ln\left(\mathrm{e}^{x}\right)\arctan\left(\mathrm{e}^{x}\right)+\frac{i\ln(\mathrm{e}^{x})\ln(1+i\mathrm{e}^{x})}{2}-\frac{i\ln(\mathrm{e}^{x})\ln(1-i\mathrm{e}^{x})}{2}+\frac{i\,\mathrm{dilog}(1+i\mathrm{e}^{x})}{2}-\frac{i\,\mathrm{dilog}(1-i\mathrm{e}^{x})}{2}$	59
default	$\ln\left(\mathrm{e}^{x}\right)\arctan\left(\mathrm{e}^{x}\right)+\frac{i\ln(\mathrm{e}^{x})\ln(1+i\mathrm{e}^{x})}{2}-\frac{i\ln(\mathrm{e}^{x})\ln(1-i\mathrm{e}^{x})}{2}+\frac{i\,\mathrm{dilog}(1+i\mathrm{e}^{x})}{2}-\frac{i\,\mathrm{dilog}(1-i\mathrm{e}^{x})}{2}$	59
risch	$-\frac{ix\ln(1+i\mathrm{e}^{x})}{2}-\frac{i\ln(-i(-\mathrm{e}^{x}+i))\ln(-i\mathrm{e}^{x})}{2}+\frac{i\ln(-i(-\mathrm{e}^{x}+i))x}{2}-\frac{i\,\mathrm{dilog}(-i\mathrm{e}^{x})}{2}-\frac{i\,\mathrm{dilog}(1-i\mathrm{e}^{x})}{2}$	69

```
[In] int(arctan(exp(x)),x,method=_RETURNVERBOSE)
[Out] x*arctan(exp(x))+1/2*I*x*ln(1+I*exp(x))-1/2*I*x*ln(1-I*exp(x))+1/2*I*dilog(
1+I*exp(x))-1/2*I*dilog(1-I*exp(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \arctan\left(e^{x}\right)\,dx = x\arctan\left(e^{x}\right)+\frac{1}{2}i\,x\log\left(i\,e^{x}+1\right)-\frac{1}{2}i\,x\log\left(-i\,e^{x}+1\right)-\frac{1}{2}i\,\mathrm{Li}_2(i\,e^{x})+\frac{1}{2}i\,\mathrm{Li}_2(-i\,e^{x})$$

```
[In] integrate(arctan(exp(x)),x, algorithm="fricas")
[Out] x*arctan(e^x) + 1/2*I*x*log(I*e^x + 1) - 1/2*I*x*log(-I*e^x + 1) - 1/2*I*di
log(I*e^x) + 1/2*I*dilog(-I*e^x)
```

Sympy [F]

$$\int \arctan\left(e^{x}\right)\,dx = \int \operatorname{atan}\left(e^{x}\right)dx$$

```
[In] integrate(atan(exp(x)),x)
[Out] Integral(atan(exp(x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \arctan\left(e^{x}\right)\,dx = x\arctan\left(e^{x}\right) - \frac{1}{4}\,\pi\log\left(e^{(2\,x)} + 1\right) - \frac{1}{2}i\,\mathrm{Li}_2(i\,e^{x} + 1) + \frac{1}{2}i\,\mathrm{Li}_2(-i\,e^{x} + 1)$$

```
[In] integrate(arctan(exp(x)),x, algorithm="maxima")
[Out] x*arctan(e^x) - 1/4*pi*log(e^(2*x) + 1) - 1/2*I*dilog(I*e^x + 1) + 1/2*I*di
log(-I*e^x + 1)
```

Giac [F]

$$\int \arctan\left(e^{x}\right)\,dx = \int \arctan\left(e^{x}\right)\,dx$$

```
[In] integrate(arctan(exp(x)),x, algorithm="giac")
[Out] integrate(arctan(e^x), x)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \arctan\left(e^{x}\right)\,dx = \frac{\mathrm{polylog}(2, -\mathrm{e}^{x}\,1\mathrm{i})\,\,1\mathrm{i}}{2} - \frac{\mathrm{polylog}(2, \mathrm{e}^{x}\,1\mathrm{i})\,\,1\mathrm{i}}{2}$$

```
[In] int(atan(exp(x)),x)
[Out] (polylog(2, -exp(x)*1i)*1i)/2 - (polylog(2, exp(x)*1i)*1i)/2
```

3.111 $\int x \arctan(e^x)\, dx$

Optimal result

Integrand size = 6, antiderivative size = 63

$$\int x \arctan(e^x)\, dx = \frac{1}{2} i x \operatorname{PolyLog}(2, -ie^x) - \frac{1}{2} i x \operatorname{PolyLog}(2, ie^x) - \frac{1}{2} i \operatorname{PolyLog}(3, -ie^x) + \frac{1}{2} i \operatorname{PolyLog}(3, ie^x)$$

```
[Out] 1/2*I*x*polylog(2,-I*exp(x))-1/2*I*x*polylog(2,I*exp(x))-1/2*I*polylog(3,-I
*exp(x))+1/2*I*polylog(3,I*exp(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5251, 2611, 2320, 6724}

$$\int x \arctan(e^x)\, dx = \frac{1}{2} i x \operatorname{PolyLog}(2, -ie^x) - \frac{1}{2} i x \operatorname{PolyLog}(2, ie^x) - \frac{1}{2} i \operatorname{PolyLog}(3, -ie^x) + \frac{1}{2} i \operatorname{PolyLog}(3, ie^x)$$

```
[In] Int[x*ArcTan[E^x],x]
```

```
[Out] (I/2)*x*PolyLog[2, (-I)*E^x] - (I/2)*x*PolyLog[2, I*E^x] - (I/2)*PolyLog[3,
 (-I)*E^x] + (I/2)*PolyLog[3, I*E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}i\int x\log\left(1-ie^x\right)\,dx-\frac{1}{2}i\int x\log\left(1+ie^x\right)\,dx\\
&= \frac{1}{2}ix\,\text{PolyLog}\left(2,-ie^x\right)-\frac{1}{2}ix\,\text{PolyLog}\left(2,ie^x\right)\\
&\quad -\frac{1}{2}i\int \text{PolyLog}\left(2,-ie^x\right)\,dx+\frac{1}{2}i\int \text{PolyLog}\left(2,ie^x\right)\,dx\\
&= \frac{1}{2}ix\,\text{PolyLog}\left(2,-ie^x\right)-\frac{1}{2}ix\,\text{PolyLog}\left(2,ie^x\right)\\
&\quad -\frac{1}{2}i\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^x\right)+\frac{1}{2}i\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^x\right)\\
&= \frac{1}{2}ix\,\text{PolyLog}\left(2,-ie^x\right)-\frac{1}{2}ix\,\text{PolyLog}\left(2,ie^x\right)-\frac{1}{2}i\,\text{PolyLog}\left(3,-ie^x\right)+\frac{1}{2}i\,\text{PolyLog}\left(3,ie^x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int x \arctan\left(e^x\right)\,dx = \frac{1}{2}i(x\,\mathrm{PolyLog}\left(2,-ie^x\right)-x\,\mathrm{PolyLog}\left(2,ie^x\right)-\mathrm{PolyLog}\left(3,-ie^x\right)+\mathrm{PolyLog}\left(3,ie^x\right))$$

```
[In] Integrate[x*ArcTan[E^x],x]
[Out] (I/2)*(x*PolyLog[2, (-I)*E^x] - x*PolyLog[2, I*E^x] - PolyLog[3, (-I)*E^x]
+ PolyLog[3, I*E^x])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{ix\,\mathrm{polylog}(2,-ie^x)}{2}-\frac{ix\,\mathrm{polylog}(2,ie^x)}{2}-\frac{i\,\mathrm{polylog}(3,-ie^x)}{2}+\frac{i\,\mathrm{polylog}(3,ie^x)}{2}$	44

```
[In] int(x*arctan(exp(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*I*x*polylog(2,-I*exp(x))-1/2*I*x*polylog(2,I*exp(x))-1/2*I*polylog(3,-I
*exp(x))+1/2*I*polylog(3,I*exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\begin{aligned}\int x \arctan\left(e^x\right)\,dx = &\ \frac{1}{2}\,x^2\arctan\left(e^x\right)+\frac{1}{4}i\,x^2\log\left(i\,e^x+1\right)\\ &-\frac{1}{4}i\,x^2\log\left(-i\,e^x+1\right)-\frac{1}{2}i\,x\mathrm{Li}_2(i\,e^x)+\frac{1}{2}i\,x\mathrm{Li}_2(-i\,e^x)\\ &+\frac{1}{2}i\,\mathrm{polylog}(3,i\,e^x)-\frac{1}{2}i\,\mathrm{polylog}(3,-i\,e^x)\end{aligned}$$

```
[In] integrate(x*arctan(exp(x)),x, algorithm="fricas")
[Out] 1/2*x^2*arctan(e^x) + 1/4*I*x^2*log(I*e^x + 1) - 1/4*I*x^2*log(-I*e^x + 1)
- 1/2*I*x*dilog(I*e^x) + 1/2*I*x*dilog(-I*e^x) + 1/2*I*polylog(3, I*e^x) -
1/2*I*polylog(3, -I*e^x)
```

Sympy [F]

$$\int x\arctan \left(e^{x}\right)\,dx=\int x\,\mathrm{atan}\left(e^{x}\right)dx$$

```
[In] integrate(x*atan(exp(x)),x)
[Out] Integral(x*atan(exp(x)), x)
```

Maxima [F]

$$\int x\arctan \left(e^{x}\right)\,dx=\int x\arctan \left(e^{x}\right)\,dx$$

```
[In] integrate(x*arctan(exp(x)),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(e^x) - integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

Giac [F]

$$\int x\arctan \left(e^{x}\right)\,dx=\int x\arctan \left(e^{x}\right)\,dx$$

```
[In] integrate(x*arctan(exp(x)),x, algorithm="giac")
[Out] integrate(x*arctan(e^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\arctan \left(e^{x}\right)\,dx=\int x\,\mathrm{atan}(\mathrm{e}^{x})\,dx$$

```
[In] int(x*atan(exp(x)),x)
[Out] int(x*atan(exp(x)), x)
```

3.112 $\int x^2 \arctan(e^x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 91

$$\int x^2 \arctan(e^x)\,dx = \frac{1}{2}ix^2\,\text{PolyLog}(2,-ie^x) - \frac{1}{2}ix^2\,\text{PolyLog}(2,ie^x) - ix\,\text{PolyLog}(3,-ie^x) + ix\,\text{PolyLog}(3,ie^x) + i\,\text{PolyLog}(4,-ie^x) - i\,\text{PolyLog}(4,ie^x)$$

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(x))-1/2*I*x^2*polylog(2,I*exp(x))-I*x*polylog(3,
-I*exp(x))+I*x*polylog(3,I*exp(x))+I*polylog(4,-I*exp(x))-I*polylog(4,I*exp
(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5251, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan(e^x)\,dx = \frac{1}{2}ix^2\,\text{PolyLog}(2,-ie^x) - \frac{1}{2}ix^2\,\text{PolyLog}(2,ie^x) - ix\,\text{PolyLog}(3,-ie^x) + ix\,\text{PolyLog}(3,ie^x) + i\,\text{PolyLog}(4,-ie^x) - i\,\text{PolyLog}(4,ie^x)$$

```
[In] Int[x^2*ArcTan[E^x],x]
```

```
[Out] (I/2)*x^2*PolyLog[2, (-I)*E^x] - (I/2)*x^2*PolyLog[2, I*E^x] - I*x*PolyLog[
3, (-I)*E^x] + I*x*PolyLog[3, I*E^x] + I*PolyLog[4, (-I)*E^x] - I*PolyLog[4
, I*E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}i\int x^2\log\left(1-ie^x\right)\,dx-\frac{1}{2}i\int x^2\log\left(1+ie^x\right)\,dx\\
&= \frac{1}{2}ix^2\,\text{PolyLog}\left(2,-ie^x\right)-\frac{1}{2}ix^2\,\text{PolyLog}\left(2,ie^x\right)\\
&\quad -i\int x\,\text{PolyLog}\left(2,-ie^x\right)\,dx+i\int x\,\text{PolyLog}\left(2,ie^x\right)\,dx\\
&= \frac{1}{2}ix^2\,\text{PolyLog}\left(2,-ie^x\right)-\frac{1}{2}ix^2\,\text{PolyLog}\left(2,ie^x\right)-ix\,\text{PolyLog}\left(3,-ie^x\right)\\
&\quad +ix\,\text{PolyLog}\left(3,ie^x\right)+i\int \text{PolyLog}\left(3,-ie^x\right)\,dx-i\int \text{PolyLog}\left(3,ie^x\right)\,dx
\end{aligned}$$

$$= \frac{1}{2} i x^2 \operatorname{PolyLog}(2, -ie^x) - \frac{1}{2} i x^2 \operatorname{PolyLog}(2, ie^x) - ix \operatorname{PolyLog}(3, -ie^x) + ix \operatorname{PolyLog}(3, ie^x) + i\text{Subst}\left(\int \frac{\operatorname{PolyLog}(3, -ix)}{x} dx, x, e^x\right) - i\text{Subst}\left(\int \frac{\operatorname{PolyLog}(3, ix)}{x} dx, x, e^x\right)$$

$$= \frac{1}{2} i x^2 \operatorname{PolyLog}(2, -ie^x) - \frac{1}{2} i x^2 \operatorname{PolyLog}(2, ie^x) - ix \operatorname{PolyLog}(3, -ie^x) + ix \operatorname{PolyLog}(3, ie^x) + i \operatorname{PolyLog}(4, -ie^x) - i \operatorname{PolyLog}(4, ie^x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int x^2 \arctan(e^x)\, dx = \frac{1}{2} i \big(x^2 \operatorname{PolyLog}(2, -ie^x) - x^2 \operatorname{PolyLog}(2, ie^x) + 2(-x \operatorname{PolyLog}(3, -ie^x) + x \operatorname{PolyLog}(3, ie^x) + \operatorname{PolyLog}(4, -ie^x) - \operatorname{PolyLog}(4, ie^x))\big)$$

```
[In] Integrate[x^2*ArcTan[E^x],x]
```

```
[Out] (I/2)*(x^2*PolyLog[2, (-I)*E^x] - x^2*PolyLog[2, I*E^x] + 2*(-(x*PolyLog[3,
 (-I)*E^x]) + x*PolyLog[3, I*E^x] + PolyLog[4, (-I)*E^x] - PolyLog[4, I*E^x
]))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result
risch	$\frac{ix^2 \operatorname{polylog}(2,-ie^x)}{2} - \frac{ix^2 \operatorname{polylog}(2,ie^x)}{2} - ix \operatorname{polylog}(3, -ie^x) + ix \operatorname{polylog}(3, ie^x) + i \operatorname{polylog}(4, -ie^x) -$

```
[In] int(x^2*arctan(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(x))-1/2*I*x^2*polylog(2,I*exp(x))-I*x*polylog(3,
-I*exp(x))+I*x*polylog(3,I*exp(x))+I*polylog(4,-I*exp(x))-I*polylog(4,I*exp
(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int x^2 \arctan(e^x) \, dx = \frac{1}{3} x^3 \arctan(e^x) + \frac{1}{6} i x^3 \log(i e^x + 1) - \frac{1}{6} i x^3 \log(-i e^x + 1) - \frac{1}{2} i x^2 \mathrm{Li}_2(i e^x) + \frac{1}{2} i x^2 \mathrm{Li}_2(-i e^x) + i x \mathrm{polylog}(3, i e^x) - i x \mathrm{polylog}(3, -i e^x) - i \,\mathrm{polylog}(4, i e^x) + i \,\mathrm{polylog}(4, -i e^x)$$

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="fricas")
[Out] 1/3*x^3*arctan(e^x) + 1/6*I*x^3*log(I*e^x + 1) - 1/6*I*x^3*log(-I*e^x + 1)
- 1/2*I*x^2*dilog(I*e^x) + 1/2*I*x^2*dilog(-I*e^x) + I*x*polylog(3, I*e^x)
- I*x*polylog(3, -I*e^x) - I*polylog(4, I*e^x) + I*polylog(4, -I*e^x)
```

Sympy [F]

$$\int x^2 \arctan(e^x) \, dx = \int x^2 \operatorname{atan}(e^x) \, dx$$

```
[In] integrate(x**2*atan(exp(x)),x)
[Out] Integral(x**2*atan(exp(x)), x)
```

Maxima [F]

$$\int x^2 \arctan(e^x) \, dx = \int x^2 \arctan(e^x) \, dx$$

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="maxima")
[Out] 1/3*x^3*arctan(e^x) - integrate(1/3*x^3*e^x/(e^(2*x) + 1), x)
```

Giac [F]

$$\int x^2 \arctan(e^x)\,dx = \int x^2 \arctan(e^x)\,dx$$

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="giac")
[Out] integrate(x^2*arctan(e^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(e^x)\,dx = \int x^2\,\mathrm{atan}(\mathrm{e}^x)\,dx$$

```
[In] int(x^2*atan(exp(x)),x)
[Out] int(x^2*atan(exp(x)), x)
```

3.113 $\int \arctan\left(e^{a+bx}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 45

$$\int \arctan\left(e^{a+bx}\right)\,dx = \frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b} - \frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

```
[Out] 1/2*I*polylog(2,-I*exp(b*x+a))/b-1/2*I*polylog(2,I*exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 4940, 2438}

$$\int \arctan\left(e^{a+bx}\right)\,dx = \frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b} - \frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

```
[In] Int[ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*PolyLog[2, I*E^(a + b*x)])/
b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\arctan(x)}{x}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{2b} - \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{a+bx}\right)}{2b}\\ &= \frac{i\,\text{PolyLog}\left(2, -ie^{a+bx}\right)}{2b} - \frac{i\,\text{PolyLog}\left(2, ie^{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.84

$$\int \arctan\left(e^{a+bx}\right)\,dx = x\arctan\left(e^{a+bx}\right) - \frac{i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{2b}$$

```
[In] Integrate[ArcTan[E^(a + b*x)],x]
[Out] x*ArcTan[E^(a + b*x)] - ((I/2)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(
a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(35) = 70.

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{\ln(\mathrm{e}^{bx+a})\arctan(\mathrm{e}^{bx+a})+\frac{i\ln(\mathrm{e}^{bx+a})\ln(1+i\mathrm{e}^{bx+a})}{2}-\frac{i\ln(\mathrm{e}^{bx+a})\ln(1-i\mathrm{e}^{bx+a})}{2}+\frac{i\,\mathrm{dilog}(1+i\mathrm{e}^{bx+a})}{2}-\frac{i\,\mathrm{dilog}(1-i\mathrm{e}^{bx+a})}{2}}{b}$
default	$\frac{\ln(\mathrm{e}^{bx+a})\arctan(\mathrm{e}^{bx+a})+\frac{i\ln(\mathrm{e}^{bx+a})\ln(1+i\mathrm{e}^{bx+a})}{2}-\frac{i\ln(\mathrm{e}^{bx+a})\ln(1-i\mathrm{e}^{bx+a})}{2}+\frac{i\,\mathrm{dilog}(1+i\mathrm{e}^{bx+a})}{2}-\frac{i\,\mathrm{dilog}(1-i\mathrm{e}^{bx+a})}{2}}{b}$
parts	$x\arctan\left(\mathrm{e}^{bx+a}\right)-\frac{-\frac{i(bx+a)\ln(1+i\mathrm{e}^{bx+a})}{2}+\frac{i(bx+a)\ln(1-i\mathrm{e}^{bx+a})}{2}-\frac{i\,\mathrm{dilog}(1+i\mathrm{e}^{bx+a})}{2}+\frac{i\,\mathrm{dilog}(1-i\mathrm{e}^{bx+a})}{2}-a\,\mathrm{arcta}}{b}$
risch	$-\frac{ix\ln(1+i\mathrm{e}^{bx+a})}{2}+\frac{i\ln(-i(-\mathrm{e}^{bx+a}+i))x}{2}-\frac{ia\ln(1+i\mathrm{e}^{bx+a})}{2b}-\frac{i\ln(-i(-\mathrm{e}^{bx+a}+i))\ln(-i\mathrm{e}^{bx+a})}{2b}+\frac{i\ln(-i(-}{}$

```
[In] int(arctan(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(ln(exp(b*x+a))*arctan(exp(b*x+a))+1/2*I*ln(exp(b*x+a))*ln(1+I*exp(b*x+
a))-1/2*I*ln(exp(b*x+a))*ln(1-I*exp(b*x+a))+1/2*I*dilog(1+I*exp(b*x+a))-1/2
*I*dilog(1-I*exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\int \arctan\left(e^{a+bx}\right)\,dx$$
$$= \frac{2\,bx\arctan\left(e^{(bx+a)}\right)+i\,a\log\left(e^{(bx+a)}+i\right)-i\,a\log\left(e^{(bx+a)}-i\right)+(i\,bx+i\,a)\log\left(i\,e^{(bx+a)}+1\right)+(-i\,bx}{2\,b}$$

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(e^(b*x + a)) + I*a*log(e^(b*x + a) + I) - I*a*log(e^(b*x
+ a) - I) + (I*b*x + I*a)*log(I*e^(b*x + a) + 1) + (-I*b*x - I*a)*log(-I*e^
(b*x + a) + 1) - I*dilog(I*e^(b*x + a)) + I*dilog(-I*e^(b*x + a)))/b
```

Sympy [F]

$$\int \arctan\left(e^{a+bx}\right)\,dx = \int \operatorname{atan}\left(e^{a+bx}\right)dx$$

```
[In] integrate(atan(exp(b*x+a)),x)
```

```
[Out] Integral(atan(exp(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \arctan\left(e^{a+bx}\right)\,dx = \frac{(bx+a)\arctan\left(e^{(bx+a)}\right)}{b} - \frac{\pi\log\left(e^{(2\,bx+2\,a)}+1\right) + 2i\,\mathrm{Li}_2\left(i\,e^{(bx+a)}+1\right) - 2i\,\mathrm{Li}_2\left(-i\,e^{(bx+a)}+1\right)}{4\,b}$$

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arctan(e^(b*x + a))/b - 1/4*(pi*log(e^(2*b*x + 2*a) + 1) + 2*I*di
log(I*e^(b*x + a) + 1) - 2*I*dilog(-I*e^(b*x + a) + 1))/b
```

Giac [F]

$$\int \arctan\left(e^{a+bx}\right)\,dx = \int \arctan\left(e^{(bx+a)}\right)\,dx$$

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(e^(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \arctan\left(e^{a+bx}\right)\,dx = -\frac{\mathrm{Li}_2\left(1-\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b} + \frac{\mathrm{Li}_2\left(1+\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}$$

```
[In] int(atan(exp(a + b*x)),x)
```

```
[Out] (dilog(exp(b*x)*exp(a)*1i + 1)*1i)/(2*b) - (dilog(1 - exp(b*x)*exp(a)*1i)*1
i)/(2*b)
```

3.114 $\int x \arctan \left(e^{a+bx}\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 91

$$\int x \arctan \left(e^{a+bx}\right) \, dx = \frac{ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{2b} - \frac{ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{2b} - \frac{i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{2b^2} + \frac{i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{2b^2}$$

```
[Out] 1/2*I*x*polylog(2,-I*exp(b*x+a))/b-1/2*I*x*polylog(2,I*exp(b*x+a))/b-1/2*I*
polylog(3,-I*exp(b*x+a))/b^2+1/2*I*polylog(3,I*exp(b*x+a))/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5251, 2611, 2320, 6724}

$$\int x \arctan \left(e^{a+bx}\right) \, dx = -\frac{i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{2b^2} + \frac{i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{2b^2} + \frac{ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{2b} - \frac{ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

```
[In] Int[x*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x*PolyLog[2, I*E^(a + b*x
)])/b - ((I/2)*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + ((I/2)*PolyLog[3, I*E^(a
 + b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int x\log\left(1-ie^{a+bx}\right)\,dx-\frac{1}{2}i\int x\log\left(1+ie^{a+bx}\right)\,dx\\
&= \frac{ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}\\
&\quad-\frac{i\int\text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{2b}+\frac{i\int\text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{2b}\\
&= \frac{ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}\\
&\quad-\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{a+bx}\right)}{2b^2}+\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{a+bx}\right)}{2b^2}\\
&= \frac{ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}\\
&\quad-\frac{i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{2b^2}+\frac{i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.78

$$\int x\arctan\left(e^{a+bx}\right)\,dx = \frac{i\left(bx\,\text{PolyLog}\left(2,-ie^{a+bx}\right)-bx\,\text{PolyLog}\left(2,ie^{a+bx}\right)-\text{PolyLog}\left(3,-ie^{a+bx}\right)+\text{PolyLog}\left(3,ie^{a+bx}\right)\right)}{2b^2}$$

```
[In] Integrate[x*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*(b*x*PolyLog[2, (-I)*E^(a + b*x)] - b*x*PolyLog[2, I*E^(a + b*x)] -
PolyLog[3, (-I)*E^(a + b*x)] + PolyLog[3, I*E^(a + b*x)]))/b^2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(71) = 142.

Time = 0.38 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.84

method	result
risch	$-\frac{ia^2\ln(1-ie^{bx+a})}{2b^2}+\frac{ix\,\text{polylog}(2,-ie^{bx+a})}{2b}+\frac{i\,\text{dilog}(-i(e^{bx+a}+i))a}{2b^2}+\frac{i\,\text{dilog}(-ie^{bx+a})a}{2b^2}-\frac{ix\,\text{polylog}(2,ie^{bx+a})}{2b}-\frac{i\,\text{po}}{}$

```
[In] int(x*arctan(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^2*a^2*ln(1-I*exp(b*x+a))+1/2*I*x*polylog(2,-I*exp(b*x+a))/b+1/2*I/
b^2*dilog(-I*(exp(b*x+a)+I))*a+1/2*I/b^2*dilog(-I*exp(b*x+a))*a-1/2*I*x*pol
ylog(2,I*exp(b*x+a))/b-1/2*I*polylog(3,-I*exp(b*x+a))/b^2-1/2*I/b*ln(-I*(-e
xp(b*x+a)+I))*a*x+1/2*I/b*ln(-I*(exp(b*x+a)+I))*a*x+1/2*I/b^2*ln(-I*exp(b*x
+a))*ln(-I*(-exp(b*x+a)+I))*a-1/2*I/b*ln(1-I*exp(b*x+a))*a*x+1/2*I*polylog(
3,I*exp(b*x+a))/b^2+1/2*I/b^2*polylog(2,-I*exp(b*x+a))*a+1/2*I/b*ln(1+I*exp
(b*x+a))*a*x-1/2*I/b^2*polylog(2,I*exp(b*x+a))*a-1/2*I/b^2*ln(-I*(-exp(b*x+
a)+I))*a^2+1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*a^2+1/2*I/b^2*a^2*ln(1+I*exp(b*x
+a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(61) = 122.

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.66

$$\int x \arctan \left(e^{a+bx}\right) \, dx$$
$$= \frac{2\,b^2x^2 \arctan\left(e^{(bx+a)}\right) - 2i\,bx\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) + 2i\,bx\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right) - i\,a^2 \log\left(e^{(bx+a)} + i\right) + i\,a^2 \log\left(e^{(bx+a)} -\right.}{}$$

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan(e^(b*x + a)) - 2*I*b*x*dilog(I*e^(b*x + a)) + 2*I*b*x
*dilog(-I*e^(b*x + a)) - I*a^2*log(e^(b*x + a) + I) + I*a^2*log(e^(b*x + a)
 - I) + (I*b^2*x^2 - I*a^2)*log(I*e^(b*x + a) + 1) + (-I*b^2*x^2 + I*a^2)*l
og(-I*e^(b*x + a) + 1) + 2*I*polylog(3, I*e^(b*x + a)) - 2*I*polylog(3, -I*
e^(b*x + a)))/b^2
```

Sympy [F]

$$\int x \arctan \left(e^{a+bx}\right) \, dx = \int x \operatorname{atan}\left(e^{a}e^{bx}\right) dx$$

```
[In] integrate(x*atan(exp(b*x+a)),x)
```

```
[Out] Integral(x*atan(exp(a)*exp(b*x)), x)
```

Maxima [F]

$$\int x \arctan \left(e^{a+bx}\right) \, dx = \int x \arctan \left(e^{(bx+a)}\right) \, dx$$

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(e^(b*x + a)) - b*integrate(1/2*x^2*e^(b*x + a)/(e^(2*b*x + 2
*a) + 1), x)
```

Giac [F]

$$\int x \arctan\left(e^{a+bx}\right)\,dx = \int x \arctan\left(e^{(bx+a)}\right)\,dx$$

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctan(e^(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \arctan\left(e^{a+bx}\right)\,dx = \int x\,\mathrm{atan}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

```
[In] int(x*atan(exp(a + b*x)),x)
[Out] int(x*atan(exp(a + b*x)), x)
```

3.115 $\int x^2 \arctan\left(e^{a+bx}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 133

$$\int x^2 \arctan\left(e^{a+bx}\right) dx = \frac{ix^2 \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{2b} - \frac{ix^2 \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{2b} - \frac{ix \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2} + \frac{ix \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^2} + \frac{i \operatorname{PolyLog}\left(4,-ie^{a+bx}\right)}{b^3} - \frac{i \operatorname{PolyLog}\left(4,ie^{a+bx}\right)}{b^3}$$

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(b*x+a))/b-1/2*I*x^2*polylog(2,I*exp(b*x+a))/b-I*
x*polylog(3,-I*exp(b*x+a))/b^2+I*x*polylog(3,I*exp(b*x+a))/b^2+I*polylog(4,
-I*exp(b*x+a))/b^3-I*polylog(4,I*exp(b*x+a))/b^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5251, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan\left(e^{a+bx}\right) dx = \frac{i \operatorname{PolyLog}\left(4,-ie^{a+bx}\right)}{b^3} - \frac{i \operatorname{PolyLog}\left(4,ie^{a+bx}\right)}{b^3} - \frac{ix \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2} + \frac{ix \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^2} + \frac{ix^2 \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{2b} - \frac{ix^2 \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

```
[In] Int[x^2*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x^2*PolyLog[2, I*E^(a +
 b*x)])/b - (I*x*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[3, I*E^(a
```

```
 + b*x)])/b^2 + (I*PolyLog[4, (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[4, I*E^(a
 + b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}i\int x^2\log\left(1-ie^{a+bx}\right)\,dx-\frac{1}{2}i\int x^2\log\left(1+ie^{a+bx}\right)\,dx\\&=\frac{ix^2\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{ix^2\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{2b}\\&\quad-\frac{i\int x\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b}+\frac{i\int x\operatorname{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b}\end{aligned}$$

$$= \frac{ix^2 \operatorname{PolyLog}(2, -ie^{a+bx})}{2b} - \frac{ix^2 \operatorname{PolyLog}(2, ie^{a+bx})}{2b} - \frac{ix \operatorname{PolyLog}(3, -ie^{a+bx})}{b^2} + \frac{ix \operatorname{PolyLog}(3, ie^{a+bx})}{b^2} + \frac{i \int \operatorname{PolyLog}(3, -ie^{a+bx})\, dx}{b^2} - \frac{i \int \operatorname{PolyLog}(3, ie^{a+bx})\, dx}{b^2}$$

$$= \frac{ix^2 \operatorname{PolyLog}(2, -ie^{a+bx})}{2b} - \frac{ix^2 \operatorname{PolyLog}(2, ie^{a+bx})}{2b} - \frac{ix \operatorname{PolyLog}(3, -ie^{a+bx})}{b^2} + \frac{ix \operatorname{PolyLog}(3, ie^{a+bx})}{b^2} + \frac{i\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-ix)}{x}\, dx, x, e^{a+bx}\right)}{b^3} - \frac{i\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,ix)}{x}\, dx, x, e^{a+bx}\right)}{b^3}$$

$$= \frac{ix^2 \operatorname{PolyLog}(2, -ie^{a+bx})}{2b} - \frac{ix^2 \operatorname{PolyLog}(2, ie^{a+bx})}{2b} - \frac{ix \operatorname{PolyLog}(3, -ie^{a+bx})}{b^2} + \frac{ix \operatorname{PolyLog}(3, ie^{a+bx})}{b^2} + \frac{i \operatorname{PolyLog}(4, -ie^{a+bx})}{b^3} - \frac{i \operatorname{PolyLog}(4, ie^{a+bx})}{b^3}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int x^2 \arctan\left(e^{a+bx}\right) dx = \frac{i\left(b^2x^2 \operatorname{PolyLog}(2, -ie^{a+bx}) - b^2x^2 \operatorname{PolyLog}(2, ie^{a+bx}) + 2\left(-bx \operatorname{PolyLog}(3, -ie^{a+bx}) + bx \operatorname{PolyLog}(3, ie^{a\cdots}\right.\right.}{2b^3}$$

```
[In] Integrate[x^2*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*(b^2*x^2*PolyLog[2, (-I)*E^(a + b*x)] - b^2*x^2*PolyLog[2, I*E^(a +
b*x)] + 2*(-(b*x*PolyLog[3, (-I)*E^(a + b*x)]) + b*x*PolyLog[3, I*E^(a + b*
x)] + PolyLog[4, (-I)*E^(a + b*x)] - PolyLog[4, I*E^(a + b*x)])))/b^3
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(111) = 222.

Time = 0.42 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.06

method	result
risch	$-\frac{i\,\mathrm{dilog}(-i\mathrm{e}^{bx+a})a^2}{2b^3} + \frac{i\ln(1-i\mathrm{e}^{bx+a})a^3}{2b^3} - \frac{i\,\mathrm{dilog}(-i(\mathrm{e}^{bx+a}+i))a^2}{2b^3} + \frac{ix^2\,\mathrm{polylog}(2,-i\mathrm{e}^{bx+a})}{2b} + \frac{ix\,\mathrm{polylog}(3,i\mathrm{e}^{bx+a})}{b^2} + \frac{i\ln\cdots}{}$

```
[In] int(x^2*arctan(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

[Out] -1/2*I/b^3*dilog(-I*exp(b*x+a))*a^2+1/2*I/b^3*ln(1-I*exp(b*x+a))*a^3-1/2*I/b^3*dilog(-I*(exp(b*x+a)+I))*a^2+1/2*I*x^2*polylog(2,-I*exp(b*x+a))/b+I*x*polylog(3,I*exp(b*x+a))/b^2+1/2*I/b^2*ln(1-I*exp(b*x+a))*x*a^2-1/2*I/b^3*a^3*ln(1+I*exp(b*x+a))-1/2*I/b^3*ln(-I*exp(b*x+a))*ln(-I*(-exp(b*x+a)+I))*a^2+1/2*I/b^2*ln(-I*(-exp(b*x+a)+I))*a^2*x-I*polylog(4,I*exp(b*x+a))/b^3-1/2*I/b^2*ln(1+I*exp(b*x+a))*a^2*x+1/2*I/b^3*polylog(2,I*exp(b*x+a))*a^2+1/2*I/b^3*ln(-I*(-exp(b*x+a)+I))*a^3-1/2*I/b^3*ln(-I*(exp(b*x+a)+I))*a^3-1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*x*a^2+I*polylog(4,-I*exp(b*x+a))/b^3-1/2*I/b^3*polylog(2,-I*exp(b*x+a))*a^2-I*x*polylog(3,-I*exp(b*x+a))/b^2-1/2*I*x^2*polylog(2,I*exp(b*x+a))/b

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.41

$$\int x^2 \arctan\left(e^{a+bx}\right)\,dx = \frac{2\,b^3x^3\arctan\left(e^{(bx+a)}\right) - 3i\,b^2x^2\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) + 3i\,b^2x^2\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right) + i\,a^3\log\left(e^{(bx+a)}+i\right) - i\,a^3\log\left(e^{(b}\right.}{}$$

[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="fricas")

[Out] 1/6*(2*b^3*x^3*arctan(e^(b*x + a)) - 3*I*b^2*x^2*dilog(I*e^(b*x + a)) + 3*I*b^2*x^2*dilog(-I*e^(b*x + a)) + I*a^3*log(e^(b*x + a) + I) - I*a^3*log(e^(b*x + a) - I) + 6*I*b*x*polylog(3, I*e^(b*x + a)) - 6*I*b*x*polylog(3, -I*e^(b*x + a)) + (I*b^3*x^3 + I*a^3)*log(I*e^(b*x + a) + 1) + (-I*b^3*x^3 - I*a^3)*log(-I*e^(b*x + a) + 1) - 6*I*polylog(4, I*e^(b*x + a)) + 6*I*polylog(4, -I*e^(b*x + a)))/b^3

Sympy [F]

$$\int x^2 \arctan\left(e^{a+bx}\right)\,dx = \int x^2 \operatorname{atan}\left(e^a e^{bx}\right)dx$$

[In] integrate(x**2*atan(exp(b*x+a)),x)

[Out] Integral(x**2*atan(exp(a)*exp(b*x)), x)

Maxima [F]

$$\int x^2 \arctan\left(e^{a+bx}\right) \, dx = \int x^2 \arctan\left(e^{(bx+a)}\right) \, dx$$

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="maxima")
[Out] 1/3*x^3*arctan(e^(b*x + a)) - b*integrate(1/3*x^3*e^(b*x + a)/(e^(2*b*x + 2
*a) + 1), x)
```

Giac [F]

$$\int x^2 \arctan\left(e^{a+bx}\right) \, dx = \int x^2 \arctan\left(e^{(bx+a)}\right) \, dx$$

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arctan(e^(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan\left(e^{a+bx}\right) \, dx = \int x^2 \operatorname{atan}\left(e^{a+b\,x}\right) \, dx$$

```
[In] int(x^2*atan(exp(a + b*x)),x)
[Out] int(x^2*atan(exp(a + b*x)), x)
```

3.116 $\int \arctan\left(a+bf^{c+dx}\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 196

$$\int \arctan\left(a+bf^{c+dx}\right) \, dx = -\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)} + \frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)} + \frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bf^{c+dx})}\right)}{2d\log(f)} - \frac{i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{2d\log(f)}$$

```
[Out] -arctan(a+b*f^(d*x+c))*ln(2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+arctan(a+b*f^(d*
x+c))*ln(2*b*f^(d*x+c)/(I-a)/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+1/2*I*polylog(2
,1-2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)-1/2*I*polylog(2,1-2*b*f^(d*x+c)/(I-a)/(
1-I*(a+b*f^(d*x+c))))/d/ln(f)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

$= \{2320, 5155, 4966, 2449, 2352, 2497\}$

$$\int \arctan\left(a+bf^{c+dx}\right)\,dx = -\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)} + \frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(-a+i)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)} + \frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(bf^{c+dx}+a)}\right)}{2d\log(f)} - \frac{i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(bf^{c+dx}+a))}\right)}{2d\log(f)}$$

```
[In] Int[ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcTan[a + b*f^(c + d*x)]*Log[2/(1 - I*(a + b*f^(c + d*x)))])/(d*Log[f]))
) + (ArcTan[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b
*f^(c + d*x))))])/(d*Log[f]) + ((I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*f^(c +
 d*x)))])/(d*Log[f]) - ((I/2)*PolyLog[2, 1 - (2*b*f^(c + d*x))/((I - a)*(1
- I*(a + b*f^(c + d*x))))])/(d*Log[f])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\arctan(a+bx)}{x}\,dx, x, f^{c+dx}\right)}{d\log(f)}\\
&= \frac{\text{Subst}\left(\int \frac{\arctan(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bf^{c+dx}\right)}{bd\log(f)}\\
&= -\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, a+bf^{c+dx}\right)}{d\log(f)}-\frac{\text{Subst}\left(\int \frac{\log\left(\frac{2\left(-\frac{a}{b}+\frac{x}{b}\right)}{\left(\frac{i}{b}-\frac{a}{b}\right)(1-ix)}\right)}{1+x^2}\,dx, x, a+bf^{c+dx}\right)}{d\log(f)}\\
&= -\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}\\
&\quad-\frac{i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{2d\log(f)}+\frac{i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}\\
&= -\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}\\
&\quad+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bf^{c+dx})}\right)}{2d\log(f)}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{2d\log(f)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.85

$$\int \arctan \left(a+bf^{c+dx}\right)\,dx = x\arctan \left(a+bf^{c+dx}\right) - \frac{b\left(dx\log(f)\left(\log\left(1+\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\log\left(1+\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)+\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\text{PolyLog}\left(2,-\frac{b^2f^{c+d}}{ab+\sqrt{-}}\right.\right.}{2\sqrt{-b^2}d\log(f)}$$

```
[In] Integrate[ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] x*ArcTan[a + b*f^(c + d*x)] - (b*(d*x*Log[f]*(Log[1 + (b^2*f^(c + d*x))/(a*
b - Sqrt[-b^2])] - Log[1 + (b^2*f^(c + d*x))/(a*b + Sqrt[-b^2])]) + PolyLog
[2, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] - PolyLog[2, -((b^2*f^(c + d*x
))/(a*b + Sqrt[-b^2]))]))/(2*Sqrt[-b^2]*d*Log[f])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\ln\left(-b\,f^{dx+c}\right)\arctan\left(a+b\,f^{dx+c}\right)-\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i+b\,f^{dx+c}+a}{i+a}\right)}{2}+\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i-b\,f^{dx+c}-a}{i-a}\right)}{2}-\frac{i\,\text{dilog}\left(\frac{i+b\,f^{dx+c}}{i+a}\right.}{2}}{d\ln(f)}$
default	$\frac{\ln\left(-b\,f^{dx+c}\right)\arctan\left(a+b\,f^{dx+c}\right)-\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i+b\,f^{dx+c}+a}{i+a}\right)}{2}+\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i-b\,f^{dx+c}-a}{i-a}\right)}{2}-\frac{i\,\text{dilog}\left(\frac{i+b\,f^{dx+c}}{i+a}\right.}{2}}{d\ln(f)}$
risch	$-\frac{ix\ln\left(1+i\left(a+b\,f^{dx+c}\right)\right)}{2}+\frac{i\,\text{dilog}\left(\frac{b\,f^{dx}f^{c}+a-i}{a-i}\right)}{2\ln(f)d}+\frac{i\ln\left(\frac{b\,f^{dx}f^{c}+a-i}{a-i}\right)x}{2}+\frac{i\ln\left(\frac{b\,f^{dx}f^{c}+a-i}{a-i}\right)c}{2d}-\frac{ic\ln\left(if^{dx}f^{c}b\right.}{2d}$

```
[In] int(arctan(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/ln(f)*(ln(-b*f^(d*x+c))*arctan(a+b*f^(d*x+c))-1/2*I*ln(-b*f^(d*x+c))*ln
((I+b*f^(d*x+c)+a)/(I+a))+1/2*I*ln(-b*f^(d*x+c))*ln((I-b*f^(d*x+c)-a)/(I-a)
)-1/2*I*dilog((I+b*f^(d*x+c)+a)/(I+a))+1/2*I*dilog((I-b*f^(d*x+c)-a)/(I-a))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08

$$\int \arctan \left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{2\,dx\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)+i\,c\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)-i\,c\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)+\left(i\,dx\right.}{}$$

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x*arctan(b*f^(d*x + c) + a)*log(f) + I*c*log(b*f^(d*x + c) + a + I
)*log(f) - I*c*log(b*f^(d*x + c) + a - I)*log(f) + (I*d*x + I*c)*log(f)*log
((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (-I*d*x - I*c)*log(f)*log
((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + I*dilog(-(a^2 + (a*b + I*
b)*f^(d*x + c) + 1)/(a^2 + 1) + 1) - I*dilog(-(a^2 + (a*b - I*b)*f^(d*x + c
) + 1)/(a^2 + 1) + 1))/(d*log(f))
```

Sympy [F]

$$\int \arctan \left(a+bf^{c+dx}\right)\,dx=\int \operatorname{atan}\left(a+bf^{c+dx}\right)dx$$

```
[In] integrate(atan(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(atan(a + b*f**(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96

$$\int \arctan \left(a+bf^{c+dx}\right)\,dx=\frac{(dx+c)\arctan\left(bf^{dx+c}+a\right)}{d}$$
$$-\frac{2\,(dx+c)\arctan\left(\frac{b^2f^{dx+c}+ab}{b}\right)\log\left(f\right)+\left(\pi-\arctan\left(\frac{1}{a}\right)\right)\log\left(b^2f^{2\,dx+2\,c}+2\,abf^{dx+c}+a^2+1\right)-\arctan}{2\,d\log\left(f\right)}$$

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)*arctan(b*f^(d*x + c) + a)/d - 1/2*(2*(d*x + c)*arctan((b^2*f^(d*x
 + c) + a*b)/b)*log(f) + (pi - arctan(1/a))*log(b^2*f^(2*d*x + 2*c) + 2*a*b
*f^(d*x + c) + a^2 + 1) - arctan(b*f^(d*x + c) + a)*log(b^2*f^(2*d*x + 2*c)
/(a^2 + 1)) + I*dilog((I*b*f^(d*x + c) + I*a + 1)/(I*a + 1)) - I*dilog((I*b
*f^(d*x + c) + I*a - 1)/(I*a - 1)))/(d*log(f))
```

Giac [F]

$$\int \arctan\left(a+bf^{c+dx}\right)\,dx=\int \arctan\left(bf^{dx+c}+a\right)\,dx$$

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="giac")
[Out] integrate(arctan(b*f^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \arctan\left(a+bf^{c+dx}\right)\,dx=\int \mathrm{atan}\left(a+b\,f^{c+d\,x}\right)\,dx$$

```
[In] int(atan(a + b*f^(c + d*x)),x)
[Out] int(atan(a + b*f^(c + d*x)), x)
```

3.117 $\int x \arctan\left(a+bf^{c+dx}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 232

$$\int x\arctan\left(a+bf^{c+dx}\right)\,dx=\frac{1}{2}x^2\arctan\left(a+bf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right)+\frac{1}{4}ix^2\log\left(1+\frac{ibf^{c+dx}}{1+ia}\right)-\frac{ix\,\mathrm{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)}+\frac{ix\,\mathrm{PolyLog}\left(2,-\frac{ibf^{c+dx}}{1+ia}\right)}{2d\log(f)}+\frac{i\,\mathrm{PolyLog}\left(3,\frac{ibf^{c+dx}}{1-ia}\right)}{2d^2\log^2(f)}-\frac{i\,\mathrm{PolyLog}\left(3,-\frac{ibf^{c+dx}}{1+ia}\right)}{2d^2\log^2(f)}$$

```
[Out] 1/2*x^2*arctan(a+b*f^(d*x+c))-1/4*I*x^2*ln(1-I*b*f^(d*x+c)/(1-I*a))+1/4*I*x
^2*ln(1+I*b*f^(d*x+c)/(1+I*a))-1/2*I*x*polylog(2,I*b*f^(d*x+c)/(1-I*a))/d/l
n(f)+1/2*I*x*polylog(2,-I*b*f^(d*x+c)/(1+I*a))/d/ln(f)+1/2*I*polylog(3,I*b*
f^(d*x+c)/(1-I*a))/d^2/ln(f)^2-1/2*I*polylog(3,-I*b*f^(d*x+c)/(1+I*a))/d^2/
ln(f)^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used

$= \{5251, 2612, 2611, 2320, 6724\}$

$$\int x \arctan \left(a+bf^{c+dx}\right) \, dx = -\frac{i \operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{2d^2\log^2(f)}+\frac{i \operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{2d^2\log^2(f)}+\frac{ix \operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}-\frac{ix \operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{2d\log(f)}+\frac{1}{4}ix^2\log\left(-ia-ibf^{c+dx}+1\right)-\frac{1}{4}ix^2\log\left(ia+ibf^{c+dx}+1\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{-a+i}\right)-\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{a+i}\right)$$

```
[In] Int[x*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/4)*x^2*Log[1 - I*a - I*b*f^(c + d*x)] - (I/4)*x^2*Log[1 + I*a + I*b*f^(c
 + d*x)] + (I/4)*x^2*Log[1 - (b*f^(c + d*x))/(I - a)] - (I/4)*x^2*Log[1 + (
b*f^(c + d*x))/(I + a)] + ((I/2)*x*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(d*
Log[f]) - ((I/2)*x*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) - ((I
/2)*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) + ((I/2)*PolyLog[3,
 -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int x\log\left(1-ia-ibf^{c+dx}\right)\,dx-\frac{1}{2}i\int x\log\left(1+ia+ibf^{c+dx}\right)\,dx\\
&= \frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)\\
&\quad -\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{2}i\int x\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right)\,dx-\frac{1}{2}i\int x\log\left(1+\frac{ibf^{c+dx}}{1+ia}\right)\,dx\\
&= \frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)\\
&\quad -\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{ix\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}-\frac{ix\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}\\
&\quad +\frac{i\int \text{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)\,dx}{2d\log(f)}-\frac{i\int \text{PolyLog}\left(2,-\frac{ibf^{c+dx}}{1+ia}\right)\,dx}{2d\log(f)}\\
&= \frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)\\
&\quad -\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{ix\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}-\frac{ix\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}\\
&\quad -\frac{i\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{bx}{i-a}\right)}{x}\,dx,x,f^{c+dx}\right)}{2d^2\log^2(f)}+\frac{i\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-\frac{bx}{i+a}\right)}{x}\,dx,x,f^{c+dx}\right)}{2d^2\log^2(f)}\\
&= \frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)\\
&\quad +\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)-\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{ix\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}-\frac{i\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{2d^2\log^2(f)}+\frac{i\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{2d^2\log^2(f)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.02

$$\int x \arctan \left(a+bf^{c+dx}\right) dx$$
$$= \frac{i\left(d^2x^2 \log^2(f) \log \left(1-ia-ibf^{c+dx}\right)-d^2x^2 \log^2(f) \log \left(1+ia+ibf^{c+dx}\right)-d^2x^2 \log^2(f) \log \left(\frac{i+a+bf^{c+dx}}{i+a}\right)\right.}{}$$

```
[In] Integrate[x*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] ((I/4)*(d^2*x^2*Log[f]^2*Log[1 - I*a - I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2*
Log[1 + I*a + I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2*Log[(I + a + b*f^(c + d*x
))/(I + a)] + d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(-I + a)] + 2*d*x*Lo
g[f]*PolyLog[2, (b*f^(c + d*x))/(I - a)] - 2*d*x*Log[f]*PolyLog[2, -((b*f^(
c + d*x))/(I + a))] - 2*PolyLog[3, (b*f^(c + d*x))/(I - a)] + 2*PolyLog[3,
-((b*f^(c + d*x))/(I + a))]))/(d^2*Log[f]^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 671 vs. 2(200) = 400.

Time = 0.88 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.90

method	result
risch	$\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-i a-1}\right) x^2}{4}-\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-i a+1}\right) x^2}{4}+\frac{i c \ln\left(\frac{b f^{dx} f^{c}+a+i}{i+a}\right) x}{2d}+\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-i a-1}\right) c^2}{4d^2}-\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-i a+1}\right) c^2}{4d^2}+\frac{i c \operatorname{dilog}}{}$

```
[In] int(x*arctan(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x^2-1/4*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c
)*x^2+1/2*I/d*c*ln((b*f^(d*x)*f^c+a+I)/(I+a))*x+1/4*I/d^2*ln(1-I*b/(-I*a-1)
*f^(d*x)*f^c)*c^2-1/4*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^2+1/2*I/ln(f)/d
^2*c*dilog((b*f^(d*x)*f^c+a+I)/(I+a))+1/4*I*x^2*ln(1-I*(a+b*f^(d*x+c)))-1/2
*I/ln(f)/d^2*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c-1/2*I/d^2*c^2*ln((b*f^(d*
x)*f^c+a-I)/(a-I))+1/4*I/d^2*c^2*ln(I*f^(d*x)*f^c*b+I*a+1)-1/2*I/ln(f)^2/d^
2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)+1/2*I/ln(f)^2/d^2*polylog(3,I*b/(1-I*
a)*f^(d*x)*f^c)-1/2*I/d*c*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x-1/2*I/ln(f)/d*pol
ylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*x+1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a+I)/(I+a
))-1/4*I/d^2*c^2*ln(1-I*a-I*f^(d*x)*f^c*b)-1/2*I/ln(f)/d^2*c*dilog((b*f^(d*
x)*f^c+a-I)/(a-I))+1/2*I/ln(f)/d^2*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c-1/
4*I*x^2*ln(1+I*(a+b*f^(d*x+c)))+1/2*I/ln(f)/d*polylog(2,I*b/(-I*a-1)*f^(d*x
)*f^c)*x-1/2*I/d*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c*x+1/2*I/d*ln(1-I*b/(-I*a-1
)*f^(d*x)*f^c)*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.31

$$\int x \arctan\left(a+bf^{c+dx}\right)\,dx$$
$$= \frac{2\,d^2x^2\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)^2 - i\,c^2\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)^2 + i\,c^2\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)^2 + \cdots}{\cdots}$$

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*x^2*arctan(b*f^(d*x + c) + a)*log(f)^2 - I*c^2*log(b*f^(d*x + c)
 + a + I)*log(f)^2 + I*c^2*log(b*f^(d*x + c) + a - I)*log(f)^2 + 2*I*d*x*di
log(-(a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) - 2*I*d*x*di
log(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + (I*d^2*x^2
 - I*c^2)*log(f)^2*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (-I
*d^2*x^2 + I*c^2)*log(f)^2*log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1
)) - 2*I*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) + 2*I*polylog(3, -(
a*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^2*log(f)^2)
```

Sympy [F]

$$\int x \arctan\left(a+bf^{c+dx}\right)\,dx = \int x \operatorname{atan}\left(a+bf^{c+dx}\right)dx$$

```
[In] integrate(x*atan(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(x*atan(a + b*f**(c + d*x)), x)
```

Maxima [F]

$$\int x \arctan\left(a+bf^{c+dx}\right)\,dx = \int x \arctan\left(bf^{dx+c}+a\right)\,dx$$

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*d*f^c*integrate(1/2*f^(d*x)*x^2/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f
^c + a^2 + 1), x)*log(f) + 1/2*x^2*arctan(b*f^(d*x)*f^c + a)
```

Giac [F]

$$\int x \arctan \left(a + b f^{c+dx}\right) \, dx = \int x \arctan \left(b f^{dx+c} + a\right) \, dx$$

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
[Out] integrate(x*arctan(b*f^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \arctan \left(a + b f^{c+dx}\right) \, dx = \int x \operatorname{atan}\left(a + b\, f^{c+d\,x}\right) \, dx$$

```
[In] int(x*atan(a + b*f^(c + d*x)),x)
[Out] int(x*atan(a + b*f^(c + d*x)), x)
```

3.118 $\int x^2 \arctan\left(a+bf^{c+dx}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 302

$$\int x^2 \arctan\left(a+bf^{c+dx}\right)\,dx = \frac{1}{3}x^3\arctan\left(a+bf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right) + \frac{1}{6}ix^3\log\left(1+\frac{ibf^{c+dx}}{1+ia}\right) - \frac{ix^2\,\text{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)} + \frac{ix^2\,\text{PolyLog}\left(2,-\frac{ibf^{c+dx}}{1+ia}\right)}{2d\log(f)} + \frac{ix\,\text{PolyLog}\left(3,\frac{ibf^{c+dx}}{1-ia}\right)}{d^2\log^2(f)} - \frac{ix\,\text{PolyLog}\left(3,-\frac{ibf^{c+dx}}{1+ia}\right)}{d^2\log^2(f)} - \frac{i\,\text{PolyLog}\left(4,\frac{ibf^{c+dx}}{1-ia}\right)}{d^3\log^3(f)} + \frac{i\,\text{PolyLog}\left(4,-\frac{ibf^{c+dx}}{1+ia}\right)}{d^3\log^3(f)}$$

```
[Out] 1/3*x^3*arctan(a+b*f^(d*x+c))-1/6*I*x^3*ln(1-I*b*f^(d*x+c)/(1-I*a))+1/6*I*x
^3*ln(1+I*b*f^(d*x+c)/(1+I*a))-1/2*I*x^2*polylog(2,I*b*f^(d*x+c)/(1-I*a))/d
/ln(f)+1/2*I*x^2*polylog(2,-I*b*f^(d*x+c)/(1+I*a))/d/ln(f)+I*x*polylog(3,I*
b*f^(d*x+c)/(1-I*a))/d^2/ln(f)^2-I*x*polylog(3,-I*b*f^(d*x+c)/(1+I*a))/d^2/
ln(f)^2-I*polylog(4,I*b*f^(d*x+c)/(1-I*a))/d^3/ln(f)^3+I*polylog(4,-I*b*f^(
d*x+c)/(1+I*a))/d^3/ln(f)^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5251, 2612, 2611, 6744, 2320, 6724}

$$\int x^2 \arctan\left(a+bf^{c+dx}\right) dx = \frac{i \operatorname{PolyLog}\left(4, \frac{bf^{c+dx}}{i-a}\right)}{d^3 \log^3(f)} - \frac{i \operatorname{PolyLog}\left(4, -\frac{bf^{c+dx}}{a+i}\right)}{d^3 \log^3(f)} - \frac{ix \operatorname{PolyLog}\left(3, \frac{bf^{c+dx}}{i-a}\right)}{d^2 \log^2(f)} + \frac{ix \operatorname{PolyLog}\left(3, -\frac{bf^{c+dx}}{a+i}\right)}{d^2 \log^2(f)} + \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{bf^{c+dx}}{i-a}\right)}{2d \log(f)} - \frac{ix^2 \operatorname{PolyLog}\left(2, -\frac{bf^{c+dx}}{a+i}\right)}{2d \log(f)} + \frac{1}{6} i x^3 \log\left(-ia-ibf^{c+dx}+1\right) - \frac{1}{6} i x^3 \log\left(ia+ibf^{c+dx}+1\right) + \frac{1}{6} i x^3 \log\left(1-\frac{bf^{c+dx}}{-a+i}\right) - \frac{1}{6} i x^3 \log\left(1+\frac{bf^{c+dx}}{a+i}\right)$$

```
[In] Int[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/6)*x^3*Log[1 - I*a - I*b*f^(c + d*x)] - (I/6)*x^3*Log[1 + I*a + I*b*f^(c
 + d*x)] + (I/6)*x^3*Log[1 - (b*f^(c + d*x))/(I - a)] - (I/6)*x^3*Log[1 + (
b*f^(c + d*x))/(I + a)] + ((I/2)*x^2*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(
d*Log[f]) - ((I/2)*x^2*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) -
 (I*x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) + (I*x*PolyLog[3,
 -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2) + (I*PolyLog[4, (b*f^(c + d*x)
)/(I - a)])/(d^3*Log[f]^3) - (I*PolyLog[4, -((b*f^(c + d*x))/(I + a))])/(d^
3*Log[f]^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}i\int x^2\log\left(1-ia-ibf^{c+dx}\right)\,dx-\frac{1}{2}i\int x^2\log\left(1+ia+ibf^{c+dx}\right)\,dx\\
&= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)\\
&\quad-\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{2}i\int x^2\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right)\,dx-\frac{1}{2}i\int x^2\log\left(1+\frac{ibf^{c+dx}}{1+ia}\right)\,dx\\
&= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)\\
&\quad-\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{ix^2\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}-\frac{ix^2\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}\\
&\quad+\frac{i\int x\,\text{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)\,dx}{d\log(f)}-\frac{i\int x\,\text{PolyLog}\left(2,-\frac{ibf^{c+dx}}{1+ia}\right)\,dx}{d\log(f)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) \\
&\quad+\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)-\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)} \\
&\quad-\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}-\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}+\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)} \\
&\quad-\frac{i\int\operatorname{PolyLog}\left(3,\frac{ibf^{c+dx}}{1-ia}\right)dx}{d^2\log^2(f)}+\frac{i\int\operatorname{PolyLog}\left(3,-\frac{ibf^{c+dx}}{1+ia}\right)dx}{d^2\log^2(f)} \\
&= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) \\
&\quad+\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)-\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)} \\
&\quad-\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}-\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}+\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)} \\
&\quad+\frac{i\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,\frac{bx}{i-a}\right)}{x}dx,x,f^{c+dx}\right)}{d^3\log^3(f)}-\frac{i\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,-\frac{bx}{i+a}\right)}{x}dx,x,f^{c+dx}\right)}{d^3\log^3(f)} \\
&= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) \\
&\quad+\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)-\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)} \\
&\quad-\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}-\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)} \\
&\quad+\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)}+\frac{i\operatorname{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)}-\frac{i\operatorname{PolyLog}\left(4,-\frac{bf^{c+dx}}{i+a}\right)}{d^3\log^3(f)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.99

$$
\int x^2\arctan\left(a+bf^{c+dx}\right)dx
$$

$$
=\frac{i\left(d^3x^3\log^3(f)\log\left(1-ia-ibf^{c+dx}\right)-d^3x^3\log^3(f)\log\left(1+ia+ibf^{c+dx}\right)-d^3x^3\log^3(f)\log\left(\frac{i+a+bf^{c+dx}}{i+a}\right)\right.}{}
$$

```
[In] Integrate[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

[Out] ((I/6)*(d^3*x^3*Log[f]^3*Log[1 - I*a - I*b*f^(c + d*x)] - d^3*x^3*Log[f]^3*Log[1 + I*a + I*b*f^(c + d*x)] - d^3*x^3*Log[f]^3*Log[(I + a + b*f^(c + d*x))/(I + a)] + d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(-I + a)] + 3*d^2*x^2*Log[f]^2*PolyLog[2, (b*f^(c + d*x))/(I - a)] - 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(I + a))] - 6*d*x*Log[f]*PolyLog[3, (b*f^(c + d*x))/(I - a)] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x))/(I + a))] + 6*PolyLog[4, (b*f^(c + d*x))/(I - a)] - 6*PolyLog[4, -((b*f^(c + d*x))/(I + a))]))/(d^3*Log[f]^3)

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(268) = 536$.

Time = 1.25 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.51

method	result
risch	$-\frac{i\ln\left(1-\frac{ib\,f^{dx} f^{c}}{-ia-1}\right)c^{3}}{3d^{3}}+\frac{i\ln\left(1-\frac{ib\,f^{dx} f^{c}}{-ia-1}\right)x^{3}}{6}-\frac{i\ln\left(1-\frac{ib\,f^{dx} f^{c}}{-ia+1}\right)x^{3}}{6}-\frac{ic^{3}\ln\left(if^{dx}f^{c}b+ia+1\right)}{6d^{3}}+\frac{i\ln\left(1-\frac{ib\,f^{dx} f^{c}}{-ia+1}\right)x\,c^{2}}{2d^{2}}-$

[In] int(x^2*arctan(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/3*I/d^3*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*c^3+1/6*I*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x^3-1/6*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^3-1/6*I/d^3*c^3*ln(I*f^(d*x)*f^c*b+I*a+1)+1/2*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x*c^2-1/6*I*x^3*ln(1+I*(a+b*f^(d*x+c)))-1/2*I/d^3*c^3*ln((b*f^(d*x)*f^c+a+I)/(I+a))+1/2*I/d^3*c^3*ln((b*f^(d*x)*f^c+a-I)/(a-I))+I/d^2/ln(f)^2*polylog(3,I*b/(1-I*a)*f^(d*x)*f^c)*x-I/d^3/ln(f)^3*polylog(4,I*b/(1-I*a)*f^(d*x)*f^c)+1/2*I/d/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x^2-I/d^2/ln(f)^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)*x+1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x+1/2*I/d^3/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c^2+I/d^3/ln(f)^3*polylog(4,I*b/(-I*a-1)*f^(d*x)*f^c)+1/2*I/d^3/ln(f)*c^2*dilog((b*f^(d*x)*f^c+a-I)/(a-I))-1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a+I)/(I+a))*x-1/2*I/d^3/ln(f)*c^2*dilog((b*f^(d*x)*f^c+a+I)/(I+a))+1/3*I/d^3*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^3-1/2*I/d^3/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c^2-1/2*I/d/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*x^2+1/6*I/d^3*c^3*ln(1-I*a-I*f^(d*x)*f^c*b)+1/6*I*x^3*ln(1-I*(a+b*f^(d*x+c)))-1/2*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x*c^2

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.25

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) dx$$
$$= \frac{2 d^3x^3 \arctan\left(bf^{dx+c} + a\right) \log\left(f\right)^3 + 3i d^2x^2 \mathrm{Li}_2\left(-\frac{a^2+(ab+ib)f^{dx+c}+1}{a^2+1} + 1\right) \log\left(f\right)^2 - 3i d^2x^2 \mathrm{Li}_2\left(-\frac{a^2+(ab-ib)f}{a^2+1}\right.}{}$$

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^3*x^3*arctan(b*f^(d*x + c) + a)*log(f)^3 + 3*I*d^2*x^2*dilog(-(a^2
 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 - 3*I*d^2*x^2*dilog
(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 + I*c^3*log(b
*f^(d*x + c) + a + I)*log(f)^3 - I*c^3*log(b*f^(d*x + c) + a - I)*log(f)^3
+ (I*d^3*x^3 + I*c^3)*log(f)^3*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2
 + 1)) + (-I*d^3*x^3 - I*c^3)*log(f)^3*log((a^2 + (a*b - I*b)*f^(d*x + c) +
 1)/(a^2 + 1)) - 6*I*d*x*log(f)*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 +
1)) + 6*I*d*x*log(f)*polylog(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1)) + 6*I*p
olylog(4, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) - 6*I*polylog(4, -(a*b - I*b)
*f^(d*x + c)/(a^2 + 1)))/(d^3*log(f)^3)
```

Sympy [F]

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) dx = \int x^2 \operatorname{atan}\left(a + bf^{c+dx}\right) dx$$

```
[In] integrate(x**2*atan(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(x**2*atan(a + b*f**(c + d*x)), x)
```

Maxima [F]

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) dx = \int x^2 \arctan\left(bf^{dx+c} + a\right) dx$$

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*d*f^c*integrate(1/3*f^(d*x)*x^3/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f
^c + a^2 + 1), x)*log(f) + 1/3*x^3*arctan(b*f^(d*x)*f^c + a)
```

Giac [F]

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) \, dx = \int x^2 \arctan\left(bf^{dx+c} + a\right) \, dx$$

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
[Out] integrate(x^2*arctan(b*f^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) \, dx = \int x^2 \operatorname{atan}\left(a + b\, f^{c+d\,x}\right) \, dx$$

```
[In] int(x^2*atan(a + b*f^(c + d*x)),x)
[Out] int(x^2*atan(a + b*f^(c + d*x)), x)
```

3.119 $\int e^{-x} \arctan(e^x)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int e^{-x} \arctan(e^x)\, dx = x - e^{-x} \arctan(e^x) - \frac{1}{2} \log\left(1 + e^{2x}\right)$$

```
[Out] x-arctan(exp(x))/exp(x)-1/2*ln(1+exp(2*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2225, 5315, 2320, 36, 29, 31}

$$\int e^{-x} \arctan(e^x)\, dx = -e^{-x} \arctan(e^x) + x - \frac{1}{2} \log\left(e^{2x} + 1\right)$$

```
[In] Int[ArcTan[E^x]/E^x,x]
```

```
[Out] x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -e^{-x}\arctan\left(e^x\right)+\int\frac{1}{1+e^{2x}}\,dx\\ &= -e^{-x}\arctan\left(e^x\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{x(1+x)}\,dx,x,e^{2x}\right)\\ &= -e^{-x}\arctan\left(e^x\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{x}\,dx,x,e^{2x}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x}\,dx,x,e^{2x}\right)\\ &= x-e^{-x}\arctan\left(e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^{-x} \arctan\left(e^x\right) dx = x - e^{-x} \arctan\left(e^x\right) - \frac{1}{2} \log\left(1 + e^{2x}\right)$$

```
[In] Integrate[ArcTan[E^x]/E^x,x]
[Out] x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\arctan\left(\mathrm{e}^{x}\right) \mathrm{e}^{-x}+\ln\left(\mathrm{e}^{x}\right)-\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}$	23
default	$-\arctan\left(\mathrm{e}^{x}\right) \mathrm{e}^{-x}+\ln\left(\mathrm{e}^{x}\right)-\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}$	23
parallelrisch	$\frac{\left(-\ln\left(1+\mathrm{e}^{2 x}\right) \mathrm{e}^{x}+2 x\, \mathrm{e}^{x}-2 \arctan\left(\mathrm{e}^{x}\right)\right) \mathrm{e}^{-x}}{2}$	29
risch	$\frac{i \mathrm{e}^{-x} \ln\left(1+i \mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}+x-\frac{i \ln\left(1-i \mathrm{e}^{x}\right) \mathrm{e}^{-x}}{2}$	42

```
[In] int(arctan(exp(x))/exp(x),x,method=_RETURNVERBOSE)
[Out] -arctan(exp(x))/exp(x)+ln(exp(x))-1/2*ln(exp(x)^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int e^{-x} \arctan\left(e^x\right) dx = \frac{1}{2} \left(2\, x e^{x} - e^{x} \log\left(e^{(2\,x)} + 1\right) - 2\, \arctan\left(e^{x}\right)\right) e^{(-x)}$$

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="fricas")
[Out] 1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) - 2*arctan(e^x))*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int e^{-x}\arctan\left(e^{x}\right)\,dx = x - \frac{\log\left(e^{2x}+1\right)}{2} - e^{-x}\operatorname{atan}\left(e^{x}\right)$$

```
[In] integrate(atan(exp(x))/exp(x),x)
[Out] x - log(exp(2*x) + 1)/2 - exp(-x)*atan(exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int e^{-x}\arctan\left(e^{x}\right)\,dx = -\arctan\left(e^{x}\right)e^{(-x)} - \frac{1}{2}\log\left(e^{(-2\,x)}+1\right)$$

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="maxima")
[Out] -arctan(e^x)*e^(-x) - 1/2*log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int e^{-x}\arctan\left(e^{x}\right)\,dx = -\arctan\left(e^{x}\right)e^{(-x)} + x - \frac{1}{2}\log\left(e^{(2\,x)}+1\right)$$

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="giac")
[Out] -arctan(e^x)*e^(-x) + x - 1/2*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int e^{-x}\arctan\left(e^{x}\right)\,dx = x - \frac{\ln\left(\mathrm{e}^{2\,x}+1\right)}{2} - \mathrm{atan}\left(\mathrm{e}^{x}\right)\,\mathrm{e}^{-x}$$

```
[In] int(atan(exp(x))*exp(-x),x)
[Out] x - log(exp(2*x) + 1)/2 - atan(exp(x))*exp(-x)
```

3.120 $\int \frac{\arctan(x)}{(-1+x)^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 45

$$\int \frac{\arctan(x)}{(-1+x)^3} dx = \frac{1}{4(1-x)} - \frac{\arctan(x)}{2(1-x)^2} - \frac{1}{4}\log(1-x) + \frac{1}{8}\log\left(1+x^2\right)$$

```
[Out] 1/4/(1-x)-1/2*arctan(x)/(1-x)^2-1/4*ln(1-x)+1/8*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4972, 724, 815, 266}

$$\int \frac{\arctan(x)}{(-1+x)^3} dx = -\frac{\arctan(x)}{2(1-x)^2} + \frac{1}{8}\log\left(x^2+1\right) + \frac{1}{4(1-x)} - \frac{1}{4}\log(1-x)$$

```
[In] Int[ArcTan[x]/(-1 + x)^3,x]
```

```
[Out] 1/(4*(1 - x)) - ArcTan[x]/(2*(1 - x)^2) - Log[1 - x]/4 + Log[1 + x^2]/8
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan(x)}{2(1-x)^2}+\frac{1}{2}\int \frac{1}{(-1+x)^2\,(1+x^2)}\,dx\\
&= \frac{1}{4(1-x)}-\frac{\arctan(x)}{2(1-x)^2}+\frac{1}{4}\int \frac{-1-x}{(-1+x)\,(1+x^2)}\,dx\\
&= \frac{1}{4(1-x)}-\frac{\arctan(x)}{2(1-x)^2}+\frac{1}{4}\int \left(\frac{1}{1-x}+\frac{x}{1+x^2}\right)dx\\
&= \frac{1}{4(1-x)}-\frac{\arctan(x)}{2(1-x)^2}-\frac{1}{4}\log(1-x)+\frac{1}{4}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{4(1-x)}-\frac{\arctan(x)}{2(1-x)^2}-\frac{1}{4}\log(1-x)+\frac{1}{8}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(x)}{(-1+x)^3}\,dx=\frac{1}{8}\left(-\frac{2}{-1+x}-\frac{4\arctan(x)}{(-1+x)^2}-2\log(1-x)+\log\left(1+x^2\right)\right)$$

```
[In] Integrate[ArcTan[x]/(-1 + x)^3,x]
[Out] (-2/(-1 + x) - (4*ArcTan[x])/(-1 + x)^2 - 2*Log[1 - x] + Log[1 + x^2])/8
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result
default	$-\frac{\arctan(x)}{2(x-1)^2}+\frac{\ln(x^2+1)}{8}-\frac{1}{4(x-1)}-\frac{\ln(x-1)}{4}$
parts	$-\frac{\arctan(x)}{2(x-1)^2}+\frac{\ln(x^2+1)}{8}-\frac{1}{4(x-1)}-\frac{\ln(x-1)}{4}$
parallelrisch	$-\frac{2\ln(x-1)x^2-\ln(x^2+1)x^2-2-4\ln(x-1)x+2\ln(x^2+1)x+2\ln(x-1)-\ln(x^2+1)+2x+4\arctan(x)}{8(x-1)^2}$
risch	$\frac{i\ln(ix+1)}{4(x-1)^2}-\frac{i(-2i\ln(x-1)x^2+i\ln(x^2+1)x^2+4i\ln(x-1)x-2i\ln(x^2+1)x-2i\ln(x-1)+i\ln(x^2+1)-2ix+2i+2\ln(-ix+1))}{8(x-1)^2}$

```
[In] int(arctan(x)/(x-1)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/(x-1)^2*arctan(x)+1/8*ln(x^2+1)-1/4/(x-1)-1/4*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{\arctan(x)}{(-1+x)^3}\,dx = \frac{(x^2-2\,x+1)\log(x^2+1)-2\,(x^2-2\,x+1)\log(x-1)-2\,x-4\,\arctan(x)+2}{8\,(x^2-2\,x+1)}$$

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="fricas")
[Out] 1/8*((x^2 - 2*x + 1)*log(x^2 + 1) - 2*(x^2 - 2*x + 1)*log(x - 1) - 2*x - 4*
arctan(x) + 2)/(x^2 - 2*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(31) = 62.

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.40

$$\int \frac{\arctan(x)}{(-1+x)^3}\,dx = -\frac{2x^2\log(x-1)}{8x^2-16x+8}+\frac{x^2\log(x^2+1)}{8x^2-16x+8}+\frac{4x\log(x-1)}{8x^2-16x+8}-\frac{2x\log(x^2+1)}{8x^2-16x+8}-\frac{2x}{8x^2-16x+8}-\frac{2\log(x-1)}{8x^2-16x+8}+\frac{\log(x^2+1)}{8x^2-16x+8}-\frac{4\,\mathrm{atan}(x)}{8x^2-16x+8}+\frac{2}{8x^2-16x+8}$$

```
[In] integrate(atan(x)/(-1+x)**3,x)
```

```
[Out] -2*x**2*log(x - 1)/(8*x**2 - 16*x + 8) + x**2*log(x**2 + 1)/(8*x**2 - 16*x
+ 8) + 4*x*log(x - 1)/(8*x**2 - 16*x + 8) - 2*x*log(x**2 + 1)/(8*x**2 - 16*
x + 8) - 2*x/(8*x**2 - 16*x + 8) - 2*log(x - 1)/(8*x**2 - 16*x + 8) + log(x
**2 + 1)/(8*x**2 - 16*x + 8) - 4*atan(x)/(8*x**2 - 16*x + 8) + 2/(8*x**2 -
16*x + 8)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(x)}{(-1+x)^3} \, dx = -\frac{1}{4(x-1)} - \frac{\arctan(x)}{2(x-1)^2} + \frac{1}{8} \log(x^2+1) - \frac{1}{4} \log(x-1)$$

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(x)}{(-1+x)^3} \, dx = -\frac{1}{4(x-1)} - \frac{\arctan(x)}{2(x-1)^2} + \frac{1}{8} \log(x^2+1) - \frac{1}{4} \log(|x-1|)$$

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(abs(x -
 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(x)}{(-1+x)^3} \, dx = \frac{\ln(x^2+1)}{8} - \frac{\ln(x-1)}{4} - \frac{\frac{x}{4} + \frac{\mathrm{atan}(x)}{2} - \frac{1}{4}}{(x-1)^2}$$

```
[In] int(atan(x)/(x - 1)^3,x)
```

```
[Out] log(x^2 + 1)/8 - log(x - 1)/4 - (x/4 + atan(x)/2 - 1/4)/(x - 1)^2
```

3.121 $\int \frac{\arctan(1+2x)}{(4+3x)^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{\arctan(1+2x)}{(4+3x)^3} dx = -\frac{1}{34(4+3x)} + \frac{8}{867}\arctan(1+2x) - \frac{\arctan(1+2x)}{6(4+3x)^2} + \frac{5}{289}\log(4+3x) - \frac{5}{578}\log\left(1+2x+2x^2\right)$$

```
[Out] -1/34/(4+3*x)+8/867*arctan(1+2*x)-1/6*arctan(1+2*x)/(4+3*x)^2+5/289*ln(4+3*
x)-5/578*ln(2*x^2+2*x+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5153, 2007, 723, 814, 648, 631, 210, 642}

$$\int \frac{\arctan(1+2x)}{(4+3x)^3} dx = -\frac{\arctan(2x+1)}{6(3x+4)^2} + \frac{8}{867}\arctan(2x+1) - \frac{5}{578}\log\left(2x^2+2x+1\right) - \frac{1}{34(3x+4)} + \frac{5}{289}\log(3x+4)$$

```
[In] Int[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

```
[Out] -1/34*1/(4 + 3*x) + (8*ArcTan[1 + 2*x])/867 - ArcTan[1 + 2*x]/(6*(4 + 3*x)^
2) + (5*Log[4 + 3*x])/289 - (5*Log[1 + 2*x + 2*x^2])/578
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan(1+2x)}{6(4+3x)^2}+\frac{1}{3}\int\frac{1}{(4+3x)^2(1+(1+2x)^2)}\,dx\\
&= -\frac{\arctan(1+2x)}{6(4+3x)^2}+\frac{1}{3}\int\frac{1}{(4+3x)^2(2+4x+4x^2)}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\arctan(1+2x)}{6(4+3x)^2}+\frac{1}{102}\int\frac{4-12x}{(4+3x)(2+4x+4x^2)}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\arctan(1+2x)}{6(4+3x)^2}+\frac{1}{102}\int\left(\frac{90}{17(4+3x)}-\frac{2(7+30x)}{17(1+2x+2x^2)}\right)dx\\
&= -\frac{1}{34(4+3x)}-\frac{\arctan(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{1}{867}\int\frac{7+30x}{1+2x+2x^2}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\arctan(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)\\
&\quad -\frac{5}{578}\int\frac{2+4x}{1+2x+2x^2}\,dx+\frac{8}{867}\int\frac{1}{1+2x+2x^2}\,dx\\
&= -\frac{1}{34(4+3x)}-\frac{\arctan(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)\\
&\quad -\frac{5}{578}\log\left(1+2x+2x^2\right)-\frac{8}{867}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+2x\right)\\
&= -\frac{1}{34(4+3x)}+\frac{8}{867}\arctan(1+2x)-\frac{\arctan(1+2x)}{6(4+3x)^2}\\
&\quad +\frac{5}{289}\log(4+3x)-\frac{5}{578}\log\left(1+2x+2x^2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

$$\int \frac{\arctan(1+2x)}{(4+3x)^3} dx$$
$$= \frac{-289\arctan(1+2x)+(4+3x)(-51-(15-8i)(4+3x)\log(i+(1+i)x)-(15+8i)(4+3x)\log(1+(}{1734(4+3x)^2}$$

```
[In] Integrate[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

```
[Out] (-289*ArcTan[1 + 2*x] + (4 + 3*x)*(-51 - (15 - 8*I)*(4 + 3*x)*Log[I + (1 +
I)*x] - (15 + 8*I)*(4 + 3*x)*Log[1 + (1 + I)*x] + 120*Log[4 + 3*x] + 90*x*L
og[4 + 3*x]))/(1734*(4 + 3*x)^2)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{2\arctan(1+2x)}{3(8+6x)^2}-\frac{1}{17(8+6x)}+\frac{5\ln(8+6x)}{289}-\frac{5\ln\left((1+2x)^2+1\right)}{578}+\frac{8\arctan(1+2x)}{867}$
default	$-\frac{2\arctan(1+2x)}{3(8+6x)^2}-\frac{1}{17(8+6x)}+\frac{5\ln(8+6x)}{289}-\frac{5\ln\left((1+2x)^2+1\right)}{578}+\frac{8\arctan(1+2x)}{867}$
parts	$-\frac{1}{34(4+3x)}+\frac{8\arctan(1+2x)}{867}-\frac{\arctan(1+2x)}{6(4+3x)^2}+\frac{5\ln(4+3x)}{289}-\frac{5\ln(2x^2+2x+1)}{578}$
parallelrisch	$\frac{810\ln(\frac{4}{3}+x)x^2-405\ln(x^2+x+\frac{1}{2})x^2+432\arctan(1+2x)x^2-612+2160\ln(\frac{4}{3}+x)x-1080\ln(x^2+x+\frac{1}{2})x+1152\arctan(1}{5202(4+3x)^2}$
risch	$\frac{i\ln(1+i(1+2x))}{12(4+3x)^2}-\frac{i(-270i\ln(2x+1-i)x^2-720i\ln(2x+1-i)x-306ix+960i\ln(4+3x)+1440i\ln(4+3x)x-720i\ln(2x+}{}$

```
[In] int(arctan(1+2*x)/(4+3*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(8+6*x)^2*arctan(1+2*x)-1/17/(8+6*x)+5/289*ln(8+6*x)-5/578*ln((1+2*x)^
2+1)+8/867*arctan(1+2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.20

$$\int \frac{\arctan(1+2x)}{(4+3x)^3}\,dx = \frac{(48\,x^2+128\,x-11)\arctan\left(2\,x+1\right)-5\left(9\,x^2+24\,x+16\right)\log\left(2\,x^2+2\,x+1\right)+10\left(9\,x^2+24\,x+16\right)\log\left(\right.}{578\left(9\,x^2+24\,x+16\right)}$$

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="fricas")
```

```
[Out] 1/578*((48*x^2 + 128*x - 11)*arctan(2*x + 1) - 5*(9*x^2 + 24*x + 16)*log(2*
x^2 + 2*x + 1) + 10*(9*x^2 + 24*x + 16)*log(3*x + 4) - 51*x - 68)/(9*x^2 +
24*x + 16)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.48

$$\begin{aligned}\int \frac{\arctan(1+2x)}{(4+3x)^3}\,dx &= \frac{90x^2\log\left(3x+4\right)}{5202x^2+13872x+9248} - \frac{45x^2\log\left(2x^2+2x+1\right)}{5202x^2+13872x+9248} \\ &+ \frac{48x^2\operatorname{atan}\left(2x+1\right)}{5202x^2+13872x+9248} + \frac{240x\log\left(3x+4\right)}{5202x^2+13872x+9248} \\ &- \frac{120x\log\left(2x^2+2x+1\right)}{5202x^2+13872x+9248} + \frac{128x\operatorname{atan}\left(2x+1\right)}{5202x^2+13872x+9248} \\ &- \frac{51x}{5202x^2+13872x+9248} + \frac{160\log\left(3x+4\right)}{5202x^2+13872x+9248} \\ &- \frac{80\log\left(2x^2+2x+1\right)}{5202x^2+13872x+9248} - \frac{11\operatorname{atan}\left(2x+1\right)}{5202x^2+13872x+9248} \\ &- \frac{68}{5202x^2+13872x+9248}\end{aligned}$$

```
[In] integrate(atan(1+2*x)/(4+3*x)**3,x)
```

```
[Out] 90*x**2*log(3*x + 4)/(5202*x**2 + 13872*x + 9248) - 45*x**2*log(2*x**2 + 2*
x + 1)/(5202*x**2 + 13872*x + 9248) + 48*x**2*atan(2*x + 1)/(5202*x**2 + 13
872*x + 9248) + 240*x*log(3*x + 4)/(5202*x**2 + 13872*x + 9248) - 120*x*log
(2*x**2 + 2*x + 1)/(5202*x**2 + 13872*x + 9248) + 128*x*atan(2*x + 1)/(5202
*x**2 + 13872*x + 9248) - 51*x/(5202*x**2 + 13872*x + 9248) + 160*log(3*x +
 4)/(5202*x**2 + 13872*x + 9248) - 80*log(2*x**2 + 2*x + 1)/(5202*x**2 + 13
872*x + 9248) - 11*atan(2*x + 1)/(5202*x**2 + 13872*x + 9248) - 68/(5202*x*
*2 + 13872*x + 9248)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\arctan(1+2x)}{(4+3x)^3} dx = -\frac{1}{34(3x+4)} - \frac{\arctan(2x+1)}{6(3x+4)^2} + \frac{8}{867}\arctan(2x+1) - \frac{5}{578}\log(2x^2+2x+1) + \frac{5}{289}\log(3x+4)$$

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="maxima")
```

```
[Out] -1/34/(3*x + 4) - 1/6*arctan(2*x + 1)/(3*x + 4)^2 + 8/867*arctan(2*x + 1) -
 5/578*log(2*x^2 + 2*x + 1) + 5/289*log(3*x + 4)
```

Giac [F]

$$\int \frac{\arctan(1+2x)}{(4+3x)^3} dx = \int \frac{\arctan(2x+1)}{(3x+4)^3} dx$$

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(1+2x)}{(4+3x)^3} dx = \frac{5\ln\left(x+\frac{4}{3}\right)}{289} - \frac{5\ln\left(x^2+x+\frac{1}{2}\right)}{578} + \frac{8\,\mathrm{atan}(2x+1)}{867} - \frac{\frac{3x}{34}+\frac{\mathrm{atan}(2x+1)}{6}+\frac{2}{17}}{(3x+4)^2}$$

```
[In] int(atan(2*x + 1)/(3*x + 4)^3,x)
```

```
[Out] (5*log(x + 4/3))/289 - (5*log(x + x^2 + 1/2))/578 + (8*atan(2*x + 1))/867 -
 ((3*x)/34 + atan(2*x + 1)/6 + 2/17)/(3*x + 4)^2
```

3.122 $\int \arctan\left(\sqrt{1+x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \arctan\left(\sqrt{1+x}\right)\,dx=-\sqrt{1+x}+2\arctan\left(\sqrt{1+x}\right)+x\arctan\left(\sqrt{1+x}\right)$$

```
[Out] 2*arctan((1+x)^(1/2))+x*arctan((1+x)^(1/2))-(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5311, 81, 65, 209}

$$\int \arctan\left(\sqrt{1+x}\right)\,dx=x\arctan\left(\sqrt{x+1}\right)+2\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

```
[In] Int[ArcTan[Sqrt[1 + x]],x]
```

```
[Out] -Sqrt[1 + x] + 2*ArcTan[Sqrt[1 + x]] + x*ArcTan[Sqrt[1 + x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan\left(\sqrt{1+x}\right)-\int\frac{x}{\sqrt{1+x}(4+2x)}\,dx\\
&= -\sqrt{1+x}+x\arctan\left(\sqrt{1+x}\right)+2\int\frac{1}{\sqrt{1+x}(4+2x)}\,dx\\
&= -\sqrt{1+x}+x\arctan\left(\sqrt{1+x}\right)+4\text{Subst}\left(\int\frac{1}{2+2x^2}\,dx,x,\sqrt{1+x}\right)\\
&= -\sqrt{1+x}+2\arctan\left(\sqrt{1+x}\right)+x\arctan\left(\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int\arctan\left(\sqrt{1+x}\right)\,dx=-\sqrt{1+x}+(2+x)\arctan\left(\sqrt{1+x}\right)$$

```
[In] Integrate[ArcTan[Sqrt[1 + x]],x]
[Out] -Sqrt[1 + x] + (2 + x)*ArcTan[Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$(1+x)\arctan\left(\sqrt{1+x}\right)-\sqrt{1+x}+\arctan\left(\sqrt{1+x}\right)$	25
default	$(1+x)\arctan\left(\sqrt{1+x}\right)-\sqrt{1+x}+\arctan\left(\sqrt{1+x}\right)$	25
parts	$2\arctan\left(\sqrt{1+x}\right)+x\arctan\left(\sqrt{1+x}\right)-\sqrt{1+x}$	25

```
[In] int(arctan((1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] (1+x)*arctan((1+x)^(1/2))-(1+x)^(1/2)+arctan((1+x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \arctan\left(\sqrt{1+x}\right)\,dx=(x+2)\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="fricas")
[Out] (x + 2)*arctan(sqrt(x + 1)) - sqrt(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \arctan\left(\sqrt{1+x}\right)\,dx=x\,\mathrm{atan}\left(\sqrt{x+1}\right)-\sqrt{x+1}+2\,\mathrm{atan}\left(\sqrt{x+1}\right)$$

```
[In] integrate(atan((1+x)**(1/2)),x)
[Out] x*atan(sqrt(x + 1)) - sqrt(x + 1) + 2*atan(sqrt(x + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \arctan\left(\sqrt{1+x}\right)\,dx = (x+1)\arctan\left(\sqrt{x+1}\right) - \sqrt{x+1} + \arctan\left(\sqrt{x+1}\right)$$

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="maxima")
[Out] (x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \arctan\left(\sqrt{1+x}\right)\,dx = (x+1)\arctan\left(\sqrt{x+1}\right) - \sqrt{x+1} + \arctan\left(\sqrt{x+1}\right)$$

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="giac")
[Out] (x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \arctan\left(\sqrt{1+x}\right)\,dx = \mathrm{atan}\left(\sqrt{x+1}\right) - \sqrt{x+1} + \mathrm{atan}\left(\sqrt{x+1}\right)\,(x+1)$$

```
[In] int(atan((x + 1)^(1/2)),x)
[Out] atan((x + 1)^(1/2)) - (x + 1)^(1/2) + atan((x + 1)^(1/2))*(x + 1)
```

3.123 $\int \frac{1}{(1+x^2)(2+\arctan(x))} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 5

$$\int \frac{1}{(1+x^2)(2+\arctan(x))} \, dx = \log(2+\arctan(x))$$

```
[Out] ln(2+arctan(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5002}

$$\int \frac{1}{(1+x^2)(2+\arctan(x))} \, dx = \log(\arctan(x)+2)$$

```
[In] Int[1/((1 + x^2)*(2 + ArcTan[x])),x]
```

```
[Out] Log[2 + ArcTan[x]]
```

Rule 5002

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\text{integral} = \log(2+\arctan(x))$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)(2+\arctan(x))}\,dx = \log(2+\arctan(x))$$

```
[In] Integrate[1/((1 + x^2)*(2 + ArcTan[x])),x]
[Out] Log[2 + ArcTan[x]]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln(2+\arctan(x))$	6
default	$\ln(2+\arctan(x))$	6
parallelrisch	$\ln(2+\arctan(x))$	6
risch	$\ln(-\ln(-ix+1)+\ln(ix+1)+4i)$	21

```
[In] int(1/(x^2+1)/(2+arctan(x)),x,method=_RETURNVERBOSE)
[Out] ln(2+arctan(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)(2+\arctan(x))}\,dx = \log(\arctan(x)+2)$$

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="fricas")
[Out] log(arctan(x) + 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\,(2+\arctan(x))}\,dx = \log\left(\operatorname{atan}\left(x\right)+2\right)$$

```
[In] integrate(1/(x**2+1)/(2+atan(x)),x)
[Out] log(atan(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\,(2+\arctan(x))}\,dx = \log\left(\arctan\left(x\right)+2\right)$$

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="maxima")
[Out] log(arctan(x) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\,(2+\arctan(x))}\,dx = \log\left(\arctan\left(x\right)+2\right)$$

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="giac")
[Out] log(arctan(x) + 2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\,(2+\arctan(x))}\,dx = \ln\left(\operatorname{atan}(x)+2\right)$$

```
[In] int(1/((x^2 + 1)*(atan(x) + 2)),x)
[Out] log(atan(x) + 2)
```

3.124 $\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log(1-2\arctan(x))}{2ab}$$

```
[Out] -1/2*ln(1-2*arctan(x))/a/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5002}

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log(1-2\arctan(x))}{2ab}$$

```
[In] Int[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]
```

```
[Out] -1/2*Log[1 - 2*ArcTan[x]]/(a*b)
```

Rule 5002

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\text{integral} = -\frac{\log(1-2\arctan(x))}{2ab}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log(-1+2\arctan(x))}{2ab}$$

```
[In] Integrate[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]
[Out] -1/2*Log[-1 + 2*ArcTan[x]]/(a*b)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$-\frac{\ln\left(\arctan(x)-\frac{1}{2}\right)}{2ab}$	14
default	$-\frac{\ln(2b\arctan(x)-b)}{2ab}$	19
risch	$-\frac{\ln(-i-\ln(-ix+1)+\ln(ix+1))}{2ab}$	29

```
[In] int(1/(a*x^2+a)/(b-2*b*arctan(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(arctan(x)-1/2)/a/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log(2\arctan(x)-1)}{2\,ab}$$

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="fricas")
[Out] -1/2*log(2*arctan(x) - 1)/(a*b)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log\left(\operatorname{atan}(x)-\frac{1}{2}\right)}{2ab}$$

```
[In] integrate(1/(a*x**2+a)/(b-2*b*atan(x)),x)
[Out] -log(atan(x) - 1/2)/(2*a*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log\left(|2\arctan(x)-1|\right)}{2\,ab}$$

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="maxima")
[Out] -1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log\left(|2\arctan(x)-1|\right)}{2\,ab}$$

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="giac")
[Out] -1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\ln\left(2\operatorname{atan}(x)-1\right)}{2\,a\,b}$$

```
[In] int(1/((a + a*x^2)*(b - 2*b*atan(x))),x)
[Out] -log(2*atan(x) - 1)/(2*a*b)
```

3.125 $\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2(1+x^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 18

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2(1+x^2)}\,dx=\frac{1}{1+x}+\frac{\arctan(x)^2}{2}+\log(1+x)$$

```
[Out] 1/(1+x)+1/2*arctan(x)^2+ln(1+x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6857, 45, 5004}

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2(1+x^2)}\,dx=\frac{\arctan(x)^2}{2}+\frac{1}{x+1}+\log(x+1)$$

```
[In] Int[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] (1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x}{(1+x)^2}+\frac{\arctan(x)}{1+x^2}\right)dx\\
&= \int \frac{x}{(1+x)^2}\,dx+\int \frac{\arctan(x)}{1+x^2}\,dx\\
&= \frac{\arctan(x)^2}{2}+\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx\\
&= \frac{1}{1+x}+\frac{\arctan(x)^2}{2}+\log(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2\,(1+x^2)}\,dx=\frac{1}{1+x}+\frac{\arctan(x)^2}{2}+\log(1+x)$$

```
[In] Integrate[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]
[Out] (1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{1}{1+x}+\frac{\arctan(x)^2}{2}+\ln(1+x)$	17
parts	$\frac{1}{1+x}+\frac{\arctan(x)^2}{2}+\ln(1+x)$	17
parallelrisch	$\frac{\arctan(x)^2x+2\ln(1+x)x+2+\arctan(x)^2+2\ln(1+x)}{2+2x}$	33
risch	$-\frac{\ln(ix+1)^2}{8}+\frac{\ln(-ix+1)\ln(ix+1)}{4}+\frac{-\ln(-ix+1)^2x+8\ln(1+x)x-\ln(-ix+1)^2+8\ln(1+x)+8}{8+8x}$	74

```
[In] int((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/(1+x)+1/2*arctan(x)^2+ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2\left(1+x^2\right)}\,dx=\frac{(x+1)\arctan\left(x\right)^2+2\,(x+1)\log\left(x+1\right)+2}{2\,(x+1)}$$

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] 1/2*((x + 1)*arctan(x)^2 + 2*(x + 1)*log(x + 1) + 2)/(x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2\left(1+x^2\right)}\,dx=\frac{2x\log\left(x+1\right)}{2x+2}+\frac{x\operatorname{atan}^2\left(x\right)}{2x+2}+\frac{2\log\left(x+1\right)}{2x+2}+\frac{\operatorname{atan}^2\left(x\right)}{2x+2}+\frac{2}{2x+2}$$

```
[In] integrate((x+x**3+(1+x)**2*atan(x))/(1+x)**2/(x**2+1),x)
[Out] 2*x*log(x + 1)/(2*x + 2) + x*atan(x)**2/(2*x + 2) + 2*log(x + 1)/(2*x + 2)
+ atan(x)**2/(2*x + 2) + 2/(2*x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2\left(1+x^2\right)}\,dx=\frac{1}{2}\arctan\left(x\right)^2+\frac{1}{x+1}+\log\left(x+1\right)$$

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x)^2 + 1/(x + 1) + log(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 5.78

$$\int \frac{x + x^3 + (1+x)^2 \arctan(x)}{(1+x)^2 (1+x^2)} \, dx$$
$$= \frac{(x+1)\left(\frac{1}{x+1} - 1\right) \arctan\left((x+1)\left(\frac{1}{x+1} - 1\right)\right)^2 + 2\,(x+1)\left(\frac{1}{x+1} - 1\right) \log\left(-(x+1)\left(\frac{1}{x+1} - 1\right) + 1\right) - \arctan}{2\left((x+1)\left(\frac{1}{x+1} - 1\right) - 1\right)}$$

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*((x + 1)*(1/(x + 1) - 1)*arctan((x + 1)*(1/(x + 1) - 1))^2 + 2*(x + 1)*
(1/(x + 1) - 1)*log(-(x + 1)*(1/(x + 1) - 1) + 1) - arctan((x + 1)*(1/(x +
1) - 1))^2 - 2*log(-(x + 1)*(1/(x + 1) - 1) + 1) - 2)/((x + 1)*(1/(x + 1) -
 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x + x^3 + (1+x)^2 \arctan(x)}{(1+x)^2 (1+x^2)} \, dx = \ln(x+1) + \frac{1}{x+1} + \frac{\operatorname{atan}(x)^2}{2}$$

```
[In] int((x + atan(x)*(x + 1)^2 + x^3)/((x^2 + 1)*(x + 1)^2),x)
```

```
[Out] log(x + 1) + 1/(x + 1) + atan(x)^2/2
```

3.126 $\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}+\frac{\arctan\left(\sqrt{x}\right)}{8}-\frac{1}{8}x^4\arctan\left(\sqrt{x}\right)$$

```
[Out] 1/24*x^(3/2)-1/40*x^(5/2)+1/56*x^(7/2)+1/16*Pi*x^4+1/8*arctan(x^(1/2))-1/8*
x^4*arctan(x^(1/2))-1/8*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5267, 30, 4946, 52, 65, 209}

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = -\frac{1}{8}x^4\arctan\left(\sqrt{x}\right)+\frac{\arctan\left(\sqrt{x}\right)}{8}+\frac{x^{7/2}}{56}-\frac{x^{5/2}}{40}+\frac{x^{3/2}}{24}+\frac{\pi x^4}{16}-\frac{\sqrt{x}}{8}$$

```
[In] Int[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] -1/8*Sqrt[x] + x^(3/2)/24 - x^(5/2)/40 + x^(7/2)/56 + (Pi*x^4)/16 + ArcTan[
Sqrt[x]]/8 - (x^4*ArcTan[Sqrt[x]])/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int x^3 \arctan\left(\sqrt{x}\right)\, dx\right)+\frac{1}{4}\pi\int x^3\, dx\\
&= \frac{\pi x^4}{16}-\frac{1}{8}x^4\arctan\left(\sqrt{x}\right)+\frac{1}{16}\int\frac{x^{7/2}}{1+x}\, dx\\
&= \frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\arctan\left(\sqrt{x}\right)-\frac{1}{16}\int\frac{x^{5/2}}{1+x}\, dx
\end{aligned}
$$

$$= -\frac{x^{5/2}}{40} + \frac{x^{7/2}}{56} + \frac{\pi x^4}{16} - \frac{1}{8}x^4 \arctan\left(\sqrt{x}\right) + \frac{1}{16}\int \frac{x^{3/2}}{1+x}\,dx$$

$$= \frac{x^{3/2}}{24} - \frac{x^{5/2}}{40} + \frac{x^{7/2}}{56} + \frac{\pi x^4}{16} - \frac{1}{8}x^4 \arctan\left(\sqrt{x}\right) - \frac{1}{16}\int \frac{\sqrt{x}}{1+x}\,dx$$

$$= -\frac{\sqrt{x}}{8} + \frac{x^{3/2}}{24} - \frac{x^{5/2}}{40} + \frac{x^{7/2}}{56} + \frac{\pi x^4}{16} - \frac{1}{8}x^4 \arctan\left(\sqrt{x}\right) + \frac{1}{16}\int \frac{1}{\sqrt{x}(1+x)}\,dx$$

$$= -\frac{\sqrt{x}}{8} + \frac{x^{3/2}}{24} - \frac{x^{5/2}}{40} + \frac{x^{7/2}}{56} + \frac{\pi x^4}{16} - \frac{1}{8}x^4 \arctan\left(\sqrt{x}\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{8} + \frac{x^{3/2}}{24} - \frac{x^{5/2}}{40} + \frac{x^{7/2}}{56} + \frac{\pi x^4}{16} + \frac{\arctan\left(\sqrt{x}\right)}{8} - \frac{1}{8}x^4 \arctan\left(\sqrt{x}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int -x^3 \arctan\left(\sqrt{x} - \sqrt{1+x}\right)\,dx = \frac{\arctan\left(\sqrt{x}\right)}{8} - \frac{1}{840}\sqrt{x}\Big(105 - 35x + 21x^2 - 15x^3 + 210x^{7/2} \arctan\left(\sqrt{x} - \sqrt{1+x}\right)\Big)$$

```
[In] Integrate[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] ArcTan[Sqrt[x]]/8 - (Sqrt[x]*(105 - 35*x + 21*x^2 - 15*x^3 + 210*x^(7/2)*Ar
cTan[Sqrt[x] - Sqrt[1 + x]]))/840
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{x^4 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{4} + \frac{x^{\frac{7}{2}}}{56} - \frac{x^{\frac{5}{2}}}{40} + \frac{x^{\frac{3}{2}}}{24} - \frac{\sqrt{x}}{8} + \frac{\arctan\left(\sqrt{x}\right)}{8}$	45
parts	$-\frac{x^4 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{4} + \frac{x^{\frac{7}{2}}}{56} - \frac{x^{\frac{5}{2}}}{40} + \frac{x^{\frac{3}{2}}}{24} - \frac{\sqrt{x}}{8} + \frac{\arctan\left(\sqrt{x}\right)}{8}$	45

```
[In] int(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^4*arctan(x^(1/2)-(1+x)^(1/2))+1/56*x^(7/2)-1/40*x^(5/2)+1/24*x^(3/2)
-1/8*x^(1/2)+1/8*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{1}{4}\left(x^4-1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right) + \frac{1}{840}\left(15\,x^3-21\,x^2+35\,x-105\right)\sqrt{x}$$

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/4*(x^4 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/840*(15*x^3 - 21*x^2 + 35*x
 - 105)*sqrt(x)
```

Sympy [F(-1)]

Timed out.

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \text{Timed out}$$

```
[In] integrate(-x**3*atan(x**(1/2)-(1+x)**(1/2)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{1}{4}\,x^4\arctan\left(\sqrt{x+1}-\sqrt{x}\right) + \frac{1}{56}\,x^{\frac{7}{2}} - \frac{1}{40}\,x^{\frac{5}{2}} + \frac{1}{24}\,x^{\frac{3}{2}} - \frac{1}{8}\,\sqrt{x} + \frac{1}{8}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
[Out] 1/4*x^4*arctan(sqrt(x + 1) - sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/24*
x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = -\frac{1}{4}\,x^4\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{56}\,x^{\frac{7}{2}} - \frac{1}{40}\,x^{\frac{5}{2}}+\frac{1}{24}\,x^{\frac{3}{2}}-\frac{1}{8}\,\sqrt{x}+\frac{1}{8}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*x^4*arctan(-sqrt(x + 1) + sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/2
4*x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{x^{3/2}}{24}-\frac{\sqrt{x}}{8}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56} + \frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\,\left(\frac{x^5}{2}+\frac{x^4}{2}\right)}{2\,x+2}+\frac{\ln\left(\frac{{\left(-1+\sqrt{x}\,1{}\mathrm{i}\right)}^2}{x+1}\right)\,1{}\mathrm{i}}{16}$$

```
[In] int(x^3*atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/16 - x^(1/2)/8 + x^(3/2)/24 - x^(5/2)/
40 + x^(7/2)/56 + (atan((x + 1)^(1/2) - x^(1/2))*(x^4/2 + x^5/2))/(2*x + 2)
```

3.127 $\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right) dx = \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{\arctan\left(\sqrt{x}\right)}{6}-\frac{1}{6}x^3\arctan\left(\sqrt{x}\right)$$

```
[Out] -1/18*x^(3/2)+1/30*x^(5/2)+1/12*Pi*x^3-1/6*arctan(x^(1/2))-1/6*x^3*arctan(x
^(1/2))+1/6*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5267, 30, 4946, 52, 65, 209}

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right) dx = -\frac{1}{6}x^3\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{6}+\frac{x^{5/2}}{30}-\frac{x^{3/2}}{18}+\frac{\pi x^3}{12}+\frac{\sqrt{x}}{6}$$

```
[In] Int[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] Sqrt[x]/6 - x^(3/2)/18 + x^(5/2)/30 + (Pi*x^3)/12 - ArcTan[Sqrt[x]]/6 - (x^
3*ArcTan[Sqrt[x]])/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int x^2\arctan\left(\sqrt{x}\right)\,dx\right)+\frac{1}{4}\pi\int x^2\,dx\\
&= \frac{\pi x^3}{12}-\frac{1}{6}x^3\arctan\left(\sqrt{x}\right)+\frac{1}{12}\int\frac{x^{5/2}}{1+x}\,dx\\
&= \frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\arctan\left(\sqrt{x}\right)-\frac{1}{12}\int\frac{x^{3/2}}{1+x}\,dx
\end{aligned}
$$

$$= -\frac{x^{3/2}}{18} + \frac{x^{5/2}}{30} + \frac{\pi x^3}{12} - \frac{1}{6}x^3 \arctan\left(\sqrt{x}\right) + \frac{1}{12}\int \frac{\sqrt{x}}{1+x}\,dx$$

$$= \frac{\sqrt{x}}{6} - \frac{x^{3/2}}{18} + \frac{x^{5/2}}{30} + \frac{\pi x^3}{12} - \frac{1}{6}x^3 \arctan\left(\sqrt{x}\right) - \frac{1}{12}\int \frac{1}{\sqrt{x}(1+x)}\,dx$$

$$= \frac{\sqrt{x}}{6} - \frac{x^{3/2}}{18} + \frac{x^{5/2}}{30} + \frac{\pi x^3}{12} - \frac{1}{6}x^3 \arctan\left(\sqrt{x}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}}{6} - \frac{x^{3/2}}{18} + \frac{x^{5/2}}{30} + \frac{\pi x^3}{12} - \frac{\arctan\left(\sqrt{x}\right)}{6} - \frac{1}{6}x^3 \arctan\left(\sqrt{x}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int -x^2 \arctan\left(\sqrt{x} - \sqrt{1+x}\right)\,dx = \frac{1}{90}\left(-15\arctan\left(\sqrt{x}\right) - \sqrt{x}\left(-15 + 5x - 3x^2 + 30x^{5/2}\arctan\left(\sqrt{x} - \sqrt{1+x}\right)\right)\right)$$

```
[In] Integrate[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
[Out] (-15*ArcTan[Sqrt[x]] - Sqrt[x]*(-15 + 5*x - 3*x^2 + 30*x^(5/2)*ArcTan[Sqrt[
x] - Sqrt[1 + x]]))/90
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{3} + \frac{x^{\frac{5}{2}}}{30} - \frac{x^{\frac{3}{2}}}{18} + \frac{\sqrt{x}}{6} - \frac{\arctan\left(\sqrt{x}\right)}{6}$	40
parts	$-\frac{x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{3} + \frac{x^{\frac{5}{2}}}{30} - \frac{x^{\frac{3}{2}}}{18} + \frac{\sqrt{x}}{6} - \frac{\arctan\left(\sqrt{x}\right)}{6}$	40

```
[In] int(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/3*x^3*arctan(x^(1/2)-(1+x)^(1/2))+1/30*x^(5/2)-1/18*x^(3/2)+1/6*x^(1/2)-
1/6*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right) dx = \frac{1}{3}\left(x^3+1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right) + \frac{1}{90}\left(3x^2-5x+15\right)\sqrt{x}$$

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/3*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/90*(3*x^2 - 5*x + 15)*sqrt(
x)
```

Sympy [A] (verification not implemented)

Time = 175.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right) dx = \frac{x^{\frac{5}{2}}}{30} - \frac{x^{\frac{3}{2}}}{18} + \frac{\sqrt{x}}{6} - \frac{x^3 \operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{3} - \frac{\operatorname{atan}\left(\sqrt{x}\right)}{6}$$

```
[In] integrate(-x**2*atan(x**(1/2)-(1+x)**(1/2)),x)
[Out] x**(5/2)/30 - x**(3/2)/18 + sqrt(x)/6 - x**3*atan(sqrt(x) - sqrt(x + 1))/3
- atan(sqrt(x))/6
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right) dx = \frac{1}{3}x^3 \arctan\left(\sqrt{x+1}-\sqrt{x}\right) + \frac{1}{30}x^{\frac{5}{2}} - \frac{1}{18}x^{\frac{3}{2}} + \frac{1}{6}\sqrt{x} - \frac{1}{6}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
[Out] 1/3*x^3*arctan(sqrt(x + 1) - sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/6*s
qrt(x) - 1/6*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = -\frac{1}{3}\,x^3\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{30}\,x^{\frac{5}{2}} - \frac{1}{18}\,x^{\frac{3}{2}}+\frac{1}{6}\,\sqrt{x}-\frac{1}{6}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*x^3*arctan(-sqrt(x + 1) + sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/6
*sqrt(x) - 1/6*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30} + \frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\,\left(\frac{2\,x^4}{3}+\frac{2\,x^3}{3}\right)}{2\,x+2}+\frac{\ln\left(\frac{{\left(\sqrt{x}-\mathrm{i}\right)}^2}{x+1}\right)\,1\mathrm{i}}{12}$$

```
[In] int(x^2*atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (log((x^(1/2) - 1i)^2/(x + 1))*1i)/12 + x^(1/2)/6 - x^(3/2)/18 + x^(5/2)/30
 + (atan((x + 1)^(1/2) - x^(1/2))*((2*x^3)/3 + (2*x^4)/3))/(2*x + 2)
```

3.128 $\int -x \arctan \left(\sqrt{x}-\sqrt{1+x}\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int -x \arctan \left(\sqrt{x}-\sqrt{1+x}\right) \, dx=-\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}+\frac{\arctan \left(\sqrt{x}\right)}{4}-\frac{1}{4} x^2 \arctan \left(\sqrt{x}\right)$$

```
[Out] 1/12*x^(3/2)+1/8*Pi*x^2+1/4*arctan(x^(1/2))-1/4*x^2*arctan(x^(1/2))-1/4*x^(
1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5267, 30, 4946, 52, 65, 209}

$$\int -x \arctan \left(\sqrt{x}-\sqrt{1+x}\right) \, dx=-\frac{1}{4} x^2 \arctan \left(\sqrt{x}\right)+\frac{\arctan \left(\sqrt{x}\right)}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{\sqrt{x}}{4}$$

```
[In] Int[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] -1/4*Sqrt[x] + x^(3/2)/12 + (Pi*x^2)/8 + ArcTan[Sqrt[x]]/4 - (x^2*ArcTan[Sq
rt[x]])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int x\arctan\left(\sqrt{x}\right)\,dx\right)+\frac{1}{4}\pi\int x\,dx\\
&= \frac{\pi x^2}{8}-\frac{1}{4}x^2\arctan\left(\sqrt{x}\right)+\frac{1}{8}\int\frac{x^{3/2}}{1+x}\,dx\\
&= \frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2\arctan\left(\sqrt{x}\right)-\frac{1}{8}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2\arctan\left(\sqrt{x}\right)+\frac{1}{8}\int\frac{1}{\sqrt{x}(1+x)}\,dx
\end{aligned}
$$

$$= -\frac{\sqrt{x}}{4} + \frac{x^{3/2}}{12} + \frac{\pi x^2}{8} - \frac{1}{4}x^2 \arctan\left(\sqrt{x}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{4} + \frac{x^{3/2}}{12} + \frac{\pi x^2}{8} + \frac{\arctan\left(\sqrt{x}\right)}{4} - \frac{1}{4}x^2 \arctan\left(\sqrt{x}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int -x \arctan\left(\sqrt{x} - \sqrt{1+x}\right)\,dx = \frac{1}{12}\Big(3\arctan\left(\sqrt{x}\right) - \sqrt{x}\Big(3 - x + 6x^{3/2}\arctan\left(\sqrt{x} - \sqrt{1+x}\right)\Big)\Big)$$

```
[In] Integrate[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
[Out] (3*ArcTan[Sqrt[x]] - Sqrt[x]*(3 - x + 6*x^(3/2)*ArcTan[Sqrt[x] - Sqrt[1 + x
]]))/12
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{x^2\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{2}+\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}+\frac{\arctan\left(\sqrt{x}\right)}{4}$	35
parts	$-\frac{x^2\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{2}+\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}+\frac{\arctan\left(\sqrt{x}\right)}{4}$	35

```
[In] int(-x*arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2*arctan(x^(1/2)-(1+x)^(1/2))+1/12*x^(3/2)-1/4*x^(1/2)+1/4*arctan(x^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.56

$$\int -x \arctan\left(\sqrt{x} - \sqrt{1+x}\right)\,dx = \frac{1}{2}\left(x^2 - 1\right)\arctan\left(\sqrt{x+1} - \sqrt{x}\right) + \frac{1}{12}(x-3)\sqrt{x}$$

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*(x - 3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 34.99 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int -x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}-\frac{x^2\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{2}+\frac{\operatorname{atan}\left(\sqrt{x}\right)}{4}$$

```
[In] integrate(-x*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] x**(3/2)/12 - sqrt(x)/4 - x**2*atan(sqrt(x) - sqrt(x + 1))/2 + atan(sqrt(x)
)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int -x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{1}{2}\,x^2\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{12}\,x^{\frac{3}{2}}-\frac{1}{4}\,\sqrt{x}+\frac{1}{4}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4*ar
ctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int -x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=-\frac{1}{2}\,x^2\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{12}\,x^{\frac{3}{2}}-\frac{1}{4}\,\sqrt{x}+\frac{1}{4}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x^2*arctan(-sqrt(x + 1) + sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4*
arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int -x \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{x^{3/2}}{12}-\frac{\sqrt{x}}{4}+\frac{\operatorname{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\,\left(x^3+x^2\right)}{2\,x+2} + \frac{\ln\left(\frac{{\left(-1+\sqrt{x}\,1{}\mathrm{i}\right)}^2}{x+1}\right)\,1{}\mathrm{i}}{8}$$

```
[In] int(x*atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/8 - x^(1/2)/4 + x^(3/2)/12 + (atan((x
+ 1)^(1/2) - x^(1/2))*(x^2 + x^3))/(2*x + 2)
```

3.129 $\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2}x\arctan\left(\sqrt{x}\right)$$

```
[Out] 1/4*Pi*x-1/2*arctan(x^(1/2))-1/2*x*arctan(x^(1/2))+1/2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5267, 8, 4930, 52, 65, 209}

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=-\frac{1}{2}x\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\pi x}{4}+\frac{\sqrt{x}}{2}$$

```
[In] Int[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
```

```
[Out] Sqrt[x]/2 + (Pi*x)/4 - ArcTan[Sqrt[x]]/2 - (x*ArcTan[Sqrt[x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \arctan\left(\sqrt{x}\right)\,dx\right)+\frac{1}{4}\pi\int 1\,dx\\
&= \frac{\pi x}{4}-\frac{1}{2}x\arctan\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\arctan\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{1}{\sqrt{x}(1+x)}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\arctan\left(\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2}x\arctan\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{\sqrt{x}}{2}-(1+x)\arctan\left(\sqrt{x}-\sqrt{1+x}\right)$$

```
[In] Integrate[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
[Out] Sqrt[x]/2 - (1 + x)*ArcTan[Sqrt[x] - Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
default	$-x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)+\frac{\sqrt{x}}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2}$	28
parts	$-x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)+\frac{\sqrt{x}}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2}$	28

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -x*arctan(x^(1/2)-(1+x)^(1/2))+1/2*x^(1/2)-1/2*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{\sqrt{x}}{2} - x\,\mathrm{atan}\left(\sqrt{x}-\sqrt{x+1}\right) - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}$$

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2)),x)
[Out] sqrt(x)/2 - x*atan(sqrt(x) - sqrt(x + 1)) - atan(sqrt(x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = x\arctan\left(\sqrt{x+1}-\sqrt{x}\right) + \frac{1}{2}\sqrt{x} - \frac{1}{2}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = -x\arctan\left(-\sqrt{x+1}+\sqrt{x}\right) + \frac{1}{2}\sqrt{x} - \frac{1}{2}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
[Out] -x*arctan(-sqrt(x + 1) + sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = x\,\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right) + \frac{\sqrt{x}}{2} - \frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{4}$$

```
[In] int(atan((x + 1)^(1/2) - x^(1/2)),x)
[Out] x*atan((x + 1)^(1/2) - x^(1/2)) - (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/4 +
x^(1/2)/2
```

3.130 $\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = \frac{1}{4}\pi\log(x)-\frac{1}{2}i\,\text{PolyLog}\left(2,-i\sqrt{x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,i\sqrt{x}\right)$$

```
[Out] 1/4*Pi*ln(x)-1/2*I*polylog(2,-I*x^(1/2))+1/2*I*polylog(2,I*x^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5267, 29, 4944, 4940, 2438}

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = -\frac{1}{2}i\,\text{PolyLog}\left(2,-i\sqrt{x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,i\sqrt{x}\right)+\frac{1}{4}\pi\log(x)$$

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
```

```
[Out] (Pi*Log[x])/4 - (I/2)*PolyLog[2, (-I)*Sqrt[x]] + (I/2)*PolyLog[2, I*Sqrt[x]
]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x}\,dx\\
&= \frac{1}{4}\pi\log(x)-\text{Subst}\left(\int \frac{\arctan(x)}{x}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{4}\pi\log(x)-\frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,\sqrt{x}\right)+\frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{4}\pi\log(x)-\frac{1}{2}i\,\text{PolyLog}\left(2,-i\sqrt{x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,i\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.00

$$\begin{aligned}
\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx &= -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\log(x)\\
&\quad+\frac{1}{4}i\left(\left(\log\left(1-i\sqrt{x}\right)-\log\left(1+i\sqrt{x}\right)\right)\log(x)\right.\\
&\quad\left.-2\,\text{PolyLog}\left(2,-i\sqrt{x}\right)+2\,\text{PolyLog}\left(2,i\sqrt{x}\right)\right)
\end{aligned}$$

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
[Out] -(ArcTan[Sqrt[x] - Sqrt[1 + x]]*Log[x]) + (I/4)*((Log[1 - I*Sqrt[x]] - Log[
1 + I*Sqrt[x]])*Log[x] - 2*PolyLog[2, (-I)*Sqrt[x]] + 2*PolyLog[2, I*Sqrt[x
]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(28) = 56$.

Time = 0.76 (sec) , antiderivative size = 374, normalized size of antiderivative = 8.90

method	result
default	$-2\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\ln\left(1+\frac{\left(1+i\left(\sqrt{x}-\sqrt{1+x}\right)\right)^4}{\left(\left(\sqrt{x}-\sqrt{1+x}\right)^2+1\right)^2}\right)+\frac{i\,\mathrm{polylog}\left(2,-\frac{\left(1+i\left(\sqrt{x}-\sqrt{1+x}\right)\right)^4}{\left(\left(\sqrt{x}-\sqrt{1+x}\right)^2+1\right)^2}\right)}{2}+2\arctan\left(\sqrt{\right.$

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctan(x^(1/2)-(1+x)^(1/2))*ln(1+(1+I*(x^(1/2)-(1+x)^(1/2)))^4/((x^(1/2)
-(1+x)^(1/2))^2+1)^2)+1/2*I*polylog(2,-(1+I*(x^(1/2)-(1+x)^(1/2)))^4/((x^(1
/2)-(1+x)^(1/2))^2+1)^2)+2*arctan(x^(1/2)-(1+x)^(1/2))*ln(1-(1+I*(x^(1/2)-(
1+x)^(1/2)))/((x^(1/2)-(1+x)^(1/2))^2+1)^(1/2))-2*I*polylog(2,(1+I*(x^(1/2)
-(1+x)^(1/2)))/((x^(1/2)-(1+x)^(1/2))^2+1)^(1/2))+2*arctan(x^(1/2)-(1+x)^(1
/2))*ln(1+(1+I*(x^(1/2)-(1+x)^(1/2)))^2/((x^(1/2)-(1+x)^(1/2))^2+1))-I*poly
log(2,-(1+I*(x^(1/2)-(1+x)^(1/2)))^2/((x^(1/2)-(1+x)^(1/2))^2+1))+2*arctan(
x^(1/2)-(1+x)^(1/2))*ln(1+(1+I*(x^(1/2)-(1+x)^(1/2)))/((x^(1/2)-(1+x)^(1/2)
)^2+1)^(1/2))-2*I*polylog(2,-(1+I*(x^(1/2)-(1+x)^(1/2)))/((x^(1/2)-(1+x)^(1
/2))^2+1)^(1/2))
```

Fricas [F]

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx=\int -\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(sqrt(x + 1) - sqrt(x))/x, x)
```

Sympy [F]

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx=-\int \frac{\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{x}\,dx$$

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x,x)
```

```
[Out] -Integral(atan(sqrt(x) - sqrt(x + 1))/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = \frac{1}{4}\pi\log\left(x+1\right)+\arctan\left(\sqrt{x+1}-\sqrt{x}\right)\log\left(x\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(i\sqrt{x}+1\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(-i\sqrt{x}+1\right)$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] 1/4*pi*log(x + 1) + arctan(sqrt(x + 1) - sqrt(x))*log(x) + 1/2*I*dilog(I*sq
rt(x) + 1) - 1/2*I*dilog(-I*sqrt(x) + 1)
```

Giac [F]

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = \int -\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctan(-sqrt(x + 1) + sqrt(x))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = \int \frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{x}\,dx$$

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x,x)
```

```
[Out] int(atan((x + 1)^(1/2) - x^(1/2))/x, x)
```

3.131 $\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2} \, dx = -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2x}$$

```
[Out] -1/4*Pi/x+1/2*arctan(x^(1/2))+1/2*arctan(x^(1/2))/x+1/2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5267, 30, 4946, 53, 65, 209}

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2} \, dx = \frac{\arctan\left(\sqrt{x}\right)}{2x}+\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{1}{2\sqrt{x}}-\frac{\pi}{4x}$$

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
```

```
[Out] -1/4*Pi/x + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x]]/(2*x)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^2}\,dx\\
&= -\frac{\pi}{4x}+\frac{\arctan\left(\sqrt{x}\right)}{2x}-\frac{1}{4}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2x}+\frac{1}{4}\int \frac{1}{\sqrt{x}(1+x)}\,dx\\
&= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx=\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}$$

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
[Out] 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x] - Sqrt[1 + x]]/x
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

method	result	size
default	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{\operatorname{arctanh}\left(\sqrt{1+x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\ln\left(1+\sqrt{1+x}\right)}{4}+\frac{\ln\left(\sqrt{1+x}-1\right)}{4}$	57
parts	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{\operatorname{arctanh}\left(\sqrt{1+x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\ln\left(1+\sqrt{1+x}\right)}{4}+\frac{\ln\left(\sqrt{1+x}-1\right)}{4}$	57

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2)-(1+x)^(1/2))/x+1/2/x^(1/2)+1/2*arctanh((1+x)^(1/2))+1/2*arct
an(x^(1/2))-1/4*ln(1+(1+x)^(1/2))+1/4*ln((1+x)^(1/2)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx=-\frac{2\,(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-\sqrt{x}}{2\,x}$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="fricas")
[Out] -1/2*(2*(x + 1)*arctan(sqrt(x + 1) - sqrt(x)) - sqrt(x))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(31) = 62$.

Time = 88.10 (sec) , antiderivative size = 537, normalized size of antiderivative = 13.10

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = -\frac{2x^{\frac{5}{2}}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{x^{\frac{5}{2}}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} - \frac{4x^{\frac{3}{2}}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{x^{\frac{3}{2}}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} - \frac{2\sqrt{x}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{2x^3\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} - \frac{x^2\sqrt{x+1}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{4x^2\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} - \frac{x\sqrt{x+1}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{2x\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2}$$

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**2,x)
```

```
[Out] -2*x**(5/2)*sqrt(x + 1)*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1
) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) + x**(5/2)/(-2*x**(5/2)*sqrt(
x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) - 4*x**(3/2)*sqrt(x + 1)
*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x +
 1) + 2*x**3 + 2*x**2) + x**(3/2)/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqr
t(x + 1) + 2*x**3 + 2*x**2) - 2*sqrt(x)*sqrt(x + 1)*atan(sqrt(x) - sqrt(x +
 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) +
 2*x**3*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*s
qrt(x + 1) + 2*x**3 + 2*x**2) - x**2*sqrt(x + 1)/(-2*x**(5/2)*sqrt(x + 1) -
 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) + 4*x**2*atan(sqrt(x) - sqrt(x +
 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) -
 x*sqrt(x + 1)/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 +
 2*x**2) + 2*x*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**
(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = -\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="maxima")
[Out] -arctan(sqrt(x + 1) - sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = \frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="giac")
[Out] arctan(-sqrt(x + 1) + sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = -\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)-\frac{\sqrt{x}}{2}}{x}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{4}$$

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x^2,x)
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/4 - (atan((x + 1)^(1/2) - x^(1/2)) - x
^(1/2)/2)/x
```

3.132 $\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{4}+\frac{\arctan\left(\sqrt{x}\right)}{4x^2}$$

```
[Out] -1/8*Pi/x^2+1/12/x^(3/2)-1/4*arctan(x^(1/2))+1/4*arctan(x^(1/2))/x^2-1/4/x^
(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5267, 30, 4946, 53, 65, 209}

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = \frac{\arctan\left(\sqrt{x}\right)}{4x^2}-\frac{\arctan\left(\sqrt{x}\right)}{4}+\frac{1}{12x^{3/2}}-\frac{\pi}{8x^2}-\frac{1}{4\sqrt{x}}$$

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
```

```
[Out] -1/8*Pi/x^2 + 1/(12*x^(3/2)) - 1/(4*Sqrt[x]) - ArcTan[Sqrt[x]]/4 + ArcTan[S
qrt[x]]/(4*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^3}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{\arctan\left(\sqrt{x}\right)}{4x^2}-\frac{1}{8}\int \frac{1}{x^{5/2}(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}+\frac{\arctan\left(\sqrt{x}\right)}{4x^2}+\frac{1}{8}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{4x^2}-\frac{1}{8}\int \frac{1}{\sqrt{x}(1+x)}\,dx
\end{aligned}$$

$$= -\frac{\pi}{8x^2} + \frac{1}{12x^{3/2}} - \frac{1}{4\sqrt{x}} + \frac{\arctan\left(\sqrt{x}\right)}{4x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\pi}{8x^2} + \frac{1}{12x^{3/2}} - \frac{1}{4\sqrt{x}} - \frac{\arctan\left(\sqrt{x}\right)}{4} + \frac{\arctan\left(\sqrt{x}\right)}{4x^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx$$
$$= -\frac{\sqrt{x}(-1+3x) + 3x^2\arctan\left(\sqrt{x}\right) - 6\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{12x^2}$$

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
[Out] -1/12*(Sqrt[x]*(-1 + 3*x) + 3*x^2*ArcTan[Sqrt[x]] - 6*ArcTan[Sqrt[x] - Sqrt
[1 + x]])/x^2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{2x^2}+\frac{1}{12x^{\frac{3}{2}}}-\frac{1}{4\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{4}$	35
parts	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{2x^2}+\frac{1}{12x^{\frac{3}{2}}}-\frac{1}{4\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{4}$	35

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x^(1/2)-(1+x)^(1/2))/x^2+1/12/x^(3/2)-1/4/x^(1/2)-1/4*arctan(x^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = \frac{6\,(x^2-1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-(3\,x-1)\sqrt{x}}{12\,x^2}$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/12*(6*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) - (3*x - 1)*sqrt(x))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = -\frac{1}{4\,\sqrt{x}}-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{2\,x^2}+\frac{1}{12\,x^{\frac{3}{2}}}-\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="maxima")
[Out] -1/4/sqrt(x) - 1/2*arctan(sqrt(x + 1) - sqrt(x))/x^2 + 1/12/x^(3/2) - 1/4*a
rctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = -\frac{3\,x-1}{12\,x^{\frac{3}{2}}}+\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{2\,x^2}-\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="giac")
[Out] -1/12*(3*x - 1)/x^(3/2) + 1/2*arctan(-sqrt(x + 1) + sqrt(x))/x^2 - 1/4*arct
an(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int -\frac{\arctan \left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = -\frac{\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{12}+\frac{x^{3/2}}{4}}{x^2}+\frac{\ln\left(\frac{\left(\sqrt{x}-\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{8}$$

[In] int(atan((x + 1)^(1/2) - x^(1/2))/x^3,x)

[Out] (log((x^(1/2) - 1i)^2/(x + 1))*1i)/8 - (atan((x + 1)^(1/2) - x^(1/2))/2 - x^(1/2)/12 + x^(3/2)/4)/x^2

3.133 $\int -\frac{\arctan(\sqrt{x}-\sqrt{1+x})}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{6}+\frac{\arctan\left(\sqrt{x}\right)}{6x^3}$$

```
[Out] -1/12*Pi/x^3+1/30/x^(5/2)-1/18/x^(3/2)+1/6*arctan(x^(1/2))+1/6*arctan(x^(1/
2))/x^3+1/6/x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5267, 30, 4946, 53, 65, 209}

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = \frac{\arctan\left(\sqrt{x}\right)}{6x^3}+\frac{\arctan\left(\sqrt{x}\right)}{6}-\frac{1}{18x^{3/2}}+\frac{1}{30x^{5/2}}-\frac{\pi}{12x^3}+\frac{1}{6\sqrt{x}}$$

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
```

```
[Out] -1/12*Pi/x^3 + 1/(30*x^(5/2)) - 1/(18*x^(3/2)) + 1/(6*Sqrt[x]) + ArcTan[Sqr
t[x]]/6 + ArcTan[Sqrt[x]]/(6*x^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\int\frac{\arctan\left(\sqrt{x}\right)}{x^4}\,dx\right)+\frac{1}{4}\pi\int\frac{1}{x^4}\,dx$$

$$= -\frac{\pi}{12x^3}+\frac{\arctan\left(\sqrt{x}\right)}{6x^3}-\frac{1}{12}\int\frac{1}{x^{7/2}(1+x)}\,dx$$

$$= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}+\frac{\arctan\left(\sqrt{x}\right)}{6x^3}+\frac{1}{12}\int\frac{1}{x^{5/2}(1+x)}\,dx$$

$$= -\frac{\pi}{12x^3} + \frac{1}{30x^{5/2}} - \frac{1}{18x^{3/2}} + \frac{\arctan\left(\sqrt{x}\right)}{6x^3} - \frac{1}{12}\int \frac{1}{x^{3/2}(1+x)}\,dx$$

$$= -\frac{\pi}{12x^3} + \frac{1}{30x^{5/2}} - \frac{1}{18x^{3/2}} + \frac{1}{6\sqrt{x}} + \frac{\arctan\left(\sqrt{x}\right)}{6x^3} + \frac{1}{12}\int \frac{1}{\sqrt{x}(1+x)}\,dx$$

$$= -\frac{\pi}{12x^3} + \frac{1}{30x^{5/2}} - \frac{1}{18x^{3/2}} + \frac{1}{6\sqrt{x}} + \frac{\arctan\left(\sqrt{x}\right)}{6x^3} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\pi}{12x^3} + \frac{1}{30x^{5/2}} - \frac{1}{18x^{3/2}} + \frac{1}{6\sqrt{x}} + \frac{\arctan\left(\sqrt{x}\right)}{6} + \frac{\arctan\left(\sqrt{x}\right)}{6x^3}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = \frac{1}{90}\left(-\frac{-3+5x-15x^2}{x^{5/2}} + 15\arctan\left(\sqrt{x}\right) + \frac{30\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\right)$$

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
[Out] (-((-3 + 5*x - 15*x^2)/x^(5/2)) + 15*ArcTan[Sqrt[x]] + (30*ArcTan[Sqrt[x] -
 Sqrt[1 + x]])/x^3)/90
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{3x^3}+\frac{1}{30x^{\frac{5}{2}}}-\frac{1}{18x^{\frac{3}{2}}}+\frac{1}{6\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{6}$	40
parts	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{3x^3}+\frac{1}{30x^{\frac{5}{2}}}-\frac{1}{18x^{\frac{3}{2}}}+\frac{1}{6\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{6}$	40

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x,method=_RETURNVERBOSE)
[Out] 1/3*arctan(x^(1/2)-(1+x)^(1/2))/x^3+1/30/x^(5/2)-1/18/x^(3/2)+1/6/x^(1/2)+1
/6*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = -\frac{30\,(x^3+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-(15\,x^2-5\,x+3)\sqrt{x}}{90\,x^3}$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="fricas")
[Out] -1/90*(30*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) - (15*x^2 - 5*x + 3)*sqrt
(x))/x^3
```

Sympy [F(-1)]

Timed out.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = \frac{1}{6\sqrt{x}}-\frac{1}{18\,x^{\frac{3}{2}}}-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{3\,x^3}+\frac{1}{30\,x^{\frac{5}{2}}}+\frac{1}{6}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="maxima")
[Out] 1/6/sqrt(x) - 1/18/x^(3/2) - 1/3*arctan(sqrt(x + 1) - sqrt(x))/x^3 + 1/30/x
^(5/2) + 1/6*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = \frac{15\,x^2-5\,x+3}{90\,x^{\frac{5}{2}}}+\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{3\,x^3}+\frac{1}{6}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/90*(15*x^2 - 5*x + 3)/x^(5/2) + 1/3*arctan(-sqrt(x + 1) + sqrt(x))/x^3 +
1/6*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = -\frac{\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{3}-\frac{\sqrt{x}}{30}+\frac{x^{3/2}}{18}-\frac{x^{5/2}}{6}}{x^3}+\frac{\ln\left(\frac{{\left(-1+\sqrt{x}\,1\mathrm{i}\right)}^2}{x+1}\right)\,1\mathrm{i}}{12}$$

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x^4,x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/12 - (atan((x + 1)^(1/2) - x^(1/2))/3
- x^(1/2)/30 + x^(3/2)/18 - x^(5/2)/6)/x^3
```

3.134 $\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 63

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] arctan(c*x/(-c^2*x^2+a)^(1/2))^(1+m)*(-c^2*x^2+a)^(1/2)/c/(1+m)/(d-c^2*d*x^
2/a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5265, 5263}

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{m+1}}{c(m+1)\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqrt
[d - (c^2*d*x^2)/a])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
```

```
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a-c^2x^2}\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$= \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqrt
[d - (c^2*d*x^2)/a])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^{1+m}\sqrt{-c^2x^2+a}}{c(1+m)\sqrt{\frac{d\left(-c^2x^2+a\right)}{a}}}$	59

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] arctan(c*x/(-c^2*x^2+a)^(1/2))^(1+m)/c/(1+m)*(-c^2*x^2+a)^(1/2)/(d*(-c^2*x^
2+a)/a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(57) = 114$.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = -\frac{\sqrt{-c^2x^2+a}\,a\left(-\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)\right)^m\sqrt{-\frac{c^2dx^2-ad}{a}}\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)}{acdm+acd-(c^3dm+c^3d)x^2}$$

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*(-arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))^m*sqr
t(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))/(a*c*d
*m + a*c*d - (c^3*d*m + c^3*d)*x^2)
```

Sympy [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\operatorname{atan}^m\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**m/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))**m/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^m}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^m/sqrt(-c^2*d*x^2/a + d), x)
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^{m+1}\sqrt{a-c^2\,x^2}}{c\,(m+1)\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

```
[In] int(atan((c*x)/(a - c^2*x^2)^(1/2))^m/(d - (c^2*d*x^2)/a)^(1/2),x)
```

```
[Out] (atan((c*x)/(a - c^2*x^2)^(1/2))^(m + 1)*(a - c^2*x^2)^(1/2))/(c*(m + 1)*(d
 - (c^2*d*x^2)/a)^(1/2))
```

3.135 $\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}} dx$

Optimal result

Integrand size = 39, antiderivative size = 59

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}} dx = \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] 1/3*arctan(c*x/(-c^2*x^2+a)^(1/2))^3*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(
1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5265, 5263}

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}} dx = \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
```

```
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a-c^2x^2}\int\frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$= \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int\frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{\frac{d\left(-c^2x^2+a\right)}{a}}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^3 a}{3\sqrt{-c^2x^2+a}\,dc}$	57

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/3/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*arctan(c*x/(-c^2*x^2+a)
^(1/2))^3*a
```

Fricas [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x
^2 - a))^2/(c^2*d*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\operatorname{atan}^2\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))**2/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^2/sqrt(-c^2*d*x^2/a + d), x)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^2/sqrt(-c^2*d*x^2/a + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^2}{\sqrt{d-\frac{c^2\,d\,x^2}{a}}}\,dx$$

```
[In] int(atan((c*x)/(a - c^2*x^2)^(1/2))^2/(d - (c^2*d*x^2)/a)^(1/2),x)
```

```
[Out] int(atan((c*x)/(a - c^2*x^2)^(1/2))^2/(d - (c^2*d*x^2)/a)^(1/2), x)
```

3.136 $\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 59

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] 1/2*arctan(c*x/(-c^2*x^2+a)^(1/2))^2*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(
1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {5265, 5263}

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
```

```
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a-c^2x^2}\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{\frac{d\left(-c^2x^2+a\right)}{a}}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2 a}{2\sqrt{-c^2x^2+a}\,dc}$	57

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*arctan(c*x/(-c^2*x^2+a)
^(1/2))^2*a
```

Fricas [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^
2 - a))/(c^2*d*x^2 - a*d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \text{Timed out}$$

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))/sqrt(-c^2*d*x^2/a + d), x)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="giac")
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))/sqrt(-c^2*d*x^2/a + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)}{\sqrt{d-\frac{c^2\,d\,x^2}{a}}}\,dx$$

```
[In] int(atan((c*x)/(a - c^2*x^2)^(1/2))/(d - (c^2*d*x^2)/a)^(1/2),x)
[Out] int(atan((c*x)/(a - c^2*x^2)^(1/2))/(d - (c^2*d*x^2)/a)^(1/2), x)
```

3.137 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 55

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\frac{\sqrt{a-c^2x^2}\log\left(\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] ln(arctan(c*x/(-c^2*x^2+a)^(1/2)))*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(1/
2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5265, 5261}

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\frac{\sqrt{a-c^2x^2}\log\left(\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
```

```
[Out] (Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*d
*x^2)/a])
```

Rule 5261

```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1/c)*Log[ArcTan[c*(x/Sqrt[a + b*x^2])]], x] /;
```

```
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}\\ &= \frac{\sqrt{a-c^2x^2}\log\left(\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\frac{\sqrt{a-c^2x^2}\log\left(\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
[Out] (Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*d
*x^2)/a])
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{\frac{d\left(-c^2x^2+a\right)}{a}}\,\ln\left(\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)\right)a}{\sqrt{-c^2x^2+a}\,dc}$	55

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*ln(arctan(c*x/(-c^2*x^2+a
)^(1/2)))*a
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx = -\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}\log\left(2\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)\right)}{c^3dx^2-acd}$$

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)*log(2*arctan(sqrt(-c^2*x^2
 + a)*c*x/(c^2*x^2 - a)))/(c^3*d*x^2 - a*c*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx = \int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\,\mathrm{atan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx = \int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}\,dx$$

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}\,dx$$

[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\frac{\ln\left(\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)\right)\,\sqrt{a-c^2\,x^2}}{c\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

[In] int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))*(d - (c^2*d*x^2)/a)^(1/2)),x)

[Out] (log(atan((c*x)/(a - c^2*x^2)^(1/2)))*(a - c^2*x^2)^(1/2))/(c*(d - (c^2*d*x^2)/a)^(1/2))

3.138 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 57

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

```
[Out] -(-c^2*x^2+a)^(1/2)/c/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5265, 5263}

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=-\frac{\sqrt{a-c^2x^2}}{c\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^
2]]))
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$= -\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx = -\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]
[Out] -(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^
2]]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\sqrt{\frac{d\left(-c^2x^2+a\right)}{a}}\,a}{\sqrt{-c^2x^2+a}\,dc\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}$	57

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x,method=_RETU
RNVERBOSE)
[Out] -1/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*a/arctan(c*x/(-c^2*x^2+a
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=-\frac{\sqrt{-c^2x^2+a}a\sqrt{-\frac{c^2dx^2-ad}{a}}}{(c^3dx^2-acd)\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)}$$

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*arcta
n(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}^2\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**2),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=-\frac{\sqrt{a}}{c\sqrt{d}\arctan\left(cx,\sqrt{-c^2x^2+a}\right)}$$

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a)))
```

Giac [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}\,dx$$

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=-\frac{\sqrt{a-c^2\,x^2}}{c\,\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

```
[In] int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))^2*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

```
[Out] -(a - c^2*x^2)^(1/2)/(c*atan((c*x)/(a - c^2*x^2)^(1/2))*(d - (c^2*d*x^2)/a)
^(1/2))
```

3.139 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 59

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = -\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

```
[Out] -1/2*(-c^2*x^2+a)^(1/2)/c/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^
(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5265, 5263}

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = -\frac{\sqrt{a-c^2x^2}}{2c\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
```

```
[Out] -1/2*Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2
*x^2]]^2)
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$= -\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = -\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
```

```
[Out] -1/2*Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2
*x^2]]^2)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{\frac{d\left(-c^2x^2+a\right)}{a}}\,a}{2\sqrt{-c^2x^2+a}\,dc\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}$	57

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*a/arctan(c*x/(-c^2*x^2
+a)^(1/2))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = \frac{\sqrt{-c^2x^2+a}a\sqrt{-\frac{c^2dx^2-ad}{a}}}{2\,(c^3dx^2-acd)\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)^2}$$

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*ar
ctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = \int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}^3\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**3/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**3),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = -\frac{\sqrt{a}}{2\,c\sqrt{d}\arctan\left(cx,\sqrt{-c^2x^2+a}\right)^2}$$

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a))^2)
```

Giac [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = \int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^3}\,dx$$

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = -\frac{\sqrt{a-c^2\,x^2}}{2\,c\,\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^2\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

```
[In] int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))^3*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

```
[Out] -(a - c^2*x^2)^(1/2)/(2*c*atan((c*x)/(a - c^2*x^2)^(1/2))^2*(d - (c^2*d*x^2
)/a)^(1/2))
```

3.140 $\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 72

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

```
[Out] arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^(1+m)*(-a*e^2/b-e^2*x^2)^(1/2)/e/(1+m)
/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5265, 5263}

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^{m+1}}{e(m+1)\sqrt{a+bx^2}}$$

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^(1
 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2\left(a+bx^2\right)}{b}}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^(1
 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

Maple [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^m}{\sqrt{x^2b+a}}dx$$

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.83

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bx^2+a}\left(-\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)\right)^m\sqrt{-\frac{be^2x^2+ae^2}{b}}\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}{aem+(bem+be)x^2+ae}$$

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)*(-arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))
)^m*sqrt(-(b*e^2*x^2 + a*e^2)/b)*arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b
*e*x^2 + a*e))/(a*e*m + (b*e*m + b*e)*x^2 + a*e)
```

Sympy [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{\operatorname{atan}^m\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**m/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**m/sqrt(a + b*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagin
ary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx=\int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^m}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="giac")
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^m/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx=\int \frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^m}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^m/(a + b*x^2)^(1/2),x)
[Out] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^m/(a + b*x^2)^(1/2), x)
```

3.141 $\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 68

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}{3e\sqrt{a+bx^2}}$$

```
[Out] 1/3*arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^
2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5265, 5263}

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3)
/(3*e*Sqrt[a + b*x^2])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int\frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$
$$=\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}{3e\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int\frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2\left(a+bx^2\right)}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^3)
/(3*e*Sqrt[a + b*x^2])
```

Maple [F]

$$\int\frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^2}{\sqrt{x^2b+a}}dx$$

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))^2/sqrt(b*
x^2 + a), x)
```

Sympy [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{\operatorname{atan}^2\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2/sqrt(a + b*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagin
ary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^2}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2/(a + b*x^2)^(1/2),x)
```

```
[Out] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2/(a + b*x^2)^(1/2), x)
```

3.142 $\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 68

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{2e\sqrt{a+bx^2}}$$

```
[Out] 1/2*arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^
2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5265, 5263}

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2)
/(2*e*Sqrt[a + b*x^2])
```

<u>Rule 5263</u>

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int\frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{2e\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int\frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2\left(a+bx^2\right)}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^2)
/(2*e*Sqrt[a + b*x^2])
```

Maple [F]

$$\int\frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)}{\sqrt{x^2b+a}}dx$$

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))/sqrt(b*x
^2 + a), x)
```

Sympy [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \int \frac{\operatorname{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))/sqrt(a + b*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagin
ary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

Giac [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="giac")
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))/(a + b*x^2)^(1/2),x)
[Out] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))/(a + b*x^2)^(1/2), x)
```

3.143 $\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}dx$

Optimal result

Integrand size = 40, antiderivative size = 64

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\log\left(\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)}{e\sqrt{a+bx^2}}$$

```
[Out] ln(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2)))*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^2+
a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5265, 5261}

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\log\left(\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*Log[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]
]])/(e*Sqrt[a + b*x^2])
```

Rule 5261

```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1/c)*Log[ArcTan[c*(x/Sqrt[a + b*x^2])]], x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\log\left(\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)}{e\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\log\left(\arctan\left(\frac{ex}{\sqrt{-\frac{e^2\left(a+bx^2\right)}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x]
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*Log[ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]
]])/(e*Sqrt[a + b*x^2])
```

Maple [F]

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)\sqrt{x^2b+a}}dx$$

[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)

[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}\log\left(2\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)\right)}{bex^2+ae}$$

[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="fricas")

[Out] sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)*log(2*arctan(b*x*sqrt(-(b*e^2*
x^2 + a*e^2)/b)/(b*e*x^2 + a*e)))/(b*e*x^2 + a*e)

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \int \frac{1}{\sqrt{a+bx^2}\operatorname{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)

[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagin
ary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}\,dx$$

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)\,\sqrt{b\,x^2+a}}\,dx$$

```
[In] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))*(a + b*x^2)^(1/2)), x)
```

3.144 $\int \frac{1}{\sqrt{a+bx^2} \arctan \left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2} \, dx$

Optimal result

Integrand size = 40, antiderivative size = 66

$$\int \frac{1}{\sqrt{a+bx^2} \arctan \left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2} \, dx = -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{e\sqrt{a+bx^2} \arctan \left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}$$

```
[Out] -(-a*e^2/b-e^2*x^2)^(1/2)/e/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^
(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5265, 5263}

$$\int \frac{1}{\sqrt{a+bx^2} \arctan \left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2} \, dx = -\frac{\sqrt{e^2 (-x^2)-\frac{ae^2}{b}}}{e\sqrt{a+bx^2} \arctan \left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[-((a*e^2)/b) - e^2*x^2]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^
2)/b) - e^2*x^2]]))
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}dx = \frac{e\sqrt{a+bx^2}}{b\sqrt{-\frac{e^2(a+bx^2)}{b}}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)}$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2),
x]
[Out] (e*Sqrt[a + b*x^2])/(b*Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^
2*(a + b*x^2))/b)]])
```

Maple [F]

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^2\sqrt{x^2b+a}}dx$$

[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)

[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}}{(bex^2+ae)\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}$$

[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)/((b*e*x^2 + a*e)*arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e)))

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,\operatorname{atan}^2\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)

[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagin
ary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}\,dx$$

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^2\,\sqrt{b\,x^2+a}}\,dx$$

```
[In] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2*(a + b*x^2)^(1/2)), x)
```

3.145 $\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 68

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{2e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}$$

```
[Out] -1/2*(-a*e^2/b-e^2*x^2)^(1/2)/e/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x
^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5265, 5263}

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = -\frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}{2e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3),x]
```

```
[Out] -1/2*Sqrt[-((a*e^2)/b) - e^2*x^2]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a
*e^2)/b) - e^2*x^2]]^2)
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{2e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = -\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}}{2e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^2}$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3),
x]
```

```
[Out] -1/2*Sqrt[-((e^2*(a + b*x^2))/b)]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((e
^2*(a + b*x^2))/b)]]^2)
```

Maple [F]

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^3\sqrt{x^2b+a}}dx$$

[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)

[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx=-\frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}}{2\left(bex^2+ae\right)\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)^2}$$

[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] -1/2*sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)/((b*e*x^2 + a*e)*arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))^2)

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx=\int \frac{1}{\sqrt{a+bx^2}\operatorname{atan}^3\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**3/(b*x**2+a)**(1/2),x)

[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**3), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagin
ary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^3}\,dx$$

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^3\,\sqrt{b\,x^2+a}}\,dx$$

```
[In] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^3*(a + b*x^2)^(1/2)), x)
```

3.146 $\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \frac{i\log(d(a+bx))\operatorname{PolyLog}(2,-ic(a+bx))}{2b} - \frac{i\log(d(a+bx))\operatorname{PolyLog}(2,ic(a+bx))}{2b} - \frac{i\operatorname{PolyLog}(3,-ic(a+bx))}{2b} + \frac{i\operatorname{PolyLog}(3,ic(a+bx))}{2b}$$

```
[Out] 1/2*I*ln(d*(b*x+a))*polylog(2,-I*c*(b*x+a))/b-1/2*I*ln(d*(b*x+a))*polylog(2
,I*c*(b*x+a))/b-1/2*I*polylog(3,-I*c*(b*x+a))/b+1/2*I*polylog(3,I*c*(b*x+a)
)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4940, 2438, 5317, 2494, 2481, 2421, 6724}

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \frac{i\operatorname{PolyLog}(2,-ic(a+bx))\log(d(a+bx))}{2b} - \frac{i\operatorname{PolyLog}(2,ic(a+bx))\log(d(a+bx))}{2b} - \frac{i\operatorname{PolyLog}(3,-ic(a+bx))}{2b} + \frac{i\operatorname{PolyLog}(3,ic(a+bx))}{2b}$$

```
[In] Int[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
```

```
[Out] ((I/2)*Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)])/b - ((I/2)*Log[d*(a +
 b*x)]*PolyLog[2, I*c*(a + b*x)])/b - ((I/2)*PolyLog[3, (-I)*c*(a + b*x)])/
b + ((I/2)*PolyLog[3, I*c*(a + b*x)])/b
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5317

```
Int[(ArcTan[v_]*Log[w_])/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*v]*(Log[w]/(a + b*x)), x], x] - Dist[I/2, Int[Log[1 + I*v]*(Log[w]
/(a + b*x)), x], x] /; FreeQ[{a, b}, x] && LinearQ[v, x] && LinearQ[w, x] &
& EqQ[Simplify[D[v/(a + b*x), x]], 0] && EqQ[Simplify[D[w/(a + b*x), x]], 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}i\int\frac{\log(d(a+bx))\log(1-ic(a+bx))}{a+bx}\,dx \\
&\quad-\frac{1}{2}i\int\frac{\log(d(a+bx))\log(1+ic(a+bx))}{a+bx}\,dx \\
&= \frac{1}{2}i\int\frac{\log(d(a+bx))\log(1-iac-ibcx)}{a+bx}\,dx-\frac{1}{2}i\int\frac{\log(d(a+bx))\log(1+iac+ibcx)}{a+bx}\,dx \\
&= \frac{i\text{Subst}\left(\int\frac{\log(dx)\log\left(\frac{iabc+b(1-iac)}{b}-icx\right)}{x}\,dx,x,a+bx\right)}{2b} \\
&\quad-\frac{i\text{Subst}\left(\int\frac{\log(dx)\log\left(\frac{-iabc+b(1+iac)}{b}+icx\right)}{x}\,dx,x,a+bx\right)}{2b} \\
&= \frac{i\log(d(a+bx))\,\text{PolyLog}(2,-ic(a+bx))}{2b}-\frac{i\log(d(a+bx))\,\text{PolyLog}(2,ic(a+bx))}{2b} \\
&\quad-\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-icx)}{x}\,dx,x,a+bx\right)}{2b}+\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,icx)}{x}\,dx,x,a+bx\right)}{2b} \\
&= \frac{i\log(d(a+bx))\,\text{PolyLog}(2,-ic(a+bx))}{2b}-\frac{i\log(d(a+bx))\,\text{PolyLog}(2,ic(a+bx))}{2b} \\
&\quad-\frac{i\,\text{PolyLog}(3,-ic(a+bx))}{2b}+\frac{i\,\text{PolyLog}(3,ic(a+bx))}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int\frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \frac{i(\log(d(a+bx))\,\text{PolyLog}(2,-ic(a+bx))-\log(d(a+bx))\,\text{PolyLog}(2,ic(a+bx))-\text{PolyLog}(3,-ic(a+b}{2b}$$

```
[In] Integrate[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
[Out] ((I/2)*(Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)] - Log[d*(a + b*x)]*Po
lyLog[2, I*c*(a + b*x)] - PolyLog[3, (-I)*c*(a + b*x)] + PolyLog[3, I*c*(a
+ b*x)]))/b
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.95 (sec) , antiderivative size = 1087, normalized size of antiderivative = 10.76

method	result	size
risch	Expression too large to display	1087

```
[In] int(arctan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2*dilog(-I*(c*(b*x+a)+I))+1/4/b*Pi*csg
n(I*(b*x+a))*csgn(I*d*(b*x+a))^2*dilog(-I*(c*(b*x+a)+I))-1/4/b*Pi*csgn(I*d)
*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))*dilog(-I*(c*(b*x+a)+I))-1/4/b*ln(-I*(-c*
(b*x+a)+I))*ln(b*x+a)*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2-1/4/b*ln(-I*(-c*(b*x
+a)+I))*ln(b*x+a)*Pi*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))^2+1/4/b*ln(-I*(-c*(b
*x+a)+I))*ln(-I*c*(b*x+a))*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2+1/4/b*ln(-I*(-c
*(b*x+a)+I))*ln(-I*c*(b*x+a))*Pi*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))^2-1/4/b*
dilog(-I*c*(b*x+a))*Pi*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))-1/4*I*(-
I*Pi*ln(b*x+a)*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))+I*Pi*ln(b*x+a)*c
sgn(I*d)*csgn(I*d*(b*x+a))^2+I*Pi*ln(b*x+a)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a
))^2-I*Pi*ln(b*x+a)*csgn(I*d*(b*x+a))^3+2*ln(d)*ln(b*x+a)+ln(b*x+a)^2)/b*ln
(1+I*c*(b*x+a))-1/4/b*Pi*csgn(I*d*(b*x+a))^3*dilog(-I*(c*(b*x+a)+I))+1/2*I*
polylog(3,I*c*(b*x+a))/b-1/4/b*dilog(-I*c*(b*x+a))*Pi*csgn(I*d*(b*x+a))^3+1
/4*I/b*ln(b*x+a)^2*ln(1+I*c*(b*x+a))+1/2*I/b*ln(b*x+a)*polylog(2,-I*c*(b*x+
a))-1/2*I/b*dilog(-I*c*(b*x+a))*ln(d)+1/4/b*ln(-I*(-c*(b*x+a)+I))*ln(b*x+a)
*Pi*csgn(I*d*(b*x+a))^3-1/4/b*ln(-I*(-c*(b*x+a)+I))*ln(-I*c*(b*x+a))*Pi*csg
n(I*d*(b*x+a))^3+1/4/b*dilog(-I*c*(b*x+a))*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2
+1/4/b*dilog(-I*c*(b*x+a))*Pi*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))^2+1/2*I/b*l
n(-I*(-c*(b*x+a)+I))*ln(b*x+a)*ln(d)-1/2*I/b*ln(-I*(-c*(b*x+a)+I))*ln(-I*c*
(b*x+a))*ln(d)-1/4*I/b*ln(b*x+a)^2*ln(1-I*c*(b*x+a))+1/4*I/b*ln(-I*(c*(b*x+
a)+I))*ln(b*x+a)^2-1/2*I/b*ln(d)*dilog(-I*(c*(b*x+a)+I))+1/4/b*ln(-I*(-c*(b
*x+a)+I))*ln(b*x+a)*Pi*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))-1/4/b*ln
(-I*(-c*(b*x+a)+I))*ln(-I*c*(b*x+a))*Pi*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*
(b*x+a))-1/2*I/b*ln(b*x+a)*polylog(2,I*c*(b*x+a))-1/2*I*polylog(3,-I*c*(b*x
+a))/b
```

Fricas [F]

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \int \frac{\arctan\left((bx+a)c\right)\log\left((bx+a)d\right)}{bx+a}\,dx$$

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*c*x + a*c)*log(b*d*x + a*d)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \int \frac{\log(ad+bdx)\,\mathrm{atan}\,(ac+bcx)}{a+bx}\,dx$$

```
[In] integrate(atan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x)
[Out] Integral(log(a*d + b*d*x)*atan(a*c + b*c*x)/(a + b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \int \frac{\arctan\left((bx+a)c\right)\log\left((bx+a)d\right)}{bx+a}\,dx$$

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="giac")
[Out] integrate(arctan((b*x + a)*c)*log((b*x + a)*d)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \int \frac{\mathrm{atan}(c\,(a+b\,x))\,\ln\left(d\,(a+b\,x)\right)}{a+b\,x}\,dx$$

```
[In] int((atan(c*(a + b*x))*log(d*(a + b*x)))/(a + b*x),x)
[Out] int((atan(c*(a + b*x))*log(d*(a + b*x)))/(a + b*x), x)
```

3.147 $\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx = \frac{e^{ac+bcx} \arctan(\sinh(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(sinh(c*(b*x+a)))/b/c-ln(1+exp(2*c*(b*x+a)))/b/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2225, 5315, 2320, 12, 266}

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx = \frac{e^{ac+bcx} \arctan(\sinh(c(a+bx)))}{bc} - \frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sinh[c*(a + b*x)]])/(b*c) - Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^x \arctan(\sinh(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int e^x \text{sech}(x)\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\sinh(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\sinh(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int e^{c(a+bx)}\arctan(\sinh(ac+bcx))\,dx = -\frac{e^{c(a+bx)}\arctan\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)+\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
[Out] -((E^(c*(a + b*x))*ArcTan[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] + Log[
1 + E^(2*c*(a + b*x))])/(b*c))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.11 (sec) , antiderivative size = 1299, normalized size of antiderivative = 27.06

method	result	size
risch	Expression too large to display	1299

```
[In] int(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+
a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c
*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(
b*x+a))*(exp(c*(b*x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*
exp(c*(b*x+a))+1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))*csgn(I*(exp(c*(b*x+a))
-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))^2*csgn(I*(exp
(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(
b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1
/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^2*exp(c*(
b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a))*(exp
(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*
csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a
)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*P
i*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*
(b*x+a))-I)^2)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x
+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/b/
c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))+1/4/b/c*
Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*c
sgn(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a)
)*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+
I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)
^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^
2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*ex
p(c*(b*x+a))-I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)+2*a/b+1/4/b/c*Pi*csg
n(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(e
xp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*
csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*exp(c*
(b*x+a))+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)+1/2/b/c*exp(c*(b*x+a))*P
i-ln(1+exp(2*c*(b*x+a)))/b/c
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx = \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan(\sinh(bcx+ac)) - \log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="fricas")
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)) - log(2*
cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\mathrm{atan}\,(\sinh(ac+bcx))\,dx$$

```
[In] integrate(exp(c*(b*x+a))*atan(sinh(b*c*x+a*c)),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(sinh(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx = \frac{\arctan(\sinh(bcx+ac))\,e^{((bx+a)c)}}{bc} - \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="maxima")
[Out] arctan(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int e^{c(a+bx)}\arctan(\sinh(ac+bcx))\,dx = \frac{\left(\arctan\left(\frac{1}{2}\,e^{(bcx+ac)}-\frac{1}{2}\,e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(1/2*e^(b*c*x + a*c) - 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*lo
g(e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int e^{c(a+bx)}\arctan(\sinh(ac+bcx))\,dx = \frac{e^{b\,c\,x}\,e^{a\,c}\,\mathrm{atan}\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}\right)}{b\,c}-\frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}+1\right)}{b\,c}$$

```
[In] int(exp(c*(a + b*x))*atan(sinh(a*c + b*c*x)),x)
```

```
[Out] (exp(b*c*x)*exp(a*c)*atan((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))
/2))/(b*c) - log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c)
```

3.148 $\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\cosh(c(a+bx)))}{bc} - \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc} - \frac{(1+\sqrt{2})\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(cosh(c*(b*x+a)))/b/c-1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c-1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2225, 5315, 2320, 12, 1261, 646, 31}

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\cosh(c(a+bx)))}{bc} - \frac{(1-\sqrt{2})\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc} - \frac{(1+\sqrt{2})\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Cosh[c*(a + b*x)]])/(b*c) - ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) - ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^x \arctan(\cosh(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\cosh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{e^x \sinh(x)}{1+\cosh^2(x)}\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\cosh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\cosh(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\cosh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\cosh(c(a+bx)))}{bc} - \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc}\\
&\quad - \frac{\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3+2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc}\\
&= \frac{e^{ac+bcx}\arctan(\cosh(c(a+bx)))}{bc} - \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc}\\
&\quad - \frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\int e^{c(a+bx)}\arctan(\cosh(ac+bcx))\,dx$$
$$= \frac{-4c(a+bx)+2e^{c(a+bx)}\arctan\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right)+\text{RootSum}\left[1+6\#1^2+\#1^4\&, \frac{ac+bcx-\log\left(e^{c(a+bx)}\right.}{}\right.}{2bc}$$

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

```
[Out] (-4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(2*E^(c*
(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b
*x)) - #1] + 7*a*c*#1^2 + 7*b*c*x*#1^2 - 7*Log[E^(c*(a + b*x)) - #1]*#1^2)/
(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.59 (sec) , antiderivative size = 1371, normalized size of antiderivative = 13.31

method	result	size
risch	Expression too large to display	1371

```
[In] int(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))+1/4/b/c
*Pi*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*exp(-c*(b*x+a))*(ex
p(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*ex
p(-c*(b*x+a)))*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*exp(-c*(
b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*Pi*
csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*
x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b
*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*
x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*
c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c
*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))+1/4/b/
c*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-e
xp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*e
xp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+
a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(2*c*(b
*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*
exp(c*(b*x+a))))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c
*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+
2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(
2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c
*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))+1/4/b/
c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(e
xp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))-1/4/
b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csg
n(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))-
1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*
exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp
(c*(b*x+a))))^2*exp(c*(b*x+a))+1/2/b/c*exp(c*(b*x+a))*Pi+1/2/b/c*2^(1/2)*ln
(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)-1/2/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(1+2^(
1/2))^2)+2*a/b+1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*
x+a)))-1/2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)-1/2/b/c*ln(exp(2*c*(b*x+a
))+(1+2^(1/2))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx$$
$$= \frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\cosh\left(bcx+ac\right)\right)+\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}-4\right)\cosh(bcx+ac)\cdots}{\cosh(bc\cdots}\right.}{2\,b\cdots}$$

[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="fricas")

[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)) + sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) - 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) - 3)*sinh(b*c*x + a*c)^2 + 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)) - log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)

Sympy [F]

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{atan}\left(\cosh\left(ac+bcx\right)\right)dx$$

[In] integrate(exp(c*(b*x+a))*atan(cosh(b*c*x+a*c)),x)

[Out] exp(a*c)*Integral(exp(b*c*x)*atan(cosh(a*c + b*c*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx = \frac{\arctan\left(\cosh\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc} - \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2\,bcx-2\,ac)}-3}{2\sqrt{2}+e^{(-2\,bcx-2\,ac)}+3}\right)}{2\,bc} - \frac{2\left(bcx+ac\right)}{bc} - \frac{\log\left(6\,e^{(-2\,bcx-2\,ac)}+e^{(-4\,bcx-4\,ac)}+1\right)}{2\,bc}$$

[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="maxima")

```
[Out] arctan(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*log(-(2*sqrt(
2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-2*b*c*x - 2*a*c) + 3))/(b*c
) - 2*(b*c*x + a*c)/(b*c) - 1/2*log(6*e^(-2*b*c*x - 2*a*c) + e^(-4*b*c*x -
4*a*c) + 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx = \frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)+2\arctan\left(\frac{1}{2}e^{(bcx+ac)}+\frac{1}{2}e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bc}\right.\right.}{2\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*e
^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) + 2*ar
ctan(1/2*e^(b*c*x + a*c) + 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*log(e
^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx = \frac{\ln\left(-8\,\mathrm{e}^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c} - \frac{\ln\left(2\,\sqrt{2}-8\,\mathrm{e}^{2\,c\,(a+b\,x)}+6\,\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c} + \frac{\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}\right)}{b\,c}$$

```
[In] int(exp(c*(a + b*x))*atan(cosh(a*c + b*c*x)),x)
```

```
[Out] (log(- 8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^
(1/2) - 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)) + 6*2^(1/2)*exp
(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) + (exp(a*c + b*c*x)*atan((exp(b*c*x
)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

3.149 $\int e^{c(a+bx)} \arctan(\tanh(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\int e^{c(a+bx)} \arctan(\tanh(ac+bcx))\,dx = \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc} + \frac{\log\left(1+e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(tanh(c*(b*x+a)))/b/c-1/2*arctan(-1+exp(b*c*x+a*c)*2^(
1/2))/b/c*2^(1/2)-1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1
+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1+exp(2*c*(b*x
+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules

used = {2225, 5315, 12, 2281, 303, 1176, 631, 210, 1179, 642}

$$\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx = \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} - \frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc} + \frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
```

```
[Out] ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c) - ArcTan[1 + Sqrt[2]*E^(a
*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Tanh[c*(a + b*x)]])/(b
*c) - Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
+ Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^x \arctan(\tanh(x))\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2e^{3x}}{1+e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$

$$
\begin{aligned}
&= \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{e^{3x}}{1+e^{4x}}\,dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^{ac+bcx}\right)}{bc} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^{ac+bcx}\right)}{2\sqrt{2}bc} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^{ac+bcx}\right)}{2\sqrt{2}bc} \\
&= \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} - \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} \\
&\quad + \frac{\log\left(1+\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} \\
&= \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} \\
&\quad - \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} + \frac{\log\left(1+\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$
\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx
= \frac{2e^{c(a+bx)}\arctan\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right) + \text{RootSum}\left[1+\#1^4\&, \frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}
$$

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
[Out] (2*E^(c*(a + b*x))*ArcTan[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b*x)) - #1])/#1 & ])/(
2*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.38 (sec) , antiderivative size = 1355, normalized size of antiderivative = 7.53

method	result	size
risch	Expression too large to display	1355

```
[In] int(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(
2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csg
n(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*
csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*csgn((1-I)*(exp(2*c*(b*x+
a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*
(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a
))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+e
xp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(
1+exp(2*c*(b*x+a))))*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^
2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))
))*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4
/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csgn((1+I)*(exp(2
*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(
2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1+I
)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))-1/4/b/c*Pi*cs
gn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/
c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))
-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+
a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^3*exp(
c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-I)-1/4*I/b/c*ln(exp
(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(exp(c*(b*x+a))+
1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(exp(c*(b*x+a))-1/2*I*2^(1/2
)-1/2*2^(1/2))*2^(1/2)-1/4*I/b/c*ln(exp(c*(b*x+a))+1/2*I*2^(1/2)-1/2*2^(1/2
))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4
/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-1/4/b/c*ln(exp(c*
(b*x+a))-1/2*I*2^(1/2)-1/2*2^(1/2))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))+1/2*I
*2^(1/2)-1/2*2^(1/2))*2^(1/2)+1/4/b/c*exp(c*(b*x+a))*Pi+1/2*I/b/c*exp(c*(b*
x+a))*ln(exp(2*c*(b*x+a))+I)-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I
*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*ex
p(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a
))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)} \arctan(\tanh(ac+bcx))\,dx =$$

$$-\frac{bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}+\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-i\,bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(i\,b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="fricas")
[Out] -1/2*(b*c*(-1/(b^4*c^4))^(1/4)*log(b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*
x + a*c) + sinh(b*c*x + a*c)) - I*b*c*(-1/(b^4*c^4))^(1/4)*log(I*b^3*c^3*(-
1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + I*b*c*(-1/(b^
4*c^4))^(1/4)*log(-I*b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sin
h(b*c*x + a*c)) - b*c*(-1/(b^4*c^4))^(1/4)*log(-b^3*c^3*(-1/(b^4*c^4))^(3/4
) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - 2*(cosh(b*c*x + a*c) + sinh(b*
c*x + a*c))*arctan(sinh(b*c*x + a*c)/cosh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)} \arctan(\tanh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{atan}\left(\tanh\left(ac+bcx\right)\right)dx$$

```
[In] integrate(exp(c*(b*x+a))*atan(tanh(b*c*x+a*c)),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(tanh(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\begin{aligned}\int e^{c(a+bx)} \arctan(\tanh(ac+bcx))\,dx &= \frac{\arctan\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} \\ &\quad - \frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bcx+ac)}\right)\right)}{2\,bc} \\ &\quad - \frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bcx+ac)}\right)\right)}{2\,bc} \\ &\quad + \frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc} \\ &\quad - \frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc}\end{aligned}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) + 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) - 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Giac [F]

$$\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx=\int \arctan\left(\tanh\left(bcx+ac\right)\right)e^{((bx+a)c)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx$$
$$=\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1}{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4-4\mathrm{i})+\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,(-1-\mathrm{i})+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4+4\mathrm{i})-\mathrm{e}^{b\,c\,x}\,\mathrm{e}\cdots\right.}{4\,b\,c\cdots}$$

```
[In] int(exp(c*(a + b*x))*atan(tanh(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(- 2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*log(exp
(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 + 4i))*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*atan((exp(2*b*
c*x)*exp(2*a*c) - 1)/(exp(2*b*c*x)*exp(2*a*c) + 1)))/(4*b*c)
```

3.150 $\int e^{c(a+bx)} \arctan(\coth(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\begin{aligned}\int e^{c(a+bx)} \arctan(\coth(ac+bcx))\,dx = & -\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}+\frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} \\ & +\frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc} \\ & +\frac{\log\left(1+e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc} \\ & -\frac{\log\left(1+e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc}\end{aligned}$$

```
[Out] exp(b*c*x+a*c)*arctan(coth(c*(b*x+a)))/b/c+1/2*arctan(-1+exp(b*c*x+a*c)*2^(
1/2))/b/c*2^(1/2)+1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1
+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1+exp(2*c*(b*x
+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {2225, 5315, 12, 2281, 303, 1176, 631, 210, 1179, 642}

$$\int e^{c(a+bx)}\arctan(\coth(ac+bcx))\,dx = -\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}+\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc}+\frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc}+\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}-\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c)) + ArcTan[1 + Sqrt[2]*E
^(a*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Coth[c*(a + b*x)]])
/(b*c) + Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*
c) - Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^x \arctan(\coth(x))\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2e^{3x}}{-1-e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{e^{3x}}{-1-e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1-x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc} - \frac{\text{Subst}\left(\int \frac{1+x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^{ac+bcx}\right)}{2\sqrt{2}bc} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

$$= \frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc} + \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} - \frac{\log\left(1+\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}$$

$$= -\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc} + \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} - \frac{\log\left(1+\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)}\arctan(\coth(ac+bcx))\,dx = \frac{2e^{c(a+bx)}\arctan\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+\#1^4\&,\frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
```

```
[Out] (2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a + b*x)) - #1])/#1 & ]
)/(2*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 1355, normalized size of antiderivative = 7.53

method	result	size
risch	Expression too large to display	1355

```
[In] int(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*
c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csg
n(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*
csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-
1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a
))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a
))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp
(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(
exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^
2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1
))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4
/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2
*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a)
)*ln(exp(2*c*(b*x+a))+I)-1/4/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2
))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4
/b/c*ln(exp(c*(b*x+a))-1/2*I*2^(1/2)-1/2*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp(c*
(b*x+a))+1/2*I*2^(1/2)-1/2*2^(1/2))*2^(1/2)+1/4/b/c*exp(c*(b*x+a))*Pi-1/4/b
```

```
/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a)
)-1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*
(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp
(c*(b*x+a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1)
)^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))
-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(
b*x+a))-1))^3*exp(c*(b*x+a))+1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-I
)+1/4*I/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)-1/4*I/b/c*
ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-1/4*I/b/c*ln(exp(c*(b*
x+a))-1/2*I*2^(1/2)-1/2*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(exp(c*(b*x+a))+1/2*I*
2^(1/2)-1/2*2^(1/2))*2^(1/2)-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I
/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*ex
p(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I/(exp(2*c*(b*x+a
))-1))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)} \arctan(\coth(ac+bcx))\,dx$$
$$= \frac{bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}+\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-i\,bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(i\,b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}+\text{co}\cdots\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*(-1/(b^4*c^4))^(1/4)*log(b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x
 + a*c) + sinh(b*c*x + a*c)) - I*b*c*(-1/(b^4*c^4))^(1/4)*log(I*b^3*c^3*(-1
/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + I*b*c*(-1/(b^4
*c^4))^(1/4)*log(-I*b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh
(b*c*x + a*c)) - b*c*(-1/(b^4*c^4))^(1/4)*log(-b^3*c^3*(-1/(b^4*c^4))^(3/4)
 + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + 2*(cosh(b*c*x + a*c) + sinh(b*c
*x + a*c))*arctan(cosh(b*c*x + a*c)/sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)} \arctan(\coth(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{atan}\left(\coth\left(ac+bcx\right)\right)dx$$

```
[In] integrate(exp(c*(b*x+a))*atan(coth(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(coth(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\int e^{c(a+bx)}\arctan(\coth(ac+bcx))\,dx = \frac{\arctan\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} + \frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bcx+ac)}\right)\right)}{2\,bc} + \frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bcx+ac)}\right)\right)}{2\,bc} - \frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc} + \frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) - 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) + 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Giac [F]

$$\int e^{c(a+bx)}\arctan(\coth(ac+bcx))\,dx = \int \arctan\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)}\arctan(\coth(ac+bcx))\,dx = \frac{4\,e^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{e^{2\,b\,c\,x}\,e^{2\,a\,c}+1}{e^{2\,b\,c\,x}\,e^{2\,a\,c}-1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4-4\mathrm{i})-e^{b\,c\,x}\,e^{a\,c}\,8\mathrm{i}\right)\,(-1-\mathrm{i})+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4+4\mathrm{i})+e^{b\,c\,x}\cdots\right.}{4\,b\cdots}$$

```
[In] int(exp(c*(a + b*x))*atan(coth(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(exp(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 - 4i))*(1 - 1i) - 2^(1/2)*log(- 2^(
1/2)*(4 + 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*atan((exp(2*b*
c*x)*exp(2*a*c) + 1)/(exp(2*b*c*x)*exp(2*a*c) - 1)))/(4*b*c)
```

3.151 $\int e^{c(a+bx)} \arctan(\textbf{sech}(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c(a+bx)} \arctan(\operatorname{sech}(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\operatorname{sech}(c(a+bx)))}{bc} + \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(sech(c*(b*x+a)))/b/c+1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c+1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2225, 5315, 2320, 12, 1261, 646, 31}

$$\int e^{c(a+bx)} \arctan(\operatorname{sech}(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\operatorname{sech}(c(a+bx)))}{bc} + \frac{(1-\sqrt{2})\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sech[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^x \arctan(\operatorname{sech}(x))\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{e^x \operatorname{sech}(x)\tanh(x)}{1+\operatorname{sech}^2(x)}\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\operatorname{sech}(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\arctan(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc} + \frac{\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3+2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc}$$

$$= \frac{e^{ac+bcx}\arctan(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc} + \frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$\int e^{c(a+bx)}\arctan(\operatorname{sech}(ac+bcx))\,dx = \frac{4c(a+bx)+2e^{c(a+bx)}\arctan\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+6\#1^2+\#1^4\&, \frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)-7ac\#1}{1+3}\right.}{2bc}$$

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

```
[Out] (4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(
a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a +
 b*x)) - #1] - 7*a*c*#1^2 - 7*b*c*x*#1^2 + 7*Log[E^(c*(a + b*x)) - #1]*#1^2
)/(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.97 (sec) , antiderivative size = 838, normalized size of antiderivative = 8.14

method	result	size
risch	Expression too large to display	838

```
[In] int(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))-1/4/b/c
*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I/(1+exp(2*c*(b*x+a)))
)*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*exp
(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*(
-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+
a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I
*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(-I*
exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(e
xp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+
1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(
c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(-exp(
2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))-1
/4/b/c*Pi*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*(exp(2*c*(b*x
+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*P
i*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*ex
p(c*(b*x+a))-1/2/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)+1/2/b/c*2^(
1/2)*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)-2*a/b+1/2*I/b/c*exp(c*(b*x+a))*ln(e
xp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))+1/2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)-
1)^2)+1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(86) = 172$.

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.68

$$\int e^{c(a+bx)}\arctan(\operatorname{sech}(ac+bcx))\,dx$$

$$= \frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^2+2\,\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2+1}\right)+\sqrt{2}\log\left(\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c)
+ sinh(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x
```

```
+ a*c) + sinh(b*c*x + a*c)^2 + 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*
c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2
*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 + 2*sqrt(2) + 3)/(cosh(b*c*x + a*c)^2 + s
inh(b*c*x + a*c)^2 + 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2
 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b
*c*x + a*c)^2)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\arctan(\operatorname{sech}(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{atan}\left(\operatorname{sech}\left(ac+bcx\right)\right)dx$$

```
[In] integrate(exp(c*(b*x+a))*atan(sech(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(sech(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.64

$$\int e^{c(a+bx)}\arctan(\operatorname{sech}(ac+bcx))\,dx = \frac{\arctan\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} - \frac{3\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{\left(2\,bcx+2\,ac\right)}-3}{2\sqrt{2}+e^{\left(2\,bcx+2\,ac\right)}+3}\right)}{8\,bc} + \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{\left(-2\,bcx-2\,ac\right)}-3}{2\sqrt{2}+e^{\left(-2\,bcx-2\,ac\right)}+3}\right)}{8\,bc} + \frac{\log\left(e^{\left(4\,bcx+4\,ac\right)}+6\,e^{\left(2\,bcx+2\,ac\right)}+1\right)}{2\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 3/8*sqrt(2)*log(-(2*sqrt(
2) - e^(2*b*c*x + 2*a*c) - 3)/(2*sqrt(2) + e^(2*b*c*x + 2*a*c) + 3))/(b*c)
+ 1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-
2*b*c*x - 2*a*c) + 3))/(b*c) + 1/2*log(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x +
 2*a*c) + 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)} \arctan(\operatorname{sech}(ac+bcx))\,dx =$$

$$-\frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)-2\,\arctan\left(\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bcx+4\,ac}\right.\right.}{2\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*
e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) - 2*a
rctan(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)))*e^(b*c*x) - e^(-a*c)*log(e^(4
*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)} \arctan(\operatorname{sech}(ac+bcx))\,dx = \frac{e^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{b\,c} + \frac{\ln\left(8\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c} - \frac{\ln\left(8\,e^{2\,c\,(a+b\,x)}+2\,\sqrt{2}+6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}$$

```
[In] int(atan(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*atan(1/((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))
/2)))/(b*c) + (log(8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a
+ b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(8*exp(2*c*(a + b*x)) + 2*2^(1/2) + 6
*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

3.152 $\int e^{c(a+bx)} \arctan(\mathbf{csch}(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int e^{c(a+bx)} \arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(csch(c*(b*x+a)))/b/c+ln(1+exp(2*c*(b*x+a)))/b/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2225, 5315, 2320, 12, 266}

$$\int e^{c(a+bx)} \arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Csch[c*(a + b*x)]])/(b*c) + Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^x \arctan(\operatorname{csch}(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int e^x \operatorname{sech}(x)\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\operatorname{csch}(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\arctan(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$
\int e^{c(a+bx)}\arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{c(a+bx)}\arctan\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right)+\log\left(1+e^{2c(a+bx)}\right)}{bc}
$$

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] + Log
[1 + E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.02 (sec) , antiderivative size = 885, normalized size of antiderivative = 18.83

method	result	size
risch	Expression too large to display	885

```
[In] int(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)-1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+
a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+
1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I
*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(
I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2
*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I/(exp(2*c*(b*
x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+
1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a
))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*(exp(c*(b*x+a))-I)^2/(exp
(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)^
3*exp(c*(b*x+a))-1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))*csgn(I*(exp(c*(b*x+a
))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))^2*csgn(I*(e
xp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2/(
exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^
2)*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b
/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+
a))+1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^2*ex
p(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-2*a/b
+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)+ln(1+exp(2*c*(b*x+a)))/b/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int e^{c(a+bx)} \arctan(\operatorname{csch}(ac+bcx))\,dx$$

$$= \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan\left(\frac{2(\cosh(bcx+ac)+\sinh(bcx+ac))}{\cosh(bcx+ac)^2+2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2-1}\right)+\log\left(\frac{\ldots}{\cosh(\ldots}\right.}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c) + sinh
(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
 + sinh(b*c*x + a*c)^2 - 1)) + log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) -
 sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\arctan(\operatorname{csch}(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{atan}\left(\operatorname{csch}\left(ac+bcx\right)\right)dx$$

```
[In] integrate(exp(c*(b*x+a))*atan(csch(b*c*x+a*c)),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(csch(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int e^{c(a+bx)}\arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{\arctan\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} + \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="maxima")
[Out] arctan(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int e^{c(a+bx)}\arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{\left(\arctan\left(\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}\right)e^{(bcx)}+e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="giac")
[Out] (arctan(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))*e^(b*c*x) + e^(-a*c)*log(e^
(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)}\arctan(\operatorname{csch}(ac+bcx))\,dx=\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1\right)}{b\,c}+\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\mathrm{atan}\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{b\,c}$$

```
[In] int(atan(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c) + (exp(b*c*x)*exp(a*c)*atan(1/((exp(
b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2)))/(b*c)
```

3.153 $\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx = ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{ibd\,\text{PolyLog}(2,-icx^n)}{2n}+\frac{ibe\log(fx^m)\,\text{PolyLog}(2,-icx^n)}{2n}-\frac{ibd\,\text{PolyLog}(2,icx^n)}{2n}-\frac{ibe\log(fx^m)\,\text{PolyLog}(2,icx^n)}{2n}-\frac{ibem\,\text{PolyLog}(3,-icx^n)}{2n^2}+\frac{ibem\,\text{PolyLog}(3,icx^n)}{2n^2}$$

```
[Out] a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m+1/2*I*b*d*polylog(2,-I*c*x^n)/n+1/2*I*b*e*l
n(f*x^m)*polylog(2,-I*c*x^n)/n-1/2*I*b*d*polylog(2,I*c*x^n)/n-1/2*I*b*e*ln(
f*x^m)*polylog(2,I*c*x^n)/n-1/2*I*b*e*m*polylog(3,-I*c*x^n)/n^2+1/2*I*b*e*m
*polylog(3,I*c*x^n)/n^2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2338, 6874, 4944, 4940, 2438, 5127, 5125, 2421, 6724}

$$\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx = ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{ibd\,\text{PolyLog}(2,-icx^n)}{2n}-\frac{ibd\,\text{PolyLog}(2,icx^n)}{2n}+\frac{ibe\,\text{PolyLog}(2,-icx^n)\log(fx^m)}{2n}-\frac{ibe\,\text{PolyLog}(2,icx^n)\log(fx^m)}{2n}-\frac{ibem\,\text{PolyLog}(3,-icx^n)}{2n^2}+\frac{ibem\,\text{PolyLog}(3,icx^n)}{2n^2}$$

```
[In] Int[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + ((I/2)*b*d*PolyLog[2, (-I)*c*x^n])/
n + ((I/2)*b*e*Log[f*x^m]*PolyLog[2, (-I)*c*x^n])/n - ((I/2)*b*d*PolyLog[2,
 I*c*x^n])/n - ((I/2)*b*e*Log[f*x^m]*PolyLog[2, I*c*x^n])/n - ((I/2)*b*e*m*
PolyLog[3, (-I)*c*x^n])/n^2 + ((I/2)*b*e*m*PolyLog[3, I*c*x^n])/n^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5125

```
Int[(ArcTan[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Dis
t[I/2, Int[Log[d*x^m]*(Log[1 - I*c*x^n]/x), x], x] - Dist[I/2, Int[Log[d*x^
m]*(Log[1 + I*c*x^n]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 5127

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcTan[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_), x
_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*ArcT
```

```
an[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\arctan(cx^n))}{x}+\frac{e(a+b\arctan(cx^n))\log(fx^m)}{x}\right)dx\\
&= d\int \frac{a+b\arctan(cx^n)}{x}\,dx+e\int \frac{(a+b\arctan(cx^n))\log(fx^m)}{x}\,dx\\
&= (ae)\int \frac{\log(fx^m)}{x}\,dx+(be)\int \frac{\arctan(cx^n)\log(fx^m)}{x}\,dx+\frac{d\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x}\,dx,x,x^n\right)}{n}\\
&= ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{1}{2}(ibe)\int \frac{\log(fx^m)\log(1-icx^n)}{x}\,dx\\
&\quad -\frac{1}{2}(ibe)\int \frac{\log(fx^m)\log(1+icx^n)}{x}\,dx\\
&\quad +\frac{(ibd)\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,x^n\right)}{2n}-\frac{(ibd)\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx,x,x^n\right)}{2n}\\
&= ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{ibd\,\text{PolyLog}(2,-icx^n)}{2n}\\
&\quad +\frac{ibe\log(fx^m)\,\text{PolyLog}(2,-icx^n)}{2n}\\
&\quad -\frac{ibd\,\text{PolyLog}(2,icx^n)}{2n}-\frac{ibe\log(fx^m)\,\text{PolyLog}(2,icx^n)}{2n}\\
&\quad -\frac{(ibem)\int \frac{\text{PolyLog}(2,-icx^n)}{x}\,dx}{2n}+\frac{(ibem)\int \frac{\text{PolyLog}(2,icx^n)}{x}\,dx}{2n}
\end{aligned}$$

$$= ad\log(x) + \frac{ae\log^2(fx^m)}{2m} + \frac{ibd\,\text{PolyLog}(2,-icx^n)}{2n} + \frac{ibe\log(fx^m)\,\text{PolyLog}(2,-icx^n)}{2n} - \frac{ibd\,\text{PolyLog}(2,icx^n)}{2n} - \frac{ibe\log(fx^m)\,\text{PolyLog}(2,icx^n)}{2n} - \frac{ibem\,\text{PolyLog}(3,-icx^n)}{2n^2} + \frac{ibem\,\text{PolyLog}(3,icx^n)}{2n^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx = -\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)}{n^2} + \frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)(d+e\log(fx^m))}{n} + \frac{1}{2}a\log(x)\left(2d-em\log(x)+2e\log(fx^m)\right)$$

```
[In] Integrate[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] -((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, -(c^2
*x^(2*n))])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, -(
c^2*x^(2*n))]*(d + e*Log[f*x^m]))/n + (a*Log[x]*(2*d - e*m*Log[x] + 2*e*Log
[f*x^m]))/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 212.62 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.36

method	result
risch	$\frac{\left(-\frac{ie\pi\ \text{csgn}(if)\,\text{csgn}(ix^m)\,\text{csgn}(if\,x^m)}{4}+\frac{ie\pi\ \text{csgn}(if)\,\text{csgn}(if\,x^m)^2}{4}+\frac{ie\pi\ \text{csgn}(ix^m)\,\text{csgn}(if\,x^m)^2}{4}-\frac{ie\pi\,\text{csgn}(if\,x^m)^3}{4}+\frac{e\ln(f)}{2}+\frac{d}{2}\right)(-ib\,\text{dilog}}{n}$

```
[In] int((a+b*arctan(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*I*e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I*e*Pi*csgn(I*f)*csgn(
I*f*x^m)^2+1/4*I*e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I*e*Pi*csgn(I*f*x^m)^
3+1/2*e*ln(f)+1/2*d)/n*(-I*b*dilog(1-I*c*x^n)+2*ln(x^n)*a+I*b*dilog(1+I*c*x
```

```
^n))+1/2*I*e*b*ln(x)^2*ln(-I*(c*x^n+I))*m+1/2*I*b*e*m*polylog(3,I*c*x^n)/n^
2-1/2*I*e*b/n*dilog(-I*(c*x^n+I))*ln(x^m)-1/2*I*e*b*ln(x)^2*ln(-I*(-c*x^n+I
))*m+1/2*I*e*b*ln(x)*ln(1-I*c*x^n)*ln(x^m)+1/2*I*e*b*m/n*ln(x)*polylog(2,-I
*c*x^n)-1/2*I*e*b/n*dilog(-I*c*x^n)*ln(x^m)-1/2*I*e*b/n*ln(-I*(-c*x^n+I))*l
n(-I*c*x^n)*ln(x^m)+1/2*e*a/m*ln(x^m)^2-1/2*I*e*b*ln(x)*ln(x^m)*ln(1+I*c*x^
n)-1/2*I*e*b*ln(x)^2*ln(1-I*c*x^n)*m+1/2*I*e*b*ln(x)^2*ln(1+I*c*x^n)*m+1/2*
I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*m*ln(x)-1/2*I*e*b*ln(x)*ln(-I*(c*x^n
+I))*ln(x^m)+1/2*I*e*b/n*dilog(-I*(c*x^n+I))*m*ln(x)-1/2*I*b*e*m*polylog(3,
-I*c*x^n)/n^2-1/2*I*e*b*m/n*ln(x)*polylog(2,I*c*x^n)+1/2*I*e*b/n*dilog(-I*c
*x^n)*m*ln(x)+1/2*I*e*b*ln(x)*ln(x^m)*ln(-I*(-c*x^n+I))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(121) = 242$.

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.53

$$\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx = \frac{2\,aemn^2\log(x)^2+2i\,bem\text{polylog}(3,i\,cx^n)-2i\,bem\text{polylog}(3,-i\,cx^n)+2\left(bemn^2\log(x)^2+2\left(ben^2\log(f)\right.\right.}{}$$

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*e*m*n^2*log(x)^2 + 2*I*b*e*m*polylog(3, I*c*x^n) - 2*I*b*e*m*polyl
og(3, -I*c*x^n) + 2*(b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(
x))*arctan(c*x^n) - 2*(I*b*e*m*n*log(x) + I*b*e*n*log(f) + I*b*d*n)*dilog(I
*c*x^n) - 2*(-I*b*e*m*n*log(x) - I*b*e*n*log(f) - I*b*d*n)*dilog(-I*c*x^n)
+ (I*b*e*m*n^2*log(x)^2 - 2*(-I*b*e*n^2*log(f) - I*b*d*n^2)*log(x))*log(I*c
*x^n + 1) + (-I*b*e*m*n^2*log(x)^2 - 2*(I*b*e*n^2*log(f) + I*b*d*n^2)*log(x
))*log(-I*c*x^n + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x))/n^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*atan(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^n))\left(d+e\log(fx^m)\right)}{x}\,dx=\int \frac{(b\arctan(cx^n)+a)(e\log(fx^m)+d)}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/2*(b*e*m*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(b*e*log(f) + b*d)*log(x))*arctan(c*x^n) - integrate(-1/2*(b*c*e
*m*n*x^n*log(x)^2 - 2*b*c*e*n*x^n*log(x)*log(x^m) - 2*(b*c*e*log(f) + b*c*d
)*n*x^n*log(x))/(c^2*x*x^(2*n) + x), x)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^n))\left(d+e\log(fx^m)\right)}{x}\,dx=\int \frac{(b\arctan(cx^n)+a)(e\log(fx^m)+d)}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^n))\left(d+e\log(fx^m)\right)}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x^n))\,\left(d+e\,\ln(f\,x^m)\right)}{x}\,dx$$

```
[In] int(((a + b*atan(c*x^n))*(d + e*log(f*x^m)))/x,x)
```

```
[Out] int(((a + b*atan(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                 end if
             else #result contains complex but optimal is not
                 if debug then
                         print("result contains complex but optimal is not");
                 fi;
                 return "C","Result contains complex when optimal does not.";
             fi;
         else # result do not contain complex
              # this assumes optimal do not as well. No check is needed here.
             if debug then
                    print("result do not contain complex, this assumes optimal do not as well")
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A"," ";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
             fi;
         fi;
     else #ExpnType(result) > ExpnType(optimal)
         if debug then
             print("ExpnType(result) > ExpnType(optimal)");
         fi;
         return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
     fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```