Computer Algebra Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.4-Inverse-cotangent/154-5.4.1-Inverse-cotangent-functions

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 11:56am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [234]. This is test number [154].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (234)	0.00 (0)
Mathematica	97.44 (228)	2.56 (6)
Maple	97.44 (228)	2.56 (6)
Fricas	71.79 (168)	28.21 (66)
Maxima	61.11 (143)	38.89 (91)
Giac	47.44 (111)	52.56 (123)
Mupad	46.15 (108)	53.85 (126)
Sympy	34.62 (81)	65.38 (153)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

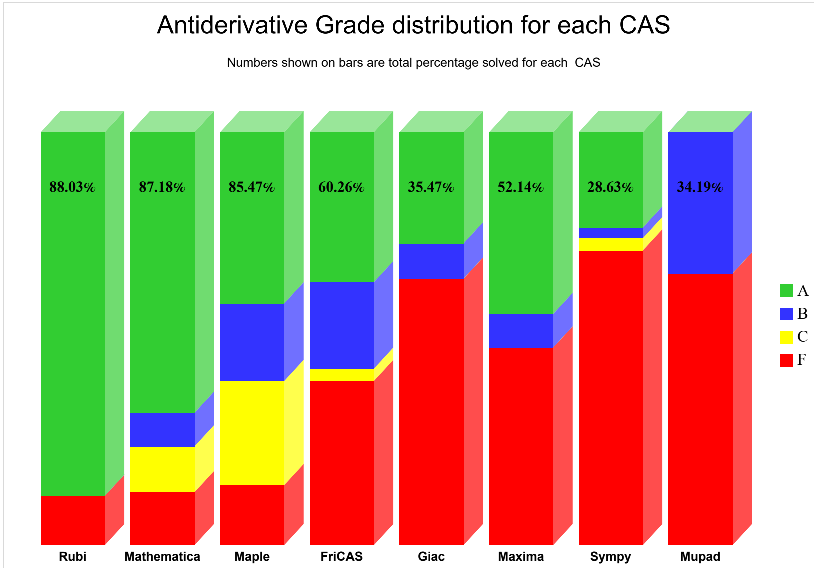
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

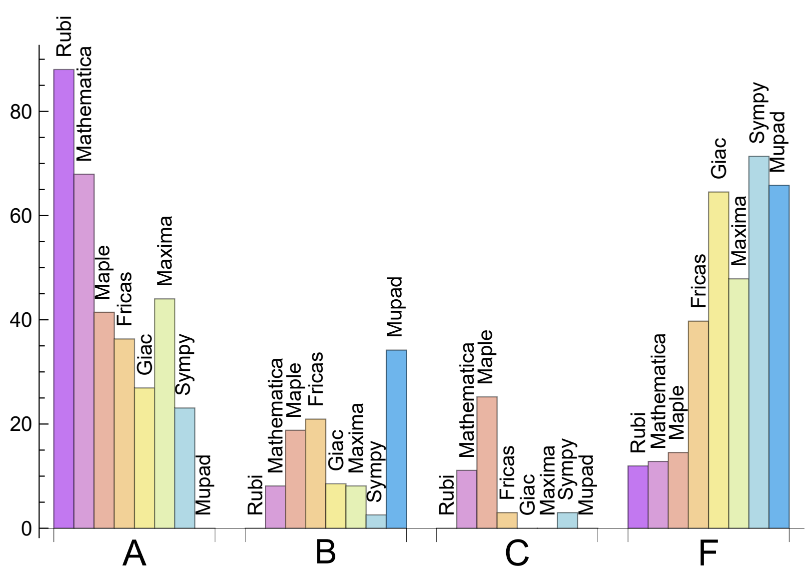
System	% A grade	% B grade	% C grade	% F grade
Rubi	88.034	0.000	0.000	11.966
Mathematica	67.949	8.120	11.111	12.821
Maxima	44.017	8.120	0.000	47.863
Maple	41.453	18.803	25.214	14.530
Fricas	36.325	20.940	2.991	39.744
Giac	26.923	8.547	0.000	64.530
Sympy	23.077	2.564	2.991	71.368
Mupad	0.000	34.188	0.000	65.812

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	6	100.00	0.00	0.00
Maple	6	100.00	0.00	0.00
Fricas	66	98.48	0.00	1.52
Maxima	91	79.12	2.20	18.68
Giac	123	92.68	5.69	1.63
Mupad	126	0.00	100.00	0.00
Sympy	153	52.94	30.72	16.34

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.17
Fricas	0.29
Mathematica	0.70
Mupad	0.96
Maxima	2.14
Giac	2.24
Maple	4.48
Sympy	5.35

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	66.30	1.09	35.00	1.00
Sympy	79.58	1.37	39.00	0.97
Rubi	135.68	1.00	91.00	1.00
Mathematica	173.53	1.39	90.00	1.00
Giac	216.16	2.18	38.00	1.03
Maxima	230.59	2.18	68.00	1.00
Fricas	240.14	1.75	67.50	1.20
Maple	733.20	3.99	128.00	1.16

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

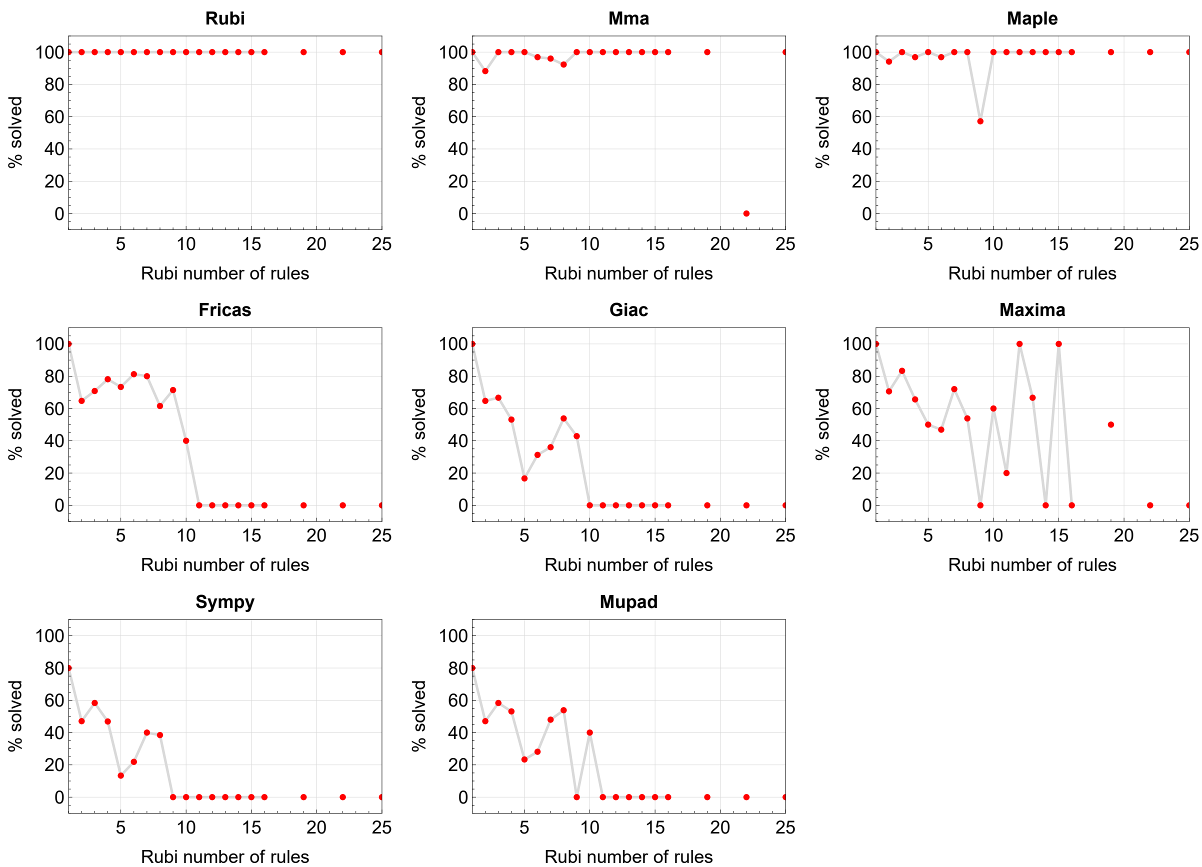


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

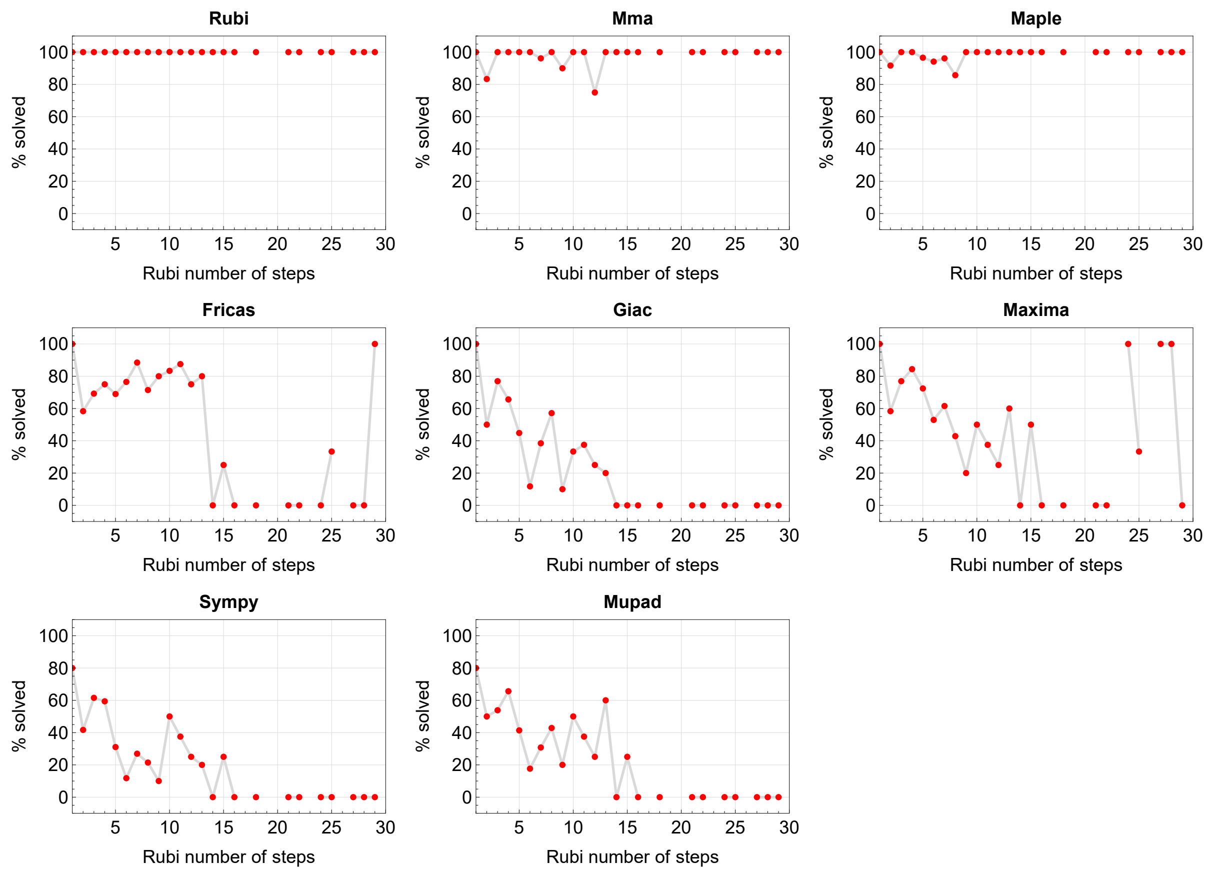


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

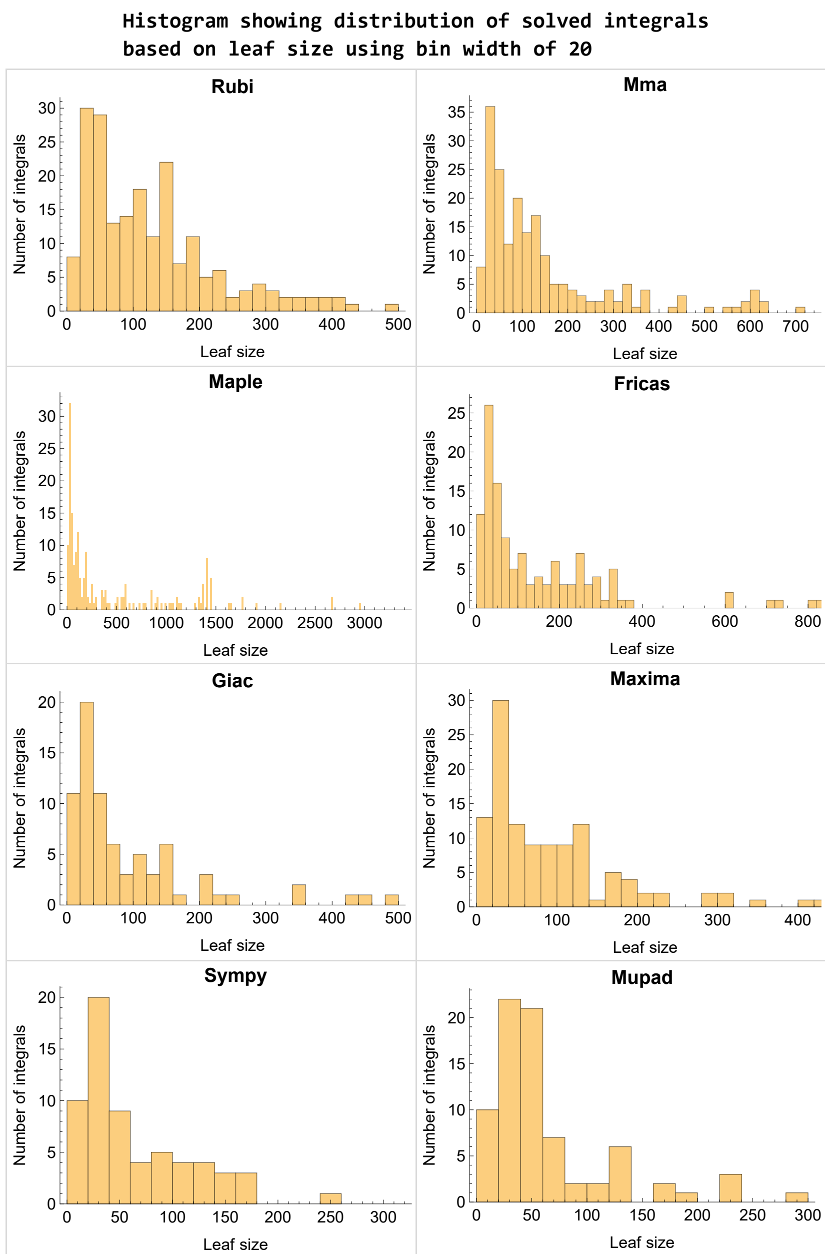


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

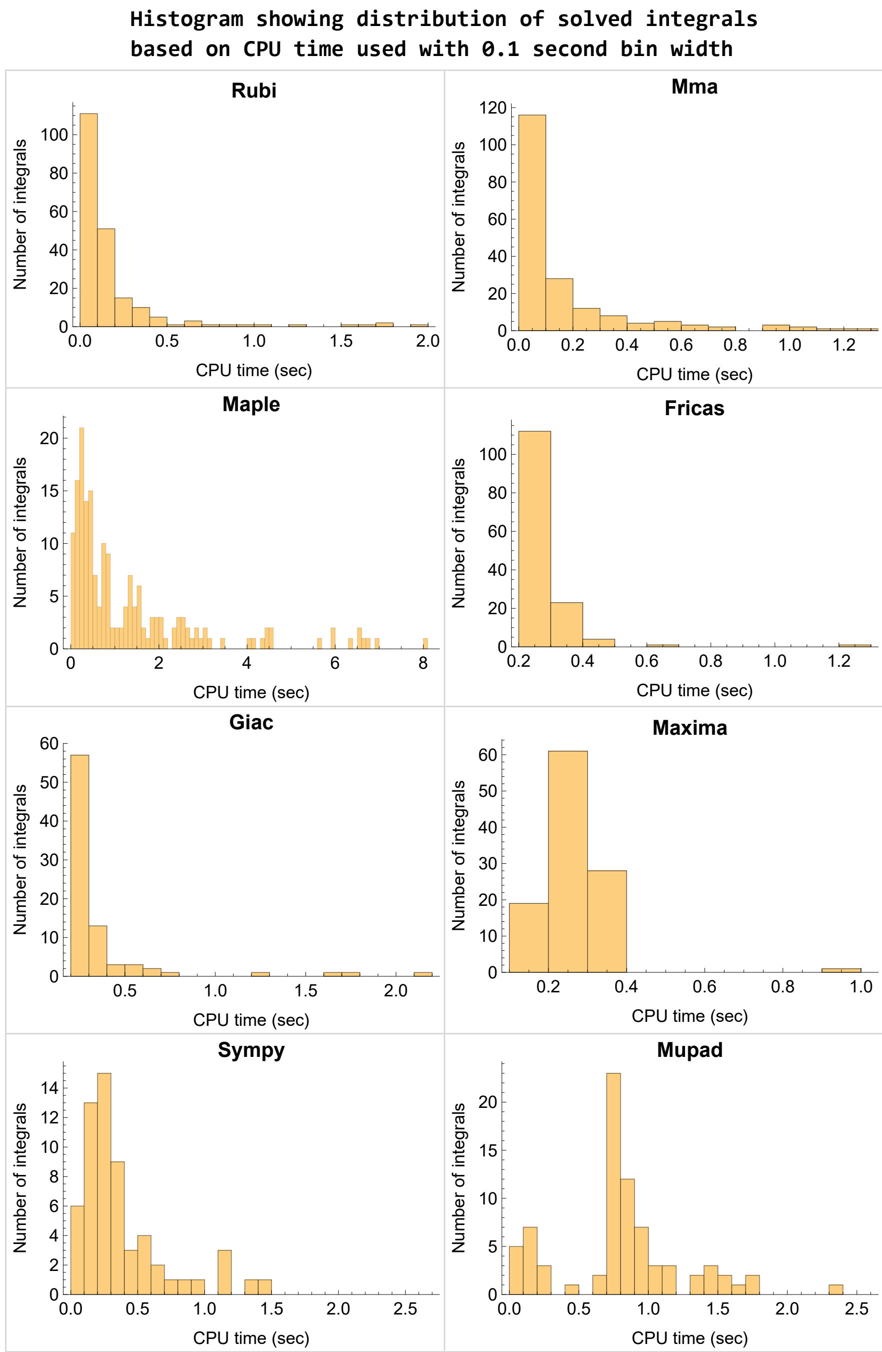


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

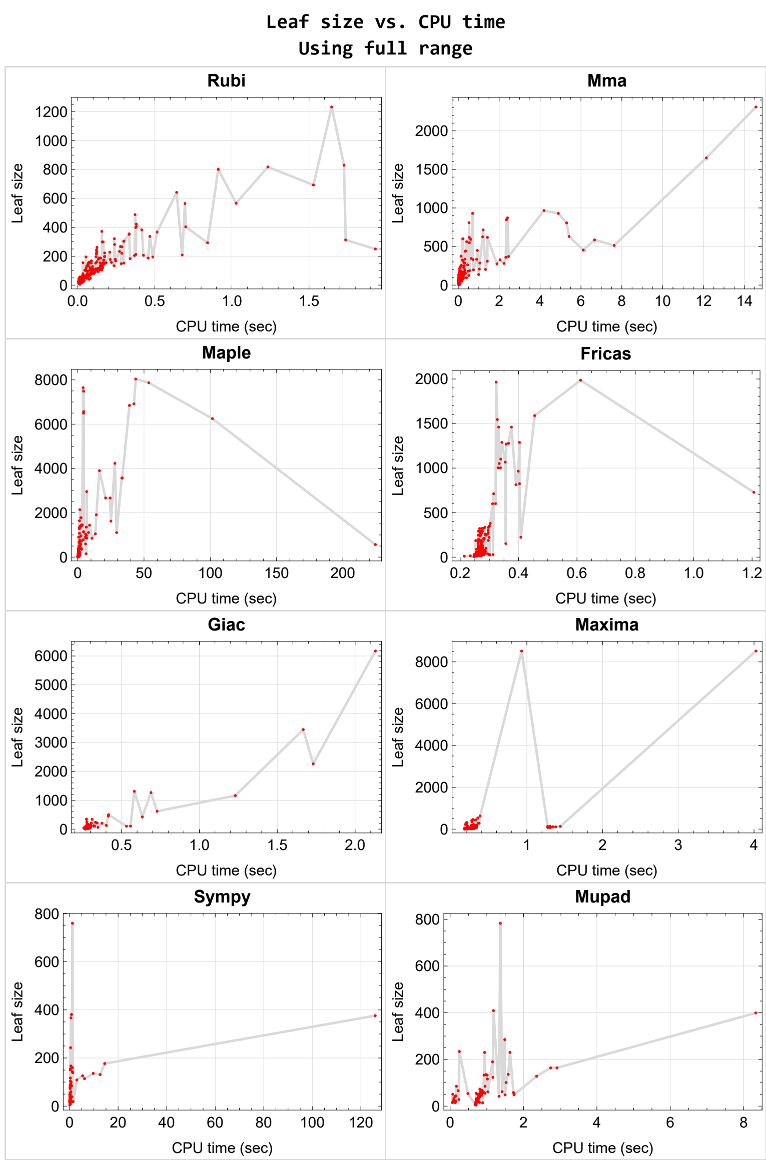


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{34, 35, 59, 60, 116, 117, 120, 121, 128, 147, 148, 151, 155, 156, 161, 165, 169, 174, 178, 182, 187, 191, 195, 199, 204, 208, 212, 216}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {116, 117, 120, 121}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {58, 141, 160, 164, 168, 173, 177, 181}

Maple {18, 24, 26, 29, 30, 31, 32, 33, 112, 139, 141, 144, 145, 158, 159, 162, 163, 166, 167, 171, 172, 175, 176, 179, 180, 183, 184, 185, 188, 189, 192, 193, 196, 197, 200, 201, 202, 205, 206, 209, 210, 213, 214, 217, 229, 230, 231, 232, 233, 234}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

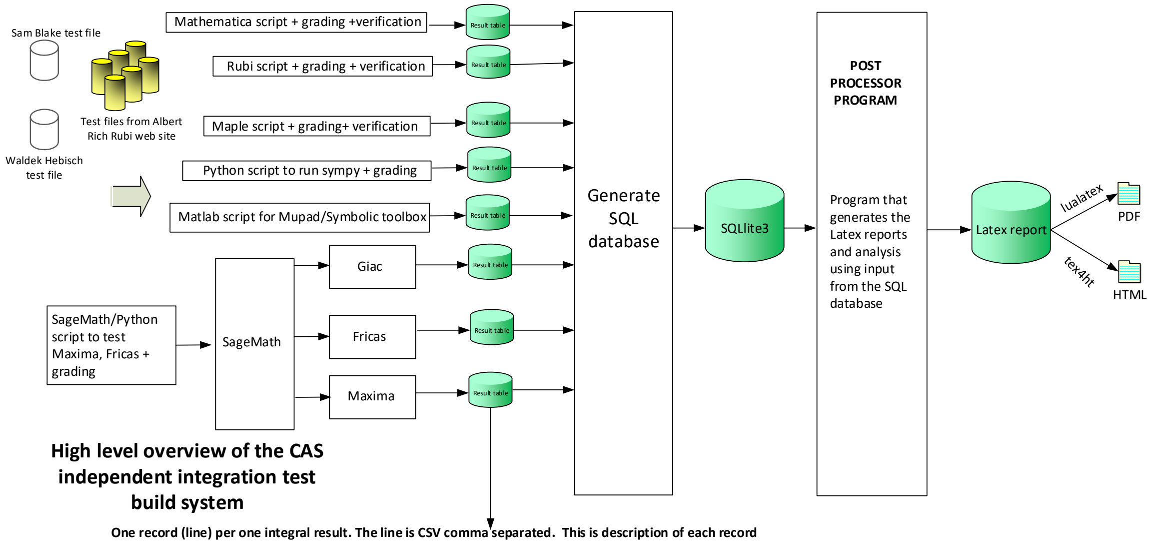
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 152, 153, 154, 157, 158, 159, 160, 162, 163, 164, 166, 167, 168, 170, 171, 172, 173, 175, 176, 177, 179, 180, 181, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 102, 107, 109, 110, 111, 112, 114, 115, 118, 119, 122, 124, 125, 126, 127, 132, 136, 137, 138, 140, 142, 143, 146, 149, 150, 154, 157, 158, 159, 162, 163, 166, 167, 170, 171, 172, 175, 176, 179, 180, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234 }

B grade { 46, 48, 50, 103, 108, 116, 117, 120, 121, 133, 141, 160, 164, 168, 173, 177, 181, 183, 200 }

C grade { 9, 11, 61, 62, 63, 64, 79, 89, 90, 99, 100, 101, 104, 105, 106, 123, 129, 130, 131, 134, 135, 217, 230, 231, 232, 233 }

F normal fail { 113, 139, 144, 145, 152, 153 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 20, 22, 37, 39, 41, 43, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 65, 66, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 114, 115, 118, 119, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 138, 140, 149, 157, 170, 219, 220, 224, 227, 228 }

B grade { 17, 19, 21, 23, 25, 27, 28, 38, 40, 42, 44, 58, 88, 97, 113, 129, 130, 136, 137, 142, 143, 152, 153, 154, 160, 164, 168, 173, 177, 181, 186, 190, 194, 198, 203, 207, 211, 215, 218, 221, 222, 223, 225, 226 }

C grade { 18, 24, 26, 29, 30, 31, 32, 33, 47, 49, 67, 68, 69, 77, 98, 111, 112, 139, 141, 144, 145, 150, 158, 159, 162, 163, 166, 167, 171, 172, 175, 176, 179, 180, 183, 184, 185, 188, 189, 192, 193, 196, 197, 200, 201, 202, 205, 206, 209, 210, 213, 214, 217, 229, 230, 231, 232, 233, 234 }

F normal fail { 36, 61, 62, 63, 64, 146 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 39, 41, 43, 45, 51, 52, 53, 54, 55, 56, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 104, 105, 106, 122, 123, 125, 129, 130, 131, 132, 134, 149, 150, 157, 170, 175, 176, 177, 179, 180, 181, 217, 219, 220, 223, 224, 225, 226, 227, 228, 234 }

B grade { 61, 62, 63, 64, 135, 158, 159, 160, 162, 163, 164, 166, 167, 168, 171, 172, 173, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 218, 221, 222, 229, 230, 233 }

C grade { 80, 81, 82, 83, 84, 231, 232 }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 65, 66, 77, 88, 98, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 124, 126, 127, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154 }

F(-1) timedout fail { }

F(-2) exception fail { 128 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 39, 41, 43, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 109, 123, 125, 129, 130, 131, 132, 134, 135, 149, 150, 157, 170, 192, 193, 194, 196, 197, 198, 209, 210, 211, 213, 214, 215, 218, 221, 224, 227, 228, 229, 230, 231, 232, 233, 234 }

B grade { 7, 77, 88, 98, 107, 108, 110, 122, 124, 126, 127, 160, 162, 163, 164, 166, 167, 168, 173 }

C grade { }

F normal fail { 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 38, 40, 42, 44, 47, 49, 65, 66, 97, 111, 112, 114, 115, 118, 119, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154, 158, 159, 171, 172, 183, 184, 185, 186, 188, 189, 190, 200, 201, 202, 203, 205, 206, 207, 217, 219, 220, 222, 223, 225, 226 }

F(-1) timedout fail { 33, 174 }

F(-2) exception fail { 59, 61, 62, 63, 64, 113, 128, 165, 169, 175, 176, 177, 178, 179, 180, 181, 182 }

Giac

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 20, 22, 31, 41, 43, 45, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 126, 150, 157, 170, 227, 228, 229, 230, 233, 234 }

B grade { 6, 99, 100, 101, 102, 104, 105, 106, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 149 }

C grade { }

F normal fail { 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 36, 37, 38, 39, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 65, 66, 72, 73, 77, 97, 98, 103, 108, 109, 114, 115, 118, 119, 133, 136, 137, 138, 139, 141, 142, 143, 146, 152, 153, 154, 158, 159, 160, 162, 163, 164, 166, 167, 168, 171, 172, 173, 175, 176, 177, 179, 180, 181, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 231, 232 }

F(-1) timedout fail { 107, 110, 113, 140, 144, 145, 200 }

F(-2) exception fail { 111, 112 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 17, 20, 22, 37, 39, 41, 43, 45, 51, 52, 53, 54, 55, 56, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 99, 100, 101, 102, 104, 105, 106, 122, 123, 125, 129, 130, 131, 132, 134, 135, 138, 149, 150, 157, 170, 227, 228, 229, 230, 231, 232, 233, 234 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 13, 15, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 88, 92, 97, 98, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 124, 126, 127, 133, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154, 158, 159, 160, 162, 163, 164, 166, 167, 168, 171, 172, 173, 175, 176, 177, 179, 180, 181, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 39, 41, 43, 45, 51, 52, 53, 54, 55, 56, 70, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94, 96, 99, 100, 101, 102, 123, 132, 149, 170, 227, 228 }

B grade { 71, 89, 90, 91, 95, 157 }

C grade { 104, 105, 106, 122, 125, 130, 131 }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 46, 47, 48, 49, 50, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 77, 88, 97, 98, 114, 124, 126, 127, 137, 138, 142, 143, 152, 153, 154, 159, 160, 173, 183, 184, 185, 186, 190, 200, 201, 202, 203, 207, 218, 219, 220, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 234 }

F(-1) timedout fail { 58, 103, 107, 108, 109, 110, 111, 112, 113, 115, 118, 119, 129, 133, 134, 135, 136, 139, 140, 141, 144, 145, 146, 147, 148, 150, 158, 161, 165, 169, 171, 172, 174, 178, 182, 188, 189, 191, 195, 199, 204, 205, 206, 212, 216, 217, 233 }

F(-2) exception fail { 72, 162, 163, 164, 166, 167, 168, 175, 176, 177, 179, 180, 181, 192, 193, 194, 196, 197, 198, 209, 210, 211, 213, 214, 215 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	47	41	48	59	55
N.S.	1	1.00	1.00	0.86	0.92	0.80	0.94	1.16	1.08
time (sec)	N/A	0.017	0.004	0.170	0.264	0.259	0.292	0.276	0.948

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	46	45	46	74	56
N.S.	1	1.00	1.00	0.94	0.94	0.92	0.94	1.51	1.14
time (sec)	N/A	0.026	0.011	0.124	0.179	0.260	0.257	0.269	0.873

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	36	38	32	39	51	46
N.S.	1	1.00	1.00	0.88	0.93	0.78	0.95	1.24	1.12
time (sec)	N/A	0.015	0.003	0.164	0.287	0.260	0.228	0.275	0.859

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	36	37	37	64	49
N.S.	1	1.00	1.00	0.95	0.92	0.95	0.95	1.64	1.26
time (sec)	N/A	0.019	0.008	0.108	0.180	0.294	0.193	0.266	0.806

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	28	23	31	36	39
N.S.	1	1.00	1.00	0.81	0.90	0.74	1.00	1.16	1.26
time (sec)	N/A	0.009	0.003	0.146	0.274	0.301	0.165	0.290	0.805

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	24	24	24	45	22
N.S.	1	1.00	1.00	0.96	1.00	1.00	1.00	1.88	0.92
time (sec)	N/A	0.005	0.002	0.103	0.195	0.254	0.087	0.273	0.151

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	33	56	0	0	38	0
N.S.	1	1.00	1.00	0.89	1.51	0.00	0.00	1.03	0.00
time (sec)	N/A	0.018	0.004	0.126	0.305	0.000	0.000	0.277	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	30	31	24	32	28
N.S.	1	1.00	1.00	0.97	1.00	1.03	0.80	1.07	0.93
time (sec)	N/A	0.013	0.003	0.099	0.185	0.256	0.102	0.270	0.238

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	36	26	23	24	24	40	44
N.S.	1	1.00	1.16	0.84	0.74	0.77	0.77	1.29	1.42
time (sec)	N/A	0.011	0.004	0.153	0.303	0.253	0.177	0.277	0.815

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	42	42	43	39	44	58
N.S.	1	1.00	0.96	0.91	0.91	0.93	0.85	0.96	1.26
time (sec)	N/A	0.020	0.010	0.122	0.186	0.254	0.201	0.271	0.927

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	36	35	37	33	32	51	47
N.S.	1	1.00	0.88	0.85	0.90	0.80	0.78	1.24	1.15
time (sec)	N/A	0.013	0.003	0.171	0.271	0.250	0.221	0.275	0.837

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	79	90	95	78	104	0	85
N.S.	1	1.00	0.76	0.87	0.91	0.75	1.00	0.00	0.82
time (sec)	N/A	0.157	0.019	0.279	0.308	0.277	0.357	0.000	0.952

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	95	205	0	0	0	0	0
N.S.	1	1.00	0.70	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.402	0.487	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	61	70	77	60	78	0	66
N.S.	1	1.00	0.76	0.88	0.96	0.75	0.98	0.00	0.82
time (sec)	N/A	0.105	0.016	0.291	0.280	0.260	0.266	0.000	0.219

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	76	185	0	0	0	0	0
N.S.	1	1.00	0.68	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.211	0.464	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	44	57	40	54	0	44
N.S.	1	1.00	0.79	0.83	1.08	0.75	1.02	0.00	0.83
time (sec)	N/A	0.052	0.012	0.260	0.282	0.258	0.196	0.000	0.157

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	130	0	0	0	0	55
N.S.	1	1.00	0.84	1.94	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.053	0.067	0.499	0.000	0.000	0.000	0.000	0.709

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	116	132	891	0	0	0	0	0
N.S.	1	1.00	1.14	7.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.047	5.668	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	222	0	0	0	0	0
N.S.	1	1.00	0.97	3.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.033	0.573	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	64	56	57	53	60	50
N.S.	1	1.00	0.95	1.08	0.95	0.97	0.90	1.02	0.85
time (sec)	N/A	0.066	0.015	0.237	0.275	0.266	0.182	0.282	0.753

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	96	251	0	0	0	0	0
N.S.	1	1.00	0.85	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	0.187	0.718	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	81	85	95	78	80	91	73
N.S.	1	1.00	0.91	0.96	1.07	0.88	0.90	1.02	0.82
time (sec)	N/A	0.113	0.015	0.287	0.306	0.283	0.232	0.274	0.810

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	125	1104	0	0	0	0	0
N.S.	1	1.00	0.64	5.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.487	0.519	29.340	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	205	205	187	1108	0	0	0	0	0
N.S.	1	1.00	0.91	5.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.369	0.553	8.098	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	96	857	0	0	0	0	0
N.S.	1	1.00	0.65	5.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	0.284	6.969	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	157	157	152	1036	0	0	0	0	0
N.S.	1	1.00	0.97	6.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.358	5.939	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	76	592	0	0	0	0	0
N.S.	1	1.00	0.74	5.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.068	5.903	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	90	187	0	0	0	0	0
N.S.	1	1.00	0.94	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.087	1.154	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	178	178	180	982	0	0	0	0	0
N.S.	1	1.00	1.01	5.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.064	6.607	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	93	93	83	1441	0	0	0	0	0
N.S.	1	1.00	0.89	15.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	0.053	8.941	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	105	90	2956	0	0	0	29	0
N.S.	1	1.00	0.86	28.15	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.145	0.123	6.764	0.000	0.000	0.000	0.284	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	167	167	151	1622	0	0	0	0	0
N.S.	1	1.00	0.90	9.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.194	25.043	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	152	152	126	852	0	0	0	0	0
N.S.	1	1.00	0.83	5.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.221	10.742	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	221	12	10	12	12
N.S.	1	1.00	1.20	1.00	22.10	1.20	1.00	1.20	1.20
time (sec)	N/A	0.010	0.642	0.771	2.291	0.267	1.954	0.291	0.691

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	183	12	10	12	12
N.S.	1	1.00	1.20	1.00	18.30	1.20	1.00	1.20	1.20
time (sec)	N/A	0.009	0.642	0.527	1.569	0.259	0.979	0.297	0.695

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	52	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.015	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	32	34	35	31	34	0	32
N.S.	1	1.00	0.80	0.85	0.88	0.78	0.85	0.00	0.80
time (sec)	N/A	0.074	0.021	0.480	0.278	0.271	0.138	0.000	0.740

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	126	0	0	0	0	0
N.S.	1	1.00	0.76	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.185	0.738	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	24	19	19	0	19
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.00	0.83
time (sec)	N/A	0.036	0.014	0.417	0.263	0.256	0.099	0.000	0.703

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	39	98	0	0	0	0	0
N.S.	1	1.00	0.81	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.068	0.353	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	8	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	1.00	0.75
time (sec)	N/A	0.009	0.004	0.517	0.179	0.250	0.319	0.265	0.681

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	117	0	0	0	0	0
N.S.	1	1.00	0.88	2.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.103	0.343	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	32	29	29	22	26	26
N.S.	1	1.00	1.00	1.07	0.97	0.97	0.73	0.87	0.87
time (sec)	N/A	0.042	0.012	0.354	0.265	0.255	0.140	0.272	0.098

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	178	0	0	0	0	0
N.S.	1	1.00	0.76	2.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.168	0.746	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	55	47	42	39	35
N.S.	1	1.00	1.00	0.91	1.17	1.00	0.89	0.83	0.74
time (sec)	N/A	0.082	0.017	0.415	0.280	0.258	0.251	0.268	0.103

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	626	261	189	0	0	0	0
N.S.	1	1.00	3.04	1.27	0.92	0.00	0.00	0.00	0.00
time (sec)	N/A	0.426	1.192	0.917	0.298	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	309	134	0	0	0	0	0
N.S.	1	1.00	1.64	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	1.421	0.764	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	592	183	187	0	0	0	0
N.S.	1	1.00	3.23	1.00	1.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.609	1.168	0.301	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	328	197	0	0	0	0	0
N.S.	1	1.00	1.47	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.175	2.045	0.816	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	619	257	183	0	0	0	0
N.S.	1	1.00	2.92	1.21	0.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	1.424	0.845	0.317	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	8	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.60	1.00
time (sec)	N/A	0.014	0.060	0.286	0.177	0.247	0.092	0.268	0.709

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	17	15	13
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.31	1.15	1.00
time (sec)	N/A	0.017	0.008	0.645	0.187	0.272	0.688	0.269	0.696

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	212	245	226	237	367	347	234
N.S.	1	1.00	0.87	1.00	0.93	0.97	1.50	1.42	0.96
time (sec)	N/A	0.122	0.057	0.515	0.189	0.268	0.567	0.275	0.250

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	149	167	159	167	243	252	190
N.S.	1	1.00	0.89	0.99	0.95	0.99	1.45	1.50	1.13
time (sec)	N/A	0.089	0.044	0.454	0.215	0.262	0.414	0.278	1.154

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	97	105	103	108	151	171	116
N.S.	1	1.00	0.89	0.96	0.94	0.99	1.39	1.57	1.06
time (sec)	N/A	0.092	0.030	0.432	0.214	0.263	0.310	0.279	1.014

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	57	53	57	73	99	62
N.S.	1	1.00	1.16	0.98	0.91	0.98	1.26	1.71	1.07
time (sec)	N/A	0.047	0.010	0.182	0.219	0.256	0.222	0.270	0.830

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	716	392	528	0	0	0	0
N.S.	1	1.00	1.78	0.97	1.31	0.00	0.00	0.00	0.00
time (sec)	N/A	0.700	1.217	1.316	0.352	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	801	801	806	2141	628	0	0	0	0
N.S.	1	1.00	1.01	2.67	0.78	0.00	0.00	0.00	0.00
time (sec)	N/A	0.911	5.297	1.592	0.382	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	0	16	15	16	16
N.S.	1	1.00	1.12	0.88	0.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.014	4.298	0.765	0.000	0.260	4.389	0.286	0.725

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	16
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.016	2.879	0.710	0.919	0.265	1.440	0.286	0.704

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	169	0	0	349	0	59	0
N.S.	1	1.00	2.56	0.00	0.00	5.29	0.00	0.89	0.00
time (sec)	N/A	0.069	0.183	0.000	0.000	0.300	0.000	0.302	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	262	0	0	712	0	126	0
N.S.	1	1.00	1.96	0.00	0.00	5.31	0.00	0.94	0.00
time (sec)	N/A	0.236	0.457	0.000	0.000	0.315	0.000	0.303	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	208	345	0	0	1278	0	208	0
N.S.	1	1.00	1.66	0.00	0.00	6.14	0.00	1.00	0.00
time (sec)	N/A	0.677	0.632	0.000	0.000	0.366	0.000	0.306	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	450	0	0	1986	0	340	0
N.S.	1	1.00	1.54	0.00	0.00	6.78	0.00	1.16	0.00
time (sec)	N/A	0.842	0.931	0.000	0.000	0.613	0.000	0.312	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	136	117	0	0	0	0	0
N.S.	1	1.00	0.70	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.997	0.870	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	89	99	0	0	0	0	0
N.S.	1	1.00	0.57	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.130	0.773	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	21	55	31	29	0	33	0
N.S.	1	1.00	0.60	1.57	0.89	0.83	0.00	0.94	0.00
time (sec)	N/A	0.017	0.032	0.568	0.270	0.298	0.000	0.281	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	37	101	63	52	0	55	0
N.S.	1	1.00	0.47	1.28	0.80	0.66	0.00	0.70	0.00
time (sec)	N/A	0.034	0.042	1.299	0.274	0.262	0.000	0.301	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	47	130	93	70	0	83	0
N.S.	1	1.00	0.40	1.10	0.79	0.59	0.00	0.70	0.00
time (sec)	N/A	0.054	0.046	1.407	0.271	0.270	0.000	0.300	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	25	24	26	21	31	32	22
N.S.	1	1.00	0.78	0.75	0.81	0.66	0.97	1.00	0.69
time (sec)	N/A	0.019	0.015	0.335	0.296	0.253	0.185	0.277	0.080

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	37	39	39	88	40	27
N.S.	1	1.00	0.82	0.84	0.89	0.89	2.00	0.91	0.61
time (sec)	N/A	0.022	0.019	0.359	0.264	0.255	0.257	0.277	0.734

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	35	38	26	0	0	22
N.S.	1	1.00	0.82	1.03	1.12	0.76	0.00	0.00	0.65
time (sec)	N/A	0.012	0.012	0.748	0.277	0.282	0.000	0.000	0.734

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	46	65	75	40	0	0	51
N.S.	1	1.00	0.82	1.16	1.34	0.71	0.00	0.00	0.91
time (sec)	N/A	0.034	0.020	0.964	0.283	0.257	0.000	0.000	0.072

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	39	38	39	39	40	35
N.S.	1	1.00	1.00	0.95	0.93	0.95	0.95	0.98	0.85
time (sec)	N/A	0.019	0.012	0.241	0.208	0.279	0.357	0.272	0.769

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	34	27	36	38	31
N.S.	1	1.00	1.00	0.84	0.92	0.73	0.97	1.03	0.84
time (sec)	N/A	0.016	0.006	0.286	0.291	0.263	0.274	0.269	0.709

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	28	28	31	47	27
N.S.	1	1.00	1.00	0.90	0.90	0.90	1.00	1.52	0.87
time (sec)	N/A	0.007	0.006	0.162	0.209	0.247	0.139	0.267	0.707

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	57	68	0	0	0	0
N.S.	1	1.00	1.00	1.54	1.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.008	0.276	0.331	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	32	37	29	32	31
N.S.	1	1.00	1.00	0.91	0.94	1.09	0.85	0.94	0.91
time (sec)	N/A	0.013	0.006	0.129	0.214	0.279	0.213	0.276	0.769

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	38	31	27	28	29	29	32
N.S.	1	1.00	1.09	0.89	0.77	0.80	0.83	0.83	0.91
time (sec)	N/A	0.014	0.006	0.191	0.261	0.301	0.273	0.280	0.727

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	136	112	137	122	136	156	54
N.S.	1	1.00	0.89	0.74	0.90	0.80	0.89	1.03	0.36
time (sec)	N/A	0.077	0.033	0.395	0.290	0.272	9.749	0.286	0.484

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	133	109	135	111	126	153	52
N.S.	1	1.00	0.89	0.73	0.90	0.74	0.84	1.02	0.35
time (sec)	N/A	0.070	0.026	0.267	0.285	0.266	5.369	0.288	0.790

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	102	97	120	96	109	144	42
N.S.	1	1.00	0.77	0.73	0.91	0.73	0.83	1.09	0.32
time (sec)	N/A	0.059	0.034	0.261	0.282	0.274	2.978	0.278	0.139

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	105	98	123	107	114	135	44
N.S.	1	1.00	0.78	0.73	0.91	0.79	0.84	1.00	0.33
time (sec)	N/A	0.062	0.032	0.220	0.262	0.278	6.182	0.285	0.775

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	146	106	133	129	131	149	52
N.S.	1	1.00	0.97	0.71	0.89	0.86	0.87	0.99	0.35
time (sec)	N/A	0.068	0.038	0.300	0.270	0.263	12.597	0.282	0.845

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	32	31	27	39	33	31
N.S.	1	1.00	0.78	0.63	0.61	0.53	0.76	0.65	0.61
time (sec)	N/A	0.010	0.012	0.039	0.288	0.252	0.972	0.280	0.760

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	33	27	26	20	32	28	26
N.S.	1	1.00	0.79	0.64	0.62	0.48	0.76	0.67	0.62
time (sec)	N/A	0.007	0.010	0.039	0.264	0.270	0.525	0.285	0.763

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	12	19	14	16
N.S.	1	1.00	1.00	0.77	0.73	0.55	0.86	0.64	0.73
time (sec)	N/A	0.004	0.014	0.057	0.269	0.254	0.344	0.268	0.700

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	61	35	0	0	19	0
N.S.	1	1.00	1.00	1.97	1.13	0.00	0.00	0.61	0.00
time (sec)	N/A	0.025	0.006	0.224	0.299	0.000	0.000	0.284	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	29	18	17	19	92	19	17
N.S.	1	1.00	1.26	0.78	0.74	0.83	4.00	0.83	0.74
time (sec)	N/A	0.009	0.009	0.044	0.265	0.266	0.492	0.277	0.742

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	34	27	26	27	160	26	24
N.S.	1	1.00	0.81	0.64	0.62	0.64	3.81	0.62	0.57
time (sec)	N/A	0.009	0.009	0.047	0.268	0.252	1.139	0.275	0.760

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	29	25	24	24	85	39	24
N.S.	1	1.00	0.81	0.69	0.67	0.67	2.36	1.08	0.67
time (sec)	N/A	0.011	0.013	0.042	0.183	0.270	0.847	0.258	0.729

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	19	24	30	0
N.S.	1	1.00	0.86	0.69	0.66	0.66	0.83	1.03	0.00
time (sec)	N/A	0.008	0.010	0.038	0.190	0.265	0.486	0.291	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	17	18	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.94	1.00	0.78
time (sec)	N/A	0.006	0.010	0.039	0.209	0.259	0.111	0.284	0.886

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	25	20	16	20
N.S.	1	1.00	1.00	0.86	0.82	1.14	0.91	0.73	0.91
time (sec)	N/A	0.006	0.010	0.040	0.201	0.264	0.331	0.265	0.752

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	26	25	33	143	23	27
N.S.	1	1.00	0.78	0.70	0.68	0.89	3.86	0.62	0.73
time (sec)	N/A	0.010	0.014	0.049	0.199	0.275	1.138	0.274	0.751

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	13	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.76	0.88
time (sec)	N/A	0.004	0.001	0.192	0.203	0.251	0.066	0.264	0.060

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	40	83	0	63	0	0	0
N.S.	1	1.00	0.85	1.77	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.028	0.014	0.543	0.000	0.287	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	57	68	0	0	0	0
N.S.	1	1.00	1.00	1.54	1.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.007	0.418	0.334	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	95	131	104	92	155	617	133
N.S.	1	1.00	0.90	1.24	0.98	0.87	1.46	5.82	1.25
time (sec)	N/A	0.078	0.061	0.295	0.294	0.263	0.381	0.728	0.922

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	114	102	85	73	117	423	101
N.S.	1	1.00	1.42	1.28	1.06	0.91	1.46	5.29	1.26
time (sec)	N/A	0.063	0.032	0.243	0.306	0.272	0.309	0.633	1.525

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	90	63	68	55	78	210	61
N.S.	1	1.00	1.50	1.05	1.13	0.92	1.30	3.50	1.02
time (sec)	N/A	0.042	0.027	0.262	0.269	0.278	0.256	0.344	1.027

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	44	30	29	43	46	111	42
N.S.	1	1.00	1.33	0.91	0.88	1.30	1.39	3.36	1.27
time (sec)	N/A	0.009	0.010	0.207	0.181	0.278	0.139	0.321	1.333

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	256	103	133	0	0	0	0
N.S.	1	1.00	2.13	0.86	1.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.153	0.389	0.338	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	66	61	77	64	167	498	62
N.S.	1	1.00	1.06	0.98	1.24	1.03	2.69	8.03	1.00
time (sec)	N/A	0.030	0.050	0.240	0.289	0.282	0.539	0.417	1.416

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	92	83	112	99	381	1309	230
N.S.	1	1.00	0.97	0.87	1.18	1.04	4.01	13.78	2.42
time (sec)	N/A	0.065	0.087	0.310	0.290	0.289	0.798	0.582	1.631

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	126	115	165	142	760	3449	285
N.S.	1	1.00	0.98	0.89	1.28	1.10	5.89	26.74	2.21
time (sec)	N/A	0.097	0.108	0.305	0.273	0.272	1.184	1.667	1.490

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	563	591	8519	0	0	0	0
N.S.	1	1.00	0.88	0.92	13.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.642	0.413	2.008	4.025	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	325	187	283	0	0	0	0
N.S.	1	1.00	2.14	1.23	1.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.200	0.788	0.359	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	422	514	296	280	0	0	0	0
N.S.	1	1.25	1.52	0.88	0.83	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	7.625	0.672	0.371	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	735	818	930	728	8518	0	0	0	0
N.S.	1	1.11	1.27	0.99	11.59	0.00	0.00	0.00	0.00
time (sec)	N/A	1.233	0.699	2.795	0.931	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	693	693	618	364	0	0	0	0	0
N.S.	1	1.00	0.89	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.529	0.538	0.427	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	830	830	809	388	0	0	0	0	0
N.S.	1	1.00	0.97	0.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.727	0.528	0.471	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-2)	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	0	959	0	0	0	0	0
N.S.	1	1.00	0.00	2.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.514	0.000	2.493	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	127	118	0	0	0	0	0
N.S.	1	1.00	0.96	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.133	1.284	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	138	156	0	0	0	0	0
N.S.	1	1.00	0.64	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	0.088	1.305	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	177	26	28	28	29	28	28
N.S.	1	1.00	6.32	0.93	1.00	1.00	1.04	1.00	1.00
time (sec)	N/A	0.028	0.302	0.747	0.275	0.265	0.801	0.440	0.820

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	180	31	33	33	31	33	33
N.S.	1	1.00	5.45	0.94	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.039	0.141	0.783	0.297	0.285	5.943	0.451	0.838

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	202	160	0	0	0	0	0
N.S.	1	1.00	1.08	0.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	1.332	1.520	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	207	202	0	0	0	0	0
N.S.	1	1.00	0.74	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	1.030	1.433	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	198	33	35	44	36	35	35
N.S.	1	1.00	5.66	0.94	1.00	1.26	1.03	1.00	1.00
time (sec)	N/A	0.092	0.741	0.884	0.357	0.284	3.454	0.496	0.885

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	200	38	40	49	37	40	40
N.S.	1	1.00	5.00	0.95	1.00	1.22	0.92	1.00	1.00
time (sec)	N/A	0.129	0.322	0.853	0.373	0.265	24.399	0.513	0.941

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	42	93	81	99	203	85
N.S.	1	1.00	0.81	0.81	1.79	1.56	1.90	3.90	1.63
time (sec)	N/A	0.028	0.017	0.433	0.255	0.266	0.620	0.374	0.170

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	141	36	52	33	56	100	49
N.S.	1	1.00	3.62	0.92	1.33	0.85	1.44	2.56	1.26
time (sec)	N/A	0.015	0.044	0.247	0.278	0.261	0.380	0.327	1.741

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	55	112	0	0	100	0
N.S.	1	1.00	0.84	1.22	2.49	0.00	0.00	2.22	0.00
time (sec)	N/A	0.029	0.007	0.475	0.328	0.000	0.000	0.532	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	41	53	59	139	238	57
N.S.	1	1.00	0.89	0.87	1.13	1.26	2.96	5.06	1.21
time (sec)	N/A	0.023	0.014	0.467	0.197	0.265	1.333	0.333	0.931

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	37	64	0	0	26	0
N.S.	1	1.00	1.00	1.06	1.83	0.00	0.00	0.74	0.00
time (sec)	N/A	0.028	0.006	0.564	0.318	0.000	0.000	0.297	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	55	122	0	0	103	0
N.S.	1	1.00	0.84	1.22	2.71	0.00	0.00	2.29	0.00
time (sec)	N/A	0.034	0.007	0.563	0.320	0.000	0.000	0.558	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	0	0	17	18	18
N.S.	1	1.00	1.11	0.89	0.00	0.00	0.94	1.00	1.00
time (sec)	N/A	0.013	4.040	0.446	0.000	0.000	3.132	0.304	1.148

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	157	496	341	325	0	2265	783
N.S.	1	1.00	0.67	2.13	1.46	1.39	0.00	9.72	3.36
time (sec)	N/A	0.271	0.201	0.883	0.276	0.280	0.000	1.731	1.367

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	118	294	216	206	376	1161	409
N.S.	1	1.00	0.77	1.91	1.40	1.34	2.44	7.54	2.66
time (sec)	N/A	0.146	0.114	0.676	0.297	0.279	125.905	1.231	1.179

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	163	113	113	110	177	451	136
N.S.	1	1.00	1.68	1.16	1.16	1.13	1.82	4.65	1.40
time (sec)	N/A	0.088	0.070	0.353	0.271	0.273	14.479	0.415	1.583

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	49	35	34	52	51	116	48
N.S.	1	1.00	1.29	0.92	0.89	1.37	1.34	3.05	1.26
time (sec)	N/A	0.016	0.010	0.297	0.182	0.268	0.165	0.321	1.498

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	336	197	0	0	0	0	0
N.S.	1	1.00	2.07	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.328	0.753	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	118	161	177	223	0	1264	128
N.S.	1	1.00	0.77	1.05	1.16	1.46	0.00	8.26	0.84
time (sec)	N/A	0.088	0.162	0.717	0.266	0.408	0.000	0.689	2.356

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	180	245	410	728	0	6173	399
N.S.	1	1.00	0.79	1.07	1.80	3.19	0.00	27.07	1.75
time (sec)	N/A	0.208	0.473	1.422	0.273	1.206	0.000	2.130	8.331

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	586	1072	0	0	0	0	0
N.S.	1	1.00	1.53	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.416	6.667	1.340	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	286	427	0	0	0	0	0
N.S.	1	1.00	1.30	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	1.054	0.884	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	118	184	0	0	0	0	123
N.S.	1	1.00	1.16	1.80	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.082	0.150	0.843	0.000	0.000	0.000	0.000	1.167

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	261	261	0	1911	0	0	0	0	0
N.S.	1	1.00	0.00	7.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	0.000	14.056	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	567	567	454	784	0	0	0	0	0
N.S.	1	1.00	0.80	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.027	6.116	4.178	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	565	565	2309	6248	0	0	0	0	0
N.S.	1	1.00	4.09	11.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	14.560	101.639	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	630	1051	0	0	0	0	0
N.S.	1	1.00	1.87	3.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	5.420	13.322	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	228	395	0	0	0	0	0
N.S.	1	1.00	1.59	2.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.256	2.461	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	372	372	0	3903	0	0	0	0	0
N.S.	1	1.00	0.00	10.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.000	16.365	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	1233	1233	0	4229	0	0	0	0	0
N.S.	1	1.00	0.00	3.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.647	0.000	28.029	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	162	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.304	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	618	36	0	22	22
N.S.	1	1.00	1.10	1.00	30.90	1.80	0.00	1.10	1.10
time (sec)	N/A	0.045	4.821	0.469	5.113	0.260	0.000	0.660	0.843

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	880	52	0	22	22
N.S.	1	1.00	1.10	1.00	44.00	2.60	0.00	1.10	1.10
time (sec)	N/A	0.044	0.520	0.551	7.835	0.283	0.000	0.721	0.796

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	37	35	51	60	127	230
N.S.	1	1.00	0.88	0.88	0.83	1.21	1.43	3.02	5.48
time (sec)	N/A	0.032	0.014	0.340	0.194	0.271	0.568	0.403	0.940

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	40	149	38	56	0	60	58
N.S.	1	1.00	0.89	3.31	0.84	1.24	0.00	1.33	1.29
time (sec)	N/A	0.034	0.027	6.300	0.197	0.282	0.000	0.348	1.726

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	34	38	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.85	0.95	0.98
time (sec)	N/A	0.034	0.151	1.408	0.884	0.274	3.353	0.416	1.195

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	488	488	0	1640	0	0	0	0	0
N.S.	1	1.00	0.00	3.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	0.000	1.319	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	0	903	0	0	0	0	0
N.S.	1	1.00	0.00	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.000	0.701	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	93	363	0	0	0	0	0
N.S.	1	1.00	0.95	3.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.029	0.431	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	38	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.95	0.98
time (sec)	N/A	0.033	0.220	0.767	0.449	0.260	2.832	0.378	0.892

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	163	91	126	38	39
N.S.	1	1.00	1.05	0.90	4.08	2.28	3.15	0.95	0.98
time (sec)	N/A	0.030	1.029	0.909	0.500	0.275	6.396	0.530	1.693

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	20	17	15	48	30	21
N.S.	1	1.00	1.12	1.25	1.06	0.94	3.00	1.88	1.31
time (sec)	N/A	0.007	0.008	0.304	0.195	0.235	0.069	0.274	0.087

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	403	403	371	8038	0	1965	0	0	0
N.S.	1	1.00	0.92	19.95	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	0.379	2.459	43.734	0.000	0.323	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	305	305	281	7646	0	1545	0	0	0
N.S.	1	1.00	0.92	25.07	0.00	5.07	0.00	0.00	0.00
time (sec)	N/A	0.300	2.238	4.089	0.000	0.327	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	549	1138	433	1101	0	0	0
N.S.	1	1.00	2.77	5.75	2.19	5.56	0.00	0.00	0.00
time (sec)	N/A	0.175	0.542	4.354	0.324	0.339	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	1.13	1.13
time (sec)	N/A	0.099	0.332	0.139	227.568	0.258	0.000	1.510	0.900

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	154	136	1448	310	321	0	0	0
N.S.	1	1.00	0.88	9.40	2.01	2.08	0.00	0.00	0.00
time (sec)	N/A	0.178	0.181	2.878	0.210	0.274	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1413	218	270	0	0	0
N.S.	1	1.00	0.89	11.49	1.77	2.20	0.00	0.00	0.00
time (sec)	N/A	0.148	0.081	2.549	0.202	0.273	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	967	562	455	199	0	0	0
N.S.	1	1.00	11.38	6.61	5.35	2.34	0.00	0.00	0.00
time (sec)	N/A	0.086	4.192	2.062	0.286	0.282	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	36	0	21	22
N.S.	1	1.00	1.10	0.90	0.00	1.71	0.00	1.00	1.05
time (sec)	N/A	0.068	0.276	0.205	0.000	0.246	0.000	1.106	1.054

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	155	141	1449	312	321	0	0	0
N.S.	1	1.00	0.91	9.35	2.01	2.07	0.00	0.00	0.00
time (sec)	N/A	0.164	0.187	3.056	0.204	0.267	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	124	111	1414	220	270	0	0	0
N.S.	1	1.00	0.90	11.40	1.77	2.18	0.00	0.00	0.00
time (sec)	N/A	0.145	0.067	2.635	0.201	0.268	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	847	595	450	201	0	0	0
N.S.	1	1.00	9.85	6.92	5.23	2.34	0.00	0.00	0.00
time (sec)	N/A	0.087	2.352	2.142	0.294	0.263	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	36	0	22	22
N.S.	1	1.00	1.09	0.91	0.00	1.64	0.00	1.00	1.00
time (sec)	N/A	0.138	0.693	0.228	0.000	0.259	0.000	1.151	1.237

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	17	10	10	19	10	16
N.S.	1	1.00	1.12	1.06	0.62	0.62	1.19	0.62	1.00
time (sec)	N/A	0.005	0.006	0.654	0.175	0.214	0.060	0.274	0.799

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	399	399	360	7868	0	1589	0	0	0
N.S.	1	1.00	0.90	19.72	0.00	3.98	0.00	0.00	0.00
time (sec)	N/A	0.376	2.325	53.611	0.000	0.455	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	303	303	275	7488	0	1289	0	0	0
N.S.	1	1.00	0.91	24.71	0.00	4.25	0.00	0.00	0.00
time (sec)	N/A	0.298	1.892	4.436	0.000	0.403	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	1649	1146	532	965	0	0	0
N.S.	1	1.00	8.33	5.79	2.69	4.87	0.00	0.00	0.00
time (sec)	N/A	0.174	12.133	4.579	0.355	0.399	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	0	17	0	17	17
N.S.	1	1.00	1.13	1.00	0.00	1.13	0.00	1.13	1.13
time (sec)	N/A	0.099	0.300	0.194	0.000	0.257	0.000	3.495	0.908

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	154	140	1448	0	174	0	0	0
N.S.	1	1.00	0.91	9.40	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.183	0.161	3.115	0.000	0.276	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1413	0	152	0	0	0
N.S.	1	1.00	0.89	11.49	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.162	0.093	2.616	0.000	0.272	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	929	587	0	116	0	0	0
N.S.	1	1.00	10.93	6.91	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.096	4.892	1.566	0.000	0.261	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	26	0	42	0	28	25
N.S.	1	1.00	1.10	1.24	0.00	2.00	0.00	1.33	1.19
time (sec)	N/A	0.092	0.326	0.099	0.000	0.254	0.000	2.381	1.434

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	155	136	1449	0	174	0	0	0
N.S.	1	1.00	0.88	9.35	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.183	0.165	3.421	0.000	0.271	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	124	110	1414	0	152	0	0	0
N.S.	1	1.00	0.89	11.40	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.157	0.091	2.865	0.000	0.262	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	872	630	0	116	0	0	0
N.S.	1	1.00	10.14	7.33	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.095	2.405	1.367	0.000	0.267	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	25	0	42	0	27	25
N.S.	1	1.00	1.09	1.14	0.00	1.91	0.00	1.23	1.14
time (sec)	N/A	0.095	0.365	0.059	0.000	0.272	0.000	2.724	1.544

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	299	600	3570	0	1460	0	0	0
N.S.	1	1.00	2.01	11.94	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	0.165	0.233	33.571	0.000	0.376	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	375	2668	0	1002	0	0	0
N.S.	1	1.00	1.64	11.65	0.00	4.38	0.00	0.00	0.00
time (sec)	N/A	0.124	0.135	24.430	0.000	0.338	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	159	159	237	1776	0	600	0	0	0
N.S.	1	1.00	1.49	11.17	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.080	0.095	2.572	0.000	0.311	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	91	184	0	334	0	0	0
N.S.	1	1.00	1.25	2.52	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.033	0.023	1.737	0.000	0.297	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	3	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	0.20	1.13
time (sec)	N/A	0.030	0.675	0.240	1.561	0.270	3.143	90.586	0.832

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	355	355	438	6916	0	1289	0	0	0
N.S.	1	1.00	1.23	19.48	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	0.334	0.347	42.430	0.000	0.343	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	267	267	330	6566	0	1067	0	0	0
N.S.	1	1.00	1.24	24.59	0.00	4.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.723	4.506	0.000	0.354	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	288	352	0	825	0	0	0
N.S.	1	1.00	1.66	2.02	0.00	4.74	0.00	0.00	0.00
time (sec)	N/A	0.170	0.275	3.026	0.000	0.403	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	1.13	1.13
time (sec)	N/A	0.083	3.872	0.155	1.047	0.244	0.000	0.578	0.883

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	142	134	1405	129	292	0	0	0
N.S.	1	1.00	0.94	9.89	0.91	2.06	0.00	0.00	0.00
time (sec)	N/A	0.165	0.147	2.531	1.275	0.267	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	113	103	1369	107	246	0	0	0
N.S.	1	1.00	0.91	12.12	0.95	2.18	0.00	0.00	0.00
time (sec)	N/A	0.133	0.059	1.962	1.382	0.288	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	544	80	186	0	0	0
N.S.	1	1.00	0.90	6.89	1.01	2.35	0.00	0.00	0.00
time (sec)	N/A	0.080	0.099	1.833	1.314	0.270	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	81	36	0	19	20
N.S.	1	1.00	1.11	0.89	4.26	1.89	0.00	1.00	1.05
time (sec)	N/A	0.082	3.107	0.105	0.642	0.250	0.000	0.370	1.063

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	145	134	1409	129	292	0	0	0
N.S.	1	1.00	0.92	9.72	0.89	2.01	0.00	0.00	0.00
time (sec)	N/A	0.158	0.158	2.473	1.441	0.276	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	116	103	1373	106	246	0	0	0
N.S.	1	1.00	0.89	11.84	0.91	2.12	0.00	0.00	0.00
time (sec)	N/A	0.132	0.087	2.032	1.347	0.270	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	517	80	186	0	0	0
N.S.	1	1.00	0.87	6.30	0.98	2.27	0.00	0.00	0.00
time (sec)	N/A	0.083	0.086	1.855	1.276	0.271	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	73	36	0	19	20
N.S.	1	1.00	1.09	0.91	3.32	1.64	0.00	0.86	0.91
time (sec)	N/A	0.086	3.190	0.128	0.628	0.250	0.000	0.375	1.055

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	299	600	3570	0	1460	0	0	0
N.S.	1	1.00	2.01	11.94	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	0.159	0.219	33.246	0.000	0.332	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	375	2668	0	1002	0	0	0
N.S.	1	1.00	1.64	11.65	0.00	4.38	0.00	0.00	0.00
time (sec)	N/A	0.123	0.134	21.014	0.000	0.330	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	159	159	237	1776	0	600	0	0	0
N.S.	1	1.00	1.49	11.17	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.074	0.100	2.316	0.000	0.321	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	91	184	0	334	0	0	0
N.S.	1	1.00	1.23	2.49	0.00	4.51	0.00	0.00	0.00
time (sec)	N/A	0.033	0.024	1.514	0.000	0.284	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	3	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	0.20	1.13
time (sec)	N/A	0.026	0.702	0.199	1.528	0.285	0.000	105.033	0.818

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	351	351	442	6844	0	1269	0	0	0
N.S.	1	1.00	1.26	19.50	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	0.333	0.359	39.007	0.000	0.358	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	265	265	334	6494	0	1051	0	0	0
N.S.	1	1.00	1.26	24.51	0.00	3.97	0.00	0.00	0.00
time (sec)	N/A	0.287	0.900	4.400	0.000	0.334	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	287	352	0	813	0	0	0
N.S.	1	1.00	1.65	2.02	0.00	4.67	0.00	0.00	0.00
time (sec)	N/A	0.168	0.306	2.918	0.000	0.391	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.081	3.839	0.142	1.054	0.261	170.894	0.544	0.859

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	142	134	1404	129	292	0	0	0
N.S.	1	1.00	0.94	9.89	0.91	2.06	0.00	0.00	0.00
time (sec)	N/A	0.161	0.155	1.944	1.303	0.263	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	113	103	1368	107	246	0	0	0
N.S.	1	1.00	0.91	12.11	0.95	2.18	0.00	0.00	0.00
time (sec)	N/A	0.143	0.073	1.542	1.339	0.261	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	544	80	186	0	0	0
N.S.	1	1.00	0.90	6.89	1.01	2.35	0.00	0.00	0.00
time (sec)	N/A	0.091	0.105	1.670	1.303	0.262	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	73	36	0	19	20
N.S.	1	1.00	1.11	0.89	3.84	1.89	0.00	1.00	1.05
time (sec)	N/A	0.064	3.090	0.123	0.620	0.255	0.000	0.337	1.019

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	145	134	1410	129	292	0	0	0
N.S.	1	1.00	0.92	9.72	0.89	2.01	0.00	0.00	0.00
time (sec)	N/A	0.171	0.155	2.308	1.308	0.272	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	116	103	1374	106	246	0	0	0
N.S.	1	1.00	0.89	11.84	0.91	2.12	0.00	0.00	0.00
time (sec)	N/A	0.142	0.076	1.911	1.285	0.269	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	517	80	186	0	0	0
N.S.	1	1.00	0.87	6.30	0.98	2.27	0.00	0.00	0.00
time (sec)	N/A	0.087	0.093	1.872	1.283	0.253	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	81	36	0	19	20
N.S.	1	1.00	1.09	0.91	3.68	1.64	0.00	0.86	0.91
time (sec)	N/A	0.087	3.140	0.135	0.612	0.254	0.000	0.337	1.006

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	187	187	132	566	0	250	0	0	0
N.S.	1	1.00	0.71	3.03	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.457	0.258	224.515	0.000	0.288	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	59	53	34	40	0	0	0
N.S.	1	1.00	1.69	1.51	0.97	1.14	0.00	0.00	0.00
time (sec)	N/A	0.019	0.019	0.820	0.308	0.284	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	58	50	0	65	0	0	0
N.S.	1	1.00	0.82	0.70	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.035	0.014	1.042	0.000	0.280	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	92	76	0	87	0	0	0
N.S.	1	1.00	0.89	0.74	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.054	0.014	1.694	0.000	0.255	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	83	95	63	103	0	0	0
N.S.	1	1.00	1.63	1.86	1.24	2.02	0.00	0.00	0.00
time (sec)	N/A	0.022	0.036	0.353	0.320	0.272	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	83	355	0	151	0	0	0
N.S.	1	1.00	0.81	3.45	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.046	0.013	0.802	0.000	0.356	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	133	413	0	187	0	0	0
N.S.	1	1.00	0.88	2.74	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.073	0.017	0.803	0.000	0.296	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	167	162	189	212	0	0	0
N.S.	1	1.00	0.85	0.83	0.96	1.08	0.00	0.00	0.00
time (sec)	N/A	0.118	0.114	1.250	0.339	0.283	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	250	678	0	304	0	0	0
N.S.	1	1.00	1.00	2.71	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	1.930	0.288	1.044	0.000	0.296	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	313	764	0	378	0	0	0
N.S.	1	1.00	1.00	2.44	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	1.738	0.228	1.543	0.000	0.303	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	19	28	19	21	22
N.S.	1	1.00	1.00	0.93	0.70	1.04	0.70	0.78	0.81
time (sec)	N/A	0.016	0.016	0.127	0.235	0.313	1.482	0.280	0.105

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	17	15	12	18	15
N.S.	1	1.00	1.00	0.82	1.00	0.88	0.71	1.06	0.88
time (sec)	N/A	0.029	0.091	0.299	0.230	0.264	0.217	0.279	0.140

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	47	47	61	1281	47	131	0	66	65
N.S.	1	1.00	1.30	27.26	1.00	2.79	0.00	1.40	1.38
time (sec)	N/A	0.058	0.060	1.248	0.310	0.276	0.000	0.293	0.877

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	103	146	1354	131	276	0	154	133
N.S.	1	1.00	1.42	13.15	1.27	2.68	0.00	1.50	1.29
time (sec)	N/A	0.117	0.084	6.524	0.331	0.296	0.000	0.298	1.009

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1323	167	258	0	0	164
N.S.	1	1.00	0.49	7.35	0.93	1.43	0.00	0.00	0.91
time (sec)	N/A	0.136	0.067	1.479	0.310	0.293	0.000	0.000	2.913

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1323	167	258	0	0	164
N.S.	1	1.00	0.49	7.35	0.93	1.43	0.00	0.00	0.91
time (sec)	N/A	0.134	0.065	1.320	0.309	0.285	0.000	0.000	2.739

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	103	145	855	169	221	0	154	135
N.S.	1	1.00	1.41	8.30	1.64	2.15	0.00	1.50	1.31
time (sec)	N/A	0.102	0.091	6.551	0.342	0.299	0.000	0.285	0.975

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	48	48	59	903	48	75	0	65	68
N.S.	1	1.00	1.23	18.81	1.00	1.56	0.00	1.35	1.42
time (sec)	N/A	0.061	0.053	1.352	0.310	0.284	0.000	0.284	0.863

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [50] had the largest ratio of [1.26699999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	8	0.375
2	A	4	3	1.00	8	0.375
3	A	4	3	1.00	8	0.375
4	A	4	3	1.00	8	0.375
5	A	3	3	1.00	6	0.500
6	A	2	2	1.00	4	0.500
7	A	3	2	1.00	8	0.250
8	A	5	5	1.00	8	0.625
9	A	3	3	1.00	8	0.375
10	A	4	3	1.00	8	0.375
11	A	4	3	1.00	8	0.375
12	A	15	7	1.00	10	0.700
13	A	14	9	1.00	10	0.900
14	A	10	7	1.00	10	0.700
15	A	9	8	1.00	10	0.800
16	A	5	5	1.00	8	0.625
17	A	5	5	1.00	6	0.833
18	A	6	5	1.00	10	0.500
19	A	4	4	1.00	10	0.400
20	A	8	7	1.00	10	0.700
21	A	8	7	1.00	10	0.700
22	A	13	8	1.00	10	0.800
23	A	33	11	1.00	10	1.100
24	A	22	11	1.00	10	1.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	18	10	1.00	10	1.000
26	A	11	9	1.00	10	0.900
27	A	8	8	1.00	8	1.000
28	A	5	6	1.00	6	1.000
29	A	8	6	1.00	10	0.600
30	A	5	6	1.00	10	0.600
31	A	7	6	1.00	10	0.600
32	A	14	11	1.00	10	1.100
33	A	16	8	1.00	10	0.800
34	N/A	0	0	1.00	10	0.000
35	N/A	0	0	1.00	10	0.000
36	A	2	2	1.00	8	0.250
37	A	9	7	1.00	13	0.538
38	A	8	8	1.00	13	0.615
39	A	4	4	1.00	13	0.308
40	A	4	4	1.00	11	0.364
41	A	1	1	1.00	10	0.100
42	A	3	3	1.00	13	0.231
43	A	7	7	1.00	13	0.538
44	A	7	7	1.00	13	0.538
45	A	12	8	1.00	13	0.615
46	A	28	15	1.00	15	1.000
47	A	10	5	1.00	13	0.385
48	A	25	13	1.00	12	1.083
49	A	15	7	1.00	15	0.467
50	A	31	19	1.00	15	1.267
51	A	1	1	1.00	12	0.083
52	A	1	1	1.00	12	0.083
53	A	4	4	1.00	14	0.286
54	A	4	4	1.00	14	0.286
55	A	5	5	1.00	14	0.357
56	A	5	4	1.00	12	0.333
57	A	27	13	1.00	14	0.929
58	A	24	12	1.00	14	0.857
59	N/A	0	0	1.00	16	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	N/A	0	0	1.00	16	0.000
61	A	5	6	1.00	16	0.375
62	A	7	9	1.00	16	0.562
63	A	8	9	1.00	16	0.562
64	A	8	9	1.00	16	0.562
65	A	3	3	1.00	14	0.214
66	A	2	2	1.00	14	0.143
67	A	1	1	1.00	14	0.071
68	A	2	2	1.00	14	0.143
69	A	3	2	1.00	14	0.143
70	A	3	3	1.00	11	0.273
71	A	4	3	1.00	11	0.273
72	A	2	2	1.00	10	0.200
73	A	4	4	1.00	12	0.333
74	A	4	3	1.00	10	0.300
75	A	4	4	1.00	10	0.400
76	A	2	2	1.00	8	0.250
77	A	4	3	1.00	10	0.300
78	A	5	5	1.00	10	0.500
79	A	4	4	1.00	10	0.400
80	A	11	8	1.00	10	0.800
81	A	11	8	1.00	10	0.800
82	A	10	7	1.00	6	1.167
83	A	10	7	1.00	10	0.700
84	A	11	8	1.00	10	0.800
85	A	6	4	1.00	10	0.400
86	A	5	4	1.00	8	0.500
87	A	4	4	1.00	6	0.667
88	A	4	3	1.00	10	0.300
89	A	4	4	1.00	10	0.400
90	A	5	4	1.00	10	0.400
91	A	3	2	1.00	12	0.167
92	A	3	2	1.00	12	0.167
93	A	2	2	1.00	12	0.167
94	A	4	4	1.00	12	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	2	1.00	12	0.167
96	A	3	3	1.00	4	0.750
97	A	4	3	1.00	10	0.300
98	A	4	3	1.00	10	0.300
99	A	7	6	1.00	10	0.600
100	A	7	6	1.00	10	0.600
101	A	7	6	1.00	8	0.750
102	A	3	3	1.00	6	0.500
103	A	5	5	1.00	10	0.500
104	A	7	7	1.00	10	0.700
105	A	8	7	1.00	10	0.700
106	A	8	7	1.00	10	0.700
107	A	15	5	1.00	16	0.312
108	A	5	5	1.00	14	0.357
109	A	37	10	1.25	16	0.625
110	A	57	11	1.11	16	0.688
111	A	55	16	1.00	18	0.889
112	A	65	19	1.00	18	1.056
113	A	12	8	1.00	19	0.421
114	A	2	2	1.00	28	0.071
115	A	3	3	1.00	33	0.091
116	N/A	0	0	1.00	28	0.000
117	N/A	0	0	1.00	33	0.000
118	A	4	4	1.00	35	0.114
119	A	5	5	1.00	40	0.125
120	N/A	0	0	1.00	35	0.000
121	N/A	0	0	1.00	40	0.000
122	A	5	4	1.00	14	0.286
123	A	4	4	1.00	12	0.333
124	A	4	3	1.00	14	0.214
125	A	6	6	1.00	14	0.429
126	A	5	4	1.00	12	0.333
127	A	5	4	1.00	19	0.210
128	N/A	0	0	1.00	18	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	7	6	1.00	18	0.333
130	A	7	6	1.00	18	0.333
131	A	7	6	1.00	16	0.375
132	A	4	3	1.00	10	0.300
133	A	5	5	1.00	18	0.278
134	A	8	8	1.00	18	0.444
135	A	9	8	1.00	18	0.444
136	A	16	13	1.00	20	0.650
137	A	13	10	1.00	18	0.556
138	A	6	6	1.00	12	0.500
139	A	2	2	1.00	20	0.100
140	A	25	25	1.00	20	1.250
141	A	21	14	1.00	20	0.700
142	A	15	11	1.00	18	0.611
143	A	6	7	1.00	12	0.583
144	A	2	2	1.00	20	0.100
145	A	35	22	1.00	20	1.100
146	A	6	4	1.00	18	0.222
147	N/A	0	0	1.00	20	0.000
148	N/A	0	0	1.00	20	0.000
149	A	4	4	1.00	12	0.333
150	A	4	4	1.00	14	0.286
151	N/A	0	0	1.00	40	0.000
152	A	9	7	1.00	40	0.175
153	A	7	6	1.00	40	0.150
154	A	4	4	1.00	38	0.105
155	N/A	0	0	1.00	40	0.000
156	N/A	0	0	1.00	40	0.000
157	A	2	2	1.00	7	0.286
158	A	11	6	1.00	15	0.400
159	A	9	5	1.00	13	0.385
160	A	7	4	1.00	11	0.364
161	N/A	0	0	1.00	15	0.000
162	A	7	7	1.00	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	A	6	6	1.00	19	0.316
164	A	5	5	1.00	17	0.294
165	N/A	0	0	1.00	21	0.000
166	A	7	7	1.00	22	0.318
167	A	6	6	1.00	20	0.300
168	A	5	5	1.00	18	0.278
169	N/A	0	0	1.00	22	0.000
170	A	2	2	1.00	7	0.286
171	A	11	6	1.00	15	0.400
172	A	9	5	1.00	13	0.385
173	A	7	4	1.00	11	0.364
174	N/A	0	0	1.00	15	0.000
175	A	7	7	1.00	21	0.333
176	A	6	6	1.00	19	0.316
177	A	5	5	1.00	17	0.294
178	N/A	0	0	1.00	21	0.000
179	A	7	7	1.00	22	0.318
180	A	6	6	1.00	20	0.300
181	A	5	5	1.00	18	0.278
182	N/A	0	0	1.00	22	0.000
183	A	12	6	1.00	15	0.400
184	A	10	6	1.00	15	0.400
185	A	8	5	1.00	13	0.385
186	A	6	4	1.00	7	0.571
187	N/A	0	0	1.00	15	0.000
188	A	11	6	1.00	15	0.400
189	A	9	5	1.00	13	0.385
190	A	7	4	1.00	11	0.364
191	N/A	0	0	1.00	15	0.000
192	A	7	7	1.00	19	0.368
193	A	6	6	1.00	17	0.353
194	A	5	5	1.00	15	0.333
195	N/A	0	0	1.00	19	0.000
196	A	7	7	1.00	22	0.318

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	6	6	1.00	20	0.300
198	A	5	5	1.00	18	0.278
199	N/A	0	0	1.00	22	0.000
200	A	12	6	1.00	15	0.400
201	A	10	6	1.00	15	0.400
202	A	8	5	1.00	13	0.385
203	A	6	4	1.00	7	0.571
204	N/A	0	0	1.00	15	0.000
205	A	11	6	1.00	15	0.400
206	A	9	5	1.00	13	0.385
207	A	7	4	1.00	11	0.364
208	N/A	0	0	1.00	15	0.000
209	A	7	7	1.00	19	0.368
210	A	6	6	1.00	17	0.353
211	A	5	5	1.00	15	0.333
212	N/A	0	0	1.00	19	0.000
213	A	7	7	1.00	22	0.318
214	A	6	6	1.00	20	0.300
215	A	5	5	1.00	18	0.278
216	N/A	0	0	1.00	22	0.000
217	A	13	9	1.00	24	0.375
218	A	4	3	1.00	4	0.750
219	A	7	4	1.00	6	0.667
220	A	9	5	1.00	8	0.625
221	A	4	3	1.00	8	0.375
222	A	7	4	1.00	10	0.400
223	A	9	5	1.00	12	0.417
224	A	6	6	1.00	12	0.500
225	A	25	8	1.00	14	0.571
226	A	29	9	1.00	16	0.562
227	A	5	6	1.00	10	0.600
228	A	1	1	1.00	19	0.053
229	A	5	5	1.00	20	0.250
230	A	8	7	1.00	20	0.350

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	13	10	1.00	20	0.500
232	A	13	10	1.00	20	0.500
233	A	8	7	1.00	20	0.350
234	A	5	5	1.00	20	0.250

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^5 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 51

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{x}{6a^5} - \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6 \cot^{-1}(ax) - \frac{\arctan(ax)}{6a^6}$$

```
[Out] 1/6*x/a^5-1/18*x^3/a^3+1/30*x^5/a+1/6*x^6*arccot(a*x)-1/6*arctan(a*x)/a^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4947, 308, 209}

$$\int x^5 \cot^{-1}(ax)\,dx = -\frac{\arctan(ax)}{6a^6} + \frac{x}{6a^5} - \frac{x^3}{18a^3} + \frac{1}{6}x^6 \cot^{-1}(ax) + \frac{x^5}{30a}$$

```
[In] Int[x^5*ArcCot[a*x],x]
```

```
[Out] x/(6*a^5) - x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCot[a*x])/6 - ArcTan[a*x]/(
6*a^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\cot^{-1}(ax)+\frac{1}{6}a\int\frac{x^6}{1+a^2x^2}\,dx\\
&= \frac{1}{6}x^6\cot^{-1}(ax)+\frac{1}{6}a\int\left(\frac{1}{a^6}-\frac{x^2}{a^4}+\frac{x^4}{a^2}-\frac{1}{a^6\left(1+a^2x^2\right)}\right)dx\\
&= \frac{x}{6a^5}-\frac{x^3}{18a^3}+\frac{x^5}{30a}+\frac{1}{6}x^6\cot^{-1}(ax)-\frac{\int\frac{1}{1+a^2x^2}\,dx}{6a^5}\\
&= \frac{x}{6a^5}-\frac{x^3}{18a^3}+\frac{x^5}{30a}+\frac{1}{6}x^6\cot^{-1}(ax)-\frac{\arctan(ax)}{6a^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^5\cot^{-1}(ax)\,dx=\frac{x}{6a^5}-\frac{x^3}{18a^3}+\frac{x^5}{30a}+\frac{1}{6}x^6\cot^{-1}(ax)-\frac{\arctan(ax)}{6a^6}$$

```
[In] Integrate[x^5*ArcCot[a*x],x]
```

```
[Out] x/(6*a^5) - x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCot[a*x])/6 - ArcTan[a*x]/(
6*a^6)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{18}+\frac{ax}{6}-\frac{\arctan(ax)}{6}}{a^6}$	44
default	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{18}+\frac{ax}{6}-\frac{\arctan(ax)}{6}}{a^6}$	44
parallelrisch	$\frac{15a^6x^6\operatorname{arccot}(ax)+3a^5x^5-5a^3x^3+15ax+15\operatorname{arccot}(ax)}{90a^6}$	45
parts	$\frac{x^6\operatorname{arccot}(ax)}{6}+\frac{a\left(\frac{\frac{1}{5}a^4x^5-\frac{1}{3}a^2x^3+x}{a^6}-\frac{\arctan(ax)}{a^7}\right)}{6}$	46
risch	$\frac{ix^6\ln(iax+1)}{12}-\frac{ix^6\ln(-iax+1)}{12}+\frac{x^6\pi}{12}+\frac{x^5}{30a}-\frac{x^3}{18a^3}+\frac{x}{6a^5}-\frac{\arctan(ax)}{6a^6}$	67

```
[In] int(x^5*arccot(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^6*(1/6*a^6*x^6*arccot(a*x)+1/30*a^5*x^5-1/18*a^3*x^3+1/6*a*x-1/6*arctan
(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^5\cot^{-1}(ax)\,dx=\frac{3\,a^5x^5-5\,a^3x^3+15\,ax+15\,(a^6x^6+1)\operatorname{arccot}(ax)}{90\,a^6}$$

```
[In] integrate(x^5*arccot(a*x),x, algorithm="fricas")
[Out] 1/90*(3*a^5*x^5 - 5*a^3*x^3 + 15*a*x + 15*(a^6*x^6 + 1)*arccot(a*x))/a^6
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int x^5\cot^{-1}(ax)\,dx=\begin{cases}\frac{x^6\operatorname{acot}(ax)}{6}+\frac{x^5}{30a}-\frac{x^3}{18a^3}+\frac{x}{6a^5}+\frac{\operatorname{acot}(ax)}{6a^6} & \text{for } a\neq 0\\ \frac{\pi x^6}{12} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*acot(a*x),x)
[Out] Piecewise((x**6*acot(a*x)/6 + x**5/(30*a) - x**3/(18*a**3) + x/(6*a**5) + a
cot(a*x)/(6*a**6), Ne(a, 0)), (pi*x**6/12, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{1}{6}\,x^6 \operatorname{arccot}(ax) + \frac{1}{90}\,a\left(\frac{3\,a^4x^5 - 5\,a^2x^3 + 15\,x}{a^6} - \frac{15\,\arctan(ax)}{a^7}\right)$$

```
[In] integrate(x^5*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccot(a*x) + 1/90*a*((3*a^4*x^5 - 5*a^2*x^3 + 15*x)/a^6 - 15*arcta
n(a*x)/a^7)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{1}{90}\left(\frac{15\,x^6\arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^5\left(\frac{5}{a^2x^2} - \frac{15}{a^4x^4} - 3\right)}{a^2} + \frac{15\,\arctan\left(\frac{1}{ax}\right)}{a^7}\right)a$$

```
[In] integrate(x^5*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/90*(15*x^6*arctan(1/(a*x))/a - x^5*(5/(a^2*x^2) - 15/(a^4*x^4) - 3)/a^2 +
 15*arctan(1/(a*x))/a^7)*a
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int x^5 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi\,x^6}{12} & \text{if } a=0 \\ \frac{x^6\operatorname{acot}(a\,x)}{6} - \frac{\frac{\operatorname{atan}(a\,x)}{6} - \frac{a\,x}{6} + \frac{a^3\,x^3}{18} - \frac{a^5\,x^5}{30}}{a^6} & \text{if } a\neq 0 \end{cases}$$

```
[In] int(x^5*acot(a*x),x)
```

```
[Out] piecewise(a == 0, (x^6*pi)/12, a ~= 0, - (atan(a*x)/6 - (a*x)/6 + (a^3*x^3)
/18 - (a^5*x^5)/30)/a^6 + (x^6*acot(a*x))/6)
```

3.2 $\int x^4 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int x^4 \cot^{-1}(ax)\,dx = -\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5 \cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{10a^5}$$

```
[Out] -1/10*x^2/a^3+1/20*x^4/a+1/5*x^5*arccot(a*x)+1/10*ln(a^2*x^2+1)/a^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4947, 272, 45}

$$\int x^4 \cot^{-1}(ax)\,dx = -\frac{x^2}{10a^3} + \frac{\log(a^2x^2+1)}{10a^5} + \frac{1}{5}x^5 \cot^{-1}(ax) + \frac{x^4}{20a}$$

```
[In] Int[x^4*ArcCot[a*x],x]
```

```
[Out] -1/10*x^2/a^3 + x^4/(20*a) + (x^5*ArcCot[a*x])/5 + Log[1 + a^2*x^2]/(10*a^5
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\cot^{-1}(ax) + \frac{1}{5}a\int\frac{x^5}{1+a^2x^2}\,dx \\
&= \frac{1}{5}x^5\cot^{-1}(ax) + \frac{1}{10}a\text{Subst}\left(\int\frac{x^2}{1+a^2x}\,dx, x, x^2\right) \\
&= \frac{1}{5}x^5\cot^{-1}(ax) + \frac{1}{10}a\text{Subst}\left(\int\left(-\frac{1}{a^4}+\frac{x}{a^2}+\frac{1}{a^4(1+a^2x)}\right)\,dx, x, x^2\right) \\
&= -\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{10a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x^4\cot^{-1}(ax)\,dx = -\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{10a^5}$$

```
[In] Integrate[x^4*ArcCot[a*x],x]
[Out] -1/10*x^2/a^3 + x^4/(20*a) + (x^5*ArcCot[a*x])/5 + Log[1 + a^2*x^2]/(10*a^5
)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{a^5x^5\,\mathrm{arccot}(ax)}{5}+\frac{a^4x^4}{20}-\frac{a^2x^2}{10}+\frac{\ln\left(a^2x^2+1\right)}{10}}{a^5}$	46
default	$\frac{\frac{a^5x^5\,\mathrm{arccot}(ax)}{5}+\frac{a^4x^4}{20}-\frac{a^2x^2}{10}+\frac{\ln\left(a^2x^2+1\right)}{10}}{a^5}$	46
parallelrisch	$\frac{4a^5x^5\,\mathrm{arccot}(ax)+a^4x^4-2a^2x^2+2+2\ln\left(a^2x^2+1\right)}{20a^5}$	47
parts	$\frac{x^5\,\mathrm{arccot}(ax)}{5}+\frac{a\left(\frac{\frac{1}{2}a^2x^4-x^2}{2a^4}+\frac{\ln\left(a^2x^2+1\right)}{2a^6}\right)}{5}$	49
risch	$\frac{ix^5\ln(iax+1)}{10}-\frac{ix^5\ln(-iax+1)}{10}+\frac{x^5\pi}{10}+\frac{x^4}{20a}-\frac{x^2}{10a^3}+\frac{\ln(-a^2x^2-1)}{10a^5}$	68

```
[In] int(x^4*arccot(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^5*(1/5*a^5*x^5*arccot(a*x)+1/20*a^4*x^4-1/10*a^2*x^2+1/10*ln(a^2*x^2+1)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int x^4\cot^{-1}(ax)\,dx=\frac{4\,a^5x^5\,\mathrm{arccot}\,(ax)+a^4x^4-2\,a^2x^2+2\,\log\left(a^2x^2+1\right)}{20\,a^5}$$

```
[In] integrate(x^4*arccot(a*x),x, algorithm="fricas")
[Out] 1/20*(4*a^5*x^5*arccot(a*x) + a^4*x^4 - 2*a^2*x^2 + 2*log(a^2*x^2 + 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int x^4\cot^{-1}(ax)\,dx=\begin{cases}\frac{x^5\,\mathrm{acot}\,(ax)}{5}+\frac{x^4}{20a}-\frac{x^2}{10a^3}+\frac{\log\left(a^2x^2+1\right)}{10a^5} & \text{for } a\neq 0\\ \frac{\pi x^5}{10} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**4*acot(a*x),x)
[Out] Piecewise((x**5*acot(a*x)/5 + x**4/(20*a) - x**2/(10*a**3) + log(a**2*x**2
+ 1)/(10*a**5), Ne(a, 0)), (pi*x**5/10, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int x^4 \cot^{-1}(ax)\,dx = \frac{1}{5}\,x^5\,\mathrm{arccot}\left(ax\right) + \frac{1}{20}\,a\left(\frac{a^2x^4 - 2\,x^2}{a^4} + \frac{2\,\log\left(a^2x^2 + 1\right)}{a^6}\right)$$

```
[In] integrate(x^4*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccot(a*x) + 1/20*a*((a^2*x^4 - 2*x^2)/a^4 + 2*log(a^2*x^2 + 1)/a^
6)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int x^4 \cot^{-1}(ax)\,dx$$
$$= \frac{1}{20}\left(\frac{4\,x^5\,\arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^4\left(\frac{2}{a^2x^2} - \frac{3}{a^4x^4} - 1\right)}{a^2} + \frac{2\,\log\left(\frac{1}{a^2x^2} + 1\right)}{a^6} - \frac{2\,\log\left(\frac{1}{a^2x^2}\right)}{a^6}\right)a$$

```
[In] integrate(x^4*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/20*(4*x^5*arctan(1/(a*x))/a - x^4*(2/(a^2*x^2) - 3/(a^4*x^4) - 1)/a^2 + 2
*log(1/(a^2*x^2) + 1)/a^6 - 2*log(1/(a^2*x^2))/a^6)*a
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int x^4 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi\,x^5}{10} & \text{if } a = 0 \\ \frac{2\,\ln\left(a^2\,x^2+1\right)-2\,a^2\,x^2+a^4\,x^4}{20\,a^5} + \frac{x^5\,\mathrm{acot}\left(a\,x\right)}{5} & \text{if } a \neq 0 \end{cases}$$

```
[In] int(x^4*acot(a*x),x)
```

```
[Out] piecewise(a == 0, (x^5*pi)/10, a ~= 0, (2*log(a^2*x^2 + 1) - 2*a^2*x^2 + a^
4*x^4)/(20*a^5) + (x^5*acot(a*x))/5)
```

3.3 $\int x^3 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^3 \cot^{-1}(ax)\,dx = -\frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4\cot^{-1}(ax) + \frac{\arctan(ax)}{4a^4}$$

```
[Out] -1/4*x/a^3+1/12*x^3/a+1/4*x^4*arccot(a*x)+1/4*arctan(a*x)/a^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4947, 308, 209}

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{\arctan(ax)}{4a^4} - \frac{x}{4a^3} + \frac{1}{4}x^4\cot^{-1}(ax) + \frac{x^3}{12a}$$

```
[In] Int[x^3*ArcCot[a*x],x]
```

```
[Out] -1/4*x/a^3 + x^3/(12*a) + (x^4*ArcCot[a*x])/4 + ArcTan[a*x]/(4*a^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^4\cot^{-1}(ax)+\frac{1}{4}a\int\frac{x^4}{1+a^2x^2}\,dx\\ &= \frac{1}{4}x^4\cot^{-1}(ax)+\frac{1}{4}a\int\left(-\frac{1}{a^4}+\frac{x^2}{a^2}+\frac{1}{a^4(1+a^2x^2)}\right)dx\\ &= -\frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\cot^{-1}(ax)+\frac{\int\frac{1}{1+a^2x^2}\,dx}{4a^3}\\ &= -\frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\cot^{-1}(ax)+\frac{\arctan(ax)}{4a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x^3\cot^{-1}(ax)\,dx=-\frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\cot^{-1}(ax)+\frac{\arctan(ax)}{4a^4}$$

```
[In] Integrate[x^3*ArcCot[a*x],x]
[Out] -1/4*x/a^3 + x^3/(12*a) + (x^4*ArcCot[a*x])/4 + ArcTan[a*x]/(4*a^4)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{a^4x^4\,\mathrm{arccot}(ax)}{4}+\frac{a^3x^3}{12}-\frac{ax}{4}+\frac{\arctan(ax)}{4}}{a^4}$	36
default	$\frac{\frac{a^4x^4\,\mathrm{arccot}(ax)}{4}+\frac{a^3x^3}{12}-\frac{ax}{4}+\frac{\arctan(ax)}{4}}{a^4}$	36
parallelrisch	$\frac{3a^4x^4\,\mathrm{arccot}(ax)+a^3x^3-3ax-3\,\mathrm{arccot}(ax)}{12a^4}$	36
parts	$\frac{x^4\,\mathrm{arccot}(ax)}{4}+\frac{a\left(\frac{\frac{1}{3}a^2x^3-x}{a^4}+\frac{\arctan(ax)}{a^5}\right)}{4}$	39
risch	$\frac{ix^4\ln(iax+1)}{8}-\frac{ix^4\ln(-iax+1)}{8}+\frac{\pi x^4}{8}+\frac{x^3}{12a}-\frac{x}{4a^3}+\frac{\arctan(ax)}{4a^4}$	59

```
[In] int(x^3*arccot(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^4*(1/4*a^4*x^4*arccot(a*x)+1/12*a^3*x^3-1/4*a*x+1/4*arctan(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{a^3x^3 - 3\,ax + 3\,(a^4x^4 - 1)\,\mathrm{arccot}\,(ax)}{12\,a^4}$$

```
[In] integrate(x^3*arccot(a*x),x, algorithm="fricas")
[Out] 1/12*(a^3*x^3 - 3*a*x + 3*(a^4*x^4 - 1)*arccot(a*x))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^3 \cot^{-1}(ax)\,dx = \begin{cases} \frac{x^4\,\mathrm{acot}\,(ax)}{4} + \frac{x^3}{12a} - \frac{x}{4a^3} - \frac{\mathrm{acot}\,(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*acot(a*x),x)
[Out] Piecewise((x**4*acot(a*x)/4 + x**3/(12*a) - x/(4*a**3) - acot(a*x)/(4*a**4)
, Ne(a, 0)), (pi*x**4/8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{1}{4}\,x^4 \operatorname{arccot}\left(ax\right) + \frac{1}{12}\,a\left(\frac{a^2x^3 - 3\,x}{a^4} + \frac{3\,\arctan\left(ax\right)}{a^5}\right)$$

```
[In] integrate(x^3*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccot(a*x) + 1/12*a*((a^2*x^3 - 3*x)/a^4 + 3*arctan(a*x)/a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{1}{12}\left(\frac{3\,x^4 \arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^3\left(\frac{3}{a^2x^2} - 1\right)}{a^2} - \frac{3\,\arctan\left(\frac{1}{ax}\right)}{a^5}\right)a$$

```
[In] integrate(x^3*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/12*(3*x^4*arctan(1/(a*x))/a - x^3*(3/(a^2*x^2) - 1)/a^2 - 3*arctan(1/(a*x
))/a^5)*a
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int x^3 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi\,x^4}{8} & \text{if } a = 0 \\ \frac{3\,\mathrm{atan}(a\,x) - 3\,a\,x + a^3\,x^3}{12\,a^4} + \frac{x^4\,\mathrm{acot}(a\,x)}{4} & \text{if } a \neq 0 \end{cases}$$

```
[In] int(x^3*acot(a*x),x)
```

```
[Out] piecewise(a == 0, (x^4*pi)/8, a ~= 0, (3*atan(a*x) - 3*a*x + a^3*x^3)/(12*a
^4) + (x^4*acot(a*x))/4)
```

3.4 $\int x^2 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{x^2}{6a} + \frac{1}{3}x^3 \cot^{-1}(ax) - \frac{\log(1+a^2x^2)}{6a^3}$$

```
[Out] 1/6*x^2/a+1/3*x^3*arccot(a*x)-1/6*ln(a^2*x^2+1)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4947, 272, 45}

$$\int x^2 \cot^{-1}(ax)\,dx = -\frac{\log(a^2x^2+1)}{6a^3} + \frac{1}{3}x^3 \cot^{-1}(ax) + \frac{x^2}{6a}$$

```
[In] Int[x^2*ArcCot[a*x],x]
```

```
[Out] x^2/(6*a) + (x^3*ArcCot[a*x])/3 - Log[1 + a^2*x^2]/(6*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}x^3\cot^{-1}(ax) + \frac{1}{3}a\int \frac{x^3}{1+a^2x^2}\,dx \\ &= \frac{1}{3}x^3\cot^{-1}(ax) + \frac{1}{6}a\text{Subst}\left(\int \frac{x}{1+a^2x}\,dx, x, x^2\right) \\ &= \frac{1}{3}x^3\cot^{-1}(ax) + \frac{1}{6}a\text{Subst}\left(\int \left(\frac{1}{a^2} - \frac{1}{a^2(1+a^2x)}\right)dx, x, x^2\right) \\ &= \frac{x^2}{6a} + \frac{1}{3}x^3\cot^{-1}(ax) - \frac{\log(1+a^2x^2)}{6a^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int x^2\cot^{-1}(ax)\,dx = \frac{x^2}{6a} + \frac{1}{3}x^3\cot^{-1}(ax) - \frac{\log(1+a^2x^2)}{6a^3}$$

```
[In] Integrate[x^2*ArcCot[a*x],x]
[Out] x^2/(6*a) + (x^3*ArcCot[a*x])/3 - Log[1 + a^2*x^2]/(6*a^3)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{-2a^3x^3\,\mathrm{arccot}(ax)-a^2x^2+\ln(a^2x^2+1)}{6a^3}$	37
derivativedivides	$\frac{\frac{a^3x^3\,\mathrm{arccot}(ax)}{3}+\frac{a^2x^2}{6}-\frac{\ln(a^2x^2+1)}{6}}{a^3}$	38
default	$\frac{\frac{a^3x^3\,\mathrm{arccot}(ax)}{3}+\frac{a^2x^2}{6}-\frac{\ln(a^2x^2+1)}{6}}{a^3}$	38
parts	$\frac{x^3\,\mathrm{arccot}(ax)}{3}+\frac{a\left(\frac{x^2}{2a^2}-\frac{\ln(a^2x^2+1)}{2a^4}\right)}{3}$	38
risch	$\frac{ix^3\ln(iax+1)}{6}-\frac{ix^3\ln(-iax+1)}{6}+\frac{\pi x^3}{6}+\frac{x^2}{6a}-\frac{\ln(-a^2x^2-1)}{6a^3}$	60

```
[In] int(x^2*arccot(a*x),x,method=_RETURNVERBOSE)
[Out] -1/6*(-2*a^3*x^3*arccot(a*x)-a^2*x^2+ln(a^2*x^2+1))/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x^2\cot^{-1}(ax)\,dx=\frac{2\,a^3x^3\,\mathrm{arccot}\,(ax)+a^2x^2-\log\left(a^2x^2+1\right)}{6\,a^3}$$

```
[In] integrate(x^2*arccot(a*x),x, algorithm="fricas")
[Out] 1/6*(2*a^3*x^3*arccot(a*x) + a^2*x^2 - log(a^2*x^2 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x^2\cot^{-1}(ax)\,dx=\begin{cases}\frac{x^3\,\mathrm{acot}\,(ax)}{3}+\frac{x^2}{6a}-\frac{\log\left(a^2x^2+1\right)}{6a^3} & \text{for } a\neq 0\\ \frac{\pi x^3}{6} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*acot(a*x),x)
[Out] Piecewise((x**3*acot(a*x)/3 + x**2/(6*a) - log(a**2*x**2 + 1)/(6*a**3), Ne(
a, 0)), (pi*x**3/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{1}{3}\,x^3 \operatorname{arccot}\left(ax\right) + \frac{1}{6}\,a\left(\frac{x^2}{a^2} - \frac{\log\left(a^2x^2+1\right)}{a^4}\right)$$

```
[In] integrate(x^2*arccot(a*x),x, algorithm="maxima")
[Out] 1/3*x^3*arccot(a*x) + 1/6*a*(x^2/a^2 - log(a^2*x^2 + 1)/a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{1}{6}\left(\frac{2\,x^3 \arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^2\left(\frac{1}{a^2x^2}-1\right)}{a^2} - \frac{\log\left(\frac{1}{a^2x^2}+1\right)}{a^4} + \frac{\log\left(\frac{1}{a^2x^2}\right)}{a^4}\right)a$$

```
[In] integrate(x^2*arccot(a*x),x, algorithm="giac")
[Out] 1/6*(2*x^3*arctan(1/(a*x))/a - x^2*(1/(a^2*x^2) - 1)/a^2 - log(1/(a^2*x^2)
+ 1)/a^4 + log(1/(a^2*x^2))/a^4)*a
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int x^2 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi\,x^3}{6} & \text{if } a=0 \\ \frac{\frac{x^2}{2}-\frac{\ln\left(a^2\,x^2+1\right)}{2\,a^2}}{3\,a} + \frac{x^3\,\mathrm{acot}(a\,x)}{3} & \text{if } a\neq 0 \end{cases}$$

```
[In] int(x^2*acot(a*x),x)
[Out] piecewise(a == 0, (x^3*pi)/6, a ~= 0, (x^2/2 - log(a^2*x^2 + 1)/(2*a^2))/(3
*a) + (x^3*acot(a*x))/3)
```

3.5 $\int x \cot^{-1}(ax) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 31

$$\int x \cot^{-1}(ax) \, dx = \frac{x}{2a} + \frac{1}{2}x^2 \cot^{-1}(ax) - \frac{\arctan(ax)}{2a^2}$$

```
[Out] 1/2*x/a+1/2*x^2*arccot(a*x)-1/2*arctan(a*x)/a^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4947, 327, 209}

$$\int x \cot^{-1}(ax) \, dx = -\frac{\arctan(ax)}{2a^2} + \frac{1}{2}x^2 \cot^{-1}(ax) + \frac{x}{2a}$$

```
[In] Int[x*ArcCot[a*x],x]
```

```
[Out] x/(2*a) + (x^2*ArcCot[a*x])/2 - ArcTan[a*x]/(2*a^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\cot^{-1}(ax)+\frac{1}{2}a\int\frac{x^2}{1+a^2x^2}\,dx\\ &= \frac{x}{2a}+\frac{1}{2}x^2\cot^{-1}(ax)-\frac{\int\frac{1}{1+a^2x^2}\,dx}{2a}\\ &= \frac{x}{2a}+\frac{1}{2}x^2\cot^{-1}(ax)-\frac{\arctan(ax)}{2a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x\cot^{-1}(ax)\,dx=\frac{x}{2a}+\frac{1}{2}x^2\cot^{-1}(ax)-\frac{\arctan(ax)}{2a^2}$$

```
[In] Integrate[x*ArcCot[a*x],x]
[Out] x/(2*a) + (x^2*ArcCot[a*x])/2 - ArcTan[a*x]/(2*a^2)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{\operatorname{arccot}(ax)a^2x^2+ax+\operatorname{arccot}(ax)}{2a^2}$	25
derivativedivides	$\frac{\frac{\operatorname{arccot}(ax)a^2x^2}{2}+\frac{ax}{2}-\frac{\arctan(ax)}{2}}{a^2}$	28
default	$\frac{\frac{\operatorname{arccot}(ax)a^2x^2}{2}+\frac{ax}{2}-\frac{\arctan(ax)}{2}}{a^2}$	28
parts	$\frac{x^2\operatorname{arccot}(ax)}{2}+\frac{a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{2}$	29
risch	$\frac{ix^2\ln(iax+1)}{4}-\frac{ix^2\ln(-iax+1)}{4}+\frac{\pi x^2}{4}+\frac{x}{2a}-\frac{\arctan(ax)}{2a^2}$	51

```
[In] int(x*arccot(a*x),x,method=_RETURNVERBOSE)
[Out] 1/2*(arccot(a*x)*a^2*x^2+a*x+arccot(a*x))/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x\cot^{-1}(ax)\,dx = \frac{ax + (a^2x^2 + 1)\operatorname{arccot}(ax)}{2\,a^2}$$

```
[In] integrate(x*arccot(a*x),x, algorithm="fricas")
[Out] 1/2*(a*x + (a^2*x^2 + 1)*arccot(a*x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x\cot^{-1}(ax)\,dx = \begin{cases} \frac{x^2\operatorname{acot}(ax)}{2} + \frac{x}{2a} + \frac{\operatorname{acot}(ax)}{2a^2} & \text{for } a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*acot(a*x),x)
[Out] Piecewise((x**2*acot(a*x)/2 + x/(2*a) + acot(a*x)/(2*a**2), Ne(a, 0)), (pi*
x**2/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x\cot^{-1}(ax)\,dx = \frac{1}{2}\,x^2\operatorname{arccot}(ax) + \frac{1}{2}\,a\left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right)$$

```
[In] integrate(x*arccot(a*x),x, algorithm="maxima")
[Out] 1/2*x^2*arccot(a*x) + 1/2*a*(x/a^2 - arctan(a*x)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int x\cot^{-1}(ax)\,dx = \frac{1}{2}\left(\frac{x^2\arctan\left(\frac{1}{ax}\right)}{a} + \frac{x}{a^2} + \frac{\arctan\left(\frac{1}{ax}\right)}{a^3}\right)a$$

```
[In] integrate(x*arccot(a*x),x, algorithm="giac")
[Out] 1/2*(x^2*arctan(1/(a*x))/a + x/a^2 + arctan(1/(a*x))/a^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int x\cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi x^2}{4} & \text{if } a=0 \\ \frac{x-\frac{\mathrm{atan}(a\,x)}{a}}{2\,a}+\frac{x^2\,\mathrm{acot}(a\,x)}{2} & \text{if } a\neq 0 \end{cases}$$

```
[In] int(x*acot(a*x),x)
[Out] piecewise(a == 0, (x^2*pi)/4, a ~= 0, (x - atan(a*x)/a)/(2*a) + (x^2*acot(a
*x))/2)
```

3.6 $\int \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \cot^{-1}(ax)\,dx = x\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{2a}$$

```
[Out] x*arccot(a*x)+1/2*ln(a^2*x^2+1)/a
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4931, 266}

$$\int \cot^{-1}(ax)\,dx = \frac{\log(a^2x^2+1)}{2a} + x\cot^{-1}(ax)$$

```
[In] Int[ArcCot[a*x],x]
```

```
[Out] x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\cot^{-1}(ax) + a\int \frac{x}{1+a^2x^2}\,dx \\ &= x\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = x\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{2a}$$

```
[In] Integrate[ArcCot[a*x],x]
[Out] x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
parts	$x\ \mathrm{arccot}\left(ax\right)+\frac{\ln\left(a^{2} x^{2}+1\right)}{2a}$	23
derivativedivides	$\frac{\mathrm{arccot}\left(ax\right) a x+\frac{\ln\left(a^{2} x^{2}+1\right)}{2}}{a}$	25
default	$\frac{\mathrm{arccot}\left(ax\right) a x+\frac{\ln\left(a^{2} x^{2}+1\right)}{2}}{a}$	25
parallelrisch	$\frac{2\ \mathrm{arccot}\left(ax\right) a x+\ln\left(a^{2} x^{2}+1\right)}{2a}$	25
risch	$\frac{i x \ln\left(i a x+1\right)}{2}-\frac{i x \ln\left(-i a x+1\right)}{2}+\frac{\pi x}{2}+\frac{\ln\left(-a^{2} x^{2}-1\right)}{2a}$	46

```
[In] int(arccot(a*x),x,method=_RETURNVERBOSE)
[Out] x*arccot(a*x)+1/2*ln(a^2*x^2+1)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = \frac{2\,ax\,\mathrm{arccot}\,(ax) + \log\left(a^2x^2 + 1\right)}{2\,a}$$

```
[In] integrate(arccot(a*x),x, algorithm="fricas")
[Out] 1/2*(2*a*x*arccot(a*x) + log(a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = \begin{cases} x\,\mathrm{acot}\,(ax) + \frac{\log\left(a^2x^2+1\right)}{2a} & \text{for } a \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(acot(a*x),x)
[Out] Piecewise((x*acot(a*x) + log(a**2*x**2 + 1)/(2*a), Ne(a, 0)), (pi*x/2, True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = \frac{2\,ax\,\mathrm{arccot}\,(ax) + \log\left(a^2x^2 + 1\right)}{2\,a}$$

```
[In] integrate(arccot(a*x),x, algorithm="maxima")
[Out] 1/2*(2*a*x*arccot(a*x) + log(a^2*x^2 + 1))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \cot^{-1}(ax)\,dx = \frac{1}{2}\,a\left(\frac{2\,x\arctan\left(\frac{1}{ax}\right)}{a} + \frac{\log\left(\frac{1}{a^2x^2}+1\right)}{a^2} - \frac{\log\left(\frac{1}{a^2x^2}\right)}{a^2}\right)$$

```
[In] integrate(arccot(a*x),x, algorithm="giac")
[Out] 1/2*a*(2*x*arctan(1/(a*x))/a + log(1/(a^2*x^2) + 1)/a^2 - log(1/(a^2*x^2))/
a^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cot^{-1}(ax)\,dx = x\,\mathrm{acot}(a\,x) + \frac{\ln\left(a^2\,x^2+1\right)}{2\,a}$$

```
[In] int(acot(a*x),x)
[Out] x*acot(a*x) + log(a^2*x^2 + 1)/(2*a)
```

3.7 $\int \frac{\cot^{-1}(ax)}{x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{\cot^{-1}(ax)}{x} \, dx = -\frac{1}{2} i \operatorname{PolyLog}\left(2, -\frac{i}{ax}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, \frac{i}{ax}\right)$$

```
[Out] -1/2*I*polylog(2,-I/a/x)+1/2*I*polylog(2,I/a/x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4941, 2438}

$$\int \frac{\cot^{-1}(ax)}{x} \, dx = \frac{1}{2} i \operatorname{PolyLog}\left(2, \frac{i}{ax}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, -\frac{i}{ax}\right)$$

```
[In] Int[ArcCot[a*x]/x,x]
```

```
[Out] (-1/2*I)*PolyLog[2, (-I)/(a*x)] + (I/2)*PolyLog[2, I/(a*x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}i\int\frac{\log\left(1-\frac{i}{ax}\right)}{x}\,dx - \frac{1}{2}i\int\frac{\log\left(1+\frac{i}{ax}\right)}{x}\,dx$$
$$= -\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{ax}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{ax}\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int\frac{\cot^{-1}(ax)}{x}\,dx = -\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{ax}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{ax}\right)$$

```
[In] Integrate[ArcCot[a*x]/x,x]
[Out] (-1/2*I)*PolyLog[2, (-I)/(a*x)] + (I/2)*PolyLog[2, I/(a*x)]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	siz
risch	$\frac{\pi\ln(-iax)}{2}+\frac{i\,\text{dilog}(-iax+1)}{2}-\frac{i\,\text{dilog}(iax+1)}{2}$	33
derivativedivides	$\ln(ax)\,\text{arccot}(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i\,\text{dilog}(iax+1)}{2}+\frac{i\,\text{dilog}(-iax+1)}{2}$	63
default	$\ln(ax)\,\text{arccot}(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i\,\text{dilog}(iax+1)}{2}+\frac{i\,\text{dilog}(-iax+1)}{2}$	63
parts	$\ln(x)\,\text{arccot}(ax)+a\left(-\frac{i\ln(x)(-\ln(-iax+1)+\ln(iax+1))}{2a}-\frac{i(\text{dilog}(iax+1)-\text{dilog}(-iax+1))}{2a}\right)$	64

```
[In] int(arccot(a*x)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*Pi*ln(-I*a*x)+1/2*I*dilog(1-I*a*x)-1/2*I*dilog(1+I*a*x)
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \int \frac{\operatorname{arccot}(ax)}{x}\,dx$$

```
[In] integrate(arccot(a*x)/x,x, algorithm="fricas")
[Out] integral(arccot(a*x)/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \int \frac{\operatorname{acot}(ax)}{x}\,dx$$

```
[In] integrate(acot(a*x)/x,x)
[Out] Integral(acot(a*x)/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(23) = 46$.

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \frac{1}{4}\pi \log\left(a^2x^2+1\right) - \arctan(ax)\log(ax) + \operatorname{arccot}(ax)\log(x) + \arctan(ax)\log(x) + \frac{1}{2}i\,\mathrm{Li}_2(i\,ax+1) - \frac{1}{2}i\,\mathrm{Li}_2(-i\,ax+1)$$

```
[In] integrate(arccot(a*x)/x,x, algorithm="maxima")
[Out] 1/4*pi*log(a^2*x^2 + 1) - arctan(a*x)*log(a*x) + arccot(a*x)*log(x) + arcta
n(a*x)*log(x) + 1/2*I*dilog(I*a*x + 1) - 1/2*I*dilog(-I*a*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = -\frac{1}{2}\left(\frac{x^2\arctan\left(\frac{1}{ax}\right)}{a} + \frac{x}{a^2} + \frac{\arctan\left(\frac{1}{ax}\right)}{a^3}\right)a^2$$

```
[In] integrate(arccot(a*x)/x,x, algorithm="giac")
[Out] -1/2*(x^2*arctan(1/(a*x))/a + x/a^2 + arctan(1/(a*x))/a^3)*a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \int \frac{\mathrm{acot}(a\,x)}{x}\,dx$$

```
[In] int(acot(a*x)/x,x)
```

```
[Out] int(acot(a*x)/x, x)
```

3.8 $\int \frac{\cot^{-1}(ax)}{x^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = -\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1+a^2x^2\right)$$

```
[Out] -arccot(a*x)/x-a*ln(x)+1/2*a*ln(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4947, 272, 36, 29, 31}

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = \frac{1}{2}a\log\left(a^2x^2+1\right) - a\log(x) - \frac{\cot^{-1}(ax)}{x}$$

```
[In] Int[ArcCot[a*x]/x^2,x]
```

```
[Out] -(ArcCot[a*x]/x) - a*Log[x] + (a*Log[1 + a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax)}{x} - a\int \frac{1}{x\,(1+a^2x^2)}\,dx \\
&= -\frac{\cot^{-1}(ax)}{x} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\,(1+a^2x)}\,dx, x, x^2\right) \\
&= -\frac{\cot^{-1}(ax)}{x} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}a^3\text{Subst}\left(\int \frac{1}{1+a^2x}\,dx, x, x^2\right) \\
&= -\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1+a^2x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = -\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1+a^2x^2\right)$$

```
[In] Integrate[ArcCot[a*x]/x^2,x]
[Out] -(ArcCot[a*x]/x) - a*Log[x] + (a*Log[1 + a^2*x^2])/2
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
parts	$-\frac{\operatorname{arccot}(ax)}{x}-a\left(\ln\left(x\right)-\frac{\ln(a^{2}x^{2}+1)}{2}\right)$	29
parallelrisch	$-\frac{2a\ln(x)x-a\ln(a^{2}x^{2}+1)x+2\,\operatorname{arccot}(ax)}{2x}$	33
derivativedivides	$a\left(-\frac{\operatorname{arccot}(ax)}{ax}+\frac{\ln(a^{2}x^{2}+1)}{2}-\ln\left(ax\right)\right)$	34
default	$a\left(-\frac{\operatorname{arccot}(ax)}{ax}+\frac{\ln(a^{2}x^{2}+1)}{2}-\ln\left(ax\right)\right)$	34
risch	$-\frac{i\ln(iax+1)}{2x}-\frac{2a\ln(x)x-a\ln(a^{2}x^{2}+1)x-i\ln(-iax+1)+\pi}{2x}$	54

```
[In] int(arccot(a*x)/x^2,x,method=_RETURNVERBOSE)
[Out] -arccot(a*x)/x-a*(ln(x)-1/2*ln(a^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx=\frac{ax\log\left(a^2x^2+1\right)-2\,ax\log\left(x\right)-2\,\operatorname{arccot}\left(ax\right)}{2\,x}$$

```
[In] integrate(arccot(a*x)/x^2,x, algorithm="fricas")
[Out] 1/2*(a*x*log(a^2*x^2 + 1) - 2*a*x*log(x) - 2*arccot(a*x))/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx=-a\log\left(x\right)+\frac{a\log\left(a^2x^2+1\right)}{2}-\frac{\operatorname{acot}\left(ax\right)}{x}$$

```
[In] integrate(acot(a*x)/x**2,x)
[Out] -a*log(x) + a*log(a**2*x**2 + 1)/2 - acot(a*x)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = \frac{1}{2}\,a\big(\log\left(a^2x^2+1\right)-\log\left(x^2\right)\big)-\frac{\operatorname{arccot}\left(ax\right)}{x}$$

```
[In] integrate(arccot(a*x)/x^2,x, algorithm="maxima")
[Out] 1/2*a*(log(a^2*x^2 + 1) - log(x^2)) - arccot(a*x)/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = -\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\frac{1}{ax}\right)}{ax}-\log\left(\frac{1}{a^2x^2}+1\right)\right)$$

```
[In] integrate(arccot(a*x)/x^2,x, algorithm="giac")
[Out] -1/2*a*(2*arctan(1/(a*x))/(a*x) - log(1/(a^2*x^2) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = \frac{a\left(\ln\left(a^2\,x^2+1\right)-2\,\ln\left(x\right)\right)}{2}-\frac{\mathrm{acot}(a\,x)}{x}$$

```
[In] int(acot(a*x)/x^2,x)
[Out] (a*(log(a^2*x^2 + 1) - 2*log(x)))/2 - acot(a*x)/x
```

3.9 $\int \frac{\cot^{-1}(ax)}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int \frac{\cot^{-1}(ax)}{x^3} dx = \frac{a}{2x} - \frac{\cot^{-1}(ax)}{2x^2} + \frac{1}{2}a^2 \arctan(ax)$$

```
[Out] 1/2*a/x-1/2*arccot(a*x)/x^2+1/2*a^2*arctan(a*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4947, 331, 209}

$$\int \frac{\cot^{-1}(ax)}{x^3} dx = \frac{1}{2}a^2 \arctan(ax) - \frac{\cot^{-1}(ax)}{2x^2} + \frac{a}{2x}$$

```
[In] Int[ArcCot[a*x]/x^3,x]
```

```
[Out] a/(2*x) - ArcCot[a*x]/(2*x^2) + (a^2*ArcTan[a*x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cot^{-1}(ax)}{2x^2} - \frac{1}{2}a\int \frac{1}{x^2(1+a^2x^2)}\,dx\\ &= \frac{a}{2x} - \frac{\cot^{-1}(ax)}{2x^2} + \frac{1}{2}a^3\int \frac{1}{1+a^2x^2}\,dx\\ &= \frac{a}{2x} - \frac{\cot^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\arctan(ax)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = -\frac{\cot^{-1}(ax)}{2x^2} + \frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{2x}$$

```
[In] Integrate[ArcCot[a*x]/x^3,x]
[Out] -1/2*ArcCot[a*x]/x^2 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/(2*x
)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$-\frac{\operatorname{arccot}(ax)a^2x^2-ax+\operatorname{arccot}(ax)}{2x^2}$	26
parts	$-\frac{\operatorname{arccot}(ax)}{2x^2}-\frac{a\left(-\frac{1}{x}-a\arctan(ax)\right)}{2}$	27
derivativedivides	$a^2\left(-\frac{\operatorname{arccot}(ax)}{2a^2x^2}+\frac{\arctan(ax)}{2}+\frac{1}{2ax}\right)$	32
default	$a^2\left(-\frac{\operatorname{arccot}(ax)}{2a^2x^2}+\frac{\arctan(ax)}{2}+\frac{1}{2ax}\right)$	32
risch	$-\frac{i\ln(iax+1)}{4x^2}-\frac{-ia^2\ln(-ax-i)x^2+ia^2\ln(-ax+i)x^2-i\ln(-iax+1)-2ax+\pi}{4x^2}$	72

```
[In] int(arccot(a*x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(arccot(a*x)*a^2*x^2-a*x+arccot(a*x))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx=\frac{ax-(a^2x^2+1)\operatorname{arccot}(ax)}{2\,x^2}$$

```
[In] integrate(arccot(a*x)/x^3,x, algorithm="fricas")
[Out] 1/2*(a*x - (a^2*x^2 + 1)*arccot(a*x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx=-\frac{a^2\operatorname{acot}(ax)}{2}+\frac{a}{2x}-\frac{\operatorname{acot}(ax)}{2x^2}$$

```
[In] integrate(acot(a*x)/x**3,x)
[Out] -a**2*acot(a*x)/2 + a/(2*x) - acot(a*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \frac{1}{2}\left(a\arctan\left(ax\right) + \frac{1}{x}\right)a - \frac{\operatorname{arccot}\left(ax\right)}{2\,x^2}$$

```
[In] integrate(arccot(a*x)/x^3,x, algorithm="maxima")
[Out] 1/2*(a*arctan(a*x) + 1/x)*a - 1/2*arccot(a*x)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \frac{1}{2}\left(a\left(\frac{1}{ax} - \arctan\left(\frac{1}{ax}\right)\right) - \frac{\arctan\left(\frac{1}{ax}\right)}{ax^2}\right)a$$

```
[In] integrate(arccot(a*x)/x^3,x, algorithm="giac")
[Out] 1/2*(a*(1/(a*x) - arctan(1/(a*x))) - arctan(1/(a*x))/(a*x^2))*a
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \begin{cases} -\frac{\pi}{4\,x^2} & \text{if} \ \ a=0 \\ \frac{a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}}{2\,a} - \frac{\mathrm{acot}(a\,x)}{2\,x^2} & \text{if} \ \ a\neq 0 \end{cases}$$

```
[In] int(acot(a*x)/x^3,x)
[Out] piecewise(a == 0, -pi/(4*x^2), a ~= 0, (a^3*atan(a*x) + a^2/x)/(2*a) - acot
(a*x)/(2*x^2))
```

3.10 $\int \frac{\cot^{-1}(ax)}{x^4} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \frac{\cot^{-1}(ax)}{x^4} \, dx = \frac{a}{6x^2} - \frac{\cot^{-1}(ax)}{3x^3} + \frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log \left(1 + a^2x^2\right)$$

```
[Out] 1/6*a/x^2-1/3*arccot(a*x)/x^3+1/3*a^3*ln(x)-1/6*a^3*ln(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4947, 272, 46}

$$\int \frac{\cot^{-1}(ax)}{x^4} \, dx = \frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log \left(a^2x^2 + 1\right) - \frac{\cot^{-1}(ax)}{3x^3} + \frac{a}{6x^2}$$

```
[In] Int[ArcCot[a*x]/x^4,x]
```

```
[Out] a/(6*x^2) - ArcCot[a*x]/(3*x^3) + (a^3*Log[x])/3 - (a^3*Log[1 + a^2*x^2])/6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax)}{3x^3} - \frac{1}{3}a\int \frac{1}{x^3(1+a^2x^2)}\,dx \\
&= -\frac{\cot^{-1}(ax)}{3x^3} - \frac{1}{6}a\text{Subst}\left(\int \frac{1}{x^2(1+a^2x)}\,dx, x, x^2\right) \\
&= -\frac{\cot^{-1}(ax)}{3x^3} - \frac{1}{6}a\text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{a^2}{x} + \frac{a^4}{1+a^2x}\right)\,dx, x, x^2\right) \\
&= \frac{a}{6x^2} - \frac{\cot^{-1}(ax)}{3x^3} + \frac{1}{3}a^3\log(x) - \frac{1}{6}a^3\log\left(1+a^2x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = -\frac{\cot^{-1}(ax)}{3x^3} - \frac{1}{6}a\left(-\frac{1}{x^2} - 2a^2\log(x) + a^2\log\left(1+a^2x^2\right)\right)$$

```
[In] Integrate[ArcCot[a*x]/x^4,x]
[Out] -1/3*ArcCot[a*x]/x^3 - (a*(-x^(-2) - 2*a^2*Log[x] + a^2*Log[1 + a^2*x^2]))/
6
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
parts	$-\frac{\operatorname{arccot}(ax)}{3x^3}-\frac{a\left(-\frac{1}{2x^2}-a^2\ln(x)+\frac{a^2\ln\left(a^2x^2+1\right)}{2}\right)}{3}$	42
derivativedivides	$a^3\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}+\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}-\frac{\ln\left(a^2x^2+1\right)}{6}\right)$	44
default	$a^3\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}+\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}-\frac{\ln\left(a^2x^2+1\right)}{6}\right)$	44
parallelrisch	$\frac{2a^3\ln(x)x^3-a^3\ln\left(a^2x^2+1\right)x^3-a^3x^3+ax-2\operatorname{arccot}(ax)}{6x^3}$	52
risch	$-\frac{i\ln(iax+1)}{6x^3}-\frac{-2a^3\ln(x)x^3+a^3\ln\left(-a^2x^2-1\right)x^3-i\ln(-iax+1)-ax+\pi}{6x^3}$	66

```
[In] int(arccot(a*x)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*arccot(a*x)/x^3-1/3*a*(-1/2/x^2-a^2*ln(x)+1/2*a^2*ln(a^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int\frac{\cot^{-1}(ax)}{x^4}\,dx=-\frac{a^3x^3\log\left(a^2x^2+1\right)-2\,a^3x^3\log\left(x\right)-ax+2\operatorname{arccot}\left(ax\right)}{6\,x^3}$$

```
[In] integrate(arccot(a*x)/x^4,x, algorithm="fricas")
[Out] -1/6*(a^3*x^3*log(a^2*x^2 + 1) - 2*a^3*x^3*log(x) - a*x + 2*arccot(a*x))/x^
3
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int\frac{\cot^{-1}(ax)}{x^4}\,dx=\frac{a^3\log\left(x\right)}{3}-\frac{a^3\log\left(a^2x^2+1\right)}{6}+\frac{a}{6x^2}-\frac{\operatorname{acot}\left(ax\right)}{3x^3}$$

```
[In] integrate(acot(a*x)/x**4,x)
[Out] a**3*log(x)/3 - a**3*log(a**2*x**2 + 1)/6 + a/(6*x**2) - acot(a*x)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = -\frac{1}{6}\left(a^2\log\left(a^2x^2+1\right) - a^2\log\left(x^2\right) - \frac{1}{x^2}\right)a - \frac{\operatorname{arccot}(ax)}{3\,x^3}$$

```
[In] integrate(arccot(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(a^2*log(a^2*x^2 + 1) - a^2*log(x^2) - 1/x^2)*a - 1/3*arccot(a*x)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = \frac{1}{6}\left(a^2\left(\frac{1}{a^2x^2} - \log\left(\frac{1}{a^2x^2}+1\right)\right) - \frac{2\arctan\left(\frac{1}{ax}\right)}{ax^3}\right)a$$

```
[In] integrate(arccot(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(a^2*(1/(a^2*x^2) - log(1/(a^2*x^2) + 1)) - 2*arctan(1/(a*x))/(a*x^3))*
a
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = \begin{cases} -\frac{\pi}{6\,x^3} & \text{if } a=0 \\ \frac{a^4\,\ln(x) - \frac{a^4\,\ln\left(a^2\,x^2+1\right)}{2} + \frac{a^2}{2\,x^2}}{3\,a} - \frac{\mathrm{acot}(a\,x)}{3\,x^3} & \text{if } a\neq 0 \end{cases}$$

```
[In] int(acot(a*x)/x^4,x)
```

```
[Out] piecewise(a == 0, -pi/(6*x^3), a ~= 0, (a^4*log(x) - (a^4*log(a^2*x^2 + 1))
/2 + a^2/(2*x^2))/(3*a) - acot(a*x)/(3*x^3))
```

3.11 $\int \frac{\cot^{-1}(ax)}{x^5} dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{\cot^{-1}(ax)}{x^5} dx = \frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\cot^{-1}(ax)}{4x^4} - \frac{1}{4}a^4 \arctan(ax)$$

```
[Out] 1/12*a/x^3-1/4*a^3/x-1/4*arccot(a*x)/x^4-1/4*a^4*arctan(a*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4947, 331, 209}

$$\int \frac{\cot^{-1}(ax)}{x^5} dx = -\frac{1}{4}a^4 \arctan(ax) - \frac{a^3}{4x} - \frac{\cot^{-1}(ax)}{4x^4} + \frac{a}{12x^3}$$

```
[In] Int[ArcCot[a*x]/x^5,x]
```

```
[Out] a/(12*x^3) - a^3/(4*x) - ArcCot[a*x]/(4*x^4) - (a^4*ArcTan[a*x])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cot^{-1}(ax)}{4x^4} - \frac{1}{4}a\int \frac{1}{x^4(1+a^2x^2)}\,dx\\ &= \frac{a}{12x^3} - \frac{\cot^{-1}(ax)}{4x^4} + \frac{1}{4}a^3\int \frac{1}{x^2(1+a^2x^2)}\,dx\\ &= \frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\cot^{-1}(ax)}{4x^4} - \frac{1}{4}a^5\int \frac{1}{1+a^2x^2}\,dx\\ &= \frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\cot^{-1}(ax)}{4x^4} - \frac{1}{4}a^4\arctan(ax)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = -\frac{\cot^{-1}(ax)}{4x^4} + \frac{a\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-a^2x^2\right)}{12x^3}$$

```
[In] Integrate[ArcCot[a*x]/x^5,x]
[Out] -1/4*ArcCot[a*x]/x^4 + (a*Hypergeometric2F1[-3/2, 1, -1/2, -(a^2*x^2)])/(12
*x^3)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
parts	$-\frac{\operatorname{arccot}(ax)}{4x^4}-\frac{a\left(-\frac{1}{3x^3}+\frac{a^2}{x}+a^3\arctan(ax)\right)}{4}$	35
parallelrisch	$\frac{3a^4x^4\operatorname{arccot}(ax)-3a^3x^3+ax-3\operatorname{arccot}(ax)}{12x^4}$	36
derivativedivides	$a^4\left(-\frac{\operatorname{arccot}(ax)}{4a^4x^4}-\frac{\arctan(ax)}{4}+\frac{1}{12a^3x^3}-\frac{1}{4ax}\right)$	40
default	$a^4\left(-\frac{\operatorname{arccot}(ax)}{4a^4x^4}-\frac{\arctan(ax)}{4}+\frac{1}{12a^3x^3}-\frac{1}{4ax}\right)$	40
risch	$-\frac{i\ln(iax+1)}{8x^4}-\frac{-3ia^4\ln(-ax+i)x^4+3ia^4\ln(-ax-i)x^4+6a^3x^3-3i\ln(-iax+1)-2ax+3\pi}{24x^4}$	82

```
[In] int(arccot(a*x)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*arccot(a*x)/x^4-1/4*a*(-1/3/x^3+a^2/x+a^3*arctan(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx=-\frac{3\,a^3x^3-ax-3\,(a^4x^4-1)\operatorname{arccot}(ax)}{12\,x^4}$$

```
[In] integrate(arccot(a*x)/x^5,x, algorithm="fricas")
[Out] -1/12*(3*a^3*x^3 - a*x - 3*(a^4*x^4 - 1)*arccot(a*x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx=\frac{a^4\operatorname{acot}(ax)}{4}-\frac{a^3}{4x}+\frac{a}{12x^3}-\frac{\operatorname{acot}(ax)}{4x^4}$$

```
[In] integrate(acot(a*x)/x**5,x)
[Out] a**4*acot(a*x)/4 - a**3/(4*x) + a/(12*x**3) - acot(a*x)/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = -\frac{1}{12}\left(3\,a^3\arctan\left(ax\right) + \frac{3\,a^2x^2 - 1}{x^3}\right)a - \frac{\operatorname{arccot}\left(ax\right)}{4\,x^4}$$

```
[In] integrate(arccot(a*x)/x^5,x, algorithm="maxima")
[Out] -1/12*(3*a^3*arctan(a*x) + (3*a^2*x^2 - 1)/x^3)*a - 1/4*arccot(a*x)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = -\frac{1}{12}\left(a^3\left(\frac{3}{ax} - \frac{1}{a^3x^3} - 3\arctan\left(\frac{1}{ax}\right)\right) + \frac{3\arctan\left(\frac{1}{ax}\right)}{ax^4}\right)a$$

```
[In] integrate(arccot(a*x)/x^5,x, algorithm="giac")
[Out] -1/12*(a^3*(3/(a*x) - 1/(a^3*x^3) - 3*arctan(1/(a*x))) + 3*arctan(1/(a*x))/
(a*x^4))*a
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = \begin{cases} -\frac{\pi}{8\,x^4} & \text{if } a=0 \\ -\frac{a^4\,\mathrm{atan}(a\,x)}{4} - \frac{\frac{\mathrm{acot}(a\,x)}{4} - \frac{a\,x}{12} + \frac{a^3\,x^3}{4}}{x^4} & \text{if } a\neq 0 \end{cases}$$

```
[In] int(acot(a*x)/x^5,x)
[Out] piecewise(a == 0, -pi/(8*x^4), a ~= 0, - (a^4*atan(a*x))/4 - (acot(a*x)/4 -
 (a*x)/12 + (a^3*x^3)/4)/x^4)
```

3.12 $\int x^5 \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 104

$$\int x^5 \cot^{-1}(ax)^2 \, dx = -\frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x \cot^{-1}(ax)}{3a^5} - \frac{x^3 \cot^{-1}(ax)}{9a^3} + \frac{x^5 \cot^{-1}(ax)}{15a} + \frac{\cot^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \cot^{-1}(ax)^2 + \frac{23 \log(1 + a^2x^2)}{90a^6}$$

```
[Out] -4/45*x^2/a^4+1/60*x^4/a^2+1/3*x*arccot(a*x)/a^5-1/9*x^3*arccot(a*x)/a^3+1/
15*x^5*arccot(a*x)/a+1/6*arccot(a*x)^2/a^6+1/6*x^6*arccot(a*x)^2+23/90*ln(a
^2*x^2+1)/a^6
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4947, 5037, 272, 45, 4931, 266, 5005}

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \frac{\cot^{-1}(ax)^2}{6a^6} + \frac{x \cot^{-1}(ax)}{3a^5} - \frac{4x^2}{45a^4} - \frac{x^3 \cot^{-1}(ax)}{9a^3} + \frac{x^4}{60a^2} + \frac{23 \log(a^2x^2 + 1)}{90a^6} + \frac{1}{6}x^6 \cot^{-1}(ax)^2 + \frac{x^5 \cot^{-1}(ax)}{15a}$$

```
[In] Int[x^5*ArcCot[a*x]^2,x]
```

```
[Out] (-4*x^2)/(45*a^4) + x^4/(60*a^2) + (x*ArcCot[a*x])/(3*a^5) - (x^3*ArcCot[a*
x])/(9*a^3) + (x^5*ArcCot[a*x])/(15*a) + ArcCot[a*x]^2/(6*a^6) + (x^6*ArcCo
t[a*x]^2)/6 + (23*Log[1 + a^2*x^2])/(90*a^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\cot^{-1}(ax)^2+\frac{1}{3}a\int\frac{x^6\cot^{-1}(ax)}{1+a^2x^2}\,dx\\
&= \frac{1}{6}x^6\cot^{-1}(ax)^2+\frac{\int x^4\cot^{-1}(ax)\,dx}{3a}-\frac{\int\frac{x^4\cot^{-1}(ax)}{1+a^2x^2}\,dx}{3a}\\
&= \frac{x^5\cot^{-1}(ax)}{15a}+\frac{1}{6}x^6\cot^{-1}(ax)^2+\frac{1}{15}\int\frac{x^5}{1+a^2x^2}\,dx-\frac{\int x^2\cot^{-1}(ax)\,dx}{3a^3}+\frac{\int\frac{x^2\cot^{-1}(ax)}{1+a^2x^2}\,dx}{3a^3}\\
&= -\frac{x^3\cot^{-1}(ax)}{9a^3}+\frac{x^5\cot^{-1}(ax)}{15a}+\frac{1}{6}x^6\cot^{-1}(ax)^2+\frac{1}{30}\text{Subst}\left(\int\frac{x^2}{1+a^2x}\,dx,x,x^2\right)\\
&\quad+\frac{\int\cot^{-1}(ax)\,dx}{3a^5}-\frac{\int\frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx}{3a^5}-\frac{\int\frac{x^3}{1+a^2x^2}\,dx}{9a^2}\\
&= \frac{x\cot^{-1}(ax)}{3a^5}-\frac{x^3\cot^{-1}(ax)}{9a^3}+\frac{x^5\cot^{-1}(ax)}{15a}+\frac{\cot^{-1}(ax)^2}{6a^6}\\
&\quad+\frac{1}{6}x^6\cot^{-1}(ax)^2+\frac{1}{30}\text{Subst}\left(\int\left(-\frac{1}{a^4}+\frac{x}{a^2}+\frac{1}{a^4(1+a^2x)}\right)dx,x,x^2\right)\\
&\quad+\frac{\int\frac{x}{1+a^2x^2}\,dx}{3a^4}-\frac{\text{Subst}\left(\int\frac{x}{1+a^2x}\,dx,x,x^2\right)}{18a^2}\\
&= -\frac{x^2}{30a^4}+\frac{x^4}{60a^2}+\frac{x\cot^{-1}(ax)}{3a^5}-\frac{x^3\cot^{-1}(ax)}{9a^3}+\frac{x^5\cot^{-1}(ax)}{15a}+\frac{\cot^{-1}(ax)^2}{6a^6}\\
&\quad+\frac{1}{6}x^6\cot^{-1}(ax)^2+\frac{\log(1+a^2x^2)}{5a^6}-\frac{\text{Subst}\left(\int\left(\frac{1}{a^2}-\frac{1}{a^2(1+a^2x)}\right)dx,x,x^2\right)}{18a^2}\\
&= -\frac{4x^2}{45a^4}+\frac{x^4}{60a^2}+\frac{x\cot^{-1}(ax)}{3a^5}-\frac{x^3\cot^{-1}(ax)}{9a^3}+\frac{x^5\cot^{-1}(ax)}{15a}\\
&\quad+\frac{\cot^{-1}(ax)^2}{6a^6}+\frac{1}{6}x^6\cot^{-1}(ax)^2+\frac{23\log(1+a^2x^2)}{90a^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int x^5\cot^{-1}(ax)^2\,dx =\frac{-16a^2x^2+3a^4x^4+4ax(15-5a^2x^2+3a^4x^4)\cot^{-1}(ax)+30(1+a^6x^6)\cot^{-1}(ax)^2+46\log(1+a^2x^2)}{180a^6}$$

```
[In] Integrate[x^5*ArcCot[a*x]^2,x]
```

```
[Out] (-16*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 - 5*a^2*x^2 + 3*a^4*x^4)*ArcCot[a*x] +
 30*(1 + a^6*x^6)*ArcCot[a*x]^2 + 46*Log[1 + a^2*x^2])/(180*a^6)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{30a^6x^6\,\mathrm{arccot}(ax)^2+12a^5x^5\,\mathrm{arccot}(ax)+3a^4x^4-20a^3x^3\,\mathrm{arccot}(ax)+16-16a^2x^2+60\,\mathrm{arccot}(ax)ax+30\,\mathrm{arccot}(ax)^2+46}{180a^6}$
parts	$\frac{x^6\,\mathrm{arccot}(ax)^2}{6}+\frac{\frac{a^5x^5\,\mathrm{arccot}(ax)}{5}-\frac{a^3x^3\,\mathrm{arccot}(ax)}{3}+\mathrm{arccot}(ax)ax-\mathrm{arccot}(ax)\arctan(ax)+\frac{a^4x^4}{20}-\frac{4a^2x^2}{15}+\frac{23\ln\left(a^2x^2+\right.}{30}}{3a^6}$
derivativedivides	$\frac{\frac{a^6x^6\,\mathrm{arccot}(ax)^2}{6}+\frac{a^5x^5\,\mathrm{arccot}(ax)}{15}-\frac{a^3x^3\,\mathrm{arccot}(ax)}{9}+\frac{\mathrm{arccot}(ax)ax}{3}-\frac{\mathrm{arccot}(ax)\arctan(ax)}{3}+\frac{a^4x^4}{60}-\frac{4a^2x^2}{45}+\frac{23\ln\left(a^2x^2+1\right)}{90}-}{a^6}$
default	$\frac{\frac{a^6x^6\,\mathrm{arccot}(ax)^2}{6}+\frac{a^5x^5\,\mathrm{arccot}(ax)}{15}-\frac{a^3x^3\,\mathrm{arccot}(ax)}{9}+\frac{\mathrm{arccot}(ax)ax}{3}-\frac{\mathrm{arccot}(ax)\arctan(ax)}{3}+\frac{a^4x^4}{60}-\frac{4a^2x^2}{45}+\frac{23\ln\left(a^2x^2+1\right)}{90}-}{a^6}$
risch	$-\frac{(a^6x^6+1)\ln(iax+1)^2}{24a^6}+\frac{(15i\pi\,a^6x^6+15x^6\ln(-iax+1)a^6+6ia^5x^5-10ia^3x^3+30iax+15\ln(-iax+1))\ln(iax+1)}{180a^6}+$

```
[In] int(x^5*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/180*(30*a^6*x^6*arccot(a*x)^2+12*a^5*x^5*arccot(a*x)+3*a^4*x^4-20*a^3*x^3
*arccot(a*x)+16-16*a^2*x^2+60*arccot(a*x)*a*x+30*arccot(a*x)^2+46*ln(a^2*x^
2+1))/a^6
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

$$\int x^5\cot^{-1}(ax)^2\,dx$$
$$=\frac{3\,a^4x^4-16\,a^2x^2+30\,(a^6x^6+1)\,\mathrm{arccot}\,(ax)^2+4\,(3\,a^5x^5-5\,a^3x^3+15\,ax)\,\mathrm{arccot}\,(ax)+46\,\log\left(a^2x^2+1\right.}{180\,a^6}$$

```
[In] integrate(x^5*arccot(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(3*a^4*x^4 - 16*a^2*x^2 + 30*(a^6*x^6 + 1)*arccot(a*x)^2 + 4*(3*a^5*x
^5 - 5*a^3*x^3 + 15*a*x)*arccot(a*x) + 46*log(a^2*x^2 + 1))/a^6
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int x^5 \cot^{-1}(ax)^2 \, dx$$
$$= \begin{cases} \frac{x^6 \operatorname{acot}^2(ax)}{6} + \frac{x^5 \operatorname{acot}(ax)}{15a} + \frac{x^4}{60a^2} - \frac{x^3 \operatorname{acot}(ax)}{9a^3} - \frac{4x^2}{45a^4} + \frac{x \operatorname{acot}(ax)}{3a^5} + \frac{23 \log(a^2x^2+1)}{90a^6} + \frac{\operatorname{acot}^2(ax)}{6a^6} & \text{for } a \neq 0 \\ \frac{\pi^2 x^6}{24} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5*acot(a*x)**2,x)
```

```
[Out] Piecewise((x**6*acot(a*x)**2/6 + x**5*acot(a*x)/(15*a) + x**4/(60*a**2) - x
**3*acot(a*x)/(9*a**3) - 4*x**2/(45*a**4) + x*acot(a*x)/(3*a**5) + 23*log(a
**2*x**2 + 1)/(90*a**6) + acot(a*x)**2/(6*a**6), Ne(a, 0)), (pi**2*x**6/24,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \frac{1}{6} x^6 \operatorname{arccot}(ax)^2 + \frac{1}{45} a \left(\frac{3a^4x^5 - 5a^2x^3 + 15x}{a^6} - \frac{15 \arctan(ax)}{a^7} \right) \operatorname{arccot}(ax) + \frac{3a^4x^4 - 16a^2x^2 - 30 \arctan(ax)^2 + 46 \log(a^2x^2 + 1)}{180a^6}$$

```
[In] integrate(x^5*arccot(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccot(a*x)^2 + 1/45*a*((3*a^4*x^5 - 5*a^2*x^3 + 15*x)/a^6 - 15*arc
tan(a*x)/a^7)*arccot(a*x) + 1/180*(3*a^4*x^4 - 16*a^2*x^2 - 30*arctan(a*x)^
2 + 46*log(a^2*x^2 + 1))/a^6
```

Giac [F]

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \int x^5 \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^5*arccot(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*arccot(a*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

$$\int x^5 \cot^{-1}(ax)^2 \, dx$$
$$= \frac{x^6 \operatorname{acot}(a\,x)^2}{6} + \frac{\frac{23\,\ln(a^2\,x^2+1)}{90} - \frac{4\,a^2\,x^2}{45} + \frac{a^4\,x^4}{60} + \frac{\operatorname{acot}(a\,x)^2}{6} - \frac{a^3\,x^3\,\operatorname{acot}(a\,x)}{9} + \frac{a^5\,x^5\,\operatorname{acot}(a\,x)}{15} + \frac{a\,x\,\operatorname{acot}(a\,x)}{3}}{a^6}$$

```
[In] int(x^5*acot(a*x)^2,x)
```

```
[Out] (x^6*acot(a*x)^2)/6 + ((23*log(a^2*x^2 + 1))/90 - (4*a^2*x^2)/45 + (a^4*x^4
)/60 + acot(a*x)^2/6 - (a^3*x^3*acot(a*x))/9 + (a^5*x^5*acot(a*x))/15 + (a*
x*acot(a*x))/3)/a^6
```

3.13 $\int x^4 \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 135

$$\int x^4 \cot^{-1}(ax)^2 \, dx = -\frac{3x}{10a^4} + \frac{x^3}{30a^2} - \frac{x^2 \cot^{-1}(ax)}{5a^3} + \frac{x^4 \cot^{-1}(ax)}{10a} + \frac{i \cot^{-1}(ax)^2}{5a^5} + \frac{1}{5} x^5 \cot^{-1}(ax)^2 + \frac{3 \arctan(ax)}{10a^5} - \frac{2 \cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{5a^5} + \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{5a^5}$$

```
[Out] -3/10*x/a^4+1/30*x^3/a^2-1/5*x^2*arccot(a*x)/a^3+1/10*x^4*arccot(a*x)/a+1/5
*I*arccot(a*x)^2/a^5+1/5*x^5*arccot(a*x)^2+3/10*arctan(a*x)/a^5-2/5*arccot(
a*x)*ln(2/(1+I*a*x))/a^5+1/5*I*polylog(2,1-2/(1+I*a*x))/a^5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {4947, 5037, 308, 209, 327, 5041, 4965, 2449, 2352}

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \frac{3 \arctan(ax)}{10a^5} + \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{iax+1}\right)}{5a^5} + \frac{i \cot^{-1}(ax)^2}{5a^5} - \frac{2 \log\left(\frac{2}{1+iax}\right) \cot^{-1}(ax)}{5a^5} - \frac{3x}{10a^4} - \frac{x^2 \cot^{-1}(ax)}{5a^3} + \frac{x^3}{30a^2} + \frac{1}{5} x^5 \cot^{-1}(ax)^2 + \frac{x^4 \cot^{-1}(ax)}{10a}$$

```
[In] Int[x^4*ArcCot[a*x]^2,x]
```

```
[Out] (-3*x)/(10*a^4) + x^3/(30*a^2) - (x^2*ArcCot[a*x])/(5*a^3) + (x^4*ArcCot[a*
x])/(10*a) + ((I/5)*ArcCot[a*x]^2)/a^5 + (x^5*ArcCot[a*x]^2)/5 + (3*ArcTan[
```

```
a*x])/(10*a^5) - (2*ArcCot[a*x]*Log[2/(1 + I*a*x)])/(5*a^5) + ((I/5)*PolyLo
g[2, 1 - 2/(1 + I*a*x)])/a^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{1}{5}(2a)\int\frac{x^5\cot^{-1}(ax)}{1+a^2x^2}\,dx\\
&= \frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2\int x^3\cot^{-1}(ax)\,dx}{5a}-\frac{2\int\frac{x^3\cot^{-1}(ax)}{1+a^2x^2}\,dx}{5a}\\
&= \frac{x^4\cot^{-1}(ax)}{10a}+\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{1}{10}\int\frac{x^4}{1+a^2x^2}\,dx-\frac{2\int x\cot^{-1}(ax)\,dx}{5a^3}+\frac{2\int\frac{x\cot^{-1}(ax)}{1+a^2x^2}\,dx}{5a^3}\\
&= -\frac{x^2\cot^{-1}(ax)}{5a^3}+\frac{x^4\cot^{-1}(ax)}{10a}+\frac{i\cot^{-1}(ax)^2}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^2\\
&\quad+\frac{1}{10}\int\left(-\frac{1}{a^4}+\frac{x^2}{a^2}+\frac{1}{a^4\left(1+a^2x^2\right)}\right)dx-\frac{2\int\frac{\cot^{-1}(ax)}{i-ax}\,dx}{5a^4}-\frac{\int\frac{x^2}{1+a^2x^2}\,dx}{5a^2}\\
&= -\frac{3x}{10a^4}+\frac{x^3}{30a^2}-\frac{x^2\cot^{-1}(ax)}{5a^3}+\frac{x^4\cot^{-1}(ax)}{10a}+\frac{i\cot^{-1}(ax)^2}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^2\\
&\quad-\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{5a^5}+\frac{\int\frac{1}{1+a^2x^2}\,dx}{10a^4}+\frac{\int\frac{1}{1+a^2x^2}\,dx}{5a^4}-\frac{2\int\frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{5a^4}\\
&= -\frac{3x}{10a^4}+\frac{x^3}{30a^2}-\frac{x^2\cot^{-1}(ax)}{5a^3}+\frac{x^4\cot^{-1}(ax)}{10a}+\frac{i\cot^{-1}(ax)^2}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^2\\
&\quad+\frac{3\arctan(ax)}{10a^5}-\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{5a^5}+\frac{(2i)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+iax}\right)}{5a^5}\\
&= -\frac{3x}{10a^4}+\frac{x^3}{30a^2}-\frac{x^2\cot^{-1}(ax)}{5a^3}+\frac{x^4\cot^{-1}(ax)}{10a}+\frac{i\cot^{-1}(ax)^2}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^2\\
&\quad+\frac{3\arctan(ax)}{10a^5}-\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{5a^5}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int x^4 \cot^{-1}(ax)^2 \, dx$$
$$= \frac{ax(-9+a^2x^2)+6(i+a^5x^5)\cot^{-1}(ax)^2+3\cot^{-1}(ax)\left(-3-2a^2x^2+a^4x^4-4\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)+}{30a^5}$$

```
[In] Integrate[x^4*ArcCot[a*x]^2,x]
```

```
[Out] (a*x*(-9 + a^2*x^2) + 6*(I + a^5*x^5)*ArcCot[a*x]^2 + 3*ArcCot[a*x]*(-3 - 2
*a^2*x^2 + a^4*x^4 - 4*Log[1 - E^((2*I)*ArcCot[a*x])]) + (6*I)*PolyLog[2, E
^((2*I)*ArcCot[a*x])])/(30*a^5)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.52

method	result
parts	$\frac{x^5 \operatorname{arccot}(ax)^2}{5}+\frac{\frac{a^4x^4\operatorname{arccot}(ax)}{10}-\frac{\operatorname{arccot}(ax)a^2x^2}{5}+\frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{5}+\frac{a^3x^3}{30}-\frac{3ax}{10}+\frac{3\arctan(ax)}{10}-\frac{i\left(\ln(ax-i)\ln\left(a^2x\right.\right.}{}}{}$
derivativedivides	$\frac{\frac{a^5x^5\operatorname{arccot}(ax)^2}{5}+\frac{a^4x^4\operatorname{arccot}(ax)}{10}-\frac{\operatorname{arccot}(ax)a^2x^2}{5}+\frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{5}+\frac{a^3x^3}{30}-\frac{3ax}{10}+\frac{3\arctan(ax)}{10}-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2\right.\right.}{}}{}$
default	$\frac{\frac{a^5x^5\operatorname{arccot}(ax)^2}{5}+\frac{a^4x^4\operatorname{arccot}(ax)}{10}-\frac{\operatorname{arccot}(ax)a^2x^2}{5}+\frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{5}+\frac{a^3x^3}{30}-\frac{3ax}{10}+\frac{3\arctan(ax)}{10}-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2\right.\right.}{}}{}$
risch	$\frac{i\pi\ln(iax+1)x^5}{10}-\frac{i\ln(iax+1)x^2}{10a^3}+\frac{i\ln(iax+1)x^4}{20a}-\frac{\pi x^2}{10a^3}+\frac{\pi x^4}{20a}+\frac{23i\ln\left(a^2x^2+1\right)}{150a^5}+\frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{5a^5}-47i$

```
[In] int(x^4*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*arccot(a*x)^2+2/5/a^5*(1/4*a^4*x^4*arccot(a*x)-1/2*arccot(a*x)*a^2*
x^2+1/2*arccot(a*x)*ln(a^2*x^2+1)+1/12*a^3*x^3-3/4*a*x+3/4*arctan(a*x)-1/4*
I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*
ln(-1/2*I*(I+a*x)))+1/4*I*(ln(I+a*x)*ln(a^2*x^2+1)-1/2*ln(I+a*x)^2-dilog(1/
2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))))
```

Fricas [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^4*arccot(a*x)^2,x, algorithm="fricas")
[Out] integral(x^4*arccot(a*x)^2, x)
```

Sympy [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{acot}^2(ax) \, dx$$

```
[In] integrate(x**4*acot(a*x)**2,x)
[Out] Integral(x**4*acot(a*x)**2, x)
```

Maxima [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^4*arccot(a*x)^2,x, algorithm="maxima")
[Out] 1/20*x^5*arctan2(1, a*x)^2 - 1/80*x^5*log(a^2*x^2 + 1)^2 + integrate(1/80*(
60*a^2*x^6*arctan2(1, a*x)^2 + 4*a^2*x^6*log(a^2*x^2 + 1) + 8*a*x^5*arctan2
(1, a*x) + 60*x^4*arctan2(1, a*x)^2 + 5*(a^2*x^6 + x^4)*log(a^2*x^2 + 1)^2)
/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^4*arccot(a*x)^2,x, algorithm="giac")
[Out] integrate(x^4*arccot(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{acot}(a\, x)^2 \, dx$$

```
[In] int(x^4*acot(a*x)^2,x)
[Out] int(x^4*acot(a*x)^2, x)
```

3.14 $\int x^3 \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \frac{x^2}{12a^2} - \frac{x \cot^{-1}(ax)}{2a^3} + \frac{x^3 \cot^{-1}(ax)}{6a} - \frac{\cot^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \cot^{-1}(ax)^2 - \frac{\log(1+a^2x^2)}{3a^4}$$

```
[Out] 1/12*x^2/a^2-1/2*x*arccot(a*x)/a^3+1/6*x^3*arccot(a*x)/a-1/4*arccot(a*x)^2/
a^4+1/4*x^4*arccot(a*x)^2-1/3*ln(a^2*x^2+1)/a^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4947, 5037, 272, 45, 4931, 266, 5005}

$$\int x^3 \cot^{-1}(ax)^2 \, dx = -\frac{\cot^{-1}(ax)^2}{4a^4} - \frac{x \cot^{-1}(ax)}{2a^3} + \frac{x^2}{12a^2} - \frac{\log(a^2x^2+1)}{3a^4} + \frac{1}{4}x^4 \cot^{-1}(ax)^2 + \frac{x^3 \cot^{-1}(ax)}{6a}$$

```
[In] Int[x^3*ArcCot[a*x]^2,x]
```

```
[Out] x^2/(12*a^2) - (x*ArcCot[a*x])/(2*a^3) + (x^3*ArcCot[a*x])/(6*a) - ArcCot[a
*x]^2/(4*a^4) + (x^4*ArcCot[a*x]^2)/4 - Log[1 + a^2*x^2]/(3*a^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x^4\cot^{-1}(ax)^2 + \frac{1}{2}a\int \frac{x^4\cot^{-1}(ax)}{1+a^2x^2}\,dx$$

$$
\begin{aligned}
&= \frac{1}{4}x^4\cot^{-1}(ax)^2 + \frac{\int x^2\cot^{-1}(ax)\,dx}{2a} - \frac{\int \frac{x^2\cot^{-1}(ax)}{1+a^2x^2}\,dx}{2a} \\
&= \frac{x^3\cot^{-1}(ax)}{6a} + \frac{1}{4}x^4\cot^{-1}(ax)^2 + \frac{1}{6}\int \frac{x^3}{1+a^2x^2}\,dx - \frac{\int \cot^{-1}(ax)\,dx}{2a^3} + \frac{\int \frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx}{2a^3} \\
&= -\frac{x\cot^{-1}(ax)}{2a^3} + \frac{x^3\cot^{-1}(ax)}{6a} - \frac{\cot^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^2 \\
&\quad + \frac{1}{12}\mathrm{Subst}\left(\int \frac{x}{1+a^2x}\,dx, x, x^2\right) - \frac{\int \frac{x}{1+a^2x^2}\,dx}{2a^2} \\
&= -\frac{x\cot^{-1}(ax)}{2a^3} + \frac{x^3\cot^{-1}(ax)}{6a} - \frac{\cot^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^2 \\
&\quad - \frac{\log(1+a^2x^2)}{4a^4} + \frac{1}{12}\mathrm{Subst}\left(\int \left(\frac{1}{a^2} - \frac{1}{a^2(1+a^2x)}\right)dx, x, x^2\right) \\
&= \frac{x^2}{12a^2} - \frac{x\cot^{-1}(ax)}{2a^3} + \frac{x^3\cot^{-1}(ax)}{6a} - \frac{\cot^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^2 - \frac{\log(1+a^2x^2)}{3a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$
\begin{aligned}
&\int x^3\cot^{-1}(ax)^2\,dx \\
&= \frac{a^2x^2 + 2ax(-3+a^2x^2)\cot^{-1}(ax) + 3(-1+a^4x^4)\cot^{-1}(ax)^2 - 4\log(1+a^2x^2)}{12a^4}
\end{aligned}
$$

```
[In] Integrate[x^3*ArcCot[a*x]^2,x]
```

```
[Out] (a^2*x^2 + 2*a*x*(-3 + a^2*x^2)*ArcCot[a*x] + 3*(-1 + a^4*x^4)*ArcCot[a*x]^
2 - 4*Log[1 + a^2*x^2])/(12*a^4)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{-3a^4x^4\operatorname{arccot}(ax)^2-2a^3x^3\operatorname{arccot}(ax)-a^2x^2+6\operatorname{arccot}(ax)ax+1+3\operatorname{arccot}(ax)^2+4\ln(a^2x^2+1)}{12a^4}$
parts	$\frac{x^4\operatorname{arccot}(ax)^2}{4}+\frac{\frac{a^3x^3\operatorname{arccot}(ax)}{3}-\operatorname{arccot}(ax)ax+\operatorname{arccot}(ax)\arctan(ax)+\frac{a^2x^2}{6}-\frac{2\ln(a^2x^2+1)}{3}+\frac{\arctan(ax)^2}{2}}{2a^4}$
derivativedivides	$\frac{\frac{a^4x^4\operatorname{arccot}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccot}(ax)}{6}-\frac{\operatorname{arccot}(ax)ax}{2}+\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{a^2x^2}{12}-\frac{\ln(a^2x^2+1)}{3}+\frac{\arctan(ax)^2}{4}}{a^4}$
default	$\frac{\frac{a^4x^4\operatorname{arccot}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccot}(ax)}{6}-\frac{\operatorname{arccot}(ax)ax}{2}+\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{a^2x^2}{12}-\frac{\ln(a^2x^2+1)}{3}+\frac{\arctan(ax)^2}{4}}{a^4}$
risch	$-\frac{(a^4x^4-1)\ln(iax+1)^2}{16a^4}+\frac{(3i\pi a^4x^4+3x^4\ln(-iax+1)a^4+2ia^3x^3-6iax-3\ln(-iax+1))\ln(iax+1)}{24a^4}-\frac{i\pi x^4\ln(-iax}{8}$

```
[In] int(x^3*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-3*a^4*x^4*arccot(a*x)^2-2*a^3*x^3*arccot(a*x)-a^2*x^2+6*arccot(a*x)
*a*x+1+3*arccot(a*x)^2+4*ln(a^2*x^2+1))/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x^3\cot^{-1}(ax)^2\,dx = \frac{a^2x^2+3\,(a^4x^4-1)\operatorname{arccot}(ax)^2+2\,(a^3x^3-3\,ax)\operatorname{arccot}(ax)-4\,\log(a^2x^2+1)}{12\,a^4}$$

```
[In] integrate(x^3*arccot(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(a^2*x^2 + 3*(a^4*x^4 - 1)*arccot(a*x)^2 + 2*(a^3*x^3 - 3*a*x)*arccot(
a*x) - 4*log(a^2*x^2 + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int x^3\cot^{-1}(ax)^2\,dx = \begin{cases} \frac{x^4\operatorname{acot}^2(ax)}{4}+\frac{x^3\operatorname{acot}(ax)}{6a}+\frac{x^2}{12a^2}-\frac{x\operatorname{acot}(ax)}{2a^3}-\frac{\log(a^2x^2+1)}{3a^4}-\frac{\operatorname{acot}^2(ax)}{4a^4} & \text{for } a\neq 0 \\ \frac{\pi^2x^4}{16} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*acot(a*x)**2,x)
```

```
[Out] Piecewise((x**4*acot(a*x)**2/4 + x**3*acot(a*x)/(6*a) + x**2/(12*a**2) - x*
acot(a*x)/(2*a**3) - log(a**2*x**2 + 1)/(3*a**4) - acot(a*x)**2/(4*a**4), N
e(a, 0)), (pi**2*x**4/16, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \frac{1}{4} x^4 \operatorname{arccot}(ax)^2 + \frac{1}{6} a \left(\frac{a^2x^3 - 3x}{a^4} + \frac{3 \arctan(ax)}{a^5} \right) \operatorname{arccot}(ax) + \frac{a^2x^2 + 3 \arctan(ax)^2 - 4 \log(a^2x^2 + 1)}{12 a^4}$$

```
[In] integrate(x^3*arccot(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccot(a*x)^2 + 1/6*a*((a^2*x^3 - 3*x)/a^4 + 3*arctan(a*x)/a^5)*arc
cot(a*x) + 1/12*(a^2*x^2 + 3*arctan(a*x)^2 - 4*log(a^2*x^2 + 1))/a^4
```

Giac [F]

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \int x^3 \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^3*arccot(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccot(a*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \frac{x^4 \operatorname{acot}(a\,x)^2}{4} - \frac{\frac{\ln(a^2\,x^2+1)}{3} - \frac{a^2\,x^2}{12} + \frac{\operatorname{acot}(a\,x)^2}{4} - \frac{a^3\,x^3\,\operatorname{acot}(a\,x)}{6} + \frac{a\,x\,\operatorname{acot}(a\,x)}{2}}{a^4}$$

```
[In] int(x^3*acot(a*x)^2,x)
```

```
[Out] (x^4*acot(a*x)^2)/4 - (log(a^2*x^2 + 1)/3 - (a^2*x^2)/12 + acot(a*x)^2/4 -
(a^3*x^3*acot(a*x))/6 + (a*x*acot(a*x))/2)/a^4
```

3.15 $\int x^2 \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 111

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \frac{x}{3a^2} + \frac{x^2 \cot^{-1}(ax)}{3a} - \frac{i \cot^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \cot^{-1}(ax)^2 - \frac{\arctan(ax)}{3a^3} + \frac{2 \cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{3a^3} - \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{3a^3}$$

```
[Out] 1/3*x/a^2+1/3*x^2*arccot(a*x)/a-1/3*I*arccot(a*x)^2/a^3+1/3*x^3*arccot(a*x)
^2-1/3*arctan(a*x)/a^3+2/3*arccot(a*x)*ln(2/(1+I*a*x))/a^3-1/3*I*polylog(2,
1-2/(1+I*a*x))/a^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4947, 5037, 327, 209, 5041, 4965, 2449, 2352}

$$\int x^2 \cot^{-1}(ax)^2 \, dx = -\frac{\arctan(ax)}{3a^3} - \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{iax+1}\right)}{3a^3} - \frac{i \cot^{-1}(ax)^2}{3a^3} + \frac{2 \log\left(\frac{2}{1+iax}\right) \cot^{-1}(ax)}{3a^3} + \frac{x}{3a^2} + \frac{1}{3}x^3 \cot^{-1}(ax)^2 + \frac{x^2 \cot^{-1}(ax)}{3a}$$

```
[In] Int[x^2*ArcCot[a*x]^2,x]
```

```
[Out] x/(3*a^2) + (x^2*ArcCot[a*x])/(3*a) - ((I/3)*ArcCot[a*x]^2)/a^3 + (x^3*ArcC
ot[a*x]^2)/3 - ArcTan[a*x]/(3*a^3) + (2*ArcCot[a*x]*Log[2/(1 + I*a*x)])/(3*
a^3) - ((I/3)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{1}{3}(2a)\int\frac{x^3\cot^{-1}(ax)}{1+a^2x^2}\,dx\\
&= \frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2\int x\cot^{-1}(ax)\,dx}{3a}-\frac{2\int\frac{x\cot^{-1}(ax)}{1+a^2x^2}\,dx}{3a}\\
&= \frac{x^2\cot^{-1}(ax)}{3a}-\frac{i\cot^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{1}{3}\int\frac{x^2}{1+a^2x^2}\,dx+\frac{2\int\frac{\cot^{-1}(ax)}{i-ax}\,dx}{3a^2}\\
&= \frac{x}{3a^2}+\frac{x^2\cot^{-1}(ax)}{3a}-\frac{i\cot^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\cot^{-1}(ax)^2\\
&\quad+\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{3a^3}-\frac{\int\frac{1}{1+a^2x^2}\,dx}{3a^2}+\frac{2\int\frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{3a^2}\\
&= \frac{x}{3a^2}+\frac{x^2\cot^{-1}(ax)}{3a}-\frac{i\cot^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\cot^{-1}(ax)^2-\frac{\arctan(ax)}{3a^3}\\
&\quad+\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{3a^3}-\frac{(2i)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+iax}\right)}{3a^3}\\
&= \frac{x}{3a^2}+\frac{x^2\cot^{-1}(ax)}{3a}-\frac{i\cot^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\cot^{-1}(ax)^2-\frac{\arctan(ax)}{3a^3}\\
&\quad+\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{3a^3}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{3a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$
\int x^2\cot^{-1}(ax)^2\,dx
= \frac{ax+(-i+a^3x^3)\cot^{-1}(ax)^2+\cot^{-1}(ax)\left(1+a^2x^2+2\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)-i\,\text{PolyLog}\left(2,e^{2i\cot^{-1}(ax}\right.}{3a^3}
$$

```
[In] Integrate[x^2*ArcCot[a*x]^2,x]
```

```
[Out] (a*x + (-I + a^3*x^3)*ArcCot[a*x]^2 + ArcCot[a*x]*(1 + a^2*x^2 + 2*Log[1 -
E^((2*I)*ArcCot[a*x])]) - I*PolyLog[2, E^((2*I)*ArcCot[a*x])])/(3*a^3)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.67

method	result
parts	$\frac{x^3 \operatorname{arccot}(ax)^2}{3} + \frac{\frac{\operatorname{arccot}(ax)a^2x^2}{3} - \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{3} + \frac{ax}{3} - \frac{\arctan(ax)}{3} + \frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right) - \frac{\ln(ax-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(a}{}\right.\right.}{6}}{}$
derivativedivides	$\frac{\frac{a^3x^3 \operatorname{arccot}(ax)^2}{3} + \frac{\operatorname{arccot}(ax)a^2x^2}{3} - \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{3} + \frac{ax}{3} - \frac{\arctan(ax)}{3} + \frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right) - \frac{\ln(ax-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(ax}{2}\right.\right.}{6}}{a^3}$
default	$\frac{\frac{a^3x^3 \operatorname{arccot}(ax)^2}{3} + \frac{\operatorname{arccot}(ax)a^2x^2}{3} - \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{3} + \frac{ax}{3} - \frac{\arctan(ax)}{3} + \frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right) - \frac{\ln(ax-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(ax}{2}\right.\right.}{6}}{a^3}$
risch	$\frac{\pi^2x^3}{12} + \frac{\pi x^2}{6a} + \frac{\ln(iax+1)\ln(-iax+1)x^3}{6} - \frac{i \operatorname{dilog}\left(\frac{1}{2} - \frac{iax}{2}\right)}{3a^3} - \frac{2i\ln\left(a^2x^2+1\right)}{9a^3} + \frac{5i\ln(-iax+1)}{36a^3} + \frac{11\pi}{18a^3} - \frac{\ln(-}{}$

[In] int(x^2*arccot(a*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/3*x^3*arccot(a*x)^2+2/3/a^3*(1/2*arccot(a*x)*a^2*x^2-1/2*arccot(a*x)*ln(a^2*x^2+1)+1/2*a*x-1/2*arctan(a*x)+1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x)))-1/4*I*(ln(I+a*x)*ln(a^2*x^2+1)-1/2*ln(I+a*x)^2-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))))

Fricas [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arccot}(ax)^2 \, dx$$

[In] integrate(x^2*arccot(a*x)^2,x, algorithm="fricas")

[Out] integral(x^2*arccot(a*x)^2, x)

Sympy [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{acot}^2(ax) \, dx$$

[In] integrate(x**2*acot(a*x)**2,x)

[Out] Integral(x**2*acot(a*x)**2, x)

Maxima [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^2*arccot(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/12*x^3*arctan2(1, a*x)^2 - 1/48*x^3*log(a^2*x^2 + 1)^2 + integrate(1/48*(
36*a^2*x^4*arctan2(1, a*x)^2 + 4*a^2*x^4*log(a^2*x^2 + 1) + 8*a*x^3*arctan2
(1, a*x) + 36*x^2*arctan2(1, a*x)^2 + 3*(a^2*x^4 + x^2)*log(a^2*x^2 + 1)^2)
/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^2*arccot(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{acot}(a\,x)^2 \, dx$$

```
[In] int(x^2*acot(a*x)^2,x)
```

```
[Out] int(x^2*acot(a*x)^2, x)
```

3.16 $\int x \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{x \cot^{-1}(ax)}{a} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2} x^2 \cot^{-1}(ax)^2 + \frac{\log(1 + a^2x^2)}{2a^2}$$

```
[Out] x*arccot(a*x)/a+1/2*arccot(a*x)^2/a^2+1/2*x^2*arccot(a*x)^2+1/2*ln(a^2*x^2+
1)/a^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4947, 5037, 4931, 266, 5005}

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{\log(a^2x^2 + 1)}{2a^2} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2} x^2 \cot^{-1}(ax)^2 + \frac{x \cot^{-1}(ax)}{a}$$

```
[In] Int[x*ArcCot[a*x]^2,x]
```

```
[Out] (x*ArcCot[a*x])/a + ArcCot[a*x]^2/(2*a^2) + (x^2*ArcCot[a*x]^2)/2 + Log[1 +
 a^2*x^2]/(2*a^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(ax)^2 + a\int \frac{x^2\cot^{-1}(ax)}{1+a^2x^2}\,dx \\
&= \frac{1}{2}x^2\cot^{-1}(ax)^2 + \frac{\int \cot^{-1}(ax)\,dx}{a} - \frac{\int \frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx}{a} \\
&= \frac{x\cot^{-1}(ax)}{a} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\cot^{-1}(ax)^2 + \int \frac{x}{1+a^2x^2}\,dx \\
&= \frac{x\cot^{-1}(ax)}{a} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\cot^{-1}(ax)^2 + \frac{\log\left(1+a^2x^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{2ax \cot^{-1}(ax) + (1 + a^2x^2) \cot^{-1}(ax)^2 + \log(1 + a^2x^2)}{2a^2}$$

```
[In] Integrate[x*ArcCot[a*x]^2,x]
```

```
[Out] (2*a*x*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a*x]^2 + Log[1 + a^2*x^2])/(2*a^2
)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{a^2x^2 \operatorname{arccot}(ax)^2+2 \operatorname{arccot}(ax)ax+\operatorname{arccot}(ax)^2+\ln(a^2x^2+1)}{2a^2}$
parts	$\frac{x^2 \operatorname{arccot}(ax)^2}{2}+\frac{-\operatorname{arccot}(ax)\arctan(ax)+\operatorname{arccot}(ax)ax+\frac{\ln\left(a^2x^2+1\right)}{2}-\frac{\arctan(ax)^2}{2}}{a^2}$
derivativedivides	$\frac{\frac{a^2x^2 \operatorname{arccot}(ax)^2}{2}-\operatorname{arccot}(ax)\arctan(ax)+\operatorname{arccot}(ax)ax+\frac{\ln\left(a^2x^2+1\right)}{2}-\frac{\arctan(ax)^2}{2}}{a^2}$
default	$\frac{\frac{a^2x^2 \operatorname{arccot}(ax)^2}{2}-\operatorname{arccot}(ax)\arctan(ax)+\operatorname{arccot}(ax)ax+\frac{\ln\left(a^2x^2+1\right)}{2}-\frac{\arctan(ax)^2}{2}}{a^2}$
risch	$-\frac{(a^2x^2+1)\ln(iax+1)^2}{8a^2}+\frac{(i\pi a^2x^2+x^2\ln(-iax+1)a^2+2iax+\ln(-iax+1))\ln(iax+1)}{4a^2}-\frac{i\pi x^2\ln(-iax+1)}{4}+\frac{\pi^2x^2}{8}$

```
[In] int(x*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*x^2*arccot(a*x)^2+2*arccot(a*x)*a*x+arccot(a*x)^2+ln(a^2*x^2+1))/a
^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{2\,ax \operatorname{arccot}(ax) + (a^2x^2 + 1) \operatorname{arccot}(ax)^2 + \log(a^2x^2 + 1)}{2\,a^2}$$

```
[In] integrate(x*arccot(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x*arccot(a*x) + (a^2*x^2 + 1)*arccot(a*x)^2 + log(a^2*x^2 + 1))/a^
2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int x\cot^{-1}(ax)^2\,dx = \begin{cases} \frac{x^2\operatorname{acot}^2(ax)}{2} + \frac{x\operatorname{acot}(ax)}{a} + \frac{\log(a^2x^2+1)}{2a^2} + \frac{\operatorname{acot}^2(ax)}{2a^2} & \text{for } a \neq 0 \\ \frac{\pi^2x^2}{8} & \text{otherwise} \end{cases}$$

[In] integrate(x*acot(a*x)**2,x)

[Out] Piecewise((x**2*acot(a*x)**2/2 + x*acot(a*x)/a + log(a**2*x**2 + 1)/(2*a**2
) + acot(a*x)**2/(2*a**2), Ne(a, 0)), (pi**2*x**2/8, True))

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int x\cot^{-1}(ax)^2\,dx = \frac{1}{2}x^2\operatorname{arccot}(ax)^2 + a\left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right)\operatorname{arccot}(ax) - \frac{\arctan(ax)^2 - \log(a^2x^2+1)}{2a^2}$$

[In] integrate(x*arccot(a*x)^2,x, algorithm="maxima")

[Out] 1/2*x^2*arccot(a*x)^2 + a*(x/a^2 - arctan(a*x)/a^3)*arccot(a*x) - 1/2*(arct
an(a*x)^2 - log(a^2*x^2 + 1))/a^2

Giac [F]

$$\int x\cot^{-1}(ax)^2\,dx = \int x\operatorname{arccot}(ax)^2\,dx$$

[In] integrate(x*arccot(a*x)^2,x, algorithm="giac")

[Out] integrate(x*arccot(a*x)^2, x)

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{x^2 \operatorname{acot}(a\,x)^2}{2} + \frac{\frac{\operatorname{acot}(a\,x)^2}{2} + a\,x\operatorname{acot}(a\,x) + \frac{\ln(a^2\,x^2+1)}{2}}{a^2}$$

```
[In] int(x*acot(a*x)^2,x)
```

```
[Out] (x^2*acot(a*x)^2)/2 + (log(a^2*x^2 + 1)/2 + acot(a*x)^2/2 + a*x*acot(a*x))/
a^2
```

3.17 $\int \cot^{-1}(ax)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 67

$$\int \cot^{-1}(ax)^2\,dx = \frac{i\cot^{-1}(ax)^2}{a} + x\cot^{-1}(ax)^2 - \frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a} + \frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a}$$

```
[Out] I*arccot(a*x)^2/a+x*arccot(a*x)^2-2*arccot(a*x)*ln(2/(1+I*a*x))/a+I*polylog
(2,1-2/(1+I*a*x))/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4931, 5041, 4965, 2449, 2352}

$$\int \cot^{-1}(ax)^2\,dx = \frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a} + x\cot^{-1}(ax)^2 + \frac{i\cot^{-1}(ax)^2}{a} - \frac{2\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}$$

```
[In] Int[ArcCot[a*x]^2,x]
```

```
[Out] (I*ArcCot[a*x]^2)/a + x*ArcCot[a*x]^2 - (2*ArcCot[a*x]*Log[2/(1 + I*a*x)])/
a + (I*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\cot^{-1}(ax)^2 + (2a)\int \frac{x\cot^{-1}(ax)}{1+a^2x^2}\,dx \\
&= \frac{i\cot^{-1}(ax)^2}{a} + x\cot^{-1}(ax)^2 - 2\int \frac{\cot^{-1}(ax)}{i-ax}\,dx \\
&= \frac{i\cot^{-1}(ax)^2}{a} + x\cot^{-1}(ax)^2 - \frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a} - 2\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx \\
&= \frac{i\cot^{-1}(ax)^2}{a} + x\cot^{-1}(ax)^2 - \frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a} + \frac{(2i)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+iax}\right)}{a} \\
&= \frac{i\cot^{-1}(ax)^2}{a} + x\cot^{-1}(ax)^2 - \frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a} + \frac{i\,\text{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \cot^{-1}(ax)^2\,dx$$
$$= \frac{\cot^{-1}(ax)\left((i+ax)\cot^{-1}(ax)-2\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)+i\,\mathrm{PolyLog}\left(2,e^{2i\cot^{-1}(ax)}\right)}{a}$$

```
[In] Integrate[ArcCot[a*x]^2,x]
```

```
[Out] (ArcCot[a*x]*((I + a*x)*ArcCot[a*x] - 2*Log[1 - E^((2*I)*ArcCot[a*x])]) + I
*PolyLog[2, E^((2*I)*ArcCot[a*x])])/a
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.94

method	result
derivativedivides	$\frac{\operatorname{arccot}(ax)^2(ax-i)-2\operatorname{arccot}(ax)\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-2\operatorname{arccot}(ax)\ln\left(1+\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+2i\operatorname{arccot}(ax)^2+2i\operatorname{polylog}\left(2,\right.}{a}$
default	$\frac{\operatorname{arccot}(ax)^2(ax-i)-2\operatorname{arccot}(ax)\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-2\operatorname{arccot}(ax)\ln\left(1+\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+2i\operatorname{arccot}(ax)^2+2i\operatorname{polylog}\left(2,\right.}{a}$
risch	$\frac{i\pi^2}{4a}-\frac{i\ln(-iax+1)\pi x}{2}+\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{a}-\frac{i\ln(-iax+1)^2}{4a}+\frac{i\ln(iax+1)^2}{4a}+\frac{i\ln(a^2x^2+1)}{2a}+$

```
[In] int(arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccot(a*x)^2*(a*x-I)-2*arccot(a*x)*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))-2*
arccot(a*x)*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*arccot(a*x)^2+2*I*polylog(2
,(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \cot^{-1}(ax)^2 \, dx = \int \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(arccot(a*x)^2,x, algorithm="fricas")
[Out] integral(arccot(a*x)^2, x)
```

Sympy [F]

$$\int \cot^{-1}(ax)^2 \, dx = \int \operatorname{acot}^2(ax) \, dx$$

```
[In] integrate(acot(a*x)**2,x)
[Out] Integral(acot(a*x)**2, x)
```

Maxima [F]

$$\int \cot^{-1}(ax)^2 \, dx = \int \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(arccot(a*x)^2,x, algorithm="maxima")
[Out] 1/4*x*arctan2(1, a*x)^2 + 12*a^2*integrate(1/16*x^2*arctan(1/(a*x))^2/(a^2*
x^2 + 1), x) + a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
+ 4*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 1/16*x*log(
a^2*x^2 + 1)^2 + 1/4*arctan(a*x)^3/a + 3/4*arctan(a*x)^2*arctan(1/(a*x))/a
+ 3/4*arctan(a*x)*arctan(1/(a*x))^2/a + 8*a*integrate(1/16*x*arctan(1/(a*x)
)/(a^2*x^2 + 1), x) + integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int \cot^{-1}(ax)^2 \, dx = \int \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(arccot(a*x)^2,x, algorithm="giac")
[Out] integrate(arccot(a*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \cot^{-1}(ax)^2\,dx = \frac{-2\,\ln\left(1-\mathrm{e}^{\mathrm{acot}(a\,x)\,2\mathrm{i}}\right)\,\mathrm{acot}(a\,x)+\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{acot}(a\,x)\,2\mathrm{i}}\right)\,1\mathrm{i}+\mathrm{acot}(a\,x)^2\,1\mathrm{i}}{a} + x\,\mathrm{acot}(a\,x)^2$$

```
[In] int(acot(a*x)^2,x)
```

```
[Out] (polylog(2, exp(acot(a*x)*2i))*1i - 2*log(1 - exp(acot(a*x)*2i))*acot(a*x)
+ acot(a*x)^2*1i)/a + x*acot(a*x)^2
```

3.18 $\int \frac{\cot^{-1}(ax)^2}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 116

$$\int \frac{\cot^{-1}(ax)^2}{x} \, dx = 2\cot^{-1}(ax)^2 \coth^{-1}\left(1-\frac{2}{1+iax}\right) - i\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2i}{i+ax}\right) + i\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right) - \frac{1}{2}\text{PolyLog}\left(3,1-\frac{2i}{i+ax}\right) + \frac{1}{2}\text{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right)$$

```
[Out] 2*arccot(a*x)^2*arccoth(1-2/(1+I*a*x))-I*arccot(a*x)*polylog(2,1-2*I/(I+a*x
))+I*arccot(a*x)*polylog(2,1-2*a*x/(I+a*x))-1/2*polylog(3,1-2*I/(I+a*x))+1/
2*polylog(3,1-2*a*x/(I+a*x))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4943, 5109, 5005, 5113, 6745}

$$\int \frac{\cot^{-1}(ax)^2}{x} \, dx = -\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2i}{ax+i}\right) + \frac{1}{2}\text{PolyLog}\left(3,1-\frac{2ax}{ax+i}\right) - i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax) + i\,\text{PolyLog}\left(2,1-\frac{2ax}{ax+i}\right)\cot^{-1}(ax) + 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

```
[In] Int[ArcCot[a*x]^2/x,x]
```

```
[Out] 2*ArcCot[a*x]^2*ArcCoth[1 - 2/(1 + I*a*x)] - I*ArcCot[a*x]*PolyLog[2, 1 - (
2*I)/(I + a*x)] + I*ArcCot[a*x]*PolyLog[2, 1 - (2*a*x)/(I + a*x)] - PolyLog
[3, 1 - (2*I)/(I + a*x)]/2 + PolyLog[3, 1 - (2*a*x)/(I + a*x)]/2
```

Rule 4943

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Dist[2*b*c*p, Int[(a + b*
ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5109

```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegrand
[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c
, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)
))^2, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)+(4a)\int\frac{\cot^{-1}(ax)\coth^{-1}\left(1-\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx$$

$$
\begin{aligned}
&= 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)\\
&\quad-(2a)\int\frac{\cot^{-1}(ax)\log\left(\frac{2i}{i+ax}\right)}{1+a^2x^2}\,dx+(2a)\int\frac{\cot^{-1}(ax)\log\left(\frac{2ax}{i+ax}\right)}{1+a^2x^2}\,dx\\
&= 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)-i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2i}{i+ax}\right)\\
&\quad+i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right)\\
&\quad-(ia)\int\frac{\mathrm{PolyLog}\left(2,1-\frac{2i}{i+ax}\right)}{1+a^2x^2}\,dx+(ia)\int\frac{\mathrm{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right)}{1+a^2x^2}\,dx\\
&= 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)-i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2i}{i+ax}\right)\\
&\quad+i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right)\\
&\quad-\frac{1}{2}\,\mathrm{PolyLog}\left(3,1-\frac{2i}{i+ax}\right)+\frac{1}{2}\,\mathrm{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$
\begin{aligned}
\int\frac{\cot^{-1}(ax)^2}{x}\,dx &= -\frac{2}{3}i\cot^{-1}(ax)^3-\cot^{-1}(ax)^2\log\left(1-e^{-2i\cot^{-1}(ax)}\right)\\
&\quad+\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)-i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right)\\
&\quad-i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)\\
&\quad-\frac{1}{2}\,\mathrm{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right)+\frac{1}{2}\,\mathrm{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)
\end{aligned}
$$

```
[In] Integrate[ArcCot[a*x]^2/x,x]
```

```
[Out] ((-2*I)/3)*ArcCot[a*x]^3 - ArcCot[a*x]^2*Log[1 - E^((-2*I)*ArcCot[a*x])] +
ArcCot[a*x]^2*Log[1 + E^((2*I)*ArcCot[a*x])] - I*ArcCot[a*x]*PolyLog[2, E^(
(-2*I)*ArcCot[a*x])] - I*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] - P
olyLog[3, E^((-2*I)*ArcCot[a*x])]/2 + PolyLog[3, -E^((2*I)*ArcCot[a*x])]/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.67 (sec) , antiderivative size = 891, normalized size of antiderivative = 7.68

method	result
derivativedivides	$\ln\left(ax\right)\operatorname{arccot}\left(ax\right)^{2}+\frac{i\pi\left(\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}\right)}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)-\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}}\right.\right.}{}$
default	$\ln\left(ax\right)\operatorname{arccot}\left(ax\right)^{2}+\frac{i\pi\left(\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}\right)}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)-\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}}\right.\right.}{}$
parts	Expression too large to display

```
[In] int(arccot(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(a*x)*arccot(a*x)^2+1/2*I*Pi*(csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(1
+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2
*x^2+1)))-csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1
)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I
+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2+csgn(I/((I+a*x)^2/(a^2*
x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3-csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+
(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*
x^2+1)))^2+csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn
(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-csgn(1/((I+a*x)^2/(
a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)
*(1+(I+a*x)^2/(a^2*x^2+1)))^2-1)*arccot(a*x)^2+arccot(a*x)^2*ln((I+a*x)^2/(
a^2*x^2+1)-1)-arccot(a*x)^2*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*arccot(a*x)
*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(I+a*x)/(a^2*x^2+1)^(1/2)
)-arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*arccot(a*x)*polylog(2,-
(I+a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(I+a*x)/(a^2*x^2+1)^(1/2))-I*arccot
(a*x)*polylog(2,-(I+a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(I+a*x)^2/(a^2*x^2+1
))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arccot}\left(ax\right)^{2}}{x}\,dx$$

```
[In] integrate(arccot(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^2}{x} \, dx = \int \frac{\operatorname{acot}^2{(ax)}}{x} \, dx$$

```
[In] integrate(acot(a*x)**2/x,x)
[Out] Integral(acot(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^2}{x} \, dx = \int \frac{\operatorname{arccot}{(ax)}^2}{x} \, dx$$

```
[In] integrate(arccot(a*x)^2/x,x, algorithm="maxima")
[Out] integrate(arccot(a*x)^2/x, x)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^2}{x} \, dx = \int \frac{\operatorname{arccot}{(ax)}^2}{x} \, dx$$

```
[In] integrate(arccot(a*x)^2/x,x, algorithm="giac")
[Out] integrate(arccot(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^2}{x} \, dx = \int \frac{\operatorname{acot}(a\,x)^2}{x} \, dx$$

```
[In] int(acot(a*x)^2/x,x)
[Out] int(acot(a*x)^2/x, x)
```

3.19 $\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = -ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - 2a\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) - ia\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)$$

```
[Out] -I*a*arccot(a*x)^2-arccot(a*x)^2/x-2*a*arccot(a*x)*ln(2-2/(1-I*a*x))-I*a*po
lylog(2,-1+2/(1-I*a*x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4947, 5045, 4989, 2497}

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = -ia\,\mathrm{PolyLog}\left(2,\frac{2}{1-iax}-1\right) - ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - 2a\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)$$

```
[In] Int[ArcCot[a*x]^2/x^2,x]
```

```
[Out] (-I)*a*ArcCot[a*x]^2 - ArcCot[a*x]^2/x - 2*a*ArcCot[a*x]*Log[2 - 2/(1 - I*a
*x)] - I*a*PolyLog[2, -1 + 2/(1 - I*a*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax)^2}{x} - (2a)\int \frac{\cot^{-1}(ax)}{x\,(1+a^2x^2)}\,dx \\
&= -ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - (2ia)\int \frac{\cot^{-1}(ax)}{x(i+ax)}\,dx \\
&= -ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - 2a\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) - (2a^2)\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx \\
&= -ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - 2a\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) \\
&\quad - ia\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = a\left(i\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{ax} - 2\cot^{-1}(ax)\log\left(1+e^{2i\cot^{-1}(ax)}\right) + i\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)\right)$$

```
[In] Integrate[ArcCot[a*x]^2/x^2,x]
```

```
[Out] a*(I*ArcCot[a*x]^2 - ArcCot[a*x]^2/(a*x) - 2*ArcCot[a*x]*Log[1 + E^((2*I)*A
rcCot[a*x])] + I*PolyLog[2, -E^((2*I)*ArcCot[a*x])])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(62) = 124.

Time = 0.57 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.36

method	result
parts	$-\frac{\operatorname{arccot}(ax)^2}{x} - 2a\left(-\frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{2} + \operatorname{arccot}(ax)\ln(ax) - \frac{i\ln(ax)\ln(iax+1)}{2} + \frac{i\ln(ax)\ln(\cdot}{2}\right.$
derivativedivides	$a\left(-\frac{\operatorname{arccot}(ax)^2}{ax} + \operatorname{arccot}(ax)\ln(a^2x^2+1) - 2\operatorname{arccot}(ax)\ln(ax) + i\ln(ax)\ln(iax+1\right.$
default	$a\left(-\frac{\operatorname{arccot}(ax)^2}{ax} + \operatorname{arccot}(ax)\ln(a^2x^2+1) - 2\operatorname{arccot}(ax)\ln(ax) + i\ln(ax)\ln(iax+1\right.$

```
[In] int(arccot(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arccot(a*x)^2/x-2*a*(-1/2*arccot(a*x)*ln(a^2*x^2+1)+arccot(a*x)*ln(a*x)-1/
2*I*ln(a*x)*ln(1+I*a*x)+1/2*I*ln(a*x)*ln(1-I*a*x)-1/2*I*dilog(1+I*a*x)+1/2*
I*dilog(1-I*a*x)+1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*
I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x)))-1/4*I*(ln(I+a*x)*ln(a^2*x^2+1)-1/2
*ln(I+a*x)^2-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arccot}\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arccot(a*x)^2/x^2,x, algorithm="fricas")
[Out] integral(arccot(a*x)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acot}^2\left(ax\right)}{x^2}\,dx$$

```
[In] integrate(acot(a*x)**2/x**2,x)
[Out] Integral(acot(a*x)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arccot}\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arccot(a*x)^2/x^2,x, algorithm="maxima")
[Out] 1/16*(4*(3*a*arctan(a*x)*arctan(1/(a*x))^2 + (arctan(a*x)^3/a + 3*arctan(a*
x)^2*arctan(1/(a*x))/a)*a^2 + 4*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(
a^2*x^4 + x^2), x) - 16*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^4 +
x^2), x) - 32*a*integrate(1/16*x*arctan(1/(a*x))/(a^2*x^4 + x^2), x) + 48*i
ntegrate(1/16*arctan(1/(a*x))^2/(a^2*x^4 + x^2), x) + 4*integrate(1/16*log(
a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x - 4*arctan2(1, a*x)^2 + log(a^2*x^2 +
 1)^2)/x
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arccot}\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arccot(a*x)^2/x^2,x, algorithm="giac")
[Out] integrate(arccot(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\mathrm{acot}(a\,x)^2}{x^2}\,dx$$

```
[In] int(acot(a*x)^2/x^2,x)
```

```
[Out] int(acot(a*x)^2/x^2, x)
```

3.20 $\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = \frac{a\cot^{-1}(ax)}{x} - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{1}{2}a^2\log\left(1+a^2x^2\right)$$

```
[Out] a*arccot(a*x)/x-1/2*a^2*arccot(a*x)^2-1/2*arccot(a*x)^2/x^2+a^2*ln(x)-1/2*a
^2*ln(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4947, 5039, 272, 36, 29, 31, 5005}

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = -\frac{1}{2}a^2\log\left(a^2x^2+1\right) + a^2\log(x) - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + \frac{a\cot^{-1}(ax)}{x}$$

```
[In] Int[ArcCot[a*x]^2/x^3,x]
```

```
[Out] (a*ArcCot[a*x])/x - (a^2*ArcCot[a*x]^2)/2 - ArcCot[a*x]^2/(2*x^2) + a^2*Log
[x] - (a^2*Log[1 + a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax)^2}{2x^2} - a\int \frac{\cot^{-1}(ax)}{x^2\left(1+a^2x^2\right)}\,dx \\
&= -\frac{\cot^{-1}(ax)^2}{2x^2} - a\int \frac{\cot^{-1}(ax)}{x^2}\,dx + a^3\int \frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx \\
&= \frac{a\cot^{-1}(ax)}{x} - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + a^2\int \frac{1}{x\left(1+a^2x^2\right)}\,dx
\end{aligned}$$

$$
\begin{aligned}
&=\frac{a\cot^{-1}(ax)}{x}-\frac{1}{2}a^2\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{2x^2}+\frac{1}{2}a^2\text{Subst}\left(\int\frac{1}{x\left(1+a^2x\right)}\,dx,x,x^2\right)\\
&=\frac{a\cot^{-1}(ax)}{x}-\frac{1}{2}a^2\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{2x^2}\\
&\quad+\frac{1}{2}a^2\text{Subst}\left(\int\frac{1}{x}\,dx,x,x^2\right)-\frac{1}{2}a^4\text{Subst}\left(\int\frac{1}{1+a^2x}\,dx,x,x^2\right)\\
&=\frac{a\cot^{-1}(ax)}{x}-\frac{1}{2}a^2\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{2x^2}+a^2\log(x)-\frac{1}{2}a^2\log\left(1+a^2x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$
\int\frac{\cot^{-1}(ax)^2}{x^3}\,dx=\frac{a\cot^{-1}(ax)}{x}+\frac{\left(-1-a^2x^2\right)\cot^{-1}(ax)^2}{2x^2}+a^2\log(x)-\frac{1}{2}a^2\log\left(1+a^2x^2\right)
$$

```
[In] Integrate[ArcCot[a*x]^2/x^3,x]
```

```
[Out] (a*ArcCot[a*x])/x + ((-1 - a^2*x^2)*ArcCot[a*x]^2)/(2*x^2) + a^2*Log[x] - (
a^2*Log[1 + a^2*x^2])/2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arccot}(ax)^2}{2a^2x^2}+\frac{\operatorname{arccot}(ax)}{ax}+\operatorname{arccot}(ax)\arctan(ax)+\ln(ax)-\frac{\ln(a^2x^2+1)}{2}+\frac{\arctan(ax)^2}{2}\right)$
default	$a^2\left(-\frac{\operatorname{arccot}(ax)^2}{2a^2x^2}+\frac{\operatorname{arccot}(ax)}{ax}+\operatorname{arccot}(ax)\arctan(ax)+\ln(ax)-\frac{\ln(a^2x^2+1)}{2}+\frac{\arctan(ax)^2}{2}\right)$
parallelrisch	$\frac{-a^2x^2\operatorname{arccot}(ax)^2+2a^2\ln(x)x^2-a^2\ln(a^2x^2+1)x^2+2\operatorname{arccot}(ax)ax-\operatorname{arccot}(ax)^2}{2x^2}$
parts	$-\frac{\operatorname{arccot}(ax)^2}{2x^2}-a^2\left(-\frac{\operatorname{arccot}(ax)}{ax}-\operatorname{arccot}(ax)\arctan(ax)-\ln(ax)+\frac{\ln(a^2x^2+1)}{2}-\frac{\arctan(ax)^2}{2}\right.$
risch	$\frac{(a^2x^2+1)\ln(iax+1)^2}{8x^2}-\frac{i\left(-ix^2\ln(-iax+1)a^2-2ax+\pi-i\ln(-iax+1)\right)\ln(iax+1)}{4x^2}-\frac{2ia^2\ln((-\pi a+6ia)x+6+i\pi)\pi x^2}{}$

```
[In] int(arccot(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2/a^2/x^2*arccot(a*x)^2+1/a/x*arccot(a*x)+arccot(a*x)*arctan(a*x)+l
n(a*x)-1/2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = -\frac{a^2x^2\log\left(a^2x^2+1\right)-2\,a^2x^2\log\left(x\right)-2\,ax\,\mathrm{arccot}\left(ax\right)+\left(a^2x^2+1\right)\mathrm{arccot}\left(ax\right)^2}{2\,x^2}$$

```
[In] integrate(arccot(a*x)^2/x^3,x, algorithm="fricas")
[Out] -1/2*(a^2*x^2*log(a^2*x^2 + 1) - 2*a^2*x^2*log(x) - 2*a*x*arccot(a*x) + (a^
2*x^2 + 1)*arccot(a*x)^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = a^2\log\left(x\right)-\frac{a^2\log\left(a^2x^2+1\right)}{2}-\frac{a^2\,\mathrm{acot}^2\left(ax\right)}{2}+\frac{a\,\mathrm{acot}\left(ax\right)}{x}-\frac{\mathrm{acot}^2\left(ax\right)}{2x^2}$$

```
[In] integrate(acot(a*x)**2/x**3,x)
[Out] a**2*log(x) - a**2*log(a**2*x**2 + 1)/2 - a**2*acot(a*x)**2/2 + a*acot(a*x)
/x - acot(a*x)**2/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = \frac{1}{2}\left(\arctan\left(ax\right)^2-\log\left(a^2x^2+1\right)+2\,\log\left(x\right)\right)a^2+\left(a\arctan\left(ax\right)+\frac{1}{x}\right)a\,\mathrm{arccot}\left(ax\right)-\frac{\mathrm{arccot}\left(ax\right)^2}{2\,x^2}$$

```
[In] integrate(arccot(a*x)^2/x^3,x, algorithm="maxima")
[Out] 1/2*(arctan(a*x)^2 - log(a^2*x^2 + 1) + 2*log(x))*a^2 + (a*arctan(a*x) + 1/
x)*a*arccot(a*x) - 1/2*arccot(a*x)^2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx$$
$$= -\frac{1}{2}\left(\left(\arctan\left(\frac{1}{ax}\right)^2 - \frac{2\arctan\left(\frac{1}{ax}\right)}{ax} + \log\left(\frac{1}{a^2x^2}+1\right)\right)a + \frac{\arctan\left(\frac{1}{ax}\right)^2}{ax^2}\right)a$$

```
[In] integrate(arccot(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*((arctan(1/(a*x))^2 - 2*arctan(1/(a*x))/(a*x) + log(1/(a^2*x^2) + 1))*
a + arctan(1/(a*x))^2/(a*x^2))*a
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = a^2\,\ln(x) - \mathrm{acot}(a\,x)^2\left(\frac{a^2}{2} + \frac{1}{2\,x^2}\right) - \frac{a^2\,\ln\left(a^2\,x^2+1\right)}{2} + \frac{a\,\mathrm{acot}(a\,x)}{x}$$

```
[In] int(acot(a*x)^2/x^3,x)
```

```
[Out] a^2*log(x) - acot(a*x)^2*(a^2/2 + 1/(2*x^2)) - (a^2*log(a^2*x^2 + 1))/2 + (
a*acot(a*x))/x
```

3.21 $\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = -\frac{a^2}{3x}+\frac{a\cot^{-1}(ax)}{3x^2}+\frac{1}{3}ia^3\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{3x^3}-\frac{1}{3}a^3\arctan(ax)+\frac{2}{3}a^3\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)+\frac{1}{3}ia^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)$$

```
[Out] -1/3*a^2/x+1/3*a*arccot(a*x)/x^2+1/3*I*a^3*arccot(a*x)^2-1/3*arccot(a*x)^2/
x^3-1/3*a^3*arctan(a*x)+2/3*a^3*arccot(a*x)*ln(2-2/(1-I*a*x))+1/3*I*a^3*pol
ylog(2,-1+2/(1-I*a*x))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4947, 5039, 331, 209, 5045, 4989, 2497}

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = -\frac{1}{3}a^3\arctan(ax)+\frac{1}{3}ia^3\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+\frac{1}{3}ia^3\cot^{-1}(ax)^2+\frac{2}{3}a^3\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)-\frac{a^2}{3x}-\frac{\cot^{-1}(ax)^2}{3x^3}+\frac{a\cot^{-1}(ax)}{3x^2}$$

```
[In] Int[ArcCot[a*x]^2/x^4,x]
```

```
[Out] -1/3*a^2/x + (a*ArcCot[a*x])/(3*x^2) + (I/3)*a^3*ArcCot[a*x]^2 - ArcCot[a*x
]^2/(3*x^3) - (a^3*ArcTan[a*x])/3 + (2*a^3*ArcCot[a*x]*Log[2 - 2/(1 - I*a*x
)])/3 + (I/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
```

```
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax)^2}{3x^3} - \frac{1}{3}(2a)\int \frac{\cot^{-1}(ax)}{x^3(1+a^2x^2)}\,dx \\
&= -\frac{\cot^{-1}(ax)^2}{3x^3} - \frac{1}{3}(2a)\int \frac{\cot^{-1}(ax)}{x^3}\,dx + \frac{1}{3}\left(2a^3\right)\int \frac{\cot^{-1}(ax)}{x(1+a^2x^2)}\,dx \\
&= \frac{a\cot^{-1}(ax)}{3x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{3x^3} \\
&\quad + \frac{1}{3}a^2\int \frac{1}{x^2(1+a^2x^2)}\,dx + \frac{1}{3}\left(2ia^3\right)\int \frac{\cot^{-1}(ax)}{x(i+ax)}\,dx \\
&= -\frac{a^2}{3x} + \frac{a\cot^{-1}(ax)}{3x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^2 \\
&\quad - \frac{\cot^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) \\
&\quad - \frac{1}{3}a^4\int \frac{1}{1+a^2x^2}\,dx + \frac{1}{3}\left(2a^4\right)\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx \\
&= -\frac{a^2}{3x} + \frac{a\cot^{-1}(ax)}{3x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{3x^3} - \frac{1}{3}a^3\arctan(ax) \\
&\quad + \frac{2}{3}a^3\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) + \frac{1}{3}ia^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx$$
$$= \frac{-a^2x^2 + \left(-1-ia^3x^3\right)\cot^{-1}(ax)^2 + ax\cot^{-1}(ax)\left(1+a^2x^2+2a^2x^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)\right) - ia^3x^3\,\text{PolyI}}{3x^3}$$

```
[In] Integrate[ArcCot[a*x]^2/x^4,x]
```

```
[Out] (-(a^2*x^2) + (-1 - I*a^3*x^3)*ArcCot[a*x]^2 + a*x*ArcCot[a*x]*(1 + a^2*x^2
 + 2*a^2*x^2*Log[1 + E^((2*I)*ArcCot[a*x])]) - I*a^3*x^3*PolyLog[2, -E^((2*
I)*ArcCot[a*x])])/(3*x^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(95) = 190.

Time = 0.72 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.22

method	result
parts	$-\frac{\operatorname{arccot}(ax)^2}{3x^3} - \frac{2a^3\left(\frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{2} - \frac{\operatorname{arccot}(ax)}{2a^2x^2} - \operatorname{arccot}(ax)\ln(ax) - \frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right) - \frac{\ln(ax-i)^2}{2} - \operatorname{dilog}\left(-\right.\right.}{4}\right.}{}$
derivativedivides	$a^3\left(-\frac{\operatorname{arccot}(ax)^2}{3a^3x^3} - \frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3} + \frac{\operatorname{arccot}(ax)}{3a^2x^2} + \frac{2\operatorname{arccot}(ax)\ln(ax)}{3} + \frac{i\left(\ln(ax-i)\ln(a^2x^2+1) - \frac{\ln(}{}\right.}{}\right.$
default	$a^3\left(-\frac{\operatorname{arccot}(ax)^2}{3a^3x^3} - \frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3} + \frac{\operatorname{arccot}(ax)}{3a^2x^2} + \frac{2\operatorname{arccot}(ax)\ln(ax)}{3} + \frac{i\left(\ln(ax-i)\ln(a^2x^2+1) - \frac{\ln(}{}\right.}{}\right.$

```
[In] int(arccot(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arccot(a*x)^2/x^3-2/3*a^3*(1/2*arccot(a*x)*ln(a^2*x^2+1)-1/2*arccot(a*
x)/a^2/x^2-arccot(a*x)*ln(a*x)-1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)
^2-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x)))+1/4*I*(ln(I+a*x)*ln(
a^2*x^2+1)-1/2*ln(I+a*x)^2-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))
)+1/2*arctan(a*x)+1/2/a/x+1/2*I*ln(a*x)*ln(1+I*a*x)-1/2*I*ln(a*x)*ln(1-I*a*
x)+1/2*I*dilog(1+I*a*x)-1/2*I*dilog(1-I*a*x))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^4}\,dx$$

```
[In] integrate(arccot(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{acot}^2(ax)}{x^4}\,dx$$

```
[In] integrate(acot(a*x)**2/x**4,x)
```

```
[Out] Integral(acot(a*x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^4}\,dx$$

```
[In] integrate(arccot(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/48*(48*x^3*integrate(1/48*(36*a^2*x^2*arctan2(1, a*x)^2 - 4*a^2*x^2*log(a
^2*x^2 + 1) - 8*a*x*arctan2(1, a*x) + 3*(a^2*x^2 + 1)*log(a^2*x^2 + 1)^2 +
36*arctan2(1, a*x)^2)/(a^2*x^6 + x^4), x) - 4*arctan2(1, a*x)^2 + log(a^2*x
^2 + 1)^2)/x^3
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^4}\,dx$$

```
[In] integrate(arccot(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{acot}(a\,x)^2}{x^4}\,dx$$

```
[In] int(acot(a*x)^2/x^4,x)
```

```
[Out] int(acot(a*x)^2/x^4, x)
```

3.22 $\int \frac{\cot^{-1}(ax)^2}{x^5} dx$

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \frac{\cot^{-1}(ax)^2}{x^5} dx = -\frac{a^2}{12x^2} + \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} - \frac{2}{3}a^4\log(x) + \frac{1}{3}a^4\log\left(1+a^2x^2\right)$$

```
[Out] -1/12*a^2/x^2+1/6*a*arccot(a*x)/x^3-1/2*a^3*arccot(a*x)/x+1/4*a^4*arccot(a*
x)^2-1/4*arccot(a*x)^2/x^4-2/3*a^4*ln(x)+1/3*a^4*ln(a^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4947, 5039, 272, 46, 36, 29, 31, 5005}

$$\int \frac{\cot^{-1}(ax)^2}{x^5} dx = -\frac{2}{3}a^4\log(x) + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{a^3\cot^{-1}(ax)}{2x} - \frac{a^2}{12x^2} + \frac{1}{3}a^4\log\left(a^2x^2+1\right) - \frac{\cot^{-1}(ax)^2}{4x^4} + \frac{a\cot^{-1}(ax)}{6x^3}$$

```
[In] Int[ArcCot[a*x]^2/x^5,x]
```

```
[Out] -1/12*a^2/x^2 + (a*ArcCot[a*x])/(6*x^3) - (a^3*ArcCot[a*x])/(2*x) + (a^4*Ar
cCot[a*x]^2)/4 - ArcCot[a*x]^2/(4*x^4) - (2*a^4*Log[x])/3 + (a^4*Log[1 + a^
2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax)^2}{4x^4} - \frac{1}{2}a\int \frac{\cot^{-1}(ax)}{x^4(1+a^2x^2)}\,dx \\
&= -\frac{\cot^{-1}(ax)^2}{4x^4} - \frac{1}{2}a\int \frac{\cot^{-1}(ax)}{x^4}\,dx + \frac{1}{2}a^3\int \frac{\cot^{-1}(ax)}{x^2(1+a^2x^2)}\,dx \\
&= \frac{a\cot^{-1}(ax)}{6x^3} - \frac{\cot^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^2\int \frac{1}{x^3(1+a^2x^2)}\,dx \\
&\quad + \frac{1}{2}a^3\int \frac{\cot^{-1}(ax)}{x^2}\,dx - \frac{1}{2}a^5\int \frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx \\
&= \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} \\
&\quad + \frac{1}{12}a^2\text{Subst}\left(\int \frac{1}{x^2(1+a^2x)}\,dx, x, x^2\right) - \frac{1}{2}a^4\int \frac{1}{x(1+a^2x^2)}\,dx \\
&= \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} \\
&\quad + \frac{1}{12}a^2\text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{a^2}{x} + \frac{a^4}{1+a^2x}\right)dx, x, x^2\right) \\
&\quad - \frac{1}{4}a^4\text{Subst}\left(\int \frac{1}{x(1+a^2x)}\,dx, x, x^2\right) \\
&= -\frac{a^2}{12x^2} + \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} - \frac{1}{6}a^4\log(x) \\
&\quad + \frac{1}{12}a^4\log\left(1+a^2x^2\right) - \frac{1}{4}a^4\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{4}a^6\text{Subst}\left(\int \frac{1}{1+a^2x}\,dx, x, x^2\right) \\
&= -\frac{a^2}{12x^2} + \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 \\
&\quad - \frac{\cot^{-1}(ax)^2}{4x^4} - \frac{2}{3}a^4\log(x) + \frac{1}{3}a^4\log\left(1+a^2x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\begin{aligned}
\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = &-\frac{a^2}{12x^2} - \frac{a(-1+3a^2x^2)\cot^{-1}(ax)}{6x^3} \\
&+ \frac{(-1+a^4x^4)\cot^{-1}(ax)^2}{4x^4} - \frac{2}{3}a^4\log(x) + \frac{1}{3}a^4\log\left(1+a^2x^2\right)
\end{aligned}$$

```
[In] Integrate[ArcCot[a*x]^2/x^5,x]
[Out] -1/12*a^2/x^2 - (a*(-1 + 3*a^2*x^2)*ArcCot[a*x])/(6*x^3) + ((-1 + a^4*x^4)*
ArcCot[a*x]^2)/(4*x^4) - (2*a^4*Log[x])/3 + (a^4*Log[1 + a^2*x^2])/3
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
parts	$-\frac{\operatorname{arccot}(ax)^2}{4x^4}-\frac{a^4\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}+\frac{\operatorname{arccot}(ax)}{ax}+\operatorname{arccot}(ax)\arctan(ax)+\frac{1}{6a^2x^2}+\frac{4\ln(ax)}{3}-\frac{2\ln\left(a^2x^2+1\right)}{3}+\frac{\arctan(ax)^2}{2}\right)}{2}$
derivativedivides	$a^4\left(-\frac{\operatorname{arccot}(ax)^2}{4a^4x^4}+\frac{\operatorname{arccot}(ax)}{6a^3x^3}-\frac{\operatorname{arccot}(ax)}{2ax}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}-\frac{1}{12a^2x^2}-\frac{2\ln(ax)}{3}+\frac{\ln(a^2x^2+1)}{3}\right.$
default	$a^4\left(-\frac{\operatorname{arccot}(ax)^2}{4a^4x^4}+\frac{\operatorname{arccot}(ax)}{6a^3x^3}-\frac{\operatorname{arccot}(ax)}{2ax}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}-\frac{1}{12a^2x^2}-\frac{2\ln(ax)}{3}+\frac{\ln(a^2x^2+1)}{3}\right.$
parallelrisch	$-\frac{-3a^4x^4\operatorname{arccot}(ax)^2+8a^4\ln(x)x^4-4a^4\ln(a^2x^2+1)x^4-a^4x^4+6a^3x^3\operatorname{arccot}(ax)+a^2x^2-2\operatorname{arccot}(ax)ax+3\operatorname{arccot}(ax}{12x^4}$
risch	$-\frac{(a^4x^4-1)\ln(iax+1)^2}{16x^4}-\frac{i(3ia^4\ln(-iax+1)x^4+6a^3x^3-3i\ln(-iax+1)-2ax+3\pi)\ln(iax+1)}{24x^4}-\frac{-6ia^4\ln((-\pi a+8}{}$

[In] int(arccot(a*x)^2/x^5,x,method=_RETURNVERBOSE)

[Out] -1/4*arccot(a*x)^2/x^4-1/2*a^4*(-1/3/a^3/x^3*arccot(a*x)+1/a/x*arccot(a*x)+arccot(a*x)*arctan(a*x)+1/6/a^2/x^2+4/3*ln(a*x)-2/3*ln(a^2*x^2+1)+1/2*arctan(a*x)^2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = \frac{4\,a^4x^4\log\left(a^2x^2+1\right)-8\,a^4x^4\log\left(x\right)-a^2x^2+3\left(a^4x^4-1\right)\operatorname{arccot}\left(ax\right)^2-2\left(3\,a^3x^3-ax\right)\operatorname{arccot}\left(ax\right)}{12\,x^4}$$

[In] integrate(arccot(a*x)^2/x^5,x, algorithm="fricas")

[Out] 1/12*(4*a^4*x^4*log(a^2*x^2 + 1) - 8*a^4*x^4*log(x) - a^2*x^2 + 3*(a^4*x^4 - 1)*arccot(a*x)^2 - 2*(3*a^3*x^3 - a*x)*arccot(a*x))/x^4

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = -\frac{2a^4\log(x)}{3} + \frac{a^4\log(a^2x^2+1)}{3} + \frac{a^4\operatorname{acot}^2(ax)}{4} - \frac{a^3\operatorname{acot}(ax)}{2x} - \frac{a^2}{12x^2} + \frac{a\operatorname{acot}(ax)}{6x^3} - \frac{\operatorname{acot}^2(ax)}{4x^4}$$

```
[In] integrate(acot(a*x)**2/x**5,x)
[Out] -2*a**4*log(x)/3 + a**4*log(a**2*x**2 + 1)/3 + a**4*acot(a*x)**2/4 - a**3*a
cot(a*x)/(2*x) - a**2/(12*x**2) + a*acot(a*x)/(6*x**3) - acot(a*x)**2/(4*x*
*4)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = -\frac{1}{6}\left(3\,a^3\arctan(ax) + \frac{3\,a^2x^2-1}{x^3}\right)a\operatorname{arccot}(ax) - \frac{\left(3\,a^2x^2\arctan(ax)^2 - 4\,a^2x^2\log(a^2x^2+1) + 8\,a^2x^2\log(x) + 1\right)a^2}{12\,x^2} - \frac{\operatorname{arccot}(ax)^2}{4\,x^4}$$

```
[In] integrate(arccot(a*x)^2/x^5,x, algorithm="maxima")
[Out] -1/6*(3*a^3*arctan(a*x) + (3*a^2*x^2 - 1)/x^3)*a*arccot(a*x) - 1/12*(3*a^2*
x^2*arctan(a*x)^2 - 4*a^2*x^2*log(a^2*x^2 + 1) + 8*a^2*x^2*log(x) + 1)*a^2/
x^2 - 1/4*arccot(a*x)^2/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = \frac{1}{12}\left(\left(3\arctan\left(\frac{1}{ax}\right)^2 - \frac{6\arctan\left(\frac{1}{ax}\right)}{ax} - \frac{1}{a^2x^2} + \frac{2\arctan\left(\frac{1}{ax}\right)}{a^3x^3} + 4\log\left(\frac{1}{a^2x^2}+1\right)\right)a^3 - \frac{3\arctan\left(\frac{1}{ax}\right)}{ax^4}\right.$$

```
[In] integrate(arccot(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/12*((3*arctan(1/(a*x))^2 - 6*arctan(1/(a*x))/(a*x) - 1/(a^2*x^2) + 2*arct
an(1/(a*x))/(a^3*x^3) + 4*log(1/(a^2*x^2) + 1))*a^3 - 3*arctan(1/(a*x))^2/(
a*x^4))*a
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = \mathrm{acot}(a\,x)^2\left(\frac{a^4}{4}-\frac{1}{4\,x^4}\right)-\frac{2\,a^4\,\ln\left(x\right)}{3}+\frac{a^4\,\ln\left(a^2\,x^2+1\right)}{3}-\frac{a^2}{12\,x^2}-\frac{a^2\,\mathrm{acot}(a\,x)\,\left(\frac{a\,x^2}{2}-\frac{1}{6\,a}\right)}{x^3}$$

```
[In] int(acot(a*x)^2/x^5,x)
```

```
[Out] acot(a*x)^2*(a^4/4 - 1/(4*x^4)) - (2*a^4*log(x))/3 + (a^4*log(a^2*x^2 + 1))
/3 - a^2/(12*x^2) - (a^2*acot(a*x)*((a*x^2)/2 - 1/(6*a)))/x^3
```

3.23 $\int x^5 \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 194

$$\begin{aligned}\int x^5 \cot^{-1}(ax)^3 \, dx = & -\frac{19x}{60a^5}+\frac{x^3}{60a^3}-\frac{4x^2\cot^{-1}(ax)}{15a^4}+\frac{x^4\cot^{-1}(ax)}{20a^2}+\frac{23i\cot^{-1}(ax)^2}{30a^6} \\ & +\frac{x\cot^{-1}(ax)^2}{2a^5}-\frac{x^3\cot^{-1}(ax)^2}{6a^3}+\frac{x^5\cot^{-1}(ax)^2}{10a} \\ & +\frac{\cot^{-1}(ax)^3}{6a^6}+\frac{1}{6}x^6\cot^{-1}(ax)^3+\frac{19\arctan(ax)}{60a^6} \\ & -\frac{23\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{15a^6}+\frac{23i\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{30a^6}\end{aligned}$$

```
[Out] -19/60*x/a^5+1/60*x^3/a^3-4/15*x^2*arccot(a*x)/a^4+1/20*x^4*arccot(a*x)/a^2
+23/30*I*arccot(a*x)^2/a^6+1/2*x*arccot(a*x)^2/a^5-1/6*x^3*arccot(a*x)^2/a^
3+1/10*x^5*arccot(a*x)^2/a+1/6*arccot(a*x)^3/a^6+1/6*x^6*arccot(a*x)^3+19/6
0*arctan(a*x)/a^6-23/15*arccot(a*x)*ln(2/(1+I*a*x))/a^6+23/30*I*polylog(2,1
-2/(1+I*a*x))/a^6
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules

used = {4947, 5037, 308, 209, 327, 5041, 4965, 2449, 2352, 4931, 5005}

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \frac{19 \arctan(ax)}{60a^6} + \frac{23i \operatorname{PolyLog}\left(2, 1 - \frac{2}{iax+1}\right)}{30a^6} + \frac{\cot^{-1}(ax)^3}{6a^6} + \frac{23i \cot^{-1}(ax)^2}{30a^6} - \frac{23 \log\left(\frac{2}{1+iax}\right) \cot^{-1}(ax)}{15a^6} - \frac{19x}{60a^5} + \frac{x \cot^{-1}(ax)^2}{2a^5} - \frac{4x^2 \cot^{-1}(ax)}{15a^4} + \frac{x^3}{60a^3} - \frac{x^3 \cot^{-1}(ax)^2}{6a^3} + \frac{x^4 \cot^{-1}(ax)}{20a^2} + \frac{1}{6}x^6 \cot^{-1}(ax)^3 + \frac{x^5 \cot^{-1}(ax)^2}{10a}$$

```
[In] Int[x^5*ArcCot[a*x]^3,x]
```

```
[Out] (-19*x)/(60*a^5) + x^3/(60*a^3) - (4*x^2*ArcCot[a*x])/(15*a^4) + (x^4*ArcCo
t[a*x])/(20*a^2) + (((23*I)/30)*ArcCot[a*x]^2)/a^6 + (x*ArcCot[a*x]^2)/(2*a
^5) - (x^3*ArcCot[a*x]^2)/(6*a^3) + (x^5*ArcCot[a*x]^2)/(10*a) + ArcCot[a*x
]^3/(6*a^6) + (x^6*ArcCot[a*x]^3)/6 + (19*ArcTan[a*x])/(60*a^6) - (23*ArcCo
t[a*x]*Log[2/(1 + I*a*x)])/(15*a^6) + (((23*I)/30)*PolyLog[2, 1 - 2/(1 + I*
a*x)])/a^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x^6 \cot^{-1}(ax)^3 + \frac{1}{2}a \int \frac{x^6 \cot^{-1}(ax)^2}{1+a^2x^2}\,dx$$

$$= \frac{1}{6}x^6 \cot^{-1}(ax)^3 + \frac{\int x^4 \cot^{-1}(ax)^2\,dx}{2a} - \frac{\int \frac{x^4 \cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{2a}$$

$$= \frac{x^5 \cot^{-1}(ax)^2}{10a} + \frac{1}{6}x^6 \cot^{-1}(ax)^3 + \frac{1}{5}\int \frac{x^5 \cot^{-1}(ax)}{1+a^2x^2}\,dx - \frac{\int x^2 \cot^{-1}(ax)^2\,dx}{2a^3} + \frac{\int \frac{x^2 \cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{2a^3}$$

$$= -\frac{x^3 \cot^{-1}(ax)^2}{6a^3} + \frac{x^5 \cot^{-1}(ax)^2}{10a} + \frac{1}{6}x^6 \cot^{-1}(ax)^3 + \frac{\int \cot^{-1}(ax)^2\,dx}{2a^5} - \frac{\int \frac{\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{2a^5} + \frac{\int x^3 \cot^{-1}(ax)\,dx}{5a^2} - \frac{\int \frac{x^3 \cot^{-1}(ax)}{1+a^2x^2}\,dx}{5a^2} - \frac{\int \frac{x^3 \cot^{-1}(ax)}{1+a^2x^2}\,dx}{3a^2}$$

$$= \frac{x^4 \cot^{-1}(ax)}{20a^2} + \frac{x \cot^{-1}(ax)^2}{2a^5} - \frac{x^3 \cot^{-1}(ax)^2}{6a^3} + \frac{x^5 \cot^{-1}(ax)^2}{10a} + \frac{\cot^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6 \cot^{-1}(ax)^3 - \frac{\int x \cot^{-1}(ax)\,dx}{5a^4} + \frac{\int \frac{x \cot^{-1}(ax)}{1+a^2x^2}\,dx}{5a^4} - \frac{\int x \cot^{-1}(ax)\,dx}{3a^4} + \frac{\int \frac{x \cot^{-1}(ax)}{1+a^2x^2}\,dx}{3a^4} + \frac{\int \frac{x \cot^{-1}(ax)}{1+a^2x^2}\,dx}{a^4} + \frac{\int \frac{x^4}{1+a^2x^2}\,dx}{20a}$$

$$= -\frac{4x^2 \cot^{-1}(ax)}{15a^4} + \frac{x^4 \cot^{-1}(ax)}{20a^2} + \frac{23i \cot^{-1}(ax)^2}{30a^6} + \frac{x \cot^{-1}(ax)^2}{2a^5} - \frac{x^3 \cot^{-1}(ax)^2}{6a^3} + \frac{x^5 \cot^{-1}(ax)^2}{10a} + \frac{\cot^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6 \cot^{-1}(ax)^3 - \frac{\int \frac{\cot^{-1}(ax)}{i-ax}\,dx}{5a^5} - \frac{\int \frac{\cot^{-1}(ax)}{i-ax}\,dx}{3a^5} - \frac{\int \frac{\cot^{-1}(ax)}{i-ax}\,dx}{a^5} - \frac{\int \frac{x^2}{1+a^2x^2}\,dx}{10a^3} - \frac{\int \frac{x^2}{1+a^2x^2}\,dx}{6a^3} + \frac{\int \left(-\frac{1}{a^4} + \frac{x^2}{a^2} + \frac{1}{a^4(1+a^2x^2)}\right)dx}{20a}$$

$$= -\frac{19x}{60a^5} + \frac{x^3}{60a^3} - \frac{4x^2 \cot^{-1}(ax)}{15a^4} + \frac{x^4 \cot^{-1}(ax)}{20a^2} + \frac{23i \cot^{-1}(ax)^2}{30a^6} + \frac{x \cot^{-1}(ax)^2}{2a^5} - \frac{x^3 \cot^{-1}(ax)^2}{6a^3} + \frac{x^5 \cot^{-1}(ax)^2}{10a} + \frac{\cot^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6 \cot^{-1}(ax)^3 - \frac{23 \cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{15a^6} + \frac{\int \frac{1}{1+a^2x^2}\,dx}{20a^5} + \frac{\int \frac{1}{1+a^2x^2}\,dx}{10a^5} + \frac{\int \frac{1}{1+a^2x^2}\,dx}{6a^5} - \frac{\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{5a^5} - \frac{\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{3a^5} - \frac{\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{a^5}$$

$$= -\frac{19x}{60a^5} + \frac{x^3}{60a^3} - \frac{4x^2\cot^{-1}(ax)}{15a^4} + \frac{x^4\cot^{-1}(ax)}{20a^2} + \frac{23i\cot^{-1}(ax)^2}{30a^6} + \frac{x\cot^{-1}(ax)^2}{2a^5} - \frac{x^3\cot^{-1}(ax)^2}{6a^3} + \frac{x^5\cot^{-1}(ax)^2}{10a} + \frac{\cot^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6\cot^{-1}(ax)^3 + \frac{19\arctan(ax)}{60a^6} - \frac{23\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{15a^6} + \frac{i\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+iax}\right)}{5a^6} + \frac{i\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+iax}\right)}{3a^6} + \frac{i\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+iax}\right)}{a^6}$$

$$= -\frac{19x}{60a^5} + \frac{x^3}{60a^3} - \frac{4x^2\cot^{-1}(ax)}{15a^4} + \frac{x^4\cot^{-1}(ax)}{20a^2} + \frac{23i\cot^{-1}(ax)^2}{30a^6} + \frac{x\cot^{-1}(ax)^2}{2a^5} - \frac{x^3\cot^{-1}(ax)^2}{6a^3} + \frac{x^5\cot^{-1}(ax)^2}{10a} + \frac{\cot^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6\cot^{-1}(ax)^3 + \frac{19\arctan(ax)}{60a^6} - \frac{23\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{15a^6} + \frac{23i\,\text{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{30a^6}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\int x^5\cot^{-1}(ax)^3\,dx$$

$$= \frac{ax(-19+a^2x^2) + 2(23i+15ax-5a^3x^3+3a^5x^5)\cot^{-1}(ax)^2 + 10(1+a^6x^6)\cot^{-1}(ax)^3 + \cot^{-1}(ax)\left(-19\right.}{60a^6}$$

```
[In] Integrate[x^5*ArcCot[a*x]^3,x]
```

```
[Out] (a*x*(-19 + a^2*x^2) + 2*(23*I + 15*a*x - 5*a^3*x^3 + 3*a^5*x^5)*ArcCot[a*x
]^2 + 10*(1 + a^6*x^6)*ArcCot[a*x]^3 + ArcCot[a*x]*(-19 - 16*a^2*x^2 + 3*a^
4*x^4 - 92*Log[1 - E^((2*I)*ArcCot[a*x])]) + (46*I)*PolyLog[2, E^((2*I)*Arc
Cot[a*x])])/(60*a^6)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1103 vs. 2(164) = 328.

Time = 29.34 (sec) , antiderivative size = 1104, normalized size of antiderivative = 5.69

method	result	size
risch	Expression too large to display	1104
parts	Expression too large to display	2453
derivativedivides	Expression too large to display	2455
default	Expression too large to display	2455

[In] int(x^5*arccot(a*x)^3,x,method=_RETURNVERBOSE)

[Out] 23/120*I/a^6*Pi^2+8929/57600*I/a^6*ln(a^2*x^2+1)-1/4*I/a^5*Pi*ln(1-I*a*x)*x-61/1920*I/a^2*ln(1-I*a*x)*x^4+151/960*I/a^4*ln(1-I*a*x)*x^2+37/480*I/a^6*Pi*arctan(a*x)+1/40/a^2*Pi*x^4+1/40/a*Pi^2*x^5+1/12*I/a^3*Pi*ln(1-I*a*x)*x^3+23/30*I/a^6*dilog(1/2-1/2*I*a*x)-1291/3600*I/a^6*ln(1-I*a*x)-1/8/a^6*Pi^2*arctan(a*x)+331/960/a^6*Pi*ln(a^2*x^2+1)-1/240*(-15*I*x^6*ln(1-I*a*x)*a^6+15*Pi*a^6*x^6+6*a^5*x^5-10*a^3*x^3-15*I*ln(1-I*a*x)+30*a*x+15*Pi-46*I)/a^6*ln(1+I*a*x)^2+1/48/a^3*ln(1-I*a*x)^2*x^3+7/1440/a^3*ln(1-I*a*x)*x^3-1/16/a^5*ln(1-I*a*x)^2*x-1/1200/a*ln(1-I*a*x)*x^5-37/480/a^5*ln(1-I*a*x)*x-1/80/a*ln(1-I*a*x)^2*x^5-1/3*I/a^6+23/30*I/a^6*ln(1/2-1/2*I*a*x)*ln(1/2+1/2*I*a*x)-23/30*I/a^6*ln(1-I*a*x)*ln(1/2+1/2*I*a*x)+1/48*x^6*Pi^3+1/48/a^6*Pi^3-19/120/a^6*Pi+(-1/16*I*(a^6*x^6+1)/a^6*ln(1-I*a*x)^2+1/120*x*(15*Pi*a^5*x^5+6*a^4*x^4-10*a^2*x^2+30)/a^5*ln(1-I*a*x)-1/240*(-15*I*Pi^2*a^6*x^6-12*I*Pi*a^5*x^5-6*I*a^4*x^4+20*I*Pi*a^3*x^3+32*I*a^2*x^2-60*I*Pi*a*x-30*ln(1-I*a*x)*Pi-92*I*ln(1-I*a*x))/a^6)*ln(1+I*a*x)-1/16*I*Pi^2*ln(1-I*a*x)*x^6-1/20*I/a*Pi*ln(1-I*a*x)*x^5-1/32*I/a^4*ln(1-I*a*x)^2*x^2+1/64*I/a^2*ln(1-I*a*x)^2*x^4-1/48*I*(a^6*x^6+1)/a^6*ln(1+I*a*x)^3-1/24/a^3*Pi^2*x^3+1/8/a^5*Pi^2*x-1/16/a^6*Pi*ln(1-I*a*x)^2+37/480/a^6*Pi*ln(1-I*a*x)-1/16*Pi*ln(1-I*a*x)^2*x^6-2/15/a^4*Pi*x^2+1/48*I/a^6*ln(1-I*a*x)^3-49/320*I/a^6*ln(1-I*a*x)^2+1/48*I*ln(1-I*a*x)^3*x^6-1/96*I*ln(1-I*a*x)^2*x^6+1/288*I*ln(1-I*a*x)*x^6-1/50*I/a^6*(1-I*a*x)^5*ln(1-I*a*x)+1/8*I/a^6*(1-I*a*x)^3*ln(1-I*a*x)^2-3/32*I/a^6*(1-I*a*x)^2*ln(1-I*a*x)^2+3/32*I/a^6*(1-I*a*x)^2*ln(1-I*a*x)+7/128*I/a^6*(1-I*a*x)^4*ln(1-I*a*x)-1/96*I/a^6*(1-I*a*x)^6*ln(1-I*a*x)^2-1/12*I/a^6*(1-I*a*x)^3*ln(1-I*a*x)+1/20*I/a^6*(1-I*a*x)^5*ln(1-I*a*x)^2+1/288*I/a^6*(1-I*a*x)^6*ln(1-I*a*x)-7/64*I/a^6*(1-I*a*x)^4*ln(1-I*a*x)^2-19/60*x/a^5+1/60*x^3/a^3+18049/28800*arctan(a*x)/a^6

Fricas [F]

$$\int x^5 \cot^{-1}(ax)^3\,dx = \int x^5 \operatorname{arccot}(ax)^3\,dx$$

[In] integrate(x^5*arccot(a*x)^3,x, algorithm="fricas")

[Out] integral(x^5*arccot(a*x)^3, x)

Sympy [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{acot}^3(ax) \, dx$$

[In] integrate(x**5*acot(a*x)**3,x)

[Out] Integral(x**5*acot(a*x)**3, x)

Maxima [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arccot}(ax)^3 \, dx$$

[In] integrate(x^5*arccot(a*x)^3,x, algorithm="maxima")

[Out] 1/480*(40*a^6*x^6*arctan2(1, a*x)^3 + 12*a^5*x^5*arctan2(1, a*x)^2 - 20*a^3*x^3*arctan2(1, a*x)^2 + 20*(5760*a^7*integrate(1/480*x^7*arctan(1/(a*x))^3/(a^7*x^2 + a^5), x) + 1440*a^6*integrate(1/480*x^6*arctan(1/(a*x))^2/(a^7*x^2 + a^5), x) + 360*a^6*integrate(1/480*x^6*log(a^2*x^2 + 1)^2/(a^7*x^2 + a^5), x) + 288*a^6*integrate(1/480*x^6*log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x) + 5760*a^5*integrate(1/480*x^5*arctan(1/(a*x))^3/(a^7*x^2 + a^5), x) + 576*a^5*integrate(1/480*x^5*arctan(1/(a*x))/(a^7*x^2 + a^5), x) - 480*a^4*integrate(1/480*x^4*log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x) - 960*a^3*integrate(1/480*x^3*arctan(1/(a*x))/(a^7*x^2 + a^5), x) + 1440*a^2*integrate(1/480*x^2*log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x) + 2880*a*integrate(1/480*x*arctan(1/(a*x))/(a^7*x^2 + a^5), x) + arctan(a*x)^3/a^6 + 3*arctan(a*x)^2*arctan(1/(a*x))/a^6 + 3*arctan(a*x)*arctan(1/(a*x))^2/a^6 + 360*integrate(1/480*log(a^2*x^2 + 1)^2/(a^7*x^2 + a^5), x))*a^6 + 60*a*x*arctan2(1, a*x)^2 + 40*arctan2(1, a*x)^3 - (3*a^5*x^5 - 5*a^3*x^3 + 15*a*x)*log(a^2*x^2 + 1)^2)/a^6

Giac [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arccot}(ax)^3 \, dx$$

[In] integrate(x^5*arccot(a*x)^3,x, algorithm="giac")

[Out] integrate(x^5*arccot(a*x)^3, x)

Mupad [F(-1)]

Timed out.

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{acot}(a\,x)^3 \, dx$$

[In] int(x^5*acot(a*x)^3,x)

[Out] int(x^5*acot(a*x)^3, x)

3.24 $\int x^4 \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 205

$$\begin{aligned}\int x^4 \cot^{-1}(ax)^3 \, dx =& \frac{x^2}{20a^3} - \frac{9x\cot^{-1}(ax)}{10a^4} + \frac{x^3\cot^{-1}(ax)}{10a^2} - \frac{9\cot^{-1}(ax)^2}{20a^5} \\ &- \frac{3x^2\cot^{-1}(ax)^2}{10a^3} + \frac{3x^4\cot^{-1}(ax)^2}{20a} + \frac{i\cot^{-1}(ax)^3}{5a^5} \\ &+ \frac{1}{5}x^5\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a^5} - \frac{\log(1+a^2x^2)}{2a^5} \\ &+ \frac{3i\cot^{-1}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^5} - \frac{3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{10a^5}\end{aligned}$$

```
[Out] 1/20*x^2/a^3-9/10*x*arccot(a*x)/a^4+1/10*x^3*arccot(a*x)/a^2-9/20*arccot(a*
x)^2/a^5-3/10*x^2*arccot(a*x)^2/a^3+3/20*x^4*arccot(a*x)^2/a+1/5*I*arccot(a
*x)^3/a^5+1/5*x^5*arccot(a*x)^3-3/5*arccot(a*x)^2*ln(2/(1+I*a*x))/a^5-1/2*l
n(a^2*x^2+1)/a^5+3/5*I*arccot(a*x)*polylog(2,1-2/(1+I*a*x))/a^5-3/10*polylo
g(3,1-2/(1+I*a*x))/a^5
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules

used = {4947, 5037, 272, 45, 4931, 266, 5005, 5041, 4965, 5115, 6745}

$$\int x^4\cot^{-1}(ax)^3\,dx = -\frac{3\,\mathrm{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{10a^5}+\frac{3i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)\cot^{-1}(ax)}{5a^5} + \frac{i\cot^{-1}(ax)^3}{5a^5}-\frac{9\cot^{-1}(ax)^2}{20a^5}-\frac{3\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)^2}{5a^5} - \frac{9x\cot^{-1}(ax)}{10a^4}+\frac{x^2}{20a^3}-\frac{3x^2\cot^{-1}(ax)^2}{10a^3}+\frac{x^3\cot^{-1}(ax)}{10a^2} - \frac{\log\left(a^2x^2+1\right)}{2a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3x^4\cot^{-1}(ax)^2}{20a}$$

```
[In] Int[x^4*ArcCot[a*x]^3,x]
```

```
[Out] x^2/(20*a^3) - (9*x*ArcCot[a*x])/(10*a^4) + (x^3*ArcCot[a*x])/(10*a^2) - (9
*ArcCot[a*x]^2)/(20*a^5) - (3*x^2*ArcCot[a*x]^2)/(10*a^3) + (3*x^4*ArcCot[a
*x]^2)/(20*a) + ((I/5)*ArcCot[a*x]^3)/a^5 + (x^5*ArcCot[a*x]^3)/5 - (3*ArcC
ot[a*x]^2*Log[2/(1 + I*a*x)])/(5*a^5) - Log[1 + a^2*x^2]/(2*a^5) + (((3*I)/
5)*ArcCot[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^5 - (3*PolyLog[3, 1 - 2/(1
+ I*a*x)])/(10*a^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{1}{5}(3a)\int\frac{x^5\cot^{-1}(ax)^2}{1+a^2x^2}\,dx\\
&= \frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3\int x^3\cot^{-1}(ax)^2\,dx}{5a}-\frac{3\int\frac{x^3\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{5a}\\
&= \frac{3x^4\cot^{-1}(ax)^2}{20a}+\frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3}{10}\int\frac{x^4\cot^{-1}(ax)}{1+a^2x^2}\,dx\\
&\quad-\frac{3\int x\cot^{-1}(ax)^2\,dx}{5a^3}+\frac{3\int\frac{x\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{5a^3}\\
&= -\frac{3x^2\cot^{-1}(ax)^2}{10a^3}+\frac{3x^4\cot^{-1}(ax)^2}{20a}+\frac{i\cot^{-1}(ax)^3}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^3\\
&\quad-\frac{3\int\frac{\cot^{-1}(ax)^2}{i-ax}\,dx}{5a^4}+\frac{3\int x^2\cot^{-1}(ax)\,dx}{10a^2}-\frac{3\int\frac{x^2\cot^{-1}(ax)}{1+a^2x^2}\,dx}{10a^2}-\frac{3\int\frac{x^2\cot^{-1}(ax)}{1+a^2x^2}\,dx}{5a^2}\\
&= \frac{x^3\cot^{-1}(ax)}{10a^2}-\frac{3x^2\cot^{-1}(ax)^2}{10a^3}+\frac{3x^4\cot^{-1}(ax)^2}{20a}+\frac{i\cot^{-1}(ax)^3}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^3\\
&\quad-\frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a^5}-\frac{3\int\cot^{-1}(ax)\,dx}{10a^4}+\frac{3\int\frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx}{10a^4}\\
&\quad-\frac{3\int\cot^{-1}(ax)\,dx}{5a^4}+\frac{3\int\frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx}{5a^4}-\frac{6\int\frac{\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{5a^4}+\frac{\int\frac{x^3}{1+a^2x^2}\,dx}{10a}\\
&= -\frac{9x\cot^{-1}(ax)}{10a^4}+\frac{x^3\cot^{-1}(ax)}{10a^2}-\frac{9\cot^{-1}(ax)^2}{20a^5}-\frac{3x^2\cot^{-1}(ax)^2}{10a^3}\\
&\quad+\frac{3x^4\cot^{-1}(ax)^2}{20a}+\frac{i\cot^{-1}(ax)^3}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^3-\frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a^5}\\
&\quad+\frac{3i\cot^{-1}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^5}+\frac{(3i)\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{5a^4}\\
&\quad-\frac{3\int\frac{x}{1+a^2x^2}\,dx}{10a^3}-\frac{3\int\frac{x}{1+a^2x^2}\,dx}{5a^3}+\frac{\operatorname{Subst}\left(\int\frac{x}{1+a^2x}\,dx,x,x^2\right)}{20a}\\
&= -\frac{9x\cot^{-1}(ax)}{10a^4}+\frac{x^3\cot^{-1}(ax)}{10a^2}-\frac{9\cot^{-1}(ax)^2}{20a^5}-\frac{3x^2\cot^{-1}(ax)^2}{10a^3}\\
&\quad+\frac{3x^4\cot^{-1}(ax)^2}{20a}+\frac{i\cot^{-1}(ax)^3}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^3-\frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a^5}\\
&\quad-\frac{9\log\left(1+a^2x^2\right)}{20a^5}+\frac{3i\cot^{-1}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^5}\\
&\quad-\frac{3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{10a^5}+\frac{\operatorname{Subst}\left(\int\left(\frac{1}{a^2}-\frac{1}{a^2(1+a^2x)}\right)\,dx,x,x^2\right)}{20a}
\end{aligned}
$$

$$= \frac{x^2}{20a^3} - \frac{9x\cot^{-1}(ax)}{10a^4} + \frac{x^3\cot^{-1}(ax)}{10a^2} - \frac{9\cot^{-1}(ax)^2}{20a^5} - \frac{3x^2\cot^{-1}(ax)^2}{10a^3} + \frac{3x^4\cot^{-1}(ax)^2}{20a} + \frac{i\cot^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a^5} - \frac{\log(1+a^2x^2)}{2a^5} + \frac{3i\cot^{-1}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^5} - \frac{3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{10a^5}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.91

$$\int x^4\cot^{-1}(ax)^3\,dx$$

$$= \frac{2 + i\pi^3 + 2a^2x^2 - 36ax\cot^{-1}(ax) + 4a^3x^3\cot^{-1}(ax) - 18\cot^{-1}(ax)^2 - 12a^2x^2\cot^{-1}(ax)^2 + 6a^4x^4\cot^{-1}(a}{}$$

```
[In] Integrate[x^4*ArcCot[a*x]^3,x]
```

```
[Out] (2 + I*Pi^3 + 2*a^2*x^2 - 36*a*x*ArcCot[a*x] + 4*a^3*x^3*ArcCot[a*x] - 18*A
rcCot[a*x]^2 - 12*a^2*x^2*ArcCot[a*x]^2 + 6*a^4*x^4*ArcCot[a*x]^2 - (8*I)*A
rcCot[a*x]^3 + 8*a^5*x^5*ArcCot[a*x]^3 - 24*ArcCot[a*x]^2*Log[1 - E^((-2*I)
*ArcCot[a*x])] + 40*Log[1/Sqrt[1 + 1/(a^2*x^2)]] + 40*Log[1/(a*x)] - (24*I)
*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x])] - 12*PolyLog[3, E^((-2*I)*A
rcCot[a*x])])/(40*a^5)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.10 (sec) , antiderivative size = 1108, normalized size of antiderivative = 5.40

method	result	size
derivativedivides	Expression too large to display	1108
default	Expression too large to display	1108
parts	Expression too large to display	1110

```
[In] int(x^4*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arccot(a*x)^3+3/20*a^4*x^4*arccot(a*x)^2-3/10*a^2*x^2*ar
ccot(a*x)^2+3/10*arccot(a*x)^2*ln(a^2*x^2+1)-3/5*arccot(a*x)^2*ln((I+a*x)/(
a^2*x^2+1)^(1/2))+3/5*arccot(a*x)^2*ln((I+a*x)^2/(a^2*x^2+1)-1)+1/20*I*(-3*
arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^3*P
i-3*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)
^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*Pi-3*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x
```

```
^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)
+3*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)*
csgn(I*(I+a*x)^2/(a^2*x^2+1))*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)+3*Pi*c
sgn(I*(I+a*x)^2/(a^2*x^2+1))^3*arccot(a*x)^2-6*Pi*csgn(I*(I+a*x)/(a^2*x^2+1
)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*arccot(a*x)^2+3*Pi*csgn(I*(I+a*x)/
(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^2-3*arccot(a
*x)^2*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-
1)^2)+6*arccot(a*x)^2*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^
2/(a^2*x^2+1)-1)^2)^2-3*arccot(a*x)^2*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2
)^3+6*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^
2)^2*Pi+4*arccot(a*x)^3-6*Pi*arccot(a*x)^2-I*a^2*x^2+12*I*arccot(a*x)^2*ln(
2)+18*I*arccot(a*x)*a*x-20*arccot(a*x)+9*I*arccot(a*x)^2-2*I*arccot(a*x)*a^
3*x^3-I)+ln((I+a*x)/(a^2*x^2+1)^(1/2)-1)+ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))-3/
5*arccot(a*x)^2*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+6/5*I*arccot(a*x)*polylog(2
,(I+a*x)/(a^2*x^2+1)^(1/2))-6/5*polylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))-3/5*ar
ccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+6/5*I*arccot(a*x)*polylog(2,-(I
+a*x)/(a^2*x^2+1)^(1/2))-6/5*polylog(3,-(I+a*x)/(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int x^4 \cot^{-1}(ax)^3\,dx = \int x^4 \operatorname{arccot}(ax)^3\,dx$$

```
[In] integrate(x^4*arccot(a*x)^3,x, algorithm="fricas")
[Out] integral(x^4*arccot(a*x)^3, x)
```

Sympy [F]

$$\int x^4 \cot^{-1}(ax)^3\,dx = \int x^4 \operatorname{acot}^3(ax)\,dx$$

```
[In] integrate(x**4*acot(a*x)**3,x)
[Out] Integral(x**4*acot(a*x)**3, x)
```

Maxima [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arccot}(ax)^3 \, dx$$

```
[In] integrate(x^4*arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/40*x^5*arctan2(1, a*x)^3 - 3/160*x^5*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 +
 integrate(1/160*(140*a^2*x^6*arctan2(1, a*x)^3 + 12*a^2*x^6*arctan2(1, a*x
)*log(a^2*x^2 + 1) + 12*a*x^5*arctan2(1, a*x)^2 + 140*x^4*arctan2(1, a*x)^3
 + 3*(5*a^2*x^6*arctan2(1, a*x) - a*x^5 + 5*x^4*arctan2(1, a*x))*log(a^2*x^
2 + 1)^2)/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arccot}(ax)^3 \, dx$$

```
[In] integrate(x^4*arccot(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4*arccot(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{acot}(a\,x)^3 \, dx$$

```
[In] int(x^4*acot(a*x)^3,x)
```

```
[Out] int(x^4*acot(a*x)^3, x)
```

3.25 $\int x^3 \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 148

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \frac{x}{4a^3} + \frac{x^2 \cot^{-1}(ax)}{4a^2} - \frac{i \cot^{-1}(ax)^2}{a^4} - \frac{3x \cot^{-1}(ax)^2}{4a^3} + \frac{x^3 \cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \cot^{-1}(ax)^3 - \frac{\arctan(ax)}{4a^4} + \frac{2\cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{a^4} - \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{a^4}$$

```
[Out] 1/4*x/a^3+1/4*x^2*arccot(a*x)/a^2-I*arccot(a*x)^2/a^4-3/4*x*arccot(a*x)^2/a
^3+1/4*x^3*arccot(a*x)^2/a-1/4*arccot(a*x)^3/a^4+1/4*x^4*arccot(a*x)^3-1/4*
arctan(a*x)/a^4+2*arccot(a*x)*ln(2/(1+I*a*x))/a^4-I*polylog(2,1-2/(1+I*a*x)
)/a^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4947, 5037, 327, 209, 5041, 4965, 2449, 2352, 4931, 5005}

$$\int x^3 \cot^{-1}(ax)^3 \, dx = -\frac{\arctan(ax)}{4a^4} - \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{iax+1}\right)}{a^4} - \frac{\cot^{-1}(ax)^3}{4a^4} - \frac{i \cot^{-1}(ax)^2}{a^4} + \frac{2\log\left(\frac{2}{1+iax}\right) \cot^{-1}(ax)}{a^4} + \frac{x}{4a^3} - \frac{3x \cot^{-1}(ax)^2}{4a^3} + \frac{x^2 \cot^{-1}(ax)}{4a^2} + \frac{1}{4}x^4 \cot^{-1}(ax)^3 + \frac{x^3 \cot^{-1}(ax)^2}{4a}$$

```
[In] Int[x^3*ArcCot[a*x]^3,x]
```

```
[Out] x/(4*a^3) + (x^2*ArcCot[a*x])/(4*a^2) - (I*ArcCot[a*x]^2)/a^4 - (3*x*ArcCot
[a*x]^2)/(4*a^3) + (x^3*ArcCot[a*x]^2)/(4*a) - ArcCot[a*x]^3/(4*a^4) + (x^4
*ArcCot[a*x]^3)/4 - ArcTan[a*x]/(4*a^4) + (2*ArcCot[a*x]*Log[2/(1 + I*a*x)]
)/a^4 - (I*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\cot^{-1}(ax)^3 + \frac{1}{4}(3a)\int \frac{x^4\cot^{-1}(ax)^2}{1+a^2x^2}\,dx \\
&= \frac{1}{4}x^4\cot^{-1}(ax)^3 + \frac{3\int x^2\cot^{-1}(ax)^2\,dx}{4a} - \frac{3\int \frac{x^2\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{4a} \\
&= \frac{x^3\cot^{-1}(ax)^2}{4a} + \frac{1}{4}x^4\cot^{-1}(ax)^3 + \frac{1}{2}\int \frac{x^3\cot^{-1}(ax)}{1+a^2x^2}\,dx \\
&\quad - \frac{3\int \cot^{-1}(ax)^2\,dx}{4a^3} + \frac{3\int \frac{\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{4a^3} \\
&= -\frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^3 \\
&\quad + \frac{\int x\cot^{-1}(ax)\,dx}{2a^2} - \frac{\int \frac{x\cot^{-1}(ax)}{1+a^2x^2}\,dx}{2a^2} - \frac{3\int \frac{x\cot^{-1}(ax)}{1+a^2x^2}\,dx}{2a^2} \\
&= \frac{x^2\cot^{-1}(ax)}{4a^2} - \frac{i\cot^{-1}(ax)^2}{a^4} - \frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} \\
&\quad + \frac{1}{4}x^4\cot^{-1}(ax)^3 + \frac{\int \frac{\cot^{-1}(ax)}{i-ax}\,dx}{2a^3} + \frac{3\int \frac{\cot^{-1}(ax)}{i-ax}\,dx}{2a^3} + \frac{\int \frac{x^2}{1+a^2x^2}\,dx}{4a}
\end{aligned}
$$

$$= \frac{x}{4a^3} + \frac{x^2\cot^{-1}(ax)}{4a^2} - \frac{i\cot^{-1}(ax)^2}{a^4} - \frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^3 + \frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4} - \frac{\int \frac{1}{1+a^2x^2}\,dx}{4a^3} + \frac{\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{2a^3} + \frac{3\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{2a^3}$$

$$= \frac{x}{4a^3} + \frac{x^2\cot^{-1}(ax)}{4a^2} - \frac{i\cot^{-1}(ax)^2}{a^4} - \frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^3 - \frac{\arctan(ax)}{4a^4} + \frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4} - \frac{i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+iax}\right)}{2a^4} - \frac{(3i)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+iax}\right)}{2a^4}$$

$$= \frac{x}{4a^3} + \frac{x^2\cot^{-1}(ax)}{4a^2} - \frac{i\cot^{-1}(ax)^2}{a^4} - \frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^3 - \frac{\arctan(ax)}{4a^4} + \frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^4} - \frac{i\,\text{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a^4}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

$$\int x^3\cot^{-1}(ax)^3\,dx = \frac{ax + (-4i - 3ax + a^3x^3)\cot^{-1}(ax)^2 + (-1 + a^4x^4)\cot^{-1}(ax)^3 + \cot^{-1}(ax)\left(1 + a^2x^2 + 8\log\left(1 - e^{2i\cot^{-1}\right.\right.}{4a^4}$$

```
[In] Integrate[x^3*ArcCot[a*x]^3,x]
```

```
[Out] (a*x + (-4*I - 3*a*x + a^3*x^3)*ArcCot[a*x]^2 + (-1 + a^4*x^4)*ArcCot[a*x]^
3 + ArcCot[a*x]*(1 + a^2*x^2 + 8*Log[1 - E^((2*I)*ArcCot[a*x])]) - (4*I)*Po
lyLog[2, E^((2*I)*ArcCot[a*x])])/(4*a^4)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 856 vs. 2(130) = 260.

Time = 6.97 (sec) , antiderivative size = 857, normalized size of antiderivative = 5.79

method	result
risch	$\frac{\pi x^2}{8a^2}+\frac{\pi^2x^3}{16a}+\frac{\pi}{8a^4}-\frac{\pi^3}{32a^4}+\frac{3i\ln(-iax+1)^2x^2}{64a^2}-\frac{i(a^4x^4-1)\ln(iax+1)^3}{32a^4}+\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{a^4}-$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x^3*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*I/a^3*Pi*ln(1-I*a*x)*x+3/64*I/a^2*ln(1-I*a*x)^2*x^2-1/32*I*(a^4*x^4-1)/
a^4*ln(1+I*a*x)^3+1/8/a^2*Pi*x^2+1/16/a*Pi^2*x^3+1/8/a^4*Pi-1/32/a^4*Pi^3+(
-3/32*I*(a^4*x^4-1)/a^4*ln(1-I*a*x)^2+1/16*x*(3*Pi*a^3*x^3+2*a^2*x^2-6)/a^3
*ln(1-I*a*x)+1/32*(3*I*Pi^2*a^4*x^4+4*I*Pi*a^3*x^3+4*I*a^2*x^2-12*I*Pi*a*x-
6*ln(1-I*a*x)*Pi-16*I*ln(1-I*a*x))/a^4)*ln(1+I*a*x)+I/a^4*ln(1/2+1/2*I*a*x)
*ln(1-I*a*x)-I/a^4*ln(1/2-1/2*I*a*x)*ln(1/2+1/2*I*a*x)-3/32*I*Pi^2*ln(1-I*a
*x)*x^4-1/8*I/a*Pi*ln(1-I*a*x)*x^3-1/4*I/a^4*Pi^2-319/1536*I/a^4*ln(a^2*x^2
+1)+1/32*x^4*Pi^3-57/128/a^4*Pi*ln(a^2*x^2+1)-21/128*I/a^2*x^2*ln(1-I*a*x)-
7/64*I/a^4*Pi*arctan(a*x)+23/48*I/a^4*ln(1-I*a*x)-I/a^4*dilog(1/2-1/2*I*a*x
)+3/16/a^4*Pi^2*arctan(a*x)-1/32*(-3*I*x^4*ln(1-I*a*x)*a^4+3*Pi*a^4*x^4+2*a
^3*x^3+3*I*ln(1-I*a*x)-6*a*x-3*Pi+8*I)/a^4*ln(1+I*a*x)^2-1/32/a*ln(1-I*a*x)
^2*x^3-1/192/a*ln(1-I*a*x)*x^3+3/32/a^4*Pi*ln(1-I*a*x)^2-7/64/a^4*Pi*ln(1-I
*a*x)-3/32*Pi*ln(1-I*a*x)^2*x^4+3/32/a^3*ln(1-I*a*x)^2*x+7/64/a^3*ln(1-I*a*
x)*x-3/16/a^3*Pi^2*x+1/4*x/a^3-511/768*arctan(a*x)/a^4+1/32*I*ln(1-I*a*x)^3
*x^4-3/32*I/a^4*(1-I*a*x)^2*ln(1-I*a*x)+3/32*I/a^4*(1-I*a*x)^2*ln(1-I*a*x)^
2+1/24*I/a^4*(1-I*a*x)^3*ln(1-I*a*x)-1/16*I/a^4*(1-I*a*x)^3*ln(1-I*a*x)^2-3
/256*I/a^4*(1-I*a*x)^4*ln(1-I*a*x)+3/128*I/a^4*(1-I*a*x)^4*ln(1-I*a*x)^2-1/
32*I/a^4*ln(1-I*a*x)^3+25/128*I/a^4*ln(1-I*a*x)^2-3/128*I*ln(1-I*a*x)^2*x^4
+3/256*I*ln(1-I*a*x)*x^4+1/4*I/a^4
```

Fricas [F]

$$\int x^3\cot^{-1}(ax)^3\,dx=\int x^3\operatorname{arccot}(ax)^3\,dx$$

```
[In] integrate(x^3*arccot(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*arccot(a*x)^3, x)
```

Sympy [F]

$$\int x^3 \cot^{-1}(ax)^3\,dx = \int x^3\,\mathrm{acot}^3\,(ax)\,dx$$

```
[In] integrate(x**3*acot(a*x)**3,x)
```

```
[Out] Integral(x**3*acot(a*x)**3, x)
```

Maxima [F]

$$\int x^3 \cot^{-1}(ax)^3\,dx = \int x^3\,\mathrm{arccot}\,(ax)^3\,dx$$

```
[In] integrate(x^3*arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(8*a^4*x^4*arctan2(1, a*x)^3 + 4*a^3*x^3*arctan2(1, a*x)^2 + 4*(512*a^
5*integrate(1/64*x^5*arctan(1/(a*x))^3/(a^5*x^2 + a^3), x) + 192*a^4*integr
ate(1/64*x^4*arctan(1/(a*x))^2/(a^5*x^2 + a^3), x) + 48*a^4*integrate(1/64*
x^4*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) + 64*a^4*integrate(1/64*x^4*log(
a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 512*a^3*integrate(1/64*x^3*arctan(1/(a*x
))^3/(a^5*x^2 + a^3), x) + 128*a^3*integrate(1/64*x^3*arctan(1/(a*x))/(a^5*
x^2 + a^3), x) - 192*a^2*integrate(1/64*x^2*log(a^2*x^2 + 1)/(a^5*x^2 + a^3
), x) - 384*a*integrate(1/64*x*arctan(1/(a*x))/(a^5*x^2 + a^3), x) - arctan
(a*x)^3/a^4 - 3*arctan(a*x)^2*arctan(1/(a*x))/a^4 - 3*arctan(a*x)*arctan(1/
(a*x))^2/a^4 - 48*integrate(1/64*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x))*a^
4 - 12*a*x*arctan2(1, a*x)^2 - 8*arctan2(1, a*x)^3 - (a^3*x^3 - 3*a*x)*log(
a^2*x^2 + 1)^2)/a^4
```

Giac [F]

$$\int x^3 \cot^{-1}(ax)^3\,dx = \int x^3\,\mathrm{arccot}\,(ax)^3\,dx$$

```
[In] integrate(x^3*arccot(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccot(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cot^{-1}(ax)^3\, dx = \int x^3\, \mathrm{acot}(a\, x)^3\, dx$$

```
[In] int(x^3*acot(a*x)^3,x)
[Out] int(x^3*acot(a*x)^3, x)
```

3.26 $\int x^2 \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 157

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \frac{x \cot^{-1}(ax)}{a^2} + \frac{\cot^{-1}(ax)^2}{2a^3} + \frac{x^2 \cot^{-1}(ax)^2}{2a} - \frac{i \cot^{-1}(ax)^3}{3a^3} + \frac{1}{3} x^3 \cot^{-1}(ax)^3 + \frac{\cot^{-1}(ax)^2 \log\left(\frac{2}{1+iax}\right)}{a^3} + \frac{\log(1+a^2x^2)}{2a^3} - \frac{i \cot^{-1}(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a^3} + \frac{\operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{2a^3}$$

```
[Out] x*arccot(a*x)/a^2+1/2*arccot(a*x)^2/a^3+1/2*x^2*arccot(a*x)^2/a-1/3*I*arcco
t(a*x)^3/a^3+1/3*x^3*arccot(a*x)^3+arccot(a*x)^2*ln(2/(1+I*a*x))/a^3+1/2*ln
(a^2*x^2+1)/a^3-I*arccot(a*x)*polylog(2,1-2/(1+I*a*x))/a^3+1/2*polylog(3,1-
2/(1+I*a*x))/a^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {4947, 5037, 4931, 266, 5005, 5041, 4965, 5115, 6745}

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \frac{\operatorname{PolyLog}\left(3, 1-\frac{2}{iax+1}\right)}{2a^3} - \frac{i \operatorname{PolyLog}\left(2, 1-\frac{2}{iax+1}\right) \cot^{-1}(ax)}{a^3} - \frac{i \cot^{-1}(ax)^3}{3a^3} + \frac{\cot^{-1}(ax)^2}{2a^3} + \frac{\log\left(\frac{2}{1+iax}\right) \cot^{-1}(ax)^2}{a^3} + \frac{x \cot^{-1}(ax)}{a^2} + \frac{\log(a^2x^2+1)}{2a^3} + \frac{1}{3} x^3 \cot^{-1}(ax)^3 + \frac{x^2 \cot^{-1}(ax)^2}{2a}$$

```
[In] Int[x^2*ArcCot[a*x]^3,x]
```

```
[Out] (x*ArcCot[a*x])/a^2 + ArcCot[a*x]^2/(2*a^3) + (x^2*ArcCot[a*x]^2)/(2*a) - (
(I/3)*ArcCot[a*x]^3)/a^3 + (x^3*ArcCot[a*x]^3)/3 + (ArcCot[a*x]^2*Log[2/(1
+ I*a*x)])/a^3 + Log[1 + a^2*x^2]/(2*a^3) - (I*ArcCot[a*x]*PolyLog[2, 1 - 2
/(1 + I*a*x)])/a^3 + PolyLog[3, 1 - 2/(1 + I*a*x)]/(2*a^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(ax)^3 + a\int \frac{x^3\cot^{-1}(ax)^2}{1+a^2x^2}\,dx \\
&= \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\int x\cot^{-1}(ax)^2\,dx}{a} - \frac{\int \frac{x\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{a} \\
&= \frac{x^2\cot^{-1}(ax)^2}{2a} - \frac{i\cot^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\int \frac{\cot^{-1}(ax)^2}{i-ax}\,dx}{a^2} + \int \frac{x^2\cot^{-1}(ax)}{1+a^2x^2}\,dx \\
&= \frac{x^2\cot^{-1}(ax)^2}{2a} - \frac{i\cot^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3} \\
&\quad + \frac{\int \cot^{-1}(ax)\,dx}{a^2} - \frac{\int \frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx}{a^2} + \frac{2\int \frac{\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{a^2} \\
&= \frac{x\cot^{-1}(ax)}{a^2} + \frac{\cot^{-1}(ax)^2}{2a^3} + \frac{x^2\cot^{-1}(ax)^2}{2a} - \frac{i\cot^{-1}(ax)^3}{3a^3} \\
&\quad + \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3} \\
&\quad - \frac{i\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3} - \frac{i\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{a^2} + \frac{\int \frac{x}{1+a^2x^2}\,dx}{a} \\
&= \frac{x\cot^{-1}(ax)}{a^2} + \frac{\cot^{-1}(ax)^2}{2a^3} + \frac{x^2\cot^{-1}(ax)^2}{2a} - \frac{i\cot^{-1}(ax)^3}{3a^3} \\
&\quad + \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3} + \frac{\log\left(1+a^2x^2\right)}{2a^3} \\
&\quad - \frac{i\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3} + \frac{\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

$$\int x^2 \cot^{-1}(ax)^3 \, dx$$

$$= \frac{-i\pi^3 + 24ax\cot^{-1}(ax) + 12\cot^{-1}(ax)^2 + 12a^2x^2\cot^{-1}(ax)^2 + 8i\cot^{-1}(ax)^3 + 8a^3x^3\cot^{-1}(ax)^3 + 24\cot}{}$$

```
[In] Integrate[x^2*ArcCot[a*x]^3,x]
```

```
[Out] ((-I)*Pi^3 + 24*a*x*ArcCot[a*x] + 12*ArcCot[a*x]^2 + 12*a^2*x^2*ArcCot[a*x]
^2 + (8*I)*ArcCot[a*x]^3 + 8*a^3*x^3*ArcCot[a*x]^3 + 24*ArcCot[a*x]^2*Log[1
 - E^((-2*I)*ArcCot[a*x])] - 24*Log[1/Sqrt[1 + 1/(a^2*x^2)]] - 24*Log[1/(a*
x)] + (24*I)*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x])] + 12*PolyLog[3,
 E^((-2*I)*ArcCot[a*x])])/(24*a^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.94 (sec) , antiderivative size = 1036, normalized size of antiderivative = 6.60

method	result	size
parts	Expression too large to display	1036
derivativedivides	Expression too large to display	1038
default	Expression too large to display	1038

```
[In] int(x^2*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arccot(a*x)^3+1/a^3*(1/2*a^2*x^2*arccot(a*x)^2-1/2*arccot(a*x)^2*ln
(a^2*x^2+1)+arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-arccot(a*x)^2*ln((I
+a*x)^2/(a^2*x^2+1)-1)-1/12*I*arccot(a*x)*(-3*arccot(a*x)*Pi*csgn(I*((I+a*x
)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)+6*arccot(a*x)*Pi*
csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2-3*a
rccot(a*x)*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3-3*arccot(a*x)*Pi*csgn(I
/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*
x^2+1)-1)^2)^2+3*arccot(a*x)*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*
(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^
2+1))-3*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-
1)^2)^3-3*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1
)-1)^2)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))+3*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(
a^2*x^2+1))^3-6*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*csgn(I*(I+a*
x)/(a^2*x^2+1)^(1/2))+3*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I
```

```
*(I+a*x)/(a^2*x^2+1)^(1/2))^2+6*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)
/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2+4*arccot(a*x)^2-6*Pi*arccot(a*x)+6*I*arccot
(a*x)+12*I*arccot(a*x)*ln(2)-12+12*I*a*x)-ln((I+a*x)/(a^2*x^2+1)^(1/2)-1)-l
n(1+(I+a*x)/(a^2*x^2+1)^(1/2))+arccot(a*x)^2*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2)
)-2*I*arccot(a*x)*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,(I+a*x)/
(a^2*x^2+1)^(1/2))+arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))-2*I*arccot
(a*x)*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(I+a*x)/(a^2*x^2+1
)^(1/2)))
```

Fricas [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arccot}(ax)^3 \, dx$$

```
[In] integrate(x^2*arccot(a*x)^3,x, algorithm="fricas")
[Out] integral(x^2*arccot(a*x)^3, x)
```

Sympy [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{acot}^3(ax) \, dx$$

```
[In] integrate(x**2*acot(a*x)**3,x)
[Out] Integral(x**2*acot(a*x)**3, x)
```

Maxima [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arccot}(ax)^3 \, dx$$

```
[In] integrate(x^2*arccot(a*x)^3,x, algorithm="maxima")
[Out] 1/24*x^3*arctan2(1, a*x)^3 - 1/32*x^3*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 +
integrate(1/32*(28*a^2*x^4*arctan2(1, a*x)^3 + 4*a^2*x^4*arctan2(1, a*x)*lo
g(a^2*x^2 + 1) + 4*a*x^3*arctan2(1, a*x)^2 + 28*x^2*arctan2(1, a*x)^3 + (3*
a^2*x^4*arctan2(1, a*x) - a*x^3 + 3*x^2*arctan2(1, a*x))*log(a^2*x^2 + 1)^2
)/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int x^2 \cot^{-1}(ax)^3\, dx = \int x^2 \operatorname{arccot}\left(ax\right)^3\, dx$$

[In] integrate(x^2*arccot(a*x)^3,x, algorithm="giac")

[Out] integrate(x^2*arccot(a*x)^3, x)

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(ax)^3\, dx = \int x^2 \operatorname{acot}(a\,x)^3\, dx$$

[In] int(x^2*acot(a*x)^3,x)

[Out] int(x^2*acot(a*x)^3, x)

3.27 $\int x \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 103

$$\int x \cot^{-1}(ax)^3 \, dx = \frac{3i \cot^{-1}(ax)^2}{2a^2} + \frac{3x \cot^{-1}(ax)^2}{2a} + \frac{\cot^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \cot^{-1}(ax)^3 - \frac{3 \cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{a^2} + \frac{3i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{2a^2}$$

```
[Out] 3/2*I*arccot(a*x)^2/a^2+3/2*x*arccot(a*x)^2/a+1/2*arccot(a*x)^3/a^2+1/2*x^2
*arccot(a*x)^3-3*arccot(a*x)*ln(2/(1+I*a*x))/a^2+3/2*I*polylog(2,1-2/(1+I*a
*x))/a^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4947, 5037, 4931, 5041, 4965, 2449, 2352, 5005}

$$\int x \cot^{-1}(ax)^3 \, dx = \frac{3i \operatorname{PolyLog}\left(2, 1 - \frac{2}{iax+1}\right)}{2a^2} + \frac{\cot^{-1}(ax)^3}{2a^2} + \frac{3i \cot^{-1}(ax)^2}{2a^2} - \frac{3 \log\left(\frac{2}{1+iax}\right) \cot^{-1}(ax)}{a^2} + \frac{1}{2}x^2 \cot^{-1}(ax)^3 + \frac{3x \cot^{-1}(ax)^2}{2a}$$

```
[In] Int[x*ArcCot[a*x]^3,x]
```

```
[Out] (((3*I)/2)*ArcCot[a*x]^2)/a^2 + (3*x*ArcCot[a*x]^2)/(2*a) + ArcCot[a*x]^3/(
2*a^2) + (x^2*ArcCot[a*x]^3)/2 - (3*ArcCot[a*x]*Log[2/(1 + I*a*x)])/a^2 + (
((3*I)/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```

<u>Rule 2352</u>

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{1}{2}(3a)\int\frac{x^2\cot^{-1}(ax)^2}{1+a^2x^2}\,dx\\
&= \frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3\int\cot^{-1}(ax)^2\,dx}{2a}-\frac{3\int\frac{\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{2a}\\
&= \frac{3x\cot^{-1}(ax)^2}{2a}+\frac{\cot^{-1}(ax)^3}{2a^2}+\frac{1}{2}x^2\cot^{-1}(ax)^3+3\int\frac{x\cot^{-1}(ax)}{1+a^2x^2}\,dx\\
&= \frac{3i\cot^{-1}(ax)^2}{2a^2}+\frac{3x\cot^{-1}(ax)^2}{2a}+\frac{\cot^{-1}(ax)^3}{2a^2}+\frac{1}{2}x^2\cot^{-1}(ax)^3-\frac{3\int\frac{\cot^{-1}(ax)}{i-ax}\,dx}{a}\\
&= \frac{3i\cot^{-1}(ax)^2}{2a^2}+\frac{3x\cot^{-1}(ax)^2}{2a}+\frac{\cot^{-1}(ax)^3}{2a^2}+\frac{1}{2}x^2\cot^{-1}(ax)^3\\
&\quad-\frac{3\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2}-\frac{3\int\frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx}{a}\\
&= \frac{3i\cot^{-1}(ax)^2}{2a^2}+\frac{3x\cot^{-1}(ax)^2}{2a}+\frac{\cot^{-1}(ax)^3}{2a^2}+\frac{1}{2}x^2\cot^{-1}(ax)^3\\
&\quad-\frac{3\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2}+\frac{(3i)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+iax}\right)}{a^2}\\
&= \frac{3i\cot^{-1}(ax)^2}{2a^2}+\frac{3x\cot^{-1}(ax)^2}{2a}+\frac{\cot^{-1}(ax)^3}{2a^2}+\frac{1}{2}x^2\cot^{-1}(ax)^3\\
&\quad-\frac{3\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2}+\frac{3i\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int x\cot^{-1}(ax)^3\,dx$$
$$=\frac{\cot^{-1}(ax)\left(3(i+ax)\cot^{-1}(ax)+(1+a^2x^2)\cot^{-1}(ax)^2-6\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)+3i\,\text{PolyLog}\left(2,e^{2i\cot^{-1}(ax)}\right)}{2a^2}$$

```
[In] Integrate[x*ArcCot[a*x]^3,x]
[Out] (ArcCot[a*x]*(3*(I + a*x)*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a*x]^2 - 6*Log
[1 - E^((2*I)*ArcCot[a*x])]) + (3*I)*PolyLog[2, E^((2*I)*ArcCot[a*x])])/(2*
a^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(89) = 178$.

Time = 5.90 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.75

method	result
risch	$\frac{3i\pi \arctan(ax)}{16a^2} + \frac{3i \ln\left(\frac{1}{2}-\frac{iax}{2}\right) \ln\left(\frac{1}{2}+\frac{iax}{2}\right)}{2a^2} - \frac{3i \ln\left(\frac{1}{2}+\frac{iax}{2}\right) \ln(-iax+1)}{2a^2} - \frac{i(a^2x^2+1) \ln(iax+1)^3}{16a^2} - \frac{3i\pi^2 \ln(-ia}{16}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(x*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/16*I/a^2*Pi*arctan(a*x)+(-3/16*I*(a^2*x^2+1)/a^2*ln(1-I*a*x)^2+3/8*x*(Pi*
a*x+2)/a*ln(1-I*a*x)+3/16*(I*Pi^2*a^2*x^2+4*I*Pi*a*x+4*I*ln(1-I*a*x)+2*ln(1
-I*a*x)*Pi)/a^2)*ln(1+I*a*x)+1/16/a^2*Pi^3+3/8*I/a^2*Pi^2+1/16*Pi^3*x^2+3/8
*Pi^2*x/a-3/16*Pi*ln(1-I*a*x)^2*x^2-3/16/a*ln(1-I*a*x)^2*x-3/16/a*ln(1-I*a*
x)*x-3/16/a^2*Pi*ln(1-I*a*x)^2+3/16/a^2*Pi*ln(1-I*a*x)-3/16*(-I*x^2*ln(1-I*
a*x)*a^2+Pi*a^2*x^2-I*ln(1-I*a*x)+2*a*x-2*I+Pi)/a^2*ln(1+I*a*x)^2+21/32*arc
tan(a*x)/a^2+3/2*I/a^2*ln(1/2-1/2*I*a*x)*ln(1/2+1/2*I*a*x)-3/2*I/a^2*ln(1/2
+1/2*I*a*x)*ln(1-I*a*x)-1/16*I*(a^2*x^2+1)/a^2*ln(1+I*a*x)^3-3/16*I*Pi^2*ln
(1-I*a*x)*x^2-3/4*I/a*Pi*ln(1-I*a*x)*x+3/32*I/a^2*(1-I*a*x)^2*ln(1-I*a*x)-3
/32*I/a^2*(1-I*a*x)^2*ln(1-I*a*x)^2+1/16*I*ln(1-I*a*x)^3*x^2-9/32*I/a^2*ln(
1-I*a*x)^2+1/16*I/a^2*ln(1-I*a*x)^3-3/32*I*ln(1-I*a*x)^2*x^2+3/32*I*ln(1-I*
a*x)*x^2-3/4*I/a^2*ln(1-I*a*x)+3/2*I/a^2*dilog(1/2-1/2*I*a*x)+21/64*I/a^2*l
n(a^2*x^2+1)+21/32/a^2*Pi*ln(a^2*x^2+1)-3/8*Pi^2/a^2*arctan(a*x)
```

Fricas [F]

$$\int x \cot^{-1}(ax)^3 \, dx = \int x \operatorname{arccot}(ax)^3 \, dx$$

```
[In] integrate(x*arccot(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arccot(a*x)^3, x)
```

Sympy [F]

$$\int x \cot^{-1}(ax)^3 \, dx = \int x \operatorname{acot}^3(ax) \, dx$$

[In] integrate(x*acot(a*x)**3,x)

[Out] Integral(x*acot(a*x)**3, x)

Maxima [F]

$$\int x \cot^{-1}(ax)^3 \, dx = \int x \operatorname{arccot}(ax)^3 \, dx$$

[In] integrate(x*arccot(a*x)^3,x, algorithm="maxima")

[Out] 1/32*(8*a^2*x^2*arctan2(1, a*x)^3 + 12*a*x*arctan2(1, a*x)^2 - 3*a*x*log(a^2*x^2 + 1)^2 + 4*(128*a^3*integrate(1/32*x^3*arctan(1/(a*x))^3/(a^3*x^2 + a), x) + 96*a^2*integrate(1/32*x^2*arctan(1/(a*x))^2/(a^3*x^2 + a), x) + 24*a^2*integrate(1/32*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) + 96*a^2*integrate(1/32*x^2*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 128*a*integrate(1/32*x*arctan(1/(a*x))^3/(a^3*x^2 + a), x) + 192*a*integrate(1/32*x*arctan(1/(a*x))/(a^3*x^2 + a), x) + arctan(a*x)^3/a^2 + 3*arctan(a*x)^2*arctan(1/(a*x))/a^2 + 3*arctan(a*x)*arctan(1/(a*x))^2/a^2 + 24*integrate(1/32*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x))*a^2 + 8*arctan2(1, a*x)^3)/a^2

Giac [F]

$$\int x \cot^{-1}(ax)^3 \, dx = \int x \operatorname{arccot}(ax)^3 \, dx$$

[In] integrate(x*arccot(a*x)^3,x, algorithm="giac")

[Out] integrate(x*arccot(a*x)^3, x)

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(ax)^3\,dx = \int x\,\mathrm{acot}(a\,x)^3\,dx$$

```
[In] int(x*acot(a*x)^3,x)
[Out] int(x*acot(a*x)^3, x)
```

3.28 $\int \cot^{-1}(ax)^3\,dx$

Optimal result

Integrand size = 6, antiderivative size = 96

$$\int \cot^{-1}(ax)^3\,dx = \frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} + \frac{3i\cot^{-1}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a} - \frac{3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a}$$

```
[Out] I*arccot(a*x)^3/a+x*arccot(a*x)^3-3*arccot(a*x)^2*ln(2/(1+I*a*x))/a+3*I*arc
cot(a*x)*polylog(2,1-2/(1+I*a*x))/a-3/2*polylog(3,1-2/(1+I*a*x))/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4931, 5041, 4965, 5005, 5115, 6745}

$$\int \cot^{-1}(ax)^3\,dx = -\frac{3\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a} + \frac{3i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)\cot^{-1}(ax)}{a} + x\cot^{-1}(ax)^3 + \frac{i\cot^{-1}(ax)^3}{a} - \frac{3\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)^2}{a}$$

```
[In] Int[ArcCot[a*x]^3,x]
```

```
[Out] (I*ArcCot[a*x]^3)/a + x*ArcCot[a*x]^3 - (3*ArcCot[a*x]^2*Log[2/(1 + I*a*x)]
)/a + ((3*I)*ArcCot[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - (3*PolyLog[3, 1
 - 2/(1 + I*a*x)])/(2*a)
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\cot^{-1}(ax)^3+(3a)\int\frac{x\cot^{-1}(ax)^2}{1+a^2x^2}\,dx\\ &= \frac{i\cot^{-1}(ax)^3}{a}+x\cot^{-1}(ax)^3-3\int\frac{\cot^{-1}(ax)^2}{i-ax}\,dx\\ &= \frac{i\cot^{-1}(ax)^3}{a}+x\cot^{-1}(ax)^3-\frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-6\int\frac{\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx\end{aligned}$$

$$= \frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} + \frac{3i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a} + 3i\int\frac{\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{1+a^2x^2}dx$$

$$= \frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} + \frac{3i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a} - \frac{3\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int\cot^{-1}(ax)^3\,dx = -\frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(1-e^{-2i\cot^{-1}(ax)}\right)}{a} - \frac{3i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right)}{a} - \frac{3\,\mathrm{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right)}{2a}$$

```
[In] Integrate[ArcCot[a*x]^3,x]
```

```
[Out] ((-I)*ArcCot[a*x]^3)/a + x*ArcCot[a*x]^3 - (3*ArcCot[a*x]^2*Log[1 - E^((-2*
I)*ArcCot[a*x])])/a - ((3*I)*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x])]
)/a - (3*PolyLog[3, E^((-2*I)*ArcCot[a*x])])/(2*a)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(89) = 178.

Time = 1.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{\operatorname{arccot}(ax)^3(ax-i)+2i\operatorname{arccot}(ax)^3-3\operatorname{arccot}(ax)^2\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+6i\operatorname{arccot}(ax)\operatorname{polylog}\left(2,\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-6\operatorname{polylog}\left(3,\right.}{a}$
default	$\frac{\operatorname{arccot}(ax)^3(ax-i)+2i\operatorname{arccot}(ax)^3-3\operatorname{arccot}(ax)^2\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+6i\operatorname{arccot}(ax)\operatorname{polylog}\left(2,\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-6\operatorname{polylog}\left(3,\right.}{a}$

```
[In] int(arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccot(a*x)^3*(a*x-I)+2*I*arccot(a*x)^3-3*arccot(a*x)^2*ln(1-(I+a*x)/(
a^2*x^2+1)^(1/2))+6*I*arccot(a*x)*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-6*po
lylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))-3*arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)
```

```
^(1/2))+6*I*arccot(a*x)*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-
(I+a*x)/(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \cot^{-1}(ax)^3\,dx = \int \operatorname{arccot}\left(ax\right)^3\,dx$$

```
[In] integrate(arccot(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3, x)
```

Sympy [F]

$$\int \cot^{-1}(ax)^3\,dx = \int \operatorname{acot}^3\left(ax\right)dx$$

```
[In] integrate(acot(a*x)**3,x)
```

```
[Out] Integral(acot(a*x)**3, x)
```

Maxima [F]

$$\int \cot^{-1}(ax)^3\,dx = \int \operatorname{arccot}\left(ax\right)^3\,dx$$

```
[In] integrate(arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/8*x*arctan2(1, a*x)^3 - 3/32*x*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 + 21/16
*arctan(a*x)^2*arctan(1/(a*x))^2/a + 7/8*arctan(a*x)*arctan(1/(a*x))^3/a +
28*a^2*integrate(1/32*x^2*arctan(1/(a*x))^3/(a^2*x^2 + 1), x) + 3*a^2*integ
rate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12*a^2
*integrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 12
*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^2 + 1), x) - 3*a*integrate(1/3
2*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*(a*arctan(a*x)^4 + 4*a*arct
an(a*x)^3*arctan(1/(a*x)))/a^2 + 3*integrate(1/32*arctan(1/(a*x))*log(a^2*x
^2 + 1)^2/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int \cot^{-1}(ax)^3\,dx = \int \operatorname{arccot}\left(ax\right)^3\,dx$$

```
[In] integrate(arccot(a*x)^3,x, algorithm="giac")
[Out] integrate(arccot(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(ax)^3\,dx = \int \operatorname{acot}(a\,x)^3\,dx$$

```
[In] int(acot(a*x)^3,x)
[Out] int(acot(a*x)^3, x)
```

3.29 $\int \frac{\cot^{-1}(ax)^3}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 178

$$
\begin{aligned}
\int \frac{\cot^{-1}(ax)^3}{x}\,dx = {} & 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right) \\
& -\frac{3}{2}i\cot^{-1}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2i}{i+ax}\right) \\
& +\frac{3}{2}i\cot^{-1}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right) \\
& -\frac{3}{2}\cot^{-1}(ax)\operatorname{PolyLog}\left(3,1-\frac{2i}{i+ax}\right) \\
& +\frac{3}{2}\cot^{-1}(ax)\operatorname{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right) \\
& +\frac{3}{4}i\operatorname{PolyLog}\left(4,1-\frac{2i}{i+ax}\right)-\frac{3}{4}i\operatorname{PolyLog}\left(4,1-\frac{2ax}{i+ax}\right)
\end{aligned}
$$

```
[Out] 2*arccot(a*x)^3*arccoth(1-2/(1+I*a*x))-3/2*I*arccot(a*x)^2*polylog(2,1-2*I/
(I+a*x))+3/2*I*arccot(a*x)^2*polylog(2,1-2*a*x/(I+a*x))-3/2*arccot(a*x)*pol
ylog(3,1-2*I/(I+a*x))+3/2*arccot(a*x)*polylog(3,1-2*a*x/(I+a*x))+3/4*I*poly
log(4,1-2*I/(I+a*x))-3/4*I*polylog(4,1-2*a*x/(I+a*x))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4943, 5109, 5005, 5113, 5117, 6745}

$$
\begin{aligned}
\int \frac{\cot^{-1}(ax)^3}{x}\,dx = {} & \frac{3}{4} i \operatorname{PolyLog}\left(4,1-\frac{2i}{ax+i}\right)-\frac{3}{4} i \operatorname{PolyLog}\left(4,1-\frac{2ax}{ax+i}\right) \\
& -\frac{3}{2} i \operatorname{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)^2 \\
& +\frac{3}{2} i \operatorname{PolyLog}\left(2,1-\frac{2ax}{ax+i}\right)\cot^{-1}(ax)^2 \\
& -\frac{3}{2} \operatorname{PolyLog}\left(3,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax) \\
& +\frac{3}{2} \operatorname{PolyLog}\left(3,1-\frac{2ax}{ax+i}\right)\cot^{-1}(ax) \\
& +2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)
\end{aligned}
$$

```
[In] Int[ArcCot[a*x]^3/x,x]
```

```
[Out] 2*ArcCot[a*x]^3*ArcCoth[1 - 2/(1 + I*a*x)] - ((3*I)/2)*ArcCot[a*x]^2*PolyLo
g[2, 1 - (2*I)/(I + a*x)] + ((3*I)/2)*ArcCot[a*x]^2*PolyLog[2, 1 - (2*a*x)/
(I + a*x)] - (3*ArcCot[a*x]*PolyLog[3, 1 - (2*I)/(I + a*x)])/2 + (3*ArcCot[
a*x]*PolyLog[3, 1 - (2*a*x)/(I + a*x)])/2 + ((3*I)/4)*PolyLog[4, 1 - (2*I)/
(I + a*x)] - ((3*I)/4)*PolyLog[4, 1 - (2*a*x)/(I + a*x)]
```

Rule 4943

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Dist[2*b*c*p, Int[(a + b*
ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5109

```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegrand
[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c
```

```
, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)
))^2, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5117

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)+(6a)\int\frac{\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx\\
&= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)\\
&\quad-(3a)\int\frac{\cot^{-1}(ax)^2\log\left(\frac{2i}{i+ax}\right)}{1+a^2x^2}\,dx+(3a)\int\frac{\cot^{-1}(ax)^2\log\left(\frac{2ax}{i+ax}\right)}{1+a^2x^2}\,dx\\
&= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}i\cot^{-1}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2i}{i+ax}\right)\\
&\quad+\frac{3}{2}i\cot^{-1}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right)\\
&\quad-(3ia)\int\frac{\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2i}{i+ax}\right)}{1+a^2x^2}\,dx\\
&\quad+(3ia)\int\frac{\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right)}{1+a^2x^2}\,dx
\end{aligned}$$

$$= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}i\cot^{-1}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2i}{i+ax}\right)$$
$$+\frac{3}{2}i\cot^{-1}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right)$$
$$-\frac{3}{2}\cot^{-1}(ax)\,\text{PolyLog}\left(3,1-\frac{2i}{i+ax}\right)+\frac{3}{2}\cot^{-1}(ax)\,\text{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right)$$
$$-\frac{1}{2}(3a)\int\frac{\text{PolyLog}\left(3,1-\frac{2i}{i+ax}\right)}{1+a^2x^2}\,dx+\frac{1}{2}(3a)\int\frac{\text{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right)}{1+a^2x^2}\,dx$$
$$= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}i\cot^{-1}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2i}{i+ax}\right)$$
$$+\frac{3}{2}i\cot^{-1}(ax)^2\,\text{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right)$$
$$-\frac{3}{2}\cot^{-1}(ax)\,\text{PolyLog}\left(3,1-\frac{2i}{i+ax}\right)+\frac{3}{2}\cot^{-1}(ax)\,\text{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right)$$
$$+\frac{3}{4}i\,\text{PolyLog}\left(4,1-\frac{2i}{i+ax}\right)-\frac{3}{4}i\,\text{PolyLog}\left(4,1-\frac{2ax}{i+ax}\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int\frac{\cot^{-1}(ax)^3}{x}\,dx=\frac{1}{64}i\Big(\pi^4-32\cot^{-1}(ax)^4+64i\cot^{-1}(ax)^3\log\left(1-e^{-2i\cot^{-1}(ax)}\right)$$
$$-64i\cot^{-1}(ax)^3\log\left(1+e^{2i\cot^{-1}(ax)}\right)$$
$$-96\cot^{-1}(ax)^2\,\text{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right)$$
$$-96\cot^{-1}(ax)^2\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)$$
$$+96i\cot^{-1}(ax)\,\text{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right)$$
$$-96i\cot^{-1}(ax)\,\text{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)$$
$$+48\,\text{PolyLog}\left(4,e^{-2i\cot^{-1}(ax)}\right)+48\,\text{PolyLog}\left(4,-e^{2i\cot^{-1}(ax)}\right)\Big)$$

```
[In] Integrate[ArcCot[a*x]^3/x,x]
```

```
[Out] (I/64)*(Pi^4 - 32*ArcCot[a*x]^4 + (64*I)*ArcCot[a*x]^3*Log[1 - E^((-2*I)*Ar
cCot[a*x])] - (64*I)*ArcCot[a*x]^3*Log[1 + E^((2*I)*ArcCot[a*x])] - 96*ArcC
ot[a*x]^2*PolyLog[2, E^((-2*I)*ArcCot[a*x])] - 96*ArcCot[a*x]^2*PolyLog[2,
-E^((2*I)*ArcCot[a*x])] + (96*I)*ArcCot[a*x]*PolyLog[3, E^((-2*I)*ArcCot[a*
x])] - (96*I)*ArcCot[a*x]*PolyLog[3, -E^((2*I)*ArcCot[a*x])] + 48*PolyLog[4
, E^((-2*I)*ArcCot[a*x])] + 48*PolyLog[4, -E^((2*I)*ArcCot[a*x])])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.61 (sec) , antiderivative size = 982, normalized size of antiderivative = 5.52

method	result	size
derivativedivides	Expression too large to display	982
default	Expression too large to display	982
parts	Expression too large to display	1417

```
[In] int(arccot(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(a*x)*arccot(a*x)^3+1/2*I*Pi*(csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(1
+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2
*x^2+1)))-csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1
)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I
+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2+csgn(I/((I+a*x)^2/(a^2*
x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3-csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+
(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*
x^2+1)))^2+csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn
(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-csgn(1/((I+a*x)^2/(
a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)
*(1+(I+a*x)^2/(a^2*x^2+1)))^2-1)*arccot(a*x)^3+arccot(a*x)^3*ln((I+a*x)^2/(
a^2*x^2+1)-1)-arccot(a*x)^3*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+3*I*arccot(a*x)
^2*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-6*arccot(a*x)*polylog(3,(I+a*x)/(a^
2*x^2+1)^(1/2))-6*I*polylog(4,(I+a*x)/(a^2*x^2+1)^(1/2))-arccot(a*x)^3*ln(1
+(I+a*x)/(a^2*x^2+1)^(1/2))+3*I*arccot(a*x)^2*polylog(2,-(I+a*x)/(a^2*x^2+1
)^(1/2))-6*arccot(a*x)*polylog(3,-(I+a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,
-(I+a*x)/(a^2*x^2+1)^(1/2))-3/2*I*arccot(a*x)^2*polylog(2,-(I+a*x)^2/(a^2*x
^2+1))+3/2*arccot(a*x)*polylog(3,-(I+a*x)^2/(a^2*x^2+1))+3/4*I*polylog(4,-(
I+a*x)^2/(a^2*x^2+1))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x}\,dx$$

```
[In] integrate(arccot(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x}\,dx$$

```
[In] integrate(acot(a*x)**3/x,x)
[Out] Integral(acot(a*x)**3/x, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x}\,dx$$

```
[In] integrate(arccot(a*x)^3/x,x, algorithm="maxima")
[Out] integrate(arccot(a*x)^3/x, x)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x}\,dx$$

```
[In] integrate(arccot(a*x)^3/x,x, algorithm="giac")
[Out] integrate(arccot(a*x)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{acot}(a\,x)^3}{x}\,dx$$

```
[In] int(acot(a*x)^3/x,x)
[Out] int(acot(a*x)^3/x, x)
```

3.30 $\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 93

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) - 3ia\cot^{-1}(ax)\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)$$

```
[Out] -I*a*arccot(a*x)^3-arccot(a*x)^3/x-3*a*arccot(a*x)^2*ln(2-2/(1-I*a*x))-3*I*
a*arccot(a*x)*polylog(2,-1+2/(1-I*a*x))-3/2*a*polylog(3,-1+2/(1-I*a*x))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4947, 5045, 4989, 5005, 5113, 6745}

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = -\frac{3}{2}a\,\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right) - 3ia\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax) - ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)^2$$

```
[In] Int[ArcCot[a*x]^3/x^2,x]
```

```
[Out] (-I)*a*ArcCot[a*x]^3 - ArcCot[a*x]^3/x - 3*a*ArcCot[a*x]^2*Log[2 - 2/(1 - I
*a*x)] - (3*I)*a*ArcCot[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] - (3*a*PolyLog[
3, -1 + 2/(1 - I*a*x)])/2
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = -\frac{\cot^{-1}(ax)^3}{x} - (3a)\int \frac{\cot^{-1}(ax)^2}{x\,(1+a^2x^2)}\,dx$$

$$= -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - (3ia)\int \frac{\cot^{-1}(ax)^2}{x(i+ax)}\,dx$$

$$= -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) - (6a^2)\int \frac{\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx$$

$$= -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) - 3ia\cot^{-1}(ax)\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) - (3ia^2)\int \frac{\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx$$

$$= -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) - 3ia\cot^{-1}(ax)\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \frac{(-1+iax)\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right) + 3ia\cot^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)$$

```
[In] Integrate[ArcCot[a*x]^3/x^2,x]
```

```
[Out] ((-1 + I*a*x)*ArcCot[a*x]^3)/x - 3*a*ArcCot[a*x]^2*Log[1 + E^((2*I)*ArcCot[
a*x])] + (3*I)*a*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] - (3*a*Poly
Log[3, -E^((2*I)*ArcCot[a*x])])/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.94 (sec) , antiderivative size = 1441, normalized size of antiderivative = 15.49

method	result	size
parts	Expression too large to display	1441
derivativedivides	Expression too large to display	1444
default	Expression too large to display	1444

```
[In] int(arccot(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arccot(a*x)^3/x-3*a*(-1/2*arccot(a*x)^2*ln(a^2*x^2+1)+arccot(a*x)^2*ln(a*x
)+arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-1/3*I*arccot(a*x)^3+1/4*(-2*I
*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(
I+a*x)^2/(a^2*x^2+1)))^2+I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((
I+a*x)^2/(a^2*x^2+1)-1)^2)-I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3+2*I*Pi*csgn
(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(
a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-2*I*Pi*csgn(I*(1+(I+a*x)^2/(a^2*x^
2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2+2*I*Pi
*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2+I*Pi*csg
n(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a
^2*x^2+1)-1)^2)^2-2*I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^
2/(a^2*x^2+1)-1)^2)^2-2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/
(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2
+2*I*Pi*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-I*Pi*
csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I
+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)+I*Pi*csgn(I*(I+a*x)^2/(a^2
*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2+2*I*Pi
*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+I*Pi*csgn(I*
(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^3+2*I*Pi*csgn(I/((I+a*x)
^2/(a^2*x^2+1)-1))*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2
*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-I*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2)
)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))+I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^
3-2*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2-2*I*Pi
*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+4*ln(2))*arc
cot(a*x)^2-I*arccot(a*x)*polylog(2,-(I+a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(
I+a*x)^2/(a^2*x^2+1)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^2}\,dx$$

```
[In] integrate(arccot(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3/x^2, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x^2}\,dx$$

```
[In] integrate(acot(a*x)**3/x**2,x)
```

```
[Out] Integral(acot(a*x)**3/x**2, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^2}\,dx$$

```
[In] integrate(arccot(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/32*(4*arctan2(1, a*x)^3 - 3*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 - (28*a*a
rctan(a*x)*arctan(1/(a*x))^3 + 7*(6*arctan(a*x)^2*arctan(1/(a*x))^2/a + (a*
arctan(a*x)^4 + 4*a*arctan(a*x)^3*arctan(1/(a*x)))/a^2)*a^2 + 96*a^2*integr
ate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 384*a
^2*integrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x)
- 384*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^4 + x^2), x) + 96*a*integ
rate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 896*integrate(1/32*arc
tan(1/(a*x))^3/(a^2*x^4 + x^2), x) + 96*integrate(1/32*arctan(1/(a*x))*log(
a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arccot}\left(ax\right)^3}{x^2}\,dx$$

```
[In] integrate(arccot(a*x)^3/x^2,x, algorithm="giac")
[Out] integrate(arccot(a*x)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{acot}(a\,x)^3}{x^2}\,dx$$

```
[In] int(acot(a*x)^3/x^2,x)
[Out] int(acot(a*x)^3/x^2, x)
```

3.31 $\int \frac{\cot^{-1}(ax)^3}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 105

$$\int \frac{\cot^{-1}(ax)^3}{x^3} dx = \frac{3}{2}ia^2\cot^{-1}(ax)^2 + \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + 3a^2\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) + \frac{3}{2}ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)$$

```
[Out] 3/2*I*a^2*arccot(a*x)^2+3/2*a*arccot(a*x)^2/x-1/2*a^2*arccot(a*x)^3-1/2*arc
cot(a*x)^3/x^2+3*a^2*arccot(a*x)*ln(2-2/(1-I*a*x))+3/2*I*a^2*polylog(2,-1+2
/(1-I*a*x))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4947, 5039, 5045, 4989, 2497, 5005}

$$\int \frac{\cot^{-1}(ax)^3}{x^3} dx = \frac{3}{2}ia^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right) - \frac{1}{2}a^2\cot^{-1}(ax)^3 + \frac{3}{2}ia^2\cot^{-1}(ax)^2 + 3a^2\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax) - \frac{\cot^{-1}(ax)^3}{2x^2} + \frac{3a\cot^{-1}(ax)^2}{2x}$$

```
[In] Int[ArcCot[a*x]^3/x^3,x]
```

```
[Out] ((3*I)/2)*a^2*ArcCot[a*x]^2 + (3*a*ArcCot[a*x]^2)/(2*x) - (a^2*ArcCot[a*x]^
3)/2 - ArcCot[a*x]^3/(2*x^2) + 3*a^2*ArcCot[a*x]*Log[2 - 2/(1 - I*a*x)] + (
(3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\cot^{-1}(ax)^3}{2x^2} - \frac{1}{2}(3a)\int \frac{\cot^{-1}(ax)^2}{x^2\,(1+a^2x^2)}\,dx$$

$$= -\frac{\cot^{-1}(ax)^3}{2x^2} - \frac{1}{2}(3a)\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx + \frac{1}{2}(3a^3)\int \frac{\cot^{-1}(ax)^2}{1+a^2x^2}\,dx$$
$$= \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + (3a^2)\int \frac{\cot^{-1}(ax)}{x(1+a^2x^2)}\,dx$$
$$= \frac{3}{2}ia^2\cot^{-1}(ax)^2 + \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + (3ia^2)\int \frac{\cot^{-1}(ax)}{x(i+ax)}\,dx$$
$$= \frac{3}{2}ia^2\cot^{-1}(ax)^2 + \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + 3a^2\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) + (3a^3)\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx$$
$$= \frac{3}{2}ia^2\cot^{-1}(ax)^2 + \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + 3a^2\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) + \frac{3}{2}ia^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = -\frac{\cot^{-1}(ax)\left(3iax(i+ax)\cot^{-1}(ax) + (1+a^2x^2)\cot^{-1}(ax)^2 - 6a^2x^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)\right)}{2x^2} - \frac{3}{2}ia^2\,\mathrm{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)$$

```
[In] Integrate[ArcCot[a*x]^3/x^3,x]
```

```
[Out] -1/2*(ArcCot[a*x]*((3*I)*a*x*(I + a*x)*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a
*x]^2 - 6*a^2*x^2*Log[1 + E^((2*I)*ArcCot[a*x])]))/x^2 - ((3*I)/2)*a^2*Poly
Log[2, -E^((2*I)*ArcCot[a*x])]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.76 (sec) , antiderivative size = 2956, normalized size of antiderivative = 28.15

method	result	size
parts	Expression too large to display	2956
derivativedivides	Expression too large to display	2957
default	Expression too large to display	2957

[In] int(arccot(a*x)^3/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*arccot(a*x)^3/x^2-3/2*a^2*(1/2*Pi*arccot(a*x)^2-1/2*I*Pi*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+1/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2+1)))-1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))+1/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2+1)))-1/4*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))+1/2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^2*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))+1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^3*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2+1)))+1/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2+1)))-1/2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^3*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))-1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^2*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2+1)))-arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))-arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))-arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))-1/2*Pi*dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))-1/2*Pi*dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+1/4*Pi*polylog(2,-(I+a*x)^2/(a^2*x^2+1))+I*arccot(a*x)^2+1/3*arccot(a*x)^3-1/4*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))+1/8*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2+1)))+1/2*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*a

```
rccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^
(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))+1/4*Pi*csgn(I*(I+a*x)^2/(a^2*x
^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*(I*arccot(
a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^
2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2
+1)^(1/2)))-1/4*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*
x^2+1))^2*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+poly
log(2,-(I+a*x)^2/(a^2*x^2+1)))-1/4*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*c
sgn(I*(I+a*x)^2/(a^2*x^2+1))*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2
))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x
^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))-1/x*arccot(a*x)^2/a+I*ar
ccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-1/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+
1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*(2*I*arccot(a
*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x
^2+1)))-1/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+
1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1)
)+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2+1)))+1/4*Pi*csgn(I/((I+a*x)
^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^
2*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+
a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a
*x)/(a^2*x^2+1)^(1/2)))+I*dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*
(I+a*x)/(a^2*x^2+1)^(1/2))+1/2*I*polylog(2,-(I+a*x)^2/(a^2*x^2+1))-arccot(a
*x)^2*arctan(a*x))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^3}\,dx$$

```
[In] integrate(arccot(a*x)^3/x^3,x, algorithm="fricas")
[Out] integral(arccot(a*x)^3/x^3, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x^3}\,dx$$

```
[In] integrate(acot(a*x)**3/x**3,x)
[Out] Integral(acot(a*x)**3/x**3, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arccot}\left(ax\right)^3}{x^3}\,dx$$

```
[In] integrate(arccot(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/32*(8*a^2*x^2*arctan2(1, a*x)^3 - 12*a*x*arctan2(1, a*x)^2 + 3*a*x*log(a
^2*x^2 + 1)^2 + 4*(3*a^2*arctan(a*x)*arctan(1/(a*x))^2 + (arctan(a*x)^3/a +
 3*arctan(a*x)^2*arctan(1/(a*x))/a)*a^3 + 24*a^3*integrate(1/32*x^3*log(a^2
*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 96*a^3*integrate(1/32*x^3*log(a^2*x^2 + 1
)/(a^2*x^5 + x^3), x) - 128*a^2*integrate(1/32*x^2*arctan(1/(a*x))^3/(a^2*x
^5 + x^3), x) - 192*a^2*integrate(1/32*x^2*arctan(1/(a*x))/(a^2*x^5 + x^3),
 x) + 96*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^5 + x^3), x) + 24*a*in
tegrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 128*integrate(1/32*
arctan(1/(a*x))^3/(a^2*x^5 + x^3), x))*x^2 + 8*arctan2(1, a*x)^3)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = -\frac{1}{2}\,a\arctan\left(\frac{1}{ax}\right)^3 - \frac{\arctan\left(\frac{1}{ax}\right)^3}{2\,x^2}$$

```
[In] integrate(arccot(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] -1/2*a*arctan(1/(a*x))^3 - 1/2*arctan(1/(a*x))^3/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{acot}(a\,x)^3}{x^3}\,dx$$

```
[In] int(acot(a*x)^3/x^3,x)
```

```
[Out] int(acot(a*x)^3/x^3, x)
```

3.32 $\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 167

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = -\frac{a^2\cot^{-1}(ax)}{x}+\frac{1}{2}a^3\cot^{-1}(ax)^2+\frac{a\cot^{-1}(ax)^2}{2x^2}+\frac{1}{3}ia^3\cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^3}{3x^3}-a^3\log(x)+\frac{1}{2}a^3\log\left(1+a^2x^2\right)+a^3\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)+ia^3\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)+\frac{1}{2}a^3\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)$$

```
[Out] -a^2*arccot(a*x)/x+1/2*a^3*arccot(a*x)^2+1/2*a*arccot(a*x)^2/x^2+1/3*I*a^3*
arccot(a*x)^3-1/3*arccot(a*x)^3/x^3-a^3*ln(x)+1/2*a^3*ln(a^2*x^2+1)+a^3*arc
cot(a*x)^2*ln(2-2/(1-I*a*x))+I*a^3*arccot(a*x)*polylog(2,-1+2/(1-I*a*x))+1/
2*a^3*polylog(3,-1+2/(1-I*a*x))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules

used = {4947, 5039, 272, 36, 29, 31, 5005, 5045, 4989, 5113, 6745}

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \frac{1}{2}a^3\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)+ia^3\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax)$$
$$-a^3\log(x)+\frac{1}{3}ia^3\cot^{-1}(ax)^3+\frac{1}{2}a^3\cot^{-1}(ax)^2$$
$$+a^3\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)^2-\frac{a^2\cot^{-1}(ax)}{x}$$
$$+\frac{1}{2}a^3\log\left(a^2x^2+1\right)-\frac{\cot^{-1}(ax)^3}{3x^3}+\frac{a\cot^{-1}(ax)^2}{2x^2}$$

```
[In] Int[ArcCot[a*x]^3/x^4,x]
```

```
[Out] -((a^2*ArcCot[a*x])/x) + (a^3*ArcCot[a*x]^2)/2 + (a*ArcCot[a*x]^2)/(2*x^2)
+ (I/3)*a^3*ArcCot[a*x]^3 - ArcCot[a*x]^3/(3*x^3) - a^3*Log[x] + (a^3*Log[1
 + a^2*x^2])/2 + a^3*ArcCot[a*x]^2*Log[2 - 2/(1 - I*a*x)] + I*a^3*ArcCot[a*
x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (a^3*PolyLog[3, -1 + 2/(1 - I*a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = -\frac{\cot^{-1}(ax)^3}{3x^3} - a\int \frac{\cot^{-1}(ax)^2}{x^3\,(1+a^2x^2)}\,dx$$

$$
\begin{aligned}
&= -\frac{\cot^{-1}(ax)^3}{3x^3} - a\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx + a^3\int \frac{\cot^{-1}(ax)^2}{x\,(1+a^2x^2)}\,dx \\
&= \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} \\
&\quad + a^2\int \frac{\cot^{-1}(ax)}{x^2\,(1+a^2x^2)}\,dx + \left(ia^3\right)\int \frac{\cot^{-1}(ax)^2}{x(i+ax)}\,dx \\
&= \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} + a^3\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\
&\quad + a^2\int \frac{\cot^{-1}(ax)}{x^2}\,dx - a^4\int \frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx + \left(2a^4\right)\int \frac{\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx \\
&= -\frac{a^2\cot^{-1}(ax)}{x} + \frac{1}{2}a^3\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} \\
&\quad + a^3\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) + ia^3\cot^{-1}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\
&\quad - a^3\int \frac{1}{x\,(1+a^2x^2)}\,dx + \left(ia^4\right)\int \frac{\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx \\
&= -\frac{a^2\cot^{-1}(ax)}{x} + \frac{1}{2}a^3\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} \\
&\quad + a^3\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) + ia^3\cot^{-1}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\
&\quad + \frac{1}{2}a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) - \frac{1}{2}a^3\operatorname{Subst}\left(\int \frac{1}{x\,(1+a^2x)}\,dx,x,x^2\right) \\
&= -\frac{a^2\cot^{-1}(ax)}{x} + \frac{1}{2}a^3\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 \\
&\quad - \frac{\cot^{-1}(ax)^3}{3x^3} + a^3\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\
&\quad + ia^3\cot^{-1}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) + \frac{1}{2}a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) \\
&\quad - \frac{1}{2}a^3\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right) + \frac{1}{2}a^5\operatorname{Subst}\left(\int \frac{1}{1+a^2x}\,dx,x,x^2\right) \\
&= -\frac{a^2\cot^{-1}(ax)}{x} + \frac{1}{2}a^3\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} \\
&\quad - a^3\log(x) + \frac{1}{2}a^3\log\left(1+a^2x^2\right) + a^3\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\
&\quad + ia^3\cot^{-1}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) + \frac{1}{2}a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \frac{1}{6}\left(-\frac{6a^2\cot^{-1}(ax)}{x}+3a^3\cot^{-1}(ax)^2+\frac{3a\cot^{-1}(ax)^2}{x^2}-2ia^3\cot^{-1}(ax)^3-\frac{2\cot^{-1}(ax)^3}{x^3}+6a^3\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)-6a^3\log\left(\frac{1}{\sqrt{1+\frac{1}{a^2x^2}}}\right)-6ia^3\cot^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)+3a^3\,\text{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)\right)$$

```
[In] Integrate[ArcCot[a*x]^3/x^4,x]
```

```
[Out] ((-6*a^2*ArcCot[a*x])/x + 3*a^3*ArcCot[a*x]^2 + (3*a*ArcCot[a*x]^2)/x^2 - (
2*I)*a^3*ArcCot[a*x]^3 - (2*ArcCot[a*x]^3)/x^3 + 6*a^3*ArcCot[a*x]^2*Log[1
+ E^((2*I)*ArcCot[a*x])] - 6*a^3*Log[1/Sqrt[1 + 1/(a^2*x^2)]] - (6*I)*a^3*A
rcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] + 3*a^3*PolyLog[3, -E^((2*I)*
ArcCot[a*x])])/6
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.04 (sec) , antiderivative size = 1622, normalized size of antiderivative = 9.71

method	result	size
parts	Expression too large to display	1622
derivativedivides	Expression too large to display	1623
default	Expression too large to display	1623

```
[In] int(arccot(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arccot(a*x)^3/x^3-a^3*(-1/2/a^2/x^2*arccot(a*x)^2-arccot(a*x)^2*ln(a*x
)+1/2*arccot(a*x)^2*ln(a^2*x^2+1)-arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2
))+I*arccot(a*x)*polylog(2,-(I+a*x)^2/(a^2*x^2+1))-1/2*polylog(3,-(I+a*x)^2
/(a^2*x^2+1))+1/12*arccot(a*x)*(4*I*arccot(a*x)^2*a*x-3*I*arccot(a*x)*Pi*cs
gn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)*a*x+6
*I*arccot(a*x)*Pi*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*
x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*a*x+3*I*arccot(a*x)*Pi*csgn(I*(I+a*x
)^2/(a^2*x^2+1))^3*a*x-6*I*arccot(a*x)*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*
```

```
(1+(I+a*x)^2/(a^2*x^2+1))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a
^2*x^2+1)))*a*x-3*I*arccot(a*x)*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3*a*
x-6*I*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*csgn(I*(I+a*x)/(a^2*x^
2+1)^(1/2))*a*x+6*I*arccot(a*x)*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I
*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*a*x+6*I*arccot(a*x)*Pi*csgn(1/((I+a*x)^2/(a
^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3*a*x-12*I*a*x-6*I*arccot(a*x)*Pi*c
sgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x
)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*a*x+6*I*arccot(a*x)*Pi*
csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*csgn(I/((I+a*
x)^2/(a^2*x^2+1)-1))*a*x-3*I*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((
I+a*x)^2/(a^2*x^2+1)-1)^2)^3*a*x-6*I*arccot(a*x)*Pi*csgn(1/((I+a*x)^2/(a^2*
x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*a*x+6*I*arccot(a*x)*Pi*csgn(I/((I+a*
x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1
)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*a*x+3*I*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(
a^2*x^2+1))*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*a*x-6*I*arccot(a*x)*Pi*csgn
(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3*a*x-3*I*arccot(a*
x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*csgn(I/((
I+a*x)^2/(a^2*x^2+1)-1)^2)*a*x-3*I*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2
+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*a*x+3*I*ar
ccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn
(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*a*x-12*arccot
(a*x)*ln(2)*a*x-6*arccot(a*x)*a*x+12+6*I*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a
^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*a*x)/a/x+ln(1+(I+a*x)^2/(a^2*x^2+1
)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^4}\,dx$$

```
[In] integrate(arccot(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3/x^4, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x^4}\,dx$$

```
[In] integrate(acot(a*x)**3/x**4,x)
```

```
[Out] Integral(acot(a*x)**3/x**4, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^4}\,dx$$

```
[In] integrate(arccot(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(96*x^3*integrate(1/32*(28*a^2*x^2*arctan2(1, a*x)^3 - 4*a^2*x^2*arcta
n2(1, a*x)*log(a^2*x^2 + 1) - 4*a*x*arctan2(1, a*x)^2 + 28*arctan2(1, a*x)^
3 + (3*a^2*x^2*arctan2(1, a*x) + a*x + 3*arctan2(1, a*x))*log(a^2*x^2 + 1)^
2)/(a^2*x^6 + x^4), x) - 4*arctan2(1, a*x)^3 + 3*arctan2(1, a*x)*log(a^2*x^
2 + 1)^2)/x^3
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^4}\,dx$$

```
[In] integrate(arccot(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\mathrm{acot}(a\,x)^3}{x^4}\,dx$$

```
[In] int(acot(a*x)^3/x^4,x)
```

```
[Out] int(acot(a*x)^3/x^4, x)
```

3.33 $\int \frac{\cot^{-1}(ax)^3}{x^5} dx$

Optimal result

Integrand size = 10, antiderivative size = 152

$$\int \frac{\cot^{-1}(ax)^3}{x^5} dx = \frac{a^3}{4x} - \frac{a^2\cot^{-1}(ax)}{4x^2} - ia^4\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{3a^3\cot^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{4x^4} + \frac{1}{4}a^4\arctan(ax) - 2a^4\cot^{-1}(ax)\log\left(2 - \frac{2}{1-iax}\right) - ia^4\operatorname{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right)$$

```
[Out] 1/4*a^3/x-1/4*a^2*arccot(a*x)/x^2-I*a^4*arccot(a*x)^2+1/4*a*arccot(a*x)^2/x
^3-3/4*a^3*arccot(a*x)^2/x+1/4*a^4*arccot(a*x)^3-1/4*arccot(a*x)^3/x^4+1/4*
a^4*arctan(a*x)-2*a^4*arccot(a*x)*ln(2-2/(1-I*a*x))-I*a^4*polylog(2,-1+2/(1
-I*a*x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4947, 5039, 331, 209, 5045, 4989, 2497, 5005}

$$\int \frac{\cot^{-1}(ax)^3}{x^5} dx = \frac{1}{4}a^4\arctan(ax) - ia^4\operatorname{PolyLog}\left(2, \frac{2}{1-iax} - 1\right) + \frac{1}{4}a^4\cot^{-1}(ax)^3 - ia^4\cot^{-1}(ax)^2 - 2a^4\log\left(2 - \frac{2}{1-iax}\right)\cot^{-1}(ax) + \frac{a^3}{4x} - \frac{3a^3\cot^{-1}(ax)^2}{4x} - \frac{a^2\cot^{-1}(ax)}{4x^2} - \frac{\cot^{-1}(ax)^3}{4x^4} + \frac{a\cot^{-1}(ax)^2}{4x^3}$$

```
[In] Int[ArcCot[a*x]^3/x^5,x]
```

```
[Out] a^3/(4*x) - (a^2*ArcCot[a*x])/(4*x^2) - I*a^4*ArcCot[a*x]^2 + (a*ArcCot[a*x
]^2)/(4*x^3) - (3*a^3*ArcCot[a*x]^2)/(4*x) + (a^4*ArcCot[a*x]^3)/4 - ArcCot
[a*x]^3/(4*x^4) + (a^4*ArcTan[a*x])/4 - 2*a^4*ArcCot[a*x]*Log[2 - 2/(1 - I*
a*x)] - I*a^4*PolyLog[2, -1 + 2/(1 - I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax)^3}{4x^4} - \frac{1}{4}(3a)\int \frac{\cot^{-1}(ax)^2}{x^4\left(1+a^2x^2\right)}\,dx \\
&= -\frac{\cot^{-1}(ax)^3}{4x^4} - \frac{1}{4}(3a)\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx + \frac{1}{4}\left(3a^3\right)\int \frac{\cot^{-1}(ax)^2}{x^2\left(1+a^2x^2\right)}\,dx \\
&= \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{\cot^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\cot^{-1}(ax)}{x^3\left(1+a^2x^2\right)}\,dx \\
&\quad + \frac{1}{4}\left(3a^3\right)\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx - \frac{1}{4}\left(3a^5\right)\int \frac{\cot^{-1}(ax)^2}{1+a^2x^2}\,dx \\
&= \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{3a^3\cot^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{4x^4} \\
&\quad + \frac{1}{2}a^2\int \frac{\cot^{-1}(ax)}{x^3}\,dx - \frac{1}{2}a^4\int \frac{\cot^{-1}(ax)}{x\left(1+a^2x^2\right)}\,dx - \frac{1}{2}\left(3a^4\right)\int \frac{\cot^{-1}(ax)}{x\left(1+a^2x^2\right)}\,dx \\
&= -\frac{a^2\cot^{-1}(ax)}{4x^2} - ia^4\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{3a^3\cot^{-1}(ax)^2}{4x} \\
&\quad + \frac{1}{4}a^4\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{4x^4} - \frac{1}{4}a^3\int \frac{1}{x^2\left(1+a^2x^2\right)}\,dx \\
&\quad - \frac{1}{2}\left(ia^4\right)\int \frac{\cot^{-1}(ax)}{x(i+ax)}\,dx - \frac{1}{2}\left(3ia^4\right)\int \frac{\cot^{-1}(ax)}{x(i+ax)}\,dx \\
&= \frac{a^3}{4x} - \frac{a^2\cot^{-1}(ax)}{4x^2} - ia^4\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{3a^3\cot^{-1}(ax)^2}{4x} \\
&\quad + \frac{1}{4}a^4\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{4x^4} - 2a^4\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) \\
&\quad + \frac{1}{4}a^5\int \frac{1}{1+a^2x^2}\,dx - \frac{1}{2}a^5\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx - \frac{1}{2}\left(3a^5\right)\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{1+a^2x^2}\,dx
\end{aligned}
$$

$$= \frac{a^3}{4x} - \frac{a^2\cot^{-1}(ax)}{4x^2} - ia^4\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{3a^3\cot^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{4x^4} + \frac{1}{4}a^4\arctan(ax) - 2a^4\cot^{-1}(ax)\log\left(2 - \frac{2}{1-iax}\right) - ia^4\,\text{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right)$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx = \frac{a^3x^3 + (ax - 3a^3x^3 + 4ia^4x^4)\cot^{-1}(ax)^2 + (-1 + a^4x^4)\cot^{-1}(ax)^3 - a^2x^2\cot^{-1}(ax)\left(1 + a^2x^2 + 8a^2x^2\,\mathrm{l}\right.}{4x^4}$$

```
[In] Integrate[ArcCot[a*x]^3/x^5,x]
```

```
[Out] (a^3*x^3 + (a*x - 3*a^3*x^3 + (4*I)*a^4*x^4)*ArcCot[a*x]^2 + (-1 + a^4*x^4)
*ArcCot[a*x]^3 - a^2*x^2*ArcCot[a*x]*(1 + a^2*x^2 + 8*a^2*x^2*Log[1 + E^((2
*I)*ArcCot[a*x])]) + (4*I)*a^4*x^4*PolyLog[2, -E^((2*I)*ArcCot[a*x])])/(4*x
^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.74 (sec) , antiderivative size = 852, normalized size of antiderivative = 5.61

method	result	size
parts	Expression too large to display	852
derivativedivides	Expression too large to display	855
default	Expression too large to display	855

```
[In] int(arccot(a*x)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arccot(a*x)^3/x^4-3/4*a^4*(-1/3/a^3/x^3*arccot(a*x)^2+1/x*arccot(a*x)^
2/a+arccot(a*x)^2*arctan(a*x)-1/3*arccot(a*x)^3-1/2*Pi*arccot(a*x)^2+8/3*ar
ccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+8/3*arccot(a*x)*ln(1-I*(I+a*x)/
(a^2*x^2+1)^(1/2))-1/3*I/a/x*(a*x-I)-1/4*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2
))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^2+1/4*Pi*csgn(I/((I+a*x)^2/(
a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*ar
ccot(a*x)^2+1/2*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*
x^2+1))^2*arccot(a*x)^2+1/4*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)
^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2-1/4*Pi*csgn(I*(I+
```

```
a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*arccot(a*x)^2-I*arccot(a*x)
^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-1/2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^3
*arccot(a*x)^2-1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a
^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))*arccot(a
*x)^2+1/2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2-1/4*Pi*csgn(
I*(I+a*x)^2/(a^2*x^2+1))^3*arccot(a*x)^2-8/3*I*dilog(1+I*(I+a*x)/(a^2*x^2+1
)^(1/2))-2/3*arccot(a*x)*(I+a*x)/a/x+1/3*arccot(a*x)*(I+a*x)^2/a^2/x^2+2/3*
arccot(a*x)*(I+a*x)*(a*x-I)/a^2/x^2-8/3*I*dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/
2))-4/3*I*arccot(a*x)^2)
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^5}\,dx$$

```
[In] integrate(arccot(a*x)^3/x^5,x, algorithm="fricas")
[Out] integral(arccot(a*x)^3/x^5, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x^5}\,dx$$

```
[In] integrate(acot(a*x)**3/x**5,x)
[Out] Integral(acot(a*x)**3/x**5, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(arccot(a*x)^3/x^5,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5} \, dx = \int \frac{\operatorname{arccot}(ax)^3}{x^5} \, dx$$

[In] integrate(arccot(a*x)^3/x^5,x, algorithm="giac")

[Out] integrate(arccot(a*x)^3/x^5, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^5} \, dx = \int \frac{\operatorname{acot}(a\,x)^3}{x^5} \, dx$$

[In] int(acot(a*x)^3/x^5,x)

[Out] int(acot(a*x)^3/x^5, x)

3.34 $\int x^m \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \cot^{-1}(ax)^3 \, dx = \text{Int}\left(x^m \cot^{-1}(ax)^3, x\right)$$

```
[Out] Unintegrable(x^m*arccot(a*x)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \cot^{-1}(ax)^3 \, dx$$

```
[In] Int[x^m*ArcCot[a*x]^3,x]
[Out] Defer[Int][x^m*ArcCot[a*x]^3, x]
```

Rubi steps

$$\text{integral} = \int x^m \cot^{-1}(ax)^3 \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \cot^{-1}(ax)^3 \, dx$$

```
[In] Integrate[x^m*ArcCot[a*x]^3,x]
[Out] Integrate[x^m*ArcCot[a*x]^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.77 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arccot}(ax)^3 \, dx$$

```
[In] int(x^m*arccot(a*x)^3,x)
[Out] int(x^m*arccot(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{arccot}(ax)^3 \, dx$$

```
[In] integrate(x^m*arccot(a*x)^3,x, algorithm="fricas")
[Out] integral(x^m*arccot(a*x)^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{acot}^3(ax) \, dx$$

```
[In] integrate(x**m*acot(a*x)**3,x)
[Out] Integral(x**m*acot(a*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 22.10

$$\int x^m \cot^{-1}(ax)^3\,dx = \int x^m \operatorname{arccot}(ax)^3\,dx$$

```
[In] integrate(x^m*arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*x*x^m*arctan2(1, a*x)^3 - 3*x*x^m*arctan2(1, a*x)*log(a^2*x^2 + 1)^
2 + 32*(m + 1)*integrate(1/32*(12*a^2*x^2*x^m*arctan2(1, a*x)*log(a^2*x^2 +
 1) + 3*((a^2*m*arctan2(1, a*x) + a^2*arctan2(1, a*x))*x^2 - a*x + m*arctan
2(1, a*x) + arctan2(1, a*x))*x^m*log(a^2*x^2 + 1)^2 + 4*(3*a*x*arctan2(1, a
*x)^2 + 7*m*arctan2(1, a*x)^3 + 7*(a^2*m*arctan2(1, a*x)^3 + a^2*arctan2(1,
 a*x)^3)*x^2 + 7*arctan2(1, a*x)^3)*x^m)/((a^2*m + a^2)*x^2 + m + 1), x))/(
m + 1)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3\,dx = \int x^m \operatorname{arccot}(ax)^3\,dx$$

```
[In] integrate(x^m*arccot(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccot(a*x)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3\,dx = \int x^m \operatorname{acot}(a\,x)^3\,dx$$

```
[In] int(x^m*acot(a*x)^3,x)
```

```
[Out] int(x^m*acot(a*x)^3, x)
```

3.35 $\int x^m \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \cot^{-1}(ax)^2 \, dx = \operatorname{Int}\left(x^m \cot^{-1}(ax)^2, x\right)$$

```
[Out] Unintegrable(x^m*arccot(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \cot^{-1}(ax)^2 \, dx$$

```
[In] Int[x^m*ArcCot[a*x]^2,x]
[Out] Defer[Int][x^m*ArcCot[a*x]^2, x]
```

Rubi steps

$$\text{integral} = \int x^m \cot^{-1}(ax)^2 \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \cot^{-1}(ax)^2 \, dx$$

```
[In] Integrate[x^m*ArcCot[a*x]^2,x]
[Out] Integrate[x^m*ArcCot[a*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arccot}(ax)^2 \, dx$$

```
[In] int(x^m*arccot(a*x)^2,x)
[Out] int(x^m*arccot(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^m*arccot(a*x)^2,x, algorithm="fricas")
[Out] integral(x^m*arccot(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{acot}^2(ax) \, dx$$

```
[In] integrate(x**m*acot(a*x)**2,x)
[Out] Integral(x**m*acot(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 183, normalized size of antiderivative = 18.30

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^m*arccot(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*x*x^m*arctan2(1, a*x)^2 - x*x^m*log(a^2*x^2 + 1)^2 + 16*(m + 1)*int
egrate(1/16*(4*a^2*x^2*x^m*log(a^2*x^2 + 1) + ((a^2*m + a^2)*x^2 + m + 1)*x
^m*log(a^2*x^2 + 1)^2 + 4*(3*(a^2*m*arctan2(1, a*x)^2 + a^2*arctan2(1, a*x)
^2)*x^2 + 2*a*x*arctan2(1, a*x) + 3*m*arctan2(1, a*x)^2 + 3*arctan2(1, a*x)
^2)*x^m)/((a^2*m + a^2)*x^2 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{arccot}(ax)^2 \, dx$$

```
[In] integrate(x^m*arccot(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccot(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{acot}(a\,x)^2 \, dx$$

```
[In] int(x^m*acot(a*x)^2,x)
```

```
[Out] int(x^m*acot(a*x)^2, x)
```

3.36 $\int x^m \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int x^m \cot^{-1}(ax)\,dx = \frac{x^{1+m}\cot^{-1}(ax)}{1+m} + \frac{ax^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}$$

```
[Out] x^(1+m)*arccot(a*x)/(1+m)+a*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-a^2*x
^2)/(m^2+3*m+2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4947, 371}

$$\int x^m \cot^{-1}(ax)\,dx = \frac{ax^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2} + \frac{x^{m+1}\cot^{-1}(ax)}{m+1}$$

```
[In] Int[x^m*ArcCot[a*x],x]
```

```
[Out] (x^(1 + m)*ArcCot[a*x])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[1, (2 + m)
/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{1+m}\cot^{-1}(ax)}{1+m}+\frac{a\int \frac{x^{1+m}}{1+a^2x^2}\,dx}{1+m}\\ &= \frac{x^{1+m}\cot^{-1}(ax)}{1+m}+\frac{ax^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\begin{aligned}&\int x^m\cot^{-1}(ax)\,dx\\ &= \frac{x^{1+m}\left((2+m)\cot^{-1}(ax)+ax\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},-a^2x^2\right)\right)}{(1+m)(2+m)}\end{aligned}$$

```
[In] Integrate[x^m*ArcCot[a*x],x]
[Out] (x^(1 + m)*((2 + m)*ArcCot[a*x] + a*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2
, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

Maple [F]

$$\int x^m\,\text{arccot}\,(ax)\,dx$$

```
[In] int(x^m*arccot(a*x),x)
[Out] int(x^m*arccot(a*x),x)
```

Fricas [F]

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{arccot}(ax)\,dx$$

```
[In] integrate(x^m*arccot(a*x),x, algorithm="fricas")
[Out] integral(x^m*arccot(a*x), x)
```

Sympy [F]

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{acot}(ax)\,dx$$

```
[In] integrate(x**m*acot(a*x),x)
[Out] Integral(x**m*acot(a*x), x)
```

Maxima [F]

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{arccot}(ax)\,dx$$

```
[In] integrate(x^m*arccot(a*x),x, algorithm="maxima")
[Out] (x*x^m*arctan2(1, a*x) + (a*m + a)*integrate(x*x^m/((a^2*m + a^2)*x^2 + m +
 1), x))/(m + 1)
```

Giac [F]

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{arccot}(ax)\,dx$$

```
[In] integrate(x^m*arccot(a*x),x, algorithm="giac")
[Out] integrate(x^m*arccot(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m\,\mathrm{acot}(a\,x)\;dx$$

```
[In] int(x^m*acot(a*x),x)
[Out] int(x^m*acot(a*x), x)
```

3.37 $\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx = \frac{x^2}{6} - x \cot^{-1}(x) + \frac{1}{3} x^3 \cot^{-1}(x) - \frac{1}{2} \cot^{-1}(x)^2 - \frac{2}{3} \log \left(1+x^2\right)$$

```
[Out] 1/6*x^2-x*arccot(x)+1/3*x^3*arccot(x)-1/2*arccot(x)^2-2/3*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5037, 4947, 272, 45, 4931, 266, 5005}

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx = \frac{1}{3} x^3 \cot^{-1}(x) + \frac{x^2}{6} - \frac{2}{3} \log \left(x^2+1\right) - x \cot^{-1}(x) - \frac{1}{2} \cot^{-1}(x)^2$$

```
[In] Int[(x^4*ArcCot[x])/(1 + x^2),x]
```

```
[Out] x^2/6 - x*ArcCot[x] + (x^3*ArcCot[x])/3 - ArcCot[x]^2/2 - (2*Log[1 + x^2])/
3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2\cot^{-1}(x)\,dx-\int\frac{x^2\cot^{-1}(x)}{1+x^2}\,dx\\ &=\frac{1}{3}x^3\cot^{-1}(x)+\frac{1}{3}\int\frac{x^3}{1+x^2}\,dx-\int\cot^{-1}(x)\,dx+\int\frac{\cot^{-1}(x)}{1+x^2}\,dx\\ &=-x\cot^{-1}(x)+\frac{1}{3}x^3\cot^{-1}(x)-\frac{1}{2}\cot^{-1}(x)^2+\frac{1}{6}\text{Subst}\left(\int\frac{x}{1+x}\,dx,x,x^2\right)-\int\frac{x}{1+x^2}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -x\cot^{-1}(x) + \frac{1}{3}x^3\cot^{-1}(x) - \frac{1}{2}\cot^{-1}(x)^2 \\
&\quad - \frac{1}{2}\log\left(1+x^2\right) + \frac{1}{6}\text{Subst}\left(\int\left(1+\frac{1}{-1-x}\right)dx, x, x^2\right) \\
&= \frac{x^2}{6} - x\cot^{-1}(x) + \frac{1}{3}x^3\cot^{-1}(x) - \frac{1}{2}\cot^{-1}(x)^2 - \frac{2}{3}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$
\int \frac{x^4\cot^{-1}(x)}{1+x^2}\,dx = \frac{1}{6}\left(x^2 + 2x\left(-3+x^2\right)\cot^{-1}(x) - 3\cot^{-1}(x)^2 - 4\log\left(1+x^2\right)\right)
$$

```
[In] Integrate[(x^4*ArcCot[x])/(1 + x^2),x]
[Out] (x^2 + 2*x*(-3 + x^2)*ArcCot[x] - 3*ArcCot[x]^2 - 4*Log[1 + x^2])/6
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{x^3\,\mathrm{arccot}(x)}{3}+\frac{x^2}{6}-x\,\mathrm{arccot}\left(x\right)-\frac{\mathrm{arccot}(x)^2}{2}-\frac{2\ln\left(x^2+1\right)}{3}-\frac{1}{3}$
default	$\frac{x^3\,\mathrm{arccot}(x)}{3}-x\,\mathrm{arccot}\left(x\right)+\mathrm{arccot}\left(x\right)\arctan\left(x\right)+\frac{x^2}{6}-\frac{2\ln\left(x^2+1\right)}{3}+\frac{\arctan(x)^2}{2}$
parts	$\frac{x^3\,\mathrm{arccot}(x)}{3}-x\,\mathrm{arccot}\left(x\right)+\mathrm{arccot}\left(x\right)\arctan\left(x\right)+\frac{x^2}{6}-\frac{2\ln\left(x^2+1\right)}{3}+\frac{\arctan(x)^2}{2}$
risch	$\frac{\ln(ix+1)^2}{8}+\left(\frac{ix^3}{6}-\frac{ix}{2}-\frac{\ln(-ix+1)}{4}\right)\ln\left(ix+1\right)+\frac{\ln(-ix+1)^2}{8}-\frac{ix^3\ln(-ix+1)}{6}+\frac{i\ln(-ix+1)x}{2}+\frac{\pi x^3}{6}-\pi$

```
[In] int(x^4*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arccot(x)+1/6*x^2-x*arccot(x)-1/2*arccot(x)^2-2/3*ln(x^2+1)-1/3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2}\,dx = \frac{1}{6}\,x^2 + \frac{1}{3}\,\left(x^3 - 3\,x\right)\operatorname{arccot}\left(x\right) - \frac{1}{2}\,\operatorname{arccot}\left(x\right)^2 - \frac{2}{3}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^4*arccot(x)/(x^2+1),x, algorithm="fricas")
[Out] 1/6*x^2 + 1/3*(x^3 - 3*x)*arccot(x) - 1/2*arccot(x)^2 - 2/3*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2}\,dx = \frac{x^3 \operatorname{acot}\left(x\right)}{3} + \frac{x^2}{6} - x \operatorname{acot}\left(x\right) - \frac{2\log\left(x^2+1\right)}{3} - \frac{\operatorname{acot}^2\left(x\right)}{2}$$

```
[In] integrate(x**4*acot(x)/(x**2+1),x)
[Out] x**3*acot(x)/3 + x**2/6 - x*acot(x) - 2*log(x**2 + 1)/3 - acot(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2}\,dx = \frac{1}{6}\,x^2 + \frac{1}{3}\,\left(x^3 - 3\,x + 3\,\arctan\left(x\right)\right)\operatorname{arccot}\left(x\right) + \frac{1}{2}\,\arctan\left(x\right)^2 - \frac{2}{3}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^4*arccot(x)/(x^2+1),x, algorithm="maxima")
[Out] 1/6*x^2 + 1/3*(x^3 - 3*x + 3*arctan(x))*arccot(x) + 1/2*arctan(x)^2 - 2/3*l
og(x^2 + 1)
```

Giac [F]

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x^4 \operatorname{arccot}(x)}{x^2+1}\,dx$$

```
[In] integrate(x^4*arccot(x)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccot(x)/(x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2}\,dx = \frac{x^3\,\mathrm{acot}(x)}{3} - \frac{2\,\ln\left(x^2+1\right)}{3} - \frac{\mathrm{acot}(x)^2}{2} - x\,\mathrm{acot}(x) + \frac{x^2}{6}$$

```
[In] int((x^4*acot(x))/(x^2 + 1),x)
```

```
[Out] (x^3*acot(x))/3 - (2*log(x^2 + 1))/3 - acot(x)^2/2 - x*acot(x) + x^2/6
```

3.38 $\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx = \frac{x}{2} + \frac{1}{2} x^2 \cot^{-1}(x) - \frac{1}{2} i \cot^{-1}(x)^2 - \frac{\arctan(x)}{2} + \cot^{-1}(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \,\mathrm{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right)$$

```
[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*I*arccot(x)^2-1/2*arctan(x)+arccot(x)*ln(2/(1+I
*x))-1/2*I*polylog(2,1-2/(1+I*x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {5037, 4947, 327, 209, 5041, 4965, 2449, 2352}

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx = -\frac{\arctan(x)}{2} - \frac{1}{2} i \,\mathrm{PolyLog}\left(2, 1 - \frac{2}{ix+1}\right) + \frac{1}{2} x^2 \cot^{-1}(x) + \frac{x}{2} - \frac{1}{2} i \cot^{-1}(x)^2 + \log\left(\frac{2}{1+ix}\right) \cot^{-1}(x)$$

```
[In] Int[(x^3*ArcCot[x])/(1 + x^2),x]
```

```
[Out] x/2 + (x^2*ArcCot[x])/2 - (I/2)*ArcCot[x]^2 - ArcTan[x]/2 + ArcCot[x]*Log[2
/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
```

```
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\cot^{-1}(x)\,dx - \int \frac{x\cot^{-1}(x)}{1+x^2}\,dx \\
&= \frac{1}{2}x^2\cot^{-1}(x) - \frac{1}{2}i\cot^{-1}(x)^2 + \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx + \int \frac{\cot^{-1}(x)}{i-x}\,dx \\
&= \frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}i\cot^{-1}(x)^2+\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx \\
&= \frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}i\cot^{-1}(x)^2-\frac{\arctan(x)}{2} \\
&\quad +\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)-i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+ix}\right) \\
&= \frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}i\cot^{-1}(x)^2-\frac{\arctan(x)}{2} \\
&\quad +\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{x^3\cot^{-1}(x)}{1+x^2}\,dx = \frac{1}{2}\Big(x - i\cot^{-1}(x)^2 + \cot^{-1}(x)\left(1+x^2+2\log\left(1-e^{2i\cot^{-1}(x)}\right)\right) - i\,\text{PolyLog}\left(2,e^{2i\cot^{-1}(x)}\right)\Big)$$

```
[In] Integrate[(x^3*ArcCot[x])/(1 + x^2),x]
[Out] (x - I*ArcCot[x]^2 + ArcCot[x]*(1 + x^2 + 2*Log[1 - E^((2*I)*ArcCot[x])]) -
 I*PolyLog[2, E^((2*I)*ArcCot[x])])/2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(53) = 106$.

Time = 0.74 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

method	result
default	$\frac{x^2 \operatorname{arccot}(x)}{2}-\frac{\operatorname{arccot}(x)\ln(x^2+1)}{2}+\frac{x}{2}-\frac{\arctan(x)}{2}+\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(i+x)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right)\right)}{4}$
parts	$\frac{x^2 \operatorname{arccot}(x)}{2}-\frac{\operatorname{arccot}(x)\ln(x^2+1)}{2}+\frac{x}{2}-\frac{\arctan(x)}{2}+\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(i+x)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right)\right)}{4}$
risch	$\frac{\pi x^2}{4}+\frac{\pi}{4}-\frac{\pi\ln(x^2+1)}{4}-\frac{i\ln(-ix+1)x^2}{4}+\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4}-\frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}+\frac{x}{2}+\frac{i\operatorname{dilog}\left(\frac{1}{2}+\frac{ix}{2}\right)}{4}+\frac{i\ln(-ix+1)}{8}$

```
[In] int(x^3*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arccot(x)-1/2*arccot(x)*ln(x^2+1)+1/2*x-1/2*arctan(x)+1/4*I*(ln(x-I
)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(I+x))-ln(x-I)*ln(-1/2*I*(I+x)))-1/4
*I*(ln(I+x)*ln(x^2+1)-1/2*ln(I+x)^2-dilog(1/2*I*(x-I))-ln(I+x)*ln(1/2*I*(x-
I)))
```

Fricas [F]

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x^3 \operatorname{arccot}(x)}{x^2+1}\,dx$$

```
[In] integrate(x^3*arccot(x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(x^3*arccot(x)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x^3 \operatorname{acot}(x)}{x^2+1}\,dx$$

```
[In] integrate(x**3*acot(x)/(x**2+1),x)
```

```
[Out] Integral(x**3*acot(x)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x^3 \operatorname{arccot}(x)}{x^2+1} \, dx$$

```
[In] integrate(x^3*arccot(x)/(x^2+1),x, algorithm="maxima")
[Out] integrate(x^3*arccot(x)/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x^3 \operatorname{arccot}(x)}{x^2+1} \, dx$$

```
[In] integrate(x^3*arccot(x)/(x^2+1),x, algorithm="giac")
[Out] integrate(x^3*arccot(x)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x^3 \operatorname{acot}(x)}{x^2+1} \, dx$$

```
[In] int((x^3*acot(x))/(x^2 + 1),x)
[Out] int((x^3*acot(x))/(x^2 + 1), x)
```

3.39 $\int \frac{x^2 \cot^{-1}(x)}{1+x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} dx = x \cot^{-1}(x) + \frac{1}{2} \cot^{-1}(x)^2 + \frac{1}{2} \log \left(1+x^2\right)$$

```
[Out] x*arccot(x)+1/2*arccot(x)^2+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5037, 4931, 266, 5005}

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} dx = \frac{1}{2} \log \left(x^2+1\right) + \frac{1}{2} \cot^{-1}(x)^2 + x \cot^{-1}(x)$$

```
[In] Int[(x^2*ArcCot[x])/(1 + x^2),x]
```

```
[Out] x*ArcCot[x] + ArcCot[x]^2/2 + Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \cot^{-1}(x)\,dx - \int \frac{\cot^{-1}(x)}{1+x^2}\,dx\\ &= x\cot^{-1}(x) + \frac{1}{2}\cot^{-1}(x)^2 + \int \frac{x}{1+x^2}\,dx\\ &= x\cot^{-1}(x) + \frac{1}{2}\cot^{-1}(x)^2 + \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2\cot^{-1}(x)}{1+x^2}\,dx = x\cot^{-1}(x) + \frac{1}{2}\cot^{-1}(x)^2 + \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(x^2*ArcCot[x])/(1 + x^2),x]
[Out] x*ArcCot[x] + ArcCot[x]^2/2 + Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
parallelrisch	$x\,\mathrm{arccot}\left(x\right)+\frac{\mathrm{arccot}\left(x\right)^{2}}{2}+\frac{\ln\left(x^{2}+1\right)}{2}$
default	$-\mathrm{arccot}\left(x\right)\arctan\left(x\right)+x\,\mathrm{arccot}\left(x\right)+\frac{\ln\left(x^{2}+1\right)}{2}-\frac{\arctan\left(x\right)^{2}}{2}$
parts	$-\mathrm{arccot}\left(x\right)\arctan\left(x\right)+x\,\mathrm{arccot}\left(x\right)+\frac{\ln\left(x^{2}+1\right)}{2}-\frac{\arctan\left(x\right)^{2}}{2}$
risch	$-\frac{\ln\left(ix+1\right)^{2}}{8}+\left(\frac{ix}{2}+\frac{\ln\left(-ix+1\right)}{4}\right)\ln\left(ix+1\right)-\frac{\ln\left(-ix+1\right)^{2}}{8}-\frac{i\ln\left(-ix+1\right)x}{2}+\frac{\pi x}{2}+\frac{\ln\left(x^{2}+1\right)}{2}-\frac{\pi\arctan\left(x\right)}{2}$

```
[In] int(x^2*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] x*arccot(x)+1/2*arccot(x)^2+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int\frac{x^{2}\cot^{-1}(x)}{1+x^{2}}\,dx=x\,\mathrm{arccot}\left(x\right)+\frac{1}{2}\,\mathrm{arccot}\left(x\right)^{2}+\frac{1}{2}\,\log\left(x^{2}+1\right)$$

```
[In] integrate(x^2*arccot(x)/(x^2+1),x, algorithm="fricas")
[Out] x*arccot(x) + 1/2*arccot(x)^2 + 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int\frac{x^{2}\cot^{-1}(x)}{1+x^{2}}\,dx=x\,\mathrm{acot}\left(x\right)+\frac{\log\left(x^{2}+1\right)}{2}+\frac{\mathrm{acot}^{2}\left(x\right)}{2}$$

```
[In] integrate(x**2*acot(x)/(x**2+1),x)
[Out] x*acot(x) + log(x**2 + 1)/2 + acot(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} dx = (x - \arctan(x)) \operatorname{arccot}(x) - \frac{1}{2} \arctan(x)^2 + \frac{1}{2} \log(x^2+1)$$

```
[In] integrate(x^2*arccot(x)/(x^2+1),x, algorithm="maxima")
[Out] (x - arctan(x))*arccot(x) - 1/2*arctan(x)^2 + 1/2*log(x^2 + 1)
```

Giac [F]

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} dx = \int \frac{x^2 \operatorname{arccot}(x)}{x^2+1} dx$$

```
[In] integrate(x^2*arccot(x)/(x^2+1),x, algorithm="giac")
[Out] integrate(x^2*arccot(x)/(x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} dx = \frac{\operatorname{acot}(x)^2}{2} + x \operatorname{acot}(x) + \frac{\ln(x^2+1)}{2}$$

```
[In] int((x^2*acot(x))/(x^2 + 1),x)
[Out] log(x^2 + 1)/2 + acot(x)^2/2 + x*acot(x)
```

3.40 $\int \frac{x \cot^{-1}(x)}{1+x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{x \cot^{-1}(x)}{1+x^2} \, dx = \frac{1}{2} i \cot^{-1}(x)^2 - \cot^{-1}(x) \log \left(\frac{2}{1+ix}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right)$$

```
[Out] 1/2*I*arccot(x)^2-arccot(x)*ln(2/(1+I*x))+1/2*I*polylog(2,1-2/(1+I*x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5041, 4965, 2449, 2352}

$$\int \frac{x \cot^{-1}(x)}{1+x^2} \, dx = \frac{1}{2} i \operatorname{PolyLog}\left(2, 1 - \frac{2}{ix+1}\right) + \frac{1}{2} i \cot^{-1}(x)^2 - \log \left(\frac{2}{1+ix}\right) \cot^{-1}(x)$$

```
[In] Int[(x*ArcCot[x])/(1 + x^2),x]
```

```
[Out] (I/2)*ArcCot[x]^2 - ArcCot[x]*Log[2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1
 + I*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2} i \cot^{-1}(x)^2 - \int \frac{\cot^{-1}(x)}{i-x}\,dx \\
&= \frac{1}{2} i \cot^{-1}(x)^2 - \cot^{-1}(x)\log\left(\frac{2}{1+ix}\right) - \int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx \\
&= \frac{1}{2} i \cot^{-1}(x)^2 - \cot^{-1}(x)\log\left(\frac{2}{1+ix}\right) + i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+ix}\right) \\
&= \frac{1}{2} i \cot^{-1}(x)^2 - \cot^{-1}(x)\log\left(\frac{2}{1+ix}\right) + \frac{1}{2} i \,\text{PolyLog}\left(2, 1-\frac{2}{1+ix}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx = -\cot^{-1}(x)\log\left(1-e^{2i\cot^{-1}(x)}\right) + \frac{1}{2} i\left(\cot^{-1}(x)^2 + \text{PolyLog}\left(2, e^{2i\cot^{-1}(x)}\right)\right)$$

```
[In] Integrate[(x*ArcCot[x])/(1 + x^2),x]
[Out] -(ArcCot[x]*Log[1 - E^((2*I)*ArcCot[x])]) + (I/2)*(ArcCot[x]^2 + PolyLog[2,
 E^((2*I)*ArcCot[x])])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(40) = 80$.

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.04

method	result
risch	$\frac{\pi \ln\left(-2+(-ix+1)^{2}+2ix\right)}{4}-\frac{i\ln(-ix+1)^{2}}{8}-\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4}+\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}+\frac{i\ln(ix+1)^{2}}{8}+\frac{i\ln\left(\frac{1}{2}-\frac{ix}{2}\right)\ln(ix+1)}{4}$
default	$\frac{\mathrm{arccot}(x)\ln(x^{2}+1)}{2}-\frac{i\left(\ln(x-i)\ln(x^{2}+1)-\frac{\ln(x-i)^{2}}{2}-\mathrm{dilog}\left(-\frac{i(i+x)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right)\right)}{4}+\frac{i\left(\ln(i+x)\ln(x^{2}+1)-\frac{\ln(i+x)^{2}}{2}\right.}{}$
parts	$\frac{\mathrm{arccot}(x)\ln(x^{2}+1)}{2}-\frac{i\left(\ln(x-i)\ln(x^{2}+1)-\frac{\ln(x-i)^{2}}{2}-\mathrm{dilog}\left(-\frac{i(i+x)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right)\right)}{4}+\frac{i\left(\ln(i+x)\ln(x^{2}+1)-\frac{\ln(i+x)^{2}}{2}\right.}{}$

```
[In] int(x*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/4*Pi*ln(-2+(1-I*x)^2+2*I*x)-1/8*I*ln(1-I*x)^2-1/4*I*ln(1/2+1/2*I*x)*ln(1-
I*x)+1/4*I*dilog(1/2-1/2*I*x)+1/8*I*ln(1+I*x)^2+1/4*I*ln(1/2-1/2*I*x)*ln(1+
I*x)-1/4*I*dilog(1/2+1/2*I*x)
```

Fricas [F]

$$\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x\,\mathrm{arccot}\,(x)}{x^2+1}\,dx$$

```
[In] integrate(x*arccot(x)/(x^2+1),x, algorithm="fricas")
[Out] integral(x*arccot(x)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x\,\mathrm{acot}\,(x)}{x^2+1}\,dx$$

```
[In] integrate(x*acot(x)/(x**2+1),x)
[Out] Integral(x*acot(x)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x \operatorname{arccot}(x)}{x^2+1} \, dx$$

```
[In] integrate(x*arccot(x)/(x^2+1),x, algorithm="maxima")
[Out] integrate(x*arccot(x)/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{x \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x \operatorname{arccot}(x)}{x^2+1} \, dx$$

```
[In] integrate(x*arccot(x)/(x^2+1),x, algorithm="giac")
[Out] integrate(x*arccot(x)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x \operatorname{acot}(x)}{x^2+1} \, dx$$

```
[In] int((x*acot(x))/(x^2 + 1),x)
[Out] int((x*acot(x))/(x^2 + 1), x)
```

3.41 $\int \frac{\cot^{-1}(x)}{1+x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{\cot^{-1}(x)}{1+x^2} \, dx = -\frac{1}{2}\cot^{-1}(x)^2$$

```
[Out] -1/2*arccot(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5005}

$$\int \frac{\cot^{-1}(x)}{1+x^2} \, dx = -\frac{1}{2}\cot^{-1}(x)^2$$

```
[In] Int[ArcCot[x]/(1 + x^2),x]
```

```
[Out] -1/2*ArcCot[x]^2
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}\cot^{-1}(x)^2$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{1}{2}\cot^{-1}(x)^2$$

```
[In] Integrate[ArcCot[x]/(1 + x^2),x]
[Out] -1/2*ArcCot[x]^2
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\operatorname{arccot}(x)^2}{2}$	7
default	$-\frac{\operatorname{arccot}(x)^2}{2}$	7
parts	$\operatorname{arccot}(x)\arctan(x)+\frac{\arctan(x)^2}{2}$	13
risch	$\frac{\ln(ix+1)^2}{8}-\frac{\ln(-ix+1)\ln(ix+1)}{4}+\frac{\ln(-ix+1)^2}{8}+\frac{\pi\arctan(x)}{2}$	45

```
[In] int(arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*arccot(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{1}{2}\operatorname{arccot}(x)^2$$

```
[In] integrate(arccot(x)/(x^2+1),x, algorithm="fricas")
[Out] -1/2*arccot(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{\operatorname{acot}^2(x)}{2}$$

```
[In] integrate(acot(x)/(x**2+1),x)
[Out] -acot(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{1}{2}\,\operatorname{arccot}(x)^2$$

```
[In] integrate(arccot(x)/(x^2+1),x, algorithm="maxima")
[Out] -1/2*arccot(x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{1}{2}\,\arctan\left(\frac{1}{x}\right)^2$$

```
[In] integrate(arccot(x)/(x^2+1),x, algorithm="giac")
[Out] -1/2*arctan(1/x)^2
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{\operatorname{acot}(x)^2}{2}$$

```
[In] int(acot(x)/(x^2 + 1),x)
[Out] -acot(x)^2/2
```

3.42 $\int \frac{\cot^{-1}(x)}{x(1+x^2)} dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\cot^{-1}(x)}{x(1+x^2)} dx = \frac{1}{2}i\cot^{-1}(x)^2 + \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)$$

```
[Out] 1/2*I*arccot(x)^2+arccot(x)*ln(2-2/(1-I*x))+1/2*I*polylog(2,-1+2/(1-I*x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5045, 4989, 2497}

$$\int \frac{\cot^{-1}(x)}{x(1+x^2)} dx = \frac{1}{2}i\,\text{PolyLog}\left(2,\frac{2}{1-ix}-1\right) + \frac{1}{2}i\cot^{-1}(x)^2 + \log\left(2-\frac{2}{1-ix}\right)\cot^{-1}(x)$$

```
[In] Int[ArcCot[x]/(x*(1 + x^2)),x]
```

```
[Out] (I/2)*ArcCot[x]^2 + ArcCot[x]*Log[2 - 2/(1 - I*x)] + (I/2)*PolyLog[2, -1 +
2/(1 - I*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}i\cot^{-1}(x)^2 + i\int \frac{\cot^{-1}(x)}{x(i+x)}\,dx \\ &= \frac{1}{2}i\cot^{-1}(x)^2 + \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) + \int \frac{\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx \\ &= \frac{1}{2}i\cot^{-1}(x)^2 + \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(x)}{x\,(1+x^2)}\,dx = -\frac{1}{2}i\cot^{-1}(x)^2 + \cot^{-1}(x)\log\left(1+e^{2i\cot^{-1}(x)}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(x)}\right)$$

```
[In] Integrate[ArcCot[x]/(x*(1 + x^2)),x]
[Out] (-1/2*I)*ArcCot[x]^2 + ArcCot[x]*Log[1 + E^((2*I)*ArcCot[x])] - (I/2)*PolyL
og[2, -E^((2*I)*ArcCot[x])]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(41) = 82.

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.39

method	result
risch	$-\frac{\pi \ln(x^2+1)}{4}+\frac{\pi \ln(-ix)}{2}+\frac{i\ln(-ix+1)^2}{8}+\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4}-\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}+\frac{i\,\mathrm{dilog}(-ix+1)}{2}-\frac{i\ln(ix+1)^2}{8}-$
default	$\mathrm{arccot}\,(x)\ln\,(x)-\frac{\mathrm{arccot}(x)\ln(x^2+1)}{2}-\frac{i\ln(x)\ln(ix+1)}{2}+\frac{i\ln(x)\ln(-ix+1)}{2}-\frac{i\,\mathrm{dilog}(ix+1)}{2}+\frac{i\,\mathrm{dilog}(-ix+1)}{2}+$
parts	$\mathrm{arccot}\,(x)\ln\,(x)-\frac{\mathrm{arccot}(x)\ln(x^2+1)}{2}-\frac{i\ln(x)\ln(ix+1)}{2}+\frac{i\ln(x)\ln(-ix+1)}{2}-\frac{i\,\mathrm{dilog}(ix+1)}{2}+\frac{i\,\mathrm{dilog}(-ix+1)}{2}+$

[In] int(arccot(x)/x/(x^2+1),x,method=_RETURNVERBOSE)

[Out] -1/4*Pi*ln(x^2+1)+1/2*Pi*ln(-I*x)+1/8*I*ln(1-I*x)^2+1/4*I*ln(1/2+1/2*I*x)*ln(1-I*x)-1/4*I*dilog(1/2-1/2*I*x)+1/2*I*dilog(1-I*x)-1/8*I*ln(1+I*x)^2-1/4*I*ln(1/2-1/2*I*x)*ln(1+I*x)+1/4*I*dilog(1/2+1/2*I*x)-1/2*I*dilog(1+I*x)

Fricas [F]

$$\int \frac{\cot^{-1}(x)}{x\,(1+x^2)}\,dx=\int \frac{\mathrm{arccot}\,(x)}{(x^2+1)x}\,dx$$

[In] integrate(arccot(x)/x/(x^2+1),x, algorithm="fricas")

[Out] integral(arccot(x)/(x^3 + x), x)

Sympy [F]

$$\int \frac{\cot^{-1}(x)}{x\,(1+x^2)}\,dx=\int \frac{\mathrm{acot}\,(x)}{x\,(x^2+1)}\,dx$$

[In] integrate(acot(x)/x/(x**2+1),x)

[Out] Integral(acot(x)/(x*(x**2 + 1)), x)

Maxima [F]

$$\int \frac{\cot^{-1}(x)}{x\,(1+x^2)}\,dx=\int \frac{\mathrm{arccot}\,(x)}{(x^2+1)x}\,dx$$

[In] integrate(arccot(x)/x/(x^2+1),x, algorithm="maxima")

[Out] integrate(arccot(x)/((x^2 + 1)*x), x)

Giac [F]

$$\int \frac{\cot^{-1}(x)}{x\,(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)x}\,dx$$

```
[In] integrate(arccot(x)/x/(x^2+1),x, algorithm="giac")
[Out] integrate(arccot(x)/((x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{x\,(1+x^2)}\,dx = \int \frac{\operatorname{acot}(x)}{x\,(x^2+1)}\,dx$$

```
[In] int(acot(x)/(x*(x^2 + 1)),x)
[Out] int(acot(x)/(x*(x^2 + 1)), x)
```

3.43 $\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx = -\frac{\cot^{-1}(x)}{x}+\frac{1}{2}\cot^{-1}(x)^2-\log(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -arccot(x)/x+1/2*arccot(x)^2-ln(x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5039, 4947, 272, 36, 29, 31, 5005}

$$\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx = \frac{1}{2}\log\left(x^2+1\right)-\log(x)+\frac{1}{2}\cot^{-1}(x)^2-\frac{\cot^{-1}(x)}{x}$$

```
[In] Int[ArcCot[x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcCot[x]/x) + ArcCot[x]^2/2 - Log[x] + Log[1 + x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cot^{-1}(x)}{x^2}\,dx - \int \frac{\cot^{-1}(x)}{1+x^2}\,dx \\
&= -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \int \frac{1}{x\,(1+x^2)}\,dx \\
&= -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right) \\
&= -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) \\
&= -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \log(x) + \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{x^2 (1+x^2)}\, dx = -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \log(x) + \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[ArcCot[x]/(x^2*(1 + x^2)),x]
[Out] -(ArcCot[x]/x) + ArcCot[x]^2/2 - Log[x] + Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

method	result
parallelrisch	$-\frac{-\operatorname{arccot}(x)^2 x+2\ln(x)x-\ln(x^2+1)x+2\operatorname{arccot}(x)}{2x}$
default	$-\frac{\operatorname{arccot}(x)}{x}-\operatorname{arccot}(x)\arctan(x)-\ln(x)+\frac{\ln(x^2+1)}{2}-\frac{\arctan(x)^2}{2}$
parts	$-\frac{\operatorname{arccot}(x)}{x}-\operatorname{arccot}(x)\arctan(x)-\ln(x)+\frac{\ln(x^2+1)}{2}-\frac{\arctan(x)^2}{2}$
risch	$-\frac{\ln(ix+1)^2}{8}+\frac{(\ln(-ix+1)x-2i)\ln(ix+1)}{4x}-\frac{-2i\ln((-\pi+6i)x+6+i\pi)\pi x+2i\ln((-\pi-6i)x+6-i\pi)\pi x+\ln(-ix+1)^2x-4i\ln}{8}$

```
[In] int(arccot(x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*(-arccot(x)^2*x+2*ln(x)*x-ln(x^2+1)*x+2*arccot(x))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(x)}{x^2 (1+x^2)}\, dx = \frac{x \operatorname{arccot}(x)^2 + x \log(x^2+1) - 2\, x \log(x) - 2 \operatorname{arccot}(x)}{2\, x}$$

```
[In] integrate(arccot(x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] 1/2*(x*arccot(x)^2 + x*log(x^2 + 1) - 2*x*log(x) - 2*arccot(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{\cot^{-1}(x)}{x^2\left(1+x^2\right)}\,dx = -\log\left(x\right) + \frac{\log\left(x^2+1\right)}{2} + \frac{\operatorname{acot}^2\left(x\right)}{2} - \frac{\operatorname{acot}\left(x\right)}{x}$$

```
[In] integrate(acot(x)/x**2/(x**2+1),x)
[Out] -log(x) + log(x**2 + 1)/2 + acot(x)**2/2 - acot(x)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(x)}{x^2\left(1+x^2\right)}\,dx = -\left(\frac{1}{x} + \arctan\left(x\right)\right)\operatorname{arccot}\left(x\right) - \frac{1}{2}\arctan\left(x\right)^2 + \frac{1}{2}\log\left(x^2+1\right) - \log\left(x\right)$$

```
[In] integrate(arccot(x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -(1/x + arctan(x))*arccot(x) - 1/2*arctan(x)^2 + 1/2*log(x^2 + 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\cot^{-1}(x)}{x^2\left(1+x^2\right)}\,dx = \frac{1}{2}\arctan\left(\frac{1}{x}\right)^2 - \frac{\arctan\left(\frac{1}{x}\right)}{x} + \frac{1}{2}\log\left(\frac{1}{x^2}+1\right)$$

```
[In] integrate(arccot(x)/x^2/(x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(1/x)^2 - arctan(1/x)/x + 1/2*log(1/x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\cot^{-1}(x)}{x^2\left(1+x^2\right)}\,dx = \frac{\ln\left(x^2+1\right)}{2} - \ln\left(x\right) - \frac{\operatorname{acot}(x)}{x} + \frac{\operatorname{acot}(x)^2}{2}$$

```
[In] int(acot(x)/(x^2*(x^2 + 1)),x)
[Out] log(x^2 + 1)/2 - log(x) - acot(x)/x + acot(x)^2/2
```

3.44 $\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx = \frac{1}{2x} - \frac{\cot^{-1}(x)}{2x^2} - \frac{1}{2}i\cot^{-1}(x)^2 + \frac{\arctan(x)}{2} - \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)$$

```
[Out] 1/2/x-1/2*arccot(x)/x^2-1/2*I*arccot(x)^2+1/2*arctan(x)-arccot(x)*ln(2-2/(1
-I*x))-1/2*I*polylog(2,-1+2/(1-I*x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5039, 4947, 331, 209, 5045, 4989, 2497}

$$\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx = \frac{\arctan(x)}{2} - \frac{1}{2}i\,\text{PolyLog}\left(2,\frac{2}{1-ix}-1\right) - \frac{\cot^{-1}(x)}{2x^2} + \frac{1}{2x} - \frac{1}{2}i\cot^{-1}(x)^2 - \log\left(2-\frac{2}{1-ix}\right)\cot^{-1}(x)$$

```
[In] Int[ArcCot[x]/(x^3*(1 + x^2)),x]
```

```
[Out] 1/(2*x) - ArcCot[x]/(2*x^2) - (I/2)*ArcCot[x]^2 + ArcTan[x]/2 - ArcCot[x]*L
og[2 - 2/(1 - I*x)] - (I/2)*PolyLog[2, -1 + 2/(1 - I*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot^{-1}(x)}{x^3}\,dx - \int \frac{\cot^{-1}(x)}{x\left(1+x^2\right)}\,dx \\
&= -\frac{\cot^{-1}(x)}{2x^2} - \frac{1}{2}i\cot^{-1}(x)^2 - i\int \frac{\cot^{-1}(x)}{x(i+x)}\,dx - \frac{1}{2}\int \frac{1}{x^2\left(1+x^2\right)}\,dx \\
&= \frac{1}{2x} - \frac{\cot^{-1}(x)}{2x^2} - \frac{1}{2}i\cot^{-1}(x)^2 - \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) \\
&\quad + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \int \frac{\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx \\
&= \frac{1}{2x} - \frac{\cot^{-1}(x)}{2x^2} - \frac{1}{2}i\cot^{-1}(x)^2 + \frac{\arctan(x)}{2} \\
&\quad - \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$
\int \frac{\cot^{-1}(x)}{x^3\left(1+x^2\right)}\,dx = \frac{1}{2}\left(\frac{1}{x} + i\cot^{-1}(x)^2 + \cot^{-1}(x)\left(-1-\frac{1}{x^2}-2\log\left(1+e^{2i\cot^{-1}(x)}\right)\right) + i\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(x)}\right)\right)
$$

```
[In] Integrate[ArcCot[x]/(x^3*(1 + x^2)),x]
[Out] (x^(-1) + I*ArcCot[x]^2 + ArcCot[x]*(-1 - x^(-2) - 2*Log[1 + E^((2*I)*ArcCo
t[x])]) + I*PolyLog[2, -E^((2*I)*ArcCot[x])])/2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(58) = 116.

Time = 0.75 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.47

method	result
default	$-\frac{\operatorname{arccot}(x)}{2x^2} - \operatorname{arccot}(x)\ln(x) + \frac{\operatorname{arccot}(x)\ln(x^2+1)}{2} - \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(i+x)}{2}\right) - \ln(x-i)\ln\left(-\frac{i(i-}{2}\right.\right.}{4}$
parts	$-\frac{\operatorname{arccot}(x)}{2x^2} - \operatorname{arccot}(x)\ln(x) + \frac{\operatorname{arccot}(x)\ln(x^2+1)}{2} - \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(i+x)}{2}\right) - \ln(x-i)\ln\left(-\frac{i(i-}{2}\right.\right.}{4}$
risch	$\frac{\pi\ln(x^2+1)}{4} - \frac{\pi}{4x^2} - \frac{\pi\ln(-ix)}{2} + \frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4} - \frac{i\operatorname{dilog}\left(\frac{1}{2}+\frac{ix}{2}\right)}{4} + \frac{1}{2x} + \frac{i\operatorname{dilog}(ix+1)}{2} - \frac{i\ln(-ix+1)^2}{8} - \frac{i\operatorname{dilog}(-ix+1}{2}$

[In] int(arccot(x)/x^3/(x^2+1),x,method=_RETURNVERBOSE)

[Out] -1/2*arccot(x)/x^2-arccot(x)*ln(x)+1/2*arccot(x)*ln(x^2+1)-1/4*I*(ln(x-I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(I+x))-ln(x-I)*ln(-1/2*I*(I+x)))+1/4*I*(ln(I+x)*ln(x^2+1)-1/2*ln(I+x)^2-dilog(1/2*I*(x-I))-ln(I+x)*ln(1/2*I*(x-I)))+1/2/x+1/2*arctan(x)+1/2*I*ln(x)*ln(1+I*x)-1/2*I*ln(x)*ln(1-I*x)+1/2*I*dilog(1+I*x)-1/2*I*dilog(1-I*x)

Fricas [F]

$$\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)x^3}\,dx$$

[In] integrate(arccot(x)/x^3/(x^2+1),x, algorithm="fricas")

[Out] integral(arccot(x)/(x^5 + x^3), x)

Sympy [F]

$$\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx = \int \frac{\operatorname{acot}(x)}{x^3(x^2+1)}\,dx$$

[In] integrate(acot(x)/x**3/(x**2+1),x)

[Out] Integral(acot(x)/(x**3*(x**2 + 1)), x)

Maxima [F]

$$\int \frac{\cot^{-1}(x)}{x^3\,(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)x^3}\,dx$$

```
[In] integrate(arccot(x)/x^3/(x^2+1),x, algorithm="maxima")
[Out] integrate(arccot(x)/((x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\cot^{-1}(x)}{x^3\,(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)x^3}\,dx$$

```
[In] integrate(arccot(x)/x^3/(x^2+1),x, algorithm="giac")
[Out] integrate(arccot(x)/((x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{x^3\,(1+x^2)}\,dx = \int \frac{\operatorname{acot}(x)}{x^3\,(x^2+1)}\,dx$$

```
[In] int(acot(x)/(x^3*(x^2 + 1)),x)
[Out] int(acot(x)/(x^3*(x^2 + 1)), x)
```

3.45 $\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = \frac{1}{6x^2} - \frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 + \frac{4\log(x)}{3} - \frac{2}{3}\log\left(1+x^2\right)$$

```
[Out] 1/6/x^2-1/3*arccot(x)/x^3+arccot(x)/x-1/2*arccot(x)^2+4/3*ln(x)-2/3*ln(x^2+
1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5039, 4947, 272, 46, 36, 29, 31, 5005}

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = -\frac{\cot^{-1}(x)}{3x^3} + \frac{1}{6x^2} - \frac{2}{3}\log\left(x^2+1\right) + \frac{4\log(x)}{3} - \frac{1}{2}\cot^{-1}(x)^2 + \frac{\cot^{-1}(x)}{x}$$

```
[In] Int[ArcCot[x]/(x^4*(1 + x^2)),x]
```

```
[Out] 1/(6*x^2) - ArcCot[x]/(3*x^3) + ArcCot[x]/x - ArcCot[x]^2/2 + (4*Log[x])/3
- (2*Log[1 + x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(x)}{x^4}\,dx - \int \frac{\cot^{-1}(x)}{x^2\,(1+x^2)}\,dx$$
$$= -\frac{\cot^{-1}(x)}{3x^3} - \frac{1}{3}\int \frac{1}{x^3\,(1+x^2)}\,dx - \int \frac{\cot^{-1}(x)}{x^2}\,dx + \int \frac{\cot^{-1}(x)}{1+x^2}\,dx$$

$$= -\frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 - \frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, x^2\right) + \int \frac{1}{x(1+x^2)}\,dx$$

$$= -\frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 - \frac{1}{6}\text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{1}{1+x}\right)dx, x, x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right)$$

$$= \frac{1}{6x^2} - \frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 + \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)$$

$$= \frac{1}{6x^2} - \frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 + \frac{4\log(x)}{3} - \frac{2}{3}\log\left(1+x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = \frac{1}{6x^2} - \frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 + \frac{4\log(x)}{3} - \frac{2}{3}\log\left(1+x^2\right)$$

```
[In] Integrate[ArcCot[x]/(x^4*(1 + x^2)),x]
[Out] 1/(6*x^2) - ArcCot[x]/(3*x^3) + ArcCot[x]/x - ArcCot[x]^2/2 + (4*Log[x])/3
- (2*Log[1 + x^2])/3
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\operatorname{arccot}(x)}{3x^3} + \frac{\operatorname{arccot}(x)}{x} + \operatorname{arccot}(x)\arctan(x) + \frac{1}{6x^2} + \frac{4\ln(x)}{3} - \frac{2\ln(x^2+1)}{3} + \frac{\arctan(x)^2}{2}$
parts	$-\frac{\operatorname{arccot}(x)}{3x^3} + \frac{\operatorname{arccot}(x)}{x} + \operatorname{arccot}(x)\arctan(x) + \frac{1}{6x^2} + \frac{4\ln(x)}{3} - \frac{2\ln(x^2+1)}{3} + \frac{\arctan(x)^2}{2}$
parallelrisch	$\frac{-3\operatorname{arccot}(x)^2x^3+8\ln(x)x^3-4\ln(x^2+1)x^3+6x^2\operatorname{arccot}(x)+x-2\operatorname{arccot}(x)}{6x^3}$
risch	$\frac{\ln(ix+1)^2}{8} - \frac{(3\ln(-ix+1)x^3-6ix^2+2i)\ln(ix+1)}{12x^3} + \frac{-6i\ln((-\pi+8i)x+8+i\pi)\pi x^3+6i\ln((-\pi-8i)x+8-i\pi)\pi x^3+3\ln(-ix+}{}$

```
[In] int(arccot(x)/x^4/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/3*arccot(x)/x^3+arccot(x)/x+arccot(x)*arctan(x)+1/6/x^2+4/3*ln(x)-2/3*ln
(x^2+1)+1/2*arctan(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = -\frac{3\,x^3\operatorname{arccot}(x)^2 + 4\,x^3\log(x^2+1) - 8\,x^3\log(x) - 2\,(3\,x^2-1)\operatorname{arccot}(x) - x}{6\,x^3}$$

```
[In] integrate(arccot(x)/x^4/(x^2+1),x, algorithm="fricas")
[Out] -1/6*(3*x^3*arccot(x)^2 + 4*x^3*log(x^2 + 1) - 8*x^3*log(x) - 2*(3*x^2 - 1)
*arccot(x) - x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = \frac{4\log(x)}{3} - \frac{2\log(x^2+1)}{3} - \frac{\operatorname{acot}^2(x)}{2} + \frac{\operatorname{acot}(x)}{x} + \frac{1}{6x^2} - \frac{\operatorname{acot}(x)}{3x^3}$$

```
[In] integrate(acot(x)/x**4/(x**2+1),x)
[Out] 4*log(x)/3 - 2*log(x**2 + 1)/3 - acot(x)**2/2 + acot(x)/x + 1/(6*x**2) - ac
ot(x)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = \frac{1}{3}\left(\frac{3\,x^2-1}{x^3} + 3\arctan(x)\right)\operatorname{arccot}(x) + \frac{3\,x^2\arctan(x)^2 - 4\,x^2\log(x^2+1) + 8\,x^2\log(x) + 1}{6\,x^2}$$

```
[In] integrate(arccot(x)/x^4/(x^2+1),x, algorithm="maxima")
[Out] 1/3*((3*x^2 - 1)/x^3 + 3*arctan(x))*arccot(x) + 1/6*(3*x^2*arctan(x)^2 - 4*
x^2*log(x^2 + 1) + 8*x^2*log(x) + 1)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{x}\right)^2 + \frac{\arctan\left(\frac{1}{x}\right)}{x} + \frac{1}{6\,x^2} - \frac{\arctan\left(\frac{1}{x}\right)}{3\,x^3} - \frac{2}{3}\log\left(\frac{1}{x^2}+1\right)$$

```
[In] integrate(arccot(x)/x^4/(x^2+1),x, algorithm="giac")
[Out] -1/2*arctan(1/x)^2 + arctan(1/x)/x + 1/6/x^2 - 1/3*arctan(1/x)/x^3 - 2/3*lo
g(1/x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = \frac{4\,\ln(x)}{3} - \frac{2\,\ln(x^2+1)}{3} - \frac{\mathrm{acot}(x)^2}{2} + \frac{1}{6\,x^2} + \frac{\mathrm{acot}(x)\,\left(x^2-\frac{1}{3}\right)}{x^3}$$

```
[In] int(acot(x)/(x^4*(x^2 + 1)),x)
[Out] (4*log(x))/3 - (2*log(x^2 + 1))/3 - acot(x)^2/2 + 1/(6*x^2) + (acot(x)*(x^2
 - 1/3))/x^3
```

3.46 $\int \frac{x^2 \cot^{-1}(cx)}{1+x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 206

$$\begin{aligned}\int \frac{x^2 \cot^{-1}(cx)}{1+x^2} \, dx = & x \cot^{-1}(cx) - \frac{1}{2} i \arctan(x) \log \left(1 - \frac{i}{cx}\right) \\ & + \frac{1}{2} i \arctan(x) \log \left(1 + \frac{i}{cx}\right) + \frac{1}{2} i \arctan(x) \log \left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right) \\ & - \frac{1}{2} i \arctan(x) \log \left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right) \\ & + \frac{\log(1+c^2x^2)}{2c} + \frac{1}{4} \text{PolyLog}\left(2, 1 + \frac{2i(i-cx)}{(1-c)(1-ix)}\right) \\ & - \frac{1}{4} \text{PolyLog}\left(2, 1 + \frac{2i(i+cx)}{(1+c)(1-ix)}\right)\end{aligned}$$

```
[Out] x*arccot(c*x)-1/2*I*arctan(x)*ln(1-I/c/x)+1/2*I*arctan(x)*ln(1+I/c/x)+1/2*I
*arctan(x)*ln(-2*I*(I-c*x)/(1-c)/(1-I*x))-1/2*I*arctan(x)*ln(-2*I*(I+c*x)/(
1+c)/(1-I*x))+1/2*ln(c^2*x^2+1)/c+1/4*polylog(2,1+2*I*(I-c*x)/(1-c)/(1-I*x)
)-1/4*polylog(2,1+2*I*(I+c*x)/(1+c)/(1-I*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used

$= \{5037, 4931, 266, 5029, 209, 2520, 6820, 12, 4996, 4940, 2438, 4966, 2449, 2352, 2497\}$

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx = -\frac{1}{2} i \arctan(x) \log\left(1-\frac{i}{cx}\right) + \frac{1}{2} i \arctan(x) \log\left(1+\frac{i}{cx}\right) + \frac{1}{2} i \arctan(x) \log\left(-\frac{2i(-cx+i)}{(1-c)(1-ix)}\right) - \frac{1}{2} i \arctan(x) \log\left(-\frac{2i(cx+i)}{(c+1)(1-ix)}\right) + \frac{\log\left(c^2x^2+1\right)}{2c} + \frac{1}{4}\operatorname{PolyLog}\left(2, \frac{2i(i-cx)}{(1-c)(1-ix)}+1\right) - \frac{1}{4}\operatorname{PolyLog}\left(2, \frac{2i(cx+i)}{(c+1)(1-ix)}+1\right) + x\cot^{-1}(cx)$$

```
[In] Int[(x^2*ArcCot[c*x])/(1 + x^2),x]
```

```
[Out] x*ArcCot[c*x] - (I/2)*ArcTan[x]*Log[1 - I/(c*x)] + (I/2)*ArcTan[x]*Log[1 +
I/(c*x)] + (I/2)*ArcTan[x]*Log[((-2*I)*(I - c*x))/((1 - c)*(1 - I*x))] - (I
/2)*ArcTan[x]*Log[((-2*I)*(I + c*x))/((1 + c)*(1 - I*x))] + Log[1 + c^2*x^2
]/(2*c) + PolyLog[2, 1 + ((2*I)*(I - c*x))/((1 - c)*(1 - I*x))]/4 - PolyLog
[2, 1 + ((2*I)*(I + c*x))/((1 + c)*(1 - I*x))]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
```

```
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5029

```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I/(c*x)]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I/(c*x)]/(d + e
*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \cot^{-1}(cx)\,dx - \int \frac{\cot^{-1}(cx)}{1+x^2}\,dx \\
&= x\cot^{-1}(cx) - \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{1+x^2}\,dx + \frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{1+x^2}\,dx + c\int \frac{x}{1+c^2x^2}\,dx \\
&= x\cot^{-1}(cx) - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) \\
&\quad + \frac{\log\left(1+c^2x^2\right)}{2c} - \frac{\int \frac{\arctan(x)}{\left(1-\frac{i}{cx}\right)x^2}\,dx}{2c} - \frac{\int \frac{\arctan(x)}{\left(1+\frac{i}{cx}\right)x^2}\,dx}{2c} \\
&= x\cot^{-1}(cx) - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) \\
&\quad + \frac{\log\left(1+c^2x^2\right)}{2c} - \frac{\int \frac{c\arctan(x)}{x(-i+cx)}\,dx}{2c} - \frac{\int \frac{c\arctan(x)}{x(i+cx)}\,dx}{2c} \\
&= x\cot^{-1}(cx) - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) \\
&\quad + \frac{\log\left(1+c^2x^2\right)}{2c} - \frac{1}{2}\int \frac{\arctan(x)}{x(-i+cx)}\,dx - \frac{1}{2}\int \frac{\arctan(x)}{x(i+cx)}\,dx \\
&= x\cot^{-1}(cx) - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) + \frac{\log\left(1+c^2x^2\right)}{2c} \\
&\quad - \frac{1}{2}\int \left(\frac{i\arctan(x)}{x} - \frac{ic\arctan(x)}{-i+cx}\right)dx - \frac{1}{2}\int \left(-\frac{i\arctan(x)}{x} + \frac{ic\arctan(x)}{i+cx}\right)dx
\end{aligned}
$$

$$
\begin{aligned}
&= x\cot^{-1}(cx) - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) \\
&\quad + \frac{\log(1+c^2x^2)}{2c} + \frac{1}{2}(ic)\int\frac{\arctan(x)}{-i+cx}\,dx - \frac{1}{2}(ic)\int\frac{\arctan(x)}{i+cx}\,dx \\
&= x\cot^{-1}(cx) - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) \\
&\quad + \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right) - \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right) \\
&\quad + \frac{\log(1+c^2x^2)}{2c} - \frac{1}{2}i\int\frac{\log\left(\frac{2(-i+cx)}{(-i+ic)(1-ix)}\right)}{1+x^2}\,dx + \frac{1}{2}i\int\frac{\log\left(\frac{2(i+cx)}{(i+ic)(1-ix)}\right)}{1+x^2}\,dx \\
&= x\cot^{-1}(cx) - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) \\
&\quad + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right) \\
&\quad - \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right) + \frac{\log(1+c^2x^2)}{2c} \\
&\quad + \frac{1}{4}\operatorname{PolyLog}\left(2,1+\frac{2i(i-cx)}{(1-c)(1-ix)}\right) - \frac{1}{4}\operatorname{PolyLog}\left(2,1+\frac{2i(i+cx)}{(1+c)(1-ix)}\right)
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 626 vs. $2(206) = 412$.

Time = 1.19 (sec) , antiderivative size = 626, normalized size of antiderivative = 3.04

$$
\int\frac{x^2\cot^{-1}(cx)}{1+x^2}\,dx = \frac{cx\cot^{-1}(cx) - \log\left(\frac{1}{c\sqrt{1+\frac{1}{c^2x^2}}x}\right) + \frac{1}{4}\sqrt{-c^2}\left(2\arccos\left(\frac{1+c^2}{-1+c^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right) - 4\cot^{-1}(cx)\operatorname{arctanh}\left(\frac{cx}{\sqrt{-c^2}}\right.\right.}{}
$$

```
[In] Integrate[(x^2*ArcCot[c*x])/(1 + x^2),x]
```

```
[Out] (c*x*ArcCot[c*x] - Log[1/(c*Sqrt[1 + 1/(c^2*x^2)]*x)] + (Sqrt[-c^2]*(2*ArcC
os[(1 + c^2)/(-1 + c^2)]*ArcTanh[Sqrt[-c^2]/(c*x)] - 4*ArcCot[c*x]*ArcTanh[
(c*x)/Sqrt[-c^2]] - (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2
]/(c*x)])*Log[(-2*(c^2 + I*Sqrt[-c^2])*(-I + c*x))/((-1 + c^2)*(Sqrt[-c^2]
- c*x))] - (ArcCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)])
*Log[((2*I)*(I*c^2 + Sqrt[-c^2])*(I + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))
] + (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] + (2*I)
*ArcTanh[(c*x)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2])/(Sqrt[-1 + c^2]*E^(I*A
rcCot[c*x])*Sqrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + (ArcCos[(1 +
```

```
c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] - (2*I)*ArcTanh[(c*x)/S
qrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2]*E^(I*ArcCot[c*x]))/(Sqrt[-1 + c^2]*Sqrt
[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + I*(-PolyLog[2, ((1 + c^2 - (
2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))] + Pol
yLog[2, ((1 + c^2 + (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqrt
[-c^2] - c*x))])))/4)/c
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.27

method	result
risch	$\frac{i\ln(icx+1)x}{2}+\frac{\pi x}{2}-\frac{i\ln(-icx+1)x}{2}-\frac{\pi\arctan(x)}{2}+\frac{i\pi}{2c}-\frac{i\arctan(cx)}{2c}+\frac{\ln(c^2x^2+1)}{4c}-\frac{1}{c}+\frac{\ln(-icx+1)\ln(}{4}$
derivativedivides	$-\operatorname{arccot}(cx)\arctan(x)c^3+\operatorname{arccot}(cx)c^3x+c^3\left(\frac{\ln(c^2x^2+1)}{2c}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2}-\frac{\arctan(x)^2}{2}-\frac{\operatorname{polylog}\left(2,\frac{(c-1)}{(x^2+}\right.}{4}\right.$
default	$-\operatorname{arccot}(cx)\arctan(x)c^3+\operatorname{arccot}(cx)c^3x+c^3\left(\frac{\ln(c^2x^2+1)}{2c}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2}-\frac{\arctan(x)^2}{2}-\frac{\operatorname{polylog}\left(2,\frac{(c-1)}{(x^2+}\right.}{4}\right.$
parts	$-\operatorname{arccot}(cx)\arctan(x)+x\operatorname{arccot}(cx)+c\left(\frac{\ln(c^2x^2+1)}{2c^2}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c}-\operatorname{arct}\right.$

```
[In] int(x^2*arccot(c*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*ln(1+I*c*x)*x+1/2*Pi*x-1/2*I*ln(1-I*c*x)*x-1/2*Pi*arctan(x)+1/2*I/c*P
i-1/2*I/c*arctan(c*x)+1/4*ln(c^2*x^2+1)/c-1/c+1/4*ln(1-I*c*x)*ln((-c-I*c*x)
/(-c-1))+1/4*dilog((-c-I*c*x)/(-c-1))-1/4*ln(1-I*c*x)*ln((c-I*c*x)/(c-1))-1
/4*dilog((c-I*c*x)/(c-1))+1/2/c*ln(1+I*c*x)+1/4*ln(1+I*c*x)*ln((-c+I*c*x)/(
-c-1))+1/4*dilog((-c+I*c*x)/(-c-1))-1/4*ln(1+I*c*x)*ln((c+I*c*x)/(c-1))-1/4
*dilog((c+I*c*x)/(c-1))
```

Fricas [F]

$$\int \frac{x^2\cot^{-1}(cx)}{1+x^2}\,dx=\int \frac{x^2\operatorname{arccot}(cx)}{x^2+1}\,dx$$

```
[In] integrate(x^2*arccot(c*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(x^2*arccot(c*x)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x^2 \operatorname{acot}(cx)}{x^2+1}\,dx$$

```
[In] integrate(x**2*acot(c*x)/(x**2+1),x)
[Out] Integral(x**2*acot(c*x)/(x**2 + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx = (x - \arctan(x))\operatorname{arccot}(cx) - \frac{4\,c\arctan(cx)\arctan(x) - 4\,c\arctan(x)\arctan\left(\frac{cx}{c-1}, -\frac{1}{c-1}\right) + c\log(x^2+1)\log\left(\frac{c^2x^2+1}{c^2+2\,c+1}\right) - c\log(x^2 -}{8}$$

```
[In] integrate(x^2*arccot(c*x)/(x^2+1),x, algorithm="maxima")
[Out] (x - arctan(x))*arccot(c*x) - 1/8*(4*c*arctan(c*x)*arctan(x) - 4*c*arctan(x
)*arctan2(c*x/(c - 1), -1/(c - 1)) + c*log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2
+ 2*c + 1)) - c*log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 - 2*c + 1)) + 2*c*dilog
((I*c*x + c)/(c + 1)) + 2*c*dilog(-(I*c*x - c)/(c + 1)) - 2*c*dilog((I*c*x
+ c)/(c - 1)) - 2*c*dilog(-(I*c*x - c)/(c - 1)) - 4*log(c^2*x^2 + 1))/c
```

Giac [F]

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x^2 \operatorname{arccot}(cx)}{x^2+1}\,dx$$

```
[In] integrate(x^2*arccot(c*x)/(x^2+1),x, algorithm="giac")
[Out] integrate(x^2*arccot(c*x)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x^2\,\mathrm{acot}(c\,x)}{x^2+1}\,dx$$

```
[In] int((x^2*acot(c*x))/(x^2 + 1),x)
[Out] int((x^2*acot(c*x))/(x^2 + 1), x)
```

3.47 $\int \frac{x \cot^{-1}(cx)}{1+x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 188

$$\int \frac{x \cot^{-1}(cx)}{1+x^2} dx = -\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right) + \frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) + \frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right) - \frac{1}{2}i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right) + \frac{1}{4}i\,\mathrm{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right) + \frac{1}{4}i\,\mathrm{PolyLog}\left(2,1+\frac{2ic(i+x)}{(1+c)(1-icx)}\right)$$

```
[Out] -arccot(c*x)*ln(2/(1-I*c*x))+1/2*arccot(c*x)*ln(2*I*c*(I-x)/(1-c)/(1-I*c*x)
)+1/2*arccot(c*x)*ln(-2*I*c*(I+x)/(1+c)/(1-I*c*x))-1/2*I*polylog(2,1-2/(1-I
*c*x))+1/4*I*polylog(2,1-2*I*c*(I-x)/(1-c)/(1-I*c*x))+1/4*I*polylog(2,1+2*I
*c*(I+x)/(1+c)/(1-I*c*x))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used

$= \{5049, 4967, 2449, 2352, 2497\}$

$$\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx = -\frac{1}{2}i\,\text{PolyLog}\left(2, 1-\frac{2}{1-icx}\right) + \frac{1}{4}i\,\text{PolyLog}\left(2, 1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right) + \frac{1}{4}i\,\text{PolyLog}\left(2, \frac{2ic(x+i)}{(c+1)(1-icx)}+1\right) + \log\left(\frac{2}{1-icx}\right)\left(-\cot^{-1}(cx)\right) + \frac{1}{2}\log\left(\frac{2ic(-x+i)}{(1-c)(1-icx)}\right)\cot^{-1}(cx) + \frac{1}{2}\log\left(-\frac{2ic(x+i)}{(c+1)(1-icx)}\right)\cot^{-1}(cx)$$

```
[In] Int[(x*ArcCot[c*x])/(1 + x^2),x]
```

```
[Out] -(ArcCot[c*x]*Log[2/(1 - I*c*x)]) + (ArcCot[c*x]*Log[((2*I)*c*(I - x))/((1
- c)*(1 - I*c*x))])/2 + (ArcCot[c*x]*Log[((-2*I)*c*(I + x))/((1 + c)*(1 - I
*c*x))])/2 - (I/2)*PolyLog[2, 1 - 2/(1 - I*c*x)] + (I/4)*PolyLog[2, 1 - ((2
*I)*c*(I - x))/((1 - c)*(1 - I*c*x))] + (I/4)*PolyLog[2, 1 + ((2*I)*c*(I +
x))/((1 + c)*(1 - I*c*x))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5049

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcCot[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\cot^{-1}(cx)}{2(i-x)}+\frac{\cot^{-1}(cx)}{2(i+x)}\right) dx \\
&= -\left(\frac{1}{2}\int \frac{\cot^{-1}(cx)}{i-x}\,dx\right)+\frac{1}{2}\int \frac{\cot^{-1}(cx)}{i+x}\,dx \\
&= -\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)+\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) \\
&\quad +\frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right)-2\left(\frac{1}{2}c\int \frac{\log\left(\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx\right) \\
&\quad +\frac{1}{2}c\int \frac{\log\left(\frac{2c(i-x)}{(-i+ic)(1-icx)}\right)}{1+c^2x^2}\,dx+\frac{1}{2}c\int \frac{\log\left(\frac{2c(i+x)}{(i+ic)(1-icx)}\right)}{1+c^2x^2}\,dx \\
&= -\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)+\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) \\
&\quad +\frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right) \\
&\quad +\frac{1}{4}i\,\text{PolyLog}\left(2,1+\frac{2ic(i+x)}{(1+c)(1-icx)}\right) \\
&\quad -2\left(\frac{1}{2}i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-icx}\right)\right) \\
&= -\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)+\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) \\
&\quad +\frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right) \\
&\quad +\frac{1}{4}i\,\text{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,1+\frac{2ic(i+x)}{(1+c)(1-icx)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.64

$$\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx = \frac{1}{2}\left(-i\cot^{-1}(cx)^2-2i\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\arctan\left(\frac{\sqrt{c^2}}{cx}\right)-2\cot^{-1}(cx)\log\left(1-e^{2i\cot^{-1}(cx)}\right)+\left(\cot^{-1}(cx)-\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\right)\log\left(\frac{-1+\left(-1+2\sqrt{c^2}\right)e^{2i\cot^{-1}(cx)}-c^2\left(-1+e^{2i\cot^{-1}(cx)}\right)}{-1+c^2}\right)+\left(\cot^{-1}(cx)+\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\right)\log\left(-\frac{1+\left(1+2\sqrt{c^2}\right)e^{2i\cot^{-1}(cx)}+c^2\left(-1+e^{2i\cot^{-1}(cx)}\right)}{-1+c^2}\right)+i\left(\cot^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\cot^{-1}(cx)}\right)\right)-\frac{1}{2}i\left(\text{PolyLog}\left(2,\frac{\left(1+c^2-2\sqrt{c^2}\right)e^{2i\cot^{-1}(cx)}}{-1+c^2}\right)+\text{PolyLog}\left(2,\frac{\left(1+c^2+2\sqrt{c^2}\right)e^{2i\cot^{-1}(cx)}}{-1+c^2}\right)\right)\right)$$

```
[In] Integrate[(x*ArcCot[c*x])/(1 + x^2),x]
```

```
[Out] ((-I)*ArcCot[c*x]^2 - (2*I)*ArcSin[Sqrt[(1 - c^2)^(-1)]]*ArcTan[Sqrt[c^2]/(
c*x)] - 2*ArcCot[c*x]*Log[1 - E^((2*I)*ArcCot[c*x])] + (ArcCot[c*x] - ArcSi
n[Sqrt[(1 - c^2)^(-1)]])*Log[(-1 + (-1 + 2*Sqrt[c^2])*E^((2*I)*ArcCot[c*x])
 - c^2*(-1 + E^((2*I)*ArcCot[c*x])))/(-1 + c^2)] + (ArcCot[c*x] + ArcSin[Sq
rt[(1 - c^2)^(-1)]])*Log[-((1 + (1 + 2*Sqrt[c^2])*E^((2*I)*ArcCot[c*x]) + c
^2*(-1 + E^((2*I)*ArcCot[c*x])))/(-1 + c^2))] + I*(ArcCot[c*x]^2 + PolyLog[
2, E^((2*I)*ArcCot[c*x])]) - (I/2)*(PolyLog[2, ((1 + c^2 - 2*Sqrt[c^2])*E^(
(2*I)*ArcCot[c*x]))/(-1 + c^2)] + PolyLog[2, ((1 + c^2 + 2*Sqrt[c^2])*E^((2
*I)*ArcCot[c*x]))/(-1 + c^2)]))/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

method	result
parts	$\frac{\ln(x^2+1)\,\mathrm{arccot}(cx)}{2} + \frac{\sum_{\_\alpha=\mathrm{RootOf}(\_Z^2c^2+1)} \frac{\ln(x-\_\alpha)\ln(x^2+1)-\ln(x-\_\alpha)\ln\left(\frac{c\_\alpha+x}{\_\alpha(1+c)}\right)-\ln(x-\_\alpha)\ln\left(\frac{c\_\alpha-x}{\_\alpha(c-1)}\right)-\mathrm{d}\cdots}{\_\alpha}}{4c}$
risch	$\frac{\pi\ln\left(-c^2+(-icx+1)^2-1+2icx\right)}{4}-\frac{i\,\mathrm{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4}-\frac{i\ln(-icx+1)\ln\left(\frac{-icx-c}{-c-1}\right)}{4}-\frac{i\,\mathrm{dilog}\left(\frac{-icx+c}{c-1}\right)}{4}-\frac{i\ln(-icx\cdots}{\cdots}$
derivativedivides	$\frac{\frac{c^2\ln(c^2x^2+c^2)\,\mathrm{arccot}(cx)}{2}+\frac{c^2\left(-\frac{i\left(\ln(cx-i)\ln(c^2x^2+c^2)-i\left(-i\,\mathrm{dilog}\left(\frac{i(cx-i)+c-1}{c-1}\right)-i\ln(cx-i)\ln\left(\frac{i(cx-i)+c-1}{c-1}\right)\right)-i\left(-i\,\mathrm{dil}\cdots\right.\right.}{2}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{c^2\ln(c^2x^2+c^2)\,\mathrm{arccot}(cx)}{2}+\frac{c^2\left(-\frac{i\left(\ln(cx-i)\ln(c^2x^2+c^2)-i\left(-i\,\mathrm{dilog}\left(\frac{i(cx-i)+c-1}{c-1}\right)-i\ln(cx-i)\ln\left(\frac{i(cx-i)+c-1}{c-1}\right)\right)-i\left(-i\,\mathrm{dil}\cdots\right.\right.}{2}\right.}{\cdots}}{\cdots}$

```
[In] int(x*arccot(c*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x^2+1)*arccot(c*x)+1/4/c*sum(1/_alpha*(ln(x-_alpha)*ln(x^2+1)-ln(x-_
alpha)*ln((_alpha*c+x)/_alpha/(1+c))-ln(x-_alpha)*ln((_alpha*c-x)/_alpha/(c
-1))-dilog((_alpha*c+x)/_alpha/(1+c))-dilog((_alpha*c-x)/_alpha/(c-1))),_al
pha=RootOf(_Z^2*c^2+1))
```

Fricas [F]

$$\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x\,\mathrm{arccot}\,(cx)}{x^2+1}\,dx$$

```
[In] integrate(x*arccot(c*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(x*arccot(c*x)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x\,\mathrm{acot}\,(cx)}{x^2+1}\,dx$$

```
[In] integrate(x*acot(c*x)/(x**2+1),x)
```

```
[Out] Integral(x*acot(c*x)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x\,\mathrm{arccot}\,(cx)}{x^2+1}\,dx$$

```
[In] integrate(x*arccot(c*x)/(x^2+1),x, algorithm="maxima")
[Out] integrate(x*arccot(c*x)/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x\,\mathrm{arccot}\,(cx)}{x^2+1}\,dx$$

```
[In] integrate(x*arccot(c*x)/(x^2+1),x, algorithm="giac")
[Out] integrate(x*arccot(c*x)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x\,\mathrm{acot}(c\,x)}{x^2+1}\,dx$$

```
[In] int((x*acot(c*x))/(x^2 + 1),x)
[Out] int((x*acot(c*x))/(x^2 + 1), x)
```

3.48 $\int \frac{\cot^{-1}(cx)}{1+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 183

$$\begin{aligned}\int \frac{\cot^{-1}(cx)}{1+x^2} \, dx = {} & \frac{1}{2} i \arctan(x) \log\left(1-\frac{i}{cx}\right) - \frac{1}{2} i \arctan(x) \log\left(1+\frac{i}{cx}\right) \\ & - \frac{1}{2} i \arctan(x) \log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right) \\ & + \frac{1}{2} i \arctan(x) \log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right) \\ & - \frac{1}{4} \text{PolyLog}\left(2, 1+\frac{2i(i-cx)}{(1-c)(1-ix)}\right) \\ & + \frac{1}{4} \text{PolyLog}\left(2, 1+\frac{2i(i+cx)}{(1+c)(1-ix)}\right)\end{aligned}$$

```
[Out] 1/2*I*arctan(x)*ln(1-I/c/x)-1/2*I*arctan(x)*ln(1+I/c/x)-1/2*I*arctan(x)*ln(
-2*I*(I-c*x)/(1-c)/(1-I*x))+1/2*I*arctan(x)*ln(-2*I*(I+c*x)/(1+c)/(1-I*x))-
1/4*polylog(2,1+2*I*(I-c*x)/(1-c)/(1-I*x))+1/4*polylog(2,1+2*I*(I+c*x)/(1+c
)/(1-I*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules

used = {5029, 209, 2520, 266, 6820, 12, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) - \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(-cx+i)}{(1-c)(1-ix)}\right) + \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(cx+i)}{(c+1)(1-ix)}\right) - \frac{1}{4}\text{PolyLog}\left(2,\frac{2i(i-cx)}{(1-c)(1-ix)}+1\right) + \frac{1}{4}\text{PolyLog}\left(2,\frac{2i(cx+i)}{(c+1)(1-ix)}+1\right)$$

```
[In] Int[ArcCot[c*x]/(1 + x^2),x]
```

```
[Out] (I/2)*ArcTan[x]*Log[1 - I/(c*x)] - (I/2)*ArcTan[x]*Log[1 + I/(c*x)] - (I/2)
*ArcTan[x]*Log[((-2*I)*(I - c*x))/((1 - c)*(1 - I*x))] + (I/2)*ArcTan[x]*Lo
g[((-2*I)*(I + c*x))/((1 + c)*(1 - I*x))] - PolyLog[2, 1 + ((2*I)*(I - c*x)
)/((1 - c)*(1 - I*x))]/4 + PolyLog[2, 1 + ((2*I)*(I + c*x))/((1 + c)*(1 - I
*x))]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5029

```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I/(c*x)]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I/(c*x)]/(d + e
```

```
*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{1+x^2}\,dx-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{1+x^2}\,dx\\
&= \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right)+\frac{\int \frac{\arctan(x)}{\left(1-\frac{i}{cx}\right)x^2}\,dx}{2c}+\frac{\int \frac{\arctan(x)}{\left(1+\frac{i}{cx}\right)x^2}\,dx}{2c}\\
&= \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right)+\frac{\int \frac{c\arctan(x)}{x(-i+cx)}\,dx}{2c}+\frac{\int \frac{c\arctan(x)}{x(i+cx)}\,dx}{2c}\\
&= \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right)\\
&\quad+\frac{1}{2}\int \frac{\arctan(x)}{x(-i+cx)}\,dx+\frac{1}{2}\int \frac{\arctan(x)}{x(i+cx)}\,dx\\
&= \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right)\\
&\quad+\frac{1}{2}\int \left(\frac{i\arctan(x)}{x}-\frac{ic\arctan(x)}{-i+cx}\right)dx+\frac{1}{2}\int \left(-\frac{i\arctan(x)}{x}+\frac{ic\arctan(x)}{i+cx}\right)dx\\
&= \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right)\\
&\quad-\frac{1}{2}(ic)\int \frac{\arctan(x)}{-i+cx}\,dx+\frac{1}{2}(ic)\int \frac{\arctan(x)}{i+cx}\,dx\\
&= \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right)\\
&\quad-\frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right)+\frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right)\\
&\quad+\frac{1}{2}i\int \frac{\log\left(\frac{2(-i+cx)}{(-i+ic)(1-ix)}\right)}{1+x^2}\,dx-\frac{1}{2}i\int \frac{\log\left(\frac{2(i+cx)}{(i+ic)(1-ix)}\right)}{1+x^2}\,dx\\
&= \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right)\\
&\quad-\frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right)+\frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right)\\
&\quad-\frac{1}{4}\operatorname{PolyLog}\left(2,1+\frac{2i(i-cx)}{(1-c)(1-ix)}\right)+\frac{1}{4}\operatorname{PolyLog}\left(2,1+\frac{2i(i+cx)}{(1+c)(1-ix)}\right)
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 592 vs. $2(183) = 366$.

Time = 0.61 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.23

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx$$
$$= \frac{c\left(2\arccos\left(\frac{1+c^2}{-1+c^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right) - 4\cot^{-1}(cx)\operatorname{arctanh}\left(\frac{cx}{\sqrt{-c^2}}\right) - \left(\arccos\left(\frac{1+c^2}{-1+c^2}\right) - 2i\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right)\right.\right.}{}$$

```
[In] Integrate[ArcCot[c*x]/(1 + x^2),x]
```

```
[Out] (c*(2*ArcCos[(1 + c^2)/(-1 + c^2)]*ArcTanh[Sqrt[-c^2]/(c*x)] - 4*ArcCot[c*x
]*ArcTanh[(c*x)/Sqrt[-c^2]] - (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh
[Sqrt[-c^2]/(c*x)])*Log[(-2*(c^2 + I*Sqrt[-c^2])*(-I + c*x))/((-1 + c^2)*(S
qrt[-c^2] - c*x))] - (ArcCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^
2]/(c*x)])*Log[((2*I)*(I*c^2 + Sqrt[-c^2])*(I + c*x))/((-1 + c^2)*(Sqrt[-c^
2] - c*x))] + (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2]/(c*x
)] + (2*I)*ArcTanh[(c*x)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2])/(Sqrt[-1 + c
^2]*E^(I*ArcCot[c*x])*Sqrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + (A
rcCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] - (2*I)*ArcTa
nh[(c*x)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2]*E^(I*ArcCot[c*x]))/(Sqrt[-1 +
 c^2]*Sqrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + I*(-PolyLog[2, ((1
 + c^2 - (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*
x))] + PolyLog[2, ((1 + c^2 + (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 +
c^2)*(Sqrt[-c^2] - c*x))])))/(4*Sqrt[-c^2])
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\pi \arctan(x)}{2} - \frac{\ln(-icx+1)\ln\left(\frac{-icx-c}{-c-1}\right)}{4} - \frac{\operatorname{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4} + \frac{\ln(-icx+1)\ln\left(\frac{-icx+c}{c-1}\right)}{4} + \frac{\operatorname{dilog}\left(\frac{-icx+c}{c-1}\right)}{4} - \frac{\ln(icx$
parts	$\operatorname{arccot}(cx)\arctan(x) + c\left(\frac{\arctan(cx)\arctan(x)}{c} - \frac{ic^2\ln\left(1-\frac{(c-1)(icx+1)^2}{(c^2x^2+1)(-c-1)}\right)\arctan(cx)}{2+2c} + \frac{ic\arctan(cx)\ln\left(1-\right.}{2-}\right.$
derivativedivides	$\frac{c\arctan(x)\operatorname{arccot}(cx)+c^2\left(\frac{\arctan(cx)\arctan(x)}{c} - \frac{ic^2\ln\left(1-\frac{(c-1)(icx+1)^2}{(c^2x^2+1)(-c-1)}\right)\arctan(cx)}{2+2c} + \frac{ic\arctan(cx)\ln\left(1-\frac{(c-1)(icx+1)^2}{(c^2x^2+1)(-c-}\right.}{2+2c}\right.}{}$
default	$\frac{c\arctan(x)\operatorname{arccot}(cx)+c^2\left(\frac{\arctan(cx)\arctan(x)}{c} - \frac{ic^2\ln\left(1-\frac{(c-1)(icx+1)^2}{(c^2x^2+1)(-c-1)}\right)\arctan(cx)}{2+2c} + \frac{ic\arctan(cx)\ln\left(1-\frac{(c-1)(icx+1)^2}{(c^2x^2+1)(-c-}\right.}{2+2c}\right.}{}$

```
[In] int(arccot(c*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*Pi*arctan(x)-1/4*ln(1-I*c*x)*ln((-c-I*c*x)/(-c-1))-1/4*dilog((-c-I*c*x)
/(-c-1))+1/4*ln(1-I*c*x)*ln((c-I*c*x)/(c-1))+1/4*dilog((c-I*c*x)/(c-1))-1/4
*ln(1+I*c*x)*ln((-c+I*c*x)/(-c-1))-1/4*dilog((-c+I*c*x)/(-c-1))+1/4*ln(1+I*
c*x)*ln((c+I*c*x)/(c-1))+1/4*dilog((c+I*c*x)/(c-1))
```

Fricas [F]

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{\operatorname{arccot}(cx)}{x^2+1}\,dx$$

```
[In] integrate(arccot(c*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arccot(c*x)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{\operatorname{acot}(cx)}{x^2+1}\,dx$$

```
[In] integrate(acot(c*x)/(x**2+1),x)
[Out] Integral(acot(c*x)/(x**2 + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = -\frac{1}{8}c\left(\frac{8\arctan(cx)\arctan(x)}{c} - \frac{4\arctan(cx)\arctan(x) - 4\arctan(x)\arctan\left(\frac{cx}{c-1}, -\frac{1}{c-1}\right) + \log(x^2+\right.$$
$$+ \operatorname{arccot}(cx)\arctan(x) + \arctan(cx)\arctan(x)$$

```
[In] integrate(arccot(c*x)/(x^2+1),x, algorithm="maxima")
[Out] -1/8*c*(8*arctan(c*x)*arctan(x)/c - (4*arctan(c*x)*arctan(x) - 4*arctan(x)*
arctan2(c*x/(c - 1), -1/(c - 1)) + log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 + 2*
c + 1)) - log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 - 2*c + 1)) + 2*dilog((I*c*x
+ c)/(c + 1)) + 2*dilog(-(I*c*x - c)/(c + 1)) - 2*dilog((I*c*x + c)/(c - 1)
) - 2*dilog(-(I*c*x - c)/(c - 1)))/c) + arccot(c*x)*arctan(x) + arctan(c*x)
*arctan(x)
```

Giac [F]

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{\operatorname{arccot}(cx)}{x^2+1}\,dx$$

```
[In] integrate(arccot(c*x)/(x^2+1),x, algorithm="giac")
[Out] integrate(arccot(c*x)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{\operatorname{acot}(c\,x)}{x^2+1}\,dx$$

```
[In] int(acot(c*x)/(x^2 + 1),x)
[Out] int(acot(c*x)/(x^2 + 1), x)
```

3.49 $\int \frac{\cot^{-1}(cx)}{x(1+x^2)} dx$

Optimal result

Integrand size = 15, antiderivative size = 223

$$\begin{aligned}\int \frac{\cot^{-1}(cx)}{x(1+x^2)} dx =& \cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)-\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right)\\&-\frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right)-\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-\frac{i}{cx}\right)\\&+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,\frac{i}{cx}\right)+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\\&-\frac{1}{4}i\,\mathrm{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right)\\&-\frac{1}{4}i\,\mathrm{PolyLog}\left(2,1+\frac{2ic(i+x)}{(1+c)(1-icx)}\right)\end{aligned}$$

```
[Out] arccot(c*x)*ln(2/(1-I*c*x))-1/2*arccot(c*x)*ln(2*I*c*(I-x)/(1-c)/(1-I*c*x))
-1/2*arccot(c*x)*ln(-2*I*c*(I+x)/(1+c)/(1-I*c*x))-1/2*I*polylog(2,-I/c/x)+1
/2*I*polylog(2,I/c/x)+1/2*I*polylog(2,1-2/(1-I*c*x))-1/4*I*polylog(2,1-2*I*
c*(I-x)/(1-c)/(1-I*c*x))-1/4*I*polylog(2,1+2*I*c*(I+x)/(1+c)/(1-I*c*x))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used

$= \{5049, 4941, 2438, 4967, 2449, 2352, 2497\}$

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx = -\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{cx}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{cx}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right) - \frac{1}{4}i\,\text{PolyLog}\left(2,\frac{2ic(x+i)}{(c+1)(1-icx)}+1\right) + \log\left(\frac{2}{1-icx}\right)\cot^{-1}(cx)-\frac{1}{2}\log\left(\frac{2ic(-x+i)}{(1-c)(1-icx)}\right)\cot^{-1}(cx) - \frac{1}{2}\log\left(-\frac{2ic(x+i)}{(c+1)(1-icx)}\right)\cot^{-1}(cx)$$

```
[In] Int[ArcCot[c*x]/(x*(1 + x^2)),x]
```

```
[Out] ArcCot[c*x]*Log[2/(1 - I*c*x)] - (ArcCot[c*x]*Log[((2*I)*c*(I - x))/((1 - c
)*(1 - I*c*x))])/2 - (ArcCot[c*x]*Log[((-2*I)*c*(I + x))/((1 + c)*(1 - I*c*
x))])/2 - (I/2)*PolyLog[2, (-I)/(c*x)] + (I/2)*PolyLog[2, I/(c*x)] + (I/2)*
PolyLog[2, 1 - 2/(1 - I*c*x)] - (I/4)*PolyLog[2, 1 - ((2*I)*c*(I - x))/((1
- c)*(1 - I*c*x))] - (I/4)*PolyLog[2, 1 + ((2*I)*c*(I + x))/((1 + c)*(1 - I
*c*x))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5049

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcCot[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\cot^{-1}(cx)}{x} - \frac{x\cot^{-1}(cx)}{1+x^2}\right) dx \\
&= \int \frac{\cot^{-1}(cx)}{x}\,dx - \int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx \\
&= \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{x}\,dx - \frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{x}\,dx - \int \left(-\frac{\cot^{-1}(cx)}{2(i-x)} + \frac{\cot^{-1}(cx)}{2(i+x)}\right) dx \\
&= -\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{cx}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{cx}\right) + \frac{1}{2}\int \frac{\cot^{-1}(cx)}{i-x}\,dx - \frac{1}{2}\int \frac{\cot^{-1}(cx)}{i+x}\,dx \\
&= \cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right) - \frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) \\
&\quad - \frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{cx}\right) \\
&\quad + \frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{cx}\right) + 2\left(\frac{1}{2}c\int \frac{\log\left(\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx\right) \\
&\quad - \frac{1}{2}c\int \frac{\log\left(\frac{2c(i-x)}{(-i+ic)(1-icx)}\right)}{1+c^2x^2}\,dx - \frac{1}{2}c\int \frac{\log\left(\frac{2c(i+x)}{(i+ic)(1-icx)}\right)}{1+c^2x^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right) - \frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) \\
&\quad - \frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{cx}\right) \\
&\quad + \frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{cx}\right) - \frac{1}{4}i\,\text{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right) \\
&\quad - \frac{1}{4}i\,\text{PolyLog}\left(2,1+\frac{2ic(i+x)}{(1+c)(1-icx)}\right) \\
&\quad + 2\left(\frac{1}{2}i\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-icx}\right)\right) \\
&= \cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right) - \frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) \\
&\quad - \frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{cx}\right) \\
&\quad + \frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{cx}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right) \\
&\quad - \frac{1}{4}i\,\text{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right) - \frac{1}{4}i\,\text{PolyLog}\left(2,1+\frac{2ic(i+x)}{(1+c)(1-icx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.47

$$
\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx
$$

$$
= \frac{1}{2}\left(-i\left(\cot^{-1}(cx)\left(\cot^{-1}(cx)+2i\log\left(1+e^{2i\cot^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\cot^{-1}(cx)}\right)\right)\right.
$$

$$
+\frac{(-1+c)(1+c)\left(i\cot^{-1}(cx)^2+2i\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\arctan\left(\frac{\sqrt{c^2}}{cx}\right)-\left(\cot^{-1}(cx)-\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\right)\log\left(\right.\right.}{}
$$

```
[In] Integrate[ArcCot[c*x]/(x*(1 + x^2)),x]
```

```
[Out] ((-I)*(ArcCot[c*x]*(ArcCot[c*x] + (2*I)*Log[1 + E^((2*I)*ArcCot[c*x])]) + P
olyLog[2, -E^((2*I)*ArcCot[c*x])]) + ((-1 + c)*(1 + c)*(I*ArcCot[c*x]^2 + (
2*I)*ArcSin[Sqrt[(1 - c^2)^(-1)]]*ArcTan[Sqrt[c^2]/(c*x)] - (ArcCot[c*x] -
ArcSin[Sqrt[(1 - c^2)^(-1)]])*Log[(-1 + (-1 + 2*Sqrt[c^2])*E^((2*I)*ArcCot[
c*x]) - c^2*(-1 + E^((2*I)*ArcCot[c*x])))/(-1 + c^2)] - (ArcCot[c*x] + ArcS
in[Sqrt[(1 - c^2)^(-1)]])*Log[-((1 + (1 + 2*Sqrt[c^2])*E^((2*I)*ArcCot[c*x]
) + c^2*(-1 + E^((2*I)*ArcCot[c*x])))/(-1 + c^2))] + (I/2)*(PolyLog[2, ((1
```

```
+ c^2 - 2*Sqrt[c^2])*E^((2*I)*ArcCot[c*x]))/(-1 + c^2)] + PolyLog[2, ((1 +
c^2 + 2*Sqrt[c^2])*E^((2*I)*ArcCot[c*x]))/(-1 + c^2)])))/(-1 + c^2))/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

method	result
parts	$\operatorname{arccot}(cx)\ln(x)-\frac{\ln(x^2+1)\operatorname{arccot}(cx)}{2}+\frac{c\left(-\frac{i\ln(x)(\ln(icx+1)-\ln(-icx+1))}{c}-\frac{i(\operatorname{dilog}(icx+1)-\operatorname{dilog}(-icx+1))}{c}-\cdots\right.}{\cdots}$
risch	$-\frac{\pi\ln(c^2x^2+c^2)}{4}+\frac{\pi\ln(-icx)}{2}+\frac{i\ln(-icx+1)\ln\left(\frac{-icx-c}{-c-1}\right)}{4}+\frac{i\operatorname{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4}+\frac{i\ln(-icx+1)\ln\left(\frac{-icx+c}{c-1}\right)}{4}+\cdots$
derivativedivides	$-\frac{\operatorname{arccot}(cx)\ln(c^2x^2+c^2)}{2}+\operatorname{arccot}(cx)\ln(cx)+\frac{c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{c^2}+\frac{i\ln(cx)\ln(-icx+1)}{c^2}-\frac{i\operatorname{dilog}(icx+1)}{c^2}+\cdots\right.}{\cdots}$
default	$-\frac{\operatorname{arccot}(cx)\ln(c^2x^2+c^2)}{2}+\operatorname{arccot}(cx)\ln(cx)+\frac{c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{c^2}+\frac{i\ln(cx)\ln(-icx+1)}{c^2}-\frac{i\operatorname{dilog}(icx+1)}{c^2}+\cdots\right.}{\cdots}$

```
[In] int(arccot(c*x)/x/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arccot(c*x)*ln(x)-1/2*ln(x^2+1)*arccot(c*x)+1/2*c*(-I*ln(x)*(ln(1+I*c*x)-ln
(1-I*c*x))/c-I*(dilog(1+I*c*x)-dilog(1-I*c*x))/c-1/2/c^2*sum(1/_alpha*(ln(x
-_alpha)*ln(x^2+1)-ln(x-_alpha)*ln((_alpha*c+x)/_alpha/(1+c))-ln(x-_alpha)*
ln((_alpha*c-x)/_alpha/(c-1))-dilog((_alpha*c+x)/_alpha/(1+c))-dilog((_alph
a*c-x)/_alpha/(c-1))),_alpha=RootOf(_Z^2*c^2+1)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx=\int \frac{\operatorname{arccot}(cx)}{(x^2+1)x}\,dx$$

```
[In] integrate(arccot(c*x)/x/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arccot(c*x)/(x^3 + x), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx = \int \frac{\operatorname{acot}(cx)}{x(x^2+1)}\,dx$$

```
[In] integrate(acot(c*x)/x/(x**2+1),x)
[Out] Integral(acot(c*x)/(x*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(cx)}{(x^2+1)x}\,dx$$

```
[In] integrate(arccot(c*x)/x/(x^2+1),x, algorithm="maxima")
[Out] integrate(arccot(c*x)/((x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(cx)}{(x^2+1)x}\,dx$$

```
[In] integrate(arccot(c*x)/x/(x^2+1),x, algorithm="giac")
[Out] integrate(arccot(c*x)/((x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx = \int \frac{\operatorname{acot}(c\,x)}{x\,(x^2+1)}\,dx$$

```
[In] int(acot(c*x)/(x*(x^2 + 1)),x)
[Out] int(acot(c*x)/(x*(x^2 + 1)), x)
```

3.50 $\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 212

$$\begin{aligned}\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx =& -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right)\\ &-c\log(x)+\frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right)\\ &-\frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right)\\ &+\frac{1}{2}c\log\left(1+c^2x^2\right)+\frac{1}{4}\operatorname{PolyLog}\left(2,1+\frac{2i(i-cx)}{(1-c)(1-ix)}\right)\\ &-\frac{1}{4}\operatorname{PolyLog}\left(2,1+\frac{2i(i+cx)}{(1+c)(1-ix)}\right)\end{aligned}$$

```
[Out] -arccot(c*x)/x-1/2*I*arctan(x)*ln(1-I/c/x)+1/2*I*arctan(x)*ln(1+I/c/x)-c*ln
(x)+1/2*I*arctan(x)*ln(-2*I*(I-c*x)/(1-c)/(1-I*x))-1/2*I*arctan(x)*ln(-2*I*
(I+c*x)/(1+c)/(1-I*x))+1/2*c*ln(c^2*x^2+1)+1/4*polylog(2,1+2*I*(I-c*x)/(1-c
)/(1-I*x))-1/4*polylog(2,1+2*I*(I+c*x)/(1+c)/(1-I*x))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.267, Rules used = {5039, 4947, 272, 36, 29, 31, 5029, 209, 2520, 266, 6820, 12, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx = -\frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(-cx+i)}{(1-c)(1-ix)}\right) - \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(cx+i)}{(c+1)(1-ix)}\right) + \frac{1}{2}c\log\left(c^2x^2+1\right)+\frac{1}{4}\operatorname{PolyLog}\left(2,\frac{2i(i-cx)}{(1-c)(1-ix)}+1\right) - \frac{1}{4}\operatorname{PolyLog}\left(2,\frac{2i(cx+i)}{(c+1)(1-ix)}+1\right)-c\log(x)-\frac{\cot^{-1}(cx)}{x}$$

```
[In] Int[ArcCot[c*x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcCot[c*x]/x) - (I/2)*ArcTan[x]*Log[1 - I/(c*x)] + (I/2)*ArcTan[x]*Log[1
 + I/(c*x)] - c*Log[x] + (I/2)*ArcTan[x]*Log[((-2*I)*(I - c*x))/((1 - c)*(1
 - I*x))] - (I/2)*ArcTan[x]*Log[((-2*I)*(I + c*x))/((1 + c)*(1 - I*x))] + (
c*Log[1 + c^2*x^2])/2 + PolyLog[2, 1 + ((2*I)*(I - c*x))/((1 - c)*(1 - I*x)
)]/4 - PolyLog[2, 1 + ((2*I)*(I + c*x))/((1 + c)*(1 - I*x))]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5029

```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I/(c*x)]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I/(c*x)]/(d + e
*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(cx)}{x^2}\,dx - \int \frac{\cot^{-1}(cx)}{1+x^2}\,dx$$

$$= -\frac{\cot^{-1}(cx)}{x} - \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{1+x^2}\,dx + \frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{1+x^2}\,dx - c\int \frac{1}{x\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{\cot^{-1}(cx)}{x} - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) - \frac{\int \frac{\arctan(x)}{\left(1-\frac{i}{cx}\right)x^2}\,dx}{2c} - \frac{\int \frac{\arctan(x)}{\left(1+\frac{i}{cx}\right)x^2}\,dx}{2c} - \frac{1}{2}c\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx, x, x^2\right)$$

$$= -\frac{\cot^{-1}(cx)}{x} - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) - \frac{\int \frac{c\arctan(x)}{x(-i+cx)}\,dx}{2c} - \frac{\int \frac{c\arctan(x)}{x(i+cx)}\,dx}{2c} - \frac{1}{2}c\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}c^3\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, x^2\right)$$

$$= -\frac{\cot^{-1}(cx)}{x} - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) - c\log(x) + \frac{1}{2}c\log\left(1+c^2x^2\right) - \frac{1}{2}\int \frac{\arctan(x)}{x(-i+cx)}\,dx - \frac{1}{2}\int \frac{\arctan(x)}{x(i+cx)}\,dx$$

$$= -\frac{\cot^{-1}(cx)}{x} - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) - c\log(x) + \frac{1}{2}c\log\left(1+c^2x^2\right) - \frac{1}{2}\int \left(\frac{i\arctan(x)}{x} - \frac{ic\arctan(x)}{-i+cx}\right)dx - \frac{1}{2}\int \left(-\frac{i\arctan(x)}{x} + \frac{ic\arctan(x)}{i+cx}\right)dx$$

$$= -\frac{\cot^{-1}(cx)}{x} - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) - c\log(x) + \frac{1}{2}c\log\left(1+c^2x^2\right) + \frac{1}{2}(ic)\int \frac{\arctan(x)}{-i+cx}\,dx - \frac{1}{2}(ic)\int \frac{\arctan(x)}{i+cx}\,dx$$

$$= -\frac{\cot^{-1}(cx)}{x} - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) - c\log(x) + \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right) - \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right) + \frac{1}{2}c\log\left(1+c^2x^2\right) - \frac{1}{2}i\int \frac{\log\left(\frac{2(-i+cx)}{(-i+ic)(1-ix)}\right)}{1+x^2}\,dx + \frac{1}{2}i\int \frac{\log\left(\frac{2(i+cx)}{(i+ic)(1-ix)}\right)}{1+x^2}\,dx$$

$$= -\frac{\cot^{-1}(cx)}{x} - \frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right) - c\log(x) + \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right) - \frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right) + \frac{1}{2}c\log\left(1+c^2x^2\right) + \frac{1}{4}\text{PolyLog}\left(2, 1+\frac{2i(i-cx)}{(1-c)(1-ix)}\right) - \frac{1}{4}\text{PolyLog}\left(2, 1+\frac{2i(i+cx)}{(1+c)(1-ix)}\right)$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 619 vs. 2(212) = 424.

Time = 1.42 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.92

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx = -\frac{\cot^{-1}(cx)}{x} - c\log\left(\frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}\right)$$
$$-\frac{c\left(2\arccos\left(\frac{1+c^2}{-1+c^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right) - 4\cot^{-1}(cx)\operatorname{arctanh}\left(\frac{cx}{\sqrt{-c^2}}\right) - \left(\arccos\left(\frac{1+c^2}{-1+c^2}\right) - 2i\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right.\right.\right.}{}$$

```
[In] Integrate[ArcCot[c*x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcCot[c*x]/x) - c*Log[1/Sqrt[1 + 1/(c^2*x^2)]] - (c*(2*ArcCos[(1 + c^2)/
(-1 + c^2)]*ArcTanh[Sqrt[-c^2]/(c*x)] - 4*ArcCot[c*x]*ArcTanh[(c*x)/Sqrt[-c
^2]] - (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)])*Log
[(-2*(c^2 + I*Sqrt[-c^2])*(-I + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))] - (A
rcCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)])*Log[((2*I)*(
I*c^2 + Sqrt[-c^2])*(I + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))] + (ArcCos[(
1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] + (2*I)*ArcTanh[(c*x
)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2])/(Sqrt[-1 + c^2]*E^(I*ArcCot[c*x])*S
qrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + (ArcCos[(1 + c^2)/(-1 + c
^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] - (2*I)*ArcTanh[(c*x)/Sqrt[-c^2]])*L
og[(Sqrt[2]*Sqrt[-c^2]*E^(I*ArcCot[c*x]))/(Sqrt[-1 + c^2]*Sqrt[-1 - c^2 + (
-1 + c^2)*Cos[2*ArcCot[c*x]]])] + I*(-PolyLog[2, ((1 + c^2 - (2*I)*Sqrt[-c^
2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))] + PolyLog[2, ((1 +
 c^2 + (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x)
)])))/(4*Sqrt[-c^2])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\pi \arctan(x)}{2}-\frac{\pi}{2x}+\frac{\ln(-icx+1)\ln\left(\frac{-icx-c}{-c-1}\right)}{4}+\frac{\operatorname{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4}-\frac{\ln(-icx+1)\ln\left(\frac{-icx+c}{c-1}\right)}{4}-\frac{\operatorname{dilog}\left(\frac{-icx+c}{c-1}\right)}{4}$
parts	$-\frac{\operatorname{arccot}(cx)}{x}-\operatorname{arccot}(cx)\arctan(x)+c\left(-\ln(x)+\frac{\ln(c^2x^2+1)}{2}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c}\right.$
derivativedivides	$c\left(-\frac{\operatorname{arccot}(cx)\arctan(x)}{c}-\frac{\operatorname{arccot}(cx)}{cx}+c^3\left(-\frac{\ln(x)-\frac{\ln(c^2x^2+1)}{2}}{c^3}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c^4}-\right.\right.$ ar
default	$c\left(-\frac{\operatorname{arccot}(cx)\arctan(x)}{c}-\frac{\operatorname{arccot}(cx)}{cx}+c^3\left(-\frac{\ln(x)-\frac{\ln(c^2x^2+1)}{2}}{c^3}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c^4}-\right.\right.$ ar

```
[In] int(arccot(c*x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*Pi*arctan(x)-1/2*Pi/x+1/4*ln(1-I*c*x)*ln((-c-I*c*x)/(-c-1))+1/4*dilog(
(-c-I*c*x)/(-c-1))-1/4*ln(1-I*c*x)*ln((c-I*c*x)/(c-1))-1/4*dilog((c-I*c*x)/
(c-1))-1/2*c*ln(-I*c*x)+1/2*c*ln(1-I*c*x)+1/2*I*ln(1-I*c*x)/x+1/4*ln(1+I*c*
x)*ln((-c+I*c*x)/(-c-1))+1/4*dilog((-c+I*c*x)/(-c-1))-1/4*ln(1+I*c*x)*ln((c
+I*c*x)/(c-1))-1/4*dilog((c+I*c*x)/(c-1))-1/2*c*ln(I*c*x)+1/2*c*ln(1+I*c*x)
-1/2*I*ln(1+I*c*x)/x
```

Fricas [F]

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx=\int \frac{\operatorname{arccot}(cx)}{(x^2+1)x^2}\,dx$$

```
[In] integrate(arccot(c*x)/x^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arccot(c*x)/(x^4 + x^2), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(cx)}{x^2 (1+x^2)} \, dx = \int \frac{\operatorname{acot}(cx)}{x^2 (x^2+1)} \, dx$$

```
[In] integrate(acot(c*x)/x**2/(x**2+1),x)
[Out] Integral(acot(c*x)/(x**2*(x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{\cot^{-1}(cx)}{x^2 (1+x^2)} \, dx = & -\left(\frac{1}{x} + \arctan(x)\right) \operatorname{arccot}(cx) - \frac{1}{2} \arctan(cx) \arctan(x) \\ & + \frac{1}{2} \arctan(x) \arctan\left(\frac{cx}{c-1}, -\frac{1}{c-1}\right) + \frac{1}{2} c \log(c^2x^2+1) \\ & - c\log(x) - \frac{1}{8} \log(x^2+1) \log\left(\frac{c^2x^2+1}{c^2+2c+1}\right) \\ & + \frac{1}{8} \log(x^2+1) \log\left(\frac{c^2x^2+1}{c^2-2c+1}\right) - \frac{1}{4} \operatorname{Li}_2\left(\frac{icx+c}{c+1}\right) \\ & - \frac{1}{4} \operatorname{Li}_2\left(-\frac{icx-c}{c+1}\right) + \frac{1}{4} \operatorname{Li}_2\left(\frac{icx+c}{c-1}\right) + \frac{1}{4} \operatorname{Li}_2\left(-\frac{icx-c}{c-1}\right)\end{aligned}$$

```
[In] integrate(arccot(c*x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -(1/x + arctan(x))*arccot(c*x) - 1/2*arctan(c*x)*arctan(x) + 1/2*arctan(x)*
arctan2(c*x/(c - 1), -1/(c - 1)) + 1/2*c*log(c^2*x^2 + 1) - c*log(x) - 1/8*
log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 + 2*c + 1)) + 1/8*log(x^2 + 1)*log((c^2
*x^2 + 1)/(c^2 - 2*c + 1)) - 1/4*dilog((I*c*x + c)/(c + 1)) - 1/4*dilog(-(I
*c*x - c)/(c + 1)) + 1/4*dilog((I*c*x + c)/(c - 1)) + 1/4*dilog(-(I*c*x - c
)/(c - 1))
```

Giac [F]

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(cx)}{(x^2+1)x^2}\,dx$$

```
[In] integrate(arccot(c*x)/x^2/(x^2+1),x, algorithm="giac")
[Out] integrate(arccot(c*x)/((x^2 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx = \int \frac{\operatorname{acot}(c\,x)}{x^2\,(x^2+1)}\,dx$$

```
[In] int(acot(c*x)/(x^2*(x^2 + 1)),x)
[Out] int(acot(c*x)/(x^2*(x^2 + 1)), x)
```

3.51 $\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 5

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\cot^{-1}(x)\right)$$

```
[Out] -ln(arccot(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5003}

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\cot^{-1}(x)\right)$$

```
[In] Int[1/((1 + x^2)*ArcCot[x]),x]
```

```
[Out] -Log[ArcCot[x]]
```

Rule 5003

```
Int[1/(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[-Log[RemoveContent[a + b*ArcCot[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\text{integral} = -\log\left(\cot^{-1}(x)\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\cot^{-1}(x)\right)$$

```
[In] Integrate[1/((1 + x^2)*ArcCot[x]),x]
[Out] -Log[ArcCot[x]]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\ln\left(\operatorname{arccot}\left(x\right)\right)$	6
default	$-\ln\left(\operatorname{arccot}\left(x\right)\right)$	6
parallelrisch	$-\ln\left(\operatorname{arccot}\left(x\right)\right)$	6
risch	$-\ln\left(\ln\left(ix+1\right)+i(i\ln\left(-ix+1\right)-\pi)\right)$	29

```
[In] int(1/(x^2+1)/arccot(x),x,method=_RETURNVERBOSE)
[Out] -ln(arccot(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\operatorname{arccot}\left(x\right)\right)$$

```
[In] integrate(1/(x^2+1)/arccot(x),x, algorithm="fricas")
[Out] -log(arccot(x))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\operatorname{acot}\left(x\right)\right)$$

```
[In] integrate(1/(x**2+1)/acot(x),x)
[Out] -log(acot(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\operatorname{arccot}\left(x\right)\right)$$

```
[In] integrate(1/(x^2+1)/arccot(x),x, algorithm="maxima")
[Out] -log(arccot(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\left|\arctan\left(\frac{1}{x}\right)\right|\right)$$

```
[In] integrate(1/(x^2+1)/arccot(x),x, algorithm="giac")
[Out] -log(abs(arctan(1/x)))
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\ln\left(\operatorname{acot}(x)\right)$$

```
[In] int(1/(acot(x)*(x^2 + 1)),x)
[Out] -log(acot(x))
```

3.52 $\int \frac{\cot^{-1}(x)^n}{1+x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\cot^{-1}(x)^n}{1+x^2} \, dx = -\frac{\cot^{-1}(x)^{1+n}}{1+n}$$

```
[Out] -arccot(x)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5005}

$$\int \frac{\cot^{-1}(x)^n}{1+x^2} \, dx = -\frac{\cot^{-1}(x)^{n+1}}{n+1}$$

```
[In] Int[ArcCot[x]^n/(1 + x^2),x]
[Out] -(ArcCot[x]^(1 + n)/(1 + n))
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{\cot^{-1}(x)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\frac{\cot^{-1}(x)^{1+n}}{1+n}$$

```
[In] Integrate[ArcCot[x]^n/(1 + x^2),x]
[Out] -(ArcCot[x]^(1 + n)/(1 + n))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\operatorname{arccot}(x)^{n+1}}{n+1}$	14
default	$-\frac{\operatorname{arccot}(x)^{n+1}}{n+1}$	14
risch	$-\frac{(\pi-i\ln(-i(i+x))+i\ln(-i(i-x)))(\pi-i\ln(-i(i+x))+i\ln(-i(i-x)))^{n}\left(\frac{1}{2}\right)^{n}}{2(n+1)}$	65

```
[In] int(arccot(x)^n/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -arccot(x)^(n+1)/(n+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\frac{\operatorname{arccot}(x)^n \operatorname{arccot}(x)}{n+1}$$

```
[In] integrate(arccot(x)^n/(x^2+1),x, algorithm="fricas")
[Out] -arccot(x)^n*arccot(x)/(n + 1)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\begin{cases} \frac{\operatorname{acot}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{acot}(x)) & \text{otherwise} \end{cases}$$

```
[In] integrate(acot(x)**n/(x**2+1),x)
[Out] -Piecewise((acot(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(acot(x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\frac{\operatorname{arccot}(x)^{n+1}}{n+1}$$

```
[In] integrate(arccot(x)^n/(x^2+1),x, algorithm="maxima")
[Out] -arccot(x)^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\frac{\arctan\left(\frac{1}{x}\right)^{n+1}}{n+1}$$

```
[In] integrate(arccot(x)^n/(x^2+1),x, algorithm="giac")
[Out] -arctan(1/x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\frac{\operatorname{acot}(x)^{n+1}}{n+1}$$

```
[In] int(acot(x)^n/(x^2 + 1),x)
[Out] -acot(x)^(n + 1)/(n + 1)
```

3.53 $\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 244

$$\begin{aligned}\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx &= \frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2-180a^2cd+35d^2)\,x^4}{1260a^5}\\&\quad+\frac{(36a^2c-7d)\,d^3x^6}{378a^3}+\frac{d^4x^8}{72a}+c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)\\&\quad+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\frac{1}{9}d^4x^9\cot^{-1}(ax)\\&\quad+\frac{(315a^8c^4-420a^6c^3d+378a^4c^2d^2-180a^2cd^3+35d^4)\log\left(1+a^2x^2\right)}{630a^9}\end{aligned}$$

```
[Out] 1/630*d*(420*a^6*c^3-378*a^4*c^2*d+180*a^2*c*d^2-35*d^3)*x^2/a^7+1/1260*d^2
*(378*a^4*c^2-180*a^2*c*d+35*d^2)*x^4/a^5+1/378*(36*a^2*c-7*d)*d^3*x^6/a^3+
1/72*d^4*x^8/a+c^4*x*arccot(a*x)+4/3*c^3*d*x^3*arccot(a*x)+6/5*c^2*d^2*x^5*
arccot(a*x)+4/7*c*d^3*x^7*arccot(a*x)+1/9*d^4*x^9*arccot(a*x)+1/630*(315*a^
8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+35*d^4)*ln(a^2*x^2+1)/a^9
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{200, 5033, 1824, 266\}$

$$\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx$$
$$= \frac{d^3x^6(36a^2c-7d)}{378a^3} + \frac{d^2x^4(378a^4c^2-180a^2cd+35d^2)}{1260a^5} + \frac{dx^2(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)}{630a^7} + \frac{(315a^8c^4-420a^6c^3d+378a^4c^2d^2-180a^2cd^3+35d^4)\log\left(a^2x^2+1\right)}{630a^9} + c^4x\cot^{-1}(ax) + \frac{4}{3}c^3dx^3\cot^{-1}(ax) + \frac{6}{5}c^2d^2x^5\cot^{-1}(ax) + \frac{4}{7}cd^3x^7\cot^{-1}(ax) + \frac{1}{9}d^4x^9\cot^{-1}(ax) + \frac{d^4x^8}{72a}$$

```
[In] Int[(c + d*x^2)^4*ArcCot[a*x],x]
```

```
[Out] (d*(420*a^6*c^3 - 378*a^4*c^2*d + 180*a^2*c*d^2 - 35*d^3)*x^2)/(630*a^7) +
(d^2*(378*a^4*c^2 - 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) + ((36*a^2*c - 7*
d)*d^3*x^6)/(378*a^3) + (d^4*x^8)/(72*a) + c^4*x*ArcCot[a*x] + (4*c^3*d*x^3
*ArcCot[a*x])/3 + (6*c^2*d^2*x^5*ArcCot[a*x])/5 + (4*c*d^3*x^7*ArcCot[a*x])
/7 + (d^4*x^9*ArcCot[a*x])/9 + ((315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*
d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/(630*a^9)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\text{integral} = c^4 x \cot^{-1}(ax) + \frac{4}{3}c^3 d x^3 \cot^{-1}(ax) + \frac{6}{5}c^2 d^2 x^5 \cot^{-1}(ax) + \frac{4}{7}c d^3 x^7 \cot^{-1}(ax) + \frac{1}{9}d^4 x^9 \cot^{-1}(ax) + a\int \frac{c^4 x + \frac{4}{3}c^3 d x^3 + \frac{6}{5}c^2 d^2 x^5 + \frac{4}{7}c d^3 x^7 + \frac{d^4 x^9}{9}}{1+a^2x^2}\,dx$$

$$= c^4 x \cot^{-1}(ax) + \frac{4}{3}c^3 d x^3 \cot^{-1}(ax) + \frac{6}{5}c^2 d^2 x^5 \cot^{-1}(ax) + \frac{4}{7}c d^3 x^7 \cot^{-1}(ax) + \frac{1}{9}d^4 x^9 \cot^{-1}(ax) + a\int \left(\frac{d(420a^6c^3 - 378a^4c^2d + 180a^2cd^2 - 35d^3)\,x}{315a^8} + \frac{d^2(378a^4c^2 - 180a^2cd + 35d^2)\,x^3}{315a^6} + \frac{(36a^2c - 7d)\,d^3x^5}{63a^4} + \frac{d^4x^7}{9a^2} + \frac{(315a^8c^4 - 420a^6c^3d + 378a^4c^2d^2 - 180a^2cd^3 + 35d^4)\,x}{315a^8\,(1+a^2x^2)}\right) dx$$

$$= \frac{d(420a^6c^3 - 378a^4c^2d + 180a^2cd^2 - 35d^3)\,x^2}{630a^7} + \frac{d^2(378a^4c^2 - 180a^2cd + 35d^2)\,x^4}{1260a^5} + \frac{(36a^2c - 7d)\,d^3x^6}{378a^3} + \frac{d^4x^8}{72a} + c^4 x \cot^{-1}(ax) + \frac{4}{3}c^3 d x^3 \cot^{-1}(ax) + \frac{6}{5}c^2 d^2 x^5 \cot^{-1}(ax) + \frac{4}{7}c d^3 x^7 \cot^{-1}(ax) + \frac{1}{9}d^4 x^9 \cot^{-1}(ax) + \frac{(315a^8c^4 - 420a^6c^3d + 378a^4c^2d^2 - 180a^2cd^3 + 35d^4)\int \frac{x}{1+a^2x^2}\,dx}{315a^7}$$

$$= \frac{d(420a^6c^3 - 378a^4c^2d + 180a^2cd^2 - 35d^3)\,x^2}{630a^7} + \frac{d^2(378a^4c^2 - 180a^2cd + 35d^2)\,x^4}{1260a^5} + \frac{(36a^2c - 7d)\,d^3x^6}{378a^3} + \frac{d^4x^8}{72a} + c^4 x \cot^{-1}(ax) + \frac{4}{3}c^3 d x^3 \cot^{-1}(ax) + \frac{6}{5}c^2 d^2 x^5 \cot^{-1}(ax) + \frac{4}{7}c d^3 x^7 \cot^{-1}(ax) + \frac{1}{9}d^4 x^9 \cot^{-1}(ax) + \frac{(315a^8c^4 - 420a^6c^3d + 378a^4c^2d^2 - 180a^2cd^3 + 35d^4)\log(1+a^2x^2)}{630a^9}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

$$\int (c+dx^2)^4 \cot^{-1}(ax)\,dx = \frac{a^2dx^2(-420d^3 + 30a^2d^2(72c + 7dx^2) - 4a^4d(1134c^2 + 270cdx^2 + 35d^2x^4) + 3a^6(1680c^3 + 756c^2dx^2 + 240$$

```
[In] Integrate[(c + d*x^2)^4*ArcCot[a*x],x]
[Out] (a^2*d*x^2*(-420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) - 4*a^4*d*(1134*c^2 + 27
0*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4 +
```

```
35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c
*d^3*x^6 + 35*d^4*x^8)*ArcCot[a*x] + 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*
a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/(7560*a^9)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00

method	result
parts	$\frac{d^4x^9 \operatorname{arccot}(ax)}{9} + \frac{4c\,d^3x^7 \operatorname{arccot}(ax)}{7} + \frac{6c^2d^2x^5 \operatorname{arccot}(ax)}{5} + \frac{4c^3d\,x^3 \operatorname{arccot}(ax)}{3} + c^4x \operatorname{arccot}(ax) + \frac{a\left(\frac{d(}{}\right.}{}$
derivativedivides	$\frac{\operatorname{arccot}(ax)c^4ax + \frac{4a \operatorname{arccot}(ax)c^3d\,x^3}{3} + \frac{6a \operatorname{arccot}(ax)c^2d^2x^5}{5} + \frac{4a \operatorname{arccot}(ax)c\,d^3x^7}{7} + \frac{a \operatorname{arccot}(ax)d^4x^9}{9} + \frac{210c^3a^8d\,x^2 + \frac{189c^2a^8d^2}{2}}{}}{}$
default	$\frac{\operatorname{arccot}(ax)c^4ax + \frac{4a \operatorname{arccot}(ax)c^3d\,x^3}{3} + \frac{6a \operatorname{arccot}(ax)c^2d^2x^5}{5} + \frac{4a \operatorname{arccot}(ax)c\,d^3x^7}{7} + \frac{a \operatorname{arccot}(ax)d^4x^9}{9} + \frac{210c^3a^8d\,x^2 + \frac{189c^2a^8d^2}{2}}{}}{}$
parallelrisch	$\frac{840x^9 \operatorname{arccot}(ax)a^9d^4 + 4320x^7 \operatorname{arccot}(ax)a^9c\,d^3 + 105d^4a^8x^8 + 9072x^5 \operatorname{arccot}(ax)a^9c^2d^2 + 720c\,a^8d^3x^6 + 10080x^3 \operatorname{arccot}(a}{}$
risch	$-\frac{d^4x^6}{54a^3} + \frac{d^4x^4}{36a^5} - \frac{d^4x^2}{18a^7} + \frac{\ln(-a^2x^2-1)c^4}{2a} + \frac{\ln(-a^2x^2-1)d^4}{18a^9} + \frac{\pi d^4x^9}{18} + \frac{\pi c^4x}{2} + \frac{2c\,d^3x^6}{21a} + \frac{3c^2d^2x^4}{10a} + \frac{2c}{}$

```
[In] int((d*x^2+c)^4*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*d^4*x^9*arccot(a*x)+4/7*c*d^3*x^7*arccot(a*x)+6/5*c^2*d^2*x^5*arccot(a*
x)+4/3*c^3*d*x^3*arccot(a*x)+c^4*x*arccot(a*x)+1/315*a*(1/2*d/a^8*(35/4*a^6
*d^3*x^8+60*a^6*c*d^2*x^6+189*a^6*c^2*d*x^4+420*a^6*c^3*x^2-35/3*a^4*d^3*x^
6-90*a^4*c*d^2*x^4-378*a^4*c^2*d*x^2+35/2*a^2*d^3*x^4+180*a^2*c*d^2*x^2-35*
d^3*x^2)+1/2*(315*a^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+35*d^
4)/a^10*ln(a^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97

$$\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx$$
$$= \frac{105\,a^8d^4x^8 + 20\,(36\,a^8cd^3 - 7\,a^6d^4)x^6 + 6\,(378\,a^8c^2d^2 - 180\,a^6cd^3 + 35\,a^4d^4)x^4 + 12\,(420\,a^8c^3d - 378\,a^6c^2d}{}$$

```
[In] integrate((d*x^2+c)^4*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 - 7*a^6*d^4)*x^6 + 6*(378*a^8*c^
2*d^2 - 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d - 378*a^6*c^2*d
^2 + 180*a^4*c*d^3 - 35*a^2*d^4)*x^2 + 24*(35*a^9*d^4*x^9 + 180*a^9*c*d^3*x
```

```
^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*arccot(a*x) +
 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4
)*log(a^2*x^2 + 1))/a^9
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.50

$$\int (c+dx^2)^4 \cot^{-1}(ax)\,dx = \begin{cases} c^4x\,\mathrm{acot}\,(ax) + \frac{4c^3dx^3\,\mathrm{acot}\,(ax)}{3} + \frac{6c^2d^2x^5\,\mathrm{acot}\,(ax)}{5} + \frac{4cd^3x^7\,\mathrm{acot}\,(ax)}{7} + \frac{d^4x^9\,\mathrm{acot}\,(ax)}{9} + \frac{c^4\log\left(x^2+\frac{1}{a^2}\right)}{2a} + \frac{2c^3dx^2}{3a} + \frac{3c^2 \\ \frac{\pi\left(c^4x+\frac{4c^3dx^3}{3}+\frac{6c^2d^2x^5}{5}+\frac{4cd^3x^7}{7}+\frac{d^4x^9}{9}\right)}{2} \end{cases}$$

```
[In] integrate((d*x**2+c)**4*acot(a*x),x)
```

```
[Out] Piecewise((c**4*x*acot(a*x) + 4*c**3*d*x**3*acot(a*x)/3 + 6*c**2*d**2*x**5*
acot(a*x)/5 + 4*c*d**3*x**7*acot(a*x)/7 + d**4*x**9*acot(a*x)/9 + c**4*log(
x**2 + a**(-2))/(2*a) + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*x**4/(10*a) + 2*c
*d**3*x**6/(21*a) + d**4*x**8/(72*a) - 2*c**3*d*log(x**2 + a**(-2))/(3*a**3
) - 3*c**2*d**2*x**2/(5*a**3) - c*d**3*x**4/(7*a**3) - d**4*x**6/(54*a**3)
+ 3*c**2*d**2*log(x**2 + a**(-2))/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*
x**4/(36*a**5) - 2*c*d**3*log(x**2 + a**(-2))/(7*a**7) - d**4*x**2/(18*a**7
) + d**4*log(x**2 + a**(-2))/(18*a**9), Ne(a, 0)), (pi*(c**4*x + 4*c**3*d*x
**3/3 + 6*c**2*d**2*x**5/5 + 4*c*d**3*x**7/7 + d**4*x**9/9)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int (c+dx^2)^4 \cot^{-1}(ax)\,dx = \frac{1}{7560}\,a\left(\frac{105\,a^6d^4x^8 + 20\,(36\,a^6cd^3 - 7\,a^4d^4)x^6 + 6\,(378\,a^6c^2d^2 - 180\,a^4cd^3 + 35\,a^2d^4)x^4 + 12\,(420\,a^6c^3d -}{a^8}\right. + \frac{1}{315}\left(35\,d^4x^9 + 180\,cd^3x^7 + 378\,c^2d^2x^5 + 420\,c^3dx^3 + 315\,c^4x\right)\mathrm{arccot}\,(ax)$$

```
[In] integrate((d*x^2+c)^4*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 - 7*a^4*d^4)*x^6 + 6*(378*a^6
*c^2*d^2 - 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d - 378*a^4*c^
2*d^2 + 180*a^2*c*d^3 - 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 - 420*a^6*c^3*d
+ 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 + 1)/a^10) + 1/315*
(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c^4*x)*
arccot(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.42

$$\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx$$
$$=\frac{1}{7560}\left(\frac{24\left(35\,d^4+\frac{180\,cd^3}{x^2}+\frac{378\,c^2d^2}{x^4}+\frac{420\,c^3d}{x^6}+\frac{315\,c^4}{x^8}\right)x^9\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(105\,d^4+\frac{720\,cd^3}{x^2}+\frac{2268\,c^2d^2}{x^4}-\frac{140\,d^4}{a^2x^2}\right.}{}\right.$$

```
[In] integrate((d*x^2+c)^4*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/7560*(24*(35*d^4 + 180*c*d^3/x^2 + 378*c^2*d^2/x^4 + 420*c^3*d/x^6 + 315*
c^4/x^8)*x^9*arctan(1/(a*x))/a + (105*d^4 + 720*c*d^3/x^2 + 2268*c^2*d^2/x^
4 - 140*d^4/(a^2*x^2) + 5040*c^3*d/x^6 - 1080*c*d^3/(a^2*x^4) + 7875*c^4/x^
8 - 4536*c^2*d^2/(a^2*x^6) + 210*d^4/(a^4*x^4) - 10500*c^3*d/(a^2*x^8) + 21
60*c*d^3/(a^4*x^6) + 9450*c^2*d^2/(a^4*x^8) - 420*d^4/(a^6*x^6) - 4500*c*d^
3/(a^6*x^8) + 875*d^4/(a^8*x^8))*x^8/a^2 + 12*(315*a^8*c^4 - 420*a^6*c^3*d
+ 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(1/(a^2*x^2) + 1)/a^10 - 12*
(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*lo
g(1/(a^2*x^2))/a^10)*a
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

$$\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx$$
$$=\operatorname{acot}(a\,x)\left(c^4\,x+\frac{4\,c^3\,d\,x^3}{3}+\frac{6\,c^2\,d^2\,x^5}{5}+\frac{4\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{9}\right)$$
$$-x^2\left(\frac{\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{a^2}+\frac{6\,c^2\,d^2}{5\,a}}{2\,a^2}-\frac{2\,c^3\,d}{3\,a}\right)-x^6\left(\frac{d^4}{54\,a^3}-\frac{2\,c\,d^3}{21\,a}\right)+x^4\left(\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{4\,a^2}+\frac{3\,c^2\,d^2}{10\,a}\right)$$
$$+\frac{\ln\left(a^2\,x^2+1\right)\,\left(315\,a^8\,c^4-420\,a^6\,c^3\,d+378\,a^4\,c^2\,d^2-180\,a^2\,c\,d^3+35\,d^4\right)}{630\,a^9}+\frac{d^4\,x^8}{72\,a}$$

```
[In] int(acot(a*x)*(c + d*x^2)^4,x)
```

```
[Out] acot(a*x)*(c^4*x + (d^4*x^9)/9 + (4*c^3*d*x^3)/3 + (4*c*d^3*x^7)/7 + (6*c^2
*d^2*x^5)/5) - x^2*(((d^4/(9*a^3) - (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5*a
))/(2*a^2) - (2*c^3*d)/(3*a)) - x^6*(d^4/(54*a^3) - (2*c*d^3)/(21*a)) + x^4
*((d^4/(9*a^3) - (4*c*d^3)/(7*a))/(4*a^2) + (3*c^2*d^2)/(10*a)) + (log(a^2*
x^2 + 1)*(35*d^4 + 315*a^8*c^4 - 180*a^2*c*d^3 - 420*a^6*c^3*d + 378*a^4*c^
2*d^2))/(630*a^9) + (d^4*x^8)/(72*a)
```

3.54 $\int (c+dx^2)^3 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 168

$$\int (c+dx^2)^3 \cot^{-1}(ax)\,dx = \frac{d(35a^4c^2-21a^2cd+5d^2)\,x^2}{70a^5}+\frac{(21a^2c-5d)\,d^2x^4}{140a^3} + \frac{d^3x^6}{42a}+c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax) + \frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax) + \frac{(35a^6c^3-35a^4c^2d+21a^2cd^2-5d^3)\log(1+a^2x^2)}{70a^7}$$

```
[Out] 1/70*d*(35*a^4*c^2-21*a^2*c*d+5*d^2)*x^2/a^5+1/140*(21*a^2*c-5*d)*d^2*x^4/a
^3+1/42*d^3*x^6/a+c^3*x*arccot(a*x)+c^2*d*x^3*arccot(a*x)+3/5*c*d^2*x^5*arc
cot(a*x)+1/7*d^3*x^7*arccot(a*x)+1/70*(35*a^6*c^3-35*a^4*c^2*d+21*a^2*c*d^2
-5*d^3)*ln(a^2*x^2+1)/a^7
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {200, 5033, 1824, 266}

$$\int (c+dx^2)^3 \cot^{-1}(ax)\,dx = \frac{d^2x^4(21a^2c-5d)}{140a^3}+\frac{dx^2(35a^4c^2-21a^2cd+5d^2)}{70a^5} + \frac{(35a^6c^3-35a^4c^2d+21a^2cd^2-5d^3)\log(a^2x^2+1)}{70a^7} + c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax) + \frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)+\frac{d^3x^6}{42a}$$

```
[In] Int[(c + d*x^2)^3*ArcCot[a*x],x]
```

```
[Out] (d*(35*a^4*c^2 - 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + ((21*a^2*c - 5*d)*d^2*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcCot[a*x] + c^2*d*x^3*ArcCot[a*
x] + (3*c*d^2*x^5*ArcCot[a*x])/5 + (d^3*x^7*ArcCot[a*x])/7 + ((35*a^6*c^3 -
 35*a^4*c^2*d + 21*a^2*c*d^2 - 5*d^3)*Log[1 + a^2*x^2])/(70*a^7)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)\\
&\quad+\frac{1}{7}d^3x^7\cot^{-1}(ax)+a\int\frac{c^3x+c^2dx^3+\frac{3}{5}cd^2x^5+\frac{d^3x^7}{7}}{1+a^2x^2}\,dx\\
&= c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)\\
&\quad+a\int\left(\frac{d(35a^4c^2-21a^2cd+5d^2)\,x}{35a^6}+\frac{(21a^2c-5d)\,d^2x^3}{35a^4}+\frac{d^3x^5}{7a^2}\right.\\
&\qquad\left.+\frac{(35a^6c^3-35a^4c^2d+21a^2cd^2-5d^3)\,x}{35a^6\,(1+a^2x^2)}\right)dx\\
&= \frac{d(35a^4c^2-21a^2cd+5d^2)\,x^2}{70a^5}+\frac{(21a^2c-5d)\,d^2x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)\\
&\quad+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)+\frac{(35a^6c^3-35a^4c^2d+21a^2cd^2-5d^3)\int\frac{x}{1+a^2x^2}\,dx}{35a^5}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{d(35a^4c^2 - 21a^2cd + 5d^2)\,x^2}{70a^5} + \frac{(21a^2c - 5d)\,d^2x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x\cot^{-1}(ax) \\
&\quad + c^2dx^3\cot^{-1}(ax) + \frac{3}{5}cd^2x^5\cot^{-1}(ax) + \frac{1}{7}d^3x^7\cot^{-1}(ax) \\
&\quad + \frac{(35a^6c^3 - 35a^4c^2d + 21a^2cd^2 - 5d^3)\log(1 + a^2x^2)}{70a^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

$$
\int (c + dx^2)^3 \cot^{-1}(ax)\,dx
$$
$$
= \frac{a^2dx^2(30d^2 - 3a^2d(42c + 5dx^2) + a^4(210c^2 + 63cdx^2 + 10d^2x^4)) + 12a^7x(35c^3 + 35c^2dx^2 + 21cd^2x^4 + 5a}{420a^7}
$$

```
[In] Integrate[(c + d*x^2)^3*ArcCot[a*x],x]
```

```
[Out] (a^2*d*x^2*(30*d^2 - 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2 +
 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6)
*ArcCot[a*x] + 6*(35*a^6*c^3 - 35*a^4*c^2*d + 21*a^2*c*d^2 - 5*d^3)*Log[1 +
 a^2*x^2])/(420*a^7)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
parts	$\frac{d^3x^7\operatorname{arccot}(ax)}{7} + \frac{3c\,d^2x^5\operatorname{arccot}(ax)}{5} + c^2d\,x^3\operatorname{arccot}(ax) + c^3x\operatorname{arccot}(ax) + \frac{a\left(\frac{d\left(\frac{5}{3}a^4d^2x^6 + \frac{21}{2}a^4c\right.}{}\right.}{}$
derivativedivides	$\frac{\operatorname{arccot}(ax)c^3ax + a\operatorname{arccot}(ax)c^2d\,x^3 + \frac{3a\operatorname{arccot}(ax)c\,d^2x^5}{5} + \frac{a\operatorname{arccot}(ax)d^3x^7}{7} + \frac{\frac{35c^2a^6d\,x^2}{2} + \frac{21c\,a^6d^2x^4}{4} - \frac{21c\,a^4d^2x^2}{2} + \frac{5d^3a}{6}}{}}{a}$
default	$\frac{\operatorname{arccot}(ax)c^3ax + a\operatorname{arccot}(ax)c^2d\,x^3 + \frac{3a\operatorname{arccot}(ax)c\,d^2x^5}{5} + \frac{a\operatorname{arccot}(ax)d^3x^7}{7} + \frac{\frac{35c^2a^6d\,x^2}{2} + \frac{21c\,a^6d^2x^4}{4} - \frac{21c\,a^4d^2x^2}{2} + \frac{5d^3a}{6}}{}}{a}$
parallelrisch	$\frac{60x^7\operatorname{arccot}(ax)a^7d^3 + 252x^5\operatorname{arccot}(ax)a^7c\,d^2 + 10d^3a^6x^6 + 420x^3\operatorname{arccot}(ax)a^7c^2d + 63c\,a^6d^2x^4 + 420x\operatorname{arccot}(ax)a^7c^3 -}{}$
risch	$-\frac{ic^2d\,x^3\ln(-iax+1)}{2} - \frac{ic^3x\ln(-iax+1)}{2} + \frac{\pi\,d^3x^7}{14} - \frac{3ic\,d^2x^5\ln(-iax+1)}{10} + \frac{3\pi c\,d^2x^5}{10} + \frac{i(5d^3x^7 + 21c\,d^2x^5 +}{}$

```
[In] int((d*x^2+c)^3*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*d^3*x^7*arccot(a*x)+3/5*c*d^2*x^5*arccot(a*x)+c^2*d*x^3*arccot(a*x)+c^3
*x*arccot(a*x)+1/35*a*(1/2*d/a^6*(5/3*a^4*d^2*x^6+21/2*a^4*c*d*x^4+35*a^4*c
^2*x^2-5/2*a^2*d^2*x^4-21*a^2*c*d*x^2+5*d^2*x^2)+1/2*(35*a^6*c^3-35*a^4*c^2
*d+21*a^2*c*d^2-5*d^3)/a^8*ln(a^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int (c+dx^2)^3 \cot^{-1}(ax)\,dx$$
$$= \frac{10\,a^6d^3x^6 + 3\,(21\,a^6cd^2 - 5\,a^4d^3)x^4 + 6\,(35\,a^6c^2d - 21\,a^4cd^2 + 5\,a^2d^3)x^2 + 12\,(5\,a^7d^3x^7 + 21\,a^7cd^2x^5 + 35\,c}{420\,a^7}$$

```
[In] integrate((d*x^2+c)^3*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 - 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
- 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 12*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35
*a^7*c^2*d*x^3 + 35*a^7*c^3*x)*arccot(a*x) + 6*(35*a^6*c^3 - 35*a^4*c^2*d +
 21*a^2*c*d^2 - 5*d^3)*log(a^2*x^2 + 1))/a^7
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.45

$$\int (c+dx^2)^3 \cot^{-1}(ax)\,dx$$
$$= \begin{cases} c^3x\,\mathrm{acot}\,(ax) + c^2dx^3\,\mathrm{acot}\,(ax) + \frac{3cd^2x^5\,\mathrm{acot}\,(ax)}{5} + \frac{d^3x^7\,\mathrm{acot}\,(ax)}{7} + \frac{c^3\log\left(x^2+\frac{1}{a^2}\right)}{2a} + \frac{c^2dx^2}{2a} + \frac{3cd^2x^4}{20a} + \frac{d^3x^6}{42a} - \frac{c^2d\,l}{} \\ \frac{\pi\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)}{2} \end{cases}$$

```
[In] integrate((d*x**2+c)**3*acot(a*x),x)
```

```
[Out] Piecewise((c**3*x*acot(a*x) + c**2*d*x**3*acot(a*x) + 3*c*d**2*x**5*acot(a*
x)/5 + d**3*x**7*acot(a*x)/7 + c**3*log(x**2 + a**(-2))/(2*a) + c**2*d*x**2
/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) - c**2*d*log(x**2 + a**(-2
))/(2*a**3) - 3*c*d**2*x**2/(10*a**3) - d**3*x**4/(28*a**3) + 3*c*d**2*log(
x**2 + a**(-2))/(10*a**5) + d**3*x**2/(14*a**5) - d**3*log(x**2 + a**(-2))/
(14*a**7), Ne(a, 0)), (pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x*
*7/7)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int \left(c+dx^2\right)^3 \cot^{-1}(ax)\,dx$$
$$= \frac{1}{420}\,a\left(\frac{10\,a^4d^3x^6+3\,(21\,a^4cd^2-5\,a^2d^3)x^4+6\,(35\,a^4c^2d-21\,a^2cd^2+5\,d^3)x^2}{a^6}+\frac{6\,(35\,a^6c^3-35\,a^4c^2d+2\right.$$
$$+\frac{1}{35}\left(5\,d^3x^7+21\,cd^2x^5+35\,c^2dx^3+35\,c^3x\right)\operatorname{arccot}(ax)$$

```
[In] integrate((d*x^2+c)^3*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 - 5*a^2*d^3)*x^4 + 6*(35*a^4*c^2
*d - 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 - 35*a^4*c^2*d + 21*a^2
*c*d^2 - 5*d^3)*log(a^2*x^2 + 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35
*c^2*d*x^3 + 35*c^3*x)*arccot(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.50

$$\int \left(c+dx^2\right)^3 \cot^{-1}(ax)\,dx$$
$$= \frac{1}{420}\left(\frac{12\left(5\,d^3+\frac{21\,cd^2}{x^2}+\frac{35\,c^2d}{x^4}+\frac{35\,c^3}{x^6}\right)x^7\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(10\,d^3+\frac{63\,cd^2}{x^2}+\frac{210\,c^2d}{x^4}-\frac{15\,d^3}{a^2x^2}+\frac{385\,c^3}{x^6}-\frac{126\,cd^2}{a^2x^4}-\right.}{a^2}\right.$$

```
[In] integrate((d*x^2+c)^3*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/420*(12*(5*d^3 + 21*c*d^2/x^2 + 35*c^2*d/x^4 + 35*c^3/x^6)*x^7*arctan(1/(
a*x))/a + (10*d^3 + 63*c*d^2/x^2 + 210*c^2*d/x^4 - 15*d^3/(a^2*x^2) + 385*c
^3/x^6 - 126*c*d^2/(a^2*x^4) - 385*c^2*d/(a^2*x^6) + 30*d^3/(a^4*x^4) + 231
*c*d^2/(a^4*x^6) - 55*d^3/(a^6*x^6))*x^6/a^2 + 6*(35*a^6*c^3 - 35*a^4*c^2*d
 + 21*a^2*c*d^2 - 5*d^3)*log(1/(a^2*x^2) + 1)/a^8 - 6*(35*a^6*c^3 - 35*a^4*
c^2*d + 21*a^2*c*d^2 - 5*d^3)*log(1/(a^2*x^2))/a^8)*a
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.13

$$\int \left(c+dx^2\right)^3 \cot^{-1}(ax)\,dx = c^3\,x\,\mathrm{acot}(a\,x)+\frac{d^3\,x^7\,\mathrm{acot}(a\,x)}{7}+\frac{c^3\,\ln\left(a^2\,x^2+1\right)}{2\,a}-\frac{d^3\,\ln\left(a^2\,x^2+1\right)}{14\,a^7}+\frac{d^3\,x^6}{42\,a}-\frac{d^3\,x^4}{28\,a^3}+\frac{d^3\,x^2}{14\,a^5}-\frac{c^2\,d\,\ln\left(a^2\,x^2+1\right)}{2\,a^3}+\frac{3\,c\,d^2\,\ln\left(a^2\,x^2+1\right)}{10\,a^5}+\frac{c^2\,d\,x^2}{2\,a}+\frac{3\,c\,d^2\,x^4}{20\,a}-\frac{3\,c\,d^2\,x^2}{10\,a^3}+c^2\,d\,x^3\,\mathrm{acot}(a\,x)+\frac{3\,c\,d^2\,x^5\,\mathrm{acot}(a\,x)}{5}$$

```
[In] int(acot(a*x)*(c + d*x^2)^3,x)
```

```
[Out] c^3*x*acot(a*x) + (d^3*x^7*acot(a*x))/7 + (c^3*log(a^2*x^2 + 1))/(2*a) - (d
^3*log(a^2*x^2 + 1))/(14*a^7) + (d^3*x^6)/(42*a) - (d^3*x^4)/(28*a^3) + (d^
3*x^2)/(14*a^5) - (c^2*d*log(a^2*x^2 + 1))/(2*a^3) + (3*c*d^2*log(a^2*x^2 +
 1))/(10*a^5) + (c^2*d*x^2)/(2*a) + (3*c*d^2*x^4)/(20*a) - (3*c*d^2*x^2)/(1
0*a^3) + c^2*d*x^3*acot(a*x) + (3*c*d^2*x^5*acot(a*x))/5
```

3.55 $\int (c+dx^2)^2 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 109

$$\int (c+dx^2)^2 \cot^{-1}(ax)\,dx = \frac{(10a^2c-3d)\,dx^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)+\frac{(15a^4c^2-10a^2cd+3d^2)\log(1+a^2x^2)}{30a^5}$$

```
[Out] 1/30*(10*a^2*c-3*d)*d*x^2/a^3+1/20*d^2*x^4/a+c^2*x*arccot(a*x)+2/3*c*d*x^3*
arccot(a*x)+1/5*d^2*x^5*arccot(a*x)+1/30*(15*a^4*c^2-10*a^2*c*d+3*d^2)*ln(a
^2*x^2+1)/a^5
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {200, 5033, 1608, 1261, 712}

$$\int (c+dx^2)^2 \cot^{-1}(ax)\,dx = \frac{dx^2(10a^2c-3d)}{30a^3}+\frac{(15a^4c^2-10a^2cd+3d^2)\log(a^2x^2+1)}{30a^5}+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)+\frac{d^2x^4}{20a}$$

```
[In] Int[(c + d*x^2)^2*ArcCot[a*x],x]
```

```
[Out] ((10*a^2*c - 3*d)*d*x^2)/(30*a^3) + (d^2*x^4)/(20*a) + c^2*x*ArcCot[a*x] +
(2*c*d*x^3*ArcCot[a*x])/3 + (d^2*x^5*ArcCot[a*x])/5 + ((15*a^4*c^2 - 10*a^2
*c*d + 3*d^2)*Log[1 + a^2*x^2])/(30*a^5)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\text{integral} = c^2x\cot^{-1}(ax) + \frac{2}{3}cdx^3\cot^{-1}(ax) + \frac{1}{5}d^2x^5\cot^{-1}(ax) + a\int \frac{c^2x + \frac{2}{3}cdx^3 + \frac{d^2x^5}{5}}{1+a^2x^2}\,dx$$

$$= c^2x\cot^{-1}(ax) + \frac{2}{3}cdx^3\cot^{-1}(ax) + \frac{1}{5}d^2x^5\cot^{-1}(ax) + a\int \frac{x\left(c^2 + \frac{2}{3}cdx^2 + \frac{d^2x^4}{5}\right)}{1+a^2x^2}\,dx$$

$$= c^2x\cot^{-1}(ax) + \frac{2}{3}cdx^3\cot^{-1}(ax) + \frac{1}{5}d^2x^5\cot^{-1}(ax) + \frac{1}{2}a\text{Subst}\left(\int \frac{c^2 + \frac{2cdx}{3} + \frac{d^2x^2}{5}}{1+a^2x}\,dx, x, x^2\right)$$

$$= c^2x\cot^{-1}(ax) + \frac{2}{3}cdx^3\cot^{-1}(ax) + \frac{1}{5}d^2x^5\cot^{-1}(ax) + \frac{1}{2}a\text{Subst}\left(\int \left(\frac{(10a^2c-3d)\,d}{15a^4} + \frac{d^2x}{5a^2} + \frac{15a^4c^2 - 10a^2cd + 3d^2}{15a^4\,(1+a^2x)}\right)\,dx, x, x^2\right)$$

$$= \frac{(10a^2c-3d)\,dx^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax) + \frac{1}{5}d^2x^5\cot^{-1}(ax)+\frac{(15a^4c^2-10a^2cd+3d^2)\log(1+a^2x^2)}{30a^5}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int (c+dx^2)^2\cot^{-1}(ax)\,dx = \frac{a^2dx^2(-6d+a^2(20c+3dx^2))+4a^5x(15c^2+10cdx^2+3d^2x^4)\cot^{-1}(ax)+(30a^4c^2-20a^2cd+6d^2)\log(1}{60a^5}$$

```
[In] Integrate[(c + d*x^2)^2*ArcCot[a*x],x]
```

```
[Out] (a^2*d*x^2*(-6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 + 3
*d^2*x^4)*ArcCot[a*x] + (30*a^4*c^2 - 20*a^2*c*d + 6*d^2)*Log[1 + a^2*x^2])
/(60*a^5)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

method	result
parts	$\frac{d^2x^5\,\mathrm{arccot}(ax)}{5}+\frac{2cd\,x^3\,\mathrm{arccot}(ax)}{3}+c^2x\,\mathrm{arccot}(ax)+\frac{a\left(\frac{d\left(\frac{3}{2}a^2d\,x^4+10a^2c\,x^2-3d\,x^2\right)}{2a^4}+\frac{\left(15a^4c^2-10a^2cd+3d\right.}{2a^6}\right.}{15}$
derivativedivides	$\frac{\mathrm{arccot}(ax)c^2ax+\frac{2a\,\mathrm{arccot}(ax)cd\,x^3}{3}+\frac{a\,\mathrm{arccot}(ax)d^2x^5}{5}+\frac{5c\,a^4d\,x^2+\frac{3d^2a^4x^4}{4}-\frac{3a^2d^2x^2}{2}+\frac{\left(15a^4c^2-10a^2cd+3d^2\right)\ln\left(a^2x^2+1\right)}{2}}{15a^4}}{a}$
default	$\frac{\mathrm{arccot}(ax)c^2ax+\frac{2a\,\mathrm{arccot}(ax)cd\,x^3}{3}+\frac{a\,\mathrm{arccot}(ax)d^2x^5}{5}+\frac{5c\,a^4d\,x^2+\frac{3d^2a^4x^4}{4}-\frac{3a^2d^2x^2}{2}+\frac{\left(15a^4c^2-10a^2cd+3d^2\right)\ln\left(a^2x^2+1\right)}{2}}{15a^4}}{a}$
parallelrisch	$\frac{12x^5\,\mathrm{arccot}(ax)a^5d^2+40x^3\,\mathrm{arccot}(ax)a^5cd+3d^2a^4x^4+60c^2\,\mathrm{arccot}(ax)x\,a^5+20c\,a^4d\,x^2+30\ln\left(a^2x^2+1\right)a^4c^2-6a^2d^2x^2}{60a^5}$
risch	$\frac{i(3d^2x^5+10cd\,x^3+15c^2x)\ln(iax+1)}{30}-\frac{id^2x^5\ln(-iax+1)}{10}+\frac{\pi d^2x^5}{10}-\frac{icd\,x^3\ln(-iax+1)}{3}+\frac{\pi cd\,x^3}{3}-\frac{ic^2x\ln(-}{2}$

```
[In] int((d*x^2+c)^2*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*d^2*x^5*arccot(a*x)+2/3*c*d*x^3*arccot(a*x)+c^2*x*arccot(a*x)+1/15*a*(1
/2*d/a^4*(3/2*a^2*d*x^4+10*a^2*c*x^2-3*d*x^2)+1/2*(15*a^4*c^2-10*a^2*c*d+3*
d^2)/a^6*ln(a^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \left(c+dx^2\right)^2 \cot^{-1}(ax)\,dx = \frac{3\,a^4d^2x^4 + 2\,(10\,a^4cd - 3\,a^2d^2)x^2 + 4\,(3\,a^5d^2x^5 + 10\,a^5cdx^3 + 15\,a^5c^2x)\,\text{arccot}\,(ax) + 2\,(15\,a^4c^2 - 10\,a^2cd -}{60\,a^5}$$

```
[In] integrate((d*x^2+c)^2*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(3*a^5*d^2*x^5 + 1
0*a^5*c*d*x^3 + 15*a^5*c^2*x)*arccot(a*x) + 2*(15*a^4*c^2 - 10*a^2*c*d + 3*
d^2)*log(a^2*x^2 + 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \left(c+dx^2\right)^2 \cot^{-1}(ax)\,dx = \begin{cases} c^2x\,\text{acot}\,(ax) + \frac{2cdx^3\,\text{acot}\,(ax)}{3} + \frac{d^2x^5\,\text{acot}\,(ax)}{5} + \frac{c^2\log\left(x^2+\frac{1}{a^2}\right)}{2a} + \frac{cdx^2}{3a} + \frac{d^2x^4}{20a} - \frac{cd\log\left(x^2+\frac{1}{a^2}\right)}{3a^3} - \frac{d^2x^2}{10a^3} + \frac{d^2\log\left(x^2+\right.}{10a^5} \\ \frac{\pi\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right)}{2} \end{cases}$$

```
[In] integrate((d*x**2+c)**2*acot(a*x),x)
```

```
[Out] Piecewise((c**2*x*acot(a*x) + 2*c*d*x**3*acot(a*x)/3 + d**2*x**5*acot(a*x)/
5 + c**2*log(x**2 + a**(-2))/(2*a) + c*d*x**2/(3*a) + d**2*x**4/(20*a) - c*
d*log(x**2 + a**(-2))/(3*a**3) - d**2*x**2/(10*a**3) + d**2*log(x**2 + a**(
-2))/(10*a**5), Ne(a, 0)), (pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2, Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \left(c+dx^2\right)^2 \cot^{-1}(ax)\,dx = \frac{1}{60}\,a\left(\frac{3\,a^2d^2x^4 + 2\,(10\,a^2cd - 3\,d^2)x^2}{a^4} + \frac{2\,(15\,a^4c^2 - 10\,a^2cd + 3\,d^2)\log\left(a^2x^2+1\right)}{a^6}\right) + \frac{1}{15}\left(3\,d^2x^5 + 10\,cdx^3 + 15\,c^2x\right)\text{arccot}\,(ax)$$

```
[In] integrate((d*x^2+c)^2*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d - 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 -
10*a^2*c*d + 3*d^2)*log(a^2*x^2 + 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3 +
15*c^2*x)*arccot(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.57

$$\int \left(c+dx^2\right)^2 \cot^{-1}(ax)\,dx = \frac{1}{60}\left(\frac{4\left(3\,d^2+\frac{10\,cd}{x^2}+\frac{15\,c^2}{x^4}\right)x^5\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(3\,d^2+\frac{20\,cd}{x^2}+\frac{45\,c^2}{x^4}-\frac{6\,d^2}{a^2x^2}-\frac{30\,cd}{a^2x^4}+\frac{9\,d^2}{a^4x^4}\right)x^4}{a^2}+\frac{2\,(15\,a^4c^2-\right.$$

```
[In] integrate((d*x^2+c)^2*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/60*(4*(3*d^2 + 10*c*d/x^2 + 15*c^2/x^4)*x^5*arctan(1/(a*x))/a + (3*d^2 +
20*c*d/x^2 + 45*c^2/x^4 - 6*d^2/(a^2*x^2) - 30*c*d/(a^2*x^4) + 9*d^2/(a^4*x
^4))*x^4/a^2 + 2*(15*a^4*c^2 - 10*a^2*c*d + 3*d^2)*log(1/(a^2*x^2) + 1)/a^6
 - 2*(15*a^4*c^2 - 10*a^2*c*d + 3*d^2)*log(1/(a^2*x^2))/a^6)*a
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int \left(c+dx^2\right)^2 \cot^{-1}(ax)\,dx = \frac{a^4\left(\frac{c^2\,\ln\left(a^2\,x^2+1\right)}{2}+\frac{d^2\,x^4}{20}+\frac{c\,d\,x^2}{3}\right)-a^2\left(\frac{d^2\,x^2}{10}+\frac{c\,d\,\ln\left(a^2\,x^2+1\right)}{3}\right)+\frac{d^2\,\ln\left(a^2\,x^2+1\right)}{10}}{a^5} + c^2\,x\,\mathrm{acot}(a\,x)+\frac{d^2\,x^5\,\mathrm{acot}(a\,x)}{5}+\frac{2\,c\,d\,x^3\,\mathrm{acot}(a\,x)}{3}$$

```
[In] int(acot(a*x)*(c + d*x^2)^2,x)
```

```
[Out] (a^4*((c^2*log(a^2*x^2 + 1))/2 + (d^2*x^4)/20 + (c*d*x^2)/3) - a^2*((d^2*x^
2)/10 + (c*d*log(a^2*x^2 + 1))/3) + (d^2*log(a^2*x^2 + 1))/10)/a^5 + c^2*x*
acot(a*x) + (d^2*x^5*acot(a*x))/5 + (2*c*d*x^3*acot(a*x))/3
```

3.56 $\int (c+dx^2) \cot^{-1}(ax) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (c+dx^2) \cot^{-1}(ax) \, dx = \frac{dx^2}{6a} + cx \cot^{-1}(ax) + \frac{1}{3} dx^3 \cot^{-1}(ax) + \frac{(3a^2c-d) \log (1+a^2x^2)}{6a^3}$$

```
[Out] 1/6*d*x^2/a+c*x*arccot(a*x)+1/3*d*x^3*arccot(a*x)+1/6*(3*a^2*c-d)*ln(a^2*x^
2+1)/a^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5033, 1607, 455, 45}

$$\int (c+dx^2) \cot^{-1}(ax) \, dx = \frac{(3a^2c-d) \log (a^2x^2+1)}{6a^3} + cx \cot^{-1}(ax) + \frac{1}{3} dx^3 \cot^{-1}(ax) + \frac{dx^2}{6a}$$

```
[In] Int[(c + d*x^2)*ArcCot[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcCot[a*x] + (d*x^3*ArcCot[a*x])/3 + ((3*a^2*c - d)*Lo
g[1 + a^2*x^2])/(6*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+a\int\frac{cx+\frac{dx^3}{3}}{1+a^2x^2}\,dx\\
&= cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+a\int\frac{x\left(c+\frac{dx^2}{3}\right)}{1+a^2x^2}\,dx\\
&= cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+\frac{1}{2}a\text{Subst}\left(\int\frac{c+\frac{dx}{3}}{1+a^2x}\,dx,x,x^2\right)\\
&= cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+\frac{1}{2}a\text{Subst}\left(\int\left(\frac{d}{3a^2}+\frac{3a^2c-d}{3a^2\left(1+a^2x\right)}\right)\,dx,x,x^2\right)\\
&= \frac{dx^2}{6a}+cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+\frac{(3a^2c-d)\log\left(1+a^2x^2\right)}{6a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int\left(c+dx^2\right)\cot^{-1}(ax)\,dx=\frac{dx^2}{6a}+cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+\frac{c\log\left(1+a^2x^2\right)}{2a}-\frac{d\log\left(1+a^2x^2\right)}{6a^3}$$

```
[In] Integrate[(c + d*x^2)*ArcCot[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcCot[a*x] + (d*x^3*ArcCot[a*x])/3 + (c*Log[1 + a^2*x^
2])/(2*a) - (d*Log[1 + a^2*x^2])/(6*a^3)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result
parts	$\frac{d\,x^3 \operatorname{arccot}(ax)}{3}+cx \operatorname{arccot}(ax)+\frac{a\left(\frac{d\,x^2}{2a^2}+\frac{\left(3a^2c-d\right)\ln\left(a^2x^2+1\right)}{2a^4}\right)}{3}$
derivativedivides	$\frac{\operatorname{arccot}(ax)cax+\frac{a \operatorname{arccot}(ax)d\,x^3}{3}+\frac{\frac{a^2d\,x^2}{2}+\frac{\left(3a^2c-d\right)\ln\left(a^2x^2+1\right)}{2}}{3a^2}}{a}$
default	$\frac{\operatorname{arccot}(ax)cax+\frac{a \operatorname{arccot}(ax)d\,x^3}{3}+\frac{\frac{a^2d\,x^2}{2}+\frac{\left(3a^2c-d\right)\ln\left(a^2x^2+1\right)}{2}}{3a^2}}{a}$
parallelrisch	$\frac{2x^3 \operatorname{arccot}(ax)a^3d+6x \operatorname{arccot}(ax)a^3c+a^2d\,x^2+3\ln(a^2x^2+1)a^2c-\ln(a^2x^2+1)d}{6a^3}$
risch	$\frac{i(d\,x^3+3cx)\ln(iax+1)}{6}-\frac{id\,x^3\ln(-iax+1)}{6}+\frac{\pi d\,x^3}{6}-\frac{icx\ln(-iax+1)}{2}+\frac{\pi cx}{2}+\frac{d\,x^2}{6a}+\frac{\ln(-a^2x^2-1)c}{2a}-\frac{\ln(-}{}$

[In] int((d*x^2+c)*arccot(a*x),x,method=_RETURNVERBOSE)

[Out] 1/3*d*x^3*arccot(a*x)+c*x*arccot(a*x)+1/3*a*(1/2*d/a^2*x^2+1/2*(3*a^2*c-d)/a^4*ln(a^2*x^2+1))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int (c+dx^2)\cot^{-1}(ax)\,dx = \frac{a^2dx^2+2\,(a^3dx^3+3\,a^3cx)\operatorname{arccot}(ax)+(3\,a^2c-d)\log(a^2x^2+1)}{6\,a^3}$$

[In] integrate((d*x^2+c)*arccot(a*x),x, algorithm="fricas")

[Out] 1/6*(a^2*d*x^2 + 2*(a^3*d*x^3 + 3*a^3*c*x)*arccot(a*x) + (3*a^2*c - d)*log(a^2*x^2 + 1))/a^3

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int (c+dx^2)\cot^{-1}(ax)\,dx$$
$$=\begin{cases} cx\,\text{acot}\,(ax)+\frac{dx^3\,\text{acot}\,(ax)}{3}+\frac{c\log\left(x^2+\frac{1}{a^2}\right)}{2a}+\frac{dx^2}{6a}-\frac{d\log\left(x^2+\frac{1}{a^2}\right)}{6a^3} & \text{for } a\neq 0\\ \frac{\pi\left(cx+\frac{dx^3}{3}\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate((d*x**2+c)*acot(a*x),x)
[Out] Piecewise((c*x*acot(a*x) + d*x**3*acot(a*x)/3 + c*log(x**2 + a**(-2))/(2*a)
 + d*x**2/(6*a) - d*log(x**2 + a**(-2))/(6*a**3), Ne(a, 0)), (pi*(c*x + d*x
**3/3)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int (c+dx^2)\cot^{-1}(ax)\,dx$$
$$=\frac{1}{6}\,a\left(\frac{dx^2}{a^2}+\frac{(3\,a^2c-d)\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}\left(dx^3+3\,cx\right)\text{arccot}\,(ax)$$

```
[In] integrate((d*x^2+c)*arccot(a*x),x, algorithm="maxima")
[Out] 1/6*a*(d*x^2/a^2 + (3*a^2*c - d)*log(a^2*x^2 + 1)/a^4) + 1/3*(d*x^3 + 3*c*x
)*arccot(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.71

$$\int (c+dx^2)\cot^{-1}(ax)\,dx$$
$$=\frac{1}{6}\left(\frac{2\left(d+\frac{3\,c}{x^2}\right)x^3\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(d+\frac{3\,c}{x^2}-\frac{d}{a^2x^2}\right)x^2}{a^2}+\frac{(3\,a^2c-d)\log\left(\frac{1}{a^2x^2}+1\right)}{a^4}-\frac{(3\,a^2c-d)\log\left(\frac{1}{a^2x^2}\right)}{a^4}\right)a$$

```
[In] integrate((d*x^2+c)*arccot(a*x),x, algorithm="giac")
[Out] 1/6*(2*(d + 3*c/x^2)*x^3*arctan(1/(a*x))/a + (d + 3*c/x^2 - d/(a^2*x^2))*x^
2/a^2 + (3*a^2*c - d)*log(1/(a^2*x^2) + 1)/a^4 - (3*a^2*c - d)*log(1/(a^2*x
^2))/a^4)*a
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int (c+dx^2) \cot^{-1}(ax)\,dx = \frac{d\,x^3\,\mathrm{acot}(a\,x)}{3} - \frac{\frac{d\,\ln\left(a^2\,x^2+1\right)}{6} - a^2\,\left(\frac{c\,\ln\left(a^2\,x^2+1\right)}{2} + \frac{d\,x^2}{6}\right)}{a^3} + c\,x\,\mathrm{acot}(a\,x)$$

```
[In] int(acot(a*x)*(c + d*x^2),x)
```

```
[Out] (d*x^3*acot(a*x))/3 - ((d*log(a^2*x^2 + 1))/6 - a^2*((c*log(a^2*x^2 + 1))/2
 + (d*x^2)/6))/a^3 + c*x*acot(a*x)
```

3.57 $\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 403

$$\begin{aligned}\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx =&\ \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}}-\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}}\\&-\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{2i\sqrt{c}\sqrt{d}(i-ax)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}}\\&+\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2i\sqrt{c}\sqrt{d}(i+ax)}{\left(a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}}\\&-\frac{\operatorname{PolyLog}\left(2,1-\frac{2i\sqrt{c}\sqrt{d}(i-ax)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{4\sqrt{c}\sqrt{d}}\\&+\frac{\operatorname{PolyLog}\left(2,1+\frac{2i\sqrt{c}\sqrt{d}(i+ax)}{\left(a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{4\sqrt{c}\sqrt{d}}\end{aligned}$$

```
[Out] 1/2*I*arctan(x*d^(1/2)/c^(1/2))*ln(1-I/a/x)/c^(1/2)/d^(1/2)-1/2*I*arctan(x*
d^(1/2)/c^(1/2))*ln(1+I/a/x)/c^(1/2)/d^(1/2)-1/2*I*arctan(x*d^(1/2)/c^(1/2)
)*ln(2*I*(I-a*x)*c^(1/2)*d^(1/2)/(a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^(1/2)))
/c^(1/2)/d^(1/2)+1/2*I*arctan(x*d^(1/2)/c^(1/2))*ln(-2*I*(I+a*x)*c^(1/2)*d^
(1/2)/(a*c^(1/2)+d^(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*polylo
g(2,1-2*I*(I-a*x)*c^(1/2)*d^(1/2)/(a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^(1/2))
)/c^(1/2)/d^(1/2)+1/4*polylog(2,1+2*I*(I+a*x)*c^(1/2)*d^(1/2)/(a*c^(1/2)+d^
(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5029, 211, 2520, 12, 266, 6820, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \frac{i\log\left(1-\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}} - \frac{i\log\left(1+\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}} - \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{2i\sqrt{c}\sqrt{d}(-ax+i)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}} + \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2i\sqrt{c}\sqrt{d}(ax+i)}{\left(a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}} - \frac{\text{PolyLog}\left(2,1-\frac{2i\sqrt{c}\sqrt{d}(i-ax)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left(2,\frac{2i\sqrt{c}\sqrt{d}(ax+i)}{\left(\sqrt{c}a+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}+1\right)}{4\sqrt{c}\sqrt{d}}$$

```
[In] Int[ArcCot[a*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 - I/(a*x)])/(Sqrt[c]*Sqrt[d]) - ((
I/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 + I/(a*x)])/(Sqrt[c]*Sqrt[d]) - ((I/
2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[((2*I)*Sqrt[c]*Sqrt[d]*(I - a*x))/((a*Sq
rt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))])/(Sqrt[c]*Sqrt[d]) + ((I/2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]]*Log[((-2*I)*Sqrt[c]*Sqrt[d]*(I + a*x))/((a*Sqrt[c]
+ Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))])/(Sqrt[c]*Sqrt[d]) - PolyLog[2, 1 - ((
2*I)*Sqrt[c]*Sqrt[d]*(I - a*x))/((a*Sqrt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]
*x))]/(4*Sqrt[c]*Sqrt[d]) + PolyLog[2, 1 + ((2*I)*Sqrt[c]*Sqrt[d]*(I + a*x)
)/((a*Sqrt[c] + Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))]/(4*Sqrt[c]*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5029

```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I/(c*x)]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I/(c*x)]/(d + e
*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{ax}\right)}{c+dx^2}\,dx-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{ax}\right)}{c+dx^2}\,dx\\
&= \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}}-\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}}\\
&\quad+\frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(1-\frac{i}{ax}\right)x^2}\,dx}{2a}+\frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(1+\frac{i}{ax}\right)x^2}\,dx}{2a}\\
&= \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}}-\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}}\\
&\quad+\frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\left(1-\frac{i}{ax}\right)x^2}\,dx}{2a\sqrt{c}\sqrt{d}}+\frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\left(1+\frac{i}{ax}\right)x^2}\,dx}{2a\sqrt{c}\sqrt{d}}\\
&= \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}}-\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}}\\
&\quad+\frac{\int \frac{a\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(-i+ax)}\,dx}{2a\sqrt{c}\sqrt{d}}+\frac{\int \frac{a\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(i+ax)}\,dx}{2a\sqrt{c}\sqrt{d}}
\end{aligned}$$

$$= \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} - \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(-i+ax)}\,dx}{2\sqrt{c}\sqrt{d}} + \frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(i+ax)}\,dx}{2\sqrt{c}\sqrt{d}}$$

$$= \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} - \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\int \left(\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x} - \frac{ia\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{-i+ax}\right)dx}{2\sqrt{c}\sqrt{d}} + \frac{\int \left(-\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x} + \frac{ia\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{i+ax}\right)dx}{2\sqrt{c}\sqrt{d}}$$

$$= \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} - \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} - \frac{(ia)\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{-i+ax}\,dx}{2\sqrt{c}\sqrt{d}} + \frac{(ia)\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{i+ax}\,dx}{2\sqrt{c}\sqrt{d}}$$

$$= \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} - \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} - \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{2i\sqrt{c}\sqrt{d}(i-ax)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}} + \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2i\sqrt{c}\sqrt{d}(i+ax)}{\left(a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}} + \frac{i\int \frac{\log\left(\frac{2\sqrt{d}(-i+ax)}{\sqrt{c}\left(ia-\frac{i\sqrt{d}}{\sqrt{c}}\right)\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}\right)}{1+\frac{dx^2}{c}}\,dx}{2c} - \frac{i\int \frac{\log\left(\frac{2\sqrt{d}(i+ax)}{\sqrt{c}\left(ia+\frac{i\sqrt{d}}{\sqrt{c}}\right)\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}\right)}{1+\frac{dx^2}{c}}\,dx}{2c}$$

$$= \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} - \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}}$$
$$- \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{2i\sqrt{c}\sqrt{d}(i-ax)}{(a\sqrt{c}-\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}\right)}{2\sqrt{c}\sqrt{d}}$$
$$+ \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2i\sqrt{c}\sqrt{d}(i+ax)}{(a\sqrt{c}+\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}\right)}{2\sqrt{c}\sqrt{d}}$$
$$- \frac{\text{PolyLog}\left(2,1-\frac{2i\sqrt{c}\sqrt{d}(i-ax)}{(a\sqrt{c}-\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left(2,1+\frac{2i\sqrt{c}\sqrt{d}(i+ax)}{(a\sqrt{c}+\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}\right)}{4\sqrt{c}\sqrt{d}}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.78

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx$$
$$= \frac{a\left(-2\arccos\left(\frac{a^2c+d}{a^2c-d}\right)\text{arctanh}\left(\frac{ac}{\sqrt{-a^2cdx}}\right) - 4\cot^{-1}(ax)\text{arctanh}\left(\frac{adx}{\sqrt{-a^2cd}}\right) - \left(\arccos\left(\frac{a^2c+d}{a^2c-d}\right) - 2i\text{arctanh}\left(\right.\right.\right.}{}$$

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2),x]
```

```
[Out] (a*(-2*ArcCos[(a^2*c + d)/(a^2*c - d)]*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)]
- 4*ArcCot[a*x]*ArcTanh[(a*d*x)/Sqrt[-(a^2*c*d)]] - (ArcCos[(a^2*c + d)/(a^
2*c - d)] - (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)])*Log[((2*I)*d*(I*a^2*
c + Sqrt[-(a^2*c*d)])*(I + a*x))/((a^2*c - d)*(Sqrt[-(a^2*c*d)] - a*d*x))]
- (ArcCos[(a^2*c + d)/(a^2*c - d)] + (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*
x)])*Log[(2*d*(a^2*c + I*Sqrt[-(a^2*c*d)])*(-I + a*x))/((a^2*c - d)*(-Sqrt[
-(a^2*c*d)] + a*d*x))] + (ArcCos[(a^2*c + d)/(a^2*c - d)] + (2*I)*ArcTanh[(
a*c)/(Sqrt[-(a^2*c*d)]*x)] + (2*I)*ArcTanh[(a*d*x)/Sqrt[-(a^2*c*d)]])*Log[(
Sqrt[2]*Sqrt[-(a^2*c*d)])/(Sqrt[a^2*c - d]*E^(I*ArcCot[a*x])*Sqrt[-(a^2*c)
- d + (a^2*c - d)*Cos[2*ArcCot[a*x]]])] + (ArcCos[(a^2*c + d)/(a^2*c - d)]
- (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)] - (2*I)*ArcTanh[(a*d*x)/Sqrt[-(
a^2*c*d)]])*Log[(Sqrt[2]*Sqrt[-(a^2*c*d)]*E^(I*ArcCot[a*x]))/(Sqrt[a^2*c -
d]*Sqrt[-(a^2*c) - d + (a^2*c - d)*Cos[2*ArcCot[a*x]]])] + I*(-PolyLog[2, (
(a^2*c + d - (2*I)*Sqrt[-(a^2*c*d)])*(Sqrt[-(a^2*c*d)] + a*d*x))/((a^2*c -
d)*(Sqrt[-(a^2*c*d)] - a*d*x))] + PolyLog[2, ((a^2*c + d + (2*I)*Sqrt[-(a^2
*c*d)])*(Sqrt[-(a^2*c*d)] + a*d*x))/((a^2*c - d)*(Sqrt[-(a^2*c*d)] - a*d*x)
)])))/(4*Sqrt[-(a^2*c*d)])
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.97

method	result
risch	$\frac{i\pi \operatorname{arctanh}\left(\frac{2(-iax+1)d-2d}{2a\sqrt{cd}}\right)}{2\sqrt{cd}} - \frac{\ln(-iax+1)\ln\left(\frac{a\sqrt{cd}-(-iax+1)d+d}{a\sqrt{cd}+d}\right)}{4\sqrt{cd}} + \frac{\ln(-iax+1)\ln\left(\frac{a\sqrt{cd}+(-iax+1)d-d}{a\sqrt{cd}-d}\right)}{4\sqrt{cd}} - \frac{\text{dil}}{}$
derivativedivides	$\frac{-\frac{i\sqrt{a^2cd}\,\operatorname{arccot}(ax)\ln\left(1-\frac{(a^2c-d)(ax+i)^2}{(a^2x^2+1)(a^2c+2\sqrt{a^2cd}+d)}\right)}{2cd} - \frac{\sqrt{a^2cd}\,\operatorname{arccot}(ax)^2}{2cd} - \frac{\sqrt{a^2cd}\,\operatorname{polylog}\left(2, \frac{(a^2c-d)(ax+i)^2}{(a^2x^2+1)(a^2c+2\sqrt{a^2cd}+d}\right.}{4cd}}{}$
default	$\frac{-\frac{i\sqrt{a^2cd}\,\operatorname{arccot}(ax)\ln\left(1-\frac{(a^2c-d)(ax+i)^2}{(a^2x^2+1)(a^2c+2\sqrt{a^2cd}+d)}\right)}{2cd} - \frac{\sqrt{a^2cd}\,\operatorname{arccot}(ax)^2}{2cd} - \frac{\sqrt{a^2cd}\,\operatorname{polylog}\left(2, \frac{(a^2c-d)(ax+i)^2}{(a^2x^2+1)(a^2c+2\sqrt{a^2cd}+d}\right.}{4cd}}{}$

```
[In] int(arccot(a*x)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*Pi/(c*d)^(1/2)*arctanh(1/2*(2*(1-I*a*x)*d-2*d)/a/(c*d)^(1/2))-1/4*ln(
1-I*a*x)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))+1/
4*ln(1-I*a*x)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2)-d
))-1/4/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))+1
/4/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2)-d))-1/4*l
n(1+I*a*x)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2)+d))+
1/4*ln(1+I*a*x)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1+I*a*x)*d-d)/(a*(c*d)^(1/2)
-d))-1/4/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2)+d))
+1/4/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)+(1+I*a*x)*d-d)/(a*(c*d)^(1/2)-d))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{arccot}(ax)}{dx^2+c}\,dx$$

```
[In] integrate(arccot(a*x)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)/(d*x^2 + c), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{acot}(ax)}{c+dx^2}\,dx$$

```
[In] integrate(acot(a*x)/(d*x**2+c),x)
[Out] Integral(acot(a*x)/(c + d*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.31

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = -\frac{a\left(\frac{8\arctan(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{a} - \frac{4\arctan(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)+4\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\arctan\left(-\frac{a\sqrt{d}x}{a\sqrt{c}-\sqrt{d}},-\frac{\sqrt{d}}{a\sqrt{c}-\sqrt{d}}\right)+\log\left(dx^2+c\right)\log\left(\frac{a^2cd+\left(a^4cd\cdots}{a^4c^2+\cdots}\right.}{\cdots}\right.}{\cdots} + \frac{\operatorname{arccot}(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}} + \frac{\arctan(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c),x, algorithm="maxima")
[Out] -1/8*a*(8*arctan(a*x)*arctan(d*x/sqrt(c*d))/a - (4*arctan(a*x)*arctan(sqrt(
d)*x/sqrt(c)) + 4*arctan(sqrt(d)*x/sqrt(c))*arctan2(-a*sqrt(d)*x/(a*sqrt(c)
 - sqrt(d)), -sqrt(d)/(a*sqrt(c) - sqrt(d))) + log(d*x^2 + c)*log((a^2*c*d
+ (a^4*c*d + a^2*d^2)*x^2 + 2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(d) + d^2)/(a^4
*c^2 + 6*a^2*c*d + 4*(a^3*c + a*d)*sqrt(c)*sqrt(d) + d^2)) - log(d*x^2 + c)
*log((a^2*c*d + (a^4*c*d + a^2*d^2)*x^2 - 2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(
d) + d^2)/(a^4*c^2 + 6*a^2*c*d - 4*(a^3*c + a*d)*sqrt(c)*sqrt(d) + d^2)) +
2*dilog((a^2*c + I*a*d*x + (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c + 2*a*sqrt
(c)*sqrt(d) + d)) + 2*dilog((a^2*c - I*a*d*x - (I*a^2*x - a)*sqrt(c)*sqrt(d
))/(a^2*c + 2*a*sqrt(c)*sqrt(d) + d)) - 2*dilog((a^2*c + I*a*d*x - (I*a^2*x
 + a)*sqrt(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c)*sqrt(d) + d)) - 2*dilog((a^2*c
- I*a*d*x + (I*a^2*x - a)*sqrt(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c)*sqrt(d) + d
)))/a)/sqrt(c*d) + arccot(a*x)*arctan(d*x/sqrt(c*d))/sqrt(c*d) + arctan(a*x
)*arctan(d*x/sqrt(c*d))/sqrt(c*d)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{arccot}(ax)}{dx^2+c}\,dx$$

```
[In] integrate(arccot(a*x)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(arccot(a*x)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{acot}(a\,x)}{d\,x^2+c}\,dx$$

```
[In] int(acot(a*x)/(c + d*x^2),x)
[Out] int(acot(a*x)/(c + d*x^2), x)
```

3.58 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 801

$$
\begin{aligned}
\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx &= \frac{x\cot^{-1}(ax)}{2c(c+dx^2)} + \frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} \\
&\quad - \frac{ia\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} \\
&\quad + \frac{ia\log\left(-\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} \\
&\quad + \frac{ia\log\left(-\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} \\
&\quad - \frac{ia\log\left(\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{a\log(1+a^2x^2)}{4c(a^2c-d)} \\
&\quad - \frac{a\log(c+dx^2)}{4c(a^2c-d)} - \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} \\
&\quad + \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} - \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}+i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} \\
&\quad + \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}+i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}
\end{aligned}
$$

```
[Out] 1/2*x*arccot(a*x)/c/(d*x^2+c)+1/4*a*ln(a^2*x^2+1)/c/(a^2*c-d)-1/4*a*ln(d*x^
2+c)/c/(a^2*c-d)+1/2*arccot(a*x)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+
1/8*I*a*ln(-(1+x*(-a^2)^(1/2))*d^(1/2)/(I*(-a^2)^(1/2)*c^(1/2)-d^(1/2)))*ln
(1-I*x*d^(1/2)/c^(1/2))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)-1/8*I*a*ln((1-x*(-a^2)
^(1/2))*d^(1/2)/(I*(-a^2)^(1/2)*c^(1/2)+d^(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2))
/c^(3/2)/(-a^2)^(1/2)/d^(1/2)+1/8*I*a*ln(-(1-x*(-a^2)^(1/2))*d^(1/2)/(I*(-a
^2)^(1/2)*c^(1/2)-d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/(-a^2)^(1/2)/
d^(1/2)-1/8*I*a*ln((1+x*(-a^2)^(1/2))*d^(1/2)/(I*(-a^2)^(1/2)*c^(1/2)+d^(1/
2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)-1/8*I*a*polylog
(2,(-a^2)^(1/2)*(c^(1/2)-I*x*d^(1/2))/((-a^2)^(1/2)*c^(1/2)-I*d^(1/2)))/c^(
3/2)/(-a^2)^(1/2)/d^(1/2)+1/8*I*a*polylog(2,(-a^2)^(1/2)*(c^(1/2)-I*x*d^(1/
2))/((-a^2)^(1/2)*c^(1/2)+I*d^(1/2)))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)-1/8*I*a*
polylog(2,(-a^2)^(1/2)*(c^(1/2)+I*x*d^(1/2))/((-a^2)^(1/2)*c^(1/2)-I*d^(1/2
)))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)+1/8*I*a*polylog(2,(-a^2)^(1/2)*(c^(1/2)+I*
x*d^(1/2))/((-a^2)^(1/2)*c^(1/2)+I*d^(1/2)))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 801, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules

used = {205, 211, 5033, 6857, 455, 36, 31, 5028, 2456, 2441, 2440, 2438}

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\cot^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{x\cot^{-1}(ax)}{2c\,(dx^2+c)} - \frac{ia\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\log\left(-\frac{\sqrt{d}\left(\sqrt{-a^2}x+1\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\log\left(-\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(\frac{i\sqrt{d}x}{\sqrt{c}}+1\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} - \frac{ia\log\left(\frac{\sqrt{d}\left(\sqrt{-a^2}x+1\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(\frac{i\sqrt{d}x}{\sqrt{c}}+1\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{a\log\left(a^2x^2+1\right)}{4c\,(a^2c-d)} - \frac{a\log\left(dx^2+c\right)}{4c\,(a^2c-d)} - \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} - \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(i\sqrt{d}x+\sqrt{c}\right)}{\sqrt{-a^2}\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(i\sqrt{d}x+\sqrt{c}\right)}{\sqrt{-a^2}\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (x*ArcCot[a*x])/(2*c*(c + d*x^2)) + (ArcCot[a*x]*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*Sqrt[d]) - ((I/8)*a*Log[(Sqrt[d]*(1 - Sqrt[-a^2]*x))/(I*Sqrt[
-a^2]*Sqrt[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/
2)*Sqrt[d]) + ((I/8)*a*Log[-((Sqrt[d]*(1 + Sqrt[-a^2]*x))/(I*Sqrt[-a^2]*Sqr
t[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[
d]) + ((I/8)*a*Log[-((Sqrt[d]*(1 - Sqrt[-a^2]*x))/(I*Sqrt[-a^2]*Sqrt[c] - S
qrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) - ((
I/8)*a*Log[(Sqrt[d]*(1 + Sqrt[-a^2]*x))/(I*Sqrt[-a^2]*Sqrt[c] + Sqrt[d])]*L
og[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) + (a*Log[1 + a^
2*x^2])/(4*c*(a^2*c - d)) - (a*Log[c + d*x^2])/(4*c*(a^2*c - d)) - ((I/8)*a
*PolyLog[2, (Sqrt[-a^2]*(Sqrt[c] - I*Sqrt[d]*x))/(Sqrt[-a^2]*Sqrt[c] - I*Sq
rt[d])])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) + ((I/8)*a*PolyLog[2, (Sqrt[-a^2]*(Sq
rt[c] - I*Sqrt[d]*x))/(Sqrt[-a^2]*Sqrt[c] + I*Sqrt[d])])/(Sqrt[-a^2]*c^(3/2
)*Sqrt[d]) - ((I/8)*a*PolyLog[2, (Sqrt[-a^2]*(Sqrt[c] + I*Sqrt[d]*x))/(Sqrt
```

```
[-a^2]*Sqrt[c] - I*Sqrt[d])])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) + ((I/8)*a*PolyL
og[2, (Sqrt[-a^2]*(Sqrt[c] + I*Sqrt[d]*x))/(Sqrt[-a^2]*Sqrt[c] + I*Sqrt[d])
])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\cot^{-1}(ax)}{2c\,(c+dx^2)} + \frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + a\int \frac{\frac{x}{2c(c+dx^2)} + \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}}{1+a^2x^2}\,dx \\
&= \frac{x\cot^{-1}(ax)}{2c\,(c+dx^2)} + \frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} \\
&\quad + a\int \left(\frac{x}{2c\,(1+a^2x^2)\,(c+dx^2)} + \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}\,(1+a^2x^2)}\right) dx
\end{aligned}
$$

$$=\frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)}+\frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{a\int\frac{x}{(1+a^2x^2)(c+dx^2)}\,dx}{2c}+\frac{a\int\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{1+a^2x^2}\,dx}{2c^{3/2}\sqrt{d}}$$

$$=\frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)}+\frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{a\text{Subst}\left(\int\frac{1}{(1+a^2x)(c+dx)}\,dx,x,x^2\right)}{4c}+\frac{(ia)\int\frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1+a^2x^2}\,dx}{4c^{3/2}\sqrt{d}}-\frac{(ia)\int\frac{\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1+a^2x^2}\,dx}{4c^{3/2}\sqrt{d}}$$

$$=\frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)}+\frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{a^3\text{Subst}\left(\int\frac{1}{1+a^2x}\,dx,x,x^2\right)}{4c\left(a^2c-d\right)}+\frac{(ia)\int\left(\frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{2\left(1-\sqrt{-a^2}x\right)}+\frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{2\left(1+\sqrt{-a^2}x\right)}\right)dx}{4c^{3/2}\sqrt{d}}-\frac{(ia)\int\left(\frac{\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{2\left(1-\sqrt{-a^2}x\right)}+\frac{\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{2\left(1+\sqrt{-a^2}x\right)}\right)dx}{4c^{3/2}\sqrt{d}}-\frac{(ad)\text{Subst}\left(\int\frac{1}{c+dx}\,dx,x,x^2\right)}{4c\left(a^2c-d\right)}$$

$$=\frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)}+\frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{a\log\left(1+a^2x^2\right)}{4c\left(a^2c-d\right)}-\frac{a\log\left(c+dx^2\right)}{4c\left(a^2c-d\right)}+\frac{(ia)\int\frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1-\sqrt{-a^2}x}\,dx}{8c^{3/2}\sqrt{d}}+\frac{(ia)\int\frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1+\sqrt{-a^2}x}\,dx}{8c^{3/2}\sqrt{d}}-\frac{(ia)\int\frac{\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1-\sqrt{-a^2}x}\,dx}{8c^{3/2}\sqrt{d}}-\frac{(ia)\int\frac{\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1+\sqrt{-a^2}x}\,dx}{8c^{3/2}\sqrt{d}}$$

$$= \frac{x\cot^{-1}(ax)}{2c\,(c+dx^2)} + \frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{ia\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$
$$+ \frac{ia\log\left(-\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$
$$+ \frac{ia\log\left(-\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$
$$- \frac{ia\log\left(\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{a\log\left(1+a^2x^2\right)}{4c\,(a^2c-d)}$$
$$- \frac{a\log\left(c+dx^2\right)}{4c\,(a^2c-d)} + \frac{a\int\frac{\log\left(-\frac{i\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{\sqrt{c}\left(\sqrt{-a^2}-\frac{i\sqrt{d}}{\sqrt{c}}\right)}\right)}{1-\frac{i\sqrt{d}x}{\sqrt{c}}}dx}{8\sqrt{-a^2}c^2} + \frac{a\int\frac{\log\left(\frac{i\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{\sqrt{c}\left(\sqrt{-a^2}+\frac{i\sqrt{d}}{\sqrt{c}}\right)}\right)}{1+\frac{i\sqrt{d}x}{\sqrt{c}}}dx}{8\sqrt{-a^2}c^2}$$
$$- \frac{a\int\frac{\log\left(-\frac{i\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{\sqrt{c}\left(-\sqrt{-a^2}-\frac{i\sqrt{d}}{\sqrt{c}}\right)}\right)}{1-\frac{i\sqrt{d}x}{\sqrt{c}}}dx}{8\sqrt{-a^2}c^2} - \frac{a\int\frac{\log\left(\frac{i\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{\sqrt{c}\left(-\sqrt{-a^2}+\frac{i\sqrt{d}}{\sqrt{c}}\right)}\right)}{1+\frac{i\sqrt{d}x}{\sqrt{c}}}dx}{8\sqrt{-a^2}c^2}$$

$$=\frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)}+\frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}-\frac{ia\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$
$$+\frac{ia\log\left(-\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$
$$+\frac{ia\log\left(-\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$
$$-\frac{ia\log\left(\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}+\frac{a\log\left(1+a^2x^2\right)}{4c\left(a^2c-d\right)}$$
$$-\frac{a\log\left(c+dx^2\right)}{4c\left(a^2c-d\right)}-\frac{(ia)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-a^2}x}{-\sqrt{-a^2}-\frac{i\sqrt{d}}{\sqrt{c}}}\right)}{x}dx,x,1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$
$$+\frac{(ia)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-a^2}x}{\sqrt{-a^2}-\frac{i\sqrt{d}}{\sqrt{c}}}\right)}{x}dx,x,1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$
$$+\frac{(ia)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-a^2}x}{-\sqrt{-a^2}+\frac{i\sqrt{d}}{\sqrt{c}}}\right)}{x}dx,x,1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$
$$-\frac{(ia)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-a^2}x}{\sqrt{-a^2}+\frac{i\sqrt{d}}{\sqrt{c}}}\right)}{x}dx,x,1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$

$$
\begin{aligned}
&= \frac{x\cot^{-1}(ax)}{2c\,(c+dx^2)} + \frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{ia\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} \\
&\quad + \frac{ia\log\left(-\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} \\
&\quad + \frac{ia\log\left(-\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} \\
&\quad - \frac{ia\log\left(\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{a\log\left(1+a^2x^2\right)}{4c\,(a^2c-d)} - \frac{a\log\left(c+dx^2\right)}{4c\,(a^2c-d)} \\
&\quad - \frac{ia\operatorname{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\operatorname{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} \\
&\quad - \frac{ia\operatorname{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}+i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\operatorname{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}+i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 5.30 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.01

$$
\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = -\frac{a\left(\frac{2\log\left(\frac{a^2c+d+\left(-a^2c+d\right)\cos\left(2\cot^{-1}(ax)\right)}{a^2c+d}\right)}{a^2c-d} + \frac{2\arccos\left(\frac{a^2c+d}{a^2c-d}\right)\operatorname{arctanh}\left(\frac{ac}{\sqrt{-a^2cd}x}\right)+4\cot^{-1}(ax)\operatorname{arctanh}\left(\frac{adx}{\sqrt{-a^2cd}}\right)+\left(\arccos\left(\frac{a^2}{a^2}\cdots\right.\right.}{\cdots}\right.}{\cdots}
$$

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^2,x]
```

```
[Out] -1/8*(a*((2*Log[(a^2*c + d + (-(a^2*c) + d)*Cos[2*ArcCot[a*x]])/(a^2*c + d)
])/(a^2*c - d) + (2*ArcCos[(a^2*c + d)/(a^2*c - d)]*ArcTanh[(a*c)/(Sqrt[-(a
^2*c*d)]*x)] + 4*ArcCot[a*x]*ArcTanh[(a*d*x)/Sqrt[-(a^2*c*d)]] + (ArcCos[(a
^2*c + d)/(a^2*c - d)] - (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)])*Log[((2
*I)*d*(I*a^2*c + Sqrt[-(a^2*c*d)])*(I + a*x))/((a^2*c - d)*(Sqrt[-(a^2*c*d)
] - a*d*x))] + (ArcCos[(a^2*c + d)/(a^2*c - d)] + (2*I)*ArcTanh[(a*c)/(Sqrt
[-(a^2*c*d)]*x)])*Log[(2*d*(a^2*c + I*Sqrt[-(a^2*c*d)])*(-I + a*x))/((a^2*c
 - d)*(-Sqrt[-(a^2*c*d)] + a*d*x))] - (ArcCos[(a^2*c + d)/(a^2*c - d)] + (2
*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)] + (2*I)*ArcTanh[(a*d*x)/Sqrt[-(a^2*
c*d)]])*Log[(Sqrt[2]*Sqrt[-(a^2*c*d)])/(Sqrt[a^2*c - d]*E^(I*ArcCot[a*x])*S
```

```
qrt[-(a^2*c) - d + (a^2*c - d)*Cos[2*ArcCot[a*x]]])] - (ArcCos[(a^2*c + d)/
(a^2*c - d)] - (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)] - (2*I)*ArcTanh[(a
*d*x)/Sqrt[-(a^2*c*d)]])*Log[(Sqrt[2]*Sqrt[-(a^2*c*d)]*E^(I*ArcCot[a*x]))/(
Sqrt[a^2*c - d]*Sqrt[-(a^2*c) - d + (a^2*c - d)*Cos[2*ArcCot[a*x]]])] + I*(
PolyLog[2, ((a^2*c + d - (2*I)*Sqrt[-(a^2*c*d)])*(Sqrt[-(a^2*c*d)] + a*d*x)
)/((a^2*c - d)*(Sqrt[-(a^2*c*d)] - a*d*x))] - PolyLog[2, ((a^2*c + d + (2*I
)*Sqrt[-(a^2*c*d)])*(Sqrt[-(a^2*c*d)] + a*d*x))/((a^2*c - d)*(Sqrt[-(a^2*c*
d)] - a*d*x))]))/Sqrt[-(a^2*c*d)] - (4*ArcCot[a*x]*Sin[2*ArcCot[a*x]])/(a^2
*c + d + (-(a^2*c) + d)*Cos[2*ArcCot[a*x]])))/c
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2140 vs. 2(593) = 1186.

Time = 1.59 (sec) , antiderivative size = 2141, normalized size of antiderivative = 2.67

method	result	size
risch	Expression too large to display	2141
derivativedivides	Expression too large to display	2275
default	Expression too large to display	2275

```
[In] int(arccot(a*x)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*a^4*ln(1-I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)*x-1/4*I*a^4*ln(1+I*a*x)/
(a^2*c-d)/(-a^2*d*x^2-a^2*c)*x-1/8*a^4*ln(1-I*a*x)*c/(a^2*c-d)/(-a^2*d*x^2-
a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2)-d))+1/8*
a^4*ln(1-I*a*x)*c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2
)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))+1/8*a^2*ln(1-I*a*x)/(a^2*c-d)/(-a^2*d*x
^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2)-d))*d
-1/4*a^3*ln(1-I*a*x)/c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)*d*x^2-1/8*a^2*ln(1-I*a*
x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)-(1-I*a*x)*d+d
)/(a*(c*d)^(1/2)+d))*d-1/4*a^3*ln(1+I*a*x)/c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)*d
*x^2+1/8*a^4*ln(1+I*a*x)*c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(
c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2)+d))-1/8*a^4*ln(1+I*a*x)*c/(a^2*c-d
)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1+I*a*x)*d-d)/(a*(c*d)^
(1/2)-d))-1/8*a^2*ln(1+I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((
a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2)+d))*d+1/8*a^2*ln(1+I*a*x)/(a^2*
c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1+I*a*x)*d-d)/(a*(c*
d)^(1/2)-d))*d+1/8*a^4*ln(1-I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)
*ln((a*(c*d)^(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))*d*x^2-1/8/c/(c*d)^(1/2
)*dilog((a*(c*d)^(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))+1/8/c/(c*d)^(1/2)*
dilog((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2)-d))-1/8/c/(c*d)^(1/2)*di
log((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2)+d))+1/8/c/(c*d)^(1/2)*dilo
g((a*(c*d)^(1/2)+(1+I*a*x)*d-d)/(a*(c*d)^(1/2)-d))-1/4*a^3*ln(1-I*a*x)/(a^2
```

```
*c-d)/(-a^2*d*x^2-a^2*c)-1/8*a/c/(a^2*c-d)*ln((1-I*a*x)^2*d-a^2*c-2*(1-I*a*
x)*d+d)-1/4*a^2/(a^2*c-d)/(c*d)^(1/2)*arctanh(1/2*(2*(1-I*a*x)*d-2*d)/a/(c*
d)^(1/2))-1/4*a^2*Pi/c/(-a^2*d*x^2-a^2*c)*x+1/4*I*Pi/c/(c*d)^(1/2)*arctanh(
1/2*(2*(1-I*a*x)*d-2*d)/a/(c*d)^(1/2))-1/8*a/c/(a^2*c-d)*ln((1+I*a*x)^2*d-a
^2*c-2*(1+I*a*x)*d+d)-1/4*a^2/(a^2*c-d)/(c*d)^(1/2)*arctanh(1/2*(2*(1+I*a*x
)*d-2*d)/a/(c*d)^(1/2))-1/4*a^3*ln(1+I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)-1/
4*I*a^2*ln(1-I*a*x)/c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)*d*x+1/8*a^4*ln(1+I*a*x)/
(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(
a*(c*d)^(1/2)+d))*d*x^2-1/8*a^4*ln(1+I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c
*d)^(1/2)*ln((a*(c*d)^(1/2)+(1+I*a*x)*d-d)/(a*(c*d)^(1/2)-d))*d*x^2-1/8*a^4
*ln(1-I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1-
I*a*x)*d-d)/(a*(c*d)^(1/2)-d))*d*x^2+1/4*I*a^2*ln(1+I*a*x)/c/(a^2*c-d)/(-a^
2*d*x^2-a^2*c)*d*x-1/8*a^2*ln(1+I*a*x)/c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)
^(1/2)*ln((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2)+d))*d^2*x^2+1/8*a^2*
ln(1-I*a*x)/c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1
-I*a*x)*d-d)/(a*(c*d)^(1/2)-d))*d^2*x^2+1/8*a^2*ln(1+I*a*x)/c/(a^2*c-d)/(-a
^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1+I*a*x)*d-d)/(a*(c*d)^(1/2)
-d))*d^2*x^2-1/8*a^2*ln(1-I*a*x)/c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)
*ln((a*(c*d)^(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))*d^2*x^2
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{arccot}(ax)}{(dx^2+c)^2}\,dx$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
[Out] integral(arccot(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(acot(a*x)/(d*x**2+c)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 628, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{cdx^2+c^2} + \frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c}\right)\operatorname{arccot}(ax) + \frac{\left(4\,acd\log\left(a^2x^2+1\right) - 4\,acd\log\left(dx^2+c\right) + \left(4\left(a^2c-d\right)\arctan\left(ax\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) + 4\left(a^2c-d\right)\operatorname{arctan}\right.\right.}{}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))*arccot(a*x) +
 1/16*(4*a*c*d*log(a^2*x^2 + 1) - 4*a*c*d*log(d*x^2 + c) + (4*(a^2*c - d)*a
rctan(a*x)*arctan(sqrt(d)*x/sqrt(c)) + 4*(a^2*c - d)*arctan(sqrt(d)*x/sqrt(
c))*arctan2(-a*sqrt(d)*x/(a*sqrt(c) - sqrt(d)), -sqrt(d)/(a*sqrt(c) - sqrt(
d))) + (a^2*c - d)*log(d*x^2 + c)*log((a^2*c*d + (a^4*c*d + a^2*d^2)*x^2 +
2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(d) + d^2)/(a^4*c^2 + 6*a^2*c*d + 4*(a^3*c
+ a*d)*sqrt(c)*sqrt(d) + d^2)) - (a^2*c - d)*log(d*x^2 + c)*log((a^2*c*d +
(a^4*c*d + a^2*d^2)*x^2 - 2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(d) + d^2)/(a^4*c
^2 + 6*a^2*c*d - 4*(a^3*c + a*d)*sqrt(c)*sqrt(d) + d^2)) + 2*(a^2*c - d)*di
log((a^2*c + I*a*d*x + (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c + 2*a*sqrt(c)*
sqrt(d) + d)) + 2*(a^2*c - d)*dilog((a^2*c - I*a*d*x - (I*a^2*x - a)*sqrt(c
)*sqrt(d))/(a^2*c + 2*a*sqrt(c)*sqrt(d) + d)) - 2*(a^2*c - d)*dilog((a^2*c
+ I*a*d*x - (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c)*sqrt(d) + d
)) - 2*(a^2*c - d)*dilog((a^2*c - I*a*d*x + (I*a^2*x - a)*sqrt(c)*sqrt(d))/
(a^2*c - 2*a*sqrt(c)*sqrt(d) + d)))*sqrt(c)*sqrt(d))*a/(a^3*c^3*d - a*c^2*d
^2)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{arccot}(ax)}{(dx^2+c)^2}\,dx$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\mathrm{acot}(a\,x)}{(d\,x^2+c)^2}\,dx$$

```
[In] int(acot(a*x)/(c + d*x^2)^2,x)
[Out] int(acot(a*x)/(c + d*x^2)^2, x)
```

3.59 $\int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx = \text{Int}\left(\sqrt{c+dx^2} \cot^{-1}(ax), x\right)$$

```
[Out] Unintegrable((d*x^2+c)^(1/2)*arccot(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx = \int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx$$

```
[In] Int[Sqrt[c + d*x^2]*ArcCot[a*x],x]
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcCot[a*x], x]
```

Rubi steps

$$\text{integral} = \int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx$$

Mathematica [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx^2}\cot^{-1}(ax)\,dx = \int \sqrt{c+dx^2}\cot^{-1}(ax)\,dx$$

```
[In] Integrate[Sqrt[c + d*x^2]*ArcCot[a*x],x]
[Out] Integrate[Sqrt[c + d*x^2]*ArcCot[a*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{d\,x^2+c}\;\mathrm{arccot}\,(ax)\,dx$$

```
[In] int((d*x^2+c)^(1/2)*arccot(a*x),x)
[Out] int((d*x^2+c)^(1/2)*arccot(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\cot^{-1}(ax)\,dx = \int \sqrt{dx^2+c}\,\mathrm{arccot}\,(ax)\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)*arccot(a*x),x, algorithm="fricas")
[Out] integral(sqrt(d*x^2 + c)*arccot(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx^2}\cot^{-1}(ax)\,dx = \int \sqrt{c+dx^2}\,\mathrm{acot}\,(ax)\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)*acot(a*x),x)
[Out] Integral(sqrt(c + d*x**2)*acot(a*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*x^2+c)^(1/2)*arccot(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx = \int \sqrt{dx^2+c}\operatorname{arccot}(ax) \, dx$$

```
[In] integrate((d*x^2+c)^(1/2)*arccot(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*arccot(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx = \int \operatorname{acot}(a\,x)\,\sqrt{d\,x^2+c} \, dx$$

```
[In] int(acot(a*x)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(acot(a*x)*(c + d*x^2)^(1/2), x)
```

3.60 $\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \operatorname{Int}\left(\frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable(arccot(a*x)/(d*x^2+c)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

```
[In] Int[ArcCot[a*x]/Sqrt[c + d*x^2],x]
```

```
[Out] Defer[Int][ArcCot[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

```
[In] Integrate[ArcCot[a*x]/Sqrt[c + d*x^2],x]
[Out] Integrate[ArcCot[a*x]/Sqrt[c + d*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arccot}(ax)}{\sqrt{d\,x^2+c}}dx$$

```
[In] int(arccot(a*x)/(d*x^2+c)^(1/2),x)
[Out] int(arccot(a*x)/(d*x^2+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{arccot}(ax)}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(arccot(a*x)/sqrt(d*x^2 + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{acot}(ax)}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate(acot(a*x)/(d*x**2+c)**(1/2),x)
[Out] Integral(acot(a*x)/sqrt(c + d*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{arccot}(ax)}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(arccot(a*x)/sqrt(d*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{arccot}(ax)}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arccot(a*x)/sqrt(d*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{acot}(a\,x)}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(acot(a*x)/(c + d*x^2)^(1/2),x)
[Out] int(acot(a*x)/(c + d*x^2)^(1/2), x)
```

3.61 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx=\frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{c\sqrt{a^2c-d}}$$

```
[Out] -arctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c/(a^2*c-d)^(1/2)+x*arccot(a*x)
/c/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {197, 5033, 12, 455, 65, 214}

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx=\frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{c\sqrt{a^2c-d}}$$

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCot[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*
c - d]]/(c*Sqrt[a^2*c - d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}} + a\int \frac{x}{c\left(1+a^2x^2\right)\sqrt{c+dx^2}}\,dx \\
&= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}} + \frac{a\int \frac{x}{\left(1+a^2x^2\right)\sqrt{c+dx^2}}\,dx}{c} \\
&= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int \frac{1}{\left(1+a^2x\right)\sqrt{c+dx}}\,dx, x, x^2\right)}{2c} \\
&= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int \frac{1}{1-\frac{a^2c}{d}+\frac{a^2x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{cd} \\
&= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{c\sqrt{a^2c-d}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.56

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{\frac{2x\cot^{-1}(ax)}{\sqrt{c+dx^2}} + \frac{-\log\left(\frac{4ac\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{\sqrt{a^2c-d}(i+ax)}\right)-\log\left(\frac{4ac\left(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{\sqrt{a^2c-d}(-i+ax)}\right)}{\sqrt{a^2c-d}}}{2c}$$

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*x*ArcCot[a*x])/Sqrt[c + d*x^2] + (-Log[(4*a*c*(a*c - I*d*x + Sqrt[a^2*c
 - d]*Sqrt[c + d*x^2]))/(Sqrt[a^2*c - d]*(I + a*x))] - Log[(4*a*c*(a*c + I*
d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/(Sqrt[a^2*c - d]*(-I + a*x))])/Sqrt
[a^2*c - d])/(2*c)
```

Maple [F]

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^2+c)^{\frac{3}{2}}}dx$$

```
[In] int(arccot(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arccot(a*x)/(d*x^2+c)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(58) = 116.

Time = 0.30 (sec) , antiderivative size = 349, normalized size of antiderivative = 5.29

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \left[\frac{4\,(a^2c-d)\sqrt{dx^2+c}x\operatorname{arccot}(ax)+\sqrt{a^2c-d}(dx^2+c)\log\left(\frac{a^4d^2x^4+8\,a^4c^2-8\,a^2cd+2\,(4\,a^4cd}{}\right.}{4\,(a^2c^3-c^2d+(a^2c^2d-cd^2)x^2)}\right.$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*c - d)*sqrt(d*x^2 + c)*x*arccot(a*x) + sqrt(a^2*c - d)*(d*x^2
+ c)*log((a^4*d^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x
^2 - 4*(a^3*d*x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(
a^4*x^4 + 2*a^2*x^2 + 1)))/(a^2*c^3 - c^2*d + (a^2*c^2*d - c*d^2)*x^2), 1/2
*(2*(a^2*c - d)*sqrt(d*x^2 + c)*x*arccot(a*x) - sqrt(-a^2*c + d)*(d*x^2 + c
)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 + c)/(a
^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)))/(a^2*c^3 - c^2*d + (a^2*c^2*d - c
*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(acot(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{x\arctan\left(\frac{1}{ax}\right)}{\sqrt{dx^2+c}c} + \frac{\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{\sqrt{-a^2c+d}c}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] x*arctan(1/(a*x))/(sqrt(d*x^2 + c)*c) + arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*
c + d))/(sqrt(-a^2*c + d)*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\mathrm{acot}(a\,x)}{(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int(acot(a*x)/(c + d*x^2)^(3/2),x)
[Out] int(acot(a*x)/(c + d*x^2)^(3/2), x)
```

3.62 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{a}{3c(a^2c-d)\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c-2d)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{3c^2(a^2c-d)^{3/2}}$$

```
[Out] 1/3*x*arccot(a*x)/c/(d*x^2+c)^(3/2)-1/3*(3*a^2*c-2*d)*arctanh(a*(d*x^2+c)^(
1/2)/(a^2*c-d)^(1/2))/c^2/(a^2*c-d)^(3/2)+1/3*a/c/(a^2*c-d)/(d*x^2+c)^(1/2)
+2/3*x*arccot(a*x)/c^2/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {198, 197, 5033, 6820, 12, 585, 79, 65, 214}

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = -\frac{(3a^2c-2d)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{3c^2(a^2c-d)^{3/2}} + \frac{a}{3c(a^2c-d)\sqrt{c+dx^2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] a/(3*c*(a^2*c - d)*Sqrt[c + d*x^2]) + (x*ArcCot[a*x])/(3*c*(c + d*x^2)^(3/2
)) + (2*x*ArcCot[a*x])/(3*c^2*Sqrt[c + d*x^2]) - ((3*a^2*c - 2*d)*ArcTanh[(
a*Sqrt[c + d*x^2])/Sqrt[a^2*c - d]])/(3*c^2*(a^2*c - d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\text{integral} = \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + a\int \frac{\frac{x}{3c(c+dx^2)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}}{1+a^2x^2}\,dx$$

$$= \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + a\int \frac{x(3c+2dx^2)}{3c^2\,(1+a^2x^2)\,(c+dx^2)^{3/2}}\,dx$$

$$= \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + \frac{a\int \frac{x(3c+2dx^2)}{(1+a^2x^2)(c+dx^2)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int \frac{3c+2dx}{(1+a^2x)(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c^2}$$

$$= \frac{a}{3c\,(a^2c-d)\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + \frac{(a(3a^2c-2d))\,\text{Subst}\left(\int \frac{1}{(1+a^2x)\sqrt{c+dx}}\,dx, x, x^2\right)}{6c^2\,(a^2c-d)}$$

$$= \frac{a}{3c\,(a^2c-d)\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + \frac{(a(3a^2c-2d))\,\text{Subst}\left(\int \frac{1}{1-\frac{a^2c}{d}+\frac{a^2x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{3c^2\,(a^2c-d)\,d}$$

$$= \frac{a}{3c\,(a^2c-d)\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c-2d)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{3c^2\,(a^2c-d)^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.96

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx =$$

$$-\frac{-\frac{2ac}{(a^2c-d)\sqrt{c+dx^2}}-\frac{2x(3c+2dx^2)\cot^{-1}(ax)}{(c+dx^2)^{3/2}}+\frac{(3a^2c-2d)\log\left(\frac{12ac^2\sqrt{a^2c-d}\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(3a^2c-2d)(i+ax)}\right)}{(a^2c-d)^{3/2}}+\frac{(3a^2c-2d)\log\left(\frac{12ac^2\sqrt{a^2c-d}}{(3}\right.}{(a^2c-}}{6c^2}$$

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] -1/6*((-2*a*c)/((a^2*c - d)*Sqrt[c + d*x^2]) - (2*x*(3*c + 2*d*x^2)*ArcCot[
a*x])/(c + d*x^2)^(3/2) + ((3*a^2*c - 2*d)*Log[(12*a*c^2*Sqrt[a^2*c - d]*(a
*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((3*a^2*c - 2*d)*(I + a*x))]
)/(a^2*c - d)^(3/2) + ((3*a^2*c - 2*d)*Log[(12*a*c^2*Sqrt[a^2*c - d]*(a*c +
 I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((3*a^2*c - 2*d)*(-I + a*x))])/(
a^2*c - d)^(3/2))/c^2
```

Maple [F]

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^2+c)^{\frac{5}{2}}}dx$$

```
[In] int(arccot(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arccot(a*x)/(d*x^2+c)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(114) = 228.

Time = 0.31 (sec) , antiderivative size = 712, normalized size of antiderivative = 5.31

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \left[\frac{(3\,a^2c^3+(3\,a^2cd^2-2\,d^3)x^4-2\,c^2d+2\,(3\,a^2c^2d-2\,cd^2)x^2)\sqrt{a^2c-d}\log\left(\frac{a^4d^2x^4+8\,a^4c}{}\right.}{}\right.$$

$$-\frac{(3\,a^2c^3+(3\,a^2cd^2-2\,d^3)x^4-2\,c^2d+2\,(3\,a^2c^2d-2\,cd^2)x^2)\sqrt{-a^2c+d}\arctan\left(-\frac{(a^2dx^2+2\,a^2c-d)\sqrt{-a^2c+d}\sqrt{dx}}{2\,(a^3c^2-acd+(a^3cd-ad^2)x^2)}\right.}{6\,(a^4c^6-2\,a^2c^5d+c^4d^2+(a^4c^4d^2-}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*a^2*c^3 + (3*a^2*c*d^2 - 2*d^3)*x^4 - 2*c^2*d + 2*(3*a^2*c^2*d -
2*c*d^2)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*
(4*a^4*c*d - 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)
*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*(a^3*c^3 - a*c^2*d +
 (a^3*c^2*d - a*c*d^2)*x^2 + (2*(a^4*c^2*d - 2*a^2*c*d^2 + d^3)*x^3 + 3*(a^
4*c^3 - 2*a^2*c^2*d + c*d^2)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(a^4*c^6 - 2*
a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 - 2*a^2*c^3*d^3 + c^2*d^4)*x^4 + 2*(a^4*
c^5*d - 2*a^2*c^4*d^2 + c^3*d^3)*x^2), -1/6*((3*a^2*c^3 + (3*a^2*c*d^2 - 2*
d^3)*x^4 - 2*c^2*d + 2*(3*a^2*c^2*d - 2*c*d^2)*x^2)*sqrt(-a^2*c + d)*arctan
(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 -
 a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(a^3*c^3 - a*c^2*d + (a^3*c^2*d - a*c*
d^2)*x^2 + (2*(a^4*c^2*d - 2*a^2*c*d^2 + d^3)*x^3 + 3*(a^4*c^3 - 2*a^2*c^2*
d + c*d^2)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(a^4*c^6 - 2*a^2*c^5*d + c^4*d^
2 + (a^4*c^4*d^2 - 2*a^2*c^3*d^3 + c^2*d^4)*x^4 + 2*(a^4*c^5*d - 2*a^2*c^4*
d^2 + c^3*d^3)*x^2)]
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(acot(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{1}{3}\,a\left(\frac{(3\,a^2c-2\,d)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{(a^2c^3-c^2d)\sqrt{-a^2c+d}a}+\frac{1}{(a^2c^2-cd)\sqrt{dx^2+c}}\right) + \frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)\arctan\left(\frac{1}{ax}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*a^2*c - 2*d)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*c + d))/((a^2*c^3
 - c^2*d)*sqrt(-a^2*c + d)*a) + 1/((a^2*c^2 - c*d)*sqrt(d*x^2 + c))) + 1/3*
x*(2*d*x^2/c^2 + 3/c)*arctan(1/(a*x))/(d*x^2 + c)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\mathrm{acot}(a\,x)}{(d\,x^2+c)^{5/2}}\,dx$$

```
[In] int(acot(a*x)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(acot(a*x)/(c + d*x^2)^(5/2), x)
```

3.63 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 208

$$\begin{aligned}\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx &= \frac{a}{15c\,(a^2c-d)\,(c+dx^2)^{3/2}} \\ &+ \frac{a(7a^2c-4d)}{15c^2\,(a^2c-d)^2\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} \\ &+ \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{(15a^4c^2-20a^2cd+8d^2)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3\,(a^2c-d)^{5/2}}\end{aligned}$$

```
[Out] 1/15*a/c/(a^2*c-d)/(d*x^2+c)^(3/2)+1/5*x*arccot(a*x)/c/(d*x^2+c)^(5/2)+4/15
*x*arccot(a*x)/c^2/(d*x^2+c)^(3/2)-1/15*(15*a^4*c^2-20*a^2*c*d+8*d^2)*arcta
nh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c^3/(a^2*c-d)^(5/2)+1/15*a*(7*a^2*c-4
*d)/c^2/(a^2*c-d)^2/(d*x^2+c)^(1/2)+8/15*x*arccot(a*x)/c^3/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {198, 197, 5033, 6820, 12, 6847, 911, 1275, 214}

$$\begin{aligned}\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx &= \frac{a(7a^2c-4d)}{15c^2\,(a^2c-d)^2\sqrt{c+dx^2}} + \frac{a}{15c\,(a^2c-d)\,(c+dx^2)^{3/2}} \\ &- \frac{(15a^4c^2-20a^2cd+8d^2)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3\,(a^2c-d)^{5/2}} \\ &+ \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} + \frac{4x\cot^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}\end{aligned}$$

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] a/(15*c*(a^2*c - d)*(c + d*x^2)^(3/2)) + (a*(7*a^2*c - 4*d))/(15*c^2*(a^2*c
 - d)^2*Sqrt[c + d*x^2]) + (x*ArcCot[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*A
rcCot[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcCot[a*x])/(15*c^3*Sqrt[c +
 d*x^2]) - ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2])/S
qrt[a^2*c - d]])/(15*c^3*(a^2*c - d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}+\frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} \\
&\quad +a\int\frac{\frac{x}{5c(c+dx^2)^{5/2}}+\frac{4x}{15c^2(c+dx^2)^{3/2}}+\frac{8x}{15c^3\sqrt{c+dx^2}}}{1+a^2x^2}\,dx \\
&= \frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}+\frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+a\int\frac{x(15c^2+20cdx^2+8d^2x^4)}{15c^3\left(1+a^2x^2\right)\left(c+dx^2\right)^{5/2}}\,dx \\
&= \frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}+\frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{a\int\frac{x(15c^2+20cdx^2+8d^2x^4)}{(1+a^2x^2)(c+dx^2)^{5/2}}\,dx}{15c^3} \\
&= \frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}+\frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{a\text{Subst}\left(\int\frac{15c^2+20cdx+8d^2x^2}{(1+a^2x)(c+dx)^{5/2}}\,dx,x,x^2\right)}{30c^3} \\
&= \frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}+\frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} \\
&\quad +\frac{a\text{Subst}\left(\int\frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{-a^2c+d}{d}+\frac{a^2x^2}{d}\right)}\,dx,x,\sqrt{c+dx^2}\right)}{15c^3d} \\
&= \frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}+\frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} \\
&\quad +\frac{a\text{Subst}\left(\int\left(\frac{3c^2d}{(-a^2c+d)x^4}-\frac{c(7a^2c-4d)d}{(-a^2c+d)^2x^2}+\frac{d(15a^4c^2-20a^2cd+8d^2)}{(-a^2c+d)^2(-a^2c+d+a^2x^2)}\right)dx,x,\sqrt{c+dx^2}\right)}{15c^3d}
\end{aligned}$$

$$= \frac{a}{15c\left(a^2c-d\right)\left(c+dx^2\right)^{3/2}} + \frac{a(7a^2c-4d)}{15c^2\left(a^2c-d\right)^2\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} + \frac{(a(15a^4c^2-20a^2cd+8d^2))\,\text{Subst}\left(\int \frac{1}{-a^2c+d+a^2x^2}\,dx, x, \sqrt{c+dx^2}\right)}{15c^3\left(a^2c-d\right)^2}$$

$$= \frac{a}{15c\left(a^2c-d\right)\left(c+dx^2\right)^{3/2}} + \frac{a(7a^2c-4d)}{15c^2\left(a^2c-d\right)^2\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{(15a^4c^2-20a^2cd+8d^2)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3\left(a^2c-d\right)^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.66

$$\int \frac{\cot^{-1}(ax)}{\left(c+dx^2\right)^{7/2}}\,dx =$$

$$-\frac{-\frac{2ac\left(-d\left(5c+4dx^2\right)+a^2c\left(8c+7dx^2\right)\right)}{\left(-a^2c+d\right)^2\left(c+dx^2\right)^{3/2}} - \frac{2x\left(15c^2+20cdx^2+8d^2x^4\right)\cot^{-1}(ax)}{\left(c+dx^2\right)^{5/2}} + \frac{\left(15a^4c^2-20a^2cd+8d^2\right)\log\left(\frac{60ac^3\left(a^2c-d\right)^{3/2}\left(ac-idx+\sqrt{a^2c\cdots}\right.}{\left(15a^4c^2-20a^2cd+8d^2\right)(i\cdots}\right.}{\left(a^2c-d\right)^{5/2}}\cdots}{30c^3}$$

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] -1/30*((-2*a*c*(-(d*(5*c + 4*d*x^2)) + a^2*c*(8*c + 7*d*x^2)))/((-(a^2*c) +
 d)^2*(c + d*x^2)^(3/2)) - (2*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcCot[a*
x])/(c + d*x^2)^(5/2) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*Log[(60*a*c^3*(a
^2*c - d)^(3/2)*(a*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((15*a^4*c
^2 - 20*a^2*c*d + 8*d^2)*(I + a*x))])/(a^2*c - d)^(5/2) + ((15*a^4*c^2 - 20
*a^2*c*d + 8*d^2)*Log[(60*a*c^3*(a^2*c - d)^(3/2)*(a*c + I*d*x + Sqrt[a^2*c
 - d]*Sqrt[c + d*x^2]))/((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*(-I + a*x))])/(a
^2*c - d)^(5/2))/c^3
```

Maple [F]

$$\int \frac{\operatorname{arccot}(ax)}{\left(d\,x^2+c\right)^{\frac{7}{2}}}dx$$

```
[In] int(arccot(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arccot(a*x)/(d*x^2+c)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. 2(180) = 360.

Time = 0.37 (sec) , antiderivative size = 1278, normalized size of antiderivative = 6.14

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2
 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x
^2 + 1)) + 4*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2 - 11*
a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*c^2*d^3
)*x^2 + (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4 - d^5)*x^5 + 20*(a^6*
c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 + 15*(a^6*c^5 - 3*a^4*c^
4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(a^6*c^9 -
3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3 - 3*a^4*c^5*d^4 + 3*a^
2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 -
 c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 - c^5*d^4)*x^2
), -1/30*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d
^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3
*(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2*c + d)*arctan(-
1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 - a
*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2
+ (7*a^5*c^3*d^2 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3
*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4
 - d^5)*x^5 + 20*(a^6*c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 +
15*(a^6*c^5 - 3*a^4*c^4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arccot(a*x))*sqrt(d
*x^2 + c))/(a^6*c^9 - 3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3
- 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6
*d^3 + 3*a^2*c^5*d^4 - c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*
c^6*d^3 - c^5*d^4)*x^2)]
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(acot(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{1}{15}\,a\left(\frac{(15\,a^4c^2-20\,a^2cd+8\,d^2)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{(a^4c^5-2\,a^2c^4d+c^3d^2)\sqrt{-a^2c+d}a}+\frac{7\,(dx^2+c)a^2c+a^2c^2-4\,(dx^2+}{(a^4c^4-2\,a^2c^3d+c^2d^2)(dx^2}\right.$$
$$+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\frac{5\,d}{c^2}\right)+\frac{15}{c}\right)x\arctan\left(\frac{1}{ax}\right)}{15\,(dx^2+c)^{\frac{5}{2}}}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*a*((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^
2*c + d))/((a^4*c^5 - 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c + d)*a) + (7*(d*x^
2 + c)*a^2*c + a^2*c^2 - 4*(d*x^2 + c)*d - c*d)/((a^4*c^4 - 2*a^2*c^3*d + c
^2*d^2)*(d*x^2 + c)^(3/2))) + 1/15*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 15/c)
*x*arctan(1/(a*x))/(d*x^2 + c)^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\mathrm{acot}(a\,x)}{(d\,x^2+c)^{7/2}}\,dx$$

```
[In] int(acot(a*x)/(c + d*x^2)^(7/2),x)
[Out] int(acot(a*x)/(c + d*x^2)^(7/2), x)
```

3.64 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 293

$$\begin{aligned}\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx &= \frac{a}{35c(a^2c-d)(c+dx^2)^{5/2}}+\frac{a(11a^2c-6d)}{105c^2(a^2c-d)^2(c+dx^2)^{3/2}}\\&+\frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3(a^2c-d)^3\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{7c(c+dx^2)^{7/2}}+\frac{6x\cot^{-1}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{8x\cot^{-1}(ax)}{35c^3(c+dx^2)^{3/2}}\\&+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}}-\frac{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4(a^2c-d)^{7/2}}\end{aligned}$$

```
[Out] 1/35*a/c/(a^2*c-d)/(d*x^2+c)^(5/2)+1/105*a*(11*a^2*c-6*d)/c^2/(a^2*c-d)^2/(
d*x^2+c)^(3/2)+1/7*x*arccot(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arccot(a*x)/c^2/(
d*x^2+c)^(5/2)+8/35*x*arccot(a*x)/c^3/(d*x^2+c)^(3/2)-1/35*(35*a^6*c^3-70*a
^4*c^2*d+56*a^2*c*d^2-16*d^3)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c^
4/(a^2*c-d)^(7/2)+1/35*a*(19*a^4*c^2-22*a^2*c*d+8*d^2)/c^3/(a^2*c-d)^3/(d*x
^2+c)^(1/2)+16/35*x*arccot(a*x)/c^4/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used

= {198, 197, 5033, 6820, 12, 6847, 1633, 65, 214}

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{a(11a^2c-6d)}{105c^2(a^2c-d)^2(c+dx^2)^{3/2}} + \frac{a}{35c(a^2c-d)(c+dx^2)^{5/2}} + \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3(a^2c-d)^3\sqrt{c+dx^2}} - \frac{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\,\mathrm{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4(a^2c-d)^{7/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + \frac{8x\cot^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{6x\cot^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{x\cot^{-1}(ax)}{7c(c+dx^2)^{7/2}}$$

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] a/(35*c*(a^2*c - d)*(c + d*x^2)^(5/2)) + (a*(11*a^2*c - 6*d))/(105*c^2*(a^2
*c - d)^2*(c + d*x^2)^(3/2)) + (a*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2))/(35*c^
3*(a^2*c - d)^3*Sqrt[c + d*x^2]) + (x*ArcCot[a*x])/(7*c*(c + d*x^2)^(7/2))
+ (6*x*ArcCot[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcCot[a*x])/(35*c^3*
(c + d*x^2)^(3/2)) + (16*x*ArcCot[a*x])/(35*c^4*Sqrt[c + d*x^2]) - ((35*a^6
*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2])/Sq
rt[a^2*c - d]])/(35*c^4*(a^2*c - d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1633

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}+\frac{6x\cot^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}}+\frac{8x\cot^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}}+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} \\
&\quad+a\int\frac{\frac{x}{7c(c+dx^2)^{7/2}}+\frac{6x}{35c^2(c+dx^2)^{5/2}}+\frac{8x}{35c^3(c+dx^2)^{3/2}}+\frac{16x}{35c^4\sqrt{c+dx^2}}}{1+a^2x^2}\,dx \\
&= \frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}+\frac{6x\cot^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}}+\frac{8x\cot^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} \\
&\quad+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+a\int\frac{x(35c^3+70c^2dx^2+56cd^2x^4+16d^3x^6)}{35c^4\,(1+a^2x^2)\,(c+dx^2)^{7/2}}\,dx \\
&= \frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}+\frac{6x\cot^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}}+\frac{8x\cot^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} \\
&\quad+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{a\int\frac{x(35c^3+70c^2dx^2+56cd^2x^4+16d^3x^6)}{(1+a^2x^2)(c+dx^2)^{7/2}}\,dx}{35c^4}
\end{aligned}$$

$$= \frac{x\cot^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int \frac{35c^3+70c^2dx+56cd^2x^2+16d^3x^3}{(1+a^2x)(c+dx)^{7/2}}\,dx, x, x^2\right)}{70c^4}$$

$$= \frac{x\cot^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int \left(-\frac{5c^3d}{(a^2c-d)(c+dx)^{7/2}} - \frac{c^2(11a^2c-6d)d}{(-a^2c+d)^2(c+dx)^{5/2}} + \frac{cd(19a^4c^2-22a^2cd+8d^2)}{(-a^2c+d)^3(c+dx)^{3/2}} + \frac{35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3}{(a^2c-d)^3(1+a^2x)\sqrt{c+dx}}\right.\right.}{70c^4}$$

$$= \frac{a}{35c\left(a^2c-d\right)\left(c+dx^2\right)^{5/2}} + \frac{a(11a^2c-6d)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}} + \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + \frac{\left(a(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\right)\text{Subst}\left(\int \frac{1}{(1+a^2x)\sqrt{c+}}\right.}{70c^4\left(a^2c-d\right)^3}$$

$$= \frac{a}{35c\left(a^2c-d\right)\left(c+dx^2\right)^{5/2}} + \frac{a(11a^2c-6d)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}} + \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + \frac{\left(a(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\right)\text{Subst}\left(\int \frac{1}{1-\frac{a^2c}{d}+\frac{a^2x^2}{d}}\right.}{35c^4\left(a^2c-d\right)^3 d}$$

$$= \frac{a}{35c\left(a^2c-d\right)\left(c+dx^2\right)^{5/2}} + \frac{a(11a^2c-6d)}{105c^2\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}} + \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3\left(a^2c-d\right)^3\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4\left(a^2c-d\right)^{7/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.54

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{\frac{2ac\left(3c^2(-a^2c+d)^2+c(11a^2c-6d)(a^2c-d)(c+dx^2)+3(19a^4c^2-22a^2cd+8d^2)(c+dx^2)^2\right)}{(a^2c-d)^3(c+dx^2)^{5/2}}+\frac{6x(35c^3+70c^2dx^2+56cd}{(c+dx}}{}$$

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] ((2*a*c*(3*c^2*(-(a^2*c) + d)^2 + c*(11*a^2*c - 6*d)*(a^2*c - d)*(c + d*x^2
) + 3*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2)*(c + d*x^2)^2))/((a^2*c - d)^3*(c +
 d*x^2)^(5/2)) + (6*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^6)*A
rcCot[a*x])/(c + d*x^2)^(7/2) - (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^
2 - 16*d^3)*Log[(140*a*c^4*(a^2*c - d)^(5/2)*(a*c - I*d*x + Sqrt[a^2*c - d]
*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*(I
+ a*x))])/(a^2*c - d)^(7/2) - (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2
- 16*d^3)*Log[(140*a*c^4*(a^2*c - d)^(5/2)*(a*c + I*d*x + Sqrt[a^2*c - d]*S
qrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*(-I +
 a*x))])/(a^2*c - d)^(7/2))/(210*c^4)
```

Maple [F]

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

```
[In] int(arccot(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arccot(a*x)/(d*x^2+c)^(9/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. 2(257) = 514.

Time = 0.61 (sec) , antiderivative size = 1986, normalized size of antiderivative = 6.78

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(3*(35*a^6*c^7 - 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 - 7
0*a^4*c^2*d^5 + 56*a^2*c*d^6 - 16*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d^3
 - 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 - 70
```

```
*a^4*c^4*d^3 + 56*a^2*c^3*d^4 - 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*a^4*
c^5*d^2 + 56*a^2*c^4*d^3 - 16*c^3*d^4)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2*x^
4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 +
2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x^2
+ 1)) + 4*(71*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*d^3 + 3*
(19*a^7*c^4*d^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6 + (182*a
^7*c^5*d^2 - 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^4 + (196*a
^7*c^6*d - 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2 + 3*(16*(a
^8*c^4*d^3 - 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)*x^7 + 56*(a
^8*c^5*d^2 - 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c*d^6)*x^5 + 7
0*(a^8*c^6*d - 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4 + c^2*d^5)*x^3
 + 35*(a^8*c^7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3 + c^3*d^4)*x)*
arccot(a*x))*sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*a^4*c^10*d^2 - 4
*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 - 4*a
^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5
- 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6*c^9*d^3 + 6*a^4*c^
8*d^4 - 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d - 4*a^6*c^10*d^2 + 6*a
^4*c^9*d^3 - 4*a^2*c^8*d^4 + c^7*d^5)*x^2), -1/210*(3*(35*a^6*c^7 - 70*a^4*
c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 - 70*a^4*c^2*d^5 + 56*a^2*c*d^6 -
16*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 - 70*a^4*c^3*d^4 + 56*a^2*c^2*
d^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 - 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 -
 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*a^4*c^5*d^2 + 56*a^2*c^4*d^3 - 16*c
^3*d^4)*x^2)*sqrt(-a^2*c + d)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a
^2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(7
1*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*d^3 + 3*(19*a^7*c^4*
d^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6 + (182*a^7*c^5*d^2 -
 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^4 + (196*a^7*c^6*d - 4
34*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2 + 3*(16*(a^8*c^4*d^3 -
 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)*x^7 + 56*(a^8*c^5*d^2 -
 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c*d^6)*x^5 + 70*(a^8*c^6*d
 - 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4 + c^2*d^5)*x^3 + 35*(a^8*c
^7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3 + c^3*d^4)*x)*arccot(a*x))
*sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*a^4*c^10*d^2 - 4*a^2*c^9*d^3
 + c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 - 4*a^2*c^5*d^7 +
 c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5 - 4*a^2*c^6*
d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6*c^9*d^3 + 6*a^4*c^8*d^4 - 4*a^
2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d - 4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 -
 4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(acot(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2)**(9/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.16

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{1}{105}\,a\left(\frac{3\,(35\,a^6c^3-70\,a^4c^2d+56\,a^2cd^2-16\,d^3)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{(a^6c^7-3\,a^4c^6d+3\,a^2c^5d^2-c^4d^3)\sqrt{-a^2c+d}a} + \frac{57\,(dx^2+c)^2a^4c^2 \cdots}{\cdots}\right) + \frac{\left(2\left(4\,x^2\left(\frac{2\,d^3x^2}{c^4}+\frac{7\,d^2}{c^3}\right)+\frac{35\,d}{c^2}\right)x^2+\frac{35}{c}\right)x\arctan\left(\frac{1}{ax}\right)}{35\,(dx^2+c)^{\frac{7}{2}}}$$

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*a*(3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*arctan(sqrt(
d*x^2 + c)*a/sqrt(-a^2*c + d))/((a^6*c^7 - 3*a^4*c^6*d + 3*a^2*c^5*d^2 - c^
4*d^3)*sqrt(-a^2*c + d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*a^4
*c^3 + 3*a^4*c^4 - 66*(d*x^2 + c)^2*a^2*c*d - 17*(d*x^2 + c)*a^2*c^2*d - 6*
a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((a^6*c
^6 - 3*a^4*c^5*d + 3*a^2*c^4*d^2 - c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/35*(2*(
4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*arctan(1/(a*x))
/(d*x^2 + c)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\mathrm{acot}(a\,x)}{(d\,x^2+c)^{9/2}}\,dx$$

```
[In] int(acot(a*x)/(c + d*x^2)^(9/2),x)
[Out] int(acot(a*x)/(c + d*x^2)^(9/2), x)
```

3.65 $\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 195

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx=\frac{1}{2}\sqrt{a+ax^2}+\frac{1}{2}x\sqrt{a+ax^2}\cot^{-1}(x)-\frac{ia\sqrt{1+x^2}\cot^{-1}(x)\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}-\frac{ia\sqrt{1+x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{2\sqrt{a+ax^2}}+\frac{ia\sqrt{1+x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{2\sqrt{a+ax^2}}$$

```
[Out] -I*a*arccot(x)*arctan((1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)^
(1/2)-1/2*I*a*polylog(2,-I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^
2+a)^(1/2)+1/2*I*a*polylog(2,I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(
a*x^2+a)^(1/2)+1/2*(a*x^2+a)^(1/2)+1/2*x*arccot(x)*(a*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

$= \{4999, 5011, 5007\}$

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx = -\frac{ia\sqrt{x^2+1}\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)\cot^{-1}(x)}{\sqrt{ax^2+a}} - \frac{ia\sqrt{x^2+1}\,\text{PolyLog}\left(2,-\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)}{2\sqrt{ax^2+a}} + \frac{ia\sqrt{x^2+1}\,\text{PolyLog}\left(2,\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)}{2\sqrt{ax^2+a}} + \frac{1}{2}\sqrt{ax^2+a} + \frac{1}{2}x\sqrt{ax^2+a}\cot^{-1}(x)$$

```
[In] Int[Sqrt[a + a*x^2]*ArcCot[x],x]
```

```
[Out] Sqrt[a + a*x^2]/2 + (x*Sqrt[a + a*x^2]*ArcCot[x])/2 - (I*a*Sqrt[1 + x^2]*Ar
cCot[x]*ArcTan[Sqrt[1 + I*x]/Sqrt[1 - I*x]])/Sqrt[a + a*x^2] - ((I/2)*a*Sqr
t[1 + x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*x])/Sqrt[1 - I*x]])/Sqrt[a + a*x^2]
+ ((I/2)*a*Sqrt[1 + x^2]*PolyLog[2, (I*Sqrt[1 + I*x])/Sqrt[1 - I*x]])/Sqrt[
a + a*x^2]
```

Rule 4999

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)
), Int[(d + e*x^2)^(q - 1)*(a + b*ArcCot[c*x]), x], x] + Simp[x*(d + e*x^2)
^q*((a + b*ArcCot[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ
[e, c^2*d] && GtQ[q, 0]
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5011

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCot[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\sqrt{a+ax^2}+\frac{1}{2}x\sqrt{a+ax^2}\cot^{-1}(x)+\frac{1}{2}a\int\frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx\\
&= \frac{1}{2}\sqrt{a+ax^2}+\frac{1}{2}x\sqrt{a+ax^2}\cot^{-1}(x)+\frac{\left(a\sqrt{1+x^2}\right)\int\frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx}{2\sqrt{a+ax^2}}\\
&= \frac{1}{2}\sqrt{a+ax^2}+\frac{1}{2}x\sqrt{a+ax^2}\cot^{-1}(x)-\frac{ia\sqrt{1+x^2}\cot^{-1}(x)\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}\\
&\quad -\frac{ia\sqrt{1+x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{2\sqrt{a+ax^2}}+\frac{ia\sqrt{1+x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{2\sqrt{a+ax^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

$$
\int\sqrt{a+ax^2}\cot^{-1}(x)\,dx = -\frac{(a(1+x^2))^{3/2}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(x)\right)-\cot^{-1}(x)\csc^2\left(\frac{1}{2}\cot^{-1}(x)\right)+4\cot^{-1}(x)\log\left(1-e^{i\cot^{-1}(x)}\right)-4\,\text{c}\ldots}{\ldots}
$$

```
[In] Integrate[Sqrt[a + a*x^2]*ArcCot[x],x]
```

```
[Out] -1/8*((a*(1 + x^2))^(3/2)*(-2*Cot[ArcCot[x]/2] - ArcCot[x]*Csc[ArcCot[x]/2]
^2 + 4*ArcCot[x]*Log[1 - E^(I*ArcCot[x])] - 4*ArcCot[x]*Log[1 + E^(I*ArcCot
[x])] + (4*I)*PolyLog[2, -E^(I*ArcCot[x])] - (4*I)*PolyLog[2, E^(I*ArcCot[x
])] + ArcCot[x]*Sec[ArcCot[x]/2]^2 - 2*Tan[ArcCot[x]/2]))/(a*(1 + x^(-2))^(
3/2)*x^3)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

method	result
default	$\frac{\sqrt{a(i+x)(x-i)}\,(x\,\text{arccot}(x)+1)}{2}-\frac{i\sqrt{a(i+x)(x-i)}\left(i\,\text{arccot}(x)\ln\left(\frac{i+x}{\sqrt{x^2+1}}+1\right)-i\,\text{arccot}(x)\ln\left(1-\frac{i+x}{\sqrt{x^2+1}}\right)+\text{polylog}\left(2,-\frac{i+x}{\sqrt{x^2+1}}\ldots\right.\right.}{2\sqrt{x^2+1}}$

```
[In] int((a*x^2+a)^(1/2)*arccot(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*(I+x)*(x-I))^(1/2)*(x*arccot(x)+1)-1/2*I*(a*(I+x)*(x-I))^(1/2)*(I*ar
ccot(x)*ln((I+x)/(x^2+1)^(1/2)+1)-I*arccot(x)*ln(1-(I+x)/(x^2+1)^(1/2))+pol
ylog(2,-(I+x)/(x^2+1)^(1/2))-polylog(2,(I+x)/(x^2+1)^(1/2)))/(x^2+1)^(1/2)
```

Fricas [F]

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx = \int \sqrt{ax^2+a}\,\mathrm{arccot}\,(x)\;dx$$

```
[In] integrate((a*x^2+a)^(1/2)*arccot(x),x, algorithm="fricas")
[Out] integral(sqrt(a*x^2 + a)*arccot(x), x)
```

Sympy [F]

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx = \int \sqrt{a\,(x^2+1)}\,\mathrm{acot}\,(x)\,dx$$

```
[In] integrate((a*x**2+a)**(1/2)*acot(x),x)
[Out] Integral(sqrt(a*(x**2 + 1))*acot(x), x)
```

Maxima [F]

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx = \int \sqrt{ax^2+a}\,\mathrm{arccot}\,(x)\;dx$$

```
[In] integrate((a*x^2+a)^(1/2)*arccot(x),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^2 + a)*arccot(x), x)
```

Giac [F]

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx = \int \sqrt{ax^2+a}\,\mathrm{arccot}\,(x)\;dx$$

```
[In] integrate((a*x^2+a)^(1/2)*arccot(x),x, algorithm="giac")
[Out] integrate(sqrt(a*x^2 + a)*arccot(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx=\int \mathrm{acot}(x)\,\sqrt{a\,x^2+a}\,dx$$

```
[In] int(acot(x)*(a + a*x^2)^(1/2),x)
[Out] int(acot(x)*(a + a*x^2)^(1/2), x)
```

3.66 $\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 155

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}} \, dx = -\frac{2i\sqrt{1+x^2}\cot^{-1}(x)\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}} - \frac{i\sqrt{1+x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}} + \frac{i\sqrt{1+x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}$$

```
[Out] -2*I*arccot(x)*arctan((1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)^
(1/2)-I*polylog(2,-I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)^(
1/2)+I*polylog(2,I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5011, 5007}

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}} \, dx = -\frac{2i\sqrt{x^2+1}\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)\cot^{-1}(x)}{\sqrt{ax^2+a}} - \frac{i\sqrt{x^2+1}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)}{\sqrt{ax^2+a}} + \frac{i\sqrt{x^2+1}\operatorname{PolyLog}\left(2,\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)}{\sqrt{ax^2+a}}$$

```
[In] Int[ArcCot[x]/Sqrt[a + a*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + x^2]*ArcCot[x]*ArcTan[Sqrt[1 + I*x]/Sqrt[1 - I*x]])/Sqrt[a
 + a*x^2] - (I*Sqrt[1 + x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*x])/Sqrt[1 - I*x]]
```

```
)/Sqrt[a + a*x^2] + (I*Sqrt[1 + x^2]*PolyLog[2, (I*Sqrt[1 + I*x])/Sqrt[1 -
I*x]])/Sqrt[a + a*x^2]
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5011

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCot[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+x^2}\int \frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx}{\sqrt{a+ax^2}}\\
&= -\frac{2i\sqrt{1+x^2}\cot^{-1}(x)\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}\\
&\quad -\frac{i\sqrt{1+x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}+\frac{i\sqrt{1+x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$
\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = -\frac{\sqrt{a(1+x^2)}\left(\cot^{-1}(x)\left(\log\left(1-e^{i\cot^{-1}(x)}\right)-\log\left(1+e^{i\cot^{-1}(x)}\right)\right)+i\,\text{PolyLog}\left(2,-e^{i\cot^{-1}(x)}\right)-i\,\text{Pol}\right.}{a\sqrt{1+\frac{1}{x^2}}\,x}
$$

```
[In] Integrate[ArcCot[x]/Sqrt[a + a*x^2],x]
```

```
[Out] -((Sqrt[a*(1 + x^2)]*(ArcCot[x]*(Log[1 - E^(I*ArcCot[x])] - Log[1 + E^(I*Ar
cCot[x])]) + I*PolyLog[2, -E^(I*ArcCot[x])] - I*PolyLog[2, E^(I*ArcCot[x])]
))/(a*Sqrt[1 + x^(-2)]*x))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{i\left(i \operatorname{arccot}(x) \ln\left(\frac{i+x}{\sqrt{x^{2}+1}}+1\right)-i \operatorname{arccot}(x) \ln\left(1-\frac{i+x}{\sqrt{x^{2}+1}}\right)+\operatorname{polylog}\left(2,-\frac{i+x}{\sqrt{x^{2}+1}}\right)-\operatorname{polylog}\left(2,\frac{i+x}{\sqrt{x^{2}+1}}\right)\right)\sqrt{a(i+x)(x-i)}}{\sqrt{x^{2}+1}\,a}$	99

[In] int(arccot(x)/(a*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -I*(I*arccot(x)*ln((I+x)/(x^2+1)^(1/2)+1)-I*arccot(x)*ln(1-(I+x)/(x^2+1)^(1/2))+polylog(2,-(I+x)/(x^2+1)^(1/2))-polylog(2,(I+x)/(x^2+1)^(1/2)))*(a*(I+x)*(x-I))^(1/2)/(x^2+1)^(1/2)/a

Fricas [F]

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\operatorname{arccot}(x)}{\sqrt{ax^2+a}}\,dx$$

[In] integrate(arccot(x)/(a*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral(arccot(x)/sqrt(a*x^2 + a), x)

Sympy [F]

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\operatorname{acot}(x)}{\sqrt{a(x^2+1)}}\,dx$$

[In] integrate(acot(x)/(a*x**2+a)**(1/2),x)

[Out] Integral(acot(x)/sqrt(a*(x**2 + 1)), x)

Maxima [F]

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\operatorname{arccot}(x)}{\sqrt{ax^2+a}}\,dx$$

[In] integrate(arccot(x)/(a*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(arccot(x)/sqrt(a*x^2 + a), x)

Giac [F]

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\operatorname{arccot}(x)}{\sqrt{ax^2+a}}\,dx$$

[In] integrate(arccot(x)/(a*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(arccot(x)/sqrt(a*x^2 + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\operatorname{acot}(x)}{\sqrt{a\,x^2+a}}\,dx$$

[In] int(acot(x)/(a + a*x^2)^(1/2),x)

[Out] int(acot(x)/(a + a*x^2)^(1/2), x)

3.67 $\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = -\frac{1}{a\sqrt{a+ax^2}}+\frac{x\cot^{-1}(x)}{a\sqrt{a+ax^2}}$$

```
[Out] -1/a/(a*x^2+a)^(1/2)+x*arccot(x)/a/(a*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5015}

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \frac{x\cot^{-1}(x)}{a\sqrt{ax^2+a}}-\frac{1}{a\sqrt{ax^2+a}}$$

```
[In] Int[ArcCot[x]/(a + a*x^2)^(3/2),x]
```

```
[Out] -(1/(a*Sqrt[a + a*x^2])) + (x*ArcCot[x])/(a*Sqrt[a + a*x^2])
```

Rule 5015

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCot[c*x])/(d*Sq
rt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\text{integral} = -\frac{1}{a\sqrt{a+ax^2}}+\frac{x\cot^{-1}(x)}{a\sqrt{a+ax^2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \frac{-1+x\cot^{-1}(x)}{a\sqrt{a(1+x^2)}}$$

```
[In] Integrate[ArcCot[x]/(a + a*x^2)^(3/2),x]
[Out] (-1 + x*ArcCot[x])/(a*Sqrt[a*(1 + x^2)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

method	result	size
risch	$\frac{ix\ln(ix+1)}{2a\sqrt{a(x^2+1)}}+\frac{-ix\ln(-ix+1)+\pi x-2}{2a\sqrt{a(x^2+1)}}$	55
default	$\frac{(\operatorname{arccot}(x)+i)(i+x)\sqrt{a(i+x)(x-i)}}{2(x^2+1)a^2}+\frac{\sqrt{a(i+x)(x-i)}\,(x-i)(\operatorname{arccot}(x)-i)}{2(x^2+1)a^2}$	68

```
[In] int(arccot(x)/(a*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I/a*x/(a*(x^2+1))^(1/2)*ln(1+I*x)+1/2/a*(-I*x*ln(1-I*x)+Pi*x-2)/(a*(x^2
+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \frac{\sqrt{ax^2+a}(x\operatorname{arccot}(x)-1)}{a^2x^2+a^2}$$

```
[In] integrate(arccot(x)/(a*x^2+a)^(3/2),x, algorithm="fricas")
[Out] sqrt(a*x^2 + a)*(x*arccot(x) - 1)/(a^2*x^2 + a^2)
```

Sympy [F]

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \int \frac{\operatorname{acot}(x)}{(a(x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(acot(x)/(a*x**2+a)**(3/2),x)
[Out] Integral(acot(x)/(a*(x**2 + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \frac{x\operatorname{arccot}(x)}{\sqrt{ax^2+a}a} - \frac{1}{\sqrt{ax^2+a}a}$$

```
[In] integrate(arccot(x)/(a*x^2+a)^(3/2),x, algorithm="maxima")
[Out] x*arccot(x)/(sqrt(a*x^2 + a)*a) - 1/(sqrt(a*x^2 + a)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \frac{x\arctan\left(\frac{1}{x}\right)}{\sqrt{ax^2+a}a} - \frac{1}{\sqrt{ax^2+a}a}$$

```
[In] integrate(arccot(x)/(a*x^2+a)^(3/2),x, algorithm="giac")
[Out] x*arctan(1/x)/(sqrt(a*x^2 + a)*a) - 1/(sqrt(a*x^2 + a)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \int \frac{\operatorname{acot}(x)}{(a\,x^2+a)^{3/2}}\,dx$$

```
[In] int(acot(x)/(a + a*x^2)^(3/2),x)
[Out] int(acot(x)/(a + a*x^2)^(3/2), x)
```

3.68 $\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = -\frac{1}{9a\,(a+ax^2)^{3/2}} - \frac{2}{3a^2\sqrt{a+ax^2}} + \frac{x\cot^{-1}(x)}{3a\,(a+ax^2)^{3/2}} + \frac{2x\cot^{-1}(x)}{3a^2\sqrt{a+ax^2}}$$

```
[Out] -1/9/a/(a*x^2+a)^(3/2)+1/3*x*arccot(x)/a/(a*x^2+a)^(3/2)-2/3/a^2/(a*x^2+a)^
(1/2)+2/3*x*arccot(x)/a^2/(a*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5017, 5015}

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = -\frac{2}{3a^2\sqrt{ax^2+a}} + \frac{2x\cot^{-1}(x)}{3a^2\sqrt{ax^2+a}} - \frac{1}{9a\,(ax^2+a)^{3/2}} + \frac{x\cot^{-1}(x)}{3a\,(ax^2+a)^{3/2}}$$

```
[In] Int[ArcCot[x]/(a + a*x^2)^(5/2),x]
```

```
[Out] -1/9*1/(a*(a + a*x^2)^(3/2)) - 2/(3*a^2*Sqrt[a + a*x^2]) + (x*ArcCot[x])/(3
*a*(a + a*x^2)^(3/2)) + (2*x*ArcCot[x])/(3*a^2*Sqrt[a + a*x^2])
```

Rule 5015

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCot[c*x])/(d*Sq
rt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5017

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3
)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCot[c*x]), x], x] - Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a
, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{9a\left(a+ax^2\right)^{3/2}}+\frac{x\cot^{-1}(x)}{3a\left(a+ax^2\right)^{3/2}}+\frac{2\int\frac{\cot^{-1}(x)}{\left(a+ax^2\right)^{3/2}}\,dx}{3a}\\ &= -\frac{1}{9a\left(a+ax^2\right)^{3/2}}-\frac{2}{3a^2\sqrt{a+ax^2}}+\frac{x\cot^{-1}(x)}{3a\left(a+ax^2\right)^{3/2}}+\frac{2x\cot^{-1}(x)}{3a^2\sqrt{a+ax^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int\frac{\cot^{-1}(x)}{\left(a+ax^2\right)^{5/2}}\,dx=\frac{-7-6x^2+\left(9x+6x^3\right)\cot^{-1}(x)}{9a\left(a\left(1+x^2\right)\right)^{3/2}}$$

```
[In] Integrate[ArcCot[x]/(a + a*x^2)^(5/2),x]
[Out] (-7 - 6*x^2 + (9*x + 6*x^3)*ArcCot[x])/(9*a*(a*(1 + x^2))^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

method	result
risch	$\frac{ix\left(2x^2+3\right)\ln(ix+1)}{6a^2\left(x^2+1\right)\sqrt{a\left(x^2+1\right)}}+\frac{-6ix^3\ln(-ix+1)+6\pi x^3-9ix\ln(-ix+1)+9\pi x-12x^2-14}{18a^2\left(x^2+1\right)\sqrt{a\left(x^2+1\right)}}$
default	$-\frac{\left(i+3\operatorname{arccot}(x)\right)\left(x^3+3ix^2-3x-i\right)\sqrt{a(i+x)(x-i)}}{72\left(x^2+1\right)^2a^3}+\frac{3(\operatorname{arccot}(x)+i)(i+x)\sqrt{a(i+x)(x-i)}}{8a^3\left(x^2+1\right)}+\frac{3\sqrt{a(i+x)(x-i)}\,(x-i)(\operatorname{arccot}(x)-i)}{8a^3\left(x^2+1\right)}$

```
[In] int(arccot(x)/(a*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/6*I/a^2*x*(2*x^2+3)/(x^2+1)/(a*(x^2+1))^(1/2)*ln(1+I*x)+1/18/a^2*(-6*I*x^
3*ln(1-I*x)+6*Pi*x^3-9*I*x*ln(1-I*x)+9*Pi*x-12*x^2-14)/(x^2+1)/(a*(x^2+1))^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = -\frac{\sqrt{ax^2+a}(6\,x^2-3\,(2\,x^3+3\,x)\,\mathrm{arccot}\,(x)+7)}{9\,(a^3x^4+2\,a^3x^2+a^3)}$$

```
[In] integrate(arccot(x)/(a*x^2+a)^(5/2),x, algorithm="fricas")
[Out] -1/9*sqrt(a*x^2 + a)*(6*x^2 - 3*(2*x^3 + 3*x)*arccot(x) + 7)/(a^3*x^4 + 2*a
^3*x^2 + a^3)
```

Sympy [F]

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = \int \frac{\mathrm{acot}\,(x)}{(a\,(x^2+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(acot(x)/(a*x**2+a)**(5/2),x)
[Out] Integral(acot(x)/(a*(x**2 + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{ax^2+a}a^2}+\frac{x}{(ax^2+a)^{\frac{3}{2}}a}\right)\mathrm{arccot}\,(x) - \frac{2}{3\,\sqrt{ax^2+a}a^2} - \frac{1}{9\,(ax^2+a)^{\frac{3}{2}}a}$$

```
[In] integrate(arccot(x)/(a*x^2+a)^(5/2),x, algorithm="maxima")
[Out] 1/3*(2*x/(sqrt(a*x^2 + a)*a^2) + x/((a*x^2 + a)^(3/2)*a))*arccot(x) - 2/3/(
sqrt(a*x^2 + a)*a^2) - 1/9/((a*x^2 + a)^(3/2)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = \frac{x\left(\frac{2\,x^2}{a}+\frac{3}{a}\right)\arctan\left(\frac{1}{x}\right)}{3\,(ax^2+a)^{\frac{3}{2}}} - \frac{6\,ax^2+7\,a}{9\,(ax^2+a)^{\frac{3}{2}}a^2}$$

```
[In] integrate(arccot(x)/(a*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*x*(2*x^2/a + 3/a)*arctan(1/x)/(a*x^2 + a)^(3/2) - 1/9*(6*a*x^2 + 7*a)/(
(a*x^2 + a)^(3/2)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = \int \frac{\operatorname{acot}(x)}{(a\,x^2+a)^{5/2}}\,dx$$

```
[In] int(acot(x)/(a + a*x^2)^(5/2),x)
[Out] int(acot(x)/(a + a*x^2)^(5/2), x)
```

3.69 $\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 118

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = -\frac{1}{25a(a+ax^2)^{5/2}}-\frac{4}{45a^2(a+ax^2)^{3/2}}-\frac{8}{15a^3\sqrt{a+ax^2}}+\frac{x\cot^{-1}(x)}{5a(a+ax^2)^{5/2}}+\frac{4x\cot^{-1}(x)}{15a^2(a+ax^2)^{3/2}}+\frac{8x\cot^{-1}(x)}{15a^3\sqrt{a+ax^2}}$$

```
[Out] -1/25/a/(a*x^2+a)^(5/2)-4/45/a^2/(a*x^2+a)^(3/2)+1/5*x*arccot(x)/a/(a*x^2+a
)^(5/2)+4/15*x*arccot(x)/a^2/(a*x^2+a)^(3/2)-8/15/a^3/(a*x^2+a)^(1/2)+8/15*
x*arccot(x)/a^3/(a*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5017, 5015}

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = -\frac{8}{15a^3\sqrt{ax^2+a}}+\frac{8x\cot^{-1}(x)}{15a^3\sqrt{ax^2+a}}-\frac{4}{45a^2(ax^2+a)^{3/2}}+\frac{4x\cot^{-1}(x)}{15a^2(ax^2+a)^{3/2}}-\frac{1}{25a(ax^2+a)^{5/2}}+\frac{x\cot^{-1}(x)}{5a(ax^2+a)^{5/2}}$$

```
[In] Int[ArcCot[x]/(a + a*x^2)^(7/2),x]
```

```
[Out] -1/25*1/(a*(a + a*x^2)^(5/2)) - 4/(45*a^2*(a + a*x^2)^(3/2)) - 8/(15*a^3*Sq
rt[a + a*x^2]) + (x*ArcCot[x])/(5*a*(a + a*x^2)^(5/2)) + (4*x*ArcCot[x])/(1
5*a^2*(a + a*x^2)^(3/2)) + (8*x*ArcCot[x])/(15*a^3*Sqrt[a + a*x^2])
```

Rule 5015

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCot[c*x])/(d*Sq
rt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5017

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3
)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCot[c*x]), x], x] - Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a
, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{25a\left(a+ax^2\right)^{5/2}}+\frac{x\cot^{-1}(x)}{5a\left(a+ax^2\right)^{5/2}}+\frac{4\int\frac{\cot^{-1}(x)}{\left(a+ax^2\right)^{5/2}}\,dx}{5a}\\
&= -\frac{1}{25a\left(a+ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a+ax^2\right)^{3/2}}+\frac{x\cot^{-1}(x)}{5a\left(a+ax^2\right)^{5/2}}\\
&\quad+\frac{4x\cot^{-1}(x)}{15a^2\left(a+ax^2\right)^{3/2}}+\frac{8\int\frac{\cot^{-1}(x)}{\left(a+ax^2\right)^{3/2}}\,dx}{15a^2}\\
&= -\frac{1}{25a\left(a+ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a+ax^2\right)^{3/2}}-\frac{8}{15a^3\sqrt{a+ax^2}}\\
&\quad+\frac{x\cot^{-1}(x)}{5a\left(a+ax^2\right)^{5/2}}+\frac{4x\cot^{-1}(x)}{15a^2\left(a+ax^2\right)^{3/2}}+\frac{8x\cot^{-1}(x)}{15a^3\sqrt{a+ax^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int\frac{\cot^{-1}(x)}{\left(a+ax^2\right)^{7/2}}\,dx=\frac{-149-260x^2-120x^4+15x(15+20x^2+8x^4)\cot^{-1}(x)}{225a\left(a\left(1+x^2\right)\right)^{5/2}}$$

```
[In] Integrate[ArcCot[x]/(a + a*x^2)^(7/2),x]
[Out] (-149 - 260*x^2 - 120*x^4 + 15*x*(15 + 20*x^2 + 8*x^4)*ArcCot[x])/(225*a*(a
*(1 + x^2))^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.10

method	result
risch	$\frac{ix(8x^4+20x^2+15)\ln(ix+1)}{30a^3(x^2+1)^2\sqrt{a(x^2+1)}}+\frac{-120ix^5\ln(-ix+1)+120\pi x^5-300ix^3\ln(-ix+1)+300\pi x^3-240x^4-225ix\ln(-ix+1)+225\pi x-520}{450a^3(x^2+1)^2\sqrt{a(x^2+1)}}$
default	$\frac{(i+5\operatorname{arccot}(x))(x^5+5ix^4-10x^3-10ix^2+5x+i)\sqrt{a(i+x)(x-i)}}{800(x^2+1)^3a^4}+\frac{5(\operatorname{arccot}(x)+i)(i+x)\sqrt{a(i+x)(x-i)}}{16(x^2+1)a^4}+\frac{5\sqrt{a(i+x)(x-i)}(x-i)(}{16(x^2+1)a}$

```
[In] int(arccot(x)/(a*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*I/a^3*x*(8*x^4+20*x^2+15)/(x^2+1)^2/(a*(x^2+1))^(1/2)*ln(1+I*x)+1/450/
a^3*(-120*I*x^5*ln(1-I*x)+120*Pi*x^5-300*I*x^3*ln(1-I*x)+300*Pi*x^3-240*x^4
-225*I*x*ln(1-I*x)+225*Pi*x-520*x^2-298)/(x^2+1)^2/(a*(x^2+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = -\frac{(120\,x^4+260\,x^2-15\,(8\,x^5+20\,x^3+15\,x)\operatorname{arccot}(x)+149)\sqrt{ax^2+a}}{225\,(a^4x^6+3\,a^4x^4+3\,a^4x^2+a^4)}$$

```
[In] integrate(arccot(x)/(a*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/225*(120*x^4 + 260*x^2 - 15*(8*x^5 + 20*x^3 + 15*x)*arccot(x) + 149)*sqr
t(a*x^2 + a)/(a^4*x^6 + 3*a^4*x^4 + 3*a^4*x^2 + a^4)
```

Sympy [F]

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = \int \frac{\operatorname{acot}(x)}{(a\,(x^2+1))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(acot(x)/(a*x**2+a)**(7/2),x)
```

```
[Out] Integral(acot(x)/(a*(x**2 + 1))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = \frac{1}{15}\left(\frac{8\,x}{\sqrt{ax^2+a}a^3}+\frac{4\,x}{(ax^2+a)^{\frac{3}{2}}a^2}+\frac{3\,x}{(ax^2+a)^{\frac{5}{2}}a}\right)\operatorname{arccot}(x) - \frac{8}{15\sqrt{ax^2+a}a^3} - \frac{4}{45\,(ax^2+a)^{\frac{3}{2}}a^2} - \frac{1}{25\,(ax^2+a)^{\frac{5}{2}}a}$$

```
[In] integrate(arccot(x)/(a*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/(sqrt(a*x^2 + a)*a^3) + 4*x/((a*x^2 + a)^(3/2)*a^2) + 3*x/((a*x^2
 + a)^(5/2)*a))*arccot(x) - 8/15/(sqrt(a*x^2 + a)*a^3) - 4/45/((a*x^2 + a)^
(3/2)*a^2) - 1/25/((a*x^2 + a)^(5/2)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = \frac{\left(4\,x^2\left(\frac{2\,x^2}{a}+\frac{5}{a}\right)+\frac{15}{a}\right)x\arctan\left(\frac{1}{x}\right)}{15\,(ax^2+a)^{\frac{5}{2}}} - \frac{120\,(ax^2+a)^2+20\,(ax^2+a)a+9\,a^2}{225\,(ax^2+a)^{\frac{5}{2}}a^3}$$

```
[In] integrate(arccot(x)/(a*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*x^2*(2*x^2/a + 5/a) + 15/a)*x*arctan(1/x)/(a*x^2 + a)^(5/2) - 1/225
*(120*(a*x^2 + a)^2 + 20*(a*x^2 + a)*a + 9*a^2)/((a*x^2 + a)^(5/2)*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = \int \frac{\operatorname{acot}(x)}{(a\,x^2+a)^{7/2}}\,dx$$

```
[In] int(acot(x)/(a + a*x^2)^(7/2),x)
```

```
[Out] int(acot(x)/(a + a*x^2)^(7/2), x)
```

3.70 $\int \frac{x \cot^{-1}(x)}{(1+x^2)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x \cot^{-1}(x)}{(1+x^2)^2} dx = -\frac{x}{4(1+x^2)} - \frac{\cot^{-1}(x)}{2(1+x^2)} - \frac{\arctan(x)}{4}$$

```
[Out] -1/4*x/(x^2+1)-1/2*arccot(x)/(x^2+1)-1/4*arctan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5051, 205, 209}

$$\int \frac{x \cot^{-1}(x)}{(1+x^2)^2} dx = -\frac{\arctan(x)}{4} - \frac{x}{4(x^2+1)} - \frac{\cot^{-1}(x)}{2(x^2+1)}$$

```
[In] Int[(x*ArcCot[x])/(1 + x^2)^2,x]
```

```
[Out] -1/4*x/(1 + x^2) - ArcCot[x]/(2*(1 + x^2)) - ArcTan[x]/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5051

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])^p/(2*e*(q +
1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcCot[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cot^{-1}(x)}{2(1+x^2)} - \frac{1}{2}\int \frac{1}{(1+x^2)^2}\,dx \\ &= -\frac{x}{4(1+x^2)} - \frac{\cot^{-1}(x)}{2(1+x^2)} - \frac{1}{4}\int \frac{1}{1+x^2}\,dx \\ &= -\frac{x}{4(1+x^2)} - \frac{\cot^{-1}(x)}{2(1+x^2)} - \frac{\arctan(x)}{4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = -\frac{x+2\cot^{-1}(x)+\arctan(x)+x^2\arctan(x)}{4+4x^2}$$

```
[In] Integrate[(x*ArcCot[x])/(1 + x^2)^2,x]
[Out] -((x + 2*ArcCot[x] + ArcTan[x] + x^2*ArcTan[x])/(4 + 4*x^2))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$\frac{x^2 \operatorname{arccot}(x)-x-\operatorname{arccot}(x)}{4x^2+4}$	24
default	$-\frac{x}{4(x^2+1)}-\frac{\operatorname{arccot}(x)}{2(x^2+1)}-\frac{\arctan(x)}{4}$	27
parts	$-\frac{x}{4(x^2+1)}-\frac{\operatorname{arccot}(x)}{2(x^2+1)}-\frac{\arctan(x)}{4}$	27
risch	$-\frac{i\ln(ix+1)}{4(x^2+1)}-\frac{-2i\ln(-ix+1)+i\ln(i+x)+i\ln(i+x)x^2-i\ln(x-i)-i\ln(x-i)x^2+2\pi+2x}{8(i+x)(x-i)}$	88

```
[In] int(x*arccot(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(x^2*arccot(x)-x-arccot(x))/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{(x^2-1)\operatorname{arccot}(x)-x}{4(x^2+1)}$$

```
[In] integrate(x*arccot(x)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 - 1)*arccot(x) - x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{x^2\operatorname{acot}(x)}{4x^2+4}-\frac{x}{4x^2+4}-\frac{\operatorname{acot}(x)}{4x^2+4}$$

```
[In] integrate(x*acot(x)/(x**2+1)**2,x)
[Out] x**2*acot(x)/(4*x**2 + 4) - x/(4*x**2 + 4) - acot(x)/(4*x**2 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = -\frac{x}{4\,(x^2+1)} - \frac{\operatorname{arccot}(x)}{2\,(x^2+1)} - \frac{1}{4}\arctan(x)$$

```
[In] integrate(x*arccot(x)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/4*x/(x^2 + 1) - 1/2*arccot(x)/(x^2 + 1) - 1/4*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = -\frac{\arctan\left(\frac{1}{x}\right)}{2\,(x^2+1)} - \frac{1}{4\,x\left(\frac{1}{x^2}+1\right)} + \frac{1}{4}\arctan\left(\frac{1}{x}\right)$$

```
[In] integrate(x*arccot(x)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*arctan(1/x)/(x^2 + 1) - 1/4/(x*(1/x^2 + 1)) + 1/4*arctan(1/x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{\mathrm{acot}(x)}{4} - \frac{\frac{x}{4}+\frac{\mathrm{acot}(x)}{2}}{x^2+1}$$

```
[In] int((x*acot(x))/(x^2 + 1)^2,x)
[Out] acot(x)/4 - (x/4 + acot(x)/2)/(x^2 + 1)
```

3.71 $\int \frac{x \cot^{-1}(x)}{(1+x^2)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{x \cot^{-1}(x)}{(1+x^2)^3} dx = -\frac{x}{16(1+x^2)^2} - \frac{3x}{32(1+x^2)} - \frac{\cot^{-1}(x)}{4(1+x^2)^2} - \frac{3\arctan(x)}{32}$$

```
[Out] -1/16*x/(x^2+1)^2-3/32*x/(x^2+1)-1/4*arccot(x)/(x^2+1)^2-3/32*arctan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5051, 205, 209}

$$\int \frac{x \cot^{-1}(x)}{(1+x^2)^3} dx = -\frac{3\arctan(x)}{32} - \frac{3x}{32(x^2+1)} - \frac{x}{16(x^2+1)^2} - \frac{\cot^{-1}(x)}{4(x^2+1)^2}$$

```
[In] Int[(x*ArcCot[x])/(1 + x^2)^3,x]
```

```
[Out] -1/16*x/(1 + x^2)^2 - (3*x)/(32*(1 + x^2)) - ArcCot[x]/(4*(1 + x^2)^2) - (3
*ArcTan[x])/32
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5051

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])^p/(2*e*(q +
1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcCot[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(x)}{4(1+x^2)^2} - \frac{1}{4}\int \frac{1}{(1+x^2)^3}\,dx \\
&= -\frac{x}{16(1+x^2)^2} - \frac{\cot^{-1}(x)}{4(1+x^2)^2} - \frac{3}{16}\int \frac{1}{(1+x^2)^2}\,dx \\
&= -\frac{x}{16(1+x^2)^2} - \frac{3x}{32(1+x^2)} - \frac{\cot^{-1}(x)}{4(1+x^2)^2} - \frac{3}{32}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{x}{16(1+x^2)^2} - \frac{3x}{32(1+x^2)} - \frac{\cot^{-1}(x)}{4(1+x^2)^2} - \frac{3\arctan(x)}{32}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx = -\frac{x(5+3x^2)+8\cot^{-1}(x)+3(1+x^2)^2\arctan(x)}{32(1+x^2)^2}$$

```
[In] Integrate[(x*ArcCot[x])/(1 + x^2)^3,x]
[Out] -1/32*(x*(5 + 3*x^2) + 8*ArcCot[x] + 3*(1 + x^2)^2*ArcTan[x])/(1 + x^2)^2
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result
default	$-\frac{x}{16(x^2+1)^2}-\frac{3x}{32(x^2+1)}-\frac{\operatorname{arccot}(x)}{4(x^2+1)^2}-\frac{3\arctan(x)}{32}$
parallelrisch	$\frac{3\ \operatorname{arccot}(x)x^4-3x^3+6x^2\operatorname{arccot}(x)-5x-5\ \operatorname{arccot}(x)}{32(x^2+1)^2}$
parts	$-\frac{x}{16(x^2+1)^2}-\frac{3x}{32(x^2+1)}-\frac{\operatorname{arccot}(x)}{4(x^2+1)^2}-\frac{3\arctan(x)}{32}$
risch	$-\frac{i\ln(ix+1)}{8(x^2+1)^2}-\frac{-8i\ln(-ix+1)-6i\ln(x-i)x^2-3i\ln(x-i)-3i\ln(x-i)x^4+6i\ln(i+x)x^2+3i\ln(i+x)+3i\ln(i+x)x^4+6x^3+8\pi+}{64(i+x)(x^2+1)(x-i)}$

```
[In] int(x*arccot(x)/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] -1/16*x/(x^2+1)^2-3/32*x/(x^2+1)-1/4*arccot(x)/(x^2+1)^2-3/32*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx=-\frac{3\,x^3-(3\,x^4+6\,x^2-5)\operatorname{arccot}(x)+5\,x}{32\,(x^4+2\,x^2+1)}$$

```
[In] integrate(x*arccot(x)/(x^2+1)^3,x, algorithm="fricas")
[Out] -1/32*(3*x^3 - (3*x^4 + 6*x^2 - 5)*arccot(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(39) = 78.

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx=\frac{3x^4\operatorname{acot}(x)}{32x^4+64x^2+32}-\frac{3x^3}{32x^4+64x^2+32}+\frac{6x^2\operatorname{acot}(x)}{32x^4+64x^2+32}-\frac{5x}{32x^4+64x^2+32}-\frac{5\operatorname{acot}(x)}{32x^4+64x^2+32}$$

```
[In] integrate(x*acot(x)/(x**2+1)**3,x)
[Out] 3*x**4*acot(x)/(32*x**4 + 64*x**2 + 32) - 3*x**3/(32*x**4 + 64*x**2 + 32) +
 6*x**2*acot(x)/(32*x**4 + 64*x**2 + 32) - 5*x/(32*x**4 + 64*x**2 + 32) - 5
*acot(x)/(32*x**4 + 64*x**2 + 32)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx = -\frac{3\,x^3+5\,x}{32\,(x^4+2\,x^2+1)} - \frac{\operatorname{arccot}(x)}{4\,(x^2+1)^2} - \frac{3}{32}\arctan(x)$$

```
[In] integrate(x*arccot(x)/(x^2+1)^3,x, algorithm="maxima")
[Out] -1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arccot(x)/(x^2 + 1)^2 - 3/32*ar
ctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx = -\frac{\frac{3}{x}+\frac{5}{x^3}}{32\,\left(\frac{1}{x^2}+1\right)^2} - \frac{\arctan\left(\frac{1}{x}\right)}{4\,(x^2+1)^2} + \frac{3}{32}\arctan\left(\frac{1}{x}\right)$$

```
[In] integrate(x*arccot(x)/(x^2+1)^3,x, algorithm="giac")
[Out] -1/32*(3/x + 5/x^3)/(1/x^2 + 1)^2 - 1/4*arctan(1/x)/(x^2 + 1)^2 + 3/32*arct
an(1/x)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx = -\frac{3\,\mathrm{atan}(x)}{32} - \frac{\frac{5\,x}{32}+\frac{\mathrm{acot}(x)}{4}+\frac{3\,x^3}{32}}{(x^2+1)^2}$$

```
[In] int((x*acot(x))/(x^2 + 1)^3,x)
[Out] - (3*atan(x))/32 - ((5*x)/32 + acot(x)/4 + (3*x^3)/32)/(x^2 + 1)^2
```

3.72 $\int \frac{\cot^{-1}(x)}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2} \, dx = -\frac{1}{4(1+x^2)} + \frac{x\cot^{-1}(x)}{2(1+x^2)} - \frac{1}{4}\cot^{-1}(x)^2$$

```
[Out] -1/4/(x^2+1)+1/2*x*arccot(x)/(x^2+1)-1/4*arccot(x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5013, 267}

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2} \, dx = -\frac{1}{4(x^2+1)} + \frac{x\cot^{-1}(x)}{2(x^2+1)} - \frac{1}{4}\cot^{-1}(x)^2$$

```
[In] Int[ArcCot[x]/(1 + x^2)^2,x]
[Out] -1/4*1/(1 + x^2) + (x*ArcCot[x])/(2*(1 + x^2)) - ArcCot[x]^2/4
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5013

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcCot[c*x])^p/(2*d*(d + e*x^2))), x] + (Dist[b*c*(p
```

```
/2), Int[x*((a + b*ArcCot[c*x])^(p - 1)/(d + e*x^2)^2), x], x] - Simp[(a +
b*ArcCot[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\cot^{-1}(x)}{2(1+x^2)} - \frac{1}{4}\cot^{-1}(x)^2 + \frac{1}{2}\int\frac{x}{(1+x^2)^2}\,dx\\ &= -\frac{1}{4(1+x^2)} + \frac{x\cot^{-1}(x)}{2(1+x^2)} - \frac{1}{4}\cot^{-1}(x)^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int\frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx = -\frac{1-2x\cot^{-1}(x)+(1+x^2)\cot^{-1}(x)^2}{4(1+x^2)}$$

```
[In] Integrate[ArcCot[x]/(1 + x^2)^2,x]
[Out] -1/4*(1 - 2*x*ArcCot[x] + (1 + x^2)*ArcCot[x]^2)/(1 + x^2)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result
default	$\frac{x\ \text{arccot}(x)}{2x^2+2}+\frac{\text{arccot}(x)\arctan(x)}{2}-\frac{1}{4(x^2+1)}+\frac{\arctan(x)^2}{4}$
parts	$\frac{x\ \text{arccot}(x)}{2x^2+2}+\frac{\text{arccot}(x)\arctan(x)}{2}-\frac{1}{4(x^2+1)}+\frac{\arctan(x)^2}{4}$
risch	$\frac{\ln(ix+1)^2}{16}-\frac{(x^2\ln(-ix+1)-2ix+\ln(-ix+1))\ln(ix+1)}{8(x^2+1)}+\frac{x^2\ln(-ix+1)^2+\ln(-ix+1)^2+2i\pi\ln(i+x)+2i\pi\ln(i+x)x^2-2i\pi\ln(x-i)}{16(i+x)(x-i)}$

```
[In] int(arccot(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x*arccot(x)/(x^2+1)+1/2*arccot(x)*arctan(x)-1/4/(x^2+1)+1/4*arctan(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx = -\frac{(x^2+1)\operatorname{arccot}(x)^2 - 2\,x\operatorname{arccot}(x) + 1}{4\,(x^2+1)}$$

```
[In] integrate(arccot(x)/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/4*((x^2 + 1)*arccot(x)^2 - 2*x*arccot(x) + 1)/(x^2 + 1)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx = \text{Exception raised: RecursionError}$$

```
[In] integrate(acot(x)/(x**2+1)**2,x)
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded in com
parison
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{x^2+1} + \arctan(x)\right)\operatorname{arccot}(x) + \frac{(x^2+1)\arctan(x)^2 - 1}{4\,(x^2+1)}$$

```
[In] integrate(arccot(x)/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2*(x/(x^2 + 1) + arctan(x))*arccot(x) + 1/4*((x^2 + 1)*arctan(x)^2 - 1)/(
x^2 + 1)
```

Giac [F]

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)^2}\,dx$$

```
[In] integrate(arccot(x)/(x^2+1)^2,x, algorithm="giac")
[Out] integrate(arccot(x)/(x^2 + 1)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{\frac{x\,\mathrm{acot}(x)}{2}-\frac{1}{4}}{x^2+1} - \frac{\mathrm{acot}(x)^2}{4}$$

```
[In] int(acot(x)/(x^2 + 1)^2,x)
[Out] ((x*acot(x))/2 - 1/4)/(x^2 + 1) - acot(x)^2/4
```

3.73 $\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2} dx = -\frac{x}{4(1+x^2)} - \frac{\cot^{-1}(x)}{2(1+x^2)} + \frac{x\cot^{-1}(x)^2}{2(1+x^2)} - \frac{1}{6}\cot^{-1}(x)^3 - \frac{\arctan(x)}{4}$$

```
[Out] -1/4*x/(x^2+1)-1/2*arccot(x)/(x^2+1)+1/2*x*arccot(x)^2/(x^2+1)-1/6*arccot(x
)^3-1/4*arctan(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5013, 5051, 205, 209}

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2} dx = -\frac{\arctan(x)}{4} - \frac{x}{4(x^2+1)} + \frac{x\cot^{-1}(x)^2}{2(x^2+1)} - \frac{\cot^{-1}(x)}{2(x^2+1)} - \frac{1}{6}\cot^{-1}(x)^3$$

```
[In] Int[ArcCot[x]^2/(1 + x^2)^2,x]
```

```
[Out] -1/4*x/(1 + x^2) - ArcCot[x]/(2*(1 + x^2)) + (x*ArcCot[x]^2)/(2*(1 + x^2))
- ArcCot[x]^3/6 - ArcTan[x]/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5013

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcCot[c*x])^p/(2*d*(d + e*x^2))), x] + (Dist[b*c*(p
/2), Int[x*((a + b*ArcCot[c*x])^(p - 1)/(d + e*x^2)^2), x], x] - Simp[(a +
b*ArcCot[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5051

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])^p/(2*e*(q +
1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcCot[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\cot^{-1}(x)^2}{2(1+x^2)} - \frac{1}{6}\cot^{-1}(x)^3 + \int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx \\
&= -\frac{\cot^{-1}(x)}{2(1+x^2)} + \frac{x\cot^{-1}(x)^2}{2(1+x^2)} - \frac{1}{6}\cot^{-1}(x)^3 - \frac{1}{2}\int \frac{1}{(1+x^2)^2}\,dx \\
&= -\frac{x}{4(1+x^2)} - \frac{\cot^{-1}(x)}{2(1+x^2)} + \frac{x\cot^{-1}(x)^2}{2(1+x^2)} - \frac{1}{6}\cot^{-1}(x)^3 - \frac{1}{4}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{x}{4(1+x^2)} - \frac{\cot^{-1}(x)}{2(1+x^2)} + \frac{x\cot^{-1}(x)^2}{2(1+x^2)} - \frac{1}{6}\cot^{-1}(x)^3 - \frac{\arctan(x)}{4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = -\frac{6\cot^{-1}(x) - 6x\cot^{-1}(x)^2 + 2(1+x^2)\cot^{-1}(x)^3 + 3(x+(1+x^2)\arctan(x))}{12(1+x^2)}$$

```
[In] Integrate[ArcCot[x]^2/(1 + x^2)^2,x]
[Out] -1/12*(6*ArcCot[x] - 6*x*ArcCot[x]^2 + 2*(1 + x^2)*ArcCot[x]^3 + 3*(x + (1
+ x^2)*ArcTan[x]))/(1 + x^2)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

method	result
default	$\frac{\operatorname{arccot}(x)^{2} x}{2x^{2}+2}+\frac{\operatorname{arccot}(x)^{2} \arctan(x)}{2}-\frac{\pi \operatorname{arccot}(x)^{2}}{4}+\frac{\operatorname{arccot}(x)^{3}}{3}+\frac{x^{2} \operatorname{arccot}(x)}{2x^{2}+2}-\frac{x}{4(x^{2}+1)}-\frac{\operatorname{arccot}(x)}{4}$
parts	$\frac{\operatorname{arccot}(x)^{2} x}{2x^{2}+2}+\frac{\operatorname{arccot}(x)^{2} \arctan(x)}{2}-\frac{\pi \operatorname{arccot}(x)^{2}}{4}+\frac{\operatorname{arccot}(x)^{3}}{3}+\frac{x^{2} \operatorname{arccot}(x)}{2x^{2}+2}-\frac{x}{4(x^{2}+1)}-\frac{\operatorname{arccot}(x)}{4}$
risch	$\frac{i \ln(ix+1)^{3}}{48}+\frac{\left(-i x^{2} \ln(-ix+1)+\pi x^{2}-i \ln(-ix+1)+\pi-2x\right) \ln(ix+1)^{2}}{16x^{2}+16}-\frac{\left(-i x^{2} \ln(-ix+1)^{2}-i \ln(-ix+1)^{2}-4 \ln(-ix+1) x+2 \ln\right.}{16(i+x)(}$

```
[In] int(arccot(x)^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*arccot(x)^2/(x^2+1)+1/2*arccot(x)^2*arctan(x)-1/4*Pi*arccot(x)^2+1/3*
arccot(x)^3+1/2*x^2*arccot(x)/(x^2+1)-1/4*x/(x^2+1)-1/4*arccot(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = -\frac{2\,(x^2+1)\operatorname{arccot}(x)^3 - 6\,x\operatorname{arccot}(x)^2 - 3\,(x^2-1)\operatorname{arccot}(x) + 3\,x}{12\,(x^2+1)}$$

```
[In] integrate(arccot(x)^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(x^2 + 1)*arccot(x)^3 - 6*x*arccot(x)^2 - 3*(x^2 - 1)*arccot(x) +
3*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = \int \frac{\operatorname{acot}^2(x)}{(x^2+1)^2}\,dx$$

```
[In] integrate(acot(x)**2/(x**2+1)**2,x)
```

```
[Out] Integral(acot(x)**2/(x**2 + 1)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{x^2+1}+\arctan(x)\right)\operatorname{arccot}(x)^2 + \frac{\left((x^2+1)\arctan(x)^2-1\right)\operatorname{arccot}(x)}{2\,(x^2+1)} + \frac{2\,(x^2+1)\arctan(x)^3-3\,(x^2+1)\arctan(x)-3\,x}{12\,(x^2+1)}$$

```
[In] integrate(arccot(x)^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(x^2 + 1) + arctan(x))*arccot(x)^2 + 1/2*((x^2 + 1)*arctan(x)^2 - 1)
*arccot(x)/(x^2 + 1) + 1/12*(2*(x^2 + 1)*arctan(x)^3 - 3*(x^2 + 1)*arctan(x
) - 3*x)/(x^2 + 1)
```

Giac [F]

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = \int \frac{\operatorname{arccot}(x)^2}{(x^2+1)^2}\,dx$$

```
[In] integrate(arccot(x)^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arccot(x)^2/(x^2 + 1)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = \frac{x\,\mathrm{acot}(x)^2}{2\,(x^2+1)} - \frac{\mathrm{acot}(x)^3}{6} - \frac{x}{4\,(x^2+1)} - \frac{\mathrm{acot}(x)}{2\,(x^2+1)} - \frac{\mathrm{atan}(x)}{4}$$

```
[In] int(acot(x)^2/(x^2 + 1)^2,x)
```

```
[Out] (x*acot(x)^2)/(2*(x^2 + 1)) - acot(x)^3/6 - x/(4*(x^2 + 1)) - acot(x)/(2*(x
^2 + 1)) - atan(x)/4
```

3.74 $\int x^5 \cot^{-1}(ax^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int x^5 \cot^{-1}(ax^2)\,dx = \frac{x^4}{12a} + \frac{1}{6}x^6 \cot^{-1}(ax^2) - \frac{\log(1+a^2x^4)}{12a^3}$$

```
[Out] 1/12*x^4/a+1/6*x^6*arccot(a*x^2)-1/12*ln(a^2*x^4+1)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4947, 272, 45}

$$\int x^5 \cot^{-1}(ax^2)\,dx = -\frac{\log(a^2x^4+1)}{12a^3} + \frac{x^4}{12a} + \frac{1}{6}x^6 \cot^{-1}(ax^2)$$

```
[In] Int[x^5*ArcCot[a*x^2],x]
```

```
[Out] x^4/(12*a) + (x^6*ArcCot[a*x^2])/6 - Log[1 + a^2*x^4]/(12*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\cot^{-1}\left(ax^2\right)+\frac{1}{3}a\int\frac{x^7}{1+a^2x^4}\,dx\\
&= \frac{1}{6}x^6\cot^{-1}\left(ax^2\right)+\frac{1}{12}a\text{Subst}\left(\int\frac{x}{1+a^2x}\,dx,x,x^4\right)\\
&= \frac{1}{6}x^6\cot^{-1}\left(ax^2\right)+\frac{1}{12}a\text{Subst}\left(\int\left(\frac{1}{a^2}-\frac{1}{a^2\left(1+a^2x\right)}\right)\,dx,x,x^4\right)\\
&= \frac{x^4}{12a}+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)-\frac{\log\left(1+a^2x^4\right)}{12a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x^5\cot^{-1}\left(ax^2\right)\,dx=\frac{x^4}{12a}+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)-\frac{\log\left(1+a^2x^4\right)}{12a^3}$$

```
[In] Integrate[x^5*ArcCot[a*x^2],x]
[Out] x^4/(12*a) + (x^6*ArcCot[a*x^2])/6 - Log[1 + a^2*x^4]/(12*a^3)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{-2x^{6} \operatorname{arccot}\left(a\,x^{2}\right) a^{3}-a^{2} x^{4}+\ln\left(a^{2} x^{4}+1\right)}{12 a^{3}}$	39
default	$\frac{x^{6} \operatorname{arccot}\left(a\,x^{2}\right)}{6}+\frac{a\left(\frac{x^{4}}{4 a^{2}}-\frac{\ln\left(a^{2} x^{4}+1\right)}{4 a^{4}}\right)}{3}$	40
parts	$\frac{x^{6} \operatorname{arccot}\left(a\,x^{2}\right)}{6}+\frac{a\left(\frac{x^{4}}{4 a^{2}}-\frac{\ln\left(a^{2} x^{4}+1\right)}{4 a^{4}}\right)}{3}$	40
risch	$\frac{i x^{6} \ln\left(i a\,x^{2}+1\right)}{12}-\frac{i x^{6} \ln\left(-i a\,x^{2}+1\right)}{12}+\frac{\pi\,x^{6}}{12}+\frac{x^{4}}{12 a}-\frac{\ln\left(-a^{2} x^{4}-1\right)}{12 a^{3}}$	64

```
[In] int(x^5*arccot(a*x^2),x,method=_RETURNVERBOSE)
[Out] -1/12*(-2*x^6*arccot(a*x^2)*a^3-a^2*x^4+ln(a^2*x^4+1))/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^5 \cot^{-1}\left(ax^2\right)\,dx=\frac{2\,a^3x^6\operatorname{arccot}\left(ax^2\right)+a^2x^4-\log\left(a^2x^4+1\right)}{12\,a^3}$$

```
[In] integrate(x^5*arccot(a*x^2),x, algorithm="fricas")
[Out] 1/12*(2*a^3*x^6*arccot(a*x^2) + a^2*x^4 - log(a^2*x^4 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^5 \cot^{-1}\left(ax^2\right)\,dx=\begin{cases}\frac{x^6\operatorname{acot}\left(ax^2\right)}{6}+\frac{x^4}{12a}-\frac{\log\left(a^2x^4+1\right)}{12a^3} & \text{for}\ a\neq 0\\ \frac{\pi x^6}{12} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*acot(a*x**2),x)
[Out] Piecewise((x**6*acot(a*x**2)/6 + x**4/(12*a) - log(a**2*x**4 + 1)/(12*a**3)
, Ne(a, 0)), (pi*x**6/12, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int x^5 \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{6}\,x^6 \operatorname{arccot}\left(ax^2\right) + \frac{1}{12}\left(\frac{x^4}{a^2} - \frac{\log\left(a^2x^4+1\right)}{a^4}\right)a$$

```
[In] integrate(x^5*arccot(a*x^2),x, algorithm="maxima")
[Out] 1/6*x^6*arccot(a*x^2) + 1/12*(x^4/a^2 - log(a^2*x^4 + 1)/a^4)*a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int x^5 \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{6}\,x^6 \arctan\left(\frac{1}{ax^2}\right) + \frac{1}{12}\left(\frac{x^4}{a^2} - \frac{\log\left(a^2x^4+1\right)}{a^4}\right)a$$

```
[In] integrate(x^5*arccot(a*x^2),x, algorithm="giac")
[Out] 1/6*x^6*arctan(1/(a*x^2)) + 1/12*(x^4/a^2 - log(a^2*x^4 + 1)/a^4)*a
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int x^5 \cot^{-1}\left(ax^2\right)\,dx = \frac{x^6\,\mathrm{acot}(a\,x^2)}{6} - \frac{\ln\left(a^2\,x^4+1\right)}{12\,a^3} + \frac{x^4}{12\,a}$$

```
[In] int(x^5*acot(a*x^2),x)
[Out] (x^6*acot(a*x^2))/6 - log(a^2*x^4 + 1)/(12*a^3) + x^4/(12*a)
```

3.75 $\int x^3 \cot^{-1}(ax^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int x^3 \cot^{-1}(ax^2)\,dx = \frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}(ax^2) - \frac{\arctan(ax^2)}{4a^2}$$

```
[Out] 1/4*x^2/a+1/4*x^4*arccot(a*x^2)-1/4*arctan(a*x^2)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4947, 281, 327, 209}

$$\int x^3 \cot^{-1}(ax^2)\,dx = -\frac{\arctan(ax^2)}{4a^2} + \frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}(ax^2)$$

```
[In] Int[x^3*ArcCot[a*x^2],x]
```

```
[Out] x^2/(4*a) + (x^4*ArcCot[a*x^2])/4 - ArcTan[a*x^2]/(4*a^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\cot^{-1}\left(ax^2\right)+\frac{1}{2}a\int\frac{x^5}{1+a^2x^4}\,dx\\
&= \frac{1}{4}x^4\cot^{-1}\left(ax^2\right)+\frac{1}{4}a\text{Subst}\left(\int\frac{x^2}{1+a^2x^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{4a}+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)-\frac{\text{Subst}\left(\int\frac{1}{1+a^2x^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{x^2}{4a}+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)-\frac{\arctan\left(ax^2\right)}{4a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x^3\cot^{-1}\left(ax^2\right)\,dx=\frac{x^2}{4a}+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)-\frac{\arctan\left(ax^2\right)}{4a^2}$$

```
[In] Integrate[x^3*ArcCot[a*x^2],x]
[Out] x^2/(4*a) + (x^4*ArcCot[a*x^2])/4 - ArcTan[a*x^2]/(4*a^2)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{\operatorname{arccot}\left(a\,x^{2}\right) a^{2} x^{4}+a\,x^{2}+\operatorname{arccot}\left(a\,x^{2}\right)}{4 a^{2}}$	31
default	$\frac{x^{4} \operatorname{arccot}\left(a\,x^{2}\right)}{4}+\frac{a \left(\frac{x^{2}}{2 a^{2}}-\frac{\arctan \left(a\,x^{2}\right)}{2 a^{3}}\right)}{2}$	36
parts	$\frac{x^{4} \operatorname{arccot}\left(a\,x^{2}\right)}{4}+\frac{a \left(\frac{x^{2}}{2 a^{2}}-\frac{\arctan \left(a\,x^{2}\right)}{2 a^{3}}\right)}{2}$	36
risch	$\frac{i x^{4} \ln \left(i a\,x^{2}+1\right)}{8}-\frac{i x^{4} \ln \left(-i a\,x^{2}+1\right)}{8}+\frac{\pi\,x^{4}}{8}+\frac{x^{2}}{4 a}-\frac{\arctan \left(a\,x^{2}\right)}{4 a^{2}}+\frac{1}{8 \pi\,a^{2}}$	67

```
[In] int(x^3*arccot(a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/4*(arccot(a*x^2)*a^2*x^4+a*x^2+arccot(a*x^2))/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int x^3 \cot^{-1}\left(ax^2\right)\, dx = \frac{ax^{2} + \left(a^{2} x^{4} + 1\right) \operatorname{arccot}\left(ax^{2}\right)}{4\,a^{2}}$$

```
[In] integrate(x^3*arccot(a*x^2),x, algorithm="fricas")
[Out] 1/4*(a*x^2 + (a^2*x^4 + 1)*arccot(a*x^2))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int x^3 \cot^{-1}\left(ax^2\right)\, dx = \begin{cases} \frac{x^{4} \operatorname{acot}{\left(ax^{2} \right)}}{4} + \frac{x^{2}}{4a} + \frac{\operatorname{acot}{\left(ax^{2} \right)}}{4a^{2}} & \text{for}\: a \neq 0 \\ \frac{\pi x^{4}}{8} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*acot(a*x**2),x)
[Out] Piecewise((x**4*acot(a*x**2)/4 + x**2/(4*a) + acot(a*x**2)/(4*a**2), Ne(a,
0)), (pi*x**4/8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^3 \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{4}\,x^4\,\mathrm{arccot}\left(ax^2\right) + \frac{1}{4}\,a\left(\frac{x^2}{a^2} - \frac{\arctan\left(ax^2\right)}{a^3}\right)$$

```
[In] integrate(x^3*arccot(a*x^2),x, algorithm="maxima")
[Out] 1/4*x^4*arccot(a*x^2) + 1/4*a*(x^2/a^2 - arctan(a*x^2)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^3 \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{4}\left(\frac{x^4\arctan\left(\frac{1}{ax^2}\right)}{a} + \frac{x^2}{a^2} + \frac{\arctan\left(\frac{1}{ax^2}\right)}{a^3}\right)a$$

```
[In] integrate(x^3*arccot(a*x^2),x, algorithm="giac")
[Out] 1/4*(x^4*arctan(1/(a*x^2))/a + x^2/a^2 + arctan(1/(a*x^2))/a^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int x^3 \cot^{-1}\left(ax^2\right)\,dx = \frac{x^4\,\mathrm{acot}(a\,x^2)}{4} - \frac{\mathrm{atan}(a\,x^2)}{4\,a^2} + \frac{x^2}{4\,a}$$

```
[In] int(x^3*acot(a*x^2),x)
[Out] (x^4*acot(a*x^2))/4 - atan(a*x^2)/(4*a^2) + x^2/(4*a)
```

3.76 $\int x \cot^{-1}(ax^2) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int x \cot^{-1}(ax^2) \, dx = \frac{1}{2}x^2 \cot^{-1}(ax^2) + \frac{\log(1 + a^2x^4)}{4a}$$

```
[Out] 1/2*x^2*arccot(a*x^2)+1/4*ln(a^2*x^4+1)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4947, 266}

$$\int x \cot^{-1}(ax^2) \, dx = \frac{\log(a^2x^4 + 1)}{4a} + \frac{1}{2}x^2 \cot^{-1}(ax^2)$$

```
[In] Int[x*ArcCot[a*x^2],x]
```

```
[Out] (x^2*ArcCot[a*x^2])/2 + Log[1 + a^2*x^4]/(4*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\cot^{-1}\left(ax^2\right)+a\int \frac{x^3}{1+a^2x^4}\,dx\\ &= \frac{1}{2}x^2\cot^{-1}\left(ax^2\right)+\frac{\log\left(1+a^2x^4\right)}{4a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x\cot^{-1}\left(ax^2\right)\,dx=\frac{1}{2}x^2\cot^{-1}\left(ax^2\right)+\frac{\log\left(1+a^2x^4\right)}{4a}$$

```
[In] Integrate[x*ArcCot[a*x^2],x]
[Out] (x^2*ArcCot[a*x^2])/2 + Log[1 + a^2*x^4]/(4*a)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
parts	$\frac{x^{2}\operatorname{arccot}\left(a\,x^{2}\right)}{2}+\frac{\ln\left(a^{2}x^{4}+1\right)}{4a}$	28
parallelrisch	$\frac{2\operatorname{arccot}\left(a\,x^{2}\right)a\,x^{2}+\ln\left(a^{2}x^{4}+1\right)}{4a}$	29
derivativedivides	$\frac{\operatorname{arccot}\left(a\,x^{2}\right)a\,x^{2}+\frac{\ln\left(a^{2}x^{4}+1\right)}{2}}{2a}$	30
default	$\frac{\operatorname{arccot}\left(a\,x^{2}\right)a\,x^{2}+\frac{\ln\left(a^{2}x^{4}+1\right)}{2}}{2a}$	30
risch	$\frac{ix^{2}\ln\left(ia\,x^{2}+1\right)}{4}-\frac{ix^{2}\ln\left(-ia\,x^{2}+1\right)}{4}+\frac{\pi\,x^{2}}{4}+\frac{\ln\left(-a^{2}x^{4}-1\right)}{4a}$	56

```
[In] int(x*arccot(a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*arccot(a*x^2)+1/4*ln(a^2*x^4+1)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x\cot^{-1}\left(ax^2\right)\,dx = \frac{2\,ax^2\,\mathrm{arccot}\left(ax^2\right) + \log\left(a^2x^4+1\right)}{4\,a}$$

```
[In] integrate(x*arccot(a*x^2),x, algorithm="fricas")
[Out] 1/4*(2*a*x^2*arccot(a*x^2) + log(a^2*x^4 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x\cot^{-1}\left(ax^2\right)\,dx = \begin{cases} \frac{x^2\,\mathrm{acot}\left(ax^2\right)}{2} + \frac{\log\left(a^2x^4+1\right)}{4a} & \text{for}\ a\neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*acot(a*x**2),x)
[Out] Piecewise((x**2*acot(a*x**2)/2 + log(a**2*x**4 + 1)/(4*a), Ne(a, 0)), (pi*x
**2/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x\cot^{-1}\left(ax^2\right)\,dx = \frac{2\,ax^2\,\mathrm{arccot}\left(ax^2\right) + \log\left(a^2x^4+1\right)}{4\,a}$$

```
[In] integrate(x*arccot(a*x^2),x, algorithm="maxima")
[Out] 1/4*(2*a*x^2*arccot(a*x^2) + log(a^2*x^4 + 1))/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int x \cot^{-1}\left(ax^2\right) \, dx = \frac{1}{4} \left(\frac{2\,x^2 \arctan\left(\frac{1}{ax^2}\right)}{a} + \frac{\log\left(\frac{1}{a^2x^4}+1\right)}{a^2} - \frac{\log\left(\frac{1}{a^2x^4}\right)}{a^2} \right) a$$

```
[In] integrate(x*arccot(a*x^2),x, algorithm="giac")
[Out] 1/4*(2*x^2*arctan(1/(a*x^2))/a + log(1/(a^2*x^4) + 1)/a^2 - log(1/(a^2*x^4)
)/a^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int x \cot^{-1}\left(ax^2\right) \, dx = \frac{x^2\,\mathrm{acot}(a\,x^2)}{2} + \frac{\ln\left(a^2\,x^4+1\right)}{4\,a}$$

```
[In] int(x*acot(a*x^2),x)
[Out] (x^2*acot(a*x^2))/2 + log(a^2*x^4 + 1)/(4*a)
```

3.77 $\int \frac{\cot^{-1}(ax^2)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{\cot^{-1}(ax^2)}{x} dx = -\frac{1}{4}i \operatorname{PolyLog}\left(2, -\frac{i}{ax^2}\right) + \frac{1}{4}i \operatorname{PolyLog}\left(2, \frac{i}{ax^2}\right)$$

```
[Out] -1/4*I*polylog(2,-I/a/x^2)+1/4*I*polylog(2,I/a/x^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4945, 4941, 2438}

$$\int \frac{\cot^{-1}(ax^2)}{x} dx = \frac{1}{4}i \operatorname{PolyLog}\left(2, \frac{i}{ax^2}\right) - \frac{1}{4}i \operatorname{PolyLog}\left(2, -\frac{i}{ax^2}\right)$$

```
[In] Int[ArcCot[a*x^2]/x,x]
[Out] (-1/4*I)*PolyLog[2, (-I)/(a*x^2)] + (I/4)*PolyLog[2, I/(a*x^2)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\cot^{-1}(ax)}{x}\,dx, x, x^2\right)\\ &= \frac{1}{4}i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{ax}\right)}{x}\,dx, x, x^2\right) - \frac{1}{4}i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{ax}\right)}{x}\,dx, x, x^2\right)\\ &= -\frac{1}{4}i\,\text{PolyLog}\left(2, -\frac{i}{ax^2}\right) + \frac{1}{4}i\,\text{PolyLog}\left(2, \frac{i}{ax^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = -\frac{1}{4}i\,\text{PolyLog}\left(2, -\frac{i}{ax^2}\right) + \frac{1}{4}i\,\text{PolyLog}\left(2, \frac{i}{ax^2}\right)$$

```
[In] Integrate[ArcCot[a*x^2]/x,x]
[Out] (-1/4*I)*PolyLog[2, (-I)/(a*x^2)] + (I/4)*PolyLog[2, I/(a*x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

method	result
default	$\ln(x)\,\text{arccot}(a\,x^2) + \frac{\sum_{\_R1=\text{RootOf}(a^2\_Z^4+1)} \frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}}{2a}$
parts	$\ln(x)\,\text{arccot}(a\,x^2) + \frac{\sum_{\_R1=\text{RootOf}(a^2\_Z^4+1)} \frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}}{2a}$
risch	$\frac{\pi\ln(x)}{2} + \frac{i\ln(x)\ln(1-ix\sqrt{-ia})}{2} + \frac{i\ln(x)\ln(1+ix\sqrt{-ia})}{2} - \frac{i\ln(x)\ln(-ia\,x^2+1)}{2} + \frac{i\,\text{dilog}(1-ix\sqrt{-ia})}{2} + \frac{i\,\text{dilog}(1+ix\sqrt{-ia})}{2}$

```
[In] int(arccot(a*x^2)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*arccot(a*x^2)+1/2/a*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1)),_R1=RootOf(_Z^4*a^2+1))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax^2)}{x} dx = \int \frac{\operatorname{arccot}(ax^2)}{x} dx$$

```
[In] integrate(arccot(a*x^2)/x,x, algorithm="fricas")
[Out] integral(arccot(a*x^2)/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax^2)}{x} dx = \int \frac{\operatorname{acot}(ax^2)}{x} dx$$

```
[In] integrate(acot(a*x**2)/x,x)
[Out] Integral(acot(a*x**2)/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(23) = 46$.

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{\cot^{-1}(ax^2)}{x} dx = \frac{1}{8}\pi \log(a^2x^4+1) - \frac{1}{2}\arctan(ax^2)\log(ax^2) + \operatorname{arccot}(ax^2)\log(x) + \arctan(ax^2)\log(x) + \frac{1}{4}i\operatorname{Li}_2(i\,ax^2+1) - \frac{1}{4}i\operatorname{Li}_2(-i\,ax^2+1)$$

```
[In] integrate(arccot(a*x^2)/x,x, algorithm="maxima")
[Out] 1/8*pi*log(a^2*x^4 + 1) - 1/2*arctan(a*x^2)*log(a*x^2) + arccot(a*x^2)*log(
x) + arctan(a*x^2)*log(x) + 1/4*I*dilog(I*a*x^2 + 1) - 1/4*I*dilog(-I*a*x^2
 + 1)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \int \frac{\operatorname{arccot}(ax^2)}{x}\,dx$$

```
[In] integrate(arccot(a*x^2)/x,x, algorithm="giac")
[Out] integrate(arccot(a*x^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \int \frac{\operatorname{acot}(a\,x^2)}{x}\,dx$$

```
[In] int(acot(a*x^2)/x,x)
[Out] int(acot(a*x^2)/x, x)
```

3.78 $\int \frac{\cot^{-1}(ax^2)}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \frac{\cot^{-1}(ax^2)}{x^3} dx = -\frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1+a^2x^4\right)$$

```
[Out] -1/2*arccot(a*x^2)/x^2-a*ln(x)+1/4*a*ln(a^2*x^4+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4947, 272, 36, 29, 31}

$$\int \frac{\cot^{-1}(ax^2)}{x^3} dx = \frac{1}{4}a\log\left(a^2x^4+1\right) - \frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x)$$

```
[In] Int[ArcCot[a*x^2]/x^3,x]
[Out] -1/2*ArcCot[a*x^2]/x^2 - a*Log[x] + (a*Log[1 + a^2*x^4])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}\left(ax^2\right)}{2x^2} - a\int \frac{1}{x\left(1+a^2x^4\right)}\,dx \\
&= -\frac{\cot^{-1}\left(ax^2\right)}{2x^2} - \frac{1}{4}a\text{Subst}\left(\int \frac{1}{x\left(1+a^2x\right)}\,dx, x, x^4\right) \\
&= -\frac{\cot^{-1}\left(ax^2\right)}{2x^2} - \frac{1}{4}a\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) + \frac{1}{4}a^3\text{Subst}\left(\int \frac{1}{1+a^2x}\,dx, x, x^4\right) \\
&= -\frac{\cot^{-1}\left(ax^2\right)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1+a^2x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(ax^2\right)}{x^3}\,dx = -\frac{\cot^{-1}\left(ax^2\right)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1+a^2x^4\right)$$

```
[In] Integrate[ArcCot[a*x^2]/x^3,x]
[Out] -1/2*ArcCot[a*x^2]/x^2 - a*Log[x] + (a*Log[1 + a^2*x^4])/4
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{2x^{2}}-a\left(\ln\left(x\right)-\frac{\ln\left(a^{2}x^{4}+1\right)}{4}\right)$	31
parts	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{2x^{2}}-a\left(\ln\left(x\right)-\frac{\ln\left(a^{2}x^{4}+1\right)}{4}\right)$	31
parallelrisch	$-\frac{4a\ln\left(x\right)x^{2}-a\ln\left(a^{2}x^{4}+1\right)x^{2}+2\,\operatorname{arccot}\left(a\,x^{2}\right)}{4x^{2}}$	39
risch	$-\frac{i\ln\left(ia\,x^{2}+1\right)}{4x^{2}}-\frac{4a\ln\left(x\right)x^{2}-a\ln\left(a^{2}x^{4}+1\right)x^{2}-i\ln\left(-ia\,x^{2}+1\right)+\pi}{4x^{2}}$	62

```
[In] int(arccot(a*x^2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*arccot(a*x^2)/x^2-a*(ln(x)-1/4*ln(a^2*x^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}\left(ax^{2}\right)}{x^{3}}\,dx=\frac{ax^{2}\log\left(a^{2}x^{4}+1\right)-4\,ax^{2}\log\left(x\right)-2\,\operatorname{arccot}\left(ax^{2}\right)}{4\,x^{2}}$$

```
[In] integrate(arccot(a*x^2)/x^3,x, algorithm="fricas")
[Out] 1/4*(a*x^2*log(a^2*x^4 + 1) - 4*a*x^2*log(x) - 2*arccot(a*x^2))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}\left(ax^{2}\right)}{x^{3}}\,dx=-a\log\left(x\right)+\frac{a\log\left(a^{2}x^{4}+1\right)}{4}-\frac{\operatorname{acot}\left(ax^{2}\right)}{2x^{2}}$$

```
[In] integrate(acot(a*x**2)/x**3,x)
[Out] -a*log(x) + a*log(a**2*x**4 + 1)/4 - acot(a*x**2)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = \frac{1}{4}\,a\left(\log\left(a^2x^4+1\right)-\log\left(x^4\right)\right)-\frac{\operatorname{arccot}\left(ax^2\right)}{2\,x^2}$$

```
[In] integrate(arccot(a*x^2)/x^3,x, algorithm="maxima")
[Out] 1/4*a*(log(a^2*x^4 + 1) - log(x^4)) - 1/2*arccot(a*x^2)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = -\frac{1}{4}\,a\left(\frac{2\,\arctan\left(\frac{1}{ax^2}\right)}{ax^2}-\log\left(\frac{1}{a^2x^4}+1\right)\right)$$

```
[In] integrate(arccot(a*x^2)/x^3,x, algorithm="giac")
[Out] -1/4*a*(2*arctan(1/(a*x^2))/(a*x^2) - log(1/(a^2*x^4) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = \frac{a\,\ln\left(-a^2\,x^4-1\right)}{4}-\frac{\operatorname{acot}(a\,x^2)}{2\,x^2}-a\,\ln\left(x\right)$$

```
[In] int(acot(a*x^2)/x^3,x)
[Out] (a*log(- a^2*x^4 - 1))/4 - acot(a*x^2)/(2*x^2) - a*log(x)
```

3.79 $\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 35

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4} + \frac{1}{4}a^2\arctan(ax^2)$$

```
[Out] 1/4*a/x^2-1/4*arccot(a*x^2)/x^4+1/4*a^2*arctan(a*x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4947, 281, 331, 209}

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{1}{4}a^2\arctan(ax^2) + \frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4}$$

```
[In] Int[ArcCot[a*x^2]/x^5,x]
[Out] a/(4*x^2) - ArcCot[a*x^2]/(4*x^4) + (a^2*ArcTan[a*x^2])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax^2)}{4x^4} - \frac{1}{2}a\int \frac{1}{x^3(1+a^2x^4)}\,dx \\
&= -\frac{\cot^{-1}(ax^2)}{4x^4} - \frac{1}{4}a\text{Subst}\left(\int \frac{1}{x^2(1+a^2x^2)}\,dx, x, x^2\right) \\
&= \frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4} + \frac{1}{4}a^3\text{Subst}\left(\int \frac{1}{1+a^2x^2}\,dx, x, x^2\right) \\
&= \frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4} + \frac{1}{4}a^2\arctan(ax^2)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = -\frac{\cot^{-1}(ax^2)}{4x^4} + \frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^4\right)}{4x^2}$$

```
[In] Integrate[ArcCot[a*x^2]/x^5,x]
[Out] -1/4*ArcCot[a*x^2]/x^4 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^4)])/(4
*x^2)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{4x^{4}}-\frac{a\left(-\frac{a\arctan\left(a\,x^{2}\right)}{2}-\frac{1}{2x^{2}}\right)}{2}$	31
parts	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{4x^{4}}-\frac{a\left(-\frac{a\arctan\left(a\,x^{2}\right)}{2}-\frac{1}{2x^{2}}\right)}{2}$	31
parallelrisch	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)a^{2}x^{4}-a\,x^{2}+\operatorname{arccot}\left(a\,x^{2}\right)}{4x^{4}}$	32
risch	$-\frac{i\ln\left(ia\,x^{2}+1\right)}{8x^{4}}-\frac{ia^{2}\ln\left(-a\,x^{2}+i\right)x^{4}-ia^{2}\ln\left(-a\,x^{2}-i\right)x^{4}-2a\,x^{2}-i\ln\left(-ia\,x^{2}+1\right)+\pi}{8x^{4}}$	82

```
[In] int(arccot(a*x^2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*arccot(a*x^2)/x^4-1/2*a*(-1/2*a*arctan(a*x^2)-1/2/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}\left(ax^2\right)}{x^5}\,dx=\frac{ax^2-\left(a^2x^4+1\right)\operatorname{arccot}\left(ax^2\right)}{4\,x^4}$$

```
[In] integrate(arccot(a*x^2)/x^5,x, algorithm="fricas")
[Out] 1/4*(a*x^2 - (a^2*x^4 + 1)*arccot(a*x^2))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}\left(ax^2\right)}{x^5}\,dx=-\frac{a^2\operatorname{acot}\left(ax^2\right)}{4}+\frac{a}{4x^2}-\frac{\operatorname{acot}\left(ax^2\right)}{4x^4}$$

```
[In] integrate(acot(a*x**2)/x**5,x)
[Out] -a**2*acot(a*x**2)/4 + a/(4*x**2) - acot(a*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{1}{4}\left(a\arctan\left(ax^2\right) + \frac{1}{x^2}\right)a - \frac{\operatorname{arccot}\left(ax^2\right)}{4\,x^4}$$

```
[In] integrate(arccot(a*x^2)/x^5,x, algorithm="maxima")
[Out] 1/4*(a*arctan(a*x^2) + 1/x^2)*a - 1/4*arccot(a*x^2)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{1}{4}\left(a\arctan\left(ax^2\right) + \frac{1}{x^2}\right)a - \frac{\arctan\left(\frac{1}{ax^2}\right)}{4\,x^4}$$

```
[In] integrate(arccot(a*x^2)/x^5,x, algorithm="giac")
[Out] 1/4*(a*arctan(a*x^2) + 1/x^2)*a - 1/4*arctan(1/(a*x^2))/x^4
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{a\,x^2 - \operatorname{acot}(a\,x^2) + a^2\,x^4\,\operatorname{atan}(a\,x^2)}{4\,x^4}$$

```
[In] int(acot(a*x^2)/x^5,x)
[Out] (a*x^2 - acot(a*x^2) + a^2*x^4*atan(a*x^2))/(4*x^4)
```

3.80 $\int x^4 \cot^{-1}(ax^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 152

$$\int x^4 \cot^{-1}(ax^2)\,dx = \frac{2x^3}{15a} + \frac{1}{5}x^5\cot^{-1}(ax^2) + \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}} - \frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{10\sqrt{2}a^{5/2}} + \frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{10\sqrt{2}a^{5/2}}$$

```
[Out] 2/15*x^3/a+1/5*x^5*arccot(a*x^2)-1/10*arctan(-1+x*2^(1/2)*a^(1/2))/a^(5/2)*
2^(1/2)-1/10*arctan(1+x*2^(1/2)*a^(1/2))/a^(5/2)*2^(1/2)-1/20*ln(1+a*x^2-x*
2^(1/2)*a^(1/2))/a^(5/2)*2^(1/2)+1/20*ln(1+a*x^2+x*2^(1/2)*a^(1/2))/a^(5/2)
*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4947, 327, 303, 1176, 631, 210, 1179, 642}

$$\int x^4 \cot^{-1}(ax^2)\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}} - \frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{5\sqrt{2}a^{5/2}} - \frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{10\sqrt{2}a^{5/2}} + \frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{10\sqrt{2}a^{5/2}} + \frac{2x^3}{15a} + \frac{1}{5}x^5\cot^{-1}(ax^2)$$

```
[In] Int[x^4*ArcCot[a*x^2],x]
```

```
[Out] (2*x^3)/(15*a) + (x^5*ArcCot[a*x^2])/5 + ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(5*S
qrt[2]*a^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(5*Sqrt[2]*a^(5/2)) - Log[1
```

```
 - Sqrt[2]*Sqrt[a]*x + a*x^2]/(10*Sqrt[2]*a^(5/2)) + Log[1 + Sqrt[2]*Sqrt[a
]*x + a*x^2]/(10*Sqrt[2]*a^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\cot^{-1}\left(ax^2\right)+\frac{1}{5}(2a)\int\frac{x^6}{1+a^2x^4}\,dx\\
&= \frac{2x^3}{15a}+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)-\frac{2\int\frac{x^2}{1+a^2x^4}\,dx}{5a}\\
&= \frac{2x^3}{15a}+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)+\frac{\int\frac{1-ax^2}{1+a^2x^4}\,dx}{5a^2}-\frac{\int\frac{1+ax^2}{1+a^2x^4}\,dx}{5a^2}\\
&= \frac{2x^3}{15a}+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)-\frac{\int\frac{1}{\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{10a^3}\\
&\quad-\frac{\int\frac{1}{\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{10a^3}-\frac{\int\frac{\frac{\sqrt{2}}{\sqrt{a}}+2x}{-\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{10\sqrt{2}a^{5/2}}-\frac{\int\frac{\frac{\sqrt{2}}{\sqrt{a}}-2x}{-\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{10\sqrt{2}a^{5/2}}\\
&= \frac{2x^3}{15a}+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)-\frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{10\sqrt{2}a^{5/2}}+\frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{10\sqrt{2}a^{5/2}}\\
&\quad-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}}+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}}\\
&= \frac{2x^3}{15a}+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)+\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}}\\
&\quad-\frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{10\sqrt{2}a^{5/2}}+\frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{10\sqrt{2}a^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int x^4 \cot^{-1}(ax^2) \, dx = \frac{8a^{3/2}x^3 + 12a^{5/2}x^5 \cot^{-1}(ax^2) + 6\sqrt{2}\arctan(1-\sqrt{2}\sqrt{a}x) - 6\sqrt{2}\arctan(1+\sqrt{2}\sqrt{a}x) - 3\sqrt{2}\log(1-\sqrt{}}{60a^{5/2}}$$

```
[In] Integrate[x^4*ArcCot[a*x^2],x]
```

```
[Out] (8*a^(3/2)*x^3 + 12*a^(5/2)*x^5*ArcCot[a*x^2] + 6*Sqrt[2]*ArcTan[1 - Sqrt[2
]*Sqrt[a]*x] - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] - 3*Sqrt[2]*Log[1 -
Sqrt[2]*Sqrt[a]*x + a*x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2])/
(60*a^(5/2))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x^5 \operatorname{arccot}(a x^2)}{5}+\frac{2a\left(\frac{x^3}{3a^2}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^4\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{5}$	112
parts	$\frac{x^5 \operatorname{arccot}(a x^2)}{5}+\frac{2a\left(\frac{x^3}{3a^2}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^4\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{5}$	112

```
[In] int(x^4*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*arccot(a*x^2)+2/5*a*(1/3*x^3/a^2-1/8/a^4/(1/a^2)^(1/4)*2^(1/2)*(ln(
(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1
/a^2)^(1/2)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^
(1/4)*x-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int x^4 \cot^{-1}\left(ax^2\right) dx$$
$$= \frac{6\,ax^5 \operatorname{arccot}\left(ax^2\right) + 4\,x^3 - 3\,a\left(-\frac{1}{a^{10}}\right)^{\frac{1}{4}} \log\left(a^7\left(-\frac{1}{a^{10}}\right)^{\frac{3}{4}} + x\right) + 3i\,a\left(-\frac{1}{a^{10}}\right)^{\frac{1}{4}} \log\left(i\,a^7\left(-\frac{1}{a^{10}}\right)^{\frac{3}{4}} + x\right) - 3i\,a(}{30\,a}$$

```
[In] integrate(x^4*arccot(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/30*(6*a*x^5*arccot(a*x^2) + 4*x^3 - 3*a*(-1/a^10)^(1/4)*log(a^7*(-1/a^10)
^(3/4) + x) + 3*I*a*(-1/a^10)^(1/4)*log(I*a^7*(-1/a^10)^(3/4) + x) - 3*I*a*
(-1/a^10)^(1/4)*log(-I*a^7*(-1/a^10)^(3/4) + x) + 3*a*(-1/a^10)^(1/4)*log(-
a^7*(-1/a^10)^(3/4) + x))/a
```

Sympy [A] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int x^4 \cot^{-1}\left(ax^2\right) dx$$
$$= \begin{cases} \frac{x^5 \operatorname{acot}\left(ax^2\right)}{5} + \frac{2x^3}{15a} - \frac{\sqrt[4]{-\frac{1}{a^2}} \operatorname{acot}\left(ax^2\right)}{5a^2} - \frac{\log\left(x - \sqrt[4]{-\frac{1}{a^2}}\right)}{5a^3 \sqrt[4]{-\frac{1}{a^2}}} + \frac{\log\left(x^2 + \sqrt{-\frac{1}{a^2}}\right)}{10a^3 \sqrt[4]{-\frac{1}{a^2}}} - \frac{\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{5a^3 \sqrt[4]{-\frac{1}{a^2}}} & \text{for } a \neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*acot(a*x**2),x)
```

```
[Out] Piecewise((x**5*acot(a*x**2)/5 + 2*x**3/(15*a) - (-1/a**2)**(1/4)*acot(a*x*
*2)/(5*a**2) - log(x - (-1/a**2)**(1/4))/(5*a**3*(-1/a**2)**(1/4)) + log(x*
*2 + sqrt(-1/a**2))/(10*a**3*(-1/a**2)**(1/4)) - atan(x/(-1/a**2)**(1/4))/(
5*a**3*(-1/a**2)**(1/4)), Ne(a, 0)), (pi*x**5/10, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int x^4 \cot^{-1}\left(ax^2\right)\, dx = \frac{1}{5}\, x^5 \operatorname{arccot}\left(ax^2\right) + \frac{1}{60}\, a \left(\frac{8\, x^3}{a^2} - \frac{3 \left(\frac{2\,\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\,ax + \sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\,\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\,ax - \sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{2} \log\left(ax^2 + \sqrt{2}\sqrt{a}x + 1\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{2} \log\left(ax^2 - \sqrt{2}\right.}{a^{\frac{3}{2}}}}{a^2}\right.$$

```
[In] integrate(x^4*arccot(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccot(a*x^2) + 1/60*a*(8*x^3/a^2 - 3*(2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/a^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*
(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a^(3/2) - sqrt(2)*log(a*x^2 + sqrt(2)*sq
rt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2))/
a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int x^4 \cot^{-1}\left(ax^2\right)\, dx = \frac{1}{5}\, x^5 \arctan\left(\frac{1}{ax^2}\right) + \frac{1}{60}\, a \left(\frac{8\, x^3}{a^2} - \frac{6\,\sqrt{2} \arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2 |a|^{\frac{3}{2}}} - \frac{6\,\sqrt{2} \arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2 |a|^{\frac{3}{2}}} + \frac{3\,\sqrt{2}\sqrt{|}}{}\right.$$

```
[In] integrate(x^4*arccot(a*x^2),x, algorithm="giac")
```

```
[Out] 1/5*x^5*arctan(1/(a*x^2)) + 1/60*a*(8*x^3/a^2 - 6*sqrt(2)*arctan(1/2*sqrt(2
)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*abs(a)^(3/2)) - 6*sqrt(2)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*abs(a)^
(3/2)) + 3*sqrt(2)*sqrt(abs(a))*log(x^2 + sqrt(2)*x/sqrt(abs(a)) + 1/abs(a)
)/a^4 - 3*sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*abs(a)^
(3/2)))
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

$$\int x^4 \cot^{-1}\left(ax^2\right)\,dx = \frac{x^5\,\mathrm{acot}(a\,x^2)}{5} + \frac{2\,x^3}{15\,a} - \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\right)}{5\,a^{5/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,a^{5/2}}$$

```
[In] int(x^4*acot(a*x^2),x)
[Out] (x^5*acot(a*x^2))/5 + (2*x^3)/(15*a) - ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*
x))/(5*a^(5/2)) - ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x*1i)*1i)/(5*a^(5/2))
```

3.81 $\int x^2 \cot^{-1}(ax^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 150

$$\int x^2 \cot^{-1}(ax^2)\,dx = \frac{2x}{3a} + \frac{1}{3}x^3\cot^{-1}(ax^2) + \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}} + \frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}a^{3/2}} - \frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}a^{3/2}}$$

```
[Out] 2/3*x/a+1/3*x^3*arccot(a*x^2)-1/6*arctan(-1+x*2^(1/2)*a^(1/2))/a^(3/2)*2^(1
/2)-1/6*arctan(1+x*2^(1/2)*a^(1/2))/a^(3/2)*2^(1/2)+1/12*ln(1+a*x^2-x*2^(1/
2)*a^(1/2))/a^(3/2)*2^(1/2)-1/12*ln(1+a*x^2+x*2^(1/2)*a^(1/2))/a^(3/2)*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {4947, 327, 217, 1179, 642, 1176, 631, 210}

$$\int x^2 \cot^{-1}(ax^2)\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}} - \frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{3\sqrt{2}a^{3/2}} + \frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{6\sqrt{2}a^{3/2}} - \frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{6\sqrt{2}a^{3/2}} + \frac{1}{3}x^3\cot^{-1}(ax^2) + \frac{2x}{3a}$$

```
[In] Int[x^2*ArcCot[a*x^2],x]
```

```
[Out] (2*x)/(3*a) + (x^3*ArcCot[a*x^2])/3 + ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(3*Sqrt
[2]*a^(3/2)) - ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(3*Sqrt[2]*a^(3/2)) + Log[1 -
```

```
Sqrt[2]*Sqrt[a]*x + a*x^2]/(6*Sqrt[2]*a^(3/2)) - Log[1 + Sqrt[2]*Sqrt[a]*x
+ a*x^2]/(6*Sqrt[2]*a^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}\left(ax^2\right)+\frac{1}{3}(2a)\int\frac{x^4}{1+a^2x^4}\,dx\\
&= \frac{2x}{3a}+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)-\frac{2\int\frac{1}{1+a^2x^4}\,dx}{3a}\\
&= \frac{2x}{3a}+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)-\frac{\int\frac{1-ax^2}{1+a^2x^4}\,dx}{3a}-\frac{\int\frac{1+ax^2}{1+a^2x^4}\,dx}{3a}\\
&= \frac{2x}{3a}+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)-\frac{\int\frac{1}{\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{6a^2}\\
&\quad-\frac{\int\frac{1}{\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{6a^2}+\frac{\int\frac{\frac{\sqrt{2}}{\sqrt{a}}+2x}{-\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{6\sqrt{2}a^{3/2}}+\frac{\int\frac{\frac{\sqrt{2}}{\sqrt{a}}-2x}{-\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{6\sqrt{2}a^{3/2}}\\
&= \frac{2x}{3a}+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)+\frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}a^{3/2}}-\frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}a^{3/2}}\\
&\quad-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}}+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}}\\
&= \frac{2x}{3a}+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)+\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}}\\
&\quad+\frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}a^{3/2}}-\frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int x^2 \cot^{-1}\left(ax^2\right)\,dx$$
$$= \frac{8\sqrt{a}x + 4a^{3/2}x^3\cot^{-1}\left(ax^2\right) + 2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right) - 2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \sqrt{2}\log\left(1-\sqrt{2}\sqrt{\right.}}{12a^{3/2}}$$

```
[In] Integrate[x^2*ArcCot[a*x^2],x]
```

```
[Out] (8*Sqrt[a]*x + 4*a^(3/2)*x^3*ArcCot[a*x^2] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*S
qrt[a]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] + Sqrt[2]*Log[1 - Sqrt[
2]*Sqrt[a]*x + a*x^2] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2])/(12*a^(
3/2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x^3 \operatorname{arccot}(a x^2)}{3}+\frac{2a\left(\frac{x}{a^2}-\frac{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2}\right)}{3}$	109
parts	$\frac{x^3 \operatorname{arccot}(a x^2)}{3}+\frac{2a\left(\frac{x}{a^2}-\frac{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2}\right)}{3}$	109

```
[In] int(x^2*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arccot(a*x^2)+2/3*a*(x/a^2-1/8/a^2*(1/a^2)^(1/4)*2^(1/2)*(ln((x^2+(
1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^
(1/2)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*
x-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int x^2 \cot^{-1}(ax^2)\,dx$$
$$= \frac{2\,ax^3\operatorname{arccot}(ax^2) - a\left(-\frac{1}{a^6}\right)^{\frac14}\log\left(a\left(-\frac{1}{a^6}\right)^{\frac14}+x\right) - i\,a\left(-\frac{1}{a^6}\right)^{\frac14}\log\left(i\,a\left(-\frac{1}{a^6}\right)^{\frac14}+x\right) + i\,a\left(-\frac{1}{a^6}\right)^{\frac14}\log\left(-i\,a\right.}{6\,a}$$

```
[In] integrate(x^2*arccot(a*x^2),x, algorithm="fricas")
[Out] 1/6*(2*a*x^3*arccot(a*x^2) - a*(-1/a^6)^(1/4)*log(a*(-1/a^6)^(1/4) + x) - I
*a*(-1/a^6)^(1/4)*log(I*a*(-1/a^6)^(1/4) + x) + I*a*(-1/a^6)^(1/4)*log(-I*a
*(-1/a^6)^(1/4) + x) + a*(-1/a^6)^(1/4)*log(-a*(-1/a^6)^(1/4) + x) + 4*x)/a
```

Sympy [A] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int x^2 \cot^{-1}(ax^2)\,dx$$
$$= \begin{cases} \frac{x^3\operatorname{acot}(ax^2)}{3} + \frac{\left(-\frac{1}{a^2}\right)^{\frac34}\operatorname{acot}(ax^2)}{3} + \frac{2x}{3a} + \frac{\sqrt[4]{-\frac{1}{a^2}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{3a} - \frac{\sqrt[4]{-\frac{1}{a^2}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{6a} - \frac{\sqrt[4]{-\frac{1}{a^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right.}{3a} \\ \frac{\pi x^3}{6} \end{cases}$$

```
[In] integrate(x**2*acot(a*x**2),x)
[Out] Piecewise((x**3*acot(a*x**2)/3 + (-1/a**2)**(3/4)*acot(a*x**2)/3 + 2*x/(3*a
) + (-1/a**2)**(1/4)*log(x - (-1/a**2)**(1/4))/(3*a) - (-1/a**2)**(1/4)*log
(x**2 + sqrt(-1/a**2))/(6*a) - (-1/a**2)**(1/4)*atan(x/(-1/a**2)**(1/4))/(3
*a), Ne(a, 0)), (pi*x**3/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int x^2 \cot^{-1}(ax^2)\,dx = \frac{1}{3}\,x^3\operatorname{arccot}(ax^2)$$
$$+\frac{1}{12}\,a\left(\frac{8\,x}{a^2} - \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}} - \frac{\sqrt{2}\log\left(ax^2-\sqrt{2}\sqrt{a}x+\right.}{\sqrt{a}}}{a^2}\right.$$

```
[In] integrate(x^2*arccot(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot(a*x^2) + 1/12*a*(8*x/a^2 - (2*sqrt(2)*arctan(1/2*sqrt(2)*(2*
a*x + sqrt(2)*sqrt(a))/sqrt(a))/sqrt(a) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a
*x - sqrt(2)*sqrt(a))/sqrt(a))/sqrt(a) + sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a
)*x + 1)/sqrt(a) - sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a))/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.02

$$\int x^2 \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{3}\,x^3\arctan\left(\frac{1}{ax^2}\right) + \frac{1}{12}\,a\left(\frac{8\,x}{a^2} - \frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2\sqrt{|a|}} - \frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2\sqrt{|a|}} - \frac{\sqrt{2}\log}{}\right.$$

```
[In] integrate(x^2*arccot(a*x^2),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(1/(a*x^2)) + 1/12*a*(8*x/a^2 - 2*sqrt(2)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*sqrt(abs(a))) - 2*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*sqrt(abs(
a))) - sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*sqrt(abs(a
))) + sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*sqrt(abs(a)
)))
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

$$\int x^2 \cot^{-1}\left(ax^2\right)\,dx = \frac{x^3\,\mathrm{acot}(a\,x^2)}{3} + \frac{2\,x}{3\,a} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\right)\,1\mathrm{i}}{3\,a^{3/2}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\,1\mathrm{i}\right)}{3\,a^{3/2}}$$

```
[In] int(x^2*acot(a*x^2),x)
```

```
[Out] (x^3*acot(a*x^2))/3 + (2*x)/(3*a) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x)*
1i)/(3*a^(3/2)) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x*1i))/(3*a^(3/2))
```

3.82 $\int \cot^{-1}(ax^2)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 132

$$\int \cot^{-1}(ax^2)\,dx = x\cot^{-1}(ax^2) - \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} + \frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}} - \frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}}$$

```
[Out] x*arccot(a*x^2)+1/2*arctan(-1+x*2^(1/2)*a^(1/2))*2^(1/2)/a^(1/2)+1/2*arctan
(1+x*2^(1/2)*a^(1/2))*2^(1/2)/a^(1/2)+1/4*ln(1+a*x^2-x*2^(1/2)*a^(1/2))*2^(
1/2)/a^(1/2)-1/4*ln(1+a*x^2+x*2^(1/2)*a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {4931, 303, 1176, 631, 210, 1179, 642}

$$\int \cot^{-1}(ax^2)\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} + \frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}} + \frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}} - \frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}} + x\cot^{-1}(ax^2)$$

```
[In] Int[ArcCot[a*x^2],x]
```

```
[Out] x*ArcCot[a*x^2] - ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]) + ArcTan[
1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]) + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^
2]/(2*Sqrt[2]*Sqrt[a]) - Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2]/(2*Sqrt[2]*Sqrt
[a])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\text{integral} = x\cot^{-1}\left(ax^2\right) + (2a)\int \frac{x^2}{1+a^2x^4}\,dx$$

$$= x\cot^{-1}\left(ax^2\right) - \int \frac{1-ax^2}{1+a^2x^4}\,dx + \int \frac{1+ax^2}{1+a^2x^4}\,dx$$

$$= x\cot^{-1}\left(ax^2\right) + \frac{\int \frac{1}{\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{2a} + \frac{\int \frac{1}{\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{2a} + \frac{\int \frac{\frac{\sqrt{2}}{\sqrt{a}}+2x}{-\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{2\sqrt{2}\sqrt{a}} + \frac{\int \frac{\frac{\sqrt{2}}{\sqrt{a}}-2x}{-\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{2\sqrt{2}\sqrt{a}}$$

$$= x\cot^{-1}\left(ax^2\right) + \frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}} - \frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}$$

$$= x\cot^{-1}\left(ax^2\right) - \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} + \frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}} - \frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \cot^{-1}\left(ax^2\right)\,dx = x\cot^{-1}\left(ax^2\right) + \frac{-2\arctan\left(1-\sqrt{2}\sqrt{a}x\right) + 2\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right) - \log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}}$$

```
[In] Integrate[ArcCot[a*x^2],x]
[Out] x*ArcCot[a*x^2] + (-2*ArcTan[1 - Sqrt[2]*Sqrt[a]*x] + 2*ArcTan[1 + Sqrt[2]*
Sqrt[a]*x] + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Log[1 + Sqrt[2]*Sqrt[a]*x
 + a*x^2])/(2*Sqrt[2]*Sqrt[a])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

method	result	size
default	$x \operatorname{arccot}\left(a\,x^{2}\right)+\frac{\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}$	97
parts	$x \operatorname{arccot}\left(a\,x^{2}\right)+\frac{\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}$	97

```
[In] int(arccot(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccot(a*x^2)+1/4/a/(1/a^2)^(1/4)*2^(1/2)*(ln((x^2-(1/a^2)^(1/4)*x*2^(1/2
)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2)))+2*arctan(2^(1
/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73

$$\int \cot^{-1}\left(ax^{2}\right)\,dx = x \operatorname{arccot}\left(ax^{2}\right)+\frac{1}{2}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^{2}}\right)^{\frac{3}{4}}+x\right) - \frac{1}{2}i\left(-\frac{1}{a^{2}}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^{2}}\right)^{\frac{3}{4}}+x\right) + \frac{1}{2}i\left(-\frac{1}{a^{2}}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^{2}}\right)^{\frac{3}{4}}+x\right) - \frac{1}{2}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^{2}}\right)^{\frac{3}{4}}+x\right)$$

```
[In] integrate(arccot(a*x^2),x, algorithm="fricas")
```

```
[Out] x*arccot(a*x^2) + 1/2*(-1/a^2)^(1/4)*log(a*(-1/a^2)^(3/4) + x) - 1/2*I*(-1/
a^2)^(1/4)*log(I*a*(-1/a^2)^(3/4) + x) + 1/2*I*(-1/a^2)^(1/4)*log(-I*a*(-1/
a^2)^(3/4) + x) - 1/2*(-1/a^2)^(1/4)*log(-a*(-1/a^2)^(3/4) + x)
```

Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int \cot^{-1}\left(ax^2\right)\,dx$$
$$= \begin{cases} x\,\mathrm{acot}\left(ax^2\right) + \sqrt[4]{-\frac{1}{a^2}}\,\mathrm{acot}\left(ax^2\right) + \frac{\log\left(x - \sqrt[4]{-\frac{1}{a^2}}\right)}{a\sqrt[4]{-\frac{1}{a^2}}} - \frac{\log\left(x^2 + \sqrt{-\frac{1}{a^2}}\right)}{2a\sqrt[4]{-\frac{1}{a^2}}} + \frac{\mathrm{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{a\sqrt[4]{-\frac{1}{a^2}}} & \text{for } a \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(acot(a*x**2),x)
```

```
[Out] Piecewise((x*acot(a*x**2) + (-1/a**2)**(1/4)*acot(a*x**2) + log(x - (-1/a**
2)**(1/4))/(a*(-1/a**2)**(1/4)) - log(x**2 + sqrt(-1/a**2))/(2*a*(-1/a**2)*
*(1/4)) + atan(x/(-1/a**2)**(1/4))/(a*(-1/a**2)**(1/4)), Ne(a, 0)), (pi*x/2
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \cot^{-1}\left(ax^2\right)\,dx$$
$$= \frac{1}{4}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{2}\,\mathrm{l}}{}\right.$$
$$+ x\,\mathrm{arccot}\left(ax^2\right)$$

```
[In] integrate(arccot(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/a^(3
/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a^(3/
2) - sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a*x^2
 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)) + x*arccot(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.09

$$\int \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{4}\,a\left(\frac{2\,\sqrt{2}\sqrt{|a|}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2}+\frac{2\,\sqrt{2}\sqrt{|a|}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2}-\frac{\sqrt{2}\sqrt{|a}}{}\right.$$
$$\left.+\,x\arctan\left(\frac{1}{ax^2}\right)\right.$$

```
[In] integrate(arccot(a*x^2),x, algorithm="giac")
```

```
[Out] 1/4*a*(2*sqrt(2)*sqrt(abs(a))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)
))*sqrt(abs(a)))/a^2 + 2*sqrt(2)*sqrt(abs(a))*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)/sqrt(abs(a)))*sqrt(abs(a)))/a^2 - sqrt(2)*sqrt(abs(a))*log(x^2 + sqrt(
2)*x/sqrt(abs(a)) + 1/abs(a))/a^2 + sqrt(2)*sqrt(abs(a))*log(x^2 - sqrt(2)*
x/sqrt(abs(a)) + 1/abs(a))/a^2) + x*arctan(1/(a*x^2))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.32

$$\int \cot^{-1}\left(ax^2\right)\,dx = x\,\mathrm{acot}\left(a\,x^2\right)+\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\right)}{\sqrt{a}} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{a}\,x\right)}{\sqrt{a}}$$

```
[In] int(acot(a*x^2),x)
```

```
[Out] x*acot(a*x^2) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x))/a^(1/2) - ((-1)^(1/
4)*atanh((-1)^(1/4)*a^(1/2)*x))/a^(1/2)
```

3.83 $\int \frac{\cot^{-1}(ax^2)}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 135

$$\int \frac{\cot^{-1}(ax^2)}{x^2} dx = -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} - \frac{\sqrt{a}\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} + \frac{\sqrt{a}\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}} - \frac{\sqrt{a}\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}}$$

```
[Out] -arccot(a*x^2)/x-1/2*arctan(-1+x*2^(1/2)*a^(1/2))*a^(1/2)*2^(1/2)-1/2*arcta
n(1+x*2^(1/2)*a^(1/2))*a^(1/2)*2^(1/2)+1/4*ln(1+a*x^2-x*2^(1/2)*a^(1/2))*a^
(1/2)*2^(1/2)-1/4*ln(1+a*x^2+x*2^(1/2)*a^(1/2))*a^(1/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4947, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\cot^{-1}(ax^2)}{x^2} dx = \frac{\sqrt{a}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} - \frac{\sqrt{a}\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}} + \frac{\sqrt{a}\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}} - \frac{\sqrt{a}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}} - \frac{\cot^{-1}(ax^2)}{x}$$

```
[In] Int[ArcCot[a*x^2]/x^2,x]
```

```
[Out] -(ArcCot[a*x^2]/x) + (Sqrt[a]*ArcTan[1 - Sqrt[2]*Sqrt[a]*x])/Sqrt[2] - (Sqr
t[a]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x])/Sqrt[2] + (Sqrt[a]*Log[1 - Sqrt[2]*Sqrt
```

```
[a]*x + a*x^2])/(2*Sqrt[2]) - (Sqrt[a]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2])/
(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax^2)}{x} - (2a)\int \frac{1}{1+a^2x^4}\,dx \\
&= -\frac{\cot^{-1}(ax^2)}{x} - a\int \frac{1-ax^2}{1+a^2x^4}\,dx - a\int \frac{1+ax^2}{1+a^2x^4}\,dx \\
&= -\frac{\cot^{-1}(ax^2)}{x} - \frac{1}{2}\int \frac{1}{\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx - \frac{1}{2}\int \frac{1}{\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx \\
&\quad + \frac{\sqrt{a}\int \frac{\frac{\sqrt{2}}{\sqrt{a}}+2x}{-\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{2\sqrt{2}} + \frac{\sqrt{a}\int \frac{\frac{\sqrt{2}}{\sqrt{a}}-2x}{-\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{2\sqrt{2}} \\
&= -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}} - \frac{\sqrt{a}\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}} \\
&\quad - \frac{\sqrt{a}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} + \frac{\sqrt{a}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} \\
&= -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} - \frac{\sqrt{a}\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} \\
&\quad + \frac{\sqrt{a}\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}} - \frac{\sqrt{a}\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$
\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\left(2\arctan\left(1-\sqrt{2}\sqrt{a}x\right) - 2\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right) - \log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)\right)}{2\sqrt{2}}
$$

```
[In] Integrate[ArcCot[a*x^2]/x^2,x]
```

```
[Out] -(ArcCot[a*x^2]/x) + (Sqrt[a]*(2*ArcTan[1 - Sqrt[2]*Sqrt[a]*x] - 2*ArcTan[1
 + Sqrt[2]*Sqrt[a]*x] + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Log[1 + Sqrt[2
]*Sqrt[a]*x + a*x^2]))/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{\operatorname{arccot}(a\,x^2)}{x}-\frac{a\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{4}$	98
parts	$-\frac{\operatorname{arccot}(a\,x^2)}{x}-\frac{a\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{4}$	98

```
[In] int(arccot(a*x^2)/x^2,x,method=_RETURNVERBOSE)
[Out] -arccot(a*x^2)/x-1/4*a*(1/a^2)^(1/4)*2^(1/2)*(ln((x^2+(1/a^2)^(1/4)*x*2^(1/
2)+(1/a^2)^(1/2))/(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2)))+2*arctan(2^(
1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{(-a^2)^{\frac{1}{4}}x\log\left(ax+(-a^2)^{\frac{1}{4}}\right)+i(-a^2)^{\frac{1}{4}}x\log\left(ax+i(-a^2)^{\frac{1}{4}}\right)-i(-a^2)^{\frac{1}{4}}x\log\left(ax-i(-a^2)^{\frac{1}{4}}\right)-(-a}{2x}$$

```
[In] integrate(arccot(a*x^2)/x^2,x, algorithm="fricas")
[Out] -1/2*((-a^2)^(1/4)*x*log(a*x + (-a^2)^(1/4)) + I*(-a^2)^(1/4)*x*log(a*x + I
*(-a^2)^(1/4)) - I*(-a^2)^(1/4)*x*log(a*x - I*(-a^2)^(1/4)) - (-a^2)^(1/4)*
x*log(a*x - (-a^2)^(1/4)) + 2*arccot(a*x^2))/x
```

Sympy [A] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = \begin{cases} a^2\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\operatorname{acot}(ax^2)+a\sqrt[4]{-\frac{1}{a^2}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)-\frac{a\sqrt[4]{-\frac{1}{a^2}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{2}-a\sqrt[4]{-\frac{1}{a^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)- \\ -\frac{\pi}{2x} \end{cases}$$

```
[In] integrate(acot(a*x**2)/x**2,x)
```

```
[Out] Piecewise((a**2*(-1/a**2)**(3/4)*acot(a*x**2) + a*(-1/a**2)**(1/4)*log(x -
(-1/a**2)**(1/4)) - a*(-1/a**2)**(1/4)*log(x**2 + sqrt(-1/a**2))/2 - a*(-1/
a**2)**(1/4)*atan(x/(-1/a**2)**(1/4)) - acot(a*x**2)/x, Ne(a, 0)), (-pi/(2*
x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax^2)}{x^2} dx = -\frac{1}{4} a \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}} - \cdots \right) - \frac{\operatorname{arccot}(ax^2)}{x}$$

```
[In] integrate(arccot(a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/sqr
t(a) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/sqrt
(a) + sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(a) - sqrt(2)*log(a*x^
2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a)) - arccot(a*x^2)/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax^2)}{x^2} dx = -\frac{1}{4} a \left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{\sqrt{|a|}} + \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{\sqrt{|a|}} + \frac{\sqrt{2}\log\left(x^2+\cdots\right.}{\sqrt{|a|}} \cdots \right) - \frac{\arctan\left(\frac{1}{ax^2}\right)}{x}$$

```
[In] integrate(arccot(a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(
a)))/sqrt(abs(a)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)
```

```
))*sqrt(abs(a)))/sqrt(abs(a)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(a)) +
1/abs(a))/sqrt(abs(a)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a
))/sqrt(abs(a))) - arctan(1/(a*x^2))/x
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.33

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{\mathrm{acot}(a\,x^2)}{x} + (-1)^{1/4}\,\sqrt{a}\,\mathrm{atan}\Big((-1)^{1/4}\,\sqrt{a}\,x\Big)\ 1\mathrm{i} + (-1)^{1/4}\,\sqrt{a}\,\mathrm{atanh}\Big((-1)^{1/4}\,\sqrt{a}\,x\Big)\ 1\mathrm{i}$$

```
[In] int(acot(a*x^2)/x^2,x)
```

```
[Out] (-1)^(1/4)*a^(1/2)*atan((-1)^(1/4)*a^(1/2)*x)*1i - acot(a*x^2)/x + (-1)^(1/
4)*a^(1/2)*atanh((-1)^(1/4)*a^(1/2)*x)*1i
```

3.84 $\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 150

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} - \frac{a^{3/2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} + \frac{a^{3/2}\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} + \frac{a^{3/2}\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}} - \frac{a^{3/2}\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}}$$

```
[Out] 2/3*a/x-1/3*arccot(a*x^2)/x^3+1/6*a^(3/2)*arctan(-1+x*2^(1/2)*a^(1/2))*2^(1
/2)+1/6*a^(3/2)*arctan(1+x*2^(1/2)*a^(1/2))*2^(1/2)+1/12*a^(3/2)*ln(1+a*x^2
-x*2^(1/2)*a^(1/2))*2^(1/2)-1/12*a^(3/2)*ln(1+a*x^2+x*2^(1/2)*a^(1/2))*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4947, 331, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = -\frac{a^{3/2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} + \frac{a^{3/2}\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{3\sqrt{2}} + \frac{a^{3/2}\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{6\sqrt{2}} - \frac{a^{3/2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{6\sqrt{2}} - \frac{\cot^{-1}(ax^2)}{3x^3} + \frac{2a}{3x}$$

```
[In] Int[ArcCot[a*x^2]/x^4,x]
```

```
[Out] (2*a)/(3*x) - ArcCot[a*x^2]/(3*x^3) - (a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[a]*x
])/(3*Sqrt[2]) + (a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[a]*x])/(3*Sqrt[2]) + (a^(
3/2)*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2])/(6*Sqrt[2]) - (a^(3/2)*Log[1 + Sqr
t[2]*Sqrt[a]*x + a*x^2])/(6*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(ax^2)}{3x^3} - \frac{1}{3}(2a)\int \frac{1}{x^2(1+a^2x^4)}\,dx \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} + \frac{1}{3}(2a^3)\int \frac{x^2}{1+a^2x^4}\,dx \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} - \frac{1}{3}a^2\int \frac{1-ax^2}{1+a^2x^4}\,dx + \frac{1}{3}a^2\int \frac{1+ax^2}{1+a^2x^4}\,dx \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} + \frac{1}{6}a\int \frac{1}{\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx + \frac{1}{6}a\int \frac{1}{\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx \\
&\quad + \frac{a^{3/2}\int \frac{\frac{\sqrt{2}}{\sqrt{a}}+2x}{-\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{6\sqrt{2}} + \frac{a^{3/2}\int \frac{\frac{\sqrt{2}}{\sqrt{a}}-2x}{-\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{6\sqrt{2}} \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} + \frac{a^{3/2}\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}} - \frac{a^{3/2}\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}} \\
&\quad + \frac{a^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} - \frac{a^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} - \frac{a^{3/2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} + \frac{a^{3/2}\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} \\
&\quad + \frac{a^{3/2}\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}} - \frac{a^{3/2}\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx$$
$$= \frac{-4\cot^{-1}(ax^2) + ax^2\left(8 - 2\sqrt{2}\sqrt{a}x\arctan\left(1-\sqrt{2}\sqrt{a}x\right) + 2\sqrt{2}\sqrt{a}x\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \sqrt{2}\sqrt{a}x\log\right.}{12x^3}$$

```
[In] Integrate[ArcCot[a*x^2]/x^4,x]
```

```
[Out] (-4*ArcCot[a*x^2] + a*x^2*(8 - 2*Sqrt[2]*Sqrt[a]*x*ArcTan[1 - Sqrt[2]*Sqrt[
a]*x] + 2*Sqrt[2]*Sqrt[a]*x*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] + Sqrt[2]*Sqrt[a]
*x*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Sqrt[2]*Sqrt[a]*x*Log[1 + Sqrt[2]*S
qrt[a]*x + a*x^2]))/(12*x^3)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\operatorname{arccot}(a\,x^2)}{3x^3}-\frac{2a\left(-\frac{1}{x}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{3}$	106
parts	$-\frac{\operatorname{arccot}(a\,x^2)}{3x^3}-\frac{2a\left(-\frac{1}{x}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{3}$	106

```
[In] int(arccot(a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arccot(a*x^2)/x^3-2/3*a*(-1/x-1/8/(1/a^2)^(1/4)*2^(1/2)*(ln((x^2-(1/a^
2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2
)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1)
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{(-a^6)^{\frac{1}{4}}\,x^3\log\left(a^5x+(-a^6)^{\frac{3}{4}}\right)-i\,(-a^6)^{\frac{1}{4}}\,x^3\log\left(a^5x+i\,(-a^6)^{\frac{3}{4}}\right)+i\,(-a^6)^{\frac{1}{4}}\,x^3\log\left(a^5x-i\,(-a^6)^{\frac{3}{4}}\right)-(-}{6\,x^3}$$

```
[In] integrate(arccot(a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((-a^6)^(1/4)*x^3*log(a^5*x + (-a^6)^(3/4)) - I*(-a^6)^(1/4)*x^3*log(a^
5*x + I*(-a^6)^(3/4)) + I*(-a^6)^(1/4)*x^3*log(a^5*x - I*(-a^6)^(3/4)) - (-
a^6)^(1/4)*x^3*log(a^5*x - (-a^6)^(3/4)) + 4*a*x^2 - 2*arccot(a*x^2))/x^3
```

Sympy [A] (verification not implemented)

Time = 12.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \begin{cases} \frac{a^2\sqrt[4]{-\frac{1}{a^2}}\operatorname{acot}(ax^2)}{3}+\frac{a\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{3\sqrt[4]{-\frac{1}{a^2}}}-\frac{a\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{6\sqrt[4]{-\frac{1}{a^2}}}+\frac{a\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{3\sqrt[4]{-\frac{1}{a^2}}}+\frac{2a}{3x}-\frac{\operatorname{acot}(ax^2)}{3x^3} & \text{for } a\neq 0 \\ -\frac{\pi}{6x^3} & \text{otherwise} \end{cases}$$

```
[In] integrate(acot(a*x**2)/x**4,x)
```

```
[Out] Piecewise((a**2*(-1/a**2)**(1/4)*acot(a*x**2)/3 + a*log(x - (-1/a**2)**(1/4
))/(3*(-1/a**2)**(1/4)) - a*log(x**2 + sqrt(-1/a**2))/(6*(-1/a**2)**(1/4))
+ a*atan(x/(-1/a**2)**(1/4))/(3*(-1/a**2)**(1/4)) + 2*a/(3*x) - acot(a*x**2
)/(3*x**3), Ne(a, 0)), (-pi/(6*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{1}{12}\left(a^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{a^{\frac{3}{2}}}+ \right.\right.$$
$$-\frac{\operatorname{arccot}(ax^2)}{3\,x^3}$$

```
[In] integrate(arccot(a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*(a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/
a^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a
^(3/2) - sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a
*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)) + 8/x)*a - 1/3*arccot(a*x^2)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{1}{12}\left(\frac{2\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{|a|^{\frac{3}{2}}}+\frac{2\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{|a|^{\frac{3}{2}}}-\sqrt{2}\sqrt{|a|}\log\left(\right.\right.$$
$$-\frac{\arctan\left(\frac{1}{ax^2}\right)}{3\,x^3}$$

```
[In] integrate(arccot(a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(ab
s(a)))/abs(a)^(3/2) + 2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(
abs(a)))*sqrt(abs(a)))/abs(a)^(3/2) - sqrt(2)*sqrt(abs(a))*log(x^2 + sqrt(2
)*x/sqrt(abs(a)) + 1/abs(a)) + sqrt(2)*a^2*log(x^2 - sqrt(2)*x/sqrt(abs(a))
 + 1/abs(a))/abs(a)^(3/2) + 8/x)*a - 1/3*arctan(1/(a*x^2))/x^3
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{2\,a}{3\,x} - \frac{\mathrm{acot}(a\,x^2)}{3\,x^3} + \frac{(-1)^{1/4}\,a^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\right)}{3} + \frac{(-1)^{1/4}\,a^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

```
[In] int(acot(a*x^2)/x^4,x)
```

```
[Out] (2*a)/(3*x) - acot(a*x^2)/(3*x^3) + ((-1)^(1/4)*a^(3/2)*atan((-1)^(1/4)*a^(
1/2)*x))/3 + ((-1)^(1/4)*a^(3/2)*atan((-1)^(1/4)*a^(1/2)*x*1i)*1i)/3
```

3.85 $\int x^2 \cot^{-1}(\sqrt{x})\, dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x^2 \cot^{-1}(\sqrt{x})\, dx = \frac{\sqrt{x}}{3} - \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \cot^{-1}(\sqrt{x}) - \frac{\arctan(\sqrt{x})}{3}$$

```
[Out] -1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccot(x^(1/2))-1/3*arctan(x^(1/2))+1/3*x
^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4947, 52, 65, 209}

$$\int x^2 \cot^{-1}(\sqrt{x})\, dx = -\frac{\arctan(\sqrt{x})}{3} + \frac{x^{5/2}}{15} - \frac{x^{3/2}}{9} + \frac{1}{3}x^3 \cot^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{3}$$

```
[In] Int[x^2*ArcCot[Sqrt[x]],x]
```

```
[Out] Sqrt[x]/3 - x^(3/2)/9 + x^(5/2)/15 + (x^3*ArcCot[Sqrt[x]])/3 - ArcTan[Sqrt[
x]]/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int\frac{x^{5/2}}{1+x}\,dx\\
&= \frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int\frac{x^{3/2}}{1+x}\,dx\\
&= -\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= \frac{\sqrt{x}}{3}-\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int\frac{1}{\sqrt{x}(1+x)}\,dx\\
&= \frac{\sqrt{x}}{3}-\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{3}-\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{45}\left(\sqrt{x}\left(15-5x+3x^2\right)+15x^3\cot^{-1}\left(\sqrt{x}\right)-15\arctan\left(\sqrt{x}\right)\right)$$

```
[In] Integrate[x^2*ArcCot[Sqrt[x]],x]
[Out] (Sqrt[x]*(15 - 5*x + 3*x^2) + 15*x^3*ArcCot[Sqrt[x]] - 15*ArcTan[Sqrt[x]])/
45
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^3\,\mathrm{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}$	32
default	$-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^3\,\mathrm{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}$	32
parts	$-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^3\,\mathrm{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}$	32

```
[In] int(x^2*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccot(x^(1/2))-1/3*arctan(x^(1/2))+1/3*x
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{3}\left(x^3+1\right)\mathrm{arccot}\left(\sqrt{x}\right)+\frac{1}{45}\left(3\,x^2-5\,x+15\right)\sqrt{x}$$

```
[In] integrate(x^2*arccot(x^(1/2)),x, algorithm="fricas")
[Out] 1/3*(x^3 + 1)*arccot(sqrt(x)) + 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{x^{\frac{5}{2}}}{15} - \frac{x^{\frac{3}{2}}}{9} + \frac{\sqrt{x}}{3} + \frac{x^3 \operatorname{acot}\left(\sqrt{x}\right)}{3} - \frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}$$

```
[In] integrate(x**2*acot(x**(1/2)),x)
```

```
[Out] x**(5/2)/15 - x**(3/2)/9 + sqrt(x)/3 + x**3*acot(sqrt(x))/3 - atan(sqrt(x))
/3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{3}\,x^3 \operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{15}\,x^{\frac{5}{2}} - \frac{1}{9}\,x^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{x} - \frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^2*arccot(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot(sqrt(x)) + 1/15*x^(5/2) - 1/9*x^(3/2) + 1/3*sqrt(x) - 1/3*ar
ctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{3}\,x^3 \arctan\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{45}\,x^{\frac{5}{2}}\left(\frac{5}{x} - \frac{15}{x^2} - 3\right) + \frac{1}{3}\,\arctan\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] integrate(x^2*arccot(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(1/sqrt(x)) - 1/45*x^(5/2)*(5/x - 15/x^2 - 3) + 1/3*arctan(1/
sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{x^3\,\mathrm{acot}\left(\sqrt{x}\right)}{3} - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{3} + \frac{\sqrt{x}}{3} - \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15}$$

```
[In] int(x^2*acot(x^(1/2)),x)
```

```
[Out] (x^3*acot(x^(1/2)))/3 - atan(x^(1/2))/3 + x^(1/2)/3 - x^(3/2)/9 + x^(5/2)/1
5
```

3.86 $\int x \cot^{-1}\left(\sqrt{x}\right) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x \cot^{-1}\left(\sqrt{x}\right) \, dx = -\frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \cot^{-1}\left(\sqrt{x}\right) + \frac{\arctan\left(\sqrt{x}\right)}{2}$$

```
[Out] 1/6*x^(3/2)+1/2*x^2*arccot(x^(1/2))+1/2*arctan(x^(1/2))-1/2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4947, 52, 65, 209}

$$\int x \cot^{-1}\left(\sqrt{x}\right) \, dx = \frac{\arctan\left(\sqrt{x}\right)}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \cot^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x}}{2}$$

```
[In] Int[x*ArcCot[Sqrt[x]],x]
```

```
[Out] -1/2*Sqrt[x] + x^(3/2)/6 + (x^2*ArcCot[Sqrt[x]])/2 + ArcTan[Sqrt[x]]/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{x^{3/2}}{1+x}\,dx\\
&= \frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= -\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{1}{\sqrt{x}(1+x)}\,dx\\
&= -\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)+\frac{\arctan\left(\sqrt{x}\right)}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{6}\left((-3+x)\sqrt{x}+3x^2\cot^{-1}\left(\sqrt{x}\right)+3\arctan\left(\sqrt{x}\right)\right)$$

```
[In] Integrate[x*ArcCot[Sqrt[x]],x]
[Out] ((-3 + x)*Sqrt[x] + 3*x^2*ArcCot[Sqrt[x]] + 3*ArcTan[Sqrt[x]])/6
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{6}+\frac{x^{2} \operatorname{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}$	27
default	$\frac{x^{\frac{3}{2}}}{6}+\frac{x^{2} \operatorname{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}$	27
parts	$\frac{x^{\frac{3}{2}}}{6}+\frac{x^{2} \operatorname{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}$	27

```
[In] int(x*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/6*x^(3/2)+1/2*x^2*arccot(x^(1/2))+1/2*arctan(x^(1/2))-1/2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int x \cot^{-1}\left(\sqrt{x}\right) \, dx = \frac{1}{2} \left(x^{2} - 1\right) \operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{6} \left(x - 3\right) \sqrt{x}$$

```
[In] integrate(x*arccot(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*arccot(sqrt(x)) + 1/6*(x - 3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int x \cot^{-1}\left(\sqrt{x}\right) \, dx = \frac{x^{\frac{3}{2}}}{6} - \frac{\sqrt{x}}{2} + \frac{x^{2} \operatorname{acot}\left(\sqrt{x}\right)}{2} + \frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

```
[In] integrate(x*acot(x**(1/2)),x)
[Out] x**(3/2)/6 - sqrt(x)/2 + x**2*acot(sqrt(x))/2 + atan(sqrt(x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2 \operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{6}\,x^{\frac{3}{2}} - \frac{1}{2}\,\sqrt{x} + \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x*arccot(x^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2*arccot(sqrt(x)) + 1/6*x^(3/2) - 1/2*sqrt(x) + 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int x \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2 \arctan\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{6}\,x^{\frac{3}{2}}\left(\frac{3}{x} - 1\right) - \frac{1}{2}\,\arctan\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] integrate(x*arccot(x^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2*arctan(1/sqrt(x)) - 1/6*x^(3/2)*(3/x - 1) - 1/2*arctan(1/sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{\operatorname{atan}\left(\sqrt{x}\right)}{2} + \frac{x^2 \operatorname{acot}\left(\sqrt{x}\right)}{2} - \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6}$$

```
[In] int(x*acot(x^(1/2)),x)
[Out] atan(x^(1/2))/2 + (x^2*acot(x^(1/2)))/2 - x^(1/2)/2 + x^(3/2)/6
```

3.87 $\int \cot^{-1}\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx=\sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)$$

```
[Out] x*arccot(x^(1/2))-arctan(x^(1/2))+x^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4931, 52, 65, 209}

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx=-\arctan\left(\sqrt{x}\right)+\sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)$$

```
[In] Int[ArcCot[Sqrt[x]],x]
```

```
[Out] Sqrt[x] + x*ArcCot[Sqrt[x]] - ArcTan[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= \sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{1}{\sqrt{x}(1+x)}\,dx\\
&= \sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int\cot^{-1}\left(\sqrt{x}\right)\,dx=\sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)$$

```
[In] Integrate[ArcCot[Sqrt[x]],x]
[Out] Sqrt[x] + x*ArcCot[Sqrt[x]] - ArcTan[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$x \operatorname{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)+\sqrt{x}$	17
default	$x \operatorname{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)+\sqrt{x}$	17
parts	$x \operatorname{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)+\sqrt{x}$	17

```
[In] int(arccot(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arccot(x^(1/2))-arctan(x^(1/2))+x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx=(x+1)\operatorname{arccot}\left(\sqrt{x}\right)+\sqrt{x}$$

```
[In] integrate(arccot(x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arccot(sqrt(x)) + sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx=\sqrt{x}+x\operatorname{acot}\left(\sqrt{x}\right)-\operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(acot(x**(1/2)),x)
[Out] sqrt(x) + x*acot(sqrt(x)) - atan(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{arccot}\left(\sqrt{x}\right) + \sqrt{x} - \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arccot(x^(1/2)),x, algorithm="maxima")
[Out] x*arccot(sqrt(x)) + sqrt(x) - arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = x\arctan\left(\frac{1}{\sqrt{x}}\right) + \sqrt{x} + \arctan\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] integrate(arccot(x^(1/2)),x, algorithm="giac")
[Out] x*arctan(1/sqrt(x)) + sqrt(x) + arctan(1/sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{acot}\left(\sqrt{x}\right) - \mathrm{atan}\left(\sqrt{x}\right) + \sqrt{x}$$

```
[In] int(acot(x^(1/2)),x)
[Out] x*acot(x^(1/2)) - atan(x^(1/2)) + x^(1/2)
```

3.88 $\int \frac{\cot^{-1}(\sqrt{x})}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{\cot^{-1}(\sqrt{x})}{x} \, dx = -i \operatorname{PolyLog}\left(2, -\frac{i}{\sqrt{x}}\right) + i \operatorname{PolyLog}\left(2, \frac{i}{\sqrt{x}}\right)$$

```
[Out] -I*polylog(2,-I/x^(1/2))+I*polylog(2,I/x^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4945, 4941, 2438}

$$\int \frac{\cot^{-1}(\sqrt{x})}{x} \, dx = i \operatorname{PolyLog}\left(2, \frac{i}{\sqrt{x}}\right) - i \operatorname{PolyLog}\left(2, -\frac{i}{\sqrt{x}}\right)$$

```
[In] Int[ArcCot[Sqrt[x]]/x,x]
[Out] (-I)*PolyLog[2, (-I)/Sqrt[x]] + I*PolyLog[2, I/Sqrt[x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, \sqrt{x}\right)\\ &= i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, \sqrt{x}\right)\\ &= -i\,\text{PolyLog}\left(2, -\frac{i}{\sqrt{x}}\right) + i\,\text{PolyLog}\left(2, \frac{i}{\sqrt{x}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = -i\,\text{PolyLog}\left(2, -\frac{i}{\sqrt{x}}\right) + i\,\text{PolyLog}\left(2, \frac{i}{\sqrt{x}}\right)$$

```
[In] Integrate[ArcCot[Sqrt[x]]/x,x]
[Out] (-I)*PolyLog[2, (-I)/Sqrt[x]] + I*PolyLog[2, I/Sqrt[x]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

method	result
derivativedivides	$\ln(x)\,\text{arccot}\left(\sqrt{x}\right) - \frac{i\ln(x)\ln(1+i\sqrt{x})}{2} + \frac{i\ln(x)\ln(1-i\sqrt{x})}{2} - i\,\text{dilog}\left(1+i\sqrt{x}\right) + i\,\text{dilog}\left(1-i\right.$
default	$\ln(x)\,\text{arccot}\left(\sqrt{x}\right) - \frac{i\ln(x)\ln(1+i\sqrt{x})}{2} + \frac{i\ln(x)\ln(1-i\sqrt{x})}{2} - i\,\text{dilog}\left(1+i\sqrt{x}\right) + i\,\text{dilog}\left(1-i\right.$
parts	$\ln(x)\,\text{arccot}\left(\sqrt{x}\right) - \frac{i\ln(x)\ln(1+i\sqrt{x})}{2} + \frac{i\ln(x)\ln(1-i\sqrt{x})}{2} - i\,\text{dilog}\left(1+i\sqrt{x}\right) + i\,\text{dilog}\left(1-i\right.$

```
[In] int(arccot(x^(1/2))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*arccot(x^(1/2))-1/2*I*ln(x)*ln(1+I*x^(1/2))+1/2*I*ln(x)*ln(1-I*x^(1/2
))-I*dilog(1+I*x^(1/2))+I*dilog(1-I*x^(1/2))
```

Fricas [F]

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(arccot(x^(1/2))/x,x, algorithm="fricas")
[Out] integral(arccot(sqrt(x))/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{acot}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(acot(x**(1/2))/x,x)
[Out] Integral(acot(sqrt(x))/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \frac{1}{2}\pi\log\left(x+1\right) + \operatorname{arccot}\left(\sqrt{x}\right)\log\left(x\right) + i\operatorname{Li}_2\left(i\sqrt{x}+1\right) - i\operatorname{Li}_2\left(-i\sqrt{x}+1\right)$$

```
[In] integrate(arccot(x^(1/2))/x,x, algorithm="maxima")
[Out] 1/2*pi*log(x + 1) + arccot(sqrt(x))*log(x) + I*dilog(I*sqrt(x) + 1) - I*dil
og(-I*sqrt(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = -x\arctan\left(\frac{1}{\sqrt{x}}\right) - \sqrt{x} - \arctan\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] integrate(arccot(x^(1/2))/x,x, algorithm="giac")
[Out] -x*arctan(1/sqrt(x)) - sqrt(x) - arctan(1/sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{acot}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] int(acot(x^(1/2))/x,x)
[Out] int(acot(x^(1/2))/x, x)
```

3.89 $\int \frac{\cot^{-1}(\sqrt{x})}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 23

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^2} dx = \frac{1}{\sqrt{x}} - \frac{\cot^{-1}(\sqrt{x})}{x} + \arctan(\sqrt{x})$$

```
[Out] -arccot(x^(1/2))/x+arctan(x^(1/2))+1/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4947, 53, 65, 209}

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^2} dx = \arctan(\sqrt{x}) + \frac{1}{\sqrt{x}} - \frac{\cot^{-1}(\sqrt{x})}{x}$$

```
[In] Int[ArcCot[Sqrt[x]]/x^2,x]
```

```
[Out] 1/Sqrt[x] - ArcCot[Sqrt[x]]/x + ArcTan[Sqrt[x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\int \frac{1}{x^{3/2}(1+x)}\,dx \\
&= \frac{1}{\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{x} + \frac{1}{2}\int \frac{1}{\sqrt{x}(1+x)}\,dx \\
&= \frac{1}{\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{x} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\
&= \frac{1}{\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{x} + \arctan\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\cot^{-1}\left(\sqrt{x}\right)}{x} + \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -x\right)}{\sqrt{x}}$$

```
[In] Integrate[ArcCot[Sqrt[x]]/x^2,x]
[Out] -(ArcCot[Sqrt[x]]/x) + Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{\sqrt{x}}$	18
default	$-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{\sqrt{x}}$	18
parts	$-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{\sqrt{x}}$	18

```
[In] int(arccot(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] -arccot(x^(1/2))/x+arctan(x^(1/2))+1/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx=-\frac{(x+1)\operatorname{arccot}\left(\sqrt{x}\right)-\sqrt{x}}{x}$$

```
[In] integrate(arccot(x^(1/2))/x^2,x, algorithm="fricas")
[Out] -((x + 1)*arccot(sqrt(x)) - sqrt(x))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(20) = 40.

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx=-\frac{x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}+\frac{x^2}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}+\frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

```
[In] integrate(acot(x**(1/2))/x**2,x)
[Out] -x**(5/2)*acot(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*acot(sqrt(x))/(x
**(5/2) + x**(3/2)) - sqrt(x)*acot(sqrt(x))/(x**(5/2) + x**(3/2)) + x**2/(x
**(5/2) + x**(3/2)) + x/(x**(5/2) + x**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x} + \frac{1}{\sqrt{x}} + \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arccot(x^(1/2))/x^2,x, algorithm="maxima")
[Out] -arccot(sqrt(x))/x + 1/sqrt(x) + arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{x} + \frac{1}{\sqrt{x}} - \arctan\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] integrate(arccot(x^(1/2))/x^2,x, algorithm="giac")
[Out] -arctan(1/sqrt(x))/x + 1/sqrt(x) - arctan(1/sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \operatorname{atan}\left(\sqrt{x}\right) - \frac{\operatorname{acot}\left(\sqrt{x}\right)}{x} + \frac{1}{\sqrt{x}}$$

```
[In] int(acot(x^(1/2))/x^2,x)
[Out] atan(x^(1/2)) - acot(x^(1/2))/x + 1/x^(1/2)
```

3.90 $\int \frac{\cot^{-1}(\sqrt{x})}{x^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3} \, dx = \frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{\arctan\left(\sqrt{x}\right)}{2}$$

```
[Out] 1/6/x^(3/2)-1/2*arccot(x^(1/2))/x^2-1/2*arctan(x^(1/2))-1/2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4947, 53, 65, 209}

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3} \, dx = -\frac{\arctan\left(\sqrt{x}\right)}{2} + \frac{1}{6x^{3/2}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{2\sqrt{x}}$$

```
[In] Int[ArcCot[Sqrt[x]]/x^3,x]
```

```
[Out] 1/(6*x^(3/2)) - 1/(2*Sqrt[x]) - ArcCot[Sqrt[x]]/(2*x^2) - ArcTan[Sqrt[x]]/2
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{4}\int \frac{1}{x^{5/2}(1+x)}\,dx \\
&= \frac{1}{6x^{3/2}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{4}\int \frac{1}{x^{3/2}(1+x)}\,dx \\
&= \frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{4}\int \frac{1}{\sqrt{x}(1+x)}\,dx \\
&= \frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\
&= \frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{\arctan\left(\sqrt{x}\right)}{2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x\right)}{6x^{3/2}}$$

[In] Integrate[ArcCot[Sqrt[x]]/x^3,x]

[Out] -1/2*ArcCot[Sqrt[x]]/x^2 + Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2))

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{1}{6x^{\frac{3}{2}}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^2}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2\sqrt{x}}$	27
default	$\frac{1}{6x^{\frac{3}{2}}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^2}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2\sqrt{x}}$	27
parts	$\frac{1}{6x^{\frac{3}{2}}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^2}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2\sqrt{x}}$	27

[In] int(arccot(x^(1/2))/x^3,x,method=_RETURNVERBOSE)

[Out] 1/6/x^(3/2)-1/2*arccot(x^(1/2))/x^2-1/2*arctan(x^(1/2))-1/2/x^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\left(x^2-1\right)\operatorname{arccot}\left(\sqrt{x}\right)-(3\,x-1)\sqrt{x}}{6\,x^2}$$

[In] integrate(arccot(x^(1/2))/x^3,x, algorithm="fricas")

[Out] 1/6*(3*(x^2 - 1)*arccot(sqrt(x)) - (3*x - 1)*sqrt(x))/x^2

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(36) = 72$.

Time = 1.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3x^{\frac{7}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} + \frac{3x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3x^3}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{2x^2}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} + \frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

```
[In] integrate(acot(x**(1/2))/x**3,x)
```

```
[Out] 3*x**(7/2)*acot(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*acot(sqrt(x
))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*acot(sqrt(x))/(6*x**(7/2) + 6*x**
(5/2)) - 3*sqrt(x)*acot(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**3/(6*x**(
7/2) + 6*x**(5/2)) - 2*x**2/(6*x**(7/2) + 6*x**(5/2)) + x/(6*x**(7/2) + 6*x
**(5/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2\,x^2} - \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arccot(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*x - 1)/x^(3/2) - 1/2*arccot(sqrt(x))/x^2 - 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{1}{2\,\sqrt{x}} - \frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{2\,x^2} + \frac{1}{6\,x^{\frac{3}{2}}} + \frac{1}{2}\,\arctan\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] integrate(arccot(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/2/sqrt(x) - 1/2*arctan(1/sqrt(x))/x^2 + 1/6/x^(3/2) + 1/2*arctan(1/sqrt(
x))
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2} - \frac{x-\frac{1}{3}}{2\,x^{3/2}} - \frac{\operatorname{acot}\left(\sqrt{x}\right)}{2\,x^2}$$

```
[In] int(acot(x^(1/2))/x^3,x)
[Out] - atan(x^(1/2))/2 - (x - 1/3)/(2*x^(3/2)) - acot(x^(1/2))/(2*x^2)
```

3.91 $\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right) dx = -\frac{x}{5} + \frac{x^2}{10} + \frac{2}{5}x^{5/2} \cot^{-1}\left(\sqrt{x}\right) + \frac{1}{5}\log(1+x)$$

```
[Out] -1/5*x+1/10*x^2+2/5*x^(5/2)*arccot(x^(1/2))+1/5*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4947, 45}

$$\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right) dx = \frac{2}{5}x^{5/2} \cot^{-1}\left(\sqrt{x}\right) + \frac{x^2}{10} - \frac{x}{5} + \frac{1}{5}\log(x+1)$$

```
[In] Int[x^(3/2)*ArcCot[Sqrt[x]],x]
```

```
[Out] -1/5*x + x^2/10 + (2*x^(5/2)*ArcCot[Sqrt[x]])/5 + Log[1 + x]/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\int\frac{x^2}{1+x}\,dx\\ &= \frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\int\left(-1+x+\frac{1}{1+x}\right)\,dx\\ &= -\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{10}\left((-2+x)x+4x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+2\log(1+x)\right)$$

```
[In] Integrate[x^(3/2)*ArcCot[Sqrt[x]],x]
[Out] ((-2 + x)*x + 4*x^(5/2)*ArcCot[Sqrt[x]] + 2*Log[1 + x])/10
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{x}{5}+\frac{x^2}{10}+\frac{2x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{5}+\frac{\ln(1+x)}{5}$	25
default	$-\frac{x}{5}+\frac{x^2}{10}+\frac{2x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{5}+\frac{\ln(1+x)}{5}$	25

```
[In] int(x^(3/2)*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/5*x+1/10*x^2+2/5*x^(5/2)*arccot(x^(1/2))+1/5*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right)\, dx = \frac{2}{5}\, x^{\frac{5}{2}} \operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{10}\, x^2 - \frac{1}{5}\, x + \frac{1}{5}\, \log\left(x + 1\right)$$

```
[In] integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="fricas")
[Out] 2/5*x^(5/2)*arccot(sqrt(x)) + 1/10*x^2 - 1/5*x + 1/5*log(x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(29) = 58.

Time = 0.85 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right)\, dx = \frac{4x^{\frac{7}{2}} \operatorname{acot}\left(\sqrt{x}\right)}{10x + 10} + \frac{4x^{\frac{5}{2}} \operatorname{acot}\left(\sqrt{x}\right)}{10x + 10} + \frac{x^3}{10x + 10} - \frac{x^2}{10x + 10} + \frac{2x \log\left(x + 1\right)}{10x + 10} + \frac{2 \log\left(x + 1\right)}{10x + 10} + \frac{2}{10x + 10}$$

```
[In] integrate(x**(3/2)*acot(x**(1/2)),x)
[Out] 4*x**(7/2)*acot(sqrt(x))/(10*x + 10) + 4*x**(5/2)*acot(sqrt(x))/(10*x + 10)
 + x**3/(10*x + 10) - x**2/(10*x + 10) + 2*x*log(x + 1)/(10*x + 10) + 2*log
(x + 1)/(10*x + 10) + 2/(10*x + 10)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right)\, dx = \frac{2}{5}\, x^{\frac{5}{2}} \operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{10}\, x^2 - \frac{1}{5}\, x + \frac{1}{5}\, \log\left(x + 1\right)$$

```
[In] integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="maxima")
[Out] 2/5*x^(5/2)*arccot(sqrt(x)) + 1/10*x^2 - 1/5*x + 1/5*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{10}\,x^2\left(\frac{2}{x}-\frac{3}{x^2}-1\right)+\frac{1}{5}\,\log\left(x\right)+\frac{1}{5}\,\log\left(\frac{1}{x}+1\right)$$

```
[In] integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="giac")
[Out] 2/5*x^(5/2)*arctan(1/sqrt(x)) - 1/10*x^2*(2/x - 3/x^2 - 1) + 1/5*log(x) + 1
/5*log(1/x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{\ln\left(x+1\right)}{5}-\frac{x}{5}+\frac{2\,x^{5/2}\,\mathrm{acot}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}$$

```
[In] int(x^(3/2)*acot(x^(1/2)),x)
[Out] log(x + 1)/5 - x/5 + (2*x^(5/2)*acot(x^(1/2)))/5 + x^2/10
```

3.92 $\int \sqrt{x} \cot^{-1}\left(\sqrt{x}\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \sqrt{x} \cot^{-1}\left(\sqrt{x}\right) \, dx = \frac{x}{3} + \frac{2}{3} x^{3/2} \cot^{-1}\left(\sqrt{x}\right) - \frac{1}{3} \log(1+x)$$

```
[Out] 1/3*x+2/3*x^(3/2)*arccot(x^(1/2))-1/3*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4947, 45}

$$\int \sqrt{x} \cot^{-1}\left(\sqrt{x}\right) \, dx = \frac{2}{3} x^{3/2} \cot^{-1}\left(\sqrt{x}\right) + \frac{x}{3} - \frac{1}{3} \log(x+1)$$

```
[In] Int[Sqrt[x]*ArcCot[Sqrt[x]],x]
```

```
[Out] x/3 + (2*x^(3/2)*ArcCot[Sqrt[x]])/3 - Log[1 + x]/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\int\frac{x}{1+x}\,dx\\
&= \frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\int\left(1+\frac{1}{-1-x}\right)dx\\
&= \frac{x}{3}+\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\log(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{3}\left(x+2x^{3/2}\cot^{-1}\left(\sqrt{x}\right)-\log(1+x)\right)$$

```
[In] Integrate[Sqrt[x]*ArcCot[Sqrt[x]],x]
[Out] (x + 2*x^(3/2)*ArcCot[Sqrt[x]] - Log[1 + x])/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{x}{3}+\frac{2x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\ln(1+x)}{3}$	20
default	$\frac{x}{3}+\frac{2x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\ln(1+x)}{3}$	20

```
[In] int(x^(1/2)*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/3*x+2/3*x^(3/2)*arccot(x^(1/2))-1/3*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\,\operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{3}\,x - \frac{1}{3}\,\log\left(x+1\right)$$

```
[In] integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="fricas")
[Out] 2/3*x^(3/2)*arccot(sqrt(x)) + 1/3*x - 1/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{3} + \frac{x}{3} - \frac{\log\left(x+1\right)}{3}$$

```
[In] integrate(x**(1/2)*acot(x**(1/2)),x)
[Out] 2*x**(3/2)*acot(sqrt(x))/3 + x/3 - log(x + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\,\operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{3}\,x - \frac{1}{3}\,\log\left(x+1\right)$$

```
[In] integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="maxima")
[Out] 2/3*x^(3/2)*arccot(sqrt(x)) + 1/3*x - 1/3*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\,\arctan\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{3}\,x\left(\frac{1}{x}-1\right) - \frac{1}{3}\,\log\left(x\right) - \frac{1}{3}\,\log\left(\frac{1}{x}+1\right)$$

```
[In] integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="giac")
[Out] 2/3*x^(3/2)*arctan(1/sqrt(x)) - 1/3*x*(1/x - 1) - 1/3*log(x) - 1/3*log(1/x
+ 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x} \cot^{-1}\left(\sqrt{x}\right) \, dx = \int \sqrt{x} \operatorname{acot}\left(\sqrt{x}\right) \, dx$$

```
[In] int(x^(1/2)*acot(x^(1/2)),x)
[Out] int(x^(1/2)*acot(x^(1/2)), x)
```

3.93 $\int \frac{\cot^{-1}(\sqrt{x})}{\sqrt{x}} dx$

Optimal result

Integrand size = 12, antiderivative size = 18

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} dx = 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right) + \log(1+x)$$

```
[Out] ln(1+x)+2*x^(1/2)*arccot(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4947, 31}

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} dx = \log(x+1) + 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)$$

```
[In] Int[ArcCot[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcCot[Sqrt[x]] + Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)+\int \frac{1}{1+x}\,dx \\ &= 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)+\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)+\log(1+x)$$

```
[In] Integrate[ArcCot[Sqrt[x]]/Sqrt[x],x]
[Out] 2*Sqrt[x]*ArcCot[Sqrt[x]] + Log[1 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\ln\left(1+x\right)+2\sqrt{x}\ \operatorname{arccot}\left(\sqrt{x}\right)$	15
default	$\ln\left(1+x\right)+2\sqrt{x}\ \operatorname{arccot}\left(\sqrt{x}\right)$	15

```
[In] int(arccot(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(1+x)+2*x^(1/2)*arccot(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\operatorname{arccot}\left(\sqrt{x}\right)+\log\left(x+1\right)$$

```
[In] integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x)*arccot(sqrt(x)) + log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\operatorname{acot}\left(\sqrt{x}\right) + \log\left(x+1\right)$$

```
[In] integrate(acot(x**(1/2))/x**(1/2),x)
[Out] 2*sqrt(x)*acot(sqrt(x)) + log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right) + \log\left(x+1\right)$$

```
[In] integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*arccot(sqrt(x)) + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\frac{1}{\sqrt{x}}\right) + \log\left(x\right) + \log\left(\frac{1}{x}+1\right)$$

```
[In] integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x)*arctan(1/sqrt(x)) + log(x) + log(1/x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \ln\left(x+1\right) + 2\sqrt{x}\,\operatorname{acot}\left(\sqrt{x}\right)$$

```
[In] int(acot(x^(1/2))/x^(1/2),x)
[Out] log(x + 1) + 2*x^(1/2)*acot(x^(1/2))
```

3.94 $\int \frac{\cot^{-1}(\sqrt{x})}{x^{3/2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}} \, dx = -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x) + \log(1+x)$$

```
[Out] -ln(x)+ln(1+x)-2*arccot(x^(1/2))/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4947, 36, 29, 31}

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}} \, dx = -\log(x) + \log(x+1) - \frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

```
[In] Int[ArcCot[Sqrt[x]]/x^(3/2),x]
[Out] (-2*ArcCot[Sqrt[x]])/Sqrt[x] - Log[x] + Log[1 + x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \int \frac{1}{x(1+x)}\,dx\\ &= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \int \frac{1}{x}\,dx + \int \frac{1}{1+x}\,dx\\ &= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x) + \log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x) + \log(1+x)$$

```
[In] Integrate[ArcCot[Sqrt[x]]/x^(3/2),x]
[Out] (-2*ArcCot[Sqrt[x]])/Sqrt[x] - Log[x] + Log[1 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\ln\left(x\right)+\ln\left(1+x\right)-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{\sqrt{x}}$	19
default	$-\ln\left(x\right)+\ln\left(1+x\right)-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{\sqrt{x}}$	19

```
[In] int(arccot(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+ln(1+x)-2*arccot(x^(1/2))/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \frac{x\log\left(x+1\right) - x\log\left(x\right) - 2\sqrt{x}\operatorname{arccot}\left(\sqrt{x}\right)}{x}$$

```
[In] integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="fricas")
[Out] (x*log(x + 1) - x*log(x) - 2*sqrt(x)*arccot(sqrt(x)))/x
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\log\left(x\right) + \log\left(x+1\right) - \frac{2\operatorname{acot}\left(\sqrt{x}\right)}{\sqrt{x}}$$

```
[In] integrate(acot(x**(1/2))/x**(3/2),x)
[Out] -log(x) + log(x + 1) - 2*acot(sqrt(x))/sqrt(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\ \operatorname{arccot}\left(\sqrt{x}\right)}{\sqrt{x}} + \log\left(x+1\right) - \log\left(x\right)$$

```
[In] integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="maxima")
[Out] -2*arccot(sqrt(x))/sqrt(x) + log(x + 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\,\arctan\left(\frac{1}{\sqrt{x}}\right)}{\sqrt{x}} + \log\left(\frac{1}{x} + 1\right)$$

```
[In] integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="giac")
[Out] -2*arctan(1/sqrt(x))/sqrt(x) + log(1/x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \ln\left(x+1\right) - 2\,\ln\left(\sqrt{x}\right) - \frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{\sqrt{x}}$$

```
[In] int(acot(x^(1/2))/x^(3/2),x)
[Out] log(x + 1) - 2*log(x^(1/2)) - (2*acot(x^(1/2)))/x^(1/2)
```

3.95 $\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{1}{3x} - \frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}} + \frac{\log(x)}{3} - \frac{1}{3}\log(1+x)$$

```
[Out] 1/3/x-2/3*arccot(x^(1/2))/x^(3/2)+1/3*ln(x)-1/3*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4947, 46}

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}} + \frac{1}{3x} + \frac{\log(x)}{3} - \frac{1}{3}\log(x+1)$$

```
[In] Int[ArcCot[Sqrt[x]]/x^(5/2),x]
```

```
[Out] 1/(3*x) - (2*ArcCot[Sqrt[x]])/(3*x^(3/2)) + Log[x]/3 - Log[1 + x]/3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{1}{3}\int\frac{1}{x^2(1+x)}\,dx\\ &= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{1}{3}\int\left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx\\ &= \frac{1}{3x}-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{\log(x)}{3}-\frac{1}{3}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int\frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx=\frac{1}{3}\left(\frac{1}{x}-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\log(x)-\log(1+x)\right)$$

```
[In] Integrate[ArcCot[Sqrt[x]]/x^(5/2),x]
[Out] (x^(-1) - (2*ArcCot[Sqrt[x]])/x^(3/2) + Log[x] - Log[1 + x])/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{1}{3x}-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln(x)}{3}-\frac{\ln(1+x)}{3}$	26
default	$\frac{1}{3x}-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln(x)}{3}-\frac{\ln(1+x)}{3}$	26

```
[In] int(arccot(x^(1/2))/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3/x-2/3*arccot(x^(1/2))/x^(3/2)+1/3*ln(x)-1/3*ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{x^2 \log\left(x+1\right) - x^2 \log\left(x\right) + 2\sqrt{x}\,\mathrm{arccot}\left(\sqrt{x}\right) - x}{3\,x^2}$$

```
[In] integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="fricas")
[Out] -1/3*(x^2*log(x + 1) - x^2*log(x) + 2*sqrt(x)*arccot(sqrt(x)) - x)/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(31) = 62.

Time = 1.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.86

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2x^{\frac{3}{2}}\,\mathrm{acot}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{2\sqrt{x}\,\mathrm{acot}\left(\sqrt{x}\right)}{3x^3+3x^2} + \frac{x^3\log\left(x\right)}{3x^3+3x^2} - \frac{x^3\log\left(x+1\right)}{3x^3+3x^2} + \frac{x^2\log\left(x\right)}{3x^3+3x^2} - \frac{x^2\log\left(x+1\right)}{3x^3+3x^2} + \frac{x^2}{3x^3+3x^2} + \frac{x}{3x^3+3x^2}$$

```
[In] integrate(acot(x**(1/2))/x**(5/2),x)
[Out] -2*x**(3/2)*acot(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*acot(sqrt(x))/(3*x*
*3 + 3*x**2) + x**3*log(x)/(3*x**3 + 3*x**2) - x**3*log(x + 1)/(3*x**3 + 3*
x**2) + x**2*log(x)/(3*x**3 + 3*x**2) - x**2*log(x + 1)/(3*x**3 + 3*x**2) +
 x**2/(3*x**3 + 3*x**2) + x/(3*x**3 + 3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}} + \frac{1}{3\,x} - \frac{1}{3}\log\left(x+1\right) + \frac{1}{3}\log\left(x\right)$$

```
[In] integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="maxima")
[Out] -2/3*arccot(sqrt(x))/x^(3/2) + 1/3/x - 1/3*log(x + 1) + 1/3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2\arctan\left(\frac{1}{\sqrt{x}}\right)}{3\,x^{\frac{3}{2}}} + \frac{1}{3\,x} - \frac{1}{3}\log\left(\frac{1}{x}+1\right)$$

```
[In] integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="giac")
[Out] -2/3*arctan(1/sqrt(x))/x^(3/2) + 1/3/x - 1/3*log(1/x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{2\,\ln\left(\sqrt{x}\right)}{3} - \frac{\ln\left(x+1\right)}{3} - \frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{3\,x^{3/2}} + \frac{1}{3\,x}$$

```
[In] int(acot(x^(1/2))/x^(5/2),x)
[Out] (2*log(x^(1/2)))/3 - log(x + 1)/3 - (2*acot(x^(1/2)))/(3*x^(3/2)) + 1/(3*x)
```

3.96 $\int \cot^{-1}\left(\frac{1}{x}\right) \, dx$

Optimal result

Integrand size = 4, antiderivative size = 17

$$\int \cot^{-1}\left(\frac{1}{x}\right) \, dx = x\cot^{-1}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] x*arccot(1/x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4931, 269, 266}

$$\int \cot^{-1}\left(\frac{1}{x}\right) \, dx = x\cot^{-1}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] Int[ArcCot[x^(-1)],x]
```

```
[Out] x*ArcCot[x^(-1)] - Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= x\cot^{-1}\left(\frac{1}{x}\right) - \int \frac{1}{\left(1+\frac{1}{x^2}\right)x}\,dx \\ &= x\cot^{-1}\left(\frac{1}{x}\right) - \int \frac{x}{1+x^2}\,dx \\ &= x\cot^{-1}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(1+x^2\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\cot^{-1}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[ArcCot[x^(-1)],x]
[Out] x*ArcCot[x^(-1)] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result
parallelrisch	$x\ \operatorname{arccot}\left(\frac{1}{x}\right)-\frac{\ln\left(x^{2}+1\right)}{2}$
parts	$x\ \operatorname{arccot}\left(\frac{1}{x}\right)-\frac{\ln\left(x^{2}+1\right)}{2}$
derivativedivides	$x\ \operatorname{arccot}\left(\frac{1}{x}\right)+\ln\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{1}{x^{2}}+1\right)}{2}$
default	$x\ \operatorname{arccot}\left(\frac{1}{x}\right)+\ln\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{1}{x^{2}}+1\right)}{2}$
risch	$\frac{i x\ln(i+x)}{2}-\frac{i\ln(x-i)x}{2}-\frac{\pi\ \mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}\left(i\left(-\mathrm{RootOf}\left(\_Z^{2}+1,\mathrm{index}=1\right)+x\right)\right)\mathrm{csgn}\left(\frac{i\left(-\mathrm{RootOf}\left(\_Z^{2}+1,\mathrm{index}\right.\right.}{x}\right.}{4}$

```
[In] int(arccot(1/x),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccot(1/x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{arccot}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(arccot(1/x),x, algorithm="fricas")
[Out] x*arccot(1/x) - 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{acot}\left(\frac{1}{x}\right) - \frac{\log\left(x^2+1\right)}{2}$$

```
[In] integrate(acot(1/x),x)
[Out] x*acot(1/x) - log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{arccot}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(arccot(1/x),x, algorithm="maxima")
[Out] x*arccot(1/x) - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(arccot(1/x),x, algorithm="giac")
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{acot}\left(\frac{1}{x}\right) - \frac{\ln\left(x^2+1\right)}{2}$$

```
[In] int(acot(1/x),x)
[Out] x*acot(1/x) - log(x^2 + 1)/2
```

3.97 $\int \frac{\cot^{-1}(ax^n)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{\cot^{-1}(ax^n)}{x} dx = -\frac{i \operatorname{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right)}{2n} + \frac{i \operatorname{PolyLog}\left(2, \frac{ix^{-n}}{a}\right)}{2n}$$

```
[Out] -1/2*I*polylog(2,-I/a/(x^n))/n+1/2*I*polylog(2,I/a/(x^n))/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4945, 4941, 2438}

$$\int \frac{\cot^{-1}(ax^n)}{x} dx = \frac{i \operatorname{PolyLog}\left(2, \frac{ix^{-n}}{a}\right)}{2n} - \frac{i \operatorname{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right)}{2n}$$

```
[In] Int[ArcCot[a*x^n]/x,x]
```

```
[Out] ((-1/2*I)*PolyLog[2, (-I)/(a*x^n)])/n + ((I/2)*PolyLog[2, I/(a*x^n)])/n
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
```

```
- I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(ax)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{ax}\right)}{x}\,dx, x, x^n\right)}{2n} - \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{ax}\right)}{x}\,dx, x, x^n\right)}{2n}\\
&= -\frac{i\,\text{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right)}{2n} + \frac{i\,\text{PolyLog}\left(2, \frac{ix^{-n}}{a}\right)}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = -\frac{i\left(\text{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right) - \text{PolyLog}\left(2, \frac{ix^{-n}}{a}\right)\right)}{2n}$$

```
[In] Integrate[ArcCot[a*x^n]/x,x]
[Out] ((-1/2*I)*(PolyLog[2, (-I)/(a*x^n)] - PolyLog[2, I/(a*x^n)]))/n
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(39) = 78.

Time = 0.54 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{\ln(a\,x^n)\,\text{arccot}(a\,x^n)-\frac{i\ln(a\,x^n)\ln(1+ix^n a)}{2}+\frac{i\ln(a\,x^n)\ln(1-ix^n a)}{2}-\frac{i\,\text{dilog}(1+ix^n a)}{2}+\frac{i\,\text{dilog}(1-ix^n a)}{2}}{n}$
default	$\frac{\ln(a\,x^n)\,\text{arccot}(a\,x^n)-\frac{i\ln(a\,x^n)\ln(1+ix^n a)}{2}+\frac{i\ln(a\,x^n)\ln(1-ix^n a)}{2}-\frac{i\,\text{dilog}(1+ix^n a)}{2}+\frac{i\,\text{dilog}(1-ix^n a)}{2}}{n}$
risch	$\frac{i\ln(x)\ln(1+ix^n a)}{2}+\frac{\pi\ln(x)}{2}+\frac{i\,\text{dilog}(1-ix^n a)}{2n}-\frac{i\ln(-i(-a\,x^n+i))\ln(x)}{2}+\frac{i\ln(-i(-a\,x^n+i))\ln(-ix^n a)}{2n}+$

```
[In] int(arccot(a*x^n)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(ln(a*x^n)*arccot(a*x^n)-1/2*I*ln(a*x^n)*ln(1+I*x^n*a)+1/2*I*ln(a*x^n)*
ln(1-I*x^n*a)-1/2*I*dilog(1+I*x^n*a)+1/2*I*dilog(1-I*x^n*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \frac{2\,n\,\text{arccot}\,(ax^n)\log(x) - i\,n\log(i\,ax^n+1)\log(x) + i\,n\log(-i\,ax^n+1)\log(x) + i\,\text{Li}_2(i\,ax^n) - i\,\text{Li}_2(-i\,ax}{2\,n}$$

```
[In] integrate(arccot(a*x^n)/x,x, algorithm="fricas")
[Out] 1/2*(2*n*arccot(a*x^n)*log(x) - I*n*log(I*a*x^n + 1)*log(x) + I*n*log(-I*a*
x^n + 1)*log(x) + I*dilog(I*a*x^n) - I*dilog(-I*a*x^n))/n
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \int \frac{\text{acot}\,(ax^n)}{x}\,dx$$

```
[In] integrate(acot(a*x**n)/x,x)
[Out] Integral(acot(a*x**n)/x, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \int \frac{\text{arccot}\,(ax^n)}{x}\,dx$$

```
[In] integrate(arccot(a*x^n)/x,x, algorithm="maxima")
[Out] a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(1/(a*x^n))*log(x)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax^n)}{x} \, dx = \int \frac{\operatorname{arccot}(ax^n)}{x} \, dx$$

```
[In] integrate(arccot(a*x^n)/x,x, algorithm="giac")
[Out] integrate(arccot(a*x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax^n)}{x} \, dx = \int \frac{\operatorname{acot}(a\,x^n)}{x} \, dx$$

```
[In] int(acot(a*x^n)/x,x)
[Out] int(acot(a*x^n)/x, x)
```

3.98 $\int \frac{\cot^{-1}(ax^5)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{\cot^{-1}(ax^5)}{x} dx = -\frac{1}{10} i \operatorname{PolyLog}\left(2, -\frac{i}{ax^5}\right) + \frac{1}{10} i \operatorname{PolyLog}\left(2, \frac{i}{ax^5}\right)$$

```
[Out] -1/10*I*polylog(2,-I/a/x^5)+1/10*I*polylog(2,I/a/x^5)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4945, 4941, 2438}

$$\int \frac{\cot^{-1}(ax^5)}{x} dx = \frac{1}{10} i \operatorname{PolyLog}\left(2, \frac{i}{ax^5}\right) - \frac{1}{10} i \operatorname{PolyLog}\left(2, -\frac{i}{ax^5}\right)$$

```
[In] Int[ArcCot[a*x^5]/x,x]
```

```
[Out] (-1/10*I)*PolyLog[2, (-I)/(a*x^5)] + (I/10)*PolyLog[2, I/(a*x^5)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{\cot^{-1}(ax)}{x}\,dx, x, x^5\right)\\ &= \frac{1}{10}i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{ax}\right)}{x}\,dx, x, x^5\right)-\frac{1}{10}i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{ax}\right)}{x}\,dx, x, x^5\right)\\ &= -\frac{1}{10}i\,\text{PolyLog}\left(2,-\frac{i}{ax^5}\right)+\frac{1}{10}i\,\text{PolyLog}\left(2,\frac{i}{ax^5}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(ax^5\right)}{x}\,dx = -\frac{1}{10}i\,\text{PolyLog}\left(2,-\frac{i}{ax^5}\right)+\frac{1}{10}i\,\text{PolyLog}\left(2,\frac{i}{ax^5}\right)$$

```
[In] Integrate[ArcCot[a*x^5]/x,x]
[Out] (-1/10*I)*PolyLog[2, (-I)/(a*x^5)] + (I/10)*PolyLog[2, I/(a*x^5)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

method	result
default	$\ln\left(x\right)\operatorname{arccot}\left(a\,x^{5}\right)+\frac{\sum_{\_R1=\operatorname{RootOf}\left(a^{2}\_Z^{10}+1\right)}\frac{\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^{5}}}{2a}$
parts	$\ln\left(x\right)\operatorname{arccot}\left(a\,x^{5}\right)+\frac{\sum_{\_R1=\operatorname{RootOf}\left(a^{2}\_Z^{10}+1\right)}\frac{\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^{5}}}{2a}$
risch	$\frac{\pi\ln\left(x\right)}{2}+\frac{i\left(\sum_{\_R1=\operatorname{RootOf}\left(\_Z^{5}a+\operatorname{RootOf}\left(\_Z^{2}+1,\mathrm{index}=1\right)\right)}\left(\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{i\ln\left(x\right)\ln\left(-ia\,x^{5}+1\right)}{2}$

```
[In] int(arccot(a*x^5)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arccot(a*x^5)+1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1)),_R1=RootOf(_Z^10*a^2+1))
```

Fricas [F]

$$\int\frac{\cot^{-1}\left(ax^{5}\right)}{x}\,dx=\int\frac{\operatorname{arccot}\left(ax^{5}\right)}{x}\,dx$$

```
[In] integrate(arccot(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x^5)/x, x)
```

Sympy [F]

$$\int\frac{\cot^{-1}\left(ax^{5}\right)}{x}\,dx=\int\frac{\operatorname{acot}\left(ax^{5}\right)}{x}\,dx$$

```
[In] integrate(acot(a*x**5)/x,x)
```

```
[Out] Integral(acot(a*x**5)/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(23) = 46$.

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \frac{1}{20}\,\pi\log\left(a^2x^{10}+1\right) - \frac{1}{5}\,\arctan\left(ax^5\right)\log\left(ax^5\right) + \operatorname{arccot}\left(ax^5\right)\log\left(x\right) + \arctan\left(ax^5\right)\log\left(x\right) + \frac{1}{10}i\,\mathrm{Li}_2\left(i\,ax^5+1\right) - \frac{1}{10}i\,\mathrm{Li}_2\left(-i\,ax^5+1\right)$$

```
[In] integrate(arccot(a*x^5)/x,x, algorithm="maxima")
[Out] 1/20*pi*log(a^2*x^10 + 1) - 1/5*arctan(a*x^5)*log(a*x^5) + arccot(a*x^5)*lo
g(x) + arctan(a*x^5)*log(x) + 1/10*I*dilog(I*a*x^5 + 1) - 1/10*I*dilog(-I*a
*x^5 + 1)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arccot}(ax^5)}{x}\,dx$$

```
[In] integrate(arccot(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arccot(a*x^5)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \int \frac{\mathrm{acot}(a\,x^5)}{x}\,dx$$

```
[In] int(acot(a*x^5)/x,x)
[Out] int(acot(a*x^5)/x, x)
```

3.99 $\int x^3 \cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 106

$$\int x^3 \cot^{-1}(a+bx)\,dx = -\frac{(1-6a^2)\,x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4\cot^{-1}(a+bx) + \frac{(1-6a^2+a^4)\arctan(a+bx)}{4b^4} + \frac{a(1-a^2)\log\left(1+(a+bx)^2\right)}{2b^4}$$

```
[Out] -1/4*(-6*a^2+1)*x/b^3-1/2*a*(b*x+a)^2/b^4+1/12*(b*x+a)^3/b^4+1/4*x^4*arccot
(b*x+a)+1/4*(a^4-6*a^2+1)*arctan(b*x+a)/b^4+1/2*a*(-a^2+1)*ln(1+(b*x+a)^2)/
b^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\int x^3 \cot^{-1}(a+bx)\,dx = \frac{a(1-a^2)\log\left((a+bx)^2+1\right)}{2b^4} - \frac{(1-6a^2)\,x}{4b^3} + \frac{(a^4-6a^2+1)\arctan(a+bx)}{4b^4} + \frac{(a+bx)^3}{12b^4} - \frac{a(a+bx)^2}{2b^4} + \frac{1}{4}x^4\cot^{-1}(a+bx)$$

```
[In] Int[x^3*ArcCot[a + b*x],x]
```

```
[Out] -1/4*((1 - 6*a^2)*x)/b^3 - (a*(a + b*x)^2)/(2*b^4) + (a + b*x)^3/(12*b^4) +
 (x^4*ArcCot[a + b*x])/4 + ((1 - 6*a^2 + a^4)*ArcTan[a + b*x])/(4*b^4) + (a
*(1 - a^2)*Log[1 + (a + b*x)^2])/(2*b^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \cot^{-1}(x)\,dx, x, a+bx\right)}{b}\\ &= \frac{1}{4}x^4\cot^{-1}(a+bx)+\frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{1+x^2}\,dx, x, a+bx\right)\end{aligned}$$

$$= \frac{1}{4}x^4 \cot^{-1}(a+bx) + \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1-6a^2}{b^4} - \frac{4ax}{b^4} + \frac{x^2}{b^4} + \frac{1-6a^2+a^4+4a(1-a^2)x}{b^4(1+x^2)}\right) dx, x, a + bx\right)$$

$$= -\frac{(1-6a^2)x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4 \cot^{-1}(a+bx) + \frac{\text{Subst}\left(\int \frac{1-6a^2+a^4+4a(1-a^2)x}{1+x^2} dx, x, a+bx\right)}{4b^4}$$

$$= -\frac{(1-6a^2)x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4 \cot^{-1}(a+bx) + \frac{(a(1-a^2))\,\text{Subst}\left(\int \frac{x}{1+x^2} dx, x, a+bx\right)}{b^4} + \frac{(1-6a^2+a^4)\,\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, a+bx\right)}{4b^4}$$

$$= -\frac{(1-6a^2)x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4 \cot^{-1}(a+bx) + \frac{(1-6a^2+a^4)\arctan(a+bx)}{4b^4} + \frac{a(1-a^2)\log\left(1+(a+bx)^2\right)}{2b^4}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int x^3 \cot^{-1}(a+bx)\,dx = \frac{6(-1+6a^2)bx - 12a(a+bx)^2 + 2(a+bx)^3 + 6b^4x^4\cot^{-1}(a+bx) - 3i(-i+a)^4\log(i-a-bx) + 3i(i+}{24b^4}$$

```
[In] Integrate[x^3*ArcCot[a + b*x],x]
```

```
[Out] (6*(-1 + 6*a^2)*b*x - 12*a*(a + b*x)^2 + 2*(a + b*x)^3 + 6*b^4*x^4*ArcCot[a
 + b*x] - (3*I)*(-I + a)^4*Log[I - a - b*x] + (3*I)*(I + a)^4*Log[I + a + b
*x])/(24*b^4)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

method	result
parallelrisch	$-\frac{-3\operatorname{arccot}(bx+a)x^4b^4-b^3x^3+3a\,b^2x^2+3\operatorname{arccot}(bx+a)a^4+6a^3\ln(b^2x^2+2abx+a^2+1)-9a^2bx-18\operatorname{arccot}(bx+a)a^2+}{12b^4}$
parts	$\frac{x^4\operatorname{arccot}(bx+a)}{4}+\frac{b\left(\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x-x}{b^4}+\frac{\frac{\left(-4a^3b+4ab\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(-3a^4-2a^2+1-\frac{\left(-4a^3b+4ab\right)a}{b}\right)\text{arc}}{b}}{b^4}\right.}{4}$
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)a^4}{4}-\operatorname{arccot}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccot}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccot}(bx+a)a(bx+a)^3+\frac{\operatorname{arccot}(bx+a)(bx+a)^4}{4}+\frac{3a^2(}{}}{b^4}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)a^4}{4}-\operatorname{arccot}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccot}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccot}(bx+a)a(bx+a)^3+\frac{\operatorname{arccot}(bx+a)(bx+a)^4}{4}+\frac{3a^2(}{}}{b^4}$
risch	$\frac{ix^4\ln(1+i(bx+a))}{8}-\frac{ix^4\ln(1-i(bx+a))}{8}+\frac{\pi x^4}{8}+\frac{x^3}{12b}+\frac{a^4\arctan(bx+a)}{4b^4}-\frac{a\,x^2}{4b^2}-\frac{a^3\ln(b^2x^2+2abx+a^2+1)}{2b^4}$

```
[In] int(x^3*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-3*arccot(b*x+a)*x^4*b^4-b^3*x^3+3*a*b^2*x^2+3*arccot(b*x+a)*a^4+6*a
^3*ln(b^2*x^2+2*a*b*x+a^2+1)-9*a^2*b*x-18*arccot(b*x+a)*a^2+15*a^3-6*a*ln(b
^2*x^2+2*a*b*x+a^2+1)+3*b*x+3*arccot(b*x+a)-9*a)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int x^3\cot^{-1}(a+bx)\,dx$$
$$=\frac{3\,b^4x^4\operatorname{arccot}\left(bx+a\right)+b^3x^3-3\,ab^2x^2+3\,(3\,a^2-1)bx+3\,(a^4-6\,a^2+1)\arctan\left(bx+a\right)-6\,(a^3-a)\,\text{lo}}{12\,b^4}$$

```
[In] integrate(x^3*arccot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4*arccot(b*x + a) + b^3*x^3 - 3*a*b^2*x^2 + 3*(3*a^2 - 1)*b*x
 + 3*(a^4 - 6*a^2 + 1)*arctan(b*x + a) - 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*x
+ a^2 + 1))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int x^3 \cot^{-1}(a+bx)\,dx$$
$$= \begin{cases} -\frac{a^4 \operatorname{acot}(a+bx)}{4b^4} - \frac{a^3 \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{3a^2x}{4b^3} + \frac{3a^2 \operatorname{acot}(a+bx)}{2b^4} - \frac{ax^2}{4b^2} + \frac{a \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{x^4 \operatorname{acot}(a+bx)}{4} + \frac{x}{12} \\ \frac{x^4 \operatorname{acot}(a)}{4} \end{cases}$$

```
[In] integrate(x**3*acot(b*x+a),x)
```

```
[Out] Piecewise((-a**4*acot(a + b*x)/(4*b**4) - a**3*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(2*b**4) + 3*a**2*x/(4*b**3) + 3*a**2*acot(a + b*x)/(2*b**4) - a*x*
*2/(4*b**2) + a*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b**4) + x**4*acot(a
+ b*x)/4 + x**3/(12*b) - x/(4*b**3) - acot(a + b*x)/(4*b**4), Ne(b, 0)), (x
**4*acot(a)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int x^3 \cot^{-1}(a+bx)\,dx = \frac{1}{4}x^4 \operatorname{arccot}(bx+a)$$
$$+ \frac{1}{12}b\left(\frac{b^2x^3 - 3\,abx^2 + 3\,(3\,a^2-1)x}{b^4} + \frac{3\,(a^4-6\,a^2+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5} - \frac{6\,(a^3-a)\log(b^2x^2+2\,abx+}{b^5}\right.$$

```
[In] integrate(x^3*arccot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccot(b*x + a) + 1/12*b*((b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 - 1)*x)/b
^4 + 3*(a^4 - 6*a^2 + 1)*arctan((b^2*x + a*b)/b)/b^5 - 6*(a^3 - a)*log(b^2*
x^2 + 2*a*b*x + a^2 + 1)/b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(92) = 184.

Time = 0.73 (sec) , antiderivative size = 617, normalized size of antiderivative = 5.82

$$\int x^3 \cot^{-1}(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*arccot(b*x+a),x, algorithm="giac")
```

```
[Out] 1/192*(96*a^3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 72*a^2*a
rctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 + 24*a*arctan(1/(b*x + a)
)*tan(1/2*arctan(1/(b*x + a)))^7 + 3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(
b*x + a)))^8 + 96*a^3*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan
(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/
(b*x + a)))^4 - 96*a^3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 +
 144*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 - 144*a^2*tan(1
/2*arctan(1/(b*x + a)))^5 - 72*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x
+ a)))^5 - 24*a*tan(1/2*arctan(1/(b*x + a)))^6 - 12*arctan(1/(b*x + a))*tan
(1/2*arctan(1/(b*x + a)))^6 - 2*tan(1/2*arctan(1/(b*x + a)))^7 - 96*a*log(1
6*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/
2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 + 72*a^2*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 144*a^2*tan(1/2*arctan(1/(
b*x + a)))^3 + 72*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 - 48
*a*tan(1/2*arctan(1/(b*x + a)))^4 - 30*arctan(1/(b*x + a))*tan(1/2*arctan(1
/(b*x + a)))^4 + 30*tan(1/2*arctan(1/(b*x + a)))^5 - 24*a*arctan(1/(b*x + a
))*tan(1/2*arctan(1/(b*x + a))) - 24*a*tan(1/2*arctan(1/(b*x + a)))^2 - 12*
arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 30*tan(1/2*arctan(1/(b
*x + a)))^3 + 3*arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b^4*
tan(1/2*arctan(1/(b*x + a)))^4)
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\int x^3 \cot^{-1}(a+bx)\,dx = \frac{\mathrm{atan}(a+b\,x)}{4\,b^4} + \frac{x^4\,\mathrm{acot}(a+b\,x)}{4} - \frac{x}{4\,b^3} + \frac{x^3}{12\,b} - \frac{a^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4} - \frac{3\,a^2\,\mathrm{atan}(a+b\,x)}{2\,b^4} + \frac{a^4\,\mathrm{atan}(a+b\,x)}{4\,b^4} - \frac{a\,x^2}{4\,b^2} + \frac{3\,a^2\,x}{4\,b^3} + \frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4}$$

```
[In] int(x^3*acot(a + b*x),x)
```

```
[Out] atan(a + b*x)/(4*b^4) + (x^4*acot(a + b*x))/4 - x/(4*b^3) + x^3/(12*b) - (a
^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4) - (3*a^2*atan(a + b*x))/(2*b^4
) + (a^4*atan(a + b*x))/(4*b^4) - (a*x^2)/(4*b^2) + (3*a^2*x)/(4*b^3) + (a*
log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4)
```

3.100 $\int x^2 \cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x^2 \cot^{-1}(a+bx)\,dx = -\frac{ax}{b^2} + \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3 \cot^{-1}(a+bx) + \frac{a(3-a^2)\arctan(a+bx)}{3b^3} - \frac{(1-3a^2)\log\left(1+(a+bx)^2\right)}{6b^3}$$

```
[Out] -a*x/b^2+1/6*(b*x+a)^2/b^3+1/3*x^3*arccot(b*x+a)+1/3*a*(-a^2+3)*arctan(b*x+
a)/b^3-1/6*(-3*a^2+1)*ln(1+(b*x+a)^2)/b^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\int x^2 \cot^{-1}(a+bx)\,dx = \frac{a(3-a^2)\arctan(a+bx)}{3b^3} - \frac{(1-3a^2)\log\left((a+bx)^2+1\right)}{6b^3} + \frac{(a+bx)^2}{6b^3} - \frac{ax}{b^2} + \frac{1}{3}x^3 \cot^{-1}(a+bx)$$

```
[In] Int[x^2*ArcCot[a + b*x],x]
```

```
[Out] -((a*x)/b^2) + (a + b*x)^2/(6*b^3) + (x^3*ArcCot[a + b*x])/3 + (a*(3 - a^2)
*ArcTan[a + b*x])/(3*b^3) - ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(6*b^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \cot^{-1}(x)\,dx, x, a+bx\right)}{b}\\
&= \frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{1+x^2}\,dx, x, a+bx\right)\\
&= \frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{1}{3}\text{Subst}\left(\int \left(-\frac{3a}{b^3}+\frac{x}{b^3}+\frac{a(3-a^2)-(1-3a^2)\,x}{b^3\,(1+x^2)}\right)\,dx, x, a+bx\right)\\
&= -\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{\text{Subst}\left(\int \frac{a(3-a^2)-(1-3a^2)x}{1+x^2}\,dx, x, a+bx\right)}{3b^3}
\end{aligned}
$$

$$= -\frac{ax}{b^2} + \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3\cot^{-1}(a+bx) - \frac{(1-3a^2)\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{3b^3} + \frac{(a(3-a^2))\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, a+bx\right)}{3b^3}$$

$$= -\frac{ax}{b^2} + \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3\cot^{-1}(a+bx) + \frac{a(3-a^2)\arctan(a+bx)}{3b^3} - \frac{(1-3a^2)\log\left(1+(a+bx)^2\right)}{6b^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int x^2\cot^{-1}(a+bx)\,dx = \frac{\frac{1}{3}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^3\cot^{-1}(a+bx) + \frac{1}{3}b\left(-\frac{3ax}{b^2}+\frac{(a+bx)^2}{2b^3}-\frac{(1+ia)^3\log(i-a-bx)}{2b^3}-\frac{(1-ia)^3\log(i+a+bx)}{2b^3}\right)}{b}$$

```
[In] Integrate[x^2*ArcCot[a + b*x],x]
```

```
[Out] ((b*(-(a/b) + (a + b*x)/b)^3*ArcCot[a + b*x])/3 + (b*((-3*a*x)/b^2 + (a + b
*x)^2/(2*b^3) - ((1 + I*a)^3*Log[I - a - b*x])/(2*b^3) - ((1 - I*a)^3*Log[I
 + a + b*x])/(2*b^3)))/3)/b
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

method	result
parallelrisch	$\frac{2\operatorname{arccot}(bx+a)x^3b^3+b^2x^2+2\operatorname{arccot}(bx+a)a^3+3a^2\ln(b^2x^2+2abx+a^2+1)-4abx-6a\operatorname{arccot}(bx+a)+7a^2-1-\ln(b^2x^2+2}{6b^3}$
derivativedivides	$\frac{-\frac{\operatorname{arccot}(bx+a)a^3}{3}+\operatorname{arccot}(bx+a)a^2(bx+a)-\operatorname{arccot}(bx+a)a(bx+a)^2+\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(-3a^2+1)}{}}{b^3}$
default	$\frac{-\frac{\operatorname{arccot}(bx+a)a^3}{3}+\operatorname{arccot}(bx+a)a^2(bx+a)-\operatorname{arccot}(bx+a)a(bx+a)^2+\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(-3a^2+1)}{}}{b^3}$
parts	$\frac{x^3\operatorname{arccot}(bx+a)}{3}+\frac{b\left(-\frac{-\frac{1}{2}x^2b+2ax}{b^3}+\frac{\frac{(3a^2b-b)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{\left(2a^3+2a-\frac{(3a^2b-b)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^3}\right)}{3}$
risch	$\frac{ix^3\ln(1+i(bx+a))}{6}-\frac{ix^3\ln(1-i(bx+a))}{6}+\frac{\pi x^3}{6}-\frac{a^3\arctan(bx+a)}{3b^3}+\frac{x^2}{6b}+\frac{a^2\ln(b^2x^2+2abx+a^2+1)}{2b^3}-\frac{2ax}{3b^2}+$

```
[In] int(x^2*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*arccot(b*x+a)*x^3*b^3+b^2*x^2+2*arccot(b*x+a)*a^3+3*a^2*ln(b^2*x^2+2
*a*b*x+a^2+1)-4*a*b*x-6*a*arccot(b*x+a)+7*a^2-1-ln(b^2*x^2+2*a*b*x+a^2+1))/
b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}(a+bx)\,dx = \frac{2\,b^3x^3 \operatorname{arccot}(bx+a) + b^2x^2 - 4\,abx - 2\,(a^3-3\,a)\arctan(bx+a) + (3\,a^2-1)\log(b^2x^2+2\,abx+a^2+1)}{6\,b^3}$$

```
[In] integrate(x^2*arccot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arccot(b*x + a) + b^2*x^2 - 4*a*b*x - 2*(a^3 - 3*a)*arctan(b
*x + a) + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

$$\int x^2 \cot^{-1}(a+bx)\,dx = \begin{cases} \frac{a^3 \operatorname{acot}(a+bx)}{3b^3} + \frac{a^2\log(a^2+2abx+b^2x^2+1)}{2b^3} - \frac{2ax}{3b^2} - \frac{a \operatorname{acot}(a+bx)}{b^3} + \frac{x^3 \operatorname{acot}(a+bx)}{3} + \frac{x^2}{6b} - \frac{\log(a^2+2abx+b^2x^2+1)}{6b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{acot}(a)}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*acot(b*x+a),x)
```

```
[Out] Piecewise((a**3*acot(a + b*x)/(3*b**3) + a**2*log(a**2 + 2*a*b*x + b**2*x**
2 + 1)/(2*b**3) - 2*a*x/(3*b**2) - a*acot(a + b*x)/b**3 + x**3*acot(a + b*x
)/3 + x**2/(6*b) - log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*b**3), Ne(b, 0)),
 (x**3*acot(a)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int x^2 \cot^{-1}(a+bx)\,dx = \frac{1}{3}\,x^3 \operatorname{arccot}\left(bx+a\right) + \frac{1}{6}\,b\left(\frac{bx^2-4\,ax}{b^3} - \frac{2\left(a^3-3\,a\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4} + \frac{\left(3\,a^2-1\right)\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^4}\right)$$

```
[In] integrate(x^2*arccot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot(b*x + a) + 1/6*b*((b*x^2 - 4*a*x)/b^3 - 2*(a^3 - 3*a)*arctan
((b^2*x + a*b)/b)/b^4 + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(70) = 140.

Time = 0.63 (sec) , antiderivative size = 423, normalized size of antiderivative = 5.29

$$\int x^2 \cot^{-1}(a+bx)\,dx = -\frac{12\,a^2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 6\,a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^5 + \arctan\left(\frac{1}{bx+a}\right)\mathrm{ta}}{}$$

```
[In] integrate(x^2*arccot(b*x+a),x, algorithm="giac")
```

```
[Out] -1/24*(12*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 6*a*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + arctan(1/(b*x + a))*tan(1/
2*arctan(1/(b*x + a)))^6 + 12*a^2*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(ta
n(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1
/2*arctan(1/(b*x + a)))^3 - 12*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*
x + a)))^2 + 12*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 - 12*a
*tan(1/2*arctan(1/(b*x + a)))^4 - 3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b
*x + a)))^4 - tan(1/2*arctan(1/(b*x + a)))^5 - 4*log(16*tan(1/2*arctan(1/(b
*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a))
)^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 6*a*arctan(1/(b*x + a))*tan(1/2*
arctan(1/(b*x + a))) + 12*a*tan(1/2*arctan(1/(b*x + a)))^2 + 3*arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 -
 arctan(1/(b*x + a)) - tan(1/2*arctan(1/(b*x + a))))/(b^3*tan(1/2*arctan(1/
(b*x + a)))^3)
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int x^2 \cot^{-1}(a+bx)\,dx = \frac{x^3\,\mathrm{acot}(a+b\,x)}{3} - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b^3} + \frac{x^2}{6\,b} + \frac{a^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^3} - \frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b^3} + \frac{a\,\mathrm{atan}(a+b\,x)}{b^3} - \frac{2\,a\,x}{3\,b^2}$$

```
[In] int(x^2*acot(a + b*x),x)
```

```
[Out] (x^3*acot(a + b*x))/3 - log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b^3) + x^2/(6*b
) + (a^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^3) - (a^3*atan(a + b*x))/(3
*b^3) + (a*atan(a + b*x))/b^3 - (2*a*x)/(3*b^2)
```

3.101 $\int x \cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x\cot^{-1}(a+bx)\,dx=\frac{x}{2b}+\frac{1}{2}x^2\cot^{-1}(a+bx)-\frac{(1-a^2)\arctan(a+bx)}{2b^2}-\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

```
[Out] 1/2*x/b+1/2*x^2*arccot(b*x+a)-1/2*(-a^2+1)*arctan(b*x+a)/b^2-1/2*a*ln(1+(b*
x+a)^2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\int x\cot^{-1}(a+bx)\,dx=-\frac{(1-a^2)\arctan(a+bx)}{2b^2}-\frac{a\log\left((a+bx)^2+1\right)}{2b^2}+\frac{1}{2}x^2\cot^{-1}(a+bx)+\frac{x}{2b}$$

```
[In] Int[x*ArcCot[a + b*x],x]
```

```
[Out] x/(2*b) + (x^2*ArcCot[a + b*x])/2 - ((1 - a^2)*ArcTan[a + b*x])/(2*b^2) - (
a*Log[1 + (a + b*x)^2])/(2*b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)\cot^{-1}(x)\,dx, x, a+bx\right)}{b}\\
&= \frac{1}{2}x^2\cot^{-1}(a+bx)+\frac{1}{2}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{1+x^2}\,dx, x, a+bx\right)\\
&= \frac{1}{2}x^2\cot^{-1}(a+bx)+\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{1-a^2+2ax}{b^2\left(1+x^2\right)}\right)\,dx, x, a+bx\right)\\
&= \frac{x}{2b}+\frac{1}{2}x^2\cot^{-1}(a+bx)-\frac{\text{Subst}\left(\int \frac{1-a^2+2ax}{1+x^2}\,dx, x, a+bx\right)}{2b^2}
\end{aligned}
$$

$$= \frac{x}{2b} + \frac{1}{2}x^2 \cot^{-1}(a+bx) - \frac{a\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b^2} - \frac{(1-a^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, a+bx\right)}{2b^2}$$

$$= \frac{x}{2b} + \frac{1}{2}x^2 \cot^{-1}(a+bx) - \frac{(1-a^2)\arctan(a+bx)}{2b^2} - \frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int x\cot^{-1}(a+bx)\,dx$$
$$= \frac{2bx + 2b^2x^2\cot^{-1}(a+bx) - i(-i+a)^2\log(i-a-bx) - i\log(i+a+bx) - 2a\log(i+a+bx) + ia^2\log(i}{4b^2}$$

```
[In] Integrate[x*ArcCot[a + b*x],x]
```

```
[Out] (2*b*x + 2*b^2*x^2*ArcCot[a + b*x] - I*(-I + a)^2*Log[I - a - b*x] - I*Log[
I + a + b*x] - 2*a*Log[I + a + b*x] + I*a^2*Log[I + a + b*x])/(4*b^2)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2} - \operatorname{arccot}(bx+a)a(bx+a) + \frac{bx}{2} + \frac{a}{2} - \frac{a\ln\left(1+(bx+a)^2\right)}{2} - \frac{\arctan(bx+a)}{2}}{b^2}$
default	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2} - \operatorname{arccot}(bx+a)a(bx+a) + \frac{bx}{2} + \frac{a}{2} - \frac{a\ln\left(1+(bx+a)^2\right)}{2} - \frac{\arctan(bx+a)}{2}}{b^2}$
parallelrisch	$-\frac{-\operatorname{arccot}(bx+a)b^2x^2 + \operatorname{arccot}(bx+a)a^2 + a\ln(b^2x^2+2abx+a^2+1) - bx - \operatorname{arccot}(bx+a) + 2a}{2b^2}$
parts	$\frac{x^2 \operatorname{arccot}(bx+a)}{2} + \frac{b\left(\frac{x}{b^2} + \frac{-\frac{a\ln(b^2x^2+2abx+a^2+1)}{b} + \frac{(a^2-1)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^2}\right)}{2}$
risch	$\frac{ix^2\ln(1+i(bx+a))}{4} - \frac{ix^2\ln(1-i(bx+a))}{4} + \frac{\pi x^2}{4} + \frac{a^2\arctan(bx+a)}{2b^2} - \frac{a\ln(b^2x^2+2abx+a^2+1)}{2b^2} + \frac{x}{2b} - \frac{\arctan(b}{2b^2}$

```
[In] int(x*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*(b*x+a)^2*arccot(b*x+a)-arccot(b*x+a)*a*(b*x+a)+1/2*b*x+1/2*a-1/
2*a*ln(1+(b*x+a)^2)-1/2*arctan(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x\cot^{-1}(a+bx)\,dx = \frac{b^2x^2\,\mathrm{arccot}\,(bx+a)+bx+(a^2-1)\arctan\,(bx+a)-a\log\,(b^2x^2+2\,abx+a^2+1)}{2\,b^2}$$

```
[In] integrate(x*arccot(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2*arccot(b*x + a) + b*x + (a^2 - 1)*arctan(b*x + a) - a*log(b^2*
x^2 + 2*a*b*x + a^2 + 1))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int x\cot^{-1}(a+bx)\,dx = \begin{cases} -\frac{a^2\,\mathrm{acot}\,(a+bx)}{2b^2}-\frac{a\log\,(a^2+2abx+b^2x^2+1)}{2b^2}+\frac{x^2\,\mathrm{acot}\,(a+bx)}{2}+\frac{x}{2b}+\frac{\mathrm{acot}\,(a+bx)}{2b^2} & \text{for } b\neq 0\\ \frac{x^2\,\mathrm{acot}\,(a)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*acot(b*x+a),x)
[Out] Piecewise((-a**2*acot(a + b*x)/(2*b**2) - a*log(a**2 + 2*a*b*x + b**2*x**2
+ 1)/(2*b**2) + x**2*acot(a + b*x)/2 + x/(2*b) + acot(a + b*x)/(2*b**2), Ne
(b, 0)), (x**2*acot(a)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int x\cot^{-1}(a+bx)\,dx = \frac{1}{2}\,x^2\,\mathrm{arccot}\,(bx+a) + \frac{1}{2}\,b\left(\frac{x}{b^2}+\frac{(a^2-1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3}-\frac{a\log\,(b^2x^2+2\,abx+a^2+1)}{b^3}\right)$$

```
[In] integrate(x*arccot(b*x+a),x, algorithm="maxima")
[Out] 1/2*x^2*arccot(b*x + a) + 1/2*b*(x/b^2 + (a^2 - 1)*arctan((b^2*x + a*b)/b)/
b^3 - a*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(50) = 100$.

Time = 0.34 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.50

$$\int x \cot^{-1}(a + bx)\, dx$$
$$= \frac{4\, a \arctan\left(\frac{1}{bx+a}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{bx+a}\right)\right)^3 + \arctan\left(\frac{1}{bx+a}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{bx+a}\right)\right)^4 + 4\, a \log\left(\frac{16 \tan}{\tan\left(\frac{1}{2} \arctan\left(\frac{1}{bx+a}\right.\right.}\right.}{}$$

```
[In] integrate(x*arccot(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(4*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 4*a*log(16*tan(1/2*arctan(1/(b*x +
a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 +
 1))*tan(1/2*arctan(1/(b*x + a)))^2 - 4*a*arctan(1/(b*x + a))*tan(1/2*arcta
n(1/(b*x + a))) + 2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*
tan(1/2*arctan(1/(b*x + a)))^3 + arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(
b*x + a))))/(b^2*tan(1/2*arctan(1/(b*x + a)))^2)
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int x \cot^{-1}(a+bx)\, dx = \frac{x^2 \operatorname{acot}(a + b\, x)}{2} + \frac{\frac{\operatorname{acot}(a+b\, x)}{2} + \frac{b\, x}{2} - \frac{a^2 \operatorname{acot}(a+b\, x)}{2} - \frac{a\, \ln\left(a^2+2\, a\, b\, x+b^2\, x^2+1\right)}{2}}{b^2}$$

```
[In] int(x*acot(a + b*x),x)
```

```
[Out] (x^2*acot(a + b*x))/2 + (acot(a + b*x)/2 + (b*x)/2 - (a^2*acot(a + b*x))/2
- (a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2)/b^2
```

3.102 $\int \cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 33

$$\int \cot^{-1}(a+bx)\,dx = \frac{(a+bx)\cot^{-1}(a+bx)}{b} + \frac{\log\left(1+(a+bx)^2\right)}{2b}$$

```
[Out] (b*x+a)*arccot(b*x+a)/b+1/2*ln(1+(b*x+a)^2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5148, 4931, 266}

$$\int \cot^{-1}(a+bx)\,dx = \frac{\log\left((a+bx)^2+1\right)}{2b} + \frac{(a+bx)\cot^{-1}(a+bx)}{b}$$

```
[In] Int[ArcCot[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcCot[a + b*x])/b + Log[1 + (a + b*x)^2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \cot^{-1}(x)\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\cot^{-1}(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\cot^{-1}(a+bx)}{b} + \frac{\log\left(1+(a+bx)^2\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \cot^{-1}(a+bx)\,dx = x\cot^{-1}(a+bx) + \frac{-2a\arctan(a+bx) + \log\left(1+a^2+2abx+b^2x^2\right)}{2b}$$

```
[In] Integrate[ArcCot[a + b*x],x]
[Out] x*ArcCot[a + b*x] + (-2*a*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^2
])/(2*b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{(bx+a)\operatorname{arccot}(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
default	$\frac{(bx+a)\operatorname{arccot}(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
parallelrisch	$\frac{2x\ \operatorname{arccot}(bx+a)b^2+2\ \operatorname{arccot}(bx+a)ab+\ln(b^2x^2+2abx+a^2+1)b}{2b^2}$	49
parts	$x\ \operatorname{arccot}(bx+a)+b\left(\frac{\ln(b^2x^2+2abx+a^2+1)}{2b^2}-\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b^2}\right)$	59
risch	$\frac{ix\ln(1+i(bx+a))}{2}-\frac{ix\ln(1-i(bx+a))}{2}+\frac{\pi x}{2}-\frac{a\arctan(bx+a)}{b}+\frac{\ln(b^2x^2+2abx+a^2+1)}{2b}$	71

```
[In] int(arccot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((b*x+a)*arccot(b*x+a)+1/2*ln(1+(b*x+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \cot^{-1}(a+bx)\,dx = \frac{2\,bx\operatorname{arccot}(bx+a)-2\,a\arctan(bx+a)+\log(b^2x^2+2\,abx+a^2+1)}{2\,b}$$

```
[In] integrate(arccot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arccot(b*x + a) - 2*a*arctan(b*x + a) + log(b^2*x^2 + 2*a*b*x +
a^2 + 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \cot^{-1}(a+bx)\,dx = \begin{cases} \frac{a\operatorname{acot}(a+bx)}{b}+x\operatorname{acot}(a+bx)+\frac{\log(a^2+2abx+b^2x^2+1)}{2b} & \text{for } b\neq 0 \\ x\operatorname{acot}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(acot(b*x+a),x)
```

```
[Out] Piecewise(((a*acot(a + b*x)/b + x*acot(a + b*x) + log(a**2 + 2*a*b*x + b**2*
x**2 + 1)/(2*b), Ne(b, 0)), (x*acot(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \cot^{-1}(a+bx)\,dx = \frac{2\,(bx+a)\,\mathrm{arccot}\,(bx+a) + \log\left((bx+a)^2+1\right)}{2\,b}$$

```
[In] integrate(arccot(b*x+a),x, algorithm="maxima")
[Out] 1/2*(2*(b*x + a)*arccot(b*x + a) + log((b*x + a)^2 + 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \cot^{-1}(a+bx)\,dx =$$

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)^2 + \log\left(\frac{16\,\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)^2}{\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 2\,\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,\arctan\left(\right.\right.}{2\,b\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)}$$

```
[In] integrate(arccot(b*x+a),x, algorithm="giac")
[Out] -1/2*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + log(16*tan(1/2*a
rctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/
(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) - arctan(1/(b*x + a)))/(b*
tan(1/2*arctan(1/(b*x + a))))
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \cot^{-1}(a+bx)\,dx = \frac{\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2} + a\,\mathrm{acot}(a+b\,x)}{b} + x\,\mathrm{acot}(a+b\,x)$$

```
[In] int(acot(a + b*x),x)
[Out] (log(a^2 + b^2*x^2 + 2*a*b*x + 1)/2 + a*acot(a + b*x))/b + x*acot(a + b*x)
```

3.103 $\int \frac{\cot^{-1}(a+bx)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 120

$$\int \frac{\cot^{-1}(a+bx)}{x} dx = -\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right) + \cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

```
[Out] -arccot(b*x+a)*ln(2/(1-I*(b*x+a)))+arccot(b*x+a)*ln(2*b*x/(I-a)/(1-I*(b*x+a
)))-1/2*I*polylog(2,1-2/(1-I*(b*x+a)))+1/2*I*polylog(2,1-2*b*x/(I-a)/(1-I*(
b*x+a)))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5156, 4967, 2449, 2352, 2497}

$$\int \frac{\cot^{-1}(a+bx)}{x} dx = -\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right) + \log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right) + \log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

```
[In] Int[ArcCot[a + b*x]/x,x]
```

```
[Out] -(ArcCot[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcCot[a + b*x]*Log[(2*b*x)/(
(I - a)*(1 - I*(a + b*x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] + (
I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$
$$-\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}dx,x,a+bx\right)+\text{Subst}\left(\int\frac{\log\left(\frac{2\left(-\frac{a}{b}+\frac{x}{b}\right)}{\left(\frac{i}{b}-\frac{a}{b}\right)(1-ix)}\right)}{1+x^2}dx,x,a+bx\right)$$

$$= -\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$
$$+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)-i\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx,x,\frac{1}{1-i(a+bx)}\right)$$

$$= -\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$
$$-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 256 vs. 2(120) = 240.

Time = 0.15 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.13

$$\int\frac{\cot^{-1}(a+bx)}{x}dx = \left(\cot^{-1}(a+bx)+\arctan(a+bx)\right)\log(x)$$
$$+\arctan(a+bx)\left(\log\left(\frac{1}{\sqrt{1+(a+bx)^2}}\right)-\log(-\sin(\arctan(a)-\arctan(a+bx)))\right)$$
$$+\frac{1}{2}\left(\frac{1}{4}i(\pi-2\arctan(a+bx))^2+i(\arctan(a)-\arctan(a+bx))^2\right.$$
$$-(\pi-2\arctan(a+bx))\log\left(1+e^{-2i\arctan(a+bx)}\right)$$
$$+2(\arctan(a)-\arctan(a+bx))\log\left(1-e^{2i(-\arctan(a)+\arctan(a+bx))}\right)$$
$$+(\pi-2\arctan(a+bx))\log\left(\frac{2}{\sqrt{1+(a+bx)^2}}\right)+2(-\arctan(a)$$
$$+\arctan(a+bx))\log(-2\sin(\arctan(a)-\arctan(a+bx)))$$
$$+i\,\text{PolyLog}\left(2,-e^{-2i\arctan(a+bx)}\right)$$
$$\left.+i\,\text{PolyLog}\left(2,e^{2i(-\arctan(a)+\arctan(a+bx))}\right)\right)$$

```
[In] Integrate[ArcCot[a + b*x]/x,x]
```

```
[Out] (ArcCot[a + b*x] + ArcTan[a + b*x])*Log[x] + ArcTan[a + b*x]*(Log[1/Sqrt[1
+ (a + b*x)^2]] - Log[-Sin[ArcTan[a] - ArcTan[a + b*x]]]) + ((I/4)*(Pi - 2*
ArcTan[a + b*x])^2 + I*(ArcTan[a] - ArcTan[a + b*x])^2 - (Pi - 2*ArcTan[a +
 b*x])*Log[1 + E^((-2*I)*ArcTan[a + b*x])] + 2*(ArcTan[a] - ArcTan[a + b*x]
)*Log[1 - E^((2*I)*(-ArcTan[a] + ArcTan[a + b*x]))] + (Pi - 2*ArcTan[a + b*
x])*Log[2/Sqrt[1 + (a + b*x)^2]] + 2*(-ArcTan[a] + ArcTan[a + b*x])*Log[-2*
Sin[ArcTan[a] - ArcTan[a + b*x]]] + I*PolyLog[2, -E^((-2*I)*ArcTan[a + b*x]
)] + I*PolyLog[2, E^((2*I)*(-ArcTan[a] + ArcTan[a + b*x]))])/2
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\pi \ln(-ibx)}{2} - \frac{i \ln(-ibx-ia+1) \ln\left(-\frac{ixb}{ia-1}\right)}{2} - \frac{i \operatorname{dilog}\left(-\frac{ixb}{ia-1}\right)}{2} + \frac{i \ln(ibx+ia+1) \ln\left(\frac{ixb}{-ia-1}\right)}{2} + \frac{i \operatorname{dilog}\left(\frac{ixb}{-ia-1}\right)}{2}$
parts	$\ln(x) \operatorname{arccot}(bx+a) + b\left(-\frac{i \ln(x)\left(\ln\left(\frac{-bx-a+i}{i-a}\right)-\ln\left(\frac{bx+a+i}{i+a}\right)\right)}{2b} - \frac{i\left(\operatorname{dilog}\left(\frac{-bx-a+i}{i-a}\right)-\operatorname{dilog}\left(\frac{bx+a+i}{i+a}\right)\right)}{2b}\right.$
derivativedivides	$\ln(-bx) \operatorname{arccot}(bx+a) + \frac{i \ln(-bx) \ln\left(\frac{bx+a+i}{i+a}\right)}{2} - \frac{i \ln(-bx) \ln\left(\frac{-bx-a+i}{i-a}\right)}{2} + \frac{i \operatorname{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2} -$
default	$\ln(-bx) \operatorname{arccot}(bx+a) + \frac{i \ln(-bx) \ln\left(\frac{bx+a+i}{i+a}\right)}{2} - \frac{i \ln(-bx) \ln\left(\frac{-bx-a+i}{i-a}\right)}{2} + \frac{i \operatorname{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2} -$

```
[In] int(arccot(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*Pi*ln(-I*b*x)-1/2*I*ln(1-I*a-I*b*x)*ln(-I*x*b/(I*a-1))-1/2*I*dilog(-I*x
*b/(I*a-1))+1/2*I*ln(1+I*a+I*b*x)*ln(I*x*b/(-I*a-1))+1/2*I*dilog(I*x*b/(-I*
a-1))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{x} \, dx = \int \frac{\operatorname{arccot}(bx+a)}{x} \, dx$$

```
[In] integrate(arccot(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(b*x + a)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{x} \, dx = \text{Timed out}$$

```
[In] integrate(acot(b*x+a)/x,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{x} \, dx = &\frac{1}{2} \arctan\left(\frac{bx}{a^2+1}, -\frac{abx}{a^2+1}\right) \log\left(b^2x^2+2\,abx+a^2+1\right) \\ &- \frac{1}{2} \arctan(bx+a) \log\left(\frac{b^2x^2}{a^2+1}\right) \\ &+ \operatorname{arccot}(bx+a) \log(x) + \arctan\left(\frac{b^2x+ab}{b}\right) \log(x) \\ &+ \frac{1}{2} i \operatorname{Li}_2\left(\frac{i\,bx+i\,a+1}{i\,a+1}\right) - \frac{1}{2} i \operatorname{Li}_2\left(\frac{i\,bx+i\,a-1}{i\,a-1}\right)\end{aligned}$$

```
[In] integrate(arccot(b*x+a)/x,x, algorithm="maxima")
[Out] 1/2*arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*log(b^2*x^2 + 2*a*b*x + a^2 +
1) - 1/2*arctan(b*x + a)*log(b^2*x^2/(a^2 + 1)) + arccot(b*x + a)*log(x) +
arctan((b^2*x + a*b)/b)*log(x) + 1/2*I*dilog((I*b*x + I*a + 1)/(I*a + 1)) -
 1/2*I*dilog((I*b*x + I*a - 1)/(I*a - 1))
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{x} \, dx = \int \frac{\operatorname{arccot}(bx+a)}{x} \, dx$$

```
[In] integrate(arccot(b*x+a)/x,x, algorithm="giac")
[Out] integrate(arccot(b*x + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx=\int \frac{\mathrm{acot}(a+b\,x)}{x}\,dx$$

```
[In] int(acot(a + b*x)/x,x)
```

```
[Out] int(acot(a + b*x)/x, x)
```

3.104 $\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = -\frac{\cot^{-1}(a+bx)}{x} + \frac{ab\arctan(a+bx)}{1+a^2} - \frac{b\log(x)}{1+a^2} + \frac{b\log\left(1+(a+bx)^2\right)}{2(1+a^2)}$$

```
[Out] -arccot(b*x+a)/x+a*b*arctan(b*x+a)/(a^2+1)-b*ln(x)/(a^2+1)+1/2*b*ln(1+(b*x+
a)^2)/(a^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5154, 378, 720, 31, 649, 209, 266}

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = \frac{ab\arctan(a+bx)}{a^2+1} - \frac{b\log(x)}{a^2+1} + \frac{b\log\left((a+bx)^2+1\right)}{2(a^2+1)} - \frac{\cot^{-1}(a+bx)}{x}$$

```
[In] Int[ArcCot[a + b*x]/x^2,x]
```

```
[Out] -(ArcCot[a + b*x]/x) + (a*b*ArcTan[a + b*x])/(1 + a^2) - (b*Log[x])/(1 + a^
2) + (b*Log[1 + (a + b*x)^2])/(2*(1 + a^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cot^{-1}(a+bx)}{x} - b\int \frac{1}{x\left(1+(a+bx)^2\right)}\,dx\\ &= -\frac{\cot^{-1}(a+bx)}{x} - b\text{Subst}\left(\int \frac{1}{(-a+x)\left(1+x^2\right)}\,dx, x, a+bx\right)\\ &= -\frac{\cot^{-1}(a+bx)}{x} - \frac{b\text{Subst}\left(\int \frac{1}{-a+x}\,dx, x, a+bx\right)}{1+a^2} - \frac{b\text{Subst}\left(\int \frac{-a-x}{1+x^2}\,dx, x, a+bx\right)}{1+a^2}\end{aligned}$$

$$= -\frac{\cot^{-1}(a+bx)}{x} - \frac{b\log(x)}{1+a^2} + \frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{1+a^2} + \frac{(ab)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, a+bx\right)}{1+a^2}$$
$$= -\frac{\cot^{-1}(a+bx)}{x} + \frac{ab\arctan(a+bx)}{1+a^2} - \frac{b\log(x)}{1+a^2} + \frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = -\frac{\cot^{-1}(a+bx)}{x} + \frac{b(-2\log(x)+(1-ia)\log(i-a-bx)+(1+ia)\log(i+a+bx))}{2\left(1+a^2\right)}$$

```
[In] Integrate[ArcCot[a + b*x]/x^2,x]
[Out] -(ArcCot[a + b*x]/x) + (b*(-2*Log[x] + (1 - I*a)*Log[I - a - b*x] + (1 + I*
a)*Log[I + a + b*x]))/(2*(1 + a^2))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
derivativedivides	$b\left(-\frac{\operatorname{arccot}(bx+a)}{bx}-\frac{\ln(-bx)}{a^2+1}+\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}\right)$
default	$b\left(-\frac{\operatorname{arccot}(bx+a)}{bx}-\frac{\ln(-bx)}{a^2+1}+\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}\right)$
parts	$-\frac{\operatorname{arccot}(bx+a)}{x}-b\left(\frac{\ln(x)}{a^2+1}-\frac{b\left(\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{2b}+\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{a^2+1}\right)$
parallelrisch	$-\frac{2x\ \operatorname{arccot}(bx+a)a^2b^2+2b^2\ln(x)ax-b^2\ln\left(b^2x^2+2abx+a^2+1\right)ax+2\ \operatorname{arccot}(bx+a)a^3b+2\ \operatorname{arccot}(bx+a)ab}{2xab(a^2+1)}$
risch	$-\frac{i\ln(1+i(bx+a))}{2x}-\frac{-ia^2\ln(1-i(bx+a))-i\ln(1-i(bx+a))+\pi a^2+\pi+2b\ln(x)x-xb\ln\left((iab+3b)x+ia^2+3i+2a\right)-ixb}{2x(i+a}$

```
[In] int(arccot(b*x+a)/x^2,x,method=_RETURNVERBOSE)
[Out] b*(-1/b/x*arccot(b*x+a)-1/(a^2+1)*ln(-b*x)+1/(a^2+1)*(1/2*ln(1+(b*x+a)^2)+a
*arctan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = \frac{2\,abx\arctan(bx+a)+bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,bx\log(x)-2\,(a^2+1)\,\mathrm{arccot}\,(bx+a)}{2\,(a^2+1)x}$$

```
[In] integrate(arccot(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x*arctan(b*x + a) + b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*b*x
*log(x) - 2*(a^2 + 1)*arccot(b*x + a))/((a^2 + 1)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.69

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = \begin{cases} -\frac{ib\,\mathrm{acot}\,(bx-i)}{2}-\frac{\mathrm{acot}\,(bx-i)}{x}+\frac{i}{2x} & \text{for}\ a=-i \\ \frac{ib\,\mathrm{acot}\,(bx+i)}{2}-\frac{\mathrm{acot}\,(bx+i)}{x}-\frac{i}{2x} & \text{for}\ a=i \\ -\frac{2a^2\,\mathrm{acot}\,(a+bx)}{2a^2x+2x}-\frac{2abx\,\mathrm{acot}\,(a+bx)}{2a^2x+2x}-\frac{2bx\log(x)}{2a^2x+2x}+\frac{bx\log\left(a^2+2abx+b^2x^2+1\right)}{2a^2x+2x}-\frac{2\,\mathrm{acot}\,(a+bx)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

```
[In] integrate(acot(b*x+a)/x**2,x)
```

```
[Out] Piecewise((-I*b*acot(b*x - I)/2 - acot(b*x - I)/x + I/(2*x), Eq(a, -I)), (I
*b*acot(b*x + I)/2 - acot(b*x + I)/x - I/(2*x), Eq(a, I)), (-2*a**2*acot(a
+ b*x)/(2*a**2*x + 2*x) - 2*a*b*x*acot(a + b*x)/(2*a**2*x + 2*x) - 2*b*x*lo
g(x)/(2*a**2*x + 2*x) + b*x*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a**2*x +
 2*x) - 2*acot(a + b*x)/(2*a**2*x + 2*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = \frac{1}{2}\,b\left(\frac{2\,a\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1}+\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1}-\frac{2\,\log\left(x\right)}{a^2+1}\right)-\frac{\operatorname{arccot}\left(bx+a\right)}{x}$$

```
[In] integrate(arccot(b*x+a)/x^2,x, algorithm="maxima")
[Out] 1/2*b*(2*a*arctan((b^2*x + a*b)/b)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2
+ 1)/(a^2 + 1) - 2*log(x)/(a^2 + 1)) - arccot(b*x + a)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(60) = 120$.

Time = 0.42 (sec) , antiderivative size = 498, normalized size of antiderivative = 8.03

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = -\frac{\left(2\,a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+2\,a\log\left(\frac{4\left(4\,a^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+4\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3+\tan\cdots\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\cdots\right)\right)}\right)\right)}{\cdots}$$

```
[In] integrate(arccot(b*x+a)/x^2,x, algorithm="giac")
[Out] -1/2*(2*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 2*a*log(4*(4
*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 +
tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1
/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*
arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) + log(4*(4*a^2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2
*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arct
an(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(
1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*a*arctan(1/(b*x +
a)) - 4*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - log(4*(4*a^2*tan
(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*
arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arcta
n(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1
/(b*x + a)))^2 + 1)))*b/(2*a^3*tan(1/2*arctan(1/(b*x + a))) + a^2*tan(1/2*a
rctan(1/(b*x + a)))^2 - a^2 + 2*a*tan(1/2*arctan(1/(b*x + a))) + tan(1/2*ar
ctan(1/(b*x + a)))^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx$$
$$= -\frac{\mathrm{acot}(a+b\,x)}{x} - \frac{b\,x\,\ln(x) - \frac{b\,x\,\ln(a^2+2\,a\,b\,x+b^2\,x^2+1)}{2} + a\,b\,x\,\mathrm{acot}(a+b\,x)}{x\,(a^2+1)}$$

```
[In] int(acot(a + b*x)/x^2,x)
```

```
[Out] - acot(a + b*x)/x - (b*x*log(x) - (b*x*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2
+ a*b*x*acot(a + b*x))/(x*(a^2 + 1))
```

3.105 $\int \frac{\cot^{-1}(a+bx)}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 95

$$\int \frac{\cot^{-1}(a+bx)}{x^3} dx = \frac{b}{2(1+a^2)x} - \frac{\cot^{-1}(a+bx)}{2x^2} + \frac{(1-a^2)b^2\arctan(a+bx)}{2(1+a^2)^2} + \frac{ab^2\log(x)}{(1+a^2)^2} - \frac{ab^2\log(1+(a+bx)^2)}{2(1+a^2)^2}$$

```
[Out] 1/2*b/(a^2+1)/x-1/2*arccot(b*x+a)/x^2+1/2*(-a^2+1)*b^2*arctan(b*x+a)/(a^2+1
)^2+a*b^2*ln(x)/(a^2+1)^2-1/2*a*b^2*ln(1+(b*x+a)^2)/(a^2+1)^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5154, 378, 724, 815, 649, 209, 266}

$$\int \frac{\cot^{-1}(a+bx)}{x^3} dx = \frac{(1-a^2)b^2\arctan(a+bx)}{2(a^2+1)^2} + \frac{ab^2\log(x)}{(a^2+1)^2} - \frac{ab^2\log((a+bx)^2+1)}{2(a^2+1)^2} + \frac{b}{2(a^2+1)x} - \frac{\cot^{-1}(a+bx)}{2x^2}$$

```
[In] Int[ArcCot[a + b*x]/x^3,x]
```

```
[Out] b/(2*(1 + a^2)*x) - ArcCot[a + b*x]/(2*x^2) + ((1 - a^2)*b^2*ArcTan[a + b*x
])/(2*(1 + a^2)^2) + (a*b^2*Log[x])/(1 + a^2)^2 - (a*b^2*Log[1 + (a + b*x)^
2])/(2*(1 + a^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\cot^{-1}(a+bx)}{2x^2} - \frac{1}{2}b\int \frac{1}{x^2\left(1+(a+bx)^2\right)}\,dx$$

$$= -\frac{\cot^{-1}(a+bx)}{2x^2} - \frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(1+x^2\right)}\,dx, x, a+bx\right)$$

$$= \frac{b}{2\left(1+a^2\right)x} - \frac{\cot^{-1}(a+bx)}{2x^2} - \frac{b^2\text{Subst}\left(\int \frac{-a-x}{(-a+x)(1+x^2)}\,dx, x, a+bx\right)}{2\left(1+a^2\right)}$$

$$= \frac{b}{2\left(1+a^2\right)x} - \frac{\cot^{-1}(a+bx)}{2x^2} - \frac{b^2\text{Subst}\left(\int \left(\frac{2a}{(1+a^2)(a-x)} + \frac{-1+a^2+2ax}{(1+a^2)(1+x^2)}\right)\,dx, x, a+bx\right)}{2\left(1+a^2\right)}$$

$$= \frac{b}{2\left(1+a^2\right)x} - \frac{\cot^{-1}(a+bx)}{2x^2} + \frac{ab^2\log(x)}{\left(1+a^2\right)^2} - \frac{b^2\text{Subst}\left(\int \frac{-1+a^2+2ax}{1+x^2}\,dx, x, a+bx\right)}{2\left(1+a^2\right)^2}$$

$$= \frac{b}{2\left(1+a^2\right)x} - \frac{\cot^{-1}(a+bx)}{2x^2} + \frac{ab^2\log(x)}{\left(1+a^2\right)^2} - \frac{\left(ab^2\right)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{\left(1+a^2\right)^2} + \frac{\left(\left(1-a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, a+bx\right)}{2\left(1+a^2\right)^2}$$

$$= \frac{b}{2\left(1+a^2\right)x} - \frac{\cot^{-1}(a+bx)}{2x^2} + \frac{\left(1-a^2\right)b^2\arctan(a+bx)}{2\left(1+a^2\right)^2} + \frac{ab^2\log(x)}{\left(1+a^2\right)^2} - \frac{ab^2\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = \frac{-2\cot^{-1}(a+bx) + \frac{bx\left(4abx\log(x)+i(i+a)^2bx\log(i-a-bx)+(-i+a)(2(i+a)+(-1-ia)bx\log(i+a+bx))\right)}{\left(1+a^2\right)^2}}{4x^2}$$

```
[In] Integrate[ArcCot[a + b*x]/x^3,x]
```

```
[Out] (-2*ArcCot[a + b*x] + (b*x*(4*a*b*x*Log[x] + I*(I + a)^2*b*x*Log[I - a - b*
x] + (-I + a)*(2*(I + a) + (-1 - I*a)*b*x*Log[I + a + b*x])))/(1 + a^2)^2)/
(4*x^2)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arccot}(bx+a)}{2b^2x^2}-\frac{a\ln\left(1+(bx+a)^2\right)+(a^2-1)\arctan(bx+a)}{2(a^2+1)^2}+\frac{1}{2(a^2+1)bx}+\frac{a\ln(-bx)}{(a^2+1)^2}\right)$
default	$b^2\left(-\frac{\operatorname{arccot}(bx+a)}{2b^2x^2}-\frac{a\ln\left(1+(bx+a)^2\right)+(a^2-1)\arctan(bx+a)}{2(a^2+1)^2}+\frac{1}{2(a^2+1)bx}+\frac{a\ln(-bx)}{(a^2+1)^2}\right)$
parts	$-\frac{\operatorname{arccot}(bx+a)}{2x^2}-\frac{b\left(-\frac{1}{(a^2+1)x}-\frac{2ab\ln(x)}{(a^2+1)^2}+\frac{b^2\left(\frac{a\ln(b^2x^2+2abx+a^2+1)}{b}+\frac{(a^2-1)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{(a^2+1)^2}\right)}{2}$
parallelrisch	$\frac{x^2\operatorname{arccot}(bx+a)a^2b^2+2b^2a\ln(x)x^2-b^2a\ln(b^2x^2+2abx+a^2+1)x^2-\operatorname{arccot}(bx+a)b^2x^2-2a\,b^2x^2-\operatorname{arccot}(bx+a)a^4+a^2bx-}{2x^2(a^4+2a^2+1)}$
risch	$-\frac{i\ln(1+i(bx+a))}{4x^2}+\frac{ia^4\ln(1-i(bx+a))+2ia^2\ln(1-i(bx+a))+i\ln(1-i(bx+a))+4b^2a\ln(-x)x^2-\pi a^4+2a^2bx-2\pi a^2+2}{}$

[In] int(arccot(b*x+a)/x^3,x,method=_RETURNVERBOSE)

[Out] b^2*(-1/2/b^2/x^2*arccot(b*x+a)-1/2/(a^2+1)^2*(a*ln(1+(b*x+a)^2)+(a^2-1)*ar
ctan(b*x+a))+1/2/(a^2+1)/b/x+1/(a^2+1)^2*a*ln(-b*x))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx =$$
$$-\frac{(a^2-1)b^2x^2\arctan(bx+a)+ab^2x^2\log(b^2x^2+2\,abx+a^2+1)-2\,ab^2x^2\log(x)-(a^2+1)bx+(a^4+2}{2\,(a^4+2\,a^2+1)x^2}$$

[In] integrate(arccot(b*x+a)/x^3,x, algorithm="fricas")

[Out] -1/2*((a^2 - 1)*b^2*x^2*arctan(b*x + a) + a*b^2*x^2*log(b^2*x^2 + 2*a*b*x +
 a^2 + 1) - 2*a*b^2*x^2*log(x) - (a^2 + 1)*b*x + (a^4 + 2*a^2 + 1)*arccot(b
*x + a))/((a^4 + 2*a^2 + 1)*x^2)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.01

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = \begin{cases} -\frac{b^2 \operatorname{acot}(bx-i)}{8} + \frac{b}{8x} - \frac{\operatorname{acot}(bx-i)}{2x^2} + \frac{i}{8x^2} \\ -\frac{b^2 \operatorname{acot}(bx+i)}{8} + \frac{b}{8x} - \frac{\operatorname{acot}(bx+i)}{2x^2} - \frac{i}{8x^2} \\ -\frac{a^4 \operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2b^2x^2 \operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2} - \frac{2a^2 \operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{2ab^2x^2 \log(x)}{2a^4x^2+4a^2x^2+2x^2} - \frac{ab^2x^2 \log(a^2+2ab}{2a^4x^2+4a^2x}\end{cases}$$

```
[In] integrate(acot(b*x+a)/x**3,x)
```

```
[Out] Piecewise((-b**2*acot(b*x - I)/8 + b/(8*x) - acot(b*x - I)/(2*x**2) + I/(8*
x**2), Eq(a, -I)), (-b**2*acot(b*x + I)/8 + b/(8*x) - acot(b*x + I)/(2*x**2
) - I/(8*x**2), Eq(a, I)), (-a**4*acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2
+ 2*x**2) + a**2*b**2*x**2*acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**
2) + a**2*b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a**2*acot(a + b*x)/(
2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*log(x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) - a*b**2*x**2*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a
**4*x**2 + 4*a**2*x**2 + 2*x**2) - b**2*x**2*acot(a + b*x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) + b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - acot(a +
b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = -\frac{1}{2}\left(\frac{(a^2-1)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2a^2+1} + \frac{ab\log(b^2x^2+2abx+a^2+1)}{a^4+2a^2+1} - \frac{2ab\log(x)}{a^4+2a^2+1} - \frac{1}{(a^2+1)x}\right)b - \frac{\operatorname{arccot}(bx+a)}{2x^2}$$

```
[In] integrate(arccot(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a^2 - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + a*b*log(b^2*x
^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(x)/(a^4 + 2*a^2 + 1)
- 1/((a^2 + 1)*x))*b - 1/2*arccot(b*x + a)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1309 vs. $2(85) = 170$.

Time = 0.58 (sec) , antiderivative size = 1309, normalized size of antiderivative = 13.78

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate(arccot(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^3*b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + a^2*b*arc
tan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 4*a^3*b*log(4*(4*a^2*tan(
1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*a
rctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan
(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/
(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a^2*b*log(4*(4*a^2*t
an(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/
2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arc
tan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan
(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + a*b*log(4*(4*a^2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2
*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arct
an(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(
1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 - 4*a^3*b*arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a))) - 14*a^2*b*arctan(1/(b*x + a))*tan(1/2
*arctan(1/(b*x + a)))^2 + 2*a^2*b*tan(1/2*arctan(1/(b*x + a)))^3 - 4*a*b*ar
ctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + a*b*tan(1/2*arctan(1/(b*
x + a)))^4 - b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 - 4*a^2*b
*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x +
a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a)))
- 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2
*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) - 2*a*b*
log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a
)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) -
 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*
tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 + a^2*b
*arctan(1/(b*x + a)) - 2*a^2*b*tan(1/2*arctan(1/(b*x + a))) + 4*a*b*arctan(
1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - 6*a*b*tan(1/2*arctan(1/(b*x + a
)))^2 - 2*b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*b*tan(1/
2*arctan(1/(b*x + a)))^3 + a*b*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2
+ 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a
*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(
1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)) + a*b -
 b*arctan(1/(b*x + a)) + 2*b*tan(1/2*arctan(1/(b*x + a))))*b/(4*a^6*tan(1/2
*arctan(1/(b*x + a)))^2 + 4*a^5*tan(1/2*arctan(1/(b*x + a)))^3 + a^4*tan(1/
```

```
2*arctan(1/(b*x + a)))^4 - 4*a^5*tan(1/2*arctan(1/(b*x + a))) + 6*a^4*tan(1
/2*arctan(1/(b*x + a)))^2 + 8*a^3*tan(1/2*arctan(1/(b*x + a)))^3 + 2*a^2*ta
n(1/2*arctan(1/(b*x + a)))^4 + a^4 - 8*a^3*tan(1/2*arctan(1/(b*x + a))) + 4
*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 + 2*a^2
- 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.42

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = \frac{\operatorname{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,(a^2+1)-a^2\,b^2}}\right)\,\left(b^3-a^2\,b^3\right)}{\sqrt{b^2}\,\left(2\,a^4+4\,a^2+2\right)} - \frac{a\,b^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,{\left(a^2+1\right)}^2} - \frac{\operatorname{acot}(a+b\,x)\,\left(\frac{a^2}{2}+\frac{1}{2}\right)-\frac{b\,x}{2}+\frac{b^2\,x^2\,\operatorname{acot}(a+b\,x)}{2}-\frac{x^3\,\left(b^3-3\,a^2\,b^3\right)}{2\,\left(a^4+2\,a^2+1\right)}+\frac{a\,b^4\,x^4}{{\left(a^2+1\right)}^2}+a\,b\,x\,\operatorname{acot}(a+b\,x)}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4+x^2} + \frac{a\,b^2\,\ln\left(x\right)}{{\left(a^2+1\right)}^2}$$

```
[In] int(acot(a + b*x)/x^3,x)
```

```
[Out] (atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))*(b^3 - a^2*b^3
))/((b^2)^(1/2)*(4*a^2 + 2*a^4 + 2)) - (a*b^2*log(a^2 + b^2*x^2 + 2*a*b*x +
 1))/(2*(a^2 + 1)^2) - (acot(a + b*x)*(a^2/2 + 1/2) - (b*x)/2 + (b^2*x^2*ac
ot(a + b*x))/2 - (x^3*(b^3 - 3*a^2*b^3))/(2*(2*a^2 + a^4 + 1)) + (a*b^4*x^4
)/(a^2 + 1)^2 + a*b*x*acot(a + b*x))/(x^2 + a^2*x^2 + b^2*x^4 + 2*a*b*x^3)
+ (a*b^2*log(x))/(a^2 + 1)^2
```

3.106 $\int \frac{\cot^{-1}(a+bx)}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 129

$$\int \frac{\cot^{-1}(a+bx)}{x^4} dx = \frac{b}{6(1+a^2)x^2} - \frac{2ab^2}{3(1+a^2)^2 x} - \frac{\cot^{-1}(a+bx)}{3x^3} - \frac{a(3-a^2)b^3\arctan(a+bx)}{3(1+a^2)^3} + \frac{(1-3a^2)b^3\log(x)}{3(1+a^2)^3} - \frac{(1-3a^2)b^3\log(1+(a+bx)^2)}{6(1+a^2)^3}$$

```
[Out] 1/6*b/(a^2+1)/x^2-2/3*a*b^2/(a^2+1)^2/x-1/3*arccot(b*x+a)/x^3-1/3*a*(-a^2+3
)*b^3*arctan(b*x+a)/(a^2+1)^3+1/3*(-3*a^2+1)*b^3*ln(x)/(a^2+1)^3-1/6*(-3*a^
2+1)*b^3*ln(1+(b*x+a)^2)/(a^2+1)^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5154, 378, 724, 815, 649, 209, 266}

$$\int \frac{\cot^{-1}(a+bx)}{x^4} dx = -\frac{a(3-a^2)b^3\arctan(a+bx)}{3(a^2+1)^3} + \frac{(1-3a^2)b^3\log(x)}{3(a^2+1)^3} - \frac{(1-3a^2)b^3\log((a+bx)^2+1)}{6(a^2+1)^3} - \frac{2ab^2}{3(a^2+1)^2 x} + \frac{b}{6(a^2+1)x^2} - \frac{\cot^{-1}(a+bx)}{3x^3}$$

```
[In] Int[ArcCot[a + b*x]/x^4,x]
```

```
[Out] b/(6*(1 + a^2)*x^2) - (2*a*b^2)/(3*(1 + a^2)^2*x) - ArcCot[a + b*x]/(3*x^3)
 - (a*(3 - a^2)*b^3*ArcTan[a + b*x])/(3*(1 + a^2)^3) + ((1 - 3*a^2)*b^3*Log
[x])/(3*(1 + a^2)^3) - ((1 - 3*a^2)*b^3*Log[1 + (a + b*x)^2])/(6*(1 + a^2)^
3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
```

```
+ d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^{-1}(a+bx)}{3x^3} - \frac{1}{3}b\int \frac{1}{x^3\,(1+(a+bx)^2)}\,dx \\
&= -\frac{\cot^{-1}(a+bx)}{3x^3} - \frac{1}{3}b^3\text{Subst}\left(\int \frac{1}{(-a+x)^3\,(1+x^2)}\,dx, x, a+bx\right) \\
&= \frac{b}{6\,(1+a^2)\,x^2} - \frac{\cot^{-1}(a+bx)}{3x^3} - \frac{b^3\text{Subst}\left(\int \frac{-a-x}{(-a+x)^2(1+x^2)}\,dx, x, a+bx\right)}{3\,(1+a^2)} \\
&= \frac{b}{6\,(1+a^2)\,x^2} - \frac{\cot^{-1}(a+bx)}{3x^3} \\
&\quad - \frac{b^3\text{Subst}\left(\int \left(-\frac{2a}{(1+a^2)(a-x)^2} + \frac{1-3a^2}{(1+a^2)^2(a-x)} + \frac{a(3-a^2)+(1-3a^2)x}{(1+a^2)^2(1+x^2)}\right)\,dx, x, a+bx\right)}{3\,(1+a^2)} \\
&= \frac{b}{6\,(1+a^2)\,x^2} - \frac{2ab^2}{3\,(1+a^2)^2\,x} - \frac{\cot^{-1}(a+bx)}{3x^3} \\
&\quad + \frac{(1-3a^2)\,b^3\log(x)}{3\,(1+a^2)^3} - \frac{b^3\text{Subst}\left(\int \frac{a(3-a^2)+(1-3a^2)x}{1+x^2}\,dx, x, a+bx\right)}{3\,(1+a^2)^3} \\
&= \frac{b}{6\,(1+a^2)\,x^2} - \frac{2ab^2}{3\,(1+a^2)^2\,x} - \frac{\cot^{-1}(a+bx)}{3x^3} \\
&\quad + \frac{(1-3a^2)\,b^3\log(x)}{3\,(1+a^2)^3} - \frac{((1-3a^2)\,b^3)\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{3\,(1+a^2)^3} \\
&\quad - \frac{(a(3-a^2)\,b^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, a+bx\right)}{3\,(1+a^2)^3} \\
&= \frac{b}{6\,(1+a^2)\,x^2} - \frac{2ab^2}{3\,(1+a^2)^2\,x} - \frac{\cot^{-1}(a+bx)}{3x^3} - \frac{a(3-a^2)\,b^3\arctan(a+bx)}{3\,(1+a^2)^3} \\
&\quad + \frac{(1-3a^2)\,b^3\log(x)}{3\,(1+a^2)^3} - \frac{(1-3a^2)\,b^3\log\left(1+(a+bx)^2\right)}{6\,(1+a^2)^3}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx$$
$$= \frac{-2(1+a^2)^3\cot^{-1}(a+bx)+2(1-3a^2)\,b^3x^3\log(x)+(-1+ia)^3b^3x^3\log(i-a-bx)+(-i+a)bx((i+a}{6\,(1+a^2)^3\,x^3}$$

```
[In] Integrate[ArcCot[a + b*x]/x^4,x]
```

```
[Out] (-2*(1 + a^2)^3*ArcCot[a + b*x] + 2*(1 - 3*a^2)*b^3*x^3*Log[x] + (-1 + I*a)
^3*b^3*x^3*Log[I - a - b*x] + (-I + a)*b*x*((I + a)*(1 + a^2 - 4*a*b*x) + I
*(-I + a)^2*b^2*x^2*Log[I + a + b*x]))/(6*(1 + a^2)^3*x^3)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.89

method	result
derivativedivides	$b^3\left(-\frac{\operatorname{arccot}(bx+a)}{3b^3x^3}+\frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3}+\frac{1}{6(a^2+1)b^2x^2}-\frac{2a}{3(a^2+1)^2bx}+\frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2}+(a^3-3a)\ a}{3(a^2+1)^3}\right.$
default	$b^3\left(-\frac{\operatorname{arccot}(bx+a)}{3b^3x^3}+\frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3}+\frac{1}{6(a^2+1)b^2x^2}-\frac{2a}{3(a^2+1)^2bx}+\frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2}+(a^3-3a)\ a}{3(a^2+1)^3}\right.$
parts	$-\frac{\operatorname{arccot}(bx+a)}{3x^3}-\frac{b\left(-\frac{1}{2\left(a^2+1\right)x^2}+\frac{b^2\left(3a^2-1\right)\ln(x)}{\left(a^2+1\right)^3}+\frac{2ba}{\left(a^2+1\right)^2x}-\frac{b^3\left(\frac{\left(3a^2b-b\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(4a^3-4a-\frac{(3a}{}\right.}{}\right.}{\left(a^2+1\right)^3}\right.}{3}$
parallelrisch	$-\frac{2x^3\operatorname{arccot}(bx+a)a^3b^3+6\ln(x)x^3a^2b^3-3\ln\left(b^2x^2+2abx+a^2+1\right)x^3a^2b^3-6x^3\operatorname{arccot}(bx+a)a\,b^3-7a^2b^3x^3-2b^3\ln(x)x^3}{}$
risch	$-\frac{i\ln(1+i(bx+a))}{6x^3}-\frac{ix^3\ln\left(\left(-a^7b-5ia^6b-27a^5b+41ia^4b+29a^3b-15ia^2b-9ab+3ib\right)x-a^8-32a^6-4ia^7+70a^4+68ia^5}{}$

```
[In] int(arccot(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3/b^3/x^3*arccot(b*x+a)+1/3*(-3*a^2+1)/(a^2+1)^3*ln(-b*x)+1/6/(a^2+
1)/b^2/x^2-2/3/(a^2+1)^2*a/b/x+1/3/(a^2+1)^3*(1/2*(3*a^2-1)*ln(1+(b*x+a)^2)
+(a^3-3*a)*arctan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{2\,(a^3-3\,a)b^3x^3\arctan\left(bx+a\right)+(3\,a^2-1)b^3x^3\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(3\,a^2-1)b^3x^3\log\left(x\right)-4\,(}{6\,(a^6+3\,a^4+3\,a^2+1)x^3}$$

```
[In] integrate(arccot(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^3 - 3*a)*b^3*x^3*arctan(b*x + a) + (3*a^2 - 1)*b^3*x^3*log(b^2*x^
2 + 2*a*b*x + a^2 + 1) - 2*(3*a^2 - 1)*b^3*x^3*log(x) - 4*(a^3 + a)*b^2*x^2
 + (a^4 + 2*a^2 + 1)*b*x - 2*(a^6 + 3*a^4 + 3*a^2 + 1)*arccot(b*x + a))/((a
^6 + 3*a^4 + 3*a^2 + 1)*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.89

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \begin{cases} \frac{ib^3\operatorname{acot}\left(bx-i\right)}{24}-\frac{ib^2}{24x}+\frac{b}{24x^2}-\frac{\operatorname{acot}\left(bx-i\right)}{3x^3}+\frac{i}{18x^3} \\ -\frac{ib^3\operatorname{acot}\left(bx+i\right)}{24}+\frac{ib^2}{24x}+\frac{b}{24x^2}-\frac{\operatorname{acot}\left(bx+i\right)}{3x^3}-\frac{i}{18x^3} \\ -\frac{2a^6\operatorname{acot}\left(a+bx\right)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{a^4bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{6a^4\operatorname{acot}\left(a+bx\right)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{2a^3b^3x^3\operatorname{acot}\left(a+bx\right)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{}{6a^6} \end{cases}$$

```
[In] integrate(acot(b*x+a)/x**4,x)
```

```
[Out] Piecewise((I*b**3*acot(b*x - I)/24 - I*b**2/(24*x) + b/(24*x**2) - acot(b*x
 - I)/(3*x**3) + I/(18*x**3), Eq(a, -I)), (-I*b**3*acot(b*x + I)/24 + I*b**
2/(24*x) + b/(24*x**2) - acot(b*x + I)/(3*x**3) - I/(18*x**3), Eq(a, I)), (
-2*a**6*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
+ a**4*b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a**4*ac
ot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**3*b
**3*x**3*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
 - 4*a**3*b**2*x**2/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) -
6*a**2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3
) + 3*a**2*b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 2*a**2*b*x/(6*a**6*x**3 + 18*a**4*x**3
 + 18*a**2*x**3 + 6*x**3) - 6*a**2*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**
```

```
3 + 18*a**2*x**3 + 6*x**3) + 6*a*b**3*x**3*acot(a + b*x)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 4*a*b**2*x**2/(6*a**6*x**3 + 18*a**4*x
**3 + 18*a**2*x**3 + 6*x**3) + 2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x*
*3 + 18*a**2*x**3 + 6*x**3) - b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)
/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + b*x/(6*a**6*x**3 +
18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*acot(a + b*x)/(6*a**6*x**3 + 18*a
**4*x**3 + 18*a**2*x**3 + 6*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.28

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{1}{6}\left(\frac{2\,(a^3-3\,a)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1}+\frac{(3\,a^2-1)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{2\,(3\,a^2-1)b^2\log\left(x\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{4}{(a\right. - \frac{\operatorname{arccot}\left(bx+a\right)}{3\,x^3}$$

```
[In] integrate(arccot(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(a^3 - 3*a)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a^2 + 1) +
(3*a^2 - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1)
- 2*(3*a^2 - 1)*b^2*log(x)/(a^6 + 3*a^4 + 3*a^2 + 1) - (4*a*b*x - a^2 - 1)/
((a^4 + 2*a^2 + 1)*x^2))*b - 1/3*arccot(b*x + a)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3449 vs. 2(115) = 230.

Time = 1.67 (sec) , antiderivative size = 3449, normalized size of antiderivative = 26.74

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(arccot(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(24*a^5*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 12*a^
4*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 2*a^3*b^2*arctan
(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 + 24*a^5*b^2*log(4*(4*a^2*tan(
1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*a
rctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan
(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/
```

(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 36*a^4*b^2*log(4*(4*a^
2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan
(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*
arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arc
tan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 + 18*a^3*b^2*log(4
*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3
 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1
/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^5 + 3*a^2*b^2*
log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a
)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) -
 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*
tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^6 - 24*a^
5*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 120*a^4*b^2*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + 24*a^4*b^2*tan(1/2*arctan(
1/(b*x + a)))^4 - 78*a^3*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)
))^4 + 22*a^3*b^2*tan(1/2*arctan(1/(b*x + a)))^5 - 36*a^2*b^2*arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 5*a^2*b^2*tan(1/2*arctan(1/(b*x + a
)))^6 - 6*a*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 - 36*a^4
*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*
x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a
))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4
 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 -
44*a^3*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan
(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b
*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x +
a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))
)^3 - 21*a^2*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*
arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arcta
n(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(
b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x
 + a)))^4 - 6*a*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1
/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*ar
ctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(
1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(
b*x + a)))^5 - b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/
2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arc
tan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1
/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b
*x + a)))^6 + 12*a^4*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) -
 24*a^4*b^2*tan(1/2*arctan(1/(b*x + a)))^2 + 78*a^3*b^2*arctan(1/(b*x + a))
*tan(1/2*arctan(1/(b*x + a)))^2 - 100*a^3*b^2*tan(1/2*arctan(1/(b*x + a)))^
3 + 24*a^2*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 - 67*a^2*
b^2*tan(1/2*arctan(1/(b*x + a)))^4 + 18*a*b^2*arctan(1/(b*x + a))*tan(1/2*a
rctan(1/(b*x + a)))^4 - 14*a*b^2*tan(1/2*arctan(1/(b*x + a)))^5 - b^2*tan(1

/2*arctan(1/(b*x + a)))^6 + 18*a^3*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) + 21*a^2*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 + 12*a*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 3*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 - 2*a^3*b^2*arctan(1/(b*x + a)) + 22*a^3*b^2*tan(1/2*arctan(1/(b*x + a))) - 36*a^2*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) + 67*a^2*b^2*tan(1/2*arctan(1/(b*x + a)))^2 - 18*a*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 20*a*b^2*tan(1/2*arctan(1/(b*x + a)))^3 - 16*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 - b^2*tan(1/2*arctan(1/(b*x + a)))^4 - 3*a^2*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)) - 6*a*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) - 3*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - 5*a^2*b^2 + 6*a*b^2*arctan(1/(b*x + a)) - 14*a*b^2*tan(1/2*arctan(1/(b*x + a))) + b^2*tan(1/2*arctan(1/(b*x + a)))^2 + b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)) + b^2)*b/(8*a^9*tan(1/2*arctan(1/(b*x + a)))^3 + 12*a^8*tan(1/2*arctan(1/(b*x + a)))^4 + 6*a^7*tan(1/2*arctan(1/(b*x + a)))^5 + a^6*tan(1/2*arctan(1/(b*x + a)))^6 - 12*a^8*tan(1/2*arctan(1/(b*x + a)))^2 + 12*a^7*tan(1/2*arctan(1/(b*x + a)))^3 + 33*a^6*tan(1/2*arctan(1/(b*x + a)))^4 + 18*a^5*tan(1/2*arctan(1/(b*x + a)))^5 + 3*a^4*tan(1/2*arctan(1/(b*x + a)))^6 + 6*a^7*tan(1/2*arctan(1/(b*x + a))) - 33*a^6*tan(1/2*arctan(1/(b*x + a)))^2 - 12*a^5*tan(1/2*arctan(1/(b*x + a)))^3 + 27*a^4*ta

```
n(1/2*arctan(1/(b*x + a)))^4 + 18*a^3*tan(1/2*arctan(1/(b*x + a)))^5 + 3*a^
2*tan(1/2*arctan(1/(b*x + a)))^6 - a^6 + 18*a^5*tan(1/2*arctan(1/(b*x + a))
) - 27*a^4*tan(1/2*arctan(1/(b*x + a)))^2 - 28*a^3*tan(1/2*arctan(1/(b*x +
a)))^3 + 3*a^2*tan(1/2*arctan(1/(b*x + a)))^4 + 6*a*tan(1/2*arctan(1/(b*x +
 a)))^5 + tan(1/2*arctan(1/(b*x + a)))^6 - 3*a^4 + 18*a^3*tan(1/2*arctan(1/
(b*x + a))) - 3*a^2*tan(1/2*arctan(1/(b*x + a)))^2 - 12*a*tan(1/2*arctan(1/
(b*x + a)))^3 - 3*tan(1/2*arctan(1/(b*x + a)))^4 - 3*a^2 + 6*a*tan(1/2*arct
an(1/(b*x + a))) + 3*tan(1/2*arctan(1/(b*x + a)))^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.21

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{\ln(x)\,\left(\frac{b^3}{3}-a^2\,b^3\right)}{a^6+3\,a^4+3\,a^2+1} - \frac{\mathrm{acot}(a+b\,x)\,\left(\frac{a^2}{3}+\frac{1}{3}\right)-\frac{b\,x}{6}+\frac{b^2\,x^2\,\mathrm{acot}(a+b\,x)}{3}-\frac{x^3\,\left(b^3-7\,a^2\,b^3\right)}{6\,\left(a^4+2\,a^2+1\right)}+\frac{a\,b^2\,x^2}{3\,\left(a^2+1\right)}+\frac{2\,a\,b^4\,x^4}{3\,{\left(a^2+1\right)}^2}+\frac{2\,a\,b\,x\,\mathrm{acot}(a+b\,x)}{3}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5+x^3} + \frac{b^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)\,\left(3\,a^2-1\right)}{6\,\left(a^6+3\,a^4+3\,a^2+1\right)} + \frac{a\,\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(a^2-3\right)\,{\left(b^2\right)}^{3/2}}{3\,\left(a^6+3\,a^4+3\,a^2+1\right)}$$

```
[In] int(acot(a + b*x)/x^4,x)
```

```
[Out] (log(x)*(b^3/3 - a^2*b^3))/(3*a^2 + 3*a^4 + a^6 + 1) - (acot(a + b*x)*(a^2/
3 + 1/3) - (b*x)/6 + (b^2*x^2*acot(a + b*x))/3 - (x^3*(b^3 - 7*a^2*b^3))/(6
*(2*a^2 + a^4 + 1)) + (a*b^2*x^2)/(3*(a^2 + 1)) + (2*a*b^4*x^4)/(3*(a^2 + 1
)^2) + (2*a*b*x*acot(a + b*x))/3)/(x^3 + a^2*x^3 + b^2*x^5 + 2*a*b*x^4) + (
b^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1)*(3*a^2 - 1))/(6*(3*a^2 + 3*a^4 + a^6 +
 1)) + (a*atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))*(a^2
- 3)*(b^2)^(3/2))/(3*(3*a^2 + 3*a^4 + a^6 + 1))
```

3.107 $\int \frac{\cot^{-1}(a+bx)}{c+dx^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 642

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = &-\frac{\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{b\left(i\sqrt{c}-\sqrt{d}x\right)}{\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}\\
&+\frac{\log\left(-\frac{i-a-bx}{a+bx}\right)\log\left(\frac{ib\left(\sqrt{c}+i\sqrt{d}x\right)}{\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}\\
&-\frac{\log\left(-\frac{i-a-bx}{a+bx}\right)\log\left(\frac{b\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(b\sqrt{c}+(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}\\
&+\frac{\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{b\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(b\sqrt{c}+i(i+a)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}\\
&+\frac{\operatorname{PolyLog}\left(2,-\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(i-a-bx)}{\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}\\
&-\frac{\operatorname{PolyLog}\left(2,-\frac{\left(b\sqrt{c}+ia\sqrt{d}\right)(i-a-bx)}{\left(b\sqrt{c}+(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}\\
&-\frac{\operatorname{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(i+a+bx)}{\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}\\
&+\frac{\operatorname{PolyLog}\left(2,\frac{\left(b\sqrt{c}+ia\sqrt{d}\right)(i+a+bx)}{\left(b\sqrt{c}+i(i+a)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}\end{aligned}$$

```
[Out] -1/4*ln((I+a+b*x)/(b*x+a))*ln(-b*(I*c^(1/2)-x*d^(1/2))/(b*x+a)/(b*c^(1/2)+(
1-I*a)*d^(1/2)))/c^(1/2)/d^(1/2)+1/4*ln((-I+a+b*x)/(b*x+a))*ln(I*b*(c^(1/2)
+I*x*d^(1/2))/(b*x+a)/(b*c^(1/2)-(1+I*a)*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*ln((
-I+a+b*x)/(b*x+a))*ln(b*(I*c^(1/2)+x*d^(1/2))/(b*x+a)/(b*c^(1/2)+(1+I*a)*d^
(1/2)))/c^(1/2)/d^(1/2)+1/4*ln((I+a+b*x)/(b*x+a))*ln(-b*(I*c^(1/2)+x*d^(1/2
))/(b*x+a)/(b*c^(1/2)+I*(I+a)*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*polylog(2,(I+a+
b*x)*(b*c^(1/2)-I*a*d^(1/2))/(b*x+a)/(b*c^(1/2)+(1-I*a)*d^(1/2)))/c^(1/2)/d
^(1/2)+1/4*polylog(2,-(I-a-b*x)*(b*c^(1/2)-I*a*d^(1/2))/(b*x+a)/(b*c^(1/2)-
(1+I*a)*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*polylog(2,-(I-a-b*x)*(b*c^(1/2)+I*a*d
^(1/2))/(b*x+a)/(b*c^(1/2)+(1+I*a)*d^(1/2)))/c^(1/2)/d^(1/2)+1/4*polylog(2,
(I+a+b*x)*(b*c^(1/2)+I*a*d^(1/2))/(b*x+a)/(b*c^(1/2)+I*(I+a)*d^(1/2)))/c^(1
/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used

$= \{5160, 2576, 2404, 2354, 2438\}$

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \frac{\text{PolyLog}\left(2,-\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(-a-bx+i)}{\left(b\sqrt{c}-(ia+1)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\text{PolyLog}\left(2,-\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(-a-bx+i)}{\left(\sqrt{d}(ia+1)+b\sqrt{c}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(a+bx+i)}{\left(\sqrt{d}(1-ia)+b\sqrt{c}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left(2,\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(a+bx+i)}{\left(i\sqrt{d}(a+i)+b\sqrt{c}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(-\frac{b\left(-\sqrt{d}x+i\sqrt{c}\right)}{(a+bx)\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)\log\left(\frac{ib\left(\sqrt{c}+i\sqrt{d}x\right)}{(a+bx)\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)\log\left(\frac{b\left(\sqrt{d}x+i\sqrt{c}\right)}{(a+bx)\left(b\sqrt{c}+(1+ia)\sqrt{d}\right)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(-\frac{b\left(\sqrt{d}x+i\sqrt{c}\right)}{(a+bx)\left(b\sqrt{c}+i(a+i)\sqrt{d}\right)}\right)}{4\sqrt{c}\sqrt{d}}$$

```
[In] Int[ArcCot[a + b*x]/(c + d*x^2),x]
```

```
[Out] -1/4*(Log[(I + a + b*x)/(a + b*x)]*Log[-((b*(I*Sqrt[c] - Sqrt[d]*x))/((b*Sq
rt[c] + (1 - I*a)*Sqrt[d])*(a + b*x)))])/(Sqrt[c]*Sqrt[d]) + (Log[-((I - a
- b*x)/(a + b*x))]*Log[(I*b*(Sqrt[c] + I*Sqrt[d]*x))/((b*Sqrt[c] - (1 + I*a
)*Sqrt[d])*(a + b*x))])/(4*Sqrt[c]*Sqrt[d]) - (Log[-((I - a - b*x)/(a + b*x
))]*Log[(b*(I*Sqrt[c] + Sqrt[d]*x))/((b*Sqrt[c] + (1 + I*a)*Sqrt[d])*(a + b
*x))])/(4*Sqrt[c]*Sqrt[d]) + (Log[(I + a + b*x)/(a + b*x)]*Log[-((b*(I*Sqrt
[c] + Sqrt[d]*x))/((b*Sqrt[c] + I*(I + a)*Sqrt[d])*(a + b*x)))])/(4*Sqrt[c]
*Sqrt[d]) + PolyLog[2, -(((b*Sqrt[c] - I*a*Sqrt[d])*(I - a - b*x))/((b*Sqrt
[c] - (1 + I*a)*Sqrt[d])*(a + b*x)))]/(4*Sqrt[c]*Sqrt[d]) - PolyLog[2, -(((
b*Sqrt[c] + I*a*Sqrt[d])*(I - a - b*x))/((b*Sqrt[c] + (1 + I*a)*Sqrt[d])*(a
 + b*x)))]/(4*Sqrt[c]*Sqrt[d]) - PolyLog[2, ((b*Sqrt[c] - I*a*Sqrt[d])*(I +
 a + b*x))/((b*Sqrt[c] + (1 - I*a)*Sqrt[d])*(a + b*x))]/(4*Sqrt[c]*Sqrt[d])
```

```
 + PolyLog[2, ((b*Sqrt[c] + I*a*Sqrt[d])*(I + a + b*x))/((b*Sqrt[c] + I*(I
+ a)*Sqrt[d])*(a + b*x))]/(4*Sqrt[c]*Sqrt[d])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$\text{integral} = \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+dx^2}\,dx - \frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+dx^2}\,dx$$

$$= -\left(\frac{1}{2}b\text{Subst}\left(\int \frac{\log(x)}{b^2c+(-i+a)^2d-(2b^2c+2a(-i+a)d)\,x+(b^2c+a^2d)\,x^2}\,dx, x, \frac{-i+a+bx}{a+bx}\right)\right) - \frac{1}{2}b\text{Subst}\left(\int \frac{\log(x)}{b^2c+(i+a)^2d-(2b^2c+2a(i+a)d)\,x+(b^2c+a^2d)\,x^2}\,dx, x, \frac{i+a+bx}{a+bx}\right)$$

$$= -\left(\frac{1}{2}b\text{Subst}\left(\int\left(\frac{(b^2c+a^2d)\log(x)}{b\sqrt{c}\sqrt{d}\left(2b^2c-2b\sqrt{c}\sqrt{d}+2iad+2a^2d-2\left(b^2c+a^2d\right)x\right)}+\frac{}{b\sqrt{c}\sqrt{d}\left(-2b^2c\right.}\right.\right.\right.$$

$$-\frac{1}{2}b\text{Subst}\left(\int\left(\frac{(b^2c+a^2d)\log(x)}{b\sqrt{c}\sqrt{d}\left(2b^2c-2b\sqrt{c}\sqrt{d}-2iad+2a^2d-2\left(b^2c+a^2d\right)x\right)}\right.\right.$$

$$\left.\left.+\frac{(b^2c+a^2d)\log(x)}{b\sqrt{c}\sqrt{d}\left(-2b^2c-2b\sqrt{c}\sqrt{d}+2iad-2a^2d+2\left(b^2c+a^2d\right)x\right)}\right)dx,x,\frac{-i+a+bx}{a+bx}\right)$$

$$= -\frac{(b^2c+a^2d)\,\text{Subst}\left(\int\frac{\log(x)}{2b^2c-2b\sqrt{c}\sqrt{d}-2iad+2a^2d-2(b^2c+a^2d)x}\,dx,x,\frac{-i+a+bx}{a+bx}\right)}{2\sqrt{c}\sqrt{d}}$$

$$-\frac{(b^2c+a^2d)\,\text{Subst}\left(\int\frac{\log(x)}{2b^2c-2b\sqrt{c}\sqrt{d}+2iad+2a^2d-2(b^2c+a^2d)x}\,dx,x,\frac{i+a+bx}{a+bx}\right)}{2\sqrt{c}\sqrt{d}}$$

$$-\frac{(b^2c+a^2d)\,\text{Subst}\left(\int\frac{\log(x)}{-2b^2c-2b\sqrt{c}\sqrt{d}-2iad-2a^2d+2(b^2c+a^2d)x}\,dx,x,\frac{i+a+bx}{a+bx}\right)}{2\sqrt{c}\sqrt{d}}$$

$$-\frac{(b^2c+a^2d)\,\text{Subst}\left(\int\frac{\log(x)}{-2b^2c-2b\sqrt{c}\sqrt{d}+2iad-2a^2d+2(b^2c+a^2d)x}\,dx,x,\frac{-i+a+bx}{a+bx}\right)}{2\sqrt{c}\sqrt{d}}$$

$$= -\frac{\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{b\left(i\sqrt{c}-\sqrt{d}x\right)}{\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}$$
$$+\frac{\log\left(-\frac{i-a-bx}{a+bx}\right)\log\left(\frac{ib\left(\sqrt{c}+i\sqrt{d}x\right)}{\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}$$
$$-\frac{\log\left(-\frac{i-a-bx}{a+bx}\right)\log\left(\frac{b\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(b\sqrt{c}+(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}$$
$$+\frac{\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{b\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(b\sqrt{c}+i(i+a)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}$$
$$+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{2\left(b^2c+a^2d\right)x}{-2b^2c-2b\sqrt{c}\sqrt{d}-2iad-2a^2d}\right)}{x}\,dx,x,\frac{i+a+bx}{a+bx}\right)}{4\sqrt{c}\sqrt{d}}$$
$$+\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{2\left(b^2c+a^2d\right)x}{-2b^2c-2b\sqrt{c}\sqrt{d}+2iad-2a^2d}\right)}{x}\,dx,x,\frac{-i+a+bx}{a+bx}\right)}{4\sqrt{c}\sqrt{d}}$$
$$-\frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{2\left(b^2c+a^2d\right)x}{2b^2c-2b\sqrt{c}\sqrt{d}-2iad+2a^2d}\right)}{x}\,dx,x,\frac{-i+a+bx}{a+bx}\right)}{4\sqrt{c}\sqrt{d}}$$
$$-\frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{2\left(b^2c+a^2d\right)x}{2b^2c-2b\sqrt{c}\sqrt{d}+2iad+2a^2d}\right)}{x}\,dx,x,\frac{i+a+bx}{a+bx}\right)}{4\sqrt{c}\sqrt{d}}$$

$$= -\frac{\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{b\left(i\sqrt{c}-\sqrt{d}x\right)}{\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\log\left(-\frac{i-a-bx}{a+bx}\right)\log\left(\frac{ib\left(\sqrt{c}+i\sqrt{d}x\right)}{\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}$$
$$- \frac{\log\left(-\frac{i-a-bx}{a+bx}\right)\log\left(\frac{b\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(b\sqrt{c}+(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{b\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(b\sqrt{c}+i(i+a)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}$$
$$+ \frac{\text{PolyLog}\left(2,-\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(i-a-bx)}{\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\text{PolyLog}\left(2,-\frac{\left(b\sqrt{c}+ia\sqrt{d}\right)(i-a-bx)}{\left(b\sqrt{c}+(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}$$
$$- \frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(i+a+bx)}{\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}+ia\sqrt{d}\right)(i+a+bx)}{\left(b\sqrt{c}+i(i+a)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx =$$
$$-\frac{i\left(\log\left(\frac{\sqrt{d}(-i+a+bx)}{b\sqrt{-c}+(-i+a)\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}x\right)-\log\left(\frac{-i+a+bx}{a+bx}\right)\log\left(\sqrt{-c}-\sqrt{d}x\right)-\log\left(\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)\log\right.}{}$$

```
[In] Integrate[ArcCot[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/4*I)*(Log[(Sqrt[d]*(-I + a + b*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])]*Lo
g[Sqrt[-c] - Sqrt[d]*x] - Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt
[d]*x] - Log[(Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]*Log[Sq
rt[-c] - Sqrt[d]*x] + Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[d]*x
] - Log[-((Sqrt[d]*(-I + a + b*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d]))]*Log[Sq
rt[-c] + Sqrt[d]*x] + Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*
x] + Log[-((Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))]*Log[Sqr
t[-c] + Sqrt[d]*x] - Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*x]
 + PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])] -
 PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])] - Po
lyLog[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d])] + Poly
Log[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])]))/(Sqrt[-
c]*Sqrt[d])
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{ib\pi \arctan\left(\frac{2iad+2(-ibx-ia+1)d-2d}{2\sqrt{-b^2cd}}\right)}{2\sqrt{-b^2cd}}-\frac{\ln(-ibx-ia+1)\ln\left(\frac{iad-b\sqrt{cd}+(-ibx-ia+1)d-d}{iad-b\sqrt{cd}-d}\right)\sqrt{cd}}{4cd}+$ $\ln(-ibx-ia+1)\ln($
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(arccot(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*b*Pi/(-b^2*c*d)^(1/2)*arctan(1/2*(2*I*a*d+2*(1-I*a-I*b*x)*d-2*d)/(-b
^2*c*d)^(1/2))-1/4*ln(1-I*a-I*b*x)/c/d*ln((I*a*d-b*(c*d)^(1/2)+(1-I*a-I*b*x
)*d-d)/(I*a*d-b*(c*d)^(1/2)-d))*(c*d)^(1/2)+1/4*ln(1-I*a-I*b*x)/c/d*ln((I*a
*d+b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d+b*(c*d)^(1/2)-d))*(c*d)^(1/2)-1/
4/c/d*dilog((I*a*d-b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d-b*(c*d)^(1/2)-d)
)*(c*d)^(1/2)+1/4/c/d*dilog((I*a*d+b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d+
b*(c*d)^(1/2)-d))*(c*d)^(1/2)-1/4*ln(1+I*a+I*b*x)/c/d*ln((I*a*d+b*(c*d)^(1/
2)-(1+I*a+I*b*x)*d+d)/(I*a*d+b*(c*d)^(1/2)+d))*(c*d)^(1/2)+1/4*ln(1+I*a+I*b
*x)/c/d*ln((I*a*d-b*(c*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I*a*d-b*(c*d)^(1/2)+d))
*(c*d)^(1/2)-1/4/c/d*(c*d)^(1/2)*dilog((I*a*d+b*(c*d)^(1/2)-(1+I*a+I*b*x)*d
+d)/(I*a*d+b*(c*d)^(1/2)+d))+1/4/c/d*(c*d)^(1/2)*dilog((I*a*d-b*(c*d)^(1/2)
-(1+I*a+I*b*x)*d+d)/(I*a*d-b*(c*d)^(1/2)+d))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{dx^2+c}\,dx$$

```
[In] integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccot(b*x + a)/(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \text{Timed out}$$

```
[In] integrate(acot(b*x+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8519 vs. $2(456) = 912$.

Time = 4.03 (sec) , antiderivative size = 8519, normalized size of antiderivative = 13.27

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/8*b*(8*arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/b - (4*arctan(sqrt(
d)*x/sqrt(c))*arctan2((2*a*b^2*c*d + (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a
^2 + 3)*b^2*d)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c
^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a^2 + 1)*b*d
)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + (2*a*b^2*d
*x + b^3*c + 3*(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^
2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (
a^2 + 1)*b*d)*sqrt(c)*sqrt(d))) + 4*arctan(sqrt(d)*x/sqrt(c))*arctan2((2*a*
b^2*c*d - (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^2 + 3)*b^2*d)*x)*sqrt(c)*s
qrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d +
(a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d)), ((a^2 +
 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - (2*a*b^2*d*x + b^3*c + 3*(a^2 + 1)*b*
d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3
)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(
d))) + log(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 + 22*a^2 + 21)*b
^20*c^10*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d^3 + 33*(5*a^8 +
 260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*(a^10 + 61*a^8 +
570*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(21*a^12 + 1386*a^10
 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 2
2*(21*a^14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 169455*a^6 + 201773*a^4 +
 121771*a^2 + 29393)*b^10*c^5*d^7 + 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a
^10 + 9438*a^8 + 13728*a^6 + 11492*a^4 + 5168*a^2 + 969)*b^8*c^4*d^8 + 33*(
5*a^18 + 285*a^16 + 3220*a^14 + 15876*a^12 + 42966*a^10 + 70070*a^8 + 70980
*a^6 + 43860*a^4 + 15181*a^2 + 2261)*b^6*c^3*d^9 + 55*(a^20 + 46*a^18 + 465
*a^16 + 2184*a^14 + 5922*a^12 + 10164*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^
4 + 1102*a^2 + 133)*b^4*c^2*d^10 + 11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^1
6 + 2730*a^14 + 4662*a^12 + 5502*a^10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211
*a^2 + 21)*b^2*c*d^11 + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 7
92*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^
12 + (b^24*c^11*d + 11*(a^2 + 21)*b^22*c^10*d^2 + 55*(a^4 + 38*a^2 + 133)*b
^20*c^9*d^3 + 33*(5*a^6 + 255*a^4 + 1615*a^2 + 2261)*b^18*c^8*d^4 + 330*(a^
8 + 60*a^6 + 510*a^4 + 1292*a^2 + 969)*b^16*c^7*d^5 + 22*(21*a^10 + 1365*a^
8 + 13650*a^6 + 46410*a^4 + 62985*a^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 +
 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c
```

^5*d^7 + 330*(a^14 + 63*a^12 + 693*a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 +
4199*a^2 + 969)*b^10*c^4*d^8 + 33*(5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^
10 + 30030*a^8 + 40040*a^6 + 30940*a^4 + 12920*a^2 + 2261)*b^8*c^3*d^9 + 55
*(a^18 + 45*a^16 + 420*a^14 + 1764*a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 +
3060*a^4 + 969*a^2 + 133)*b^6*c^2*d^10 + 11*(a^20 + 30*a^18 + 225*a^16 + 8
40*a^14 + 1890*a^12 + 2772*a^10 + 2730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 +
21)*b^4*c*d^11 + (a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^1
2 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b^2*d^12)*x^2 + 2*(
11*(a^2 + 1)*b^21*c^10*d + 110*(a^4 + 8*a^2 + 7)*b^19*c^9*d^2 + 33*(15*a^6
+ 205*a^4 + 589*a^2 + 399)*b^17*c^8*d^3 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 6
46*a^2 + 323)*b^15*c^7*d^4 + 110*(21*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 +
6137*a^2 + 2261)*b^13*c^6*d^5 + 4*(693*a^12 + 15708*a^10 + 105105*a^8 + 30
8880*a^6 + 449735*a^4 + 319124*a^2 + 88179)*b^11*c^5*d^6 + 110*(21*a^14 + 4
83*a^12 + 3465*a^10 + 11583*a^8 + 20735*a^6 + 20553*a^4 + 10659*a^2 + 2261)
*b^9*c^4*d^7 + 264*(5*a^16 + 110*a^14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6
890*a^6 + 4930*a^4 + 1938*a^2 + 323)*b^7*c^3*d^8 + 33*(15*a^18 + 295*a^16 +
2044*a^14 + 7308*a^12 + 15554*a^10 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 29
83*a^2 + 399)*b^5*c^2*d^9 + 110*(a^20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*
a^12 + 1008*a^10 + 966*a^8 + 624*a^6 + 261*a^4 + 64*a^2 + 7)*b^3*c*d^10 + 1
1*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 3
30*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11 + (11*b^23*c^10*d + 110*(a^2
+ 7)*b^21*c^9*d^2 + 33*(15*a^4 + 190*a^2 + 399)*b^19*c^8*d^3 + 264*(5*a^6
+ 85*a^4 + 323*a^2 + 323)*b^17*c^7*d^4 + 110*(21*a^8 + 420*a^6 + 2142*a^4 +
3876*a^2 + 2261)*b^15*c^6*d^5 + 4*(693*a^10 + 15015*a^8 + 90090*a^6 + 2187
90*a^4 + 230945*a^2 + 88179)*b^13*c^5*d^6 + 110*(21*a^12 + 462*a^10 + 3003*
a^8 + 8580*a^6 + 12155*a^4 + 8398*a^2 + 2261)*b^11*c^4*d^7 + 264*(5*a^14 +
105*a^12 + 693*a^10 + 2145*a^8 + 3575*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*
c^3*d^8 + 33*(15*a^16 + 280*a^14 + 1764*a^12 + 5544*a^10 + 10010*a^8 + 1092
0*a^6 + 7140*a^4 + 2584*a^2 + 399)*b^7*c^2*d^9 + 110*(a^18 + 15*a^16 + 84*a
^14 + 252*a^12 + 462*a^10 + 546*a^8 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c
d^10 + 11(a^20 + 10*a^18 + 45*a^16 + 120*a^14 + 210*a^12 + 252*a^10 + 210
*a^8 + 120*a^6 + 45*a^4 + 10*a^2 + 1)*b^3*d^11)*x^2 + 2*(11*a*b^22*c^10*d +
110*(a^3 + 7*a)*b^20*c^9*d^2 + 33*(15*a^5 + 190*a^3 + 399*a)*b^18*c^8*d^3
+ 264*(5*a^7 + 85*a^5 + 323*a^3 + 323*a)*b^16*c^7*d^4 + 110*(21*a^9 + 420*a
^7 + 2142*a^5 + 3876*a^3 + 2261*a)*b^14*c^6*d^5 + 4*(693*a^11 + 15015*a^9 +
90090*a^7 + 218790*a^5 + 230945*a^3 + 88179*a)*b^12*c^5*d^6 + 110*(21*a^13
+ 462*a^11 + 3003*a^9 + 8580*a^7 + 12155*a^5 + 8398*a^3 + 2261*a)*b^10*c^4
d^7 + 264(5*a^15 + 105*a^13 + 693*a^11 + 2145*a^9 + 3575*a^7 + 3315*a^5 +
1615*a^3 + 323*a)*b^8*c^3*d^8 + 33*(15*a^17 + 280*a^15 + 1764*a^13 + 5544*
a^11 + 10010*a^9 + 10920*a^7 + 7140*a^5 + 2584*a^3 + 399*a)*b^6*c^2*d^9 + 1
10*(a^19 + 15*a^17 + 84*a^15 + 252*a^13 + 462*a^11 + 546*a^9 + 420*a^7 + 20
4*a^5 + 57*a^3 + 7*a)*b^4*c*d^10 + 11*(a^21 + 10*a^19 + 45*a^17 + 120*a^15
+ 210*a^13 + 252*a^11 + 210*a^9 + 120*a^7 + 45*a^5 + 10*a^3 + a)*b^2*d^11)*
x)*sqrt(c)*sqrt(d) + 2*(a*b^23*c^11*d + 11*(a^3 + 21*a)*b^21*c^10*d^2 + 55*
(a^5 + 38*a^3 + 133*a)*b^19*c^9*d^3 + 33*(5*a^7 + 255*a^5 + 1615*a^3 + 2261

*a)*b^17*c^8*d^4 + 330*(a^9 + 60*a^7 + 510*a^5 + 1292*a^3 + 969*a)*b^15*c^7
d^5 + 22(21*a^11 + 1365*a^9 + 13650*a^7 + 46410*a^5 + 62985*a^3 + 29393*a
)*b^13*c^6*d^6 + 22*(21*a^13 + 1386*a^11 + 15015*a^9 + 60060*a^7 + 109395*a
^5 + 92378*a^3 + 29393*a)*b^11*c^5*d^7 + 330*(a^15 + 63*a^13 + 693*a^11 + 3
003*a^9 + 6435*a^7 + 7293*a^5 + 4199*a^3 + 969*a)*b^9*c^4*d^8 + 33*(5*a^17
+ 280*a^15 + 2940*a^13 + 12936*a^11 + 30030*a^9 + 40040*a^7 + 30940*a^5 + 1
2920*a^3 + 2261*a)*b^7*c^3*d^9 + 55*(a^19 + 45*a^17 + 420*a^15 + 1764*a^13
+ 4158*a^11 + 6006*a^9 + 5460*a^7 + 3060*a^5 + 969*a^3 + 133*a)*b^5*c^2*d^1
0 + 11*(a^21 + 30*a^19 + 225*a^17 + 840*a^15 + 1890*a^13 + 2772*a^11 + 2730
*a^9 + 1800*a^7 + 765*a^5 + 190*a^3 + 21*a)*b^3*c*d^11 + (a^23 + 11*a^21 +
55*a^19 + 165*a^17 + 330*a^15 + 462*a^13 + 462*a^11 + 330*a^9 + 165*a^7 + 5
5*a^5 + 11*a^3 + a)*b*d^12)*x)/(b^24*c^12 + 12*(a^2 + 23)*b^22*c^11*d + 66*
(a^4 + 42*a^2 + 161)*b^20*c^10*d^2 + 44*(5*a^6 + 285*a^4 + 1995*a^2 + 3059)
*b^18*c^9*d^3 + 99*(5*a^8 + 340*a^6 + 3230*a^4 + 9044*a^2 + 7429)*b^16*c^8*
d^4 + 264*(3*a^10 + 225*a^8 + 2550*a^6 + 9690*a^4 + 14535*a^2 + 7429)*b^14*
c^7*d^5 + 4*(231*a^12 + 18018*a^10 + 225225*a^8 + 1021020*a^6 + 2078505*a^4
 + 1939938*a^2 + 676039)*b^12*c^6*d^6 + 264*(3*a^14 + 231*a^12 + 3003*a^10
+ 15015*a^8 + 36465*a^6 + 46189*a^4 + 29393*a^2 + 7429)*b^10*c^5*d^7 + 99*(
5*a^16 + 360*a^14 + 4620*a^12 + 24024*a^10 + 64350*a^8 + 97240*a^6 + 83980*
a^4 + 38760*a^2 + 7429)*b^8*c^4*d^8 + 44*(5*a^18 + 315*a^16 + 3780*a^14 + 1
9404*a^12 + 54054*a^10 + 90090*a^8 + 92820*a^6 + 58140*a^4 + 20349*a^2 + 30
59)*b^6*c^3*d^9 + 66*(a^20 + 50*a^18 + 525*a^16 + 2520*a^14 + 6930*a^12 + 1
2012*a^10 + 13650*a^8 + 10200*a^6 + 4845*a^4 + 1330*a^2 + 161)*b^4*c^2*d^10
 + 12*(a^22 + 33*a^20 + 275*a^18 + 1155*a^16 + 2970*a^14 + 5082*a^12 + 6006
*a^10 + 4950*a^8 + 2805*a^6 + 1045*a^4 + 231*a^2 + 23)*b^2*c*d^11 + (a^24 +
 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 +
 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^12 + 8*(3*b^23*c^11 + 11*(3*a^2
 + 23)*b^21*c^10*d + 33*(5*a^4 + 70*a^2 + 161)*b^19*c^9*d^2 + 99*(5*a^6 + 9
5*a^4 + 399*a^2 + 437)*b^17*c^8*d^3 + 22*(45*a^8 + 1020*a^6 + 5814*a^4 + 11
628*a^2 + 7429)*b^15*c^7*d^4 + 6*(231*a^10 + 5775*a^8 + 39270*a^6 + 106590*
a^4 + 124355*a^2 + 52003)*b^13*c^6*d^5 + 6*(231*a^12 + 6006*a^10 + 45045*a^
8 + 145860*a^6 + 230945*a^4 + 176358*a^2 + 52003)*b^11*c^5*d^6 + 22*(45*a^1
4 + 1155*a^12 + 9009*a^10 + 32175*a^8 + 60775*a^6 + 62985*a^4 + 33915*a^2 +
 7429)*b^9*c^4*d^7 + 99*(5*a^16 + 120*a^14 + 924*a^12 + 3432*a^10 + 7150*a^
8 + 8840*a^6 + 6460*a^4 + 2584*a^2 + 437)*b^7*c^3*d^8 + 33*(5*a^18 + 105*a^
16 + 756*a^14 + 2772*a^12 + 6006*a^10 + 8190*a^8 + 7140*a^6 + 3876*a^4 + 11
97*a^2 + 161)*b^5*c^2*d^9 + 11*(3*a^20 + 50*a^18 + 315*a^16 + 1080*a^14 + 2
310*a^12 + 3276*a^10 + 3150*a^8 + 2040*a^6 + 855*a^4 + 210*a^2 + 23)*b^3*c*
d^10 + 3*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a
^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11)*sqrt(c)*sqrt(d))) -
log(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 + 22*a^2 + 21)*b^20*c^1
0*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d^3 + 33*(5*a^8 + 260*a^
6 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*(a^10 + 61*a^8 + 570*a^6
 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(21*a^12 + 1386*a^10 + 1501
5*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a

^14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 169455*a^6 + 201773*a^4 + 121771
*a^2 + 29393)*b^10*c^5*d^7 + 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 9
438*a^8 + 13728*a^6 + 11492*a^4 + 5168*a^2 + 969)*b^8*c^4*d^8 + 33*(5*a^18
+ 285*a^16 + 3220*a^14 + 15876*a^12 + 42966*a^10 + 70070*a^8 + 70980*a^6 +
43860*a^4 + 15181*a^2 + 2261)*b^6*c^3*d^9 + 55*(a^20 + 46*a^18 + 465*a^16 +
2184*a^14 + 5922*a^12 + 10164*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 110
2*a^2 + 133)*b^4*c^2*d^10 + 11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 273
0*a^14 + 4662*a^12 + 5502*a^10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 +
21)*b^2*c*d^11 + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14
+ 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^12 + (b
^24*c^11*d + 11*(a^2 + 21)*b^22*c^10*d^2 + 55*(a^4 + 38*a^2 + 133)*b^20*c^9
d^3 + 33(5*a^6 + 255*a^4 + 1615*a^2 + 2261)*b^18*c^8*d^4 + 330*(a^8 + 60*
a^6 + 510*a^4 + 1292*a^2 + 969)*b^16*c^7*d^5 + 22*(21*a^10 + 1365*a^8 + 136
50*a^6 + 46410*a^4 + 62985*a^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a
^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^5*d^7
+ 330*(a^14 + 63*a^12 + 693*a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^
2 + 969)*b^10*c^4*d^8 + 33*(5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^10 + 30
030*a^8 + 40040*a^6 + 30940*a^4 + 12920*a^2 + 2261)*b^8*c^3*d^9 + 55*(a^18
+ 45*a^16 + 420*a^14 + 1764*a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a
^4 + 969*a^2 + 133)*b^6*c^2*d^10 + 11*(a^20 + 30*a^18 + 225*a^16 + 840*a^14
+ 1890*a^12 + 2772*a^10 + 2730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^
4*c*d^11 + (a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462
*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b^2*d^12)*x^2 - 2*(11*(a^2
+ 1)*b^21*c^10*d + 110*(a^4 + 8*a^2 + 7)*b^19*c^9*d^2 + 33*(15*a^6 + 205*a
^4 + 589*a^2 + 399)*b^17*c^8*d^3 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2
+ 323)*b^15*c^7*d^4 + 110*(21*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a
^2 + 2261)*b^13*c^6*d^5 + 4*(693*a^12 + 15708*a^10 + 105105*a^8 + 308880*a^
6 + 449735*a^4 + 319124*a^2 + 88179)*b^11*c^5*d^6 + 110*(21*a^14 + 483*a^12
+ 3465*a^10 + 11583*a^8 + 20735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^
4*d^7 + 264*(5*a^16 + 110*a^14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6
+ 4930*a^4 + 1938*a^2 + 323)*b^7*c^3*d^8 + 33*(15*a^18 + 295*a^16 + 2044*a
^14 + 7308*a^12 + 15554*a^10 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2
+ 399)*b^5*c^2*d^9 + 110*(a^20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 +
1008*a^10 + 966*a^8 + 624*a^6 + 261*a^4 + 64*a^2 + 7)*b^3*c*d^10 + 11*(a^22
+ 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8
+ 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11 + (11*b^23*c^10*d + 110*(a^2 + 7)*b
^21*c^9*d^2 + 33*(15*a^4 + 190*a^2 + 399)*b^19*c^8*d^3 + 264*(5*a^6 + 85*a^
4 + 323*a^2 + 323)*b^17*c^7*d^4 + 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a
^2 + 2261)*b^15*c^6*d^5 + 4*(693*a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4
+ 230945*a^2 + 88179)*b^13*c^5*d^6 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 8
580*a^6 + 12155*a^4 + 8398*a^2 + 2261)*b^11*c^4*d^7 + 264*(5*a^14 + 105*a^1
2 + 693*a^10 + 2145*a^8 + 3575*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^3*d^8
+ 33*(15*a^16 + 280*a^14 + 1764*a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 +
7140*a^4 + 2584*a^2 + 399)*b^7*c^2*d^9 + 110*(a^18 + 15*a^16 + 84*a^14 + 2
52*a^12 + 462*a^10 + 546*a^8 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c*d^10 +

11*(a^20 + 10*a^18 + 45*a^16 + 120*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 120*a^6 + 45*a^4 + 10*a^2 + 1)*b^3*d^11)*x^2 + 2*(11*a*b^22*c^10*d + 110*(a^3 + 7*a)*b^20*c^9*d^2 + 33*(15*a^5 + 190*a^3 + 399*a)*b^18*c^8*d^3 + 264*(5*a^7 + 85*a^5 + 323*a^3 + 323*a)*b^16*c^7*d^4 + 110*(21*a^9 + 420*a^7 + 2142*a^5 + 3876*a^3 + 2261*a)*b^14*c^6*d^5 + 4*(693*a^11 + 15015*a^9 + 90090*a^7 + 218790*a^5 + 230945*a^3 + 88179*a)*b^12*c^5*d^6 + 110*(21*a^13 + 462*a^11 + 3003*a^9 + 8580*a^7 + 12155*a^5 + 8398*a^3 + 2261*a)*b^10*c^4*d^7 + 264*(5*a^15 + 105*a^13 + 693*a^11 + 2145*a^9 + 3575*a^7 + 3315*a^5 + 1615*a^3 + 323*a)*b^8*c^3*d^8 + 33*(15*a^17 + 280*a^15 + 1764*a^13 + 5544*a^11 + 10010*a^9 + 10920*a^7 + 7140*a^5 + 2584*a^3 + 399*a)*b^6*c^2*d^9 + 110*(a^19 + 15*a^17 + 84*a^15 + 252*a^13 + 462*a^11 + 546*a^9 + 420*a^7 + 204*a^5 + 57*a^3 + 7*a)*b^4*c*d^10 + 11*(a^21 + 10*a^19 + 45*a^17 + 120*a^15 + 210*a^13 + 252*a^11 + 210*a^9 + 120*a^7 + 45*a^5 + 10*a^3 + a)*b^2*d^11)*x)*sqrt(c)*sqrt(d) + 2*(a*b^23*c^11*d + 11*(a^3 + 21*a)*b^21*c^10*d^2 + 55*(a^5 + 38*a^3 + 133*a)*b^19*c^9*d^3 + 33*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*a)*b^17*c^8*d^4 + 330*(a^9 + 60*a^7 + 510*a^5 + 1292*a^3 + 969*a)*b^15*c^7*d^5 + 22*(21*a^11 + 1365*a^9 + 13650*a^7 + 46410*a^5 + 62985*a^3 + 29393*a)*b^13*c^6*d^6 + 22*(21*a^13 + 1386*a^11 + 15015*a^9 + 60060*a^7 + 109395*a^5 + 92378*a^3 + 29393*a)*b^11*c^5*d^7 + 330*(a^15 + 63*a^13 + 693*a^11 + 3003*a^9 + 6435*a^7 + 7293*a^5 + 4199*a^3 + 969*a)*b^9*c^4*d^8 + 33*(5*a^17 + 280*a^15 + 2940*a^13 + 12936*a^11 + 30030*a^9 + 40040*a^7 + 30940*a^5 + 12920*a^3 + 2261*a)*b^7*c^3*d^9 + 55*(a^19 + 45*a^17 + 420*a^15 + 1764*a^13 + 4158*a^11 + 6006*a^9 + 5460*a^7 + 3060*a^5 + 969*a^3 + 133*a)*b^5*c^2*d^10 + 11*(a^21 + 30*a^19 + 225*a^17 + 840*a^15 + 1890*a^13 + 2772*a^11 + 2730*a^9 + 1800*a^7 + 765*a^5 + 190*a^3 + 21*a)*b^3*c*d^11 + (a^23 + 11*a^21 + 55*a^19 + 165*a^17 + 330*a^15 + 462*a^13 + 462*a^11 + 330*a^9 + 165*a^7 + 55*a^5 + 11*a^3 + a)*b*d^12)*x)/(b^24*c^12 + 12*(a^2 + 23)*b^22*c^11*d + 66*(a^4 + 42*a^2 + 161)*b^20*c^10*d^2 + 44*(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*b^18*c^9*d^3 + 99*(5*a^8 + 340*a^6 + 3230*a^4 + 9044*a^2 + 7429)*b^16*c^8*d^4 + 264*(3*a^10 + 225*a^8 + 2550*a^6 + 9690*a^4 + 14535*a^2 + 7429)*b^14*c^7*d^5 + 4*(231*a^12 + 18018*a^10 + 225225*a^8 + 1021020*a^6 + 2078505*a^4 + 1939938*a^2 + 676039)*b^12*c^6*d^6 + 264*(3*a^14 + 231*a^12 + 3003*a^10 + 15015*a^8 + 36465*a^6 + 46189*a^4 + 29393*a^2 + 7429)*b^10*c^5*d^7 + 99*(5*a^16 + 360*a^14 + 4620*a^12 + 24024*a^10 + 64350*a^8 + 97240*a^6 + 83980*a^4 + 38760*a^2 + 7429)*b^8*c^4*d^8 + 44*(5*a^18 + 315*a^16 + 3780*a^14 + 19404*a^12 + 54054*a^10 + 90090*a^8 + 92820*a^6 + 58140*a^4 + 20349*a^2 + 3059)*b^6*c^3*d^9 + 66*(a^20 + 50*a^18 + 525*a^16 + 2520*a^14 + 6930*a^12 + 12012*a^10 + 13650*a^8 + 10200*a^6 + 4845*a^4 + 1330*a^2 + 161)*b^4*c^2*d^10 + 12*(a^22 + 33*a^20 + 275*a^18 + 1155*a^16 + 2970*a^14 + 5082*a^12 + 6006*a^10 + 4950*a^8 + 2805*a^6 + 1045*a^4 + 231*a^2 + 23)*b^2*c*d^11 + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^12 - 8*(3*b^23*c^11 + 11*(3*a^2 + 23)*b^21*c^10*d + 33*(5*a^4 + 70*a^2 + 161)*b^19*c^9*d^2 + 99*(5*a^6 + 95*a^4 + 399*a^2 + 437)*b^17*c^8*d^3 + 22*(45*a^8 + 1020*a^6 + 5814*a^4 + 11628*a^2 + 7429)*b^15*c^7*d^4 + 6*(231*a^10 + 5775*a^8 + 39270*a^6 + 106590*a^4 + 1

```
24355*a^2 + 52003)*b^13*c^6*d^5 + 6*(231*a^12 + 6006*a^10 + 45045*a^8 + 145
860*a^6 + 230945*a^4 + 176358*a^2 + 52003)*b^11*c^5*d^6 + 22*(45*a^14 + 115
5*a^12 + 9009*a^10 + 32175*a^8 + 60775*a^6 + 62985*a^4 + 33915*a^2 + 7429)*
b^9*c^4*d^7 + 99*(5*a^16 + 120*a^14 + 924*a^12 + 3432*a^10 + 7150*a^8 + 884
0*a^6 + 6460*a^4 + 2584*a^2 + 437)*b^7*c^3*d^8 + 33*(5*a^18 + 105*a^16 + 75
6*a^14 + 2772*a^12 + 6006*a^10 + 8190*a^8 + 7140*a^6 + 3876*a^4 + 1197*a^2
+ 161)*b^5*c^2*d^9 + 11*(3*a^20 + 50*a^18 + 315*a^16 + 1080*a^14 + 2310*a^1
2 + 3276*a^10 + 3150*a^8 + 2040*a^6 + 855*a^4 + 210*a^2 + 23)*b^3*c*d^10 +
3*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 3
30*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11)*sqrt(c)*sqrt(d))) + 2*dilog
(((a + I)*b*d*x + b^2*c + (I*b^2*x + (-I*a + 1)*b)*sqrt(c)*sqrt(d))/(b^2*c
+ 2*(-I*a + 1)*b*sqrt(c)*sqrt(d) - (a^2 + 2*I*a - 1)*d)) - 2*dilog(((a + I)
*b*d*x + b^2*c - (I*b^2*x + (-I*a + 1)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a
 + 1)*b*sqrt(c)*sqrt(d) - (a^2 + 2*I*a - 1)*d)) - 2*dilog(((a - I)*b*d*x +
b^2*c + (I*b^2*x + (-I*a - 1)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a - 1)*b*s
qrt(c)*sqrt(d) - (a^2 - 2*I*a - 1)*d)) + 2*dilog(((a - I)*b*d*x + b^2*c - (
I*b^2*x + (-I*a - 1)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a - 1)*b*sqrt(c)*sq
rt(d) - (a^2 - 2*I*a - 1)*d)))/b)/sqrt(c*d) + arccot(b*x + a)*arctan(d*x/sq
rt(c*d))/sqrt(c*d) + arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/sqrt(c*d
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \text{Timed out}$$

```
[In] integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{d\,x^2+c}\,dx$$

```
[In] int(acot(a + b*x)/(c + d*x^2),x)
[Out] int(acot(a + b*x)/(c + d*x^2), x)
```

3.108 $\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 152

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} - \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d} + \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{2d}$$

```
[Out] -arccot(b*x+a)*ln(2/(1-I*(b*x+a)))/d+arccot(b*x+a)*ln(2*b*(d*x+c)/(b*c+I*d-
a*d)/(1-I*(b*x+a)))/d-1/2*I*polylog(2,1-2/(1-I*(b*x+a)))/d+1/2*I*polylog(2,
1-2*b*(d*x+c)/(b*c+I*d-a*d)/(1-I*(b*x+a)))/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used

= {5156, 4967, 2449, 2352, 2497}

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc-ad+id)(1-i(a+bx))}\right)}{2d} + \frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d} - \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d} - \frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

```
[In] Int[ArcCot[a + b*x]/(c + d*x),x]
```

```
[Out] -((ArcCot[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcCot[a + b*x]*Log[(2*b
*(c + d*x))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d - ((I/2)*PolyLog[2, 1
 - 2/(1 - I*(a + b*x))])/d + ((I/2)*PolyLog[2, 1 - (2*b*(c + d*x))/((b*c +
I*d - a*d)*(1 - I*(a + b*x)))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d}\\
&\quad -\frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, a+bx\right)}{d}+\frac{\text{Subst}\left(\int \frac{\log\left(\frac{2\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}{\left(\frac{id}{b}+\frac{bc-ad}{b}\right)(1-ix)}\right)}{1+x^2}\,dx, x, a+bx\right)}{d}\\
&= -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d}\\
&\quad +\frac{i\,\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{2d}-\frac{i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-i(a+bx)}\right)}{d}\\
&= -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d}\\
&\quad -\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{2d}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 325 vs. $2(152) = 304$.

Time = 0.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.14

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$$

$$= \frac{(\cot^{-1}(a+bx)+\arctan(a+bx))\log(c+dx)+\arctan(a+bx)\left(\log\left(\frac{1}{\sqrt{1+(a+bx)^2}}\right)-\log\left(\sin\left(\arctan\left(\frac{bc-}{d}\right.\right.\right.\right.}{}$$

```
[In] Integrate[ArcCot[a + b*x]/(c + d*x),x]
```

```
[Out] ((ArcCot[a + b*x] + ArcTan[a + b*x])*Log[c + d*x] + ArcTan[a + b*x]*(Log[1/
Sqrt[1 + (a + b*x)^2]] - Log[Sin[ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x]]])
 + ((I/4)*(Pi - 2*ArcTan[a + b*x])^2 + I*(ArcTan[(b*c - a*d)/d] + ArcTan[a
+ b*x])^2 - (Pi - 2*ArcTan[a + b*x])*Log[1 + E^((-2*I)*ArcTan[a + b*x])] -
2*(ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x])*Log[1 - E^((2*I)*(ArcTan[(b*c -
 a*d)/d] + ArcTan[a + b*x]))] + (Pi - 2*ArcTan[a + b*x])*Log[2/Sqrt[1 + (a
+ b*x)^2]] + 2*(ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x])*Log[2*Sin[ArcTan[(
b*c - a*d)/d] + ArcTan[a + b*x]]] + I*PolyLog[2, -E^((-2*I)*ArcTan[a + b*x]
)] + I*PolyLog[2, E^((2*I)*(ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x]))])/2)/
d
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{b \ln(ad-bc-d(bx+a)) \operatorname{arccot}(bx+a)}{d} - b\Bigg(-\frac{i \ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d} - \frac{i\Big(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\operatorname{di}}{2d}}{b}$
default	$\frac{\frac{b \ln(ad-bc-d(bx+a)) \operatorname{arccot}(bx+a)}{d} - b\Bigg(-\frac{i \ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d} - \frac{i\Big(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\operatorname{di}}{2d}}{b}$
parts	$\frac{\ln(dx+c) \operatorname{arccot}(bx+a)}{d} + b\Bigg(-\frac{i \ln(dx+c)\left(\ln\left(\frac{id-ad+bc-b(dx+c)}{-ad+bc+id}\right)-\ln\left(\frac{id+ad-bc+b(dx+c)}{ad-bc+id}\right)\right)}{2db} - \frac{i\Big(\operatorname{dilog}\Big(\frac{id-ad+bc-b}{-ad+bc+}}{}$
risch	$-\frac{i \operatorname{dilog}\left(\frac{iad-ibc+(-ibx-ia+1)d-d}{iad-ibc-d}\right)}{2d} - \frac{i \ln(-ibx-ia+1) \ln\left(\frac{iad-ibc+(-ibx-ia+1)d-d}{iad-ibc-d}\right)}{2d} + \frac{\pi \ln(iad-ibc+(-ibx-ia+1)}{2d}$

```
[In] int(arccot(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b*ln(a*d-b*c-d*(b*x+a))/d*arccot(b*x+a)-b*(-1/2*I*ln(a*d-b*c-d*(b*x+a)
)*(ln((I*d+d*(b*x+a))/(a*d-b*c+I*d))-ln((I*d-d*(b*x+a))/(b*c+I*d-a*d)))/d-1
/2*I*(dilog((I*d+d*(b*x+a))/(a*d-b*c+I*d))-dilog((I*d-d*(b*x+a))/(b*c+I*d-a
*d)))/d))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{dx+c}\,dx$$

```
[In] integrate(arccot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(arccot(b*x + a)/(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \text{Timed out}$$

```
[In] integrate(acot(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(130) = 260.

Time = 0.36 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.86

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \frac{\operatorname{arccot}(bx+a)\log(dx+c)}{d} + \frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(dx+c)}{d} + \frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2\,abcd+(a^2+1)d^2}, \frac{b^2c^2-abcd+(b^2cd-abd^2)x}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)\log(b^2x^2+2\,abx+a^2+1) - \arctan(bx+a)\log\left(\frac{b^2d^2 \cdots}{b^2c^2 \cdots}\right) \cdots}{2\,d}$$

```
[In] integrate(arccot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] arccot(b*x + a)*log(d*x + c)/d + arctan((b^2*x + a*b)/b)*log(d*x + c)/d + 1
/2*(arctan2((b*d^2*x + b*c*d)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2), (b^2*c
^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)
)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - arctan(b*x + a)*log((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)) + I*dilog((I*b*d
*x + (I*a + 1)*d)/(-I*b*c + (I*a + 1)*d)) - I*dilog((I*b*d*x + (I*a - 1)*d)
/(-I*b*c + (I*a - 1)*d)))/d
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{dx+c}\,dx$$

```
[In] integrate(arccot(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(arccot(b*x + a)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx=\int \frac{\mathrm{acot}(a+b\,x)}{c+d\,x}\,dx$$

```
[In] int(acot(a + b*x)/(c + d*x),x)
[Out] int(acot(a + b*x)/(c + d*x), x)
```

3.109 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 338

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx =& \frac{\log(i-a-bx)}{2bc}+\frac{i(a+bx)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2bc}\\&+\frac{\log(i+a+bx)}{2bc}-\frac{i(a+bx)\log\left(\frac{i+a+bx}{a+bx}\right)}{2bc}\\&+\frac{id\log\left(\frac{c(i-a-bx)}{ic-ac+bd}\right)\log(d+cx)}{2c^2}-\frac{id\log\left(-\frac{i-a-bx}{a+bx}\right)\log(d+cx)}{2c^2}\\&-\frac{id\log\left(\frac{c(i+a+bx)}{(i+a)c-bd}\right)\log(d+cx)}{2c^2}+\frac{id\log\left(\frac{i+a+bx}{a+bx}\right)\log(d+cx)}{2c^2}\\&-\frac{id\,\mathrm{PolyLog}\left(2,-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2}+\frac{id\,\mathrm{PolyLog}\left(2,\frac{b(d+cx)}{ic-ac+bd}\right)}{2c^2}\end{aligned}$$

```
[Out] 1/2*ln(I-a-b*x)/b/c+1/2*I*(b*x+a)*ln((-I+a+b*x)/(b*x+a))/b/c+1/2*ln(I+a+b*x
)/b/c-1/2*I*(b*x+a)*ln((I+a+b*x)/(b*x+a))/b/c+1/2*I*d*ln(c*(I-a-b*x)/(I*c-a
*c+b*d))*ln(c*x+d)/c^2-1/2*I*d*ln((-I+a+b*x)/(b*x+a))*ln(c*x+d)/c^2-1/2*I*d
*ln(c*(I+a+b*x)/((I+a)*c-b*d))*ln(c*x+d)/c^2+1/2*I*d*ln((I+a+b*x)/(b*x+a))*
ln(c*x+d)/c^2-1/2*I*d*polylog(2,-b*(c*x+d)/((I+a)*c-b*d))/c^2+1/2*I*d*polyl
og(2,b*(c*x+d)/(I*c-a*c+b*d))/c^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.25, number of steps used = 37, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5160, 2593, 2456, 2436, 2332, 2441, 2440, 2438, 199, 45}

$$
\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = & -\frac{id\,\text{PolyLog}\left(2,\frac{c(-a-bx+i)}{(i-a)c+bd}\right)}{2c^2}+\frac{id\,\text{PolyLog}\left(2,\frac{c(a+bx+i)}{(a+i)c-bd}\right)}{2c^2} \\
& -\frac{id\left(\log\left(-\frac{-a-bx+i}{a+bx}\right)+\log(a+bx)-\log(a+bx-i)\right)\log(cx+d)}{2c^2} \\
& +\frac{id\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\log(cx+d)}{2c^2} \\
& +\frac{id\log(a+bx+i)\log\left(-\frac{b(cx+d)}{-bd+(a+i)c}\right)}{2c^2} \\
& -\frac{id\log(a+bx-i)\log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2} \\
& +\frac{ix\left(\log\left(-\frac{-a-bx+i}{a+bx}\right)+\log(a+bx)-\log(a+bx-i)\right)}{2c} \\
& -\frac{i(-a-bx+i)\log(a+bx-i)}{2bc}-\frac{i(a+bx+i)\log(a+bx+i)}{2bc} \\
& -\frac{ix\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)}{2c}
\end{aligned}
$$

```
[In] Int[ArcCot[a + b*x]/(c + d/x),x]
```

```
[Out] ((I/2)*x*(Log[-((I - a - b*x)/(a + b*x))] + Log[a + b*x] - Log[-I + a + b*x
]))/c - ((I/2)*(I - a - b*x)*Log[-I + a + b*x])/(b*c) - ((I/2)*(I + a + b*x
)*Log[I + a + b*x])/(b*c) - ((I/2)*x*(Log[a + b*x] - Log[I + a + b*x] + Log
[(I + a + b*x)/(a + b*x)]))/c - ((I/2)*d*(Log[-((I - a - b*x)/(a + b*x))] +
 Log[a + b*x] - Log[-I + a + b*x])*Log[d + c*x])/c^2 + ((I/2)*d*(Log[a + b*
x] - Log[I + a + b*x] + Log[(I + a + b*x)/(a + b*x)])*Log[d + c*x])/c^2 + (
(I/2)*d*Log[I + a + b*x]*Log[-((b*(d + c*x))/((I + a)*c - b*d))])/c^2 - ((I
/2)*d*Log[-I + a + b*x]*Log[(b*(d + c*x))/((I - a)*c + b*d)])/c^2 - ((I/2)*
d*PolyLog[2, (c*(I - a - b*x))/((I - a)*c + b*d)])/c^2 + ((I/2)*d*PolyLog[2
, (c*(I + a + b*x))/((I + a)*c - b*d)])/c^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
```

```
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+\frac{d}{x}}\,dx-\frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+\frac{d}{x}}\,dx\\
&= \frac{1}{2}i\int \frac{\log(-i+a+bx)}{c+\frac{d}{x}}\,dx-\frac{1}{2}i\int \frac{\log(i+a+bx)}{c+\frac{d}{x}}\,dx\\
&\quad -\frac{1}{2}\left(i\left(-\log(a+bx)+\log(-i+a+bx)-\log\left(\frac{-i+a+bx}{a+bx}\right)\right)\right)\int \frac{1}{c+\frac{d}{x}}\,dx\\
&\quad +\frac{1}{2}\left(i\left(-\log(a+bx)+\log(i+a+bx)-\log\left(\frac{i+a+bx}{a+bx}\right)\right)\right)\int \frac{1}{c+\frac{d}{x}}\,dx\\
&= \frac{1}{2}i\int \left(\frac{\log(-i+a+bx)}{c}-\frac{d\log(-i+a+bx)}{c(d+cx)}\right)dx\\
&\quad -\frac{1}{2}i\int \left(\frac{\log(i+a+bx)}{c}-\frac{d\log(i+a+bx)}{c(d+cx)}\right)dx\\
&\quad -\frac{1}{2}\left(i\left(-\log(a+bx)+\log(-i+a+bx)-\log\left(\frac{-i+a+bx}{a+bx}\right)\right)\right)\int \frac{x}{d+cx}\,dx\\
&\quad +\frac{1}{2}\left(i\left(-\log(a+bx)+\log(i+a+bx)-\log\left(\frac{i+a+bx}{a+bx}\right)\right)\right)\int \frac{x}{d+cx}\,dx\\
&= \frac{i\int \log(-i+a+bx)\,dx}{2c}-\frac{i\int \log(i+a+bx)\,dx}{2c}\\
&\quad -\frac{(id)\int \frac{\log(-i+a+bx)}{d+cx}\,dx}{2c}+\frac{(id)\int \frac{\log(i+a+bx)}{d+cx}\,dx}{2c}\\
&\quad -\frac{1}{2}\left(i\left(-\log(a+bx)+\log(-i+a+bx)-\log\left(\frac{-i+a+bx}{a+bx}\right)\right)\right)\int \left(\frac{1}{c}-\frac{d}{c(d+cx)}\right)dx\\
&\quad +\frac{1}{2}\left(i\left(-\log(a+bx)+\log(i+a+bx)-\log\left(\frac{i+a+bx}{a+bx}\right)\right)\right)\int \left(\frac{1}{c}-\frac{d}{c(d+cx)}\right)dx
\end{aligned}
$$

$$=\frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c}-\frac{ix\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c}-\frac{id\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)\log(d+cx)}{2c^2}+\frac{id\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)\log(d+cx)}{2c^2}+\frac{id\log(i+a+bx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2}-\frac{id\log(-i+a+bx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2}+\frac{i\text{Subst}(\int\log(x)\,dx,x,-i+a+bx)}{2bc}-\frac{i\text{Subst}(\int\log(x)\,dx,x,i+a+bx)}{2bc}+\frac{(ibd)\int\frac{\log\left(\frac{b(d+cx)}{-((-i+a)c)+bd}\right)}{-i+a+bx}\,dx}{2c^2}-\frac{(ibd)\int\frac{\log\left(\frac{b(d+cx)}{-((i+a)c)+bd}\right)}{i+a+bx}\,dx}{2c^2}$$

$$=\frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c}-\frac{i(i-a-bx)\log(-i+a+bx)}{2bc}-\frac{i(i+a+bx)\log(i+a+bx)}{2bc}-\frac{ix\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c}-\frac{id\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)\log(d+cx)}{2c^2}+\frac{id\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)\log(d+cx)}{2c^2}+\frac{id\log(i+a+bx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2}-\frac{id\log(-i+a+bx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2}+\frac{(id)\text{Subst}\left(\int\frac{\log\left(1+\frac{cx}{-((-i+a)c)+bd}\right)}{x}\,dx,x,-i+a+bx\right)}{2c^2}-\frac{(id)\text{Subst}\left(\int\frac{\log\left(1+\frac{cx}{-((i+a)c)+bd}\right)}{x}\,dx,x,i+a+bx\right)}{2c^2}$$

$$= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i(i-a-bx)\log(-i+a+bx)}{2bc} - \frac{i(i+a+bx)\log(i+a+bx)}{2bc} - \frac{ix\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c} - \frac{id\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)\log(d+cx)}{2c^2} + \frac{id\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)\log(d+cx)}{2c^2} + \frac{id\log(i+a+bx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2} - \frac{id\log(-i+a+bx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2} - \frac{id\,\mathrm{PolyLog}\left(2,\frac{c(i-a-bx)}{(i-a)c+bd}\right)}{2c^2} + \frac{id\,\mathrm{PolyLog}\left(2,\frac{c(i+a+bx)}{(i+a)c-bd}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 7.62 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.52

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$$

$$= \frac{2ac^2\cot^{-1}(a+bx) - ibcd\pi\cot^{-1}(a+bx) + 2bc^2x\cot^{-1}(a+bx) - ibcd\cot^{-1}(a+bx)^2 + abcd\cot^{-1}(a+bx}{}$$

```
[In] Integrate[ArcCot[a + b*x]/(c + d/x),x]
```

```
[Out] (2*a*c^2*ArcCot[a + b*x] - I*b*c*d*Pi*ArcCot[a + b*x] + 2*b*c^2*x*ArcCot[a
+ b*x] - I*b*c*d*ArcCot[a + b*x]^2 + a*b*c*d*ArcCot[a + b*x]^2 - b^2*d^2*Ar
cCot[a + b*x]^2 - a*b*c*d*Sqrt[1 + c^2/(a*c - b*d)^2]*E^(I*ArcTan[c/(-(a*c)
 + b*d)])*ArcCot[a + b*x]^2 + b^2*d^2*Sqrt[1 + c^2/(a*c - b*d)^2]*E^(I*ArcT
an[c/(-(a*c) + b*d)])*ArcCot[a + b*x]^2 + (2*I)*b*c*d*ArcCot[a + b*x]*ArcTa
n[c/(-(a*c) + b*d)] - b*c*d*Pi*Log[1 + E^((-2*I)*ArcCot[a + b*x])] + 2*b*c*
d*ArcCot[a + b*x]*Log[1 - E^((2*I)*ArcCot[a + b*x])] - 2*b*c*d*ArcCot[a + b
*x]*Log[1 - E^((2*I)*(ArcCot[a + b*x] + ArcTan[c/(-(a*c) + b*d)]))] - 2*b*c
*d*ArcTan[c/(-(a*c) + b*d)]*Log[1 - E^((2*I)*(ArcCot[a + b*x] + ArcTan[c/(-
(a*c) + b*d)]))] - 2*c^2*Log[(a + b*x)^(-1)] - 2*c^2*Log[1/Sqrt[1 + (a + b*
x)^(-2)]] + b*c*d*Pi*Log[1/Sqrt[1 + (a + b*x)^(-2)]] + 2*b*c*d*ArcTan[c/(-(
a*c) + b*d)]*Log[Sin[ArcCot[a + b*x] + ArcTan[c/(-(a*c) + b*d)]]] - I*b*c*d
*PolyLog[2, E^((2*I)*ArcCot[a + b*x])] + I*b*c*d*PolyLog[2, E^((2*I)*(ArcCo
t[a + b*x] + ArcTan[c/(-(a*c) + b*d)]))])/(2*b*c^3)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)}{c}-\frac{\operatorname{arccot}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))\right.}{2}}{}}{}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)}{c}-\frac{\operatorname{arccot}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))\right.}{2}}{}}{}$
parts	$\frac{\operatorname{arccot}(bx+a)x}{c}-\frac{\operatorname{arccot}(bx+a)d\ln(cx+d)}{c^2}+\frac{b\left(\frac{\ln\left(a^2c^2-2abcd+2abc(cx+d)+b^2d^2-2b^2d(cx+d)+b^2(cx+d)^2+c^2\right)}{2b^2}-\frac{a\operatorname{arct}}{}\right.}{}$
risch	$\frac{id\operatorname{dilog}\left(\frac{iac-ibd+(-ibx-ia+1)c-c}{iac-ibd-c}\right)}{2c^2}-\frac{id\operatorname{dilog}\left(\frac{-iac+ibd+(ibx+ia+1)c-c}{-iac+ibd-c}\right)}{2c^2}-\frac{id\ln(ibx+ia+1)\ln\left(\frac{-iac+ibd+(ibx+ia+1)}{-iac+ibd-c}\right.}{2c^2}$

```
[In] int(arccot(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arccot(b*x+a)/c*(b*x+a)-arccot(b*x+a)*d*b/c^2*ln(a*c-b*d-c*(b*x+a))-1/
c*(-1/2*ln(a^2*c^2-2*a*b*c*d+b^2*d^2-2*a*c*(a*c-b*d-c*(b*x+a))+2*b*d*(a*c-b
*d-c*(b*x+a))+c^2+(a*c-b*d-c*(b*x+a))^2)-b*d*(-1/2*I*ln(a*c-b*d-c*(b*x+a))*
(ln((I*c+c*(b*x+a))/(a*c-b*d+I*c))-ln((I*c-c*(b*x+a))/(I*c-a*c+b*d)))/c-1/2
*I*(dilog((I*c+c*(b*x+a))/(a*c-b*d+I*c))-dilog((I*c-c*(b*x+a))/(I*c-a*c+b*d
)))/c)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{x}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(c+d/x),x, algorithm="fricas")
```

```
[Out] integral(x*arccot(b*x + a)/(c*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \text{Timed out}$$

```
[In] integrate(acot(b*x+a)/(c+d/x),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$$

$$= \frac{2\,bcx\arctan(1,bx+a) - bd\arctan(1,bx+a)\log\left(-\frac{b^2c^2x^2+2\,b^2cdx+b^2d^2}{2\,abcd-b^2d^2-(a^2+1)c^2}\right) - 2\,ac\arctan(bx+a) + i\,bd\mathrm{Li}_2\Big(}{}$$

```
[In] integrate(arccot(b*x+a)/(c+d/x),x, algorithm="maxima")
[Out] 1/2*(2*b*c*x*arctan2(1, b*x + a) - b*d*arctan2(1, b*x + a)*log(-(b^2*c^2*x^
2 + 2*b^2*c*d*x + b^2*d^2)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - 2*a*c*a
rctan(b*x + a) + I*b*d*dilog((b*c*x + (a + I)*c)/((a + I)*c - b*d)) - I*b*d
*dilog((b*c*x + (a - I)*c)/((a - I)*c - b*d)) - (b*d*arctan2(-(b*c^2*x + b*
c*d)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2), (a*b*c*d - b^2*d^2 + (a*b*c^2 -
 b^2*c*d)*x)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - c)*log(b^2*x^2 + 2*a*
b*x + a^2 + 1))/(b*c^2)
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{x}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(c+d/x),x, algorithm="giac")
[Out] integrate(arccot(b*x + a)/(c + d/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx=\int \frac{\mathrm{acot}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

```
[In] int(acot(a + b*x)/(c + d/x),x)
[Out] int(acot(a + b*x)/(c + d/x), x)
```

3.110 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 735

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx &= \frac{\log(i-a-bx)}{2bc}+\frac{i(a+bx)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2bc}\\
&-\frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c^{3/2}}+\frac{\log(i+a+bx)}{2bc}\\
&-\frac{i(a+bx)\log\left(\frac{i+a+bx}{a+bx}\right)}{2bc}+\frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{2c^{3/2}}\\
&-\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i-a-bx)}{(i-a)\sqrt{c}+ib\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}}\\
&+\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i+a+bx)}{(i+a)\sqrt{c}-ib\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}}\\
&+\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i-a-bx)}{(i-a)\sqrt{c}-ib\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}}\\
&-\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i+a+bx)}{(i+a)\sqrt{c}+ib\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}}\\
&-\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{b(\sqrt{d}-i\sqrt{c}x)}{(1+ia)\sqrt{c}+b\sqrt{d}}\right)}{4c^{3/2}}+\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{b(\sqrt{d}-i\sqrt{c}x)}{i(i+a)\sqrt{c}+b\sqrt{d}}\right)}{4c^{3/2}}\\
&+\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,-\frac{b(\sqrt{d}+i\sqrt{c}x)}{(1+ia)\sqrt{c}-b\sqrt{d}}\right)}{4c^{3/2}}-\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{b(\sqrt{d}+i\sqrt{c}x)}{(1-ia)\sqrt{c}+b\sqrt{d}}\right)}{4c^{3/2}}\end{aligned}$$

```
[Out] 1/2*ln(I-a-b*x)/b/c+1/2*I*(b*x+a)*ln((-I+a+b*x)/(b*x+a))/b/c+1/2*ln(I+a+b*x
)/b/c-1/2*I*(b*x+a)*ln((I+a+b*x)/(b*x+a))/b/c-1/2*I*arctan(x*c^(1/2)/d^(1/2
```

```
))*ln((-I+a+b*x)/(b*x+a))*d^(1/2)/c^(3/2)+1/2*I*arctan(x*c^(1/2)/d^(1/2))*l
n((I+a+b*x)/(b*x+a))*d^(1/2)/c^(3/2)+1/4*ln(1+I*x*c^(1/2)/d^(1/2))*ln((I-a-
b*x)*c^(1/2)/((I-a)*c^(1/2)-I*b*d^(1/2)))*d^(1/2)/c^(3/2)+1/4*ln(1-I*x*c^(1
/2)/d^(1/2))*ln((I+a+b*x)*c^(1/2)/((I+a)*c^(1/2)-I*b*d^(1/2)))*d^(1/2)/c^(3
/2)-1/4*ln(1-I*x*c^(1/2)/d^(1/2))*ln((I-a-b*x)*c^(1/2)/((I-a)*c^(1/2)+I*b*d
^(1/2)))*d^(1/2)/c^(3/2)-1/4*ln(1+I*x*c^(1/2)/d^(1/2))*ln((I+a+b*x)*c^(1/2)
/((I+a)*c^(1/2)+I*b*d^(1/2)))*d^(1/2)/c^(3/2)+1/4*polylog(2,-b*(I*x*c^(1/2)
+d^(1/2))/((1+I*a)*c^(1/2)-b*d^(1/2)))*d^(1/2)/c^(3/2)-1/4*polylog(2,b*(I*x
*c^(1/2)+d^(1/2))/((1-I*a)*c^(1/2)+b*d^(1/2)))*d^(1/2)/c^(3/2)-1/4*polylog(
2,b*(-I*x*c^(1/2)+d^(1/2))/((1+I*a)*c^(1/2)+b*d^(1/2)))*d^(1/2)/c^(3/2)+1/4
*polylog(2,b*(-I*x*c^(1/2)+d^(1/2))/(I*(I+a)*c^(1/2)+b*d^(1/2)))*d^(1/2)/c^
(3/2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.11, number of steps used = 57, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules

used = {5160, 2593, 2456, 2436, 2332, 2441, 2440, 2438, 199, 327, 211}

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \frac{ix\left(\log\left(-\frac{-a-bx+i}{a+bx}\right)+\log(a+bx)-\log(a+bx-i)\right)}{2c} - \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log\left(-\frac{-a-bx+i}{a+bx}\right)+\log(a+bx)-\log(a+bx-i)\right)}{2c^{3/2}} - \frac{i(-a-bx+i)\log(a+bx-i)}{2bc} - \frac{i(a+bx+i)\log(a+bx+i)}{2bc} - \frac{ix\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)}{2c} + \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)}{2c^{3/2}} - \frac{i\sqrt{d}\log(a+bx-i)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\log(a+bx+i)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{\sqrt{-c}(a+i)+b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\log(a+bx+i)\log\left(-\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{(a+i)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\log(a+bx-i)\log\left(\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{\sqrt{-c}(i-a)+b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+i)}{(i-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+i)}{\sqrt{-c}(i-a)+b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+i)}{(a+i)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+i)}{\sqrt{-c}(a+i)+b\sqrt{d}}\right)}{4(-c)^{3/2}}$$

```
[In] Int[ArcCot[a + b*x]/(c + d/x^2),x]
```

```
[Out] ((I/2)*x*(Log[-((I - a - b*x)/(a + b*x))] + Log[a + b*x] - Log[-I + a + b*x
]))/c - ((I/2)*Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[-((I - a - b*x)/(a
+ b*x))] + Log[a + b*x] - Log[-I + a + b*x]))/c^(3/2) - ((I/2)*(I - a - b*x
)*Log[-I + a + b*x])/(b*c) - ((I/2)*(I + a + b*x)*Log[I + a + b*x])/(b*c) -
 ((I/2)*x*(Log[a + b*x] - Log[I + a + b*x] + Log[(I + a + b*x)/(a + b*x)]))
/c + ((I/2)*Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[a + b*x] - Log[I + a +
 b*x] + Log[(I + a + b*x)/(a + b*x)]))/c^(3/2) - ((I/4)*Sqrt[d]*Log[-I + a
+ b*x]*Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/((I - a)*Sqrt[-c] - b*Sqrt[d]))])/(
-c)^(3/2) + ((I/4)*Sqrt[d]*Log[I + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/
((I + a)*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) - ((I/4)*Sqrt[d]*Log[I + a + b*
```

```
x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d]))])/(-c)^
(3/2) + ((I/4)*Sqrt[d]*Log[-I + a + b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((I
 - a)*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[
-c]*(I - a - b*x))/((I - a)*Sqrt[-c] - b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqr
t[d]*PolyLog[2, (Sqrt[-c]*(I - a - b*x))/((I - a)*Sqrt[-c] + b*Sqrt[d])])/(
-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/((I + a)*Sqr
t[-c] - b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I +
a + b*x))/((I + a)*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}}\,dx-\frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}}\,dx\\
&= \frac{1}{2}i\int \frac{\log(-i+a+bx)}{c+\frac{d}{x^2}}\,dx-\frac{1}{2}i\int \frac{\log(i+a+bx)}{c+\frac{d}{x^2}}\,dx\\
&\quad -\frac{1}{2}\left(i\left(-\log(a+bx)+\log(-i+a+bx)-\log\left(\frac{-i+a+bx}{a+bx}\right)\right)\right)\int \frac{1}{c+\frac{d}{x^2}}\,dx\\
&\quad +\frac{1}{2}\left(i\left(-\log(a+bx)+\log(i+a+bx)-\log\left(\frac{i+a+bx}{a+bx}\right)\right)\right)\int \frac{1}{c+\frac{d}{x^2}}\,dx
\end{aligned}
$$

$$= \frac{1}{2}i\int \left(\frac{\log(-i+a+bx)}{c} - \frac{d\log(-i+a+bx)}{c\,(d+cx^2)}\right)dx$$
$$- \frac{1}{2}i\int \left(\frac{\log(i+a+bx)}{c} - \frac{d\log(i+a+bx)}{c\,(d+cx^2)}\right)dx$$
$$- \frac{1}{2}\left(i\left(-\log(a+bx)+\log(-i+a+bx)-\log\left(\frac{-i+a+bx}{a+bx}\right)\right)\right)\int \frac{x^2}{d+cx^2}\,dx$$
$$+ \frac{1}{2}\left(i\left(-\log(a+bx)+\log(i+a+bx)-\log\left(\frac{i+a+bx}{a+bx}\right)\right)\right)\int \frac{x^2}{d+cx^2}\,dx$$

$$= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c}$$
$$- \frac{ix\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c} + \frac{i\int\log(-i+a+bx)\,dx}{2c}$$
$$- \frac{i\int\log(i+a+bx)\,dx}{2c} - \frac{(id)\int\frac{\log(-i+a+bx)}{d+cx^2}\,dx}{2c} + \frac{(id)\int\frac{\log(i+a+bx)}{d+cx^2}\,dx}{2c}$$
$$+ \frac{\left(id\left(-\log(a+bx)+\log(-i+a+bx)-\log\left(\frac{-i+a+bx}{a+bx}\right)\right)\right)\int\frac{1}{d+cx^2}\,dx}{2c}$$
$$- \frac{\left(id\left(-\log(a+bx)+\log(i+a+bx)-\log\left(\frac{i+a+bx}{a+bx}\right)\right)\right)\int\frac{1}{d+cx^2}\,dx}{2c}$$

$$= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c}$$
$$- \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c^{3/2}}$$
$$- \frac{ix\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c}$$
$$+ \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c^{3/2}}$$
$$+ \frac{i\text{Subst}(\int\log(x)\,dx,x,-i+a+bx)}{2bc} - \frac{i\text{Subst}(\int\log(x)\,dx,x,i+a+bx)}{2bc}$$
$$- \frac{(id)\int\left(\frac{\log(-i+a+bx)}{2\sqrt{d}\left(\sqrt{d}-\sqrt{-c}x\right)}+\frac{\log(-i+a+bx)}{2\sqrt{d}\left(\sqrt{d}+\sqrt{-c}x\right)}\right)dx}{2c}$$
$$+ \frac{(id)\int\left(\frac{\log(i+a+bx)}{2\sqrt{d}\left(\sqrt{d}-\sqrt{-c}x\right)}+\frac{\log(i+a+bx)}{2\sqrt{d}\left(\sqrt{d}+\sqrt{-c}x\right)}\right)dx}{2c}$$

$$= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c^{3/2}} - \frac{i(i-a-bx)\log(-i+a+bx)}{2bc} - \frac{i(i+a+bx)\log(i+a+bx)}{2bc} - \frac{ix\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c} + \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c^{3/2}} - \frac{\left(i\sqrt{d}\right)\int\frac{\log(-i+a+bx)}{\sqrt{d}-\sqrt{-c}x}\,dx}{4c} - \frac{\left(i\sqrt{d}\right)\int\frac{\log(-i+a+bx)}{\sqrt{d}+\sqrt{-c}x}\,dx}{4c} + \frac{\left(i\sqrt{d}\right)\int\frac{\log(i+a+bx)}{\sqrt{d}-\sqrt{-c}x}\,dx}{4c} + \frac{\left(i\sqrt{d}\right)\int\frac{\log(i+a+bx)}{\sqrt{d}+\sqrt{-c}x}\,dx}{4c}$$

$$
\begin{aligned}
&= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} \\
&\quad - \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c^{3/2}} \\
&\quad - \frac{i(i-a-bx)\log(-i+a+bx)}{2bc} - \frac{i(i+a+bx)\log(i+a+bx)}{2bc} \\
&\quad - \frac{ix\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c} \\
&\quad + \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c^{3/2}} \\
&\quad - \frac{i\sqrt{d}\log(-i+a+bx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&\quad + \frac{i\sqrt{d}\log(i+a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i+a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&\quad - \frac{i\sqrt{d}\log(i+a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&\quad + \frac{i\sqrt{d}\log(-i+a+bx)\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&\quad + \frac{\left(ib\sqrt{d}\right)\int\frac{\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(-i+a)\sqrt{-c}+b\sqrt{d}}\right)}{-i+a+bx}\,dx}{4(-c)^{3/2}} - \frac{\left(ib\sqrt{d}\right)\int\frac{\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i+a)\sqrt{-c}+b\sqrt{d}}\right)}{i+a+bx}\,dx}{4(-c)^{3/2}} \\
&\quad - \frac{\left(ib\sqrt{d}\right)\int\frac{\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{-((-i+a)\sqrt{-c})+b\sqrt{d}}\right)}{-i+a+bx}\,dx}{4(-c)^{3/2}} + \frac{\left(ib\sqrt{d}\right)\int\frac{\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{-((i+a)\sqrt{-c})+b\sqrt{d}}\right)}{i+a+bx}\,dx}{4(-c)^{3/2}}
\end{aligned}
$$

$$= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c^{3/2}} - \frac{i(i-a-bx)\log(-i+a+bx)}{2bc} - \frac{i(i+a+bx)\log(i+a+bx)}{2bc} - \frac{ix\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c} + \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c^{3/2}} - \frac{i\sqrt{d}\log(-i+a+bx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\log(i+a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i+a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\log(i+a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\log(-i+a+bx)\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{\left(i\sqrt{d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-c}x}{-((-i+a)\sqrt{-c})+b\sqrt{d}}\right)}{x}\,dx,x,-i+a+bx\right)}{4(-c)^{3/2}} + \frac{\left(i\sqrt{d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-c}x}{(-i+a)\sqrt{-c}+b\sqrt{d}}\right)}{x}\,dx,x,-i+a+bx\right)}{4(-c)^{3/2}} + \frac{\left(i\sqrt{d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-c}x}{-((i+a)\sqrt{-c})+b\sqrt{d}}\right)}{x}\,dx,x,i+a+bx\right)}{4(-c)^{3/2}} - \frac{\left(i\sqrt{d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-c}x}{(i+a)\sqrt{-c}+b\sqrt{d}}\right)}{x}\,dx,x,i+a+bx\right)}{4(-c)^{3/2}}$$

$$
\begin{aligned}
&= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} \\
&- \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c^{3/2}} \\
&- \frac{i(i-a-bx)\log(-i+a+bx)}{2bc} - \frac{i(i+a+bx)\log(i+a+bx)}{2bc} \\
&- \frac{ix\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c} \\
&+ \frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log(a+bx)-\log(i+a+bx)+\log\left(\frac{i+a+bx}{a+bx}\right)\right)}{2c^{3/2}} \\
&- \frac{i\sqrt{d}\log(-i+a+bx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&+ \frac{i\sqrt{d}\log(i+a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i+a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&- \frac{i\sqrt{d}\log(i+a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&+ \frac{i\sqrt{d}\log(-i+a+bx)\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(i-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&- \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i-a-bx)}{(i-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i-a-bx)}{(i-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} \\
&- \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i+a+bx)}{(i+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i+a+bx)}{(i+a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.27

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$$

$$= \frac{4c\log(a+bx)+2c\log\left(\frac{-i+a+bx}{a+bx}\right)+2iac\log\left(\frac{-i+a+bx}{a+bx}\right)+2ibcx\log\left(\frac{-i+a+bx}{a+bx}\right)+2c\log\left(\frac{i+a+bx}{a+bx}\right)-2iac\log\left(\right.}{}$$

```
[In] Integrate[ArcCot[a + b*x]/(c + d/x^2),x]
```

```
[Out] (4*c*Log[a + b*x] + 2*c*Log[(-I + a + b*x)/(a + b*x)] + (2*I)*a*c*Log[(-I +
 a + b*x)/(a + b*x)] + (2*I)*b*c*x*Log[(-I + a + b*x)/(a + b*x)] + 2*c*Log[
```

```
(I + a + b*x)/(a + b*x)] - (2*I)*a*c*Log[(I + a + b*x)/(a + b*x)] - (2*I)*b
*c*x*Log[(I + a + b*x)/(a + b*x)] + I*b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(-I
+ a + b*x))/((-I)*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])]*Log[Sqrt[d] - Sqrt[-c
]*x] - I*b*Sqrt[-c]*Sqrt[d]*Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[d] - Sqr
t[-c]*x] - I*b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(I + a + b*x))/((I + a)*Sqrt[
-c] + b*Sqrt[d])]*Log[Sqrt[d] - Sqrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]*Log[(I +
 a + b*x)/(a + b*x)]*Log[Sqrt[d] - Sqrt[-c]*x] - I*b*Sqrt[-c]*Sqrt[d]*Log[(
Sqrt[-c]*(I - a - b*x))/(-((-I + a)*Sqrt[-c]) + b*Sqrt[d])]*Log[Sqrt[d] + S
qrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]*Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[d]
 + Sqrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(I + a + b*x))/((I + a)
*Sqrt[-c] - b*Sqrt[d])]*Log[Sqrt[d] + Sqrt[-c]*x] - I*b*Sqrt[-c]*Sqrt[d]*Lo
g[(I + a + b*x)/(a + b*x)]*Log[Sqrt[d] + Sqrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]
*PolyLog[2, (b*(Sqrt[d] - Sqrt[-c]*x))/((-I)*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt
[d])] - I*b*Sqrt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[
-c] + a*Sqrt[-c] + b*Sqrt[d])] + I*b*Sqrt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d
] + Sqrt[-c]*x))/((-I)*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])] - I*b*Sqrt[-c]*S
qrt[d]*PolyLog[2, (b*(Sqrt[d] + Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*S
qrt[d])])/(4*b*c^2)
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 728, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\pi x}{2c}+\frac{\pi a}{2bc}+\frac{i\ln(ibx+ia+1)a}{2bc}-\frac{i\ln(-ibx-ia+1)a}{2bc}+\frac{i\ln(ibx+ia+1)x}{2c}-\frac{i\ln(-ibx-ia+1)x}{2c}+\frac{i\pi}{2bc}+\frac{ib\pi d\,\mathrm{arcta}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(arccot(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*Pi/c*x+1/2/b*Pi/c*a+1/2*I/b/c*ln(1+I*a+I*b*x)*a-1/2*I/b/c*ln(1-I*a-I*b*
x)*a+1/2*I/c*ln(1+I*a+I*b*x)*x-1/2*I/c*ln(1-I*a-I*b*x)*x+1/2*I/b*Pi/c+1/2*I
*b*Pi*d/c/(-b^2*c*d)^(1/2)*arctan(1/2*(2*I*a*c+2*(1-I*a-I*b*x)*c-2*c)/(-b^2
*c*d)^(1/2))+1/2/b/c*ln(1-I*a-I*b*x)-1/b/c+1/4/c^2*ln(1-I*a-I*b*x)*(c*d)^(1
/2)*ln((I*a*c-b*(c*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c-b*(c*d)^(1/2)-c))-1/4
/c^2*ln(1-I*a-I*b*x)*(c*d)^(1/2)*ln((I*a*c+b*(c*d)^(1/2)+(1-I*a-I*b*x)*c-c)
/(I*a*c+b*(c*d)^(1/2)-c))+1/4/c^2*dilog((I*a*c-b*(c*d)^(1/2)+(1-I*a-I*b*x)*
c-c)/(I*a*c-b*(c*d)^(1/2)-c))*(c*d)^(1/2)-1/4/c^2*dilog((I*a*c+b*(c*d)^(1/2
)+(1-I*a-I*b*x)*c-c)/(I*a*c+b*(c*d)^(1/2)-c))*(c*d)^(1/2)+1/2/b/c*ln(1+I*a+
I*b*x)+1/4/c^2*ln(1+I*a+I*b*x)*(c*d)^(1/2)*ln((I*a*c+b*(c*d)^(1/2)-(1+I*a+I
*b*x)*c+c)/(I*a*c+b*(c*d)^(1/2)+c))-1/4/c^2*ln(1+I*a+I*b*x)*(c*d)^(1/2)*ln(
(I*a*c-b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c-b*(c*d)^(1/2)+c))-1/4/c^2*(c
*d)^(1/2)*dilog((I*a*c-b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c-b*(c*d)^(1/2
```

```
)+c))+1/4/c^2*(c*d)^(1/2)*dilog((I*a*c+b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*
a*c+b*(c*d)^(1/2)+c))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{x^2}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="fricas")
[Out] integral(x^2*arccot(b*x + a)/(c*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Timed out}$$

```
[In] integrate(acot(b*x+a)/(c+d/x**2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8518 vs. $2(502) = 1004$.

Time = 0.93 (sec) , antiderivative size = 8518, normalized size of antiderivative = 11.59

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="maxima")
[Out] -(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arccot(b*x + a) - 1/8*(8*a*c
*arctan(b*x + a) + (4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d + (a
*b^3*d + (a^3 + a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*
b^3*c*d + (a^2 + 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2
 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d
+ (a^4 + 2*a^2 + 1)*c^2 + (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*s
qrt(d) + (a*b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d +
(a^4 + 2*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + 4*b*a
rctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d - (a*b^3*d + (a^3 + a)*b*c +
(b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*c^2
```

)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4*(b^3*d + (a
^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 -
(2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^
3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4
*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + b*log(c*x^2 + d)*log(((a^2 + 1
)*b^22*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(a^6 + 39*a^4 + 1
71*a^2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 22
61)*b^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969
)*b^14*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a
^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10
+ 75075*a^8 + 169455*a^6 + 201773*a^4 + 121771*a^2 + 29393)*b^10*c^7*d^5 +
330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 9438*a^8 + 13728*a^6 + 11492*
a^4 + 5168*a^2 + 969)*b^8*c^8*d^4 + 33*(5*a^18 + 285*a^16 + 3220*a^14 + 158
76*a^12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 43860*a^4 + 15181*a^2 + 2261
)*b^6*c^9*d^3 + 55*(a^20 + 46*a^18 + 465*a^16 + 2184*a^14 + 5922*a^12 + 101
64*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102*a^2 + 133)*b^4*c^10*d^2 +
11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730*a^14 + 4662*a^12 + 5502*a^
10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 21)*b^2*c^11*d + (a^24 + 12*
a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495
*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*c^12 + (b^24*c*d^11 + 11*(a^2 + 21)*b
^22*c^2*d^10 + 55*(a^4 + 38*a^2 + 133)*b^20*c^3*d^9 + 33*(5*a^6 + 255*a^4 +
1615*a^2 + 2261)*b^18*c^4*d^8 + 330*(a^8 + 60*a^6 + 510*a^4 + 1292*a^2 + 9
69)*b^16*c^5*d^7 + 22*(21*a^10 + 1365*a^8 + 13650*a^6 + 46410*a^4 + 62985*a
^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6
+ 109395*a^4 + 92378*a^2 + 29393)*b^12*c^7*d^5 + 330*(a^14 + 63*a^12 + 693*
a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2 + 969)*b^10*c^8*d^4 + 33*(
5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^10 + 30030*a^8 + 40040*a^6 + 30940*
a^4 + 12920*a^2 + 2261)*b^8*c^9*d^3 + 55*(a^18 + 45*a^16 + 420*a^14 + 1764*
a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^4 + 969*a^2 + 133)*b^6*c^10
d^2 + 11(a^20 + 30*a^18 + 225*a^16 + 840*a^14 + 1890*a^12 + 2772*a^10 + 2
730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4*c^11*d + (a^22 + 11*a^20 +
55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 +
55*a^4 + 11*a^2 + 1)*b^2*c^12)*x^2 + 2*(11*(a^2 + 1)*b^21*c*d^10 + 110*(a^4
+ 8*a^2 + 7)*b^19*c^2*d^9 + 33*(15*a^6 + 205*a^4 + 589*a^2 + 399)*b^17*c^3
d^8 + 264(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 + 323)*b^15*c^4*d^7 + 110*(2
1*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^2 + 2261)*b^13*c^5*d^6 + 4*
(693*a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6 + 449735*a^4 + 319124*a^2
+ 88179)*b^11*c^6*d^5 + 110*(21*a^14 + 483*a^12 + 3465*a^10 + 11583*a^8 + 2
0735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^7*d^4 + 264*(5*a^16 + 110*a^
14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6 + 4930*a^4 + 1938*a^2 + 323
)*b^7*c^8*d^3 + 33*(15*a^18 + 295*a^16 + 2044*a^14 + 7308*a^12 + 15554*a^10
+ 20930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 + 399)*b^5*c^9*d^2 + 110*(a^
20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1008*a^10 + 966*a^8 + 624*a^
6 + 261*a^4 + 64*a^2 + 7)*b^3*c^10*d + 11*(a^22 + 11*a^20 + 55*a^18 + 165*a
^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2

+ 1)*b*c^11 + (11*b^23*c*d^10 + 110*(a^2 + 7)*b^21*c^2*d^9 + 33*(15*a^4 + 1
90*a^2 + 399)*b^19*c^3*d^8 + 264*(5*a^6 + 85*a^4 + 323*a^2 + 323)*b^17*c^4*
d^7 + 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^2 + 2261)*b^15*c^5*d^6 + 4*
(693*a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 + 230945*a^2 + 88179)*b^13*c
^6*d^5 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 8580*a^6 + 12155*a^4 + 8398*a
^2 + 2261)*b^11*c^7*d^4 + 264*(5*a^14 + 105*a^12 + 693*a^10 + 2145*a^8 + 35
75*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^8*d^3 + 33*(15*a^16 + 280*a^14 +
1764*a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 + 7140*a^4 + 2584*a^2 + 399)*
b^7*c^9*d^2 + 110*(a^18 + 15*a^16 + 84*a^14 + 252*a^12 + 462*a^10 + 546*a^8
 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c^10*d + 11*(a^20 + 10*a^18 + 45*a^1
6 + 120*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 120*a^6 + 45*a^4 + 10*a^2 +
1)*b^3*c^11)*x^2 + 2*(11*a*b^22*c*d^10 + 110*(a^3 + 7*a)*b^20*c^2*d^9 + 33*
(15*a^5 + 190*a^3 + 399*a)*b^18*c^3*d^8 + 264*(5*a^7 + 85*a^5 + 323*a^3 + 3
23*a)*b^16*c^4*d^7 + 110*(21*a^9 + 420*a^7 + 2142*a^5 + 3876*a^3 + 2261*a)*
b^14*c^5*d^6 + 4*(693*a^11 + 15015*a^9 + 90090*a^7 + 218790*a^5 + 230945*a^
3 + 88179*a)*b^12*c^6*d^5 + 110*(21*a^13 + 462*a^11 + 3003*a^9 + 8580*a^7 +
 12155*a^5 + 8398*a^3 + 2261*a)*b^10*c^7*d^4 + 264*(5*a^15 + 105*a^13 + 693
*a^11 + 2145*a^9 + 3575*a^7 + 3315*a^5 + 1615*a^3 + 323*a)*b^8*c^8*d^3 + 33
*(15*a^17 + 280*a^15 + 1764*a^13 + 5544*a^11 + 10010*a^9 + 10920*a^7 + 7140
*a^5 + 2584*a^3 + 399*a)*b^6*c^9*d^2 + 110*(a^19 + 15*a^17 + 84*a^15 + 252*
a^13 + 462*a^11 + 546*a^9 + 420*a^7 + 204*a^5 + 57*a^3 + 7*a)*b^4*c^10*d +
11*(a^21 + 10*a^19 + 45*a^17 + 120*a^15 + 210*a^13 + 252*a^11 + 210*a^9 + 1
20*a^7 + 45*a^5 + 10*a^3 + a)*b^2*c^11)*x)*sqrt(c)*sqrt(d) + 2*(a*b^23*c*d^
11 + 11*(a^3 + 21*a)*b^21*c^2*d^10 + 55*(a^5 + 38*a^3 + 133*a)*b^19*c^3*d^9
 + 33*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*a)*b^17*c^4*d^8 + 330*(a^9 + 60*a^
7 + 510*a^5 + 1292*a^3 + 969*a)*b^15*c^5*d^7 + 22*(21*a^11 + 1365*a^9 + 136
50*a^7 + 46410*a^5 + 62985*a^3 + 29393*a)*b^13*c^6*d^6 + 22*(21*a^13 + 1386
*a^11 + 15015*a^9 + 60060*a^7 + 109395*a^5 + 92378*a^3 + 29393*a)*b^11*c^7*
d^5 + 330*(a^15 + 63*a^13 + 693*a^11 + 3003*a^9 + 6435*a^7 + 7293*a^5 + 419
9*a^3 + 969*a)*b^9*c^8*d^4 + 33*(5*a^17 + 280*a^15 + 2940*a^13 + 12936*a^11
 + 30030*a^9 + 40040*a^7 + 30940*a^5 + 12920*a^3 + 2261*a)*b^7*c^9*d^3 + 55
*(a^19 + 45*a^17 + 420*a^15 + 1764*a^13 + 4158*a^11 + 6006*a^9 + 5460*a^7 +
 3060*a^5 + 969*a^3 + 133*a)*b^5*c^10*d^2 + 11*(a^21 + 30*a^19 + 225*a^17 +
 840*a^15 + 1890*a^13 + 2772*a^11 + 2730*a^9 + 1800*a^7 + 765*a^5 + 190*a^3
 + 21*a)*b^3*c^11*d + (a^23 + 11*a^21 + 55*a^19 + 165*a^17 + 330*a^15 + 462
*a^13 + 462*a^11 + 330*a^9 + 165*a^7 + 55*a^5 + 11*a^3 + a)*b*c^12)*x)/(b^2
4*d^12 + 12*(a^2 + 23)*b^22*c*d^11 + 66*(a^4 + 42*a^2 + 161)*b^20*c^2*d^10
+ 44*(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*b^18*c^3*d^9 + 99*(5*a^8 + 340*a^6
 + 3230*a^4 + 9044*a^2 + 7429)*b^16*c^4*d^8 + 264*(3*a^10 + 225*a^8 + 2550*
a^6 + 9690*a^4 + 14535*a^2 + 7429)*b^14*c^5*d^7 + 4*(231*a^12 + 18018*a^10
+ 225225*a^8 + 1021020*a^6 + 2078505*a^4 + 1939938*a^2 + 676039)*b^12*c^6*d
^6 + 264*(3*a^14 + 231*a^12 + 3003*a^10 + 15015*a^8 + 36465*a^6 + 46189*a^4
 + 29393*a^2 + 7429)*b^10*c^7*d^5 + 99*(5*a^16 + 360*a^14 + 4620*a^12 + 240
24*a^10 + 64350*a^8 + 97240*a^6 + 83980*a^4 + 38760*a^2 + 7429)*b^8*c^8*d^4
 + 44*(5*a^18 + 315*a^16 + 3780*a^14 + 19404*a^12 + 54054*a^10 + 90090*a^8

+ 92820*a^6 + 58140*a^4 + 20349*a^2 + 3059)*b^6*c^9*d^3 + 66*(a^20 + 50*a^1
8 + 525*a^16 + 2520*a^14 + 6930*a^12 + 12012*a^10 + 13650*a^8 + 10200*a^6 +
4845*a^4 + 1330*a^2 + 161)*b^4*c^10*d^2 + 12*(a^22 + 33*a^20 + 275*a^18 +
1155*a^16 + 2970*a^14 + 5082*a^12 + 6006*a^10 + 4950*a^8 + 2805*a^6 + 1045*
a^4 + 231*a^2 + 23)*b^2*c^11*d + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495
*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^
2 + 1)*c^12 + 8*(3*b^23*d^11 + 11*(3*a^2 + 23)*b^21*c*d^10 + 33*(5*a^4 + 70
*a^2 + 161)*b^19*c^2*d^9 + 99*(5*a^6 + 95*a^4 + 399*a^2 + 437)*b^17*c^3*d^8
+ 22*(45*a^8 + 1020*a^6 + 5814*a^4 + 11628*a^2 + 7429)*b^15*c^4*d^7 + 6*(2
31*a^10 + 5775*a^8 + 39270*a^6 + 106590*a^4 + 124355*a^2 + 52003)*b^13*c^5*
d^6 + 6*(231*a^12 + 6006*a^10 + 45045*a^8 + 145860*a^6 + 230945*a^4 + 17635
8*a^2 + 52003)*b^11*c^6*d^5 + 22*(45*a^14 + 1155*a^12 + 9009*a^10 + 32175*a
^8 + 60775*a^6 + 62985*a^4 + 33915*a^2 + 7429)*b^9*c^7*d^4 + 99*(5*a^16 + 1
20*a^14 + 924*a^12 + 3432*a^10 + 7150*a^8 + 8840*a^6 + 6460*a^4 + 2584*a^2
+ 437)*b^7*c^8*d^3 + 33*(5*a^18 + 105*a^16 + 756*a^14 + 2772*a^12 + 6006*a^
10 + 8190*a^8 + 7140*a^6 + 3876*a^4 + 1197*a^2 + 161)*b^5*c^9*d^2 + 11*(3*a
^20 + 50*a^18 + 315*a^16 + 1080*a^14 + 2310*a^12 + 3276*a^10 + 3150*a^8 + 2
040*a^6 + 855*a^4 + 210*a^2 + 23)*b^3*c^10*d + 3*(a^22 + 11*a^20 + 55*a^18
+ 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 +
11*a^2 + 1)*b*c^11)*sqrt(c)*sqrt(d))) - b*log(c*x^2 + d)*log(((a^2 + 1)*b^2
2*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(a^6 + 39*a^4 + 171*a^
2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b
^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^1
4*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 +
92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10 + 75
075*a^8 + 169455*a^6 + 201773*a^4 + 121771*a^2 + 29393)*b^10*c^7*d^5 + 330*
(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 9438*a^8 + 13728*a^6 + 11492*a^4 +
5168*a^2 + 969)*b^8*c^8*d^4 + 33*(5*a^18 + 285*a^16 + 3220*a^14 + 15876*a^
12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 43860*a^4 + 15181*a^2 + 2261)*b^6
*c^9*d^3 + 55*(a^20 + 46*a^18 + 465*a^16 + 2184*a^14 + 5922*a^12 + 10164*a^
10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102*a^2 + 133)*b^4*c^10*d^2 + 11*(a
^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730*a^14 + 4662*a^12 + 5502*a^10 +
4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 21)*b^2*c^11*d + (a^24 + 12*a^22
+ 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8
+ 220*a^6 + 66*a^4 + 12*a^2 + 1)*c^12 + (b^24*c*d^11 + 11*(a^2 + 21)*b^22*c
^2*d^10 + 55*(a^4 + 38*a^2 + 133)*b^20*c^3*d^9 + 33*(5*a^6 + 255*a^4 + 1615
*a^2 + 2261)*b^18*c^4*d^8 + 330*(a^8 + 60*a^6 + 510*a^4 + 1292*a^2 + 969)*b
^16*c^5*d^7 + 22*(21*a^10 + 1365*a^8 + 13650*a^6 + 46410*a^4 + 62985*a^2 +
29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109
395*a^4 + 92378*a^2 + 29393)*b^12*c^7*d^5 + 330*(a^14 + 63*a^12 + 693*a^10
+ 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2 + 969)*b^10*c^8*d^4 + 33*(5*a^1
6 + 280*a^14 + 2940*a^12 + 12936*a^10 + 30030*a^8 + 40040*a^6 + 30940*a^4 +
12920*a^2 + 2261)*b^8*c^9*d^3 + 55*(a^18 + 45*a^16 + 420*a^14 + 1764*a^12
+ 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^4 + 969*a^2 + 133)*b^6*c^10*d^2
+ 11*(a^20 + 30*a^18 + 225*a^16 + 840*a^14 + 1890*a^12 + 2772*a^10 + 2730*a

^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4*c^11*d + (a^22 + 11*a^20 + 55*a
^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^
4 + 11*a^2 + 1)*b^2*c^12)*x^2 - 2*(11*(a^2 + 1)*b^21*c*d^10 + 110*(a^4 + 8*
a^2 + 7)*b^19*c^2*d^9 + 33*(15*a^6 + 205*a^4 + 589*a^2 + 399)*b^17*c^3*d^8
+ 264*(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 + 323)*b^15*c^4*d^7 + 110*(21*a^1
0 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^2 + 2261)*b^13*c^5*d^6 + 4*(693*
a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6 + 449735*a^4 + 319124*a^2 + 881
79)*b^11*c^6*d^5 + 110*(21*a^14 + 483*a^12 + 3465*a^10 + 11583*a^8 + 20735*
a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^7*d^4 + 264*(5*a^16 + 110*a^14 +
798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6 + 4930*a^4 + 1938*a^2 + 323)*b^7
*c^8*d^3 + 33*(15*a^18 + 295*a^16 + 2044*a^14 + 7308*a^12 + 15554*a^10 + 20
930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 + 399)*b^5*c^9*d^2 + 110*(a^20 +
16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1008*a^10 + 966*a^8 + 624*a^6 + 2
61*a^4 + 64*a^2 + 7)*b^3*c^10*d + 11*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 +
 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*
b*c^11 + (11*b^23*c*d^10 + 110*(a^2 + 7)*b^21*c^2*d^9 + 33*(15*a^4 + 190*a^
2 + 399)*b^19*c^3*d^8 + 264*(5*a^6 + 85*a^4 + 323*a^2 + 323)*b^17*c^4*d^7 +
 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^2 + 2261)*b^15*c^5*d^6 + 4*(693*
a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 + 230945*a^2 + 88179)*b^13*c^6*d^
5 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 8580*a^6 + 12155*a^4 + 8398*a^2 +
2261)*b^11*c^7*d^4 + 264*(5*a^14 + 105*a^12 + 693*a^10 + 2145*a^8 + 3575*a^
6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^8*d^3 + 33*(15*a^16 + 280*a^14 + 1764*
a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 + 7140*a^4 + 2584*a^2 + 399)*b^7*c
^9*d^2 + 110*(a^18 + 15*a^16 + 84*a^14 + 252*a^12 + 462*a^10 + 546*a^8 + 42
0*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c^10*d + 11*(a^20 + 10*a^18 + 45*a^16 + 1
20*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 120*a^6 + 45*a^4 + 10*a^2 + 1)*b^
3*c^11)*x^2 + 2*(11*a*b^22*c*d^10 + 110*(a^3 + 7*a)*b^20*c^2*d^9 + 33*(15*a
^5 + 190*a^3 + 399*a)*b^18*c^3*d^8 + 264*(5*a^7 + 85*a^5 + 323*a^3 + 323*a)
*b^16*c^4*d^7 + 110*(21*a^9 + 420*a^7 + 2142*a^5 + 3876*a^3 + 2261*a)*b^14*
c^5*d^6 + 4*(693*a^11 + 15015*a^9 + 90090*a^7 + 218790*a^5 + 230945*a^3 + 8
8179*a)*b^12*c^6*d^5 + 110*(21*a^13 + 462*a^11 + 3003*a^9 + 8580*a^7 + 1215
5*a^5 + 8398*a^3 + 2261*a)*b^10*c^7*d^4 + 264*(5*a^15 + 105*a^13 + 693*a^11
 + 2145*a^9 + 3575*a^7 + 3315*a^5 + 1615*a^3 + 323*a)*b^8*c^8*d^3 + 33*(15*
a^17 + 280*a^15 + 1764*a^13 + 5544*a^11 + 10010*a^9 + 10920*a^7 + 7140*a^5
+ 2584*a^3 + 399*a)*b^6*c^9*d^2 + 110*(a^19 + 15*a^17 + 84*a^15 + 252*a^13
+ 462*a^11 + 546*a^9 + 420*a^7 + 204*a^5 + 57*a^3 + 7*a)*b^4*c^10*d + 11*(a
^21 + 10*a^19 + 45*a^17 + 120*a^15 + 210*a^13 + 252*a^11 + 210*a^9 + 120*a^
7 + 45*a^5 + 10*a^3 + a)*b^2*c^11)*x)*sqrt(c)*sqrt(d) + 2*(a*b^23*c*d^11 +
11*(a^3 + 21*a)*b^21*c^2*d^10 + 55*(a^5 + 38*a^3 + 133*a)*b^19*c^3*d^9 + 33
*(5*a^7 + 255*a^5 + 1615*a^3 + 2261*a)*b^17*c^4*d^8 + 330*(a^9 + 60*a^7 + 5
10*a^5 + 1292*a^3 + 969*a)*b^15*c^5*d^7 + 22*(21*a^11 + 1365*a^9 + 13650*a^
7 + 46410*a^5 + 62985*a^3 + 29393*a)*b^13*c^6*d^6 + 22*(21*a^13 + 1386*a^11
 + 15015*a^9 + 60060*a^7 + 109395*a^5 + 92378*a^3 + 29393*a)*b^11*c^7*d^5 +
 330*(a^15 + 63*a^13 + 693*a^11 + 3003*a^9 + 6435*a^7 + 7293*a^5 + 4199*a^3
 + 969*a)*b^9*c^8*d^4 + 33*(5*a^17 + 280*a^15 + 2940*a^13 + 12936*a^11 + 30

030*a^9 + 40040*a^7 + 30940*a^5 + 12920*a^3 + 2261*a)*b^7*c^9*d^3 + 55*(a^1
9 + 45*a^17 + 420*a^15 + 1764*a^13 + 4158*a^11 + 6006*a^9 + 5460*a^7 + 3060
*a^5 + 969*a^3 + 133*a)*b^5*c^10*d^2 + 11*(a^21 + 30*a^19 + 225*a^17 + 840*
a^15 + 1890*a^13 + 2772*a^11 + 2730*a^9 + 1800*a^7 + 765*a^5 + 190*a^3 + 21
*a)*b^3*c^11*d + (a^23 + 11*a^21 + 55*a^19 + 165*a^17 + 330*a^15 + 462*a^13
 + 462*a^11 + 330*a^9 + 165*a^7 + 55*a^5 + 11*a^3 + a)*b*c^12)*x)/(b^24*d^1
2 + 12*(a^2 + 23)*b^22*c*d^11 + 66*(a^4 + 42*a^2 + 161)*b^20*c^2*d^10 + 44*
(5*a^6 + 285*a^4 + 1995*a^2 + 3059)*b^18*c^3*d^9 + 99*(5*a^8 + 340*a^6 + 32
30*a^4 + 9044*a^2 + 7429)*b^16*c^4*d^8 + 264*(3*a^10 + 225*a^8 + 2550*a^6 +
 9690*a^4 + 14535*a^2 + 7429)*b^14*c^5*d^7 + 4*(231*a^12 + 18018*a^10 + 225
225*a^8 + 1021020*a^6 + 2078505*a^4 + 1939938*a^2 + 676039)*b^12*c^6*d^6 +
264*(3*a^14 + 231*a^12 + 3003*a^10 + 15015*a^8 + 36465*a^6 + 46189*a^4 + 29
393*a^2 + 7429)*b^10*c^7*d^5 + 99*(5*a^16 + 360*a^14 + 4620*a^12 + 24024*a^
10 + 64350*a^8 + 97240*a^6 + 83980*a^4 + 38760*a^2 + 7429)*b^8*c^8*d^4 + 44
*(5*a^18 + 315*a^16 + 3780*a^14 + 19404*a^12 + 54054*a^10 + 90090*a^8 + 928
20*a^6 + 58140*a^4 + 20349*a^2 + 3059)*b^6*c^9*d^3 + 66*(a^20 + 50*a^18 + 5
25*a^16 + 2520*a^14 + 6930*a^12 + 12012*a^10 + 13650*a^8 + 10200*a^6 + 4845
*a^4 + 1330*a^2 + 161)*b^4*c^10*d^2 + 12*(a^22 + 33*a^20 + 275*a^18 + 1155*
a^16 + 2970*a^14 + 5082*a^12 + 6006*a^10 + 4950*a^8 + 2805*a^6 + 1045*a^4 +
 231*a^2 + 23)*b^2*c^11*d + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16
 + 792*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1
)*c^12 - 8*(3*b^23*d^11 + 11*(3*a^2 + 23)*b^21*c*d^10 + 33*(5*a^4 + 70*a^2
+ 161)*b^19*c^2*d^9 + 99*(5*a^6 + 95*a^4 + 399*a^2 + 437)*b^17*c^3*d^8 + 22
*(45*a^8 + 1020*a^6 + 5814*a^4 + 11628*a^2 + 7429)*b^15*c^4*d^7 + 6*(231*a^
10 + 5775*a^8 + 39270*a^6 + 106590*a^4 + 124355*a^2 + 52003)*b^13*c^5*d^6 +
 6*(231*a^12 + 6006*a^10 + 45045*a^8 + 145860*a^6 + 230945*a^4 + 176358*a^2
 + 52003)*b^11*c^6*d^5 + 22*(45*a^14 + 1155*a^12 + 9009*a^10 + 32175*a^8 +
60775*a^6 + 62985*a^4 + 33915*a^2 + 7429)*b^9*c^7*d^4 + 99*(5*a^16 + 120*a^
14 + 924*a^12 + 3432*a^10 + 7150*a^8 + 8840*a^6 + 6460*a^4 + 2584*a^2 + 437
)*b^7*c^8*d^3 + 33*(5*a^18 + 105*a^16 + 756*a^14 + 2772*a^12 + 6006*a^10 +
8190*a^8 + 7140*a^6 + 3876*a^4 + 1197*a^2 + 161)*b^5*c^9*d^2 + 11*(3*a^20 +
 50*a^18 + 315*a^16 + 1080*a^14 + 2310*a^12 + 3276*a^10 + 3150*a^8 + 2040*a
^6 + 855*a^4 + 210*a^2 + 23)*b^3*c^10*d + 3*(a^22 + 11*a^20 + 55*a^18 + 165
*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^
2 + 1)*b*c^11)*sqrt(c)*sqrt(d))) + 2*b*dilog(((a + I)*b*c*x + b^2*d + (I*b^
2*x + (-I*a + 1)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + 1)*b*sqrt(c)*sqrt(d) + b^2*
d - (a^2 + 2*I*a - 1)*c)) - 2*b*dilog(-((a + I)*b*c*x + b^2*d - (I*b^2*x +
(-I*a + 1)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + 1)*b*sqrt(c)*sqrt(d) - b^2*d + (a
^2 + 2*I*a - 1)*c)) - 2*b*dilog(((a - I)*b*c*x + b^2*d + (I*b^2*x + (-I*a -
 1)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - 1)*b*sqrt(c)*sqrt(d) + b^2*d - (a^2 - 2*
I*a - 1)*c)) + 2*b*dilog(-((a - I)*b*c*x + b^2*d - (I*b^2*x + (-I*a - 1)*b)
*sqrt(c)*sqrt(d))/(2*(-I*a - 1)*b*sqrt(c)*sqrt(d) - b^2*d + (a^2 - 2*I*a -
1)*c)))*sqrt(c)*sqrt(d) - 4*c*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b*c^2)

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Timed out}$$

```
[In] integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

```
[In] int(acot(a + b*x)/(c + d/x^2),x)
[Out] int(acot(a + b*x)/(c + d/x^2), x)
```

3.111 $\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 693

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = & -\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d}+\frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}\\
&-\frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&-\frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d}-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2}\\
&-\frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d}+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2}\\
&-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-i-a}d}\right)}{d^2}-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{d^2}\\
&+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2}+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}\end{aligned}$$

```
[Out] -I*c*ln((-I+a+b*x)/(b*x+a))*ln(c+d*x^(1/2))/d^2+I*c*ln((I+a+b*x)/(b*x+a))*l
n(c+d*x^(1/2))/d^2-I*c*ln(c+d*x^(1/2))*ln(d*((-I-a)^(1/2)-b^(1/2)*x^(1/2))/
```

```
(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x^(1/2))*ln(d*((I-a)^(1/2)-b^(1/
2)*x^(1/2))/(d*(I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(-d*((-I-a
)^(1/2)+b^(1/2)*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x^(1/2
))*ln(-d*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*
polylog(2,b^(1/2)*(c+d*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylo
g(2,b^(1/2)*(c+d*x^(1/2))/(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(
1/2)*(c+d*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(1/2)*(c
+d*x^(1/2))/(d*(I-a)^(1/2)+c*b^(1/2)))/d^2+2*I*arctanh(b^(1/2)*x^(1/2)/(I-a
)^(1/2))*(I-a)^(1/2)/d/b^(1/2)-2*I*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a
)^(1/2)/d/b^(1/2)+I*ln((-I+a+b*x)/(b*x+a))*x^(1/2)/d-I*ln((I+a+b*x)/(b*x+a)
)*x^(1/2)/d
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.00, number of steps used = 55, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {5160, 196, 45, 2608, 2603, 12, 492, 211, 214, 2604, 2465, 266, 2463, 2441, 2440, 2438}

$$\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = &-\frac{2i\sqrt{a+i}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+i}}\right)}{\sqrt{b}d}+\frac{2i\sqrt{-a+i}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a+i}}\right)}{\sqrt{b}d} \\
&-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-i}d}\right)}{d^2}-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a-i}d}\right)}{d^2} \\
&+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2}+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2} \\
&-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(-\sqrt{b}\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}c+\sqrt{-a-i}d}\right)}{d^2} \\
&+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(-\sqrt{b}\sqrt{x}+\sqrt{-a+i}\right)}{\sqrt{b}c+\sqrt{-a+i}d}\right)}{d^2} \\
&-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{b}\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}c-\sqrt{-a-i}d}\right)}{d^2} \\
&+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{b}\sqrt{x}+\sqrt{-a+i}\right)}{\sqrt{b}c-\sqrt{-a+i}d}\right)}{d^2} \\
&-\frac{ic\log\left(-\frac{-a-bx+i}{a+bx}\right)\log\left(c+d\sqrt{x}\right)}{d^2}+\frac{ic\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(c+d\sqrt{x}\right)}{d^2} \\
&+\frac{i\sqrt{x}\log\left(-\frac{-a-bx+i}{a+bx}\right)}{d}-\frac{i\sqrt{x}\log\left(\frac{a+bx+i}{a+bx}\right)}{d}
\end{aligned}$$

```
[In] Int[ArcCot[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((-2*I)*Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*d) + ((
2*I)*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*d) - (I*c
*Log[(d*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log
[c + d*Sqrt[x]])/d^2 + (I*c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b
]*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 - (I*c*Log[-((d*(Sqrt[-I - a]
 + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2
 + (I*c*Log[-((d*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*
d))]*Log[c + d*Sqrt[x]])/d^2 + (I*Sqrt[x]*Log[-((I - a - b*x)/(a + b*x))])/
d - (I*c*Log[c + d*Sqrt[x]]*Log[-((I - a - b*x)/(a + b*x))])/d^2 - (I*Sqrt[
x]*Log[(I + a + b*x)/(a + b*x)])/d + (I*c*Log[c + d*Sqrt[x]]*Log[(I + a + b
*x)/(a + b*x)])/d^2 - (I*c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c
- Sqrt[-I - a]*d)])/d^2 - (I*c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b
]*c + Sqrt[-I - a]*d)])/d^2 + (I*c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sq
rt[b]*c - Sqrt[I - a]*d)])/d^2 + (I*c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/
(Sqrt[b]*c + Sqrt[I - a]*d)])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+d\sqrt{x}}\,dx-\frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+d\sqrt{x}}\,dx\\
&= i\text{Subst}\left(\int \frac{x\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx,x,\sqrt{x}\right)-i\text{Subst}\left(\int \frac{x\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx,x,\sqrt{x}\right)\\
&= i\text{Subst}\left(\int \left(\frac{\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{d}-\frac{c\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{d(c+dx)}\right)dx,x,\sqrt{x}\right)\\
&\quad -i\text{Subst}\left(\int \left(\frac{\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{d}-\frac{c\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{d(c+dx)}\right)dx,x,\sqrt{x}\right)
\end{aligned}
$$

$$= \frac{i\text{Subst}\left(\int \log\left(\frac{-i+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{d} - \frac{i\text{Subst}\left(\int \log\left(\frac{i+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{d} - \frac{(ic)\text{Subst}\left(\int \frac{\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx, x, \sqrt{x}\right)}{d} + \frac{(ic)\text{Subst}\left(\int \frac{\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} - \frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d} + \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2} + \frac{(ic)\text{Subst}\left(\int \frac{(a+bx^2)\left(\frac{2bx}{a+bx^2} - \frac{2bx\left(-i+a+bx^2\right)}{\left(a+bx^2\right)^2}\right)\log(c+dx)}{-i+a+bx^2}\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(ic)\text{Subst}\left(\int \frac{(a+bx^2)\left(\frac{2bx}{a+bx^2} - \frac{2bx\left(i+a+bx^2\right)}{\left(a+bx^2\right)^2}\right)\log(c+dx)}{i+a+bx^2}\,dx, x, \sqrt{x}\right)}{d^2} - \frac{i\text{Subst}\left(\int \frac{2ibx^2}{(a+bx^2)(-i+a+bx^2)}\,dx, x, \sqrt{x}\right)}{d} + \frac{i\text{Subst}\left(\int -\frac{2ibx^2}{(a+bx^2)(i+a+bx^2)}\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} - \frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d} + \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2} + \frac{(ic)\text{Subst}\left(\int \left(-\frac{2bx\log(c+dx)}{a+bx^2} + \frac{2bx\log(c+dx)}{-i+a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(ic)\text{Subst}\left(\int \left(-\frac{2bx\log(c+dx)}{a+bx^2} + \frac{2bx\log(c+dx)}{i+a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{d^2} + \frac{(2b)\text{Subst}\left(\int \frac{x^2}{(a+bx^2)(-i+a+bx^2)}\,dx, x, \sqrt{x}\right)}{d} + \frac{(2b)\text{Subst}\left(\int \frac{x^2}{(a+bx^2)(i+a+bx^2)}\,dx, x, \sqrt{x}\right)}{d}$$

$$
\begin{aligned}
&= \frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} \\
&\quad - \frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d} + \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2} \\
&\quad + \frac{(2ibc)\text{Subst}\left(\int \frac{x\log(c+dx)}{-i+a+bx^2}\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(2ibc)\text{Subst}\left(\int \frac{x\log(c+dx)}{i+a+bx^2}\,dx, x, \sqrt{x}\right)}{d^2} \\
&\quad + \frac{(2(1-ia))\text{Subst}\left(\int \frac{1}{i+a+bx^2}\,dx, x, \sqrt{x}\right)}{d} + \frac{(2(1+ia))\text{Subst}\left(\int \frac{1}{-i+a+bx^2}\,dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} + \frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d} \\
&\quad + \frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} \\
&\quad - \frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d} + \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2} \\
&\quad - \frac{(2ibc)\text{Subst}\left(\int \left(-\frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-i-a}-\sqrt{b}x\right)} + \frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-i-a}+\sqrt{b}x\right)}\right)dx, x, \sqrt{x}\right)}{d^2} \\
&\quad + \frac{(2ibc)\text{Subst}\left(\int \left(-\frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{i-a}-\sqrt{b}x\right)} + \frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{i-a}+\sqrt{b}x\right)}\right)dx, x, \sqrt{x}\right)}{d^2} \\
&= -\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} + \frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d} + \frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d} \\
&\quad - \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} - \frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d} + \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2} \\
&\quad + \frac{\left(i\sqrt{b}c\right)\text{Subst}\left(\int \frac{\log(c+dx)}{\sqrt{-i-a}-\sqrt{b}x}\,dx, x, \sqrt{x}\right)}{d^2} - \frac{\left(i\sqrt{b}c\right)\text{Subst}\left(\int \frac{\log(c+dx)}{\sqrt{i-a}-\sqrt{b}x}\,dx, x, \sqrt{x}\right)}{d^2} \\
&\quad - \frac{\left(i\sqrt{b}c\right)\text{Subst}\left(\int \frac{\log(c+dx)}{\sqrt{-i-a}+\sqrt{b}x}\,dx, x, \sqrt{x}\right)}{d^2} + \frac{\left(i\sqrt{b}c\right)\text{Subst}\left(\int \frac{\log(c+dx)}{\sqrt{i-a}+\sqrt{b}x}\,dx, x, \sqrt{x}\right)}{d^2}
\end{aligned}
$$

$$= -\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} + \frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}$$

$$- \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} + \frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$

$$- \frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$

$$+ \frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} + \frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d}$$

$$- \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} - \frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d}$$

$$+ \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2} + \frac{(ic)\text{Subst}\left(\int\frac{\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{c+dx}dx, x, \sqrt{x}\right)}{d}$$

$$- \frac{(ic)\text{Subst}\left(\int\frac{\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)}{c+dx}dx, x, \sqrt{x}\right)}{d}$$

$$+ \frac{(ic)\text{Subst}\left(\int\frac{\log\left(\frac{d\left(\sqrt{-i-a}+\sqrt{b}x\right)}{-\sqrt{b}c+\sqrt{-i-a}d}\right)}{c+dx}dx, x, \sqrt{x}\right)}{d}$$

$$- \frac{(ic)\text{Subst}\left(\int\frac{\log\left(\frac{d\left(\sqrt{i-a}+\sqrt{b}x\right)}{-\sqrt{b}c+\sqrt{i-a}d}\right)}{c+dx}dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} + \frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}$$

$$- \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} + \frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$

$$- \frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$

$$+ \frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} + \frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d}$$

$$- \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} - \frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d}$$

$$+ \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2} + \frac{(ic)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}c+\sqrt{-i-a}d}\right)}{x}dx, x, c+d\sqrt{x}\right)}{d^2}$$

$$+ \frac{(ic)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{x}dx, x, c+d\sqrt{x}\right)}{d^2}$$

$$- \frac{(ic)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}c+\sqrt{i-a}d}\right)}{x}dx, x, c+d\sqrt{x}\right)}{d^2}$$

$$- \frac{(ic)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}c+\sqrt{i-a}d}\right)}{x}dx, x, c+d\sqrt{x}\right)}{d^2}$$

$$= -\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d} + \frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}$$

$$- \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} + \frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$

$$- \frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$

$$+ \frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}$$

$$+ \frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2}$$

$$- \frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d} + \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2}$$

$$- \frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-i-a}d}\right)}{d^2} - \frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{d^2}$$

$$+ \frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2} + \frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 618, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = -\frac{i\left(\frac{2\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}} - \frac{2\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}} + c\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right) - c\log\left(\frac{d\left(\sqrt{i-a}-\right.}{\sqrt{b}c+\sqrt{i}}\right.\right.}{}$$

```
[In] Integrate[ArcCot[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((-I)*((2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (2
*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c*Log[(d*(
Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d*Sq
rt[x]] - c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]
*d)]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(-(Sqr
t[b]*c) + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[I - a] + Sqr
t[b]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt[
x]*Log[(-I + a + b*x)/(a + b*x)] + c*Log[c + d*Sqrt[x]]*Log[(-I + a + b*x)/
(a + b*x)] + d*Sqrt[x]*Log[(I + a + b*x)/(a + b*x)] - c*Log[c + d*Sqrt[x]]*
Log[(I + a + b*x)/(a + b*x)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt
```

```
[b]*c - Sqrt[-I - a]*d)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*
c + Sqrt[-I - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c -
Sqrt[I - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[
I - a]*d)]))/d^2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2\,\mathrm{arccot}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccot}(bx+a)c\ln(c+d\sqrt{x})}{d^2}+\frac{4b\left(\frac{d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\left(-4c\ldots\right.\right.}\ldots\right.}{}\ldots\right.}{}$ …
default	$\frac{2\,\mathrm{arccot}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccot}(bx+a)c\ln(c+d\sqrt{x})}{d^2}+\frac{4b\left(\frac{d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\left(-4c\ldots\right.\right.}\ldots\right.}{}\ldots\right.}{}$ …

```
[In] int(arccot(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arccot(b*x+a)/d*x^(1/2)-2*arccot(b*x+a)*c/d^2*ln(c+d*x^(1/2))+4*b/d^2*(1/
4*d^2/b*sum((_R^2-2*_R*c+c^2)/(_R^3*b-3*_R^2*b*c+_R*a*d^2+3*_R*b*c^2-a*c*d^
2-b*c^3)*ln(d*x^(1/2)-_R+c),_R=RootOf(b^2*_Z^4-4*b^2*c*_Z^3+(2*a*b*d^2+6*b^
2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d^4))
-1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*c+a*d^2+b*c^2)*(ln(c+d*x^(1/2))*ln((-d*
x^(1/2)+_R1-c)/_R1)+dilog((-d*x^(1/2)+_R1-c)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^
2*c*_Z^3+(2*a*b*d^2+6*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a
*b*c^2*d^2+b^2*c^4+d^4)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{d\sqrt{x}+c}\,dx$$

```
[In] integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
[Out] integral((d*sqrt(x)*arccot(b*x + a) - c*arccot(b*x + a))/(d^2*x - c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \text{Timed out}$$

```
[In] integrate(acot(b*x+a)/(c+d*x**(1/2)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{d\sqrt{x}+c}\,dx$$

```
[In] integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
[Out] integrate(arccot(b*x + a)/(d*sqrt(x) + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]War
ning, replacing 0 by -24, a substitution variable should perhaps be purged.
Warnin
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

```
[In] int(acot(a + b*x)/(c + d*x^(1/2)),x)
[Out] int(acot(a + b*x)/(c + d*x^(1/2)), x)
```

3.112 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}} dx$

Optimal result

Integrand size = 18, antiderivative size = 830

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} - \frac{2i\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2}$$
$$+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+ \frac{(1+ia)\log(i-a-bx)}{2bc} - \frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2}$$
$$+ \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3}$$
$$+ \frac{(1-ia)\log(i+a+bx)}{2bc} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2}$$
$$- \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3}$$
$$+ \frac{id^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{c^3} - \frac{id^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3}$$
$$+ \frac{id^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{c^3} - \frac{id^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3}$$

```
[Out] 1/2*(1+I*a)*ln(I-a-b*x)/b/c+I*d^2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)+b^(1/2
)*x^(1/2))/(c*(-I-a)^(1/2)-d*b^(1/2)))/c^3+1/2*(1-I*a)*ln(I+a+b*x)/b/c+I*d^
2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(-I-a)^(1/2)+d*b^(
1/2)))/c^3-I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(I-a)^(1/2)+d*b^(1/2)))
/c^3-I*d^2*ln((I+a+b*x)/(b*x+a))*ln(d+c*x^(1/2))/c^3-I*d*ln((-I+a+b*x)/(b*x
+a))*x^(1/2)/c^2+I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(-I-a)^(1/2)-d*b
^(1/2)))/c^3-2*I*d*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))*(I-a)^(1/2)/c^2/b^(
1/2)-I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(I-a)^(1/2)-d*b^(1/2)))/c^3-
I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(I-a)^(1/2)-d*b
^(1/2)))/c^3+I*d*ln((I+a+b*x)/(b*x+a))*x^(1/2)/c^2+1/2*I*x*ln((-I+a+b*x)/(b
*x+a))/c-1/2*I*x*ln((I+a+b*x)/(b*x+a))/c+2*I*d*arctan(b^(1/2)*x^(1/2)/(I+a)
```

```
^(1/2))*(I+a)^(1/2)/c^2/b^(1/2)+I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-
I-a)^(1/2)+d*b^(1/2)))/c^3-I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)-b^(1/2)*
x^(1/2))/(c*(I-a)^(1/2)+d*b^(1/2)))/c^3+I*d^2*ln((-I+a+b*x)/(b*x+a))*ln(d+c
*x^(1/2))/c^3
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 830, normalized size of antiderivative = 1.00, number of steps used = 65, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.056, Rules used = {5160, 196, 46, 2608, 2603, 12, 492, 211, 214, 2605, 457, 78, 2604, 2465, 266, 2463, 2441, 2440, 2438}

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \frac{i\log\left(\frac{c\left(\sqrt{-a-i}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-i}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3} - \frac{i\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3} + \frac{i\log\left(\frac{c\left(\sqrt{-a-i}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-i}c-\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3} - \frac{i\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3} + \frac{i\log\left(\sqrt{x}c+d\right)\log\left(-\frac{-a-bx+i}{a+bx}\right)d^2}{c^3} - \frac{i\log\left(\sqrt{x}c+d\right)\log\left(\frac{a+bx+i}{a+bx}\right)d^2}{c^3} + \frac{i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{-a-i}c-\sqrt{b}d}\right)d^2}{c^3} - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)d^2}{c^3} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{-a-i}c+\sqrt{b}d}\right)d^2}{c^3} - \frac{i\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(\sqrt{x}c+d\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)d^2}{c^3} + \frac{2i\sqrt{a+i}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+i}}\right)d}{\sqrt{b}c^2} - \frac{2i\sqrt{i-a}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)d}{\sqrt{b}c^2} - \frac{i\sqrt{x}\log\left(-\frac{-a-bx+i}{a+bx}\right)d}{c^2} + \frac{i\sqrt{x}\log\left(\frac{a+bx+i}{a+bx}\right)d}{c^2} + \frac{(ia+1)\log(-a-bx+i)}{2bc} + \frac{ix\log\left(-\frac{-a-bx+i}{a+bx}\right)}{2c} + \frac{(1-ia)\log(a+bx+i)}{2bc} - \frac{ix\log\left(\frac{a+bx+i}{a+bx}\right)}{2c}$$

```
[In] Int[ArcCot[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((2*I)*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2) -
 ((2*I)*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2)
 + (I*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b
]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x
]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (I*d^2*Log[(c*(S
qrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqr
t[x]])/c^3 - (I*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c
- Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 + ((1 + I*a)*Log[I - a - b*x])/(2*b*c
) - (I*d*Sqrt[x]*Log[-((I - a - b*x)/(a + b*x))])/c^2 + ((I/2)*x*Log[-((I -
 a - b*x)/(a + b*x))])/c + (I*d^2*Log[d + c*Sqrt[x]]*Log[-((I - a - b*x)/(a
 + b*x))])/c^3 + ((1 - I*a)*Log[I + a + b*x])/(2*b*c) + (I*d*Sqrt[x]*Log[(I
 + a + b*x)/(a + b*x)])/c^2 - ((I/2)*x*Log[(I + a + b*x)/(a + b*x)])/c - (I
*d^2*Log[d + c*Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)])/c^3 + (I*d^2*PolyLog[
2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d))])/c^3 - (I*d^2
*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3
+ (I*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]
)/c^3 - (I*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b
]*d)])/c^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$\text{integral} = \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}\,dx - \frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= i\text{Subst}\left(\int \frac{x^2\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \frac{x^2\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right)$$

$$= i\text{Subst}\left(\int \left(-\frac{d\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{c^2} + \frac{x\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{c} + \frac{d^2\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{c^2(d+cx)}\right)\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \left(-\frac{d\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{c^2} + \frac{x\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{c} + \frac{d^2\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{c^2(d+cx)}\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{i\text{Subst}\left(\int x\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{c} - \frac{i\text{Subst}\left(\int x\log\left(\frac{i+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{c} - \frac{(id)\text{Subst}\left(\int \log\left(\frac{-i+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{c^2} + \frac{(id)\text{Subst}\left(\int \log\left(\frac{i+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{c^2} + \frac{(id^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right)}{c^2} - \frac{(id^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2} - \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3} - \frac{i\text{Subst}\left(\int\frac{2ibx^3}{(a+bx^2)(-i+a+bx^2)}\,dx,x,\sqrt{x}\right)}{2c} + \frac{i\text{Subst}\left(\int-\frac{2ibx^3}{(a+bx^2)(i+a+bx^2)}\,dx,x,\sqrt{x}\right)}{2c} + \frac{(id)\text{Subst}\left(\int\frac{2ibx^2}{(a+bx^2)(-i+a+bx^2)}\,dx,x,\sqrt{x}\right)}{c^2} - \frac{(id)\text{Subst}\left(\int-\frac{2ibx^2}{(a+bx^2)(i+a+bx^2)}\,dx,x,\sqrt{x}\right)}{c^2} - \frac{(id^2)\,\text{Subst}\left(\int\frac{(a+bx^2)\left(\frac{2bx}{a+bx^2}-\frac{2bx\left(-i+a+bx^2\right)}{\left(a+bx^2\right)^2}\right)\log(d+cx)}{-i+a+bx^2}\,dx,x,\sqrt{x}\right)}{c^3} + \frac{(id^2)\,\text{Subst}\left(\int\frac{(a+bx^2)\left(\frac{2bx}{a+bx^2}-\frac{2bx\left(i+a+bx^2\right)}{\left(a+bx^2\right)^2}\right)\log(d+cx)}{i+a+bx^2}\,dx,x,\sqrt{x}\right)}{c^3}$$

$$= -\frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2} - \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3} + \frac{b\text{Subst}\left(\int\frac{x^3}{(a+bx^2)(-i+a+bx^2)}\,dx,x,\sqrt{x}\right)}{c} + \frac{b\text{Subst}\left(\int\frac{x^3}{(a+bx^2)(i+a+bx^2)}\,dx,x,\sqrt{x}\right)}{c} - \frac{(2bd)\text{Subst}\left(\int\frac{x^2}{(a+bx^2)(-i+a+bx^2)}\,dx,x,\sqrt{x}\right)}{c^2} - \frac{(2bd)\text{Subst}\left(\int\frac{x^2}{(a+bx^2)(i+a+bx^2)}\,dx,x,\sqrt{x}\right)}{c^2} - \frac{(id^2)\,\text{Subst}\left(\int\left(-\frac{2bx\log(d+cx)}{a+bx^2}+\frac{2bx\log(d+cx)}{-i+a+bx^2}\right)dx,x,\sqrt{x}\right)}{c^3} + \frac{(id^2)\,\text{Subst}\left(\int\left(-\frac{2bx\log(d+cx)}{a+bx^2}+\frac{2bx\log(d+cx)}{i+a+bx^2}\right)dx,x,\sqrt{x}\right)}{c^3}$$

$$= -\frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2} - \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3} + \frac{b\text{Subst}\left(\int\frac{x}{(a+bx)(-i+a+bx)}\,dx,x,x\right)}{2c} + \frac{b\text{Subst}\left(\int\frac{x}{(a+bx)(i+a+bx)}\,dx,x,x\right)}{2c} - \frac{(2(1-ia)d)\text{Subst}\left(\int\frac{1}{i+a+bx^2}\,dx,x,\sqrt{x}\right)}{c^2} - \frac{(2(1+ia)d)\text{Subst}\left(\int\frac{1}{-i+a+bx^2}\,dx,x,\sqrt{x}\right)}{c^2} - \frac{(2ibd^2)\,\text{Subst}\left(\int\frac{x\log(d+cx)}{-i+a+bx^2}\,dx,x,\sqrt{x}\right)}{c^3} + \frac{(2ibd^2)\,\text{Subst}\left(\int\frac{x\log(d+cx)}{i+a+bx^2}\,dx,x,\sqrt{x}\right)}{c^3}$$

$$= \frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} - \frac{2i\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} - \frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2} - \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3} + \frac{b\text{Subst}\left(\int\left(-\frac{ia}{b(a+bx)}+\frac{1+ia}{b(-i+a+bx)}\right)dx,x,x\right)}{2c} + \frac{b\text{Subst}\left(\int\left(\frac{ia}{b(a+bx)}+\frac{1-ia}{b(i+a+bx)}\right)dx,x,x\right)}{2c} + \frac{(2ibd^2)\,\text{Subst}\left(\int\left(-\frac{\log(d+cx)}{2\sqrt{b}\left(\sqrt{-i-a}-\sqrt{b}x\right)}+\frac{\log(d+cx)}{2\sqrt{b}\left(\sqrt{-i-a}+\sqrt{b}x\right)}\right)dx,x,\sqrt{x}\right)}{c^3} - \frac{(2ibd^2)\,\text{Subst}\left(\int\left(-\frac{\log(d+cx)}{2\sqrt{b}\left(\sqrt{i-a}-\sqrt{b}x\right)}+\frac{\log(d+cx)}{2\sqrt{b}\left(\sqrt{i-a}+\sqrt{b}x\right)}\right)dx,x,\sqrt{x}\right)}{c^3}$$

$$= \frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} - \frac{2i\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} + \frac{(1+ia)\log(i-a-bx)}{2bc}$$
$$- \frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3}$$
$$+ \frac{(1-ia)\log(i+a+bx)}{2bc} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2} - \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c}$$
$$- \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3} - \frac{\left(i\sqrt{b}d^2\right)\text{Subst}\left(\int\frac{\log(d+cx)}{\sqrt{-i-a}-\sqrt{b}x}\,dx, x, \sqrt{x}\right)}{c^3}$$
$$+ \frac{\left(i\sqrt{b}d^2\right)\text{Subst}\left(\int\frac{\log(d+cx)}{\sqrt{i-a}-\sqrt{b}x}\,dx, x, \sqrt{x}\right)}{c^3}$$
$$+ \frac{\left(i\sqrt{b}d^2\right)\text{Subst}\left(\int\frac{\log(d+cx)}{\sqrt{-i-a}+\sqrt{b}x}\,dx, x, \sqrt{x}\right)}{c^3}$$
$$- \frac{\left(i\sqrt{b}d^2\right)\text{Subst}\left(\int\frac{\log(d+cx)}{\sqrt{i-a}+\sqrt{b}x}\,dx, x, \sqrt{x}\right)}{c^3}$$

$$= \frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} - \frac{2i\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2}$$
$$+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} + \frac{(1+ia)\log(i-a-bx)}{2bc}$$
$$- \frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3}$$
$$+ \frac{(1-ia)\log(i+a+bx)}{2bc} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2} - \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c}$$
$$- \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3} - \frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}x\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{d+cx}dx,x,\sqrt{x}\right)}{c^2}$$
$$+ \frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}x\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)}{d+cx}dx,x,\sqrt{x}\right)}{c^2}$$
$$- \frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}x\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{d+cx}dx,x,\sqrt{x}\right)}{c^2}$$
$$+ \frac{(id^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}x\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)}{d+cx}dx,x,\sqrt{x}\right)}{c^2}$$

$$
\begin{aligned}
&= \frac{2i\sqrt{i+a}\,d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} - \frac{2i\sqrt{i-a}\,d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} \\
&+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} + \frac{(1+ia)\log(i-a-bx)}{2bc} \\
&- \frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3} \\
&+ \frac{(1-ia)\log(i+a+bx)}{2bc} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2} \\
&- \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3} \\
&- \frac{(id^2)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{b}x}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{x}\,dx, x, d+c\sqrt{x}\right)}{c^3} \\
&+ \frac{(id^2)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{b}x}{\sqrt{i-a}c-\sqrt{b}d}\right)}{x}\,dx, x, d+c\sqrt{x}\right)}{c^3} \\
&- \frac{(id^2)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{x}\,dx, x, d+c\sqrt{x}\right)}{c^3} \\
&+ \frac{(id^2)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{i-a}c+\sqrt{b}d}\right)}{x}\,dx, x, d+c\sqrt{x}\right)}{c^3}
\end{aligned}
$$

$$
= \frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} - \frac{2i\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2}
+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}
- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}
+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}
- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} + \frac{(1+ia)\log(i-a-bx)}{2bc}
- \frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3}
+ \frac{(1-ia)\log(i+a+bx)}{2bc} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2}
- \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3}
+ \frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{c^3} - \frac{id^2\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3}
+ \frac{id^2\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{c^3} - \frac{id^2\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3}
$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 809, normalized size of antiderivative = 0.97

$$
\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx
= \frac{4i\sqrt{i+a}\sqrt{b}cd\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right) - 4i\sqrt{i-a}\sqrt{b}cd\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right) + 2ibd^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right.}{}
$$

```
[In] Integrate[ArcCot[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((4*I)*Sqrt[I + a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]] - (4*I
)*Sqrt[I - a]*Sqrt[b]*c*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]] + (2*I)*b*
d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]*
Log[d + c*Sqrt[x]] - (2*I)*b*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(S
qrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + (2*I)*b*d^2*Log[(c*(Sqrt[-I
```

```
 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]]
- (2*I)*b*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt
[b]*d)]*Log[d + c*Sqrt[x]] + c^2*Log[I - a - b*x] + I*a*c^2*Log[I - a - b*x
] - (2*I)*b*c*d*Sqrt[x]*Log[(-I + a + b*x)/(a + b*x)] + I*b*c^2*x*Log[(-I +
 a + b*x)/(a + b*x)] + (2*I)*b*d^2*Log[d + c*Sqrt[x]]*Log[(-I + a + b*x)/(a
 + b*x)] + c^2*Log[I + a + b*x] - I*a*c^2*Log[I + a + b*x] + (2*I)*b*c*d*Sq
rt[x]*Log[(I + a + b*x)/(a + b*x)] - I*b*c^2*x*Log[(I + a + b*x)/(a + b*x)]
 - (2*I)*b*d^2*Log[d + c*Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)] + (2*I)*b*d^
2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-I - a]*c) + Sqrt[b]*d)] + (
2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d
)] - (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[I - a]*c) + S
qrt[b]*d)] - (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*
c + Sqrt[b]*d)])/(2*b*c^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.47

method	result
derivativedivides	$\frac{\operatorname{arccot}(bx+a)x}{c} - \frac{2\operatorname{arccot}(bx+a)d\sqrt{x}}{c^2} + \frac{2\operatorname{arccot}(bx+a)d^2\ln(d+c\sqrt{x})}{c^3} + \frac{4b\left(-\frac{c\left(\_R=\operatorname{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3+(2\right.\right.}{}\right.}{}$
default	$\frac{\operatorname{arccot}(bx+a)x}{c} - \frac{2\operatorname{arccot}(bx+a)d\sqrt{x}}{c^2} + \frac{2\operatorname{arccot}(bx+a)d^2\ln(d+c\sqrt{x})}{c^3} + \frac{4b\left(-\frac{c\left(\_R=\operatorname{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3+(2\right.\right.}{}\right.}{}$

```
[In] int(arccot(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] arccot(b*x+a)/c*x-2*arccot(b*x+a)/c^2*d*x^(1/2)+2*arccot(b*x+a)*d^2/c^3*ln(
d+c*x^(1/2))+4*b/c^2*(-1/8*c/b*sum((-_R^3+5*_R^2*d-7*_R*d^2+3*d^3)/(_R^3*b-
3*_R^2*b*d+_R*a*c^2+3*_R*b*d^2-a*c^2*d-b*d^3)*ln(c*x^(1/2)-_R+d),_R=RootOf(
b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2*d^3)*_
Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))+1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*d+
a*c^2+b*d^2)*(ln(d+c*x^(1/2))*ln((-c*x^(1/2)+_R1-d)/_R1)+dilog((-c*x^(1/2)+
_R1-d)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-
4*a*b*c^2*d-4*b^2*d^3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
[Out] integral((c*x*arccot(b*x + a) - d*sqrt(x)*arccot(b*x + a))/(c^2*x - d^2), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Timed out}$$

```
[In] integrate(acot(b*x+a)/(c+d/x**(1/2)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
[Out] integrate(arccot(b*x + a)/(c + d/sqrt(x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]War
ning, replacing 0 by -24, a substitution variable should perhaps be purged.
Warnin
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx=\int \frac{\mathrm{acot}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

```
[In] int(acot(a + b*x)/(c + d/x^(1/2)),x)
[Out] int(acot(a + b*x)/(c + d/x^(1/2)), x)
```

3.113 $\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 367

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} + \frac{i\,\mathrm{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}} - \frac{i\,\mathrm{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}}$$

```
[Out] arccot(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+
e*(b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arccot(e*x+d)*ln(2*e*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-
4*a*c+b^2)^(1/2)+1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^(
1/2)))/(1-I*(e*x+d))/(2*I*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(
1/2)-1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))/(1-I*
(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {632, 212, 6860, 5156, 4967, 2449, 2352, 2497}

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \frac{i\,\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2ic+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}} - \frac{i\,\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}} + \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(1-i(d+ex))\left(e\left(b-\sqrt{b^2-4ac}\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}} - \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(\sqrt{b^2-4ac}+b+2cx\right)}{(1-i(d+ex))\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Int[ArcCot[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcCot[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) +
(b - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] - (ArcCot
[d + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b + Sq
rt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] + ((I/2)*PolyLog
[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c -
 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c]
- ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*
x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b
^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{2c\cot^{-1}(d+ex)}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\cot^{-1}(d+ex)}{\sqrt{b^2-4ac}\,\left(b+\sqrt{b^2-4ac}+2cx\right)} \right) dx$$

$$= \frac{(2c)\int \frac{\cot^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\cot^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$
= \frac{(2c)\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}dx, x, d+ex\right)}{\sqrt{b^2-4ac}e} - \frac{(2c)\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}dx, x, d+ex\right)}{\sqrt{b^2-4ac}e}
$$

$$
= \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} + \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2\left(\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}\right)}{\left(\frac{2ic}{e}+\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}\right)(1-ix)}\right)}{1+x^2}dx, x, d+ex\right)}{\sqrt{b^2-4ac}} - \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2\left(\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}\right)}{\left(\frac{2ic}{e}+\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}\right)(1-ix)}\right)}{1+x^2}dx, x, d+ex\right)}{\sqrt{b^2-4ac}}
$$

$$
= \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} + \frac{i\,\text{PolyLog}\left(2, 1-\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}} - \frac{i\,\text{PolyLog}\left(2, 1-\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}}
$$

Mathematica [F]

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx$$

```
[In] Integrate[ArcCot[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] Integrate[ArcCot[d + e*x]/(a + b*x + c*x^2), x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(329) = 658$.

Time = 2.49 (sec) , antiderivative size = 959, normalized size of antiderivative = 2.61

method	result
risch	$\frac{ie\pi \arctan\left(\frac{ibe-2icd-2(-iex-id+1)c+2c}{\sqrt{-4ac\,e^2+b^2e^2}}\right)}{\sqrt{-4ac\,e^2+b^2e^2}} - \frac{e\ln(-iex-id+1)\ln\left(\frac{ibe-2icd-2(-iex-id+1)c+\sqrt{4ac\,e^2-b^2e^2}+2c}{ibe-2icd+2c+\sqrt{4ac\,e^2-b^2e^2}}\right)}{2\sqrt{4ac\,e^2-b^2e^2}} + \frac{e\,\mathrm{l}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(arccot(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] I*e*Pi/(-4*a*c*e^2+b^2*e^2)^(1/2)*arctan((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c+2
*c)/(-4*a*c*e^2+b^2*e^2)^(1/2))-1/2*e*ln(1-I*d-I*e*x)/(4*a*c*e^2-b^2*e^2)^(
1/2)*ln((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/(I*
b*e-2*I*c*d+2*c+(4*a*c*e^2-b^2*e^2)^(1/2)))+1/2*e*ln(1-I*d-I*e*x)/(4*a*c*e^
2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1
/2)+2*c)/(I*b*e-2*I*c*d+2*c-(4*a*c*e^2-b^2*e^2)^(1/2)))-1/2*e/(4*a*c*e^2-b^
2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/
2)+2*c)/(I*b*e-2*I*c*d+2*c+(4*a*c*e^2-b^2*e^2)^(1/2)))+1/2*e/(4*a*c*e^2-b^2
*e^2)^(1/2)*dilog((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1/2
)+2*c)/(I*b*e-2*I*c*d+2*c-(4*a*c*e^2-b^2*e^2)^(1/2)))-1/2*e*ln(1+I*d+I*e*x)
/(4*a*c*e^2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c-(4*a*c*e^2-b
^2*e^2)^(1/2)-2*c)/(I*b*e-2*I*c*d-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))+1/2*e*ln(
1+I*d+I*e*x)/(4*a*c*e^2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c+
(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*b*e-2*I*c*d+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c
))-1/2*e/(4*a*c*e^2-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c-(
4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*b*e-2*I*c*d-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)
)+1/2*e/(4*a*c*e^2-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c+(4
*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*b*e-2*I*c*d+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))
```

Fricas [F]

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{arccot}(ex+d)}{cx^2+bx+a}\,dx$$

```
[In] integrate(arccot(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral(arccot(e*x + d)/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \text{Timed out}$$

```
[In] integrate(acot(e*x+d)/(c*x**2+b*x+a),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(arccot(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \text{Timed out}$$

```
[In] integrate(arccot(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\mathrm{acot}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

```
[In] int(acot(d + e*x)/(a + b*x + c*x^2),x)
[Out] int(acot(d + e*x)/(a + b*x + c*x^2), x)
```

3.114 $\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{2i\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

```
[Out] -2*I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-I*poly
log(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+I*polylog(2,I*(1+I*(b*x
+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5164, 5007}

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{2i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)}{b} - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

```
[In] Int[ArcCot[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((-2*I)*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]
)/b - (I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
+ (I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5164

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)^
q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\
&= -\frac{2i\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} \\
&\quad -\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{\sqrt{1+a^2+2abx+b^2x^2}\left(\cot^{-1}(a+bx)\left(\log\left(1-e^{i\cot^{-1}(a+bx)}\right)-\log\left(1+e^{i\cot^{-1}(a+bx)}\right)\right)+i\,\text{PolyLog}\left(\right.\right.}{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}$$

```
[In] Integrate[ArcCot[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -((Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(ArcCot[a + b*x]*(Log[1 - E^(I*ArcCot[
a + b*x])] - Log[1 + E^(I*ArcCot[a + b*x])]) + I*PolyLog[2, -E^(I*ArcCot[a
+ b*x])] - I*PolyLog[2, E^(I*ArcCot[a + b*x])]))/(b*(a + b*x)*Sqrt[1 + (a +
 b*x)^(-2)]))
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-\operatorname{arccot}(bx+a)\ln\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)+i\operatorname{dilog}\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)-i\operatorname{dilog}\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)}{b}$

```
[In] int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-arccot(b*x+a)*ln((I
+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+I*dilog((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)-I*d
ilog(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

```
[In] integrate(acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(acot(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

```
[In] int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
[Out] int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.115 $\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 216

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{2i\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} - \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}$$

```
[Out] -2*I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+
a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I
*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+I*polylog(2,I*
(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used

$= \{5164, 5011, 5007\}$

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{2i\sqrt{(a+bx)^2+1}\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)}{b\sqrt{c(a+bx)^2+c}} - \frac{i\sqrt{(a+bx)^2+1}\,\text{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}} + \frac{i\sqrt{(a+bx)^2+1}\,\text{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}$$

```
[In] Int[ArcCot[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + (a + b*x)^2]*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - (I*Sqrt[1 + (a + b*x)
^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt
[c + c*(a + b*x)^2]) + (I*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a
 + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2])
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5011

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCot[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5164

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)^
q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\sqrt{1+(a+bx)^2}\,\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b\sqrt{c+c(a+bx)^2}}$$

$$= -\frac{2i\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} - \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.64

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{(1+(a+bx)^2)\left(\cot^{-1}(a+bx)\left(\log\left(1-e^{i\cot^{-1}(a+bx)}\right)-\log\left(1+e^{i\cot^{-1}(a+bx)}\right)\right)+i\,\text{PolyLog}\left(2,-e^{i\cot\cdots}\right.\right.}{b(a+bx)\sqrt{c\left(1+a^2+2abx+b^2x^2\right)}\sqrt{1+\frac{1}{(a+bx)^2}}}$$

```
[In] Integrate[ArcCot[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
[Out] -(((1 + (a + b*x)^2)*(ArcCot[a + b*x]*(Log[1 - E^(I*ArcCot[a + b*x])] - Log
[1 + E^(I*ArcCot[a + b*x])]) + I*PolyLog[2, -E^(I*ArcCot[a + b*x])] - I*Pol
yLog[2, E^(I*ArcCot[a + b*x])]))/(b*(a + b*x)*Sqrt[c*(1 + a^2 + 2*a*b*x + b
^2*x^2)]*Sqrt[1 + (a + b*x)^(-2)]))
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.72

method	result
default	$\frac{i\left(i\,\text{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\,\text{arccot}(bx+a)\ln\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)+\text{polylog}\left(2,\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-\text{polylog}\left(2,-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)\cdots\right.}{\sqrt{b^2x^2+2abx+a^2+1}\,bc}$

```
[In] int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+a)*ln((
I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-po
lylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{arccot}\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{acot}(a+b\,x)}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

```
[In] int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)
```

```
[Out] int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2), x)
```

3.116 $\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \operatorname{Int}\left(\frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

```
[Out] Unintegrable(arccot(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

```
[In] Int[ArcCot[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcCot[x]/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt[3]{1+x^2}}\,dx, x, a+bx\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(23) = 46.

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 6.32

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{6\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+a^2+2abx+b^2x^2)\left(-3+2(a+bx)\cot^{-1}(a+bx)\right)+4(a+bx)\cot^{-1}(a+\right.}{20b\left(1+a^2+2abx+b^2x^2\right)^4}$$

```
[In] Integrate[ArcCot[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a + b*x
)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1, 4/3,
11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*
HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b^2*x^2)
^(-1)])/(20*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(4/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [N/A] (verified)

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arccot}(bx+a)}{\left(b^2x^2+2abx+a^2+1\right)^{\frac{1}{3}}}dx$$

```
[In] int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas"
)
```

```
[Out] integral(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{acot}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

```
[In] integrate(acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
[Out] Integral(acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima"
)
[Out] integrate(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{1/3}}\,dx$$

```
[In] int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
[Out] int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.117 $\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \mathrm{Int}\left(\frac{\cot^{-1}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

```
[Out] Unintegrable(arccot(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$

```
[In] Int[ArcCot[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcCot[x]/(c + c*x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral} = \frac{\mathrm{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt[3]{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 180 vs. $2(25) = 50$.

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.45

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$
$$= \frac{c\Big(6\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+a^2+2abx+b^2x^2)\left(-3+2(a+bx)\cot^{-1}(a+bx)\right)+4(a+bx)\cot^{-1}(\cdots}{20b\left(c\left(1+a^2+2abx+b^2x\cdots\right.\right.}$$

```
[In] Integrate[ArcCot[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] (c*(6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a +
b*x)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1, 4/
3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma[5/
3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b^2*x
^2)^(-1)]))/(20*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))^(4/3)*Gamma[11/6]*Gamma
[7/3])
```

Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arccot}\left(bx+a\right)}{\left(\left(a^2+1\right)c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}dx$$

```
[In] int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{arccot}\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"fricas")
```

```
[Out] integral(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 5.94 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{acot}\,(a+bx)}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

```
[In] integrate(acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),x)
[Out] Integral(acot(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{arccot}\,(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"maxima")
[Out] integrate(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{arccot}\,(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"giac")
[Out] integrate(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)^{1/3}}\,dx$$

```
[In] int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3),x)
[Out] int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3), x)
```

3.118 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 187

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx = \frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)}{2b} + \frac{i\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} - \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}$$

```
[Out] I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*I*pol
ylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*I*polylog(2,I*(1+I
*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a
)*arccot(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5166, 5073, 267, 5007}

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx = \frac{i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)}{b} + \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b} - \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b} + \frac{\sqrt{(a+bx)^2+1}}{2b} + \frac{(a+bx)\sqrt{(a+bx)^2+1}\cot^{-1}(a+bx)}{2b}$$

[In] Int[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]

[Out] Sqrt[1 + (a + b*x)^2]/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcCot[a + b*x])/(2*b) + (I*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/b + ((I/2)*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b - ((I/2)*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5073

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcCot[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
+ b*ArcCot[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2*
m)), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5166

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b} - \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b}$$

$$= \frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)}{2b} + \frac{i\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\,\text{PolyLog}\left(2, -\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} - \frac{i\,\text{PolyLog}\left(2, \frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{\sqrt{(a+bx)^2\left(1+\frac{1}{(a+bx)^2}\right)}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(a+bx)\right) - \cot^{-1}(a+bx)\csc^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right) - 4\cot^{-1}(a\right.}{}$$

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x
]
```

```
[Out] -1/8*(Sqrt[(a + b*x)^2*(1 + (a + b*x)^(-2))]*(-2*Cot[ArcCot[a + b*x]/2] - A
rcCot[a + b*x]*Csc[ArcCot[a + b*x]/2]^2 - 4*ArcCot[a + b*x]*Log[1 - E^(I*Ar
cCot[a + b*x])] + 4*ArcCot[a + b*x]*Log[1 + E^(I*ArcCot[a + b*x])] - (4*I)*
PolyLog[2, -E^(I*ArcCot[a + b*x])] + (4*I)*PolyLog[2, E^(I*ArcCot[a + b*x])
] + ArcCot[a + b*x]*Sec[ArcCot[a + b*x]/2]^2 - 2*Tan[ArcCot[a + b*x]/2]))/(
b*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

method	result
default	$\frac{(\operatorname{arccot}(bx+a)bx+a\operatorname{arccot}(bx+a)+1)\sqrt{b^2x^2+2abx+a^2+1}}{2b} - \frac{i\left(i\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\operatorname{arccot}(bx+a)\ln\left(\frac{bx+a+}{\sqrt{1+(bx+}}\right.\right.}{2b}$

```
[In] int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/2*(arccot(b*x+a)*b*x+a*arccot(b*x+a)+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b-1
/2*I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+a)*l
n((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/2))
-polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))/b
```

Fricas [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**2*acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Giac [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

```
[In] int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.119 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 281

$$\begin{aligned}\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx =& \frac{\sqrt{c+c(a+bx)^2}}{2bc} \\ &+ \frac{(a+bx)\sqrt{c+c(a+bx)^2}\cot^{-1}(a+bx)}{2bc} \\ &+ \frac{i\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} \\ &+ \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}} \\ &- \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}}\end{aligned}$$

```
[Out] I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^
2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+1/2*I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-
I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-1/2*I*polylog
(2,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b
*x+a)^2)^(1/2)+1/2*(c+c*(b*x+a)^2)^(1/2)/b/c+1/2*(b*x+a)*arccot(b*x+a)*(c+c
*(b*x+a)^2)^(1/2)/b/c
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5166, 5073, 267, 5011, 5007}

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{i\sqrt{(a+bx)^2+1}\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)}{b\sqrt{c(a+bx)^2+c}} + \frac{i\sqrt{(a+bx)^2+1}\,\text{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}} - \frac{i\sqrt{(a+bx)^2+1}\,\text{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}} + \frac{\sqrt{c(a+bx)^2+c}}{2bc} + \frac{(a+bx)\sqrt{c(a+bx)^2+c}\cot^{-1}(a+bx)}{2bc}$$

```
[In] Int[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2]
,x]
```

```
[Out] Sqrt[c + c*(a + b*x)^2]/(2*b*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcCot
[a + b*x])/(2*b*c) + (I*Sqrt[1 + (a + b*x)^2]*ArcCot[a + b*x]*ArcTan[Sqrt[1
 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + ((I/2
)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*
(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - ((I/2)*Sqrt[1 + (a + b*x)^2]*Pol
yLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a
+ b*x)^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5011

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCot[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5073

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcCot[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
+ b*ArcCot[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2*
m)), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5166

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2\cot^{-1}(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\sqrt{c+c(a+bx)^2}\cot^{-1}(a+bx)}{2bc} \\
&\quad + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{2b} - \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{2b} \\
&= \frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\cot^{-1}(a+bx)}{2bc} \\
&\quad - \frac{\sqrt{1+(a+bx)^2}\,\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b\sqrt{c+c(a+bx)^2}}
\end{aligned}$$

$$= \frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\cot^{-1}(a+bx)}{2bc}$$
$$+ \frac{i\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}$$
$$+ \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}}$$
$$- \frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx =$$
$$-\frac{\sqrt{c(1+a^2+2abx+b^2x^2)}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-\cot^{-1}(a+bx)\csc^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-4\cot^{-1}(\right.}{}$$

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*
c*x^2],x]
```

```
[Out] -1/8*(Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)]*(-2*Cot[ArcCot[a + b*x]/2] - Ar
cCot[a + b*x]*Csc[ArcCot[a + b*x]/2]^2 - 4*ArcCot[a + b*x]*Log[1 - E^(I*Arc
Cot[a + b*x])] + 4*ArcCot[a + b*x]*Log[1 + E^(I*ArcCot[a + b*x])] - (4*I)*P
olyLog[2, -E^(I*ArcCot[a + b*x])] + (4*I)*PolyLog[2, E^(I*ArcCot[a + b*x])]
 + ArcCot[a + b*x]*Sec[ArcCot[a + b*x]/2]^2 - 2*Tan[ArcCot[a + b*x]/2]))/(b
*c*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.72

method	result
default	$\frac{(\text{arccot}(bx+a)bx+a\ \text{arccot}(bx+a)+1)\sqrt{c(bx+a-i)(bx+a+i)}}{2bc} - \frac{i\left(i\,\text{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\,\text{arccot}(bx+a)\ln\left(\frac{bx+}{\sqrt{1+(}}\right.\right.}{}$

```
[In] int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/2*(arccot(b*x+a)*b*x+a*arccot(b*x+a)+1)*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/b/
c-1/2*I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+a
)*ln((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/
2))-polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(I+a+b*x))^(1/
2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Fricas [F]

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2\,\mathrm{arccot}\,(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c
*x + (a^2 + 1)*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**2*acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),
x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2\,\mathrm{arccot}\,(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1
)*c), x)
```

Giac [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1
)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

```
[In] int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
,x)
```

```
[Out] int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
, x)
```

3.120 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx = \text{Int}\left(\frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+(a+bx)^2}}, x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arccot(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx = \int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx$$

```
[In] Int[((a + b*x)^2*ArcCot[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
[Out] Defer[Subst][Defer[Int][(x^2*ArcCot[x])/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2 \cot^{-1}(x)}{\sqrt[3]{1+x^2}} \, dx, x, a+bx\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(30) = 60.

Time = 0.74 (sec) , antiderivative size = 198, normalized size of antiderivative = 5.66

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \frac{3\Big(\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+(a+bx)^2)\left(3(7+(a+bx)^2)+4(a+bx)\left(-2+(a+bx)^2\right)\cot^{-1}(a+b\right.\right.}{140b\sqrt[3]{1+a^2+2ab}}$$

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)
,x]
```

```
[Out] (3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(a
 + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x]*
Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5*2^
(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1
+ a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/
3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

Maple [N/A] (verified)

Not integrable

Time = 0.88 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx$$

```
[In] int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(a+bx)^2 \operatorname{acot}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

```
[In] integrate((b*x+a)**2*acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
[Out] Integral((a + b*x)**2*acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3)
, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="maxima")
[Out] integrate((b*x + a)^2*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="giac")
[Out] integrate((b*x + a)^2*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
x)
```

Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx = \int \frac{\mathrm{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{1/3}} \, dx$$

```
[In] int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
[Out] int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.121 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \mathrm{Int}\left(\frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arccot(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$

```
[In] Int[((a + b*x)^2*ArcCot[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/
3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcCot[x])/(c + c*x^2)^(1/3), x], x, a + b*x]/
b
```

Rubi steps

$$\text{integral} = \frac{\mathrm{Subst}\left(\int \frac{x^2\cot^{-1}(x)}{\sqrt[3]{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 200 vs. $2(32) = 64$.

Time = 0.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx = \frac{3\Big(\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+(a+bx)^2)\left(3(7+(a+bx)^2)+4(a+bx)\left(-2+(a+bx)^2\right)\cot^{-1}(a+b\right.\right.}{140b\sqrt[3]{c\,(1+a^2+2a}}$$

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^
2)^(1/3),x]
```

```
[Out] (3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(a
 + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x]*
Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5*2^
(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1
+ a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))
^(1/3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\left((a^2+1)c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}dx$$

```
[In] int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x +
 (a^2 + 1)*c)^(1/3), x)
```

Sympy [N/A]

Not integrable

Time = 24.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(a+bx)^2\,\mathrm{acot}\,(a+bx)}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

```
[In] integrate((b*x+a)**2*acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),
x)
```

```
[Out] Integral((a + b*x)**2*acot(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(
1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2\,\mathrm{arccot}\,(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)
^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2\,\mathrm{arccot}\,(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)
^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\mathrm{acot}(a+b\,x)\ {(a+b\,x)}^2}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}}\,dx$$

```
[In] int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
,x)
[Out] int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
, x)
```

3.122 $\int (a+bx)^2 \cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx = \frac{(a+bx)^2}{6b} + \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} - \frac{\log\left(1+(a+bx)^2\right)}{6b}$$

```
[Out] 1/6*(b*x+a)^2/b+1/3*(b*x+a)^3*arccot(b*x+a)/b-1/6*ln(1+(b*x+a)^2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5152, 4947, 272, 45}

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx = \frac{(a+bx)^2}{6b} - \frac{\log\left((a+bx)^2+1\right)}{6b} + \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b}$$

```
[In] Int[(a + b*x)^2*ArcCot[a + b*x],x]
```

```
[Out] (a + b*x)^2/(6*b) + ((a + b*x)^3*ArcCot[a + b*x])/(3*b) - Log[1 + (a + b*x)
^2]/(6*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^2 \cot^{-1}(x)\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{x^3}{1+x^2}\,dx, x, a+bx\right)}{3b} \\
&= \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, (a+bx)^2\right)}{6b} \\
&= \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} + \frac{\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, (a+bx)^2\right)}{6b} \\
&= \frac{(a+bx)^2}{6b} + \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} - \frac{\log\left(1+(a+bx)^2\right)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx = \frac{(a+bx)^2 + 2(a+bx)^3 \cot^{-1}(a+bx) - \log\left(1+(a+bx)^2\right)}{6b}$$

```
[In] Integrate[(a + b*x)^2*ArcCot[a + b*x],x]
[Out] ((a + b*x)^2 + 2*(a + b*x)^3*ArcCot[a + b*x] - Log[1 + (a + b*x)^2])/(6*b)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}-\frac{\ln\left(1+(bx+a)^2\right)}{6}}{b}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}-\frac{\ln\left(1+(bx+a)^2\right)}{6}}{b}$
parts	$\frac{\operatorname{arccot}(bx+a)b^2x^3}{3}+\operatorname{arccot}(bx+a)\,ba\,x^2+\operatorname{arccot}(bx+a)\,a^2x+\frac{\operatorname{arccot}(bx+a)a^3}{3b}+\frac{x^2b}{6}+\frac{ax}{3}-$
parallelrisch	$-\frac{-2\operatorname{arccot}(bx+a)x^3b^4-6a\,b^3\operatorname{arccot}(bx+a)x^2-6x\operatorname{arccot}(bx+a)a^2b^2-b^3x^2-2\operatorname{arccot}(bx+a)a^3b-2a\,b^2x+5a^2b+\ln(b^2x}{6b^2}$
risch	$\frac{i(bx+a)^3\ln(1+i(bx+a))}{6b}-\frac{ib^2x^3\ln(1-i(bx+a))}{6}-\frac{iba\,x^2\ln(1-i(bx+a))}{2}+\frac{\pi\,b^2x^3}{6}-\frac{ia^2x\ln(1-i(bx+a))}{2}+\frac{b\,x^2a}{2}$

```
[In] int((b*x+a)^2*arccot(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*arccot(b*x+a)*(b*x+a)^3+1/6*(b*x+a)^2-1/6*ln(1+(b*x+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int (a+bx)^2\cot^{-1}(a+bx)\,dx$$
$$=\frac{b^2x^2-2\,a^3\arctan(bx+a)+2\,abx+2\left(b^3x^3+3\,ab^2x^2+3\,a^2bx\right)\operatorname{arccot}(bx+a)-\log\left(b^2x^2+2\,abx+a^2+\right.}{6\,b}$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="fricas")
[Out] 1/6*(b^2*x^2 - 2*a^3*arctan(b*x + a) + 2*a*b*x + 2*(b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x)*arccot(b*x + a) - log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int (a+bx)^2\cot^{-1}(a+bx)\,dx$$
$$=\begin{cases}\frac{a^3\operatorname{acot}(a+bx)}{3b}+a^2x\operatorname{acot}(a+bx)+abx^2\operatorname{acot}(a+bx)+\frac{ax}{3}+\frac{b^2x^3\operatorname{acot}(a+bx)}{3}+\frac{bx^2}{6}-\frac{\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{3b}-\frac{i\operatorname{acot}(a+bx}{3b}\\ a^2x\operatorname{acot}(a)\end{cases}$$

```
[In] integrate((b*x+a)**2*acot(b*x+a),x)
```

```
[Out] Piecewise((a**3*acot(a + b*x)/(3*b) + a**2*x*acot(a + b*x) + a*b*x**2*acot(
a + b*x) + a*x/3 + b**2*x**3*acot(a + b*x)/3 + b*x**2/6 - log(a/b + x - I/b
)/(3*b) - I*acot(a + b*x)/(3*b), Ne(b, 0)), (a**2*x*acot(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.79

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx = -\frac{1}{6}\left(\frac{2\,a^3\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2} - \frac{bx^2+2\,ax}{b} + \frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^2}\right)b + \frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\operatorname{arccot}\left(bx+a\right)$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^3*arctan((b^2*x + a*b)/b)/b^2 - (b*x^2 + 2*a*x)/b + log(b^2*x^2 +
 2*a*b*x + a^2 + 1)/b^2)*b + 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*arccot(b*x
 + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(46) = 92$.

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.90

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx = -\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^6 - 3\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 - \tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}{}$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="giac")
```

```
[Out] -1/24*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 - 3*arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^4 - tan(1/2*arctan(1/(b*x + a)))^5 - 4*
log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*t
an(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 3*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x +
a)))^3 - arctan(1/(b*x + a)) - tan(1/2*arctan(1/(b*x + a))))/(b*tan(1/2*arc
tan(1/(b*x + a)))^3)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx = \frac{a\,x}{3} - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b} + \frac{b\,x^2}{6} - \frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b} + \frac{b^2\,x^3\,\mathrm{acot}(a+b\,x)}{3} + a^2\,x\,\mathrm{acot}(a+b\,x) + a\,b\,x^2\,\mathrm{acot}(a+b\,x)$$

```
[In] int(acot(a + b*x)*(a + b*x)^2,x)
```

```
[Out] (a*x)/3 - log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b) + (b*x^2)/6 - (a^3*atan(a
+ b*x))/(3*b) + (b^2*x^3*acot(a + b*x))/3 + a^2*x*acot(a + b*x) + a*b*x^2*a
cot(a + b*x)
```

3.123 $\int (a+bx)\cot^{-1}(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int (a+bx)\cot^{-1}(a+bx)\,dx=\frac{x}{2}+\frac{(a+bx)^2\cot^{-1}(a+bx)}{2b}-\frac{\arctan(a+bx)}{2b}$$

```
[Out] 1/2*x+1/2*(b*x+a)^2*arccot(b*x+a)/b-1/2*arctan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5152, 4947, 327, 209}

$$\int (a+bx)\cot^{-1}(a+bx)\,dx=-\frac{\arctan(a+bx)}{2b}+\frac{(a+bx)^2\cot^{-1}(a+bx)}{2b}+\frac{x}{2}$$

```
[In] Int[(a + b*x)*ArcCot[a + b*x],x]
```

```
[Out] x/2 + ((a + b*x)^2*ArcCot[a + b*x])/(2*b) - ArcTan[a + b*x]/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x\cot^{-1}(x)\,dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)^2\cot^{-1}(a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, a+bx\right)}{2b}\\
&= \frac{x}{2} + \frac{(a+bx)^2\cot^{-1}(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, a+bx\right)}{2b}\\
&= \frac{x}{2} + \frac{(a+bx)^2\cot^{-1}(a+bx)}{2b} - \frac{\arctan(a+bx)}{2b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.62

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = ax\cot^{-1}(a+bx) + \frac{1}{2}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^2\cot^{-1}(a+bx) + \frac{1}{2}b\left(\frac{x}{b} - \frac{i(i-a)^2\log(i-a-bx)}{2b^2} + \frac{i(i+a)^2\log(i+a+bx)}{2b^2}\right) + \frac{a\left(-2a\arctan(a+bx)+\log\left(1+a^2+2abx+b^2x^2\right)\right)}{2b}$$

```
[In] Integrate[(a + b*x)*ArcCot[a + b*x],x]
```

[Out] a*x*ArcCot[a + b*x] + (b*(-(a/b) + (a + b*x)/b)^2*ArcCot[a + b*x])/2 + (b*(x/b - ((I/2)*(I - a)^2*Log[I - a - b*x])/b^2 + ((I/2)*(I + a)^2*Log[I + a + b*x])/b^2))/2 + (a*(-2*a*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^2]))/(2*b)

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}-\frac{\arctan(bx+a)}{2}}{b}$
default	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}-\frac{\arctan(bx+a)}{2}}{b}$
parts	$\frac{\operatorname{arccot}(bx+a)x^2 b}{2}+\operatorname{arccot}(bx+a)\,ax+\frac{b\left(\frac{x}{b}+\frac{(-a^2-1)\arctan(bx+a)}{b^2}\right)}{2}$
parallelrisch	$\frac{\operatorname{arccot}(bx+a)x^2b^3+2a\,\operatorname{arccot}(bx+a)x\,b^2+\operatorname{arccot}(bx+a)a^2b+b^2x+\operatorname{arccot}(bx+a)b-2ab}{2b^2}$
risch	$\frac{i(x^2b+2ax)\ln(1+i(bx+a))}{4}-\frac{ib\,x^2\ln(1-i(bx+a))}{4}-\frac{iax\ln(1-i(bx+a))}{2}+\frac{\pi b\,x^2}{4}+\frac{\pi ax}{2}-\frac{a^2\arctan(bx+a)}{2b}+$

[In] int((b*x+a)*arccot(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b*(1/2*(b*x+a)^2*arccot(b*x+a)+1/2*b*x+1/2*a-1/2*arctan(b*x+a))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int (a+bx)\cot^{-1}(a+bx)\,dx=\frac{bx+(b^2x^2+2\,abx+a^2+1)\operatorname{arccot}(bx+a)}{2\,b}$$

[In] integrate((b*x+a)*arccot(b*x+a),x, algorithm="fricas")

[Out] 1/2*(b*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)*arccot(b*x + a))/b

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int (a+bx)\cot^{-1}(a+bx)\,dx$$
$$= \begin{cases} \frac{a^2 \operatorname{acot}(a+bx)}{2b} + ax \operatorname{acot}(a+bx) + \frac{bx^2 \operatorname{acot}(a+bx)}{2} + \frac{x}{2} + \frac{\operatorname{acot}(a+bx)}{2b} & \text{for } b \neq 0 \\ ax \operatorname{acot}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x+a)*acot(b*x+a),x)
[Out] Piecewise((a**2*acot(a + b*x)/(2*b) + a*x*acot(a + b*x) + b*x**2*acot(a + b
*x)/2 + x/2 + acot(a + b*x)/(2*b), Ne(b, 0)), (a*x*acot(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{1}{2}b\left(\frac{x}{b} - \frac{(a^2+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2}\right) + \frac{1}{2}\left(bx^2+2\,ax\right)\operatorname{arccot}(bx+a)$$

```
[In] integrate((b*x+a)*arccot(b*x+a),x, algorithm="maxima")
[Out] 1/2*b*(x/b - (a^2 + 1)*arctan((b^2*x + a*b)/b)/b^2) + 1/2*(b*x^2 + 2*a*x)*a
rccot(b*x + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(33) = 66$.

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

$$\int (a+bx)\cot^{-1}(a+bx)\,dx$$
$$= \frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3}{8\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

```
[In] integrate((b*x+a)*arccot(b*x+a),x, algorithm="giac")
[Out] 1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x +
 a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 + ar
ctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b*tan(1/2*arctan(1/(b*
x + a)))^2)
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{x}{2}+\frac{\frac{\mathrm{acot}(a+b\,x)}{2}+\frac{a^2\,\mathrm{acot}(a+b\,x)}{2}}{b}+a\,x\,\mathrm{acot}(a+b\,x)+\frac{b\,x^2\,\mathrm{acot}(a+b\,x)}{2}$$

```
[In] int(acot(a + b*x)*(a + b*x),x)
```

```
[Out] x/2 + (acot(a + b*x)/2 + (a^2*acot(a + b*x))/2)/b + a*x*acot(a + b*x) + (b*
x^2*acot(a + b*x))/2
```

3.124 $\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=-\frac{i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2b}+\frac{i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2b}$$

```
[Out] -1/2*I*polylog(2,-I/(b*x+a))/b+1/2*I*polylog(2,I/(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5152, 4941, 2438}

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=\frac{i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2b}-\frac{i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2b}$$

```
[In] Int[ArcCot[a + b*x]/(a + b*x),x]
[Out] ((-1/2*I)*PolyLog[2, (-I)/(a + b*x)])/b + ((I/2)*PolyLog[2, I/(a + b*x)])/b
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, a+bx\right)}{b} \\ &= \frac{i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, a+bx\right)}{2b} - \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, a+bx\right)}{2b} \\ &= -\frac{i\,\text{PolyLog}\left(2, -\frac{i}{a+bx}\right)}{2b} + \frac{i\,\text{PolyLog}\left(2, \frac{i}{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = -\frac{i\left(\text{PolyLog}\left(2, -\frac{i}{a+bx}\right) - \text{PolyLog}\left(2, \frac{i}{a+bx}\right)\right)}{2b}$$

```
[In] Integrate[ArcCot[a + b*x]/(a + b*x),x]
[Out] ((-1/2*I)*(PolyLog[2, (-I)/(a + b*x)] - PolyLog[2, I/(a + b*x)]))/b
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

method	result
risch	$\frac{\pi \ln(-ibx-ia)}{2b}+\frac{i\,\text{dilog}(-ibx-ia+1)}{2b}-\frac{i\,\text{dilog}(ibx+ia+1)}{2b}$
derivativedivides	$\frac{\ln(bx+a)\,\text{arccot}(bx+a)-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\text{dilog}(1+i(bx+a))}{2}+\frac{i\,\text{dilog}(1-i(bx+a))}{2}}{b}$
default	$\frac{\ln(bx+a)\,\text{arccot}(bx+a)-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\text{dilog}(1+i(bx+a))}{2}+\frac{i\,\text{dilog}(1-i(bx+a))}{2}}{b}$
parts	$\frac{\ln(bx+a)\,\text{arccot}(bx+a)}{b}+\frac{-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\text{dilog}(1+i(bx+a))}{2}+\frac{i\,\text{dilog}(1-i(bx+a))}{2}}{b}$

```
[In] int(arccot(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

[Out] 1/2/b*Pi*ln(-I*a-I*b*x)+1/2*I/b*dilog(1-I*a-I*b*x)-1/2*I/b*dilog(1+I*a+I*b*x)

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{bx+a}\,dx$$

[In] integrate(arccot(b*x+a)/(b*x+a),x, algorithm="fricas")

[Out] integral(arccot(b*x + a)/(b*x + a), x)

Sympy [F]

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = \int \frac{\operatorname{acot}(a+bx)}{a+bx}\,dx$$

[In] integrate(acot(b*x+a)/(b*x+a),x)

[Out] Integral(acot(a + b*x)/(a + b*x), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(31) = 62.

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = \frac{\operatorname{arccot}(bx+a)\log(bx+a)}{b} + \frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(bx+a)}{b} + \frac{\arctan(bx+a,0)\log(b^2x^2+2\,abx+a^2+1) - 2\,\arctan(bx+a)\log(|bx+a|) + i\,\mathrm{Li}_2(i\,bx+i\,a+1) - i}{2\,b}$$

[In] integrate(arccot(b*x+a)/(b*x+a),x, algorithm="maxima")

[Out] arccot(b*x + a)*log(b*x + a)/b + arctan((b^2*x + a*b)/b)*log(b*x + a)/b + 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I*a + 1))/b

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(31) = 62.

Time = 0.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = -\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right.\right.}{8\,b^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

```
[In] integrate(arccot(b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x
+ a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 + a
rctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b^2*tan(1/2*arctan(1/
(b*x + a)))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{a+b\,x}\,dx$$

```
[In] int(acot(a + b*x)/(a + b*x),x)
```

```
[Out] int(acot(a + b*x)/(a + b*x), x)
```

3.125 $\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2} dx = -\frac{\cot^{-1}(a+bx)}{b(a+bx)} - \frac{\log(a+bx)}{b} + \frac{\log(1+(a+bx)^2)}{2b}$$

```
[Out] -arccot(b*x+a)/b/(b*x+a)-ln(b*x+a)/b+1/2*ln(1+(b*x+a)^2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5152, 4947, 272, 36, 29, 31}

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2} dx = -\frac{\log(a+bx)}{b} + \frac{\log((a+bx)^2+1)}{2b} - \frac{\cot^{-1}(a+bx)}{b(a+bx)}$$

```
[In] Int[ArcCot[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -(ArcCot[a + b*x]/(b*(a + b*x))) - Log[a + b*x]/b + Log[1 + (a + b*x)^2]/(2
*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\cot^{-1}(a+bx)}{b(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\cot^{-1}(a+bx)}{b(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, (a+bx)^2\right)}{2b}\\
&= -\frac{\cot^{-1}(a+bx)}{b(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (a+bx)^2\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, (a+bx)^2\right)}{2b}\\
&= -\frac{\cot^{-1}(a+bx)}{b(a+bx)} - \frac{\log(a+bx)}{b} + \frac{\log\left(1+(a+bx)^2\right)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{-\frac{\cot^{-1}(a+bx)}{a+bx} - \log(a+bx) + \frac{1}{2}\log\left(1+(a+bx)^2\right)}{b}$$

```
[In] Integrate[ArcCot[a + b*x]/(a + b*x)^2,x]
[Out] (-(ArcCot[a + b*x]/(a + b*x)) - Log[a + b*x] + Log[1 + (a + b*x)^2]/2)/b
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccot}(bx+a)}{bx+a}-\ln(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$
default	$\frac{-\frac{\operatorname{arccot}(bx+a)}{bx+a}-\ln(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$
parts	$-\frac{\operatorname{arccot}(bx+a)}{b(bx+a)}+\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{2b}-\frac{\ln(bx+a)}{b}$
parallelrisch	$-\frac{6\ln(bx+a)xa\,b^2-3b^2\ln\left(b^2x^2+2abx+a^2+1\right)ax+6\ln(bx+a)a^2b-3\ln\left(b^2x^2+2abx+a^2+1\right)a^2b+6\,\operatorname{arccot}(bx+a)ab}{6(bx+a)a\,b^2}$
risch	$-\frac{i\ln(1+i(bx+a))}{2b(bx+a)}-\frac{2\ln(-bx-a)bx-\ln\left(b^2x^2+2abx+a^2+1\right)bx+2\ln(-bx-a)a-a\ln\left(b^2x^2+2abx+a^2+1\right)-i\ln(1-i(bx+}{2(bx+a)b}$

```
[In] int(arccot(b*x+a)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-arccot(b*x+a)/(b*x+a)-ln(b*x+a)+1/2*ln(1+(b*x+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{(bx+a)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(bx+a)\log\left(bx+a\right)-2\,\operatorname{arccot}\left(bx+a\right)}{2\,(b^2x+ab)}$$

```
[In] integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*((b*x + a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(b*x + a)*log(b*x + a)
- 2*arccot(b*x + a))/(b^2*x + a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.96

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \begin{cases} -\frac{a\log\left(\frac{a}{b}+x\right)}{ab+b^2x} + \frac{a\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{ab+b^2x} + \frac{ia\operatorname{acot}(a+bx)}{ab+b^2x} - \frac{bx\log\left(\frac{a}{b}+x\right)}{ab+b^2x} + \frac{bx\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{ab+b^2x} + \frac{ibx\operatorname{acot}(a+bx)}{ab+b^2x} - \frac{\operatorname{acot}(a+bx)}{ab+b^2x} & \text{for } b \neq \\ \frac{x\operatorname{acot}(a)}{a^2} & \text{otherw} \end{cases}$$

```
[In] integrate(acot(b*x+a)/(b*x+a)**2,x)
```

```
[Out] Piecewise((-a*log(a/b + x)/(a*b + b**2*x) + a*log(a/b + x - I/b)/(a*b + b**
2*x) + I*a*acot(a + b*x)/(a*b + b**2*x) - b*x*log(a/b + x)/(a*b + b**2*x) +
 b*x*log(a/b + x - I/b)/(a*b + b**2*x) + I*b*x*acot(a + b*x)/(a*b + b**2*x)
 - acot(a + b*x)/(a*b + b**2*x), Ne(b, 0)), (x*acot(a)/a**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b} - \frac{\log\left(bx+a\right)}{b} - \frac{\operatorname{arccot}\left(bx+a\right)}{(bx+a)b}$$

```
[In] integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b - log(b*x + a)/b - arccot(b*x + a)/(
(b*x + a)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(45) = 90.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.06

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx =$$

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)^2 - \frac{\arctan\left(\frac{1}{bx+a}\right)^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 - \log\left(\frac{4\left(\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\operatorname{arct}\right.}{\tan}}{2\,b}$$

```
[In] integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(arctan(1/(b*x + a))^2 - (arctan(1/(b*x + a))^2*tan(1/2*arctan(1/(b*x
+ a)))^2 - log(4*(tan(1/2*arctan(1/(b*x + a)))^4 - 2*tan(1/2*arctan(1/(b*x
+ a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a
)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - arctan(1/(b*x + a))^2 + 4*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) + log(4*(tan(1/2*arctan(1/(b*x
 + a)))^4 - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x +
a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)))/(tan(1/2*arctan(1/(b*x + a
)))^2 - 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{\ln\left(-a^2-2\,a\,b\,x-b^2\,x^2-1\right)}{2\,b}-\frac{\ln\left(a+b\,x\right)}{b}-\frac{\mathrm{acot}(a+b\,x)}{x\,b^2+a\,b}$$

```
[In] int(acot(a + b*x)/(a + b*x)^2,x)
```

```
[Out] log(- a^2 - b^2*x^2 - 2*a*b*x - 1)/(2*b) - log(a + b*x)/b - acot(a + b*x)/(
a*b + b^2*x)
```

3.126 $\int \frac{\cot^{-1}(1+x)}{2+2x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \frac{\cot^{-1}(1+x)}{2+2x} \, dx = -\frac{1}{4} i \operatorname{PolyLog}\left(2,-\frac{i}{1+x}\right)+\frac{1}{4} i \operatorname{PolyLog}\left(2,\frac{i}{1+x}\right)$$

```
[Out] -1/4*I*polylog(2,-I/(1+x))+1/4*I*polylog(2,I/(1+x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5152, 12, 4941, 2438}

$$\int \frac{\cot^{-1}(1+x)}{2+2x} \, dx = \frac{1}{4} i \operatorname{PolyLog}\left(2,\frac{i}{x+1}\right)-\frac{1}{4} i \operatorname{PolyLog}\left(2,-\frac{i}{x+1}\right)$$

```
[In] Int[ArcCot[1 + x]/(2 + 2*x),x]
[Out] (-1/4*I)*PolyLog[2, (-I)/(1 + x)] + (I/4)*PolyLog[2, I/(1 + x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\cot^{-1}(x)}{2x}\,dx, x, 1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, 1+x\right)\\
&= \frac{1}{4}i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, 1+x\right) - \frac{1}{4}i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, 1+x\right)\\
&= -\frac{1}{4}i\,\text{PolyLog}\left(2, -\frac{i}{1+x}\right) + \frac{1}{4}i\,\text{PolyLog}\left(2, \frac{i}{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = -\frac{1}{4}i\,\text{PolyLog}\left(2, -\frac{i}{1+x}\right) + \frac{1}{4}i\,\text{PolyLog}\left(2, \frac{i}{1+x}\right)$$

```
[In] Integrate[ArcCot[1 + x]/(2 + 2*x),x]
[Out] (-1/4*I)*PolyLog[2, (-I)/(1 + x)] + (I/4)*PolyLog[2, I/(1 + x)]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result
risch	$\frac{i\,\mathrm{dilog}(-ix-i+1)}{4}+\frac{\pi\ln(-ix-i)}{4}-\frac{i\,\mathrm{dilog}(ix+i+1)}{4}$
derivativedivides	$\frac{\ln(1+x)\,\mathrm{arccot}(1+x)}{2}-\frac{i\ln(1+x)\ln(1+i(1+x))}{4}+\frac{i\ln(1+x)\ln(1-i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1-i(1+}{4}$
default	$\frac{\ln(1+x)\,\mathrm{arccot}(1+x)}{2}-\frac{i\ln(1+x)\ln(1+i(1+x))}{4}+\frac{i\ln(1+x)\ln(1-i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1-i(1+}{4}$
parts	$\frac{\ln(1+x)\,\mathrm{arccot}(1+x)}{2}-\frac{i\ln(1+x)\ln(1+i(1+x))}{4}+\frac{i\ln(1+x)\ln(1-i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1-i(1+}{4}$

```
[In] int(arccot(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
[Out] 1/4*I*dilog(-I*x+1-I)+1/4*Pi*ln(-I-I*x)-1/4*I*dilog(I*x+1+I)
```

Fricas [F]

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx=\int \frac{\operatorname{arccot}(x+1)}{2\,(x+1)}\,dx$$

```
[In] integrate(arccot(1+x)/(2+2*x),x, algorithm="fricas")
[Out] integral(1/2*arccot(x + 1)/(x + 1), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx=\frac{\int \frac{\operatorname{acot}(x+1)}{x+1}\,dx}{2}$$

```
[In] integrate(acot(1+x)/(2+2*x),x)
[Out] Integral(acot(x + 1)/(x + 1), x)/2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(21) = 42.

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\begin{aligned}\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx=&\ \frac{1}{4}\arctan(x+1,0)\log\left(x^2+2\,x+2\right)+\frac{1}{2}\operatorname{arccot}(x+1)\log(x+1)\\&+\frac{1}{2}\arctan(x+1)\log(x+1)-\frac{1}{2}\arctan(x+1)\log(|x+1|)\\&+\frac{1}{4}i\,\mathrm{Li}_2(i\,x+i+1)-\frac{1}{4}i\,\mathrm{Li}_2(-i\,x-i+1)\end{aligned}$$

```
[In] integrate(arccot(1+x)/(2+2*x),x, algorithm="maxima")
```

```
[Out] 1/4*arctan2(x + 1, 0)*log(x^2 + 2*x + 2) + 1/2*arccot(x + 1)*log(x + 1) + 1
/2*arctan(x + 1)*log(x + 1) - 1/2*arctan(x + 1)*log(abs(x + 1)) + 1/4*I*dil
og(I*x + I + 1) - 1/4*I*dilog(-I*x - I + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}(1+x)}{2+2x} dx = -\frac{1}{4}(x+1)^2 \arctan\left(\frac{1}{x+1}\right) - \frac{1}{4}x - \frac{1}{4}\arctan\left(\frac{1}{x+1}\right) - \frac{1}{4}$$

```
[In] integrate(arccot(1+x)/(2+2*x),x, algorithm="giac")
```

```
[Out] -1/4*(x + 1)^2*arctan(1/(x + 1)) - 1/4*x - 1/4*arctan(1/(x + 1)) - 1/4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(1+x)}{2+2x} dx = \int \frac{\mathrm{acot}(x+1)}{2x+2} dx$$

```
[In] int(acot(x + 1)/(2*x + 2),x)
```

```
[Out] int(acot(x + 1)/(2*x + 2), x)
```

3.127 $\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=-\frac{i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2d}+\frac{i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2d}$$

```
[Out] -1/2*I*polylog(2,-I/(b*x+a))/d+1/2*I*polylog(2,I/(b*x+a))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5152, 12, 4941, 2438}

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2d}-\frac{i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2d}$$

```
[In] Int[ArcCot[a + b*x]/((a*d)/b + d*x),x]
[Out] ((-1/2*I)*PolyLog[2, (-I)/(a + b*x)])/d + ((I/2)*PolyLog[2, I/(a + b*x)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{b\cot^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}\\
&= \frac{i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, a+bx\right)}{2d} - \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, a+bx\right)}{2d}\\
&= -\frac{i\,\text{PolyLog}\left(2, -\frac{i}{a+bx}\right)}{2d} + \frac{i\,\text{PolyLog}\left(2, \frac{i}{a+bx}\right)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = -\frac{i\left(\text{PolyLog}\left(2, -\frac{i}{a+bx}\right) - \text{PolyLog}\left(2, \frac{i}{a+bx}\right)\right)}{2d}$$

```
[In] Integrate[ArcCot[a + b*x]/((a*d)/b + d*x),x]
[Out] ((-1/2*I)*(PolyLog[2, (-I)/(a + b*x)] - PolyLog[2, I/(a + b*x)]))/d
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

method	result
risch	$\frac{\pi \ln(-ibx-ia)}{2d}+\frac{i\,\mathrm{dilog}(-ibx-ia+1)}{2d}-\frac{i\,\mathrm{dilog}(ibx+ia+1)}{2d}$
parts	$\frac{\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}}{d}$
derivativedivides	$\frac{\frac{b\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$
default	$\frac{\frac{b\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$

```
[In] int(arccot(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
[Out] 1/2/d*Pi*ln(-I*a-I*b*x)+1/2*I/d*dilog(1-I*a-I*b*x)-1/2*I/d*dilog(1+I*a+I*b*
x)
```

Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\mathrm{arccot}\,(bx+a)}{dx+\frac{ad}{b}}\,dx$$

```
[In] integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
[Out] integral(b*arccot(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{b\int \frac{\mathrm{acot}\,(a+bx)}{a+bx}\,dx}{d}$$

```
[In] integrate(acot(b*x+a)/(a*d/b+d*x),x)
[Out] b*Integral(acot(a + b*x)/(a + b*x), x)/d
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(31) = 62.

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.71

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\operatorname{arccot}(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d}+\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{d} + \frac{\arctan(bx+a,0)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\arctan(bx+a)\log\left(|bx+a|\right)+i\,\mathrm{Li}_2(i\,bx+i\,a+1)-i}{2\,d}$$

[In] integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")

[Out] arccot(b*x + a)*log(d*x + a*d/b)/d + arctan((b^2*x + a*b)/b)*log(d*x + a*d/b)/d + 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I*a + 1))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(31) = 62.

Time = 0.56 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = -\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right.}{8\,bd\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

[In] integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="giac")

[Out] -1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 + arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b*d*tan(1/2*arctan(1/(b*x + a)))^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

```
[In] int(acot(a + b*x)/(d*x + (a*d)/b),x)
[Out] int(acot(a + b*x)/(d*x + (a*d)/b), x)
```

3.128 $\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)} \, dx = \text{Int}\left((a+bx)^2 \sqrt{\cot^{-1}(a+bx)}, x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)} \, dx = \int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)} \, dx$$

```
[In] Int[(a + b*x)^2*Sqrt[ArcCot[a + b*x]],x]
[Out] Defer[Int][(a + b*x)^2*Sqrt[ArcCot[a + b*x]], x]
```

Rubi steps

$$\text{integral} = \int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx$$

```
[In] Integrate[(a + b*x)^2*Sqrt[ArcCot[a + b*x]],x]
[Out] Integrate[(a + b*x)^2*Sqrt[ArcCot[a + b*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (bx+a)^2 \sqrt{\operatorname{arccot}(bx+a)}dx$$

```
[In] int((b*x+a)^2*arccot(b*x+a)^(1/2),x)
[Out] int((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \int (a+bx)^2 \sqrt{\operatorname{acot}(a+bx)}\,dx$$

```
[In] integrate((b*x+a)**2*acot(b*x+a)**(1/2),x)
[Out] Integral((a + b*x)**2*sqrt(acot(a + b*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \int (bx+a)^2\sqrt{\operatorname{arccot}(bx+a)}\,dx$$

```
[In] integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2*sqrt(arccot(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \int \sqrt{\operatorname{acot}(a+b\,x)}\,(a+b\,x)^2\,dx$$

```
[In] int(acot(a + b*x)^(1/2)*(a + b*x)^2,x)
[Out] int(acot(a + b*x)^(1/2)*(a + b*x)^2, x)
```

3.129 $\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 233

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) dx = \frac{bf(6d^2e^2-12cdef-(1-6c^2)f^2)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{b(d^4e^4-4cd^3e^3f-6(1-c^2)d^2e^2f^2+4c(3-c^2)def^3+(1-6c^2+c^4)f^4)\arctan(c+dx)}{4d^4f} + \frac{b(de-cf)(de+f-cf)(de-(1+c)f)\log(1+(c+dx)^2)}{2d^4}$$

```
[Out] 1/4*b*f*(6*d^2*e^2-12*c*d*e*f-(-6*c^2+1)*f^2)*x/d^3+1/2*b*f^2*(-c*f+d*e)*(d
*x+c)^2/d^4+1/12*b*f^3*(d*x+c)^3/d^4+1/4*(f*x+e)^4*(a+b*arccot(d*x+c))/f+1/
4*b*(d^4*e^4-4*c*d^3*e^3*f-6*(-c^2+1)*d^2*e^2*f^2+4*c*(-c^2+3)*d*e*f^3+(c^4
-6*c^2+1)*f^4)*arctan(d*x+c)/d^4/f+1/2*b*(-c*f+d*e)*(-c*f+d*e+f)*(d*e-(1+c)
*f)*ln(1+(d*x+c)^2)/d^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{5156, 4973, 716, 649, 209, 266\}$

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{b\arctan(c+dx)\left(-6(1-c^2)\,d^2e^2f^2+4c(3-c^2)\,def^3+(c^4-6c^2+1)\,f^4-4cd^3e^3f+d^4e^4\right)}{4d^4f} + \frac{bfx(-(1-6c^2)\,f^2-12cdef+6d^2e^2)}{4d^3} + \frac{bf^2(c+dx)^2(de-cf)}{2d^4} + \frac{b(de-cf)(-cf+de+f)(de-(c+1)f)\log\left((c+dx)^2+1\right)}{2d^4} + \frac{bf^3(c+dx)^3}{12d^4}$$

```
[In] Int[(e + f*x)^3*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] (b*f*(6*d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*x)/(4*d^3) + (b*f^2*(d*e -
c*f)*(c + d*x)^2)/(2*d^4) + (b*f^3*(c + d*x)^3)/(12*d^4) + ((e + f*x)^4*(a
+ b*ArcCot[c + d*x]))/(4*f) + (b*(d^4*e^4 - 4*c*d^3*e^3*f - 6*(1 - c^2)*d^2
*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2 + c^4)*f^4)*ArcTan[c + d*x])/
(4*d^4*f) + (b*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c + d
*x)^2])/(2*d^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
```

```
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3 \left(a+b\cot^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^4}{1+x^2}\,dx, x, c+dx\right)}{4f}$$

$$= \frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{b\text{Subst}\left(\int \left(\frac{f^2(6d^2e^2-12cdef-(1-6c^2)f^2)}{d^4}+\frac{4f^3(de-cf)x}{d^4}+\frac{f^4x^2}{d^4}+\frac{d^4e^4-4cd^3e^3f-6(1-c^2)d^2e^2f^2+4c(3-c^2)def^3+}{d^4(}\right.\right.}{4f}$$

$$= \frac{bf(6d^2e^2-12cdef-(1-6c^2)f^2)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{b\text{Subst}\left(\int \frac{d^4e^4-4cd^3e^3f-6(1-c^2)d^2e^2f^2+4c(3-c^2)def^3+(1-6c^2+c^4)f^4+4f(de-cf)(de-f-cf)(de+f-cf)x}{1+x^2}\,dx, x, c+\right.}{4d^4f}$$

$$= \frac{bf(6d^2e^2-12cdef-(1-6c^2)f^2)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{(b(de-cf)(de+f-cf)(de-(1+c)f))\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{d^4} + \frac{\left(b\left(d^4e^4-4cd^3e^3f-6(1-c^2)d^2e^2f^2+4c(3-c^2)def^3+(1-6c^2+c^4)f^4\right)\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, \right.}{4d^4f}$$

$$= \frac{bf(6d^2e^2 - 12cdef - (1-6c^2)f^2)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4(a+b\cot^{-1}(c+dx))}{4f} + \frac{b(d^4e^4 - 4cd^3e^3f - 6(1-c^2)d^2e^2f^2 + 4c(3-c^2)def^3 + (1-6c^2+c^4)f^4)\arctan(c+dx)}{4d^4f} + \frac{b(de-cf)(de+f-cf)(de-(1+c)f)\log(1+(c+dx)^2)}{2d^4}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.67

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) dx$$
$$= \frac{(e+fx)^4(a+b\cot^{-1}(c+dx)) + \frac{b(6df^2(6d^2e^2-12cdef+(-1+6c^2)f^2)x+12f^3(de-cf)(c+dx)^2+2f^4(c+dx)^3-3i(de-(-i+c)f)^4\log}{6d^4}}{4f}$$

```
[In] Integrate[(e + f*x)^3*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] ((e + f*x)^4*(a + b*ArcCot[c + d*x]) + (b*(6*d*f^2*(6*d^2*e^2 - 12*c*d*e*f
+ (-1 + 6*c^2)*f^2)*x + 12*f^3*(d*e - c*f)*(c + d*x)^2 + 2*f^4*(c + d*x)^3
- (3*I)*(d*e - (-I + c)*f)^4*Log[I - c - d*x] + (3*I)*(d*e - (I + c)*f)^4*L
og[I + c + d*x]))/(6*d^4))/(4*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(221) = 442$.

Time = 0.88 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.13

method	result
parts	$\frac{a(fx+e)^4}{4f} - \frac{f^3bc\,x^2}{4d^2} + \frac{f^2be\,x^2}{2d} + \frac{3f^3b\,c^2x}{4d^3} + \frac{3fb\,e^2x}{2d} + \frac{b\,f^3\,\operatorname{arccot}(dx+c)x^4}{4} + b\,\operatorname{arccot}\left(dx+c\right)x\,e^3$
derivativedivides	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3f} - \frac{b\left(-\frac{f^3\,\operatorname{arccot}(dx+c)c^4}{4}+f^2\,\operatorname{arccot}(dx+c)c^3de+f^3\,\operatorname{arccot}(dx+c)c^3(dx+c)-\frac{3f\,\operatorname{arccot}(dx+c)c^2d^2e^2}{2}-3f^2\,\operatorname{arc}\right.}{}}{}$
default	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3f} - \frac{b\left(-\frac{f^3\,\operatorname{arccot}(dx+c)c^4}{4}+f^2\,\operatorname{arccot}(dx+c)c^3de+f^3\,\operatorname{arccot}(dx+c)c^3(dx+c)-\frac{3f\,\operatorname{arccot}(dx+c)c^2d^2e^2}{2}-3f^2\,\operatorname{arc}\right.}{}}{}$
parallelrisch	$-\frac{-42b\,c^2de\,f^2+24ac\,d^3e^3+6\ln\left(d^2x^2+2cdx+c^2+1\right)b\,c^3f^3-6\ln\left(d^2x^2+2cdx+c^2+1\right)b\,d^3e^3-6\ln\left(d^2x^2+2cdx+c^2+1\right)bc}{}$
risch	$\frac{x^4f^3a}{4} + x\,e^3a - \frac{f^3bc\,x^2}{4d^2} + \frac{f^2be\,x^2}{2d} + \frac{3f^3b\,c^2x}{4d^3} + \frac{3fb\,e^2x}{2d} - \frac{3fb\,e^2\arctan(dx+c)}{2d^2} - \frac{f^2be\ln\left(d^2x^2+2cdx+c\right.}{2d^3}$

```
[In] int((f*x+e)^3*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*(f*x+e)^4/f-1/4/d^2*f^3*b*c*x^2+1/2/d*f^2*b*e*x^2+3/4/d^3*f^3*b*c^2*x
+3/2/d*f*b*e^2*x+1/4*b*f^3*arccot(d*x+c)*x^4+b*arccot(d*x+c)*x*e^3+1/4*b/f*
arccot(d*x+c)*e^4-3/2/d^2*f*b*e^2*arctan(d*x+c)+1/4/d^4*f^3*b*c^4*arctan(d*
x+c)-3/2/d^4*f^3*b*c^2*arctan(d*x+c)-1/d*b*c*e^3*arctan(d*x+c)-1/4*b/d^4*f^
3*c+13/12*b/d^4*f^3*c^3+1/2*b/d*ln(1+(d*x+c)^2)*e^3+1/4/d^4*f^3*b*arctan(d*
x+c)+1/12/d*f^3*b*x^3-1/4/d^3*f^3*b*x+1/4/f*b*e^4*arctan(d*x+c)+3/2*b/d^3*f
^2*ln(1+(d*x+c)^2)*c^2*e-1/2*b/d^4*f^3*ln(1+(d*x+c)^2)*c^3+3/2*b*f*arccot(d
*x+c)*e^2*x^2+b*f^2*arccot(d*x+c)*e*x^3+1/2*b/d^4*f^3*ln(1+(d*x+c)^2)*c-1/2
*b/d^3*f^2*ln(1+(d*x+c)^2)*e-2/d^2*f^2*b*c*e*x-1/d^3*f^2*b*c^3*e*arctan(d*x
+c)+3/2/d^2*f*b*c^2*e^2*arctan(d*x+c)+3/d^3*f^2*b*c*e*arctan(d*x+c)-5/2*b/d
^3*f^2*c^2*e+3/2*b/d^2*f*c*e^2-3/2*b/d^2*f*ln(1+(d*x+c)^2)*c*e^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.39

$$\int (e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= \frac{3\,ad^4f^3x^4 + (12\,ad^4ef^2 + bd^3f^3)x^3 + 3\,(6\,ad^4e^2f + 2\,bd^3ef^2 - bcd^2f^3)x^2 + 3\,(4\,ad^4e^3 + 6\,bd^3e^2f - 8\,bcd^2}{}$$

```
[In] integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*f^3*x^4 + (12*a*d^4*e*f^2 + b*d^3*f^3)*x^3 + 3*(6*a*d^4*e^2*f
 + 2*b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + 3*(4*a*d^4*e^3 + 6*b*d^3*e^2*f - 8*b*
c*d^2*e*f^2 + (3*b*c^2 - b)*d*f^3)*x + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*x^3
 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x)*arccot(d*x + c) - 3*(4*b*c*d^3*e^3 -
6*(b*c^2 - b)*d^2*e^2*f + 4*(b*c^3 - 3*b*c)*d*e*f^2 - (b*c^4 - 6*b*c^2 + b)
*f^3)*arctan(d*x + c) + 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + (3*b*c^2 - b)*d*e*
f^2 - (b*c^3 - b*c)*f^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**3*(a+b*acot(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.46

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{1}{4}\,af^3x^4 + aef^2x^3 + \frac{3}{2}\,ae^2fx^2 + \frac{3}{2}\left(x^2\,\mathrm{arccot}\left(dx+c\right) + d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)be^2f + \frac{1}{2}\left(2\,x^3\,\mathrm{arccot}\left(dx+c\right) + d\left(\frac{dx^2-4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx+\right.}{d^4}\right.\right. + \frac{1}{12}\left(3\,x^4\,\mathrm{arccot}\left(dx+c\right) + d\left(\frac{d^2x^3-3\,cdx^2+3\,(3\,c^2-1)x}{d^4} + \frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5} - \frac{6\,(c^3}{}\right.\right. + ae^3x + \frac{\left(2\,(dx+c)\,\mathrm{arccot}\left(dx+c\right)+\log\left((dx+c)^2+1\right)\right)be^3}{2\,d}$$

```
[In] integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/2*(x^2*arccot(d*x + c) +
d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*x)/
d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)/d^4))*b*e*f^2 + 1/12*(3*x^4*arccot(d*x + c) + d*((d^2*x
```

```
^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d^2*x +
 c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*f^3 + a
*e^3*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b*e^3/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2265 vs. 2(216) = 432.

Time = 1.73 (sec) , antiderivative size = 2265, normalized size of antiderivative = 9.72

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(96*b*d^3*e^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 2
88*b*c*d^2*e^2*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 288*b
*c^2*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 96*b*c^3*
f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 72*b*d^2*e^2*f*arc
tan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 + 144*b*c*d*e*f^2*arctan(1/
(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 - 72*b*c^2*f^3*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^6 + 24*b*d*e*f^2*arctan(1/(d*x + c))*tan(1/2
*arctan(1/(d*x + c)))^7 - 24*b*c*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(
d*x + c)))^7 - 3*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^8 +
 96*b*d^3*e^3*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x
+ c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c
)))^4 - 288*b*c*d^2*e^2*f*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*ar
ctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arcta
n(1/(d*x + c)))^4 + 288*b*c^2*d*e*f^2*log(16*tan(1/2*arctan(1/(d*x + c)))^2
/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*t
an(1/2*arctan(1/(d*x + c)))^4 - 96*b*c^3*f^3*log(16*tan(1/2*arctan(1/(d*x +
 c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2
+ 1))*tan(1/2*arctan(1/(d*x + c)))^4 + 96*a*d^3*e^3*tan(1/2*arctan(1/(d*x +
 c)))^5 - 288*a*c*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^5 + 288*a*c^2*d*e*
f^2*tan(1/2*arctan(1/(d*x + c)))^5 - 96*a*c^3*f^3*tan(1/2*arctan(1/(d*x + c
)))^5 - 72*a*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^6 + 144*a*c*d*e*f^2*tan
(1/2*arctan(1/(d*x + c)))^6 - 72*a*c^2*f^3*tan(1/2*arctan(1/(d*x + c)))^6 +
 24*a*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^7 - 24*a*c*f^3*tan(1/2*arctan(1/
(d*x + c)))^7 - 3*a*f^3*tan(1/2*arctan(1/(d*x + c)))^8 - 96*b*d^3*e^3*arcta
n(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 288*b*c*d^2*e^2*f*arctan(1/
(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 288*b*c^2*d*e*f^2*arctan(1/(d*x
 + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 96*b*c^3*f^3*arctan(1/(d*x + c))*ta
n(1/2*arctan(1/(d*x + c)))^3 - 144*b*d^2*e^2*f*arctan(1/(d*x + c))*tan(1/2*
arctan(1/(d*x + c)))^4 + 288*b*c*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan
(1/(d*x + c)))^4 - 144*b*c^2*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x
+ c)))^4 + 144*b*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^5 - 288*b*c*d*e*f^2
```

```
*tan(1/2*arctan(1/(d*x + c)))^5 + 144*b*c^2*f^3*tan(1/2*arctan(1/(d*x + c))
)^5 - 72*b*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 72*
b*c*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 24*b*d*e*f^2*t
an(1/2*arctan(1/(d*x + c)))^6 + 24*b*c*f^3*tan(1/2*arctan(1/(d*x + c)))^6 +
 12*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 + 2*b*f^3*tan(
1/2*arctan(1/(d*x + c)))^7 - 96*a*d^3*e^3*tan(1/2*arctan(1/(d*x + c)))^3 +
288*a*c*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^3 - 288*a*c^2*d*e*f^2*tan(1/
2*arctan(1/(d*x + c)))^3 + 96*a*c^3*f^3*tan(1/2*arctan(1/(d*x + c)))^3 - 14
4*a*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^4 + 288*a*c*d*e*f^2*tan(1/2*arct
an(1/(d*x + c)))^4 - 144*a*c^2*f^3*tan(1/2*arctan(1/(d*x + c)))^4 - 96*b*d*
e*f^2*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4
 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^4 +
96*b*c*f^3*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c
)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))
^4 - 72*a*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^5 + 72*a*c*f^3*tan(1/2*arcta
n(1/(d*x + c)))^5 + 12*a*f^3*tan(1/2*arctan(1/(d*x + c)))^6 - 72*b*d^2*e^2*
f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 144*b*c*d*e*f^2*arct
an(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 72*b*c^2*f^3*arctan(1/(d*x
 + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 144*b*d^2*e^2*f*tan(1/2*arctan(1/(d
*x + c)))^3 + 288*b*c*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^3 - 144*b*c^2*f^
3*tan(1/2*arctan(1/(d*x + c)))^3 + 72*b*d*e*f^2*arctan(1/(d*x + c))*tan(1/2
*arctan(1/(d*x + c)))^3 - 72*b*c*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(
d*x + c)))^3 - 48*b*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 48*b*c*f^3*tan
(1/2*arctan(1/(d*x + c)))^4 + 30*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1
/(d*x + c)))^4 - 30*b*f^3*tan(1/2*arctan(1/(d*x + c)))^5 - 72*a*d^2*e^2*f*t
an(1/2*arctan(1/(d*x + c)))^2 + 144*a*c*d*e*f^2*tan(1/2*arctan(1/(d*x + c))
)^2 - 72*a*c^2*f^3*tan(1/2*arctan(1/(d*x + c)))^2 + 72*a*d*e*f^2*tan(1/2*ar
ctan(1/(d*x + c)))^3 - 72*a*c*f^3*tan(1/2*arctan(1/(d*x + c)))^3 + 30*a*f^3
*tan(1/2*arctan(1/(d*x + c)))^4 - 24*b*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*
arctan(1/(d*x + c))) + 24*b*c*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x
 + c))) - 24*b*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + 24*b*c*f^3*tan(1/2*
arctan(1/(d*x + c)))^2 + 12*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x
 + c)))^2 + 30*b*f^3*tan(1/2*arctan(1/(d*x + c)))^3 - 24*a*d*e*f^2*tan(1/2*
arctan(1/(d*x + c))) + 24*a*c*f^3*tan(1/2*arctan(1/(d*x + c))) + 12*a*f^3*t
an(1/2*arctan(1/(d*x + c)))^2 - 3*b*f^3*arctan(1/(d*x + c)) - 2*b*f^3*tan(1
/2*arctan(1/(d*x + c))) - 3*a*f^3)/(d^4*tan(1/2*arctan(1/(d*x + c)))^4)
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 783, normalized size of antiderivative = 3.36

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \operatorname{acot}(c+d\,x)\,\left(b\,e^3\,x+\frac{3\,b\,e^2\,f\,x^2}{2}+b\,e\,f^2\,x^3+\frac{b\,f^3\,x^4}{4}\right) + x\left(\frac{e\,(6\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+2\,a\,d^2\,e^2+3\,b\,d\,e\,f+6\,a\,f^2)}{2\,d^2} - \frac{(4\,c^2+4)\,\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{4\,d^2} + \frac{2\,c\,\left(\frac{2\,c\,\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2+4\,a\,f^3}{4\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{4\,d^2}\right)}{d}\right) - x^2\left(\frac{c\,\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d} - \frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2+4\,a\,f^3}{8\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{8\,d^2}\right) + x^3\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{12\,d}-\frac{2\,a\,c\,f^3}{3\,d}\right)+\frac{a\,f^3\,x^4}{4} + \frac{\ln(c^2+2\,c\,d\,x+d^2\,x^2+1)\,(-64\,b\,c^3\,d^4\,f^3+192\,b\,c^2\,d^5\,e\,f^2-192\,b\,c\,d^6\,e^2\,f+64\,b\,c\,d^4\,f^3+64\,b\,d^7\,e^3 -}{128\,d^8} + \frac{b\,\operatorname{atan}\left(\frac{4\,d^3\,\left(\frac{c\,(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3)}{4\,d^3}+\frac{x\,(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c}{4\,d^2}\right.}{c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3}\right.}{4\,d}$$

```
[In] int((e + f*x)^3*(a + b*acot(c + d*x)),x)
```

```
[Out] acot(c + d*x)*((b*f^3*x^4)/4 + b*e^3*x + (3*b*e^2*f*x^2)/2 + b*e*f^2*x^3) +
 x*((e*(6*a*f^2 + 6*a*c^2*f^2 + 2*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f))/(2
*d^2) - ((4*c^2 + 4)*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/
d))/(4*d^2) + (2*c*((2*c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f
^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*
d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 + 4))/(4*d^2)))/d) - x^2*((c*((f^2*(b*f +
8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 + 4
```

```
*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2) + (a*f^3*(4*c^2 + 4))
/(8*d^2)) + x^3*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(12*d) - (2*a*c*f^3)/(3*d
)) + (a*f^3*x^4)/4 + (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(64*b*d^7*e^3 - 64*b
*c^3*d^4*f^3 + 64*b*c*d^4*f^3 - 64*b*d^5*e*f^2 - 192*b*c*d^6*e^2*f + 192*b*
c^2*d^5*e*f^2))/(128*d^8) + (b*atan((4*d^3*((c*(f^3 - 6*c^2*f^3 + c^4*f^3 -
 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2
))/(4*d^3) + (x*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*
c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^2)))/(f^3 - 6*c^2*f^3 +
 c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c
^3*d*e*f^2))*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2
*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^4)
```

3.130 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 154

$$\begin{aligned}
&\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right) dx \\
&= \frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f} \\
&\quad+\frac{b(de-cf)\left(d^2e^2-2cdef-\left(3-c^2\right)f^2\right)\arctan(c+dx)}{3d^3f} \\
&\quad+\frac{b\left(3d^2e^2-6cdef-\left(1-3c^2\right)f^2\right)\log\left(1+(c+dx)^2\right)}{6d^3}
\end{aligned}$$

```
[Out] b*f*(-c*f+d*e)*x/d^2+1/6*b*f^2*(d*x+c)^2/d^3+1/3*(f*x+e)^3*(a+b*arccot(d*x+
c))/f+1/3*b*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*arctan(d*x+c)/d^3/f
+1/6*b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*ln(1+(d*x+c)^2)/d^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\begin{aligned}
&\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right) dx \\
&= \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f} \\
&\quad+\frac{b\arctan(c+dx)(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)}{3d^3f} \\
&\quad+\frac{b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left((c+dx)^2+1\right)}{6d^3}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{bfx(de-cf)}{d^2}
\end{aligned}$$

```
[In] Int[(e + f*x)^2*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] (b*f*(d*e - c*f)*x)/d^2 + (b*f^2*(c + d*x)^2)/(6*d^3) + ((e + f*x)^3*(a + b
*ArcCot[c + d*x]))/(3*f) + (b*(d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*
f^2)*ArcTan[c + d*x])/(3*d^3*f) + (b*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f
^2)*Log[1 + (c + d*x)^2])/(6*d^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\cot^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f} + \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3}{1+x^2}\,dx, x, c+dx\right)}{3f}$$

$$= \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f} + \frac{b\text{Subst}\left(\int \left(\frac{3f^2(de-cf)}{d^3}+\frac{f^3x}{d^3}+\frac{(de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cdef-(1-3c^2)f^2)x}{d^3(1+x^2)}\right)\,dx, x, c+dx\right)}{3f}$$

$$= \frac{bf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f} + \frac{b\text{Subst}\left(\int \frac{(de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cdef-(1-3c^2)f^2)x}{1+x^2}\,dx, x, c+dx\right)}{3d^3f}$$

$$= \frac{bf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f} + \frac{\left(b(3d^2e^2-6cdef-(1-3c^2)f^2)\right)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{3d^3} + \frac{\left(b(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{3d^3f}$$

$$= \frac{bf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f} + \frac{b(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)\arctan(c+dx)}{3d^3f} + \frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)\log\left(1+(c+dx)^2\right)}{6d^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

$$\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)+\frac{b\left(6df^2(de-cf)x+f^3(c+dx)^2-i(de-(-i+c)f)^3\log(i-c-dx)+i(de-(i+c)f)^3\log(i+c+dx)\right)}{2d^3}}{3f}$$

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] ((e + f*x)^3*(a + b*ArcCot[c + d*x]) + (b*(6*d*f^2*(d*e - c*f)*x + f^3*(c +
 d*x)^2 - I*(d*e - (-I + c)*f)^3*Log[I - c - d*x] + I*(d*e - (I + c)*f)^3*L
og[I + c + d*x]))/(2*d^3))/(3*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(146) = 292.

Time = 0.68 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.91

method	result
parts	$\frac{a(fx+e)^3}{3f} - \frac{2f^2bcx}{3d^2} + \frac{fbex}{d} + \frac{b\,f^2 \ln\left(1+(dx+c)^2\right)c^2}{2d^3} + \frac{b\,e^2 \ln\left(1+(dx+c)^2\right)}{2d} - \frac{f^2 b\,c^3 \arctan(dx+c)}{3d^3} + \frac{b\,e^3 \operatorname{arct}}{3}$
parallelrisch	$\frac{-f^2b-\ln\left(d^2x^2+2cdx+c^2+1\right)b\,f^2+7b\,c^2f^2-6fead-6a\,c^2efd+2x^3 \operatorname{arccot}(dx+c)b\,d^3f^2+6x \operatorname{arccot}(dx+c)b\,d^3e^2+6 \operatorname{arcco}}{}$
derivativedivides	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\left(-\frac{f^2 \operatorname{arccot}(dx+c)c^3}{3}+f \operatorname{arccot}(dx+c)c^2de+f^2 \operatorname{arccot}(dx+c)c^2(dx+c)-\operatorname{arccot}(dx+c)c\,d^2e^2-2f \operatorname{arccot}(dx\right.}{}}{}$
default	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\left(-\frac{f^2 \operatorname{arccot}(dx+c)c^3}{3}+f \operatorname{arccot}(dx+c)c^2de+f^2 \operatorname{arccot}(dx+c)c^2(dx+c)-\operatorname{arccot}(dx+c)c\,d^2e^2-2f \operatorname{arccot}(dx\right.}{}}{}$
risch	$-\frac{ifbe\,x^2 \ln(1-i(dx+c))}{2} - \frac{ib\,e^3 \ln\left(d^2x^2+2cdx+c^2+1\right)}{12f} - \frac{f^2 b \ln\left(d^2x^2+2cdx+c^2+1\right)}{6d^3} + \frac{i(fx+e)^3 b \ln(1+i(dx+c))}{6f} -$

```
[In] int((f*x+e)^2*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(f*x+e)^3/f-2/3*f^2/d^2*b*c*x+f/d*b*e*x+1/2*b/d^3*f^2*ln(1+(d*x+c)^2)
*c^2+1/2*b*e^2*ln(1+(d*x+c)^2)/d-1/3*f^2/d^3*b*c^3*arctan(d*x+c)+1/3/f*b*e^
3*arctan(d*x+c)+f^2/d^3*b*c*arctan(d*x+c)-f/d^2*b*e*arctan(d*x+c)+1/3*b*f^2
*arccot(d*x+c)*x^3+b*arccot(d*x+c)*x*e^2-5/6*b/d^3*f^2*c^2+b/d^2*f*c*e+1/3*
b/f*arccot(d*x+c)*e^3+1/6*f^2/d*b*x^2-1/6*b/d^3*f^2*ln(1+(d*x+c)^2)-b/d^2*f
*ln(1+(d*x+c)^2)*c*e+f/d^2*b*c^2*e*arctan(d*x+c)-1/d*b*c*e^2*arctan(d*x+c)+
b*f*arccot(d*x+c)*e*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.34

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= \frac{2\,ad^3f^2x^3+(6\,ad^3ef+bd^2f^2)x^2+2\,(3\,ad^3e^2+3\,bd^2ef-2\,bcdf^2)x+2\,(bd^3f^2x^3+3\,bd^3efx^2+3\,bd^3e^2x)}{}$$

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f + b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 + 3*b
*d^2*e*f - 2*b*c*d*f^2)*x + 2*(b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*e^
2*x)*arccot(d*x + c) - 2*(3*b*c*d^2*e^2 - 3*(b*c^2 - b)*d*e*f + (b*c^3 - 3*
b*c)*f^2)*arctan(d*x + c) + (3*b*d^2*e^2 - 6*b*c*d*e*f + (3*b*c^2 - b)*f^2)
*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 125.91 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.44

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= \begin{cases} ae^2x+aefx^2+\frac{af^2x^3}{3}+\frac{bc^3f^2\operatorname{acot}(c+dx)}{3d^3}-\frac{bc^2ef\operatorname{acot}(c+dx)}{d^2}+\frac{bc^2f^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3}+\frac{ibc^2f^2\operatorname{acot}(c+dx)}{d^3}+\frac{bce^2\operatorname{acot}(c+d}{d} \\ (a+b\operatorname{acot}(c))\left(e^2x+efx^2+\frac{f^2x^3}{3}\right) \end{cases}$$

```
[In] integrate((f*x+e)**2*(a+b*acot(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*acot(c + d*x
)/(3*d**3) - b*c**2*e*f*acot(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x - I/d)
/d**3 + I*b*c**2*f**2*acot(c + d*x)/d**3 + b*c*e**2*acot(c + d*x)/d - 2*b*c
*e*f*log(c/d + x - I/d)/d**2 - 2*I*b*c*e*f*acot(c + d*x)/d**2 - 2*b*c*f**2*
x/(3*d**2) - b*c*f**2*acot(c + d*x)/d**3 + b*e**2*x*acot(c + d*x) + b*e*f*x
**2*acot(c + d*x) + b*f**2*x**3*acot(c + d*x)/3 + b*e**2*log(c/d + x - I/d)
/d + I*b*e**2*acot(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) + b*e*f*acot(
c + d*x)/d**2 - b*f**2*log(c/d + x - I/d)/(3*d**3) - I*b*f**2*acot(c + d*x)
/(3*d**3), Ne(d, 0)), ((a + b*acot(c))*(e**2*x + e*f*x**2 + f**2*x**3/3), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.40

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{1}{3}af^2x^3 + aefx^2 + \left(x^2\operatorname{arccot}(dx+c) + d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)bef + \frac{1}{6}\left(2x^3\operatorname{arccot}(dx+c) + d\left(\frac{dx^2-4cx}{d^3} - \frac{2(c^3-3c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3c^2-1)\log(d^2x^2+2cdx+c^2+1)}{d^4}\right)\right)bf^2 + ae^2x + \frac{\left(2(dx+c)\operatorname{arccot}(dx+c) + \log\left((dx+c)^2+1\right)\right)be^2}{2d}$$

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 + (x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arc
tan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*e*f +
 1/6*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan
((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*
b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*
b*e^2/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. 2(142) = 284.

Time = 1.23 (sec) , antiderivative size = 1161, normalized size of antiderivative = 7.54

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(12*b*d^2*e^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - 24
*b*c*d*e*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 + 12*b*c^2*f^
2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - 6*b*d*e*f*arctan(1/(
d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 6*b*c*f^2*arctan(1/(d*x + c))*ta
n(1/2*arctan(1/(d*x + c)))^5 + b*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(
d*x + c)))^6 + 12*b*d^2*e^2*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*
arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arc
tan(1/(d*x + c)))^3 - 24*b*c*d*e*f*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(t
an(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(
```

```
1/2*arctan(1/(d*x + c)))^3 + 12*b*c^2*f^2*log(16*tan(1/2*arctan(1/(d*x + c)
))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1
))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*a*d^2*e^2*tan(1/2*arctan(1/(d*x + c)
))^4 - 24*a*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^4 + 12*a*c^2*f^2*tan(1/2*a
rctan(1/(d*x + c)))^4 - 6*a*d*e*f*tan(1/2*arctan(1/(d*x + c)))^5 + 6*a*c*f^
2*tan(1/2*arctan(1/(d*x + c)))^5 + a*f^2*tan(1/2*arctan(1/(d*x + c)))^6 - 1
2*b*d^2*e^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 24*b*c*d*e
*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 12*b*c^2*f^2*arctan
(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 12*b*d*e*f*arctan(1/(d*x + c
))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c*f^2*arctan(1/(d*x + c))*tan(1/2*
arctan(1/(d*x + c)))^3 + 12*b*d*e*f*tan(1/2*arctan(1/(d*x + c)))^4 - 12*b*c
*f^2*tan(1/2*arctan(1/(d*x + c)))^4 - 3*b*f^2*arctan(1/(d*x + c))*tan(1/2*a
rctan(1/(d*x + c)))^4 - b*f^2*tan(1/2*arctan(1/(d*x + c)))^5 - 12*a*d^2*e^2
*tan(1/2*arctan(1/(d*x + c)))^2 + 24*a*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))
^2 - 12*a*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 12*a*d*e*f*tan(1/2*arcta
n(1/(d*x + c)))^3 + 12*a*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*f^2*log
(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(
1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^3 - 3*a*f^2*t
an(1/2*arctan(1/(d*x + c)))^4 - 6*b*d*e*f*arctan(1/(d*x + c))*tan(1/2*arcta
n(1/(d*x + c))) + 6*b*c*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))
) - 12*b*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 12*b*c*f^2*tan(1/2*arctan(1
/(d*x + c)))^2 + 3*b*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2
 - 2*b*f^2*tan(1/2*arctan(1/(d*x + c)))^3 - 6*a*d*e*f*tan(1/2*arctan(1/(d*x
 + c))) + 6*a*c*f^2*tan(1/2*arctan(1/(d*x + c))) + 3*a*f^2*tan(1/2*arctan(1
/(d*x + c)))^2 - b*f^2*arctan(1/(d*x + c)) - b*f^2*tan(1/2*arctan(1/(d*x +
c))) - a*f^2)/(d^3*tan(1/2*arctan(1/(d*x + c)))^3)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.66

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx = x^2\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right) - x\left(\frac{2\,c\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d} - \frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2+3\,b\,d\,e\,f+3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2+3)}{3\,d^2}\right) + \mathrm{acot}(c+d\,x)\left(b\,e^2\,x+b\,e\,f\,x^2+\frac{b\,f^2\,x^3}{3}\right)+\frac{a\,f^2\,x^3}{3} + \frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\left(36\,b\,c^2\,d^3\,f^2-72\,b\,c\,d^4\,e\,f+36\,b\,d^5\,e^2-12\,b\,d^3\,f^2\right)}{72\,d^6} - \frac{b\,\mathrm{atan}\left(\frac{3\,d^2\left(\frac{c\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d^2}+\frac{x\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d}\right)}{c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f}\right)\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e\right.}{3\,d^3}$$

```
[In] int((e + f*x)^2*(a + b*acot(c + d*x)),x)
```

```
[Out] x^2*((f*(b*f + 6*a*c*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - x*((2*c*((f*(b*f
+ 6*a*c*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*f^2 + 3*a*c^2*f^2 +
3*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 + 3))/(3*d^
2)) + acot(c + d*x)*((b*f^2*x^3)/3 + b*e^2*x + b*e*f*x^2) + (a*f^2*x^3)/3 +
 (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(36*b*d^5*e^2 - 12*b*d^3*f^2 + 36*b*c^2*
d^3*f^2 - 72*b*c*d^4*e*f))/(72*d^6) - (b*atan((3*d^2*((c*(c^3*f^2 - 3*c*f^2
 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d^2) + (x*(c^3*f^2 - 3*c*f^2 +
3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d)))/(c^3*f^2 - 3*c*f^2 + 3*c*d^2*
e^2 + 3*d*e*f - 3*c^2*d*e*f))*(c^3*f^2 - 3*c*f^2 + 3*c*d^2*e^2 + 3*d*e*f -
3*c^2*d*e*f))/(3*d^3)
```

3.131 $\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right) \, dx = \frac{bfx}{2d} + \frac{(e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)}{2f} + \frac{b(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2d^2f} + \frac{b(de-cf)\log\left(1+(c+dx)^2\right)}{2d^2}$$

[Out] 1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*arccot(d*x+c))/f+1/2*b*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/d^2/f+1/2*b*(-c*f+d*e)*ln(1+(d*x+c)^2)/d^2

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right) \, dx = \frac{(e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)}{2f} + \frac{b\arctan(c+dx)(-cf+de+f)(de-(c+1)f)}{2d^2f} + \frac{b(de-cf)\log\left((c+dx)^2+1\right)}{2d^2} + \frac{bfx}{2d}$$

[In] Int[(e + f*x)*(a + b*ArcCot[c + d*x]),x]

[Out] (b*f*x)/(2*d) + ((e + f*x)^2*(a + b*ArcCot[c + d*x]))/(2*f) + (b*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*d^2*f) + (b*(d*e - c*f)*Log[1 + (c + d*x)^2])/(2*d^2)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\cot^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f} + \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2}{1+x^2}\,dx, x, c+dx\right)}{2f}$$

$$= \frac{(e+fx)^2 (a+b\cot^{-1}(c+dx))}{2f} + \frac{b\text{Subst}\left(\int \left(\frac{f^2}{d^2} + \frac{(de-f-cf)(de+f-cf)+2f(de-cf)x}{d^2(1+x^2)}\right) dx, x, c+dx\right)}{2f}$$

$$= \frac{bfx}{2d} + \frac{(e+fx)^2 (a+b\cot^{-1}(c+dx))}{2f} + \frac{b\text{Subst}\left(\int \frac{(de-f-cf)(de+f-cf)+2f(de-cf)x}{1+x^2} dx, x, c+dx\right)}{2d^2 f}$$

$$= \frac{bfx}{2d} + \frac{(e+fx)^2 (a+b\cot^{-1}(c+dx))}{2f} + \frac{(b(de-cf))\text{Subst}\left(\int \frac{x}{1+x^2} dx, x, c+dx\right)}{d^2} + \frac{(b(de+f-cf)(de-(1+c)f))\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c+dx\right)}{2d^2 f}$$

$$= \frac{bfx}{2d} + \frac{(e+fx)^2 (a+b\cot^{-1}(c+dx))}{2f} + \frac{b(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2d^2 f} + \frac{b(de-cf)\log(1+(c+dx)^2)}{2d^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right) dx$$
$$= aex + \frac{1}{2}afx^2 + bex\cot^{-1}(c+dx) + \frac{bf\left(\frac{1}{2}d\left(-\frac{c}{d}+\frac{c+dx}{d}\right)^2 \cot^{-1}(c+dx) + \frac{1}{2}d\left(\frac{x}{d} - \frac{i(i-c)^2\log(i-c-dx)}{2d^2} + \frac{i(i+c)^2\log(i+c+dx)}{2d^2}\right)\right)}{d} + \frac{be(-2c\arctan(c+dx)+\log(1+c^2+2cdx+d^2x^2))}{2d}$$

```
[In] Integrate[(e + f*x)*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] a*e*x + (a*f*x^2)/2 + b*e*x*ArcCot[c + d*x] + (b*f*((d*(-(c/d) + (c + d*x)/
d)^2*ArcCot[c + d*x])/2 + (d*(x/d - ((I/2)*(I - c)^2*Log[I - c - d*x])/d^2
+ ((I/2)*(I + c)^2*Log[I + c + d*x])/d^2))/2))/d + (b*e*(-2*c*ArcTan[c + d*
x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

method	result
parts	$a\left(\frac{1}{2}f\,x^{2}+ex\right)+\frac{b\left(\frac{\operatorname{arccot}(dx+c)(dx+c)^{2}f}{2d}-\frac{\operatorname{arccot}(dx+c)cf(dx+c)}{d}+\operatorname{arccot}(dx+c)e(dx+c)+\frac{f(dx+c)+\frac{(-2cf+2de)\ln\left(1\right.}{2}}{2}\right.}{d}$
derivativedivides	$-\frac{\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^{2}}{2}\right)}{d}-\frac{b\left(\operatorname{arccot}(dx+c)fc(dx+c)-\operatorname{arccot}(dx+c)ed(dx+c)-\frac{\operatorname{arccot}(dx+c)f(dx+c)^{2}}{2}-\frac{f(dx+c)}{2}+\frac{(2cf}{}\right.}{d}}{d}$
default	$-\frac{\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^{2}}{2}\right)}{d}-\frac{b\left(\operatorname{arccot}(dx+c)fc(dx+c)-\operatorname{arccot}(dx+c)ed(dx+c)-\frac{\operatorname{arccot}(dx+c)f(dx+c)^{2}}{2}-\frac{f(dx+c)}{2}+\frac{(2cf}{}\right.}{d}}{d}$
parallelrisch	$-\frac{-\operatorname{arccot}(dx+c)b\,d^{2}f\,x^{2}-a\,d^{2}f\,x^{2}-2x\,\operatorname{arccot}(dx+c)b\,d^{2}e-2a\,d^{2}ex+\operatorname{arccot}(dx+c)b\,c^{2}f-2\,\operatorname{arccot}(dx+c)bcde+bcf\ln(d}{2d^{2}}$
risch	$\frac{ib(f\,x^{2}+2ex)\ln(1+i(dx+c))}{4}-\frac{ibf\,x^{2}\ln(1-i(dx+c))}{4}-\frac{ibex\ln(1-i(dx+c))}{2}+\frac{\pi bf\,x^{2}}{4}+\frac{\pi bex}{2}+\frac{af\,x^{2}}{2}+\operatorname{arctan}$

```
[In] int((f*x+e)*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*f*x^2+e*x)+b/d*(1/2/d*arccot(d*x+c)*(d*x+c)^2*f-1/d*arccot(d*x+c)*c*
f*(d*x+c)+arccot(d*x+c)*e*(d*x+c)+1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*ln(1+
(d*x+c)^2)-f*arctan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\frac{ad^2fx^2+(2\,ad^2e+bdf)x+(bd^2fx^2+2\,bd^2ex)\operatorname{arccot}(dx+c)-(2\,bcde-(bc^2-b)f)\arctan(dx+c)+(b}{2\,d^2}$$

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*f*x^2 + (2*a*d^2*e + b*d*f)*x + (b*d^2*f*x^2 + 2*b*d^2*e*x)*arcc
ot(d*x + c) - (2*b*c*d*e - (b*c^2 - b)*f)*arctan(d*x + c) + (b*d*e - b*c*f)
*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.48 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\begin{cases} aex+\frac{afx^2}{2}-\frac{bc^2f\,\mathrm{acot}\,(c+dx)}{2d^2}+\frac{bce\,\mathrm{acot}\,(c+dx)}{d}-\frac{bcf\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2}-\frac{ibcf\,\mathrm{acot}\,(c+dx)}{d^2}+bex\,\mathrm{acot}\,(c+dx)+\frac{bfx^2\,\mathrm{acot}}{2} \\ (a+b\,\mathrm{acot}\,(c))\left(ex+\frac{fx^2}{2}\right) \end{cases}$$

```
[In] integrate((f*x+e)*(a+b*acot(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*acot(c + d*x)/(2*d**2) + b*c*e*aco
t(c + d*x)/d - b*c*f*log(c/d + x - I/d)/d**2 - I*b*c*f*acot(c + d*x)/d**2 +
 b*e*x*acot(c + d*x) + b*f*x**2*acot(c + d*x)/2 + b*e*log(c/d + x - I/d)/d
+ I*b*e*acot(c + d*x)/d + b*f*x/(2*d) + b*f*acot(c + d*x)/(2*d**2), Ne(d, 0
)), ((a + b*acot(c))*(e*x + f*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx=\frac{1}{2}\,afx^2$$
$$+\frac{1}{2}\left(x^2\,\mathrm{arccot}\,(dx+c)+d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bf$$
$$+aex+\frac{\left(2\,(dx+c)\,\mathrm{arccot}\,(dx+c)+\log\left((dx+c)^2+1\right)\right)be}{2\,d}$$

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
 + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f + a*e*x + 1/2
*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b*e/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(89) = 178$.

Time = 0.41 (sec) , antiderivative size = 451, normalized size of antiderivative = 4.65

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx =$$

$$-\frac{4\,bde\arctan\left(\frac{1}{dx+c}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^3 - 4\,bcf\arctan\left(\frac{1}{dx+c}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^3 - bf\arctan\left(\frac{1}{dx+\cdots}\right.}{\cdots}$$

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(4*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c*f*
arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - b*f*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^4 + 4*b*d*e*log(16*tan(1/2*arctan(1/(d*x + c
)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 +
1))*tan(1/2*arctan(1/(d*x + c)))^2 - 4*b*c*f*log(16*tan(1/2*arctan(1/(d*x +
 c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2
+ 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 4*a*d*e*tan(1/2*arctan(1/(d*x + c)))
^3 - 4*a*c*f*tan(1/2*arctan(1/(d*x + c)))^3 - a*f*tan(1/2*arctan(1/(d*x + c
)))^4 - 4*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) + 4*b*c*f*
arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) - 2*b*f*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*f*tan(1/2*arctan(1/(d*x + c)))^3 - 4
*a*d*e*tan(1/2*arctan(1/(d*x + c))) + 4*a*c*f*tan(1/2*arctan(1/(d*x + c)))
- 2*a*f*tan(1/2*arctan(1/(d*x + c)))^2 - b*f*arctan(1/(d*x + c)) - 2*b*f*ta
n(1/2*arctan(1/(d*x + c))) - a*f)/(d^2*tan(1/2*arctan(1/(d*x + c)))^2)
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx = a\,e\,x + \frac{a\,f\,x^2}{2} + \frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d} + \frac{b\,f\,\mathrm{acot}(c+d\,x)}{2\,d^2} + \frac{b\,f\,x^2\,\mathrm{acot}(c+d\,x)}{2} + \frac{b\,f\,x}{2\,d} + b\,e\,x\,\mathrm{acot}(c+d\,x) - \frac{b\,c^2\,f\,\mathrm{acot}(c+d\,x)}{2\,d^2} - \frac{b\,c\,f\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d^2} + \frac{b\,c\,e\,\mathrm{acot}(c+d\,x)}{d}$$

```
[In] int((e + f*x)*(a + b*acot(c + d*x)),x)
```

```
[Out] a*e*x + (a*f*x^2)/2 + (b*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*f*a
cot(c + d*x))/(2*d^2) + (b*f*x^2*acot(c + d*x))/2 + (b*f*x)/(2*d) + b*e*x*a
cot(c + d*x) - (b*c^2*f*acot(c + d*x))/(2*d^2) - (b*c*f*log(c^2 + d^2*x^2 +
 2*c*d*x + 1))/(2*d^2) + (b*c*e*acot(c + d*x))/d
```

3.132 $\int (a+b\cot^{-1}(c+dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = ax+\frac{b(c+dx)\cot^{-1}(c+dx)}{d}+\frac{b\log\left(1+(c+dx)^2\right)}{2d}$$

```
[Out] a*x+b*(d*x+c)*arccot(d*x+c)/d+1/2*b*ln(1+(d*x+c)^2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5148, 4931, 266}

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = ax+\frac{b\log\left((c+dx)^2+1\right)}{2d}+\frac{b(c+dx)\cot^{-1}(c+dx)}{d}$$

```
[In] Int[a + b*ArcCot[c + d*x],x]
```

```
[Out] a*x + (b*(c + d*x)*ArcCot[c + d*x])/d + (b*Log[1 + (c + d*x)^2])/(2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= ax + b\int \cot^{-1}(c+dx)\,dx \\
&= ax + \frac{b\text{Subst}\left(\int \cot^{-1}(x)\,dx, x, c+dx\right)}{d} \\
&= ax + \frac{b(c+dx)\cot^{-1}(c+dx)}{d} + \frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{d} \\
&= ax + \frac{b(c+dx)\cot^{-1}(c+dx)}{d} + \frac{b\log\left(1+(c+dx)^2\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$
\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = ax + bx\cot^{-1}(c+dx) + \frac{b\left(-2c\arctan(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right)\right)}{2d}
$$

```
[In] Integrate[a + b*ArcCot[c + d*x],x]
[Out] a*x + b*x*ArcCot[c + d*x] + (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d*
x + d^2*x^2]))/(2*d)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$ax+\frac{b\left(\operatorname{arccot}(dx+c)(dx+c)+\frac{\ln\left(1+(dx+c)^{2}\right)}{2}\right)}{d}$	35
parts	$ax+\frac{b\left(\operatorname{arccot}(dx+c)(dx+c)+\frac{\ln\left(1+(dx+c)^{2}\right)}{2}\right)}{d}$	35
derivativedivides	$\frac{(dx+c)a+b\left(\operatorname{arccot}(dx+c)(dx+c)+\frac{\ln\left(1+(dx+c)^{2}\right)}{2}\right)}{d}$	40
parallelrisch	$\frac{b\left(2x\operatorname{arccot}(dx+c)d^{2}+2c\operatorname{arccot}(dx+c)d+\ln\left(d^{2}x^{2}+2cdx+c^{2}+1\right)d\right)}{2d^{2}}+ax$	54
risch	$ax+\frac{ibx\ln(1+i(dx+c))}{2}-\frac{ibx\ln(1-i(dx+c))}{2}+\frac{\pi bx}{2}-\frac{bc\arctan(dx+c)}{d}+\frac{b\ln\left(d^{2}x^{2}+2cdx+c^{2}+1\right)}{2d}$	79

```
[In] int(a+b*arccot(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+b/d*(arccot(d*x+c)*(d*x+c)+1/2*ln(1+(d*x+c)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int\left(a+b\cot^{-1}(c+dx)\right)\,dx=\frac{2\,bdx\operatorname{arccot}\left(dx+c\right)+2\,adx-2\,bc\arctan\left(dx+c\right)+b\log\left(d^{2}x^{2}+2\,cdx+c^{2}+1\right)}{2\,d}$$

```
[In] integrate(a+b*arccot(d*x+c),x, algorithm="fricas")
[Out] 1/2*(2*b*d*x*arccot(d*x + c) + 2*a*d*x - 2*b*c*arctan(d*x + c) + b*log(d^2*
x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int\left(a+b\cot^{-1}(c+dx)\right)\,dx=ax+b\left(\begin{cases}\frac{c\operatorname{acot}(c+dx)}{d}+x\operatorname{acot}(c+dx)+\frac{\log\left(c^{2}+2cdx+d^{2}x^{2}+1\right)}{2d} & \text{for}\ d\neq 0\\ x\operatorname{acot}(c) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a+b*acot(d*x+c),x)
[Out] a*x + b*Piecewise((c*acot(c + d*x)/d + x*acot(c + d*x) + log(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/(2*d), Ne(d, 0)), (x*acot(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = ax + \frac{\left(2\,(dx+c)\,\operatorname{arccot}\,(dx+c)+\log\left((dx+c)^2+1\right)\right)b}{2\,d}$$

```
[In] integrate(a+b*arccot(d*x+c),x, algorithm="maxima")
[Out] a*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(36) = 72.

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.05

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = ax - \frac{\left(\arctan\left(\frac{1}{dx+c}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2+\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\operatorname{arcta}\right.}{2\,d\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)}$$

```
[In] integrate(a+b*arccot(d*x+c),x, algorithm="giac")
[Out] a*x - 1/2*(arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + log(16*tan(
1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arct
an(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c))) - arctan(1/(d*x + c))
)*b/(d*tan(1/2*arctan(1/(d*x + c))))
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = a\,x + \frac{\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2}+b\,c\,\mathrm{acot}(c+d\,x)}{d} + b\,x\,\mathrm{acot}(c+d\,x)$$

```
[In] int(a + b*acot(c + d*x),x)
[Out] a*x + ((b*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/2 + b*c*acot(c + d*x))/d + b*x*
acot(c + d*x)
```

3.133 $\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 162

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx = -\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} - \frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} + \frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}$$

```
[Out] -(a+b*arccot(d*x+c))*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))*ln(2*d*(f*x+
e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-1/2*I*b*polylog(2,1-2/(1-I*(d*x+c)))/f+1/
2*I*b*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used

$= \{5156, 4967, 2449, 2352, 2497\}$

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx = \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f} - \frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

```
[In] Int[(a + b*ArcCot[c + d*x])/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCot[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot[c
+ d*x])*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - ((I
/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f + ((I/2)*b*PolyLog[2, 1 - (2*d
*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} - \frac{b\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{f} + \frac{b\text{Subst}\left(\int \frac{\log\left(\frac{2\left(\frac{de-cf}{d}+\frac{fx}{d}\right)}{\left(\frac{if}{d}+\frac{de-cf}{d}\right)(1-ix)}\right)}{1+x^2}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} + \frac{ib\,\text{PolyLog}\left(2, 1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} - \frac{(ib)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-i(c+dx)}\right)}{f}$$

$$= -\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} - \frac{ib\,\text{PolyLog}\left(2, 1-\frac{2}{1-i(c+dx)}\right)}{2f} + \frac{ib\,\text{PolyLog}\left(2, 1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 336 vs. 2(162) = 324.

Time = 0.33 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.07

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx$$

$$= \frac{a\log(e+fx) + b\left((\cot^{-1}(c+dx)+\arctan(c+dx))\log(e+fx) + \arctan(c+dx)\left(\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right) - \log\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcCot[c + d*x])/(e + f*x),x]
```

```
[Out] (a*Log[e + f*x] + b*((ArcCot[c + d*x] + ArcTan[c + d*x])*Log[e + f*x] + Arc
Tan[c + d*x]*(Log[1/Sqrt[1 + (c + d*x)^2]] - Log[Sin[ArcTan[(d*e - c*f)/f]
+ ArcTan[c + d*x]]]) + ((I/4)*(Pi - 2*ArcTan[c + d*x])^2 + I*(ArcTan[(d*e -
 c*f)/f] + ArcTan[c + d*x])^2 - (Pi - 2*ArcTan[c + d*x])*Log[1 + E^((-2*I)*
ArcTan[c + d*x])] - 2*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x])*Log[1 - E^(
(2*I)*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))] + (Pi - 2*ArcTan[c + d*x]
)*Log[2/Sqrt[1 + (c + d*x)^2]] + 2*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]
)*Log[2*Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]] + I*PolyLog[2, -E^((-
2*I)*ArcTan[c + d*x])] + I*PolyLog[2, E^((2*I)*(ArcTan[(d*e - c*f)/f] + Arc
Tan[c + d*x]))])/2))/f
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

method	result
parts	$\frac{a\ln(fx+e)}{f}+\frac{b\Bigg(\frac{d\ln(f(dx+c)-cf+de)\operatorname{arccot}(dx+c)}{f}+d\Bigg(-\frac{i\ln(f(dx+c)-cf+de)\left(\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)-\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)\right)}{2f}-\frac{i(}{}}{d}$
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\Bigg(-\frac{\ln(cf-de-f(dx+c))\operatorname{arccot}(dx+c)}{f}-\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\right)}{2f}}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\Bigg(-\frac{\ln(cf-de-f(dx+c))\operatorname{arccot}(dx+c)}{f}-\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\right)}{2f}}{d}$
risch	$-\frac{ib\operatorname{dilog}\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}-\frac{ib\ln(-idx-ic+1)\ln\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}+\frac{\ln(icf-ide+(-idx-ic}{2f}$

```
[In] int((a+b*arccot(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(f*x+e)/f+b/d*(d*ln(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)+d*(-1/2*I*ln(f*(
d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln((I*f+f*(d*x+c))/(c*f-
d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-dilog((I*f+f*(d*x+
c))/(c*f-d*e+I*f)))/f))
```

Fricas [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\,\mathrm{arccot}\,(dx+c)+a}{fx+e}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="fricas")
[Out] integral((b*arccot(d*x + c) + a)/(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*acot(d*x+c))/(f*x+e),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\,\mathrm{arccot}\,(dx+c)+a}{fx+e}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="maxima")
[Out] 2*b*integrate(1/2*arctan2(1, d*x + c)/(f*x + e), x) + a*log(f*x + e)/f
```

Giac [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\,\mathrm{arccot}\,(dx+c)+a}{fx+e}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="giac")
[Out] integrate((b*arccot(d*x + c) + a)/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx = \int \frac{a+b\,\mathrm{acot}(c+d\,x)}{e+f\,x}\,dx$$

```
[In] int((a + b*acot(c + d*x))/(e + f*x),x)
[Out] int((a + b*acot(c + d*x))/(e + f*x), x)
```

3.134 $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 153

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)} - \frac{bd(de-cf)\arctan(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} - \frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}$$

```
[Out] (-a-b*arccot(d*x+c))/f/(f*x+e)-b*d*(-c*f+d*e)*arctan(d*x+c)/f/(d^2*e^2-2*c*
d*e*f+(c^2+1)*f^2)-b*d*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+1/2*b*d*ln
(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5154, 2007, 719, 31, 648, 632, 210, 642}

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)} - \frac{bd\arctan(c+dx)(de-cf)}{f\left((c^2+1)\,f^2-2cdef+d^2e^2\right)} + \frac{bd\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left((c^2+1)\,f^2-2cdef+d^2e^2\right)} - \frac{bd\log(e+fx)}{(c^2+1)\,f^2-2cdef+d^2e^2}$$

```
[In] Int[(a + b*ArcCot[c + d*x])/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcCot[c + d*x])/(f*(e + f*x))) - (b*d*(d*e - c*f)*ArcTan[c + d*x]
)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (b*d*Log[e + f*x])/(d^2*e^2 -
 2*c*d*e*f + (1 + c^2)*f^2) + (b*d*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
```

```
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{(bd)\int\frac{1}{(e+fx)(1+(c+dx)^2)}\,dx}{f}\\
&= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{(bd)\int\frac{1}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}\\
&= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{(bd)\int\frac{d^2e-2cdf-d^2fx}{1+c^2+2cdx+d^2x^2}\,dx}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}-\frac{(bdf)\int\frac{1}{e+fx}\,dx}{d^2e^2-2cdef+(1+c^2)f^2}\\
&= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)f^2}\\
&\quad+\frac{(bd)\int\frac{2cd+2d^2x}{1+c^2+2cdx+d^2x^2}\,dx}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)}-\frac{(bd^2(de-cf))\int\frac{1}{1+c^2+2cdx+d^2x^2}\,dx}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}\\
&= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)}\\
&\quad+\frac{(2bd^2(de-cf))\,\text{Subst}\left(\int\frac{1}{-4d^2-x^2}\,dx,x,2cd+2d^2x\right)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}\\
&= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{bd(de-cf)\arctan(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}\\
&\quad-\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

$$\int\frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx=\frac{-\frac{a+b\cot^{-1}(c+dx)}{e+fx}+\frac{bd((ide+f-icf)\log(i-c-dx)+(-ide+f+icf)\log(i+c+dx)-2f\log(d(e+fx)))}{2(d^2e^2-2cdef+(1+c^2)f^2)}}{f}$$

```
[In] Integrate[(a + b*ArcCot[c + d*x])/(e + f*x)^2,x]
```

```
[Out] (-((a + b*ArcCot[c + d*x])/(e + f*x)) + (b*d*((I*d*e + f - I*c*f)*Log[I - c
 - d*x] + ((-I)*d*e + f + I*c*f)*Log[I + c + d*x] - 2*f*Log[d*(e + f*x)]))/
(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

method	result
parts	$-\frac{a}{(fx+e)f}+\frac{b\left(-\frac{d^2\operatorname{arccot}(dx+c)}{(f(dx+c)-cf+de)f}-\frac{d^2\left(\frac{f\ln(f(dx+c)-cf+de)}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{-\frac{f\ln\left(1+(dx+c)^2\right)}{2}+(-cf+de)\arctan(dx+c)}{c^2f^2-2cdef+d^2e^2+f^2}\right)}{f}\right)}{d}$
derivativedivides	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+b\,d^2\left(\frac{\operatorname{arccot}(dx+c)}{(cf-de-f(dx+c))f}+\frac{\frac{\frac{f\ln\left(1+(dx+c)^2\right)}{2}+(cf-de)\arctan(dx+c)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}}{f}\right)}{d}$
default	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+b\,d^2\left(\frac{\operatorname{arccot}(dx+c)}{(cf-de-f(dx+c))f}+\frac{\frac{\frac{f\ln\left(1+(dx+c)^2\right)}{2}+(cf-de)\arctan(dx+c)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}}{f}\right)}{d}$
parallelrisch	$-\frac{2x\operatorname{arccot}(dx+c)bc\,d^3f^2-2x\operatorname{arccot}(dx+c)b\,d^4ef+2\ln(fx+e)xb\,d^3f^2-\ln(d^2x^2+2cdx+c^2+1)xb\,d^3f^2+2\operatorname{arccot}(dx+}{2(}$
risch	$-\frac{ib\ln(1+i(dx+c))}{2f(fx+e)}-\frac{2a\,c^2f^2+\pi b\,c^2f^2+\pi b\,d^2e^2+2f^2a+\pi b\,f^2-ib\,c^2f^2\ln(1-i(dx+c))-ib\,d^2e^2\ln(1-i(dx+c))-i\ln(}{}$

```
[In] int((a+b*arccot(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(f*x+e)/f+b/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)-d^2/f*(1/(c^2*f^
2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)*(-1/2*f*ln(1+(d*x+c)^2)+(-c*f+d*e)*arctan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.46

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{2\,ad^2e^2-4\,acdef+2\,(ac^2+a)f^2+2\,(bd^2e^2-2\,bcdef+(bc^2+b)f^2)\operatorname{arccot}(dx+c)+2\,(bd^2e^2-bcde}{2\,(d^2e^3f-2\,cde^2f^2+(c^2+1}$$

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 + a)*f^2 + 2*(b*d^2*e^2 - 2*b*c*
d*e*f + (b*c^2 + b)*f^2)*arccot(d*x + c) + 2*(b*d^2*e^2 - b*c*d*e*f + (b*d^
2*e*f - b*c*d*f^2)*x)*arctan(d*x + c) - (b*d*f^2*x + b*d*e*f)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1) + 2*(b*d*f^2*x + b*d*e*f)*log(f*x + e))/(d^2*e^3*f - 2*
c*d*e^2*f^2 + (c^2 + 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + (c^2 + 1)*f^4)
*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\cot^{-1}(c + dx)}{(e + fx)^2} dx = \text{Timed out}$$

```
[In] integrate((a+b*acot(d*x+c))/(f*x+e)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

$$\int \frac{a + b\cot^{-1}(c + dx)}{(e + fx)^2} dx = -\frac{1}{2}\left(d\left(\frac{2\,(d^2e - cdf)\arctan\left(\frac{d^2x+cd}{d}\right)}{(d^2e^2f - 2\,cdef^2 + (c^2+1)f^3)d} - \frac{\log\left(d^2x^2 + 2\,cdx + c^2 + 1\right)}{d^2e^2 - 2\,cdef + (c^2+1)f^2} + \frac{2\,\log\left(fx + e\right)}{d^2e^2 - 2\,cdef + (c^2+1)f^2}\right) + \right. - \frac{a}{f^2x + ef}$$

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
[Out] -1/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^
2 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e
*f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
 + 2*arccot(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. 2(151) = 302.

Time = 0.69 (sec) , antiderivative size = 1264, normalized size of antiderivative = 8.26

$$\int \frac{a + b\cot^{-1}(c + dx)}{(e + fx)^2} dx = \text{Too large to display}$$

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="giac")
[Out] -1/2*(2*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 2*b*c*f*
arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*d*e*log(4*(4*d^2*e
^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^
2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*
```

```
x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/
(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arct
an(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arc
tan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan
(1/(d*x + c))) - 2*b*c*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 -
8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x
+ c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(
1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arct
an(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arc
tan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arct
an(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c))) + 2*a*d*e*tan(1/2*arc
tan(1/(d*x + c)))^2 - 2*a*c*f*tan(1/2*arctan(1/(d*x + c)))^2 - b*f*log(4*(4
*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x +
 c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan
(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arc
tan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/
2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1
/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*
arctan(1/(d*x + c)))^2 - 2*b*d*e*arctan(1/(d*x + c)) + 2*b*c*f*arctan(1/(d*
x + c)) + 4*b*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) - 2*a*d*e
+ 2*a*c*f + b*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f
*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2
- 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x +
c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x
 + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*
x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x
 + c)))^2 + 1)) + 4*a*f*tan(1/2*arctan(1/(d*x + c))))*d/(2*d^3*e^3*tan(1/2*
arctan(1/(d*x + c))) - 6*c*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c))) + 6*c^2*d
*e*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*c^3*f^3*tan(1/2*arctan(1/(d*x + c))
) - d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^2 + 2*c*d*e*f^2*tan(1/2*arctan(1
/(d*x + c)))^2 - c^2*f^3*tan(1/2*arctan(1/(d*x + c)))^2 + d^2*e^2*f - 2*c*d
*e*f^2 + c^2*f^3 + 2*d*e*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*c*f^3*tan(1/2
*arctan(1/(d*x + c))) - f^3*tan(1/2*arctan(1/(d*x + c)))^2 + f^3)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a}{x\,f^2+e\,f}-\frac{b\,\mathrm{acot}(c+d\,x)}{f\,\left(e+f\,x\right)}-\frac{b\,d\,\ln\left(e+f\,x\right)}{d^2\,e^2-2\,c\,d\,e\,f+\left(c^2+1\right)\,f^2}+\frac{b\,d\,\ln\left(c+d\,x-\mathrm{i}\right)\,1\mathrm{i}}{2\,f\,\left(d\,e-c\,f+f\,1\mathrm{i}\right)}+\frac{b\,d\,\ln\left(c+d\,x+1\mathrm{i}\right)}{2\,f\,\left(f-c\,f\,1\mathrm{i}+d\,e\,1\mathrm{i}\right)}$$

```
[In] int((a + b*acot(c + d*x))/(e + f*x)^2,x)
```

```
[Out] (b*d*log(c + d*x - 1i)*1i)/(2*f*(f*1i - c*f + d*e)) - (b*acot(c + d*x))/(f*
(e + f*x)) - (b*d*log(e + f*x))/(f^2*(c^2 + 1) + d^2*e^2 - 2*c*d*e*f) - a/(
e*f + f^2*x) + (b*d*log(c + d*x + 1i))/(2*f*(f - c*f*1i + d*e*1i))
```

3.135 $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 228

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3} dx = \frac{bd}{2(d^2e^2-2cdef+(1+c^2)f^2)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2f(d^2e^2-2cdef+(1+c^2)f^2)^2} - \frac{bd^2(de-cf)\log(e+fx)}{(d^2e^2-2cdef+(1+c^2)f^2)^2} + \frac{bd^2(de-cf)\log(1+c^2+2cdx+d^2x^2)}{2(d^2e^2-2cdef+(1+c^2)f^2)^2}$$

```
[Out] 1/2*b*d/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)/(f*x+e)+1/2*(-a-b*arccot(d*x+c))/f/
(f*x+e)^2-1/2*b*d^2*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/f/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)^2-b*d^2*(-c*f+d*e)*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)
*f^2)^2+1/2*b*d^2*(-c*f+d*e)*ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+(
c^2+1)*f^2)^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

$= \{5154, 2007, 723, 814, 648, 632, 210, 642\}$

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = -\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2\arctan(c+dx)(-cf+de+f)(de-(c+1)f)}{2f\left((c^2+1)f^2-2cdef+d^2e^2\right)^2} + \frac{bd^2(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left((c^2+1)f^2-2cdef+d^2e^2\right)^2} + \frac{bd}{2(e+fx)\left((c^2+1)f^2-2cdef+d^2e^2\right)} - \frac{bd^2(de-cf)\log(e+fx)}{\left((c^2+1)f^2-2cdef+d^2e^2\right)^2}$$

```
[In] Int[(a + b*ArcCot[c + d*x])/(e + f*x)^3,x]
```

```
[Out] (b*d)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (a + b*ArcCot[c
 + d*x])/(2*f*(e + f*x)^2) - (b*d^2*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTa
n[c + d*x])/(2*f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2) - (b*d^2*(d*e - c
*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2 + (b*d^2*(d*e - c
*f)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}-\frac{(bd)\int\frac{1}{(e+fx)^2(1+(c+dx)^2)}\,dx}{2f}\\
&= -\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}-\frac{(bd)\int\frac{1}{(e+fx)^2(1+c^2+2cdx+d^2x^2)}\,dx}{2f}\\
&= \frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}\\
&\quad -\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}-\frac{(bd)\int\frac{d(de-2cf)-d^2fx}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{2f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}
\end{aligned}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{(bd)\int\left(\frac{2df^2(de-cf)}{(d^2e^2-2cdef+(1+c^2)f^2)(e+fx)} + \frac{d^2\left(d^2e^2-4cdef-(1-3c^2)f^2-2df(de-cf)x\right)}{(d^2e^2-2cdef+(1+c^2)f^2)(1+c^2+2cdx+d^2x^2)}\right)dx}{2f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2} - \frac{(bd^3)\int\frac{d^2e^2-4cdef-(1-3c^2)f^2-2df(de-cf)x}{1+c^2+2cdx+d^2x^2}dx}{2f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2} + \frac{\left(bd^2(de-cf)\right)\int\frac{2cd+2d^2x}{1+c^2+2cdx+d^2x^2}dx}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2} - \frac{\left(bd\left(4cd^2f(de-cf)+2d^2\left(d^2e^2-4cdef-\left(1-3c^2\right)f^2\right)\right)\right)\int\frac{1}{1+c^2+2cdx+d^2x^2}dx}{4f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2} + \frac{bd^2(de-cf)\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2} + \frac{\left(bd\left(4cd^2f(de-cf)+2d^2\left(d^2e^2-4cdef-\left(1-3c^2\right)f^2\right)\right)\right)\text{Subst}\left(\int\frac{1}{-4d^2-x^2}dx, x, 2cd+2d^2x\right)}{2f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de-f-cf)(de+f-cf)\arctan(c+dx)}{2f\left(d^2e^2-2cdef+f^2+c^2f^2\right)^2} - \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2} + \frac{bd^2(de-cf)\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.79

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \frac{\frac{bdf}{(d^2e^2-2cdef+(1+c^2)f^2)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2} + \frac{ibd^2\log(i-c-dx)}{2(de-(-i+c)f)^2} - \frac{ibd^2\log(i+c+dx)}{2(de-(i+c)f)^2} - \frac{2bd^2f(de-cf)\log(d(e+fx))}{(d^2e^2-2cdef+(1+c^2)f^2)^2}}{2f}$$

```
[In] Integrate[(a + b*ArcCot[c + d*x])/(e + f*x)^3,x]
```

[Out] ((b*d*f)/((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (a + b*ArcCot[c + d*x])/(e + f*x)^2 + ((I/2)*b*d^2*Log[I - c - d*x])/(d*e - (-I + c)*f)^2 - ((I/2)*b*d^2*Log[I + c + d*x])/(d*e - (I + c)*f)^2 - (2*b*d^2*f*(d*e - c*f)*Log[d*(e + f*x)])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2)/(2*f)

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.07

method	result
parts	$-\frac{a}{2(fx+e)^2 f}+\frac{b\left(-\frac{d^3 \operatorname{arccot}(dx+c)}{2(f(dx+c)-cf+de)^2 f}-\frac{d^3\left(-\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(f(dx+c)-cf+de)}-\frac{2(cf-de)f\ln(f(dx+c)-cf+}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)}\right.}{2}\right.}{d}$
derivativedivides	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\operatorname{arccot}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(dx}{\left(c^2f^2-2cdef+d^2e^2+f^2\right.}}{}\right.}{d}$
default	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\operatorname{arccot}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(dx}{\left(c^2f^2-2cdef+d^2e^2+f^2\right.}}{}\right.}{d}$
parallelrisch	$\frac{-a\,d^2f^5-2x^2\operatorname{arccot}(dx+c)bc\,d^5e\,f^4+2x\operatorname{arccot}(dx+c)b\,c^2d^4e\,f^4-4x\operatorname{arccot}(dx+c)bc\,d^5e^2f^3+4\ln(fx+e)xbc\,d^4e\,f^4-2}{}$
risch	Expression too large to display

[In] int((a+b*arccot(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)

[Out] -1/2*a/(f*x+e)^2/f+b/d*(-1/2*d^3/(f*(d*x+c)-c*f+d*e)^2/f*arccot(d*x+c)-1/2*d^3/f*(-f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(f*(d*x+c)-c*f+d*e)-2*(c*f-d*e)*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*(1/2*(2*c*f^2-2*d*e*f)*ln(1+(d*x+c)^2)+(c^2*f^2-2*c*d*e*f+d^2*e^2-f^2)*arctan(d*x+c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(220) = 440.

Time = 1.21 (sec) , antiderivative size = 728, normalized size of antiderivative = 3.19

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = -\frac{ad^4e^4-(4\,ac+b)d^3e^3f+2\,(3\,ac^2+bc+a)d^2e^2f^2-(4\,ac^3+bc^2+4\,ac+b)def^3+(ac^4+2\,ac^2+a)f^4}{}$$

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^4*e^4 - (4*a*c + b)*d^3*e^3*f + 2*(3*a*c^2 + b*c + a)*d^2*e^2*f^2
 - (4*a*c^3 + b*c^2 + 4*a*c + b)*d*e*f^3 + (a*c^4 + 2*a*c^2 + a)*f^4 - (b*d
^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 + b)*d*f^4)*x + (b*d^4*e^4 - 4*b*c*d^
3*e^3*f + 2*(3*b*c^2 + b)*d^2*e^2*f^2 - 4*(b*c^3 + b*c)*d*e*f^3 + (b*c^4 +
2*b*c^2 + b)*f^4)*arccot(d*x + c) + (b*d^4*e^4 - 2*b*c*d^3*e^3*f + (b*c^2 -
 b)*d^2*e^2*f^2 + (b*d^4*e^2*f^2 - 2*b*c*d^3*e*f^3 + (b*c^2 - b)*d^2*f^4)*x
^2 + 2*(b*d^4*e^3*f - 2*b*c*d^3*e^2*f^2 + (b*c^2 - b)*d^2*e*f^3)*x)*arctan(
d*x + c) - (b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2
 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) +
2*(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d
^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(f*x + e))/(d^4*e^6*f - 4*c*d^3*e^5*f^2 +
 2*(3*c^2 + 1)*d^2*e^4*f^3 - 4*(c^3 + c)*d*e^3*f^4 + (c^4 + 2*c^2 + 1)*e^2*
f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 + 1)*d^2*e^2*f^5 - 4*(c^3 +
 c)*d*e*f^6 + (c^4 + 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e^5*f^2 - 4*c*d^3*e^4*f^3
 + 2*(3*c^2 + 1)*d^2*e^3*f^4 - 4*(c^3 + c)*d*e^2*f^5 + (c^4 + 2*c^2 + 1)*e*
f^6)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acot(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.80

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$$
$$=\frac{1}{2}\left(d\left(\frac{(d^2e-cdf)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^4e^4-4\,cd^3e^3f+2\,(3\,c^2+1)d^2e^2f^2-4\,(c^3+c)def^3+(c^4+2\,c^2+1)f^4}-\frac{}{d^4e^4-4\,cd^3e^3f+2\,(3}\right.\right.$$
$$-\frac{a}{2\,(f^3x^2+2\,ef^2x+e^2f)}$$

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(d*((d^2*e - c*d*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^4*e^4 - 4*c*d^3
*e^3*f + 2*(3*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1
)*f^4) - 2*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^2
 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1)*f^4) - (d^4*e^2
 - 2*c*d^3*e*f + (c^2 - 1)*d^2*f^2)*arctan((d^2*x + c*d)/d)/((d^4*e^4*f - 4
*c*d^3*e^3*f^2 + 2*(3*c^2 + 1)*d^2*e^2*f^3 - 4*(c^3 + c)*d*e*f^4 + (c^4 + 2
*c^2 + 1)*f^5)*d) + 1/(d^2*e^3 - 2*c*d*e^2*f + (c^2 + 1)*e*f^2 + (d^2*e^2*f
 - 2*c*d*e*f^2 + (c^2 + 1)*f^3)*x)) - arccot(d*x + c)/(f^3*x^2 + 2*e*f^2*x
+ e^2*f))*b - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6173 vs. 2(220) = 440.

Time = 2.13 (sec) , antiderivative size = 6173, normalized size of antiderivative = 27.07

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/2*(4*b*d^4*e^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 12*b
*c*d^3*e^2*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c^2*
d^2*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c^3*d*f^
3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - b*d^3*e^2*f*arctan(1
/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 + 2*b*c*d^2*e*f^2*arctan(1/(d*x
+ c))*tan(1/2*arctan(1/(d*x + c)))^4 - b*c^2*d*f^3*arctan(1/(d*x + c))*tan(
1/2*arctan(1/(d*x + c)))^4 + 4*b*d^4*e^3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/
(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/
2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2
```

*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*
e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2
*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4
+ 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 - 1
2*b*c*d^3*e^2*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f
*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2
- 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x +
c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x
 + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*
x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x
 + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 12*b*c^2*d^2*e*f^2*log(4*(
4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x
+ c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arcta
n(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*ar
ctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1
/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(
1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2
*arctan(1/(d*x + c)))^2 - 4*b*c^3*d*f^3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(
d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2
*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*
tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e
*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*
f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 +
 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 4*
a*d^4*e^3*tan(1/2*arctan(1/(d*x + c)))^3 - 12*a*c*d^3*e^2*f*tan(1/2*arctan(
1/(d*x + c)))^3 + 12*a*c^2*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c)))^3 - 4*a*c
^3*d*f^3*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*d^3*e^2*f*log(4*(4*d^2*e^2*ta
n(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4
*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c
)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x
+ c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/
(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1
/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d
*x + c)))^3 + 8*b*c*d^2*e*f^2*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))
^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/
(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*ar
ctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2
*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/
2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2
*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c^2*d*f^
3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arcta
n(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(
1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*t
an(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*
f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f

```
^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1)
)*tan(1/2*arctan(1/(d*x + c)))^3 - a*d^3*e^2*f*tan(1/2*arctan(1/(d*x + c)))
^4 + 2*a*c*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c)))^4 - a*c^2*d*f^3*tan(1/2*a
rctan(1/(d*x + c)))^4 + b*d^2*e*f^2*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x
+ c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arc
tan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(
1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*t
an(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*
tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*t
an(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^4 - b*c*d*
f^3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arc
tan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*ta
n(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2
*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*
c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 +
 f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 +
1))*tan(1/2*arctan(1/(d*x + c)))^4 - 4*b*d^4*e^3*arctan(1/(d*x + c))*tan(1/
2*arctan(1/(d*x + c))) + 12*b*c*d^3*e^2*f*arctan(1/(d*x + c))*tan(1/2*arcta
n(1/(d*x + c))) - 12*b*c^2*d^2*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(
d*x + c))) + 4*b*c^3*d*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))
 + 14*b*d^3*e^2*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 28*b
*c*d^2*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 14*b*c^2*
d*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 2*b*d^3*e^2*f*ta
n(1/2*arctan(1/(d*x + c)))^3 + 4*b*c*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c)))
^3 - 2*b*c^2*d*f^3*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*d^2*e*f^2*arctan(1/
(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 4*b*c*d*f^3*arctan(1/(d*x + c))
*tan(1/2*arctan(1/(d*x + c)))^3 + b*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c)))^
4 - b*c*d*f^3*tan(1/2*arctan(1/(d*x + c)))^4 + b*d*f^3*arctan(1/(d*x + c))*
tan(1/2*arctan(1/(d*x + c)))^4 - 4*a*d^4*e^3*tan(1/2*arctan(1/(d*x + c))) +
 12*a*c*d^3*e^2*f*tan(1/2*arctan(1/(d*x + c))) - 12*a*c^2*d^2*e*f^2*tan(1/2
*arctan(1/(d*x + c))) + 4*a*c^3*d*f^3*tan(1/2*arctan(1/(d*x + c))) + 4*b*d^
3*e^2*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2
*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*
f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 +
 f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))
- 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))
^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^
2 + 1))*tan(1/2*arctan(1/(d*x + c))) - 8*b*c*d^2*e*f^2*log(4*(4*d^2*e^2*tan
(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*
c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)
))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x +
 c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(
d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/
(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*
x + c))) + 4*b*c^2*d*f^3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 -
```

8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c))) + 14*a*d^3*e^2*f*tan(1/2*arctan(1/(d*x + c)))^2 - 28*a*c*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + 14*a*c^2*d*f^3*tan(1/2*arctan(1/(d*x + c)))^2 - 2*b*d^2*e*f^2*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*c*d*f^3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 - 4*a*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + 4*a*c*d*f^3*tan(1/2*arctan(1/(d*x + c)))^3 + a*d*f^3*tan(1/2*arctan(1/(d*x + c)))^4 - b*d^3*e^2*f*arctan(1/(d*x + c)) + 2*b*c*d^2*e*f^2*arctan(1/(d*x + c)) - b*c^2*d*f^3*arctan(1/(d*x + c)) + 2*b*d^3*e^2*f*tan(1/2*arctan(1/(d*x + c))) - 4*b*c*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c))) + 2*b*c^2*d*f^3*tan(1/2*arctan(1/(d*x + c))) + 4*b*d^2*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) - 4*b*c*d*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) - 6*b*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + 6*b*c*d*f^3*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*d*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*d*f^3*tan(1/2*arctan(1/(d*x + c)))^3 - a*d^3*e^2*f + 2*a*c*d^2*e*f^2 - a*c^2*d*f^3 + b*d^2*e*f^2*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1)) - b*c*d*f^3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1)) + 4*a*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c))) - 4*a*c*d*f^3*tan(1/2*arctan(1/(d*x + c))) + 2*a*d*f^3*tan(1/2*arctan(1/(d*x + c)))^2 + b

```
*d^2*e*f^2 - b*c*d*f^3 + b*d*f^3*arctan(1/(d*x + c)) - 2*b*d*f^3*tan(1/2*ar
ctan(1/(d*x + c))) + a*d*f^3)*d/(4*d^6*e^6*tan(1/2*arctan(1/(d*x + c)))^2 -
 24*c*d^5*e^5*f*tan(1/2*arctan(1/(d*x + c)))^2 + 60*c^2*d^4*e^4*f^2*tan(1/2
*arctan(1/(d*x + c)))^2 - 80*c^3*d^3*e^3*f^3*tan(1/2*arctan(1/(d*x + c)))^2
 + 60*c^4*d^2*e^2*f^4*tan(1/2*arctan(1/(d*x + c)))^2 - 24*c^5*d*e*f^5*tan(1
/2*arctan(1/(d*x + c)))^2 + 4*c^6*f^6*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d^
5*e^5*f*tan(1/2*arctan(1/(d*x + c)))^3 + 20*c*d^4*e^4*f^2*tan(1/2*arctan(1/
(d*x + c)))^3 - 40*c^2*d^3*e^3*f^3*tan(1/2*arctan(1/(d*x + c)))^3 + 40*c^3*
d^2*e^2*f^4*tan(1/2*arctan(1/(d*x + c)))^3 - 20*c^4*d*e*f^5*tan(1/2*arctan(
1/(d*x + c)))^3 + 4*c^5*f^6*tan(1/2*arctan(1/(d*x + c)))^3 + d^4*e^4*f^2*ta
n(1/2*arctan(1/(d*x + c)))^4 - 4*c*d^3*e^3*f^3*tan(1/2*arctan(1/(d*x + c)))
^4 + 6*c^2*d^2*e^2*f^4*tan(1/2*arctan(1/(d*x + c)))^4 - 4*c^3*d*e*f^5*tan(1
/2*arctan(1/(d*x + c)))^4 + c^4*f^6*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d^5*
e^5*f*tan(1/2*arctan(1/(d*x + c))) - 20*c*d^4*e^4*f^2*tan(1/2*arctan(1/(d*x
 + c))) + 40*c^2*d^3*e^3*f^3*tan(1/2*arctan(1/(d*x + c))) - 40*c^3*d^2*e^2*
f^4*tan(1/2*arctan(1/(d*x + c))) + 20*c^4*d*e*f^5*tan(1/2*arctan(1/(d*x + c
))) - 4*c^5*f^6*tan(1/2*arctan(1/(d*x + c))) + 6*d^4*e^4*f^2*tan(1/2*arctan
(1/(d*x + c)))^2 - 24*c*d^3*e^3*f^3*tan(1/2*arctan(1/(d*x + c)))^2 + 36*c^2
*d^2*e^2*f^4*tan(1/2*arctan(1/(d*x + c)))^2 - 24*c^3*d*e*f^5*tan(1/2*arctan
(1/(d*x + c)))^2 + 6*c^4*f^6*tan(1/2*arctan(1/(d*x + c)))^2 - 8*d^3*e^3*f^3
*tan(1/2*arctan(1/(d*x + c)))^3 + 24*c*d^2*e^2*f^4*tan(1/2*arctan(1/(d*x +
c)))^3 - 24*c^2*d*e*f^5*tan(1/2*arctan(1/(d*x + c)))^3 + 8*c^3*f^6*tan(1/2*
arctan(1/(d*x + c)))^3 + 2*d^2*e^2*f^4*tan(1/2*arctan(1/(d*x + c)))^4 - 4*c
*d*e*f^5*tan(1/2*arctan(1/(d*x + c)))^4 + 2*c^2*f^6*tan(1/2*arctan(1/(d*x +
 c)))^4 + d^4*e^4*f^2 - 4*c*d^3*e^3*f^3 + 6*c^2*d^2*e^2*f^4 - 4*c^3*d*e*f^5
 + c^4*f^6 + 8*d^3*e^3*f^3*tan(1/2*arctan(1/(d*x + c))) - 24*c*d^2*e^2*f^4*
tan(1/2*arctan(1/(d*x + c))) + 24*c^2*d*e*f^5*tan(1/2*arctan(1/(d*x + c)))
- 8*c^3*f^6*tan(1/2*arctan(1/(d*x + c))) - 4*d*e*f^5*tan(1/2*arctan(1/(d*x
+ c)))^3 + 4*c*f^6*tan(1/2*arctan(1/(d*x + c)))^3 + f^6*tan(1/2*arctan(1/(d
*x + c)))^4 + 2*d^2*e^2*f^4 - 4*c*d*e*f^5 + 2*c^2*f^6 + 4*d*e*f^5*tan(1/2*a
rctan(1/(d*x + c))) - 4*c*f^6*tan(1/2*arctan(1/(d*x + c))) - 2*f^6*tan(1/2*
arctan(1/(d*x + c)))^2 + f^6)
```

Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.75

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \frac{b\,d\,e}{2\,(e+f\,x)^2\,(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2)} - \frac{a\,f}{2\,(e+f\,x)^2\,(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2)} - \frac{b\,\mathrm{acot}(c+d\,x)}{2\,f\,(e+f\,x)^2} - \frac{a\,c^2\,f}{2\,(e+f\,x)^2\,(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2)} - \frac{b\,d^3\,e\,\ln(e+f\,x)}{(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2)^2} + \frac{b\,c\,d^2\,f\,\ln(e+f\,x)}{(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2)^2} + \frac{a\,c\,d\,e}{(e+f\,x)^2\,(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2)} + \frac{b\,d\,f\,x}{2\,(e+f\,x)^2\,(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2)} - \frac{a\,d^2\,e^2}{2\,f\,(e+f\,x)^2\,(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2)} + \frac{b\,d^2\,\ln(c+d\,x-\mathrm{i})\,1\mathrm{i}}{4\,f\,(d\,e-c\,f+f\,1\mathrm{i})^2} - \frac{b\,d^2\,\ln(c+d\,x+1\mathrm{i})\,1\mathrm{i}}{4\,f\,(c\,f-d\,e+f\,1\mathrm{i})^2}$$

```
[In] int((a + b*acot(c + d*x))/(e + f*x)^3,x)
```

```
[Out] (b*d*e)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (a*f)/(2*(e
 + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*acot(c + d*x))/(2*f*(
e + f*x)^2) + (b*d^2*log(c + d*x - 1i)*1i)/(4*f*(f*1i - c*f + d*e)^2) - (b*
d^2*log(c + d*x + 1i)*1i)/(4*f*(f*1i + c*f - d*e)^2) - (a*c^2*f)/(2*(e + f*
x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d^3*e*log(e + f*x))/(f^2 +
 c^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 + (b*c*d^2*f*log(e + f*x))/(f^2 + c^2*f^2
 + d^2*e^2 - 2*c*d*e*f)^2 + (a*c*d*e)/((e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2
 - 2*c*d*e*f)) + (b*d*f*x)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*
e*f)) - (a*d^2*e^2)/(2*f*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f))
```

3.136 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 382

$$\begin{aligned}
&\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx \\
&= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} \\
&\quad + \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)}{3d^3} \\
&\quad + \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3} \\
&\quad - \frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3f} \\
&\quad + \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^2}{3f} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&\quad - \frac{2b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&\quad + \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&\quad + \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}
\end{aligned}$$

```
[Out] 1/3*b^2*f^2*x/d^2+2*a*b*f*(-c*f+d*e)*x/d^2+2*b^2*f*(-c*f+d*e)*(d*x+c)*arcco
t(d*x+c)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arccot(d*x+c))/d^3+1/3*I*(3*d^2*e^2-6
*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))^2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-
2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arccot(d*x+c))^2/d^3/f+1/3*(f*x+e)^3*(a+b*arcc
ot(d*x+c))^2/f-1/3*b^2*f^2*arctan(d*x+c)/d^3-2/3*b*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arccot(d*x+c))*ln(2/(1+I*(d*x+c)))/d^3+b^2*f*(-c*f+d*e)*l
```

```
n(1+(d*x+c)^2)/d^3+1/3*I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(2
,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5156, 4975, 4931, 266, 4947, 327, 209, 5105, 5005, 5041, 4965, 2449, 2352}

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 dx$$
$$= \frac{i(-(1-3c^2)f^2-6cdef+3d^2e^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3} - \frac{(de-cf)(-(3-c^2)f^2-2cdef+d^2e^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3f} - \frac{2b(-(1-3c^2)f^2-6cdef+3d^2e^2)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{3d^3} + \frac{bf^2(c+dx)^2(a+b\cot^{-1}(c+dx))}{3d^3} + \frac{2abfx(de-cf)}{d^2} + \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^2}{3f} - \frac{b^2f^2\arctan(c+dx)}{3d^3} + \frac{ib^2(-(1-3c^2)f^2-6cdef+3d^2e^2)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{3d^3} + \frac{b^2f(de-cf)\log\left((c+dx)^2+1\right)}{d^3} + \frac{2b^2f(c+dx)(de-cf)\cot^{-1}(c+dx)}{d^3} + \frac{b^2f^2x}{3d^2}$$

```
[In] Int[(e + f*x)^2*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) + (2*a*b*f*(d*e - c*f)*x)/d^2 + (2*b^2*f*(d*e - c*f)*(c
 + d*x)*ArcCot[c + d*x])/d^3 + (b*f^2*(c + d*x)^2*(a + b*ArcCot[c + d*x]))/
(3*d^3) + ((I/3)*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c
+ d*x])^2)/d^3 - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*
ArcCot[c + d*x])^2)/(3*d^3*f) + ((e + f*x)^3*(a + b*ArcCot[c + d*x])^2)/(3*
f) - (b^2*f^2*ArcTan[c + d*x])/(3*d^3) - (2*b*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))])/(3*d^3) + (b
^2*f*(d*e - c*f)*Log[1 + (c + d*x)^2])/d^3 + ((I/3)*b^2*(3*d^2*e^2 - 6*c*d*
e*f - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4975

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\cot^{-1}(x)\right)^2\, dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^2}{3f} + \frac{(2b)\text{Subst}\left(\int \left(\frac{3f^2(de-cf)\left(a+b\cot^{-1}(x)\right)}{d^3}+\frac{f^3x\left(a+b\cot^{-1}(x)\right)}{d^3}+\frac{((de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cdef-}{d^3(1+x^2)}\right.\right.}{3f}$$

$$= \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^2}{3f}$$

$$+ \frac{(2b)\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cdef-(1-3c^2)f^2)x)(a+b\cot^{-1}(x))}{1+x^2}\,dx, x, c+dx\right)}{3d^3f}$$

$$+ \frac{(2bf^2)\,\text{Subst}\left(\int x(a+b\cot^{-1}(x))\,dx, x, c+dx\right)}{3d^3}$$

$$+ \frac{(2bf(de-cf))\text{Subst}\left(\int (a+b\cot^{-1}(x))\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2abf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2 \left(a+b\cot^{-1}(c+dx)\right)}{3d^3} + \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^2}{3f}$$

$$+ \frac{(2b)\text{Subst}\left(\int \left(\frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)(a+b\cot^{-1}(x))}{1+x^2} + \frac{f(3d^2e^2-6cdef-(1-3c^2)f^2)x(a+b\cot^{-1}(x))}{1+x^2}\right) dx,\right.}{3d^3f}$$

$$+ \frac{(b^2f^2)\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, c+dx\right)}{3d^3} + \frac{(2b^2f(de-cf))\,\text{Subst}\left(\int \cot^{-1}(x)\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3}$$

$$+ \frac{bf^2(c+dx)^2 \left(a+b\cot^{-1}(c+dx)\right)}{3d^3} + \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^2}{3f}$$

$$- \frac{(b^2f^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{3d^3} + \frac{(2b^2f(de-cf))\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{d^3}$$

$$+ \frac{(2b(3d^2e^2-6cdef-(1-3c^2)f^2))\,\text{Subst}\left(\int \frac{x(a+b\cot^{-1}(x))}{1+x^2}\,dx, x, c+dx\right)}{3d^3}$$

$$+ \frac{(2b(de-cf)(d^2e^2-2cdef-(3-c^2)f^2))\,\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{1+x^2}\,dx, x, c+dx\right)}{3d^3f}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3}$$

$$+ \frac{bf^2(c+dx)^2 \left(a+b\cot^{-1}(c+dx)\right)}{3d^3}$$

$$+ \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3}$$

$$- \frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3f}$$

$$+ \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^2}{3f}$$

$$- \frac{b^2f^2\arctan(c+dx)}{3d^3} + \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3}$$

$$- \frac{(2b(3d^2e^2-6cdef-(1-3c^2)f^2))\,\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{i-x}\,dx, x, c+dx\right)}{3d^3}$$

$$
\begin{aligned}
&= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} \\
&+ \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)}{3d^3} \\
&+ \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3} \\
&- \frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3f} \\
&+ \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^2}{3f} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&- \frac{2b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&+ \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&- \frac{(2b^2(3d^2e^2-6cdef-(1-3c^2)f^2))\,\mathrm{Subst}\left(\int\frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{3d^3} \\
&= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} \\
&+ \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)}{3d^3} \\
&+ \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3} \\
&- \frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3f} \\
&+ \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^2}{3f} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&- \frac{2b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&+ \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&+ \frac{(2ib^2(3d^2e^2-6cdef-(1-3c^2)f^2))\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+i(c+dx)}\right)}{3d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} \\
&+ \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)}{3d^3} \\
&+ \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3} \\
&- \frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3f} \\
&+ \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^2}{3f} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&- \frac{2b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&+ \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&+ \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.67 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.53

$$
\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2\,dx
$$

$$
= \frac{b^2cf^2+3a^2d^3e^2x+6abd^2efx+b^2df^2x-4abcdf^2x+3a^2d^3efx^2+abd^2f^2x^2+a^2d^3f^2x^3+b^2(i+c+dx)}{}
$$

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (b^2*c*f^2 + 3*a^2*d^3*e^2*x + 6*a*b*d^2*e*f*x + b^2*d*f^2*x - 4*a*b*c*d*f^
2*x + 3*a^2*d^3*e*f*x^2 + a*b*d^2*f^2*x^2 + a^2*d^3*f^2*x^3 + b^2*(I + c +
d*x)*((I + c)^2*f^2 - (I + c)*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*
x^2))*ArcCot[c + d*x]^2 - 6*a*b*c*d^2*e^2*ArcTan[c + d*x] - 6*a*b*d*e*f*Arc
Tan[c + d*x] + 6*a*b*c^2*d*e*f*ArcTan[c + d*x] + 6*a*b*c*f^2*ArcTan[c + d*x
] - 2*a*b*c^3*f^2*ArcTan[c + d*x] - b*ArcCot[c + d*x]*(-2*a*d^3*x*(3*e^2 +
3*e*f*x + f^2*x^2) + b*f*(5*c^2*f - 6*d^2*e*x + c*d*(-6*e + 4*f*x) - f*(1 +
 d^2*x^2)) + 2*b*(3*d^2*e^2 - 6*c*d*e*f + (-1 + 3*c^2)*f^2)*Log[1 - E^((2*I
)*ArcCot[c + d*x])]) + 6*b^2*c*f^2*Log[(c + d*x)^(-1)] + 3*a*b*d^2*e^2*Log[
1 + c^2 + 2*c*d*x + d^2*x^2] - 6*a*b*c*d*e*f*Log[1 + c^2 + 2*c*d*x + d^2*x^
2] - a*b*f^2*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + 3*a*b*c^2*f^2*Log[1 + c^2 +
 2*c*d*x + d^2*x^2] + 6*b^2*c*f^2*Log[1/Sqrt[1 + (c + d*x)^(-2)]] - 6*b^2*d
*e*f*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] + I*b^2*(3*d^2*e^2 - 6*c*d
*e*f + (-1 + 3*c^2)*f^2)*PolyLog[2, E^((2*I)*ArcCot[c + d*x])])/(3*d^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(362) = 724.

Time = 1.34 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.81

method	result	size
parts	Expression too large to display	1072
derivativedivides	Expression too large to display	1087
default	Expression too large to display	1087
risch	Expression too large to display	3165

```
[In] int((f*x+e)^2*(a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*(f*x+e)^3/f+b^2/d*(1/3/d^2*f^2*arccot(d*x+c)^2*(d*x+c)^3-1/d^2*f^2*
arccot(d*x+c)^2*(d*x+c)^2*c+1/d*f*arccot(d*x+c)^2*(d*x+c)^2*e+1/d^2*f^2*arc
cot(d*x+c)^2*(d*x+c)*c^2-2/d*f*arccot(d*x+c)^2*(d*x+c)*c*e+arccot(d*x+c)^2*
(d*x+c)*e^2-1/3/d^2*f^2*arccot(d*x+c)^2*c^3+1/d*f*arccot(d*x+c)^2*c^2*e-arc
cot(d*x+c)^2*c*e^2+1/3*d/f*arccot(d*x+c)^2*e^3+2/3/d^2/f*(1/2*arccot(d*x+c)
*f^3*(d*x+c)^2-3*arccot(d*x+c)*c*f^3*(d*x+c)+3*arccot(d*x+c)*d*e*f^2*(d*x+c
)+3/2*arccot(d*x+c)*ln(1+(d*x+c)^2)*c^2*f^3-3*arccot(d*x+c)*ln(1+(d*x+c)^2)
*c*d*e*f^2+3/2*arccot(d*x+c)*ln(1+(d*x+c)^2)*d^2*e^2*f-1/2*arccot(d*x+c)*ln
(1+(d*x+c)^2)*f^3-arccot(d*x+c)*arctan(d*x+c)*c^3*f^3+3*arccot(d*x+c)*arcta
n(d*x+c)*c^2*d*e*f^2-3*arccot(d*x+c)*arctan(d*x+c)*c*d^2*e^2*f+arccot(d*x+c
)*arctan(d*x+c)*d^3*e^3+3*arccot(d*x+c)*arctan(d*x+c)*c*f^3-3*arccot(d*x+c)
*arctan(d*x+c)*d*e*f^2+1/2*f^2*(f*(d*x+c)+1/2*(-6*c*f+6*d*e)*ln(1+(d*x+c)^2
)-f*arctan(d*x+c))+1/2*f*(3*c^2*f^2-6*c*d*e*f+3*d^2*e^2-f^2)*(-1/2*I*(ln(d*
x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I
)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^
2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))+1/4*(-2*c^3*f^3+
6*c^2*d*e*f^2-6*c*d^2*e^2*f+2*d^3*e^3+6*c*f^3-6*d*e*f^2)*arctan(d*x+c)^2))+
2/3*a*b/f*arccot(d*x+c)*e^3-5/3/d^3*c^2*f^2*b*a+1/3/d*f^2*b*a*x^2-1/3*a*b/d
^3*f^2*ln(1+(d*x+c)^2)-2*a*b/d^2*f*ln(1+(d*x+c)^2)*c*e+2*b/d^2*arctan(d*x+c
)*a*c^2*e*f-2*b/d*arctan(d*x+c)*a*c*e^2+2*a*b*f*arccot(d*x+c)*e*x^2+2/d^2*c
*f*e*b*a-4/3/d^2*c*f^2*x*b*a+2/d*e*x*f*b*a+a*b/d^3*f^2*ln(1+(d*x+c)^2)*c^2+
a*b/d*ln(1+(d*x+c)^2)*e^2-2/3*b/d^3*arctan(d*x+c)*a*c^3*f^2+2/3*a*b/f*arcta
n(d*x+c)*e^3+2*b/d^3*arctan(d*x+c)*a*c*f^2-2*b/d^2*arctan(d*x+c)*a*e*f+2/3*
a*b*f^2*arccot(d*x+c)*x^3+2*a*b*arccot(d*x+c)*x*e^2
```

Fricas [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \int (fx+e)^2 (b\,\mathrm{arccot}\,(dx+c)+a)^2 \,dx$$

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x +
 b^2*e^2)*arccot(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arcco
t(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**2*(a+b*acot(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \int (fx+e)^2 (b\,\mathrm{arccot}\,(dx+c)+a)^2 \,dx$$

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f^2*x^3*arctan2(1, d*x + c)^2 + 1/4*b^2*e*f*x^2*arctan2(1, d*x + c
)^2 + 1/3*a^2*f^2*x^3 + 1/4*b^2*e^2*x*arctan2(1, d*x + c)^2 + a^2*e*f*x^2 +
 2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3
- c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*e*f + 1/3*(2*x^3*arccot(d*x
+ c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 +
 (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*f^2 + a^2*e^2*x + (
2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a*b*e^2/d - 1/48*(b^2*f
^2*x^3 + 3*b^2*e*f*x^2 + 3*b^2*e^2*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 +
integrate(1/48*(36*b^2*d^2*f^2*x^4*arctan2(1, d*x + c)^2 + 8*(9*b^2*d^2*e*f
*arctan2(1, d*x + c)^2 + (9*b^2*c*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*
x + c))*d*f^2)*x^3 + 36*(b^2*c^2*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x
 + c)^2)*e^2 + 12*(3*b^2*d^2*e^2*arctan2(1, d*x + c)^2 + 2*(6*b^2*c*arctan2
(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*e*f + 3*(b^2*c^2*arctan2(1, d*x
 + c)^2 + b^2*arctan2(1, d*x + c)^2)*f^2)*x^2 + 3*(b^2*d^2*f^2*x^4 + 2*(b^2
*d^2*e*f + b^2*c*d*f^2)*x^3 + (b^2*c^2 + b^2)*e^2 + (b^2*d^2*e^2 + 4*b^2*c*
```

```
d*e*f + (b^2*c^2 + b^2)*f^2)*x^2 + 2*(b^2*c*d*e^2 + (b^2*c^2 + b^2)*e*f)*x)
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 24*((3*b^2*c*arctan2(1, d*x + c)^2 +
b^2*arctan2(1, d*x + c))*d*e^2 + 3*(b^2*c^2*arctan2(1, d*x + c)^2 + b^2*arc
tan2(1, d*x + c)^2)*e*f)*x + 4*(b^2*d^2*f^2*x^4 + 3*b^2*c*d*e^2*x + (3*b^2*
d^2*e*f + b^2*c*d*f^2)*x^3 + 3*(b^2*d^2*e^2 + b^2*c*d*e*f)*x^2)*log(d^2*x^2
 + 2*c*d*x + c^2 + 1))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
```

Giac [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx=\int (fx+e)^2 (b\operatorname{arccot}(dx+c)+a)^2 \, dx$$

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)^2*(b*arccot(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx=\int (e+f\,x)^2\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2 \,d x$$

```
[In] int((e + f*x)^2*(a + b*acot(c + d*x))^2,x)
[Out] int((e + f*x)^2*(a + b*acot(c + d*x))^2, x)
```

3.137 $\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 220

$$\begin{aligned}
&\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx \\
&= \frac{abfx}{d}+\frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2} \\
&\quad -\frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2f} \\
&\quad +\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}-\frac{2b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad +\frac{b^2f\log\left(1+(c+dx)^2\right)}{2d^2}+\frac{ib^2(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}
\end{aligned}$$

```
[Out] a*b*f*x/d+b^2*f*(d*x+c)*arccot(d*x+c)/d^2+I*(-c*f+d*e)*(a+b*arccot(d*x+c))^
2/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a+b*arccot(d*x+c))^2/d^2/f+1/2*(f*x+e
)^2*(a+b*arccot(d*x+c))^2/f-2*b*(-c*f+d*e)*(a+b*arccot(d*x+c))*ln(2/(1+I*(d
*x+c)))/d^2+1/2*b^2*f*ln(1+(d*x+c)^2)/d^2+I*b^2*(-c*f+d*e)*polylog(2,1-2/(1
+I*(d*x+c)))/d^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5156, 4975, 4931, 266, 5105, 5005, 5041, 4965, 2449, 2352}

$$\begin{aligned}
&\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2dx \\
&= \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2} \\
&\quad -\frac{(-cf+de+f)(de-(c+1)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2f} \\
&\quad -\frac{2b(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)}{d^2} \\
&\quad +\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}+\frac{abfx}{d}+\frac{ib^2(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{d^2} \\
&\quad +\frac{b^2f\log\left((c+dx)^2+1\right)}{2d^2}+\frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2}
\end{aligned}$$

```
[In] Int[(e + f*x)*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (a*b*f*x)/d + (b^2*f*(c + d*x)*ArcCot[c + d*x])/d^2 + (I*(d*e - c*f)*(a + b
*ArcCot[c + d*x])^2)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcCot
[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcCot[c + d*x])^2)/(2*f) - (
2*b*(d*e - c*f)*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (b^
2*f*Log[1 + (c + d*x)^2])/(2*d^2) + (I*b^2*(d*e - c*f)*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/d^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4975

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\cot^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} + \frac{b\text{Subst}\left(\int \left(\frac{f^2(a+b\cot^{-1}(x))}{d^2}+\frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b\cot^{-1}(x))}{d^2(1+x^2)}\right)\,dx,x,c+dx\right)}{f}$$

$$= \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} + \frac{b\text{Subst}\left(\int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)(a+b\cot^{-1}(x))}{1+x^2}\,dx,x,c+dx\right)}{d^2f} + \frac{(bf)\text{Subst}\left(\int \left(a+b\cot^{-1}(x)\right)\,dx,x,c+dx\right)}{d^2}$$

$$= \frac{abfx}{d} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} + \frac{b\text{Subst}\left(\int \left(\frac{(de+f-cf)(de-(1+c)f)(a+b\cot^{-1}(x))}{1+x^2}-\frac{2f(-de+cf)x(a+b\cot^{-1}(x))}{1+x^2}\right)\,dx,x,c+dx\right)}{d^2f} + \frac{(b^2f)\,\text{Subst}\left(\int \cot^{-1}(x)\,dx,x,c+dx\right)}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} + \frac{(b^2f)\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,c+dx\right)}{d^2} + \frac{(2b(de-cf))\text{Subst}\left(\int \frac{x(a+b\cot^{-1}(x))}{1+x^2}\,dx,x,c+dx\right)}{d^2} + \frac{(b(de+f-cf)(de-(1+c)f))\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{1+x^2}\,dx,x,c+dx\right)}{d^2f}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2} - \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2f} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} + \frac{b^2f\log\left(1+(c+dx)^2\right)}{2d^2} - \frac{(2b(de-cf))\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{i-x}\,dx,x,c+dx\right)}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2 f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2} - \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2 f} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} - \frac{2b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} + \frac{b^2 f\log\left(1+(c+dx)^2\right)}{2d^2} - \frac{\left(2b^2(de-cf)\right)\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2 f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2} - \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2 f} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} - \frac{2b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} + \frac{b^2 f\log\left(1+(c+dx)^2\right)}{2d^2} + \frac{\left(2ib^2(de-cf)\right)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+i(c+dx)}\right)}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2 f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2} - \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2 f} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} - \frac{2b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} + \frac{b^2 f\log\left(1+(c+dx)^2\right)}{2d^2} + \frac{ib^2(de-cf)\text{PolyLog}\left(2, 1-\frac{2}{1+i(c+dx)}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.30

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

$$= \frac{2a^2cde+2abcf-a^2c^2f+2a^2d^2ex+2abdfx+a^2d^2fx^2+b^2(i+c+dx)(-((i+c)f)+d(2e+fx))\cot^{-1}(}{}$$

```
[In] Integrate[(e + f*x)*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d^
2*f*x^2 + b^2*(I + c + d*x)*(-((I + c)*f) + d*(2*e + f*x))*ArcCot[c + d*x]^
2 - 2*a*b*f*ArcTan[c + d*x] + 2*b*ArcCot[c + d*x]*(-((c + d*x)*(-(b*f) + a*
c*f - a*d*(2*e + f*x))) - 2*b*(d*e - c*f)*Log[1 - E^((2*I)*ArcCot[c + d*x])
]) - 4*a*b*d*e*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] - 2*b^2*f*Log[1/
((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] + 4*a*b*c*f*Log[1/((c + d*x)*Sqrt[1 +
 (c + d*x)^(-2)])] + (2*I)*b^2*(d*e - c*f)*PolyLog[2, E^((2*I)*ArcCot[c + d
*x])])/(2*d^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(210) = 420.

Time = 0.88 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.94

method	result
parts	$a^2\left(\frac{1}{2}f\,x^2+ex\right)+\frac{b^2\left(\frac{\operatorname{arccot}(dx+c)^2(dx+c)^2f}{2d}-\frac{\operatorname{arccot}(dx+c)^2cf(dx+c)}{d}+\operatorname{arccot}(dx+c)^2e(dx+c)+\frac{-\operatorname{arccot}(dx+c)\ln(\right.}{}$
derivativedivides	$\frac{-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arccot}(dx+c)^2fc(dx+c)-\operatorname{arccot}(dx+c)^2ed(dx+c)-\frac{\operatorname{arccot}(dx+c)^2f(dx+c)^2}{2}+\operatorname{arccot}(\right.}{}}{}$
default	$\frac{-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arccot}(dx+c)^2fc(dx+c)-\operatorname{arccot}(dx+c)^2ed(dx+c)-\frac{\operatorname{arccot}(dx+c)^2f(dx+c)^2}{2}+\operatorname{arccot}(\right.}{}}{}$
risch	Expression too large to display

```
[In] int((f*x+e)*(a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(1/2*f*x^2+e*x)+b^2/d*(1/2/d*arccot(d*x+c)^2*(d*x+c)^2*f-1/d*arccot(d*x
+c)^2*c*f*(d*x+c)+arccot(d*x+c)^2*e*(d*x+c)+1/d*(-arccot(d*x+c)*ln(1+(d*x+c
)^2)*c*f+arccot(d*x+c)*ln(1+(d*x+c)^2)*d*e-arccot(d*x+c)*arctan(d*x+c)*f+ar
ccot(d*x+c)*(d*x+c)*f+1/2*f*ln(1+(d*x+c)^2)-1/2*arctan(d*x+c)^2*f+1/2*(-2*c
*f+2*d*e)*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2
*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*
x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+
c-I))))))+2*a*b/d*(1/2/d*arccot(d*x+c)*(d*x+c)^2*f-1/d*arccot(d*x+c)*c*f*(d
*x+c)+arccot(d*x+c)*e*(d*x+c)+1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*ln(1+(d*x
+c)^2)-f*arctan(d*x+c)))
```

Fricas [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (fx+e)(b\operatorname{arccot}(dx+c)+a)^2\,dx$$

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arccot(d*x + c)^2 + 2*(a*b*f*x
 + a*b*e)*arccot(d*x + c), x)
```

Sympy [F]

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (a+b\,\mathrm{acot}\,(c+dx))^2 \,(e+fx) \, dx$$

[In] integrate((f*x+e)*(a+b*acot(d*x+c))**2,x)

[Out] Integral((a + b*acot(c + d*x))**2*(e + f*x), x)

Maxima [F]

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^2 \, dx$$

[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")

[Out] 1/8*b^2*f*x^2*arctan2(1, d*x + c)^2 + 1/4*b^2*e*x*arctan2(1, d*x + c)^2 + 1/2*a^2*f*x^2 + (x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f + a^2*e*x + (2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a*b*e/d - 1/32*(b^2*f*x^2 + 2*b^2*e*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + integrate(1/16*(12*b^2*d^2*f*x^3*arctan2(1, d*x + c)^2 + 4*(3*b^2*d^2*e*arctan2(1, d*x + c)^2 + (6*b^2*c*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*f)*x^2 + (b^2*d^2*f*x^3 + (b^2*d^2*e + 2*b^2*c*d*f)*x^2 + (b^2*c^2 + b^2)*e + (2*b^2*c*d*e + (b^2*c^2 + b^2)*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 12*(b^2*c^2*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c)^2)*e + 4*(2*(3*b^2*c*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*e + 3*(b^2*c^2*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c)^2)*f)*x + 2*(b^2*d^2*f*x^3 + 2*b^2*c*d*e*x + (2*b^2*d^2*e + b^2*c*d*f)*x^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)

Giac [F]

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^2 \, dx$$

[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="giac")

[Out] integrate((f*x + e)*(b*arccot(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (e+f\,x)\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2\,dx$$

```
[In] int((e + f*x)*(a + b*acot(c + d*x))^2,x)
[Out] int((e + f*x)*(a + b*acot(c + d*x))^2, x)
```

3.138 $\int (a+b\cot^{-1}(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int (a+b\cot^{-1}(c+dx))^2\,dx=\frac{i(a+b\cot^{-1}(c+dx))^2}{d}+\frac{(c+dx)\,(a+b\cot^{-1}(c+dx))^2}{d}-\frac{2b(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}$$

```
[Out] I*(a+b*arccot(d*x+c))^2/d+(d*x+c)*(a+b*arccot(d*x+c))^2/d-2*b*(a+b*arccot(d
*x+c))*ln(2/(1+I*(d*x+c)))/d+I*b^2*polylog(2,1-2/(1+I*(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5148, 4931, 5041, 4965, 2449, 2352}

$$\int (a+b\cot^{-1}(c+dx))^2\,dx=\frac{(c+dx)\,(a+b\cot^{-1}(c+dx))^2}{d}+\frac{i(a+b\cot^{-1}(c+dx))^2}{d}-\frac{2b\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{d}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{d}$$

```
[In] Int[(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (I*(a + b*ArcCot[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcCot[c + d*x])^2)/d -
 (2*b*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))])/d + (I*b^2*PolyLog[
2, 1 - 2/(1 + I*(c + d*x))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+b\cot^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}+\frac{(2b)\text{Subst}\left(\int \frac{x(a+b\cot^{-1}(x))}{1+x^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{i(a+b\cot^{-1}(c+dx))^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{i-x}\,dx,x,c+dx\right)}{d}$$

$$= \frac{i(a+b\cot^{-1}(c+dx))^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d} - \frac{2b(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}-\frac{(2b^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{i(a+b\cot^{-1}(c+dx))^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d} - \frac{2b(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+i(c+dx)}\right)}{d}$$

$$= \frac{i(a+b\cot^{-1}(c+dx))^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d} - \frac{2b(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{ib^2\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

$$= \frac{b^2(i+c+dx)\cot^{-1}(c+dx)^2+2b\cot^{-1}(c+dx)\left(ac+adx-b\log\left(1-e^{2i\cot^{-1}(c+dx)}\right)\right)+a\Big(ac+adx-2}{d}$$

```
[In] Integrate[(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (b^2*(I + c + d*x)*ArcCot[c + d*x]^2 + 2*b*ArcCot[c + d*x]*(a*c + a*d*x - b
*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + a*(a*c + a*d*x - 2*b*Log[1/((c + d*x
)*Sqrt[1 + (c + d*x)^(-2)])]) + I*b^2*PolyLog[2, E^((2*I)*ArcCot[c + d*x])]
)/d
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.80

method	result
parts	$a^2x+\frac{b^2\left(\operatorname{arccot}(dx+c)^2(dx+c-i)-2\operatorname{arccot}(dx+c)\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)-2\operatorname{arccot}(dx+c)\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+2i\right.}{d}$
derivativedivides	$\frac{(dx+c)a^2+b^2\left(\operatorname{arccot}(dx+c)^2(dx+c-i)-2\operatorname{arccot}(dx+c)\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)-2\operatorname{arccot}(dx+c)\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+\right.}{}$
default	$\frac{(dx+c)a^2+b^2\left(\operatorname{arccot}(dx+c)^2(dx+c-i)-2\operatorname{arccot}(dx+c)\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)-2\operatorname{arccot}(dx+c)\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+\right.}{}$
risch	$-\frac{\ln(-idx-ic+1)^2b^2x}{4}+\frac{ia^2}{d}-\frac{b^2\arctan(dx+c)\pi c}{2d}-\frac{b\arctan(dx+c)ac}{d}+\frac{\pi abc}{d}+\frac{\pi^2b^2x}{4}+\frac{b^2\ln(d^2x^2+2cdx-}{4d}$

```
[In] int((a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+b^2/d*(arccot(d*x+c)^2*(d*x+c-I)-2*arccot(d*x+c)*ln(1+(d*x+c+I)/(1+(d
*x+c)^2)^(1/2))-2*arccot(d*x+c)*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I*arc
cot(d*x+c)^2+2*I*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I*polylog(2,(d
*x+c+I)/(1+(d*x+c)^2)^(1/2)))+2*a*b/d*(arccot(d*x+c)*(d*x+c)+1/2*ln(1+(d*x+
c)^2))
```

Fricas [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int \left(b\operatorname{arccot}\left(dx+c\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int \left(a+b\operatorname{acot}\left(c+dx\right)\right)^2\,dx$$

```
[In] integrate((a+b*acot(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acot(c + d*x))**2, x)
```

Maxima [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int \left(b\operatorname{arccot}\left(dx+c\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*x*arctan2(1, d*x + c)^2 - x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 16
*integrate(1/16*(12*d^2*x^2*arctan2(1, d*x + c)^2 + 12*c^2*arctan2(1, d*x +
 c)^2 + 8*(3*c*arctan2(1, d*x + c)^2 + arctan2(1, d*x + c))*d*x + (d^2*x^2
+ 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 12*arctan2(1, d*x
 + c)^2 + 4*(d^2*x^2 + c*d*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x^2 +
2*c*d*x + c^2 + 1), x))*b^2 + a^2*x + (2*(d*x + c)*arccot(d*x + c) + log((d
*x + c)^2 + 1))*a*b/d
```

Giac [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int \left(b\operatorname{arccot}\left(dx+c\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccot(d*x + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.21

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=a^2\,x+\frac{a\,b\,\left(\ln\left({\left(c+d\,x\right)}^2+1\right)+2\,\mathrm{acot}(c+d\,x)\,\left(c+d\,x\right)\right)}{d}-\frac{2\,b^2\,\ln\left(1-{\mathrm{e}}^{\mathrm{acot}(c+d\,x)\,2{}\mathrm{i}}\right)\,\mathrm{acot}(c+d\,x)}{d}+\frac{b^2\,{\mathrm{acot}(c+d\,x)}^2\,\left(c+d\,x\right)}{d}+\frac{b^2\,\mathrm{polylog}\left(2,{\mathrm{e}}^{\mathrm{acot}(c+d\,x)\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{d}+\frac{b^2\,{\mathrm{acot}(c+d\,x)}^2\,1{}\mathrm{i}}{d}$$

```
[In] int((a + b*acot(c + d*x))^2,x)
```

```
[Out] a^2*x + (b^2*polylog(2, exp(acot(c + d*x)*2i))*1i)/d + (b^2*acot(c + d*x)^2
*1i)/d + (a*b*(log((c + d*x)^2 + 1) + 2*acot(c + d*x)*(c + d*x)))/d - (2*b^
2*log(1 - exp(acot(c + d*x)*2i))*acot(c + d*x))/d + (b^2*acot(c + d*x)^2*(c
 + d*x))/d
```

3.139 $\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 261

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx = -\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} - \frac{ib(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{f} + \frac{ib(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}$$

```
[Out] -(a+b*arccot(d*x+c))^2*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))^2*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-I*b*(a+b*arccot(d*x+c))*polylog(2,1-2
/(1-I*(d*x+c)))/f+I*b*(a+b*arccot(d*x+c))*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-
c*f)/(1-I*(d*x+c)))/f-1/2*b^2*polylog(3,1-2/(1-I*(d*x+c)))/f+1/2*b^2*polylo
g(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5156, 4969}

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx = \frac{ib(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f} - \frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f} - \frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^2}{f} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

```
[In] Int[(a + b*ArcCot[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCot[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot[
c + d*x])^2*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f -
 (I*b*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f + (I*b
*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*
(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) +
(b^2*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])
/(2*f)
```

Rule 4969

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[I
*b*(a + b*ArcCot[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] + Simp[I*b*(a
+ b*ArcCot[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^2}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\
&\quad+\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\
&\quad-\frac{ib(a+b\cot^{-1}(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{f} \\
&\quad+\frac{ib(a+b\cot^{-1}(c+dx))\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\
&\quad-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}
\end{aligned}$$

Mathematica [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx$$

```
[In] Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x),x]
[Out] Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.06 (sec) , antiderivative size = 1911, normalized size of antiderivative = 7.32

method	result	size
derivativedivides	Expression too large to display	1911
default	Expression too large to display	1911
parts	Expression too large to display	2022

```
[In] int((a+b*arccot(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*d*ln(c*f-d*e-f*(d*x+c))/f-b^2*d*(-ln(c*f-d*e-f*(d*x+c))/f*arccot(d
*x+c)^2-2/f*(-1/2*arccot(d*x+c)^2*ln(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*
x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)-1/4*I*f/(
-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)
^2))+1/2*arccot(d*x+c)^2*ln((d*x+c+I)^2/(1+(d*x+c)^2)-1)-1/2*arccot(d*x+c)^
2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*d*e*arccot(d*x+c)*polylog(2,(d*e+I*
f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))/(-2*I*f+2*c*f-2*d*e)-polyl
og(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/2*arccot(d*x+c)^2*ln(1-(d*x+c+I)/(1+
(d*x+c)^2)^(1/2))+I*arccot(d*x+c)*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))
-polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/2*I*f/(-I*f+c*f-d*e)*arccot(d*x
+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+1/2*c*f/
(-I*f+c*f-d*e)*arccot(d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^
2/(1+(d*x+c)^2))+1/4*c*f/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I
*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-1/2*f/(-I*f+c*f-d*e)*arccot(d*x+c)*polylog(2
,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-1/2*I*c*f/(-I*f+c*
f-d*e)*arccot(d*x+c)*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+
(d*x+c)^2))+I*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+1/4*I*
Pi*csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e
*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*(csg
n(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+
c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e))*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))-cs
gn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x
+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*csgn(I/((
d*x+c+I)^2/(1+(d*x+c)^2)-1))-csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*
x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e))*csgn(I*(
-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^
2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))+csgn(I*(-I*f*(d
*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d
*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))^2)*arccot(d*x+c)^2-1/2
*d*e/(-I*f+c*f-d*e)*arccot(d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+
c+I)^2/(1+(d*x+c)^2))-1/4*d*e/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+
d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))))-2*a*b*d*(-ln(c*f-d*e-f*(d*x+c))/f*arc
cot(d*x+c)-1/2*I*ln(c*f-d*e-f*(d*x+c))*(ln((I*f+f*(d*x+c))/(c*f-d*e+I*f))-l
n((I*f-f*(d*x+c))/(d*e+I*f-c*f)))/f-1/2*I*(dilog((I*f+f*(d*x+c))/(c*f-d*e+I
*f))-dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f)))/f))
```

Fricas [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^2}{fx+e}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="fricas")
[Out] integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)/(f*x + e), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acot(d*x+c))**2/(f*x+e),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^2}{fx+e}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="maxima")
[Out] a^2*log(f*x + e)/f + integrate(1/16*(12*b^2*arctan2(1, d*x + c)^2 + b^2*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan2(1, d*x + c))/(f*x + e), x)
```

Giac [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^2}{fx+e}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="giac")
[Out] integrate((b*arccot(d*x + c) + a)^2/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{e+fx}\,dx=\int \frac{\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2}{e+f\,x}\,dx$$

```
[In] int((a + b*acot(c + d*x))^2/(e + f*x),x)
[Out] int((a + b*acot(c + d*x))^2/(e + f*x), x)
```

3.140 $\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 567

$$\begin{aligned}\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx = &\frac{ib^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2}+\frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f\,(d^2e^2-2cdef+(1+c^2)\,f^2)}\\
&-\frac{(a+b\cot^{-1}(c+dx))^2}{f(e+fx)}-\frac{2abd(de-cf)\arctan(c+dx)}{f\,(f^2+(de-cf)^2)}\\
&-\frac{2abd\log(e+fx)}{f^2+(de-cf)^2}+\frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&-\frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&-\frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&+\frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2}+\frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&-\frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\\
&+\frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}\end{aligned}$$

```
[Out] I*b^2*d*arccot(d*x+c)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^2*d*(-c*f+d*e)*ar
ccot(d*x+c)^2/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arccot(d*x+c))^2/f/(f*
x+e)-2*a*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)-2*a*b*d*ln(f*x+e
)/(f^2+(-c*f+d*e)^2)+2*b^2*d*arccot(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c
```

```
*d*e*f+(c^2+1)*f^2)-2*b^2*d*arccot(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I
*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-2*b^2*d*arccot(d*x+c)*ln(2/(1+I*
(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+a*b*d*ln(1+(d*x+c)^2)/(f^2+(-c*f+
d*e)^2)+I*b^2*d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2
)-I*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)+I*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f
+(c^2+1)*f^2)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {5154, 2007, 719, 31, 648, 632, 210, 642, 6873, 5166, 720, 649, 209, 266, 6820, 12, 6857, 4967, 2449, 2352, 2497, 5105, 5005, 5041, 4965}

$$\begin{aligned}\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)^2}dx = & -\frac{2abd\arctan(c+dx)(de-cf)}{f\left((de-cf)^2+f^2\right)}-\frac{2abd\log(e+fx)}{(de-cf)^2+f^2} \\ & +\frac{abd\log\left((c+dx)^2+1\right)}{(de-cf)^2+f^2}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} \\ & +\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{(c^2+1)f^2-2cdef+d^2e^2} \\ & -\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)}{(c^2+1)f^2-2cdef+d^2e^2} \\ & +\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{(c^2+1)f^2-2cdef+d^2e^2} \\ & +\frac{ib^2d\cot^{-1}(c+dx)^2}{(c^2+1)f^2-2cdef+d^2e^2} \\ & +\frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left((c^2+1)f^2-2cdef+d^2e^2\right)} \\ & +\frac{2b^2d\log\left(\frac{2}{1-i(c+dx)}\right)\cot^{-1}(c+dx)}{(c^2+1)f^2-2cdef+d^2e^2} \\ & -\frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(de+(-c+i)f)}\right)}{(c^2+1)f^2-2cdef+d^2e^2} \\ & -\frac{2b^2d\log\left(\frac{2}{1+i(c+dx)}\right)\cot^{-1}(c+dx)}{(c^2+1)f^2-2cdef+d^2e^2}\end{aligned}$$

```
[In] Int[(a + b*ArcCot[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (I*b^2*d*ArcCot[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*
(d*e - c*f)*ArcCot[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) -
```

```
(a + b*ArcCot[c + d*x])^2/(f*(e + f*x)) - (2*a*b*d*(d*e - c*f)*ArcTan[c + d
*x])/(f*(f^2 + (d*e - c*f)^2)) - (2*a*b*d*Log[e + f*x])/(f^2 + (d*e - c*f)^
2) + (2*b^2*d*ArcCot[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2) - (2*b^2*d*ArcCot[c + d*x]*Log[(2*d*(e + f*x))/((d*e + (
I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (2*b
^2*d*ArcCot[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) + (a*b*d*Log[1 + (c + d*x)^2])/(f^2 + (d*e - c*f)^2) + (I*b^2*d*P
olyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
 (I*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x
)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5166

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}-\frac{(2bd)\int\frac{a+b\cot^{-1}(c+dx)}{(e+fx)(1+(c+dx)^2)}\,dx}{f}\\
&= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}-\frac{(2bd)\int\frac{a+b\cot^{-1}(c+dx)}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}\\
&= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}-\frac{(2b)\text{Subst}\left(\int\frac{a+b\cot^{-1}(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx,x,c+dx\right)}{f}\\
&= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}-\frac{(2b)\text{Subst}\left(\int\frac{d\left(a+b\cot^{-1}(x)\right)}{(de-cf+fx)(1+x^2)}\,dx,x,c+dx\right)}{f}
\end{aligned}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2bd)\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2bd)\text{Subst}\left(\int \left(\frac{a}{(de-cf+fx)(1+x^2)} + \frac{b\cot^{-1}(x)}{(de-cf+fx)(1+x^2)}\right)\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2abd)\text{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} - \frac{(2b^2d)\,\text{Subst}\left(\int \frac{\cot^{-1}(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2b^2d)\,\text{Subst}\left(\int \left(\frac{f^2\cot^{-1}(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)} + \frac{(de-cf-fx)\cot^{-1}(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(1+x^2)}\right)\,dx, x, c+dx\right)}{f} - \frac{(2abd)\text{Subst}\left(\int \frac{de-cf-fx}{1+x^2}\,dx, x, c+dx\right)}{f\left(f^2+(de-cf)^2\right)} - \frac{(2abdf)\text{Subst}\left(\int \frac{1}{de-cf+fx}\,dx, x, c+dx\right)}{f^2+(de-cf)^2}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{(2b^2d)\,\text{Subst}\left(\int \frac{(de-cf-fx)\cot^{-1}(x)}{1+x^2}\,dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} - \frac{(2b^2df)\,\text{Subst}\left(\int \frac{\cot^{-1}(x)}{de-cf+fx}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{(2abd)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{f^2+(de-cf)^2} - \frac{(2abd(de-cf))\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{f\left(f^2+(de-cf)^2\right)}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{(2b^2d)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{(2b^2d)\,\text{Subst}\left(\int\frac{\log\left(\frac{2(de-cf+fx)}{(de+if-cf)(1-ix)}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{(2b^2d)\,\text{Subst}\left(\int\left(\frac{de\left(1-\frac{cf}{de}\right)\cot^{-1}(x)}{1+x^2}-\frac{fx\cot^{-1}(x)}{1+x^2}\right)dx,x,c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} - \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{(2ib^2d)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{(2b^2d)\,\text{Subst}\left(\int\frac{x\cot^{-1}(x)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{(2b^2d(de-cf))\,\text{Subst}\left(\int\frac{\cot^{-1}(x)}{1+x^2}\,dx,x,c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}$$

$$= \frac{ib^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} - \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2abd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{(2b^2d)\,\text{Subst}\left(\int\frac{\cot^{-1}(x)}{i-x}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}$$

$$= \frac{ib^2 d \cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f(d^2e^2-2cdef+(1+c^2)f^2)} - \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2abd(de-cf)\arctan(c+dx)}{f(f^2+(de-cf)^2)} - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{(2b^2d)\,\mathrm{Subst}\left(\int\frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$= \frac{ib^2 d \cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f(d^2e^2-2cdef+(1+c^2)f^2)} - \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2abd(de-cf)\arctan(c+dx)}{f(f^2+(de-cf)^2)} - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{(2ib^2d)\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$
\begin{aligned}
&= \frac{ib^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f\,(d^2e^2-2cdef+(1+c^2)\,f^2)} \\
&\quad - \frac{(a+b\cot^{-1}(c+dx))^2}{f(e+fx)} - \frac{2abd(de-cf)\arctan(c+dx)}{f\,(f^2+(de-cf)^2)} \\
&\quad - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad - \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad + \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&\quad - \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.12 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.80

$$
\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx =
$$

$$
-\frac{a^2+\frac{2abf\left(\left(-cde+f+c^2f-d^2ex+cdfx\right)\cot^{-1}(c+dx)+d(e+fx)\log\left(-\frac{d(e+fx)}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}\right)\right)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^2d(e+fx)(1+(c+dx)^2)\left(\frac{e^{i\arctan\left(\frac{f}{de-c}\right.}}{(-de+cf)\sqrt{}}\right.}{}}{}
$$

```
[In] Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] -((a^2 + (2*a*b*f*((-(c*d*e) + f + c^2*f - d^2*e*x + c*d*f*x)*ArcCot[c + d*
x] + d*(e + f*x)*Log[-((d*(e + f*x))/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]))]
))/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(e + f*x)*(1 + (c + d*x)^
2)*((E^(I*ArcTan[f/(d*e - c*f)])*ArcCot[c + d*x]^2)/((-(d*e) + c*f)*Sqrt[1
+ f^2/(d*e - c*f)^2]) + ArcCot[c + d*x]^2/(d*e + d*f*x) + (f*(I*Pi*ArcCot[c
 + d*x] + Pi*Log[1 + E^((-2*I)*ArcCot[c + d*x])] + 2*ArcCot[c + d*x]*Log[1
- E^((2*I)*(ArcCot[c + d*x] + ArcTan[f/(d*e - c*f)]))] - Pi*Log[1/Sqrt[1 +
(c + d*x)^(-2)]] + 2*ArcTan[f/(-(d*e) + c*f)]*(I*ArcCot[c + d*x] - Log[1 -
E^((2*I)*(ArcCot[c + d*x] + ArcTan[f/(d*e - c*f)]))] + Log[Sin[ArcCot[c + d
*x] + ArcTan[f/(d*e - c*f)]]]) - I*PolyLog[2, E^((2*I)*(ArcCot[c + d*x] + A
rcTan[f/(d*e - c*f)])))]))/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/((c + d*x
)^2*(1 + (c + d*x)^(-2))))/(f*(e + f*x))
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.38

method	result
parts	$-\frac{a^2}{(fx+e)f}+\frac{b^2\left(-\frac{d^2\operatorname{arccot}(dx+c)^2}{(f(dx+c)-cf+de)f}-\frac{2d^2\left(\frac{\operatorname{arccot}(dx+c)f\ln(f(dx+c)-cf+de)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{\operatorname{arccot}(dx+c)f\ln\left(1+(dx+c)^2\right)}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)}-\frac{\operatorname{arccot}(d}{c^2f^2-}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arccot}(dx+c)^2}{(cf-de-f(dx+c))f}+\frac{-\frac{2\operatorname{arccot}(dx+c)f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{2\operatorname{arccot}(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{2\operatorname{ar}}{}}{}\right.}{}$
default	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arccot}(dx+c)^2}{(cf-de-f(dx+c))f}+\frac{-\frac{2\operatorname{arccot}(dx+c)f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{2\operatorname{arccot}(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{2\operatorname{ar}}{}}{}\right.}{}$

```
[In] int((a+b*arccot(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/(f*x+e)/f+b^2/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)^2-2*d^2/f*(a
rccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)-1/2*ar
ccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d*x+c)^2)-arccot(d*x+c)
/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c*f+arccot(d*x+c)/(c^2*f^2-2
*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*d*e+f^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)
*(-1/2*I*ln(f*(d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln((I*f+f
*(d*x+c))/(c*f-d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-dil
og((I*f+f*(d*x+c))/(c*f-d*e+I*f)))/f)-1/2*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)
*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c
+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1
/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))-1
/2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(c*f-d*e)*arctan(d*x+c)^2))+2*a*b/d*(-d^
```

```
2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)-d^2/f*(1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f
^2)*f*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*f*ln(1+
(d*x+c)^2)+(-c*f+d*e)*arctan(d*x+c))))
```

Fricas [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx=\int \frac{\left(b\operatorname{arccot}\left(dx+c\right)+a\right)^2}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
[Out] integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)/(f^2*x^2 + 2
*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*acot(d*x+c))**2/(f*x+e)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx=\int \frac{\left(b\operatorname{arccot}\left(dx+c\right)+a\right)^2}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
[Out] -(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2 +
(c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*f +
 (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2)) + 2
*arccot(d*x + c)/(f^2*x + e*f))*a*b - 1/16*(4*arctan2(1, d*x + c)^2 - 16*(f
^2*x + e*f)*integrate(1/16*(12*d^2*f*x^2*arctan2(1, d*x + c)^2 + 8*(3*c*arc
tan2(1, d*x + c)^2 - arctan2(1, d*x + c))*d*f*x - 8*d*e*arctan2(1, d*x + c)
 + (d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
 + 12*(c^2*arctan2(1, d*x + c)^2 + arctan2(1, d*x + c)^2)*f - 4*(d^2*f*x^2
+ c*d*e + (d^2*e + c*d*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*f^3*x^4
 + (c^2 + 1)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2*e^2*f + 4*c*d*e*f^2
 + (c^2 + 1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)*x), x) - log(d^2*x^
2 + 2*c*d*x + c^2 + 1)^2)*b^2/(f^2*x + e*f) - a^2/(f^2*x + e*f)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a + b\cot^{-1}(c + dx))^2}{(e + fx)^2} \, dx = \text{Timed out}$$

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\cot^{-1}(c + dx))^2}{(e + fx)^2} \, dx = \int \frac{(a + b\,\mathrm{acot}(c + d\,x))^2}{(e + f\,x)^2} \, dx$$

```
[In] int((a + b*acot(c + d*x))^2/(e + f*x)^2,x)
[Out] int((a + b*acot(c + d*x))^2/(e + f*x)^2, x)
```

3.141 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 dx$

Optimal result

Integrand size = 20, antiderivative size = 565

$$\begin{aligned}
&\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 dx \\
&= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3} + \frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3} \\
&+ \frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} \\
&+ \frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} + \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3} \\
&+ \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3} \\
&- \frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3f} \\
&+ \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f} - \frac{6b^2f(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&- \frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+ \frac{b^3f^2\log\left(1+(c+dx)^2\right)}{2d^3} + \frac{3ib^3f(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+ \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&- \frac{b^3(3d^2e^2-6cdef-(1-3c^2)f^2)\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^3}
\end{aligned}$$

```
[Out] a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arccot(d*x+c)/d^3+1/2*b*f^2*(a+b*arccot(d*x
+c))^2/d^3+3*I*b*f*(-c*f+d*e)*(a+b*arccot(d*x+c))^2/d^3+3*b*f*(-c*f+d*e)*(d
*x+c)*(a+b*arccot(d*x+c))^2/d^3+1/2*b*f^2*(d*x+c)^2*(a+b*arccot(d*x+c))^2/d
^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))^3/d^3-1/3
*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arccot(d*x+c))^3/d^3/f+1/
3*(f*x+e)^3*(a+b*arccot(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arccot(d*x+c))*
ln(2/(1+I*(d*x+c)))/d^3-b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(
d*x+c))^2*ln(2/(1+I*(d*x+c)))/d^3+1/2*b^3*f^2*ln(1+(d*x+c)^2)/d^3+3*I*b^3*f
*(-c*f+d*e)*polylog(2,1-2/(1+I*(d*x+c)))/d^3+I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arccot(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^3-1/2*b^3*(
3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(3,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules

used = {5156, 4975, 4931, 5041, 4965, 2449, 2352, 4947, 5037, 266, 5005, 5105, 5115, 6745}

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 dx$$

$$= \frac{ib^2(-(1-3c^2)f^2-6cdef+3d^2e^2)\,\mathrm{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\cot^{-1}(c+dx))}{d^3}$$
$$-\frac{6b^2f(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{d^3}+\frac{ab^2f^2x}{d^2}$$
$$+\frac{i(-(1-3c^2)f^2-6cdef+3d^2e^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3}$$
$$-\frac{(de-cf)\left(-(3-c^2)f^2-2cdef+d^2e^2\right)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3f}$$
$$-\frac{b(-(1-3c^2)f^2-6cdef+3d^2e^2)\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}$$
$$+\frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}$$
$$+\frac{3bf(c+dx)(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}+\frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}$$
$$+\frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f}$$
$$-\frac{b^3(-(1-3c^2)f^2-6cdef+3d^2e^2)\,\mathrm{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)}{2d^3}$$
$$+\frac{3ib^3f(de-cf)\,\mathrm{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{d^3}$$
$$+\frac{b^3f^2\log\left((c+dx)^2+1\right)}{2d^3}+\frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3}$$

```
[In] Int[(e + f*x)^2*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcCot[c + d*x])/d^3 + (b*f^2*(a + b
*ArcCot[c + d*x])^2)/(2*d^3) + ((3*I)*b*f*(d*e - c*f)*(a + b*ArcCot[c + d*x
])^2)/d^3 + (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcCot[c + d*x])^2)/d^3 + (
b*f^2*(c + d*x)^2*(a + b*ArcCot[c + d*x])^2)/(2*d^3) + ((I/3)*(3*d^2*e^2 -
6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])^3)/d^3 - ((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcCot[c + d*x])^3)/(3*d^3*f)
+ ((e + f*x)^3*(a + b*ArcCot[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a +
 b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^3 - (b*(3*d^2*e^2 - 6*c*d*e
*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d
^3 + (b^3*f^2*Log[1 + (c + d*x)^2])/(2*d^3) + ((3*I)*b^3*f*(d*e - c*f)*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))])/d^3 + (I*b^2*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
```

```
^3 - (b^3*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I
*(c + d*x))])/(2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4975

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\cot^{-1}(x)\right)^3 \,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^3}{3f} + \frac{b\text{Subst}\left(\int \left(\frac{3f^2(de-cf)\left(a+b\cot^{-1}(x)\right)^2}{d^3}+\frac{f^3x\left(a+b\cot^{-1}(x)\right)^2}{d^3}+\frac{((de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cdef-}{d^3(1+x^2)}\right.\right.}{f}$$

$$= \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^3}{3f} + \frac{b\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cdef-3f^2+c^2f^2)+f(3d^2e^2-6cdef-(1-3c^2)f^2)x)\left(a+b\cot^{-1}(x)\right)^2}{1+x^2} \,dx, x, c+dx\right)}{d^3f} + \frac{(bf^2)\,\text{Subst}\left(\int x\left(a+b\cot^{-1}(x)\right)^2 \,dx, x, c+dx\right)}{d^3} + \frac{(3bf(de-cf))\text{Subst}\left(\int \left(a+b\cot^{-1}(x)\right)^2 \,dx, x, c+dx\right)}{d^3}$$

$$= \frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} + \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3} + \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f} + \frac{b\text{Subst}\left(\int \left(\frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}+\frac{f(3d^2e^2-6cdef-(1-3c^2)f^2)x\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}\right) dx, x\right.}{d^3f} + \frac{(b^2f^2)\,\text{Subst}\left(\int \frac{x^2\left(a+b\cot^{-1}(x)\right)}{1+x^2} \,dx, x, c+dx\right)}{d^3} + \frac{(6b^2f(de-cf))\,\text{Subst}\left(\int \frac{x\left(a+b\cot^{-1}(x)\right)}{1+x^2} \,dx, x, c+dx\right)}{d^3}$$

$$= \frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}+\frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}$$
$$+\frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f}$$
$$+\frac{(b^2f^2)\,\mathrm{Subst}\left(\int\left(a+b\cot^{-1}(x)\right)\,dx,x,c+dx\right)}{d^3}$$
$$-\frac{(b^2f^2)\,\mathrm{Subst}\left(\int\frac{a+b\cot^{-1}(x)}{1+x^2}\,dx,x,c+dx\right)}{d^3}$$
$$-\frac{(6b^2f(de-cf))\,\mathrm{Subst}\left(\int\frac{a+b\cot^{-1}(x)}{i-x}\,dx,x,c+dx\right)}{d^3}$$
$$+\frac{(b(3d^2e^2-6cdef-(1-3c^2)\,f^2))\,\mathrm{Subst}\left(\int\frac{x\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}\,dx,x,c+dx\right)}{d^3}$$
$$+\frac{(b(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2))\,\mathrm{Subst}\left(\int\frac{\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}\,dx,x,c+dx\right)}{d^3f}$$

$$=\frac{ab^2f^2x}{d^2}+\frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}+\frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}$$
$$+\frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}$$
$$+\frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}$$
$$+\frac{i(3d^2e^2-6cdef-(1-3c^2)\,f^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3}$$
$$-\frac{(de-cf)\,(d^2e^2-2cdef-(3-c^2)\,f^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3f}$$
$$+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f}$$
$$-\frac{6b^2f(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3}$$
$$+\frac{(b^3f^2)\,\mathrm{Subst}\left(\int\cot^{-1}(x)\,dx,x,c+dx\right)}{d^3}$$
$$-\frac{(6b^3f(de-cf))\,\mathrm{Subst}\left(\int\frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^3}$$
$$-\frac{(b(3d^2e^2-6cdef-(1-3c^2)\,f^2))\,\mathrm{Subst}\left(\int\frac{\left(a+b\cot^{-1}(x)\right)^2}{i-x}\,dx,x,c+dx\right)}{d^3}$$

$$
\begin{aligned}
&= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3} + \frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3} \\
&+ \frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} \\
&+ \frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} \\
&+ \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3} \\
&+ \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3} \\
&- \frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3f} \\
&+ \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f} \\
&- \frac{6b^2f(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&- \frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+ \frac{(b^3f^2)\,\text{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,c+dx\right)}{d^3} \\
&+ \frac{(6ib^3f(de-cf))\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+i(c+dx)}\right)}{d^3} \\
&- \frac{(2b^2(3d^2e^2-6cdef-(1-3c^2)f^2))\,\text{Subst}\left(\int\frac{(a+b\cot^{-1}(x))\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^3}
\end{aligned}
$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3} + \frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}$$
$$+ \frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}$$
$$+ \frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}$$
$$+ \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}$$
$$+ \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3}$$
$$- \frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3f}$$
$$+ \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f}$$
$$- \frac{6b^2f(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3}$$
$$- \frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3}$$
$$+ \frac{b^3f^2\log\left(1+(c+dx)^2\right)}{2d^3} + \frac{3ib^3f(de-cf)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3}$$
$$+ \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3}$$
$$+ \frac{(ib^3(3d^2e^2-6cdef-(1-3c^2)f^2))\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3} + \frac{bf^2(a+b\cot^{-1}(c+dx))^2}{2d^3}$$
$$+ \frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}$$
$$+ \frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}$$
$$+ \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}$$
$$+ \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3}$$
$$- \frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3f}$$
$$+ \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f}$$
$$- \frac{6b^2f(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3}$$
$$- \frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3}$$
$$+ \frac{b^3f^2\log\left(1+(c+dx)^2\right)}{2d^3} + \frac{3ib^3f(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3}$$
$$+ \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3}$$
$$- \frac{b^3(3d^2e^2-6cdef-(1-3c^2)f^2)\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^3}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2309 vs. $2(565) = 1130$.

Time = 14.56 (sec) , antiderivative size = 2309, normalized size of antiderivative = 4.09

$$\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x^
2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[c +
 d*x] + ((-3*a^2*b*c*d^2*e^2 - 3*a^2*b*d*e*f + 3*a^2*b*c^2*d*e*f + 3*a^2*b*
c*f^2 - a^2*b*c^3*f^2)*ArcTan[c + d*x])/d^3 + ((3*a^2*b*d^2*e^2 - 6*a^2*b*c
*d*e*f - a^2*b*f^2 + 3*a^2*b*c^2*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*
```

d^3) - (3*a*b^2*e^2*(1 + (c + d*x)^2)*(-((c + d*x)*ArcCot[c + d*x]^2) + 2*A
rcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] - I*(ArcCot[c + d*x]^2 +
PolyLog[2, E^((2*I)*ArcCot[c + d*x])])))/(d*(c + d*x)^2*(1 + (c + d*x)^(-2)
)) + (6*a*b^2*e*f*(1 + (c + d*x)^2)*(((c + d*x)*ArcCot[c + d*x])/d^2 - (c*(
c + d*x)*ArcCot[c + d*x]^2)/d^2 + ((c + d*x)^2*(1 + (c + d*x)^(-2))*ArcCot[
c + d*x]^2)/(2*d^2) - Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])]/d^2 + (2*
c*(ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] - (I/2)*(ArcCot[c + d
*x]^2 + PolyLog[2, E^((2*I)*ArcCot[c + d*x])])))/d^2))/((c + d*x)^2*(1 + (c
 + d*x)^(-2))) - (a*b^2*f^2*x^2*(1 + (c + d*x)^2)*(-((c + d*x)*ArcCot[c + d
*x]^2) + ArcCot[c + d*x]*(-1 + 3*c*ArcCot[c + d*x]) - (1 - 6*c*ArcCot[c + d
*x] - ArcCot[c + d*x]^2 + 3*c^2*ArcCot[c + d*x]^2)/((c + d*x)*(1 + (c + d*x
)^(-2))) - (6*c*(Log[(c + d*x)^(-1)] + Log[1/Sqrt[1 + (c + d*x)^(-2)]]))/((
c + d*x)^2*(1 + (c + d*x)^(-2))) + (I*(ArcCot[c + d*x]*(ArcCot[c + d*x] + (
2*I)*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + PolyLog[2, E^((2*I)*ArcCot[c + d
*x])]))/((c + d*x)^2*(1 + (c + d*x)^(-2))) + (6*c^2*(ArcCot[c + d*x]*Log[1
- E^((2*I)*ArcCot[c + d*x])] - (I/2)*(ArcCot[c + d*x]^2 + PolyLog[2, E^((2*
I)*ArcCot[c + d*x])])))/((c + d*x)^2*(1 + (c + d*x)^(-2)))))/(d*(c + d*x)^2
*(1 + (c + d*x)^(-2))*(1/Sqrt[1 + (c + d*x)^(-2)] - c/((c + d*x)*Sqrt[1 + (
c + d*x)^(-2)]))^2) - (b^3*e^2*(1 + (c + d*x)^2)*((-1/8*I)*Pi^3 + I*ArcCot[
c + d*x]^3 - (c + d*x)*ArcCot[c + d*x]^3 + 3*ArcCot[c + d*x]^2*Log[1 - E^((
-2*I)*ArcCot[c + d*x])] + (3*I)*ArcCot[c + d*x]*PolyLog[2, E^((-2*I)*ArcCot
[c + d*x])] + (3*PolyLog[3, E^((-2*I)*ArcCot[c + d*x])])/2))/(d*(c + d*x)^2
*(1 + (c + d*x)^(-2))) + (b^3*e*f*(1 + (c + d*x)^2)*((-I)*c*Pi^3 + (12*I)*A
rcCot[c + d*x]^2 + 12*(c + d*x)*ArcCot[c + d*x]^2 + (8*I)*c*ArcCot[c + d*x]
^3 - 8*c*(c + d*x)*ArcCot[c + d*x]^3 + 4*(c + d*x)^2*(1 + (c + d*x)^(-2))*A
rcCot[c + d*x]^3 + 24*c*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x]
)] - 24*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] + (24*I)*c*ArcCo
t[c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] + (12*I)*PolyLog[2, E^((2
*I)*ArcCot[c + d*x])] + 12*c*PolyLog[3, E^((-2*I)*ArcCot[c + d*x])]))/(4*d^
2*(c + d*x)^2*(1 + (c + d*x)^(-2))) - (b^3*f^2*(1 + (c + d*x)^2)*(I*(-1 + 3
*c^2)*ArcCot[c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] + ((c + d*x)^3
*(1 + (c + d*x)^(-2))^(3/2)*(((3*I)*Pi^3)/((c + d*x)*Sqrt[1 + (c + d*x)^(-2
)]) - ((9*I)*c^2*Pi^3)/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]) - (24*ArcCot[c
+ d*x])/Sqrt[1 + (c + d*x)^(-2)] + (72*c*ArcCot[c + d*x]^2)/Sqrt[1 + (c + d
*x)^(-2)] - (48*ArcCot[c + d*x]^2)/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]) + (
(216*I)*c*ArcCot[c + d*x]^2)/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]) - (24*Arc
Cot[c + d*x]^3)/Sqrt[1 + (c + d*x)^(-2)] - (24*c^2*ArcCot[c + d*x]^3)/Sqrt[
1 + (c + d*x)^(-2)] - ((24*I)*ArcCot[c + d*x]^3)/((c + d*x)*Sqrt[1 + (c + d
*x)^(-2)]) + (96*c*ArcCot[c + d*x]^3)/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])
+ ((72*I)*c^2*ArcCot[c + d*x]^3)/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]) + 24*
ArcCot[c + d*x]*Cos[3*ArcCot[c + d*x]] - 72*c*ArcCot[c + d*x]^2*Cos[3*ArcCo
t[c + d*x]] - 8*ArcCot[c + d*x]^3*Cos[3*ArcCot[c + d*x]] + 24*c^2*ArcCot[c
+ d*x]^3*Cos[3*ArcCot[c + d*x]] - (72*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*A
rcCot[c + d*x])])/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]) + (216*c^2*ArcCot[c
+ d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])])/((c + d*x)*Sqrt[1 + (c + d*x)

```
^(-2)]) - (432*c*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])])/((c +
d*x)*Sqrt[1 + (c + d*x)^(-2)]) + (72*Log[(c + d*x)^(-1)])/((c + d*x)*Sqrt[1
 + (c + d*x)^(-2)]) + (72*Log[1/Sqrt[1 + (c + d*x)^(-2)]])/((c + d*x)*Sqrt[
1 + (c + d*x)^(-2)]) + ((288*I)*c*PolyLog[2, E^((2*I)*ArcCot[c + d*x])])/((
c + d*x)^3*(1 + (c + d*x)^(-2))^(3/2)) + (48*(-1 + 3*c^2)*PolyLog[3, E^((-2
*I)*ArcCot[c + d*x])])/((c + d*x)^3*(1 + (c + d*x)^(-2))^(3/2)) - I*Pi^3*Si
n[3*ArcCot[c + d*x]] + (3*I)*c^2*Pi^3*Sin[3*ArcCot[c + d*x]] - (72*I)*c*Arc
Cot[c + d*x]^2*Sin[3*ArcCot[c + d*x]] + (8*I)*ArcCot[c + d*x]^3*Sin[3*ArcCo
t[c + d*x]] - (24*I)*c^2*ArcCot[c + d*x]^3*Sin[3*ArcCot[c + d*x]] + 24*ArcC
ot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])]*Sin[3*ArcCot[c + d*x]] -
72*c^2*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])]*Sin[3*ArcCot[c
 + d*x]] + 144*c*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])]*Sin[3*A
rcCot[c + d*x]] - 24*Log[(c + d*x)^(-1)]*Sin[3*ArcCot[c + d*x]] - 24*Log[1/
Sqrt[1 + (c + d*x)^(-2)]]*Sin[3*ArcCot[c + d*x]]))/96))/(d^3*(c + d*x)^2*(1
 + (c + d*x)^(-2)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 101.64 (sec) , antiderivative size = 6248, normalized size of antiderivative = 11.06

method	result	size
parts	Expression too large to display	6248
derivativedivides	Expression too large to display	10834
default	Expression too large to display	10834

```
[In] int((f*x+e)^2*(a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2 (b \operatorname{arccot}(dx+c)+a)^3 \, dx$$

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x +
 b^3*e^2)*arccot(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^2)
*arccot(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arccot(d
*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**2*(a+b*acot(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2 (b \operatorname{arccot}(dx+c)+a)^3 \, dx$$

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
[Out] 1/24*b^3*f^2*x^3*arctan2(1, d*x + c)^3 + 1/8*b^3*e*f*x^2*arctan2(1, d*x + c
)^3 + 1/8*b^3*e^2*x*arctan2(1, d*x + c)^3 + 1/3*a^3*f^2*x^3 + a^3*e*f*x^2 +
 3*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3
- c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*e*f + 1/2*(2*x^3*arccot(d*
x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4
 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b*f^2 + a^3*e^2*x
 + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a^2*b*e^2/d - 1
/32*(b^3*f^2*x^3*arctan2(1, d*x + c) + 3*b^3*e*f*x^2*arctan2(1, d*x + c) +
3*b^3*e^2*x*arctan2(1, d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + integ
rate(1/32*(4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)
*d^2*f^2*x^4 + 4*(2*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x
+ c)^2)*d^2*e*f + (b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3*arctan2(1, d*x + c)
^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*f^2)*x^3 + 4*(7*b^3*arctan2(1, d*
x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arctan2(1, d*x + c)^3 +
24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*e^2 + 4*((7*b^3*arctan2(1, d*x + c)^3
+ 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*e^2 + (3*b^3*arctan2(1, d*x + c)^2 +
4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*e*f +
 (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arc
tan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*f^2)*x^2 + (3*b^3
*d^2*f^2*x^4*arctan2(1, d*x + c) + (6*b^3*d^2*e*f*arctan2(1, d*x + c) + (6*
b^3*c*arctan2(1, d*x + c) - b^3)*d*f^2)*x^3 + 3*(b^3*c^2*arctan2(1, d*x + c
) + b^3*arctan2(1, d*x + c))*e^2 + 3*(b^3*d^2*e^2*arctan2(1, d*x + c) + (4*
b^3*c*arctan2(1, d*x + c) - b^3)*d*e*f + (b^3*c^2*arctan2(1, d*x + c) + b^3
*arctan2(1, d*x + c))*f^2)*x^2 + 3*((2*b^3*c*arctan2(1, d*x + c) - b^3)*d*e
^2 + 2*(b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c))*e*f)*x)*log(
d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*((3*b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3
*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*e^2 + 2*(7*b^
```

```
3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arctan2(1
, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*e*f)*x + 4*(b^3*d^2*f^2
*x^4*arctan2(1, d*x + c) + 3*b^3*c*d*e^2*x*arctan2(1, d*x + c) + (3*b^3*d^2
*e*f*arctan2(1, d*x + c) + b^3*c*d*f^2*arctan2(1, d*x + c))*x^3 + 3*(b^3*d^
2*e^2*arctan2(1, d*x + c) + b^3*c*d*e*f*arctan2(1, d*x + c))*x^2)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
```

Giac [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2 (b\,\mathrm{arccot}\,(dx+c)+a)^3 \, dx$$

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)^2*(b*arccot(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (e+f\,x)^2 \,(a+b\,\mathrm{acot}(c+d\,x))^3 \, dx$$

```
[In] int((e + f*x)^2*(a + b*acot(c + d*x))^3,x)
[Out] int((e + f*x)^2*(a + b*acot(c + d*x))^3, x)
```

3.142 $\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^3 dx$

Optimal result

Integrand size = 18, antiderivative size = 337

$$\begin{aligned}
&\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^3 dx \\
&= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} \\
&\quad + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2} \\
&\quad - \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^3}{2d^2f} \\
&\quad + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} - \frac{3b^2f\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad - \frac{3b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} + \frac{3ib^3f\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d^2} \\
&\quad + \frac{3ib^2(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad - \frac{3b^3(de-cf)\,\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}
\end{aligned}$$

```
[Out] 3/2*I*b*f*(a+b*arccot(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*arccot(d*x+c))^2/d
^2+I*(-c*f+d*e)*(a+b*arccot(d*x+c))^3/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a
+b*arccot(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arccot(d*x+c))^3/f-3*b^2*f*(a+
b*arccot(d*x+c))*ln(2/(1+I*(d*x+c)))/d^2-3*b*(-c*f+d*e)*(a+b*arccot(d*x+c))
^2*ln(2/(1+I*(d*x+c)))/d^2+3/2*I*b^3*f*polylog(2,1-2/(1+I*(d*x+c)))/d^2+3*I
*b^2*(-c*f+d*e)*(a+b*arccot(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^2-3/2*b^
3*(-c*f+d*e)*polylog(3,1-2/(1+I*(d*x+c)))/d^2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5156, 4975, 4931, 5041, 4965, 2449, 2352, 5105, 5005, 5115, 6745}

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$
$$= \frac{3ib^2(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)\left(a+b\cot^{-1}(c+dx)\right)}{d^2} - \frac{3b^2f\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)}{d^2} + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2} - \frac{(-cf+de+f)(de-(c+1)f)\left(a+b\cot^{-1}(c+dx)\right)^3}{2d^2f} - \frac{3b(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2} + \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} - \frac{3b^3(de-cf)\operatorname{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)}{2d^2} + \frac{3ib^3f\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{2d^2}$$

```
[In] Int[(e + f*x)*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (((3*I)/2)*b*f*(a + b*ArcCot[c + d*x])^2)/d^2 + (3*b*f*(c + d*x)*(a + b*Arc
Cot[c + d*x])^2)/(2*d^2) + (I*(d*e - c*f)*(a + b*ArcCot[c + d*x])^3)/d^2 -
((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcCot[c + d*x])^3)/(2*d^2*f) + (
(e + f*x)^2*(a + b*ArcCot[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcCot[c + d
*x])*Log[2/(1 + I*(c + d*x))])/d^2 - (3*b*(d*e - c*f)*(a + b*ArcCot[c + d*x
])^2*Log[2/(1 + I*(c + d*x))])/d^2 + (((3*I)/2)*b^3*f*PolyLog[2, 1 - 2/(1 +
 I*(c + d*x))])/d^2 + ((3*I)*b^2*(d*e - c*f)*(a + b*ArcCot[c + d*x])*PolyLo
g[2, 1 - 2/(1 + I*(c + d*x))])/d^2 - (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1
 + I*(c + d*x))])/(2*d^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4975

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
```

```
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\cot^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}\\
&= \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f}\\
&\quad+\frac{(3b)\text{Subst}\left(\int \left(\frac{f^2\left(a+b\cot^{-1}(x)\right)^2}{d^2}+\frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)\left(a+b\cot^{-1}(x)\right)^2}{d^2(1+x^2)}\right)dx,x,c+dx\right)}{2f}\\
&= \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f}\\
&\quad+\frac{(3b)\text{Subst}\left(\int \frac{((de-f-cf)(de+f-cf)+2f(de-cf)x)\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}\,dx,x,c+dx\right)}{2d^2f}\\
&\quad+\frac{(3bf)\text{Subst}\left(\int \left(a+b\cot^{-1}(x)\right)^2\,dx,x,c+dx\right)}{2d^2}\\
&= \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2}+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f}\\
&\quad+\frac{(3b)\text{Subst}\left(\int \left(\frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}-\frac{2f(-de+cf)x\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}\right)dx,x,c+dx\right)}{2d^2f}\\
&\quad+\frac{(3b^2f)\,\text{Subst}\left(\int \frac{x\left(a+b\cot^{-1}(x)\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} \\
&+ \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} - \frac{(3b^2f)\,\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{i-x}\,dx, x, c+dx\right)}{d^2} \\
&+ \frac{(3b(de-cf))\text{Subst}\left(\int \frac{x\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}\,dx, x, c+dx\right)}{d^2} \\
&+ \frac{(3b(de+f-cf)(de-(1+c)f))\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}\,dx, x, c+dx\right)}{2d^2f} \\
&= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} \\
&+ \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2} \\
&- \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^3}{2d^2f} \\
&+ \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} - \frac{3b^2f(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&- \frac{(3b^3f)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d^2} \\
&- \frac{(3b(de-cf))\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)^2}{i-x}\,dx, x, c+dx\right)}{d^2} \\
&= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} \\
&+ \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2} \\
&- \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^3}{2d^2f} \\
&+ \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} - \frac{3b^2f(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&- \frac{3b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&+ \frac{(3ib^3f)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+i(c+dx)}\right)}{d^2} \\
&- \frac{(6b^2(de-cf))\,\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, c+dx\right)}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3ibf(a+b\cot^{-1}(c+dx))^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} \\
&+ \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2} \\
&- \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^3}{2d^2f} \\
&+ \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} - \frac{3b^2f(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&- \frac{3b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&+ \frac{3ib^3f\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d^2} \\
&+ \frac{3ib^2(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&+ \frac{(3ib^3(de-cf))\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3ibf(a+b\cot^{-1}(c+dx))^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} \\
&+ \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2} \\
&- \frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^3}{2d^2f} \\
&+ \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} - \frac{3b^2f(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&- \frac{3b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&+ \frac{3ib^3f\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d^2} \\
&+ \frac{3ib^2(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&- \frac{3b^3(de-cf)\,\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.42 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.87

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$

$$= \frac{a^2(2ade+3bf-2acf)(c+dx)+a^3f(c+dx)^2-3a^2b(c+dx)(cf-d(2e+fx))\cot^{-1}(c+dx)-3a^2bf\,\text{ar}}{}$$

```
[In] Integrate[(e + f*x)*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + a^3*f*(c + d*x)^2 - 3*a^2*b*(c
 + d*x)*(c*f - d*(2*e + f*x))*ArcCot[c + d*x] - 3*a^2*b*f*ArcTan[c + d*x] +
 6*a*b^2*f*((c + d*x)*ArcCot[c + d*x] + ((1 + (c + d*x)^2)*ArcCot[c + d*x]^
2)/2 - Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])]) + 3*a^2*b*(d*e - c*f)*L
og[1 + (c + d*x)^2] + 6*a*b^2*d*e*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c
+ d*x] - 2*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcC
ot[c + d*x])]) - 6*a*b^2*c*f*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c + d*x
] - 2*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcCot[c
+ d*x])]) + b^3*f*(3*(c + d*x)*ArcCot[c + d*x]^2 + (1 + (c + d*x)^2)*ArcCot
[c + d*x]^3 - 6*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] + (3*I)*
(ArcCot[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcCot[c + d*x])])) + 2*b^3*d*e*((
I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c + d*x]^3 - 3*ArcCot[c
+ d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*ArcCot[c + d*x]*PolyLo
g[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2*I)*ArcCot[c + d*x])
])/2) - 2*b^3*c*f*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c +
d*x]^3 - 3*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*Ar
cCot[c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2
*I)*ArcCot[c + d*x])])/2))/(2*d^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1050 vs. 2(316) = 632.

Time = 13.32 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.12

method	result	size
parts	Expression too large to display	1051
derivativedivides	Expression too large to display	17316
default	Expression too large to display	17316

```
[In] int((f*x+e)*(a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/2*f*x^2+e*x)+b^3/d*(1/2/d*arccot(d*x+c)^3*(d*x+c)^2*f-1/d*arccot(d*x
+c)^3*c*f*(d*x+c)+arccot(d*x+c)^3*e*(d*x+c)+3/2/d*(1/3*f*arccot(d*x+c)^3+2*
I*f*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-2/3*I*arccot(d*x+c)^3*c*f+2*I
*f*arccot(d*x+c)^2+arccot(d*x+c)^2*f*(d*x+c-I)-2*f*arccot(d*x+c)*ln(1-(d*x+
c+I)/(1+(d*x+c)^2)^(1/2))+2/3*I*arccot(d*x+c)^3*d*e-2*f*arccot(d*x+c)*ln(1+
(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+4*polylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*
c*f+4*I*d*e*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*ln(1-(
d*x+c+I)/(1+(d*x+c)^2)^(1/2))*c*f*arccot(d*x+c)^2+2*I*f*polylog(2,(d*x+c+I)
/(1+(d*x+c)^2)^(1/2))-2*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d*e*arccot(d*x+
c)^2-4*I*c*f*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-4*polyl
og(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d*e+4*I*d*e*arccot(d*x+c)*polylog(2,-(
d*x+c+I)/(1+(d*x+c)^2)^(1/2))-4*I*c*f*arccot(d*x+c)*polylog(2,-(d*x+c+I)/(1
+(d*x+c)^2)^(1/2))-4*polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d*e+4*polylog
(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*c*f+2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))
*c*f*arccot(d*x+c)^2-2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d*e*arccot(d*x+c
)^2))+3*a*b^2/d*(1/2/d*arccot(d*x+c)^2*(d*x+c)^2*f-1/d*arccot(d*x+c)^2*c*f*
(d*x+c)+arccot(d*x+c)^2*e*(d*x+c)+1/d*(-arccot(d*x+c)*ln(1+(d*x+c)^2)*c*f+a
rccot(d*x+c)*ln(1+(d*x+c)^2)*d*e-arccot(d*x+c)*arctan(d*x+c)*f+arccot(d*x+c
)*(d*x+c)*f+1/2*f*ln(1+(d*x+c)^2)-1/2*arctan(d*x+c)^2*f+1/2*(-2*c*f+2*d*e)*
(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+
I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/
2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))))+
3*a^2*b/d*(1/2/d*arccot(d*x+c)*(d*x+c)^2*f-1/d*arccot(d*x+c)*c*f*(d*x+c)+ar
ccot(d*x+c)*e*(d*x+c)+1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*ln(1+(d*x+c)^2)-f
*arctan(d*x+c)))
```

Fricas [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arccot(d*x + c)^3 + 3*(a*b^2*f
*x + a*b^2*e)*arccot(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arccot(d*x + c),
x)
```

Sympy [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (a+b\operatorname{acot}(c+dx))^3\,(e+fx)\,dx$$

```
[In] integrate((f*x+e)*(a+b*acot(d*x+c))**3,x)
[Out] Integral((a + b*acot(c + d*x))**3*(e + f*x), x)
```

Maxima [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\operatorname{arccot}(dx+c)+a)^3\,dx$$

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
[Out] 1/16*b^3*f*x^2*arctan2(1, d*x + c)^3 + 1/8*b^3*e*x*arctan2(1, d*x + c)^3 +
1/2*a^3*f*x^2 + 3/2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2
*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*f + a^3*e
*x + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a^2*b*e/d - 3
/64*(b^3*f*x^2*arctan2(1, d*x + c) + 2*b^3*e*x*arctan2(1, d*x + c))*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2 + integrate(1/64*(8*(7*b^3*arctan2(1, d*x + c)^
3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*f*x^3 + 4*(2*(7*b^3*arctan2(1, d*x
+ c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*e + (3*b^3*arctan2(1, d*x + c)
^2 + 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*
f)*x^2 + 3*(2*b^3*d^2*f*x^3*arctan2(1, d*x + c) + (2*b^3*d^2*e*arctan2(1, d
*x + c) + (4*b^3*c*arctan2(1, d*x + c) - b^3)*d*f)*x^2 + 2*(b^3*c^2*arctan2
(1, d*x + c) + b^3*arctan2(1, d*x + c))*e + 2*((2*b^3*c*arctan2(1, d*x + c)
 - b^3)*d*e + (b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c))*f)*x)
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 8*(7*b^3*arctan2(1, d*x + c)^3 + 24*a
*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan
2(1, d*x + c)^2)*c^2)*e + 8*((3*b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3*arctan
2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*e + (7*b^3*arctan2(1
, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arctan2(1, d*x + c)^
3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*f)*x + 12*(b^3*d^2*f*x^3*arctan2(1
, d*x + c) + 2*b^3*c*d*e*x*arctan2(1, d*x + c) + (2*b^3*d^2*e*arctan2(1, d*
x + c) + b^3*c*d*f*arctan2(1, d*x + c))*x^2)*log(d^2*x^2 + 2*c*d*x + c^2 +
1))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
```

Giac [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)*(b*arccot(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (e+f\,x)\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^3\,dx$$

```
[In] int((e + f*x)*(a + b*acot(c + d*x))^3,x)
[Out] int((e + f*x)*(a + b*acot(c + d*x))^3, x)
```

3.143 $\int \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 143

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d} - \frac{3b\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} + \frac{3ib^2\left(a+b\cot^{-1}(c+dx)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} - \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}$$

```
[Out] I*(a+b*arccot(d*x+c))^3/d+(d*x+c)*(a+b*arccot(d*x+c))^3/d-3*b*(a+b*arccot(d
*x+c))^2*ln(2/(1+I*(d*x+c)))/d+3*I*b^2*(a+b*arccot(d*x+c))*polylog(2,1-2/(1
+I*(d*x+c)))/d-3/2*b^3*polylog(3,1-2/(1+I*(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used

$= \{5148, 4931, 5041, 4965, 5005, 5115, 6745\}$

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx = \frac{3ib^2\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)\left(a+b\cot^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d} + \frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{d} - \frac{3b\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{d} - \frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)}{2d}$$

```
[In] Int[(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (I*(a + b*ArcCot[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcCot[c + d*x])^3)/d -
 (3*b*(a + b*ArcCot[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d + ((3*I)*b^2*(a
 + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d - (3*b^3*PolyL
og[3, 1 - 2/(1 + I*(c + d*x))])/(2*d)
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+b\cot^{-1}(x)\right)^3\, dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d} + \frac{(3b)\text{Subst}\left(\int \frac{x\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}\, dx, x, c+dx\right)}{d}$$

$$= \frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d} - \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)^2}{i-x}\, dx, x, c+dx\right)}{d}$$

$$= \frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d} - \frac{3b\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} - \frac{(6b^2)\,\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)\log\left(\frac{2}{1+ix}\right)}{1+x^2}\, dx, x, c+dx\right)}{d}$$

$$= \frac{i(a+b\cot^{-1}(c+dx))^3}{d} + \frac{(c+dx)(a+b\cot^{-1}(c+dx))^3}{d} - \frac{3b(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} + \frac{3ib^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} + \frac{(3ib^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{i(a+b\cot^{-1}(c+dx))^3}{d} + \frac{(c+dx)(a+b\cot^{-1}(c+dx))^3}{d} - \frac{3b(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} + \frac{3ib^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} - \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.59

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$

$$= \frac{2a^3(c+dx)+6a^2b(c+dx)\cot^{-1}(c+dx)+3a^2b\log\left(1+(c+dx)^2\right)+6ab^2\left(\cot^{-1}(c+dx)\left((i+c+dx)\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcCot[c + d*x] + 3*a^2*b*Log[1 + (c +
 d*x)^2] + 6*a*b^2*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c + d*x] - 2*Log[
1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcCot[c + d*x])])
+ 2*b^3*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c + d*x]^3 - 3
*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*ArcCot[c + d
*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2*I)*ArcCot
[c + d*x])])/2))/(2*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(136) = 272.

Time = 2.46 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.76

method	result
derivativedivides	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccot}(dx+c)^3(dx+c-i)+2i\operatorname{arccot}(dx+c)^3-3\operatorname{arccot}(dx+c)^2\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+6i\operatorname{arccot}(dx+c)\operatorname{poly}\right.}{}$
default	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccot}(dx+c)^3(dx+c-i)+2i\operatorname{arccot}(dx+c)^3-3\operatorname{arccot}(dx+c)^2\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+6i\operatorname{arccot}(dx+c)\operatorname{poly}\right.}{}$
parts	$a^3x+\frac{b^3\left(\operatorname{arccot}(dx+c)^3(dx+c-i)+2i\operatorname{arccot}(dx+c)^3-3\operatorname{arccot}(dx+c)^2\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+6i\operatorname{arccot}(dx+c)\operatorname{polylog}\right.}{}$

```
[In] int((a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^3+b^3*(arccot(d*x+c)^3*(d*x+c-I)+2*I*arccot(d*x+c)^3-3*arcco
t(d*x+c)^2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+6*I*arccot(d*x+c)*polylog(2,
-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-6*polylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))
-3*arccot(d*x+c)^2*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+6*I*arccot(d*x+c)*po
lylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-6*polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^
(1/2)))+3*a*b^2*(arccot(d*x+c)^2*(d*x+c-I)-2*arccot(d*x+c)*ln(1+(d*x+c+I)/(
1+(d*x+c)^2)^(1/2))-2*arccot(d*x+c)*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I
*arccot(d*x+c)^2+2*I*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I*polylog(
2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2)))+3*a^2*b*(arccot(d*x+c)*(d*x+c)+1/2*ln(1+(
d*x+c)^2)))
```

Fricas [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int \left(b\operatorname{arccot}\left(dx+c\right)+a\right)^3\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arccot
(d*x + c) + a^3, x)
```

Sympy [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int \left(a+b\,\mathrm{acot}\,(c+dx)\right)^3\,dx$$

```
[In] integrate((a+b*acot(d*x+c))**3,x)
[Out] Integral((a + b*acot(c + d*x))**3, x)
```

Maxima [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^3,x, algorithm="maxima")
[Out] 1/8*b^3*x*arctan2(1, d*x + c)^3 - 3/32*b^3*x*arctan2(1, d*x + c)*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)^2 + a^3*x + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((
d*x + c)^2 + 1))*a^2*b/d + integrate(1/32*(28*b^3*arctan2(1, d*x + c)^3 + 4
*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*x^2 + 9
6*a*b^2*arctan2(1, d*x + c)^2 + 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*a
rctan2(1, d*x + c)^2)*c^2 + 4*(3*b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3*arcta
n2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*x + 3*(b^3*d^2*x^2*
arctan2(1, d*x + c) + b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c)
 + (2*b^3*c*arctan2(1, d*x + c) - b^3)*d*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)^2 + 12*(b^3*d^2*x^2*arctan2(1, d*x + c) + b^3*c*d*x*arctan2(1, d*x + c))*
log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
```

Giac [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*arccot(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (a+b\,\mathrm{acot}(c+d\,x))^3\,dx$$

```
[In] int((a + b*acot(c + d*x))^3,x)
[Out] int((a + b*acot(c + d*x))^3, x)
```

3.144 $\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 372

$$\begin{aligned}\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx =& -\frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\ &+\frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\ &-\frac{3ib(a+b\cot^{-1}(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\ &+\frac{3ib(a+b\cot^{-1}(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\ &-\frac{3b^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\ &+\frac{3b^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\ &+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f}-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{4f}\end{aligned}$$

```
[Out] -(a+b*arccot(d*x+c))^3*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))^3*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*I*b*(a+b*arccot(d*x+c))^2*polylog
(2,1-2/(1-I*(d*x+c)))/f+3/2*I*b*(a+b*arccot(d*x+c))^2*polylog(2,1-2*d*(f*x+
e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*b^2*(a+b*arccot(d*x+c))*polylog(3,1-2
```

```
/(1-I*(d*x+c)))/f+3/2*b^2*(a+b*arccot(d*x+c))*polylog(3,1-2*d*(f*x+e)/(d*e+
I*f-c*f)/(1-I*(d*x+c)))/f+3/4*I*b^3*polylog(4,1-2/(1-I*(d*x+c)))/f-3/4*I*b^
3*polylog(4,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5156, 4971}

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$$
$$= \frac{3b^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f} - \frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{2f} + \frac{3ib(a+b\cot^{-1}(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f} + \frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f} - \frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^2}{2f} - \frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^3}{f} - \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{4f} + \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f}$$

```
[In] Int[(a + b*ArcCot[c + d*x])^3/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCot[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot[
c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f -
 (((3*I)/2)*b*(a + b*ArcCot[c + d*x])^2*PolyLog[2, 1 - 2/(1 - I*(c + d*x))]
)/f + (((3*I)/2)*b*(a + b*ArcCot[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))
/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (3*b^2*(a + b*ArcCot[c + d*x])
*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) + (3*b^2*(a + b*ArcCot[c + d*x]
)*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2
*f) + (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*(c + d*x))])/f - (((3*I)/4)*b^
3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 4971

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^3)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^3*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[3
*I*b*(a + b*ArcCot[c*x])^2*(PolyLog[2, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp
[3*I*b*(a + b*ArcCot[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1
 - I*c*x)))]/(2*e)), x] - Simp[3*b^2*(a + b*ArcCot[c*x])*(PolyLog[3, 1 - 2/
(1 - I*c*x)]/(2*e)), x] + Simp[3*b^2*(a + b*ArcCot[c*x])*(PolyLog[3, 1 - 2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x] + Simp[3*I*b^3*(PolyLog
[4, 1 - 2/(1 - I*c*x)]/(4*e)), x] - Simp[3*I*b^3*(PolyLog[4, 1 - 2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^3}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\
&\quad+\frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\
&\quad-\frac{3ib(a+b\cot^{-1}(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\
&\quad+\frac{3ib(a+b\cot^{-1}(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\
&\quad-\frac{3b^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\
&\quad+\frac{3b^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\
&\quad+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f}-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{4f}
\end{aligned}$$

Mathematica [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$$

```
[In] Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x),x]
[Out] Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.36 (sec) , antiderivative size = 3903, normalized size of antiderivative = 10.49

method	result	size
derivativedivides	Expression too large to display	3903
default	Expression too large to display	3903
parts	Expression too large to display	4139

```
[In] int((a+b*arccot(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/d*(a^3*d*ln(c*f-d*e-f*(d*x+c))/f-b^3*d*(-ln(c*f-d*e-f*(d*x+c))/f*arccot(d
*x+c)^3-3/f*(-1/3*arccot(d*x+c)^3*ln(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*
x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)-2*I*polyl
og(4,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+1/3*arccot(d*x+c)^3*ln((d*x+c+I)^2/(1+(
d*x+c)^2)-1)-1/3*arccot(d*x+c)^3*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*arcc
ot(d*x+c)^2*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-2*arccot(d*x+c)*polylo
g(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*d*e*arccot(d*x+c)^2*polylog(2,(d*e+I*f
-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))/(-2*I*f+2*c*f-2*d*e)-1/3*ar
ccot(d*x+c)^3*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*arccot(d*x+c)^2*polylog
(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-2*arccot(d*x+c)*polylog(3,-(d*x+c+I)/(1+
(d*x+c)^2)^(1/2))+1/6*I*Pi*csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+
c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^
2/(1+(d*x+c)^2)-1))*(csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2
/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e))*csgn(I/((d*x+c+I
)^2/(1+(d*x+c)^2)-1))-csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^
2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+
(d*x+c)^2)-1))*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))-csgn(I*(-I*f*(d*x+c+I)
^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2
)-I*f-c*f+d*e))*csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(
d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c
)^2)-1))+csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^
2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1)
```

```
)^2)*arccot(d*x+c)^3+1/3*c*f/(-I*f+c*f-d*e)*arccot(d*x+c)^3*ln(1-(d*e+I*f-c
*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+1/2*c*f/(-I*f+c*f-d*e)*arccot
(d*x+c)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-1
/3*I*f/(-I*f+c*f-d*e)*arccot(d*x+c)^3*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*
x+c+I)^2/(1+(d*x+c)^2))-2*I*polylog(4,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/2*I
*f/(-I*f+c*f-d*e)*arccot(d*x+c)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x
+c+I)^2/(1+(d*x+c)^2))+1/4*I*c*f/(-I*f+c*f-d*e)*polylog(4,(d*e+I*f-c*f)/(-c
*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-1/2*f/(-I*f+c*f-d*e)*arccot(d*x+c)^2
*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+1/4*f/(-
I*f+c*f-d*e)*polylog(4,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^
2))-1/3*d*e/(-I*f+c*f-d*e)*arccot(d*x+c)^3*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f
)*(d*x+c+I)^2/(1+(d*x+c)^2))-1/2*d*e/(-I*f+c*f-d*e)*arccot(d*x+c)*polylog(3
,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-I*d*e*polylog(4,(d
*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))/(-4*I*f+4*c*f-4*d*e)-
1/2*I*c*f/(-I*f+c*f-d*e)*arccot(d*x+c)^2*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-
I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))))-3*a*b^2*d*(-ln(c*f-d*e-f*(d*x+c))/f*arcco
t(d*x+c)^2-2/f*(-1/2*arccot(d*x+c)^2*ln(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*
(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)-1/4*I*
f/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x
+c)^2))+1/2*arccot(d*x+c)^2*ln((d*x+c+I)^2/(1+(d*x+c)^2)-1)-1/2*arccot(d*x+
c)^2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*d*e*arccot(d*x+c)*polylog(2,(d*e
+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))/(-2*I*f+2*c*f-2*d*e)-po
lylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/2*arccot(d*x+c)^2*ln(1-(d*x+c+I)/
(1+(d*x+c)^2)^(1/2))+I*arccot(d*x+c)*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/
2))-polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/2*I*f/(-I*f+c*f-d*e)*arccot(
d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+1/2*c
*f/(-I*f+c*f-d*e)*arccot(d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+
I)^2/(1+(d*x+c)^2))+1/4*c*f/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*
e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-1/2*f/(-I*f+c*f-d*e)*arccot(d*x+c)*polylo
g(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-1/2*I*c*f/(-I*f
+c*f-d*e)*arccot(d*x+c)*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/
(1+(d*x+c)^2))+I*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+1/4
*I*Pi*csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-
d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*(
csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d
*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e))*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))
-csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(
d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*csgn(I
/((d*x+c+I)^2/(1+(d*x+c)^2)-1))-csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*
(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e))*csgn(
I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+
I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))+csgn(I*(-I*f
*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1
+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))^2)*arccot(d*x+c)^2-
1/2*d*e/(-I*f+c*f-d*e)*arccot(d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d
```

```
*x+c+I)^2/(1+(d*x+c)^2))-1/4*d*e/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c
*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))))-3*a^2*b*d*(-ln(c*f-d*e-f*(d*x+c))/
f*arccot(d*x+c)-1/2*I*ln(c*f-d*e-f*(d*x+c))*(ln((I*f+f*(d*x+c))/(c*f-d*e+I*
f))-ln((I*f-f*(d*x+c))/(d*e+I*f-c*f)))/f-1/2*I*(dilog((I*f+f*(d*x+c))/(c*f-
d*e+I*f))-dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f)))/f))
```

Fricas [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^3}{fx+e}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="fricas")
[Out] integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcco
t(d*x + c) + a^3)/(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acot(d*x+c))**3/(f*x+e),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^3}{fx+e}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="maxima")
[Out] a^3*log(f*x + e)/f + integrate(1/32*(28*b^3*arctan2(1, d*x + c)^3 + 3*b^3*a
rctan2(1, d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan2(1,
 d*x + c)^2 + 96*a^2*b*arctan2(1, d*x + c))/(f*x + e), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(a+b\,\mathrm{acot}(c+d\,x))^3}{e+f\,x}\,dx$$

```
[In] int((a + b*acot(c + d*x))^3/(e + f*x),x)
[Out] int((a + b*acot(c + d*x))^3/(e + f*x), x)
```

3.145 $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 1233

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f(d^2e^2-2cdef+(1+c^2)f^2)}$$
$$+\frac{ib^3d\cot^{-1}(c+dx)^3}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+\frac{b^3d(de-cf)\cot^{-1}(c+dx)^3}{f(d^2e^2-2cdef+(1+c^2)f^2)}-\frac{(a+b\cot^{-1}(c+dx))^3}{f(e+fx)}$$
$$-\frac{3a^2bd(de-cf)\arctan(c+dx)}{f(f^2+(de-cf)^2)}-\frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2}$$
$$+\frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+\frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$-\frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$-\frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$-\frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$-\frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+\frac{3a^2bd\log(1+(c+dx)^2)}{2(f^2+(de-cf)^2)}$$
$$+\frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+\frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$-\frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$-\frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+\frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+\frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+\frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{}$$

```
[Out] 3*I*a*b^2*d*arccot(d*x+c)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*a*b^2*d*(-c*f
+d*e)*arccot(d*x+c)^2/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*I*a*b^2*d*polylog
(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^
2)+b^3*d*(-c*f+d*e)*arccot(d*x+c)^3/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*
arccot(d*x+c))^3/f/(f*x+e)-3*a^2*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+
d*e)^2)-3*a^2*b*d*ln(f*x+e)/(f^2+(-c*f+d*e)^2)+6*a*b^2*d*arccot(d*x+c)*ln(2
/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*b^3*d*arccot(d*x+c)^2*ln(
2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-6*a*b^2*d*arccot(d*x+c)*ln
(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3
*b^3*d*arccot(d*x+c)^2*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2
-2*c*d*e*f+(c^2+1)*f^2)-6*a*b^2*d*arccot(d*x+c)*ln(2/(1+I*(d*x+c)))/(d^2*e^
2-2*c*d*e*f+(c^2+1)*f^2)-3*b^3*d*arccot(d*x+c)^2*ln(2/(1+I*(d*x+c)))/(d^2*e
^2-2*c*d*e*f+(c^2+1)*f^2)+3/2*a^2*b*d*ln(1+(d*x+c)^2)/(f^2+(-c*f+d*e)^2)+3*
I*b^3*d*arccot(d*x+c)*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+
1)*f^2)-3*I*b^3*d*arccot(d*x+c)*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*
(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*polylog(2,1-2/(1-I*(d
*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*b^3*d*arccot(d*x+c)*polylog(2,1
-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+I*b^3*d*arccot(d*x+c)^3/(
d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3/2*b^3*d*polylog(3,1-2/(1-I*(d*x+c)))/(d^2*e^
2-2*c*d*e*f+(c^2+1)*f^2)-3/2*b^3*d*polylog(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1
-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*b^3*d*polylog(3,1-2/(1+I*(
d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 1233, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {5154, 6873, 5166, 6820, 12, 6857, 720, 31, 649, 209, 266, 4967, 2449, 2352, 2497, 5105, 5005, 5041, 4965, 4969, 5115, 6745}

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \frac{id\cot^{-1}(c+dx)^3b^3}{d^2e^2-2cdfe+(c^2+1)f^2} + \frac{d(de-cf)\cot^{-1}(c+dx)^3b^3}{f\left(d^2e^2-2cdfe+(c^2+1)f^2\right)}$$
$$+\frac{3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)b^3}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$-\frac{3d\cot^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^3}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$-\frac{3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{i(c+dx)+1}\right)b^3}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$+\frac{3id\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)b^3}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$-\frac{3id\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^3}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$+\frac{3id\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)b^3}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$+\frac{3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)b^3}{2\left(d^2e^2-2cdfe+(c^2+1)f^2\right)}$$
$$-\frac{3d\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^3}{2\left(d^2e^2-2cdfe+(c^2+1)f^2\right)}$$
$$-\frac{3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)b^3}{2\left(d^2e^2-2cdfe+(c^2+1)f^2\right)}$$
$$+\frac{3iad\cot^{-1}(c+dx)^2b^2}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$+\frac{3ad(de-cf)\cot^{-1}(c+dx)^2b^2}{f\left(d^2e^2-2cdfe+(c^2+1)f^2\right)}$$
$$+\frac{6ad\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)b^2}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$-\frac{6ad\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^2}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$-\frac{6ad\cot^{-1}(c+dx)\log\left(\frac{2}{i(c+dx)+1}\right)b^2}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$+\frac{3iad\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)b^2}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$-\frac{3iad\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+(i-c)f)(1-i(c+dx))}\right)b^2}{d^2e^2-2cdfe+(c^2+1)f^2}$$
$$+\frac{3iad\,\mathrm{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)b^2}{d^2e^2-2cdfe+(c^2+1)f^2}$$

```
[In] Int[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] ((3*I)*a*b^2*d*ArcCot[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (
3*a*b^2*d*(d*e - c*f)*ArcCot[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2)) + (I*b^3*d*ArcCot[c + d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
 + (b^3*d*(d*e - c*f)*ArcCot[c + d*x]^3)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2)) - (a + b*ArcCot[c + d*x])^3/(f*(e + f*x)) - (3*a^2*b*d*(d*e - c*f)*
ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) - (3*a^2*b*d*Log[e + f*x])/(f^2
+ (d*e - c*f)^2) + (6*a*b^2*d*ArcCot[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcCot[c + d*x]^2*Log[2/(1 -
I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (6*a*b^2*d*ArcCot[c
+ d*x]*Log[(2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcCot[c + d*x]^2*Log[(2*d*(e + f*
x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2) - (6*a*b^2*d*ArcCot[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcCot[c + d*x]^2*Log[2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*a^2*b*d*Log[1 + (c + d*x)^2
])/(2*(f^2 + (d*e - c*f)^2)) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 - I*(c +
d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcCot[c + d*x
]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2
) - ((3*I)*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I
*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*b^3*d*ArcCot[
c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + (I - c)*f)*(1 - I*(c + d*x)
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*a*b^2*d*PolyLog[2, 1 -
2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*
ArcCot[c + d*x]*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
 (1 + c^2)*f^2) + (3*b^3*d*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2)) - (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((
d*e + (I - c)*f)*(1 - I*(c + d*x)))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)) - (3*b^3*d*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
```

```
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4969

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[I
*b*(a + b*ArcCot[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] + Simp[I*b*(a
+ b*ArcCot[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5166

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\text{integral} = -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3bd)\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)(1+(c+dx)^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3bd)\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)^2}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3b)\text{Subst}\left(\int \frac{d\left(a+b\cot^{-1}(x)\right)^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3bd)\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3bd)\text{Subst}\left(\int \left(\frac{a^2}{(de-cf+fx)(1+x^2)} + \frac{2ab\cot^{-1}(x)}{(de-cf+fx)(1+x^2)} + \frac{b^2\cot^{-1}(x)^2}{(de-cf+fx)(1+x^2)}\right)\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3a^2bd)\,\text{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} - \frac{(6ab^2d)\,\text{Subst}\left(\int \frac{\cot^{-1}(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} - \frac{(3b^3d)\,\text{Subst}\left(\int \frac{\cot^{-1}(x)^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(6ab^2d)\,\text{Subst}\left(\int \left(\frac{f^2\cot^{-1}(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)} + \frac{(de-cf-fx)\cot^{-1}(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(1+x^2)}\right)\,dx, x, c+dx\right)}{f} - \frac{(3b^3d)\,\text{Subst}\left(\int \left(\frac{f^2\cot^{-1}(x)^2}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)} + \frac{(de-cf-fx)\cot^{-1}(x)^2}{(d^2e^2-2cdef+(1+c^2)f^2)(1+x^2)}\right)\,dx, x, c+dx\right)}{f} - \frac{(3a^2bd)\,\text{Subst}\left(\int \frac{de-cf-fx}{1+x^2}\,dx, x, c+dx\right)}{f\left(f^2+(de-cf)^2\right)} - \frac{(3a^2bdf)\,\text{Subst}\left(\int \frac{1}{de-cf+fx}\,dx, x, c+dx\right)}{f^2+(de-cf)^2}$$

$$
\begin{aligned}
&= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&- \frac{(6ab^2d)\,\mathrm{Subst}\left(\int \frac{(de-cf-fx)\cot^{-1}(x)}{1+x^2}\,dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} \\
&- \frac{(3b^3d)\,\mathrm{Subst}\left(\int \frac{(de-cf-fx)\cot^{-1}(x)^2}{1+x^2}\,dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} \\
&- \frac{(6ab^2df)\,\mathrm{Subst}\left(\int \frac{\cot^{-1}(x)}{de-cf+fx}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&- \frac{(3b^3df)\,\mathrm{Subst}\left(\int \frac{\cot^{-1}(x)^2}{de-cf+fx}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{(3a^2bd)\,\mathrm{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, c+dx\right)}{f^2+(de-cf)^2} \\
&- \frac{(3a^2bd(de-cf))\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{f\left(f^2+(de-cf)^2\right)}
\end{aligned}
$$

$$= -\frac{(a+b\cot^{-1}(c+dx))^3}{f(e+fx)} - \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2}$$
$$+ \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)}$$
$$+ \frac{3ib^3d\cot^{-1}(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{3ib^3d\cot^{-1}(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+ \frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)}$$
$$+ \frac{(6ab^2d)\operatorname{Subst}\left(\int\frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{(6ab^2d)\operatorname{Subst}\left(\int\frac{\log\left(\frac{2(de-cf+fx)}{(de+if-cf)(1-ix)}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{(6ab^2d)\operatorname{Subst}\left(\int\left(\frac{de\left(1-\frac{cf}{de}\right)\cot^{-1}(x)}{1+x^2}-\frac{fx\cot^{-1}(x)}{1+x^2}\right)dx,x,c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}$$
$$- \frac{(3b^3d)\operatorname{Subst}\left(\int\left(\frac{de\left(1-\frac{cf}{de}\right)\cot^{-1}(x)^2}{1+x^2}-\frac{fx\cot^{-1}(x)^2}{1+x^2}\right)dx,x,c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}$$

$$= -\frac{(a+b\cot^{-1}(c+dx))^3}{f(e+fx)} - \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2}$$

$$+ \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)}$$

$$+ \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+ \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)}$$

$$+ \frac{(6iab^2d)\,\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+ \frac{(6ab^2d)\,\mathrm{Subst}\left(\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+ \frac{(3b^3d)\,\mathrm{Subst}\left(\int \frac{x\cot^{-1}(x)^2}{1+x^2}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{(6ab^2d(de-cf))\,\mathrm{Subst}\left(\int \frac{\cot^{-1}(x)}{1+x^2}\,dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}$$

$$- \frac{(3b^3d(de-cf))\,\mathrm{Subst}\left(\int \frac{\cot^{-1}(x)^2}{1+x^2}\,dx, x, c+dx\right)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}$$

$$
\begin{aligned}
&= \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f(d^2e^2-2cdef+(1+c^2)f^2)} \\
&\quad + \frac{ib^3d\cot^{-1}(c+dx)^3}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^3d(de-cf)\cot^{-1}(c+dx)^3}{f(d^2e^2-2cdef+(1+c^2)f^2)} \\
&\quad - \frac{(a+b\cot^{-1}(c+dx))^3}{f(e+fx)} - \frac{3a^2bd(de-cf)\arctan(c+dx)}{f(f^2+(de-cf)^2)} - \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&\quad + \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&\quad - \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&\quad - \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&\quad + \frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2(f^2+(de-cf)^2)} + \frac{3iab^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&\quad + \frac{3ib^3d\cot^{-1}(c+dx)\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&\quad - \frac{3iab^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&\quad - \frac{3ib^3d\cot^{-1}(c+dx)\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&\quad + \frac{3b^3d\,\text{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)} - \frac{3b^3d\,\text{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)} \\
&\quad - \frac{(6ab^2d)\,\text{Subst}\left(\int \frac{\cot^{-1}(x)}{i-x}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{(3b^3d)\,\text{Subst}\left(\int \frac{\cot^{-1}(x)^2}{i-x}\,dx, x, c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}
\end{aligned}
$$

$$= \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f(d^2e^2-2cdef+(1+c^2)f^2)}$$

$$+ \frac{ib^3d\cot^{-1}(c+dx)^3}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^3d(de-cf)\cot^{-1}(c+dx)^3}{f(d^2e^2-2cdef+(1+c^2)f^2)}$$

$$- \frac{(a+b\cot^{-1}(c+dx))^3}{f(e+fx)} - \frac{3a^2bd(de-cf)\arctan(c+dx)}{f(f^2+(de-cf)^2)} - \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2}$$

$$+ \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+ \frac{3a^2bd\log(1+(c+dx)^2)}{2(f^2+(de-cf)^2)} + \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+ \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+ \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)} - \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)}$$

$$- \frac{(6ab^2d)\,\mathrm{Subst}\left(\int\frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$- \frac{(6b^3d)\,\mathrm{Subst}\left(\int\frac{\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)}{1+x^2}dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$
\begin{aligned}
&= \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f(d^2e^2-2cdef+(1+c^2)f^2)} \\
&+ \frac{ib^3d\cot^{-1}(c+dx)^3}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^3d(de-cf)\cot^{-1}(c+dx)^3}{f(d^2e^2-2cdef+(1+c^2)f^2)} \\
&- \frac{(a+b\cot^{-1}(c+dx))^3}{f(e+fx)} - \frac{3a^2bd(de-cf)\arctan(c+dx)}{f(f^2+(de-cf)^2)} - \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
&+ \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&- \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&- \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&- \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&+ \frac{3a^2bd\log(1+(c+dx)^2)}{2(f^2+(de-cf)^2)} + \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&+ \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&- \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&- \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&+ \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&+ \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)} - \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)} \\
&+ \frac{(6iab^2d)\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} \\
&+ \frac{(3ib^3d)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,1-\frac{2}{1+ix}\right)}{1+x^2}\,dx,x,c+dx\right)}{d^2e^2-2cdef+(1+c^2)f^2}
\end{aligned}
$$

$$= \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f(d^2e^2-2cdef+(1+c^2)f^2)}$$
$$+ \frac{ib^3d\cot^{-1}(c+dx)^3}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^3d(de-cf)\cot^{-1}(c+dx)^3}{f(d^2e^2-2cdef+(1+c^2)f^2)}$$
$$- \frac{(a+b\cot^{-1}(c+dx))^3}{f(e+fx)} - \frac{3a^2bd(de-cf)\arctan(c+dx)}{f(f^2+(de-cf)^2)} - \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2}$$
$$+ \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+ \frac{3a^2bd\log(1+(c+dx)^2)}{2(f^2+(de-cf)^2)} + \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+ \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$- \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+ \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$
$$+ \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)}$$
$$- \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)} - \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2(d^2e^2-2cdef+(1+c^2)f^2)}$$

Mathematica [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx=\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx$$

```
[In] Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 28.03 (sec) , antiderivative size = 4229, normalized size of antiderivative = 3.43

method	result	size
parts	Expression too large to display	4229
derivativedivides	Expression too large to display	4722
default	Expression too large to display	4722

```
[In] int((a+b*arccot(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3/(f*x+e)/f+b^3/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)^3-3*d^2/f*(a
rccot(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)-1/2*
arccot(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d*x+c)^2)-arccot(d*
x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c*f+arccot(d*x+c)^2/(c
^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*d*e+1/(c^2*f^2-2*c*d*e*f+d^2*e^
2+f^2)*f*arctan(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-1/3*I/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*f*arctan(d*x+c)^3+1/4/(c^2*f^2-2*c*d*e*f+d^2*e^2+
f^2)*(-2*Pi*c*f+2*Pi*d*e+4*f*ln(2)+I*f*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)
^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))
^2)^2-2*I*f*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*(1+(1+I*(d*
x+c))^2/(1+(d*x+c)^2))^2)^2+4*I*f*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)
^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f
+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+2*I*f*Pi*csgn(I*(I*f*(1+I*(d
*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^
2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*
e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^
2)))-I*f*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x
+c)^2))^2)^3+2*I*f*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))*csgn(I*(1+I
*(d*x+c))^2/(1+(d*x+c)^2))^2+I*f*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)
))^2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)-I*f*Pi*csgn(I/(1+(1+I*(d*x
+c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*
```

```
(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)-2*I*f*Pi*csgn
(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2
)))^3-I*f*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3-2*I*f*Pi*csgn(I*(I*f*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*
x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^
2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+
c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2-4*I*f*Pi+I*f*Pi*csgn(I*(1+(1+
I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3-I*f*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^
(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))+I*f*Pi*csgn(I/(1+(1+I*(d*x+c
))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c)
)^2/(1+(d*x+c)^2))^2)^2-2*I*f*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+
c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f
-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*
x+c)^2))))*arctan(d*x+c)^2-1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*arctan(d*x+c
)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-
d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)+1/(c^2*f^2-2*c*d*e*f+d^2*e^2
+f^2)*f^2/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))
^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+I/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2/(c*f-
d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(
d*e+I*f-c*f))+I/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*d*e/(c*f-d*e+I*f)*arctan(
d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))
+1/2*I/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2/(c*f-d*e+I*f)*polylog(3,(c*f-d*e
+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/(c^2*f^2-2*c*d*e*f+d^2
*e^2+f^2)*f^2*c/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+
c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2*
c/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+
I*f-c*f))-I/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2*c/(c*f-d*e+I*f)*arctan(d*x+
c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/(
c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*d*e/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*
f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2/(c^2*f^2-2*c*d*
e*f+d^2*e^2+f^2)*f*d*e/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^
2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+2/3*(c*f-d*e)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2
)*arctan(d*x+c)^3-1/2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*Pi*(-1/2*I*(ln(d*x+
c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*
ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-
dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))+1/(c^2*f^2-2*c*d*e
*f+d^2*e^2+f^2)*f^2*Pi*(-1/2*I*ln(f*(d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d
*e+I*f-c*f))-ln((I*f+f*(d*x+c))/(c*f-d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+
c))/(d*e+I*f-c*f))-dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f)))/f)))+3*a^2*b/d*(-d
^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)-d^2/f*(1/(c^2*f^2-2*c*d*e*f+d^2*e^2+
f^2)*f*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*f*ln(1
+(d*x+c)^2)+(-c*f+d*e)*arctan(d*x+c))))+3*a*b^2/d*(-d^2/(f*(d*x+c)-c*f+d*e)
/f*arccot(d*x+c)^2-2*d^2/f*(arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f
*ln(f*(d*x+c)-c*f+d*e)-1/2*arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*
```

```
ln(1+(d*x+c)^2)-arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)
*c*f+arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*d*e+f^2/(c
^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*I*ln(f*(d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x
+c))/(d*e+I*f-c*f))-ln((I*f+f*(d*x+c))/(c*f-d*e+I*f)))/f-1/2*I*(dilog((I*f-
f*(d*x+c))/(d*e+I*f-c*f))-dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f)))/f)-1/2*f/(c
^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d
*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(
ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x
+c+I)*ln(1/2*I*(d*x+c-I))))-1/2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(c*f-d*e)*a
rctan(d*x+c)^2))
```

Fricas [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{\left(b\operatorname{arccot}\left(dx+c\right)+a\right)^3}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
[Out] integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcco
t(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acot(d*x+c))**3/(f*x+e)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{\left(b\operatorname{arccot}\left(dx+c\right)+a\right)^3}{\left(fx+e\right)^2}\,dx$$

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
[Out] -3/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^
2 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e
*f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
 + 2*arccot(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) - 1/32*(4*b^3
```

```
*arctan2(1, d*x + c)^3 - 3*b^3*arctan2(1, d*x + c)*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)^2 - 32*(f^2*x + e*f)*integrate(-1/32*(12*b^3*d*e*arctan2(1, d*x +
c)^2 - 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2
*f*x^2 + 4*(3*b^3*arctan2(1, d*x + c)^2 - 2*(7*b^3*arctan2(1, d*x + c)^3 +
24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*f*x - 3*(b^3*d^2*f*x^2*arctan2(1, d*x
+ c) + b^3*d*e + (2*b^3*c*arctan2(1, d*x + c) + b^3)*d*f*x + (b^3*c^2*arcta
n2(1, d*x + c) + b^3*arctan2(1, d*x + c))*f)*log(d^2*x^2 + 2*c*d*x + c^2 +
1)^2 - 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7
*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*f + 12*(b
^3*d^2*f*x^2*arctan2(1, d*x + c) + b^3*c*d*e*arctan2(1, d*x + c) + (b^3*d^2
*e*arctan2(1, d*x + c) + b^3*c*d*f*arctan2(1, d*x + c))*x)*log(d^2*x^2 + 2*
c*d*x + c^2 + 1))/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*
x^3 + (d^2*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^2 +
 1)*e*f^2)*x), x))/(f^2*x + e*f)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{(a+b\,\mathrm{acot}(c+d\,x))^3}{(e+f\,x)^2}\,dx$$

```
[In] int((a + b*acot(c + d*x))^3/(e + f*x)^2,x)
```

```
[Out] int((a + b*acot(c + d*x))^3/(e + f*x)^2, x)
```

3.146 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 177

$$\begin{aligned}
&\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) \, dx \\
&= \frac{(e+fx)^{1+m} \left(a+b\cot^{-1}(c+dx)\right)}{f(1+m)} \\
&\quad + \frac{ibd(e+fx)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de+if-cf}\right)}{2f(de+(i-c)f)(1+m)(2+m)} \\
&\quad - \frac{ibd(e+fx)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(i+c)f}\right)}{2f(de-(i+c)f)(1+m)(2+m)}
\end{aligned}$$

```
[Out] (f*x+e)^(1+m)*(a+b*arccot(d*x+c))/f/(1+m)+1/2*I*b*d*(f*x+e)^(2+m)*hypergeom
([1, 2+m],[3+m],d*(f*x+e)/(d*e+I*f-c*f))/f/(d*e+(I-c)*f)/(1+m)/(2+m)-1/2*I*
b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(d*e-(I+c)*f))/f/(d*e-
(I+c)*f)/(1+m)/(2+m)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used

$= \{5156, 4973, 726, 70\}$

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= \frac{(e+fx)^{m+1}\left(a+b\cot^{-1}(c+dx)\right)}{f(m+1)} + \frac{ibd(e+fx)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{d(e+fx)}{de-cf+if}\right)}{2f(m+1)(m+2)(de+(-c+i)f)} - \frac{ibd(e+fx)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{d(e+fx)}{de-(c+i)f}\right)}{2f(m+1)(m+2)(de-(c+i)f)}$$

```
[In] Int[(e + f*x)^m*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcCot[c + d*x]))/(f*(1 + m)) + ((I/2)*b*d*(e + f
*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + I*f - c
*f)])/(f*(d*e + (I - c)*f)*(1 + m)*(2 + m)) - ((I/2)*b*d*(e + f*x)^(2 + m)*
Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (I + c)*f)])/(f*(d*
e - (I + c)*f)*(1 + m)*(2 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m (a+b\cot^{-1}(x))\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^{1+m}(a+b\cot^{-1}(c+dx))}{f(1+m)} + \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1+x^2}\,dx, x, c+dx\right)}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}(a+b\cot^{-1}(c+dx))}{f(1+m)} + \frac{b\text{Subst}\left(\int \left(\frac{i\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(i-x)} + \frac{i\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(i+x)}\right)dx, x, c+dx\right)}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}(a+b\cot^{-1}(c+dx))}{f(1+m)} + \frac{(ib)\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{i-x}\,dx, x, c+dx\right)}{2f(1+m)} + \frac{(ib)\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{i+x}\,dx, x, c+dx\right)}{2f(1+m)}$$

$$= \frac{(e+fx)^{1+m}(a+b\cot^{-1}(c+dx))}{f(1+m)} + \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1, 2+m, 3+m, \frac{d(e+fx)}{de+if-cf}\right)}{2f(de+(i-c)f)(1+m)(2+m)} - \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1, 2+m, 3+m, \frac{d(e+fx)}{de-(i+c)f}\right)}{2f(de-(i+c)f)(1+m)(2+m)}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx$$

$$= \frac{(e+fx)^{1+m}\left(2(a+b\cot^{-1}(c+dx)) + \frac{bd(e+fx)\left((de-(i+c)f)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(-i+c)f}\right)+(-de+(-i+c)f)\right.}{(-ide+f+icf)(de-(i+c)f)(2+m)}}{2f(1+m)}$$

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(2*(a + b*ArcCot[c + d*x]) + (b*d*(e + f*x)*((d*e - (I +
 c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (-I + c)*f)]
 + (-(d*e) + (-I + c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(
d*e - (I + c)*f)]))/(((-I)*d*e + f + I*c*f)*(d*e - (I + c)*f)*(2 + m))))/(2
*f*(1 + m))
```

Maple [F]

$$\int (fx+e)^m (a+b \operatorname{arccot}(dx+c))\, dx$$

```
[In] int((f*x+e)^m*(a+b*arccot(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*(a+b*arccot(d*x+c)),x)
```

Fricas [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\, dx = \int (b \operatorname{arccot}(dx+c)+a)(fx+e)^m\, dx$$

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arccot(d*x + c) + a)*(f*x + e)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\, dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**m*(a+b*acot(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\, dx = \int (b \operatorname{arccot}(dx+c)+a)(fx+e)^m\, dx$$

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((f*x*arctan2(1, d*x + c) + e*arctan2(1, d*x + c))*(f*x + e)^m + 2*(f*m
 + f)*integrate(1/2*((c^2*arctan2(1, d*x + c) + arctan2(1, d*x + c))*f*m +
(d^2*f*m*arctan2(1, d*x + c) + d^2*f*arctan2(1, d*x + c))*x^2 + d*e + (c^2*
```

```
arctan2(1, d*x + c) + arctan2(1, d*x + c))*f + (2*c*d*f*m*arctan2(1, d*x +
c) + (2*c*arctan2(1, d*x + c) + 1)*d*f)*x)*(f*x + e)^m/((c^2 + 1)*f*m + (d^
2*f*m + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))*b/(f*m + f)
+ (f*x + e)^(m + 1)*a/(f*(m + 1))
```

Giac [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) \, dx = \int (b \operatorname{arccot}(dx+c)+a)(fx+e)^m \, dx$$

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="giac")
[Out] integrate((b*arccot(d*x + c) + a)*(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) \, dx = \int (e+f\,x)^m \,(a+b\,\mathrm{acot}(c+d\,x)) \, dx$$

```
[In] int((e + f*x)^m*(a + b*acot(c + d*x)),x)
[Out] int((e + f*x)^m*(a + b*acot(c + d*x)), x)
```

3.147 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \operatorname{Int}\left((e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2,x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx$$

```
[In] Int[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2,x]
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCot[x])^2, x]
, x, c + d*x]/d
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\cot^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

Mathematica [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$$

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2,x]
[Out] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m \left(a+b \operatorname{arccot}(dx+c)\right)^2 dx$$

```
[In] int((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
[Out] int((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^2 (fx+e)^m \, dx$$

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
[Out] integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)*(f*x + e)^m,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**m*(a+b*acot(d*x+c))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 618, normalized size of antiderivative = 30.90

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (b\,\mathrm{arccot}\,(dx+c)+a)^2(fx+e)^m \, dx$$

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (f*x + e)^(m + 1)*a^2/(f*(m + 1)) - 1/16*((b^2*f*x + b^2*e)*(f*x + e)^m*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 4*(b^2*f*x*arctan2(1, d*x + c)^2 + b^2*e*
arctan2(1, d*x + c)^2)*(f*x + e)^m - 16*(f*m + f)*integrate(1/16*(((b^2*c^2
 + b^2)*f*m + (b^2*d^2*f*m + b^2*d^2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*
d*f*m + b^2*c*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b
^2*d^2*f*x^2 + b^2*c*d*e + (b^2*d^2*e + b^2*c*d*f)*x)*(f*x + e)^m*log(d^2*x
^2 + 2*c*d*x + c^2 + 1) + 4*(2*b^2*d*e*arctan2(1, d*x + c) + (3*b^2*arctan2
(1, d*x + c)^2 + (3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*x + c))*
c^2 + 8*a*b*arctan2(1, d*x + c))*f*m + ((3*b^2*arctan2(1, d*x + c)^2 + 8*a*
b*arctan2(1, d*x + c))*d^2*f*m + (3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arcta
n2(1, d*x + c))*d^2*f)*x^2 + (3*b^2*arctan2(1, d*x + c)^2 + (3*b^2*arctan2(
1, d*x + c)^2 + 8*a*b*arctan2(1, d*x + c))*c^2 + 8*a*b*arctan2(1, d*x + c))
*f + 2*((3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*x + c))*c*d*f*m +
 (b^2*arctan2(1, d*x + c) + (3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1,
 d*x + c))*c)*d*f)*x)*(f*x + e)^m)/((c^2 + 1)*f*m + (d^2*f*m + d^2*f)*x^2 +
 (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m + f)
```

Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (b\,\mathrm{arccot}\,(dx+c)+a)^2(fx+e)^m \, dx$$

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccot(d*x + c) + a)^2*(f*x + e)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (e+f\,x)^m\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2 \,dx$$

```
[In] int((e + f*x)^m*(a + b*acot(c + d*x))^2,x)
[Out] int((e + f*x)^m*(a + b*acot(c + d*x))^2, x)
```

3.148 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \operatorname{Int}\left((e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3, x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$$

```
[In] Int[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCot[x])^3, x]
, x, c + d*x]/d
```

Rubi steps

$$\text{integral} = \frac{\operatorname{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\cot^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$$

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3,x]
[Out] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b \operatorname{arccot}(dx+c))^3 \, dx$$

```
[In] int((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
[Out] int((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^3 (fx+e)^m \, dx$$

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
[Out] integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcco
t(d*x + c) + a^3)*(f*x + e)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**m*(a+b*acot(d*x+c))**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 7.83 (sec) , antiderivative size = 880, normalized size of antiderivative = 44.00

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 dx = \int (b\,\mathrm{arccot}\,(dx+c)+a)^3(fx+e)^m\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (f*x + e)^(m + 1)*a^3/(f*(m + 1)) - 1/32*(3*(b^3*f*x*arctan2(1, d*x + c) +
b^3*e*arctan2(1, d*x + c))*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 -
 4*(b^3*f*x*arctan2(1, d*x + c)^3 + b^3*e*arctan2(1, d*x + c)^3)*(f*x + e)^
m - 32*(f*m + f)*integrate(-1/32*(3*(b^3*d*e - (b^3*c^2*arctan2(1, d*x + c)
 + b^3*arctan2(1, d*x + c))*f*m - (b^3*d^2*f*m*arctan2(1, d*x + c) + b^3*d^
2*f*arctan2(1, d*x + c))*x^2 - (b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1
, d*x + c))*f - (2*b^3*c*d*f*m*arctan2(1, d*x + c) + (2*b^3*c*arctan2(1, d*
x + c) - b^3)*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 12*(
b^3*d^2*f*x^2*arctan2(1, d*x + c) + b^3*c*d*e*arctan2(1, d*x + c) + (b^3*d^
2*e*arctan2(1, d*x + c) + b^3*c*d*f*arctan2(1, d*x + c))*x)*(f*x + e)^m*log
(d^2*x^2 + 2*c*d*x + c^2 + 1) - 4*(3*b^3*d*e*arctan2(1, d*x + c)^2 + (7*b^3
*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(
1, d*x + c) + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2
 + 24*a^2*b*arctan2(1, d*x + c))*c^2)*f*m + ((7*b^3*arctan2(1, d*x + c)^3 +
 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c))*d^2*f*m + (
7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arc
tan2(1, d*x + c))*d^2*f)*x^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arct
an2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c) + (7*b^3*arctan2(1, d*x +
c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c))*c^2)*
f + (2*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a
^2*b*arctan2(1, d*x + c))*c*d*f*m + (3*b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3
*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(
1, d*x + c))*c)*d*f)*x)*(f*x + e)^m)/((c^2 + 1)*f*m + (d^2*f*m + d^2*f)*x^2
 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m + f)
```

Giac [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 dx = \int (b\,\mathrm{arccot}\,(dx+c)+a)^3(fx+e)^m\,dx$$

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccot(d*x + c) + a)^3*(f*x + e)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (e+f\,x)^m\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^3 \,dx$$

```
[In] int((e + f*x)^m*(a + b*acot(c + d*x))^3,x)
[Out] int((e + f*x)^m*(a + b*acot(c + d*x))^3, x)
```

3.149 $\int x^3 \cot^{-1}(a + bx^4) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int x^3 \cot^{-1}(a + bx^4) \, dx = \frac{(a + bx^4) \cot^{-1}(a + bx^4)}{4b} + \frac{\log\left(1 + (a + bx^4)^2\right)}{8b}$$

```
[Out] 1/4*(b*x^4+a)*arccot(b*x^4+a)/b+1/8*ln(1+(b*x^4+a)^2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6847, 5148, 4931, 266}

$$\int x^3 \cot^{-1}(a + bx^4) \, dx = \frac{\log\left((a + bx^4)^2 + 1\right)}{8b} + \frac{(a + bx^4) \cot^{-1}(a + bx^4)}{4b}$$

```
[In] Int[x^3*ArcCot[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcCot[a + b*x^4])/(4*b) + Log[1 + (a + b*x^4)^2]/(8*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \cot^{-1}(a+bx)\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \cot^{-1}(x)\,dx, x, a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\cot^{-1}\left(a+bx^4\right)}{4b} + \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\cot^{-1}\left(a+bx^4\right)}{4b} + \frac{\log\left(1+(a+bx^4)^2\right)}{8b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$
\int x^3 \cot^{-1}\left(a+bx^4\right)\,dx = \frac{2(a+bx^4)\cot^{-1}\left(a+bx^4\right)+\log\left(1+(a+bx^4)^2\right)}{8b}
$$

```
[In] Integrate[x^3*ArcCot[a + b*x^4],x]
[Out] (2*(a + b*x^4)*ArcCot[a + b*x^4] + Log[1 + (a + b*x^4)^2])/(8*b)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\operatorname{arccot}(b\,x^4+a)(b\,x^4+a)+\frac{\ln\left(1+(b\,x^4+a)^2\right)}{2}}{4b}$
default	$\frac{\operatorname{arccot}(b\,x^4+a)(b\,x^4+a)+\frac{\ln\left(1+(b\,x^4+a)^2\right)}{2}}{4b}$
parallelrisch	$\frac{2\operatorname{arccot}(b\,x^4+a)x^4b^2+2a\operatorname{arccot}(b\,x^4+a)b+\ln(b^2x^8+2ab\,x^4+a^2+1)b}{8b^2}$
parts	$\frac{x^4\operatorname{arccot}(b\,x^4+a)}{4}+b\left(\frac{\ln(b^2x^8+2ab\,x^4+a^2+1)}{8b^2}-\frac{a\arctan\left(\frac{2b^2x^4+2ab}{2b}\right)}{4b^2}\right)$
risch	$\frac{ix^4\ln(1+i(b\,x^4+a))}{8}-\frac{ix^4\ln(1-i(b\,x^4+a))}{8}+\frac{\pi\,x^4}{8}-\frac{a\arctan\left(\frac{b\,x^4}{a^2+1}+\frac{a^2b\,x^4}{a^2+1}+\frac{a^3}{a^2+1}+\frac{a}{a^2+1}\right)}{4b}+\frac{a\arctan(a)}{4b}+$

```
[In] int(x^3*arccot(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*(arccot(b*x^4+a)*(b*x^4+a)+1/2*ln(1+(b*x^4+a)^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^3\cot^{-1}\left(a+bx^4\right)\,dx = \frac{2\,bx^4\operatorname{arccot}\left(bx^4+a\right)-2\,a\arctan\left(bx^4+a\right)+\log\left(b^2x^8+2\,abx^4+a^2+1\right)}{8\,b}$$

```
[In] integrate(x^3*arccot(b*x^4+a),x, algorithm="fricas")
[Out] 1/8*(2*b*x^4*arccot(b*x^4 + a) - 2*a*arctan(b*x^4 + a) + log(b^2*x^8 + 2*a*
b*x^4 + a^2 + 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int x^3\cot^{-1}\left(a+bx^4\right)\,dx = \begin{cases}\frac{a\operatorname{acot}\left(a+bx^4\right)}{4b}+\frac{x^4\operatorname{acot}\left(a+bx^4\right)}{4}+\frac{\log\left(a^2+2abx^4+b^2x^8+1\right)}{8b} & \text{for}\ b\neq 0\\ \frac{x^4\operatorname{acot}(a)}{4} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3*acot(b*x**4+a),x)
[Out] Piecewise((a*acot(a + b*x**4)/(4*b) + x**4*acot(a + b*x**4)/4 + log(a**2 +
2*a*b*x**4 + b**2*x**8 + 1)/(8*b), Ne(b, 0)), (x**4*acot(a)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int x^3 \cot^{-1}\left(a+bx^4\right)\,dx=\frac{2\left(bx^4+a\right)\operatorname{arccot}\left(bx^4+a\right)+\log\left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

```
[In] integrate(x^3*arccot(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arccot(b*x^4 + a) + log((b*x^4 + a)^2 + 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(38) = 76.

Time = 0.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.02

$$\int x^3 \cot^{-1}\left(a+bx^4\right)\,dx=$$

$$-\frac{\arctan\left(\frac{1}{bx^4+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^2+\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\operatorname{arcta}\right.}{8\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)}$$

```
[In] integrate(x^3*arccot(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*(arctan(1/(b*x^4 + a))*tan(1/2*arctan(1/(b*x^4 + a)))^2 + log(16*tan(1
/2*arctan(1/(b*x^4 + a)))^2/(tan(1/2*arctan(1/(b*x^4 + a)))^4 + 2*tan(1/2*a
rctan(1/(b*x^4 + a)))^2 + 1))*tan(1/2*arctan(1/(b*x^4 + a))) - arctan(1/(b*
x^4 + a)))/(b*tan(1/2*arctan(1/(b*x^4 + a))))
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 230, normalized size of antiderivative = 5.48

$$\int x^3 \cot^{-1}\left(a+bx^4\right)\,dx=\frac{\ln\left(a^2+2\,a\,b\,x^4+b^2\,x^8+1\right)}{8\,b}+\frac{x^4\,\mathrm{acot}\left(b\,x^4+a\right)}{4}$$

$$-\frac{a\,\mathrm{atan}\left(\frac{a}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^3}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^5}{a^6+3\,a^4+3\,a^2+1}+\frac{a^7}{a^6+3\,a^4+3\,a^2+1}+\frac{b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^2\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{}{a^6}\right.}{4\,b}$$

```
[In] int(x^3*acot(a + b*x^4),x)
```

```
[Out] log(a^2 + b^2*x^8 + 2*a*b*x^4 + 1)/(8*b) + (x^4*acot(a + b*x^4))/4 - (a*ata
n(a/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^3)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^5)
```

```
/(3*a^2 + 3*a^4 + a^6 + 1) + a^7/(3*a^2 + 3*a^4 + a^6 + 1) + (b*x^4)/(3*a^2
 + 3*a^4 + a^6 + 1) + (3*a^2*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^4*b*x^
4)/(3*a^2 + 3*a^4 + a^6 + 1) + (a^6*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1)))/(4*b
)
```

3.150 $\int x^{-1+n} \cot^{-1}(a+bx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int x^{-1+n} \cot^{-1}(a+bx^n)\,dx = \frac{(a+bx^n)\cot^{-1}(a+bx^n)}{bn} + \frac{\log\left(1+(a+bx^n)^2\right)}{2bn}$$

```
[Out] (a+b*x^n)*arccot(a+b*x^n)/b/n+1/2*ln(1+(a+b*x^n)^2)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6847, 5148, 4931, 266}

$$\int x^{-1+n} \cot^{-1}(a+bx^n)\,dx = \frac{\log\left((a+bx^n)^2+1\right)}{2bn} + \frac{(a+bx^n)\cot^{-1}(a+bx^n)}{bn}$$

```
[In] Int[x^(-1 + n)*ArcCot[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcCot[a + b*x^n])/(b*n) + Log[1 + (a + b*x^n)^2]/(2*b*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \cot^{-1}(a+bx)\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \cot^{-1}(x)\,dx,x,a+bx^n\right)}{bn}\\
&= \frac{(a+bx^n)\cot^{-1}(a+bx^n)}{bn}+\frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx^n\right)}{bn}\\
&= \frac{(a+bx^n)\cot^{-1}(a+bx^n)}{bn}+\frac{\log\left(1+(a+bx^n)^2\right)}{2bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x^{-1+n}\cot^{-1}(a+bx^n)\ dx=\frac{2(a+bx^n)\cot^{-1}(a+bx^n)+\log\left(1+(a+bx^n)^2\right)}{2bn}$$

```
[In] Integrate[x^(-1 + n)*ArcCot[a + b*x^n],x]
[Out] (2*(a + b*x^n)*ArcCot[a + b*x^n] + Log[1 + (a + b*x^n)^2])/(2*b*n)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.31

method	result
risch	$\frac{i x^{n} \ln(1+i(a+b\,x^{n}))}{2n}-\frac{i x^{n} \ln(1-i(a+b\,x^{n}))}{2n}+\frac{\pi\,x^{n}}{2n}+\frac{i\ln\left(x^{n}-\frac{i-a}{b}\right)a}{2bn}-\frac{i\ln\left(\frac{i+a}{b}+x^{n}\right)a}{2bn}+\frac{\ln\left(x^{n}-\frac{i-a}{b}\right)}{2bn}+\frac{\ln\left(\frac{i+a}{b}+x^{n}\right)}{2bn}$

```
[In] int(x^(-1+n)*arccot(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/n*x^n*ln(1+I*(a+b*x^n))-1/2*I/n*x^n*ln(1-I*(a+b*x^n))+1/2/n*Pi*x^n+1/
2*I/b/n*ln(x^n-(I-a)/b)*a-1/2*I/b/n*ln((I+a)/b+x^n)*a+1/2/b/n*ln(x^n-(I-a)/
b)+1/2/b/n*ln((I+a)/b+x^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int x^{-1+n}\cot^{-1}(a+bx^n)\,dx = \frac{2\,bx^n\operatorname{arccot}(bx^n+a)-2\,a\arctan(bx^n+a)+\log\left(b^2x^{2\,n}+2\,abx^n+a^2+1\right)}{2\,bn}$$

```
[In] integrate(x^(-1+n)*arccot(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^n*arccot(b*x^n + a) - 2*a*arctan(b*x^n + a) + log(b^2*x^(2*n) +
2*a*b*x^n + a^2 + 1))/(b*n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}\cot^{-1}(a+bx^n)\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+n)*acot(a+b*x**n),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int x^{-1+n} \cot^{-1}(a+bx^n)\, dx = \frac{2\,(bx^n+a)\,\mathrm{arccot}\,(bx^n+a)+\log\left((bx^n+a)^2+1\right)}{2\,bn}$$

```
[In] integrate(x^(-1+n)*arccot(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arccot(b*x^n + a) + log((b*x^n + a)^2 + 1))/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int x^{-1+n} \cot^{-1}(a+bx^n)\, dx = \frac{b\left(\frac{2\,(bx^n+a)\arctan\left(\frac{1}{bx^n+a}\right)}{b^2}+\frac{\log\left(\frac{1}{(bx^n+a)^2}+1\right)}{b^2}-\frac{\log\left(\frac{1}{(bx^n+a)^2}\right)}{b^2}\right)}{2\,n}$$

```
[In] integrate(x^(-1+n)*arccot(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*b*(2*(b*x^n + a)*arctan(1/(b*x^n + a))/b^2 + log(1/(b*x^n + a)^2 + 1)/b
^2 - log((b*x^n + a)^(-2))/b^2)/n
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int x^{-1+n} \cot^{-1}(a+bx^n)\, dx = \frac{\frac{\ln\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1\right)}{2}+a\,\mathrm{acot}(a+b\,x^n)}{b\,n}+\frac{x^n\,\mathrm{acot}(a+b\,x^n)}{n}$$

```
[In] int(x^(n - 1)*acot(a + b*x^n),x)
```

```
[Out] (log(a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)/2 + a*acot(a + b*x^n))/(b*n) + (x^n
*acot(a + b*x^n))/n
```

3.151 $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\text{Int}\left(\frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

```
[In] Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

```
[In] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\ \mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

```
[In] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
[Out] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="fricas")
[Out] integral(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 3.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
[Out] -Integral((a + b*acot(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="maxima")
[Out] -integrate((b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="giac")
[Out] integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
[Out] -int((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.152 $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 488

$$\begin{aligned}
&\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx \\
&= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&+\frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&-\frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&+\frac{3b^2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&-\frac{3b^2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}
\end{aligned}$$

```
[Out] -2*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3*arccoth(1-2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+3/2*I*b*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*I*b*(a+b*arccot((-
c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,1-2*(-c*x+1)^(1/2)/(I+(-c*x+1)^(1/
```

```
2)/(c*x+1)^(1/2))/(c*x+1)^(1/2))/c+3/2*b^2*(a+b*arccot((-c*x+1)^(1/2)/(c*x+
1)^(1/2)))*polylog(3,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b^2*(a+b
*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2*(-c*x+1)^(1/2)/(I+(-c*
x+1)^(1/2)/(c*x+1)^(1/2))/(c*x+1)^(1/2))/c-3/4*I*b^3*polylog(4,1-2*I/(I+(-c
*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*I*b^3*polylog(4,1-2*(-c*x+1)^(1/2)/(I+(-c
*x+1)^(1/2)/(c*x+1)^(1/2))/(c*x+1)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6813, 4943, 5109, 5005, 5113, 5117, 6745}

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$
$$=\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c}-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)}{4c}+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)}{4c}$$

```
[In] Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/2)*b*(a + b*ArcCot[Sqrt[1 - c*x]/S
qrt[1 + c*x]])^2*PolyLog[2, 1 - (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c
 - (((3*I)/2)*b*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, 1
- (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/c +
 (3*b^2*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - (2*I)/(I
 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b^2*(a + b*ArcCot[Sqrt[1 - c*x
]/Sqrt[1 + c*x]])*PolyLog[3, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt
```

```
[1 - c*x]/Sqrt[1 + c*x]))])/(2*c) - (((3*I)/4)*b^3*PolyLog[4, 1 - (2*I)/(I
+ Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c + (((3*I)/4)*b^3*PolyLog[4, 1 - (2*Sqrt[
1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/c
```

Rule 4943

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Dist[2*b*c*p, Int[(a + b*
ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5109

```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegrand
[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c
, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)
))^2, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5117

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}\\
&\quad-\frac{(6b)\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^2\coth^{-1}\left(1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}\\
&\quad+\frac{(3b)\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^2\log\left(\frac{2i}{i+x}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&\quad-\frac{(3b)\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^2\log\left(\frac{2x}{i+x}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}\\
&\quad+\frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}\\
&\quad-\frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}\\
&\quad+\frac{(3ib^2)\,\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))\,\text{PolyLog}\left(2,1-\frac{2i}{i+x}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&\quad-\frac{(3ib^2)\,\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))\,\text{PolyLog}\left(2,1-\frac{2x}{i+x}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}
\end{aligned}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} + \frac{3b^2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{3b^2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} + \frac{(3b^3)\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(3,1-\frac{2i}{i+x}\right)}{1+x^2}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} - \frac{(3b^3)\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(3,1-\frac{2x}{i+x}\right)}{1+x^2}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} + \frac{3b^2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{3b^2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} - \frac{3ib^3\,\text{PolyLog}\left(4,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c} + \frac{3ib^3\,\text{PolyLog}\left(4,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}$$

Mathematica [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

```
[In] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1639 vs. $2(402) = 804$.

Time = 1.32 (sec) , antiderivative size = 1640, normalized size of antiderivative = 3.36

method	result	size
default	Expression too large to display	1640
parts	Expression too large to display	1640

```
[In] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-b^3*(-1/c*arccot((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^3*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(
1/2))+3*I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(I+(-c*x+1)^(1
/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6/c*arccot((-c*x+1)^(1/2)/(c
*x+1)^(1/2))*polylog(3,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1
)^(1/2))-6*I/c*polylog(4,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)
+1)^(1/2))+1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(I+(-c*x+1)^(1/2
)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-3/2*I/c*arccot((-c*x+1)^(1/2)/(c*x
+1)^(1/2))^2*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1
)+1))+3/2/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(I+(-c*x+1)^(1/
2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+3/4*I/c*polylog(4,-(I+(-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/c*arccot((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^3*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)
)+3*I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-(I+(-c*x+1)^(1/2)
/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6/c*arccot((-c*x+1)^(1/2)/(c*x+
1)^(1/2))*polylog(3,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^
(1/2))-6*I/c*polylog(4,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+
1)^(1/2)))-3*a*b^2*(-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(I+(-c
*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2*I/c*arccot((-c*x+1
)^(1/2)/(c*x+1)^(1/2))*polylog(2,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)
/(c*x+1)+1)^(1/2))-2/c*polylog(3,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)
```

```
/(c*x+1)+1)^(1/2))+1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(I+(-c*x
+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-I/c*arccot((-c*x+1)^(1/2)/
(c*x+1)^(1/2))*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x
+1)+1))+1/2/c*polylog(3,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+
1)+1))-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(I+(-c*x+1)^(1/2)/(c
*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2*I/c*arccot((-c*x+1)^(1/2)/(c*x+1
)^(1/2))*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(
1/2))-2/c*polylog(3,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^
(1/2)))-3*a^2*b*(-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(I+(-c*x+1)
^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+I/c*polylog(2,(I+(-c*x+1)
^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+1/c*arccot((-c*x+1)^(1/2)
/(c*x+1)^(1/2))*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1
))-1/2*I/c*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+
1))-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)
^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+I/c*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1
)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^3*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccot(sqrt
(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1))
+ a^3)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=-\int \frac{a^3}{c^2x^2-1}\,dx-\int \frac{b^3\,\mathrm{acot}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{3ab^2\,\mathrm{acot}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{3a^2b\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*acot(sqrt(-c*x + 1)/sqrt
(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*acot(sqrt(-c*x + 1)/s
qrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*acot(sqrt(-c*x + 1
)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\left(a + b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1 - c^2x^2}\,dx = \int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^3}{c^2x^2 - 1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/64*(4*(b^3*log(c*x + 1) - b^3
*log(-c*x + 1))*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^3 - 3*(b^3*log(2)^2*
log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1))*arctan2(sqrt(c*x + 1), sqrt(-c*x
 + 1)) + 64*c*integrate(-1/128*(112*b^3*arctan2(sqrt(c*x + 1), sqrt(-c*x +
1))^3 + 384*a*b^2*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^2 + 3*(b^3*log(2)^
2*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1) - 4*(b^3*log(c*x + 1) - b^3*log
(-c*x + 1))*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^2)*sqrt(c*x + 1)*sqrt(-c
*x + 1) + 12*(b^3*log(2)^2 + 32*a^2*b)*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1
)))/(c^2*x^2 - 1), x))/c
```

Giac [F]

$$\int \frac{\left(a + b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1 - c^2x^2}\,dx = \int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^3}{c^2x^2 - 1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1 - c^2x^2}\,dx = \int -\frac{\left(a + b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2 - 1}\,dx$$

```
[In] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.153 $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 321

$$\begin{aligned}
&\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx \\
&= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&\quad+\frac{ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&\quad-\frac{ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} \\
&\quad+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}
\end{aligned}$$

```
[Out] -2*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*arccoth(1-2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+I*b*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-I*b*(a+b*arccot((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))*polylog(2,1-2*(-c*x+1)^(1/2)/(I+(-c*x+1)^(1/2)/(c*x+1)^(
1/2))/(c*x+1)^(1/2))/c+1/2*b^2*polylog(3,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1
/2)))/c-1/2*b^2*polylog(3,1-2*(-c*x+1)^(1/2)/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2
))/(c*x+1)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6813, 4943, 5109, 5005, 5113, 6745}

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \frac{ib\,\text{PolyLog}\left(2,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} - \frac{2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c} + \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)}{2c} - \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)}{2c}$$

```
[In] Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcCoth[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (I*b*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])*PolyLog[2, 1 - (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c - (I*b*(a
 + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/
(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/c + (b^2*PolyLog[3, 1 -
 (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) - (b^2*PolyLog[3, 1 - (2*S
qrt[1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/(2*c)
```

Rule 4943

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Dist[2*b*c*p, Int[(a + b*
ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5109

```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegrand
[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c
, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)
))^2, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\ &= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\ &\quad -\frac{(4b)\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)\coth^{-1}\left(1-\frac{2}{1+ix}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \end{aligned}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(2b)\text{Subst}\left(\int\frac{(a+b\cot^{-1}(x))\log\left(\frac{2i}{i+x}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{(2b)\text{Subst}\left(\int\frac{(a+b\cot^{-1}(x))\log\left(\frac{2x}{i+x}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{(ib^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,1-\frac{2i}{i+x}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{(ib^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,1-\frac{2x}{i+x}\right)}{1+x^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

Mathematica [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

```
[In] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(268) = 536.

Time = 0.70 (sec) , antiderivative size = 903, normalized size of antiderivative = 2.81

method	result
default	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(-\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{2i\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}\right.$
parts	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(-\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{2i\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}\right.$

```
[In] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2*(-1/c*arccot((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^2*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(
1/2))+2*I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(I+(-c*x+1)^(1/2
)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2/c*polylog(3,(I+(-c*x+1)^(1/2
)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+1/c*arccot((-c*x+1)^(1/2)/(c*x
+1)^(1/2))^2*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-
I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2/c*polylog(3,-(I+(-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2
*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2*I/c*ar
ccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/
2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2/c*polylog(3,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1
/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))-2*a*b*(-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)
```

```
^(1/2))*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+I
/c*polylog(2,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+1
/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2
))^2/((-c*x+1)/(c*x+1)+1))-1/2*I/c*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/
2))^2/((-c*x+1)/(c*x+1)+1))-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(
I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+I/c*polylog(2,-
(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^2*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccot(sqrt(-
c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=-\int \frac{a^2}{c^2x^2-1}\,dx-\int \frac{b^2\operatorname{acot}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{2ab\operatorname{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acot(sqrt(-c*x + 1)/sqrt
(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acot(sqrt(-c*x + 1)/sqrt
(c*x + 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\left(a + b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1 - c^2x^2}\,dx = \int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^2}{c^2x^2 - 1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/32*(b^2*log(2)^2*log(c*x + 1)
 - b^2*log(2)^2*log(-c*x + 1) - 4*(b^2*log(c*x + 1) - b^2*log(-c*x + 1))*ar
ctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^2 - (b^2*(log(c*x + 1)/c - log(c*x - 1
)/c)*log(2)^2 + 64*b^2*integrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(s
qrt(c*x + 1)/sqrt(-c*x + 1))*log(c*x + 1)/(c^2*x^2 - 1), x) - 64*b^2*integr
ate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(c*x + 1)/sqrt(-c*x + 1))*
log(-c*x + 1)/(c^2*x^2 - 1), x) - 384*b^2*integrate(1/16*arctan(sqrt(c*x +
1)/sqrt(-c*x + 1))^2/(c^2*x^2 - 1), x) - 1024*a*b*integrate(1/16*arctan(sqr
t(c*x + 1)/sqrt(-c*x + 1))/(c^2*x^2 - 1), x))*c)/c
```

Giac [F]

$$\int \frac{\left(a + b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1 - c^2x^2}\,dx = \int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^2}{c^2x^2 - 1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1 - c^2x^2}\,dx = \int -\frac{\left(a + b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2 - 1}\,dx$$

```
[In] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.154 $\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 98

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}+\frac{ib\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}-\frac{ib\,\text{PolyLog}\left(2,\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}$$

```
[Out] -a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c+1/2*I*b*polylog(2,-I*(c*x+1)^(1/2)/(-
c*x+1)^(1/2))/c-1/2*I*b*polylog(2,I*(c*x+1)^(1/2)/(-c*x+1)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {212, 6813, 4941, 2438}

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}+\frac{ib\,\text{PolyLog}\left(2,-\frac{i\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}-\frac{ib\,\text{PolyLog}\left(2,\frac{i\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}$$

```
[In] Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) + ((I/2)*b*PolyLog[2, ((-I)*Sqrt[
1 + c*x])/Sqrt[1 - c*x]])/c - ((I/2)*b*PolyLog[2, (I*Sqrt[1 + c*x])/Sqrt[1
- c*x]])/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} + \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}\\
&= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} + \frac{ib\,\text{PolyLog}\left(2, -\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c} - \frac{ib\,\text{PolyLog}\left(2, \frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right) - \frac{1}{2}ib\,\text{PolyLog}\left(2, -\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right) + \frac{1}{2}ib\,\text{PolyLog}\left(2, \frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{c}$$

```
[In] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] - (I/2)*b*PolyLog[2, ((-I)*Sqrt[1 + c
*x])/Sqrt[1 - c*x]] + (I/2)*b*PolyLog[2, (I*Sqrt[1 + c*x])/Sqrt[1 - c*x]])/
c)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(78) = 156$.

Time = 0.43 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.70

method	result
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(-\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{i\operatorname{polylog}\left(2,\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right.}{}\right.$
parts	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(-\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{i\operatorname{polylog}\left(2,\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right.}{}\right.$

```
[In] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)-b*(-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^
(1/2))*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+I/
c*polylog(2,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+1/
c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2/((-c*x+1)/(c*x+1)+1))-1/2*I/c*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2
))^2/((-c*x+1)/(c*x+1)+1))-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(I
+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+I/c*polylog(2,-(
I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] integral(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\int \frac{a}{c^2x^2-1}\,dx-\int \frac{b\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*acot(sqrt(-c*x + 1)/sqrt(c*x +
 1))/(c**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*((log(c*x + 1) - log(-c*x + 1
))*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)) + 2*c*integrate(1/2*(e^(1/2*log(c
*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) - e^(1/2*log(c*x + 1) + 1/2*log(-
c*x + 1))*log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1) - (c^2*x^2 - 1)*(c*x - 1)
), x))*b/c
```

Giac [F]

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="giac")
[Out] integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
[Out] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.155 $\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \operatorname{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\ \mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
[Out] int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="fricas")
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - a), x)
```

Sympy [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{ac^2x^2-a+bc^2x^2\operatorname{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\operatorname{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acot(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - b*acot(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="maxima")
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="giac")
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=-\int \frac{1}{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

```
[In] int(-1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.156 $\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x
]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}dx$$

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccot(sqrt(-c*x + 1)/sqrt(c
*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1
))), x)
```

Sympy [N/A]

Not integrable

Time = 6.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{a^2c^2x^2-a^2+2abc^2x^2\operatorname{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\operatorname{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^2c^2x^2\operatorname{acot}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^2\operatorname{acot}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}$$

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*acot(sqrt(-c*x + 1)/sq
rt(c*x + 1)) - 2*a*b*acot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*ac
ot(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*acot(sqrt(-c*x + 1)/sqrt(c*x + 1
))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.08

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="maxima")
```

```
[Out] -2*(2*(b^2*c^2*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)) + a*b*c^2)*sqrt(c*x +
 1)*sqrt(-c*x + 1)*integrate(1/2*x/((a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2
)*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
 + 1)/((b^2*c*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)) + a*b*c)*sqrt(c*x + 1)
*sqrt(-c*x + 1))
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2)
, x)
```

Mupad [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=-\int \frac{1}{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

```
[In] int(-1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.157 $\int \cot^{-1}(\tan(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \cot^{-1}(\tan(a+bx))\,dx = -\frac{\cot^{-1}(\tan(a+bx))^2}{2b}$$

```
[Out] -1/2*(1/2*Pi-arctan(tan(b*x+a)))^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2188, 30}

$$\int \cot^{-1}(\tan(a+bx))\,dx = -\frac{\cot^{-1}(\tan(a+bx))^2}{2b}$$

```
[In] Int[ArcCot[Tan[a + b*x]],x]
```

```
[Out] -1/2*ArcCot[Tan[a + b*x]]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x\,dx, x, \cot^{-1}(\tan(a+bx))\right)}{b}$$
$$= -\frac{\cot^{-1}(\tan(a+bx))^2}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \cot^{-1}(\tan(a+bx))\,dx = \frac{bx^2}{2} + x\cot^{-1}(\tan(a+bx))$$

```
[In] Integrate[ArcCot[Tan[a + b*x]],x]
[Out] (b*x^2)/2 + x*ArcCot[Tan[a + b*x]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

method	result
default	$\frac{\pi x}{2}-\frac{\arctan(\tan(bx+a))^2}{2b}$
parts	$\frac{\pi x}{2}-\frac{\arctan(\tan(bx+a))^2}{2b}$
parallelrisch	$\frac{x^2 b}{2}-x\arctan\left(\tan\left(bx+a\right)\right)+\frac{\pi x}{2}$
derivativedivides	$\frac{\pi\arctan(\tan(bx+a))-\arctan(\tan(bx+a))^2}{2b}$
risch	$\frac{\pi x}{2}+ix\ln\left(e^{i(bx+a)}\right)+\frac{\pi x\ \mathrm{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(ie^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right)}{4}-\frac{\pi x\ \mathrm{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right.}{}$

```
[In] int(1/2*Pi-arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*Pi*x-1/2*arctan(tan(b*x+a))^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \cot^{-1}(\tan(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,(\pi - 2\,a)x$$

```
[In] integrate(1/2*pi-arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(17) = 34.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.00

$$\int \cot^{-1}(\tan(a+bx))\,dx = \frac{\pi x}{2} - \begin{cases} \frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for } b \neq 0 \\ x\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/2*pi-atan(tan(b*x+a)),x)
[Out] pi*x/2 - Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2
/(2*b), Ne(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \cot^{-1}(\tan(a+bx))\,dx = \frac{1}{2}\,\pi x - \frac{(bx+a)^2}{2\,b}$$

```
[In] integrate(1/2*pi-arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/2*pi*x - 1/2*(b*x + a)^2/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \cot^{-1}(\tan(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \pi x\left\lfloor\frac{bx+a}{\pi} + \frac{1}{2}\right\rfloor + \frac{1}{2}\,\pi x - ax$$

```
[In] integrate(1/2*pi-arctan(tan(b*x+a)),x, algorithm="giac")
[Out] -1/2*b*x^2 + pi*x*floor((b*x + a)/pi + 1/2) + 1/2*pi*x - a*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \cot^{-1}(\tan(a+bx))\,dx = \frac{\Pi\,x}{2} + \frac{b\,x^2}{2} - x\,\mathrm{atan}(\tan(a+b\,x))$$

```
[In] int(Pi/2 - atan(tan(a + b*x)),x)
[Out] (Pi*x)/2 + (b*x^2)/2 - x*atan(tan(a + b*x))
```

3.158 $\int x^2 \cot^{-1}(c+d\tan(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 403

$$\begin{aligned}\int x^2\cot^{-1}(c+d\tan(a+bx))\,dx &= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx))\\
&-\frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)\\
&+\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)\\
&-\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b}\\
&+\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}\\
&-\frac{ix\operatorname{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2}\\
&+\frac{ix\operatorname{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2}\\
&+\frac{\operatorname{PolyLog}\left(4,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^3}\\
&-\frac{\operatorname{PolyLog}\left(4,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^3}\end{aligned}$$

```
[Out] 1/3*x^3*arccot(c+d*tan(b*x+a))-1/6*I*x^3*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))+1/6*I*x^3*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))-1/4*x
^2*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b+1/4*x^2*polylog(2,-
(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b-1/4*I*x*polylog(3,-(1+I*c+d)*
exp(2*I*a+2*I*b*x)/(1+I*c-d))/b^2+1/4*I*x*polylog(3,-(c+I*(1-d))*exp(2*I*a+
```

```
2*I*b*x)/(c+I*(1+d)))/b^2+1/8*polylog(4,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*
c-d))/b^3-1/8*polylog(4,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5284, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \cot^{-1}(c+d\tan(a+bx))\,dx = \frac{\text{PolyLog}\left(4,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^3} - \frac{\text{PolyLog}\left(4,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^3} - \frac{ix\,\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2} + \frac{ix\,\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2} - \frac{x^2\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b} + \frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b} - \frac{1}{6}ix^3\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right) + \frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right) + \frac{1}{3}x^3\cot^{-1}(d\tan(a+bx)+c)$$

```
[In] Int[x^2*ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Tan[a + b*x]])/3 - (I/6)*x^3*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] + (I/6)*x^3*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] - (x^2*PolyLog[2, -(((1 + I*c + d)*E
^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) + (x^2*PolyLog[2, -(((c + I*
(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) - ((I/4)*x*PolyL
og[3, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 + ((I/
4)*x*PolyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d))
)])/b^2 + PolyLog[4, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d
))]/(8*b^3) - PolyLog[4, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I
*(1 + d)))]/(8*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5284

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (-Dist[b*((1 - I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 + Dist[b*((1 + I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*
I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) \\
&\quad -\frac{1}{3}(b(1-ic-d))\int \frac{e^{2ia+2ibx}x^3}{1-ic+d+(1-ic-d)e^{2ia+2ibx}}\,dx \\
&\quad +\frac{1}{3}(b(1+ic+d))\int \frac{e^{2ia+2ibx}x^3}{1+ic-d+(1+ic+d)e^{2ia+2ibx}}\,dx \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad +\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad -\frac{1}{2}i\int x^2\log\left(1+\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx \\
&\quad +\frac{1}{2}i\int x^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad +\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad -\frac{x^2\,\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b}+\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} \\
&\quad -\frac{\int x\,\text{PolyLog}\left(2,-\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx}{2b}+\frac{\int x\,\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx}{2b} \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad +\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad -\frac{x^2\,\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b}+\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} \\
&\quad -\frac{ix\,\text{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2}+\frac{ix\,\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2} \\
&\quad -\frac{i\int \text{PolyLog}\left(3,-\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx}{4b^2}+\frac{i\int \text{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx}{4b^2}
\end{aligned}
$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) - \frac{x^2\operatorname{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} + \frac{x^2\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2} + \frac{ix\operatorname{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2} + \frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,-\frac{(1+ic+d)x}{1+ic-d}\right)}{x}dx,x,e^{2ia+2ibx}\right)}{8b^3} - \frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,-\frac{(c-i(-1+d))x}{c+i(1+d)}\right)}{x}dx,x,e^{2ia+2ibx}\right)}{8b^3}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) - \frac{x^2\operatorname{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} + \frac{x^2\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2} + \frac{ix\operatorname{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^3} - \frac{\operatorname{PolyLog}\left(4,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.92

$$\int x^2\cot^{-1}(c+d\tan(a+bx))\,dx = \frac{8b^3x^3\cot^{-1}(c+d\tan(a+bx)) - 4ib^3x^3\log\left(1+\frac{(c+i(-1+d))e^{-2i(a+bx)}}{c-i(1+d)}\right) + 4ib^3x^3\log\left(1+\frac{(c+i(1+d))e^{-2i(a+bx)}}{i+c-id}\right) -$$

[In] Integrate[x^2*ArcCot[c + d*Tan[a + b*x]],x]

[Out] (8*b^3*x^3*ArcCot[c + d*Tan[a + b*x]] - (4*I)*b^3*x^3*Log[1 + (c + I*(-1 + d))/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + (4*I)*b^3*x^3*Log[1 + (c + I*(
1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-c - I
(1 + d))/((c - I(-1 + d))*E^((2*I)*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, (I

```
 - c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + (6*I)*b*x*PolyLog[3, (
-c - I*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] - (6*I)*b*x*PolyLog[3,
 (I - c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (-c -
I*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] - 3*PolyLog[4, (I - c - I*d
)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 43.73 (sec) , antiderivative size = 8038, normalized size of antiderivative = 19.95

method	result	size
risch	Expression too large to display	8038

```
[In] int(x^2*arccot(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1965 vs. 2(290) = 580.

Time = 0.32 (sec) , antiderivative size = 1965, normalized size of antiderivative = 4.88

$$\int x^2 \cot^{-1}(c + d\tan(a + bx))\, dx = \text{Too large to display}$$

```
[In] integrate(x^2*arccot(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arccot(d*tan(b*x + a) + c) - 6*b^2*x^2*dilog(2*((I*c*d - d
^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x
+ a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)
+ 1) + 6*b^2*x^2*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
(I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*ta
n(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^2*dilog(2*((-I*c*d - d^2
 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x +
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
 1) + 6*b^2*x^2*dilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d +
(-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) - 4*I*a^3*log(((I*c*d + d^2 + d)*
tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d
 - 1)/(tan(b*x + a)^2 + 1)) + 4*I*a^3*log(((I*c*d + d^2 - d)*tan(b*x + a)^2
 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x
 + a)^2 + 1)) - 4*I*a^3*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d
```

```
 + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1))
+ 4*I*a^3*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*
d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - 6*I*b*x*poly
log(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*
(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b
*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^
2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)
*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d
+ 1)) + 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2
- 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 +
d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) - 6*I*b*x*polylog(3,
((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2
+ 2*c*d - I*d^2 + I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 - 2*d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log(-2*((I*c*d - d^2 + d
)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) +
 d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) - 4*(
I*b^3*x^3 + I*a^3)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
 (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log(-2*((-I*c
*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*t
an(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*
d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2
- c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 +
d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 3*polylog(4, ((c^2
+ 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c
*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)
^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*tan(b*x + a) - 1)/
((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 3*polylog(4
, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*(-I*c
^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x +
a)^2 + c^2 + d^2 - 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(
b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*tan(b*x +
a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\tan(a + bx))\,dx = \text{Timed out}$$

```
[In] integrate(x**2*acot(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2 \cot^{-1}(c + d \tan(a + bx))\, dx = \int x^2 \operatorname{arccot}\left(d \tan\left(bx + a\right) + c\right)\, dx$$

[In] integrate(x^2*arccot(c+d*tan(b*x+a)),x, algorithm="maxima")

[Out] 1/6*x^3*arctan2(-(d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, -c*cos(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/6*x^3*arctan2(-(d - 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, -c*cos(2*b*x + 2*a) - (d - 1)*sin(2*b*x + 2*a) - c) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*x + 2*a)^2 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)

Giac [F]

$$\int x^2 \cot^{-1}(c + d \tan(a + bx))\, dx = \int x^2 \operatorname{arccot}\left(d \tan\left(bx + a\right) + c\right)\, dx$$

[In] integrate(x^2*arccot(c+d*tan(b*x+a)),x, algorithm="giac")

[Out] integrate(x^2*arccot(d*tan(b*x + a) + c), x)

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\tan(a + bx))\, dx = \int x^2\, \mathrm{acot}(c + d\tan(a + b\,x))\ dx$$

```
[In] int(x^2*acot(c + d*tan(a + b*x)),x)
[Out] int(x^2*acot(c + d*tan(a + b*x)), x)
```

3.159 $\int x \cot^{-1}(c+d\tan(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 305

$$\begin{aligned}\int x\cot^{-1}(c+d\tan(a+bx))\,dx &= \frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx)) \\ &- \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\ &+ \frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\ &- \frac{x\,\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} \\ &+ \frac{x\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} \\ &- \frac{i\,\text{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^2} \\ &+ \frac{i\,\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^2}\end{aligned}$$

```
[Out] 1/2*x^2*arccot(c+d*tan(b*x+a))-1/4*I*x^2*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))+1/4*I*x^2*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))-1/4*x
*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b+1/4*x*polylog(2,-(c+I
*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b-1/8*I*polylog(3,-(1+I*c+d)*exp(2*
I*a+2*I*b*x)/(1+I*c-d))/b^2+1/8*I*polylog(3,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)
/(c+I*(1+d)))/b^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5284, 2221, 2611, 2320, 6724}

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx = -\frac{i\,\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^2} + \frac{i\,\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^2} - \frac{x\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b} + \frac{x\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b} - \frac{1}{4}ix^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right) + \frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right) + \frac{1}{2}x^2\cot^{-1}(d\tan(a+bx)+c)$$

```
[In] Int[x*ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Tan[a + b*x]])/2 - (I/4)*x^2*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] + (I/4)*x^2*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] - (x*PolyLog[2, -(((1 + I*c + d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) + (x*PolyLog[2, -(((c + I*(1 -
 d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) - ((I/8)*PolyLog[3,
-(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 + ((I/8)*Pol
yLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5284

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (-Dist[b*((1 - I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 + Dist[b*((1 + I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*
I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx)) \\
&\quad -\frac{1}{2}(b(1-ic-d))\int\frac{e^{2ia+2ibx}x^2}{1-ic+d+(1-ic-d)e^{2ia+2ibx}}\,dx \\
&\quad +\frac{1}{2}(b(1+ic+d))\int\frac{e^{2ia+2ibx}x^2}{1+ic-d+(1+ic+d)e^{2ia+2ibx}}\,dx \\
&= \frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad +\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad -\frac{1}{2}i\int x\log\left(1+\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx \\
&\quad +\frac{1}{2}i\int x\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx
\end{aligned}$$

$$=\frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)+\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)-\frac{x\,\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b}+\frac{x\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}-\frac{\int\text{PolyLog}\left(2,-\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx}{4b}+\frac{\int\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx}{4b}$$

$$=\frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)+\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)-\frac{x\,\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b}+\frac{x\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}-\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-\frac{(1+ic+d)x}{1+ic-d}\right)}{x}dx,x,e^{2ia+2ibx}\right)}{8b^2}+\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-\frac{(c-i(-1+d))x}{c+i(1+d)}\right)}{x}dx,x,e^{2ia+2ibx}\right)}{8b^2}$$

$$=\frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)+\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)-\frac{x\,\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b}+\frac{x\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}-\frac{i\,\text{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^2}+\frac{i\,\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.92

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx=\frac{4b^2x^2\cot^{-1}(c+d\tan(a+bx))-2ib^2x^2\log\left(1+\frac{(c+i(-1+d))e^{-2i(a+bx)}}{c-i(1+d)}\right)+2ib^2x^2\log\left(1+\frac{(c+i(1+d))e^{-2i(a+bx)}}{i+c-id}\right)\cdots}{}$$

```
[In] Integrate[x*ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] (4*b^2*x^2*ArcCot[c + d*Tan[a + b*x]] - (2*I)*b^2*x^2*Log[1 + (c + I*(-1 +
d))/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + (2*I)*b^2*x^2*Log[1 + (c + I*(
1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] - 2*b*x*PolyLog[2, (-c - I*(1
+ d))/((c - I*(-1 + d))*E^((2*I)*(a + b*x)))] + 2*b*x*PolyLog[2, (I - c - I
*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + I*PolyLog[3, (-c - I*(1 + d))/
((I + c - I*d)*E^((2*I)*(a + b*x)))] - I*PolyLog[3, (I - c - I*d)/((c - I*(
1 + d))*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.09 (sec) , antiderivative size = 7646, normalized size of antiderivative = 25.07

method	result	size
risch	Expression too large to display	7646

```
[In] int(x*arccot(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(217) = 434.

Time = 0.33 (sec) , antiderivative size = 1545, normalized size of antiderivative = 5.07

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x*arccot(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arccot(d*tan(b*x + a) + c) - 2*b*x*dilog(2*((I*c*d - d^2 +
d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a)
+ d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1)
+ 2*b*x*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 -
2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a
)^2 + c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(2*((-I*c*d - d^2 + d)*tan(b*x
 + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/
((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) + 2*b*x*d
ilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d +
 I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*d + 1) + 1) + 2*I*a^2*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 -
c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x +
a)^2 + 1)) - 2*I*a^2*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d +
```

```
(I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) + 2
*I*a^2*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2
 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) - 2*I*a^2*log(((I
*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*
tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - 2*(-I*b^2*x^2 + I*a^2)*log(-2
*((I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 +
 I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
 - 2*d + 1)) - 2*(I*b^2*x^2 - I*a^2)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)
^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2
+ d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(I*b^2*x^2 - I*
a^2)*log(-2*((-I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*
c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 - 2*d + 1)) - 2*(-I*b^2*x^2 + I*a^2)*log(-2*((-I*c*d - d^2 -
d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a)
 - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - I
*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2
 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*
tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + I*polylog(3, ((c^2 - 2*I*c*d - d^2
 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*
tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d +
 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c
*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2
*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) - I*polylog(3, ((c^2 - 2*I*c
*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d
^2 + I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
 - 2*d + 1)))/b^2
```

Sympy [F]

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx=\int x\,\mathrm{acot}\left(c+d\tan\left(a+bx\right)\right)dx$$

```
[In] integrate(x*acot(c+d*tan(b*x+a)),x)
[Out] Integral(x*acot(c + d*tan(a + b*x)), x)
```

Maxima [F]

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx=\int x\,\mathrm{arccot}\left(d\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*arctan2(-(d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, -c*
cos(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/4*x^2*arctan2(-(d - 1)
*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, -c*cos(2*b*x + 2*a) - (d -
1)*sin(2*b*x + 2*a) - c) - 2*b*d*integrate(-(2*(c^2 + d^2 + 1)*x^2*cos(2*b*
x + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x +
 2*a)^2 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a
) - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos
(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^
4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4
*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a
)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4
 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 +
 d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*
a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^
4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*d
^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*
a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx=\int x\,\mathrm{arccot}\left(d\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot(d*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx=\int x\,\mathrm{acot}(c+d\tan(a+b\,x))\,dx$$

```
[In] int(x*acot(c + d*tan(a + b*x)),x)
```

```
[Out] int(x*acot(c + d*tan(a + b*x)), x)
```

3.160 $\int \cot^{-1}(c+d\tan(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 198

$$\int \cot^{-1}(c+d\tan(a+bx))\,dx = x\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) - \frac{\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} + \frac{\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}$$

```
[Out] x*arccot(c+d*tan(b*x+a))-1/2*I*x*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d
))+1/2*I*x*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))-1/4*polylog(2,-
(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b+1/4*polylog(2,-(c+I*(1-d))*exp(2*
I*a+2*I*b*x)/(c+I*(1+d)))/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used

$= \{5276, 2221, 2317, 2438\}$

$$\int \cot^{-1}(c+d\tan(a+bx))\,dx = -\frac{\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b} + \frac{\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b} - \frac{1}{2}ix\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right) + \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right) + x\cot^{-1}(d\tan(a+bx)+c)$$

```
[In] Int[ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Tan[a + b*x]] - (I/2)*x*Log[1 + ((1 + I*c + d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c - d)] + (I/2)*x*Log[1 + ((c + I*(1 - d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 + d))] - PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*
I)*b*x))/(1 + I*c - d))]/(4*b) + PolyLog[2, -(((c + I*(1 - d))*E^((2*I)*a +
 (2*I)*b*x))/(c + I*(1 + d)))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5276

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcC
ot[c + d*Tan[a + b*x]], x] + (-Dist[b*(1 - I*c - d), Int[x*(E^(2*I*a + 2*I*
b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] + Dist[b*(1
 + I*c + d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I
```

```
*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\cot^{-1}(c+d\tan(a+bx)) - (b(1-ic-d))\int \frac{e^{2ia+2ibx}x}{1-ic+d+(1-ic-d)e^{2ia+2ibx}}\,dx \\
&\quad + (b(1+ic+d))\int \frac{e^{2ia+2ibx}x}{1+ic-d+(1+ic+d)e^{2ia+2ibx}}\,dx \\
&= x\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad + \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) - \frac{1}{2}i\int \log\left(1+\frac{(1-ic-d)e^{2ia+2ibx}}{1-ic+d}\right)dx \\
&\quad + \frac{1}{2}i\int \log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)dx \\
&= x\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad + \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{(1-ic-d)x}{1-ic+d}\right)}{x}\,dx, x, e^{2ia+2ibx}\right)}{4b} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{(1+ic+d)x}{1+ic-d}\right)}{x}\,dx, x, e^{2ia+2ibx}\right)}{4b} \\
&= x\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad + \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad - \frac{\text{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} + \frac{\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 549 vs. $2(198) = 396$.

Time = 0.54 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.77

$$\int \cot^{-1}(c+d\tan(a+bx))\,dx = x\cot^{-1}(c+d\tan(a+bx))$$

$$+\frac{x\left(4a\sqrt{-d^2}\arctan(c+d\tan(a+bx)) - id\log(1-i\tan(a+bx))\log\left(\frac{-cd+\sqrt{-d^2}-d^2\tan(a+bx)}{-cd+id^2+\sqrt{-d^2}}\right) + id\log(1+\right.}{}$$

```
[In] Integrate[ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Tan[a + b*x]] + (x*(4*a*Sqrt[-d^2]*ArcTan[c + d*Tan[a + b*x]
] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - d^2*Tan[a + b*x]
)/(-(c*d) + I*d^2 + Sqrt[-d^2])] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - S
qrt[-d^2] + d^2*Tan[a + b*x])/(c*d + I*d^2 - Sqrt[-d^2])] + I*d*Log[1 - I*T
an[a + b*x]]*Log[(c*d + Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d - I*d^2 + Sqrt[
-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + d^2*Tan[a + b
*x])/(c*d + I*d^2 + Sqrt[-d^2])] - I*d*PolyLog[2, (d^2*(1 - I*Tan[a + b*x])
)/(I*c*d + d^2 - I*Sqrt[-d^2])] + I*d*PolyLog[2, (d^2*(1 - I*Tan[a + b*x]))
/(I*c*d + d^2 + I*Sqrt[-d^2])] + I*d*PolyLog[2, (d^2*(1 + I*Tan[a + b*x]))/
((-I)*c*d + d^2 + I*Sqrt[-d^2])] - I*d*PolyLog[2, (d^2*(1 + I*Tan[a + b*x])
)/(d^2 - I*(c*d + Sqrt[-d^2]))]))/(2*Sqrt[-d^2]*(2*a - I*Log[1 - I*Tan[a +
b*x]] + I*Log[1 + I*Tan[a + b*x]]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(168) = 336$.

Time = 4.35 (sec) , antiderivative size = 1138, normalized size of antiderivative = 5.75

method	result	size
derivativedivides	Expression too large to display	1138
default	Expression too large to display	1138
risch	Expression too large to display	4969

```
[In] int(arccot(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(d*arctan(tan(b*x+a))*arccot(c+d*tan(b*x+a))+d^2*(-1/d*arctan(d*((c+d
*tan(b*x+a))/d-c/d)+c)*arctan(-(c+d*tan(b*x+a))/d+c/d)-1/d^2*(1/2*I*d^2*ln(
1-(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*tan(b*x+a))/d-
c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)/(1+I*c+d)+1/2*
I*d*ln(1-(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*tan(b*x
+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)/(1+I*c+
d)+1/2*I*d/(c-I*d-I)*ln(1-(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/
((d*((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c*arctan(d*((c+d*tan(b*x+a)
)/d-c/d)+c)+1/2*d^2*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)^2/(1+I*c+d)+1/4*d^
2*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*tan(
b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/2*d*arctan(d*((c+d*tan(b*x+a)
)/d-c/d)+c)^2/(1+I*c+d)+1/2*d/(c-I*d-I)*c*arctan(d*((c+d*tan(b*x+a))/d-c/d)
+c)^2+1/4*d*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*
((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/4*d/(c-I*d-I)*polyl
og(2,(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*tan(b*x+a))
/d-c/d)+c)^2+1)/(I*d+I-c))*c-1/2*I*d*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)*l
n(1-(I+c+I*d)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*tan(b*x+a))/
```

```
d-c/d)+c)^2+1)/(-I*d+I-c))-1/2*d*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)^2-1/4
*d*polylog(2,(I+c+I*d)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*tan
(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1101 vs. 2(141) = 282.

Time = 0.34 (sec) , antiderivative size = 1101, normalized size of antiderivative = 5.56

$$\int \cot^{-1}(c + d\tan(a + bx))\,dx = \text{Too large to display}$$

```
[In] integrate(arccot(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arccot(d*tan(b*x + a) + c) - 2*(-I*b*x - I*a)*log(-2*((I*c*d - d
^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x
+ a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1))
 - 2*(I*b*x + I*a)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
 (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(I*b*x + I*a)*log(-2*((-I*c*d - d
^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x
 + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)
) - 2*(-I*b*x - I*a)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*
d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*I*a*log(((I*c*d + d^2 + d)*ta
n(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d -
 1)/(tan(b*x + a)^2 + 1)) + 2*I*a*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c
^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a
)^2 + 1)) - 2*I*a*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*
c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) + 2*I*
a*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*
I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - dilog(2*((I*c*d - d^
2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x +
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
 1) + dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*
c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 + 2*d + 1) + 1) - dilog(2*((-I*c*d - d^2 + d)*tan(b*x + a)^2
- c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/((c^2 +
d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) + dilog(2*((-I*c*
d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*ta
n(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d
 + 1) + 1))/b
```

Sympy [F]

$$\int \cot^{-1}(c + d\tan(a + bx))\,dx = \int \operatorname{acot}\left(c + d\tan\left(a + bx\right)\right)dx$$

```
[In] integrate(acot(c+d*tan(b*x+a)),x)
[Out] Integral(acot(c + d*tan(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(141) = 282$.

Time = 0.32 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.19

$$\int \cot^{-1}(c + d\tan(a + bx))\,dx = -\frac{d\left(\frac{8\,(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{d} - \frac{4\,(bx+a)\arctan\left(\frac{cd+\left(d^2+d\right)\tan(bx+a)}{c^2+d^2+2\,d+1}, \frac{cd\tan(bx+a)+c^2+d+1}{c^2+d^2+2\,d+1}\right) - 4\,(bx+a)\arctan\left(\frac{cd+\left(d^2-d\right)\tan(\ldots}{c^2+d^2-2\,d\ldots}\right.}{\ldots}\right.}{\ldots}$$

```
[In] integrate(arccot(c+d*tan(b*x+a)),x, algorithm="maxima")
[Out] -1/8*(d*(8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d)/d - (4*(b*x + a)*ar
ctan2((c*d + (d^2 + d)*tan(b*x + a))/(c^2 + d^2 + 2*d + 1), (c*d*tan(b*x +
a) + c^2 + d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(b*x + a)*arctan2((c*d + (d^2
- d)*tan(b*x + a))/(c^2 + d^2 - 2*d + 1), (c*d*tan(b*x + a) + c^2 - d + 1)/
(c^2 + d^2 - 2*d + 1)) + log(tan(b*x + a)^2 + 1)*log((d^2*tan(b*x + a)^2 +
2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) - log(tan(b*x + a)^2 +
 1)*log((d^2*tan(b*x + a)^2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 - 2*
d + 1)) + 2*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + 1)) - 2*dilog(-(I*d*ta
n(b*x + a) - d)/(I*c + d - 1)) + 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
 1)) - 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - 1)))/d) - 8*(b*x + a)*arc
cot(d*tan(b*x + a) + c) - 8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d))/b
```

Giac [F]

$$\int \cot^{-1}(c + d\tan(a + bx))\,dx = \int \operatorname{arccot}\left(d\tan\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(arccot(c+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot(d*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c + d\tan(a + bx))\,dx = \int \operatorname{acot}(c + d\tan(a + b\,x))\,dx$$

```
[In] int(acot(c + d*tan(a + b*x)),x)
[Out] int(acot(c + d*tan(a + b*x)), x)
```

3.161 $\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+d*tan(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c + d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x} \, dx = \int \frac{\cot^{-1}(c+d\tan(a+bx))}{x} \, dx$$

```
[In] Integrate[ArcCot[c + d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + d*Tan[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}(c+d\tan(bx+a))}{x}dx$$

```
[In] int(arccot(c+d*tan(b*x+a))/x,x)
[Out] int(arccot(c+d*tan(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x} \, dx = \int \frac{\operatorname{arccot}(d\tan(bx+a)+c)}{x} \, dx$$

```
[In] integrate(arccot(c+d*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arccot(d*tan(b*x + a) + c)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x} \, dx = \text{Timed out}$$

```
[In] integrate(acot(c+d*tan(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 227.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*tan(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arccot(d*tan(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot(d*tan(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{acot}(c+d\tan(a+b\,x))}{x}\,dx$$

```
[In] int(acot(c + d*tan(a + b*x))/x,x)
[Out] int(acot(c + d*tan(a + b*x))/x, x)
```

3.162 $\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) + \frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2} - \frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arccot(c+(1+I*c)*tan(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*I*
a+2*I*b*x))+1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(3,I
*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5280, 2215, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = -\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}+\frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}+\frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{bx^4}{12}$$

```
[In] Int[x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCot[c + (1 + I*c)*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - PolyLog[
4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5280

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{3}(bc)\int\frac{e^{2ia+2ibx}x^3}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))\\
&\quad+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{1}{2}i\int x^2\log\left(1+\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad+\frac{x^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{\int x\operatorname{PolyLog}\left(2,-\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx}{2b}\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))\\
&\quad+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{x^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}\\
&\quad+\frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}-\frac{i\int\operatorname{PolyLog}\left(3,-\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx}{4b^2}\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad+\frac{x^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}\\
&\quad-\frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,icx)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^3}
\end{aligned}
$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3 \cot^{-1}(c + (1+ic)\tan(a+bx)) + \frac{1}{6}ix^3 \log\left(1 - ice^{2ia+2ibx}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, ice^{2ia+2ibx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int x^2 \cot^{-1}(c + (1+ic)\tan(a+bx))\, dx = \frac{1}{24}\left(8x^3 \cot^{-1}(c + (1+ic)\tan(a+bx)) + 4ix^3 \log\left(1 + \frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2 \operatorname{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix \operatorname{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3 \operatorname{PolyLog}\left(4, -\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)$$

```
[In] Integrate[x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCot[c + (1 + I*c)*Tan[a + b*x]] + (4*I)*x^3*Log[1 + I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)
*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, (-I)/(c*E^
((2*I)*(a + b*x)))])/b^3)/24
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.88 (sec) , antiderivative size = 1448, normalized size of antiderivative = 9.40

method	result	size
risch	Expression too large to display	1448

```
[In] int(x^2*arccot(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*x*polylog(3,I*exp(2*I*(b*x+a))*c)/b^2+1/12*(Pi*csgn(I/(exp(2*I*(b*x+a
))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*
(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(
exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*
```

```
(b*x+a))+1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I
)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a
)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+Pi*csgn(I*exp(2*
I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1)
)*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I*exp(2*I*(b*
x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn(I*(exp
(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2
*I*(b*x+a))+1))+Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-Pi*c
sgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*(exp(2*I*(b*x+a
))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*ex
p(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp
(2*I*(b*x+a))+1))+Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+Pi
*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3-Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))
*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn(I*(exp(2*I*(
b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(
2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-Pi*csg
n((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*exp(2*I*(b*x+a))
*(c-I)/(exp(2*I*(b*x+a))+1))^3-Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b
*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn(exp(
2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+2*I*ln(c-I))*x^3-1/3*I/b^3*ln(1-
I*exp(2*I*(b*x+a))*c)*a^3+1/3*I*x^3*ln(exp(I*(b*x+a)))+1/2*I/b^3*a^3*ln(1-I
*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1
/2))+1/4*x^2*polylog(2,I*exp(2*I*(b*x+a))*c)/b-1/4/b^3*polylog(2,I*exp(2*I*
(b*x+a))*c)*a^2+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/8*pol
ylog(4,I*exp(2*I*(b*x+a))*c)/b^3-1/2*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*x*a^2
-1/6*I/b^3*a^3*ln(exp(2*I*(b*x+a))*c+I)+1/6*I*x^3*ln(1-I*exp(2*I*(b*x+a))*c
)+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/6*I*x^3*ln(exp(2*I*
(b*x+a))*c+I)+1/2/b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2/b^3*a^
2*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/12*b*x^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(107) = 214$.

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.08

$$\int x^2 \cot^{-1}(c + (1 + ic)\tan(a + bx))\,dx$$
$$= \frac{b^4x^4 - 2i\,b^3x^3\log\left(\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right)}{}$$

```
[In] integrate(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*I*b^3*x^3*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2
*I*a)/(c - I)) + 6*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*b^2*x
```

```
^2*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e^(
I*b*x + I*a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a) - I
*sqrt(4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 1
2*I*b*x*polylog(3, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - 2*(-I*b^3*x^3 - I*a^
3)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1
/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(4*I*c)*e^(I*b*
x + I*a)) - 12*polylog(4, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*acot(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*I + 2*c*exp(2*I*a
) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp
(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(107) = 214$.

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.01

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{\frac{4\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{arccot}((ic+1)\tan(bx+a)+c)}{b^2} - \frac{\left(-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(4i(bx+a)^3-9i(bx+a\right.\right.}{}}{}$$

```
[In] integrate(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccot((I*c + 1)*
tan(b*x + a) + c)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*ar
ctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*
I*(b*x + a)*a + 3*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 -
9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2*b
*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^(2
*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(-I*c - 1)/(b^2
*(c - I)))/b
```

Giac [F]

$$\int x^2 \cot^{-1}(c + (1 + ic) \tan(a + bx))\, dx = \int x^2 \operatorname{arccot}\left((i\,c + 1) \tan\left(bx + a\right) + c\right)\, dx$$

```
[In] integrate(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arccot((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + (1 + ic) \tan(a + bx))\, dx = \int x^2 \operatorname{acot}(c + \tan(a + b\,x)\ (1 + c\,1\mathrm{i}))\, dx$$

```
[In] int(x^2*acot(c + tan(a + b*x)*(c*1i + 1)),x)
[Out] int(x^2*acot(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.163 $\int x \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}$$

```
[Out] 1/6*b*x^3+1/2*x^2*arccot(c+(1+I*c)*tan(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*I*a
+2*I*b*x))+1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,I*c*ex
p(2*I*a+2*I*b*x))/b^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5280, 2215, 2221, 2611, 2320, 6724}

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{bx^3}{6}$$

```
[In] Int[x*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 -
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/
(4*b) + ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5280

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{2}(ib)\int\frac{x^2}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}(bc)\int\frac{e^{2ia+2ibx}x^2}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) - \frac{1}{2}i\int x\log\left(1+\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) + \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} - \frac{\int\text{PolyLog}\left(2,-\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx}{4b}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) + \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,icx)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^2}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) + \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

```
[In] Integrate[x*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.55 (sec) , antiderivative size = 1413, normalized size of antiderivative = 11.49

method	result	size
risch	Expression too large to display	1413

```
[In] int(x*arccot(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/b^2*polylog(3,I*exp(2*I*(b*x+a))*c)+1/8*(Pi*csgn(I/(exp(2*I*(b*x+a))+
1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(b*
x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp
(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*(b*
x+a))+1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(
exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))
-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+Pi*csgn(I*exp(2*I*(
b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*c
sgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I*exp(2*I*(b*x+a
)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn(I*(exp(2*
I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*
(b*x+a))+1))+Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn
(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*(exp(2*I*(b*x+a))*
c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*exp(2
*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*
I*(b*x+a))+1))+Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+Pi*cs
gn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3-Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*cs
gn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn(I*(exp(2*I*(b*x
+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I
*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn((
exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*exp(2*I*(b*x+a))*(c
-I)/(exp(2*I*(b*x+a))+1))^3-Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+
a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn(exp(2*I
*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+2*I*ln(c-I))*x^2+1/2*I/b*ln(1-I*exp
(2*I*(b*x+a))*c)*a*x-1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/2*I/
b*a*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/2*I*x^2*ln(exp(I*(b*x+a)))+1/4/
b*polylog(2,I*exp(2*I*(b*x+a))*c)*x+1/4/b^2*polylog(2,I*exp(2*I*(b*x+a))*c)
*a+1/4*I*ln(1-I*exp(2*I*(b*x+a))*c)*x^2-1/4*I*ln(exp(2*I*(b*x+a))*c+I)*x^2-
1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln(1-I*exp(
I*(b*x+a))*(-I*c)^(1/2))+1/4*I/b^2*a^2*ln(exp(2*I*(b*x+a))*c+I)+1/4*I/b^2*l
n(1-I*exp(2*I*(b*x+a))*c)*a^2-1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/
2))-1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/6*b*x^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(85) = 170.

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.20

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{2\,b^3x^3 - 3i\,b^2x^2\log\left(\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 2\,a^3 + 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(i\,bx\cdots}\right.}{}$$

```
[In] integrate(x*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*I*b^2*x^2*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x -
 2*I*a)/(c - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6
*b*x*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*b*
x + I*a) + I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*sqr
t(4*I*c))/c) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) +
 1) - 3*(-I*b^2*x^2 + I*a^2)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) + 6*
I*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*sqrt(4*
I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*acot(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*I + 2*c*exp(2*I*a
) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp
(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(85) = 170.

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.77

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{arccot}((i\,c+1)\tan(bx+a)+c)}{b} - \frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right)\cdots\right.}{}}{}$$

```
[In] integrate(x*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arccot((I*c + 1)*tan(b*x + a) + c)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x + 2
*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), c*
sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x +
 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(3,
 I*c*e^(2*I*b*x + 2*I*a)))*(-I*c - 1)/(b*(c - I)))/b
```

Giac [F]

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\int x\,\mathrm{arccot}\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\int x\,\mathrm{acot}(c+\tan(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

```
[In] int(x*acot(c + tan(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(x*acot(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.164 $\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arccot(c+(1+I*c)*tan(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b*x
))+1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5272, 2215, 2221, 2317, 2438}

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) + x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{bx^2}{2}$$

```
[In] Int[ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCot[c + (1 + I*c)*Tan[a + b*x]] + (I/2)*x*Log[1 - I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5272

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcC
ot[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\cot^{-1}(c+(1+ic)\tan(a+bx))+(ib)\int\frac{x}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))-(bc)\int\frac{e^{2ia+2ibx}x}{i(1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))\\
&\quad+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{1}{2}i\int\log\left(1+\frac{ce^{2ia+2ibx}}{i(1+ic)+c}\right)dx\\
&= \frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad-\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{cx}{i(1+ic)+c}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{4b}\\
&= \frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 967 vs. $2(85) = 170$.

Time = 4.19 (sec) , antiderivative size = 967, normalized size of antiderivative = 11.38

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = x\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$-\frac{}{((i+c)\cos(a+bx)+(1+ic)\sin(a+bx))\left(2bx-i\log\left(1-\frac{\sec(bx)((-i+c)\cos(a)+i(i+c)\sin(a))(\cos(a+bx)-i\sin(a\right.\right.}{2c}}$$

```
[In] Integrate[ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (1 + I*c)*Tan[a + b*x]] - (I*x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[
b*x] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a + b*
x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*((
1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*
Log[1 + I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (
Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x
]))/(2*c)] + PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] - (I + c)*Sin[a])*(Cos[
a + b*x] + I*Sin[a + b*x]))/2])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*C
os[b*x] + Sin[b*x])*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(((I
+ c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x])*(2*b*x - I*Log[1 - (Sec[b*x]*((
-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]
- I*Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] +
I*Sin[a + b*x]))/2] - (I*(-I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1
 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]) + ((I + c)
*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((I + c)*Cos[a +
 b*x] + (1 + I*c)*Sin[a + b*x]) - (2*I)*b*x*Tan[b*x] + Log[1 - (Sec[b*x]*((
-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*
Tan[b*x] - Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a +
b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - Log[1 - I*Tan[b*x]]*Tan[b*x] + Cos[a]
^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (
Log[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] -
 (2*I)*Sin[a])]*Sec[b*x]^2)/(-I + Tan[b*x]) - (Log[(Sec[b*x]*(Cos[a] - I*Si
n[a])*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(
I + Tan[b*x]))*(-I + Tan[a + b*x])*(1 - I*c + (-I + c)*Tan[a + b*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(69) = 138.

Time = 2.06 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.61

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c^2}{2i-2c}-\frac{2i\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c}{2i-2c}-\frac{\operatorname{arccot}(c+(}{}}{}$
default	$\frac{\frac{\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c^2}{2i-2c}-\frac{2i\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c}{2i-2c}-\frac{\operatorname{arccot}(c+(}{}}{}$
risch	Expression too large to display

```
[In] int(arccot(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I*c+1)*(arccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+a
)+I)*c^2-2*I*arccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+a
)+I)*c-arccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+a)+I)-a
rccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))*c^2+2*I*a
rccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))*c+arccot(
c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))+(I*c+1)^2*(1/2/
(I-c)*(-1/4*I*ln(-I+c+(I*c+1)*tan(b*x+a))^2+1/2*I*(dilog(-1/2*I*(c+(I*c+1)*
tan(b*x+a)+I))+ln(-I+c+(I*c+1)*tan(b*x+a))*ln(-1/2*I*(c+(I*c+1)*tan(b*x+a)+
I))))-1/2/(I-c)*(1/2*I*(dilog(1/2*(c+(I*c+1)*tan(b*x+a)+I)/c)+ln(-c+(I*c+1)
*tan(b*x+a)+I)*ln(1/2*(c+(I*c+1)*tan(b*x+a)+I)/c))-1/2*I*(dilog((-I+c+(I*c+
1)*tan(b*x+a))/(-2*I+2*c))+ln(-c+(I*c+1)*tan(b*x+a)+I)*ln((-I+c+(I*c+1)*tan
(b*x+a))/(-2*I+2*c))))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(60) = 120.

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.34

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$$
$$=\frac{b^2x^2-i\,bx\log\left(\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right)-a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}+1\right)+(i\,bx+i\,a)\log\left(-\right.}{}$$

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - I*b*x*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(
c - I)) - a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) + (I
*b*x + I*a)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*c*e^
(I*b*x + I*a) + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) - I*sq
rt(4*I*c))/c) + dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sqrt(4*
I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c + (1 + ic)\tan(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(acot(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*I + 2*c*exp(2*I*a
) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(I
*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(60) = 120.

Time = 0.29 (sec) , antiderivative size = 455, normalized size of antiderivative = 5.35

$$\int \cot^{-1}(c + (1 + ic)\tan(a + bx))\,dx = -\frac{(-ic-1)\left(\frac{4i(bx+a)\log\left(-\frac{2\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)}{2ic^2-2\left(c^2-2ic-1\right)\tan(bx+a)+2i}\right)}{ic+1} - \frac{i\left(4(bx+a)(\log(-ic^2+(c^2-2ic-1)\tan(bx+a)-2c+i)-\log(-\right.}{}\right.}{}$$

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((-I*c - 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x
+ a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/(I*c +
1) - I*(4*(b*x + a)*(log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c + I)
 - log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2
- 2*I*c - 1)*tan(b*x + a) - 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)
*tan(b*x + a) - I)*log(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) + 2*I*log
(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c + 1)*tan(b*x +
 a) + c + I)/c + 1) - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c
 + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(1/2*(c - I)*tan(b*x + a) -
```

```
1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c + 1)*tan(b*x + a) + c + I)/c) - 2*I*d
ilog(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)) - 8*(b*x + a)*arccot((I*c + 1)*t
an(b*x + a) + c) - 4*(b*x + a)*(c - I)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*t
an(b*x + a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/
(I*c + 1))/b
```

Giac [F]

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\int \operatorname{arccot}\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\int \operatorname{acot}\left(c+\tan\left(a+b\,x\right)\,\left(1+c\,1\mathrm{i}\right)\right)\,dx$$

```
[In] int(acot(c + tan(a + b*x)*(c*1i + 1)),x)
[Out] int(acot(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.165 $\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccot}\left(c+(ic+1)\tan\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arccot(c+(I*c+1)*tan(b*x+a))/x,x)
[Out] int(arccot(c+(I*c+1)*tan(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left(\left(i\,c+1\right)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(-1/2*I*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(acot(c+(1+I*c)*tan(b*x+a))/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot((I*c + 1)*tan(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{acot}(c+\tan(a+b\,x)\ (1+c\,1\mathrm{i}))}{x}\,dx$$

```
[In] int(acot(c + tan(a + b*x)*(c*1i + 1))/x,x)
[Out] int(acot(c + tan(a + b*x)*(c*1i + 1))/x, x)
```

3.166 $\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) -\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) -\frac{x^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} -\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} +\frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}$$

```
[Out] -1/12*b*x^4+1/3*x^3*arccot(c-(1-I*c)*tan(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*I
*a+2*I*b*x))-1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog(3
,-I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{5280, 2215, 2221, 2611, 6744, 2320, 6724\}$

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \frac{\text{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3} - \frac{ix\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} - \frac{x^2\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{bx^4}{12}$$

```
[In] Int[x^2*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcCot[c - (1 - I*c)*Tan[a + b*x]])/3 - (I/6)*x^3*Log[
1 + I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I
)*b*x)])/(4*b) - ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 +
 PolyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5280

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{3}(bc)\int\frac{e^{2ia+2ibx}x^3}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))\\
&\quad -\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}i\int x^2\log\left(1+\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)dx\\
&= -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad -\frac{x^2\,\mathrm{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{\int x\,\mathrm{PolyLog}\left(2,-\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)dx}{2b}
\end{aligned}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))$$
$$-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) - \frac{x^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$
$$-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} + \frac{i\int\operatorname{PolyLog}\left(3,-\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)dx}{4b^2}$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)$$
$$-\frac{x^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}$$
$$+\frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-icx)}{x}dx,x,e^{2ia+2ibx}\right)}{8b^3}$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))$$
$$-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) - \frac{x^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$
$$-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.91

$$\int x^2\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+i(i+c)\tan(a+bx))$$
$$-\frac{4ib^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx\operatorname{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right) + 3\operatorname{PolyLog}\left(\right.}{24b^3}$$

```
[In] Integrate[x^2*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + I*(I + c)*Tan[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 - I/(c*E^
((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] + (6*
I)*b*x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, I/(c*E^((2*I)*(
a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.06 (sec) , antiderivative size = 1449, normalized size of antiderivative = 9.35

method	result	size
risch	Expression too large to display	1449

```
[In] int(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/12*(Pi*csgn(I/(exp(2*I*
(b*x+a))+1))*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))-Pi*csgn(I/(ex
p(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*
c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*
(b*x+a))+1)*(I+c))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a
))*c-I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*
(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+Pi*csgn(I*
exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))+1)
*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I*exp(2
*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(
I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/
(exp(2*I*(b*x+a))+1))-Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^
2-Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x
+a))*c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*
x+a))+1))^2-Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2+Pi*csgn(I
*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1)
)^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^3-Pi*csgn(I/(exp(2*I*(b*x+a))+1)*
(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn(I*(exp
(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)
/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+
Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-Pi*csgn(I*exp(2*I*
(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(
b*x+a))+1))^2+Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+Pi*csg
n(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3+2*I*ln(I+c))*x^3+1/6*I/b^3
*a^3*ln(-exp(2*I*(b*x+a))*c+I)+1/3*I/b^3*ln(I*exp(2*I*(b*x+a))*c+1)*a^3-1/2
*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/2*I/b^2*ln(I*exp(2*I*(b*x
+a))*c+1)*x*a^2-1/4*x^2*polylog(2,-I*exp(2*I*(b*x+a))*c)/b+1/4/b^3*polylog(
2,-I*exp(2*I*(b*x+a))*c)*a^2-1/3*I*x^3*ln(exp(I*(b*x+a)))+1/8*polylog(4,-I*
exp(2*I*(b*x+a))*c)/b^3+1/6*I*x^3*ln(exp(2*I*(b*x+a))*c-I)-1/6*I*x^3*ln(I*e
xp(2*I*(b*x+a))*c+1)-1/4*I*x*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2-1/2*I/b^2
*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a
))*(I*c)^(1/2))-1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/2/b^3*a
^2*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/12*b*x^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(108) = 216$.

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.07

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx =$$

$$-\frac{b^4x^4 + 2i\,b^3x^3\log\left(\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) - a^4}{}$$

```
[In] integrate(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(b^4*x^4 + 2*I*b^3*x^3*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x
+ 2*I*a) - I)) + 6*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*b^2*
x^2*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a)
- I*sqrt(-4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)
) + 12*I*b*x*polylog(3, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 2*(I*b^3*x^3 +
 I*a^3)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) + 2*(I*b^3*x^3 + I*a^3)*l
og(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(-4*I*c)*
e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*acot(c-(1-I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 - 3*_t0**2*I*c*e
xp(2*I*a) + _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(108) = 216$.

Time = 0.20 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.01

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx =$$

$$-\frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arccot}((-i\,c+1)\tan(bx+a)-c)}{b^2} + \frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\right.\right.}{}}{}$$

```
[In] integrate(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccot((-I*c + 1
)*tan(b*x + a) - c)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)
*arctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2
- 6*I*(b*x + a)*a + 3*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)
^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*si
n(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*
c*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1
)/(b^2*(c + I)))/b
```

Giac [F]

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \int x^2 \operatorname{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(-(-I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \int x^2 \operatorname{acot}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

```
[In] int(x^2*acot(c + tan(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x^2*acot(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.167 $\int x \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) - \frac{x\,\mathrm{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} - \frac{i\,\mathrm{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}$$

```
[Out] -1/6*b*x^3+1/2*x^2*arccot(c-(1-I*c)*tan(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*I*
a+2*I*b*x))-1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,-I*c
*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5280, 2215, 2221, 2611, 2320, 6724}

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{i\,\mathrm{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2} - \frac{x\,\mathrm{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} - \frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) + \frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{bx^3}{6}$$

```
[In] Int[x*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcCot[c - (1 - I*c)*Tan[a + b*x]])/2 - (I/4)*x^2*Log[1
 + I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) - ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5280

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}(bc)\int\frac{e^{2ia+2ibx}x^2}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))\\
&\quad -\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}i\int x\log\left(1+\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)dx\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad -\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{\int\text{PolyLog}\left(2,-\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)dx}{4b}\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad -\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-icx)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^2}\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad -\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{i\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx\\
&= \frac{1}{2}x^2\cot^{-1}(c+i(i+c)\tan(a+bx))\\
&\quad -\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}
\end{aligned}$$

```
[In] Integrate[x*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + I*(I + c)*Tan[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.64 (sec) , antiderivative size = 1414, normalized size of antiderivative = 11.40

method	result	size
risch	Expression too large to display	1414

```
[In] int(x*arccot(c-(1-I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/8*(Pi*csgn(I/(exp(2*I*(b
*x+a))+1))*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))-Pi*csgn(I/(exp(
2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-
I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b
*x+a))+1)*(I+c))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))
*c-I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b
*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+Pi*csgn(I*ex
p(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))+1)*(
I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I*exp(2*I
*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*
exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(e
xp(2*I*(b*x+a))+1))-Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-
Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a
))*c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+
a))+1))^2-Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2+Pi*csgn(I*(
exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^
2+Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^3-Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I
+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn(I*(exp(2
*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(
exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+Pi
*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-Pi*csgn(I*exp(2*I*(b
*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*
x+a))+1))^2+Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(
exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3+2*I*ln(I+c))*x^2-1/4*I/b^2*a
^2*ln(-exp(2*I*(b*x+a))*c+I)+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2
))-1/4*I/b^2*ln(I*exp(2*I*(b*x+a))*c+1)*a^2-1/4*I*ln(I*exp(2*I*(b*x+a))*c+1
)*x^2-1/4/b*polylog(2,-I*exp(2*I*(b*x+a))*c)*x-1/4/b^2*polylog(2,-I*exp(2*I
*(b*x+a))*c)*a-1/2*I*x^2*ln(exp(I*(b*x+a)))-1/8*I/b^2*polylog(3,-I*exp(2*I*
(b*x+a))*c)+1/4*I*x^2*ln(exp(2*I*(b*x+a))*c-I)+1/2*I/b*a*ln(1-I*exp(I*(b*x+
a))*(I*c)^(1/2))*x+1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/2*I/b*l
n(I*exp(2*I*(b*x+a))*c+1)*a*x+1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2
))+1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/6*b*x^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(86) = 172.

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.18

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{2\,b^3x^3+3i\,b^2x^2\log\left(\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)+2\,a^3+6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right)+6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right)}{}$$

```
[In] integrate(x*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*x^3 + 3*I*b^2*x^2*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) +
6*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*
sqrt(-4*I*c))/c) + 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*
a) + 1) + 3*(I*b^2*x^2 - I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1)
+ 6*I*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*sq
rt(-4*I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*acot(c-(1-I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 - 3*_t0**2*I*c*e
xp(2*I*a) + _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(86) = 172.

Time = 0.20 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.77

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{arccot}((-i\,c+1)\tan(bx+a)-c)}{b}+\frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx\right.\right.}{}}{}$$

```
[In] integrate(x*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arccot((-I*c + 1)*tan(b*x + a) - c)/
b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x
+ 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a)
, -c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*
b*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polyl
og(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(c + I)))/b
```

Giac [F]

$$\int x \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \int x \operatorname{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot(-(-I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \int x \operatorname{acot}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

```
[In] int(x*acot(c + tan(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x*acot(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.168 $\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{bx^2}{2}+x\cot^{-1}(c-(1-ic)\tan(a+bx)) -\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] -1/2*b*x^2+x*arccot(c-(1-I*c)*tan(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b*
x))-1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5272, 2215, 2221, 2317, 2438}

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) +x\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{bx^2}{2}$$

```
[In] Int[ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcCot[c - (1 - I*c)*Tan[a + b*x]] - (I/2)*x*Log[1 + I*c*E
^((2*I)*a + (2*I)*b*x)] - PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5272

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcC
ot[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\cot^{-1}(c-(1-ic)\tan(a+bx))+(ib)\int\frac{x}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^2}{2}+x\cot^{-1}(c-(1-ic)\tan(a+bx))+(bc)\int\frac{e^{2ia+2ibx}x}{i(-1+ic)+c+ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^2}{2}+x\cot^{-1}(c-(1-ic)\tan(a+bx))\\
&\quad -\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}i\int\log\left(1+\frac{ce^{2ia+2ibx}}{i(-1+ic)+c}\right)\,dx\\
&= -\frac{bx^2}{2}+x\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad +\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{cx}{i(-1+ic)+c}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{4b}\\
&= -\frac{bx^2}{2}+x\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 847 vs. $2(86) = 172$.

Time = 2.35 (sec) , antiderivative size = 847, normalized size of antiderivative = 9.85

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = x\cot^{-1}(c+i(i+c)\tan(a+bx)) - \frac{ix\left(-2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+\log\left(\frac{\sec(bx)(\cos(a)\cdots}{}\right.\right.}{((-i+c)\cos(a+bx)+i(i+c)\sin(a+bx))\left(-2bx+i\log\left(1-\frac{\sec(bx)((i+c)\cos(a)+(1+ic)\sin(a))(\cos(a+bx)-i\sin\cdots}{2c}\right.\right.}$$

```
[In] Integrate[ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcCot[c + I*(I + c)*Tan[a + b*x]] - (I*x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos
[b*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a +
b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*x]*
(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]*Log
[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (Sec
[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/
(2*c)] - PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a]
)*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + S
in[b*x]))/(((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x])*(-2*b*x + I*Log
[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a
+ b*x]))/(2*c)] + (I*(I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*
Tan[b*x]]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) + ((1 + I*c)*(
Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-1 - I*c)*Cos[a
+ b*x] + (I + c)*Sin[a + b*x]) + (2*I)*b*x*Tan[b*x] - Log[1 - (Sec[b*x]*((I
 + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Ta
n[b*x] + Log[1 - I*Tan[b*x]]*Tan[b*x] - Log[1 + I*Tan[b*x]]*Tan[b*x] - (Log
[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*
x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*((I + c
)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]
) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a + b*x] + I*(I + c)*S
in[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I + Tan[a + b*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(70) = 140.

Time = 2.14 (sec) , antiderivative size = 595, normalized size of antiderivative = 6.92

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\operatorname{arccot}(c+(ic-}{}}{}$
default	$\frac{\frac{\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\operatorname{arccot}(c+(ic-}{}}{}$
risch	Expression too large to display

```
[In] int(arccot(c-(1-I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(-1+I*c)*(arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b
*x+a))*c^2+2*I*arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(
b*x+a))*c-arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b*x+a
))-arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*c^2-
2*I*arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*c+a
rccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)-(-1+I*c)
^2*(1/2/(I+c)*(1/4*I*ln(I+c+(-1+I*c)*tan(b*x+a))^2-1/2*I*((ln(I+c+(-1+I*c)*
tan(b*x+a))-ln(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a))))*ln(-1/2*I*(I-c-(-1+I*c)*t
an(b*x+a)))-dilog(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))))-1/2/(I+c)*(1/2*I*(dil
og((-I-c-(-1+I*c)*tan(b*x+a))/(-2*I-2*c))+ln(c-(-1+I*c)*tan(b*x+a)+I)*ln((-
I-c-(-1+I*c)*tan(b*x+a))/(-2*I-2*c)))-1/2*I*(dilog(-1/2*(I-c-(-1+I*c)*tan(b
*x+a))/c)+ln(c-(-1+I*c)*tan(b*x+a)+I)*ln(-1/2*(I-c-(-1+I*c)*tan(b*x+a))/c))
)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(61) = 122.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.34

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx =$$

$$-\frac{b^2x^2+i\,bx\log\left(\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)-a^2-(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}+1\right)-(-i\,bx-i\,a)\log\left(-\frac{1}{2}\sqrt{}\right.}{}$$

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 + I*b*x*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a
) - I)) - a^2 - (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) -
(-I*b*x - I*a)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*
c*e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) -
 I*sqrt(-4*I*c))/c) + dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*
sqrt(-4*I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(acot(c-(1-I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 - 3*_t0**2*I*c*e
xp(2*I*a) + _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(I*a)]
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(61) = 122.

Time = 0.29 (sec) , antiderivative size = 450, normalized size of antiderivative = 5.23

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \frac{(ic-1)\left(\frac{4i(bx+a)\log\left(-\frac{2\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)-i\right)}{2ic^2-2\left(c^2+2ic-1\right)\tan(bx+a)-4c-2i}\right)}{ic-1}+\frac{i\left(4(bx+a)\left(\log\left(-ic^2+(c^2+2ic-1)\tan(bx+a)+2c+i\right)-\log\left(-ic^2+(c\right.\right.\right.}{}\right.}{}$$

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*((I*c - 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x +
a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I))/(I*c - 1)
 + I*(4*(b*x + a)*(log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c + I) -
 log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2 +
2*I*c - 1)*tan(b*x + a) + 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*t
an(b*x + a) - I)*log(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) + 2*I*log(-I
*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c - 1)*tan(b*x + a)
 + c - I)/c + 1) - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c +
I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(-1/2*(c + I)*tan(b*x + a) + 1
```

```
/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c - 1)*tan(b*x + a) + c - I)/c) - 2*I*dil
og(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)) - 8*(b*x + a)*arccot((-I*c + 1)*ta
n(b*x + a) - c) + 4*(-I*b*x - I*a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b
*x + a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I)))/b
```

Giac [F]

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx=\int \operatorname{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot(-(-I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx=\int \operatorname{acot}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

```
[In] int(acot(c + tan(a + b*x)*(c*1i - 1)),x)
[Out] int(acot(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.169 $\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x} dx = \operatorname{Int}\left(\frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x} dx = \int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x} dx$$

```
[In] Int[ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arccot}\left(c-(-ic+1)\tan\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arccot(c-(1-I*c)*tan(b*x+a))/x,x)
[Out] int(arccot(c-(1-I*c)*tan(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(-1/2*I*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) - I)
)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(acot(c-(1-I*c)*tan(b*x+a))/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot(-(-I*c + 1)*tan(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \int \frac{\mathrm{acot}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))}{x}\,dx$$

```
[In] int(acot(c + tan(a + b*x)*(c*1i - 1))/x,x)
[Out] int(acot(c + tan(a + b*x)*(c*1i - 1))/x, x)
```

3.170 $\int \cot^{-1}(\cot(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \cot^{-1}(\cot(a+bx))\,dx = \frac{\cot^{-1}(\cot(a+bx))^2}{2b}$$

```
[Out] 1/2*arccot(cot(b*x+a))^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$\int \cot^{-1}(\cot(a+bx))\,dx = \frac{\cot^{-1}(\cot(a+bx))^2}{2b}$$

```
[In] Int[ArcCot[Cot[a + b*x]],x]
```

```
[Out] ArcCot[Cot[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x\,dx, x, \cot^{-1}(\cot(a+bx))\right)}{b}$$
$$= \frac{\cot^{-1}(\cot(a+bx))^2}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \cot^{-1}(\cot(a+bx))\,dx = -\frac{bx^2}{2} + x\cot^{-1}(\cot(a+bx))$$

```
[In] Integrate[ArcCot[Cot[a + b*x]],x]
[Out] -1/2*(b*x^2) + x*ArcCot[Cot[a + b*x]]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{x^2 b}{2} + x\ \operatorname{arccot}\left(\cot\left(bx+a\right)\right)$
parts	$x\ \operatorname{arccot}\left(\cot\left(bx+a\right)\right) + \frac{-\frac{(bx+a)^2}{2}+(bx+a)a}{b}$
derivativedivides	$\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
default	$\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
risch	$-ix\ln\left(e^{i(bx+a)}\right) - \frac{\pi x\,\operatorname{csgn}\left(ie^{i(bx+a)}\right)^2\operatorname{csgn}\left(ie^{2i(bx+a)}\right)}{4} + \frac{\pi x\ \operatorname{csgn}\left(ie^{i(bx+a)}\right)\operatorname{csgn}\left(ie^{2i(bx+a)}\right)^2}{2} - \frac{\pi x\,\operatorname{csgn}(\cdot$

```
[In] int(arccot(cot(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2*b+x*arccot(cot(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \cot^{-1}(\cot(a+bx))\,dx = \frac{1}{2}x^2b + xa$$

```
[In] integrate(arccot(cot(b*x+a)),x, algorithm="fricas")
[Out] 1/2*x^2*b + x*a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \cot^{-1}(\cot(a+bx))\,dx = \begin{cases} \frac{\operatorname{acot}^2(\cot(a+bx))}{2b} & \text{for } b \neq 0 \\ x\operatorname{acot}(\cot(a)) & \text{otherwise} \end{cases}$$

```
[In] integrate(acot(cot(b*x+a)),x)
[Out] Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \cot^{-1}(\cot(a+bx))\,dx = \frac{1}{2}bx^2 + ax$$

```
[In] integrate(arccot(cot(b*x+a)),x, algorithm="maxima")
[Out] 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \cot^{-1}(\cot(a+bx))\,dx = \frac{1}{2}bx^2 + ax$$

```
[In] integrate(arccot(cot(b*x+a)),x, algorithm="giac")
[Out] 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(\cot(a+bx))\,dx = x\,\mathrm{acot}(\cot(a+b\,x)) - \frac{b\,x^2}{2}$$

```
[In] int(acot(cot(a + b*x)),x)
[Out] x*acot(cot(a + b*x)) - (b*x^2)/2
```

3.171 $\int x^2 \cot^{-1}(c+d\cot(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 399

$$\begin{aligned}\int x^2\cot^{-1}(c+d\cot(a+bx))\,dx=&\ \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx))\\&-\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)\\&+\frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)\\&-\frac{x^2\operatorname{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b}\\&+\frac{x^2\operatorname{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}\\&-\frac{ix\operatorname{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2}\\&+\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}\\&+\frac{\operatorname{PolyLog}\left(4,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^3}\\&-\frac{\operatorname{PolyLog}\left(4,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3}\end{aligned}$$

```
[Out] 1/3*x^3*arccot(c+d*cot(b*x+a))-1/6*I*x^3*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))+1/6*I*x^3*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))-1/4*x
^2*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b+1/4*x^2*polylog(2,(c
+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b-1/4*I*x*polylog(3,(1+I*c-d)*exp
(2*I*a+2*I*b*x)/(1+I*c+d))/b^2+1/4*I*x*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*
```

```
b*x)/(c+I*(1-d)))/b^2+1/8*polylog(4,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))
/b^3-1/8*polylog(4,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5286, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \cot^{-1}(c+d\cot(a+bx))\,dx = \frac{\text{PolyLog}\left(4,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^3} - \frac{\text{PolyLog}\left(4,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3} - \frac{ix\,\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2} + \frac{ix\,\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2} - \frac{x^2\,\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b} + \frac{x^2\,\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} - \frac{1}{6}ix^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right) + \frac{1}{6}ix^3\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right) + \frac{1}{3}x^3\cot^{-1}(d\cot(a+bx)+c)$$

```
[In] Int[x^2*ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Cot[a + b*x]])/3 - (I/6)*x^3*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] + (I/6)*x^3*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] - (x^2*PolyLog[2, ((1 + I*c - d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) + (x^2*PolyLog[2, ((c + I*(1 +
d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) - ((I/4)*x*PolyLog[3,
((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 + ((I/4)*x*Poly
Log[3, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2 + Po
lyLog[4, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)]/(8*b^3) - P
olyLog[4, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]/(8*b^3
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5286

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (-Dist[b*((1 + I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 + Dist[b*((1 - I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*
I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) \\
&\quad -\frac{1}{3}(b(1+ic-d))\int\frac{e^{2ia+2ibx}x^3}{1+ic+d+(-1-ic+d)e^{2ia+2ibx}}\,dx \\
&\quad +\frac{1}{3}(b(1-ic+d))\int\frac{e^{2ia+2ibx}x^3}{1-ic-d+(-1+ic-d)e^{2ia+2ibx}}\,dx \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad +\frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\
&\quad -\frac{1}{2}i\int x^2\log\left(1+\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)dx \\
&\quad +\frac{1}{2}i\int x^2\log\left(1+\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)dx \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad +\frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)-\frac{x^2\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\
&\quad +\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}-\frac{\int x\,\text{PolyLog}\left(2,-\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)dx}{2b} \\
&\quad +\frac{\int x\,\text{PolyLog}\left(2,-\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)dx}{2b} \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad +\frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)-\frac{x^2\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\
&\quad +\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}-\frac{ix\,\text{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2} \\
&\quad +\frac{ix\,\text{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}-\frac{i\int\text{PolyLog}\left(3,-\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)dx}{4b^2} \\
&\quad +\frac{i\int\text{PolyLog}\left(3,-\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)dx}{4b^2}
\end{aligned}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) - \frac{x^2\operatorname{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} + \frac{x^2\operatorname{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2} + \frac{ix\operatorname{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2} + \frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,-\frac{(-1-ic+d)x}{1+ic+d}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^3} - \frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,\frac{(c+i(1+d))x}{c-i(-1+d)}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^3}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) - \frac{x^2\operatorname{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} + \frac{x^2\operatorname{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2} + \frac{ix\operatorname{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^3} - \frac{\operatorname{PolyLog}\left(4,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.90

$$\int x^2\cot^{-1}(c+d\cot(a+bx))\,dx = \frac{8b^3x^3\cot^{-1}(c+d\cot(a+bx)) - 4ib^3x^3\log\left(1+\frac{(-c+i(1+d))e^{-2i(a+bx)}}{c+i(-1+d)}\right) + 4ib^3x^3\log\left(1+\frac{(-c+i(-1+d))e^{-2i(a+bx)}}{c+i(1+d)}\right)}{}$$

```
[In] Integrate[x^2*ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcCot[c + d*Cot[a + b*x]] - (4*I)*b^3*x^3*Log[1 + (-c + I*(1 +
d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + (4*I)*b^3*x^3*Log[1 + (-c + I
*(-1 + d))/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, (c
 - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2
```

```
, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - (6*I)*b*x*PolyLog[
3, (c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + (6*I)*b*x*Poly
Log[3, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - 3*PolyLog[4,
(c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (I +
 c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 53.61 (sec) , antiderivative size = 7868, normalized size of antiderivative = 19.72

method	result	size
risch	Expression too large to display	7868

```
[In] int(x^2*arccot(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1589 vs. 2(283) = 566.

Time = 0.46 (sec) , antiderivative size = 1589, normalized size of antiderivative = 3.98

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*arccot(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arccot(d*cot(b*x + a) + c) - 6*b^2*x^2*dilog(-(c^2 + d^2 -
 (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*
sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I
*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^
2*dilog(-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1)
 + 6*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a)
 + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d
 + 1) + 1) + 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1
/2) - 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos
(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 4
*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x
 + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 4*I*a^3
```

```
*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a
) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 6*I*b*x*polyl
og(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2
+ I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 +
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a))/(c^2 + d^2 - 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2
 + d^2 + 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d
+ 1)) - 4*(I*b^3*x^3 + I*a^3)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*co
s(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(
c^2 + d^2 + 2*d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log((c^2 + d^2 - (c^2 - 2*I*
c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2
*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log((c^2 + d
^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 -
 I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 4*(I*b^3*x^3 + I*a
^3)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 +
2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + 3*p
olylog(4, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*
d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) - 3*polylog(4, ((c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x +
 2*a))/(c^2 + d^2 - 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos
(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 +
 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I
*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\, dx = \text{Timed out}$$

```
[In] integrate(x**2*acot(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\,dx = \int x^2 \operatorname{arccot}\left(d\cot\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(x^2*arccot(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*x^3*arctan2((d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, c*co
s(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/6*x^3*arctan2((d - 1)*co
s(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, c*cos(2*b*x + 2*a) - (d - 1)*s
in(2*b*x + 2*a) - c) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*x +
2*a)^2 - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a)
 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(
2*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4
 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*
b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)
^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4
+ d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 +
d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a
) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4
 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(c*d^
3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a
))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\,dx = \int x^2 \operatorname{arccot}\left(d\cot\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(x^2*arccot(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(d*cot(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\,dx = \int x^2\,\operatorname{acot}(c + d\cot(a + b\,x))\,dx$$

```
[In] int(x^2*acot(c + d*cot(a + b*x)),x)
```

```
[Out] int(x^2*acot(c + d*cot(a + b*x)), x)
```

3.172 $\int x \cot^{-1}(c+d\cot(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 303

$$\begin{aligned}\int x\cot^{-1}(c+d\cot(a+bx))\,dx &= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) \\ &\quad -\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\ &\quad +\frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\ &\quad -\frac{x\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\ &\quad +\frac{x\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} \\ &\quad -\frac{i\,\text{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^2} \\ &\quad +\frac{i\,\text{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}\end{aligned}$$

```
[Out] 1/2*x^2*arccot(c+d*cot(b*x+a))-1/4*I*x^2*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))+1/4*I*x^2*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))-1/4*x
*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b+1/4*x*polylog(2,(c+I*(
1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b-1/8*I*polylog(3,(1+I*c-d)*exp(2*I*a
+2*I*b*x)/(1+I*c+d))/b^2+1/8*I*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+
I*(1-d)))/b^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5286, 2221, 2611, 2320, 6724}

$$\int x\cot^{-1}(c+d\cot(a+bx))\,dx=-\frac{i\,\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^2}+\frac{i\,\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}+\frac{x\,\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)+\frac{1}{4}ix^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)+\frac{1}{2}x^2\cot^{-1}(d\cot(a+bx)+c)$$

```
[In] Int[x*ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Cot[a + b*x]])/2 - (I/4)*x^2*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] + (I/4)*x^2*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] - (x*PolyLog[2, ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) + (x*PolyLog[2, ((c + I*(1 + d))*
E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) - ((I/8)*PolyLog[3, ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 + ((I/8)*PolyLog[3, (
(c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5286

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (-Dist[b*((1 + I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 + Dist[b*((1 - I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*
I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) \\
&\quad -\frac{1}{2}(b(1+ic-d))\int \frac{e^{2ia+2ibx}x^2}{1+ic+d+(-1-ic+d)e^{2ia+2ibx}}\,dx \\
&\quad +\frac{1}{2}(b(1-ic+d))\int \frac{e^{2ia+2ibx}x^2}{1-ic-d+(-1+ic-d)e^{2ia+2ibx}}\,dx \\
&= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad +\frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\
&\quad -\frac{1}{2}i\int x\log\left(1+\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)dx \\
&\quad +\frac{1}{2}i\int x\log\left(1+\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)dx
\end{aligned}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) - \frac{x\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} + \frac{x\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} - \frac{\int \text{PolyLog}\left(2,-\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)dx}{4b} + \frac{\int \text{PolyLog}\left(2,-\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)dx}{4b}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) - \frac{x\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} + \frac{x\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} - \frac{i\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-\frac{(-1-ic+d)x}{1+ic+d}\right)}{x}dx,x,e^{2ia+2ibx}\right)}{8b^2} + \frac{i\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{(c+i(1+d))x}{c-i(-1+d)}\right)}{x}dx,x,e^{2ia+2ibx}\right)}{8b^2}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) - \frac{x\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} + \frac{x\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} - \frac{i\,\text{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^2} + \frac{i\,\text{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.91

$$\int x\cot^{-1}(c+d\cot(a+bx))\,dx = \frac{4b^2x^2\cot^{-1}(c+d\cot(a+bx)) - 2ib^2x^2\log\left(1+\frac{(-c+i(1+d))e^{-2i(a+bx)}}{c+i(-1+d)}\right) + 2ib^2x^2\log\left(1+\frac{(-c+i(-1+d))e^{-2i(a+bx}}{c+i(1+d)}\right.}{}$$

```
[In] Integrate[x*ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] (4*b^2*x^2*ArcCot[c + d*Cot[a + b*x]] - (2*I)*b^2*x^2*Log[1 + (-c + I*(1 +
d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + (2*I)*b^2*x^2*Log[1 + (-c + I
*(-1 + d))/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + 2*b*x*PolyLog[2, (c - I
*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - 2*b*x*PolyLog[2, (I + c
 - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - I*PolyLog[3, (c - I*(1 + d
))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + I*PolyLog[3, (I + c - I*d)/((c
 + I*(1 + d))*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.44 (sec) , antiderivative size = 7488, normalized size of antiderivative = 24.71

method	result	size
risch	Expression too large to display	7488

```
[In] int(x*arccot(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(213) = 426.

Time = 0.40 (sec) , antiderivative size = 1289, normalized size of antiderivative = 4.25

$$\int x\cot^{-1}(c+d\cot(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x*arccot(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arccot(d*cot(b*x + a) + c) - 2*b*x*dilog(-(c^2 + d^2 - (c^2
 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2
*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(-(c^2 + d^2
 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)
*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(-(c^2
 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) + 2*b*x*dilo
g(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d
 - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - 2*I*
a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2
*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a^2*log
(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a^2*log(-1/2*c
```

```
^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*
c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 2*I*a^2*log(-1/2*c^2 + I
*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 2*(I*b^2*x^2 - I*a^2)*log((c^2
 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 2*(-I*b^2*x^2
+ I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c
^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1))
- 2*(-I*b^2*x^2 + I*a^2)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b
*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 +
 d^2 - 2*d + 1)) - 2*(I*b^2*x^2 - I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d -
d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) -
2*d + 1)/(c^2 + d^2 - 2*d + 1)) - I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*c
os(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2
+ 2*d + 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I
*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)) + I*poly
log(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^
2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) - I*polylog(3, ((c^2 - 2*I*
c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x +
2*a))/(c^2 + d^2 - 2*d + 1)))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+d\cot(a+bx))\,dx=\text{Timed out}$$

```
[In] integrate(x*acot(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\cot^{-1}(c+d\cot(a+bx))\,dx=\int x\,\mathrm{arccot}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*arctan2((d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, c*co
s(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/4*x^2*arctan2((d - 1)*co
s(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, c*cos(2*b*x + 2*a) - (d - 1)*s
in(2*b*x + 2*a) - c) - 2*b*d*integrate((2*(c^2 + d^2 + 1)*x^2*cos(2*b*x + 2
*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x + 2*a)
^2 - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (
```

```
c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*
x + 2*a) - (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d
^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*b*x
+ 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)^2 +
 (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^
4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 + d^4
- 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) -
4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d
^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(c*d^3 +
(c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a))*s
in(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x \cot^{-1}(c + d\cot(a + bx))\,dx = \int x \operatorname{arccot}\left(d \cot\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(x*arccot(c+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccot(d*cot(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cot^{-1}(c + d\cot(a + bx))\,dx = \int x\,\mathrm{acot}(c + d\cot(a + b\,x))\,dx$$

```
[In] int(x*acot(c + d*cot(a + b*x)),x)
[Out] int(x*acot(c + d*cot(a + b*x)), x)
```

3.173 $\int \cot^{-1}(c+d\cot(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 198

$$\begin{aligned}\int \cot^{-1}(c+d\cot(a+bx))\,dx = {} & x\cot^{-1}(c+d\cot(a+bx)) \\ & -\frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\ & +\frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\ & -\frac{\operatorname{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\ & +\frac{\operatorname{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}\end{aligned}$$

```
[Out] x*arccot(c+d*cot(b*x+a))-1/2*I*x*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d
))+1/2*I*x*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))-1/4*polylog(2,(
1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b+1/4*polylog(2,(c+I*(1+d))*exp(2*I*
a+2*I*b*x)/(c+I*(1-d)))/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{5278, 2221, 2317, 2438\}$

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx = -\frac{\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b} + \frac{\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} - \frac{1}{2}ix\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right) + \frac{1}{2}ix\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right) + x\cot^{-1}(d\cot(a+bx)+c)$$

```
[In] Int[ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Cot[a + b*x]] - (I/2)*x*Log[1 - ((1 + I*c - d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c + d)] + (I/2)*x*Log[1 - ((c + I*(1 + d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 - d))] - PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)
*b*x))/(1 + I*c + d)]/(4*b) + PolyLog[2, ((c + I*(1 + d))*E^((2*I)*a + (2*I
)*b*x))/(c + I*(1 - d))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5278

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcC
ot[c + d*Cot[a + b*x]], x] + (-Dist[b*(1 + I*c - d), Int[x*(E^(2*I*a + 2*I*
b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] + Dist[b*(1
 - I*c + d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I
```

```
*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\cot^{-1}(c+d\cot(a+bx)) - (b(1+ic-d))\int \frac{e^{2ia+2ibx}x}{1+ic+d+(-1-ic+d)e^{2ia+2ibx}}\,dx \\
&\quad + (b(1-ic+d))\int \frac{e^{2ia+2ibx}x}{1-ic-d+(-1+ic-d)e^{2ia+2ibx}}\,dx \\
&= x\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad + \frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) - \frac{1}{2}i\int \log\left(1+\frac{(-1+ic-d)e^{2ia+2ibx}}{1-ic-d}\right)dx \\
&\quad + \frac{1}{2}i\int \log\left(1+\frac{(-1-ic+d)e^{2ia+2ibx}}{1+ic+d}\right)dx \\
&= x\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad + \frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{(-1+ic-d)x}{1-ic-d}\right)}{x}\,dx, x, e^{2ia+2ibx}\right)}{4b} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{(-1-ic+d)x}{1+ic+d}\right)}{x}\,dx, x, e^{2ia+2ibx}\right)}{4b} \\
&= x\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\
&\quad + \frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\
&\quad - \frac{\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} + \frac{\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1649 vs. $2(198) = 396$.

Time = 12.13 (sec) , antiderivative size = 1649, normalized size of antiderivative = 8.33

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx = x\cot^{-1}(c+d\cot(a+bx))$$
$$-\frac{d\left(4a\sqrt{-d^2}\arctan\left(\frac{cd+\tan(a+bx)+c^2\tan(a+bx)}{d}\right)+id\log(1+i\tan(a+bx))\log\left(\frac{cd-\sqrt{-d^2}+\tan(a+bx)+c^2\tan(a+bx)}{i+ic^2+cd-\sqrt{-d^2}}\right.\right.}{\frac{d\log\left(1-\frac{(1+c^2)(1-i\tan(a+bx))}{1+c^2+icd-i\sqrt{-d^2}}\right)\sec^2(a+bx)}{1-i\tan(a+bx)}-\frac{d\log\left(1-\frac{(1+c^2)(1-i\tan(a+bx))}{1+c^2+icd+i\sqrt{-d^2}}\right)\sec^2(a+\cdots}{1-i\tan(a+bx)}}$$

```
[In] Integrate[ArcCot[c + d*Cot[a + b*x]],x]

[Out] x*ArcCot[c + d*Cot[a + b*x]] - (d*(4*a*Sqrt[-d^2]*ArcTan[(c*d + Tan[a + b*x
] + c^2*Tan[a + b*x])/d] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2
] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d - Sqrt[-d^2])] + I*d*
Log[1 - I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a +
b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c
*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[
-d^2])] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*
Tan[a + b*x])/(I + I*c^2 - c*d + Sqrt[-d^2])] - I*d*PolyLog[2, ((1 + c^2)*(
1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])] + I*d*PolyLog[2, ((1
 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])] - I*d*PolyL
og[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])] +
I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-
d^2])])*((2*a)/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)]
 - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])) - (2*(a + b*x))/(b*(-1 -
 c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)]
 - 2*c*d*Sin[2*(a + b*x)]))))/((d*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/
(1 + c^2 + I*c*d - I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d
*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])]
*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) + (d*Log[(c*d + Sqrt[-d^2] + Tan[a +
b*x] + c^2*Tan[a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(
1 - I*Tan[a + b*x]) - (d*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x])
/(I + I*c^2 - c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d*
Log[1 - ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])]*
Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[1 - ((1 + c^2)*(1 + I*Tan[a +
 b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*
x]) - (d*Log[(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^
2 + c*d - Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[(c*d +
 Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[-d^2
])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (I*d*Log[1 + I*Tan[a + b*x]]*(Se
c[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*
Tan[a + b*x]) + (I*d*Log[1 - I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a +
b*x]^2))/(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x]) - (I*d*Log[1
+ I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d + Sqrt[-d^2]
+ Tan[a + b*x] + c^2*Tan[a + b*x]) + (I*(1 + c^2)*d*Log[1 - I*Tan[a + b*x]]
*Sec[a + b*x]^2)/(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x]) + (4*a*Sqrt
[-d^2]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(d*(1 + (c*d + Tan[a + b*x] +
 c^2*Tan[a + b*x])^2/d^2)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(168) = 336.

Time = 4.58 (sec) , antiderivative size = 1146, normalized size of antiderivative = 5.79

method	result	size
derivativedivides	Expression too large to display	1146
default	Expression too large to display	1146
risch	Expression too large to display	4982

```
[In] int(arccot(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arccot(c+d*cot(b*x+a))-d^2*(-1/d*arct
an(d*((c+d*cot(b*x+a))/d-c/d)+c)*arctan(-(c+d*cot(b*x+a))/d+c/d)-1/d^2*(1/2
*I*d^2*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*cot(
b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)/(1+I
*c+d)+1/2*I*d*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c
+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)+
c)/(1+I*c+d)+1/2*I*d/(c-I*d-I)*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c
/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c*arctan(d*((c+d*
cot(b*x+a))/d-c/d)+c)+1/2*d^2*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^2/(1+I*c
+d)+1/4*d^2*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*
((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/2*d*arctan(d*((c+d*
cot(b*x+a))/d-c/d)+c)^2/(1+I*c+d)+1/2*d/(c-I*d-I)*c*arctan(d*((c+d*cot(b*x+
a))/d-c/d)+c)^2+1/4*d*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+
c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/4*d/(c-I*
d-I)*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*c
ot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c-1/2*I*d*arctan(d*((c+d*cot(b*x+a))/d
-c/d)+c)*ln(1-(I+c+I*d)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*co
t(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))-1/2*d*arctan(d*((c+d*cot(b*x+a))/d-c/d
)+c)^2-1/4*d*polylog(2,(I+c+I*d)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d
*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 965 vs. 2(140) = 280.

Time = 0.40 (sec) , antiderivative size = 965, normalized size of antiderivative = 4.87

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(arccot(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arccot(d*cot(b*x + a) + c) + 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2
 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d
+ I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c
^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(
2*b*x + 2*a) + 1/2) - 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2
 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x +
2*a) - 1/2) + 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1
/2) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x
+ 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^
2 + 2*d + 1)) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)
*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)
/(c^2 + d^2 + 2*d + 1)) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d
- d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a)
 - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2
- 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b
*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - dilog(-(c^2 + d^2 - (c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
+ 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - dilog(-(c^2 + d^2 - (c^2 - 2
*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
+ 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + dilog(-(c^2 + d^2 - (c^2 + 2
*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
 + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) + dilog(-(c^2 + d^2 - (c^2 -
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1))/b
```

Sympy [F]

$$\int \cot^{-1}(c + d\cot(a + bx))\,dx = \int \operatorname{acot}\left(c + d\cot\left(a + bx\right)\right)dx$$

```
[In] integrate(acot(c+d*cot(b*x+a)),x)
```

```
[Out] Integral(acot(c + d*cot(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(140) = 280$.

Time = 0.35 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.69

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx$$
$$= \frac{d\left(\frac{8(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)}{d} - \frac{8(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)-4\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)\arctan\left(\frac{cd+\left(c^2+d+\cdots\right.}{c^2+d^2\cdots}\right.}{}\right.}{}$$

[In] integrate(arccot(c+d*cot(b*x+a)),x, algorithm="maxima")

[Out] 1/8*(d*(8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)/d - (8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d) - 4*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)*arctan2((c*d + (c^2 + d + 1)*tan(b*x + a))/(c^2 + d^2 + 2*d + 1), -(c*d*tan(b*x + a) - c^2 - d - 1)/(c^2 + d^2 + 2*d + 1)) + 4*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)*arctan2(-(c*d + (c^2 - d + 1)*tan(b*x + a))/(c^2 + d^2 - 2*d + 1), -(c*d*tan(b*x + a) - c^2 + d - 1)/(c^2 + d^2 - 2*d + 1)) - (log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) - log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 - 2*d + 1)))*log((c^2 + 1)*d^2 + 2*(c^3 + c)*d*tan(b*x + a) + (c^4 + 2*c^2 + 1)*tan(b*x + a)^2) - 2*dilog(((I*c - 1)*tan(b*x + a) + I*d)/(c + I*d + I)) + 2*dilog(((I*c + 1)*tan(b*x + a) + I*d)/(c + I*d - I)) + 2*dilog(-((I*c - 1)*tan(b*x + a) + I*d)/(c - I*d + I)) - 2*dilog(-((I*c + 1)*tan(b*x + a) + I*d)/(c - I*d - I)))/d) + 8*(b*x + a)*arccot(c + d/tan(b*x + a)) - 8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d))/b

Giac [F]

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx = \int \operatorname{arccot}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

[In] integrate(arccot(c+d*cot(b*x+a)),x, algorithm="giac")

[Out] integrate(arccot(d*cot(b*x + a) + c), x)

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c + d\cot(a + bx))\,dx = \int \mathrm{acot}(c + d\cot(a + b\,x))\;dx$$

```
[In] int(acot(c + d*cot(a + b*x)),x)
[Out] int(acot(c + d*cot(a + b*x)), x)
```

3.174 $\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+d\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+d*cot(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c + d*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c + d*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + d*Cot[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(c+d\cot\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arccot(c+d*cot(b*x+a))/x,x)
[Out] int(arccot(c+d*cot(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*cot(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arccot(d*cot(b*x + a) + c)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(acot(c+d*cot(b*x+a))/x,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(arccot(c+d*cot(b*x+a))/x,x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*cot(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot(d*cot(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\mathrm{acot}(c+d\cot(a+b\,x))}{x}\,dx$$

```
[In] int(acot(c + d*cot(a + b*x))/x,x)
[Out] int(acot(c + d*cot(a + b*x))/x, x)
```

3.175 $\int x^2 \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int x^2\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx=-\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{x^2\,\mathrm{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{ix\,\mathrm{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}+\frac{\mathrm{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}$$

```
[Out] -1/12*b*x^4+1/3*x^3*(Pi-arccot(-c-(1-I*c)*cot(b*x+a)))-1/6*I*x^3*ln(1-I*c*e
xp(2*I*a+2*I*b*x))-1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*poly
log(3,I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5282, 2215, 2221, 2611, 6744, 2320, 6724}

$$\int x^2\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$$
$$=\frac{\mathrm{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}-\frac{ix\,\mathrm{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}-\frac{x^2\,\mathrm{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{bx^4}{12}$$

```
[In] Int[x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/3 - (I/6)*x^3*Log[
1 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) - ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + PolyL
og[4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5282

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{3}(bc)\int\frac{e^{2ia+2ibx}x^3}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))\\
&\quad -\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}i\int x^2\log\left(1-\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)dx\\
&= -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad -\frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{\int x\,\text{PolyLog}\left(2,\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)dx}{2b}\\
&= -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))\\
&\quad -\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}\\
&\quad -\frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}+\frac{i\int\text{PolyLog}\left(3,\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)dx}{4b^2}\\
&= -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))\\
&\quad -\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}\\
&\quad -\frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}+\frac{\text{Subst}\left(\int\frac{\text{PolyLog}(3,icx)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^3}
\end{aligned}
$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3 \cot^{-1}(c + (1 - ic)\cot(a + bx)) - \frac{1}{6}ix^3 \log\left(1 - ice^{2ia+2ibx}\right) - \frac{x^2 \operatorname{PolyLog}\left(2, ice^{2ia+2ibx}\right)}{4b} - \frac{ix \operatorname{PolyLog}\left(3, ice^{2ia+2ibx}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4, ice^{2ia+2ibx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}(c + (1 - ic)\cot(a + bx))\,dx = \frac{1}{3}x^3 \cot^{-1}(c + (1 - ic)\cot(a + bx)) - \frac{4ib^3x^3 \log\left(1 + \frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2 \operatorname{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx \operatorname{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{c}\right) + 3 \operatorname{PolyL}}{24b^3}$$

```
[In] Integrate[x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 + I/(c*E^
((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))] +
(6*I)*b*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (-I)/(c*E
^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.12 (sec) , antiderivative size = 1448, normalized size of antiderivative = 9.40

method	result	size
risch	Expression too large to display	1448

```
[In] int(x^2*(Pi-arccot(-c-(1-I*c)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*x^3*ln(1-I*exp(2*I*(b*x+a))*c)-1/12*(Pi*csgn(I*exp(I*(b*x+a)))^2*csg
n(I*exp(2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^
2+Pi*csgn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(e
xp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))-Pi*
csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1)
)^2-Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(
exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))+Pi*csgn(I/(exp(2*I*(b*x+a))-1))
*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))-Pi*csgn(I/(exp(2*I*(b*x+a
))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))-1))
*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I*exp(2*I*(b
*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*
x+a))-1))-Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I*
(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))
```

```
^2-Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*
x+a))*c+I)/(exp(2*I*(b*x+a))-1))-Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b
*x+a))-1))^2-Pi*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(
I*(I+c)/(exp(2*I*(b*x+a))-1))^3-Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(
I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(exp(2*I*(b*x+a)
)*c+I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b
*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I*ex
p(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3-Pi*csgn(I*exp(2*I*(b*x+a))*(I+
c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^
2+Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(exp(2*I*(b
*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3+2*I*ln(I+c))*x^3-1/3*I*x^3*ln(exp(I*(b
*x+a)))+1/2*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*x*a^2-1/4*I*x*polylog(3,I*exp(
2*I*(b*x+a))*c)/b^2+1/3*I/b^3*ln(1-I*exp(2*I*(b*x+a))*c)*a^3-1/4*x^2*polylo
g(2,I*exp(2*I*(b*x+a))*c)/b+1/4/b^3*polylog(2,I*exp(2*I*(b*x+a))*c)*a^2-1/2
*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/8*polylog(4,I*exp(2*I*(b*x
+a))*c)/b^3-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/6*I*x^3*l
n(exp(2*I*(b*x+a))*c+I)-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x
-1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/6*I/b^3*a^3*ln(exp(2*I
*(b*x+a))*c+I)-1/2/b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2/b^3*a
^2*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/12*b*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.13

$$\int x^2 \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{2\,b^4x^4 - 8\,\pi b^3x^3 - 4i\,b^3x^3\log\left(\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c+i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right) - 2\,a^4 - 4i\,a^3\log\left(\frac{ce^{(2}}{}\right.}{}$$

```
[In] integrate(x^2*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/24*(2*b^4*x^4 - 8*pi*b^3*x^3 - 4*I*b^3*x^3*log((c*e^(2*I*b*x + 2*I*a) +
I)*e^(-2*I*b*x - 2*I*a)/(c + I)) + 6*b^2*x^2*dilog(I*c*e^(2*I*b*x + 2*I*a))
 - 2*a^4 - 4*I*a^3*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*I*b*x*polylog(3,
I*c*e^(2*I*b*x + 2*I*a)) + 4*(I*b^3*x^3 + I*a^3)*log(-I*c*e^(2*I*b*x + 2*I*
a) + 1) - 3*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c + (1 - ic)\cot(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*(pi-acot(-c-(1-I*c)*cot(b*x+a))),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 + 3*_t0**2*I*c*e
xp(2*I*a) - _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c + (1 - ic)\cot(a + bx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int x^2 \cot^{-1}(c + (1 - ic)\cot(a + bx))\,dx = \int (\pi - \operatorname{arccot}(-(-i\,c + 1)\cot(bx + a) - c))x^2\,dx$$

```
[In] integrate(x^2*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="giac")
```

```
[Out] integrate((pi - arccot(-(-I*c + 1)*cot(b*x + a) - c))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + (1 - ic)\cot(a + bx))\,dx = \int x^2\,(\Pi + \operatorname{acot}(c - \cot(a + b\,x)\,(-1 + c\,1\mathrm{i})))\,dx$$

```
[In] int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))),x)
```

```
[Out] int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))), x)
```

3.176 $\int x \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int x \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) - \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}$$

```
[Out] -1/6*b*x^3+1/2*x^2*(Pi-arccot(-c-(1-I*c)*cot(b*x+a)))-1/4*I*x^2*ln(1-I*c*ex
p(2*I*a+2*I*b*x))-1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3
,I*c*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5282, 2215, 2221, 2611, 2320, 6724}

$$\int x \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2} - \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} - \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) + \frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{bx^3}{6}$$

```
[In] Int[x*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/2 - (I/4)*x^2*Log[1
 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) - ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5282

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}(bc)\int\frac{e^{2ia+2ibx}x^2}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))\\
&\quad -\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}i\int x\log\left(1-\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)dx\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad -\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{\int\text{PolyLog}\left(2,\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)dx}{4b}\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad -\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,icx)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^2}\\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad -\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

```
[In] Integrate[x*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.62 (sec) , antiderivative size = 1413, normalized size of antiderivative = 11.49

method	result	size
risch	Expression too large to display	1413

```
[In] int(x*(Pi-arccot(-c-(1-I*c)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/8*(Pi*csgn(I*exp(I*(b*x+a
)))^2*csgn(I*exp(2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(
b*x+a)))^2+Pi*csgn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I
*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a)
)-1))-Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b
*x+a))-1))^2-Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I))
*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))+Pi*csgn(I/(exp(2*I*(b*
x+a))-1))*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))-Pi*csgn(I/(exp(2
*I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I/(exp(2*I*(b*
x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I*e
xp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(ex
p(2*I*(b*x+a))-1))-Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+P
i*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*
x+a))-1))^2-Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((ex
p(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))-Pi*csgn((exp(2*I*(b*x+a))*c+I)/(e
xp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2
+Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^3-Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-
1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(exp(2*
I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(e
xp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+Pi*
csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3-Pi*csgn(I*exp(2*I*(b*
x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x
+a))-1))^2+Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(e
xp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3+2*I*ln(I+c))*x^2-1/4*I/b^2*ln
(1-I*exp(2*I*(b*x+a))*c)*a^2-1/2*I*x^2*ln(exp(I*(b*x+a)))+1/2*I/b^2*a^2*ln(
1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/4*I*ln(1-I*exp(2*I*(b*x+a))*c)*x^2-1/4/b
*polylog(2,I*exp(2*I*(b*x+a))*c)*x-1/4/b^2*polylog(2,I*exp(2*I*(b*x+a))*c)*
a-1/4*I/b^2*a^2*ln(exp(2*I*(b*x+a))*c+I)+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a)
)*(-I*c)^(1/2))+1/4*I*x^2*ln(exp(2*I*(b*x+a))*c+I)-1/2*I/b*ln(1-I*exp(2*I*(
b*x+a))*c)*a*x-1/8*I/b^2*polylog(3,I*exp(2*I*(b*x+a))*c)+1/2*I/b*a*ln(1+I*e
xp(I*(b*x+a))*(-I*c)^(1/2))*x+1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/
2))+1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/6*b*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

$$\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{4\,b^3x^3-12\,\pi b^2x^2-6i\,b^2x^2\log\left(\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c+i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i}}{}\right.}{24\,b^2}$$

```
[In] integrate(x*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b^3*x^3 - 12*pi*b^2*x^2 - 6*I*b^2*x^2*log((c*e^(2*I*b*x + 2*I*a) +
 I)*e^(-2*I*b*x - 2*I*a)/(c + I)) + 4*a^3 + 6*b*x*dilog(I*c*e^(2*I*b*x + 2*
I*a)) + 6*I*a^2*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*(I*b^2*x^2 - I*a^2)*
log(-I*c*e^(2*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, I*c*e^(2*I*b*x + 2*I*a))
)/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*(pi-acot(-c-(1-I*c)*cot(b*x+a))),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 + 3*_t0**2*I*c*e
xp(2*I*a) - _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

Maxima [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int x \cot^{-1}(c + (1 - ic)\cot(a + bx))\,dx = \int (\pi - \operatorname{arccot}(-(-i\,c + 1)\cot(bx + a) - c))x\,dx$$

```
[In] integrate(x*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="giac")
```

```
[Out] integrate((pi - arccot(-(-I*c + 1)*cot(b*x + a) - c))*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cot^{-1}(c + (1 - ic)\cot(a + bx))\,dx = \int x\,(\Pi + \operatorname{acot}(c - \cot(a + b\,x)\,\,(-1 + c\,1\mathrm{i})))\,\,dx$$

```
[In] int(x*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))),x)
```

```
[Out] int(x*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))), x)
```

3.177 $\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx)) -\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] -1/2*b*x^2+x*(Pi-arccot(-c-(1-I*c)*cot(b*x+a)))-1/2*I*x*ln(1-I*c*exp(2*I*a+
2*I*b*x))-1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5274, 2215, 2221, 2317, 2438}

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) +x\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{bx^2}{2}$$

```
[In] Int[ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcCot[c + (1 - I*c)*Cot[a + b*x]] - (I/2)*x*Log[1 - I*c*E
^((2*I)*a + (2*I)*b*x)] - PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5274

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcC
ot[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\cot^{-1}(c+(1-ic)\cot(a+bx))+(ib)\int \frac{x}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx))-(bc)\int \frac{e^{2ia+2ibx}x}{-i(1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx))\\
&\quad -\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}i\int \log\left(1-\frac{ce^{2ia+2ibx}}{-i(1-ic)+c}\right)\,dx\\
&= -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)\\
&\quad +\frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{cx}{-i(1-ic)+c}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{4b}\\
&= -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 929 vs. 2(85) = 170.

Time = 4.89 (sec) , antiderivative size = 929, normalized size of antiderivative = 10.93

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = x\cot^{-1}(c+(1-ic)\cot(a+bx))$$

$$+\frac{}{(i+\cot(a+bx))(1+ic+(i+c)\cot(a+bx))\left(2ibx+\log\left(1-\frac{\sec(bx)((-i+c)\cos(a)+i(i+c)\sin(a))(\cos(a+bx)-i\sin}{2c}\right.\right.}$$

```
[In] Integrate[ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (1 - I*c)*Cot[a + b*x]] + (I*x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos
[b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I +
c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - I*L
og[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] -
(2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x
]] + I*PolyLog[2, (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b
*x] - I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] -
(I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2])*(Cos[b*x] - I*Sin[b*x]
)*(Cos[b*x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(1 + I*c + (I + c)*Cot[a + b
*x])*((2*I)*b*x + Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(C
os[a + b*x] - I*Sin[a + b*x]))/(2*c)] + Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a
] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2] + ((-I + c)*Cos[a +
 b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] +
(1 + I*c)*Sin[a + b*x]) + ((I + c)*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x
]])*Sin[a + b*x])/((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]) + 2*b*x*
Tan[b*x] + I*Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a
+ b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-1 - I*c)
*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - I*
Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Cos[a]^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + I*
Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (I*Log[(Sec[b*x]*((1 - I*c)*Cos[a +
 b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I
+ Tan[b*x]) - (I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a + b*x] +
(1 + I*c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + Tan[b*x])))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(76) = 152.

Time = 1.57 (sec) , antiderivative size = 587, normalized size of antiderivative = 6.91

method	result
default	$\pi x - \frac{-\frac{\operatorname{arccot}(-c+\cot(bx+a)(ic-1))\ln(\cot(bx+a)(ic-1)+c+i)c^2}{2i+2c}-\frac{2i\operatorname{arccot}(-c+\cot(bx+a)(ic-1))\ln(\cot(bx+a)(ic-1)+c+i)c}{2i+2c}+}{}$
parts	$\pi x - \frac{-\frac{\operatorname{arccot}(-c+\cot(bx+a)(ic-1))\ln(\cot(bx+a)(ic-1)+c+i)c^2}{2i+2c}-\frac{2i\operatorname{arccot}(-c+\cot(bx+a)(ic-1))\ln(\cot(bx+a)(ic-1)+c+i)c}{2i+2c}+}{}$
derivativedivides	$\frac{-\frac{\pi\ln\left(4c^2+4(-c+\cot(bx+a)(ic-1))c+(-c+\cot(bx+a)(ic-1))^2+1\right)c^2}{2(2i+2c)}-\frac{i\pi\ln\left(4c^2+4(-c+\cot(bx+a)(ic-1))c+(-c+\cot(bx+a)(ic-1)\right.}{2i+2c}}{}$
risch	Expression too large to display

```
[In] int(Pi-arccot(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] Pi*x-1/b/(-1+I*c)*(-arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)*
(-1+I*c)+c+I)*c^2-2*I*arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a
)*(-1+I*c)+c+I)*c+arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)*(-
1+I*c)+c+I)+arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)*(-1+I
*c)-c)*c^2+2*I*arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)*(-
1+I*c)-c)*c-arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)*(-1+I
*c)-c)+(-1+I*c)^2*(-1/2/(I+c)*(-1/4*I*ln(-I+cot(b*x+a)*(-1+I*c)-c)^2+1/2*I*
(dilog(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))+ln(-I+cot(b*x+a)*(-1+I*c)-c)*ln(-1
/2*I*(cot(b*x+a)*(-1+I*c)-c+I))))+1/2/(I+c)*(1/2*I*(dilog(-1/2*(cot(b*x+a)*
(-1+I*c)-c+I)/c)+ln(cot(b*x+a)*(-1+I*c)+c+I)*ln(-1/2*(cot(b*x+a)*(-1+I*c)-c
+I)/c))-1/2*I*(dilog((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))+ln(cot(b*x+a)*(
-1+I*c)+c+I)*ln((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.36

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{2\,b^2x^2 - 4\,\pi bx - 2i\,bx\log\left(\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c+i}\right) - 2\,a^2 + 2\,(i\,bx+i\,a)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right) - 2i}{4\,b}$$

```
[In] integrate(pi-arccot(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^2 - 4*pi*b*x - 2*I*b*x*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/(c + I)) - 2*a^2 + 2*(I*b*x + I*a)*log(-I*c*e^(2*I*b*x + 2*I
*a) + 1) - 2*I*a*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + dilog(I*c*e^(2*I*b*x
+ 2*I*a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(pi-acot(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 + 3*_t0**2*I*c*e
xp(2*I*a) - _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(I*a)]
```

Maxima [F(-2)]

Exception generated.

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(pi-arccot(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \int \pi - \operatorname{arccot}\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

```
[In] integrate(pi-arccot(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(pi - arccot(-(-I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx=\int \Pi+\operatorname{acot}(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

```
[In] int(Pi + acot(c - cot(a + b*x)*(c*1i - 1)),x)
[Out] int(Pi + acot(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.178 $\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate((Pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{\pi - \operatorname{arccot}\left(-c-(-ic+1)\cot\left(bx+a\right)\right)}{x}dx$$

```
[In] int((Pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x)
[Out] int((Pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\pi - \operatorname{arccot}\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

```
[In] integrate((pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x, algorithm="fricas")
[Out] integral(1/2*(2*pi + I*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)
/(c + I)))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate((pi-acot(-c-(1-I*c)*cot(b*x+a)))/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\pi - \operatorname{arccot}\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

```
[In] integrate((pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x, algorithm="giac")
[Out] integrate((pi - arccot(-(-I*c + 1)*cot(b*x + a) - c))/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\Pi + \mathrm{acot}(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))}{x}\,dx$$

```
[In] int((Pi + acot(c - cot(a + b*x)*(c*1i - 1)))/x,x)
[Out] int((Pi + acot(c - cot(a + b*x)*(c*1i - 1)))/x, x)
```

3.179 $\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) + \frac{x^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}$$

```
[Out] 1/12*b*x^4+1/3*x^3*(Pi-arccot(-c+(1+I*c)*cot(b*x+a)))+1/6*I*x^3*ln(1+I*c*ex
p(2*I*a+2*I*b*x))+1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*poly
log(3,-I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b
^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{5282, 2215, 2221, 2611, 6744, 2320, 6724\}$

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = -\frac{\text{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3} + \frac{ix\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} + \frac{x^2\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} + \frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) + \frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) + \frac{bx^4}{12}$$

```
[In] Int[x^2*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCot[c - (1 + I*c)*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) + ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - Po
lyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5282

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{3}(bc)\int\frac{e^{2ia+2ibx}x^3}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad-\frac{1}{2}i\int x^2\log\left(1-\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)\,dx\\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad+\frac{x^2\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{\int x\,\text{PolyLog}\left(2,\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)\,dx}{2b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \cot^{-1}(c - (1 + ic)\cot(a + bx)) \\
&\quad + \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} \\
&\quad + \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} - \frac{i \int \operatorname{PolyLog}\left(3, \frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right) dx}{4b^2} \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \cot^{-1}(c - (1 + ic)\cot(a + bx)) \\
&\quad + \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} \\
&\quad + \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, -icx)}{x} dx, x, e^{2ia+2ibx}\right)}{8b^3} \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \cot^{-1}(c - (1 + ic)\cot(a + bx)) \\
&\quad + \frac{1}{6}ix^3 \log\left(1 + ice^{2ia+2ibx}\right) + \frac{x^2 \operatorname{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} \\
&\quad + \frac{ix \operatorname{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, -ice^{2ia+2ibx}\right)}{8b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$
\int x^2 \cot^{-1}(c - (1 + ic)\cot(a + bx))\, dx = \frac{1}{24}\left(8x^3 \cot^{-1}(c + (-1 - ic)\cot(a + bx)) + 4ix^3 \log\left(1 - \frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2 \operatorname{PolyLog}\left(2, \frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix \operatorname{PolyLog}\left(3, \frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3 \operatorname{PolyLog}\left(4, \frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)
$$

```
[In] Integrate[x^2*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCot[c + (-1 - I*c)*Cot[a + b*x]] + (4*I)*x^3*Log[1 - I/(c*E^((2*I
)*(a + b*x)))] - (6*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)*x
*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, I/(c*E^((2*I)*(
a + b*x)))])/b^3)/24
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.42 (sec) , antiderivative size = 1449, normalized size of antiderivative = 9.35

method	result	size
risch	Expression too large to display	1449

```
[In] int(x^2*(Pi-arccot(-c+(I*c+1)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*x^3*ln(exp(2*I*(b*x+a))*c-I)+1/12*(Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(
I*exp(2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+
Pi*csgn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(
b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))-Pi*cs
gn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^
2+Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+a))-1)
*(c-I))-Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn
(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))-Pi*csgn(I/(exp(2*I*(b*x+a))
-1))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+Pi*csgn(I/(exp(2*I*(b*x+a))-1))*c
sgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(exp(2*I*(b*
x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+
a))-1))+Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(c
-I))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/
(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))+Pi*
csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I*(exp(2*I*(b*x
+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I/
(exp(2*I*(b*x+a))-1)*(c-I))^3-Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))*csgn(I*
exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(exp(2*I*(b*x+a))*
c-I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x
+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn((exp(2
*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(
exp(2*I*(b*x+a))-1))^3-Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1
))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(exp(2*I*(b*x
+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3+2*I*ln(c-I))*x^3+1/6*I*x^3*ln(I*exp(2*I*
(b*x+a))*c+1)-1/2*I/b^2*ln(I*exp(2*I*(b*x+a))*c+1)*x*a^2-1/6*I/b^3*a^3*ln(-
exp(2*I*(b*x+a))*c+I)+1/4*I*x*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2+1/4*x^2*
polylog(2,-I*exp(2*I*(b*x+a))*c)/b-1/4/b^3*polylog(2,-I*exp(2*I*(b*x+a))*c)
*a^2+1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/8*polylog(4,-I*exp(
2*I*(b*x+a))*c)/b^3-1/3*I/b^3*ln(I*exp(2*I*(b*x+a))*c+1)*a^3+1/2*I/b^2*a^2*
ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/3*I*x^3*ln(exp(I*(b*x+a)))+1/2*I/b^3
*a^3*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))
*(I*c)^(1/2))*x+1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2/b^3*a
^2*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/12*b*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.12

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \frac{2\,b^4x^4 + 8\,\pi b^3x^3 + 4i\,b^3x^3\log\left(\frac{(c-i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right) - 2\,a^4 - 4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)}{24}$$

```
[In] integrate(x^2*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/24*(2*b^4*x^4 + 8*pi*b^3*x^3 + 4*I*b^3*x^3*log((c - I)*e^(2*I*b*x + 2*I*a
)/(c*e^(2*I*b*x + 2*I*a) - I)) + 6*b^2*x^2*dilog(-I*c*e^(2*I*b*x + 2*I*a))
- 2*a^4 - 4*I*a^3*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*I*b*x*polylog(3, -
I*c*e^(2*I*b*x + 2*I*a)) - 4*(-I*b^3*x^3 - I*a^3)*log(I*c*e^(2*I*b*x + 2*I*
a) + 1) - 3*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*(pi-acot(-c+(1+I*c)*cot(b*x+a))),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**2*I + 2*c*exp(2*I*
a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,ex
p(I*a)]
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \int (\pi - \operatorname{arccot}((i\,c+1)\cot(bx+a)-c))x^2\,dx$$

```
[In] integrate(x^2*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="giac")
[Out] integrate((pi - arccot((I*c + 1)*cot(b*x + a) - c))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \int x^2\,(\Pi + \operatorname{acot}(c-\cot(a+b\,x)\,(1+c\,1\mathrm{i})))\,dx$$

```
[In] int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))),x)
[Out] int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))), x)
```

3.180 $\int x \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}$$

```
[Out] 1/6*b*x^3+1/2*x^2*(Pi-arccot(-c+(1+I*c)*cot(b*x+a)))+1/4*I*x^2*ln(1+I*c*exp
(2*I*a+2*I*b*x))+1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3
,-I*c*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5282, 2215, 2221, 2611, 2320, 6724}

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\frac{i\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{bx^3}{6}$$

```
[In] Int[x*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCot[c - (1 + I*c)*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 +
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5282

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}(bc)\int\frac{e^{2ia+2ibx}x^2}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad-\frac{1}{2}i\int x\log\left(1-\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)dx\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad+\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{\int\text{PolyLog}\left(2,\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)dx}{4b}\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad+\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-icx)}{x}\,dx,x,e^{2ia+2ibx}\right)}{8b^2}\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad+\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$
\begin{aligned}
&\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx\\
&=\frac{1}{2}x^2\cot^{-1}(c+(-1-ic)\cot(a+bx))\\
&\quad+\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}
\end{aligned}
$$

```
[In] Integrate[x*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
[Out] (x^2*ArcCot[c + (-1 - I*c)*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
 PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.86 (sec) , antiderivative size = 1414, normalized size of antiderivative = 11.40

method	result	size
risch	Expression too large to display	1414

```
[In] int(x*(Pi-arccot(-c+(I*c+1)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/8*(Pi*csgn(I*exp(I*(b*x+a
)))^2*csgn(I*exp(2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(
b*x+a)))^2+Pi*csgn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I
/(exp(2*I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a)
)-1))-Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b
*x+a))-1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I))*csgn(I/(exp(2*I*
(b*x+a))-1)*(c-I))-Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))
*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))-Pi*csgn(I/(exp(2
*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+Pi*csgn(I/(exp(2*I*(b*
x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(
exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(ex
p(2*I*(b*x+a))-1))+Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-P
i*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+Pi*csgn(I*exp(2*I*(b*x
+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+
a))-1))+Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I*(e
xp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2
+Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^3-Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-
I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(exp(2*
I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(e
xp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*
csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*exp(2*I*(b*x+
a))*(c-I)/(exp(2*I*(b*x+a))-1))^3-Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I
*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(e
xp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3+2*I*ln(c-I))*x^2+1/4*I/b^2*ln
(I*exp(2*I*(b*x+a))*c+1)*a^2+1/8*I/b^2*polylog(3,-I*exp(2*I*(b*x+a))*c)-1/2
*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/2*I/b*a*ln(1+I*exp(I*(b*x+a
))*(I*c)^(1/2))*x+1/4/b*polylog(2,-I*exp(2*I*(b*x+a))*c)*x+1/4/b^2*polylog(
2,-I*exp(2*I*(b*x+a))*c)*a+1/2*I*x^2*ln(exp(I*(b*x+a)))-1/4*I*x^2*ln(exp(2*
I*(b*x+a))*c-I)+1/4*I/b^2*a^2*ln(-exp(2*I*(b*x+a))*c+I)+1/4*I*ln(I*exp(2*I*
(b*x+a))*c+1)*x^2+1/2*I/b*ln(I*exp(2*I*(b*x+a))*c+1)*a*x-1/2*I/b^2*a^2*ln(1
-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/
2))-1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/6*b*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \frac{4\,b^3x^3+12\,\pi b^2x^2+6i\,b^2x^2\log\left(\frac{(c-i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)-}{24\,b^2}$$

```
[In] integrate(x*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/24*(4*b^3*x^3 + 12*pi*b^2*x^2 + 6*I*b^2*x^2*log((c - I)*e^(2*I*b*x + 2*I*
a)/(c*e^(2*I*b*x + 2*I*a) - I)) + 4*a^3 + 6*b*x*dilog(-I*c*e^(2*I*b*x + 2*I
*a)) + 6*I*a^2*log((c*e^(2*I*b*x + 2*I*a) - I)/c) - 6*(-I*b^2*x^2 + I*a^2)*
log(I*c*e^(2*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, -I*c*e^(2*I*b*x + 2*I*a))
)/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*(pi-acot(-c+(1+I*c)*cot(b*x+a))),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**2*I + 2*c*exp(2*I*
a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,ex
p(I*a)]
```

Maxima [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int x \cot^{-1}(c - (1 + ic)\cot(a + bx))\,dx = \int (\pi - \operatorname{arccot}((i\,c + 1)\cot(bx + a) - c))x\,dx$$

```
[In] integrate(x*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="giac")
[Out] integrate((pi - arccot((I*c + 1)*cot(b*x + a) - c))*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cot^{-1}(c - (1 + ic)\cot(a + bx))\,dx = \int x\,(\Pi + \operatorname{acot}(c - \cot(a + b\,x)\,(1 + c\,1\mathrm{i})))\,dx$$

```
[In] int(x*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))),x)
[Out] int(x*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))), x)
```

3.181 $\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*(Pi-arccot(-c+(1+I*c)*cot(b*x+a)))+1/2*I*x*ln(1+I*c*exp(2*I*a+2
*I*b*x))+1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5274, 2215, 2221, 2317, 2438}

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{bx^2}{2}$$

```
[In] Int[ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCot[c - (1 + I*c)*Cot[a + b*x]] + (I/2)*x*Log[1 + I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5274

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcC
ot[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\cot^{-1}(c-(1+ic)\cot(a+bx))+(ib)\int \frac{x}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+(bc)\int \frac{e^{2ia+2ibx}x}{-i(-1-ic)+c-ce^{2ia+2ibx}}\,dx\\
&= \frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad-\frac{1}{2}i\int \log\left(1-\frac{ce^{2ia+2ibx}}{-i(-1-ic)+c}\right)dx\\
&= \frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)\\
&\quad-\frac{\text{Subst}\left(\int \frac{\log\left(1-\frac{cx}{-i(-1-ic)+c}\right)}{x}\,dx,x,e^{2ia+2ibx}\right)}{4b}\\
&= \frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 872 vs. 2(86) = 172.

Time = 2.40 (sec) , antiderivative size = 872, normalized size of antiderivative = 10.14

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = x\cot^{-1}(c+(-1-ic)\cot(a+bx))$$
$$-\frac{ix\csc(a+bx)\left(2bx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+i\log\left(\frac{\sec(\cdots}{\cdots}\right.\right.}{(i+\cot(a+bx))((-i+c)\cos(a+bx)+i(i+c)\sin(a+bx))\left(-2ibx-\log\left(1-\frac{\sec(bx)((i+c)\cos(a)+(1+ic)\sin\cdots}{2\cdots}\right.\right.}$$

```
[In] Integrate[ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (-1 - I*c)*Cot[a + b*x]] - (I*x*Csc[a + b*x]*(2*b*x*Log[2*Cos[
b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I +
c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - I*L
og[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a +
b*x]))/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] +
I*PolyLog[2, (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] -
I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a]
 + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x]
 + I*Sin[b*x]))/((I + Cot[a + b*x])*((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[
a + b*x])*((-2*I)*b*x - Log[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a
])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)] - (Log[1 - I*Tan[b*x]]*((I + c)*
Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*
Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[
a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) + (Log[(Sec[b*x
]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]*S
ec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*Tan[b*x] - I*Log[1 - (Sec[b*x]*((I + c)
*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x
] + I*Log[1 - I*Tan[b*x]]*Tan[b*x] - I*Log[1 + I*Tan[b*x]]*Tan[b*x] + (I*Lo
g[1 - ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*
x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*
((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Sec[b*x]^2)/(I + T
an[b*x])))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(75) = 150$.

Time = 1.37 (sec) , antiderivative size = 630, normalized size of antiderivative = 7.33

method	result
default	$\pi x - \frac{\frac{\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)c^2}{2i-2c} - \frac{2i\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)c}{2i-2c}}{}$
parts	$\pi x - \frac{\frac{\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)c^2}{2i-2c} - \frac{2i\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)c}{2i-2c}}{}$
derivativedivides	$\frac{(ic+1)^2\left(-\frac{\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)}{2i-2c} + \frac{\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)}{2i-2c} - \right.}{}$
risch	Expression too large to display

```
[In] int(Pi-arccot(-c+(I*c+1)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] Pi*x-1/b/(I*c+1)*(arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(b
*x+a)-c+I)*c^2-2*I*arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(
b*x+a)-c+I)*c-arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(b*x+a
)-c+I)-arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x+a)-c)*c
^2+2*I*arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x+a)-c)*c
+arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x+a)-c)-(I*c+1)
^2*(-1/2/(I-c)*(1/4*I*ln(I+(I*c+1)*cot(b*x+a)-c)^2-1/2*I*((ln(I+(I*c+1)*cot
(b*x+a)-c)-ln(-1/2*I*(I+(I*c+1)*cot(b*x+a)-c)))*ln(-1/2*I*(I-(I*c+1)*cot(b*
x+a)+c))-dilog(-1/2*I*(I+(I*c+1)*cot(b*x+a)-c))))+1/2/(I-c)*(1/2*I*(dilog((
-I-(I*c+1)*cot(b*x+a)+c)/(-2*I+2*c))+ln(-(I*c+1)*cot(b*x+a)-c+I)*ln((-I-(I*
c+1)*cot(b*x+a)+c)/(-2*I+2*c)))-1/2*I*(dilog(1/2*(I-(I*c+1)*cot(b*x+a)+c)/c
)+ln(-(I*c+1)*cot(b*x+a)-c+I)*ln(1/2*(I-(I*c+1)*cot(b*x+a)+c)/c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$$
$$= \frac{2\,b^2x^2 + 4\,\pi bx + 2i\,bx\log\left(\frac{(c-i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) - 2\,a^2 - 2\,(-i\,bx - i\,a)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right) - 2i\,a\log\left(\frac{ce^{(2i\,bx}}{}\right.}{4\,b}$$

```
[In] integrate(pi-arccot(-c+(1+I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2 + 4*pi*b*x + 2*I*b*x*log((c - I)*e^(2*I*b*x + 2*I*a)/(c*e^(2
*I*b*x + 2*I*a) - I)) - 2*a^2 - 2*(-I*b*x - I*a)*log(I*c*e^(2*I*b*x + 2*I*a
) + 1) - 2*I*a*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + dilog(-I*c*e^(2*I*b*x +
 2*I*a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(pi-acot(-c+(1+I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**2*I + 2*c*exp(2*I*
a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(
I*a)]
```

Maxima [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(pi-arccot(-c+(1+I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \int \pi - \operatorname{arccot}\left((i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

```
[In] integrate(pi-arccot(-c+(1+I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(pi - arccot((I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\int \Pi+\operatorname{acot}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\;dx$$

```
[In] int(Pi + acot(c - cot(a + b*x)*(c*1i + 1)),x)
[Out] int(Pi + acot(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.182 $\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate((Pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\pi-\operatorname{arccot}\left(-c+(ic+1)\cot\left(bx+a\right)\right)}{x}dx$$

```
[In] int((Pi-arccot(-c+(I*c+1)*cot(b*x+a)))/x,x)
[Out] int((Pi-arccot(-c+(I*c+1)*cot(b*x+a)))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx=\int \frac{\pi-\operatorname{arccot}\left(\left(i\,c+1\right)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

```
[In] integrate((pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x, algorithm="fricas")
[Out] integral(1/2*(2*pi + I*log((c - I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*
a) - I)))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate((pi-acot(-c+(1+I*c)*cot(b*x+a)))/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Giac [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx = \int \frac{\pi - \operatorname{arccot}\left((i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

```
[In] integrate((pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x, algorithm="giac")
[Out] integrate((pi - arccot((I*c + 1)*cot(b*x + a) - c))/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx = \int \frac{\Pi + \mathrm{acot}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))}{x}\,dx$$

```
[In] int((Pi + acot(c - cot(a + b*x)*(c*1i + 1)))/x,x)
[Out] int((Pi + acot(c - cot(a + b*x)*(c*1i + 1)))/x, x)
```

3.183 $\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 299

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} + \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} - \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}$$

```
[Out] 1/4*(f*x+e)^4*arccot(tanh(b*x+a))/f+1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f-
1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^3*polylog(2,I*
exp(2*b*x+2*a))/b+3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2-3/8*I*
f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2-3/8*I*f^2*(f*x+e)*polylog(4,-I*
exp(2*b*x+2*a))/b^3+3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3+3/16*
```

```
I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a
))/b^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5292, 4265, 2611, 6744, 2320, 6724}

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} - \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4} - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f}$$

```
[In] Int[(e + f*x)^3*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcCot[Tanh[a + b*x]])/(4*f) + ((e + f*x)^4*ArcTan[E^(2*a + 2*
b*x)])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (((3*I)/8)*f*(e + f*x)^2
*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f*(e + f*x)^2*PolyLog[3
, I*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*a
 + 2*b*x)])/b^3 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/b
^3 + (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 - (((3*I)/16)*f^
3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5292

```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Tanh[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{b \int (e+fx)^4 \operatorname{sech}(2a+2bx)\, dx}{4f}$$

$$= \frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{4f} - \frac{1}{2}i\int(e+fx)^3\log\left(1-ie^{2a+2bx}\right)\,dx + \frac{1}{2}i\int(e+fx)^3\log\left(1+ie^{2a+2bx}\right)\,dx$$

$$= \frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{(3if)\int(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)\,dx}{4b} - \frac{(3if)\int(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)\,dx}{4b}$$

$$= \frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} - \frac{(3if^2)\int(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)\,dx}{4b^2} + \frac{(3if^2)\int(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)\,dx}{4b^2}$$

$$= \frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} - \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} + \frac{(3if^3)\int\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)\,dx}{8b^3} - \frac{(3if^3)\int\operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)\,dx}{8b^3}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} + \frac{(3if^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{16b^4} - \frac{(3if^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,ix)}{x}\,dx,x,e^{2a+2bx}\right)}{16b^4}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} + \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} - \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. $2(299) = 598$.

Time = 0.23 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.01

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\cot^{-1}(\tanh(a+bx)) + \frac{i\left(8b^4e^3x\log\left(1-ie^{2(a+bx)}\right)+12b^4e^2fx^2\log\left(1-ie^{2(a+bx)}\right)+8b^4ef^2x^3\log\left(1-ie^{2(a+bx)}\right)+2b^4f^3x^4\log\right.}{}$$

```
[In] Integrate[(e + f*x)^3*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCot[Tanh[a + b*x]])/4 + (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
```

```
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 33.57 (sec) , antiderivative size = 3570, normalized size of antiderivative = 11.94

method	result	size
risch	Expression too large to display	3570

```
[In] int((f*x+e)^3*arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b*e^3*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I/b*e^3*dilog(1-exp(b*x+a)*
(-1)^(3/4))-1/2*I*e^3*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e^3*ln(1-exp(b*x+
a)*(-1)^(3/4))*x-1/8*I*f^3*ln(1+I*exp(2*b*x+2*a))*x^4-1/8*I/f*e^4*ln(-exp(2
*b*x+2*a)+I)+1/8*I*f^3*ln(exp(2*b*x+2*a)-I)*x^4+1/2*I*ln(exp(2*b*x+2*a)-I)*
x*e^3+1/8*I/f*ln(exp(2*b*x+2*a)-I)*e^4+1/2*I/b*e^3*ln(((-I)^(1/2)-exp(b*x+a
))/(-I)^(1/2))*a+1/2*I/b*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a-1/2*I
*f^3/b^4*a^3*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+3/8*I*f^2/b^3*e*poly
log(4,I*exp(2*b*x+2*a))+1/4*I*f^3/b*polylog(2,I*exp(2*b*x+2*a))*x^3+1/4*I*f
^3/b^4*polylog(2,I*exp(2*b*x+2*a))*a^3+3/8*I*f^3/b^4*ln(1-I*exp(2*b*x+2*a))
*a^4-1/2*I*f^3/b^4*a^4*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f^3/b^4
*a^4*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/8*I*f^3/b^4*a^4*ln(exp(2*b*x+
2*a)+I)+3/8*I*f^3/b^3*polylog(4,I*exp(2*b*x+2*a))*x-3/8*I*f/b^2*e^2*polylog
(3,I*exp(2*b*x+2*a))+1/2*I*f^2*e*ln(1-I*exp(2*b*x+2*a))*x^3+3/4*I*f*e^2*ln(
1-I*exp(2*b*x+2*a))*x^2-1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)-exp(b*x+a))/(-I
)^(1/2))-3/8*I*f^3/b^2*polylog(3,I*exp(2*b*x+2*a))*x^2-1/2*I/b*a*e^3*ln(exp
(2*b*x+2*a)+I)+3/16*I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+3/2*I*f^2/b^2*e*
ln(1+I*exp(2*b*x+2*a))*a^2*x-3/2*I*f/b*e^2*ln(1+I*exp(2*b*x+2*a))*a*x-3/2*I
*f^2/b^2*a^2*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x-3/2*I*f^2/b^2*a^2*e*ln(1-exp(b
*x+a)*(-1)^(3/4))*x+3/2*I*f/b*a*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*x+3/2*I*f/b
*a*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))*x+I*f^2/b^3*e*ln(1+I*exp(2*b*x+2*a))*a^3
+3/4*I*f^2/b^2*e*polylog(3,-I*exp(2*b*x+2*a))*x-3/4*I*f/b^2*e^2*ln(1+I*exp(
2*b*x+2*a))*a^2-3/4*I*f/b*e^2*polylog(2,-I*exp(2*b*x+2*a))*x-3/4*I*f/b^2*e^
2*polylog(2,-I*exp(2*b*x+2*a))*a+1/2*I*f^2/b^3*a^3*e*ln(-exp(2*b*x+2*a)+I)-
3/4*I*f/b^2*a^2*e^2*ln(-exp(2*b*x+2*a)+I)-3/2*I*f^2/b^3*a^3*e*ln(1+exp(b*x+
a)*(-1)^(3/4))-3/2*I*f^2/b^3*a^3*e*ln(1-exp(b*x+a)*(-1)^(3/4))-3/2*I*f^2/b^
```

3*a^2*e*dilog(1+exp(b*x+a)*(-1)^(3/4))-3/2*I*f^2/b^3*a^2*e*dilog(1-exp(b*x+a)*(-1)^(3/4))+3/2*I*f/b^2*a^2*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))+3/2*I*f/b^2*a^2*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))+3/2*I*f/b^2*a*e^2*dilog(1+exp(b*x+a)*(-1)^(3/4))+3/2*I*f/b^2*a*e^2*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f^3/b^3*ln(1+I*exp(2*b*x+2*a))*a^3*x+1/2*I*f^3/b^3*a^3*ln(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*f^3/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))*x-3/4*I*f^2/b*e*polylog(2,-I*exp(2*b*x+2*a))*x^2+3/4*I*f^2/b^3*e*polylog(2,-I*exp(2*b*x+2*a))*a^2-3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a))/b^4+1/8*I/f*e^4*ln(exp(2*b*x+2*a)+I)+1/8*I*f^3*ln(1-I*exp(2*b*x+2*a))*x^4+1/2*I*e^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+1/2*I*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+1/2*I/b*e^3*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I/b*e^3*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+3/4*I*f*ln(exp(2*b*x+2*a)-I)*x^2*e^2-3/8*I*f^3/b^4*ln(1+I*exp(2*b*x+2*a))*a^4-1/4*I*f^3/b*polylog(2,-I*exp(2*b*x+2*a))*x^3-1/4*I*f^3/b^4*polylog(2,-I*exp(2*b*x+2*a))*a^3+3/8*I*f^3/b^2*polylog(3,-I*exp(2*b*x+2*a))*x^2-3/8*I*f^3/b^3*polylog(4,-I*exp(2*b*x+2*a))*x-1/2*I*f^2*e*ln(1+I*exp(2*b*x+2*a))*x^3-3/4*I*f*e^2*ln(1+I*exp(2*b*x+2*a))*x^2-1/8*I*f^3/b^4*a^4*ln(-exp(2*b*x+2*a)+I)+1/2*I*f^3/b^4*a^4*ln(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f^3/b^4*a^4*ln(1-exp(b*x+a)*(-1)^(3/4))+1/2*I*f^3/b^4*a^3*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f^3/b^4*a^3*dilog(1-exp(b*x+a)*(-1)^(3/4))-3/8*I*f^2/b^3*e*polylog(4,-I*exp(2*b*x+2*a))+3/8*I*f/b^2*e^2*polylog(3,-I*exp(2*b*x+2*a))+1/2*I/b*a*e^3*ln(-exp(2*b*x+2*a)+I)-1/2*I/b*e^3*ln(1+exp(b*x+a)*(-1)^(3/4))*a-1/2*I/b*e^3*ln(1-exp(b*x+a)*(-1)^(3/4))*a+1/2*I*f^2*ln(exp(2*b*x+2*a)-I)*x^3*e+1/16*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))+csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3+1)*(f*x+e)^4/f-3/4*I*f^2/b^3*e*polylog(2,I*exp(2*b*x+2*a))*a^2-3/4*I*f^2/b^2*e*polylog(3,I*exp(2*b*x+2*a))*x+3/4*I*f/b^2*e^2*ln(1-I*exp(2*b*x+2*a))*a^2+3/4*I*f/b*e^2*polylog(2,I*exp(2*b*x+2*a))*x+3/4*I*f/b^2*e^2*polylog(2,I*exp(2*b*x+2*a))*a-1/2*I*f^2/b^3*a^3*e*ln(exp(2*b*x+2*a)+I)+3/4*I*f/b^2*a^2*e^2*ln(exp(2*b*x+2*a)+I)+3/2*I*f^2/b^3*a^3*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+3/2*I*f^2/b^3*a^3*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+3/2*I*f^2/b^3*a^2*e*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+3/2*I*f^2/b

```
^3*a^2*e*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-3/2*I*f/b^2*a^2*e^2*ln((
(-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-3/2*I*f/b^2*a^2*e^2*ln(((-I)^(1/2)+exp(b
*x+a))/(-I)^(1/2))-3/2*I*f/b^2*a*e^2*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/
2))-3/2*I*f/b^2*a*e^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*f^3/b
^3*a^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*f^3/b^3*a^3*ln(((-I)^
(1/2)+exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f^3/b^3*ln(1-I*exp(2*b*x+2*a))*a^3*x-
I*f^2/b^3*e*ln(1-I*exp(2*b*x+2*a))*a^3+3/4*I*f^2/b*e*polylog(2,I*exp(2*b*x+
2*a))*x^2-3/2*I*f^2/b^2*e*ln(1-I*exp(2*b*x+2*a))*a^2*x+3/2*I*f/b*e^2*ln(1-I
*exp(2*b*x+2*a))*a*x+3/2*I*f^2/b^2*a^2*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1
/2))*x+3/2*I*f^2/b^2*a^2*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x-3/2*I*f
/b*a*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-3/2*I*f/b*a*e^2*ln(((-I)^
(1/2)+exp(b*x+a))/(-I)^(1/2))*x-1/8*I*(f*x+e)^4/f*ln(exp(2*b*x+2*a)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(236) = 472$.

Time = 0.38 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^3*arccot(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(-24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*
f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*f^3*p
olylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(cosh(b*x + a)/si
nh(b*x + a)) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*
b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f
^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-1/2*sqrt(4
*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2
 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(
b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3
*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (I*b^4*f^3*x
^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3
- 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(1/2*sqrt(4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b
^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*
b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1
) + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x
 - 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(1/2
*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^
4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b
```

```
^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a)
+ sinh(b*x + a)) + 1) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f
^2 + I*a^4*f^3)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-4*
I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-I*sqrt(
4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^
2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*s
inh(b*x + a)) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^
4*f^3)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 24*(-I*b*f^
3*x - I*b*e*f^2)*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- 24*(-I*b*f^3*x - I*b*e*f^2)*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + si
nh(b*x + a))) - 24*(I*b*f^3*x + I*b*e*f^2)*polylog(4, 1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a))) - 24*(I*b*f^3*x + I*b*e*f^2)*polylog(4, -1/2*sqr
t(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*e*f^
2*x + I*b^2*e^2*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))
) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*e*f^2*x + I*b^2*e^2*f)*polylog(3, -1/2*sqrt
(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*e*f^2
*x - I*b^2*e^2*f)*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*e*f^2*x - I*b^2*e^2*f)*polylog(3, -1/2*sqr
t(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \int (e+fx)^3 \operatorname{acot}(\tanh(a+bx))\,dx$$

```
[In] integrate((f*x+e)**3*acot(tanh(b*x+a)),x)
[Out] Integral((e + f*x)**3*acot(tanh(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)^3 \operatorname{arccot}(\tanh(bx+a))\,dx$$

```
[In] integrate((f*x+e)^3*arccot(tanh(b*x+a)),x, algorithm="maxima")
[Out] 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan2(e^(2*b*x + 2*a)
 + 1, e^(2*b*x + 2*a) - 1) + integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2*x
^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x
 + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)^3 \operatorname{arccot}\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate((f*x+e)^3*arccot(tanh(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{acot}(\tanh(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

```
[In] int(acot(tanh(a + b*x))*(e + f*x)^3,x)
[Out] int(acot(tanh(a + b*x))*(e + f*x)^3, x)
```

3.184 $\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 229

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} - \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}$$

```
[Out] 1/3*(f*x+e)^3*arccot(tanh(b*x+a))/f+1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f-
1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^2*polylog(2,I*
exp(2*b*x+2*a))/b+1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*I*f*
(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2-1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*
a))/b^3+1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5292, 4265, 2611, 6744, 2320, 6724}

$$\int (e+fx)^2\cot^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f} - \frac{if^2\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{if^2\operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} + \frac{if(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} - \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}$$

```
[In] Int[(e + f*x)^2*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcCot[Tanh[a + b*x]])/(3*f) + ((e + f*x)^3*ArcTan[E^(2*a + 2*
b*x)])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/4)*f*(e + f*x)*PolyL
og[3, (-I)*E^(2*a + 2*b*x)])/b^2 - ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a +
 2*b*x)])/b^2 - ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 + ((I/8)*f
^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5292

```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Tanh[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}+\frac{b\int(e+fx)^3\text{sech}(2a+2bx)\,dx}{3f}\\
&= \frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}+\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f}\\
&\quad -\frac{1}{2}i\int(e+fx)^2\log\left(1-ie^{2a+2bx}\right)\,dx+\frac{1}{2}i\int(e+fx)^2\log\left(1+ie^{2a+2bx}\right)\,dx
\end{aligned}
$$

$$= \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{(if)\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)\,dx}{2b} - \frac{(if)\int(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)\,dx}{2b}$$

$$= \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} - \frac{(if^2)\int\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)\,dx}{4b^2} + \frac{(if^2)\int\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)\,dx}{4b^2}$$

$$= \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} - \frac{(if^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3} + \frac{(if^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3}$$

$$= \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} - \frac{if^2\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{if^2\operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.64

$$\int (e+fx)^2\cot^{-1}(\tanh(a+bx))\,dx=\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\cot^{-1}(\tanh(a+bx))$$
$$+\frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^2*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[Tanh[a + b*x]])/3 + ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.43 (sec) , antiderivative size = 2668, normalized size of antiderivative = 11.65

method	result	size
risch	Expression too large to display	2668

```
[In] int((f*x+e)^2*arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*f*e*ln(1+I*exp(2*b*x+2*a))*x^2+1/6*I*f^2/b^3*a^3*ln(-exp(2*b*x+2*a)+
I)+I*f/b*e*ln(1-I*exp(2*b*x+2*a))*a*x-I*f/b*a*e*ln(((-I)^(1/2)-exp(b*x+a))/
(-I)^(1/2))*x-I*f/b*a*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+I*f/b*a*e*
ln(1+exp(b*x+a)*(-1)^(3/4))*x+I*f/b*a*e*ln(1-exp(b*x+a)*(-1)^(3/4))*x-I*f/b
*e*ln(1+I*exp(2*b*x+2*a))*a*x+1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3-1/2
*I*f^2/b^3*a^2*dilog(1-exp(b*x+a)*(-1)^(3/4))+1/4*I*f/b^2*e*polylog(3,-I*ex
p(2*b*x+2*a))+1/3*I*f^2/b^3*ln(1+I*exp(2*b*x+2*a))*a^3-1/4*I*f^2/b*polylog(
2,-I*exp(2*b*x+2*a))*x^2+1/4*I*f^2/b^3*polylog(2,-I*exp(2*b*x+2*a))*a^2+1/4
*I*f^2/b^2*polylog(3,-I*exp(2*b*x+2*a))*x-1/2*I*f^2/b^3*a^3*ln(1+exp(b*x+a)
*(-1)^(3/4))-1/2*I*f^2/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f^2/b^3*a^
2*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I/b*a*e^2*ln(-exp(2*b*x+2*a)+I)-1/2*I/
b*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*a-1/2*I/b*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))
*a+1/2*I*f*ln(exp(2*b*x+2*a)-I)*x^2*e+1/2*I/b*e^2*ln(((-I)^(1/2)-exp(b*x+a)
```

)/(-I)^(1/2))*a+1/2*I/b*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a-1/2*I/
b*a*e^2*ln(exp(2*b*x+2*a)+I)-1/6*I*f^2/b^3*a^3*ln(exp(2*b*x+2*a)+I)-1/3*I*f
^2/b^3*ln(1-I*exp(2*b*x+2*a))*a^3+1/4*I*f^2/b*polylog(2,I*exp(2*b*x+2*a))*x
^2-1/4*I*f^2/b^3*polylog(2,I*exp(2*b*x+2*a))*a^2-1/4*I*f^2/b^2*polylog(3,I*
exp(2*b*x+2*a))*x+1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+
1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f^2/b^3*a^2*
dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/
2)+exp(b*x+a))/(-I)^(1/2))-1/4*I*f/b^2*e*polylog(3,I*exp(2*b*x+2*a))+1/2*I*
f*e*ln(1-I*exp(2*b*x+2*a))*x^2-1/2*I*f^2/b^2*ln(1-I*exp(2*b*x+2*a))*a^2*x+1
/2*I*f^2/b^2*a^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f^2/b^2*a^2
*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f/b^2*e*ln(1-I*exp(2*b*x+2*
a))*a^2+1/2*I*f/b*e*polylog(2,I*exp(2*b*x+2*a))*x+1/2*I*f/b^2*e*polylog(2,I
*exp(2*b*x+2*a))*a-I*f/b^2*a^2*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-I*f
/b^2*a^2*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-I*f/b^2*a*e*dilog(((-I)^(
1/2)-exp(b*x+a))/(-I)^(1/2))-I*f/b^2*a*e*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)
^(1/2))+1/2*I*f/b^2*a^2*e*ln(exp(2*b*x+2*a)+I)-1/2*I/b*e^2*dilog(1+exp(b*x+
a)*(-1)^(3/4))-1/2*I/b*e^2*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*e^2*ln(1+ex
p(b*x+a)*(-1)^(3/4))*x-1/2*I*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/6*I*f^2*ln
(1+I*exp(2*b*x+2*a))*x^3-1/6*I/f*e^3*ln(-exp(2*b*x+2*a)+I)+1/6*I*f^2*ln(exp
(2*b*x+2*a)-I)*x^3+1/2*I*ln(exp(2*b*x+2*a)-I)*x*e^2+1/6*I/f*ln(exp(2*b*x+2*
a)-I)*e^3-1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*a))/b^3+1/2*I/b*e^2*dilog(((-I
)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I/b*e^2*dilog(((-I)^(1/2)+exp(b*x+a))/(
-I)^(1/2))+1/2*I*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+1/2*I*e^2*ln(
((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+1/6*I*f^2*ln(1-I*exp(2*b*x+2*a))*x^3+
1/6*I/f*e^3*ln(exp(2*b*x+2*a)+I)+I*f/b^2*a^2*e*ln(1+exp(b*x+a)*(-1)^(3/4))+
I*f/b^2*a^2*e*ln(1-exp(b*x+a)*(-1)^(3/4))+I*f/b^2*a*e*dilog(1+exp(b*x+a)*(-
1)^(3/4))+I*f/b^2*a*e*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f^2/b^2*a^2*ln(1
-exp(b*x+a)*(-1)^(3/4))*x-1/2*I*f/b^2*e*ln(1+I*exp(2*b*x+2*a))*a^2-1/2*I*f/
b*e*polylog(2,-I*exp(2*b*x+2*a))*x-1/2*I*f/b^2*e*polylog(2,-I*exp(2*b*x+2*a
))*a-1/2*I*f/b^2*a^2*e*ln(-exp(2*b*x+2*a)+I)+1/2*I*f^2/b^2*ln(1+I*exp(2*b*x
+2*a))*a^2*x-1/2*I*f^2/b^2*a^2*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/6*I*(f*x+e)^
3/f*ln(exp(2*b*x+2*a)+I)+1/12*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*
b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+
2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*
a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*
a)+1))^2-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*
a)+1))^2+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*
a)+1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*
x+2*a)+I)/(exp(2*b*x+2*a)+1))+csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)
+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+
2*a)-I)/(exp(2*b*x+2*a)+1))+csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1
))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(exp(2*b*x+2*a)
-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2
-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(exp(2*b*x+2*a)+I)/

```
(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csg
n((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-csgn((1-I)*(exp(2*b*x+2*a)
-I)/(exp(2*b*x+2*a)+1))^3+1)*(f*x+e)^3/f
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(180) = 360$.

Time = 0.34 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.38

$$\int (e + fx)^2 \cot^{-1}(\tanh(a + bx))\, dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*arccot(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*
polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*polylo
g(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3*
b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) - 3*(-I*b^2*
f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + s
inh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(-1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*e*f
*x + I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(
I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x
+ a) + sinh(b*x + a))) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3*e^2*x +
 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(4*I)*(cosh(b*x + a
) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3*e^2*x
+ 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(4*I)*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b^3*e^2
*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(-4*I)*(cosh(b*
x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b^3*
e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt(-4*I)*(cos
h(b*x + a) + sinh(b*x + a)) + 1) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*
f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-3*I*a*b^2*e^2
 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b
*x + a)) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(I*sqrt(-4*I) + 2
*cosh(b*x + a) + 2*sinh(b*x + a)) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*
f^2)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 6*(I*b*f^2*x
+ I*b*e*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*(I
*b*f^2*x + I*b*e*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a
))) - 6*(-I*b*f^2*x - I*b*e*f)*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + s
inh(b*x + a))) - 6*(-I*b*f^2*x - I*b*e*f)*polylog(3, -1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \int (e+fx)^2 \operatorname{acot}(\tanh(a+bx))\,dx$$

```
[In] integrate((f*x+e)**2*acot(tanh(b*x+a)),x)
[Out] Integral((e + f*x)**2*acot(tanh(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)^2 \operatorname{arccot}(\tanh(bx+a))\,dx$$

```
[In] integrate((f*x+e)^2*arccot(tanh(b*x+a)),x, algorithm="maxima")
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x +
 2*a) - 1) + integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b*e
^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)^2 \operatorname{arccot}(\tanh(bx+a))\,dx$$

```
[In] integrate((f*x+e)^2*arccot(tanh(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{acot}(\tanh(a+b\,x))\,(e+f\,x)^2\,dx$$

```
[In] int(acot(tanh(a + b*x))*(e + f*x)^2,x)
[Out] int(acot(tanh(a + b*x))*(e + f*x)^2, x)
```

3.185 $\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 159

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f} + \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} - \frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{if\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

```
[Out] 1/2*(f*x+e)^2*arccot(tanh(b*x+a))/f+1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f-
1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)*polylog(2,I*exp(
2*b*x+2*a))/b+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/8*I*f*polylog(3,I*
exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used

$= \{5292, 4265, 2611, 2320, 6724\}$

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{if\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{if\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} - \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}$$

```
[In] Int[(e + f*x)*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcCot[Tanh[a + b*x]])/(2*f) + ((e + f*x)^2*ArcTan[E^(2*a + 2*
b*x)])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I/4
)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/8)*f*PolyLog[3, (-I)*E^(
2*a + 2*b*x)])/b^2 - ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5292

```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Tanh[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{b\int(e+fx)^2\text{sech}(2a+2bx)\,dx}{2f}\\
&= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}\\
&\quad -\frac{1}{2}i\int(e+fx)\log\left(1-ie^{2a+2bx}\right)\,dx+\frac{1}{2}i\int(e+fx)\log\left(1+ie^{2a+2bx}\right)\,dx\\
&= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}\\
&\quad -\frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}\\
&\quad +\frac{(if)\int\text{PolyLog}\left(2,-ie^{2a+2bx}\right)\,dx}{4b}-\frac{(if)\int\text{PolyLog}\left(2,ie^{2a+2bx}\right)\,dx}{4b}\\
&= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}\\
&\quad -\frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}\\
&\quad +\frac{(if)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&\quad -\frac{(if)\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f}\\
&\quad -\frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}\\
&\quad +\frac{if\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}-\frac{if\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.49

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = ex\cot^{-1}(\tanh(a+bx)) + \frac{1}{2}fx^2\cot^{-1}(\tanh(a+bx)) + \frac{ie\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b} + \frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\,\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\,\text{PolyLog}\left(2,ie\right.\right.}{8b^2}$$

```
[In] Integrate[(e + f*x)*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] e*x*ArcCot[Tanh[a + b*x]] + (f*x^2*ArcCot[Tanh[a + b*x]])/2 + ((I/4)*e*(2*b
*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x))]) - PolyLog[2, (
-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^(2*(a + b*x))]))/b + ((I/8)*f*(2*b^2*
x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a + b*x))] - 2*b
*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*(a + b*x))] +
 PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.57 (sec) , antiderivative size = 1776, normalized size of antiderivative = 11.17

method	result	size
risch	Expression too large to display	1776

```
[In] int((f*x+e)*arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*ln(exp(2*b*x+2*a)-I)*x^2*f+1/2*I*ln(exp(2*b*x+2*a)-I)*e*x-1/4*I*f*ln(
1+I*exp(2*b*x+2*a))*x^2-1/2*I*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e*ln(1-
exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e/b*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*e/b
*dilog(1-exp(b*x+a)*(-1)^(3/4))+1/4*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(
exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(
2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*
b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*
b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*
b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*
b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(ex
p(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))+csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*
x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(
2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+
2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(exp(2*b*
```

```
x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)
+1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(exp(2*b*x+2*
a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))
^2-csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-csgn((1-I)*(exp(2*b*
x+2*a)-I)/(exp(2*b*x+2*a)+1))^3+1)*(1/2*f*x^2+e*x)-1/2*I*(1/2*f*x^2+e*x)*ln
(exp(2*b*x+2*a)+I)+1/2*I*f/b*ln(1-I*exp(2*b*x+2*a))*a*x-1/2*I*f/b*a*ln(((-I
)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*f/b*a*ln(((-I)^(1/2)+exp(b*x+a))/(-
I)^(1/2))*x+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/4*I*f*ln(1-I*exp(2*b
*x+2*a))*x^2+1/2*I*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+1/2*I*e*ln(((
-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+1/2*I*e/b*dilog(((-I)^(1/2)-exp(b*x+a))
/(-I)^(1/2))+1/2*I*e/b*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/4*I*f/b^
2*ln(1+I*exp(2*b*x+2*a))*a^2-1/4*I*f/b*polylog(2,-I*exp(2*b*x+2*a))*x-1/4*I
*f/b^2*polylog(2,-I*exp(2*b*x+2*a))*a-1/4*I*f/b^2*a^2*ln(-exp(2*b*x+2*a)+I)
-1/2*I*e/b*ln(1+exp(b*x+a)*(-1)^(3/4))*a-1/2*I*e/b*ln(1-exp(b*x+a)*(-1)^(3/
4))*a+1/2*I*e/b*a*ln(-exp(2*b*x+2*a)+I)+1/2*I*f/b^2*a^2*ln(1+exp(b*x+a)*(-1
)^(3/4))+1/2*I*f/b^2*a^2*ln(1-exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1+
exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1-exp(b*x+a)*(-1)^(3/4))+1/4*I*f
/b^2*ln(1-I*exp(2*b*x+2*a))*a^2+1/4*I*f/b*polylog(2,I*exp(2*b*x+2*a))*x+1/4
*I*f/b^2*polylog(2,I*exp(2*b*x+2*a))*a+1/4*I*f/b^2*a^2*ln(exp(2*b*x+2*a)+I)
+1/2*I*e/b*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*a+1/2*I*e/b*ln(((-I)^(1/2
)+exp(b*x+a))/(-I)^(1/2))*a-1/2*I*e/b*a*ln(exp(2*b*x+2*a)+I)-1/2*I*f/b^2*a^
2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f/b^2*a^2*ln(((-I)^(1/2)+exp
(b*x+a))/(-I)^(1/2))-1/2*I*f/b^2*a*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2)
)-1/2*I*f/b^2*a*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*f/b*ln(1+I*
exp(2*b*x+2*a))*a*x+1/2*I*f/b*a*ln(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*f/b*a*l
n(1-exp(b*x+a)*(-1)^(3/4))*x-1/8*I*f*polylog(3,I*exp(2*b*x+2*a))/b^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(130) = 260.

Time = 0.31 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.77

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx$$

$$= \frac{2\,(b^2fx^2+2\,b^2ex)\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)-2\,(-i\,bfx-i\,be)\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-2\,(\cdots}{\cdots}$$

```
[In] integrate((f*x+e)*arccot(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) - 2*(-I*
b*f*x - I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(-I
*b*f*x - I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(
I*b*f*x + I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*
```

```
(I*b*f*x + I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
(I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(4*I)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*
a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*
x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(b*x + a)
+ sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*
log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-2*I*a*b*e + I*
a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2*I*a*b*e +
 I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2*I*a*b*
e - I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2*I*a
*b*e - I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 2*
I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polyl
og(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polylog(3, 1/
2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polylog(3, -1/2*sqrt(
-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \int (e+fx)\operatorname{acot}(\tanh(a+bx))\,dx$$

```
[In] integrate((f*x+e)*acot(tanh(b*x+a)),x)
[Out] Integral((e + f*x)*acot(tanh(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)\operatorname{arccot}(\tanh(bx+a))\,dx$$

```
[In] integrate((f*x+e)*arccot(tanh(b*x+a)),x, algorithm="maxima")
[Out] 1/2*(f*x^2 + 2*e*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + int
egrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1),
 x)
```

Giac [F]

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)\,\mathrm{arccot}\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate((f*x+e)*arccot(tanh(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \int \mathrm{acot}(\tanh(a+b\,x))\,\left(e+f\,x\right)\,dx$$

```
[In] int(acot(tanh(a + b*x))*(e + f*x),x)
[Out] int(acot(tanh(a + b*x))*(e + f*x), x)
```

3.186 $\int \cot^{-1}(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 73

$$\int \cot^{-1}(\tanh(a+bx))\,dx = x\cot^{-1}(\tanh(a+bx)) + x\arctan\left(e^{2a+2bx}\right) - \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] x*arccot(tanh(b*x+a))+x*arctan(exp(2*b*x+2*a))-1/4*I*polylog(2,-I*exp(2*b*x
+2*a))/b+1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5288, 4265, 2317, 2438}

$$\int \cot^{-1}(\tanh(a+bx))\,dx = x\arctan\left(e^{2a+2bx}\right) - \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + x\cot^{-1}(\tanh(a+bx))$$

```
[In] Int[ArcCot[Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[Tanh[a + b*x]] + x*ArcTan[E^(2*a + 2*b*x)] - ((I/4)*PolyLog[2, (-I
)*E^(2*a + 2*b*x)])/b + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5288

```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCot[Tanh[a + b
*x]], x] + Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\cot^{-1}(\tanh(a+bx)) + b\int x\text{sech}(2a+2bx)\,dx \\
&= x\cot^{-1}(\tanh(a+bx)) + x\arctan\left(e^{2a+2bx}\right) \\
&\quad -\frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)\,dx + \frac{1}{2}i\int \log\left(1+ie^{2a+2bx}\right)\,dx \\
&= x\cot^{-1}(\tanh(a+bx)) + x\arctan\left(e^{2a+2bx}\right) \\
&\quad -\frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} + \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&= x\cot^{-1}(\tanh(a+bx)) + x\arctan\left(e^{2a+2bx}\right) \\
&\quad -\frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \cot^{-1}(\tanh(a+bx))\,dx = x\cot^{-1}(\tanh(a+bx)) + \frac{i\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b}$$

```
[In] Integrate[ArcCot[Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[Tanh[a + b*x]] + ((I/4)*(2*b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1
 + I*E^(2*(a + b*x))]) - PolyLog[2, (-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^
(2*(a + b*x))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(62) = 124.

Time = 1.74 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.52

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))\operatorname{arccot}(\tanh(bx+a))+\arctan(\tanh(bx+a))\operatorname{arctanh}(\tanh(bx+a))+\frac{\arctan(\tanh(bx+a))\ln\left(1+\frac{i(1+i\tan}{\tanh(bx}\right.}{2}}{b}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))\operatorname{arccot}(\tanh(bx+a))+\arctan(\tanh(bx+a))\operatorname{arctanh}(\tanh(bx+a))+\frac{\arctan(\tanh(bx+a))\ln\left(1+\frac{i(1+i\tan}{\tanh(bx}\right.}{2}}{b}$
risch	Expression too large to display

```
[In] int(arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(tanh(b*x+a))*arccot(tanh(b*x+a))+arctan(tanh(b*x+a))*arctanh(t
anh(b*x+a))+1/2*arctan(tanh(b*x+a))*ln(1+I*(1+I*tanh(b*x+a))^2/(tanh(b*x+a)
^2+1))-1/2*arctan(tanh(b*x+a))*ln(1-I*(1+I*tanh(b*x+a))^2/(tanh(b*x+a)^2+1)
)-1/4*I*dilog(1+I*(1+I*tanh(b*x+a))^2/(tanh(b*x+a)^2+1))+1/4*I*dilog(1-I*(1
+I*tanh(b*x+a))^2/(tanh(b*x+a)^2+1)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(56) = 112$.

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.58

$$\int \cot^{-1}(\tanh(a+bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right) + (i\,bx + i\,a)\log\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a) + \sinh(bx+a)) + 1\right) + (i\,bx + i\,a)\log\left(-\right.}{}$$

[In] integrate(arccot(tanh(b*x+a)),x, algorithm="fricas")

[Out] 1/2*(2*b*x*arctan(cosh(b*x + a)/sinh(b*x + a)) + (I*b*x + I*a)*log(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*a*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b

Sympy [F]

$$\int \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{acot}(\tanh(a+bx))\,dx$$

[In] integrate(acot(tanh(b*x+a)),x)

[Out] Integral(acot(tanh(a + b*x)), x)

Maxima [F]

$$\int \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{arccot}(\tanh(bx+a))\,dx$$

[In] integrate(arccot(tanh(b*x+a)),x, algorithm="maxima")

[Out] x*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + 2*b*integrate(x*e^(2*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)

Giac [F]

$$\int \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{arccot}\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(arccot(tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot(tanh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{acot}(\tanh(a+b\,x))\,dx$$

```
[In] int(acot(tanh(a + b*x)),x)
[Out] int(acot(tanh(a + b*x)), x)
```

3.187 $\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \operatorname{Int}\left(\frac{\cot^{-1}(\tanh(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arccot(tanh(b*x+a))/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx$$

```
[In] Int[ArcCot[Tanh[a + b*x]]/(e + f*x),x]
[Out] Defer[Int][ArcCot[Tanh[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx$$

```
[In] Integrate[ArcCot[Tanh[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcCot[Tanh[a + b*x]]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}(\tanh(bx+a))}{fx+e}dx$$

```
[In] int(arccot(tanh(b*x+a))/(f*x+e),x)
[Out] int(arccot(tanh(b*x+a))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arccot}(\tanh(bx+a))}{fx+e}\,dx$$

```
[In] integrate(arccot(tanh(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arccot(tanh(b*x + a))/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{acot}(\tanh(a+bx))}{e+fx}\,dx$$

```
[In] integrate(acot(tanh(b*x+a))/(f*x+e),x)
[Out] Integral(acot(tanh(a + b*x))/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arccot}(\tanh(bx+a))}{fx+e}\,dx$$

```
[In] integrate(arccot(tanh(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arccot(tanh(b*x + a))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 90.59 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.20

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arccot}(\tanh(bx+a))}{fx+e}\,dx$$

```
[In] integrate(arccot(tanh(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{acot}(\tanh(a+b\,x))}{e+f\,x}\,dx$$

```
[In] int(acot(tanh(a + b*x))/(e + f*x),x)
[Out] int(acot(tanh(a + b*x))/(e + f*x), x)
```

3.188 $\int x^2 \cot^{-1}(c+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 355

$$
\begin{aligned}
\int x^2 \cot^{-1}(c+d\tanh(a+bx))\,dx &= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) \\
&\quad -\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad +\frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\
&\quad -\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&\quad +\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\
&\quad +\frac{ix\operatorname{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\
&\quad -\frac{ix\operatorname{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} \\
&\quad -\frac{i\operatorname{PolyLog}\left(4,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} \\
&\quad +\frac{i\operatorname{PolyLog}\left(4,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}
\end{aligned}
$$

```
[Out] 1/3*x^3*arccot(c+d*tanh(b*x+a))-1/6*I*x^3*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))+1/6*I*x^3*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x^2*polylog(2,-(I-
c-d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x^2*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/
(I+c-d))/b+1/4*I*x*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/4*I*x*p
olylog(3,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2-1/8*I*polylog(4,-(I-c-d)*exp(
```

```
2*b*x+2*a)/(I-c+d))/b^3+1/8*I*polylog(4,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^
3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5308, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \cot^{-1}(c+d\tanh(a+bx))\,dx = -\frac{i\,\text{PolyLog}\left(4,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3} + \frac{i\,\text{PolyLog}\left(4,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3} + \frac{ix\,\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2} - \frac{ix\,\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2} - \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} + \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} - \frac{1}{6}ix^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) + \frac{1}{6}ix^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right) + \frac{1}{3}x^3\cot^{-1}(d\tanh(a+bx)+c)$$

```
[In] Int[x^2*ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Tanh[a + b*x]])/3 - (I/6)*x^3*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] + (I/6)*x^3*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] - ((I/4)*x^2*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c
+ d))])/b + ((I/4)*x^2*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c -
d))])/b + ((I/4)*x*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))]
)/b^2 - ((I/4)*x*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/
b^2 - ((I/8)*PolyLog[4, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^3
+ ((I/8)*PolyLog[4, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5308

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*
b*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
- d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{3}(b(1-i(c+d)))\int \frac{e^{2a+2bx}x^3}{i+c-d+(i+c+d)e^{2a+2bx}}\,dx + \frac{1}{3}(b(1+i(c+d)))\int \frac{e^{2a+2bx}x^3}{i-c+d+(i-c-d)e^{2a+2bx}}\,dx$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{1}{2}i\int x^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx - \frac{1}{2}i\int x^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{i\int x\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx}{2b} - \frac{i\int x\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx}{2b}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{ix\,\text{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} - \frac{ix\,\text{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} - \frac{i\int \text{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx}{4b^2} + \frac{i\int \text{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx}{4b^2}$$

$$= \frac{1}{3}x^3 \cot^{-1}(c + d \tanh(a + bx)) - \frac{1}{6}ix^3 \log\left(1 + \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}ix^3 \log\left(1 + \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix^2 \operatorname{PolyLog}\left(2, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix^2 \operatorname{PolyLog}\left(2, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} - \frac{ix \operatorname{PolyLog}\left(3, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} - \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, -\frac{(-i+c+d)x}{-i+c-d}\right)}{x} \, dx, x, e^{2a+2bx}\right)}{8b^3} + \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, -\frac{(i+c+d)x}{i+c-d}\right)}{x} \, dx, x, e^{2a+2bx}\right)}{8b^3}$$

$$= \frac{1}{3}x^3 \cot^{-1}(c + d \tanh(a + bx)) - \frac{1}{6}ix^3 \log\left(1 + \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}ix^3 \log\left(1 + \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix^2 \operatorname{PolyLog}\left(2, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix^2 \operatorname{PolyLog}\left(2, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(3, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} - \frac{ix \operatorname{PolyLog}\left(3, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} + \frac{i \operatorname{PolyLog}\left(4, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.23

$$\int x^2 \cot^{-1}(c + d \tanh(a + bx)) \, dx = \frac{1}{3}x^3 \cot^{-1}(c + d \tanh(a + bx)) + \frac{d\left(4b^3x^3 \log\left(1 + \frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right) - 4b^3x^3 \log\left(1 + \frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) + 6b^2x^2 \operatorname{PolyLog}\left(2, \frac{(1+c^2+2cd+d^2)e}{-1-c^2+d^2+2}\right.\right.}{}$$

```
[In] Integrate[x^2*ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Tanh[a + b*x]])/3 + (d*(4*b^3*x^3*Log[1 + (2*(1 + (c + d)
^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] - 4*b^3*x^3*Log[1
+ ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 6*b^2
*x^2*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 +
 2*Sqrt[-d^2])] - 6*b^2*x^2*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/
(1 + c^2 - d^2 + 2*Sqrt[-d^2]))] - 6*b*x*PolyLog[3, ((1 + c^2 + 2*c*d + d^2
```

```
)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 6*b*x*PolyLog[3, -(((
1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))] + 3*PolyLo
g[4, (-2*(1 + (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2]
)] - 3*PolyLog[4, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sq
rt[-d^2]))]))/(24*b^3*Sqrt[-d^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 42.43 (sec) , antiderivative size = 6916, normalized size of antiderivative = 19.48

method	result	size
risch	Expression too large to display	6916

```
[In] int(x^2*arccot(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(263) = 526.

Time = 0.34 (sec) , antiderivative size = 1289, normalized size of antiderivative = 3.63

$$\int x^2 \cot^{-1}(c+d\tanh(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*arccot(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan(cosh(b*x + a)/(c*cosh(b*x + a) + d*sinh(b*x + a))) -
3*I*b^2*x^2*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(c
osh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 + 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*
x^2*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x
+ a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^3*log(2*(c^2
 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a)
 + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d
^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*
c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 +
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)
*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2
*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*l
og(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh
```

```
(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -
sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a))) - 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -sqrt(-
(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
+ a))) + (-I*b^3*x^3 - I*a^3)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*l
og(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^
3 + I*a^3)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) - 6*I*polylog(4, sqrt(-(c^2 - d^2 + 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polylo
g(4, -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a)
 + sinh(b*x + a))) + 6*I*polylog(4, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*
c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*polylog(4, -sqrt(-(c
^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
a))))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\tanh(a + bx))\,dx = \text{Timed out}$$

```
[In] integrate(x**2*acot(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2 \cot^{-1}(c + d\tanh(a + bx))\,dx = \int x^2 \operatorname{arccot}\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(x^2*arccot(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2(e^(2*b*x + 2*a) + 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c
- d) + 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^
(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) -
 d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Giac [F]

$$\int x^2 \cot^{-1}(c + d\tanh(a + bx))\,dx = \int x^2 \operatorname{arccot}\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(x^2*arccot(c+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arccot(d*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\tanh(a + bx))\,dx = \int x^2 \operatorname{acot}(c + d\tanh(a + b\,x))\,dx$$

```
[In] int(x^2*acot(c + d*tanh(a + b*x)),x)
[Out] int(x^2*acot(c + d*tanh(a + b*x)), x)
```

3.189 $\int x \cot^{-1}(c+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 267

$$\begin{aligned}\int x\cot^{-1}(c+d\tanh(a+bx))\,dx=&\frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx))\\&-\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)\\&+\frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)\\&-\frac{ix\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b}\\&+\frac{ix\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}\\&+\frac{i\,\text{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2}\\&-\frac{i\,\text{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}\end{aligned}$$

```
[Out] 1/2*x^2*arccot(c+d*tanh(b*x+a))-1/4*I*x^2*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))+1/4*I*x^2*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x*polylog(2,-(I-c-
d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c
-d))/b+1/8*I*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/8*I*polylog(3
,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5308, 2221, 2611, 2320, 6724}

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx = \frac{i\,\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2} - \frac{i\,\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2} - \frac{ix\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} + \frac{ix\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} - \frac{1}{4}ix^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) + \frac{1}{4}ix^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right) + \frac{1}{2}x^2\cot^{-1}(d\tanh(a+bx)+c)$$

```
[In] Int[x*ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Tanh[a + b*x]])/2 - (I/4)*x^2*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] + (I/4)*x^2*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] - ((I/4)*x*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d))])/b + ((I/4)*x*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]
)/b + ((I/8)*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^2
- ((I/8)*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5308

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*
b*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
- d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{2}(b(1-i(c+d)))\int \frac{e^{2a+2bx}x^2}{i+c-d+(i+c+d)e^{2a+2bx}}\,dx \\
&\quad + \frac{1}{2}(b(1+i(c+d)))\int \frac{e^{2a+2bx}x^2}{i-c+d+(i-c-d)e^{2a+2bx}}\,dx \\
&= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad + \frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{1}{2}i\int x\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx \\
&\quad - \frac{1}{2}i\int x\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx
\end{aligned}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{i\int\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx}{4b} - \frac{i\int\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx}{4b}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{i\text{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{(-i+c+d)x}{-i+c-d}\right)}{x}dx,x,e^{2a+2bx}\right)}{8b^2} - \frac{i\text{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{(i+c+d)x}{i+c-d}\right)}{x}dx,x,e^{2a+2bx}\right)}{8b^2}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{i\operatorname{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} - \frac{i\operatorname{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.24

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx)) + \frac{d\left(2b^2x^2\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right) - 2b^2x^2\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) + 2bx\operatorname{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2)e}{-1-c^2+d^2+2\sqrt{}}\right.\right.}{}$$

```
[In] Integrate[x*ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Tanh[a + b*x]])/2 + (d*(2*b^2*x^2*Log[1 + (2*(1 + (c + d)
^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] - 2*b^2*x^2*Log[1
+ ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 2*b*x
*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*S
qrt[-d^2])] - 2*b*x*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2
 - d^2 + 2*Sqrt[-d^2]))] - PolyLog[3, (-2*(1 + (c + d)^2)*E^(2*(a + b*x)))/
(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + PolyLog[3, -(((1 + (c + d)^2)*E^(2*(a
 + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))]))/(8*b^2*Sqrt[-d^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.51 (sec) , antiderivative size = 6566, normalized size of antiderivative = 24.59

method	result	size
risch	Expression too large to display	6566

```
[In] int(x*arccot(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. 2(197) = 394.

Time = 0.35 (sec) , antiderivative size = 1067, normalized size of antiderivative = 4.00

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(x*arccot(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan(cosh(b*x + a)/(c*cosh(b*x + a) + d*sinh(b*x + a))) -
2*I*b*x*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(
b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(sq
rt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(
b*x + a))) + 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^
2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 +
 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2
- 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^
2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2
 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
```

```
 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-
(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*
d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c
^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1)
)) + (-I*b^2*x^2 + I*a^2)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(-
sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c
^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 -
I*a^2)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a)) + 1) + 2*I*polylog(3, sqrt(-(c^2 - d^2 + 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3,
 -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a))) - 2*I*polylog(3, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d
+ d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3, -sqrt(-(c^2 -
 d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
)/b^2
```

Sympy [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx=\text{Timed out}$$

```
[In] integrate(x*acot(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx=\int x\,\mathrm{arccot}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(e^(2*b*x + 2*a) + 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c
- d) + 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a
) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2
*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Giac [F]

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx=\int x\,\mathrm{arccot}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccot(d*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx=\int x\,\mathrm{acot}(c+d\tanh(a+b\,x))\,dx$$

```
[In] int(x*acot(c + d*tanh(a + b*x)),x)
[Out] int(x*acot(c + d*tanh(a + b*x)), x)
```

3.190 $\int \cot^{-1}(c+d\tanh(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 174

$$\begin{aligned}\int \cot^{-1}(c+d\tanh(a+bx))\,dx &= x\cot^{-1}(c+d\tanh(a+bx)) \\ &\quad -\frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\ &\quad +\frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\ &\quad -\frac{i\,\mathrm{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ &\quad +\frac{i\,\mathrm{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}\end{aligned}$$

```
[Out] x*arccot(c+d*tanh(b*x+a))-1/2*I*x*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+d))+1/2*
I*x*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*polylog(2,-(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b+1/4*I*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used

$= \{5300, 2221, 2317, 2438\}$

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx = -\frac{i\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} - \frac{1}{2}ix\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) + \frac{1}{2}ix\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right) + x\cot^{-1}(d\tanh(a+bx)+c)$$

```
[In] Int[ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Tanh[a + b*x]] - (I/2)*x*Log[1 + ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] + (I/2)*x*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] - ((I/4)*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b + ((
I/4)*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5300

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Cot[c + d*Tanh[a + b*x]], x] + (-Dist[I*b*(I - c - d), Int[x*(E^(2*a + 2*b*
x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*b*(I + c + d
), Int[x*(E^(2*a + 2*b*x)/(I + c - d + (I + c + d)*E^(2*a + 2*b*x))), x], x
]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\cot^{-1}(c+d\tanh(a+bx)) - (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(i+c+d)e^{2a+2bx}}\,dx \\
&\quad + (b(1+i(c+d)))\int \frac{e^{2a+2bx}x}{i-c+d+(i-c-d)e^{2a+2bx}}\,dx \\
&= x\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad + \frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{1}{2}i\int \log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)dx \\
&\quad - \frac{1}{2}i\int \log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)dx \\
&= x\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad + \frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) + \frac{i\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(i-c-d)x}{i-c+d}\right)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&\quad - \frac{i\,\text{Subst}\left(\int \frac{\log\left(1+\frac{(i+c+d)x}{i+c-d}\right)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&= x\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad + \frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\
&\quad - \frac{i\,\text{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.66

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx = x\cot^{-1}(c+d\tanh(a+bx)) \\ -\frac{4a\sqrt{-d^2}\arctan\left(\frac{1+c^2-d^2+(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right) - 2d(a+bx)\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right) + 2d(a+bx)\log}{4b\sqrt{-}}$$

```
[In] Integrate[ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Tanh[a + b*x]] - (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 + (1
+ c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] - 2*d*(a + b*x)*Log[1 + (2*(1
```

```
+ (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + 2*d*(a
+ b*x)*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d
^2])] + d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 -
 d^2 + 2*Sqrt[-d^2]))] - d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*
x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])])/(4*b*Sqrt[-d^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(150) = 300$.

Time = 3.03 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-d^2\left(\frac{\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\frac{i+}{}\right.}{2}}{}\right.}{}$
default	$\frac{\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-d^2\left(\frac{\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\frac{i+}{}\right.}{2}}{}\right.}{}$
risch	Expression too large to display

```
[In] int(arccot(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(1/2*arccot(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)-1/2*arccot(c+d*ta
nh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)-1/2*d^2*(1/d*(1/2*I*ln(-d*tanh(b*x+a)+d)*
ln((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I*ln(-d*tanh(b*x+a)+d)*ln((I-d*tanh(b*x
+a)-c)/(I-c-d))+1/2*I*dilog((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I*dilog((I-d*t
anh(b*x+a)-c)/(I-c-d)))-1/d*(1/2*I*ln(-d*tanh(b*x+a)-d)*ln((I+d*tanh(b*x+a)
+c)/(I+c-d))-1/2*I*ln(-d*tanh(b*x+a)-d)*ln((I-d*tanh(b*x+a)-c)/(I-c+d))+1/2
*I*dilog((I+d*tanh(b*x+a)+c)/(I+c-d))-1/2*I*dilog((I-d*tanh(b*x+a)-c)/(I-c+
d)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(128) = 256$.

Time = 0.40 (sec) , antiderivative size = 825, normalized size of antiderivative = 4.74

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx$$

$$=\frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{c\cosh(bx+a)+d\sinh(bx+a)}\right)+i\,a\log\left(2\left(c^2+2\,cd+d^2+1\right)\cosh\left(bx+a\right)+2\left(c^2+2\,cd+d^2+1\right)\mathrm{si}\right.}{}$$

```
[In] integrate(arccot(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(cosh(b*x + a)/(c*cosh(b*x + a) + d*sinh(b*x + a))) + I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 -
 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(
c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d
^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I
*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*
sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))) + (-I*b*x - I*a)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*
a)*log(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)/
(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*
a)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
a) + sinh(b*x + a)) + 1) - I*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c
*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-sqrt(-(c^2 - d^2
 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I
*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a
) + sinh(b*x + a))) + I*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx=\int \operatorname{acot}\left(c+d\tanh\left(a+bx\right)\right)dx$$

```
[In] integrate(acot(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(acot(c + d*tanh(a + b*x)), x)
```

Maxima [F]

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx=\int \operatorname{arccot}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arccot(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*d
*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2*e^(2*a)
+ e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2(e^(2*b*x + 2*a) + 1, (c*e^(2*a) +
 d*e^(2*a))*e^(2*b*x) + c - d)
```

Giac [F]

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx=\int \operatorname{arccot}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arccot(c+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot(d*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx=\int \operatorname{acot}(c+d\tanh(a+b\,x))\,dx$$

```
[In] int(acot(c + d*tanh(a + b*x)),x)
[Out] int(acot(c + d*tanh(a + b*x)), x)
```

3.191 $\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\cot^{-1}(c+d\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+d*tanh(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c + d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c + d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + d*Tanh[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(c+d\tanh\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arccot(c+d*tanh(b*x+a))/x,x)
[Out] int(arccot(c+d*tanh(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arccot(d*tanh(b*x + a) + c)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx=\text{Timed out}$$

```
[In] integrate(acot(c+d*tanh(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c + d\tanh(a + bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\tanh\left(bx + a\right) + c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arccot(d*tanh(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c + d\tanh(a + bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\tanh\left(bx + a\right) + c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*tanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot(d*tanh(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c + d\tanh(a + bx))}{x}\,dx = \int \frac{\operatorname{acot}(c + d\tanh(a + b\,x))}{x}\,dx$$

```
[In] int(acot(c + d*tanh(a + b*x))/x,x)
[Out] int(acot(c + d*tanh(a + b*x))/x, x)
```

3.192 $\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\begin{aligned}\int x^2\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = {} & \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) \\ & -\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) \\ & -\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} \\ & +\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2} \\ & -\frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}\end{aligned}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arccot(c+(I+c)*tanh(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*b
*x+2*a))-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,-I*c*
exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5304, 2215, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}&\int x^2\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx \\ &= -\frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}+\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} \\ &\quad -\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))+\frac{1}{12}ibx^4\end{aligned}$$

```
[In] Int[x^2*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcCot[c + (I + c)*Tanh[a + b*x]])/3 - (I/6)*x^3*Log[1
+ I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b +
 ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, (-I)*
c*E^(2*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{3}b\int\frac{x^3}{-i+ce^{2a+2bx}}\,dx\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{-i+ce^{2a+2bx}}\,dx\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx))\\
&\quad-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}i\int x^2\log\left(1+ice^{2a+2bx}\right)\,dx\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)\\
&\quad-\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\int x\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)\,dx}{2b}\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx))\\
&\quad-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}\\
&\quad+\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{i\int\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)\,dx}{4b^2}\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx))\\
&\quad-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}\\
&\quad+\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{i\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3}
\end{aligned}
$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) - \frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2} - \frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int x^2\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{8b^3x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - 4ib^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) + 6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right) + 6ibx$$

```
[In] Integrate[x^2*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcCot[c + (I + c)*Tanh[a + b*x]] - (4*I)*b^3*x^3*Log[1 - I/(c*E
^(2*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] + (6*I)*
b*x*PolyLog[3, I/(c*E^(2*(a + b*x)))] + (3*I)*PolyLog[4, I/(c*E^(2*(a + b*x
)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.53 (sec) , antiderivative size = 1405, normalized size of antiderivative = 9.89

method	result	size
risch	Expression too large to display	1405

```
[In] int(x^2*arccot(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I/b^3*c/(I+c)*a^4+1/12*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2
*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(
I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b
*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2
*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*c
sgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*
(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(2*exp(2*b*x+2*a)*c-2
*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2
```

```
+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)
*c-2*I)/(exp(2*b*x+2*a)+1))-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/
(exp(2*b*x+2*a)+1))^3+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2
*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+
1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*cs
gn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-csgn((2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+csgn((2*I*exp(2*b*x+
2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn((2*exp(2*b*x+2*a)*c-2*I
)/(exp(2*b*x+2*a)+1))^3+csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2
)*x^3-1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a^2*ln(1+I*exp
(b*x+a)*(I*c)^(1/2))*x+1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)-1/2*I/b^2*a^2*
ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x+1/12*I*b*c/(I+c)*x^4+1/2*I/b^2*ln(1+I*c*ex
p(2*b*x+2*a))*a^2*x+1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c-2*I)+1/3*I/b^3*ln(1+I*c
*exp(2*b*x+2*a))*a^3+1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2-1/4*I*x^2
*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a))/b^
2-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3-1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)+
2*exp(2*b*x+2*a)*c)-1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))-1/2*I/b^3*a^3*ln(1-I
*exp(b*x+a)*(I*c)^(1/2))-1/12*b/(I+c)*x^4-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a
)*(I*c)^(1/2))-1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/12/b^3/(I+
c)*a^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.06

$$\int x^2 \cot^{-1}(c + (i + c)\tanh(a + bx))\,dx$$
$$= \frac{i\,b^4x^4 + 2i\,b^3x^3\log\left(\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c+i}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) -}{}$$

```
[In] integrate(x^2*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/
(c + I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*di
log(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c)
)/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*b*x*polylog
(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(-4*
I*c)*e^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(-4*I*c)*e^(b*x
+ a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*polylog(4,
 -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*acot(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) - 3*_t0**2*I*c*exp(2*a) + _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{3}x^3 \operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right) - \frac{4}{9}\left(\frac{3\,x^4}{4i\,c-4} - \frac{4\,b^3x^3\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-i\,ce^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(-}{-2\,b^4(-i\,c+1)}\right.$$

```
[In] integrate(x^2*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot((c + I)*tanh(b*x + a) + c) - 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c - 2)))*b*(c + I)
```

Giac [F]

$$\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \int x^2 \operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x^2*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot((c + I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx=\int x^2\,\mathrm{acot}\left(c+\tanh(a+b\,x)\,\left(c+1\mathrm{i}\right)\right)\,dx$$

```
[In] int(x^2*acot(c + tanh(a + b*x)*(c + 1i)),x)
[Out] int(x^2*acot(c + tanh(a + b*x)*(c + 1i)), x)
```

3.193 $\int x \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx=\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arccot(c+(I+c)*tanh(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,-I*c*exp(2
*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5304, 2215, 2221, 2611, 2320, 6724}

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx=\frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}-\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}x^2\cot^{-1}(c+(c+i)\tanh(a+bx))+\frac{1}{6}ibx^3$$

```
[In] Int[x*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCot[c + (I + c)*Tanh[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b + ((
I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{-i+ce^{2a+2bx}}\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{-i+ce^{2a+2bx}}\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))\\
&\quad -\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}i\int x\log\left(1+ice^{2a+2bx}\right)\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\int\text{PolyLog}\left(2,-ice^{2a+2bx}\right)\,dx}{4b}\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\cot^{-1}(c+(i+c)\tanh(a+bx))-i\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)+i\,\text{PolyLo}}{8b^2}$$

```
[In] Integrate[x*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCot[c + (I + c)*Tanh[a + b*x]] - I*Log[1 - I/(c*E^(2*(a +
b*x)))]) + (2*I)*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] + I*PolyLog[3, I/(c*
E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.96 (sec) , antiderivative size = 1369, normalized size of antiderivative = 12.12

method	result	size
risch	Expression too large to display	1369

```
[In] int(x*arccot(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x+1/8*Pi*(csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*
b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+
2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*
exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*I*exp(2*b*x+2*a)+2*ex
p(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+
2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(2*
exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(ex
p(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn
((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-csgn(I*(2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c
)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp
(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a
)+1))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+cs
gn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn((2*ex
p(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+csgn((2*exp(2*b*x+2*a)*c-2*I)/(ex
p(2*b*x+2*a)+1))^2)*x^2-1/4*I*x^2*ln(1+I*c*exp(2*b*x+2*a))+1/2*I/b*a*ln(1-I
*exp(b*x+a)*(I*c)^(1/2))*x+1/8*I*polylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/2*I/b
^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^2*ln(1+I*exp(b*x+a)*(I*c)^(1
/2))*a^2+1/6*I/b^2*c/(I+c)*a^3+1/2*I/b^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*a^2
+1/6*I*b*c/(I+c)*x^3-1/4*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^2-1/4*I*x*polylog
(2,-I*c*exp(2*b*x+2*a))/b-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*
c)-1/4*I/b^2*polylog(2,-I*c*exp(2*b*x+2*a))*a-1/4*I/b^2*a^2*ln(-exp(2*b*x+2
*a)*c+I)+1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c-2*I)-1/6/b^2/(I+c)*a^3-1/6*b/(I+c)
*x^3-1/2*I/b*ln(1+I*c*exp(2*b*x+2*a))*a*x+1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*
(I*c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(83) = 166.

Time = 0.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.18

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{2i\,b^3x^3+3i\,b^2x^2\log\left(\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c+i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(b}\right.}{}$$

```
[In] integrate(x*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a
)/(c + I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b*
x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
 I*sqrt(-4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
 - 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*x
^2 - I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sqr
t(-4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*acot(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) - 3*_t0**2*I*c*exp(2*a) + _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = -\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i) + \frac{1}{2}\,x^2\,\mathrm{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)$$

```
[In] integrate(x*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2)
))*b*(c + I) + 1/2*x^2*arccot((c + I)*tanh(b*x + a) + c)
```

Giac [F]

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx=\int x\,\mathrm{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot((c + I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx=\int x\,\mathrm{acot}\left(c+\tanh(a+b\,x)\,\left(c+1\mathrm{i}\right)\right)\,dx$$

```
[In] int(x*acot(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x*acot(c + tanh(a + b*x)*(c + 1i)), x)
```

3.194 $\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{2}ibx^2 + x\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arccot(c+(I+c)*tanh(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5296, 2215, 2221, 2317, 2438}

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = -\frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + x\cot^{-1}(c+(c+i)\tanh(a+bx)) + \frac{1}{2}ibx^2$$

```
[In] Int[ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCot[c + (I + c)*Tanh[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5296

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Cot[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\cot^{-1}(c+(i+c)\tanh(a+bx))+b\int\frac{x}{-i+ce^{2a+2bx}}\,dx\\
&= \frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx))-(ibc)\int\frac{e^{2a+2bx}x}{-i+ce^{2a+2bx}}\,dx\\
&= \frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}i\int\log\left(1+ice^{2a+2bx}\right)\,dx\\
&= \frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx))\\
&\quad-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+\frac{i\text{Subst}\left(\int\frac{\log(1+icx)}{x}\,dx,x,e^{2a+2bx}\right)}{4b}\\
&= \frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)-\frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$$
$$= x\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\operatorname{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

```
[In] Integrate[ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (I + c)*Tanh[a + b*x]] - ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(65) = 130.

Time = 1.83 (sec) , antiderivative size = 544, normalized size of antiderivative = 6.89

method	result
derivativedivides	$-\frac{\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\operatorname{arccot}(c+(i}{}$
default	$-\frac{\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\operatorname{arccot}(c+(i}{}$
risch	Expression too large to display

```
[In] int(arccot(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I+c)*(-arccot(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))
+2*I*arccot(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c+arcc
ot(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c^2+arccot(c+(I
+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)-2*I*arccot(c+(I+c)*tan
h(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c-arccot(c+(I+c)*tanh(b*x+a))
/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c^2+(I+c)^2*(1/2/(I+c)*(1/4*I*ln(I+c+(
I+c)*tanh(b*x+a))^2-1/2*I*((ln(I+c+(I+c)*tanh(b*x+a))-ln(-1/2*I*(I+c+(I+c)*
tanh(b*x+a))))*ln(-1/2*I*(I-c-(I+c)*tanh(b*x+a)))-dilog(-1/2*I*(I+c+(I+c)*t
anh(b*x+a)))))-1/2/(I+c)*(1/2*I*(dilog((-I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c))
+ln(c-(I+c)*tanh(b*x+a)+I)*ln((-I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c)))-1/2*I*(
dilog(-1/2*(I-c-(I+c)*tanh(b*x+a))/c)+ln(c-(I+c)*tanh(b*x+a)+I)*ln(-1/2*(I-
c-(I+c)*tanh(b*x+a))/c)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.35

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{i\,b^2x^2+i\,bx\log\left(\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c+i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log}{}$$

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c + I)
) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-I*b*x
- I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(-4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
- I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x
+ a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(acot(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) - 3*_t0**2*I*c*exp(2*a) + _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = -2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right) + x\,\mathrm{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)$$

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dil
og(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arccot((c + I)*tanh(b*x +
a) + c)
```

Giac [F]

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \int \operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot((c + I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \int \operatorname{acot}(c+\tanh(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

```
[In] int(acot(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(acot(c + tanh(a + b*x)*(c + 1i)), x)
```

3.195 $\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+(I+c)*tanh(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c + (I + c)*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCot[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c + (I + c)*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arccot}\left(c+(i+c)\tanh\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arccot(c+(I+c)*tanh(b*x+a))/x,x)
[Out] int(arccot(c+(I+c)*tanh(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c + I))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx=\text{Timed out}$$

```
[In] integrate(acot(c+(I+c)*tanh(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 81, normalized size of antiderivative = 4.26

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/2*pi*log(x) - 1/4*(2*pi + 4*I*a - 2*arctan(1/c) + I*log(c^2 + 1)
)*log(x) + 1/2*integrate(arctan(e^(-2*b*x - 2*a)/c)/x, x) + 1/4*I*integrate
(log(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot((c + I)*tanh(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acot}(c+\tanh(a+b\,x)\,\,(c+1\mathrm{i}))}{x}\,dx$$

```
[In] int(acot(c + tanh(a + b*x)*(c + 1i))/x,x)
```

```
[Out] int(acot(c + tanh(a + b*x)*(c + 1i))/x, x)
```

3.196 $\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\begin{aligned}\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)\\&\quad+\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}+\frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}\end{aligned}$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arccot(c-(I-c)*tanh(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,I*c*e
xp(2*b*x+2*a))/b^2+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5304, 2215, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx &= \frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}\\&\quad-\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}\\&\quad+\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}\\&\quad+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)\\&\quad+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\tanh(a+bx))-\frac{1}{12}ibx^4\end{aligned}$$

```
[In] Int[x^2*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/12*I)*b*x^4 + (x^3*ArcCot[c - (I - c)*Tanh[a + b*x]])/3 + (I/6)*x^3*Log
[1 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b -
 ((I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, I*c*E^(2
*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{3}b\int\frac{x^3}{i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))\\
&\quad+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)-\frac{1}{2}i\int x^2\log\left(1-ice^{2a+2bx}\right)\,dx\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)\\
&\quad+\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{i\int x\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)\,dx}{2b}\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}\\
&\quad-\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}+\frac{i\int\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)\,dx}{4b^2}\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))\\
&\quad+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}\\
&\quad-\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}+\frac{i\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3}\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)\\
&\quad+\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}+\frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$$
$$= \frac{8b^3x^3\cot^{-1}(c+(-i+c)\tanh(a+bx))+4ib^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-6ib^2x^2\,\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-6i}{24b^3}$$

```
[In] Integrate[x^2*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcCot[c + (-I + c)*Tanh[a + b*x]] + (4*I)*b^3*x^3*Log[1 + I/(c*
E^(2*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - (6
*I)*b*x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - (3*I)*PolyLog[4, (-I)/(c*E^(
2*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.47 (sec) , antiderivative size = 1409, normalized size of antiderivative = 9.72

method	result	size
risch	Expression too large to display	1409

```
[In] int(x^2*arccot(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/12*Pi*(csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*
I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(ex
p(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)
/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I
))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I*exp(2
*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)
*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1
))^3+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+
2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*
x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))+csgn(I*(-2*I*e
xp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-csgn(I*(-2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+
2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c)/(exp(2*b*x+2*a)+1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(ex
```

```
p(2*b*x+2*a)+1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x
+2*a)+1))^2+csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3+csgn((2*exp
(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-4)*x^3+1/12*I*b*c/(I-c)*x^4+1/2*I/
b^3*a^2*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*
(-I*c)^(1/2))+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3+1/2*I/b^2*a^2*ln(1+I*
exp(b*x+a)*(-I*c)^(1/2))*x-1/6*I*x^3*ln(-2*exp(2*b*x+2*a)*c-2*I)-1/2*I/b^2*
ln(1-I*c*exp(2*b*x+2*a))*a^2*x+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2)
)*x+1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))-1/12*I/b^3*c*a^4/(I-c)+1/2*I/b^3*a^2
*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(-I*c)^
(1/2))-1/4*I*x*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/6*I/b^3*a^3*ln(exp(2*b*x
+2*a)*c+I)-1/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3-1/12/b^3/(I-c)*a^4+1/12*b
/(I-c)*x^4-1/4*I/b^3*polylog(2,I*c*exp(2*b*x+2*a))*a^2+1/4*I*x^2*polylog(2,
I*c*exp(2*b*x+2*a))/b
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(105) = 210$.

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.01

$$\int x^2 \cot^{-1}(c - (i - c)\tanh(a + bx))\, dx$$

$$= \frac{-i\, b^4 x^4 + 2i\, b^3 x^3 \log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) + 6i\, b^2 x^2 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) + 6i\, b^2 x^2 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) + i\, a^4 -}{}$$

```
[In] integrate(x^2*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*
a) + I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*dil
og(-1/2*sqrt(4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c)
 - 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*b*x*polylog(3, -
1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(4*I*c)*e
^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) +
 1) + 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*polylog(4, -1/2*s
qrt(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*acot(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) + I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{3}\,x^3 \operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right) + \frac{4}{9}\left(\frac{3\,x^4}{4i\,c+4} - \frac{4\,b^3x^3\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(i\,ce^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(i\,ce^{(}}{-2\,b^4(-i\,c-1)}\right.$$

```
[In] integrate(x^2*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot((c - I)*tanh(b*x + a) + c) + 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c + 2)))*b*(c - I)
```

Giac [F]

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \int x^2 \operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x^2*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \int x^2\,\mathrm{acot}(c+\tanh(a+b\,x)\ (c-\mathrm{i}))\ dx$$

```
[In] int(x^2*acot(c + tanh(a + b*x)*(c - 1i)),x)
[Out] int(x^2*acot(c + tanh(a + b*x)*(c - 1i)), x)
```

3.197 $\int x \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int x \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx)) +\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) +\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} -\frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arccot(c-(I-c)*tanh(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,I*c*exp(2*
b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5304, 2215, 2221, 2611, 2320, 6724}

$$\int x \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = -\frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2} +\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} +\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) +\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\tanh(a+bx))-\frac{1}{6}ibx^3$$

```
[In] Int[x*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/6*I)*b*x^3 + (x^2*ArcCot[c - (I - c)*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[
1 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - ((
I/8)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))\\
&\quad+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{1}{2}i\int x\log\left(1-ice^{2a+2bx}\right)\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\
&\quad+\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{i\int\text{PolyLog}\left(2,ice^{2a+2bx}\right)\,dx}{4b}\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\
&\quad+\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\
&\quad+\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\cot^{-1}(c+(-i+c)\tanh(a+bx))+i\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right)-2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-i\,\text{Poly}}{8b^2}$$

```
[In] Integrate[x*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
[Out] (2*b^2*x^2*(2*ArcCot[c + (-I + c)*Tanh[a + b*x]] + I*Log[1 + I/(c*E^(2*(a +
 b*x)))]) - (2*I)*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - I*PolyLog[3, (
-I)/(c*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.03 (sec) , antiderivative size = 1373, normalized size of antiderivative = 11.84

method	result	size
risch	Expression too large to display	1373

```
[In] int(x*arccot(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*x^2*ln(-2*exp(2*b*x+2*a)*c-2*I)-1/8*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+
2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1
))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*
a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*e
xp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*ex
p(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x
+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(2
*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(e
xp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csg
n((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))+csgn(I*(-2*I*exp(2*b*x+2*a)+
2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b
*x+2*a)+1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1
))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+cs
gn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3+csgn((2*exp(2*b*x+2*a)*c+
2*I)/(exp(2*b*x+2*a)+1))^2-4)*x^2-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(
1/2))+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I/b^2*polylog(2,I*c*exp(2
*b*x+2*a))*a+1/6*I*b*c/(I-c)*x^3+1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2-1/8
*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(
2*b*x+2*a)*c)-1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x-1/2*I/b^2*a*dilog
(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))*a*x-1/2*I/b^
2*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*a^2+1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))+1/
6*I/b^2*c/(I-c)*a^3-1/2*I/b*a*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x+1/6/b^2/(I-
c)*a^3+1/6*b/(I-c)*x^3+1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+I)-1/2*I/b^2*ln(1-
I*exp(b*x+a)*(-I*c)^(1/2))*a^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.12

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{-2i\,b^3x^3+3i\,b^2x^2\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right)-2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}e^{(bx+a)}\right)+3i\,c}{}$$

```
[In] integrate(x*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
2*a) + I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b*x
*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) + I
*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - 3
*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^2*x^2
+ I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*sqrt(4*
I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*acot(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) + I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \left(\frac{2\,x^3}{3i\,c+3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i) + \frac{1}{2}\,x^2\,\mathrm{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)$$

```
[In] integrate(x*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2)))
*b*(c - I) + 1/2*x^2*arccot((c - I)*tanh(b*x + a) + c)
```

Giac [F]

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx=\int x\,\mathrm{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx=\int x\,\mathrm{acot}(c+\tanh(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

```
[In] int(x*acot(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x*acot(c + tanh(a + b*x)*(c - 1i)), x)
```

3.198 $\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{i\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

```
[Out] -1/2*I*b*x^2+x*arccot(c-(I-c)*tanh(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5296, 2215, 2221, 2317, 2438}

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{i\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + x\cot^{-1}(c-(-c+i)\tanh(a+bx))-\frac{1}{2}ibx^2$$

```
[In] Int[ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/2*I)*b*x^2 + x*ArcCot[c - (I - c)*Tanh[a + b*x]] + (I/2)*x*Log[1 - I*c*
E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5296

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Cot[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\cot^{-1}(c-(i-c)\tanh(a+bx))+b\int \frac{x}{i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\tanh(a+bx))+(ibc)\int \frac{e^{2a+2bx}x}{i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\tanh(a+bx))\\
&\quad+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{1}{2}i\int \log\left(1-ice^{2a+2bx}\right)\,dx\\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\tanh(a+bx))\\
&\quad+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{i\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{4b}\\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{i\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$$
$$=x\cot^{-1}(c+(-i+c)\tanh(a+bx))+\frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

```
[In] Integrate[ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (-I + c)*Tanh[a + b*x]] + ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(68) = 136.

Time = 1.86 (sec) , antiderivative size = 517, normalized size of antiderivative = 6.30

method	result
derivativedivides	$-\frac{\operatorname{arccot}(c+\tanh(bx+a)(c-i))\ln(\tanh(bx+a)(c-i)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}(c+\tanh(bx+a)(c-i))\ln(\tanh(bx+a)(c-i)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}(c+\tanh(}{}$
default	$-\frac{\operatorname{arccot}(c+\tanh(bx+a)(c-i))\ln(\tanh(bx+a)(c-i)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}(c+\tanh(bx+a)(c-i))\ln(\tanh(bx+a)(c-i)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}(c+\tanh(}{}$
risch	Expression too large to display

```
[In] int(arccot(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(c-I)*(-arccot(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)
-2*I*arccot(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)*c+arcc
ot(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)*c^2+arccot(c+ta
nh(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)+2*I*arccot(c+tanh(b*x
+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)*c-arccot(c+tanh(b*x+a)*(c-I
))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)*c^2-(I-c)^2*(1/2/(I-c)*(-1/4*I*ln(-
I+tanh(b*x+a)*(c-I)+c)^2+1/2*I*(dilog(-1/2*I*(tanh(b*x+a)*(c-I)+c+I))+ln(-I
+tanh(b*x+a)*(c-I)+c)*ln(-1/2*I*(tanh(b*x+a)*(c-I)+c+I))))-1/2/(I-c)*(1/2*I
*(dilog(1/2*(tanh(b*x+a)*(c-I)+c+I)/c)+ln(tanh(b*x+a)*(c-I)-c+I)*ln(1/2*(ta
nh(b*x+a)*(c-I)+c+I)/c))-1/2*I*(dilog((-I+tanh(b*x+a)*(c-I)+c)/(-2*I+2*c))+
ln(tanh(b*x+a)*(c-I)-c+I)*ln((-I+tanh(b*x+a)*(c-I)+c)/(-2*I+2*c))))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.27

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$$
$$= \frac{-i\,b^2x^2 + i\,bx\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) + i\,a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{4i\,c}\right.}{}$$

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I)
) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (I*b*x + I
*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) + I*di
log(1/2*sqrt(4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(acot(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) + I of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,ex
p(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$$
$$= 2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2} - \frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right) + \mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)$$
$$+ x\,\mathrm{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)$$

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + dil
og(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arccot((c - I)*tanh(b*x + a
) + c)
```

Giac [F]

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx=\int \operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx=\int \operatorname{acot}(c+\tanh(a+b\,x)\,\,(c-\mathrm{i}))\,dx$$

```
[In] int(acot(c + tanh(a + b*x)*(c - 1i)),x)
[Out] int(acot(c + tanh(a + b*x)*(c - 1i)), x)
```

3.199 $\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c-(I-c)*tanh(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c - (I - c)*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCot[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c - (I - c)*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arccot}\left(c-(i-c)\tanh\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arccot(c-(I-c)*tanh(b*x+a))/x,x)
[Out] int(arccot(c-(I-c)*tanh(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(acot(c-(I-c)*tanh(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.32

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x - 1/4*(-4*I*a - 2*arctan(1/c) - I*log(c^2 + 1))*log(x) + 1/2*integrat
e(arctan(e^(-2*b*x - 2*a)/c)/x, x) - 1/4*I*integrate(log(c^2*e^(4*b*x + 4*a
) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot((c - I)*tanh(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx=\int \frac{\mathrm{acot}(c+\tanh(a+b\,x)\ (c-\mathrm{i}))}{x}\,dx$$

```
[In] int(acot(c + tanh(a + b*x)*(c - 1i))/x,x)
```

```
[Out] int(acot(c + tanh(a + b*x)*(c - 1i))/x, x)
```

3.200 $\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 299

$$\begin{aligned}\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = {} & \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} \\ & - \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} \\ & + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ & - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ & - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} \\ & + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\ & + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ & - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\ & - \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} \\ & + \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}\end{aligned}$$

```
[Out] 1/4*(f*x+e)^4*arccot(coth(b*x+a))/f-1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+
1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^3*polylog(2,I*
exp(2*b*x+2*a))/b-3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2+3/8*I*
f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2+3/8*I*f^2*(f*x+e)*polylog(4,-I*
exp(2*b*x+2*a))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3-3/16*
```

```
I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a
))/b^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5294, 4265, 2611, 6744, 2320, 6724}

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = -\frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} - \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} + \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4} + \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} - \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f}$$

```
[In] Int[(e + f*x)^3*ArcCot[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcCot[Coth[a + b*x]])/(4*f) - ((e + f*x)^4*ArcTan[E^(2*a + 2*
b*x)])/(4*f) + ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I
/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b - (((3*I)/8)*f*(e + f*x)^2
*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f*(e + f*x)^2*PolyLog[3
, I*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*a
 + 2*b*x)])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/b
^3 - (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 + (((3*I)/16)*f^
3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5294

```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Coth[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{b\int (e+fx)^4 \operatorname{sech}(2a+2bx)\,dx}{4f}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{1}{2} i \int (e+fx)^3 \log\left(1-ie^{2a+2bx}\right) dx - \frac{1}{2} i \int (e+fx)^3 \log\left(1+ie^{2a+2bx}\right) dx$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{(3if)\int (e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right) dx}{4b} + \frac{(3if)\int (e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right) dx}{4b}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{(3if^2)\int (e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right) dx}{4b^2} - \frac{(3if^2)\int (e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right) dx}{4b^2}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} - \frac{(3if^3)\int \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right) dx}{8b^3} + \frac{(3if^3)\int \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right) dx}{8b^3}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} - \frac{(3if^3) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,-ix)}{x} dx, x, e^{2a+2bx}\right)}{16b^4} + \frac{(3if^3) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,ix)}{x} dx, x, e^{2a+2bx}\right)}{16b^4}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} - \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} + \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. 2(299) = 598.

Time = 0.22 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.01

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\cot^{-1}(\coth(a+bx)) - \frac{i\left(8b^4e^3x\log\left(1-ie^{2(a+bx)}\right)+12b^4e^2fx^2\log\left(1-ie^{2(a+bx)}\right)+8b^4ef^2x^3\log\left(1-ie^{2(a+bx)}\right)+2b^4f^3x^4\log\left(\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^3*ArcCot[Coth[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCot[Coth[a + b*x]])/4 - (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
```

```
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 33.25 (sec) , antiderivative size = 3570, normalized size of antiderivative = 11.94

method	result	size
risch	Expression too large to display	3570

```
[In] int((f*x+e)^3*arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,-I*exp(2*b*
x+2*a))/b^4+3/2*I*f/b*e^2*ln(1+I*exp(2*b*x+2*a))*a*x+3/2*I*f^2/b^2*a^2*e*ln
(1+exp(b*x+a)*(-1)^(3/4))*x+3/2*I*f^2/b^2*a^2*e*ln(1-exp(b*x+a)*(-1)^(3/4))
*x-3/2*I*f/b*a*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*x-3/2*I*f/b*a*e^2*ln(1-exp(b
*x+a)*(-1)^(3/4))*x-3/8*I*f^3/b^3*polylog(4,I*exp(2*b*x+2*a))*x+1/2*I*f^3/b
^4*a^4*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f^3/b^4*a^4*ln(((-I)^(1
/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)-exp(b*x+a))
/(-I)^(1/2))+1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/
8*I*f^3/b^4*a^4*ln(exp(2*b*x+2*a)+I)-1/2*I*f^2*e*ln(1-I*exp(2*b*x+2*a))*x^3
-3/4*I*f*e^2*ln(1-I*exp(2*b*x+2*a))*x^2-3/8*I*f^2/b^3*e*polylog(4,I*exp(2*b
*x+2*a))+3/8*I*f/b^2*e^2*polylog(3,I*exp(2*b*x+2*a))-3/8*I*f^3/b^4*ln(1-I*e
xp(2*b*x+2*a))*a^4-1/4*I*f^3/b*polylog(2,I*exp(2*b*x+2*a))*x^3-1/4*I*f^3/b^
4*polylog(2,I*exp(2*b*x+2*a))*a^3+3/8*I*f^3/b^2*polylog(3,I*exp(2*b*x+2*a))
*x^2+1/2*I/b*a*e^3*ln(exp(2*b*x+2*a)+I)-1/2*I/b*e^3*ln(((-I)^(1/2)-exp(b*x+
a))/(-I)^(1/2))*a-1/2*I/b*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a-3/2*
I*f/b^2*a^2*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))-3/2*I*f/b^2*a^2*e^2*ln(1-exp(b*
x+a)*(-1)^(3/4))-3/2*I*f/b^2*a*e^2*dilog(1+exp(b*x+a)*(-1)^(3/4))-3/2*I*f/b
^2*a*e^2*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f^3/b^4*a^4*ln(1-exp(b*x+a)*(
-1)^(3/4))-1/2*I*f^3/b^4*a^3*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f^3/b^4*a
^3*dilog(1-exp(b*x+a)*(-1)^(3/4))+3/8*I*f^2/b^3*e*polylog(4,-I*exp(2*b*x+2*
a))+3/8*I*f^3/b^4*ln(1+I*exp(2*b*x+2*a))*a^4+1/4*I*f^3/b*polylog(2,-I*exp(2
*b*x+2*a))*x^3+1/4*I*f^3/b^4*polylog(2,-I*exp(2*b*x+2*a))*a^3-3/8*I*f^3/b^2
*polylog(3,-I*exp(2*b*x+2*a))*x^2+1/8*I*f^3/b^4*a^4*ln(-exp(2*b*x+2*a)+I)+1
/2*I*f^2*e*ln(1+I*exp(2*b*x+2*a))*x^3+3/4*I*f*e^2*ln(1+I*exp(2*b*x+2*a))*x^
2-3/8*I*f/b^2*e^2*polylog(3,-I*exp(2*b*x+2*a))+3/8*I*f^3/b^3*polylog(4,-I*e
```

xp(2*b*x+2*a))*x-1/2*I*f^3/b^4*a^4*ln(1+exp(b*x+a)*(-1)^(3/4))-I*f^2/b^3*e*
ln(1+I*exp(2*b*x+2*a))*a^3+3/4*I*f^2/b*e*polylog(2,-I*exp(2*b*x+2*a))*x^2-3
/4*I*f^2/b^3*e*polylog(2,-I*exp(2*b*x+2*a))*a^2-3/4*I*f^2/b^2*e*polylog(3,-
I*exp(2*b*x+2*a))*x+3/4*I*f/b^2*e^2*ln(1+I*exp(2*b*x+2*a))*a^2+3/4*I*f/b*e^
2*polylog(2,-I*exp(2*b*x+2*a))*x+3/4*I*f/b^2*e^2*polylog(2,-I*exp(2*b*x+2*a
))*a+1/2*I*f^3/b^3*ln(1+I*exp(2*b*x+2*a))*a^3*x-1/2*I*f^3/b^3*a^3*ln(1+exp(
b*x+a)*(-1)^(3/4))*x-1/2*I*f^3/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/2*I*
f^2/b^3*a^3*e*ln(-exp(2*b*x+2*a)+I)+3/4*I*f/b^2*a^2*e^2*ln(-exp(2*b*x+2*a)+
I)+3/2*I*f^2/b^3*a^3*e*ln(1+exp(b*x+a)*(-1)^(3/4))+3/2*I*f^2/b^3*a^3*e*ln(1
-exp(b*x+a)*(-1)^(3/4))+3/2*I*f^2/b^3*a^2*e*dilog(1+exp(b*x+a)*(-1)^(3/4))+
3/2*I*f^2/b^3*a^2*e*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f^2*ln(exp(2*b*x+2
*a)-I)*x^3*e-3/4*I*f*ln(exp(2*b*x+2*a)-I)*x^2*e^2-1/2*I/b*a*e^3*ln(-exp(2*b
*x+2*a)+I)+1/2*I/b*e^3*ln(1+exp(b*x+a)*(-1)^(3/4))*a+1/2*I/b*e^3*ln(1-exp(b
x+a)(-1)^(3/4))*a-1/16*Pi*(csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2
*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn(I*(exp(2*b*x+2*a
)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a
)+I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-
1))+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1)
)^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a
)-I)/(exp(2*b*x+2*a)-1))-csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^
2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+
I)/(exp(2*b*x+2*a)-1))-csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+
csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(
exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn
(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(exp(
2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn((1+
I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+csgn((1-I)*(exp(2*b*x+2*a)+I)/(
exp(2*b*x+2*a)-1))^3-1)*(f*x+e)^4/f+3/2*I*f^2/b^2*e*ln(1-I*exp(2*b*x+2*a))*
a^2*x-3/2*I*f/b*e^2*ln(1-I*exp(2*b*x+2*a))*a*x-3/2*I*f^2/b^2*a^2*e*ln(((-I)
^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-3/2*I*f^2/b^2*a^2*e*ln(((-I)^(1/2)+exp(b*x
+a))/(-I)^(1/2))*x+3/2*I*f/b*a*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x
+3/2*I*f/b*a*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x-3/2*I*f^2/b^2*e*l
n(1+I*exp(2*b*x+2*a))*a^2*x+1/8*I*(f*x+e)^4/f*ln(exp(2*b*x+2*a)+I)-1/2*I*e^
3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*e^3*ln(((-I)^(1/2)+exp(b*x
+a))/(-I)^(1/2))*x-1/2*I/b*e^3*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/
2*I/b*e^3*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/8*I/f*e^4*ln(exp(2*b*
x+2*a)+I)-1/8*I*f^3*ln(1-I*exp(2*b*x+2*a))*x^4+1/2*I/b*e^3*dilog(1-exp(b*x+
a)*(-1)^(3/4))+1/8*I/f*e^4*ln(-exp(2*b*x+2*a)+I)+1/8*I*f^3*ln(1+I*exp(2*b*x
+2*a))*x^4-1/8*I*f^3*ln(exp(2*b*x+2*a)-I)*x^4-1/2*I*ln(exp(2*b*x+2*a)-I)*x*
e^3-1/8*I/f*ln(exp(2*b*x+2*a)-I)*e^4+1/2*I*e^3*ln(1+exp(b*x+a)*(-1)^(3/4))*
x+1/2*I*e^3*ln(1-exp(b*x+a)*(-1)^(3/4))*x+1/2*I/b*e^3*dilog(1+exp(b*x+a)*(-
1)^(3/4))+I*f^2/b^3*e*ln(1-I*exp(2*b*x+2*a))*a^3-1/2*I*f^3/b^3*ln(1-I*exp(2
*b*x+2*a))*a^3*x+1/2*I*f^2/b^3*a^3*e*ln(exp(2*b*x+2*a)+I)-3/4*I*f/b^2*a^2*e

```
^2*ln(exp(2*b*x+2*a)+I)-3/4*I*f/b*e^2*polylog(2,I*exp(2*b*x+2*a))*x-3/4*I*f
/b^2*e^2*polylog(2,I*exp(2*b*x+2*a))*a-3/2*I*f^2/b^3*a^3*e*ln(((-I)^(1/2)-e
xp(b*x+a))/(-I)^(1/2))-3/2*I*f^2/b^3*a^3*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^
(1/2))-3/2*I*f^2/b^3*a^2*e*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-3/2*I*
f^2/b^3*a^2*e*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+3/2*I*f/b^2*a^2*e^2
*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+3/2*I*f/b^2*a^2*e^2*ln(((-I)^(1/2)+
exp(b*x+a))/(-I)^(1/2))+3/2*I*f/b^2*a*e^2*dilog(((-I)^(1/2)-exp(b*x+a))/(-I
)^(1/2))+3/2*I*f/b^2*a*e^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*
f^3/b^3*a^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f^3/b^3*a^3*ln((
(-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x-3/4*I*f^2/b*e*polylog(2,I*exp(2*b*x+2*
a))*x^2+3/4*I*f^2/b^3*e*polylog(2,I*exp(2*b*x+2*a))*a^2+3/4*I*f^2/b^2*e*pol
ylog(3,I*exp(2*b*x+2*a))*x-3/4*I*f/b^2*e^2*ln(1-I*exp(2*b*x+2*a))*a^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1460 vs. 2(236) = 472.

Time = 0.33 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^3*arccot(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2
4*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f
^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f^3*po
lylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
+ 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(sinh(b*x + a)/cos
h(b*x + a)) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^
3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*
x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 -
 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*
x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*
e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-I*b^4*f^3*x
^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3
+ 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(1/2*sqrt(4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*
b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3
*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) +
1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x
 + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(1/2
*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4
*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^
```

```
2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) + (4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2
 - I*a^4*f^3)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (4*I*a
*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-I*sqrt(4*I
) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*
f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sin
h(b*x + a)) + (-4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4
*f^3)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 24*(I*b*f^3*
x + I*b*e*f^2)*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
24*(I*b*f^3*x + I*b*e*f^2)*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(
b*x + a))) - 24*(-I*b*f^3*x - I*b*e*f^2)*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*
x + a) + sinh(b*x + a))) - 24*(-I*b*f^3*x - I*b*e*f^2)*polylog(4, -1/2*sqrt
(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*e*f^
2*x - I*b^2*e^2*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))
) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*e*f^2*x - I*b^2*e^2*f)*polylog(3, -1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*e*f^2
*x + I*b^2*e^2*f)*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))
) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*e*f^2*x + I*b^2*e^2*f)*polylog(3, -1/2*sqrt
(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \int (e+fx)^3 \operatorname{acot}(\coth(a+bx))\,dx$$

```
[In] integrate((f*x+e)**3*acot(coth(b*x+a)),x)
[Out] Integral((e + f*x)**3*acot(coth(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \int (fx+e)^3 \operatorname{arccot}(\coth(bx+a))\,dx$$

```
[In] integrate((f*x+e)^3*arccot(coth(b*x+a)),x, algorithm="maxima")
[Out] 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan((e^(2*b*x + 2*a)
 - 1)/(e^(2*b*x + 2*a) + 1)) - integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2
*x^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*b
*x + 4*a) + 1), x)
```

Giac [F(-1)]

Timed out.

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)^3*arccot(coth(b*x+a)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{acot}(\coth(a+b\,x))\,(e+f\,x)^3\,dx$$

```
[In] int(acot(coth(a + b*x))*(e + f*x)^3,x)
[Out] int(acot(coth(a + b*x))*(e + f*x)^3, x)
```

3.201 $\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 229

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f} + \frac{i(e+fx)^2\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if(e+fx)\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} + \frac{if(e+fx)\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} + \frac{if^2\,\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{if^2\,\text{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}$$

```
[Out] 1/3*(f*x+e)^3*arccot(coth(b*x+a))/f-1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+
1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^2*polylog(2,I*
exp(2*b*x+2*a))/b-1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/4*I*f*
(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2+1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*
a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5294, 4265, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int (e+fx)^2\cot^{-1}(\coth(a+bx))\,dx = & -\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f} \\ & +\frac{if^2\,\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ & -\frac{if^2\,\text{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\ & -\frac{if(e+fx)\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} \\ & +\frac{if(e+fx)\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} \\ & +\frac{i(e+fx)^2\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ & -\frac{i(e+fx)^2\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ & +\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f}\end{aligned}$$

```
[In] Int[(e + f*x)^2*ArcCot[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcCot[Coth[a + b*x]])/(3*f) - ((e + f*x)^3*ArcTan[E^(2*a + 2*
b*x)])/(3*f) + ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I
/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/4)*f*(e + f*x)*PolyL
og[3, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a +
 2*b*x)])/b^2 + ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 - ((I/8)*f
^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5294

```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Coth[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{b\int (e+fx)^3\operatorname{sech}(2a+2bx)\,dx}{3f}\\ &= \frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{3f}\\ &\quad + \frac{1}{2}i\int (e+fx)^2\log\left(1-ie^{2a+2bx}\right)\,dx - \frac{1}{2}i\int (e+fx)^2\log\left(1+ie^{2a+2bx}\right)\,dx\end{aligned}$$

$$= \frac{(e+fx)^3 \cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{(if)\int (e+fx) \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)\,dx}{2b} + \frac{(if)\int (e+fx) \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)\,dx}{2b}$$

$$= \frac{(e+fx)^3 \cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} + \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} + \frac{(if^2)\int \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)\,dx}{4b^2} - \frac{(if^2)\int \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)\,dx}{4b^2}$$

$$= \frac{(e+fx)^3 \cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} + \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} + \frac{(if^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{8b^3} - \frac{(if^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,ix)}{x}\,dx, x, e^{2a+2bx}\right)}{8b^3}$$

$$= \frac{(e+fx)^3 \cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} + \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} + \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} - \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.64

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\cot^{-1}(\coth(a+bx))$$
$$-\frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(1\right.\right.}{}$$

```
[In] Integrate[(e + f*x)^2*ArcCot[Coth[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[Coth[a + b*x]])/3 - ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 21.01 (sec) , antiderivative size = 2668, normalized size of antiderivative = 11.65

method	result	size
risch	Expression too large to display	2668

```
[In] int((f*x+e)^2*arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f^2/b^3*
a^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/4*I*f/b^2*e*polylog(3,I*exp
(2*b*x+2*a))+1/6*I*f^2/b^3*a^3*ln(exp(2*b*x+2*a)+I)+1/3*I*f^2/b^3*ln(1-I*ex
p(2*b*x+2*a))*a^3-1/4*I*f^2/b*polylog(2,I*exp(2*b*x+2*a))*x^2+1/2*I/b*a*e^2
*ln(exp(2*b*x+2*a)+I)-1/2*I/b*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*a-
1/2*I/b*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a+1/8*I*f^2*polylog(4,-I
*exp(2*b*x+2*a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3-1/6*I*f^2/b^
3*a^3*ln(-exp(2*b*x+2*a)+I)+1/2*I*f*e*ln(1+I*exp(2*b*x+2*a))*x^2-1/3*I*f^2/
b^3*ln(1+I*exp(2*b*x+2*a))*a^3+1/4*I*f^2/b*polylog(2,-I*exp(2*b*x+2*a))*x^2
-1/4*I*f^2/b^3*polylog(2,-I*exp(2*b*x+2*a))*a^2-1/4*I*f^2/b^2*polylog(3,-I*
exp(2*b*x+2*a))*x+1/2*I*f^2/b^3*a^3*ln(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f^2/b
^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))+1/2*I*f^2/b^3*a^2*dilog(1+exp(b*x+a)*(-1
)^(3/4))+1/2*I*f^2/b^3*a^2*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/4*I*f/b^2*e*pol
```

ylog(3,-I*exp(2*b*x+2*a))+1/2*I/b*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*a+1/2*I/b
*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))*a-1/2*I/b*a*e^2*ln(-exp(2*b*x+2*a)+I)-1/2*
I*f*ln(exp(2*b*x+2*a)-I)*x^2*e-1/2*I*f*e*ln(1-I*exp(2*b*x+2*a))*x^2+1/4*I*f
^2/b^3*polylog(2,I*exp(2*b*x+2*a))*a^2+1/4*I*f^2/b^2*polylog(3,I*exp(2*b*x+
2*a))*x-1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f^2/
b^3*a^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f/b^2*e*ln(1+I*exp(2*b
*x+2*a))*a^2+1/2*I*f/b*e*polylog(2,-I*exp(2*b*x+2*a))*x+1/2*I*f/b^2*e*polyl
og(2,-I*exp(2*b*x+2*a))*a-1/2*I*f^2/b^2*ln(1+I*exp(2*b*x+2*a))*a^2*x-I*f/b^
2*a^2*e*ln(1+exp(b*x+a)*(-1)^(3/4))-I*f/b^2*a^2*e*ln(1-exp(b*x+a)*(-1)^(3/4
))-I*f/b^2*a*e*dilog(1+exp(b*x+a)*(-1)^(3/4))-I*f/b^2*a*e*dilog(1-exp(b*x+a
)*(-1)^(3/4))+1/2*I*f/b^2*a^2*e*ln(-exp(2*b*x+2*a)+I)+1/2*I*f^2/b^2*a^2*ln(
1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*f^2/b^2*a^2*ln(1-exp(b*x+a)*(-1)^(3/4))*x+
1/6*I/f*e^3*ln(-exp(2*b*x+2*a)+I)+1/6*I*f^2*ln(1+I*exp(2*b*x+2*a))*x^3+1/2*
I/b*e^2*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I/b*e^2*dilog(1-exp(b*x+a)*(-1)^
(3/4))+1/2*I*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*e^2*ln(1-exp(b*x+a)*(-
1)^(3/4))*x-1/6*I*f^2*ln(exp(2*b*x+2*a)-I)*x^3-1/2*I*ln(exp(2*b*x+2*a)-I)*x
*e^2-1/6*I/f*ln(exp(2*b*x+2*a)-I)*e^3+1/6*I*(f*x+e)^3/f*ln(exp(2*b*x+2*a)+I
)-1/12*Pi*(csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(ex
p(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(
2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp(
2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))+csgn(I*(exp(2*
b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(exp(2*
b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*
a)-1))-csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*
x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)
-1))-csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-csgn(I/(exp(2*b*x+
2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+
2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(exp(2*b*x+
2*a)-I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))
csgn((1+I)(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)
+I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csg
n((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn((1+I)*(exp(2*b*x+2*a)
-I)/(exp(2*b*x+2*a)-1))^3+csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))
^3-1)*(f*x+e)^3/f+I*f/b^2*a^2*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+I*f/
b^2*a*e*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+I*f/b^2*a*e*dilog(((-I)^(
1/2)+exp(b*x+a))/(-I)^(1/2))+I*f/b^2*a^2*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^
(1/2))+1/2*I*f^2/b^2*ln(1-I*exp(2*b*x+2*a))*a^2*x-1/2*I*f^2/b^2*a^2*ln(((-I
)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*f^2/b^2*a^2*ln(((-I)^(1/2)+exp(b*x+
a))/(-I)^(1/2))*x-1/2*I*f/b^2*a^2*e*ln(exp(2*b*x+2*a)+I)-1/2*I*f/b^2*e*ln(1
-I*exp(2*b*x+2*a))*a^2-1/2*I*f/b*e*polylog(2,I*exp(2*b*x+2*a))*x-1/2*I*f/b^
2*e*polylog(2,I*exp(2*b*x+2*a))*a-1/2*I*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)
^(1/2))*x-1/2*I*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x-1/2*I/b*e^2*di
log(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I/b*e^2*dilog(((-I)^(1/2)+exp(b
*x+a))/(-I)^(1/2))-1/6*I*f^2*ln(1-I*exp(2*b*x+2*a))*x^3-1/6*I/f*e^3*ln(exp(
2*b*x+2*a)+I)+I*f/b*e*ln(1+I*exp(2*b*x+2*a))*a*x-I*f/b*a*e*ln(1+exp(b*x+a)*

```
(-1)^(3/4))*x-I*f/b*a*e*ln(1-exp(b*x+a)*(-1)^(3/4))*x+I*f/b*a*e*ln(((-I)^(1
/2)-exp(b*x+a))/(-I)^(1/2))*x+I*f/b*a*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/
2))*x-I*f/b*e*ln(1-I*exp(2*b*x+2*a))*a*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(180) = 360$.

Time = 0.33 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.38

$$\int (e + fx)^2 \cot^{-1}(\coth(a + bx))\, dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*arccot(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(-6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6
*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2
*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2*polyl
og(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3
*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) - 3*(I*b^2*
f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + s
inh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f
*x - I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(
-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x
 + a) + sinh(b*x + a))) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b^3*e^2*x
 - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(4*I)*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*I*b^3*e^2
*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt(4*I)*(cosh(b*
x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3*e
^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*b^3
*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(-4*I)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) + (3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*
f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (3*I*a*b^2*e^2
- 3*I*a^2*b*e*f + I*a^3*f^2)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*
x + a)) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(I*sqrt(-4*I) + 2
*cosh(b*x + a) + 2*sinh(b*x + a)) + (-3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3
*f^2)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 6*(-I*b*f^2*
x - I*b*e*f)*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*
(-I*b*f^2*x - I*b*e*f)*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 6*(I*b*f^2*x + I*b*e*f)*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) - 6*(I*b*f^2*x + I*b*e*f)*polylog(3, -1/2*sqrt(-4*I)*(cosh
(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \int (e+fx)^2 \operatorname{acot}(\coth(a+bx))\,dx$$

```
[In] integrate((f*x+e)**2*acot(coth(b*x+a)),x)
[Out] Integral((e + f*x)**2*acot(coth(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \int (fx+e)^2 \operatorname{arccot}(\coth(bx+a))\,dx$$

```
[In] integrate((f*x+e)^2*arccot(coth(b*x+a)),x, algorithm="maxima")
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x
+ 2*a) + 1)) - integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b
*e^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \int (fx+e)^2 \operatorname{arccot}(\coth(bx+a))\,dx$$

```
[In] integrate((f*x+e)^2*arccot(coth(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{acot}(\coth(a+b\,x))\,(e+f\,x)^2\,dx$$

```
[In] int(acot(coth(a + b*x))*(e + f*x)^2,x)
[Out] int(acot(coth(a + b*x))*(e + f*x)^2, x)
```

3.202 $\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 159

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{if\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

```
[Out] 1/2*(f*x+e)^2*arccot(coth(b*x+a))/f-1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+
1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)*polylog(2,I*exp(
2*b*x+2*a))/b-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/8*I*f*polylog(3,I*
exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used

$= \{5294, 4265, 2611, 2320, 6724\}$

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = -\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} - \frac{if\,\mathrm{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{if\,\mathrm{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} + \frac{i(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\,\mathrm{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f}$$

```
[In] Int[(e + f*x)*ArcCot[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcCot[Coth[a + b*x]])/(2*f) - ((e + f*x)^2*ArcTan[E^(2*a + 2*
b*x)])/(2*f) + ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I/4
)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/8)*f*PolyLog[3, (-I)*E^(
2*a + 2*b*x)])/b^2 + ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5294

```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Coth[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{b\int (e+fx)^2\operatorname{sech}(2a+2bx)\,dx}{2f} \\
&= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} \\
&\quad + \frac{1}{2}i\int (e+fx)\log\left(1-ie^{2a+2bx}\right)\,dx - \frac{1}{2}i\int (e+fx)\log\left(1+ie^{2a+2bx}\right)\,dx \\
&= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} \\
&\quad + \frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad - \frac{(if)\int \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)\,dx}{4b} + \frac{(if)\int \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)\,dx}{4b} \\
&= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} \\
&\quad + \frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad - \frac{(if)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2} \\
&\quad + \frac{(if)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2} \\
&= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} \\
&\quad + \frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\
&\quad - \frac{if\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{if\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.49

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = ex\cot^{-1}(\coth(a+bx))+\frac{1}{2}fx^2\cot^{-1}(\coth(a+bx)) - \frac{ie\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\operatorname{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\operatorname{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b} - \frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\operatorname{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\operatorname{PolyLog}\left(2,\right.\right.}{8b^2}$$

```
[In] Integrate[(e + f*x)*ArcCot[Coth[a + b*x]],x]
```

```
[Out] e*x*ArcCot[Coth[a + b*x]] + (f*x^2*ArcCot[Coth[a + b*x]])/2 - ((I/4)*e*(2*b
*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x))]) - PolyLog[2, (
-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^(2*(a + b*x))]))/b - ((I/8)*f*(2*b^2*
x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a + b*x))] - 2*b
*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*(a + b*x))] +
 PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x))]))/b^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.32 (sec) , antiderivative size = 1776, normalized size of antiderivative = 11.17

method	result	size
risch	Expression too large to display	1776

```
[In] int((f*x+e)*arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*f/b^2*ln(1+I*exp(2*b*x+2*a))*a^2+1/4*I*f/b*polylog(2,-I*exp(2*b*x+2*a
))*x+1/4*I*f/b^2*polylog(2,-I*exp(2*b*x+2*a))*a+1/4*I*f/b^2*a^2*ln(-exp(2*b
*x+2*a)+I)+1/2*I*e/b*ln(1+exp(b*x+a)*(-1)^(3/4))*a+1/2*I*e/b*ln(1-exp(b*x+a
)*(-1)^(3/4))*a-1/2*I*e/b*a*ln(-exp(2*b*x+2*a)+I)+1/2*I*(1/2*f*x^2+e*x)*ln(
exp(2*b*x+2*a)+I)-1/4*Pi*(csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)
-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn(I*(exp(2*b*x+2*a)-I
))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I
))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))
+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2
+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I
)/(exp(2*b*x+2*a)-1))-csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-c
sgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/
(exp(2*b*x+2*a)-1))-csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-csg
n(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+csg
n(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csg
```

```
n(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(exp
(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*
(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b
*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn((1+I)*
(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp
(2*b*x+2*a)-1))^3-1)*(1/2*f*x^2+e*x)-1/4*I*ln(exp(2*b*x+2*a)-I)*x^2*f-1/2*I
*ln(exp(2*b*x+2*a)-I)*e*x+1/4*I*f*ln(1+I*exp(2*b*x+2*a))*x^2+1/2*I*e*ln(1+e
xp(b*x+a)*(-1)^(3/4))*x+1/2*I*e*ln(1-exp(b*x+a)*(-1)^(3/4))*x+1/2*I*e/b*dil
og(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*e/b*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*
f/b*a*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/2*I*f/b*ln(1-I*exp(2*b*x+2*a))*a*x+1/
2*I*f/b*a*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f/b*a*ln(((-I)^(1/
2)+exp(b*x+a))/(-I)^(1/2))*x+1/8*I*f*polylog(3,I*exp(2*b*x+2*a))/b^2+1/2*I*
f/b*ln(1+I*exp(2*b*x+2*a))*a*x-1/2*I*f/b*a*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/
4*I*f*ln(1-I*exp(2*b*x+2*a))*x^2-1/2*I*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1
/2))*x-1/2*I*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x-1/2*I*e/b*dilog(((-
I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*e/b*dilog(((-I)^(1/2)+exp(b*x+a))/(-
I)^(1/2))-1/4*I*f/b^2*ln(1-I*exp(2*b*x+2*a))*a^2-1/4*I*f/b*polylog(2,I*exp(
2*b*x+2*a))*x-1/4*I*f/b^2*polylog(2,I*exp(2*b*x+2*a))*a-1/4*I*f/b^2*a^2*ln(
exp(2*b*x+2*a)+I)-1/2*I*e/b*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*a-1/2*I*
e/b*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a+1/2*I*e/b*a*ln(exp(2*b*x+2*a)+
I)+1/2*I*f/b^2*a^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f/b^2*a^2*l
n(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f/b^2*a*dilog(((-I)^(1/2)-exp(b
*x+a))/(-I)^(1/2))+1/2*I*f/b^2*a*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-
1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/2*I*f/b^2*a^2*ln(1+exp(b*x+a)*(-
1)^(3/4))-1/2*I*f/b^2*a^2*ln(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a*dilog(1
+exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a*dilog(1-exp(b*x+a)*(-1)^(3/4))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(130) = 260$.

Time = 0.32 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.77

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx$$
$$= \frac{2\left(b^2fx^2+2\,b^2ex\right)\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)-2\left(i\,bfx+i\,be\right)\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i}(\cosh(bx+a)+\sinh(bx+a))\right)-2\left(i\,bf\right.}{}$$

```
[In] integrate((f*x+e)*arccot(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) - 2*(I*b
*f*x + I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(I*b
*f*x + I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(-I
*b*f*x - I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(
```

```
-I*b*f*x - I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
(-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e +
I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f
*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(b*x + a)
 + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*
log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (2*I*a*b*e - I*a
^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2*I*a*b*e - I
*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2*I*a*b*e
 + I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-2*I*a
*b*e + I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + 2*
I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polyl
og(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polylog(3, 1/
2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polylog(3, -1/2*sqrt(
-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx=\int (e+fx)\operatorname{acot}(\coth(a+bx))\,dx$$

```
[In] integrate((f*x+e)*acot(coth(b*x+a)),x)
[Out] Integral((e + f*x)*acot(coth(a + b*x)), x)
```

Maxima [F]

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx=\int (fx+e)\operatorname{arccot}(\coth(bx+a))\,dx$$

```
[In] integrate((f*x+e)*arccot(coth(b*x+a)),x, algorithm="maxima")
[Out] 1/2*(f*x^2 + 2*e*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - i
ntegrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1
), x)
```

Giac [F]

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = \int (fx+e)\,\mathrm{arccot}\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate((f*x+e)*arccot(coth(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = \int \mathrm{acot}(\coth(a+b\,x))\,\left(e+f\,x\right)\,dx$$

```
[In] int(acot(coth(a + b*x))*(e + f*x),x)
[Out] int(acot(coth(a + b*x))*(e + f*x), x)
```

3.203 $\int \cot^{-1}(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 74

$$\int \cot^{-1}(\coth(a+bx))\,dx = x\cot^{-1}(\coth(a+bx)) - x\arctan\left(e^{2a+2bx}\right) + \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] x*arccot(coth(b*x+a))-x*arctan(exp(2*b*x+2*a))+1/4*I*polylog(2,-I*exp(2*b*x
+2*a))/b-1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5290, 4265, 2317, 2438}

$$\int \cot^{-1}(\coth(a+bx))\,dx = -x\arctan\left(e^{2a+2bx}\right) + \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + x\cot^{-1}(\coth(a+bx))$$

```
[In] Int[ArcCot[Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[Coth[a + b*x]] - x*ArcTan[E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, (-I
)*E^(2*a + 2*b*x)])/b - ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5290

```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCot[Coth[a + b
*x]], x] - Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\cot^{-1}(\coth(a+bx)) - b\int x\,\text{sech}(2a+2bx)\,dx \\
&= x\cot^{-1}(\coth(a+bx)) - x\arctan\left(e^{2a+2bx}\right) \\
&\quad + \frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)\,dx - \frac{1}{2}i\int \log\left(1+ie^{2a+2bx}\right)\,dx \\
&= x\cot^{-1}(\coth(a+bx)) - x\arctan\left(e^{2a+2bx}\right) \\
&\quad + \frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} - \frac{i\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} \\
&= x\cot^{-1}(\coth(a+bx)) - x\arctan\left(e^{2a+2bx}\right) \\
&\quad + \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \cot^{-1}(\coth(a+bx))\,dx = x\cot^{-1}(\coth(a+bx)) - \frac{i\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b}$$

```
[In] Integrate[ArcCot[Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[Coth[a + b*x]] - ((I/4)*(2*b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1
 + I*E^(2*(a + b*x))]) - PolyLog[2, (-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^
(2*(a + b*x))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(63) = 126.

Time = 1.51 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.49

method	result
derivativedivides	$\frac{\text{arctanh}(\coth(bx+a))\,\text{arccot}(\coth(bx+a))+\arctan(\coth(bx+a))\,\text{arctanh}(\coth(bx+a))+\frac{\arctan(\coth(bx+a))\ln\left(1+\frac{i(1+i\cot}{\coth(b}\right.}{2}}{}$
default	$\frac{\text{arctanh}(\coth(bx+a))\,\text{arccot}(\coth(bx+a))+\arctan(\coth(bx+a))\,\text{arctanh}(\coth(bx+a))+\frac{\arctan(\coth(bx+a))\ln\left(1+\frac{i(1+i\cot}{\coth(b}\right.}{2}}{}$
risch	Expression too large to display

```
[In] int(arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(coth(b*x+a))*arccot(coth(b*x+a))+arctan(coth(b*x+a))*arctanh(c
oth(b*x+a))+1/2*arctan(coth(b*x+a))*ln(1+I*(1+I*coth(b*x+a))^2/(coth(b*x+a)
^2+1))-1/2*arctan(coth(b*x+a))*ln(1-I*(1+I*coth(b*x+a))^2/(coth(b*x+a)^2+1)
)-1/4*I*dilog(1+I*(1+I*coth(b*x+a))^2/(coth(b*x+a)^2+1))+1/4*I*dilog(1-I*(1
+I*coth(b*x+a))^2/(coth(b*x+a)^2+1)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(57) = 114.

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.51

$$\int \cot^{-1}(\coth(a+bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right) + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a)+\sinh(bx+a))+1\right) + (-i\,bx - i\,a)\log\left(\right.}{}$$

```
[In] integrate(arccot(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-I*b*x - I*a)*log(1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(1/2*sqrt(-4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) + I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) - I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) + I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
+ I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{acot}(\coth(a+bx))\,dx$$

```
[In] integrate(acot(coth(b*x+a)),x)
```

```
[Out] Integral(acot(coth(a + b*x)), x)
```

Maxima [F]

$$\int \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{arccot}(\coth(bx+a))\,dx$$

```
[In] integrate(arccot(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - 2*b*integrate(x*e^(
2*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

Giac [F]

$$\int \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{arccot}\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(arccot(coth(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot(coth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{acot}(\coth(a+b\,x))\,dx$$

```
[In] int(acot(coth(a + b*x)),x)
[Out] int(acot(coth(a + b*x)), x)
```

3.204 $\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \text{Int}\left(\frac{\cot^{-1}(\coth(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arccot(coth(b*x+a))/(f*x+e),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$$

```
[In] Int[ArcCot[Coth[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcCot[Coth[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx=\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$$

```
[In] Integrate[ArcCot[Coth[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcCot[Coth[a + b*x]]/(e + f*x), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(\coth\left(bx+a\right)\right)}{fx+e}dx$$

```
[In] int(arccot(coth(b*x+a))/(f*x+e),x)
[Out] int(arccot(coth(b*x+a))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{arccot}\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

```
[In] integrate(arccot(coth(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arccot(coth(b*x + a))/(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx=\text{Timed out}$$

```
[In] integrate(acot(coth(b*x+a))/(f*x+e),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{arccot}\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

```
[In] integrate(arccot(coth(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arccot(coth(b*x + a))/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 105.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.20

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{arccot}\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

```
[In] integrate(arccot(coth(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx=\int \frac{\mathrm{acot}(\coth(a+b\,x))}{e+f\,x}\,dx$$

```
[In] int(acot(coth(a + b*x))/(e + f*x),x)
[Out] int(acot(coth(a + b*x))/(e + f*x), x)
```

3.205 $\int x^2 \cot^{-1}(c + d\coth(a + bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 351

$$\begin{aligned}\int x^2 \cot^{-1}(c+d\coth(a+bx))\,dx =& \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx)) \\ &-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\ &+\frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\ &-\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ &+\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ &+\frac{ix\operatorname{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\ &-\frac{ix\operatorname{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} \\ &-\frac{i\operatorname{PolyLog}\left(4,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} \\ &+\frac{i\operatorname{PolyLog}\left(4,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}\end{aligned}$$

```
[Out] 1/3*x^3*arccot(c+d*coth(b*x+a))-1/6*I*x^3*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))+1/6*I*x^3*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x^2*polylog(2,(I-c
-d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x^2*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I
+c-d))/b+1/4*I*x*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/4*I*x*poly
```

```
log(3,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2-1/8*I*polylog(4,(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b^3+1/8*I*polylog(4,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5310, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^2 \cot^{-1}(c+d\coth(a+bx))\,dx = &-\frac{i\,\text{PolyLog}\left(4,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}\\&+\frac{i\,\text{PolyLog}\left(4,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3}\\&+\frac{ix\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}\\&-\frac{ix\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}\\&-\frac{ix^2\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}\\&+\frac{ix^2\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}\\&-\frac{1}{6}ix^3\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)\\&+\frac{1}{6}ix^3\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)\\&+\frac{1}{3}x^3\cot^{-1}(d\coth(a+bx)+c)\end{aligned}$$

```
[In] Int[x^2*ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Coth[a + b*x]])/3 - (I/6)*x^3*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] + (I/6)*x^3*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] - ((I/4)*x^2*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d)])/b + ((I/4)*x^2*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/
b + ((I/4)*x*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 - (
(I/4)*x*PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2 - ((I/8)
*PolyLog[4, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^3 + ((I/8)*PolyLo
g[4, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5310

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b
*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c -
 d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx)) \\
&\quad+\frac{1}{3}(b(1-i(c+d)))\int\frac{e^{2a+2bx}x^3}{i+c-d+(-i-c-d)e^{2a+2bx}}\,dx \\
&\quad-\frac{1}{3}(b(1+i(c+d)))\int\frac{e^{2a+2bx}x^3}{i-c+d+(-i+c+d)e^{2a+2bx}}\,dx \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad+\frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{1}{2}i\int x^2\log\left(1+\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right)dx \\
&\quad+\frac{1}{2}i\int x^2\log\left(1+\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right)dx \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad+\frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{ix^2\,\text{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&\quad+\frac{ix^2\,\text{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}-\frac{i\int x\,\text{PolyLog}\left(2,-\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right)dx}{2b} \\
&\quad+\frac{i\int x\,\text{PolyLog}\left(2,-\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right)dx}{2b} \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad+\frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{ix^2\,\text{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&\quad+\frac{ix^2\,\text{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}+\frac{ix\,\text{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\
&\quad-\frac{ix\,\text{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2}+\frac{i\int\text{PolyLog}\left(3,-\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right)dx}{4b^2} \\
&\quad-\frac{i\int\text{PolyLog}\left(3,-\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right)dx}{4b^2}
\end{aligned}
$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix^2\,\text{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix^2\,\text{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{ix\,\text{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} - \frac{ix\,\text{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} - \frac{i\text{Subst}\left(\int\frac{\text{PolyLog}\left(3,\frac{(-i+c+d)x}{-i+c-d}\right)}{x}dx,x,e^{2a+2bx}\right)}{8b^3} + \frac{i\text{Subst}\left(\int\frac{\text{PolyLog}\left(3,\frac{(i+c+d)x}{i+c-d}\right)}{x}dx,x,e^{2a+2bx}\right)}{8b^3}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix^2\,\text{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix^2\,\text{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{ix\,\text{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} - \frac{ix\,\text{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} - \frac{i\,\text{PolyLog}\left(4,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} + \frac{i\,\text{PolyLog}\left(4,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.26

$$\int x^2\cot^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx)) - \frac{d\left(4b^3x^3\log\left(1-\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) - 4b^3x^3\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{-1-c^2+d^2+2\sqrt{-d^2}}\right) + 6b^2x^2\,\text{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2)}{1+c^2-d^2+2}\right.\right.}{}$$

```
[In] Integrate[x^2*ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Coth[a + b*x]])/3 - (d*(4*b^3*x^3*Log[1 - ((1 + (c + d)^2
)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 4*b^3*x^3*Log[1 + ((1
+ (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 6*b^2*x^2*
PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqr
t[-d^2])] - 6*b^2*x^2*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 -
c^2 + d^2 + 2*Sqrt[-d^2]))] - 6*b*x*PolyLog[3, ((1 + c^2 + 2*c*d + d^2)*E^(
```

```
2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 6*b*x*PolyLog[3, -(((1 + (c
 + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2]))] - 3*PolyLog[4,
((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 - 2*Sqrt[-d^2])] +
 3*PolyLog[4, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*
Sqrt[-d^2])]))/(24*b^3*Sqrt[-d^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 39.01 (sec) , antiderivative size = 6844, normalized size of antiderivative = 19.50

method	result	size
risch	Expression too large to display	6844

```
[In] int(x^2*arccot(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1269 vs. 2(259) = 518.

Time = 0.36 (sec) , antiderivative size = 1269, normalized size of antiderivative = 3.62

$$\int x^2 \cot^{-1}(c + d\coth(a + bx))\, dx = \text{Too large to display}$$

```
[In] integrate(x^2*arccot(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan(sinh(b*x + a)/(d*cosh(b*x + a) + c*sinh(b*x + a))) -
3*I*b^2*x^2*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 + 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^
2*dilog(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a
) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^3*log(2*(c^2 + 2
*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2
*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d +
 d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d
 + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(
b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d +
 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c
^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x +
```

```
a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))) + 6*I*b*x*polylog(3, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -sqrt((c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)
)) - 6*I*b*x*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1
))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -sqrt((c^2 - d^2 -
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + (-I
*b^3*x^3 - I*a^3)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(-sqrt((c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)
) + 1) + (I*b^3*x^3 + I*a^3)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d
+ d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 6*I*polylog(4, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, -sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 + 6*I*polylog(4, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) + 6*I*polylog(4, -sqrt((c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\coth(a + bx))\, dx = \text{Timed out}$$

```
[In] integrate(x**2*acot(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2 \cot^{-1}(c + d\coth(a + bx))\, dx = \int x^2 \operatorname{arccot}\left(d \coth\left(bx + a\right) + c\right)\, dx$$

```
[In] integrate(x^2*arccot(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2(e^(2*b*x + 2*a) - 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c
+ d) - 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^
(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) -
 d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Giac [F]

$$\int x^2 \cot^{-1}(c + d \coth(a + bx)) \, dx = \int x^2 \operatorname{arccot}(d \coth(bx + a) + c) \, dx$$

```
[In] integrate(x^2*arccot(c+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arccot(d*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d \coth(a + bx)) \, dx = \int x^2 \operatorname{acot}(c + d \coth(a + b x)) \, dx$$

```
[In] int(x^2*acot(c + d*coth(a + b*x)),x)
[Out] int(x^2*acot(c + d*coth(a + b*x)), x)
```

3.206 $\int x \cot^{-1}(c+d\coth(a+bx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 265

$$\begin{aligned}\int x\cot^{-1}(c+d\coth(a+bx))\,dx &= \frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx)) \\ &\quad -\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\ &\quad +\frac{1}{4}ix^2\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\ &\quad -\frac{ix\,\mathrm{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ &\quad +\frac{ix\,\mathrm{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ &\quad +\frac{i\,\mathrm{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} \\ &\quad -\frac{i\,\mathrm{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}\end{aligned}$$

```
[Out] 1/2*x^2*arccot(c+d*coth(b*x+a))-1/4*I*x^2*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))+1/4*I*x^2*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x*polylog(2,(I-c-d
)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d
))/b+1/8*I*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/8*I*polylog(3,(I
+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5310, 2221, 2611, 2320, 6724}

$$\int x\cot^{-1}(c+d\coth(a+bx))\,dx = \frac{i\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2} - \frac{i\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2} - \frac{ix\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} + \frac{ix\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} - \frac{1}{4}ix^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) + \frac{1}{4}ix^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right) + \frac{1}{2}x^2\cot^{-1}(d\coth(a+bx)+c)$$

```
[In] Int[x*ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Coth[a + b*x]])/2 - (I/4)*x^2*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] + (I/4)*x^2*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] - ((I/4)*x*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)
])/b + ((I/4)*x*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b +
((I/8)*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 - ((I/8)*
PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5310

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b
*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c -
 d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx)) \\
&\quad +\frac{1}{2}(b(1-i(c+d)))\int\frac{e^{2a+2bx}x^2}{i+c-d+(-i-c-d)e^{2a+2bx}}\,dx \\
&\quad -\frac{1}{2}(b(1+i(c+d)))\int\frac{e^{2a+2bx}x^2}{i-c+d+(-i+c+d)e^{2a+2bx}}\,dx \\
&= \frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&\quad +\frac{1}{4}ix^2\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{1}{2}i\int x\log\left(1+\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right)dx \\
&\quad +\frac{1}{2}i\int x\log\left(1+\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right)dx
\end{aligned}$$

$$= \frac{1}{2}x^2 \cot^{-1}(c + d \coth(a + bx)) - \frac{1}{4}ix^2 \log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) + \frac{1}{4}ix^2 \log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) - \frac{ix \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} - \frac{i \int \operatorname{PolyLog}\left(2, -\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right) dx}{4b} + \frac{i \int \operatorname{PolyLog}\left(2, -\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right) dx}{4b}$$

$$= \frac{1}{2}x^2 \cot^{-1}(c + d \coth(a + bx)) - \frac{1}{4}ix^2 \log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) + \frac{1}{4}ix^2 \log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) - \frac{ix \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, \frac{(-i+c+d)x}{-i+c-d}\right)}{x} dx, x, e^{2a+2bx}\right)}{8b^2} - \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, \frac{(i+c+d)x}{i+c-d}\right)}{x} dx, x, e^{2a+2bx}\right)}{8b^2}$$

$$= \frac{1}{2}x^2 \cot^{-1}(c + d \coth(a + bx)) - \frac{1}{4}ix^2 \log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) + \frac{1}{4}ix^2 \log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) - \frac{ix \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{ix \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} + \frac{i \operatorname{PolyLog}\left(3, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} - \frac{i \operatorname{PolyLog}\left(3, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.26

$$\int x \cot^{-1}(c + d \coth(a + bx)) \, dx = \frac{1}{2}x^2 \cot^{-1}(c + d \coth(a + bx)) - \frac{d\left(2b^2x^2 \log\left(1 - \frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) - 2b^2x^2 \log\left(1 + \frac{(1+(c+d)^2)e^{2(a+bx)}}{-1-c^2+d^2+2\sqrt{-d^2}}\right) + 2bx \operatorname{PolyLog}\left(2, \frac{(1+c^2+2cd+d^2)e^{2(a}}{1+c^2-d^2+2\sqrt{-d}}\right.\right.}{}$$

```
[In] Integrate[x*ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Coth[a + b*x]])/2 - (d*(2*b^2*x^2*Log[1 - ((1 + (c + d)^2
)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*b^2*x^2*Log[1 + ((1
+ (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 2*b*x*Poly
Log[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d
^2])] - 2*b*x*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^
2 + 2*Sqrt[-d^2]))] + PolyLog[3, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/
(1 + c^2 - d^2 - 2*Sqrt[-d^2])] - PolyLog[3, ((1 + c^2 + 2*c*d + d^2)*E^(2*
(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])]))/(8*b^2*Sqrt[-d^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.40 (sec) , antiderivative size = 6494, normalized size of antiderivative = 24.51

method	result	size
risch	Expression too large to display	6494

```
[In] int(x*arccot(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1051 vs. 2(195) = 390.

Time = 0.33 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.97

$$\int x \cot^{-1}(c + d \coth(a + bx)) \, dx = \text{Too large to display}$$

```
[In] integrate(x*arccot(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan(sinh(b*x + a)/(d*cosh(b*x + a) + c*sinh(b*x + a))) -
2*I*b*x*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b
*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(sqrt
((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) + 2*I*b*x*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*
cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 - 2*
I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log
(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b
*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c
*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^
```

```
2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 -
d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2
 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^
2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + (-I
*b^2*x^2 + I*a^2)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(-sqrt((c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)
) + 1) + (I*b^2*x^2 - I*a^2)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d
+ d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + 2*I*polylog(3, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, -sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 - 2*I*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3, -sqrt((c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+d\coth(a+bx))\,dx=\text{Timed out}$$

```
[In] integrate(x*acot(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\cot^{-1}(c+d\coth(a+bx))\,dx=\int x\,\mathrm{arccot}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(e^(2*b*x + 2*a) - 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c
+ d) - 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a
) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2
*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Giac [F]

$$\int x\cot^{-1}(c+d\coth(a+bx))\,dx = \int x\,\operatorname{arccot}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccot(d*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+d\coth(a+bx))\,dx = \int x\,\operatorname{acot}(c+d\coth(a+b\,x))\,dx$$

```
[In] int(x*acot(c + d*coth(a + b*x)),x)
[Out] int(x*acot(c + d*coth(a + b*x)), x)
```

3.207 $\int \cot^{-1}(c+d\coth(a+bx))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 174

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx = x\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{i\,\mathrm{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}$$

```
[Out] x*arccot(c+d*coth(b*x+a))-1/2*I*x*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))+1/2*
I*x*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*polylog(2,(I-c-d)*exp(2*b*x+
2*a)/(I-c+d))/b+1/4*I*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

= {5302, 2221, 2317, 2438}

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx = -\frac{i\,\mathrm{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} - \frac{1}{2}ix\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) + \frac{1}{2}ix\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right) + x\cot^{-1}(d\coth(a+bx)+c)$$

```
[In] Int[ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Coth[a + b*x]] - (I/2)*x*Log[1 - ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] + (I/2)*x*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] - ((I/4)*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b + ((I/4
)*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5302

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Cot[c + d*Coth[a + b*x]], x] + (Dist[I*b*(I - c - d), Int[x*(E^(2*a + 2*b*x
)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b*(I + c + d)
, Int[x*(E^(2*a + 2*b*x)/(I + c - d - (I + c + d)*E^(2*a + 2*b*x))), x], x]
) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$\text{integral} = x\cot^{-1}(c+d\coth(a+bx)) + (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(-i-c-d)e^{2a+2bx}}\,dx - (b(1+i(c+d)))\int \frac{e^{2a+2bx}x}{i-c+d+(-i+c+d)e^{2a+2bx}}\,dx$$

$$= x\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{1}{2}i\int \log\left(1+\frac{(-i-c-d)e^{2a+2bx}}{i+c-d}\right)dx + \frac{1}{2}i\int \log\left(1+\frac{(-i+c+d)e^{2a+2bx}}{i-c+d}\right)dx$$

$$= x\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{(-i-c-d)x}{i+c-d}\right)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} + \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{(-i+c+d)x}{i-c+d}\right)}{x}\,dx, x, e^{2a+2bx}\right)}{4b}$$

$$= x\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{i\,\text{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.65

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx = x\cot^{-1}(c+d\coth(a+bx)) - \frac{4a\sqrt{-d^2}\arctan\left(\frac{1+c^2-d^2-(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right) + 2d(a+bx)\log\left(1-\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) - 2d(a+bx)\log\left(\right.}{4b\sqrt{-d}}$$

```
[In] Integrate[ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Coth[a + b*x]] - (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 - (1
+ c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + 2*d*(a + b*x)*Log[1 - ((1 +
```

```
(c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*d*(a + b*x)
*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])]
 + d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 +
2*Sqrt[-d^2])] - d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(
-1 - c^2 + d^2 + 2*Sqrt[-d^2]))])/(4*b*Sqrt[-d^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(150) = 300.

Time = 2.92 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln\left(\right.}{2}\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln\left(\right.}{2}\right.}{}}{}$
risch	Expression too large to display

```
[In] int(arccot(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*arccot(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arccot(c+d*c
oth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*d^2*(1/d*(1/2*I*ln(-d*coth(b*x+a)+d)
*ln((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I*ln(-d*coth(b*x+a)+d)*ln((I-d*coth(b*
x+a)-c)/(I-c-d))+1/2*I*dilog((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I*dilog((I-d*
coth(b*x+a)-c)/(I-c-d)))-1/d*(1/2*I*ln(-d*coth(b*x+a)-d)*ln((I+d*coth(b*x+a
)+c)/(I+c-d))-1/2*I*ln(-d*coth(b*x+a)-d)*ln((I-d*coth(b*x+a)-c)/(I-c+d))+1/
2*I*dilog((I+d*coth(b*x+a)+c)/(I+c-d))-1/2*I*dilog((I-d*coth(b*x+a)-c)/(I-c
+d)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(128) = 256.

Time = 0.39 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.67

$$\int \cot^{-1}(c + d\coth(a + bx))\, dx$$

$$= \frac{2\, bx \arctan\left(\frac{\sinh(bx+a)}{d\cosh(bx+a)+c\sinh(bx+a)}\right) + i\, a \log\left(2\left(c^2 + 2\, cd + d^2 + 1\right)\cosh\left(bx + a\right) + 2\left(c^2 + 2\, cd + d^2 + 1\right)\right.}{}$$

```
[In] integrate(arccot(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(sinh(b*x + a)/(d*cosh(b*x + a) + c*sinh(b*x + a))) + I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c
^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2
+ 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2
 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + (-I*b*x - I*a)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 -
 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*lo
g(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (I*b*x + I*a)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - I*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2
+ 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-sqrt((c^2 - d^2 + 2*I*d +
 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(sqr
t((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*
x + a))) + I*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \cot^{-1}(c + d\coth(a + bx))\,dx = \int \operatorname{acot}\left(c + d\coth\left(a + bx\right)\right)dx$$

```
[In] integrate(acot(c+d*coth(b*x+a)),x)
```

```
[Out] Integral(acot(c + d*coth(a + b*x)), x)
```

Maxima [F]

$$\int \cot^{-1}(c + d\coth(a + bx))\,dx = \int \operatorname{arccot}\left(d\coth\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(arccot(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*
d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2*e^(2*a)
 + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2(e^(2*b*x + 2*a) - 1, (c*e^(2*a)
+ d*e^(2*a))*e^(2*b*x) - c + d)
```

Giac [F]

$$\int \cot^{-1}(c + d\coth(a + bx))\,dx = \int \operatorname{arccot}\left(d\coth\left(bx + a\right) + c\right)\,dx$$

```
[In] integrate(arccot(c+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot(d*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c + d\coth(a + bx))\,dx = \int \operatorname{acot}(c + d\coth(a + b\,x))\,dx$$

```
[In] int(acot(c + d*coth(a + b*x)),x)
[Out] int(acot(c + d*coth(a + b*x)), x)
```

3.208 $\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\text{Int}\left(\frac{\cot^{-1}(c+d\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+d*coth(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCot[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c + d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + d*Coth[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(c+d\coth\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arccot(c+d*coth(b*x+a))/x,x)
[Out] int(arccot(c+d*coth(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arccot(d*coth(b*x + a) + c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 170.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{acot}\left(c+d\coth\left(a+bx\right)\right)}{x}\,dx$$

```
[In] integrate(acot(c+d*coth(b*x+a))/x,x)
[Out] Integral(acot(c + d*coth(a + b*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*coth(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arccot(d*coth(b*x + a) + c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot(d*coth(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\mathrm{acot}(c+d\coth(a+b\,x))}{x}\,dx$$

```
[In] int(acot(c + d*coth(a + b*x))/x,x)
[Out] int(acot(c + d*coth(a + b*x))/x, x)
```

3.209 $\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\,\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arccot(c+(I+c)*coth(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*b
*x+2*a))-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,I*c*ex
p(2*b*x+2*a))/b^2-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5306, 2215, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = -\frac{i\,\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}+\frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} -\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))+\frac{1}{12}ibx^4$$

```
[In] Int[x^2*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcCot[c + (I + c)*Coth[a + b*x]])/3 - (I/6)*x^3*Log[1
- I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((
```

I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, I*c*E^(2*a + 2*b*x)])/b^3

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5306

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{3}b\int\frac{x^3}{-i-ce^{2a+2bx}}\,dx\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{-i-ce^{2a+2bx}}\,dx\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))\\
&\quad-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}i\int x^2\log\left(1-ice^{2a+2bx}\right)\,dx\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)\\
&\quad-\frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\int x\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)\,dx}{2b}\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))\\
&\quad-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)-\frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}\\
&\quad+\frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\int\text{PolyLog}\left(3,ice^{2a+2bx}\right)\,dx}{4b^2}\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))\\
&\quad-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)-\frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}\\
&\quad+\frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(3,icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3}\\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)\\
&\quad-\frac{ix^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{ix\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\,\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$$
$$=\frac{8b^3x^3\cot^{-1}(c+(i+c)\coth(a+bx))-4ib^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)+6ib^2x^2\,\mathrm{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)+6ibx}{24b^3}$$

```
[In] Integrate[x^2*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcCot[c + (I + c)*Coth[a + b*x]] - (4*I)*b^3*x^3*Log[1 + I/(c*E
^(2*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] + (6*
I)*b*x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] + (3*I)*PolyLog[4, (-I)/(c*E^(2
*(a + b*x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.94 (sec) , antiderivative size = 1404, normalized size of antiderivative = 9.89

method	result	size
risch	Expression too large to display	1404

```
[In] int(x^2*arccot(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3+1/12*Pi*(csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x
+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1)
)-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a
)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp
(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a
)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(2*exp
(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2
*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2
*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(
exp(2*b*x+2*a)-1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*
b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1
))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn(
```

```
(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn((2*exp(2
*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2
*b*x+2*a)-1))^2)*x^3-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/6*I*x^
3*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)+1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c
+2*I)-1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))+1/6*I/b^3*a^3*ln(exp(2*b*x+2*a)*c+
I)-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I/b^3*polylog(2,I*c*exp(2*
b*x+2*a))*a^2+1/12*I*b*c/(I+c)*x^4+1/4*I*x*polylog(3,I*c*exp(2*b*x+2*a))/b^
2-1/12*I/b^3*c/(I+c)*a^4-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x+1/
2*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2*x-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/
b^3-1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln(1+I*e
xp(b*x+a)*(-I*c)^(1/2))*x-1/12*b/(I+c)*x^4-1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*
(-I*c)^(1/2))-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/12/b^3/(I+
c)*a^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(105) = 210.

Time = 0.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.06

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{i\,b^4x^4 + 2i\,b^3x^3\log\left(\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c+i}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) - \cdots}{\cdots}$$

```
[In] integrate(x^2*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/
(c + I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*dil
og(-1/2*sqrt(4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c)
 + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*b*x*polylog(3, -
1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(4*I*c)*e^
(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1
) - 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*polylog(4, -1/2*sqr
t(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*acot(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) + 3*_t0**2*I*c*exp(2*a) - _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{1}{3}x^3 \operatorname{arccot}\left((c+i)\coth(bx+a)+c\right) - \frac{4}{9}\left(\frac{3x^4}{4ic-4} - \frac{4b^3x^3\log\left(-ice^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(ice^{(2bx+2a)}\right)-6bx\mathrm{Li}_3\left(ice^{(2bx+2a)}\right)+3\,\mathrm{Li}_4\left(ice^{(\ldots}{-2b^4(-ic+1)}\right.$$

```
[In] integrate(x^2*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot((c + I)*coth(b*x + a) + c) - 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c - 2)))*b*(c + I)
```

Giac [F]

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \int x^2 \operatorname{arccot}\left((c+i)\coth(bx+a)+c\right)\,dx$$

```
[In] integrate(x^2*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot((c + I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\int x^2\,\mathrm{acot}(c+\coth(a+b\,x)\,\,(c+1\mathrm{i}))\,\,dx$$

```
[In] int(x^2*acot(c + coth(a + b*x)*(c + 1i)),x)
[Out] int(x^2*acot(c + coth(a + b*x)*(c + 1i)), x)
```

3.210 $\int x \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{ix\,\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\,\mathrm{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arccot(c+(I+c)*coth(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,I*c*exp(2*b
*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5306, 2215, 2221, 2611, 2320, 6724}

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\frac{i\,\mathrm{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}-\frac{ix\,\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}x^2\cot^{-1}(c+(c+i)\coth(a+bx))+\frac{1}{6}ibx^3$$

```
[In] Int[x*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCot[c + (I + c)*Coth[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((I/8
)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5306

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{-i-ce^{2a+2bx}}\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{-i-ce^{2a+2bx}}\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))\\
&\quad -\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}i\int x\log\left(1-ice^{2a+2bx}\right)\,dx\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\int\text{PolyLog}\left(2,ice^{2a+2bx}\right)\,dx}{4b}\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)\\
&\quad -\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{2b^2x^2\left(2\cot^{-1}(c+(i+c)\coth(a+bx))-i\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)+i\,\text{PolyLo}}{8b^2}$$

```
[In] Integrate[x*ArcCot[c + (I + c)*Coth[a + b*x]],x]
[Out] (2*b^2*x^2*(2*ArcCot[c + (I + c)*Coth[a + b*x]] - I*Log[1 + I/(c*E^(2*(a +
b*x)))]) + (2*I)*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] + I*PolyLog[3, (-
I)/(c*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.54 (sec) , antiderivative size = 1368, normalized size of antiderivative = 12.11

method	result	size
risch	Expression too large to display	1368

```
[In] int(x*arccot(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c+2*I)+1/8*Pi*(csgn(I/(exp(2*b*x+2*a)-1))*csg
n(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*
a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-c
sgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1
))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)
*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*
b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1
))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(2*exp(2*
b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*
x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*ex
p(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp
(2*b*x+2*a)-1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x
+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-
csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn((2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn((2*exp(2*b*
x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*
x+2*a)-1))^2)*x^2+1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^2*
a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*c)^(1/
2))*x+1/2*I/b^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*a^2-1/4*I*x*polylog(2,I*c*e
xp(2*b*x+2*a))/b+1/6*I/b^2*c/(I+c)*a^3+1/8*I*polylog(3,I*c*exp(2*b*x+2*a))/
b^2-1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))-1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+I)
+1/6*I*b*c/(I+c)*x^3-1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2+1/2*I/b*a*ln(1+
I*exp(b*x+a)*(-I*c)^(1/2))*x-1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))*a*x+1/2*I/b^2
*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*a^2-1/6*b/(I+c)*x^3-1/6/b^2/(I+c)*a^3-1/4*
I/b^2*polylog(2,I*c*exp(2*b*x+2*a))*a-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(83) = 166.

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.18

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{2i\,b^3x^3+3i\,b^2x^2\log\left(\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c+i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right)}{}$$

```
[In] integrate(x*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a
)/(c + I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b*x
*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) + I
*sqrt(4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - 3
*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*x^2 -
I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sqrt(4*I*
c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*acot(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) + 3*_t0**2*I*c*exp(2*a) - _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx = -\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i) + \frac{1}{2}\,x^2\,\mathrm{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)$$

```
[In] integrate(x*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2))
)*b*(c + I) + 1/2*x^2*arccot((c + I)*coth(b*x + a) + c)
```

Giac [F]

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\int x\,\mathrm{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot((c + I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\int x\,\mathrm{acot}(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

```
[In] int(x*acot(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x*acot(c + coth(a + b*x)*(c + 1i)), x)
```

3.211 $\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{i\,\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arccot(c+(I+c)*coth(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5298, 2215, 2221, 2317, 2438}

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx=-\frac{i\,\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\cot^{-1}(c+(c+i)\coth(a+bx))+\frac{1}{2}ibx^2$$

```
[In] Int[ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCot[c + (I + c)*Coth[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5298

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Cot[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\cot^{-1}(c+(i+c)\coth(a+bx))+b\int\frac{x}{-i-ce^{2a+2bx}}\,dx\\
&=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))+(ibc)\int\frac{e^{2a+2bx}x}{-i-ce^{2a+2bx}}\,dx\\
&=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}i\int\log\left(1-ice^{2a+2bx}\right)\,dx\\
&=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))\\
&\quad-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{i\text{Subst}\left(\int\frac{\log(1-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{4b}\\
&=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{i\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$$
$$= x\cot^{-1}(c+(i+c)\coth(a+bx)) - \frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\mathrm{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

```
[In] Integrate[ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (I + c)*Coth[a + b*x]] - ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(65) = 130.

Time = 1.67 (sec) , antiderivative size = 544, normalized size of antiderivative = 6.89

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\operatorname{arccot}(c+(i+c)\,\mathrm{co}}{}}{}$
default	$\frac{\frac{\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\operatorname{arccot}(c+(i+c)\,\mathrm{co}}{}}{}$
risch	Expression too large to display

```
[In] int(arccot(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I+c)*(arccot(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)-
2*I*arccot(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c-arcco
t(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c^2-arccot(c+(I+
c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))+2*I*arccot(c+(I+c)*coth
(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c+arccot(c+(I+c)*coth(b*x+a))/
(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c^2+(I+c)^2*(1/2/(I+c)*(1/4*I*ln(I+c+(I
+c)*coth(b*x+a))^2-1/2*I*((ln(I+c+(I+c)*coth(b*x+a))-ln(-1/2*I*(I+c+(I+c)*c
oth(b*x+a))))*ln(-1/2*I*(I-c-(I+c)*coth(b*x+a)))-dilog(-1/2*I*(I+c+(I+c)*co
th(b*x+a)))))-1/2/(I+c)*(1/2*I*(dilog((-I-c-(I+c)*coth(b*x+a))/(-2*I-2*c))+
ln(c-(I+c)*coth(b*x+a)+I)*ln((-I-c-(I+c)*coth(b*x+a))/(-2*I-2*c)))-1/2*I*(d
ilog(-1/2*(I-c-(I+c)*coth(b*x+a))/c)+ln(c-(I+c)*coth(b*x+a)+I)*ln(-1/2*(I-c
-(I+c)*coth(b*x+a))/c)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(58) = 116$.

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.35

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{i\,b^2x^2 + i\,bx\log\left(\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c+i}\right) - i\,a^2 + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}+1\right) + (-i\,bx - i\,a)\log$$

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c + I)
) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-I*b*x -
 I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - I*
dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)))
/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(acot(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) + 3*_t0**2*I*c*exp(2*a) - _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = -2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2} - \frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right) + \mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right) + x\,\mathrm{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)$$

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + di
log(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arccot((c + I)*coth(b*x +
a) + c)
```

Giac [F]

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\int \operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot((c + I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\int \operatorname{acot}(c+\coth(a+b\,x)\,\,(c+1\mathrm{i}))\,dx$$

```
[In] int(acot(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(acot(c + coth(a + b*x)*(c + 1i)), x)
```

3.212 $\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+(I+c)*coth(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

```
[In] Int[ArcCot[c + (I + c)*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + (I + c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c + (I + c)*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + (I + c)*Coth[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arccot}\left(c+(i+c)\coth\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arccot(c+(I+c)*coth(b*x+a))/x,x)
[Out] int(arccot(c+(I+c)*coth(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c + I))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx=\text{Timed out}$$

```
[In] integrate(acot(c+(I+c)*coth(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-4*I*a + 2*arctan(1/c) - I*log(c^2 + 1))*log(x) - 1/2*integra
te(arctan(e^(-2*b*x - 2*a)/c)/x, x) + 1/4*I*integrate(log(c^2*e^(4*b*x + 4*
a) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot((c + I)*coth(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{acot}(c+\coth(a+b\,x)\ (c+1\mathrm{i}))}{x}\,dx$$

```
[In] int(acot(c + coth(a + b*x)*(c + 1i))/x,x)
```

```
[Out] int(acot(c + coth(a + b*x)*(c + 1i))/x, x)
```

3.213 $\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\begin{aligned}\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)\\ &\quad+\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}\end{aligned}$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arccot(c-(I-c)*coth(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,-I*c
*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5306, 2215, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx &= \frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}\\ &\quad-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}\\ &\quad+\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}\\ &\quad+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)\\ &\quad+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\coth(a+bx))-\frac{1}{12}ibx^4\end{aligned}$$

```
[In] Int[x^2*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (-1/12*I)*b*x^4 + (x^3*ArcCot[c - (I - c)*Coth[a + b*x]])/3 + (I/6)*x^3*Log
[1 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/
b - ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, (-
I)*c*E^(2*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5306

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{3}b\int\frac{x^3}{i-ce^{2a+2bx}}\,dx\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{3}(ibc)\int\frac{e^{2a+2bx}x^3}{i-ce^{2a+2bx}}\,dx\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx))\\
&\quad+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{1}{2}i\int x^2\log\left(1+ice^{2a+2bx}\right)\,dx\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)\\
&\quad+\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\int x\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)\,dx}{2b}\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx))\\
&\quad+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}\\
&\quad-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{i\int\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)\,dx}{4b^2}\\
&= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx))\\
&\quad+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}\\
&\quad-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{i\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^3}
\end{aligned}$$

$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1 + ice^{2a+2bx}\right) + \frac{ix^2\,\mathrm{PolyLog}\left(2, -ice^{2a+2bx}\right)}{4b} - \frac{ix\,\mathrm{PolyLog}\left(3, -ice^{2a+2bx}\right)}{4b^2} + \frac{i\,\mathrm{PolyLog}\left(4, -ice^{2a+2bx}\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int x^2\cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \frac{8b^3x^3\cot^{-1}(c+(-i+c)\coth(a+bx)) + 4ib^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - 6ib^2x^2\,\mathrm{PolyLog}\left(2, \frac{ie^{-2(a+bx)}}{c}\right) - 6ib}{24b^3}$$

```
[In] Integrate[x^2*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (8*b^3*x^3*ArcCot[c + (-I + c)*Coth[a + b*x]] + (4*I)*b^3*x^3*Log[1 - I/(c*
E^(2*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - (6*I)
*b*x*PolyLog[3, I/(c*E^(2*(a + b*x)))] - (3*I)*PolyLog[4, I/(c*E^(2*(a + b*
x)))])/(24*b^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.31 (sec) , antiderivative size = 1410, normalized size of antiderivative = 9.72

method	result	size
risch	Expression too large to display	1410

```
[In] int(x^2*arccot(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))-1/12*Pi*(csgn(I/(exp(2*b*x+
2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*
b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(ex
p(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*c
sgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*exp(2*b*x
+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/
(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3
+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)
```

```
*c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*
a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))+csgn(I*(-2*I*exp(2
*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-csgn(I*(-2*I*exp(2*b*x+
2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*ex
p(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c
)/(exp(2*b*x+2*a)-1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*
b*x+2*a)-1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a
)-1))^2+csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+csgn((2*exp(2*b
*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-4)*x^3+1/2*I/b^3*a^2*dilog(1-I*exp(b*x
+a)*(I*c)^(1/2))+1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))+1/2*I/b^3*a^3*ln(1-I*ex
p(b*x+a)*(I*c)^(1/2))-1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a))/b^2-1/12*I/b^3
*c*a^4/(I-c)-1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2-1/6*I*x^3*ln(-2*e
xp(2*b*x+2*a)*c+2*I)+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))-1/6*I/b^3
*a^3*ln(-exp(2*b*x+2*a)*c+I)+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/3
*I/b^3*ln(1+I*c*exp(2*b*x+2*a))*a^3+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^
3+1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/2*I/b^2*ln(1+I*c*ex
p(2*b*x+2*a))*a^2*x+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x+1/2*I/b^
2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x+1/12*I*b*c/(I-c)*x^4-1/12/b^3/(I-c)*
a^4+1/12*b/(I-c)*x^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.01

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$$
$$= \frac{-i\,b^4x^4 + 2i\,b^3x^3\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + i\,a^4 -}{}$$

```
[In] integrate(x^2*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*
a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*di
log(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c)
)/c) - 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*b*x*polylog
(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(-4
*I*c)*e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(-4*I*c)*e^(b*
x + a) + 1) + 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*polylog(
4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x**2*acot(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) - I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \frac{1}{3}\,x^3\,\mathrm{arccot}\left((c-i)\coth\left(bx+a\right)+c\right) + \frac{4}{9}\left(\frac{3\,x^4}{4i\,c+4} - \frac{4\,b^3x^3\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-i\,ce^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(-}{-2\,b^4(-i\,c-1)}\right.$$

```
[In] integrate(x^2*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot((c - I)*coth(b*x + a) + c) + 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c + 2)))*b*(c - I)
```

Giac [F]

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \int x^2\,\mathrm{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x^2*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot((c - I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\int x^2\,\mathrm{acot}(c+\coth(a+b\,x)\,(c-\mathrm{i}))\,dx$$

```
[In] int(x^2*acot(c + coth(a + b*x)*(c - 1i)),x)
[Out] int(x^2*acot(c + coth(a + b*x)*(c - 1i)), x)
```

3.214 $\int x \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\begin{aligned}\int x \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = &-\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))\\ &+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\ &+\frac{ix\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}\\ &-\frac{i\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}\end{aligned}$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arccot(c-(I-c)*coth(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,-I*c*exp(
2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5306, 2215, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int x \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = &-\frac{i\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}\\ &+\frac{ix\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}\\ &+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\ &+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\coth(a+bx))-\frac{1}{6}ibx^3\end{aligned}$$

```
[In] Int[x*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (-1/6*I)*b*x^3 + (x^2*ArcCot[c - (I - c)*Coth[a + b*x]])/2 + (I/4)*x^2*Log[
1 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b -
 ((I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5306

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{i-ce^{2a+2bx}}\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^2}{i-ce^{2a+2bx}}\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))\\
&\quad+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{1}{2}i\int x\log\left(1+ice^{2a+2bx}\right)\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad+\frac{ix\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\int\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)\,dx}{4b}\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad+\frac{ix\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-icx)}{x}\,dx,x,e^{2a+2bx}\right)}{8b^2}\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)\\
&\quad+\frac{ix\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$
\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx
$$
$$
=\frac{2b^2x^2\left(2\cot^{-1}(c+(-i+c)\coth(a+bx))+i\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)-2ibx\operatorname{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-i\operatorname{Poly}}{8b^2}
$$

```
[In] Integrate[x*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCot[c + (-I + c)*Coth[a + b*x]] + I*Log[1 - I/(c*E^(2*(a +
 b*x)))]) - (2*I)*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - I*PolyLog[3, I/(c
*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.91 (sec) , antiderivative size = 1374, normalized size of antiderivative = 11.84

method	result	size
risch	Expression too large to display	1374

```
[In] int(x*arccot(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b*ln(1+I*c*exp(2*b*x+2*a))*a*x-1/8*Pi*(csgn(I/(exp(2*b*x+2*a)-1))*csg
n(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*
a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)
-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp
(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2
*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp
(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn(
(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))+csgn(I*(-2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)
*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(
exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x
+2*a)-1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn
((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+csgn((2*exp(2*b*x+2*a)*c-2*
I)/(exp(2*b*x+2*a)-1))^2-4)*x^2-1/2*I/b*a*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x-
1/4*I*x^2*ln(-2*exp(2*b*x+2*a)*c+2*I)+1/4*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^
2+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/2*I/b^2*ln(1-I*exp(b*x+a)*(I*c
)^(1/2))*a^2-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))-1/8*I*polylog(3,
-I*c*exp(2*b*x+2*a))/b^2-1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x+1/4*I/b
^2*polylog(2,-I*c*exp(2*b*x+2*a))*a+1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)+1
/4*I*x^2*ln(1+I*c*exp(2*b*x+2*a))-1/2*I/b^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*
a^2+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/2*I/b^2*a*dilog(1
+I*exp(b*x+a)*(I*c)^(1/2))+1/6/b^2/(I-c)*a^3+1/6*b/(I-c)*x^3+1/6*I/b^2*c/(I
-c)*a^3+1/6*I*b*c/(I-c)*x^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.12

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx$$
$$= \frac{-2i\,b^3x^3 + 3i\,b^2x^2\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) - 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + \cdots}{}$$

```
[In] integrate(x*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
2*a) - I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b*
x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
 I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
 - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^2
*x^2 + I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*s
qrt(-4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b
^2
```

Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(x*acot(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) - I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx$$
$$= \left(\frac{2\,x^3}{3i\,c+3} - \frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right) - \mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i)$$
$$+ \frac{1}{2}\,x^2\,\mathrm{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)$$

```
[In] integrate(x*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(
-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2))
)*b*(c - I) + 1/2*x^2*arccot((c - I)*coth(b*x + a) + c)
```

Giac [F]

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\int x\operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(x*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot((c - I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\int x\operatorname{acot}(c+\coth(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

```
[In] int(x*acot(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x*acot(c + coth(a + b*x)*(c - 1i)), x)
```

3.215 $\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx=-\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+\frac{i\,\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

```
[Out] -1/2*I*b*x^2+x*arccot(c-(I-c)*coth(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5298, 2215, 2221, 2317, 2438}

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\frac{i\,\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\cot^{-1}(c-(-c+i)\coth(a+bx))-\frac{1}{2}ibx^2$$

```
[In] Int[ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (-1/2*I)*b*x^2 + x*ArcCot[c - (I - c)*Coth[a + b*x]] + (I/2)*x*Log[1 + I*c*
E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5298

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Cot[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\cot^{-1}(c-(i-c)\coth(a+bx))+b\int \frac{x}{i-ce^{2a+2bx}}\,dx\\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))-(ibc)\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}\,dx\\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))\\
&\quad+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)-\frac{1}{2}i\int \log\left(1+ice^{2a+2bx}\right)\,dx\\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))\\
&\quad+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)-\frac{i\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx,x,e^{2a+2bx}\right)}{4b}\\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+\frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$$
$$= x\cot^{-1}(c+(-i+c)\coth(a+bx))+\frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

```
[In] Integrate[ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (-I + c)*Coth[a + b*x]] + ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(68) = 136.

Time = 1.87 (sec) , antiderivative size = 517, normalized size of antiderivative = 6.30

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccot}(c+\coth(bx+a)(c-i))\ln(\coth(bx+a)(c-i)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}(c+\coth(bx+a)(c-i))\ln(\coth(bx+a)(c-i)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}(c+\coth}{}}{}$
default	$\frac{-\frac{\operatorname{arccot}(c+\coth(bx+a)(c-i))\ln(\coth(bx+a)(c-i)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}(c+\coth(bx+a)(c-i))\ln(\coth(bx+a)(c-i)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}(c+\coth}{}}{}$
risch	Expression too large to display

```
[In] int(arccot(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(c-I)*(-arccot(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)
-2*I*arccot(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)*c+arcc
ot(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)*c^2+arccot(c+co
th(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)+2*I*arccot(c+coth(b*x
+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)*c-arccot(c+coth(b*x+a)*(c-I
))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)*c^2-(I-c)^2*(1/2/(I-c)*(-1/4*I*ln(-
I+coth(b*x+a)*(c-I)+c)^2+1/2*I*(dilog(-1/2*I*(coth(b*x+a)*(c-I)+c+I))+ln(-I
+coth(b*x+a)*(c-I)+c)*ln(-1/2*I*(coth(b*x+a)*(c-I)+c+I))))-1/2/(I-c)*(1/2*I
*(dilog(1/2*(coth(b*x+a)*(c-I)+c+I)/c)+ln(coth(b*x+a)*(c-I)-c+I)*ln(1/2*(co
th(b*x+a)*(c-I)+c+I)/c))-1/2*I*(dilog((-I+coth(b*x+a)*(c-I)+c)/(-2*I+2*c))+
ln(coth(b*x+a)*(c-I)-c+I)*ln((-I+coth(b*x+a)*(c-I)+c)/(-2*I+2*c))))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(58) = 116$.

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.27

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \frac{-i\,b^2x^2 + i\,bx\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) + i\,a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{-4i\right.}{}$$

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I)
) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (I*b*x +
I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c) +
I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x +
a)))/b
```

Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

```
[In] integrate(acot(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) - I of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,ex
p(a)]
```

Maxima [A] (verification not implemented)

none

Time = 1.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = 2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2} - \frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right) + \mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right) + x\,\mathrm{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)$$

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dilo
g(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arccot((c - I)*coth(b*x + a
) + c)
```

Giac [F]

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\int \operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot((c - I)*coth(b*x + a) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\int \operatorname{acot}(c+\coth(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

```
[In] int(acot(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(acot(c + coth(a + b*x)*(c - 1i)), x)
```

3.216 $\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x} dx = \text{Int}\left(\frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}, x\right)$$

```
[Out] CannotIntegrate(arccot(c-(I-c)*coth(b*x+a))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x} dx = \int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x} dx$$

```
[In] Int[ArcCot[c - (I - c)*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCot[c - (I - c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$$

```
[In] Integrate[ArcCot[c - (I - c)*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c - (I - c)*Coth[a + b*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arccot}\left(c-(i-c)\coth\left(bx+a\right)\right)}{x}dx$$

```
[In] int(arccot(c-(I-c)*coth(b*x+a))/x,x)
[Out] int(arccot(c-(I-c)*coth(b*x+a))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I))/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx=\text{Timed out}$$

```
[In] integrate(acot(c-(I-c)*coth(b*x+a))/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.68

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x + 1/2*pi*log(x) - 1/4*(2*pi - 4*I*a - 2*arctan(1/c) - I*log(c^2 + 1))
*log(x) - 1/2*integrate(arctan(e^(-2*b*x - 2*a)/c)/x, x) - 1/4*I*integrate(
log(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot((c - I)*coth(b*x + a) + c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\mathrm{acot}(c+\coth(a+b\,x)\ (c-\mathrm{i}))}{x}\,dx$$

```
[In] int(acot(c + coth(a + b*x)*(c - 1i))/x,x)
```

```
[Out] int(acot(c + coth(a + b*x)*(c - 1i))/x, x)
```

3.217 $\int \frac{(a+b\cot^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{(a+b\cot^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx = ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{ibd\,\text{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)}{2n}-\frac{ibe\log(fx^m)\,\text{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)}{2n}+\frac{ibd\,\text{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)}{2n}+\frac{ibe\log(fx^m)\,\text{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)}{2n}-\frac{ibem\,\text{PolyLog}\left(3,-\frac{ix^{-n}}{c}\right)}{2n^2}+\frac{ibem\,\text{PolyLog}\left(3,\frac{ix^{-n}}{c}\right)}{2n^2}$$

```
[Out] a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m-1/2*I*b*d*polylog(2,-I/c/(x^n))/n-1/2*I*b*e
*ln(f*x^m)*polylog(2,-I/c/(x^n))/n+1/2*I*b*d*polylog(2,I/c/(x^n))/n+1/2*I*b
*e*ln(f*x^m)*polylog(2,I/c/(x^n))/n-1/2*I*b*e*m*polylog(3,-I/c/(x^n))/n^2+1
/2*I*b*e*m*polylog(3,I/c/(x^n))/n^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

= {2338, 6874, 4945, 4941, 2438, 5128, 5126, 2421, 6724}

$$\int \frac{(a+b\cot^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$$
$$= ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{ibd\,\text{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)}{2n}+\frac{ibd\,\text{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)}{2n}-\frac{ibe\,\text{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)\log(fx^m)}{2n}+\frac{ibe\,\text{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)\log(fx^m)}{2n}-\frac{ibem\,\text{PolyLog}\left(3,-\frac{ix^{-n}}{c}\right)}{2n^2}+\frac{ibem\,\text{PolyLog}\left(3,\frac{ix^{-n}}{c}\right)}{2n^2}$$

```
[In] Int[((a + b*ArcCot[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) - ((I/2)*b*d*PolyLog[2, (-I)/(c*x^n)]
)/n - ((I/2)*b*e*Log[f*x^m]*PolyLog[2, (-I)/(c*x^n)])/n + ((I/2)*b*d*PolyLo
g[2, I/(c*x^n)])/n + ((I/2)*b*e*Log[f*x^m]*PolyLog[2, I/(c*x^n)])/n - ((I/2
)*b*e*m*PolyLog[3, (-I)/(c*x^n)])/n^2 + ((I/2)*b*e*m*PolyLog[3, I/(c*x^n)])
/n^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5126

```
Int[(ArcCot[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Dis
t[I/2, Int[Log[d*x^m]*(Log[1 - I/(c*x^n)]/x), x], x] - Dist[I/2, Int[Log[d*
x^m]*(Log[1 + I/(c*x^n)]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 5128

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcCot[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_), x
_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*ArcC
ot[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(a+b\cot^{-1}(cx^n))}{x}+\frac{e(a+b\cot^{-1}(cx^n))\log(fx^m)}{x}\right)dx\\
&= d\int \frac{a+b\cot^{-1}(cx^n)}{x}\,dx+e\int \frac{(a+b\cot^{-1}(cx^n))\log(fx^m)}{x}\,dx\\
&= (ae)\int \frac{\log(fx^m)}{x}\,dx+(be)\int \frac{\cot^{-1}(cx^n)\log(fx^m)}{x}\,dx+\frac{d\text{Subst}\left(\int \frac{a+b\cot^{-1}(cx)}{x}\,dx,x,x^n\right)}{n}\\
&= ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{1}{2}(ibe)\int \frac{\log(fx^m)\log\left(1-\frac{ix^{-n}}{c}\right)}{x}\,dx\\
&\quad -\frac{1}{2}(ibe)\int \frac{\log(fx^m)\log\left(1+\frac{ix^{-n}}{c}\right)}{x}\,dx\\
&\quad +\frac{(ibd)\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{cx}\right)}{x}\,dx,x,x^n\right)}{2n}-\frac{(ibd)\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{cx}\right)}{x}\,dx,x,x^n\right)}{2n}
\end{aligned}$$

$$= ad\log(x) + \frac{ae\log^2(fx^m)}{2m} - \frac{ibd\,\text{PolyLog}\left(2, -\frac{ix^{-n}}{c}\right)}{2n} - \frac{ibe\log(fx^m)\,\text{PolyLog}\left(2, -\frac{ix^{-n}}{c}\right)}{2n} + \frac{ibd\,\text{PolyLog}\left(2, \frac{ix^{-n}}{c}\right)}{2n} + \frac{ibe\log(fx^m)\,\text{PolyLog}\left(2, \frac{ix^{-n}}{c}\right)}{2n} + \frac{(ibem)\int \frac{\text{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)}{x}dx}{2n} - \frac{(ibem)\int \frac{\text{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)}{x}dx}{2n}$$

$$= ad\log(x) + \frac{ae\log^2(fx^m)}{2m} - \frac{ibd\,\text{PolyLog}\left(2, -\frac{ix^{-n}}{c}\right)}{2n} - \frac{ibe\log(fx^m)\,\text{PolyLog}\left(2, -\frac{ix^{-n}}{c}\right)}{2n} + \frac{ibd\,\text{PolyLog}\left(2, \frac{ix^{-n}}{c}\right)}{2n} + \frac{ibe\log(fx^m)\,\text{PolyLog}\left(2, \frac{ix^{-n}}{c}\right)}{2n} - \frac{ibem\,\text{PolyLog}\left(3, -\frac{ix^{-n}}{c}\right)}{2n^2} + \frac{ibem\,\text{PolyLog}\left(3, \frac{ix^{-n}}{c}\right)}{2n^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.71

$$\int \frac{(a + b\cot^{-1}(cx^n))(d + e\log(fx^m))}{x}dx = \frac{bcemx^n\,{}_4F_3\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; -c^2x^{2n}\right)}{n^2} - \frac{bcx^n\,{}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; -c^2x^{2n}\right)(d + e\log(fx^m))}{n} - \frac{1}{2}\left(a + b\cot^{-1}(cx^n) + b\arctan(cx^n)\right)\log(x)\left(em\log(x) - 2(d + e\log(fx^m))\right)$$

```
[In] Integrate[((a + b*ArcCot[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] (b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, -(c^2*x
^(2*n))])/n^2 - (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, -(c^2
*x^(2*n))]*(d + e*Log[f*x^m]))/n - ((a + b*ArcCot[c*x^n] + b*ArcTan[c*x^n])
*Log[x]*(e*m*Log[x] - 2*(d + e*Log[f*x^m])))/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 224.52 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.03

method	result
risch	$\frac{\left(-\frac{ie\pi\,\mathrm{csgn}(if)\,\mathrm{csgn}(ix^m)\,\mathrm{csgn}(if\,x^m)}{4}+\frac{ie\pi\,\mathrm{csgn}(if)\,\mathrm{csgn}(if\,x^m)^2}{4}+\frac{ie\pi\,\mathrm{csgn}(ix^m)\,\mathrm{csgn}(if\,x^m)^2}{4}-\frac{ie\pi\,\mathrm{csgn}(if\,x^m)^3}{4}+\frac{e\ln(f)}{2}+\frac{d}{2}\right)((b\pi+2a)}{n}$

[In] int((a+b*arccot(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)

[Out] (-1/4*I*e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I*e*Pi*csgn(I*f)*csgn(I*f*x^m)^2+1/4*I*e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I*e*Pi*csgn(I*f*x^m)^3+1/2*e*ln(f)+1/2*d)/n*((Pi*b+2*a)*ln(x^n)-I*b*dilog(1+I*c*x^n)+I*b*dilog(1-I*c*x^n))+1/4*e/m*ln(x^m)^2*b*Pi+1/2*e/m*ln(x^m)^2*a+1/2*I*e*b*ln(x)*ln(-I*(c*x^n+I))*ln(x^m)-1/2*I*e*b/n*dilog(-I*c*x^n)*m*ln(x)-1/2*I*e*b*ln(x)*ln(1-I*c*x^n)*ln(x^m)-1/2*I*e*b*m/n^2*polylog(3,I*c*x^n)+1/2*I*e*b*ln(x)^2*ln(1-I*c*x^n)*m-1/2*I*e*b*ln(x)*ln(-I*(-c*x^n+I))*ln(x^m)-1/2*I*e*b*ln(x)^2*ln(1+I*c*x^n)*m+1/2*I*e*b*m/n*ln(x)*polylog(2,I*c*x^n)+1/2*I*e*b*ln(x)^2*ln(-I*(-c*x^n+I))*m+1/2*I*e*b*m/n^2*polylog(3,-I*c*x^n)-1/2*I*e*b/n*dilog(-I*(c*x^n+I))*m*ln(x)-1/2*I*e*b*m/n*ln(x)*polylog(2,-I*c*x^n)-1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*m*ln(x)+1/2*I*e*b/n*dilog(-I*(c*x^n+I))*ln(x^m)+1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*ln(x^m)+1/2*I*e*b*ln(x)*ln(1+I*c*x^n)*ln(x^m)-1/2*I*e*b*ln(x)^2*ln(-I*(c*x^n+I))*m+1/2*I*e*b/n*dilog(-I*c*x^n)*ln(x^m)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\cot^{-1}(cx^n))\,(d+e\log(fx^m))}{x}\,dx$$
$$= \frac{2\,aemn^2\log(x)^2-2i\,bem\mathrm{polylog}(3,i\,cx^n)+2i\,bem\mathrm{polylog}(3,-i\,cx^n)+2\left(bemn^2\log(x)^2+2\left(ben^2\log(\right.\right.}{}$$

[In] integrate((a+b*arccot(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")

[Out] 1/4*(2*a*e*m*n^2*log(x)^2 - 2*I*b*e*m*polylog(3, I*c*x^n) + 2*I*b*e*m*polylog(3, -I*c*x^n) + 2*(b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*arccot(c*x^n) - 2*(-I*b*e*m*n*log(x) - I*b*e*n*log(f) - I*b*d*n)*dilog(I*c*x^n) - 2*(I*b*e*m*n*log(x) + I*b*e*n*log(f) + I*b*d*n)*dilog(-I*c*x^n) + (-I*b*e*m*n^2*log(x)^2 - 2*(I*b*e*n^2*log(f) + I*b*d*n^2)*log(x))*log(I*c*x^n + 1) + (I*b*e*m*n^2*log(x)^2 - 2*(-I*b*e*n^2*log(f) - I*b*d*n^2)*log(x))*log(-I*c*x^n + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x))/n^2

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\cot^{-1}(cx^n))(d + e\log(fx^m))}{x} dx = \text{Timed out}$$

```
[In] integrate((a+b*acot(c*x**n))*(d+e*ln(f*x**m))/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + b\cot^{-1}(cx^n))(d + e\log(fx^m))}{x} dx = \int \frac{(b\,\mathrm{arccot}(cx^n) + a)(e\log(fx^m) + d)}{x} dx$$

```
[In] integrate((a+b*arccot(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/2*(b*e*m*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(b*e*log(f) + b*d)*log(x))*arctan(1/(c*x^n)) + integrate(-1/2*(b
*c*e*m*n*x^n*log(x)^2 - 2*b*c*e*n*x^n*log(x)*log(x^m) - 2*(b*c*e*log(f) + b
*c*d)*n*x^n*log(x))/(c^2*x*x^(2*n) + x), x)
```

Giac [F]

$$\int \frac{(a + b\cot^{-1}(cx^n))(d + e\log(fx^m))}{x} dx = \int \frac{(b\,\mathrm{arccot}(cx^n) + a)(e\log(fx^m) + d)}{x} dx$$

```
[In] integrate((a+b*arccot(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
[Out] integrate((b*arccot(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\cot^{-1}(cx^n))(d + e\log(fx^m))}{x} dx = \int \frac{(a + b\,\mathrm{acot}(c\,x^n))\,(d + e\,\ln(f\,x^m))}{x} dx$$

```
[In] int(((a + b*acot(c*x^n))*(d + e*log(f*x^m)))/x,x)
[Out] int(((a + b*acot(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

3.218 $\int \cot^{-1}(e^x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 35

$$\int \cot^{-1}(e^x)\,dx = -\frac{1}{2}i\,\text{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,ie^{-x}\right)$$

```
[Out] -1/2*I*polylog(2,-I/exp(x))+1/2*I*polylog(2,I/exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2320, 4941, 2438}

$$\int \cot^{-1}(e^x)\,dx = \frac{1}{2}i\,\text{PolyLog}\left(2,ie^{-x}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-ie^{-x}\right)$$

```
[In] Int[ArcCot[E^x],x]
[Out] (-1/2*I)*PolyLog[2, (-I)/E^x] + (I/2)*PolyLog[2, I/E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, e^x\right)\\ &= \frac{1}{2}i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, e^x\right) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, e^x\right)\\ &= -\frac{1}{2}i\,\text{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,ie^{-x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cot^{-1}\left(e^x\right)\,dx = x\cot^{-1}\left(e^x\right) + \frac{1}{2}i(x(\log(1-ie^x)-\log(1+ie^x))-\text{PolyLog}(2,-ie^x)+\text{PolyLog}(2,ie^x))$$

```
[In] Integrate[ArcCot[E^x],x]
```

```
[Out] x*ArcCot[E^x] + (I/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I
)*E^x] + PolyLog[2, I*E^x])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.

Time = 0.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

method	result	size
parts	$x \,\mathrm{arccot}\left(\mathrm{e}^{x}\right)-\frac{i x \ln\left(1+i \,\mathrm{e}^{x}\right)}{2}+\frac{i x \ln\left(1-i \,\mathrm{e}^{x}\right)}{2}-\frac{i \,\mathrm{dilog}\left(1+i \,\mathrm{e}^{x}\right)}{2}+\frac{i \,\mathrm{dilog}\left(1-i \,\mathrm{e}^{x}\right)}{2}$	53
derivativedivides	$\ln\left(\mathrm{e}^{x}\right) \mathrm{arccot}\left(\mathrm{e}^{x}\right)-\frac{i \ln\left(\mathrm{e}^{x}\right) \ln\left(1+i \,\mathrm{e}^{x}\right)}{2}+\frac{i \ln\left(\mathrm{e}^{x}\right) \ln\left(1-i \,\mathrm{e}^{x}\right)}{2}-\frac{i \,\mathrm{dilog}\left(1+i \,\mathrm{e}^{x}\right)}{2}+\frac{i \,\mathrm{dilog}\left(1-i \,\mathrm{e}^{x}\right)}{2}$	59
default	$\ln\left(\mathrm{e}^{x}\right) \mathrm{arccot}\left(\mathrm{e}^{x}\right)-\frac{i \ln\left(\mathrm{e}^{x}\right) \ln\left(1+i \,\mathrm{e}^{x}\right)}{2}+\frac{i \ln\left(\mathrm{e}^{x}\right) \ln\left(1-i \,\mathrm{e}^{x}\right)}{2}-\frac{i \,\mathrm{dilog}\left(1+i \,\mathrm{e}^{x}\right)}{2}+\frac{i \,\mathrm{dilog}\left(1-i \,\mathrm{e}^{x}\right)}{2}$	59
risch	$\frac{i x \ln\left(1+i \,\mathrm{e}^{x}\right)}{2}+\frac{\pi x}{2}+\frac{i \,\mathrm{dilog}\left(1-i \,\mathrm{e}^{x}\right)}{2}+\frac{i \ln\left(-i \,\mathrm{e}^{x}\right) \ln\left(-i \left(-\mathrm{e}^{x}+i\right)\right)}{2}-\frac{i \ln\left(-i \left(-\mathrm{e}^{x}+i\right)\right) x}{2}+\frac{i \,\mathrm{dilog}\left(-i \,\mathrm{e}^{x}\right)}{2}$	73

```
[In] int(arccot(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccot(exp(x))-1/2*I*x*ln(1+I*exp(x))+1/2*I*x*ln(1-I*exp(x))-1/2*I*dilog(
1+I*exp(x))+1/2*I*dilog(1-I*exp(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \cot^{-1}\left(e^{x}\right)\,dx = x \,\mathrm{arccot}\left(e^{x}\right) - \frac{1}{2} i\,x \log\left(i\,e^{x}+1\right) + \frac{1}{2} i\,x \log\left(-i\,e^{x}+1\right) + \frac{1}{2} i\,\mathrm{Li}_2(i\,e^{x}) - \frac{1}{2} i\,\mathrm{Li}_2(-i\,e^{x})$$

```
[In] integrate(arccot(exp(x)),x, algorithm="fricas")
```

```
[Out] x*arccot(e^x) - 1/2*I*x*log(I*e^x + 1) + 1/2*I*x*log(-I*e^x + 1) + 1/2*I*di
log(I*e^x) - 1/2*I*dilog(-I*e^x)
```

Sympy [F]

$$\int \cot^{-1}\left(e^{x}\right)\,dx = \int \operatorname{acot}\left(e^{x}\right)\,dx$$

```
[In] integrate(acot(exp(x)),x)
```

```
[Out] Integral(acot(exp(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \cot^{-1}\left(e^{x}\right)\,dx = x\,\mathrm{arccot}\left(e^{x}\right) + \frac{1}{4}\,\pi\log\left(e^{(2\,x)} + 1\right) + \frac{1}{2}i\,\mathrm{Li}_2(i\,e^{x} + 1) - \frac{1}{2}i\,\mathrm{Li}_2(-i\,e^{x} + 1)$$

```
[In] integrate(arccot(exp(x)),x, algorithm="maxima")
[Out] x*arccot(e^x) + 1/4*pi*log(e^(2*x) + 1) + 1/2*I*dilog(I*e^x + 1) - 1/2*I*di
log(-I*e^x + 1)
```

Giac [F]

$$\int \cot^{-1}\left(e^{x}\right)\,dx = \int \mathrm{arccot}\left(e^{x}\right)\,dx$$

```
[In] integrate(arccot(exp(x)),x, algorithm="giac")
[Out] integrate(arccot(e^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}\left(e^{x}\right)\,dx = \int \mathrm{acot}(\mathrm{e}^{x})\,dx$$

```
[In] int(acot(exp(x)),x)
[Out] int(acot(exp(x)), x)
```

3.219 $\int x \cot^{-1}(e^x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 71

$$\int x \cot^{-1}(e^x) \, dx = -\frac{1}{2} i x \operatorname{PolyLog}(2, -i e^{-x}) + \frac{1}{2} i x \operatorname{PolyLog}(2, i e^{-x}) - \frac{1}{2} i \operatorname{PolyLog}(3, -i e^{-x}) + \frac{1}{2} i \operatorname{PolyLog}(3, i e^{-x})$$

```
[Out] -1/2*I*x*polylog(2,-I/exp(x))+1/2*I*x*polylog(2,I/exp(x))-1/2*I*polylog(3,-
I/exp(x))+1/2*I*polylog(3,I/exp(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5252, 2611, 2320, 6724}

$$\int x \cot^{-1}(e^x) \, dx = -\frac{1}{2} i x \operatorname{PolyLog}(2, -i e^{-x}) + \frac{1}{2} i x \operatorname{PolyLog}(2, i e^{-x}) - \frac{1}{2} i \operatorname{PolyLog}(3, -i e^{-x}) + \frac{1}{2} i \operatorname{PolyLog}(3, i e^{-x})$$

```
[In] Int[x*ArcCot[E^x],x]
```

```
[Out] (-1/2*I)*x*PolyLog[2, (-I)/E^x] + (I/2)*x*PolyLog[2, I/E^x] - (I/2)*PolyLog
[3, (-I)/E^x] + (I/2)*PolyLog[3, I/E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}i\int x\log\left(1-ie^{-x}\right)\,dx-\frac{1}{2}i\int x\log\left(1+ie^{-x}\right)\,dx\\
&= -\frac{1}{2}ix\,\text{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}ix\,\text{PolyLog}\left(2,ie^{-x}\right)\\
&\quad+\frac{1}{2}i\int\text{PolyLog}\left(2,-ie^{-x}\right)\,dx-\frac{1}{2}i\int\text{PolyLog}\left(2,ie^{-x}\right)\,dx\\
&= -\frac{1}{2}ix\,\text{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}ix\,\text{PolyLog}\left(2,ie^{-x}\right)\\
&\quad-\frac{1}{2}i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{-x}\right)\\
&\quad+\frac{1}{2}i\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{-x}\right)\\
&= -\frac{1}{2}ix\,\text{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}ix\,\text{PolyLog}\left(2,ie^{-x}\right)\\
&\quad-\frac{1}{2}i\,\text{PolyLog}\left(3,-ie^{-x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(3,ie^{-x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int x \cot^{-1}(e^x)\, dx = -\frac{1}{2}i\left(x \operatorname{PolyLog}\left(2,-ie^{-x}\right) - x \operatorname{PolyLog}\left(2,ie^{-x}\right) + \operatorname{PolyLog}\left(3,-ie^{-x}\right) - \operatorname{PolyLog}\left(3,ie^{-x}\right)\right)$$

```
[In] Integrate[x*ArcCot[E^x],x]
```

```
[Out] (-1/2*I)*(x*PolyLog[2, (-I)/E^x] - x*PolyLog[2, I/E^x] + PolyLog[3, (-I)/E^
x] - PolyLog[3, I/E^x])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{\pi x^2}{4}+\frac{i \operatorname{polylog}(2,ie^x)x}{2}-\frac{i \operatorname{polylog}(3,ie^x)}{2}-\frac{ix \operatorname{polylog}(2,-ie^x)}{2}+\frac{i \operatorname{polylog}(3,-ie^x)}{2}$	50

```
[In] int(x*arccot(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi*x^2+1/2*I*polylog(2,I*exp(x))*x-1/2*I*polylog(3,I*exp(x))-1/2*I*x*po
lylog(2,-I*exp(x))+1/2*I*polylog(3,-I*exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int x \cot^{-1}(e^x)\, dx = \frac{1}{2}x^2 \operatorname{arccot}(e^x) - \frac{1}{4}i x^2 \log(i e^x+1) + \frac{1}{4}i x^2 \log(-i e^x+1) + \frac{1}{2}i x \operatorname{Li}_2(i e^x) - \frac{1}{2}i x \operatorname{Li}_2(-i e^x) - \frac{1}{2}i \operatorname{polylog}(3, i e^x) + \frac{1}{2}i \operatorname{polylog}(3, -i e^x)$$

```
[In] integrate(x*arccot(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arccot(e^x) - 1/4*I*x^2*log(I*e^x + 1) + 1/4*I*x^2*log(-I*e^x + 1)
+ 1/2*I*x*dilog(I*e^x) - 1/2*I*x*dilog(-I*e^x) - 1/2*I*polylog(3, I*e^x) +
1/2*I*polylog(3, -I*e^x)
```

Sympy [F]

$$\int x\cot^{-1}\left(e^{x}\right)\,dx=\int x\,\mathrm{acot}\left(e^{x}\right)dx$$

```
[In] integrate(x*acot(exp(x)),x)
[Out] Integral(x*acot(exp(x)), x)
```

Maxima [F]

$$\int x\cot^{-1}\left(e^{x}\right)\,dx=\int x\,\mathrm{arccot}\left(e^{x}\right)\,dx$$

```
[In] integrate(x*arccot(exp(x)),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(e^(-x)) + integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

Giac [F]

$$\int x\cot^{-1}\left(e^{x}\right)\,dx=\int x\,\mathrm{arccot}\left(e^{x}\right)\,dx$$

```
[In] integrate(x*arccot(exp(x)),x, algorithm="giac")
[Out] integrate(x*arccot(e^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}\left(e^{x}\right)\,dx=\int x\,\mathrm{acot}(\mathrm{e}^{x})\,dx$$

```
[In] int(x*acot(exp(x)),x)
[Out] int(x*acot(exp(x)), x)
```

3.220 $\int x^2 \cot^{-1}(e^x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 103

$$\begin{aligned}\int x^2 \cot^{-1}(e^x)\,dx =& -\frac{1}{2}ix^2 \operatorname{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}ix^2 \operatorname{PolyLog}\left(2,ie^{-x}\right)\\&-ix\operatorname{PolyLog}\left(3,-ie^{-x}\right)+ix\operatorname{PolyLog}\left(3,ie^{-x}\right)\\&-i\operatorname{PolyLog}\left(4,-ie^{-x}\right)+i\operatorname{PolyLog}\left(4,ie^{-x}\right)\end{aligned}$$

```
[Out] -1/2*I*x^2*polylog(2,-I/exp(x))+1/2*I*x^2*polylog(2,I/exp(x))-I*x*polylog(3
,-I/exp(x))+I*x*polylog(3,I/exp(x))-I*polylog(4,-I/exp(x))+I*polylog(4,I/ex
p(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5252, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^2 \cot^{-1}(e^x)\,dx =& -\frac{1}{2}ix^2 \operatorname{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}ix^2 \operatorname{PolyLog}\left(2,ie^{-x}\right)\\&-ix\operatorname{PolyLog}\left(3,-ie^{-x}\right)+ix\operatorname{PolyLog}\left(3,ie^{-x}\right)\\&-i\operatorname{PolyLog}\left(4,-ie^{-x}\right)+i\operatorname{PolyLog}\left(4,ie^{-x}\right)\end{aligned}$$

```
[In] Int[x^2*ArcCot[E^x],x]
```

```
[Out] (-1/2*I)*x^2*PolyLog[2, (-I)/E^x] + (I/2)*x^2*PolyLog[2, I/E^x] - I*x*PolyL
og[3, (-I)/E^x] + I*x*PolyLog[3, I/E^x] - I*PolyLog[4, (-I)/E^x] + I*PolyLo
g[4, I/E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}i\int x^2\log\left(1-ie^{-x}\right)\,dx-\frac{1}{2}i\int x^2\log\left(1+ie^{-x}\right)\,dx\\&=-\frac{1}{2}ix^2\,\text{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}ix^2\,\text{PolyLog}\left(2,ie^{-x}\right)\\&\quad+i\int x\,\text{PolyLog}\left(2,-ie^{-x}\right)\,dx-i\int x\,\text{PolyLog}\left(2,ie^{-x}\right)\,dx\end{aligned}$$

$$= -\frac{1}{2}ix^2 \text{PolyLog}\left(2, -ie^{-x}\right) + \frac{1}{2}ix^2 \text{PolyLog}\left(2, ie^{-x}\right) - ix \text{PolyLog}\left(3, -ie^{-x}\right) + ix \text{PolyLog}\left(3, ie^{-x}\right) + i\int \text{PolyLog}\left(3, -ie^{-x}\right) dx - i\int \text{PolyLog}\left(3, ie^{-x}\right) dx$$

$$= -\frac{1}{2}ix^2 \text{PolyLog}\left(2, -ie^{-x}\right) + \frac{1}{2}ix^2 \text{PolyLog}\left(2, ie^{-x}\right) - ix \text{PolyLog}\left(3, -ie^{-x}\right) + ix \text{PolyLog}\left(3, ie^{-x}\right) - i\text{Subst}\left(\int \frac{\text{PolyLog}(3, -ix)}{x} dx, x, e^{-x}\right) + i\text{Subst}\left(\int \frac{\text{PolyLog}(3, ix)}{x} dx, x, e^{-x}\right)$$

$$= -\frac{1}{2}ix^2 \text{PolyLog}\left(2, -ie^{-x}\right) + \frac{1}{2}ix^2 \text{PolyLog}\left(2, ie^{-x}\right) - ix \text{PolyLog}\left(3, -ie^{-x}\right) + ix \text{PolyLog}\left(3, ie^{-x}\right) - i \text{PolyLog}\left(4, -ie^{-x}\right) + i \text{PolyLog}\left(4, ie^{-x}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int x^2 \cot^{-1}\left(e^x\right) dx = -\frac{1}{2}i\left(x^2 \text{PolyLog}\left(2, -ie^{-x}\right) - x^2 \text{PolyLog}\left(2, ie^{-x}\right) + 2\left(x \text{PolyLog}\left(3, -ie^{-x}\right) - x \text{PolyLog}\left(3, ie^{-x}\right) + \text{PolyLog}\left(4, -ie^{-x}\right) - \text{PolyLog}\left(4, ie^{-x}\right)\right)\right)$$

```
[In] Integrate[x^2*ArcCot[E^x],x]
[Out] (-1/2*I)*(x^2*PolyLog[2, (-I)/E^x] - x^2*PolyLog[2, I/E^x] + 2*(x*PolyLog[3
, (-I)/E^x] - x*PolyLog[3, I/E^x] + PolyLog[4, (-I)/E^x] - PolyLog[4, I/E^x
]))
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

method	result
risch	$\frac{\pi x^3}{6} + \frac{ix^2 \text{polylog}(2, ie^x)}{2} - ix \text{polylog}\left(3, ie^x\right) + i \text{polylog}\left(4, ie^x\right) - \frac{ix^2 \text{polylog}(2, -ie^x)}{2} + ix \text{polylog}\left(3, -\right.$

```
[In] int(x^2*arccot(exp(x)),x,method=_RETURNVERBOSE)
[Out] 1/6*Pi*x^3+1/2*I*polylog(2,I*exp(x))*x^2-I*x*polylog(3,I*exp(x))+I*polylog(
4,I*exp(x))-1/2*I*x^2*polylog(2,-I*exp(x))+I*polylog(3,-I*exp(x))*x-I*polyl
og(4,-I*exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int x^2 \cot^{-1}(e^x) \, dx = \frac{1}{3} x^3 \operatorname{arccot}(e^x) - \frac{1}{6} i x^3 \log(i e^x + 1) + \frac{1}{6} i x^3 \log(-i e^x + 1) + \frac{1}{2} i x^2 \mathrm{Li}_2(i e^x) - \frac{1}{2} i x^2 \mathrm{Li}_2(-i e^x) - i x \mathrm{polylog}(3, i e^x) + i x \mathrm{polylog}(3, -i e^x) + i \,\mathrm{polylog}(4, i e^x) - i \,\mathrm{polylog}(4, -i e^x)$$

```
[In] integrate(x^2*arccot(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arccot(e^x) - 1/6*I*x^3*log(I*e^x + 1) + 1/6*I*x^3*log(-I*e^x + 1)
+ 1/2*I*x^2*dilog(I*e^x) - 1/2*I*x^2*dilog(-I*e^x) - I*x*polylog(3, I*e^x)
+ I*x*polylog(3, -I*e^x) + I*polylog(4, I*e^x) - I*polylog(4, -I*e^x)
```

Sympy [F]

$$\int x^2 \cot^{-1}(e^x) \, dx = \int x^2 \operatorname{acot}(e^x) \, dx$$

```
[In] integrate(x**2*acot(exp(x)),x)
```

```
[Out] Integral(x**2*acot(exp(x)), x)
```

Maxima [F]

$$\int x^2 \cot^{-1}(e^x) \, dx = \int x^2 \operatorname{arccot}(e^x) \, dx$$

```
[In] integrate(x^2*arccot(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(e^(-x)) + integrate(1/3*x^3*e^x/(e^(2*x) + 1), x)
```

Giac [F]

$$\int x^2 \cot^{-1}(e^x)\,dx = \int x^2 \operatorname{arccot}(e^x)\,dx$$

```
[In] integrate(x^2*arccot(exp(x)),x, algorithm="giac")
[Out] integrate(x^2*arccot(e^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(e^x)\,dx = \int x^2 \operatorname{acot}(e^x)\,dx$$

```
[In] int(x^2*acot(exp(x)),x)
[Out] int(x^2*acot(exp(x)), x)
```

3.221 $\int \cot^{-1}\left(e^{a+bx}\right) dx$

Optimal result

Integrand size = 8, antiderivative size = 51

$$\int \cot^{-1}\left(e^{a+bx}\right) dx = -\frac{i \operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b} + \frac{i \operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}$$

```
[Out] -1/2*I*polylog(2,-I*exp(-b*x-a))/b+1/2*I*polylog(2,I*exp(-b*x-a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 4941, 2438}

$$\int \cot^{-1}\left(e^{a+bx}\right) dx = \frac{i \operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{2b} - \frac{i \operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}$$

```
[In] Int[ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*PolyLog[2, (-I)*E^(-a - b*x)])/b + ((I/2)*PolyLog[2, I*E^(-a - b*
x)])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, e^{a+bx}\right)}{2b} - \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, e^{a+bx}\right)}{2b}\\ &= -\frac{i\,\text{PolyLog}\left(2, -ie^{-a-bx}\right)}{2b} + \frac{i\,\text{PolyLog}\left(2, ie^{-a-bx}\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx = x\cot^{-1}\left(e^{a+bx}\right) + \frac{i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{2b}$$

```
[In] Integrate[ArcCot[E^(a + b*x)],x]
[Out] x*ArcCot[E^(a + b*x)] + ((I/2)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(
a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(41) = 82.

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{\ln(\mathrm{e}^{bx+a})\operatorname{arccot}(\mathrm{e}^{bx+a})-\frac{i\ln(\mathrm{e}^{bx+a})\ln(1+i\mathrm{e}^{bx+a})}{2}+\frac{i\ln(\mathrm{e}^{bx+a})\ln(1-i\mathrm{e}^{bx+a})}{2}-\frac{i\operatorname{dilog}(1+i\mathrm{e}^{bx+a})}{2}+\frac{i\operatorname{dilog}(1-i\mathrm{e}^{bx+a})}{2}}{b}$
default	$\frac{\ln(\mathrm{e}^{bx+a})\operatorname{arccot}(\mathrm{e}^{bx+a})-\frac{i\ln(\mathrm{e}^{bx+a})\ln(1+i\mathrm{e}^{bx+a})}{2}+\frac{i\ln(\mathrm{e}^{bx+a})\ln(1-i\mathrm{e}^{bx+a})}{2}-\frac{i\operatorname{dilog}(1+i\mathrm{e}^{bx+a})}{2}+\frac{i\operatorname{dilog}(1-i\mathrm{e}^{bx+a})}{2}}{b}$
parts	$x\operatorname{arccot}(\mathrm{e}^{bx+a})+\frac{-\frac{i(bx+a)\ln(1+i\mathrm{e}^{bx+a})}{2}+\frac{i(bx+a)\ln(1-i\mathrm{e}^{bx+a})}{2}-\frac{i\operatorname{dilog}(1+i\mathrm{e}^{bx+a})}{2}+\frac{i\operatorname{dilog}(1-i\mathrm{e}^{bx+a})}{2}-a\arctan}{b}$
risch	$\frac{ix\ln(1+i\mathrm{e}^{bx+a})}{2}+\frac{\pi x}{2}+\frac{i\operatorname{dilog}(1-i\mathrm{e}^{bx+a})}{2b}-\frac{i\ln(-i(-\mathrm{e}^{bx+a}+i))x}{2}+\frac{ia\ln(1+i\mathrm{e}^{bx+a})}{2b}+\frac{i\ln(-i\mathrm{e}^{bx+a})\ln(-i(-}{2b}$

[In] int(arccot(exp(b*x+a)),x,method=_RETURNVERBOSE)

[Out] 1/b*(ln(exp(b*x+a))*arccot(exp(b*x+a))-1/2*I*ln(exp(b*x+a))*ln(1+I*exp(b*x+a))+1/2*I*ln(exp(b*x+a))*ln(1-I*exp(b*x+a))-1/2*I*dilog(1+I*exp(b*x+a))+1/2*I*dilog(1-I*exp(b*x+a)))

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.02

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx = \frac{2\,bx\operatorname{arccot}\left(e^{(bx+a)}\right)-i\,a\log\left(e^{(bx+a)}+i\right)+i\,a\log\left(e^{(bx+a)}-i\right)+(-i\,bx-i\,a)\log\left(i\,e^{(bx+a)}+1\right)+(i\,bx+}{2\,b}$$

[In] integrate(arccot(exp(b*x+a)),x, algorithm="fricas")

[Out] 1/2*(2*b*x*arccot(e^(b*x + a)) - I*a*log(e^(b*x + a) + I) + I*a*log(e^(b*x + a) - I) + (-I*b*x - I*a)*log(I*e^(b*x + a) + 1) + (I*b*x + I*a)*log(-I*e^(b*x + a) + 1) + I*dilog(I*e^(b*x + a)) - I*dilog(-I*e^(b*x + a)))/b

Sympy [F]

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx = \int \operatorname{acot}\left(e^{a+bx}\right)dx$$

[In] integrate(acot(exp(b*x+a)),x)

[Out] Integral(acot(exp(a + b*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx = \frac{(bx+a)\,\mathrm{arccot}\left(e^{(bx+a)}\right)}{b} + \frac{\pi\log\left(e^{(2\,bx+2\,a)}+1\right)+2i\,\mathrm{Li}_2\left(i\,e^{(bx+a)}+1\right)-2i\,\mathrm{Li}_2\left(-i\,e^{(bx+a)}+1\right)}{4\,b}$$

```
[In] integrate(arccot(exp(b*x+a)),x, algorithm="maxima")
[Out] (b*x + a)*arccot(e^(b*x + a))/b + 1/4*(pi*log(e^(2*b*x + 2*a) + 1) + 2*I*di
log(I*e^(b*x + a) + 1) - 2*I*dilog(-I*e^(b*x + a) + 1))/b
```

Giac [F]

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx = \int \mathrm{arccot}\left(e^{(bx+a)}\right)\,dx$$

```
[In] integrate(arccot(exp(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot(e^(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx = \int \mathrm{acot}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

```
[In] int(acot(exp(a + b*x)),x)
[Out] int(acot(exp(a + b*x)), x)
```

3.222 $\int x\cot^{-1}\left(e^{a+bx}\right)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 103

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=-\frac{ix\,\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{ix\,\text{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}-\frac{i\,\text{PolyLog}\left(3,-ie^{-a-bx}\right)}{2b^2}+\frac{i\,\text{PolyLog}\left(3,ie^{-a-bx}\right)}{2b^2}$$

```
[Out] -1/2*I*x*polylog(2,-I*exp(-b*x-a))/b+1/2*I*x*polylog(2,I*exp(-b*x-a))/b-1/2
*I*polylog(3,-I*exp(-b*x-a))/b^2+1/2*I*polylog(3,I*exp(-b*x-a))/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5252, 2611, 2320, 6724}

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=-\frac{i\,\text{PolyLog}\left(3,-ie^{-a-bx}\right)}{2b^2}+\frac{i\,\text{PolyLog}\left(3,ie^{-a-bx}\right)}{2b^2}-\frac{ix\,\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{ix\,\text{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}$$

```
[In] Int[x*ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*x*PolyLog[2, (-I)*E^(-a - b*x)])/b + ((I/2)*x*PolyLog[2, I*E^(-a
- b*x)])/b - ((I/2)*PolyLog[3, (-I)*E^(-a - b*x)])/b^2 + ((I/2)*PolyLog[3,
I*E^(-a - b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int x\log\left(1-ie^{-a-bx}\right)\,dx-\frac{1}{2}i\int x\log\left(1+ie^{-a-bx}\right)\,dx\\
&= -\frac{ix\,\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{ix\,\text{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}\\
&\quad+\frac{i\int\text{PolyLog}\left(2,-ie^{-a-bx}\right)\,dx}{2b}-\frac{i\int\text{PolyLog}\left(2,ie^{-a-bx}\right)\,dx}{2b}\\
&= -\frac{ix\,\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{ix\,\text{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}\\
&\quad-\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{-a-bx}\right)}{2b^2}+\frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{-a-bx}\right)}{2b^2}\\
&= -\frac{ix\,\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{ix\,\text{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}\\
&\quad-\frac{i\,\text{PolyLog}\left(3,-ie^{-a-bx}\right)}{2b^2}+\frac{i\,\text{PolyLog}\left(3,ie^{-a-bx}\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx = -\frac{i\left(bx\,\text{PolyLog}\left(2,-ie^{-a-bx}\right)-bx\,\text{PolyLog}\left(2,ie^{-a-bx}\right)+\text{PolyLog}\left(3,-ie^{-a-bx}\right)-\text{PolyLog}\left(3,ie^{-a-bx}\right)\right)}{2b^2}$$

```
[In] Integrate[x*ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*(b*x*PolyLog[2, (-I)*E^(-a - b*x)] - b*x*PolyLog[2, I*E^(-a - b*x
)] + PolyLog[3, (-I)*E^(-a - b*x)] - PolyLog[3, I*E^(-a - b*x)]))/b^2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(83) = 166.

Time = 0.80 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.45

method	result
risch	$-\frac{i\,\text{polylog}(3,i\mathrm{e}^{bx+a})}{2b^2}+\frac{\pi x^2}{4}+\frac{i\,\text{polylog}(3,-i\mathrm{e}^{bx+a})}{2b^2}-\frac{i\ln(-i(\mathrm{e}^{bx+a}+i))a^2}{2b^2}+\frac{ia^2\ln(1-i\mathrm{e}^{bx+a})}{2b^2}+\frac{ix\,\text{polylog}(2,i\mathrm{e}^{bx+a})}{2b}+$

```
[In] int(x*arccot(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^2*polylog(3,I*exp(b*x+a))+1/4*Pi*x^2+1/2*I/b^2*polylog(3,-I*exp(b*
x+a))-1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*a^2+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a))+
1/2*I/b*polylog(2,I*exp(b*x+a))*x+1/2*I/b*ln(1-I*exp(b*x+a))*a*x-1/2*I/b^2*
dilog(-I*(exp(b*x+a)+I))*a-1/2*I/b^2*polylog(2,-I*exp(b*x+a))*a+1/2*I/b^2*p
olylog(2,I*exp(b*x+a))*a-1/2*I/b*ln(1+I*exp(b*x+a))*a*x-1/2*I/b^2*dilog(-I*
exp(b*x+a))*a+1/2*I/b*ln(-I*(-exp(b*x+a)+I))*a*x-1/2*I/b*ln(-I*(exp(b*x+a)+
I))*a*x-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a))-1/2*I/b^2*ln(-I*exp(b*x+a))*ln(-I*
(-exp(b*x+a)+I))*a-1/2*I/b*polylog(2,-I*exp(b*x+a))*x+1/2*I/b^2*ln(-I*(-exp
(b*x+a)+I))*a^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(73) = 146$.

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.47

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx$$
$$= \frac{2\,b^2x^2\,\mathrm{arccot}\left(e^{(bx+a)}\right) + 2i\,bx\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) - 2i\,bx\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right) + i\,a^2\log\left(e^{(bx+a)} + i\right) - i\,a^2\log\left(e^{(bx+a)}\right.}{}$$

```
[In] integrate(x*arccot(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arccot(e^(b*x + a)) + 2*I*b*x*dilog(I*e^(b*x + a)) - 2*I*b*x
*dilog(-I*e^(b*x + a)) + I*a^2*log(e^(b*x + a) + I) - I*a^2*log(e^(b*x + a)
 - I) + (-I*b^2*x^2 + I*a^2)*log(I*e^(b*x + a) + 1) + (I*b^2*x^2 - I*a^2)*l
og(-I*e^(b*x + a) + 1) - 2*I*polylog(3, I*e^(b*x + a)) + 2*I*polylog(3, -I*
e^(b*x + a)))/b^2
```

Sympy [F]

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx = \int x\,\mathrm{acot}\left(e^ae^{bx}\right)dx$$

```
[In] integrate(x*acot(exp(b*x+a)),x)
```

```
[Out] Integral(x*acot(exp(a)*exp(b*x)), x)
```

Maxima [F]

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx = \int x\,\mathrm{arccot}\left(e^{(bx+a)}\right)\,dx$$

```
[In] integrate(x*arccot(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(e^(-b*x - a)) + b*integrate(1/2*x^2*e^(b*x + a)/(e^(2*b*x +
2*a) + 1), x)
```

Giac [F]

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=\int x\,\mathrm{arccot}\left(e^{(bx+a)}\right)\,dx$$

```
[In] integrate(x*arccot(exp(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccot(e^(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=\int x\,\mathrm{acot}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

```
[In] int(x*acot(exp(a + b*x)),x)
[Out] int(x*acot(exp(a + b*x)), x)
```

3.223 $\int x^2 \cot^{-1}\left(e^{a+bx}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 151

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right) dx = -\frac{ix^2 \operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b} + \frac{ix^2 \operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{2b} - \frac{ix \operatorname{PolyLog}\left(3,-ie^{-a-bx}\right)}{b^2} + \frac{ix \operatorname{PolyLog}\left(3,ie^{-a-bx}\right)}{b^2} - \frac{i \operatorname{PolyLog}\left(4,-ie^{-a-bx}\right)}{b^3} + \frac{i \operatorname{PolyLog}\left(4,ie^{-a-bx}\right)}{b^3}$$

```
[Out] -1/2*I*x^2*polylog(2,-I*exp(-b*x-a))/b+1/2*I*x^2*polylog(2,I*exp(-b*x-a))/b
-I*x*polylog(3,-I*exp(-b*x-a))/b^2+I*x*polylog(3,I*exp(-b*x-a))/b^2-I*polyl
og(4,-I*exp(-b*x-a))/b^3+I*polylog(4,I*exp(-b*x-a))/b^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5252, 2611, 6744, 2320, 6724}

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right) dx = -\frac{i \operatorname{PolyLog}\left(4,-ie^{-a-bx}\right)}{b^3} + \frac{i \operatorname{PolyLog}\left(4,ie^{-a-bx}\right)}{b^3} - \frac{ix \operatorname{PolyLog}\left(3,-ie^{-a-bx}\right)}{b^2} + \frac{ix \operatorname{PolyLog}\left(3,ie^{-a-bx}\right)}{b^2} - \frac{ix^2 \operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b} + \frac{ix^2 \operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}$$

```
[In] Int[x^2*ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*x^2*PolyLog[2, (-I)*E^(-a - b*x)])/b + ((I/2)*x^2*PolyLog[2, I*E^
(-a - b*x)])/b - (I*x*PolyLog[3, (-I)*E^(-a - b*x)])/b^2 + (I*x*PolyLog[3,
```

```
I*E^(-a - b*x)])/b^2 - (I*PolyLog[4, (-I)*E^(-a - b*x)])/b^3 + (I*PolyLog[4
, I*E^(-a - b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}i\int x^2\log\left(1-ie^{-a-bx}\right)\,dx-\frac{1}{2}i\int x^2\log\left(1+ie^{-a-bx}\right)\,dx\\ &= -\frac{ix^2\operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{ix^2\operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}\\ &\quad+\frac{i\int x\operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)\,dx}{b}-\frac{i\int x\operatorname{PolyLog}\left(2,ie^{-a-bx}\right)\,dx}{b}\end{aligned}$$

$$= -\frac{ix^2 \operatorname{PolyLog}(2, -ie^{-a-bx})}{2b} + \frac{ix^2 \operatorname{PolyLog}(2, ie^{-a-bx})}{2b} - \frac{ix \operatorname{PolyLog}(3, -ie^{-a-bx})}{b^2} + \frac{ix \operatorname{PolyLog}(3, ie^{-a-bx})}{b^2} + \frac{i \int \operatorname{PolyLog}(3, -ie^{-a-bx})\, dx}{b^2} - \frac{i \int \operatorname{PolyLog}(3, ie^{-a-bx})\, dx}{b^2}$$

$$= -\frac{ix^2 \operatorname{PolyLog}(2, -ie^{-a-bx})}{2b} + \frac{ix^2 \operatorname{PolyLog}(2, ie^{-a-bx})}{2b} - \frac{ix \operatorname{PolyLog}(3, -ie^{-a-bx})}{b^2} + \frac{ix \operatorname{PolyLog}(3, ie^{-a-bx})}{b^2} - \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-ix)}{x}\, dx, x, e^{-a-bx}\right)}{b^3} + \frac{i\text{Subst}\left(\int \frac{\operatorname{PolyLog}(3,ix)}{x}\, dx, x, e^{-a-bx}\right)}{b^3}$$

$$= -\frac{ix^2 \operatorname{PolyLog}(2, -ie^{-a-bx})}{2b} + \frac{ix^2 \operatorname{PolyLog}(2, ie^{-a-bx})}{2b} - \frac{ix \operatorname{PolyLog}(3, -ie^{-a-bx})}{b^2} + \frac{ix \operatorname{PolyLog}(3, ie^{-a-bx})}{b^2} - \frac{i \operatorname{PolyLog}(4, -ie^{-a-bx})}{b^3} + \frac{i \operatorname{PolyLog}(4, ie^{-a-bx})}{b^3}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right) dx = -\frac{i\left(b^2x^2 \operatorname{PolyLog}(2, -ie^{-a-bx}) - b^2x^2 \operatorname{PolyLog}(2, ie^{-a-bx}) + 2\left(bx \operatorname{PolyLog}(3, -ie^{-a-bx}) - bx \operatorname{PolyLog}\right.\right.}{2b^3}$$

```
[In] Integrate[x^2*ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*(b^2*x^2*PolyLog[2, (-I)*E^(-a - b*x)] - b^2*x^2*PolyLog[2, I*E^(
-a - b*x)] + 2*(b*x*PolyLog[3, (-I)*E^(-a - b*x)] - b*x*PolyLog[3, I*E^(-a
- b*x)] + PolyLog[4, (-I)*E^(-a - b*x)] - PolyLog[4, I*E^(-a - b*x)])))/b^3
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(129) = 258.

Time = 0.80 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.74

method	result
risch	$\frac{i\ln(-i(e^{bx+a}+i))a^3}{2b^3} + \frac{\pi x^3}{6} + \frac{i\ln(1+ie^{bx+a})a^2x}{2b^2} - \frac{i\,\mathrm{polylog}(3,ie^{bx+a})x}{b^2} - \frac{i\ln(1-ie^{bx+a})a^3}{2b^3} + \frac{i\,\mathrm{polylog}(3,-ie^{bx+a})x}{b^2} +$

```
[In] int(x^2*arccot(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

[Out] 1/2*I/b^3*ln(-I*(exp(b*x+a)+I))*a^3+1/6*Pi*x^3+1/2*I/b^2*ln(1+I*exp(b*x+a))
*a^2*x-I/b^2*polylog(3,I*exp(b*x+a))*x-1/2*I/b^3*ln(1-I*exp(b*x+a))*a^3+I/b
^2*polylog(3,-I*exp(b*x+a))*x+I/b^3*polylog(4,I*exp(b*x+a))+1/2*I/b^2*ln(-I
*(exp(b*x+a)+I))*x*a^2-1/2*I/b^2*ln(1-I*exp(b*x+a))*x*a^2-1/2*I/b*polylog(2
,-I*exp(b*x+a))*x^2+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a))-1/2*I/b^2*ln(-I*(-exp(
b*x+a)+I))*a^2*x-1/2*I/b^3*ln(-I*(-exp(b*x+a)+I))*a^3+1/2*I/b^3*polylog(2,-
I*exp(b*x+a))*a^2-1/2*I/b^3*polylog(2,I*exp(b*x+a))*a^2+1/2*I/b^3*dilog(-I*
exp(b*x+a))*a^2-I*polylog(4,-I*exp(b*x+a))/b^3+1/2*I/b^3*dilog(-I*(exp(b*x+
a)+I))*a^2+1/2*I/b*polylog(2,I*exp(b*x+a))*x^2+1/2*I/b^3*ln(-I*exp(b*x+a))*
ln(-I*(-exp(b*x+a)+I))*a^2

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.24

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right)\,dx = \frac{2\,b^3x^3\,\mathrm{arccot}\left(e^{(bx+a)}\right) + 3i\,b^2x^2\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) - 3i\,b^2x^2\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right) - i\,a^3\log\left(e^{(bx+a)}+i\right) + i\,a^3\log\left(e^{(bx+}\right.}{}$$

[In] integrate(x^2*arccot(exp(b*x+a)),x, algorithm="fricas")

[Out] 1/6*(2*b^3*x^3*arccot(e^(b*x + a)) + 3*I*b^2*x^2*dilog(I*e^(b*x + a)) - 3*I
*b^2*x^2*dilog(-I*e^(b*x + a)) - I*a^3*log(e^(b*x + a) + I) + I*a^3*log(e^(
b*x + a) - I) - 6*I*b*x*polylog(3, I*e^(b*x + a)) + 6*I*b*x*polylog(3, -I*e
^(b*x + a)) + (-I*b^3*x^3 - I*a^3)*log(I*e^(b*x + a) + 1) + (I*b^3*x^3 + I*
a^3)*log(-I*e^(b*x + a) + 1) + 6*I*polylog(4, I*e^(b*x + a)) - 6*I*polylog(
4, -I*e^(b*x + a)))/b^3

Sympy [F]

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right)\,dx = \int x^2\,\mathrm{acot}\left(e^a e^{bx}\right)dx$$

[In] integrate(x**2*acot(exp(b*x+a)),x)

[Out] Integral(x**2*acot(exp(a)*exp(b*x)), x)

Maxima [F]

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right)\,dx = \int x^2 \operatorname{arccot}\left(e^{(bx+a)}\right)\,dx$$

[In] integrate(x^2*arccot(exp(b*x+a)),x, algorithm="maxima")

[Out] 1/3*x^3*arctan(e^(-b*x - a)) + b*integrate(1/3*x^3*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)

Giac [F]

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right)\,dx = \int x^2 \operatorname{arccot}\left(e^{(bx+a)}\right)\,dx$$

[In] integrate(x^2*arccot(exp(b*x+a)),x, algorithm="giac")

[Out] integrate(x^2*arccot(e^(b*x + a)), x)

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right)\,dx = \int x^2 \operatorname{acot}\left(e^{a+b\,x}\right)\,dx$$

[In] int(x^2*acot(exp(a + b*x)),x)

[Out] int(x^2*acot(exp(a + b*x)), x)

3.224 $\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 196

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=-\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}-\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bf^{c+dx})}\right)}{2d\log(f)}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{2d\log(f)}$$

```
[Out] -arccot(a+b*f^(d*x+c))*ln(2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+arccot(a+b*f^(d*
x+c))*ln(2*b*f^(d*x+c)/(I-a)/(1-I*(a+b*f^(d*x+c))))/d/ln(f)-1/2*I*polylog(2
,1-2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+1/2*I*polylog(2,1-2*b*f^(d*x+c)/(I-a)/(
1-I*(a+b*f^(d*x+c))))/d/ln(f)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{2320, 5156, 4967, 2449, 2352, 2497\}$

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx = -\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(bf^{c+dx}+a)}\right)}{2d\log(f)} + \frac{i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(bf^{c+dx}+a))}\right)}{2d\log(f)} - \frac{\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)\cot^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)} + \frac{\log\left(\frac{2bf^{c+dx}}{(-a+i)(1-i(a+bf^{c+dx}))}\right)\cot^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}$$

```
[In] Int[ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcCot[a + b*f^(c + d*x)]*Log[2/(1 - I*(a + b*f^(c + d*x)))])/(d*Log[f])
) + (ArcCot[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b
*f^(c + d*x))))])/(d*Log[f]) - ((I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*f^(c +
 d*x)))])/(d*Log[f]) + ((I/2)*PolyLog[2, 1 - (2*b*f^(c + d*x))/((I - a)*(1
- I*(a + b*f^(c + d*x))))])/(d*Log[f])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(a+bx)}{x}\,dx, x, f^{c+dx}\right)}{d\log(f)}\\
&= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bf^{c+dx}\right)}{bd\log(f)}\\
&= -\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}\\
&\quad -\frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1-ix}\right)}{1+x^2}\,dx, x, a+bf^{c+dx}\right)}{d\log(f)}+\frac{\text{Subst}\left(\int \frac{\log\left(\frac{2\left(-\frac{a}{b}+\frac{x}{b}\right)}{\left(\frac{i}{b}-\frac{a}{b}\right)(1-ix)}\right)}{1+x^2}\,dx, x, a+bf^{c+dx}\right)}{d\log(f)}\\
&= -\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}\\
&\quad +\frac{i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{2d\log(f)}-\frac{i\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}\\
&= -\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}\\
&\quad -\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bf^{c+dx})}\right)}{2d\log(f)}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{2d\log(f)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.85

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx = x\cot^{-1}\left(a+bf^{c+dx}\right) + \frac{b\Big(dx\log(f)\left(\log\left(1+\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\log\left(1+\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)+\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\text{PolyLog}\left(2,-\frac{b^2f^{c\cdots}}{ab+\sqrt{\cdots}}\right.}{2\sqrt{-b^2}d\log(f)}$$

```
[In] Integrate[ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] x*ArcCot[a + b*f^(c + d*x)] + (b*(d*x*Log[f]*(Log[1 + (b^2*f^(c + d*x))/(a*
b - Sqrt[-b^2])] - Log[1 + (b^2*f^(c + d*x))/(a*b + Sqrt[-b^2])]) + PolyLog
[2, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] - PolyLog[2, -((b^2*f^(c + d*x
))/(a*b + Sqrt[-b^2]))]))/(2*Sqrt[-b^2]*d*Log[f])
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arccot}\left(a+b\,f^{dx+c}\right)+\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i+b\,f^{dx+c}+a}{i+a}\right)}{2}-\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i-b\,f^{dx+c}-a}{i-a}\right)}{2}+\frac{i\,\text{dilog}\left(\frac{i+b\,f^{dx\cdots}}{i+\cdots}\right.}{2}}{d\ln(f)}$
default	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arccot}\left(a+b\,f^{dx+c}\right)+\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i+b\,f^{dx+c}+a}{i+a}\right)}{2}-\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i-b\,f^{dx+c}-a}{i-a}\right)}{2}+\frac{i\,\text{dilog}\left(\frac{i+b\,f^{dx\cdots}}{i+\cdots}\right.}{2}}{d\ln(f)}$
risch	$\frac{ix\ln\left(1+i\left(a+b\,f^{dx+c}\right)\right)}{2}+\frac{\pi x}{2}-\frac{i\ln\left(-ib\,f^{dx+c}-ia+1\right)\ln\left(-\frac{if^{dx+c}b}{ia-1}\right)}{2d\ln(f)}-\frac{i\,\text{dilog}\left(-\frac{if^{dx+c}b}{ia-1}\right)}{2d\ln(f)}-\frac{i\,\text{dilog}\left(\frac{b\,f^{dx}f^{c\cdots}}{a-\cdots}\right.}{2\ln(f)d\cdots}$

```
[In] int(arccot(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/ln(f)*(ln(-b*f^(d*x+c))*arccot(a+b*f^(d*x+c))+1/2*I*ln(-b*f^(d*x+c))*ln
((I+b*f^(d*x+c)+a)/(I+a))-1/2*I*ln(-b*f^(d*x+c))*ln((I-b*f^(d*x+c)-a)/(I-a)
)+1/2*I*dilog((I+b*f^(d*x+c)+a)/(I+a))-1/2*I*dilog((I-b*f^(d*x+c)-a)/(I-a))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{2\,dx\,\mathrm{arccot}\left(bf^{dx+c}+a\right)\log\left(f\right)-i\,c\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)+i\,c\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)+\left(-i\,dx\right.}{}$$

```
[In] integrate(arccot(a+b*f^(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(2*d*x*arccot(b*f^(d*x + c) + a)*log(f) - I*c*log(b*f^(d*x + c) + a + I
)*log(f) + I*c*log(b*f^(d*x + c) + a - I)*log(f) + (-I*d*x - I*c)*log(f)*lo
g((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (I*d*x + I*c)*log(f)*log
((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1)) - I*dilog(-(a^2 + (a*b + I*
b)*f^(d*x + c) + 1)/(a^2 + 1) + 1) + I*dilog(-(a^2 + (a*b - I*b)*f^(d*x + c
) + 1)/(a^2 + 1) + 1))/(d*log(f))
```

Sympy [F]

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int \mathrm{acot}\left(a+bf^{c+dx}\right)dx$$

```
[In] integrate(acot(a+b*f**(d*x+c)),x)
[Out] Integral(acot(a + b*f**(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\frac{\left(dx+c\right)\mathrm{arccot}\left(bf^{dx+c}+a\right)}{d}$$
$$+\frac{2\left(dx+c\right)\arctan\left(\frac{b^2f^{dx+c}+ab}{b}\right)\log\left(f\right)+\left(\pi-\arctan\left(\frac{1}{a}\right)\right)\log\left(b^2f^{2\,dx+2\,c}+2\,abf^{dx+c}+a^2+1\right)-\arctan}{2\,d\log\left(f\right)}$$

```
[In] integrate(arccot(a+b*f^(d*x+c)),x, algorithm="maxima")
[Out] (d*x + c)*arccot(b*f^(d*x + c) + a)/d + 1/2*(2*(d*x + c)*arctan((b^2*f^(d*x
 + c) + a*b)/b)*log(f) + (pi - arctan(1/a))*log(b^2*f^(2*d*x + 2*c) + 2*a*b
*f^(d*x + c) + a^2 + 1) - arctan(b*f^(d*x + c) + a)*log(b^2*f^(2*d*x + 2*c)
/(a^2 + 1)) + I*dilog((I*b*f^(d*x + c) + I*a + 1)/(I*a + 1)) - I*dilog((I*b
*f^(d*x + c) + I*a - 1)/(I*a - 1)))/(d*log(f))
```

Giac [F]

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int \operatorname{arccot}\left(bf^{dx+c}+a\right)\,dx$$

```
[In] integrate(arccot(a+b*f^(d*x+c)),x, algorithm="giac")
[Out] integrate(arccot(b*f^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int \mathrm{acot}\left(a+b\,f^{c+d\,x}\right)\,dx$$

```
[In] int(acot(a + b*f^(c + d*x)),x)
[Out] int(acot(a + b*f^(c + d*x)), x)
```

3.225 $\int x \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 250

$$\begin{aligned}\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx = &-\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)\\&+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\&-\frac{ix\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}\\&+\frac{i\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{2d^2\log^2(f)}-\frac{i\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{2d^2\log^2(f)}\end{aligned}$$

```
[Out] -1/4*I*x^2*ln(1-b*f^(d*x+c)/(I-a))+1/4*I*x^2*ln(1+b*f^(d*x+c)/(I+a))+1/4*I*
x^2*ln(1-I/(a+b*f^(d*x+c)))-1/4*I*x^2*ln(1+I/(a+b*f^(d*x+c)))-1/2*I*x*polyl
og(2,b*f^(d*x+c)/(I-a))/d/ln(f)+1/2*I*x*polylog(2,-b*f^(d*x+c)/(I+a))/d/ln(
f)+1/2*I*polylog(3,b*f^(d*x+c)/(I-a))/d^2/ln(f)^2-1/2*I*polylog(3,-b*f^(d*x
+c)/(I+a))/d^2/ln(f)^2
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

$= \{5252, 2631, 12, 6874, 2221, 2611, 2320, 6724\}$

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx = \frac{i\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{2d^2\log^2(f)} - \frac{i\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{2d^2\log^2(f)} - \frac{ix\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)} + \frac{ix\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{2d\log(f)} - \frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{-a+i}\right) + \frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{a+i}\right) + \frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right) - \frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)$$

```
[In] Int[x*ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] (-1/4*I)*x^2*Log[1 - (b*f^(c + d*x))/(I - a)] + (I/4)*x^2*Log[1 + (b*f^(c +
 d*x))/(I + a)] + (I/4)*x^2*Log[1 - I/(a + b*f^(c + d*x))] - (I/4)*x^2*Log[
1 + I/(a + b*f^(c + d*x))] - ((I/2)*x*PolyLog[2, (b*f^(c + d*x))/(I - a)])/
(d*Log[f]) + ((I/2)*x*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) +
((I/2)*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) - ((I/2)*PolyLog
[3, -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}i\int x\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\,dx-\frac{1}{2}i\int x\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\,dx\\
&= \frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\
&\quad+\frac{1}{4}\int\frac{bdf^{c+dx}x^2\log(f)}{\left(i(1-ia)+bf^{c+dx}\right)\left(a+bf^{c+dx}\right)}\,dx\\
&\quad+\frac{1}{4}\int\frac{bdf^{c+dx}x^2\log(f)}{\left(-i(1+ia)+bf^{c+dx}\right)\left(a+bf^{c+dx}\right)}\,dx\\
&= \frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\
&\quad+\frac{1}{4}(bd\log(f))\int\frac{f^{c+dx}x^2}{\left(i(1-ia)+bf^{c+dx}\right)\left(a+bf^{c+dx}\right)}\,dx\\
&\quad+\frac{1}{4}(bd\log(f))\int\frac{f^{c+dx}x^2}{\left(-i(1+ia)+bf^{c+dx}\right)\left(a+bf^{c+dx}\right)}\,dx
\end{aligned}
$$

$$= \frac{1}{4} i x^2 \log \left(1 - \frac{i}{a + b f^{c+dx}}\right) - \frac{1}{4} i x^2 \log \left(1 + \frac{i}{a + b f^{c+dx}}\right) + \frac{1}{4}(bd \log(f)) \int \left(\frac{i f^{c+dx} x^2}{a + b f^{c+dx}} - \frac{i f^{c+dx} x^2}{-i + a + b f^{c+dx}}\right) dx + \frac{1}{4}(bd \log(f)) \int \left(-\frac{i f^{c+dx} x^2}{a + b f^{c+dx}} + \frac{i f^{c+dx} x^2}{i + a + b f^{c+dx}}\right) dx$$

$$= \frac{1}{4} i x^2 \log \left(1 - \frac{i}{a + b f^{c+dx}}\right) - \frac{1}{4} i x^2 \log \left(1 + \frac{i}{a + b f^{c+dx}}\right) - \frac{1}{4}(ibd \log(f)) \int \frac{f^{c+dx} x^2}{-i + a + b f^{c+dx}} dx + \frac{1}{4}(ibd \log(f)) \int \frac{f^{c+dx} x^2}{i + a + b f^{c+dx}} dx$$

$$= -\frac{1}{4} i x^2 \log \left(1 - \frac{b f^{c+dx}}{i - a}\right) + \frac{1}{4} i x^2 \log \left(1 + \frac{b f^{c+dx}}{i + a}\right) + \frac{1}{4} i x^2 \log \left(1 - \frac{i}{a + b f^{c+dx}}\right) - \frac{1}{4} i x^2 \log \left(1 + \frac{i}{a + b f^{c+dx}}\right) + \frac{1}{2} i \int x \log \left(1 + \frac{b f^{c+dx}}{-i + a}\right) dx - \frac{1}{2} i \int x \log \left(1 + \frac{b f^{c+dx}}{i + a}\right) dx$$

$$= -\frac{1}{4} i x^2 \log \left(1 - \frac{b f^{c+dx}}{i - a}\right) + \frac{1}{4} i x^2 \log \left(1 + \frac{b f^{c+dx}}{i + a}\right) + \frac{1}{4} i x^2 \log \left(1 - \frac{i}{a + b f^{c+dx}}\right) - \frac{1}{4} i x^2 \log \left(1 + \frac{i}{a + b f^{c+dx}}\right) - \frac{i x \operatorname{PolyLog}\left(2, \frac{b f^{c+dx}}{i-a}\right)}{2d \log(f)} + \frac{i x \operatorname{PolyLog}\left(2, -\frac{b f^{c+dx}}{i+a}\right)}{2d \log(f)} + \frac{i \int \operatorname{PolyLog}\left(2, -\frac{b f^{c+dx}}{-i+a}\right) dx}{2d \log(f)} - \frac{i \int \operatorname{PolyLog}\left(2, -\frac{b f^{c+dx}}{i+a}\right) dx}{2d \log(f)}$$

$$= -\frac{1}{4} i x^2 \log \left(1 - \frac{b f^{c+dx}}{i - a}\right) + \frac{1}{4} i x^2 \log \left(1 + \frac{b f^{c+dx}}{i + a}\right) + \frac{1}{4} i x^2 \log \left(1 - \frac{i}{a + b f^{c+dx}}\right) - \frac{1}{4} i x^2 \log \left(1 + \frac{i}{a + b f^{c+dx}}\right) - \frac{i x \operatorname{PolyLog}\left(2, \frac{b f^{c+dx}}{i-a}\right)}{2d \log(f)} + \frac{i x \operatorname{PolyLog}\left(2, -\frac{b f^{c+dx}}{i+a}\right)}{2d \log(f)} + \frac{i \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, \frac{bx}{i-a}\right)}{x} dx, x, f^{c+dx}\right)}{2d^2 \log^2(f)} - \frac{i \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, -\frac{bx}{i+a}\right)}{x} dx, x, f^{c+dx}\right)}{2d^2 \log^2(f)}$$

$$= -\frac{1}{4} i x^2 \log \left(1 - \frac{b f^{c+dx}}{i - a}\right) + \frac{1}{4} i x^2 \log \left(1 + \frac{b f^{c+dx}}{i + a}\right) + \frac{1}{4} i x^2 \log \left(1 - \frac{i}{a + b f^{c+dx}}\right) - \frac{1}{4} i x^2 \log \left(1 + \frac{i}{a + b f^{c+dx}}\right) - \frac{i x \operatorname{PolyLog}\left(2, \frac{b f^{c+dx}}{i-a}\right)}{2d \log(f)} + \frac{i x \operatorname{PolyLog}\left(2, -\frac{b f^{c+dx}}{i+a}\right)}{2d \log(f)} + \frac{i \operatorname{PolyLog}\left(3, \frac{b f^{c+dx}}{i-a}\right)}{2d^2 \log^2(f)} - \frac{i \operatorname{PolyLog}\left(3, -\frac{b f^{c+dx}}{i+a}\right)}{2d^2 \log^2(f)}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=-\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{ix\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}+\frac{i\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{2d^2\log^2(f)}-\frac{i\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{2d^2\log^2(f)}$$

```
[In] Integrate[x*ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] (-1/4*I)*x^2*Log[1 - (b*f^(c + d*x))/(I - a)] + (I/4)*x^2*Log[1 + (b*f^(c +
 d*x))/(I + a)] + (I/4)*x^2*Log[1 - I/(a + b*f^(c + d*x))] - (I/4)*x^2*Log[
1 + I/(a + b*f^(c + d*x))] - ((I/2)*x*PolyLog[2, (b*f^(c + d*x))/(I - a)])/
(d*Log[f]) + ((I/2)*x*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) +
((I/2)*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) - ((I/2)*PolyLog
[3, -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(218) = 436.

Time = 1.04 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.71

method	result
risch	$-\frac{ix^2\ln\left(1-i\left(a+b\,f^{dx+c}\right)\right)}{4}+\frac{\pi\,x^2}{4}-\frac{ic\ln\left(\frac{b\,f^{dx}\,f^{c}+a+i}{i+a}\right)x}{2d}-\frac{i\,\text{polylog}\left(2,\frac{ib\,f^{dx}\,f^{c}}{-ia-1}\right)c}{2\ln(f)d^2}+\frac{ic\ln\left(\frac{b\,f^{dx}\,f^{c}+a-i}{a-i}\right)x}{2d}+\frac{ic^2\ln(1-ia-}{4d}$

```
[In] int(x*arccot(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*x^2*ln(1-I*(a+b*f^(d*x+c)))+1/4*Pi*x^2-1/2*I/d*c*ln((b*f^(d*x)*f^c+a
+I)/(I+a))*x-1/2*I/ln(f)/d^2*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c+1/2*I/d*
c*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x+1/4*I/d^2*c^2*ln(1-I*a-I*f^(d*x)*f^c*b)-1
/2*I/ln(f)/d^2*c*dilog((b*f^(d*x)*f^c+a+I)/(I+a))+1/4*I*x^2*ln(1+I*(a+b*f^(
d*x+c)))+1/2*I/ln(f)/d^2*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c-1/4*I*ln(1-I*
b/(-I*a-1)*f^(d*x)*f^c)*x^2-1/4*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*c^2+1/
2*I/d*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c*x+1/2*I/ln(f)^2/d^2*polylog(3,I*b/(-I
*a-1)*f^(d*x)*f^c)+1/2*I/ln(f)/d*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*x-1/4*I
/d^2*c^2*ln(I*f^(d*x)*f^c*b+I*a+1)-1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a+I)/(I+
```

```
a))+1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(a-I))-1/2*I/ln(f)/d*polylog(2,I*b
/(-I*a-1)*f^(d*x)*f^c)*x-1/2*I/ln(f)^2/d^2*polylog(3,I*b/(1-I*a)*f^(d*x)*f^
c)-1/2*I/d*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*c*x+1/4*I/d^2*ln(1-I*b/(1-I*a)*f^
(d*x)*f^c)*c^2+1/4*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^2+1/2*I/ln(f)/d^2*c*di
log((b*f^(d*x)*f^c+a-I)/(a-I))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.22

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx$$

$$= \frac{2\,d^2x^2\operatorname{arccot}\left(bf^{dx+c}+a\right)\log\left(f\right)^2+i\,c^2\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)^2-i\,c^2\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)^2-}{}$$

```
[In] integrate(x*arccot(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*x^2*arccot(b*f^(d*x + c) + a)*log(f)^2 + I*c^2*log(b*f^(d*x + c)
 + a + I)*log(f)^2 - I*c^2*log(b*f^(d*x + c) + a - I)*log(f)^2 - 2*I*d*x*di
log(-(a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + 2*I*d*x*di
log(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + (-I*d^2*x^
2 + I*c^2)*log(f)^2*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (I
*d^2*x^2 - I*c^2)*log(f)^2*log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1
)) + 2*I*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) - 2*I*polylog(3, -(
a*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^2*log(f)^2)
```

Sympy [F]

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\operatorname{acot}\left(a+bf^{c+dx}\right)dx$$

```
[In] integrate(x*acot(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(x*acot(a + b*f**(c + d*x)), x)
```

Maxima [F]

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{arccot}\left(bf^{dx+c}+a\right)\,dx$$

```
[In] integrate(x*arccot(a+b*f^(d*x+c)),x, algorithm="maxima")
[Out] b*d*f^c*integrate(1/2*f^(d*x)*x^2/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f^
c + a^2 + 1), x)*log(f) + 1/2*x^2*arctan(1/(b*f^(d*x)*f^c + a))
```

Giac [F]

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{arccot}\left(bf^{dx+c}+a\right)\,dx$$

```
[In] integrate(x*arccot(a+b*f^(d*x+c)),x, algorithm="giac")
[Out] integrate(x*arccot(b*f^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{acot}\left(a+b\,f^{c+d\,x}\right)\,dx$$

```
[In] int(x*acot(a + b*f^(c + d*x)),x)
[Out] int(x*acot(a + b*f^(c + d*x)), x)
```

3.226 $\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 313

$$\begin{aligned}\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx = & -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)\\ & +\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\ & -\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}\\ & +\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)}\\ & -\frac{i\operatorname{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)}+\frac{i\operatorname{PolyLog}\left(4,-\frac{bf^{c+dx}}{i+a}\right)}{d^3\log^3(f)}\end{aligned}$$

```
[Out] -1/6*I*x^3*ln(1-b*f^(d*x+c)/(I-a))+1/6*I*x^3*ln(1+b*f^(d*x+c)/(I+a))+1/6*I*
x^3*ln(1-I/(a+b*f^(d*x+c)))-1/6*I*x^3*ln(1+I/(a+b*f^(d*x+c)))-1/2*I*x^2*pol
ylog(2,b*f^(d*x+c)/(I-a))/d/ln(f)+1/2*I*x^2*polylog(2,-b*f^(d*x+c)/(I+a))/d
/ln(f)+I*x*polylog(3,b*f^(d*x+c)/(I-a))/d^2/ln(f)^2-I*x*polylog(3,-b*f^(d*x
+c)/(I+a))/d^2/ln(f)^2-I*polylog(4,b*f^(d*x+c)/(I-a))/d^3/ln(f)^3+I*polylog
(4,-b*f^(d*x+c)/(I+a))/d^3/ln(f)^3
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5252, 2631, 12, 6874, 2221, 2611, 6744, 2320, 6724}

$$\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx = -\frac{i\operatorname{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)}+\frac{i\operatorname{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+i}\right)}{d^3\log^3(f)}+\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{d^2\log^2(f)}-\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{2d\log(f)}-\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{-a+i}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{a+i}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)$$

```
[In] Int[x^2*ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] (-1/6*I)*x^3*Log[1 - (b*f^(c + d*x))/(I - a)] + (I/6)*x^3*Log[1 + (b*f^(c +
 d*x))/(I + a)] + (I/6)*x^3*Log[1 - I/(a + b*f^(c + d*x))] - (I/6)*x^3*Log[
1 + I/(a + b*f^(c + d*x))] - ((I/2)*x^2*PolyLog[2, (b*f^(c + d*x))/(I - a)]
)/(d*Log[f]) + ((I/2)*x^2*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]
) + (I*x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) - (I*x*PolyLog
[3, -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2) - (I*PolyLog[4, (b*f^(c + d
*x))/(I - a)])/(d^3*Log[f]^3) + (I*PolyLog[4, -((b*f^(c + d*x))/(I + a))])/
(d^3*Log[f]^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}i\int x^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)dx-\frac{1}{2}i\int x^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)dx\\
&= \frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\
&\quad+\frac{1}{6}\int\frac{bdf^{c+dx}x^3\log(f)}{(i(1-ia)+bf^{c+dx})\,(a+bf^{c+dx})}dx\\
&\quad+\frac{1}{6}\int\frac{bdf^{c+dx}x^3\log(f)}{(-i(1+ia)+bf^{c+dx})\,(a+bf^{c+dx})}dx\\
&= \frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\
&\quad+\frac{1}{6}(bd\log(f))\int\frac{f^{c+dx}x^3}{(i(1-ia)+bf^{c+dx})\,(a+bf^{c+dx})}dx\\
&\quad+\frac{1}{6}(bd\log(f))\int\frac{f^{c+dx}x^3}{(-i(1+ia)+bf^{c+dx})\,(a+bf^{c+dx})}dx\\
&= \frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\
&\quad+\frac{1}{6}(bd\log(f))\int\left(\frac{if^{c+dx}x^3}{a+bf^{c+dx}}-\frac{if^{c+dx}x^3}{-i+a+bf^{c+dx}}\right)dx\\
&\quad+\frac{1}{6}(bd\log(f))\int\left(-\frac{if^{c+dx}x^3}{a+bf^{c+dx}}+\frac{if^{c+dx}x^3}{i+a+bf^{c+dx}}\right)dx\\
&= \frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\
&\quad-\frac{1}{6}(ibd\log(f))\int\frac{f^{c+dx}x^3}{-i+a+bf^{c+dx}}dx+\frac{1}{6}(ibd\log(f))\int\frac{f^{c+dx}x^3}{i+a+bf^{c+dx}}dx\\
&= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)\\
&\quad+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\
&\quad+\frac{1}{2}i\int x^2\log\left(1+\frac{bf^{c+dx}}{-i+a}\right)dx-\frac{1}{2}i\int x^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)dx\\
&= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\\
&\quad-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{ix^2\,\mathrm{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\,\mathrm{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}\\
&\quad+\frac{i\int x\,\mathrm{PolyLog}\left(2,-\frac{bf^{c+dx}}{-i+a}\right)dx}{d\log(f)}-\frac{i\int x\,\mathrm{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)dx}{d\log(f)}
\end{aligned}$$

$$= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}+\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)}-\frac{i\int\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{-i+a}\right)dx}{d^2\log^2(f)}+\frac{i\int\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)dx}{d^2\log^2(f)}$$

$$= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}+\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)}-\frac{i\text{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,\frac{bx}{i-a}\right)}{x}dx,x,f^{c+dx}\right)}{d^3\log^3(f)}+\frac{i\text{Subst}\left(\int\frac{\operatorname{PolyLog}\left(3,-\frac{bx}{i+a}\right)}{x}dx,x,f^{c+dx}\right)}{d^3\log^3(f)}$$

$$= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}+\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)}-\frac{i\operatorname{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)}+\frac{i\operatorname{PolyLog}\left(4,-\frac{bf^{c+dx}}{i+a}\right)}{d^3\log^3(f)}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx &= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)\\ &\quad+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\ &\quad-\frac{ix^2\,\mathrm{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\,\mathrm{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}\\ &\quad+\frac{ix\,\mathrm{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{ix\,\mathrm{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)}\\ &\quad-\frac{i\,\mathrm{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)}+\frac{i\,\mathrm{PolyLog}\left(4,-\frac{bf^{c+dx}}{i+a}\right)}{d^3\log^3(f)}\end{aligned}$$

```
[In] Integrate[x^2*ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] (-1/6*I)*x^3*Log[1 - (b*f^(c + d*x))/(I - a)] + (I/6)*x^3*Log[1 + (b*f^(c +
 d*x))/(I + a)] + (I/6)*x^3*Log[1 - I/(a + b*f^(c + d*x))] - (I/6)*x^3*Log[
1 + I/(a + b*f^(c + d*x))] - ((I/2)*x^2*PolyLog[2, (b*f^(c + d*x))/(I - a)]
)/(d*Log[f]) + ((I/2)*x^2*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]
) + (I*x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) - (I*x*PolyLog
[3, -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2) - (I*PolyLog[4, (b*f^(c + d
*x))/(I - a)])/(d^3*Log[f]^3) + (I*PolyLog[4, -((b*f^(c + d*x))/(I + a))])/
(d^3*Log[f]^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 763 vs. 2(281) = 562.

Time = 1.54 (sec) , antiderivative size = 764, normalized size of antiderivative = 2.44

method	result
risch	$\frac{ix^3\ln\left(1+i\left(a+b\,f^{dx+c}\right)\right)}{6}-\frac{i\ln\left(1-\frac{ib\,f^{dx}\,f^{c}}{-ia-1}\right)x^3}{6}-\frac{ix^3\ln\left(1-i\left(a+b\,f^{dx+c}\right)\right)}{6}+\frac{i\ln\left(1-\frac{ib\,f^{dx}\,f^{c}}{-ia+1}\right)x^3}{6}+\frac{\pi\,x^3}{6}-\frac{i\,\mathrm{polylog}\left(4,\frac{ib\,f}{-i}\right.}{\ln(f)^3d^3}$

```
[In] int(x^2*arccot(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/ln(f)/d^3*c^2*dilog((b*f^(d*x)*f^c+a+I)/(I+a))-I/ln(f)^2/d^2*polylog(
3,I*b/(1-I*a)*f^(d*x)*f^c)*x-1/2*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^2*x-
1/2*I/ln(f)/d^3*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c^2+1/2*I/d^2*c^2*ln((b*
f^(d*x)*f^c+a+I)/(I+a))*x+1/2*I/ln(f)/d*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*
```

```
x^2+I/ln(f)^2/d^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)*x+I/ln(f)^3/d^3*polyl
og(4,I*b/(1-I*a)*f^(d*x)*f^c)+1/2*I/d^3*c^3*ln((b*f^(d*x)*f^c+a+I)/(I+a))-1
/6*I/d^3*c^3*ln(1-I*a-I*f^(d*x)*f^c*b)-1/3*I/d^3*ln(1-I*b/(1-I*a)*f^(d*x)*f
^c)*c^3+1/6*I*x^3*ln(1+I*(a+b*f^(d*x+c)))-1/6*I*ln(1-I*b/(-I*a-1)*f^(d*x)*f
^c)*x^3-1/6*I*x^3*ln(1-I*(a+b*f^(d*x+c)))+1/6*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^
c)*x^3-1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x-1/2*I/ln(f)/d^3*c^2*di
log((b*f^(d*x)*f^c+a-I)/(a-I))+1/2*I/ln(f)/d^3*polylog(2,I*b/(-I*a-1)*f^(d*
x)*f^c)*c^2-1/2*I/ln(f)/d*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x^2+1/2*I/d^2
*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*c^2*x-1/2*I/d^3*c^3*ln((b*f^(d*x)*f^c+a-I)/
(a-I))-I/ln(f)^3/d^3*polylog(4,I*b/(-I*a-1)*f^(d*x)*f^c)+1/3*I/d^3*ln(1-I*b
/(-I*a-1)*f^(d*x)*f^c)*c^3+1/6*I/d^3*c^3*ln(I*f^(d*x)*f^c*b+I*a+1)+1/6*Pi*x
^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.21

$$\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$= \frac{2\,d^3x^3\operatorname{arccot}\left(bf^{dx+c}+a\right)\log\left(f\right)^3 - 3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)^2 + 3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b)}{a^2+}\right.}{}$$

```
[In] integrate(x^2*arccot(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^3*x^3*arccot(b*f^(d*x + c) + a)*log(f)^3 - 3*I*d^2*x^2*dilog(-(a^2
 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 + 3*I*d^2*x^2*dilog
(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 - I*c^3*log(b
*f^(d*x + c) + a + I)*log(f)^3 + I*c^3*log(b*f^(d*x + c) + a - I)*log(f)^3
+ (-I*d^3*x^3 - I*c^3)*log(f)^3*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^
2 + 1)) + (I*d^3*x^3 + I*c^3)*log(f)^3*log((a^2 + (a*b - I*b)*f^(d*x + c) +
 1)/(a^2 + 1)) + 6*I*d*x*log(f)*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 +
1)) - 6*I*d*x*log(f)*polylog(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1)) - 6*I*p
olylog(4, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) + 6*I*polylog(4, -(a*b - I*b)
*f^(d*x + c)/(a^2 + 1)))/(d^3*log(f)^3)
```

Sympy [F]

$$\int x^2 \cot^{-1}\left(a + bf^{c+dx}\right)\,dx = \int x^2 \operatorname{acot}\left(a + bf^{c+dx}\right)dx$$

[In] integrate(x**2*acot(a+b*f**(d*x+c)),x)

[Out] Integral(x**2*acot(a + b*f**(c + d*x)), x)

Maxima [F]

$$\int x^2 \cot^{-1}\left(a + bf^{c+dx}\right)\,dx = \int x^2 \operatorname{arccot}\left(bf^{dx+c} + a\right)\,dx$$

[In] integrate(x^2*arccot(a+b*f^(d*x+c)),x, algorithm="maxima")

[Out] b*d*f^c*integrate(1/3*f^(d*x)*x^3/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f^c + a^2 + 1), x)*log(f) + 1/3*x^3*arctan(1/(b*f^(d*x)*f^c + a))

Giac [F]

$$\int x^2 \cot^{-1}\left(a + bf^{c+dx}\right)\,dx = \int x^2 \operatorname{arccot}\left(bf^{dx+c} + a\right)\,dx$$

[In] integrate(x^2*arccot(a+b*f^(d*x+c)),x, algorithm="giac")

[Out] integrate(x^2*arccot(b*f^(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}\left(a + bf^{c+dx}\right)\,dx = \int x^2 \operatorname{acot}\left(a + b\,f^{c+d\,x}\right)\,dx$$

[In] int(x^2*acot(a + b*f^(c + d*x)),x)

[Out] int(x^2*acot(a + b*f^(c + d*x)), x)

3.227 $\int e^{-x} \cot^{-1}(e^x)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-x} \cot^{-1}(e^x)\, dx = -x - e^{-x} \cot^{-1}(e^x) + \frac{1}{2} \log\left(1 + e^{2x}\right)$$

```
[Out] -x-arccot(exp(x))/exp(x)+1/2*ln(1+exp(2*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2225, 5316, 2320, 36, 29, 31}

$$\int e^{-x} \cot^{-1}(e^x)\, dx = -x + \frac{1}{2} \log\left(e^{2x} + 1\right) - e^{-x} \cot^{-1}(e^x)$$

```
[In] Int[ArcCot[E^x]/E^x,x]
```

```
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -e^{-x}\cot^{-1}\left(e^x\right)-\int \frac{1}{1+e^{2x}}\,dx\\ &= -e^{-x}\cot^{-1}\left(e^x\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,e^{2x}\right)\\ &= -e^{-x}\cot^{-1}\left(e^x\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,e^{2x}\right)\\ &= -x-e^{-x}\cot^{-1}\left(e^x\right)+\frac{1}{2}\log\left(1+e^{2x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{-x} \cot^{-1}(e^x) \, dx = -x - e^{-x} \cot^{-1}(e^x) + \frac{1}{2} \log(1 + e^{2x})$$

```
[In] Integrate[ArcCot[E^x]/E^x,x]
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\operatorname{arccot}(\mathrm{e}^x)\,\mathrm{e}^{-x} - \ln(\mathrm{e}^x) + \frac{\ln(1+\mathrm{e}^{2x})}{2}$	25
default	$-\operatorname{arccot}(\mathrm{e}^x)\,\mathrm{e}^{-x} - \ln(\mathrm{e}^x) + \frac{\ln(1+\mathrm{e}^{2x})}{2}$	25
parallelrisch	$\frac{(\ln(1+\mathrm{e}^{2x})\mathrm{e}^x - 2x\,\mathrm{e}^x - 2\operatorname{arccot}(\mathrm{e}^x))\mathrm{e}^{-x}}{2}$	28
risch	$-\frac{i\mathrm{e}^{-x}\ln(1+i\mathrm{e}^x)}{2} + \frac{\ln(1+\mathrm{e}^{2x})}{2} - x + \frac{i\mathrm{e}^{-x}\ln(1-i\mathrm{e}^x)}{2} - \frac{\mathrm{e}^{-x}\pi}{2}$	51

```
[In] int(arccot(exp(x))/exp(x),x,method=_RETURNVERBOSE)
[Out] -arccot(exp(x))/exp(x)-ln(exp(x))+1/2*ln(exp(x)^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int e^{-x} \cot^{-1}(e^x) \, dx = -\frac{1}{2} \left(2 \, x e^x - e^x \log\left(e^{(2 \, x)} + 1\right) + 2 \, \operatorname{arccot}(e^x)\right) e^{(-x)}$$

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="fricas")
[Out] -1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) + 2*arccot(e^x))*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-x}\cot^{-1}\left(e^x\right)\,dx = -x + \frac{\log\left(e^{2x}+1\right)}{2} - e^{-x}\operatorname{acot}\left(e^x\right)$$

```
[In] integrate(acot(exp(x))/exp(x),x)
[Out] -x + log(exp(2*x) + 1)/2 - exp(-x)*acot(exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-x}\cot^{-1}\left(e^x\right)\,dx = -\operatorname{arccot}\left(e^x\right)e^{(-x)} + \frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="maxima")
[Out] -arccot(e^x)*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-x}\cot^{-1}\left(e^x\right)\,dx = -\arctan\left(e^{(-x)}\right)e^{(-x)} + \frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="giac")
[Out] -arctan(e^(-x))*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-x}\cot^{-1}\left(e^x\right)\,dx = \frac{\ln\left(\mathrm{e}^{2x}+1\right)}{2} - x - \operatorname{acot}(\mathrm{e}^x)\,\mathrm{e}^{-x}$$

```
[In] int(acot(exp(x))*exp(-x),x)
[Out] log(exp(2*x) + 1)/2 - x - acot(exp(x))*exp(-x)
```

3.228 $\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))} dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))} dx = \frac{\log(1-2\cot^{-1}(x))}{2ab}$$

```
[Out] 1/2*ln(1-2*arccot(x))/a/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5003}

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))} dx = \frac{\log(1-2\cot^{-1}(x))}{2ab}$$

```
[In] Int[1/((a + a*x^2)*(b - 2*b*ArcCot[x])),x]
```

```
[Out] Log[1 - 2*ArcCot[x]]/(2*a*b)
```

Rule 5003

```
Int[1/(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[-Log[RemoveContent[a + b*ArcCot[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\text{integral} = \frac{\log(1-2\cot^{-1}(x))}{2ab}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))}\,dx = \frac{\log(-1+2\cot^{-1}(x))}{2ab}$$

```
[In] Integrate[1/((a + a*x^2)*(b - 2*b*ArcCot[x])),x]
[Out] Log[-1 + 2*ArcCot[x]]/(2*a*b)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$\frac{\ln\left(\operatorname{arccot}(x)-\frac{1}{2}\right)}{2ab}$	14
default	$\frac{\ln(2b\ \operatorname{arccot}(x)-b)}{2ab}$	19
risch	$\frac{\ln(\ln(ix+1)-i(-i\ln(-ix+1)+\pi-1))}{2ab}$	34

```
[In] int(1/(a*x^2+a)/(b-2*b*arccot(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(arccot(x)-1/2)/a/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))}\,dx = \frac{\log(2\ \operatorname{arccot}(x)-1)}{2\,ab}$$

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arccot(x)),x, algorithm="fricas")
[Out] 1/2*log(2*arccot(x) - 1)/(a*b)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+ax^2)\,(b-2b\cot^{-1}(x))}\,dx = \frac{\log\left(\operatorname{acot}(x)-\frac{1}{2}\right)}{2ab}$$

```
[In] integrate(1/(a*x**2+a)/(b-2*b*acot(x)),x)
[Out] log(acot(x) - 1/2)/(2*a*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ax^2)\,(b-2b\cot^{-1}(x))}\,dx = \frac{\log\left(|2\,\arctan(1,x)-1|\right)}{2\,ab}$$

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arccot(x)),x, algorithm="maxima")
[Out] 1/2*log(abs(2*arctan2(1, x) - 1))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+ax^2)\,(b-2b\cot^{-1}(x))}\,dx = \frac{\log\left(\left|2\,\arctan\left(\frac{1}{x}\right)-1\right|\right)}{2\,ab}$$

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arccot(x)),x, algorithm="giac")
[Out] 1/2*log(abs(2*arctan(1/x) - 1))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+ax^2)\,(b-2b\cot^{-1}(x))}\,dx = \frac{\ln\left(2\,\operatorname{acot}(x)-1\right)}{2\,a\,b}$$

```
[In] int(1/((a + a*x^2)*(b - 2*b*acot(x))),x)
[Out] log(2*acot(x) - 1)/(2*a*b)
```

3.229 $\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

```
[Out] exp(b*c*x+a*c)*arccot(sinh(c*(b*x+a)))/b/c+ln(1+exp(2*c*(b*x+a)))/b/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2225, 5316, 2320, 12, 266}

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx = \frac{\log\left(e^{2c(a+bx)}+1\right)}{bc} + \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcCot[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Sinh[c*(a + b*x)]])/(b*c) + Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^x \cot^{-1}(\sinh(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int e^x \text{sech}(x)\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int e^{c(a+bx)}\cot^{-1}(\sinh(ac+bcx))\,dx = \frac{-e^{c(a+bx)}\cot^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)+\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Sinh[a*c + b*c*x]],x]
[Out] (-(E^(c*(a + b*x))*ArcCot[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2]) + Log
[1 + E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.25 (sec) , antiderivative size = 1281, normalized size of antiderivative = 27.26

method	result	size
risch	Expression too large to display	1281

```
[In] int(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/b-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn
(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(
exp(c*(b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*
(b*x+a))-I)^2)*exp(c*(b*x+a))-I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)-1/4
/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-1/4/b/c
*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*
csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*cs
gn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn
(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))
+I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^
2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2
*exp(c*(b*x+a))+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)+ln(1+exp(2*c*(b*x
+a)))/b/c+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*csgn(exp(
-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(
b*x+a))*(exp(c*(b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*
exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*csgn
(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(
exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(
b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b
*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(
I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*
exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+
a))-1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))*csgn(I*(exp(c*(b*x+a))-I)^2)^2*ex
p(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))^2*csgn(I*(exp(c*(b*x+a))
-I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(exp(c
*(b*x+a))+I)^2)*exp(c*(b*x+a))+1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I
*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(
exp(c*(b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+
a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx$$
$$= \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan\left(\frac{2(\cosh(bcx+ac)+\sinh(bcx+ac))}{\cosh(bcx+ac)^2+2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2-1}\right)+\log\left(\frac{\cdots}{\cosh(\cdots}\right.}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c) + sinh
(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
 + sinh(b*c*x + a*c)^2 - 1)) + log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) -
 sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\mathrm{acot}\,(\sinh(ac+bcx))\,dx$$

```
[In] integrate(exp(c*(b*x+a))*acot(sinh(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acot(sinh(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx = \frac{\mathrm{arccot}\,(\sinh(bcx+ac))\,e^{((bx+a)c)}}{bc} + \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx = \frac{\left(\arctan\left(\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}\right)e^{(bcx)} + e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))*e^(b*c*x) + e^(-a*c)*log(e^
(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx = \frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}+1\right)}{b\,c} + \frac{e^{b\,c\,x}\,e^{a\,c}\,\mathrm{acot}\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2} - \frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}\right)}{b\,c}$$

```
[In] int(exp(c*(a + b*x))*acot(sinh(a*c + b*c*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c) + (exp(b*c*x)*exp(a*c)*acot((exp(b*c
*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

3.230 $\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc} + \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arccot(cosh(c*(b*x+a)))/b/c+1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c+1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2225, 5316, 2320, 12, 1261, 646, 31}

$$\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx = \frac{(1-\sqrt{2})\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc} + \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcCot[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Cosh[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^x \cot^{-1}(\cosh(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{e^x\sinh(x)}{1+\cosh^2(x)}\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc}\\
&\quad + \frac{\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3+2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc}\\
&\quad + \frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$
\int e^{c(a+bx)}\cot^{-1}(\cosh(ac+bcx))\,dx
$$

$$
= \frac{4c(a+bx) + 2e^{c(a+bx)}\cot^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right) + \text{RootSum}\left[1+6\#1^2+\#1^4\&, \frac{-ac-bcx+\log\left(e^{c(a+bx)}\right.}{}\right.}{2bc}
$$

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Cosh[a*c + b*c*x]],x]
```

```
[Out] (4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcCot[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(
a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a +
 b*x)) - #1] - 7*a*c*#1^2 - 7*b*c*x*#1^2 + 7*Log[E^(c*(a + b*x)) - #1]*#1^2
)/(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.52 (sec) , antiderivative size = 1354, normalized size of antiderivative = 13.15

method	result	size
risch	Expression too large to display	1354

```
[In] int(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))-1/4/b/c*
Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*
(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c
*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(-I*exp(2*
c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2
*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp
(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))
+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))
*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*exp(-c*(b*x+a))*(exp(2
*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b
*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))-1/4/b/c*Pi
*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*(
b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c
*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(2*c
*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2
*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-ex
p(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a
))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)
))*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp
(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*
(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))-1/4/b/c
*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(ex
p(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))+1/4/b
/c*Pi*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c
*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*
(b*x+a))))*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*e
xp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(
c*(b*x+a))))^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+
1+2*I*exp(c*(b*x+a)))+1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)*2^(1/2)-1/
2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)*2^(1/2)-2*a/b+1/2/b/c*ln(exp(2*c*(
b*x+a))+(1+2^(1/2))^2)+1/2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(86) = 172$.

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.68

$$\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx$$

$$= \frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^2+2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2+1}\right)+\sqrt{2}\log}{}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c)
+ sinh(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x
+ a*c) + sinh(b*c*x + a*c)^2 + 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*
c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2
*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 + 2*sqrt(2) + 3)/(cosh(b*c*x + a*c)^2 + s
inh(b*c*x + a*c)^2 + 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2
 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b
*c*x + a*c)^2)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{acot}\left(\cosh\left(ac+bcx\right)\right)dx$$

```
[In] integrate(exp(c*(b*x+a))*acot(cosh(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acot(cosh(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$\begin{aligned}\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx &= \frac{\operatorname{arccot}\left(\cosh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} \\ &\quad + \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{\left(-2\,bcx-2\,ac\right)}-3}{2\sqrt{2}+e^{\left(-2\,bcx-2\,ac\right)}+3}\right)}{2\,bc} + \frac{2\left(bcx+ac\right)}{bc} \\ &\quad + \frac{\log\left(6\,e^{\left(-2\,bcx-2\,ac\right)}+e^{\left(-4\,bcx-4\,ac\right)}+1\right)}{2\,bc}\end{aligned}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(2*sqrt(
2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-2*b*c*x - 2*a*c) + 3))/(b*c
) + 2*(b*c*x + a*c)/(b*c) + 1/2*log(6*e^(-2*b*c*x - 2*a*c) + e^(-4*b*c*x -
4*a*c) + 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)}\cot^{-1}(\cosh(ac+bcx))\,dx = -\frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2ac)}-e^{(2bcx+4ac)}-3e^{(2ac)}}{2\sqrt{2}e^{(2ac)}+e^{(2bcx+4ac)}+3e^{(2ac)}}\right)-2\arctan\left(\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4bcx+4ac}\right.\right.}{2bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*
e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) - 2*a
rctan(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)))*e^(b*c*x) - e^(-a*c)*log(e^(4
*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int e^{c(a+bx)}\cot^{-1}(\cosh(ac+bcx))\,dx = \frac{\ln\left(8\,\mathrm{e}^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c} - \frac{\ln\left(8\,\mathrm{e}^{2\,c\,(a+b\,x)}+2\,\sqrt{2}+6\,\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c} + \frac{\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}\right)}{b\,c}$$

```
[In] int(exp(c*(a + b*x))*acot(cosh(a*c + b*c*x)),x)
```

```
[Out] (log(8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1
/2) + 1))/(2*b*c) - (log(8*exp(2*c*(a + b*x)) + 2*2^(1/2) + 6*2^(1/2)*exp(2
*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) + (exp(a*c + b*c*x)*acot((exp(b*c*x)*
exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

3.231 $\int e^{c(a+bx)} \cot^{-1}(\tanh(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\int e^{c(a+bx)} \cot^{-1}(\tanh(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\log\left(1+e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc} - \frac{\log\left(1+e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

```
[Out] exp(b*c*x+a*c)*arccot(tanh(c*(b*x+a)))/b/c+1/2*arctan(-1+exp(b*c*x+a*c)*2^(
1/2))/b/c*2^(1/2)+1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1
+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1+exp(2*c*(b*x
+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules

used = {2225, 5316, 12, 2281, 303, 1176, 631, 210, 1179, 642}

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx = -\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc} + \frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc} - \frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc} + \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcCot[Tanh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Tanh[c*(a + b*x)]])/(b*c) - ArcTan[1 - Sqrt[2]*E^(a
*c + b*c*x)]/(Sqrt[2]*b*c) + ArcTan[1 + Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b
*c) + Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
- Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^x \cot^{-1}(\tanh(x))\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2e^{3x}}{1+e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{e^{3x}}{1+e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^{ac+bcx}\right)}{bc} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^{ac+bcx}\right)}{2\sqrt{2}bc} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} - \frac{\log\left(1+\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} - \frac{\log\left(1+\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx$$

$$= \frac{2e^{c(a+bx)}\cot^{-1}\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right) + \text{RootSum}\left[1+\#1^4\&, \frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Tanh[a*c + b*c*x]],x]
```

```
[Out] (2*E^(c*(a + b*x))*ArcCot[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a + b*x)) - #1])/#1 & ]
)/(2*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 1323, normalized size of antiderivative = 7.35

method	result	size
risch	Expression too large to display	1323

```
[In] int(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(
b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(
c*(b*x+a))-1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))
^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a)
)))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*
(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+e
xp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(
1+exp(2*c*(b*x+a))))*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^
2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b
*x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c
*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*
x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))-I))*
csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))-1/4/b/c*Pi
*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*
(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c
*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2
*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn
(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*c
sgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))
))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+
a))))*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))-
1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csgn((1+I)*(ex
p(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4*Pi/b/c*exp(c*(b*
x+a))+1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-I)-1/4*I/b/c*ln(exp(c*(b
*x+a))+(1/2-1/2*I)*2^(1/2))*2^(1/2)-1/4*I/b/c*ln(exp(c*(b*x+a))-(1/2+1/2*I)
*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(exp(c*(b*x+a))+(-1/2+1/2*I)*2^(1/2))*2^(1/2)
+1/4*I/b/c*ln(exp(c*(b*x+a))+(1/2+1/2*I)*2^(1/2))*2^(1/2)-1/2*I/b/c*exp(c*(
b*x+a))*ln(exp(2*c*(b*x+a))+I)-1/4/b/c*ln(exp(c*(b*x+a))+(1/2-1/2*I)*2^(1/2
))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))-(1/2+1/2*I)*2^(1/2))*2^(1/2)+1/4/b/c*l
n(exp(c*(b*x+a))+(-1/2+1/2*I)*2^(1/2))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))+(1
/2+1/2*I)*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx = \frac{bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}+\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-i\,bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(i\,b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}+\cosh\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x, algorithm="fricas")
[Out] 1/2*(b*c*(-1/(b^4*c^4))^(1/4)*log(b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x
 + a*c) + sinh(b*c*x + a*c)) - I*b*c*(-1/(b^4*c^4))^(1/4)*log(I*b^3*c^3*(-1
/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + I*b*c*(-1/(b^4
*c^4))^(1/4)*log(-I*b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh
(b*c*x + a*c)) - b*c*(-1/(b^4*c^4))^(1/4)*log(-b^3*c^3*(-1/(b^4*c^4))^(3/4)
 + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + 2*(cosh(b*c*x + a*c) + sinh(b*c
*x + a*c))*arctan(cosh(b*c*x + a*c)/sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\operatorname{acot}\left(\tanh\left(ac+bcx\right)\right)dx$$

```
[In] integrate(exp(c*(b*x+a))*acot(tanh(b*c*x+a*c)),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*acot(tanh(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx = \frac{\operatorname{arccot}\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} + \frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bcx+ac)}\right)\right)}{2\,bc} + \frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bcx+ac)}\right)\right)}{2\,bc} - \frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc} + \frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) - 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) + 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Giac [F]

$$\int e^{c(a+bx)} \cot^{-1}(\tanh(ac+bcx))\,dx = \int \operatorname{arccot}\left(\tanh\left(bcx+ac\right)\right) e^{\left((bx+a)c\right)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)} \cot^{-1}(\tanh(ac+bcx))\,dx = \frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1}{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4-4\mathrm{i}\right)-\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,\left(-1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4+4\mathrm{i}\right)+\mathrm{e}^{b\,c\,x}\cdots\right.}{4\,b\cdots}$$

```
[In] int(exp(c*(a + b*x))*acot(tanh(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(exp(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 - 4i))*(1 - 1i) - 2^(1/2)*log(- 2^(
1/2)*(4 + 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*acot((exp(2*b*
c*x)*exp(2*a*c) - 1)/(exp(2*b*c*x)*exp(2*a*c) + 1)))/(4*b*c)
```

3.232 $\int e^{c(a+bx)} \cot^{-1}(\coth(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\int e^{c(a+bx)} \cot^{-1}(\coth(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc} + \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\log\left(1+e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc} + \frac{\log\left(1+e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

```
[Out] exp(b*c*x+a*c)*arccot(coth(c*(b*x+a)))/b/c-1/2*arctan(-1+exp(b*c*x+a*c)*2^(
1/2))/b/c*2^(1/2)-1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1
+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1+exp(2*c*(b*x
+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules

used = {2225, 5316, 12, 2281, 303, 1176, 631, 210, 1179, 642}

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx = \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}bc} - \frac{\log\left(e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc} + \frac{\log\left(e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}+1\right)}{2\sqrt{2}bc} + \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcCot[Coth[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Coth[c*(a + b*x)]])/(b*c) + ArcTan[1 - Sqrt[2]*E^(a
*c + b*c*x)]/(Sqrt[2]*b*c) - ArcTan[1 + Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b
*c) - Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
+ Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^x \cot^{-1}(\coth(x))\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2e^{3x}}{-1-e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{e^{3x}}{-1-e^{4x}}\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{1-x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc} + \frac{\text{Subst}\left(\int \frac{1+x^2}{-1-x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^{ac+bcx}\right)}{2bc} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^{ac+bcx}\right)}{2\sqrt{2}bc} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc} - \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} + \frac{\log\left(1+\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}$$

$$= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc} + \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} + \frac{\log\left(1+\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx = \frac{2e^{c(a+bx)}\cot^{-1}\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right) + \text{RootSum}\left[1+\#1^4\&, \frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Coth[a*c + b*c*x]],x]
```

```
[Out] (2*E^(c*(a + b*x))*ArcCot[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b*x)) - #1])/#1 & ])/(
2*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.32 (sec) , antiderivative size = 1323, normalized size of antiderivative = 7.35

method	result	size
risch	Expression too large to display	1323

```
[In] int(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I
*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(
I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I
)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-
I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-
1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*
c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(exp
(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c
*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I/(exp(2*c*(b*x+a))-
1))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/4/b/
c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x
+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(
b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(
b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)
/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))
-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))
-1))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a)
)-1))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*
(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2
*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)
/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x
+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*ln(
exp(2*c*(b*x+a))-I)+1/4*I/b/c*ln(exp(c*(b*x+a))+(1/2-1/2*I)*2^(1/2))*2^(1/2
)+1/4*I/b/c*ln(exp(c*(b*x+a))-(1/2+1/2*I)*2^(1/2))*2^(1/2)-1/4*I/b/c*ln(exp
(c*(b*x+a))+(-1/2+1/2*I)*2^(1/2))*2^(1/2)-1/4*I/b/c*ln(exp(c*(b*x+a))+(1/2+
1/2*I)*2^(1/2))*2^(1/2)-1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c
*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4*Pi/b/c*exp(c*(b*x+a))+1/4/b/c*ln(exp(c*(
b*x+a))+(1/2-1/2*I)*2^(1/2))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))-(1/2+1/2*I)*
2^(1/2))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))+(-1/2+1/2*I)*2^(1/2))*2^(1/2)+1/
4/b/c*ln(exp(c*(b*x+a))+(1/2+1/2*I)*2^(1/2))*2^(1/2)+1/2*I/b/c*exp(c*(b*x+a
))*ln(exp(2*c*(b*x+a))+I)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx =$$

$$-\frac{bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}+\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-i\,bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(i\,b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x, algorithm="fricas")
[Out] -1/2*(b*c*(-1/(b^4*c^4))^(1/4)*log(b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*
x + a*c) + sinh(b*c*x + a*c)) - I*b*c*(-1/(b^4*c^4))^(1/4)*log(I*b^3*c^3*(-
1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + I*b*c*(-1/(b^
4*c^4))^(1/4)*log(-I*b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sin
h(b*c*x + a*c)) - b*c*(-1/(b^4*c^4))^(1/4)*log(-b^3*c^3*(-1/(b^4*c^4))^(3/4
) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - 2*(cosh(b*c*x + a*c) + sinh(b*
c*x + a*c))*arctan(sinh(b*c*x + a*c)/cosh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{acot}\left(\coth\left(ac+bcx\right)\right)dx$$

```
[In] integrate(exp(c*(b*x+a))*acot(coth(b*c*x+a*c)),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*acot(coth(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx = \frac{\operatorname{arccot}\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} - \frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bcx+ac)}\right)\right)}{2\,bc} - \frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bcx+ac)}\right)\right)}{2\,bc} + \frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc} - \frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) + 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) - 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Giac [F]

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx = \int \operatorname{arccot}\left(\coth\left(bcx+ac\right)\right)e^{((bx+a)c)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx$$
$$= \frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1}{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4-4\mathrm{i})+\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,(-1-\mathrm{i})+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4+4\mathrm{i})-\mathrm{e}^{b\,c\,x}\,\mathrm{e}\right.}{4\,b\,c}$$

```
[In] int(exp(c*(a + b*x))*acot(coth(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(- 2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*log(exp
(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 + 4i))*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*acot((exp(2*b*
c*x)*exp(2*a*c) + 1)/(exp(2*b*c*x)*exp(2*a*c) - 1)))/(4*b*c)
```

3.233 $\int e^{c(a+bx)} \cot^{-1}(\operatorname{sech}(ac+bcx)) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{sech}(ac+bcx)) \, dx = \frac{e^{ac+bcx} \cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc} - \frac{(1-\sqrt{2}) \log \left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc} - \frac{(1+\sqrt{2}) \log \left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arccot(sech(c*(b*x+a)))/b/c-1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c-1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2225, 5316, 2320, 12, 1261, 646, 31}

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{sech}(ac+bcx)) \, dx = -\frac{(1-\sqrt{2}) \log \left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc} - \frac{(1+\sqrt{2}) \log \left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc} + \frac{e^{ac+bcx} \cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcCot[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Sech[c*(a + b*x)]])/(b*c) - ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) - ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^x \cot^{-1}(\text{sech}(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{sech}(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{e^x\text{sech}(x)\tanh(x)}{1+\text{sech}^2(x)}\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{sech}(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{sech}(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{sech}(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{sech}(c(a+bx)))}{bc} - \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc}\\
&\quad - \frac{\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3+2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{sech}(c(a+bx)))}{bc} - \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc}\\
&\quad - \frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}+e^{2ac+2bcx}\right)}{2bc}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$
\int e^{c(a+bx)}\cot^{-1}(\text{sech}(ac+bcx))\,dx
$$

$$
= \frac{-4c(a+bx)+2e^{c(a+bx)}\cot^{-1}\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+6\#1^2+\#1^4\&, \frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)+7ac\#1^2}{1+3}\right.}{2bc}
$$

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Sech[a*c + b*c*x]],x]
```

```
[Out] (-4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcCot[(2*E^(c*(a + b*x)))/(1 + E^(2*c*
(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b
*x)) - #1] + 7*a*c*#1^2 + 7*b*c*x*#1^2 - 7*Log[E^(c*(a + b*x)) - #1]*#1^2)/
(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.55 (sec) , antiderivative size = 855, normalized size of antiderivative = 8.30

method	result	size
risch	Expression too large to display	855

```
[In] int(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))+1/4/b/c
*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I/(1+exp(2*c*(b*x+a)))
)*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*exp
(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*(
-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+
a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I
*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(-I*
exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(e
xp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))-
1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(
c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(-exp(
2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))+1
/4/b/c*Pi*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*(exp(2*c*(b*x
+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*P
i*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*ex
p(c*(b*x+a))+1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+
a)))+1/2*Pi/b/c*exp(c*(b*x+a))+1/2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)*2
^(1/2)-1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)*2^(1/2)+2*a/b-1/2/b/c*ln(
exp(2*c*(b*x+a))+(2^(1/2)-1)^2)-1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(86) = 172$.

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{sech}(ac+bcx))\,dx$$
$$= \frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\cosh\left(bcx+ac\right)\right)+\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}-4\right)\mathrm{co}}{\cosh(b}\right.}{2b}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)) +
sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) - 4)*cos
```

```
h(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) - 3)*sinh(b*c*x + a*c)^2 +
2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)) - log(2*(co
sh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(
b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*acot(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.64

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\operatorname{arccot}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} + \frac{3\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2\,bcx+2\,ac)}-3}{2\sqrt{2}+e^{(2\,bcx+2\,ac)}+3}\right)}{8\,bc} - \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2\,bcx-2\,ac)}-3}{2\sqrt{2}+e^{(-2\,bcx-2\,ac)}+3}\right)}{8\,bc} - \frac{\log\left(e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{2\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 3/8*sqrt(2)*log(-(2*sqrt(
2) - e^(2*b*c*x + 2*a*c) - 3)/(2*sqrt(2) + e^(2*b*c*x + 2*a*c) + 3))/(b*c)
- 1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-
2*b*c*x - 2*a*c) + 3))/(b*c) - 1/2*log(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x +
 2*a*c) + 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2ac)}-e^{(2bcx+4ac)}-3e^{(2ac)}}{2\sqrt{2}e^{(2ac)}+e^{(2bcx+4ac)}+3e^{(2ac)}}\right)+2\arctan\left(\frac{1}{2}e^{(bcx+ac)}+\frac{1}{2}e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4bcx-}\right.\right.}{2bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*e
^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) + 2*ar
ctan(1/2*e^(b*c*x + a*c) + 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*log(e
^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{e^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{b\,c} + \frac{\ln\left(-8\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c} - \frac{\ln\left(2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}+6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c}$$

```
[In] int(acot(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*acot(1/((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))
/2)))/(b*c) + (log(- 8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(
a + b*x)))*(2^(1/2) - 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)) +
 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c)
```

3.234 $\int e^{c(a+bx)} \cot^{-1}(\textbf{csch}(ac+bcx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\operatorname{csch}(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

```
[Out] exp(b*c*x+a*c)*arccot(csch(c*(b*x+a)))/b/c-ln(1+exp(2*c*(b*x+a)))/b/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2225, 5316, 2320, 12, 266}

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\operatorname{csch}(c(a+bx)))}{bc} - \frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

```
[In] Int[E^(c*(a + b*x))*ArcCot[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Csch[c*(a + b*x)]])/(b*c) - Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int e^x \cot^{-1}(\text{csch}(x))\,dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{csch}(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int e^x \text{sech}(x)\,dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{csch}(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{csch}(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx}\cot^{-1}(\text{csch}(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int e^{c(a+bx)}\cot^{-1}(\text{csch}(ac+bcx))\,dx = \frac{e^{c(a+bx)}\cot^{-1}\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right) - \log\left(1+e^{2c(a+bx)}\right)}{bc}$$

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Csch[a*c + b*c*x]],x]
[Out] (E^(c*(a + b*x))*ArcCot[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] - Log
[1 + E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.35 (sec) , antiderivative size = 903, normalized size of antiderivative = 18.81

method	result	size
risch	Expression too large to display	903

```
[In] int(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)-I))^2*csgn(I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/2/b/c*Pi*csgn(I*(exp(
c*(b*x+a))-I))*csgn(I*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csg
n(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)-I)^2)*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(
b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*(
exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(
I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^2
*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I/(exp(2*c*(b*
x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-
1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*
c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2/(exp
(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*
csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a
))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp
(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*
csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*
Pi*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+I/b/c
*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)+1/2*Pi/b/c*exp(c*(b*x+a))+2*a/b-ln(1+e
xp(2*c*(b*x+a)))/b/c
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan(\sinh(bcx+ac))-\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)) - log(2*
cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = e^{ac} \int e^{bcx} \operatorname{acot}(\operatorname{csch}(ac+bcx))\,dx$$

```
[In] integrate(exp(c*(b*x+a))*acot(csch(b*c*x+a*c)),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*acot(csch(a*c + b*c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{\operatorname{arccot}(\operatorname{csch}(bcx+ac))\,e^{((bx+a)c)}}{bc} - \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x, algorithm="maxima")
[Out] arccot(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{\left(\arctan\left(\frac{1}{2}\,e^{(bcx+ac)} - \frac{1}{2}\,e^{(-bcx-ac)}\right)e^{(bcx)} - e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x, algorithm="giac")
[Out] (arctan(1/2*e^(b*c*x + a*c) - 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*lo
g(e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{csch}(ac+bcx))\,dx=\frac{e^{b\,c\,x}\,e^{a\,c}\,\mathrm{acot}\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{b\,c}-\frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}+1\right)}{b\,c}$$

```
[In] int(acot(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (exp(b*c*x)*exp(a*c)*acot(1/((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*
c))/2)))/(b*c) - log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func):  #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func):  #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args)))  #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:  #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```