Computer Algebra Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.6-Inverse-cosecant/159-5.6.2-Inverse-cosecant-functions

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [49]. This is test number [159].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (49)	0.00 (0)
Mathematica	100.00 (49)	0.00 (0)
Maple	73.47 (36)	26.53 (13)
Fricas	55.10 (27)	44.90 (22)
Giac	51.02 (25)	48.98 (24)
Maxima	30.61 (15)	69.39 (34)
Mupad	24.49 (12)	75.51 (37)
Sympy	24.49 (12)	75.51 (37)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

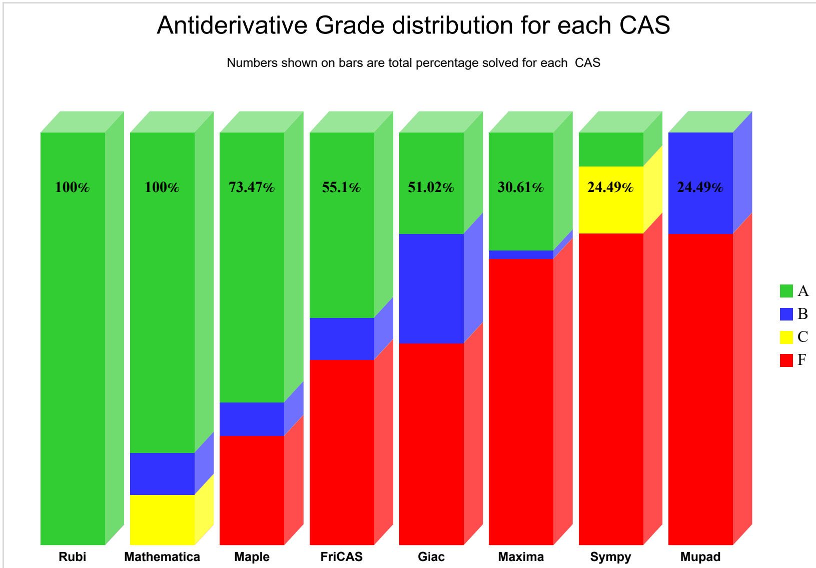
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

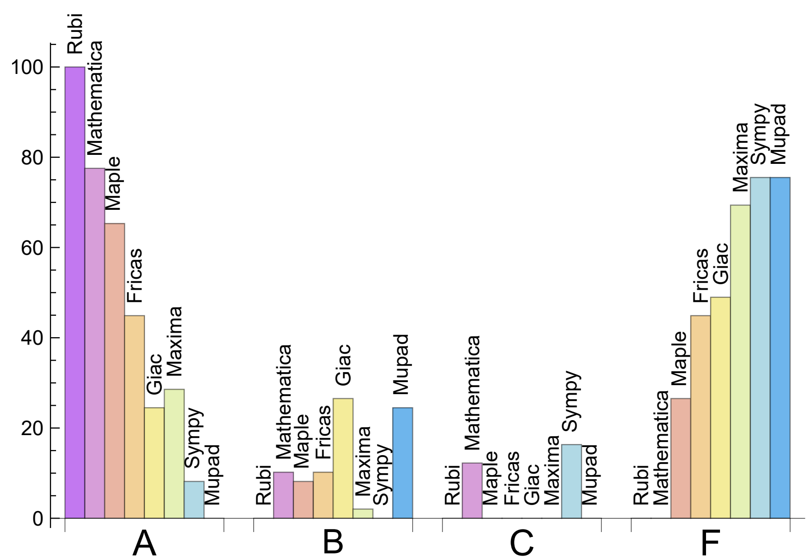
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	77.551	10.204	12.245	0.000
Maple	65.306	8.163	0.000	26.531
Fricas	44.898	10.204	0.000	44.898
Maxima	28.571	2.041	0.000	69.388
Giac	24.490	26.531	0.000	48.980
Sympy	8.163	0.000	16.327	75.510
Mupad	0.000	24.490	0.000	75.510

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	13	100.00	0.00	0.00
Fricas	22	90.91	0.00	9.09
Giac	24	91.67	0.00	8.33
Maxima	34	100.00	0.00	0.00
Mupad	37	0.00	100.00	0.00
Sympy	37	94.59	5.41	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.11
Maxima	0.22
Fricas	0.28
Giac	0.31
Mathematica	0.52
Mupad	0.92
Maple	0.93
Sympy	12.51

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	41.50	0.91	40.00	0.92
Maxima	50.80	1.20	52.00	1.14
Sympy	67.67	1.56	56.00	1.67
Fricas	121.11	1.34	51.00	0.83
Rubi	124.59	1.00	79.00	1.00
Giac	155.00	1.80	81.00	1.75
Mathematica	170.96	1.50	99.00	1.11
Maple	217.97	1.61	138.50	1.60

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

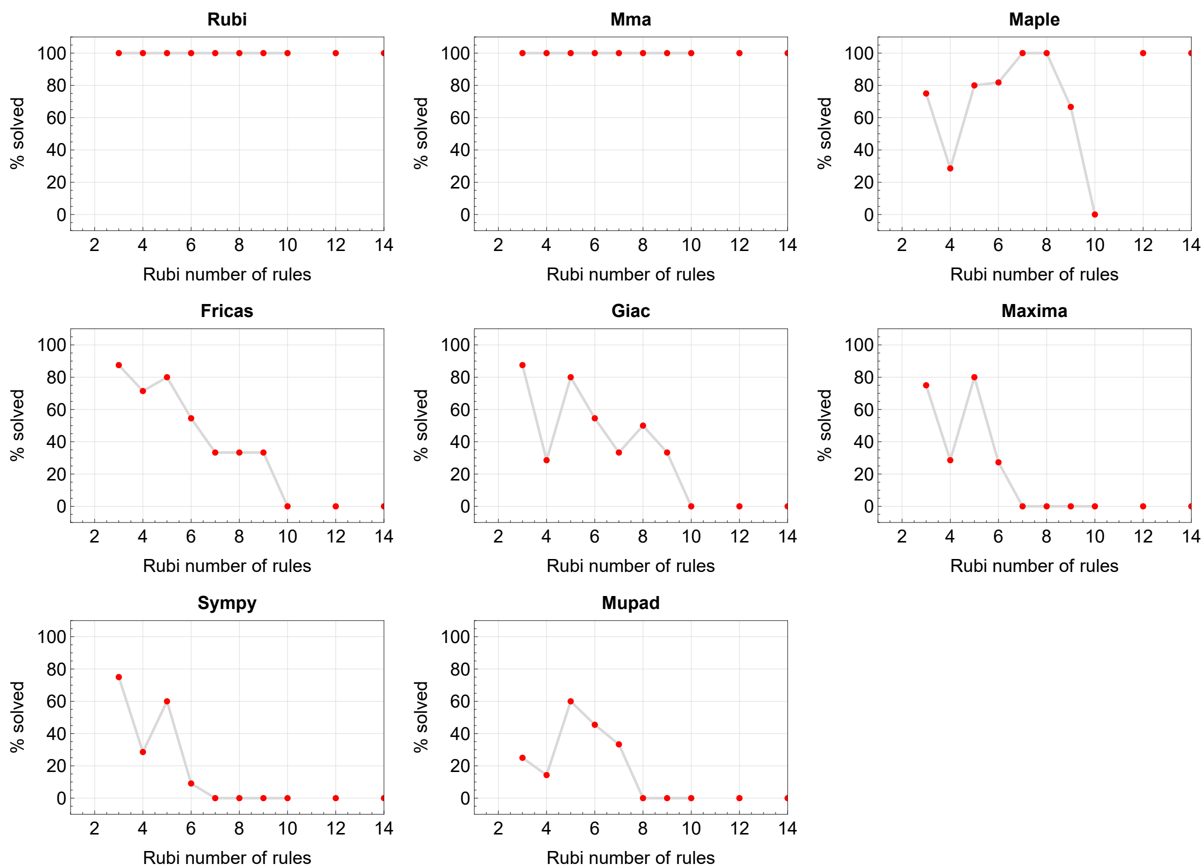


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

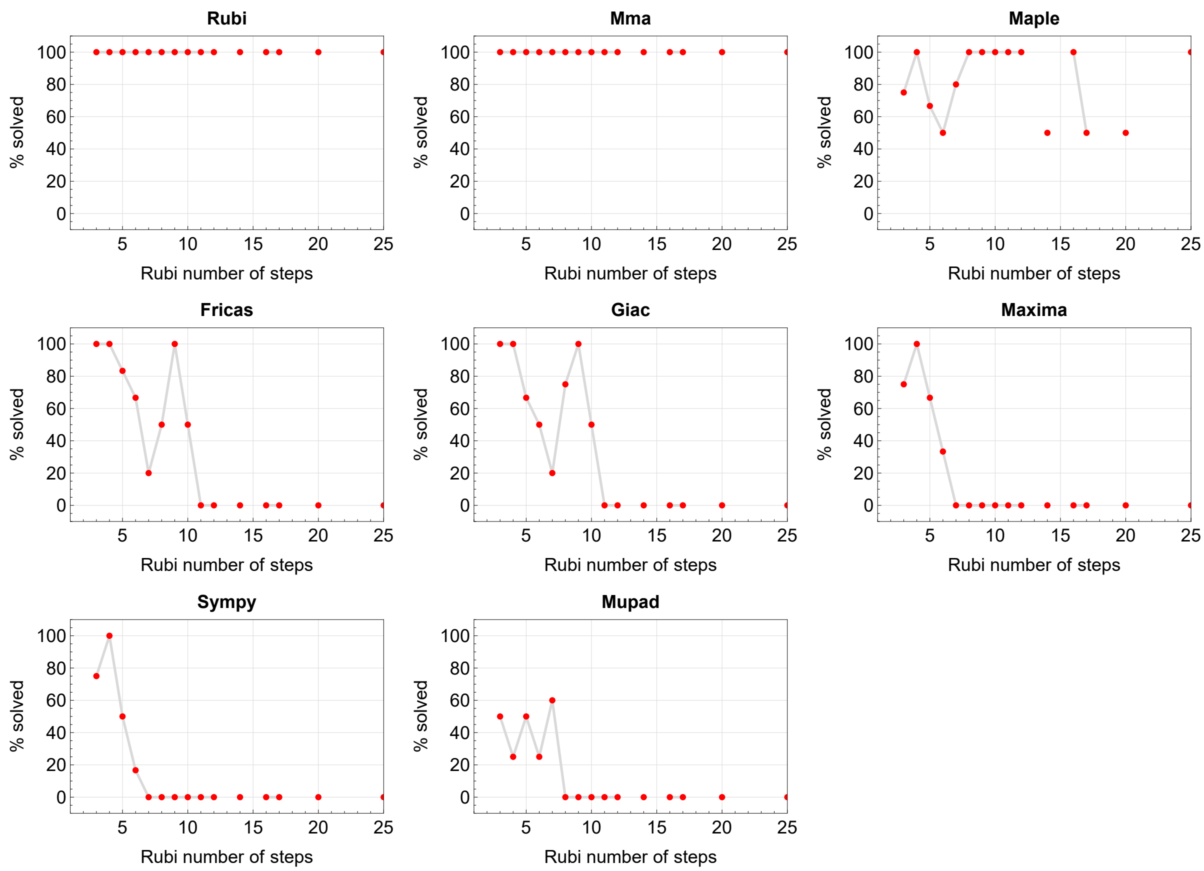


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

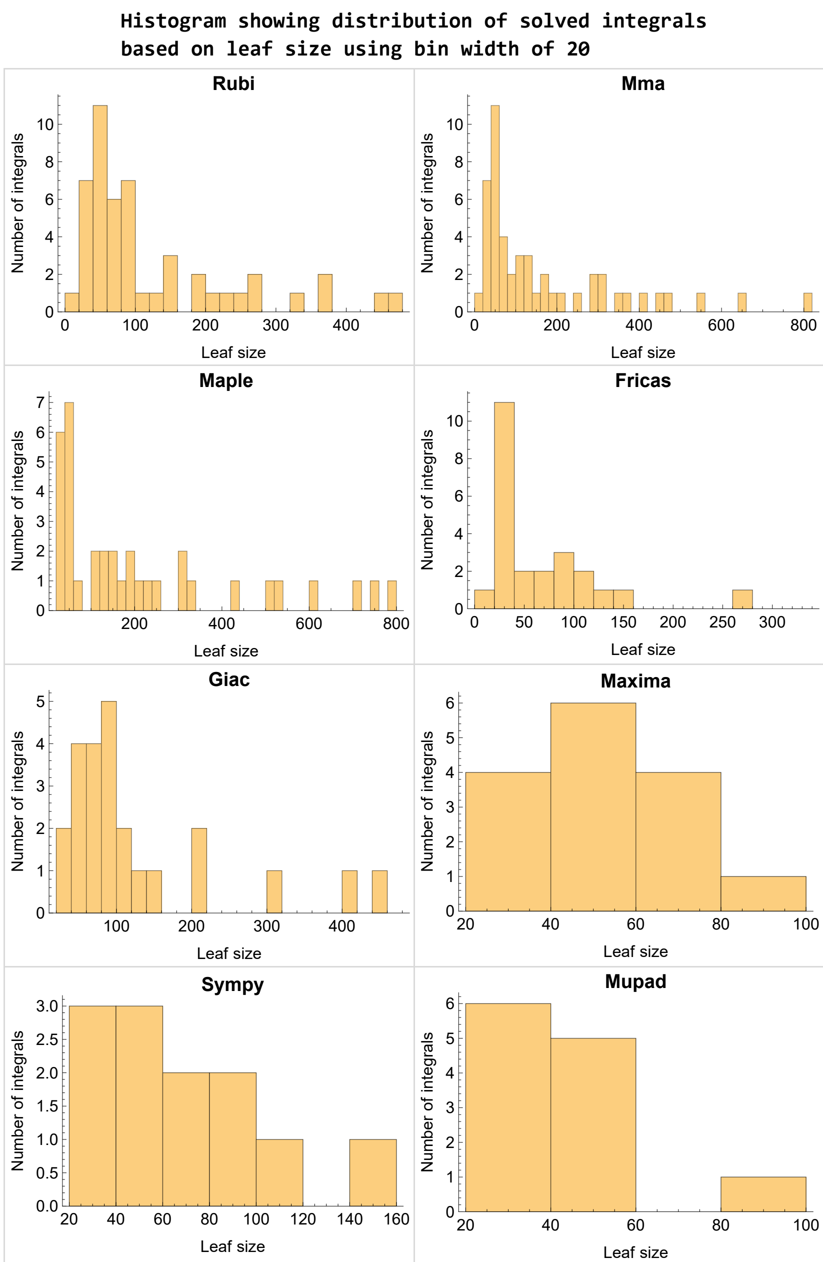


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

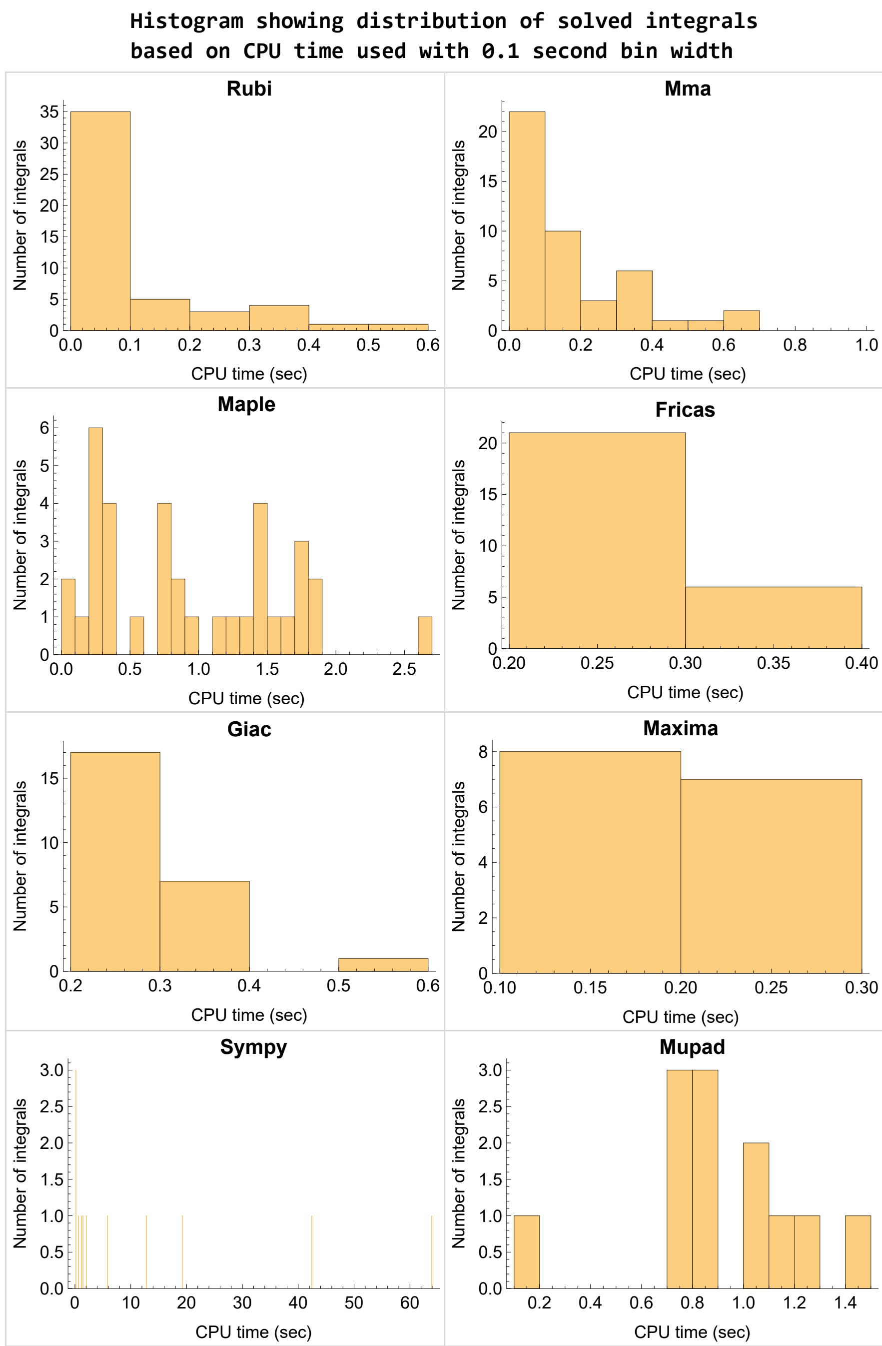


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

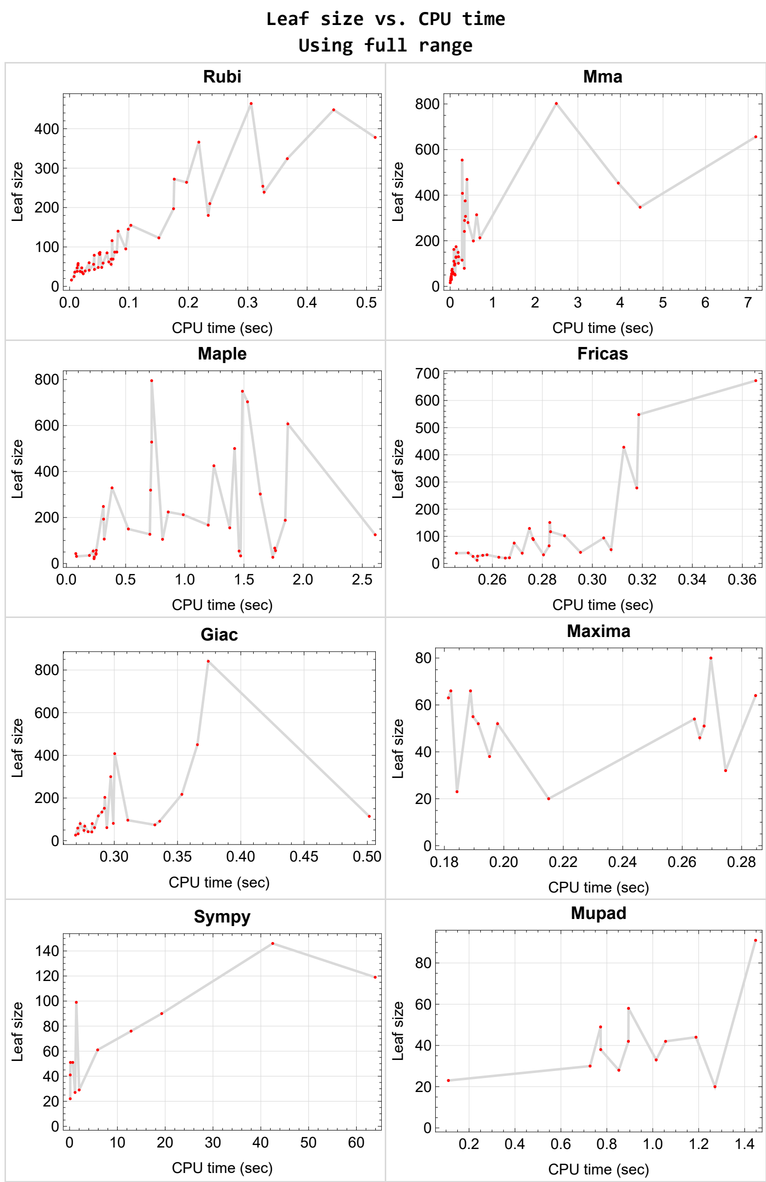


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{27, 28, 33\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

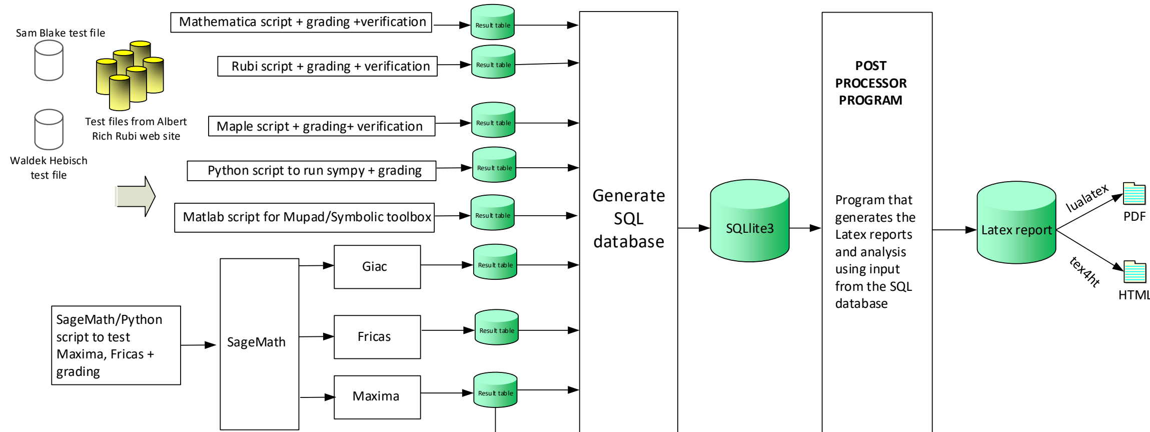
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
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2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 23
Giac 24
Mupad 24
Sympy 24

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17, 18, 19, 20, 22, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

B grade { 13, 32, 38, 39, 40 }

C grade { 16, 21, 23, 24, 25, 26 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 27, 28, 29, 30, 32, 33, 34, 35, 38, 40, 49 }

B grade { 22, 24, 25, 26 }

C grade { }

F normal fail { 1, 31, 36, 37, 39, 41, 42, 43, 44, 45, 46, 47, 48 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 4, 5, 7, 8, 9, 10, 11, 14, 15, 17, 18, 19, 20, 24, 25, 26, 45, 46, 47, 48 }

B grade { 13, 21, 23, 38, 39 }

C grade { }

F normal fail { 1, 6, 12, 22, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 42, 43, 44, 49 }

F(-1) timedout fail { }

F(-2) exception fail { 16, 40 }

Maxima

A grade { 2, 3, 4, 5, 7, 9, 10, 11, 13, 14, 15, 21, 38, 39 }

B grade { 8 }

C grade { }

F normal fail { 1, 6, 12, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade $\{\ 7, 8, 9, 10, 11, 14, 15, 20, 23, 39, 45, 49\ \}$

B grade $\{\ 2, 3, 4, 5, 13, 17, 18, 19, 21, 24, 25, 26, 38\ \}$

C grade $\{\ \}$

F normal fail $\{\ 1, 12, 16, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 46, 47, 48\ \}$

F(-1) timedout fail $\{\ \}$

F(-2) exception fail $\{\ 6, 27\ \}$

Mupad

A grade $\{\ \}$

B grade $\{\ 1, 5, 6, 7, 10, 11, 12, 13, 21, 38, 39, 40\ \}$

C grade $\{\ \}$

F normal fail $\{\ \}$

F(-1) timedout fail $\{\ 2, 3, 4, 8, 9, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 42, 43, 44, 45, 46, 47, 48, 49\ \}$

F(-2) exception fail $\{\ \}$

Sympy

A grade $\{\ 9, 10, 11, 15\ \}$

B grade $\{\ \}$

C grade $\{\ 2, 3, 4, 5, 7, 8, 13, 14\ \}$

F normal fail $\{\ 1, 6, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49\ \}$

F(-1) timedout fail $\{\ 38, 39\ \}$

F(-2) exception fail $\{\ \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	56	0	0	0	0	0	58
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.94
time (sec)	N/A	0.066	0.037	0.000	0.000	0.000	0.000	0.000	0.894

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	40	66	32	119	152	0
N.S.	1	1.00	0.69	0.69	1.14	0.55	2.05	2.62	0.00
time (sec)	N/A	0.014	0.020	0.250	0.189	0.258	63.927	0.292	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	35	35	52	27	90	116	0
N.S.	1	1.00	0.74	0.74	1.11	0.57	1.91	2.47	0.00
time (sec)	N/A	0.013	0.018	0.194	0.191	0.254	19.272	0.287	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	28	38	20	61	80	0
N.S.	1	1.00	0.75	0.78	1.06	0.56	1.69	2.22	0.00
time (sec)	N/A	0.009	0.016	0.235	0.195	0.265	5.871	0.283	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	21	20	12	29	41	20
N.S.	1	1.00	1.00	1.31	1.25	0.75	1.81	2.56	1.25
time (sec)	N/A	0.003	0.002	0.233	0.215	0.254	2.010	0.282	1.271

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	105	0	0	0	0	42
N.S.	1	1.00	0.96	1.88	0.00	0.00	0.00	0.00	0.75
time (sec)	N/A	0.070	0.039	0.812	0.000	0.000	0.000	0.000	1.056

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	44	51	21	76	32	28
N.S.	1	1.00	0.84	1.16	1.34	0.55	2.00	0.84	0.74
time (sec)	N/A	0.013	0.020	0.250	0.267	0.267	12.862	0.271	0.852

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	55	57	80	30	146	59	0
N.S.	1	1.00	1.02	1.06	1.48	0.56	2.70	1.09	0.00
time (sec)	N/A	0.014	0.035	0.252	0.270	0.256	42.491	0.271	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	42	56	54	39	51	68	0
N.S.	1	1.00	0.75	1.00	0.96	0.70	0.91	1.21	0.00
time (sec)	N/A	0.040	0.026	1.767	0.264	0.250	0.178	0.277	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	67	46	38	41	48	38
N.S.	1	1.00	0.94	1.43	0.98	0.81	0.87	1.02	0.81
time (sec)	N/A	0.021	0.023	1.760	0.266	0.246	0.147	0.276	0.773

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	23	26	22	26	23
N.S.	1	1.00	1.00	1.08	0.92	1.04	0.88	1.04	0.92
time (sec)	N/A	0.008	0.007	1.744	0.184	0.252	0.135	0.269	0.110

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	125	0	0	0	0	49
N.S.	1	1.00	0.92	2.12	0.00	0.00	0.00	0.00	0.83
time (sec)	N/A	0.056	0.033	2.609	0.000	0.000	0.000	0.000	0.773

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	93	31	52	65	27	61	30
N.S.	1	1.00	2.91	0.97	1.62	2.03	0.84	1.91	0.94
time (sec)	N/A	0.023	0.104	0.085	0.198	0.283	1.148	0.284	0.727

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	33	32	38	51	61	0
N.S.	1	1.00	0.92	0.87	0.84	1.00	1.34	1.61	0.00
time (sec)	N/A	0.018	0.020	1.472	0.275	0.272	0.667	0.294	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	69	54	64	92	99	80	0
N.S.	1	1.00	1.15	0.90	1.07	1.53	1.65	1.33	0.00
time (sec)	N/A	0.033	0.040	1.459	0.285	0.276	1.411	0.273	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	63	155	0	0	0	0	0
N.S.	1	1.00	0.91	2.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.062	1.380	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	174	329	0	151	0	408	0
N.S.	1	1.00	0.88	1.67	0.00	0.77	0.00	2.07	0.00
time (sec)	N/A	0.175	0.139	0.385	0.000	0.283	0.000	0.300	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	149	248	0	129	0	300	0
N.S.	1	1.00	0.96	1.60	0.00	0.83	0.00	1.94	0.00
time (sec)	N/A	0.103	0.187	0.312	0.000	0.275	0.000	0.297	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	129	193	0	117	0	203	0
N.S.	1	1.00	1.11	1.66	0.00	1.01	0.00	1.75	0.00
time (sec)	N/A	0.072	0.137	0.314	0.000	0.283	0.000	0.293	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	110	106	0	102	0	134	0
N.S.	1	1.00	1.39	1.34	0.00	1.29	0.00	1.70	0.00
time (sec)	N/A	0.042	0.087	0.319	0.000	0.289	0.000	0.290	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	469	43	55	75	0	81	33
N.S.	1	1.00	13.03	1.19	1.53	2.08	0.00	2.25	0.92
time (sec)	N/A	0.021	0.395	0.079	0.190	0.269	0.000	0.299	1.015

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	375	607	0	0	0	0	0
N.S.	1	1.00	1.79	2.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.353	1.871	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	115	127	0	278	0	96	0
N.S.	1	1.00	1.67	1.84	0.00	4.03	0.00	1.39	0.00
time (sec)	N/A	0.073	0.281	0.705	0.000	0.318	0.000	0.311	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	199	319	0	428	0	217	0
N.S.	1	1.00	1.62	2.59	0.00	3.48	0.00	1.76	0.00
time (sec)	N/A	0.150	0.545	0.711	0.000	0.313	0.000	0.353	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	241	528	0	548	0	450	0
N.S.	1	1.00	1.34	2.93	0.00	3.04	0.00	2.50	0.00
time (sec)	N/A	0.234	0.334	0.720	0.000	0.319	0.000	0.366	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	307	795	0	673	0	841	0
N.S.	1	1.00	1.28	3.33	0.00	2.82	0.00	3.52	0.00
time (sec)	N/A	0.328	0.360	0.720	0.000	0.365	0.000	0.374	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	453	703	0	0	0	0	0
N.S.	1	1.00	1.24	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	3.950	1.530	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	347	500	0	0	0	0	0
N.S.	1	1.00	1.28	1.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	4.462	1.421	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	213	212	0	0	0	0	0
N.S.	1	1.00	1.47	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.099	0.699	0.987	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	99	150	0	0	0	0	0
N.S.	1	1.00	1.15	1.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.109	0.523	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	324	324	408	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	0.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	802	302	0	0	0	0	0
N.S.	1	1.00	3.16	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	2.494	1.638	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	464	464	656	749	0	0	0	0	0
N.S.	1	1.00	1.41	1.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	7.179	1.487	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	314	425	0	0	0	0	0
N.S.	1	1.00	1.19	1.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.622	1.246	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	162	224	0	0	0	0	0
N.S.	1	1.00	1.16	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.087	0.860	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	448	448	554	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	0.282	0.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	378	378	289	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	0.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	127	54	63	88	0	91	44
N.S.	1	1.00	2.65	1.12	1.31	1.83	0.00	1.90	0.92
time (sec)	N/A	0.048	0.137	0.226	0.181	0.276	0.000	0.336	1.188

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	130	0	66	94	0	74	42
N.S.	1	1.00	2.71	0.00	1.38	1.96	0.00	1.54	0.88
time (sec)	N/A	0.054	0.197	0.000	0.182	0.305	0.000	0.332	0.894

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	280	188	0	0	0	0	91
N.S.	1	1.00	3.29	2.21	0.00	0.00	0.00	0.00	1.07
time (sec)	N/A	0.063	0.418	1.849	0.000	0.000	0.000	0.000	1.448

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	79	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.332	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	101	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.193	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	54	0	0	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	75	0	0	0	0	0	0
N.S.	1	1.00	1.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	33	0	0	23	0	42	0
N.S.	1	1.00	0.85	0.00	0.00	0.59	0.00	1.08	0.00
time (sec)	N/A	0.026	0.029	0.000	0.000	0.263	0.000	0.279	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	30	0	0	32	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.033	0.031	0.000	0.000	0.280	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	54	0	0	41	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.050	0.105	0.000	0.000	0.295	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	50	0	0	51	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.051	0.117	0.000	0.000	0.307	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	59	167	0	0	0	114	0
N.S.	1	1.00	0.86	2.42	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.070	0.059	1.198	0.000	0.000	0.000	0.502	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [34] had the largest ratio of [1.19999999999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	10	0.600
2	A	4	3	1.00	10	0.300
3	A	4	3	1.00	10	0.300
4	A	4	3	1.00	8	0.375
5	A	3	3	1.00	6	0.500
6	A	7	6	1.00	10	0.600
7	A	5	5	1.00	10	0.500
8	A	6	5	1.00	10	0.500
9	A	5	4	1.00	10	0.400
10	A	4	4	1.00	8	0.500
11	A	3	3	1.00	6	0.500
12	A	6	6	1.00	10	0.600
13	A	5	5	1.00	10	0.500
14	A	3	3	1.00	10	0.300
15	A	6	6	1.00	10	0.600
16	A	7	6	1.00	10	0.600
17	A	9	8	1.00	10	0.800
18	A	8	7	1.00	10	0.700
19	A	7	6	1.00	10	0.600
20	A	6	6	1.00	8	0.750
21	A	5	5	1.00	6	0.833
22	A	14	8	1.00	10	0.800
23	A	6	6	1.00	10	0.600
24	A	8	8	1.00	10	0.800

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	9	9	1.00	10	0.900
26	A	10	9	1.00	10	0.900
27	A	20	9	1.00	12	0.750
28	A	17	9	1.00	12	0.750
29	A	11	8	1.00	10	0.800
30	A	8	6	1.00	8	0.750
31	A	17	9	1.00	12	0.750
32	A	12	8	1.00	12	0.667
33	A	25	14	1.00	12	1.167
34	A	16	12	1.00	10	1.200
35	A	10	7	1.00	8	0.875
36	A	20	10	1.00	12	0.833
37	A	14	9	1.00	12	0.750
38	A	6	6	1.00	12	0.500
39	A	6	6	1.00	14	0.429
40	A	7	7	1.00	10	0.700
41	A	6	4	1.00	10	0.400
42	A	6	4	1.00	8	0.500
43	A	5	3	1.00	6	0.500
44	A	6	5	1.00	10	0.500
45	A	3	3	1.00	10	0.300
46	A	5	4	1.00	10	0.400
47	A	6	4	1.00	10	0.400
48	A	6	4	1.00	10	0.400
49	A	8	8	1.00	19	0.421

CHAPTER 3

LISTING OF INTEGRALS

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Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\csc^{-1}(ax^5)}{x}\,dx = \frac{1}{10}i\csc^{-1}(ax^5)^2 - \frac{1}{5}\csc^{-1}(ax^5)\log\left(1-e^{2i\csc^{-1}(ax^5)}\right) + \frac{1}{10}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(ax^5)}\right)$$

```
[Out] 1/10*I*arccsc(a*x^5)^2-1/5*arccsc(a*x^5)*ln(1-(I/a/x^5+(1-1/a^2/x^10)^(1/2)
)^2)+1/10*I*polylog(2,(I/a/x^5+(1-1/a^2/x^10)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5327, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{\csc^{-1}(ax^5)}{x}\,dx = \frac{1}{10}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(ax^5)}\right) + \frac{1}{10}i\csc^{-1}(ax^5)^2 - \frac{1}{5}\csc^{-1}(ax^5)\log\left(1-e^{2i\csc^{-1}(ax^5)}\right)$$

```
[In] Int[ArcCsc[a*x^5]/x,x]
```

```
[Out] (I/10)*ArcCsc[a*x^5]^2 - (ArcCsc[a*x^5]*Log[1 - E^((2*I)*ArcCsc[a*x^5])])/5
 + (I/10)*PolyLog[2, E^((2*I)*ArcCsc[a*x^5])]
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5327

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{\csc^{-1}(ax)}{x}\,dx, x, x^5\right)\\
&= -\left(\frac{1}{5}\text{Subst}\left(\int \frac{\arcsin\left(\frac{x}{a}\right)}{x}\,dx, x, \frac{1}{x^5}\right)\right)\\
&= -\left(\frac{1}{5}\text{Subst}\left(\int x\cot(x)\,dx, x, \csc^{-1}\left(ax^5\right)\right)\right)\\
&= \frac{1}{10}i\csc^{-1}\left(ax^5\right)^2 + \frac{2}{5}i\text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \csc^{-1}\left(ax^5\right)\right)
\end{aligned}
$$

$$= \frac{1}{10} i \csc^{-1}\left(ax^5\right)^2 - \frac{1}{5}\csc^{-1}\left(ax^5\right)\log\left(1 - e^{2i\csc^{-1}(ax^5)}\right) + \frac{1}{5}\text{Subst}\left(\int \log\left(1 - e^{2ix}\right)\,dx, x, \csc^{-1}\left(ax^5\right)\right)$$

$$= \frac{1}{10} i \csc^{-1}\left(ax^5\right)^2 - \frac{1}{5}\csc^{-1}\left(ax^5\right)\log\left(1 - e^{2i\csc^{-1}(ax^5)}\right) - \frac{1}{10} i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(ax^5)}\right)$$

$$= \frac{1}{10} i \csc^{-1}\left(ax^5\right)^2 - \frac{1}{5}\csc^{-1}\left(ax^5\right)\log\left(1 - e^{2i\csc^{-1}(ax^5)}\right) + \frac{1}{10} i\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(ax^5)}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{\csc^{-1}\left(ax^5\right)}{x}\,dx = \frac{1}{10} i\left(\csc^{-1}\left(ax^5\right)\left(\csc^{-1}\left(ax^5\right) + 2i\log\left(1 - e^{2i\csc^{-1}(ax^5)}\right)\right) + \text{PolyLog}\left(2, e^{2i\csc^{-1}(ax^5)}\right)\right)$$

```
[In] Integrate[ArcCsc[a*x^5]/x,x]
```

```
[Out] (I/10)*(ArcCsc[a*x^5]*(ArcCsc[a*x^5] + (2*I)*Log[1 - E^((2*I)*ArcCsc[a*x^5]
)]) + PolyLog[2, E^((2*I)*ArcCsc[a*x^5])])
```

Maple [F]

$$\int \frac{\operatorname{arccsc}\left(a\,x^5\right)}{x}dx$$

```
[In] int(arccsc(a*x^5)/x,x)
```

```
[Out] int(arccsc(a*x^5)/x,x)
```

Fricas [F]

$$\int \frac{\csc^{-1}\left(ax^5\right)}{x}\,dx = \int \frac{\operatorname{arccsc}\left(ax^5\right)}{x}\,dx$$

```
[In] integrate(arccsc(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arccsc(a*x^5)/x, x)
```

Sympy [F]

$$\int \frac{\csc^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{acsc}(ax^5)}{x}\,dx$$

```
[In] integrate(acsc(a*x**5)/x,x)
```

```
[Out] Integral(acsc(a*x**5)/x, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arccsc}(ax^5)}{x}\,dx$$

```
[In] integrate(arccsc(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] 5*a^2*integrate(sqrt(a*x^5 + 1)*sqrt(a*x^5 - 1)*log(x)/(a^4*x^11 - a^2*x),
x) - 5*I*a^2*integrate(log(x)/(a^4*x^11 - a^2*x), x) + (arctan2(1, sqrt(a*x
^5 + 1)*sqrt(a*x^5 - 1)) + I*log(a))*log(x) - 1/2*I*log(a^2*x^10)*log(x) +
1/2*I*log(a*x^5 + 1)*log(x) + 1/2*I*log(-a*x^5 + 1)*log(x) + 5/2*I*log(x)^2
 + 1/10*I*dilog(a*x^5) + 1/10*I*dilog(-a*x^5)
```

Giac [F]

$$\int \frac{\csc^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arccsc}(ax^5)}{x}\,dx$$

```
[In] integrate(arccsc(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arccsc(a*x^5)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{\csc^{-1}(ax^5)}{x}\,dx = -\frac{\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{a\,x^5}\right)\,2{}\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{a\,x^5}\right)}{5} + \frac{\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{a\,x^5}\right)\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{10} + \frac{{\mathrm{asin}\left(\frac{1}{a\,x^5}\right)}^2\,1{}\mathrm{i}}{10}$$

```
[In] int(asin(1/(a*x^5))/x,x)
```

```
[Out] (polylog(2, exp(asin(1/(a*x^5))*2i))*1i)/10 - (log(1 - exp(asin(1/(a*x^5))*
2i))*asin(1/(a*x^5)))/5 + (asin(1/(a*x^5))^2*1i)/10
```

3.2 $\int x^3 \csc^{-1}\left(\sqrt{x}\right)\,dx$

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Mathematica [A] (verified) 46
Maple [A] (verified) 47
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Sympy [C] (verification not implemented) 47
Maxima [A] (verification not implemented) 48
Giac [B] (verification not implemented) 48
Mupad [F(-1)] 49

Optimal result

Integrand size = 10, antiderivative size = 58

$$\int x^3 \csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{\sqrt{-1+x}}{4} + \frac{1}{4}(-1+x)^{3/2} + \frac{3}{20}(-1+x)^{5/2} + \frac{1}{28}(-1+x)^{7/2} + \frac{1}{4}x^4 \csc^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/4*(-1+x)^(3/2)+3/20*(-1+x)^(5/2)+1/28*(-1+x)^(7/2)+1/4*x^4*arccsc(x^(1/2)
)+1/4*(-1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5379, 12, 45}

$$\int x^3 \csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{4}x^4 \csc^{-1}\left(\sqrt{x}\right) + \frac{1}{28}(x-1)^{7/2} + \frac{3}{20}(x-1)^{5/2} + \frac{1}{4}(x-1)^{3/2} + \frac{\sqrt{x-1}}{4}$$

```
[In] Int[x^3*ArcCsc[Sqrt[x]],x]
```

```
[Out] Sqrt[-1 + x]/4 + (-1 + x)^(3/2)/4 + (3*(-1 + x)^(5/2))/20 + (-1 + x)^(7/2)/
28 + (x^4*ArcCsc[Sqrt[x]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5379

```
Int[((a_.) + ArcCsc[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCsc[u])/(d*(m + 1))), x] + Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^4\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{x^3}{2\sqrt{-1+x}}\,dx\\ &= \frac{1}{4}x^4\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\int\frac{x^3}{\sqrt{-1+x}}\,dx\\ &= \frac{1}{4}x^4\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\int\left(\frac{1}{\sqrt{-1+x}}+3\sqrt{-1+x}+3(-1+x)^{3/2}+(-1+x)^{5/2}\right)dx\\ &= \frac{\sqrt{-1+x}}{4}+\frac{1}{4}(-1+x)^{3/2}+\frac{3}{20}(-1+x)^{5/2}+\frac{1}{28}(-1+x)^{7/2}+\frac{1}{4}x^4\csc^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int x^3\csc^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{140}\sqrt{-1+x}\left(16+8x+6x^2+5x^3\right)+\frac{1}{4}x^4\csc^{-1}\left(\sqrt{x}\right)$$

```
[In] Integrate[x^3*ArcCsc[Sqrt[x]],x]
[Out] (Sqrt[-1 + x]*(16 + 8*x + 6*x^2 + 5*x^3))/140 + (x^4*ArcCsc[Sqrt[x]])/4
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

method	result	size
parts	$\frac{x^{4} \operatorname{arccsc}\left(\sqrt{x}\right)}{4}+\frac{\sqrt{\frac{x-1}{x}}\, \sqrt{x}\, \left(5 x^{3}+6 x^{2}+8 x+16\right)}{140}$	40
derivativedivides	$\frac{x^{4} \operatorname{arccsc}\left(\sqrt{x}\right)}{4}+\frac{\left(x-1\right) \left(5 x^{3}+6 x^{2}+8 x+16\right)}{140 \sqrt{\frac{x-1}{x}}\, \sqrt{x}}$	43
default	$\frac{x^{4} \operatorname{arccsc}\left(\sqrt{x}\right)}{4}+\frac{\left(x-1\right) \left(5 x^{3}+6 x^{2}+8 x+16\right)}{140 \sqrt{\frac{x-1}{x}}\, \sqrt{x}}$	43

```
[In] int(x^3*arccsc(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*arccsc(x^(1/2))+1/140*((x-1)/x)^(1/2)*x^(1/2)*(5*x^3+6*x^2+8*x+16)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int x^3 \csc^{-1}\left(\sqrt{x}\right)\, dx = \frac{1}{4}\, x^{4} \operatorname{arccsc}\left(\sqrt{x}\right) + \frac{1}{140}\, {\left(5\, x^{3} + 6\, x^{2} + 8\, x + 16\right)} \sqrt{x - 1}$$

```
[In] integrate(x^3*arccsc(x^(1/2)),x, algorithm="fricas")
[Out] 1/4*x^4*arccsc(sqrt(x)) + 1/140*(5*x^3 + 6*x^2 + 8*x + 16)*sqrt(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 63.93 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.05

$$\int x^3 \csc^{-1}\left(\sqrt{x}\right)\, dx = \frac{x^{4} \operatorname{acsc}{\left(\sqrt{x} \right)}}{4} + \frac{\begin{cases} \frac{2 x^{3} \sqrt{x - 1}}{7} + \frac{12 x^{2} \sqrt{x - 1}}{35} + \frac{16 x \sqrt{x - 1}}{35} + \frac{32 \sqrt{x - 1}}{35} & \text{for}\: \left|{x}\right| > 1 \\\frac{2 i x^{3} \sqrt{1 - x}}{7} + \frac{12 i x^{2} \sqrt{1 - x}}{35} + \frac{16 i x \sqrt{1 - x}}{35} + \frac{32 i \sqrt{1 - x}}{35} & \text{otherwise} \end{cases}}{8}$$

```
[In] integrate(x**3*acsc(x**(1/2)),x)
[Out] x**4*acsc(sqrt(x))/4 + Piecewise((2*x**3*sqrt(x - 1)/7 + 12*x**2*sqrt(x - 1
)/35 + 16*x*sqrt(x - 1)/35 + 32*sqrt(x - 1)/35, Abs(x) > 1), (2*I*x**3*sqrt
(1 - x)/7 + 12*I*x**2*sqrt(1 - x)/35 + 16*I*x*sqrt(1 - x)/35 + 32*I*sqrt(1
- x)/35, True))/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int x^3 \csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{28}\,x^{\frac{7}{2}}\left(-\frac{1}{x}+1\right)^{\frac{7}{2}} + \frac{3}{20}\,x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}} + \frac{1}{4}\,x^4 \operatorname{arccsc}\left(\sqrt{x}\right) + \frac{1}{4}\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}} + \frac{1}{4}\,\sqrt{x}\sqrt{-\frac{1}{x}+1}$$

```
[In] integrate(x^3*arccsc(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/28*x^(7/2)*(-1/x + 1)^(7/2) + 3/20*x^(5/2)*(-1/x + 1)^(5/2) + 1/4*x^4*arc
csc(sqrt(x)) + 1/4*x^(3/2)*(-1/x + 1)^(3/2) + 1/4*sqrt(x)*sqrt(-1/x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.62

$$\int x^3 \csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{3584}\,x^{\frac{7}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^7 + \frac{7}{2560}\,x^{\frac{5}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^5 + \frac{1}{4}\,x^4 \arcsin\left(\frac{1}{\sqrt{x}}\right) + \frac{7}{512}\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3 + \frac{35}{512}\,\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right) - \frac{1225\,x^3\left(\sqrt{-\frac{1}{x}+1}-1\right)^6 + 245\,x^2\left(\sqrt{-\frac{1}{x}+1}-1\right)^4 + 49\,x\left(\sqrt{-\frac{1}{x}+1}-1\right)^2 + 5}{17920\,x^{\frac{7}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^7}$$

```
[In] integrate(x^3*arccsc(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3584*x^(7/2)*(sqrt(-1/x + 1) - 1)^7 + 7/2560*x^(5/2)*(sqrt(-1/x + 1) - 1)
^5 + 1/4*x^4*arcsin(1/sqrt(x)) + 7/512*x^(3/2)*(sqrt(-1/x + 1) - 1)^3 + 35/
512*sqrt(x)*(sqrt(-1/x + 1) - 1) - 1/17920*(1225*x^3*(sqrt(-1/x + 1) - 1)^6
 + 245*x^2*(sqrt(-1/x + 1) - 1)^4 + 49*x*(sqrt(-1/x + 1) - 1)^2 + 5)/(x^(7/
2)*(sqrt(-1/x + 1) - 1)^7)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \csc^{-1}\left(\sqrt{x}\right) \, dx = \int x^3 \operatorname{asin}\left(\frac{1}{\sqrt{x}}\right) \, dx$$

```
[In] int(x^3*asin(1/x^(1/2)),x)
[Out] int(x^3*asin(1/x^(1/2)), x)
```

3.3 $\int x^2 \csc^{-1}\left(\sqrt{x}\right)\,dx$

Optimal result 50
Rubi [A] (verified) 50
Mathematica [A] (verified) 51
Maple [A] (verified) 51
Fricas [A] (verification not implemented) 52
Sympy [C] (verification not implemented) 52
Maxima [A] (verification not implemented) 53
Giac [B] (verification not implemented) 53
Mupad [F(-1)] 54

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int x^2 \csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{\sqrt{-1+x}}{3} + \frac{2}{9}(-1+x)^{3/2} + \frac{1}{15}(-1+x)^{5/2} + \frac{1}{3}x^3 \csc^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2/9*(-1+x)^(3/2)+1/15*(-1+x)^(5/2)+1/3*x^3*arccsc(x^(1/2))+1/3*(-1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5379, 12, 45}

$$\int x^2 \csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{3}x^3 \csc^{-1}\left(\sqrt{x}\right) + \frac{1}{15}(x-1)^{5/2} + \frac{2}{9}(x-1)^{3/2} + \frac{\sqrt{x-1}}{3}$$

```
[In] Int[x^2*ArcCsc[Sqrt[x]],x]
```

```
[Out] Sqrt[-1 + x]/3 + (2*(-1 + x)^(3/2))/9 + (-1 + x)^(5/2)/15 + (x^3*ArcCsc[Sqr
t[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5379

```
Int[((a_.) + ArcCsc[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCsc[u])/(d*(m + 1))), x] + Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\int\frac{x^2}{2\sqrt{-1+x}}\,dx\\
&= \frac{1}{3}x^3\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int\frac{x^2}{\sqrt{-1+x}}\,dx\\
&= \frac{1}{3}x^3\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int\left(\frac{1}{\sqrt{-1+x}}+2\sqrt{-1+x}+(-1+x)^{3/2}\right)dx\\
&= \frac{\sqrt{-1+x}}{3}+\frac{2}{9}(-1+x)^{3/2}+\frac{1}{15}(-1+x)^{5/2}+\frac{1}{3}x^3\csc^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int x^2\csc^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{45}\sqrt{-1+x}\left(8+4x+3x^2\right)+\frac{1}{3}x^3\csc^{-1}\left(\sqrt{x}\right)$$

```
[In] Integrate[x^2*ArcCsc[Sqrt[x]],x]
[Out] (Sqrt[-1 + x]*(8 + 4*x + 3*x^2))/45 + (x^3*ArcCsc[Sqrt[x]])/3
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
parts	$\frac{x^3 \operatorname{arccsc}\left(\sqrt{x}\right)}{3}+\frac{\sqrt{\frac{x-1}{x}}\,\sqrt{x}\,\left(3x^2+4x+8\right)}{45}$	35
derivativedivides	$\frac{x^3 \operatorname{arccsc}\left(\sqrt{x}\right)}{3}+\frac{(x-1)\left(3x^2+4x+8\right)}{45\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	38
default	$\frac{x^3 \operatorname{arccsc}\left(\sqrt{x}\right)}{3}+\frac{(x-1)\left(3x^2+4x+8\right)}{45\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	38

```
[In] int(x^2*arccsc(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arccsc(x^(1/2))+1/45*((x-1)/x)^(1/2)*x^(1/2)*(3*x^2+4*x+8)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int x^2 \csc^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{3}\,x^3 \operatorname{arccsc}\left(\sqrt{x}\right)+\frac{1}{45}\left(3\,x^2+4\,x+8\right)\sqrt{x-1}$$

```
[In] integrate(x^2*arccsc(x^(1/2)),x, algorithm="fricas")
[Out] 1/3*x^3*arccsc(sqrt(x)) + 1/45*(3*x^2 + 4*x + 8)*sqrt(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int x^2 \csc^{-1}\left(\sqrt{x}\right)\,dx=\frac{x^3 \operatorname{acsc}\left(\sqrt{x}\right)}{3}+\frac{\begin{cases}\frac{2x^2\sqrt{x-1}}{5}+\frac{8x\sqrt{x-1}}{15}+\frac{16\sqrt{x-1}}{15} & \text{for } |x|>1\\ \frac{2ix^2\sqrt{1-x}}{5}+\frac{8ix\sqrt{1-x}}{15}+\frac{16i\sqrt{1-x}}{15} & \text{otherwise}\end{cases}}{6}$$

```
[In] integrate(x**2*acsc(x**(1/2)),x)
[Out] x**3*acsc(sqrt(x))/3 + Piecewise((2*x**2*sqrt(x - 1)/5 + 8*x*sqrt(x - 1)/15
 + 16*sqrt(x - 1)/15, Abs(x) > 1), (2*I*x**2*sqrt(1 - x)/5 + 8*I*x*sqrt(1 -
 x)/15 + 16*I*sqrt(1 - x)/15, True))/6
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int x^2 \csc^{-1}\left(\sqrt{x}\right)\, dx = \frac{1}{15}\, x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}} + \frac{1}{3}\, x^3 \operatorname{arccsc}\left(\sqrt{x}\right) + \frac{2}{9}\, x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{x}\sqrt{-\frac{1}{x}+1}$$

```
[In] integrate(x^2*arccsc(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/15*x^(5/2)*(-1/x + 1)^(5/2) + 1/3*x^3*arccsc(sqrt(x)) + 2/9*x^(3/2)*(-1/x
 + 1)^(3/2) + 1/3*sqrt(x)*sqrt(-1/x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(31) = 62.

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.47

$$\int x^2 \csc^{-1}\left(\sqrt{x}\right)\, dx = \frac{1}{480}\, x^{\frac{5}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^5 + \frac{5}{288}\, x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3 + \frac{1}{3}\, x^3 \arcsin\left(\frac{1}{\sqrt{x}}\right) + \frac{5}{48}\,\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right) - \frac{150\, x^2\left(\sqrt{-\frac{1}{x}+1}-1\right)^4 + 25\, x\left(\sqrt{-\frac{1}{x}+1}-1\right)^2 + 3}{1440\, x^{\frac{5}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^5}$$

```
[In] integrate(x^2*arccsc(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/480*x^(5/2)*(sqrt(-1/x + 1) - 1)^5 + 5/288*x^(3/2)*(sqrt(-1/x + 1) - 1)^3
 + 1/3*x^3*arcsin(1/sqrt(x)) + 5/48*sqrt(x)*(sqrt(-1/x + 1) - 1) - 1/1440*(
150*x^2*(sqrt(-1/x + 1) - 1)^4 + 25*x*(sqrt(-1/x + 1) - 1)^2 + 3)/(x^(5/2)*
(sqrt(-1/x + 1) - 1)^5)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \csc^{-1}\left(\sqrt{x}\right)\,dx = \int x^2\,\mathrm{asin}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

```
[In] int(x^2*asin(1/x^(1/2)),x)
[Out] int(x^2*asin(1/x^(1/2)), x)
```

3.4 $\int x \csc^{-1}\left(\sqrt{x}\right) \, dx$

Optimal result 55
Rubi [A] (verified) 55
Mathematica [A] (verified) 56
Maple [A] (verified) 56
Fricas [A] (verification not implemented) 57
Sympy [C] (verification not implemented) 57
Maxima [A] (verification not implemented) 57
Giac [B] (verification not implemented) 58
Mupad [F(-1)] 58

Optimal result

Integrand size = 8, antiderivative size = 36

$$\int x \csc^{-1}\left(\sqrt{x}\right) \, dx = \frac{\sqrt{-1+x}}{2} + \frac{1}{6}(-1+x)^{3/2} + \frac{1}{2}x^2 \csc^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/6*(-1+x)^(3/2)+1/2*x^2*arccsc(x^(1/2))+1/2*(-1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5379, 12, 45}

$$\int x \csc^{-1}\left(\sqrt{x}\right) \, dx = \frac{1}{2}x^2 \csc^{-1}\left(\sqrt{x}\right) + \frac{1}{6}(x-1)^{3/2} + \frac{\sqrt{x-1}}{2}$$

```
[In] Int[x*ArcCsc[Sqrt[x]],x]
[Out] Sqrt[-1 + x]/2 + (-1 + x)^(3/2)/6 + (x^2*ArcCsc[Sqrt[x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5379

```
Int[((a_.) + ArcCsc[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCsc[u])/(d*(m + 1))), x] + Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int\frac{x}{2\sqrt{-1+x}}\,dx\\
&= \frac{1}{2}x^2\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{x}{\sqrt{-1+x}}\,dx\\
&= \frac{1}{2}x^2\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\left(\frac{1}{\sqrt{-1+x}}+\sqrt{-1+x}\right)dx\\
&= \frac{\sqrt{-1+x}}{2}+\frac{1}{6}(-1+x)^{3/2}+\frac{1}{2}x^2\csc^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int x\csc^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{6}\left(\sqrt{-1+x}(2+x)+3x^2\csc^{-1}\left(\sqrt{x}\right)\right)$$

```
[In] Integrate[x*ArcCsc[Sqrt[x]],x]
[Out] (Sqrt[-1 + x]*(2 + x) + 3*x^2*ArcCsc[Sqrt[x]])/6
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
parts	$\frac{x^2\operatorname{arccsc}\left(\sqrt{x}\right)}{2}+\frac{\sqrt{\frac{x-1}{x}}\,\sqrt{x}\,(2+x)}{6}$	28
derivativedivides	$\frac{x^2\operatorname{arccsc}\left(\sqrt{x}\right)}{2}+\frac{(x-1)(2+x)}{6\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	31
default	$\frac{x^2\operatorname{arccsc}\left(\sqrt{x}\right)}{2}+\frac{(x-1)(2+x)}{6\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	31

```
[In] int(x*arccsc(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*arccsc(x^(1/2))+1/6*((x-1)/x)^(1/2)*x^(1/2)*(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int x\csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\,\mathrm{arccsc}\left(\sqrt{x}\right) + \frac{1}{6}\,(x+2)\sqrt{x-1}$$

```
[In] integrate(x*arccsc(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2*arccsc(sqrt(x)) + 1/6*(x + 2)*sqrt(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.87 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int x\csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{x^2\,\mathrm{acsc}\left(\sqrt{x}\right)}{2} + \frac{\begin{cases}\frac{2x\sqrt{x-1}}{3}+\frac{4\sqrt{x-1}}{3} & \text{for } |x|>1\\ \frac{2ix\sqrt{1-x}}{3}+\frac{4i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}}{4}$$

```
[In] integrate(x*acsc(x**(1/2)),x)
[Out] x**2*acsc(sqrt(x))/2 + Piecewise((2*x*sqrt(x - 1)/3 + 4*sqrt(x - 1)/3, Abs(
x) > 1), (2*I*x*sqrt(1 - x)/3 + 4*I*sqrt(1 - x)/3, True))/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int x\csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{6}\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}} + \frac{1}{2}\,x^2\,\mathrm{arccsc}\left(\sqrt{x}\right) + \frac{1}{2}\,\sqrt{x}\sqrt{-\frac{1}{x}+1}$$

```
[In] integrate(x*arccsc(x^(1/2)),x, algorithm="maxima")
[Out] 1/6*x^(3/2)*(-1/x + 1)^(3/2) + 1/2*x^2*arccsc(sqrt(x)) + 1/2*sqrt(x)*sqrt(-
1/x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.22

$$\int x\csc^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{48}\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3+\frac{1}{2}\,x^2\arcsin\left(\frac{1}{\sqrt{x}}\right)+\frac{3}{16}\,\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right)-\frac{9\,x\left(\sqrt{-\frac{1}{x}+1}-1\right)^2+1}{48\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3}$$

```
[In] integrate(x*arccsc(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/48*x^(3/2)*(sqrt(-1/x + 1) - 1)^3 + 1/2*x^2*arcsin(1/sqrt(x)) + 3/16*sqrt
(x)*(sqrt(-1/x + 1) - 1) - 1/48*(9*x*(sqrt(-1/x + 1) - 1)^2 + 1)/(x^(3/2)*(
sqrt(-1/x + 1) - 1)^3)
```

Mupad [F(-1)]

Timed out.

$$\int x\csc^{-1}\left(\sqrt{x}\right)\,dx = \int x\,\mathrm{asin}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

```
[In] int(x*asin(1/x^(1/2)),x)
```

```
[Out] int(x*asin(1/x^(1/2)), x)
```

3.5 $\int \csc^{-1}\left(\sqrt{x}\right) \, dx$

Optimal result 59
Rubi [A] (verified) 59
Mathematica [A] (verified) 60
Maple [A] (verified) 60
Fricas [A] (verification not implemented) 61
Sympy [C] (verification not implemented) 61
Maxima [A] (verification not implemented) 61
Giac [B] (verification not implemented) 62
Mupad [B] (verification not implemented) 62

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \csc^{-1}\left(\sqrt{x}\right) \, dx = \sqrt{-1+x} + x\csc^{-1}\left(\sqrt{x}\right)$$

```
[Out] x*arccsc(x^(1/2))+(-1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5377, 12, 32}

$$\int \csc^{-1}\left(\sqrt{x}\right) \, dx = \sqrt{x-1} + x\csc^{-1}\left(\sqrt{x}\right)$$

```
[In] Int[ArcCsc[Sqrt[x]],x]
[Out] Sqrt[-1 + x] + x*ArcCsc[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5377

```
Int[ArcCsc[u_], x_Symbol] :> Simp[x*ArcCsc[u], x] + Dist[u/Sqrt[u^2], Int[S
implifyIntegrand[x*(D[u, x]/(u*Sqrt[u^2 - 1])), x], x], x] /; InverseFuncti
onFreeQ[u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= x\csc^{-1}\left(\sqrt{x}\right)+\int\frac{1}{2\sqrt{-1+x}}\,dx\\ &= x\csc^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int\frac{1}{\sqrt{-1+x}}\,dx\\ &= \sqrt{-1+x}+x\csc^{-1}\left(\sqrt{x}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int\csc^{-1}\left(\sqrt{x}\right)\,dx=\sqrt{-1+x}+x\csc^{-1}\left(\sqrt{x}\right)$$

```
[In] Integrate[ArcCsc[Sqrt[x]],x]
[Out] Sqrt[-1 + x] + x*ArcCsc[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

method	result	size
parts	$x\,\operatorname{arccsc}\left(\sqrt{x}\right)+\sqrt{\frac{x-1}{x}}\,\sqrt{x}$	21
derivativedivides	$x\,\operatorname{arccsc}\left(\sqrt{x}\right)+\frac{x-1}{\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	24
default	$x\,\operatorname{arccsc}\left(\sqrt{x}\right)+\frac{x-1}{\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	24

```
[In] int(arccsc(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arccsc(x^(1/2))+((x-1)/x)^(1/2)*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \csc^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{arccsc}\left(\sqrt{x}\right) + \sqrt{x-1}$$

```
[In] integrate(arccsc(x^(1/2)),x, algorithm="fricas")
[Out] x*arccsc(sqrt(x)) + sqrt(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \csc^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{acsc}\left(\sqrt{x}\right) + \frac{\begin{cases} 2\sqrt{x-1} & \text{for } |x| > 1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate(acsc(x**(1/2)),x)
[Out] x*acsc(sqrt(x)) + Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x),
True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \csc^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{arccsc}\left(\sqrt{x}\right) + \sqrt{x}\sqrt{-\frac{1}{x}+1}$$

```
[In] integrate(arccsc(x^(1/2)),x, algorithm="maxima")
[Out] x*arccsc(sqrt(x)) + sqrt(x)*sqrt(-1/x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int \csc^{-1}\left(\sqrt{x}\right)\,dx = x \arcsin\left(\frac{1}{\sqrt{x}}\right) + \frac{1}{2}\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right) - \frac{1}{2\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right)}$$

```
[In] integrate(arccsc(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*arcsin(1/sqrt(x)) + 1/2*sqrt(x)*(sqrt(-1/x + 1) - 1) - 1/2/(sqrt(x)*(sqrt
(-1/x + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \csc^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{asin}\left(\frac{1}{\sqrt{x}}\right) + \sqrt{x}\,\sqrt{1-\frac{1}{x}}$$

```
[In] int(asin(1/x^(1/2)),x)
```

```
[Out] x*asin(1/x^(1/2)) + x^(1/2)*(1 - 1/x)^(1/2)
```

3.6 $\int \frac{\csc^{-1}(\sqrt{x})}{x}\,dx$

Optimal result 63
Rubi [A] (verified) 63
Mathematica [A] (verified) 65
Maple [A] (verified) 65
Fricas [F] 66
Sympy [F] 66
Maxima [F] 66
Giac [F(-2)] 66
Mupad [B] (verification not implemented) 67

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x}\,dx = i\csc^{-1}\left(\sqrt{x}\right)^2 - 2\csc^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2i\csc^{-1}\left(\sqrt{x}\right)}\right) + i\,\text{PolyLog}\left(2, e^{2i\csc^{-1}\left(\sqrt{x}\right)}\right)$$

```
[Out] I*arccsc(x^(1/2))^2-2*arccsc(x^(1/2))*ln(1-(I/x^(1/2)+(1-1/x)^(1/2))^2)+I*p
olylog(2,(I/x^(1/2)+(1-1/x)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5327, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x}\,dx = i\,\text{PolyLog}\left(2, e^{2i\csc^{-1}\left(\sqrt{x}\right)}\right) + i\csc^{-1}\left(\sqrt{x}\right)^2 - 2\csc^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2i\csc^{-1}\left(\sqrt{x}\right)}\right)$$

```
[In] Int[ArcCsc[Sqrt[x]]/x,x]
```

```
[Out] I*ArcCsc[Sqrt[x]]^2 - 2*ArcCsc[Sqrt[x]]*Log[1 - E^((2*I)*ArcCsc[Sqrt[x]])]
+ I*PolyLog[2, E^((2*I)*ArcCsc[Sqrt[x]])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5327

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{\csc^{-1}(x)}{x}\,dx, x, \sqrt{x}\right)\\
&= -\left(2\text{Subst}\left(\int \frac{\arcsin(x)}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\left(2\text{Subst}\left(\int x\cot(x)\,dx, x, \arcsin\left(\frac{1}{\sqrt{x}}\right)\right)\right)\\
&= i\arcsin\left(\frac{1}{\sqrt{x}}\right)^2 + 4i\text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \arcsin\left(\frac{1}{\sqrt{x}}\right)\right)
\end{aligned}$$

$$= i \arcsin\left(\frac{1}{\sqrt{x}}\right)^2 - 2\arcsin\left(\frac{1}{\sqrt{x}}\right)\log\left(1 - e^{2i\arcsin\left(\frac{1}{\sqrt{x}}\right)}\right) + 2\text{Subst}\left(\int \log\left(1 - e^{2ix}\right)\,dx, x, \arcsin\left(\frac{1}{\sqrt{x}}\right)\right)$$

$$= i \arcsin\left(\frac{1}{\sqrt{x}}\right)^2 - 2\arcsin\left(\frac{1}{\sqrt{x}}\right)\log\left(1 - e^{2i\arcsin\left(\frac{1}{\sqrt{x}}\right)}\right) - i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\arcsin\left(\frac{1}{\sqrt{x}}\right)}\right)$$

$$= i \arcsin\left(\frac{1}{\sqrt{x}}\right)^2 - 2\arcsin\left(\frac{1}{\sqrt{x}}\right)\log\left(1 - e^{2i\arcsin\left(\frac{1}{\sqrt{x}}\right)}\right) + i\,\text{PolyLog}\left(2, e^{2i\arcsin\left(\frac{1}{\sqrt{x}}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x}\,dx = i\left(\csc^{-1}\left(\sqrt{x}\right)\left(\csc^{-1}\left(\sqrt{x}\right) + 2i\log\left(1 - e^{2i\csc^{-1}\left(\sqrt{x}\right)}\right)\right) + \text{PolyLog}\left(2, e^{2i\csc^{-1}\left(\sqrt{x}\right)}\right)\right)$$

```
[In] Integrate[ArcCsc[Sqrt[x]]/x,x]
```

```
[Out] I*(ArcCsc[Sqrt[x]]*(ArcCsc[Sqrt[x]] + (2*I)*Log[1 - E^((2*I)*ArcCsc[Sqrt[x]
])]) + PolyLog[2, E^((2*I)*ArcCsc[Sqrt[x]])])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.88

method	result
derivativedivides	$i\,\mathrm{arccsc}\left(\sqrt{x}\right)^2 - 2\,\mathrm{arccsc}\left(\sqrt{x}\right)\ln\left(1 - \frac{i}{\sqrt{x}} - \sqrt{1 - \frac{1}{x}}\right) + 2i\,\mathrm{polylog}\left(2, \frac{i}{\sqrt{x}} + \sqrt{1 - \frac{1}{x}}\right)$
default	$i\,\mathrm{arccsc}\left(\sqrt{x}\right)^2 - 2\,\mathrm{arccsc}\left(\sqrt{x}\right)\ln\left(1 - \frac{i}{\sqrt{x}} - \sqrt{1 - \frac{1}{x}}\right) + 2i\,\mathrm{polylog}\left(2, \frac{i}{\sqrt{x}} + \sqrt{1 - \frac{1}{x}}\right)$

```
[In] int(arccsc(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] I*arccsc(x^(1/2))^2-2*arccsc(x^(1/2))*ln(1-I/x^(1/2)-(1-1/x)^(1/2))+2*I*pol
ylog(2,I/x^(1/2)+(1-1/x)^(1/2))-2*arccsc(x^(1/2))*ln(1+I/x^(1/2)+(1-1/x)^(1
/2))+2*I*polylog(2,-I/x^(1/2)-(1-1/x)^(1/2))
```

Fricas [F]

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{arccsc}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(arccsc(x^(1/2))/x,x, algorithm="fricas")
[Out] integral(arccsc(sqrt(x))/x, x)
```

Sympy [F]

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{acsc}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(acsc(x**(1/2))/x,x)
[Out] Integral(acsc(sqrt(x))/x, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{arccsc}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(arccsc(x^(1/2))/x,x, algorithm="maxima")
[Out] integrate(arccsc(sqrt(x))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(arccsc(x^(1/2))/x,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: Inval
id series expansion: non tractable function asin at +infinity
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x}\,dx = \mathrm{polylog}\left(2, \mathrm{e}^{\mathrm{asin}\left(\frac{1}{\sqrt{x}}\right)\,2\mathrm{i}}\right)\,1\mathrm{i} + \mathrm{asin}\left(\frac{1}{\sqrt{x}}\right)^2\,1\mathrm{i} - 2\,\ln\left(1 - \mathrm{e}^{\mathrm{asin}\left(\frac{1}{\sqrt{x}}\right)\,2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] int(asin(1/x^(1/2))/x,x)
```

```
[Out] polylog(2, exp(asin(1/x^(1/2))*2i))*1i + asin(1/x^(1/2))^2*1i - 2*log(1 - e
xp(asin(1/x^(1/2))*2i))*asin(1/x^(1/2))
```

3.7 $\int \frac{\csc^{-1}(\sqrt{x})}{x^2} dx$

Optimal result . . . 68
Rubi [A] (verified) . . . 68
Mathematica [A] (verified) . . . 70
Maple [A] (verified) . . . 70
Fricas [A] (verification not implemented) . . . 70
Sympy [C] (verification not implemented) . . . 71
Maxima [A] (verification not implemented) . . . 71
Giac [A] (verification not implemented) . . . 71
Mupad [B] (verification not implemented) . . . 72

Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^2} dx = -\frac{\sqrt{-1+x}}{2x} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\arctan\left(\sqrt{-1+x}\right)$$

```
[Out] -arccsc(x^(1/2))/x-1/2*arctan((-1+x)^(1/2))-1/2*(-1+x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5379, 12, 44, 65, 209}

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^2} dx = -\frac{1}{2}\arctan\left(\sqrt{x-1}\right) - \frac{\sqrt{x-1}}{2x} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{x}$$

```
[In] Int[ArcCsc[Sqrt[x]]/x^2,x]
```

```
[Out] -1/2*Sqrt[-1 + x]/x - ArcCsc[Sqrt[x]]/x - ArcTan[Sqrt[-1 + x]]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5379

```
Int[((a_.) + ArcCsc[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCsc[u])/(d*(m + 1))), x] + Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\csc^{-1}\left(\sqrt{x}\right)}{x} - \int \frac{1}{2\sqrt{-1+x}x^2}\,dx \\
&= -\frac{\csc^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\int \frac{1}{\sqrt{-1+x}x^2}\,dx \\
&= -\frac{\sqrt{-1+x}}{2x} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{4}\int \frac{1}{\sqrt{-1+x}x}\,dx \\
&= -\frac{\sqrt{-1+x}}{2x} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\right) \\
&= -\frac{\sqrt{-1+x}}{2x} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\arctan\left(\sqrt{-1+x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\sqrt{-1+x}+2\csc^{-1}\left(\sqrt{x}\right)-x\arcsin\left(\frac{1}{\sqrt{x}}\right)}{2x}$$

```
[In] Integrate[ArcCsc[Sqrt[x]]/x^2,x]
[Out] -1/2*(Sqrt[-1 + x] + 2*ArcCsc[Sqrt[x]] - x*ArcSin[1/Sqrt[x]])/x
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

method	result	size
parts	$-\frac{\operatorname{arccsc}(\sqrt{x})}{x}-\frac{\sqrt{\frac{x-1}{x}}\left(\arctan(\sqrt{x-1})x+\sqrt{x-1}\right)}{2\sqrt{x}\sqrt{x-1}}$	44
derivativedivides	$-\frac{\operatorname{arccsc}(\sqrt{x})}{x}+\frac{\sqrt{x-1}\left(\arctan\left(\frac{1}{\sqrt{x-1}}\right)x-\sqrt{x-1}\right)}{2\sqrt{\frac{x-1}{x}}\,x^{\frac{3}{2}}}$	46
default	$-\frac{\operatorname{arccsc}(\sqrt{x})}{x}+\frac{\sqrt{x-1}\left(\arctan\left(\frac{1}{\sqrt{x-1}}\right)x-\sqrt{x-1}\right)}{2\sqrt{\frac{x-1}{x}}\,x^{\frac{3}{2}}}$	46

```
[In] int(arccsc(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] -arccsc(x^(1/2))/x-1/2*((x-1)/x)^(1/2)/x^(1/2)*(arctan((x-1)^(1/2))*x+(x-1)
^(1/2))/(x-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \frac{(x-2)\operatorname{arccsc}\left(\sqrt{x}\right)-\sqrt{x-1}}{2\,x}$$

```
[In] integrate(arccsc(x^(1/2))/x^2,x, algorithm="fricas")
[Out] 1/2*((x - 2)*arccsc(sqrt(x)) - sqrt(x - 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.86 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\begin{cases} i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right) - \frac{i}{\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{i}{x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for}\ \frac{1}{|x|} > 1 \\ -\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right) + \frac{\sqrt{1-\frac{1}{x}}}{\sqrt{x}} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{acsc}\left(\sqrt{x}\right)}{x}$$

```
[In] integrate(acsc(x**(1/2))/x**2,x)
```

```
[Out] -Piecewise((I*acosh(1/sqrt(x)) - I/(sqrt(x)*sqrt(-1 + 1/x)) + I/(x**(3/2)*s
qrt(-1 + 1/x)), 1/Abs(x) > 1), (-asin(1/sqrt(x)) + sqrt(1 - 1/x)/sqrt(x), T
rue))/2 - acsc(sqrt(x))/x
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \frac{\sqrt{x}\sqrt{-\frac{1}{x}+1}}{2\left(x\left(\frac{1}{x}-1\right)-1\right)} - \frac{\operatorname{arccsc}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\arctan\left(\sqrt{x}\sqrt{-\frac{1}{x}+1}\right)$$

```
[In] integrate(arccsc(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x)*sqrt(-1/x + 1)/(x*(1/x - 1) - 1) - arccsc(sqrt(x))/x - 1/2*arct
an(sqrt(x)*sqrt(-1/x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\left(\frac{1}{x}-1\right)\arcsin\left(\frac{1}{\sqrt{x}}\right) - \frac{\sqrt{-\frac{1}{x}+1}}{2\sqrt{x}} - \frac{1}{2}\arcsin\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] integrate(arccsc(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -(1/x - 1)*arcsin(1/sqrt(x)) - 1/2*sqrt(-1/x + 1)/sqrt(x) - 1/2*arcsin(1/sq
rt(x))
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\sqrt{1-\frac{1}{x}}}{2\,\sqrt{x}} - \frac{\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)\,\left(\frac{2}{x}-1\right)}{2}$$

```
[In] int(asin(1/x^(1/2))/x^2,x)
[Out] - (1 - 1/x)^(1/2)/(2*x^(1/2)) - (asin(1/x^(1/2))*(2/x - 1))/2
```

3.8 $\int \frac{\csc^{-1}(\sqrt{x})}{x^3}\,dx$

Optimal result 73
Rubi [A] (verified) 73
Mathematica [A] (verified) 75
Maple [A] (verified) 75
Fricas [A] (verification not implemented) 75
Sympy [C] (verification not implemented) 76
Maxima [B] (verification not implemented) 76
Giac [A] (verification not implemented) 77
Mupad [F(-1)] 77

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\sqrt{-1+x}}{8x^2} - \frac{3\sqrt{-1+x}}{16x} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{3}{16}\arctan\left(\sqrt{-1+x}\right)$$

```
[Out] -1/2*arccsc(x^(1/2))/x^2-3/16*arctan((-1+x)^(1/2))-1/8*(-1+x)^(1/2)/x^2-3/1
6*(-1+x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5379, 12, 44, 65, 209}

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{3}{16}\arctan\left(\sqrt{x-1}\right) - \frac{\sqrt{x-1}}{8x^2} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{3\sqrt{x-1}}{16x}$$

```
[In] Int[ArcCsc[Sqrt[x]]/x^3,x]
```

```
[Out] -1/8*Sqrt[-1 + x]/x^2 - (3*Sqrt[-1 + x])/(16*x) - ArcCsc[Sqrt[x]]/(2*x^2) -
 (3*ArcTan[Sqrt[-1 + x]])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5379

```
Int[((a_.) + ArcCsc[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCsc[u])/(d*(m + 1))), x] + Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !Function
OfExponentialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\csc^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{2}\int \frac{1}{2\sqrt{-1+x}x^3}\,dx \\
&= -\frac{\csc^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{4}\int \frac{1}{\sqrt{-1+x}x^3}\,dx \\
&= -\frac{\sqrt{-1+x}}{8x^2} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{3}{16}\int \frac{1}{\sqrt{-1+x}x^2}\,dx \\
&= -\frac{\sqrt{-1+x}}{8x^2} - \frac{3\sqrt{-1+x}}{16x} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{3}{32}\int \frac{1}{\sqrt{-1+x}x}\,dx \\
&= -\frac{\sqrt{-1+x}}{8x^2} - \frac{3\sqrt{-1+x}}{16x} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{3}{16}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\right) \\
&= -\frac{\sqrt{-1+x}}{8x^2} - \frac{3\sqrt{-1+x}}{16x} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{3}{16}\arctan\left(\sqrt{-1+x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \left(-\frac{1}{8x^{3/2}} - \frac{3}{16\sqrt{x}}\right)\sqrt{\frac{-1+x}{x}} - \frac{\csc^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{3}{16}\arcsin\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] Integrate[ArcCsc[Sqrt[x]]/x^3,x]
[Out] (-1/8*1/x^(3/2) - 3/(16*Sqrt[x]))*Sqrt[(-1 + x)/x] - ArcCsc[Sqrt[x]]/(2*x^2
) + (3*ArcSin[1/Sqrt[x]])/16
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\operatorname{arccsc}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{x-1}\left(3\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^2-3\sqrt{x-1}\,x-2\sqrt{x-1}\right)}{16\sqrt{\frac{x-1}{x}}\,x^{\frac{5}{2}}}$	57
default	$-\frac{\operatorname{arccsc}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{x-1}\left(3\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^2-3\sqrt{x-1}\,x-2\sqrt{x-1}\right)}{16\sqrt{\frac{x-1}{x}}\,x^{\frac{5}{2}}}$	57
parts	$-\frac{\operatorname{arccsc}\left(\sqrt{x}\right)}{2x^2}-\frac{\sqrt{\frac{x-1}{x}}\left(3\arctan\left(\sqrt{x-1}\right)x^2+3\sqrt{x-1}\,x+2\sqrt{x-1}\right)}{16x^{\frac{3}{2}}\sqrt{x-1}}$	57

```
[In] int(arccsc(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*arccsc(x^(1/2))/x^2+1/16*(x-1)^(1/2)*(3*arctan(1/(x-1)^(1/2))*x^2-3*(x
-1)^(1/2)*x-2*(x-1)^(1/2))/((x-1)/x)^(1/2)/x^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.56

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{(3\,x^2-8)\operatorname{arccsc}\left(\sqrt{x}\right)-(3\,x+2)\sqrt{x-1}}{16\,x^2}$$

```
[In] integrate(arccsc(x^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/16*((3*x^2 - 8)*arccsc(sqrt(x)) - (3*x + 2)*sqrt(x - 1))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.70

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\begin{cases} \frac{3i\,\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4} - \frac{3i}{4\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{2x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for } \frac{1}{|x|} > 1 \\ -\frac{3\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4} + \frac{3}{4\sqrt{x}\sqrt{1-\frac{1}{x}}} - \frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{2x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}}{4} - \frac{\operatorname{acsc}\left(\sqrt{x}\right)}{2x^2}$$

```
[In] integrate(acsc(x**(1/2))/x**3,x)
```

```
[Out] -Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*
x**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-
3*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1 -
1/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))/4 - acsc(sqrt(x))/(2*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{3\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}} + 5\,\sqrt{x}\sqrt{-\frac{1}{x}+1}}{16\left(x^2\left(\frac{1}{x}-1\right)^2 - 2\,x\left(\frac{1}{x}-1\right) + 1\right)} - \frac{\operatorname{arccsc}\left(\sqrt{x}\right)}{2\,x^2} - \frac{3}{16}\arctan\left(\sqrt{x}\sqrt{-\frac{1}{x}+1}\right)$$

```
[In] integrate(arccsc(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*x^(3/2)*(-1/x + 1)^(3/2) + 5*sqrt(x)*sqrt(-1/x + 1))/(x^2*(1/x - 1
)^2 - 2*x*(1/x - 1) + 1) - 1/2*arccsc(sqrt(x))/x^2 - 3/16*arctan(sqrt(x)*sq
rt(-1/x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{1}{2}\left(\frac{1}{x}-1\right)^2 \arcsin\left(\frac{1}{\sqrt{x}}\right) - \left(\frac{1}{x}-1\right)\arcsin\left(\frac{1}{\sqrt{x}}\right) + \frac{\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}}{8\sqrt{x}} - \frac{5\sqrt{-\frac{1}{x}+1}}{16\sqrt{x}} - \frac{5}{16}\arcsin\left(\frac{1}{\sqrt{x}}\right)$$

```
[In] integrate(arccsc(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(1/x - 1)^2*arcsin(1/sqrt(x)) - (1/x - 1)*arcsin(1/sqrt(x)) + 1/8*(-1/
x + 1)^(3/2)/sqrt(x) - 5/16*sqrt(-1/x + 1)/sqrt(x) - 5/16*arcsin(1/sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \int \frac{\mathrm{asin}\left(\frac{1}{\sqrt{x}}\right)}{x^3}\,dx$$

```
[In] int(asin(1/x^(1/2))/x^3,x)
```

```
[Out] int(asin(1/x^(1/2))/x^3, x)
```

3.9 $\int x^2 \csc^{-1}\left(\frac{a}{x}\right) \, dx$

Optimal result 78
Rubi [A] (verified) 78
Mathematica [A] (verified) 79
Maple [A] (verified) 80
Fricas [A] (verification not implemented) 80
Sympy [A] (verification not implemented) 80
Maxima [A] (verification not implemented) 81
Giac [A] (verification not implemented) 81
Mupad [F(-1)] 81

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int x^2 \csc^{-1}\left(\frac{a}{x}\right) \, dx = \frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}} - \frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2} + \frac{1}{3}x^3 \arcsin\left(\frac{x}{a}\right)$$

```
[Out] -1/9*a^3*(1-x^2/a^2)^(3/2)+1/3*x^3*arcsin(x/a)+1/3*a^3*(1-x^2/a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5373, 4723, 272, 45}

$$\int x^2 \csc^{-1}\left(\frac{a}{x}\right) \, dx = -\frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2} + \frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{3}x^3 \arcsin\left(\frac{x}{a}\right)$$

```
[In] Int[x^2*ArcCsc[a/x],x]
```

```
[Out] (a^3*Sqrt[1 - x^2/a^2])/3 - (a^3*(1 - x^2/a^2)^(3/2))/9 + (x^3*ArcSin[x/a])
/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5373

```
Int[ArcCsc[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSin[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^2 \arcsin\left(\frac{x}{a}\right)\,dx \\
&= \frac{1}{3}x^3\arcsin\left(\frac{x}{a}\right) - \frac{\int \frac{x^3}{\sqrt{1-\frac{x^2}{a^2}}}\,dx}{3a} \\
&= \frac{1}{3}x^3\arcsin\left(\frac{x}{a}\right) - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{6a} \\
&= \frac{1}{3}x^3\arcsin\left(\frac{x}{a}\right) - \frac{\text{Subst}\left(\int \left(\frac{a^2}{\sqrt{1-\frac{x}{a^2}}} - a^2\sqrt{1-\frac{x}{a^2}}\right)\,dx, x, x^2\right)}{6a} \\
&= \frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}} - \frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2} + \frac{1}{3}x^3\arcsin\left(\frac{x}{a}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int x^2\csc^{-1}\left(\frac{a}{x}\right)\,dx = \frac{1}{9}a\left(2a^2+x^2\right)\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{3}x^3\csc^{-1}\left(\frac{a}{x}\right)$$

```
[In] Integrate[x^2*ArcCsc[a/x],x]
[Out] (a*(2*a^2 + x^2)*Sqrt[1 - x^2/a^2])/9 + (x^3*ArcCsc[a/x])/3
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{x^{3} \operatorname{arccsc}\left(\frac{a}{x}\right)}{3}-\frac{-\frac{x^{2} a^{2} \sqrt{1-\frac{x^{2}}{a^{2}}}}{3}-\frac{2 a^{4} \sqrt{1-\frac{x^{2}}{a^{2}}}}{3}}{3 a}$	56
derivativedivides	$-a^{3}\left(-\frac{x^{3} \operatorname{arccsc}\left(\frac{a}{x}\right)}{3 a^{3}}-\frac{\left(\frac{a^{2}}{x^{2}}-1\right)\left(\frac{2 a^{2}}{x^{2}}+1\right) x^{4}}{9 \sqrt{\frac{\left(\frac{a^{2}}{x^{2}}-1\right) x^{2}}{a^{2}}}\, a^{4}}\right)$	66
default	$-a^{3}\left(-\frac{x^{3} \operatorname{arccsc}\left(\frac{a}{x}\right)}{3 a^{3}}-\frac{\left(\frac{a^{2}}{x^{2}}-1\right)\left(\frac{2 a^{2}}{x^{2}}+1\right) x^{4}}{9 \sqrt{\frac{\left(\frac{a^{2}}{x^{2}}-1\right) x^{2}}{a^{2}}}\, a^{4}}\right)$	66

```
[In] int(x^2*arccsc(a/x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arccsc(a/x)-1/3/a*(-1/3*x^2*a^2*(1-x^2/a^2)^(1/2)-2/3*a^4*(1-x^2/a^
2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.70

$$\int x^2 \csc^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{3} x^3 \operatorname{arccsc}\left(\frac{a}{x}\right) + \frac{1}{9}\left(2 a^2 x + x^3\right)\sqrt{\frac{a^2 - x^2}{x^2}}$$

```
[In] integrate(x^2*arccsc(a/x),x, algorithm="fricas")
[Out] 1/3*x^3*arccsc(a/x) + 1/9*(2*a^2*x + x^3)*sqrt((a^2 - x^2)/x^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int x^2 \csc^{-1}\left(\frac{a}{x}\right) dx = \begin{cases} \frac{2a^3\sqrt{1-\frac{x^2}{a^2}}}{9} + \frac{ax^2\sqrt{1-\frac{x^2}{a^2}}}{9} + \frac{x^3 \operatorname{acsc}\left(\frac{a}{x}\right)}{3} & \text{for } a \neq 0 \\ \tilde{\infty} x^3 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*acsc(a/x),x)
[Out] Piecewise((2*a**3*sqrt(1 - x**2/a**2)/9 + a*x**2*sqrt(1 - x**2/a**2)/9 + x*
*3*acsc(a/x)/3, Ne(a, 0)), (zoo*x**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int x^2 \csc^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{3} x^3 \operatorname{arccsc}\left(\frac{a}{x}\right) + \frac{2 a^4 \sqrt{-\frac{x^2}{a^2}+1} + a^2 x^2 \sqrt{-\frac{x^2}{a^2}+1}}{9 a}$$

```
[In] integrate(x^2*arccsc(a/x),x, algorithm="maxima")
[Out] 1/3*x^3*arccsc(a/x) + 1/9*(2*a^4*sqrt(-x^2/a^2 + 1) + a^2*x^2*sqrt(-x^2/a^2
 + 1))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int x^2 \csc^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{3} a^2 x \left(\frac{x^2}{a^2}-1\right) \arcsin\left(\frac{x}{a}\right) - \frac{1}{9} a^3 \left(-\frac{x^2}{a^2}+1\right)^{\frac{3}{2}} + \frac{1}{3} a^2 x \arcsin\left(\frac{x}{a}\right) + \frac{1}{3} a^3 \sqrt{-\frac{x^2}{a^2}+1}$$

```
[In] integrate(x^2*arccsc(a/x),x, algorithm="giac")
[Out] 1/3*a^2*x*(x^2/a^2 - 1)*arcsin(x/a) - 1/9*a^3*(-x^2/a^2 + 1)^(3/2) + 1/3*a^
2*x*arcsin(x/a) + 1/3*a^3*sqrt(-x^2/a^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \csc^{-1}\left(\frac{a}{x}\right) dx = \begin{cases} \frac{x^3 \operatorname{asin}\left(\frac{x}{a}\right)}{3} + \frac{\sqrt{a^2-x^2}\,(2a^2+x^2)}{9} & \text{if } 0<a \\ \int x^2 \operatorname{asin}\left(\frac{x}{a}\right) dx & \text{if } \neg 0<a \end{cases}$$

```
[In] int(x^2*asin(x/a),x)
[Out] piecewise(0 < a, (x^3*asin(x/a))/3 + ((a^2 - x^2)^(1/2)*(2*a^2 + x^2))/9, ~
0 < a, int(x^2*asin(x/a), x))
```

3.10 $\int x \csc^{-1}\left(\frac{a}{x}\right) \, dx$

Optimal result . . . 82
Rubi [A] (verified) . . . 82
Mathematica [A] (verified) . . . 83
Maple [A] (verified) . . . 84
Fricas [A] (verification not implemented) . . . 84
Sympy [A] (verification not implemented) . . . 85
Maxima [A] (verification not implemented) . . . 85
Giac [A] (verification not implemented) . . . 85
Mupad [B] (verification not implemented) . . . 86

Optimal result

Integrand size = 8, antiderivative size = 47

$$\int x \csc^{-1}\left(\frac{a}{x}\right) \, dx = \frac{1}{4} a x \sqrt{1 - \frac{x^2}{a^2}} - \frac{1}{4} a^2 \arcsin\left(\frac{x}{a}\right) + \frac{1}{2} x^2 \arcsin\left(\frac{x}{a}\right)$$

```
[Out] -1/4*a^2*arcsin(x/a)+1/2*x^2*arcsin(x/a)+1/4*a*x*(1-x^2/a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5373, 4723, 327, 222}

$$\int x \csc^{-1}\left(\frac{a}{x}\right) \, dx = -\frac{1}{4} a^2 \arcsin\left(\frac{x}{a}\right) + \frac{1}{4} a x \sqrt{1 - \frac{x^2}{a^2}} + \frac{1}{2} x^2 \arcsin\left(\frac{x}{a}\right)$$

```
[In] Int[x*ArcCsc[a/x],x]
```

```
[Out] (a*x*Sqrt[1 - x^2/a^2])/4 - (a^2*ArcSin[x/a])/4 + (x^2*ArcSin[x/a])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5373

```
Int[ArcCsc[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSin[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x \arcsin\left(\frac{x}{a}\right) dx \\
&= \frac{1}{2}x^2 \arcsin\left(\frac{x}{a}\right) - \frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}}\,dx}{2a} \\
&= \frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{2}x^2 \arcsin\left(\frac{x}{a}\right) - \frac{1}{4}a\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx \\
&= \frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}} - \frac{1}{4}a^2 \arcsin\left(\frac{x}{a}\right) + \frac{1}{2}x^2 \arcsin\left(\frac{x}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int x \csc^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{4}\left(ax\sqrt{1-\frac{x^2}{a^2}} + 2x^2 \csc^{-1}\left(\frac{a}{x}\right) - a^2 \arcsin\left(\frac{x}{a}\right)\right)$$

```
[In] Integrate[x*ArcCsc[a/x],x]
[Out] (a*x*Sqrt[1 - x^2/a^2] + 2*x^2*ArcCsc[a/x] - a^2*ArcSin[x/a])/4
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

method	result	size
parts	$\frac{x^{2} \operatorname{arccsc}\left(\frac{a}{x}\right)}{2}-\frac{-\frac{x a^{2} \sqrt{1-\frac{x^{2}}{a^{2}}}}{2}+\frac{a^{2} \arctan\left(\frac{\sqrt{\frac{1}{a^{2}}}\, x}{\sqrt{1-\frac{x^{2}}{a^{2}}}}\right)}{2 \sqrt{\frac{1}{a^{2}}}}}{2 a}$	67
derivativedivides	$-a^{2}\left(-\frac{x^{2} \operatorname{arccsc}\left(\frac{a}{x}\right)}{2 a^{2}}+\frac{\sqrt{\frac{a^{2}}{x^{2}}-1}\left(\frac{\arctan\left(\frac{1}{\sqrt{\frac{a^{2}}{x^{2}}-1}}\right) a^{2}}{x^{2}}-\sqrt{\frac{a^{2}}{x^{2}}-1}\right) x^{3}}{4 \sqrt{\frac{\left(\frac{a^{2}}{x^{2}}-1\right) x^{2}}{a^{2}}}\, a^{3}}\right)$	91
default	$-a^{2}\left(-\frac{x^{2} \operatorname{arccsc}\left(\frac{a}{x}\right)}{2 a^{2}}+\frac{\sqrt{\frac{a^{2}}{x^{2}}-1}\left(\frac{\arctan\left(\frac{1}{\sqrt{\frac{a^{2}}{x^{2}}-1}}\right) a^{2}}{x^{2}}-\sqrt{\frac{a^{2}}{x^{2}}-1}\right) x^{3}}{4 \sqrt{\frac{\left(\frac{a^{2}}{x^{2}}-1\right) x^{2}}{a^{2}}}\, a^{3}}\right)$	91

```
[In] int(x*arccsc(a/x),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*arccsc(a/x)-1/2/a*(-1/2*x*a^2*(1-x^2/a^2)^(1/2)+1/2*a^2/(1/a^2)^(1/
2)*arctan((1/a^2)^(1/2)*x/(1-x^2/a^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int x \csc^{-1}\left(\frac{a}{x}\right)\,dx=\frac{1}{4}\,x^{2}\sqrt{\frac{a^{2}-x^{2}}{x^{2}}}-\frac{1}{4}\left(a^{2}-2\,x^{2}\right)\operatorname{arccsc}\left(\frac{a}{x}\right)$$

```
[In] integrate(x*arccsc(a/x),x, algorithm="fricas")
[Out] 1/4*x^2*sqrt((a^2 - x^2)/x^2) - 1/4*(a^2 - 2*x^2)*arccsc(a/x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int x \csc^{-1}\left(\frac{a}{x}\right) dx = \begin{cases} -\frac{a^2 \operatorname{acsc}\left(\frac{a}{x}\right)}{4} + \frac{ax\sqrt{1-\frac{x^2}{a^2}}}{4} + \frac{x^2 \operatorname{acsc}\left(\frac{a}{x}\right)}{2} & \text{for } a \neq 0 \\ \tilde{\infty} x^2 & \text{otherwise} \end{cases}$$

```
[In] integrate(x*acsc(a/x),x)
```

```
[Out] Piecewise((-a**2*acsc(a/x)/4 + a*x*sqrt(1 - x**2/a**2)/4 + x**2*acsc(a/x)/2
, Ne(a, 0)), (zoo*x**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int x \csc^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{2} x^2 \operatorname{arccsc}\left(\frac{a}{x}\right) - \frac{a^3 \arcsin\left(\frac{x}{a}\right) - a^2 x \sqrt{-\frac{x^2}{a^2} + 1}}{4 a}$$

```
[In] integrate(x*arccsc(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccsc(a/x) - 1/4*(a^3*arcsin(x/a) - a^2*x*sqrt(-x^2/a^2 + 1))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int x \csc^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{2} a^2 \left(\frac{x^2}{a^2} - 1\right) \arcsin\left(\frac{x}{a}\right) + \frac{1}{4} a^2 \arcsin\left(\frac{x}{a}\right) + \frac{1}{4} a x \sqrt{-\frac{x^2}{a^2} + 1}$$

```
[In] integrate(x*arccsc(a/x),x, algorithm="giac")
```

```
[Out] 1/2*a^2*(x^2/a^2 - 1)*arcsin(x/a) + 1/4*a^2*arcsin(x/a) + 1/4*a*x*sqrt(-x^2
/a^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int x \csc^{-1}\left(\frac{a}{x}\right) dx = \frac{a^2 \operatorname{asin}\left(\frac{x}{a}\right) \left(\frac{2x^2}{a^2} - 1\right)}{4} + \frac{a x \sqrt{1 - \frac{x^2}{a^2}}}{4}$$

```
[In] int(x*asin(x/a),x)
[Out] (a^2*asin(x/a)*((2*x^2)/a^2 - 1))/4 + (a*x*(1 - x^2/a^2)^(1/2))/4
```

3.11 $\int \csc^{-1}\left(\frac{a}{x}\right) dx$

Optimal result 87
Rubi [A] (verified) 87
Mathematica [A] (verified) 88
Maple [A] (verified) 88
Fricas [A] (verification not implemented) 89
Sympy [A] (verification not implemented) 89
Maxima [A] (verification not implemented) 89
Giac [A] (verification not implemented) 90
Mupad [B] (verification not implemented) 90

Optimal result

Integrand size = 6, antiderivative size = 25

$$\int \csc^{-1}\left(\frac{a}{x}\right) dx = a\sqrt{1-\frac{x^2}{a^2}} + x \arcsin\left(\frac{x}{a}\right)$$

```
[Out] x*arcsin(x/a)+a*(1-x^2/a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5373, 4715, 267}

$$\int \csc^{-1}\left(\frac{a}{x}\right) dx = a\sqrt{1-\frac{x^2}{a^2}} + x \arcsin\left(\frac{x}{a}\right)$$

```
[In] Int[ArcCsc[a/x],x]
```

```
[Out] a*Sqrt[1 - x^2/a^2] + x*ArcSin[x/a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5373

```
Int[ArcCsc[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSin[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \arcsin\left(\frac{x}{a}\right)\,dx\\ &= x\arcsin\left(\frac{x}{a}\right) - \frac{\int \frac{x}{\sqrt{1-\frac{x^2}{a^2}}}\,dx}{a}\\ &= a\sqrt{1-\frac{x^2}{a^2}} + x\arcsin\left(\frac{x}{a}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \csc^{-1}\left(\frac{a}{x}\right)\,dx = a\sqrt{1-\frac{x^2}{a^2}} + x\csc^{-1}\left(\frac{a}{x}\right)$$

```
[In] Integrate[ArcCsc[a/x],x]
[Out] a*Sqrt[1 - x^2/a^2] + x*ArcCsc[a/x]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
parts	$x\ \operatorname{arccsc}\left(\frac{a}{x}\right)+a\sqrt{\frac{a^2-x^2}{a^2}}$	27
derivativedivides	$-a\left(-\frac{x\ \operatorname{arccsc}\left(\frac{a}{x}\right)}{a}-\frac{x^2\left(\frac{a^2}{x^2}-1\right)}{\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\ a^2}\right)$	52
default	$-a\left(-\frac{x\ \operatorname{arccsc}\left(\frac{a}{x}\right)}{a}-\frac{x^2\left(\frac{a^2}{x^2}-1\right)}{\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\ a^2}\right)$	52

```
[In] int(arccsc(a/x),x,method=_RETURNVERBOSE)
[Out] x*arccsc(a/x)+a*((a^2-x^2)/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \csc^{-1}\left(\frac{a}{x}\right)\,dx = x\,\mathrm{arccsc}\left(\frac{a}{x}\right) + x\sqrt{\frac{a^2-x^2}{x^2}}$$

```
[In] integrate(arccsc(a/x),x, algorithm="fricas")
[Out] x*arccsc(a/x) + x*sqrt((a^2 - x^2)/x^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \csc^{-1}\left(\frac{a}{x}\right)\,dx = \begin{cases} a\sqrt{1-\frac{x^2}{a^2}} + x\,\mathrm{acsc}\left(\frac{a}{x}\right) & \text{for } a \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(acsc(a/x),x)
[Out] Piecewise((a*sqrt(1 - x**2/a**2) + x*acsc(a/x), Ne(a, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \csc^{-1}\left(\frac{a}{x}\right)\,dx = x\,\mathrm{arccsc}\left(\frac{a}{x}\right) + a\sqrt{-\frac{x^2}{a^2}+1}$$

```
[In] integrate(arccsc(a/x),x, algorithm="maxima")
[Out] x*arccsc(a/x) + a*sqrt(-x^2/a^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \csc^{-1}\left(\frac{a}{x}\right)\,dx = a\left(\frac{x\arcsin\left(\frac{x}{a}\right)}{a} + \sqrt{-\frac{x^2}{a^2}+1}\right)$$

```
[In] integrate(arccsc(a/x),x, algorithm="giac")
[Out] a*(x*arcsin(x/a)/a + sqrt(-x^2/a^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \csc^{-1}\left(\frac{a}{x}\right)\,dx = a\sqrt{1-\frac{x^2}{a^2}} + x\,\mathrm{asin}\left(\frac{x}{a}\right)$$

```
[In] int(asin(x/a),x)
[Out] a*(1 - x^2/a^2)^(1/2) + x*asin(x/a)
```

3.12 $\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x}\,dx$

Optimal result 91
Rubi [A] (verified) 91
Mathematica [A] (verified) 93
Maple [A] (verified) 93
Fricas [F] 94
Sympy [F] 94
Maxima [F] 94
Giac [F] 94
Mupad [B] (verification not implemented) 95

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x}\,dx = -\frac{1}{2}i\arcsin\left(\frac{x}{a}\right)^2+\arcsin\left(\frac{x}{a}\right)\log\left(1-e^{2i\arcsin\left(\frac{x}{a}\right)}\right) -\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{x}{a}\right)}\right)$$

```
[Out] -1/2*I*arcsin(x/a)^2+arcsin(x/a)*ln(1-(I*x/a+(1-x^2/a^2)^(1/2))^2)-1/2*I*po
lylog(2,(I*x/a+(1-x^2/a^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5373, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x}\,dx = -\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\arcsin\left(\frac{x}{a}\right)^2 +\arcsin\left(\frac{x}{a}\right)\log\left(1-e^{2i\arcsin\left(\frac{x}{a}\right)}\right)$$

```
[In] Int[ArcCsc[a/x]/x,x]
```

```
[Out] (-1/2*I)*ArcSin[x/a]^2 + ArcSin[x/a]*Log[1 - E^((2*I)*ArcSin[x/a])] - (I/2)
*PolyLog[2, E^((2*I)*ArcSin[x/a])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5373

```
Int[ArcCsc[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSin[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\arcsin\left(\frac{x}{a}\right)}{x}\,dx \\
&= \text{Subst}\left(\int x\cot(x)\,dx, x, \arcsin\left(\frac{x}{a}\right)\right) \\
&= -\frac{1}{2}i\arcsin\left(\frac{x}{a}\right)^2 - 2i\text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \arcsin\left(\frac{x}{a}\right)\right) \\
&= -\frac{1}{2}i\arcsin\left(\frac{x}{a}\right)^2 + \arcsin\left(\frac{x}{a}\right)\log\left(1-e^{2i\arcsin\left(\frac{x}{a}\right)}\right) \\
&\quad - \text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx, x, \arcsin\left(\frac{x}{a}\right)\right)
\end{aligned}
$$

$$= -\frac{1}{2}i\arcsin\left(\frac{x}{a}\right)^2 + \arcsin\left(\frac{x}{a}\right)\log\left(1-e^{2i\arcsin\left(\frac{x}{a}\right)}\right) + \frac{1}{2}i\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2i\arcsin\left(\frac{x}{a}\right)}\right)$$

$$= -\frac{1}{2}i\arcsin\left(\frac{x}{a}\right)^2 + \arcsin\left(\frac{x}{a}\right)\log\left(1-e^{2i\arcsin\left(\frac{x}{a}\right)}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2, e^{2i\arcsin\left(\frac{x}{a}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int\frac{\csc^{-1}\left(\frac{a}{x}\right)}{x}\,dx = \csc^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2i\csc^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}i\left(\csc^{-1}\left(\frac{a}{x}\right)^2 + \text{PolyLog}\left(2, e^{2i\csc^{-1}\left(\frac{a}{x}\right)}\right)\right)$$

```
[In] Integrate[ArcCsc[a/x]/x,x]
[Out] ArcCsc[a/x]*Log[1 - E^((2*I)*ArcCsc[a/x])] - (I/2)*(ArcCsc[a/x]^2 + PolyLog
[2, E^((2*I)*ArcCsc[a/x])])
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.12

method	result
derivativedivides	$-\frac{i\,\text{arccsc}\left(\frac{a}{x}\right)^2}{2} + \text{arccsc}\left(\frac{a}{x}\right)\ln\left(1+\frac{ix}{a}+\sqrt{1-\frac{x^2}{a^2}}\right) - i\,\text{polylog}\left(2, -\frac{ix}{a}-\sqrt{1-\frac{x^2}{a^2}}\right) + \text{a}$
default	$-\frac{i\,\text{arccsc}\left(\frac{a}{x}\right)^2}{2} + \text{arccsc}\left(\frac{a}{x}\right)\ln\left(1+\frac{ix}{a}+\sqrt{1-\frac{x^2}{a^2}}\right) - i\,\text{polylog}\left(2, -\frac{ix}{a}-\sqrt{1-\frac{x^2}{a^2}}\right) + \text{a}$

```
[In] int(arccsc(a/x)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*I*arccsc(a/x)^2+arccsc(a/x)*ln(1+I*x/a+(1-x^2/a^2)^(1/2))-I*polylog(2,
-I*x/a-(1-x^2/a^2)^(1/2))+arccsc(a/x)*ln(1-I*x/a-(1-x^2/a^2)^(1/2))-I*polyl
og(2,I*x/a+(1-x^2/a^2)^(1/2))
```

Fricas [F]

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arccsc}\left(\frac{a}{x}\right)}{x}\,dx$$

```
[In] integrate(arccsc(a/x)/x,x, algorithm="fricas")
[Out] integral(arccsc(a/x)/x, x)
```

Sympy [F]

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{acsc}\left(\frac{a}{x}\right)}{x}\,dx$$

```
[In] integrate(acsc(a/x)/x,x)
[Out] Integral(acsc(a/x)/x, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arccsc}\left(\frac{a}{x}\right)}{x}\,dx$$

```
[In] integrate(arccsc(a/x)/x,x, algorithm="maxima")
[Out] integrate(arccsc(a/x)/x, x)
```

Giac [F]

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arccsc}\left(\frac{a}{x}\right)}{x}\,dx$$

```
[In] integrate(arccsc(a/x)/x,x, algorithm="giac")
[Out] integrate(arccsc(a/x)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x}\,dx = -\frac{\mathrm{polylog}\left(2, \mathrm{e}^{\mathrm{asin}\left(\frac{x}{a}\right)\,2\mathrm{i}}\right)\,1\mathrm{i}}{2} + \ln\left(1 - \mathrm{e}^{\mathrm{asin}\left(\frac{x}{a}\right)\,2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{x}{a}\right) - \frac{\mathrm{asin}\left(\frac{x}{a}\right)^2\,1\mathrm{i}}{2}$$

```
[In] int(asin(x/a)/x,x)
```

```
[Out] log(1 - exp(asin(x/a)*2i))*asin(x/a) - (polylog(2, exp(asin(x/a)*2i))*1i)/2
 - (asin(x/a)^2*1i)/2
```

3.13 $\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx$

Optimal result 96
Rubi [A] (verified) 96
Mathematica [B] (verified) 98
Maple [A] (verified) 98
Fricas [B] (verification not implemented) 98
Sympy [C] (verification not implemented) 99
Maxima [A] (verification not implemented) 99
Giac [B] (verification not implemented) 99
Mupad [B] (verification not implemented) 100

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\arcsin\left(\frac{x}{a}\right)}{x} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}$$

```
[Out] -arcsin(x/a)/x-arctanh((1-x^2/a^2)^(1/2))/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5373, 4723, 272, 65, 214}

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a} - \frac{\arcsin\left(\frac{x}{a}\right)}{x}$$

```
[In] Int[ArcCsc[a/x]/x^2,x]
```

```
[Out] -(ArcSin[x/a]/x) - ArcTanh[Sqrt[1 - x^2/a^2]]/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5373

```
Int[ArcCsc[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSin[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\arcsin\left(\frac{x}{a}\right)}{x^2}\,dx \\
&= -\frac{\arcsin\left(\frac{x}{a}\right)}{x} + \frac{\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx}{a} \\
&= -\frac{\arcsin\left(\frac{x}{a}\right)}{x} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{2a} \\
&= -\frac{\arcsin\left(\frac{x}{a}\right)}{x} - a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{x^2}{a^2}}\right) \\
&= -\frac{\arcsin\left(\frac{x}{a}\right)}{x} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.91

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\csc^{-1}\left(\frac{a}{x}\right)}{x} - \frac{\sqrt{-1+\frac{a^2}{x^2}}\,x\left(-\log\left(1-\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\,x}\right)+\log\left(1+\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\,x}\right)\right)}{2a^2\sqrt{1-\frac{x^2}{a^2}}}$$

```
[In] Integrate[ArcCsc[a/x]/x^2,x]
[Out] -(ArcCsc[a/x]/x) - (Sqrt[-1 + a^2/x^2]*x*(-Log[1 - a/(Sqrt[-1 + a^2/x^2]*x)
] + Log[1 + a/(Sqrt[-1 + a^2/x^2]*x)]))/(2*a^2*Sqrt[1 - x^2/a^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
parts	$-\frac{\operatorname{arccsc}\left(\frac{a}{x}\right)}{x}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right)}{a}$	31
derivativedivides	$-\frac{\frac{\operatorname{arccsc}\left(\frac{a}{x}\right)a}{x}+\ln\left(\frac{a}{x}+\frac{a\sqrt{1-\frac{x^2}{a^2}}}{x}\right)}{a}$	42
default	$-\frac{\frac{\operatorname{arccsc}\left(\frac{a}{x}\right)a}{x}+\ln\left(\frac{a}{x}+\frac{a\sqrt{1-\frac{x^2}{a^2}}}{x}\right)}{a}$	42

```
[In] int(arccsc(a/x)/x^2,x,method=_RETURNVERBOSE)
[Out] -arccsc(a/x)/x-1/a*arctanh(1/(1-x^2/a^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.03

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{2\,a\operatorname{arccsc}\left(\frac{a}{x}\right)+x\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}+a\right)-x\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}-a\right)}{2\,ax}$$

```
[In] integrate(arccsc(a/x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*arccsc(a/x) + x*log(x*sqrt((a^2 - x^2)/x^2) + a) - x*log(x*sqrt((
a^2 - x^2)/x^2) - a))/(a*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\operatorname{acsc}\left(\frac{a}{x}\right)}{x} + \frac{\begin{cases} -\operatorname{acosh}\left(\frac{a}{x}\right) & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{a}{x}\right) & \text{otherwise} \end{cases}}{a}$$

```
[In] integrate(acsc(a/x)/x**2,x)
```

```
[Out] -acsc(a/x)/x + Piecewise((-acosh(a/x), Abs(a**2/x**2) > 1), (I*asin(a/x), T
rue))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\frac{2\,a \operatorname{arccsc}\left(\frac{a}{x}\right)}{x} + \log\left(\sqrt{-\frac{x^2}{a^2}+1}+1\right) - \log\left(-\sqrt{-\frac{x^2}{a^2}+1}+1\right)}{2\,a}$$

```
[In] integrate(arccsc(a/x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*arccsc(a/x)/x + log(sqrt(-x^2/a^2 + 1) + 1) - log(-sqrt(-x^2/a^2
+ 1) + 1))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{a\left(\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{a} - \frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{a}\right)}{2\,|a|} - \frac{\arcsin\left(\frac{x}{a}\right)}{x}$$

```
[In] integrate(arccsc(a/x)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*a*(log(abs(a + sqrt(a^2 - x^2)))/a - log(abs(-a + sqrt(a^2 - x^2)))/a)
/abs(a) - arcsin(x/a)/x
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\operatorname{asin}\left(\frac{x}{a}\right)}{x} - \frac{\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right)}{a}$$

[In] `int(asin(x/a)/x^2,x)`

[Out] `- asin(x/a)/x - atanh(1/(1 - x^2/a^2)^(1/2))/a`

3.14 $\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx$

Optimal result 101
Rubi [A] (verified) 101
Mathematica [A] (verified) 102
Maple [A] (verified) 102
Fricas [A] (verification not implemented) 103
Sympy [C] (verification not implemented) 103
Maxima [A] (verification not implemented) 103
Giac [A] (verification not implemented) 104
Mupad [F(-1)] 104

Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx=-\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}-\frac{\arcsin\left(\frac{x}{a}\right)}{2x^2}$$

```
[Out] -1/2*arcsin(x/a)/x^2-1/2*(1-x^2/a^2)^(1/2)/a/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5373, 4723, 270}

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx=-\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}-\frac{\arcsin\left(\frac{x}{a}\right)}{2x^2}$$

```
[In] Int[ArcCsc[a/x]/x^3,x]
```

```
[Out] -1/2*Sqrt[1 - x^2/a^2]/(a*x) - ArcSin[x/a]/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
```

```
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5373

```
Int[ArcCsc[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSin[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\arcsin\left(\frac{x}{a}\right)}{x^3}\,dx\\ &= -\frac{\arcsin\left(\frac{x}{a}\right)}{2x^2}+\frac{\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx}{2a}\\ &= -\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}-\frac{\arcsin\left(\frac{x}{a}\right)}{2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{x\sqrt{1-\frac{x^2}{a^2}}+a\csc^{-1}\left(\frac{a}{x}\right)}{2ax^2}$$

```
[In] Integrate[ArcCsc[a/x]/x^3,x]
[Out] -1/2*(x*Sqrt[1 - x^2/a^2] + a*ArcCsc[a/x])/(a*x^2)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
parts	$-\frac{\operatorname{arccsc}\left(\frac{a}{x}\right)}{2x^2}-\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}$	33
derivativedivides	$-\frac{\frac{a^2\operatorname{arccsc}\left(\frac{a}{x}\right)}{2x^2}+\frac{x\left(\frac{a^2}{x^2}-1\right)}{2\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^2}$	54
default	$-\frac{\frac{a^2\operatorname{arccsc}\left(\frac{a}{x}\right)}{2x^2}+\frac{x\left(\frac{a^2}{x^2}-1\right)}{2\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^2}$	54

```
[In] int(arccsc(a/x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*arccsc(a/x)/x^2-1/2*(1-x^2/a^2)^(1/2)/a/x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{a^2 \operatorname{arccsc}\left(\frac{a}{x}\right) + x^2\sqrt{\frac{a^2-x^2}{x^2}}}{2\,a^2x^2}$$

```
[In] integrate(arccsc(a/x)/x^3,x, algorithm="fricas")
[Out] -1/2*(a^2*arccsc(a/x) + x^2*sqrt((a^2 - x^2)/x^2))/(a^2*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{\operatorname{acsc}\left(\frac{a}{x}\right)}{2x^2} + \frac{\begin{cases} -\frac{\sqrt{\frac{a^2}{x^2}-1}}{a} & \text{for}\ \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{i\sqrt{-\frac{a^2}{x^2}+1}}{a} & \text{otherwise} \end{cases}}{2a}$$

```
[In] integrate(acsc(a/x)/x**3,x)
[Out] -acsc(a/x)/(2*x**2) + Piecewise((-sqrt(a**2/x**2 - 1)/a, Abs(a**2/x**2) > 1
), (-I*sqrt(-a**2/x**2 + 1)/a, True))/(2*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{\operatorname{arccsc}\left(\frac{a}{x}\right)}{2\,x^2} - \frac{\sqrt{-\frac{x^2}{a^2}+1}}{2\,ax}$$

```
[In] integrate(arccsc(a/x)/x^3,x, algorithm="maxima")
[Out] -1/2*arccsc(a/x)/x^2 - 1/2*sqrt(-x^2/a^2 + 1)/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{a\left(\frac{a+\sqrt{a^2-x^2}}{a^2x}-\frac{x}{\left(a+\sqrt{a^2-x^2}\right)a^2}\right)}{4\,|a|}-\frac{\arcsin\left(\frac{x}{a}\right)}{2\,x^2}$$

```
[In] integrate(arccsc(a/x)/x^3,x, algorithm="giac")
[Out] -1/4*a*((a + sqrt(a^2 - x^2))/(a^2*x) - x/((a + sqrt(a^2 - x^2))*a^2))/abs(
a) - 1/2*arcsin(x/a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{\mathrm{asin}\left(\frac{x}{a}\right)}{x^3}\,dx$$

```
[In] int(asin(x/a)/x^3,x)
[Out] int(asin(x/a)/x^3, x)
```

3.15 $\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx$

Optimal result 105
Rubi [A] (verified) 105
Mathematica [A] (verified) 107
Maple [A] (verified) 107
Fricas [A] (verification not implemented) 108
Sympy [A] (verification not implemented) 108
Maxima [A] (verification not implemented) 109
Giac [A] (verification not implemented) 109
Mupad [F(-1)] 109

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\arcsin\left(\frac{x}{a}\right)}{3x^3} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3}$$

```
[Out] -1/3*arcsin(x/a)/x^3-1/6*arctanh((1-x^2/a^2)^(1/2))/a^3-1/6*(1-x^2/a^2)^(1/
2)/a/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5373, 4723, 272, 44, 65, 214}

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3} - \frac{\arcsin\left(\frac{x}{a}\right)}{3x^3}$$

```
[In] Int[ArcCsc[a/x]/x^4,x]
```

```
[Out] -1/6*Sqrt[1 - x^2/a^2]/(a*x^2) - ArcSin[x/a]/(3*x^3) - ArcTanh[Sqrt[1 - x^2
/a^2]]/(6*a^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5373

```
Int[ArcCsc[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSin[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\arcsin\left(\frac{x}{a}\right)}{x^4}\,dx \\
&= -\frac{\arcsin\left(\frac{x}{a}\right)}{3x^3} + \frac{\int \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx}{3a} \\
&= -\frac{\arcsin\left(\frac{x}{a}\right)}{3x^3} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{6a}
\end{aligned}$$

$$= -\frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\arcsin\left(\frac{x}{a}\right)}{3x^3} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{12a^3}$$

$$= -\frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\arcsin\left(\frac{x}{a}\right)}{3x^3} - \frac{\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{x^2}{a^2}}\right)}{6a}$$

$$= -\frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\arcsin\left(\frac{x}{a}\right)}{3x^3} - \frac{\text{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{a^2x\sqrt{1-\frac{x^2}{a^2}} + 2a^3\csc^{-1}\left(\frac{a}{x}\right) - x^3\log(x) + x^3\log\left(1+\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3x^3}$$

```
[In] Integrate[ArcCsc[a/x]/x^4,x]
[Out] -1/6*(a^2*x*Sqrt[1 - x^2/a^2] + 2*a^3*ArcCsc[a/x] - x^3*Log[x] + x^3*Log[1
+ Sqrt[1 - x^2/a^2]])/(a^3*x^3)
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

method	result	size
parts	$-\frac{\operatorname{arccsc}\left(\frac{a}{x}\right)}{3x^3} + \frac{-\frac{\sqrt{1-\frac{x^2}{a^2}}}{2x^2} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right)}{2a^2}}{3a}$	54
derivativedivides	$-\frac{\frac{a^3 \operatorname{arccsc}\left(\frac{a}{x}\right)}{3x^3} + \frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{a\sqrt{\frac{a^2}{x^2}-1}}{x} + \ln\left(\frac{a}{x}+\sqrt{\frac{a^2}{x^2}-1}\right)\right)x}{6\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^3}$	91
default	$-\frac{\frac{a^3 \operatorname{arccsc}\left(\frac{a}{x}\right)}{3x^3} + \frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{a\sqrt{\frac{a^2}{x^2}-1}}{x} + \ln\left(\frac{a}{x}+\sqrt{\frac{a^2}{x^2}-1}\right)\right)x}{6\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^3}$	91

```
[In] int(arccsc(a/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arccsc(a/x)/x^3+1/3/a*(-1/2/x^2*(1-x^2/a^2)^(1/2)-1/2/a^2*arctanh(1/(1
-x^2/a^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{4\,a^3 \operatorname{arccsc}\left(\frac{a}{x}\right) + x^3 \log\left(x\sqrt{\frac{a^2-x^2}{x^2}} + a\right) - x^3 \log\left(x\sqrt{\frac{a^2-x^2}{x^2}} - a\right) + 2\,ax^2\sqrt{\frac{a^2-x^2}{x^2}}}{12\,a^3x^3}$$

```
[In] integrate(arccsc(a/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(4*a^3*arccsc(a/x) + x^3*log(x*sqrt((a^2 - x^2)/x^2) + a) - x^3*log(x
*sqrt((a^2 - x^2)/x^2) - a) + 2*a*x^2*sqrt((a^2 - x^2)/x^2))/(a^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{\operatorname{acsc}\left(\frac{a}{x}\right)}{3x^3} + \frac{\begin{cases} -\frac{\sqrt{\frac{a^2}{x^2}-1}}{2ax} - \frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{2a^2} & \text{for}\ \left|\frac{a^2}{x^2}\right| > 1 \\ \frac{ia}{2x^3\sqrt{-\frac{a^2}{x^2}+1}} - \frac{i}{2ax\sqrt{-\frac{a^2}{x^2}+1}} + \frac{i\operatorname{asin}\left(\frac{a}{x}\right)}{2a^2} & \text{otherwise} \end{cases}}{3a}$$

```
[In] integrate(acsc(a/x)/x**4,x)
```

```
[Out] -acsc(a/x)/(3*x**3) + Piecewise((-sqrt(a**2/x**2 - 1)/(2*a*x) - acosh(a/x)/
(2*a**2), Abs(a**2/x**2) > 1), (I*a/(2*x**3*sqrt(-a**2/x**2 + 1)) - I/(2*a*
x*sqrt(-a**2/x**2 + 1)) + I*asin(a/x)/(2*a**2), True))/(3*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{\frac{\log\left(\frac{2\sqrt{-\frac{x^2}{a^2}+1}}{|x|}+\frac{2}{|x|}\right)}{a^2}+\frac{\sqrt{-\frac{x^2}{a^2}+1}}{x^2}}{6\,a} - \frac{\operatorname{arccsc}\left(\frac{a}{x}\right)}{3\,x^3}$$

```
[In] integrate(arccsc(a/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(log(2*sqrt(-x^2/a^2 + 1)/abs(x) + 2/abs(x))/a^2 + sqrt(-x^2/a^2 + 1)/
x^2)/a - 1/3*arccsc(a/x)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{a\left(\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{a^3}-\frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{a^3}+\frac{2\sqrt{a^2-x^2}}{a^2x^2}\right)}{12\,|a|} - \frac{\arcsin\left(\frac{x}{a}\right)}{3\,x^3}$$

```
[In] integrate(arccsc(a/x)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*a*(log(abs(a + sqrt(a^2 - x^2)))/a^3 - log(abs(-a + sqrt(a^2 - x^2)))
/a^3 + 2*sqrt(a^2 - x^2)/(a^2*x^2))/abs(a) - 1/3*arcsin(x/a)/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\operatorname{asin}\left(\frac{x}{a}\right)}{x^4}\,dx$$

```
[In] int(asin(x/a)/x^4,x)
```

```
[Out] int(asin(x/a)/x^4, x)
```

3.16 $\int \frac{\csc^{-1}(ax^n)}{x} \, dx$

Optimal result 110
Rubi [A] (verified) 110
Mathematica [C] (verified) 112
Maple [A] (verified) 112
Fricas [F(-2)] 113
Sympy [F] 113
Maxima [F] 113
Giac [F] 114
Mupad [F(-1)] 114

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \frac{\csc^{-1}(ax^n)}{x} \, dx = \frac{i \csc^{-1}(ax^n)^2}{2n} - \frac{\csc^{-1}(ax^n) \log\left(1 - e^{2i\csc^{-1}(ax^n)}\right)}{n} + \frac{i \operatorname{PolyLog}\left(2, e^{2i\csc^{-1}(ax^n)}\right)}{2n}$$

```
[Out] 1/2*I*arccsc(a*x^n)^2/n-arccsc(a*x^n)*ln(1-(I/a/(x^n)+(1-1/a^2/(x^n)^2)^(1/
2))^2)/n+1/2*I*polylog(2,(I/a/(x^n)+(1-1/a^2/(x^n)^2)^(1/2))^2)/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5327, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{\csc^{-1}(ax^n)}{x} \, dx = \frac{i \operatorname{PolyLog}\left(2, e^{2i\csc^{-1}(ax^n)}\right)}{2n} + \frac{i \csc^{-1}(ax^n)^2}{2n} - \frac{\csc^{-1}(ax^n) \log\left(1 - e^{2i\csc^{-1}(ax^n)}\right)}{n}$$

```
[In] Int[ArcCsc[a*x^n]/x,x]
```

```
[Out] ((I/2)*ArcCsc[a*x^n]^2)/n - (ArcCsc[a*x^n]*Log[1 - E^((2*I)*ArcCsc[a*x^n])]
)/n + ((I/2)*PolyLog[2, E^((2*I)*ArcCsc[a*x^n])])/n
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5327

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\csc^{-1}(ax)}{x}\,dx, x, x^n\right)}{n}$$

$$= -\frac{\text{Subst}\left(\int \frac{\arcsin\left(\frac{x}{a}\right)}{x}\,dx, x, x^{-n}\right)}{n}$$

$$= -\frac{\text{Subst}\left(\int x\cot(x)\,dx, x, \arcsin\left(\frac{x^{-n}}{a}\right)\right)}{n}$$

$$
\begin{aligned}
&= \frac{i \arcsin\left(\frac{x^{-n}}{a}\right)^2}{2n} + \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \arcsin\left(\frac{x^{-n}}{a}\right)\right)}{n} \\
&= \frac{i \arcsin\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\arcsin\left(\frac{x^{-n}}{a}\right)\log\left(1-e^{2i\arcsin\left(\frac{x^{-n}}{a}\right)}\right)}{n} \\
&\quad + \frac{\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx, x, \arcsin\left(\frac{x^{-n}}{a}\right)\right)}{n} \\
&= \frac{i \arcsin\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\arcsin\left(\frac{x^{-n}}{a}\right)\log\left(1-e^{2i\arcsin\left(\frac{x^{-n}}{a}\right)}\right)}{n} \\
&\quad - \frac{i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\arcsin\left(\frac{x^{-n}}{a}\right)}\right)}{2n} \\
&= \frac{i \arcsin\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\arcsin\left(\frac{x^{-n}}{a}\right)\log\left(1-e^{2i\arcsin\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{i\,\text{PolyLog}\left(2, e^{2i\arcsin\left(\frac{x^{-n}}{a}\right)}\right)}{2n}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$
\int \frac{\csc^{-1}(ax^n)}{x}\,dx = -\frac{x^{-n}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{x^{-2n}}{a^2}\right)}{an} + \left(\csc^{-1}(ax^n) - \arcsin\left(\frac{x^{-n}}{a}\right)\right)\log(x)
$$

```
[In] Integrate[ArcCsc[a*x^n]/x,x]
[Out] -(HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, 1/(a^2*x^(2*n))]/(a*n*x^n)
) + (ArcCsc[a*x^n] - ArcSin[1/(a*x^n)])*Log[x]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.25

method	result
derivativedivides	$\frac{\frac{i\,\mathrm{arccsc}(a\,x^n)^2}{2} - \mathrm{arccsc}(a\,x^n)\ln\left(1+\frac{ix^{-n}}{a}+\sqrt{1-\frac{x^{-2n}}{a^2}}\right) + i\,\mathrm{polylog}\left(2,-\frac{ix^{-n}}{a}-\sqrt{1-\frac{x^{-2n}}{a^2}}\right) - \mathrm{arccsc}(a\,x^n)\ln\left(1-\frac{ix^{-n}}{a}-\right.}{n}$
default	$\frac{\frac{i\,\mathrm{arccsc}(a\,x^n)^2}{2} - \mathrm{arccsc}(a\,x^n)\ln\left(1+\frac{ix^{-n}}{a}+\sqrt{1-\frac{x^{-2n}}{a^2}}\right) + i\,\mathrm{polylog}\left(2,-\frac{ix^{-n}}{a}-\sqrt{1-\frac{x^{-2n}}{a^2}}\right) - \mathrm{arccsc}(a\,x^n)\ln\left(1-\frac{ix^{-n}}{a}-\right.}{n}$

```
[In] int(arccsc(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/2*I*arccsc(a*x^n)^2-arccsc(a*x^n)*ln(1+I/a/(x^n)+(1-1/a^2/(x^n)^2)^(
1/2))+I*polylog(2,-I/a/(x^n)-(1-1/a^2/(x^n)^2)^(1/2))-arccsc(a*x^n)*ln(1-I/
a/(x^n)-(1-1/a^2/(x^n)^2)^(1/2))+I*polylog(2,I/a/(x^n)+(1-1/a^2/(x^n)^2)^(1
/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\csc^{-1}(ax^n)}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccsc(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\csc^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{acsc}(ax^n)}{x}\,dx$$

```
[In] integrate(acsc(a*x**n)/x,x)
```

```
[Out] Integral(acsc(a*x**n)/x, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arccsc}(ax^n)}{x}\,dx$$

```
[In] integrate(arccsc(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] a^2*n*integrate(sqrt(a*x^n + 1)*sqrt(a*x^n - 1)*log(x)/(a^4*x*x^(2*n) - a^2
*x), x) + arctan2(1, sqrt(a*x^n + 1)*sqrt(a*x^n - 1))*log(x)
```

Giac [F]

$$\int \frac{\csc^{-1}(ax^n)}{x} \, dx = \int \frac{\operatorname{arccsc}(ax^n)}{x} \, dx$$

```
[In] integrate(arccsc(a*x^n)/x,x, algorithm="giac")
[Out] integrate(arccsc(a*x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(ax^n)}{x} \, dx = \int \frac{\operatorname{asin}\left(\frac{1}{a\,x^n}\right)}{x} \, dx$$

```
[In] int(asin(1/(a*x^n))/x,x)
[Out] int(asin(1/(a*x^n))/x, x)
```

3.17 $\int x^4 \csc^{-1}(a+bx)\,dx$

Optimal result . . . 115
Rubi [A] (verified) . . . 115
Mathematica [A] (verified) . . . 119
Maple [A] (verified) . . . 119
Fricas [A] (verification not implemented) . . . 120
Sympy [F] . . . 120
Maxima [F] . . . 120
Giac [B] (verification not implemented) . . . 121
Mupad [F(-1)] . . . 121

Optimal result

Integrand size = 10, antiderivative size = 197

$$\int x^4 \csc^{-1}(a+bx)\,dx = -\frac{a(20+53a^2)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5} - \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} + \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{(9+58a^2)(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5} + \frac{a^5\csc^{-1}(a+bx)}{5b^5} + \frac{1}{5}x^5\csc^{-1}(a+bx) + \frac{(3+40a^2+40a^4)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{40b^5}$$

```
[Out] 1/5*a^5*arccsc(b*x+a)/b^5+1/5*x^5*arccsc(b*x+a)+1/40*(40*a^4+40*a^2+3)*arct
anh((1-1/(b*x+a)^2)^(1/2))/b^5-1/30*a*(53*a^2+20)*(b*x+a)*(1-1/(b*x+a)^2)^(
1/2)/b^5-11/60*a*x^2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3+1/20*x^3*(b*x+a)*(1-
1/(b*x+a)^2)^(1/2)/b^2+1/120*(58*a^2+9)*(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used

$= \{5367, 4512, 3867, 4141, 4133, 3855, 3852, 8\}$

$$\int x^4 \csc^{-1}(a+bx)\,dx = \frac{a^5 \csc^{-1}(a+bx)}{5b^5} - \frac{(53a^2+20)\,a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5} + \frac{(58a^2+9)\,(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5} + \frac{(40a^4+40a^2+3)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{40b^5} - \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} + \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{1}{5}x^5\csc^{-1}(a+bx)$$

```
[In] Int[x^4*ArcCsc[a + b*x],x]
```

```
[Out] -1/30*(a*(20 + 53*a^2)*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/b^5 - (11*a*x^2*
(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(60*b^3) + (x^3*(a + b*x)*Sqrt[1 - (a +
 b*x)^(-2)])/(20*b^2) + ((9 + 58*a^2)*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)])
/(120*b^5) + (a^5*ArcCsc[a + b*x])/(5*b^5) + (x^5*ArcCsc[a + b*x])/5 + ((3
+ 40*a^2 + 40*a^4)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/(40*b^5)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x\cot(x)\csc(x)(-a+\csc(x))^4\,dx, x, \csc^{-1}(a+bx)\right)}{b^5}\\
&= \frac{1}{5}x^5\csc^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\csc(x))^5\,dx, x, \csc^{-1}(a+bx)\right)}{5b^5}\\
&= \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{1}{5}x^5\csc^{-1}(a+bx)\\
&\quad - \frac{\text{Subst}\left(\int(-a+\csc(x))^2\left(-4a^3+3(1+4a^2)\csc(x)-11a\csc^2(x)\right)\,dx, x, \csc^{-1}(a+bx)\right)}{20b^5}
\end{aligned}$$

$$= -\frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} + \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{1}{5}x^5\csc^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\csc(x))\left(12a^4-a(31+48a^2)\csc(x)+(9+58a^2)\csc^2(x)\right)\,dx,x,\csc^{-1}(a+bx)\right)}{60b^5}$$

$$= -\frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} + \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{(9+58a^2)(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5} + \frac{1}{5}x^5\csc^{-1}(a+bx) - \frac{\text{Subst}\left(\int\left(-24a^5+3(3+40a^2+40a^4)\csc(x)-4a(20+53a^2)\csc^2(x)\right)\,dx,x,\csc^{-1}(a+bx)\right)}{120b^5}$$

$$= -\frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} + \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{(9+58a^2)(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5} + \frac{a^5\csc^{-1}(a+bx)}{5b^5} + \frac{1}{5}x^5\csc^{-1}(a+bx) + \frac{(a(20+53a^2))\,\text{Subst}\left(\int\csc^2(x)\,dx,x,\csc^{-1}(a+bx)\right)}{30b^5} - \frac{(3+40a^2+40a^4)\,\text{Subst}\left(\int\csc(x)\,dx,x,\csc^{-1}(a+bx)\right)}{40b^5}$$

$$= -\frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} + \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{(9+58a^2)(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5} + \frac{a^5\csc^{-1}(a+bx)}{5b^5} + \frac{1}{5}x^5\csc^{-1}(a+bx) + \frac{(3+40a^2+40a^4)\,\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{40b^5} - \frac{(a(20+53a^2))\,\text{Subst}\left(\int 1\,dx,x,(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{30b^5}$$

$$= -\frac{a(20+53a^2)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5} - \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} + \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{(9+58a^2)(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5} + \frac{a^5\csc^{-1}(a+bx)}{5b^5} + \frac{1}{5}x^5\csc^{-1}(a+bx) + \frac{(3+40a^2+40a^4)\,\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{40b^5}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.88

$$\int x^4 \csc^{-1}(a+bx)\,dx$$

$$= \frac{-\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\left(a^2(71+154a^2)+2a(31+48a^2)\,bx-9(1+4a^2)\,b^2x^2+16ab^3x^3-6b^4x^4\right)+24b^5x^5\,\mathrm{cs}}{120b^5}$$

[In] Integrate[x^4*ArcCsc[a + b*x],x]

[Out] (-(Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(a^2*(71 + 154*a^2) + 2*a*(31 + 48*a^2)*b*x - 9*(1 + 4*a^2)*b^2*x^2 + 16*a*b^3*x^3 - 6*b^4*x^4)) + 24*b^5*x^5*ArcCsc[a + b*x] + 24*a^5*ArcSin[(a + b*x)^(-1)] + 3*(3 + 40*a^2 + 40*a^4)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(120*b^5)

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccsc}(bx+a)a^5}{5}+\operatorname{arccsc}(bx+a)a^4(bx+a)-2\operatorname{arccsc}(bx+a)a^3(bx+a)^2+2\operatorname{arccsc}(bx+a)a^2(bx+a)^3-\operatorname{arccsc}(bx+a)a(bx+}{}$
default	$\frac{-\frac{\operatorname{arccsc}(bx+a)a^5}{5}+\operatorname{arccsc}(bx+a)a^4(bx+a)-2\operatorname{arccsc}(bx+a)a^3(bx+a)^2+2\operatorname{arccsc}(bx+a)a^2(bx+a)^3-\operatorname{arccsc}(bx+a)a(bx+}{}$
parts	$\frac{x^5\operatorname{arccsc}(bx+a)}{5}-\frac{\sqrt{b^2x^2+2abx+a^2-1}\left(-6x^3\sqrt{b^2x^2+2abx+a^2-1}\,b^3\sqrt{b^2}+22\sqrt{b^2}\,\sqrt{b^2x^2+2abx+a^2-1}\,a\,b^2x^2-24a^5\right.}{}$

[In] int(x^4*arccsc(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b^5*(-1/5*arccsc(b*x+a)*a^5+arccsc(b*x+a)*a^4*(b*x+a)-2*arccsc(b*x+a)*a^3*(b*x+a)^2+2*arccsc(b*x+a)*a^2*(b*x+a)^3-arccsc(b*x+a)*a*(b*x+a)^4+1/5*arccsc(b*x+a)*(b*x+a)^5+1/120*((b*x+a)^2-1)^(1/2)*(24*a^5*arctan(1/((b*x+a)^2-1)^(1/2))+120*a^4*ln(b*x+a+((b*x+a)^2-1)^(1/2))-240*a^3*((b*x+a)^2-1)^(1/2)+120*a^2*(b*x+a)*((b*x+a)^2-1)^(1/2)-40*a*(b*x+a)^2*((b*x+a)^2-1)^(1/2)+6*(b*x+a)^3*((b*x+a)^2-1)^(1/2)+120*a^2*ln(b*x+a+((b*x+a)^2-1)^(1/2))-80*a*((b*x+a)^2-1)^(1/2)+9*(b*x+a)*((b*x+a)^2-1)^(1/2)+9*ln(b*x+a+((b*x+a)^2-1)^(1/2)))/(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.77

$$\int x^4 \csc^{-1}(a+bx)\,dx = \frac{24\,b^5x^5\,\mathrm{arccsc}\left(bx+a\right) - 48\,a^5\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right) - 3\left(40\,a^4+40\,a^2+3\right)\log\left(-bx\right.}{1}$$

```
[In] integrate(x^4*arccsc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^5*x^5*arccsc(b*x + a) - 48*a^5*arctan(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1)) - 3*(40*a^4 + 40*a^2 + 3)*log(-b*x - a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1)) + (6*b^3*x^3 - 22*a*b^2*x^2 - 154*a^3 + (58*a^2 + 9)*
b*x - 71*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^5
```

Sympy [F]

$$\int x^4 \csc^{-1}(a+bx)\,dx = \int x^4 \operatorname{acsc}\left(a+bx\right)dx$$

```
[In] integrate(x**4*acsc(b*x+a),x)
```

```
[Out] Integral(x**4*acsc(a + b*x), x)
```

Maxima [F]

$$\int x^4 \csc^{-1}(a+bx)\,dx = \int x^4\,\mathrm{arccsc}\left(bx+a\right)\,dx$$

```
[In] integrate(x^4*arccsc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + integrate(1/5*(b^
2*x^6 + a*b*x^5)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 +
 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*
x + a - 1)) - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(173) = 346$.

Time = 0.30 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.07

$$\int x^4 \csc^{-1}(a+bx)\,dx =$$

$$-\frac{1}{960}b\left(\frac{192\,(bx+a)^5\left(\frac{5\,a}{bx+a}-\frac{10\,a^2}{(bx+a)^2}+\frac{10\,a^3}{(bx+a)^3}-\frac{5\,a^4}{(bx+a)^4}-1\right)\arcsin\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^6}+\frac{3\,(bx+a)}{}\right.$$

```
[In] integrate(x^4*arccsc(b*x+a),x, algorithm="giac")
```

```
[Out] -1/960*b*(192*(b*x + a)^5*(5*a/(b*x + a) - 10*a^2/(b*x + a)^2 + 10*a^3/(b*x
 + a)^3 - 5*a^4/(b*x + a)^4 - 1)*arcsin(-1/((b*x + a)*(a/(b*x + a) - 1) - a
))/b^6 + (3*(b*x + a)^4*(sqrt(-1/(b*x + a)^2 + 1) - 1)^4 + 40*(b*x + a)^3*a
*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 240*(b*x + a)^2*a^2*(sqrt(-1/(b*x + a)^
2 + 1) - 1)^2 + 960*(b*x + a)*a^3*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 24*(b*x
+ a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 360*(b*x + a)*a*(sqrt(-1/(b*x + a
)^2 + 1) - 1) + 24*(40*a^4 + 40*a^2 + 3)*log(-(sqrt(-1/(b*x + a)^2 + 1) - 1
)*abs(b*x + a)) - (120*(8*a^3 + 3*a)*(b*x + a)^3*(sqrt(-1/(b*x + a)^2 + 1)
- 1)^3 + 24*(10*a^2 + 1)*(b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 40*
(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 3)/((b*x + a)^4*(sqrt(-1/(b*x
+ a)^2 + 1) - 1)^4))/b^6)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \csc^{-1}(a+bx)\,dx = \int x^4\,\mathrm{asin}\left(\frac{1}{a+b\,x}\right)\,dx$$

```
[In] int(x^4*asin(1/(a + b*x)),x)
```

```
[Out] int(x^4*asin(1/(a + b*x)), x)
```

3.18 $\int x^3 \csc^{-1}(a+bx)\,dx$

Optimal result 122
Rubi [A] (verified) 122
Mathematica [A] (verified) 125
Maple [A] (verified) 125
Fricas [A] (verification not implemented) 126
Sympy [F] 126
Maxima [F] 126
Giac [B] (verification not implemented) 127
Mupad [F(-1)] 127

Optimal result

Integrand size = 10, antiderivative size = 155

$$\int x^3 \csc^{-1}(a+bx)\,dx = \frac{(2+17a^2)\,(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4} + \frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} - \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4} - \frac{a^4\csc^{-1}(a+bx)}{4b^4} + \frac{1}{4}x^4\csc^{-1}(a+bx) - \frac{a(1+2a^2)\,\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{2b^4}$$

```
[Out] -1/4*a^4*arccsc(b*x+a)/b^4+1/4*x^4*arccsc(b*x+a)-1/2*a*(2*a^2+1)*arctanh((1
-1/(b*x+a)^2)^(1/2))/b^4+1/12*(17*a^2+2)*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4+
1/12*x^2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2-1/3*a*(b*x+a)^2*(1-1/(b*x+a)^2)^
(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5367, 4512, 3867, 4133, 3855, 3852, 8}

$$\int x^3 \csc^{-1}(a+bx)\,dx = -\frac{a^4\csc^{-1}(a+bx)}{4b^4} - \frac{(2a^2+1)\,a\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{2b^4} + \frac{(17a^2+2)\,(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4} - \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4} + \frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{1}{4}x^4\csc^{-1}(a+bx)$$

```
[In] Int[x^3*ArcCsc[a + b*x],x]
```

```
[Out] ((2 + 17*a^2)*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(12*b^4) + (x^2*(a + b*x)
*Sqrt[1 - (a + b*x)^(-2)])/(12*b^2) - (a*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-2
)])/(3*b^4) - (a^4*ArcCsc[a + b*x])/(4*b^4) + (x^4*ArcCsc[a + b*x])/4 - (a*
(1 + 2*a^2)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/(2*b^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x\cot(x)\csc(x)(-a+\csc(x))^3\,dx,x,\csc^{-1}(a+bx)\right)}{b^4}\\
&= \frac{1}{4}x^4\csc^{-1}(a+bx)-\frac{\text{Subst}\left(\int(-a+\csc(x))^4\,dx,x,\csc^{-1}(a+bx)\right)}{4b^4}\\
&= \frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2}+\frac{1}{4}x^4\csc^{-1}(a+bx)\\
&\quad-\frac{\text{Subst}\left(\int(-a+\csc(x))\left(-3a^3+\left(2+9a^2\right)\csc(x)-8a\csc^2(x)\right)\,dx,x,\csc^{-1}(a+bx)\right)}{12b^4}\\
&= \frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2}-\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4}+\frac{1}{4}x^4\csc^{-1}(a+bx)\\
&\quad-\frac{\text{Subst}\left(\int\left(6a^4-12a\left(1+2a^2\right)\csc(x)+2\left(2+17a^2\right)\csc^2(x)\right)\,dx,x,\csc^{-1}(a+bx)\right)}{24b^4}\\
&= \frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2}-\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4}-\frac{a^4\csc^{-1}(a+bx)}{4b^4}\\
&\quad+\frac{1}{4}x^4\csc^{-1}(a+bx)+\frac{\left(a\left(1+2a^2\right)\right)\text{Subst}\left(\int\csc(x)\,dx,x,\csc^{-1}(a+bx)\right)}{2b^4}\\
&\quad-\frac{\left(2+17a^2\right)\text{Subst}\left(\int\csc^2(x)\,dx,x,\csc^{-1}(a+bx)\right)}{12b^4}\\
&= \frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2}-\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4}-\frac{a^4\csc^{-1}(a+bx)}{4b^4}+\frac{1}{4}x^4\csc^{-1}(a\\
&\quad+bx)-\frac{a\left(1+2a^2\right)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{2b^4}\\
&\quad+\frac{\left(2+17a^2\right)\text{Subst}\left(\int1\,dx,x,(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{12b^4}\\
&= \frac{\left(2+17a^2\right)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4}+\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2}-\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4}\\
&\quad-\frac{a^4\csc^{-1}(a+bx)}{4b^4}+\frac{1}{4}x^4\csc^{-1}(a+bx)-\frac{a\left(1+2a^2\right)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int x^3 \csc^{-1}(a+bx)\,dx$$
$$= \frac{\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}(2a+13a^3+2bx+9a^2bx-3ab^2x^2+b^3x^3)+3b^4x^4\csc^{-1}(a+bx)-3a^4\arcsin\left(\frac{1}{a+bx}\right)-}{12b^4}$$

```
[In] Integrate[x^3*ArcCsc[a + b*x],x]
```

```
[Out] (Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(2*a + 13*a^3 + 2*b*x + 9
*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3) + 3*b^4*x^4*ArcCsc[a + b*x] - 3*a^4*ArcSi
n[(a + b*x)^(-1)] - 6*a*(1 + 2*a^2)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 + 2*a
*b*x + b^2*x^2)/(a + b*x)^2])])/(12*b^4)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{\frac{\operatorname{arccsc}(bx+a)a^4}{4}-\operatorname{arccsc}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccsc}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccsc}(bx+a)a(bx+a)^3+\frac{\operatorname{arccsc}(bx+a)(bx+a)^4}{4}+\frac{\sqrt{(bx}}{}}{}$
default	$\frac{\frac{\operatorname{arccsc}(bx+a)a^4}{4}-\operatorname{arccsc}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccsc}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccsc}(bx+a)a(bx+a)^3+\frac{\operatorname{arccsc}(bx+a)(bx+a)^4}{4}+\frac{\sqrt{(bx}}{}}{}$
parts	$\frac{x^4\operatorname{arccsc}(bx+a)}{4}+\frac{\sqrt{b^2x^2+2abx+a^2-1}\left(x^2\sqrt{b^2x^2+2abx+a^2-1}\,b^2\sqrt{b^2}-3a^4\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{b^2}-4\sqrt{b^2}\right.}{}$

```
[In] int(x^3*arccsc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*arccsc(b*x+a)*a^4-arccsc(b*x+a)*a^3*(b*x+a)+3/2*arccsc(b*x+a)*a^
2*(b*x+a)^2-arccsc(b*x+a)*a*(b*x+a)^3+1/4*arccsc(b*x+a)*(b*x+a)^4+1/12*((b*
x+a)^2-1)^(1/2)*(-3*a^4*arctan(1/((b*x+a)^2-1)^(1/2))-12*a^3*ln(b*x+a+((b*x
+a)^2-1)^(1/2))+18*a^2*((b*x+a)^2-1)^(1/2)-6*a*(b*x+a)*((b*x+a)^2-1)^(1/2)+
(b*x+a)^2*((b*x+a)^2-1)^(1/2)-6*a*ln(b*x+a+((b*x+a)^2-1)^(1/2))+2*((b*x+a)^
2-1)^(1/2))/(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.83

$$\int x^3 \csc^{-1}(a+bx)\,dx$$
$$= \frac{3\,b^4x^4\,\mathrm{arccsc}\,(bx+a)+6\,a^4\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)+6\,(2\,a^3+a)\log\left(-bx-a+\sqrt{b^2\cdots}\right.}{12\,b^4}$$

```
[In] integrate(x^3*arccsc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4*arccsc(b*x + a) + 6*a^4*arctan(-b*x - a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 - 1)) + 6*(2*a^3 + a)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a
^2 - 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b^2*x^2 - 4*a*b*x + 13*a^2 +
2))/b^4
```

Sympy [F]

$$\int x^3 \csc^{-1}(a+bx)\,dx = \int x^3\,\mathrm{acsc}\,(a+bx)\,dx$$

```
[In] integrate(x**3*acsc(b*x+a),x)
```

```
[Out] Integral(x**3*acsc(a + b*x), x)
```

Maxima [F]

$$\int x^3 \csc^{-1}(a+bx)\,dx = \int x^3\,\mathrm{arccsc}\,(bx+a)\,dx$$

```
[In] integrate(x^3*arccsc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + integrate(1/4*(b^
2*x^5 + a*b*x^4)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 +
 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*
x + a - 1)) - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(135) = 270$.

Time = 0.30 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.94

$$\int x^3 \csc^{-1}(a+bx)\,dx =$$

$$-\frac{1}{96}b\left(\frac{24\,(bx+a)^4\left(\frac{4\,a}{bx+a}-\frac{6\,a^2}{(bx+a)^2}+\frac{4\,a^3}{(bx+a)^3}-1\right)\arcsin\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^5}-\frac{(bx+a)^3\left(\sqrt{-\frac{1}{(bx+a)^2}}\right.}{}\right.$$

```
[In] integrate(x^3*arccsc(b*x+a),x, algorithm="giac")
```

```
[Out] -1/96*b*(24*(b*x + a)^4*(4*a/(b*x + a) - 6*a^2/(b*x + a)^2 + 4*a^3/(b*x + a
)^3 - 1)*arcsin(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/b^5 - ((b*x + a)^3*(s
qrt(-1/(b*x + a)^2 + 1) - 1)^3 + 12*(b*x + a)^2*a*(sqrt(-1/(b*x + a)^2 + 1)
 - 1)^2 + 72*(b*x + a)*a^2*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 9*(b*x + a)*(sq
rt(-1/(b*x + a)^2 + 1) - 1) + 48*(2*a^3 + a)*log(-(sqrt(-1/(b*x + a)^2 + 1)
 - 1)*abs(b*x + a)) - (9*(8*a^2 + 1)*(b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1)
- 1)^2 + 12*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)/((b*x + a)^3*(s
qrt(-1/(b*x + a)^2 + 1) - 1)^3))/b^5)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \csc^{-1}(a+bx)\,dx = \int x^3\,\mathrm{asin}\left(\frac{1}{a+b\,x}\right)\,dx$$

```
[In] int(x^3*asin(1/(a + b*x)),x)
```

```
[Out] int(x^3*asin(1/(a + b*x)), x)
```

3.19 $\int x^2 \csc^{-1}(a+bx)\,dx$

Optimal result 128
Rubi [A] (verified) 128
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Sympy [F] 132
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Optimal result

Integrand size = 10, antiderivative size = 116

$$\int x^2 \csc^{-1}(a+bx)\,dx = -\frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3}+\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2}+\frac{a^3\csc^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\csc^{-1}(a+bx)+\frac{(1+6a^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{6b^3}$$

```
[Out] 1/3*a^3*arccsc(b*x+a)/b^3+1/3*x^3*arccsc(b*x+a)+1/6*(6*a^2+1)*arctanh((1-1/
(b*x+a)^2)^(1/2))/b^3-5/6*a*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3+1/6*x*(b*x+a)
*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5367, 4512, 3867, 3855, 3852, 8}

$$\int x^2 \csc^{-1}(a+bx)\,dx = \frac{a^3\csc^{-1}(a+bx)}{3b^3}+\frac{(6a^2+1)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{6b^3}-\frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3}+\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2}+\frac{1}{3}x^3\csc^{-1}(a+bx)$$

```
[In] Int[x^2*ArcCsc[a + b*x],x]
```

```
[Out] (-5*a*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(6*b^3) + (x*(a + b*x)*Sqrt[1 - (
a + b*x)^(-2)])/(6*b^2) + (a^3*ArcCsc[a + b*x])/(3*b^3) + (x^3*ArcCsc[a + b
*x])/3 + ((1 + 6*a^2)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/(6*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x\cot(x)\csc(x)(-a+\csc(x))^2\,dx, x, \csc^{-1}(a+bx)\right)}{b^3}\\
&= \frac{1}{3}x^3\csc^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\csc(x))^3\,dx, x, \csc^{-1}(a+bx)\right)}{3b^3}\\
&= \frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{1}{3}x^3\csc^{-1}(a+bx)\\
&\quad - \frac{\text{Subst}\left(\int\left(-2a^3+(1+6a^2)\csc(x)-5a\csc^2(x)\right)\,dx, x, \csc^{-1}(a+bx)\right)}{6b^3}\\
&= \frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\csc^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)\\
&\quad + \frac{(5a)\text{Subst}\left(\int\csc^2(x)\,dx, x, \csc^{-1}(a+bx)\right)}{6b^3}\\
&\quad - \frac{(1+6a^2)\,\text{Subst}\left(\int\csc(x)\,dx, x, \csc^{-1}(a+bx)\right)}{6b^3}\\
&= \frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\csc^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)\\
&\quad + \frac{(1+6a^2)\,\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{6b^3} - \frac{(5a)\text{Subst}\left(\int 1\,dx, x, (a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{6b^3}\\
&= -\frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3} + \frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\csc^{-1}(a+bx)}{3b^3}\\
&\quad + \frac{1}{3}x^3\csc^{-1}(a+bx) + \frac{(1+6a^2)\,\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{6b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11

$$
\int x^2\csc^{-1}(a+bx)\,dx
$$

$$
= \frac{(-5a^2-4abx+b^2x^2)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}} + 2b^3x^3\csc^{-1}(a+bx) + 2a^3\arcsin\left(\frac{1}{a+bx}\right) + (1+6a^2)\log\left((a+bx\right.}{6b^3}
$$

```
[In] Integrate[x^2*ArcCsc[a + b*x],x]
```

```
[Out] ((-5*a^2 - 4*a*b*x + b^2*x^2)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)
^2] + 2*b^3*x^3*ArcCsc[a + b*x] + 2*a^3*ArcSin[(a + b*x)^(-1)] + (1 + 6*a^2
)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(6
*b^3)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.66

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccsc}(bx+a)a^3}{3}+\operatorname{arccsc}(bx+a)a^2(bx+a)-\operatorname{arccsc}(bx+a)a(bx+a)^2+\frac{\operatorname{arccsc}(bx+a)(bx+a)^3}{3}-\frac{\sqrt{(bx+a)^2-1}\left(-2a^3\arctan\left(\frac{}{\sqrt{}}\right.\right.}{}}{b^3}$
default	$\frac{-\frac{\operatorname{arccsc}(bx+a)a^3}{3}+\operatorname{arccsc}(bx+a)a^2(bx+a)-\operatorname{arccsc}(bx+a)a(bx+a)^2+\frac{\operatorname{arccsc}(bx+a)(bx+a)^3}{3}-\frac{\sqrt{(bx+a)^2-1}\left(-2a^3\arctan\left(\frac{}{\sqrt{}}\right.\right.}{}}{b^3}$
parts	$\frac{x^3\operatorname{arccsc}(bx+a)}{3}-\frac{\sqrt{b^2x^2+2abx+a^2-1}\left(-2a^3\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{b^2}-x\sqrt{b^2x^2+2abx+a^2-1}\,b\sqrt{b^2}-6\ln\left(\right.\right.}{6b^3\sqrt{\frac{b^2x^2+}{(b}}}$

[In] int(x^2*arccsc(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b^3*(-1/3*arccsc(b*x+a)*a^3+arccsc(b*x+a)*a^2*(b*x+a)-arccsc(b*x+a)*a*(b*x+a)^2+1/3*arccsc(b*x+a)*(b*x+a)^3-1/6*((b*x+a)^2-1)^(1/2)*(-2*a^3*arctan(1/((b*x+a)^2-1)^(1/2))-6*a^2*ln(b*x+a+((b*x+a)^2-1)^(1/2))+6*a*((b*x+a)^2-1)^(1/2)-(b*x+a)*((b*x+a)^2-1)^(1/2)-ln(b*x+a+((b*x+a)^2-1)^(1/2)))/((((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int x^2\csc^{-1}(a+bx)\,dx$$
$$=\frac{2\,b^3x^3\operatorname{arccsc}(bx+a)-4\,a^3\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-(6\,a^2+1)\log\left(-bx-a+\sqrt{b^2\right.}{6\,b^3}$$

[In] integrate(x^2*arccsc(b*x+a),x, algorithm="fricas")

[Out] 1/6*(2*b^3*x^3*arccsc(b*x + a) - 4*a^3*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (6*a^2 + 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b*x - 5*a))/b^3

Sympy [F]

$$\int x^2 \csc^{-1}(a+bx)\,dx = \int x^2 \operatorname{acsc}(a+bx)\,dx$$

```
[In] integrate(x**2*acsc(b*x+a),x)
[Out] Integral(x**2*acsc(a + b*x), x)
```

Maxima [F]

$$\int x^2 \csc^{-1}(a+bx)\,dx = \int x^2 \operatorname{arccsc}(bx+a)\,dx$$

```
[In] integrate(x^2*arccsc(b*x+a),x, algorithm="maxima")
[Out] 1/3*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + integrate(1/3*(b^
2*x^4 + a*b*x^3)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 +
 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*
x + a - 1)) - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(100) = 200$.

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.75

$$\int x^2 \csc^{-1}(a+bx)\,dx =$$

$$-\frac{1}{24}b\left(\frac{8(bx+a)^3\left(\frac{3a}{bx+a}-\frac{3a^2}{(bx+a)^2}-1\right)\arcsin\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^4}+\frac{(bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2+\cdots}{\cdots}\right.$$

```
[In] integrate(x^2*arccsc(b*x+a),x, algorithm="giac")
[Out] -1/24*b*(8*(b*x + a)^3*(3*a/(b*x + a) - 3*a^2/(b*x + a)^2 - 1)*arcsin(-1/((
b*x + a)*(a/(b*x + a) - 1) - a))/b^4 + ((b*x + a)^2*(sqrt(-1/(b*x + a)^2 +
1) - 1)^2 + 12*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 4*(6*a^2 + 1)*l
og(-(sqrt(-1/(b*x + a)^2 + 1) - 1)*abs(b*x + a)) - (12*(b*x + a)*a*(sqrt(-1
/(b*x + a)^2 + 1) - 1) + 1)/((b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2))
/b^4)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \csc^{-1}(a+bx)\,dx = \int x^2\,\mathrm{asin}\left(\frac{1}{a+b\,x}\right)\,dx$$

```
[In] int(x^2*asin(1/(a + b*x)),x)
[Out] int(x^2*asin(1/(a + b*x)), x)
```

3.20 $\int x \csc^{-1}(a+bx)\,dx$

Optimal result 134
Rubi [A] (verified) 134
Mathematica [A] (verified) 136
Maple [A] (verified) 136
Fricas [A] (verification not implemented) 137
Sympy [F] 137
Maxima [F] 137
Giac [A] (verification not implemented) 138
Mupad [F(-1)] 138

Optimal result

Integrand size = 8, antiderivative size = 79

$$\int x\csc^{-1}(a+bx)\,dx=\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2}-\frac{a^2\csc^{-1}(a+bx)}{2b^2}+\frac{1}{2}x^2\csc^{-1}(a+bx)-\frac{a\,\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2}$$

```
[Out] -1/2*a^2*arccsc(b*x+a)/b^2+1/2*x^2*arccsc(b*x+a)-a*arctanh((1-1/(b*x+a)^2)^
(1/2))/b^2+1/2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5367, 4512, 3858, 3855, 3852, 8}

$$\int x\csc^{-1}(a+bx)\,dx=-\frac{a^2\csc^{-1}(a+bx)}{2b^2}-\frac{a\,\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2}+\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2}+\frac{1}{2}x^2\csc^{-1}(a+bx)$$

```
[In] Int[x*ArcCsc[a + b*x],x]
```

```
[Out] ((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(2*b^2) - (a^2*ArcCsc[a + b*x])/(2*b^2
) + (x^2*ArcCsc[a + b*x])/2 - (a*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x\cot(x)\csc(x)(-a+\csc(x))\,dx, x, \csc^{-1}(a+bx)\right)}{b^2}\\
&= \frac{1}{2}x^2\csc^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\csc(x))^2\,dx, x, \csc^{-1}(a+bx)\right)}{2b^2}\\
&= -\frac{a^2\csc^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\csc^{-1}(a+bx) - \frac{\text{Subst}\left(\int\csc^2(x)\,dx, x, \csc^{-1}(a+bx)\right)}{2b^2}\\
&\quad + \frac{a\text{Subst}\left(\int\csc(x)\,dx, x, \csc^{-1}(a+bx)\right)}{b^2}
\end{aligned}$$

$$= -\frac{a^2\csc^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\csc^{-1}(a+bx) - \frac{a\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2} + \frac{\operatorname{Subst}\left(\int 1\,dx, x, (a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{2b^2}$$

$$= \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2} - \frac{a^2\csc^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\csc^{-1}(a+bx) - \frac{a\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.39

$$\int x\csc^{-1}(a+bx)\,dx$$
$$= \frac{(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}} + b^2x^2\csc^{-1}(a+bx) - a^2\arcsin\left(\frac{1}{a+bx}\right) - 2a\log\left((a+bx)\left(1+\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)\right)}{2b^2}$$

```
[In] Integrate[x*ArcCsc[a + b*x],x]
```

```
[Out] ((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2] + b^2*x^2*ArcCs
c[a + b*x] - a^2*ArcSin[(a + b*x)^(-1)] - 2*a*Log[(a + b*x)*(1 + Sqrt[(-1 +
 a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(2*b^2)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{\operatorname{arccsc}(bx+a)(bx+a)^2}{2} - \operatorname{arccsc}(bx+a)a(bx+a) + \frac{\sqrt{(bx+a)^2-1}\left(-2a\ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)+\sqrt{(bx+a)^2-1}\right)}{2\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}}{b^2}$
default	$\frac{\frac{\operatorname{arccsc}(bx+a)(bx+a)^2}{2} - \operatorname{arccsc}(bx+a)a(bx+a) + \frac{\sqrt{(bx+a)^2-1}\left(-2a\ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)+\sqrt{(bx+a)^2-1}\right)}{2\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)}}{b^2}$
parts	$\frac{x^2\operatorname{arccsc}(bx+a)}{2} + \frac{\sqrt{b^2x^2+2abx+a^2-1}\left(-a^2\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{b^2} - 2a\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2-1}\,\sqrt{b^2}+ab}{\sqrt{b^2}}\right)\right.}{2b^2\sqrt{\frac{b^2x^2+2abx+a^2-1}{(bx+a)^2}}\,(bx+a)\sqrt{b^2}}$

```
[In] int(x*arccsc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arccsc(b*x+a)*(b*x+a)^2-arccsc(b*x+a)*a*(b*x+a)+1/2/(((b*x+a)^2-
1)/(b*x+a)^2)^(1/2)/(b*x+a)*((b*x+a)^2-1)^(1/2)*(-2*a*ln(b*x+a+((b*x+a)^2-1
)^(1/2))+((b*x+a)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

$$\int x \csc^{-1}(a+bx) \, dx$$
$$= \frac{b^2x^2 \operatorname{arccsc}(bx+a) + 2a^2 \arctan\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right) + 2a\log\left(-bx-a+\sqrt{b^2x^2+2a\right.}{2b^2}$$

```
[In] integrate(x*arccsc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2*arccsc(b*x + a) + 2*a^2*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1)) + 2*a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sq
rt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^2
```

Sympy [F]

$$\int x \csc^{-1}(a+bx) \, dx = \int x \operatorname{acsc}(a+bx) \, dx$$

```
[In] integrate(x*acsc(b*x+a),x)
```

```
[Out] Integral(x*acsc(a + b*x), x)
```

Maxima [F]

$$\int x \csc^{-1}(a+bx) \, dx = \int x \operatorname{arccsc}(bx+a) \, dx$$

```
[In] integrate(x*arccsc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + integrate(1/2*(b^
2*x^3 + a*b*x^2)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 +
 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*
x + a - 1)) - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.70

$$\int x \csc^{-1}(a+bx)\,dx = -\frac{1}{4}\,b\left(\frac{2\,(bx+a)^2\left(\frac{2\,a}{bx+a}-1\right)\arcsin\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^3} - \frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+4\,a\log\left(-\left(\right.\right.}{}\right.$$

```
[In] integrate(x*arccsc(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*b*(2*(b*x + a)^2*(2*a/(b*x + a) - 1)*arcsin(-1/((b*x + a)*(a/(b*x + a)
 - 1) - a))/b^3 - ((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 4*a*log(-(sqr
t(-1/(b*x + a)^2 + 1) - 1)*abs(b*x + a)) - 1/((b*x + a)*(sqrt(-1/(b*x + a)^
2 + 1) - 1)))/b^3)
```

Mupad [F(-1)]

Timed out.

$$\int x \csc^{-1}(a+bx)\,dx = \int x\,\mathrm{asin}\left(\frac{1}{a+b\,x}\right)\,dx$$

```
[In] int(x*asin(1/(a + b*x)),x)
```

```
[Out] int(x*asin(1/(a + b*x)), x)
```

3.21 $\int \csc^{-1}(a+bx)\,dx$

Optimal result 139
Rubi [A] (verified) 139
Mathematica [C] (verified) 141
Maple [A] (verified) 141
Fricas [B] (verification not implemented) 142
Sympy [F] 142
Maxima [A] (verification not implemented) 142
Giac [B] (verification not implemented) 143
Mupad [B] (verification not implemented) 143

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int \csc^{-1}(a+bx)\,dx=\frac{(a+bx)\csc^{-1}(a+bx)}{b}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}$$

```
[Out] (b*x+a)*arccsc(b*x+a)/b+arctanh((1-1/(b*x+a)^2)^(1/2))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5359, 379, 272, 65, 212}

$$\int \csc^{-1}(a+bx)\,dx=\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}+\frac{(a+bx)\csc^{-1}(a+bx)}{b}$$

```
[In] Int[ArcCsc[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcCsc[a + b*x])/b + ArcTanh[Sqrt[1 - (a + b*x)^(-2)]]/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5359

```
Int[ArcCsc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcCsc[c + d*x]
/d), x] + Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+bx)\csc^{-1}(a+bx)}{b} + \int \frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\,dx \\
&= \frac{(a+bx)\csc^{-1}(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\csc^{-1}(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{(a+bx)^2}\right)}{2b} \\
&= \frac{(a+bx)\csc^{-1}(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b} \\
&= \frac{(a+bx)\csc^{-1}(a+bx)}{b} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 469, normalized size of antiderivative = 13.03

$$\int \csc^{-1}(a+bx)\,dx = x\csc^{-1}(a+bx)$$

$$-\frac{(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\left(\sqrt[4]{-1}\left(-i+\sqrt{-1+a^2}\right)\sqrt{2i-ia^2+2\sqrt{-1+a^2}}\arctan\left(\frac{(-1)^{3/4}\sqrt{2i-ia^2+2\sqrt{}}}{a\sqrt{-1+a^2}-a\sqrt{-1+a^2+}}\right.\right.}{}$$

```
[In] Integrate[ArcCsc[a + b*x],x]
```

```
[Out] x*ArcCsc[a + b*x] - ((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x
)^2]*((-1)^(1/4)*(-I + Sqrt[-1 + a^2])*Sqrt[2*I - I*a^2 + 2*Sqrt[-1 + a^2]]
*ArcTan[((-1)^(3/4)*Sqrt[2*I - I*a^2 + 2*Sqrt[-1 + a^2]]*b*x)/(a*Sqrt[-1 +
a^2] - a*Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2])] + (-1)^(3/4)*(I + Sqrt[-1 + a
^2])*Sqrt[-2*I + I*a^2 + 2*Sqrt[-1 + a^2]]*ArcTan[((-1)^(1/4)*Sqrt[-2*I + I
*a^2 + 2*Sqrt[-1 + a^2]]*b*x)/(a*Sqrt[-1 + a^2] - a*Sqrt[-1 + a^2 + 2*a*b*x
 + b^2*x^2])] + a*(a*ArcTan[(Sqrt[-1 + a^2]*b^2*x^2)/(a^4 + a^3*b*x + b^2*x
^2 - a^2*(1 + Sqrt[-1 + a^2]*Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2]))] - Log[Sq
rt[-1 + a^2] - b*x - Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2]] + Log[b^2*(Sqrt[-1
 + a^2] + b*x - Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2])])))/(a*b*Sqrt[-1 + a^2
+ 2*a*b*x + b^2*x^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\operatorname{arccsc}(bx+a)(bx+a)+\ln\left(bx+a+(bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{b}$
default	$\frac{\operatorname{arccsc}(bx+a)(bx+a)+\ln\left(bx+a+(bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{b}$
parts	$x\operatorname{arccsc}(bx+a)+\frac{\sqrt{b^2x^2+2abx+a^2-1}\left(a\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{b^2}+\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2-1}\sqrt{b^2}}{\sqrt{b^2}}\right.\right.}{b\sqrt{\frac{b^2x^2+2abx+a^2-1}{(bx+a)^2}}(bx+a)\sqrt{b^2}}$

```
[In] int(arccsc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arccsc(b*x+a)*(b*x+a)+ln(b*x+a+(b*x+a)*(1-1/(b*x+a)^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.08

$$\int \csc^{-1}(a+bx)\,dx$$
$$= \frac{bx \operatorname{arccsc}(bx+a) - 2\,a \arctan\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 - 1}\right) - \log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2}\right.}{b}$$

```
[In] integrate(arccsc(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*arccsc(b*x + a) - 2*a*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)) - log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

Sympy [F]

$$\int \csc^{-1}(a+bx)\,dx = \int \operatorname{acsc}(a+bx)\,dx$$

```
[In] integrate(acsc(b*x+a),x)
```

```
[Out] Integral(acsc(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \csc^{-1}(a+bx)\,dx$$
$$= \frac{2\,(bx+a)\operatorname{arccsc}(bx+a) + \log\left(\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)}{2\,b}$$

```
[In] integrate(arccsc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arccsc(b*x + a) + log(sqrt(-1/(b*x + a)^2 + 1) + 1) - log(
-sqrt(-1/(b*x + a)^2 + 1) + 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(34) = 68$.

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.25

$$\int \csc^{-1}(a+bx)\,dx$$
$$=\frac{1}{2}b\left(\frac{2(bx+a)\arcsin\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^2}+\frac{\log\left(\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)}{b^2}\right)$$

```
[In] integrate(arccsc(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*b*(2*(b*x + a)*arcsin(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/b^2 + (log(
sqrt(-1/(b*x + a)^2 + 1) + 1) - log(-sqrt(-1/(b*x + a)^2 + 1) + 1))/b^2)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \csc^{-1}(a+bx)\,dx=\frac{\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{(a+b\,x)^2}}}\right)+\mathrm{asin}\left(\frac{1}{a+b\,x}\right)\,(a+b\,x)}{b}$$

```
[In] int(asin(1/(a + b*x)),x)
```

```
[Out] (atanh(1/(1 - 1/(a + b*x)^2)^(1/2)) + asin(1/(a + b*x))*(a + b*x))/b
```

3.22 $\int \frac{\csc^{-1}(a+bx)}{x} \, dx$

Optimal result 144
Rubi [A] (verified) 145
Mathematica [A] (verified) 148
Maple [B] (verified) 149
Fricas [F] 149
Sympy [F] 150
Maxima [F] 150
Giac [F] 150
Mupad [F(-1)] 150

Optimal result

Integrand size = 10, antiderivative size = 210

$$\begin{aligned}\int \frac{\csc^{-1}(a+bx)}{x} \, dx = {} & \csc^{-1}(a+bx) \log \left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\ & + \csc^{-1}(a+bx) \log \left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\ & - \csc^{-1}(a+bx) \log \left(1-e^{2i\csc^{-1}(a+bx)}\right) \\ & - i \operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\ & - i \operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)\end{aligned}$$

```
[Out] -arccsc(b*x+a)*ln(1-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)+arccsc(b*x+a)*ln(1
+I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arccsc(b*x+a)*ln
(1+I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+1/2*I*polylog(
2,(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)-I*polylog(2,-I*a*(I/(b*x+a)+(1-1/(b*
x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))-I*polylog(2,-I*a*(I/(b*x+a)+(1-1/(b*x+a)
^2)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5367, 4648, 4625, 3798, 2221, 2317, 2438, 4615}

$$\int \frac{\csc^{-1}(a+bx)}{x}\,dx = -i\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-i\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)-\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)$$

```
[In] Int[ArcCsc[a + b*x]/x,x]
```

```
[Out] ArcCsc[a + b*x]*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])] +
ArcCsc[a + b*x]*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] -
ArcCsc[a + b*x]*Log[1 - E^((2*I)*ArcCsc[a + b*x])] - I*PolyLog[2, ((-I)*a*E
^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])] - I*PolyLog[2, ((-I)*a*E^(I*ArcC
sc[a + b*x]))/(1 + Sqrt[1 - a^2])] + (I/2)*PolyLog[2, E^((2*I)*ArcCsc[a + b
*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4625

```
Int[(Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cot[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cos[c + d*x]*(Cot[c + d*x]^(n -
1)/(a + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Rule 4648

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/(Csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> In
t[(e + f*x)^m*Sin[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Sin[c + d*x]))
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\cot(x)\csc(x)}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)$$

$$= -\text{Subst}\left(\int \frac{x\cot(x)}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)$$

$$= -\left(a\text{Subst}\left(\int \frac{x\cos(x)}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)\right)$$
$$- \text{Subst}\left(\int x\cot(x)\,dx, x, \csc^{-1}(a+bx)\right)$$

$$= 2i\text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \csc^{-1}(a+bx)\right)$$
$$- a\text{Subst}\left(\int \frac{e^{ix}x}{1-\sqrt{1-a^2}+iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)$$
$$- a\text{Subst}\left(\int \frac{e^{ix}x}{1+\sqrt{1-a^2}+iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)$$

$$= \csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$- \csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)$$
$$- \text{Subst}\left(\int \log\left(1+\frac{iae^{ix}}{1-\sqrt{1-a^2}}\right)dx, x, \csc^{-1}(a+bx)\right)$$
$$- \text{Subst}\left(\int \log\left(1+\frac{iae^{ix}}{1+\sqrt{1-a^2}}\right)dx, x, \csc^{-1}(a+bx)\right)$$
$$+ \text{Subst}\left(\int \log\left(1-e^{2ix}\right)dx, x, \csc^{-1}(a+bx)\right)$$

$$= \csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$- \csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(a+bx)}\right)$$
$$+ i\text{Subst}\left(\int \frac{\log\left(1+\frac{iax}{1-\sqrt{1-a^2}}\right)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)$$
$$+ i\text{Subst}\left(\int \frac{\log\left(1+\frac{iax}{1+\sqrt{1-a^2}}\right)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)$$

$$= \csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$- \csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right) - i\,\text{PolyLog}\left(2, -\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$
$$- i\,\text{PolyLog}\left(2, -\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(a+bx)}\right)$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.79

$$\int \frac{\csc^{-1}(a+bx)}{x}\,dx$$
$$= \frac{1}{8}\Bigg(i\left(\pi-2\csc^{-1}(a+bx)\right)^2 - 32i\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\arctan\left(\frac{(1+a)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(a+bx)\right)\right)}{\sqrt{1-a^2}}\right) - 4\left(\pi - 2\csc^{-1}(a+bx) + 4\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\right)\log\left(1+\frac{i\left(-1+\sqrt{1-a^2}\right)e^{-i\csc^{-1}(a+bx)}}{a}\right)$$
$$-4\left(\pi-2\csc^{-1}(a+bx)-4\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\right)\log\left(1-\frac{i\left(1+\sqrt{1-a^2}\right)e^{-i\csc^{-1}(a+bx)}}{a}\right)$$
$$-8\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)+4\left(\pi-2\csc^{-1}(a+bx)\right)\log\left(\frac{bx}{a+bx}\right)$$
$$+8\csc^{-1}(a+bx)\log\left(\frac{bx}{a+bx}\right)+8i\left(\text{PolyLog}\left(2,-\frac{i\left(-1+\sqrt{1-a^2}\right)e^{-i\csc^{-1}(a+bx)}}{a}\right)+\text{PolyLog}\left(2,\frac{i\left(1+\sqrt{1-a^2}\right)e^{-i\csc^{-1}(a+bx)}}{a}\right)\right)$$
$$+4i\left(\csc^{-1}(a+bx)^2+\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)\right)\Bigg)$$

```
[In] Integrate[ArcCsc[a + b*x]/x,x]
```

```
[Out] (I*(Pi - 2*ArcCsc[a + b*x])^2 - (32*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Arc
Tan[((1 + a)*Cot[(Pi + 2*ArcCsc[a + b*x])/4])/Sqrt[1 - a^2]] - 4*(Pi - 2*Ar
cCsc[a + b*x] + 4*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]])*Log[1 + (I*(-1 + Sqrt[1
 - a^2]))/(a*E^(I*ArcCsc[a + b*x]))] - 4*(Pi - 2*ArcCsc[a + b*x] - 4*ArcSin
[Sqrt[(-1 + a)/a]/Sqrt[2]])*Log[1 - (I*(1 + Sqrt[1 - a^2]))/(a*E^(I*ArcCsc[
a + b*x]))] - 8*ArcCsc[a + b*x]*Log[1 - E^((2*I)*ArcCsc[a + b*x])] + 4*(Pi
- 2*ArcCsc[a + b*x])*Log[(b*x)/(a + b*x)] + 8*ArcCsc[a + b*x]*Log[(b*x)/(a
+ b*x)] + (8*I)*(PolyLog[2, ((-I)*(-1 + Sqrt[1 - a^2]))/(a*E^(I*ArcCsc[a +
b*x]))] + PolyLog[2, (I*(1 + Sqrt[1 - a^2]))/(a*E^(I*ArcCsc[a + b*x]))]) +
(4*I)*(ArcCsc[a + b*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[a + b*x])]))/8
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 606 vs. 2(269) = 538.

Time = 1.87 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.89

method	result
derivativedivides	$-\frac{i a^2 \operatorname{dilog}\left(\frac{-\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)a+\sqrt{a^2-1}+i}{i+\sqrt{a^2-1}}\right)}{a^2-1}-\frac{i a^2 \operatorname{dilog}\left(\frac{\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)a+\sqrt{a^2-1}-i}{-i+\sqrt{a^2-1}}\right)}{a^2-1}-\frac{\operatorname{arccsc}(bx}{}$
default	$-\frac{i a^2 \operatorname{dilog}\left(\frac{-\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)a+\sqrt{a^2-1}+i}{i+\sqrt{a^2-1}}\right)}{a^2-1}-\frac{i a^2 \operatorname{dilog}\left(\frac{\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)a+\sqrt{a^2-1}-i}{-i+\sqrt{a^2-1}}\right)}{a^2-1}-\frac{\operatorname{arccsc}(bx}{}$

```
[In] int(arccsc(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -I*a^2/(a^2-1)*dilog((-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a+(a^2-1)^(1/2)+I)
/(I+(a^2-1)^(1/2)))-I*a^2/(a^2-1)*dilog(((I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*
a+(a^2-1)^(1/2)-I)/(-I+(a^2-1)^(1/2)))-arccsc(b*x+a)/(a^2-1)*ln((-(I/(b*x+a
)+(1-1/(b*x+a)^2)^(1/2))*a+(a^2-1)^(1/2)+I)/(I+(a^2-1)^(1/2)))-arccsc(b*x+a
)/(a^2-1)*ln(((I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a+(a^2-1)^(1/2)-I)/(-I+(a^2
-1)^(1/2)))+a^2*arccsc(b*x+a)/(a^2-1)*ln((-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2)
)*a+(a^2-1)^(1/2)+I)/(I+(a^2-1)^(1/2)))-I*dilog(I/(b*x+a)+(1-1/(b*x+a)^2)^(
1/2))+I*dilog(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+a^2*arccsc(b*x+a)/(a^2-1)*
ln(((I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a+(a^2-1)^(1/2)-I)/(-I+(a^2-1)^(1/2))
)-arccsc(b*x+a)*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+I/(a^2-1)*dilog((-(I/
(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a+(a^2-1)^(1/2)+I)/(I+(a^2-1)^(1/2)))+I/(a^2
-1)*dilog(((I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a+(a^2-1)^(1/2)-I)/(-I+(a^2-1)
^(1/2)))
```

Fricas [F]

$$\int \frac{\csc^{-1}(a+bx)}{x}\,dx=\int \frac{\operatorname{arccsc}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(arccsc(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccsc(b*x + a)/x, x)
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{acsc}(a+bx)}{x}\,dx$$

```
[In] integrate(acsc(b*x+a)/x,x)
[Out] Integral(acsc(a + b*x)/x, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arccsc}(bx+a)}{x}\,dx$$

```
[In] integrate(arccsc(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(arccsc(b*x + a)/x, x)
```

Giac [F]

$$\int \frac{\csc^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arccsc}(bx+a)}{x}\,dx$$

```
[In] integrate(arccsc(b*x+a)/x,x, algorithm="giac")
[Out] integrate(arccsc(b*x + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{a+b\,x}\right)}{x}\,dx$$

```
[In] int(asin(1/(a + b*x))/x,x)
[Out] int(asin(1/(a + b*x))/x, x)
```

3.23 $\int \frac{\csc^{-1}(a+bx)}{x^2} \, dx$

Optimal result . . . 151
Rubi [A] (verified) . . . 151
Mathematica [C] (verified) . . . 153
Maple [A] (verified) . . . 154
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Sympy [F] . . . 155
Maxima [F] . . . 155
Giac [A] (verification not implemented) . . . 155
Mupad [F(-1)] . . . 156

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \frac{\csc^{-1}(a+bx)}{x^2} \, dx = -\frac{b\csc^{-1}(a+bx)}{a} - \frac{\csc^{-1}(a+bx)}{x} - \frac{2b\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

```
[Out] -b*arccsc(b*x+a)/a-arccsc(b*x+a)/x-2*b*arctan((a-tan(1/2*arccsc(b*x+a)))/(-
a^2+1)^(1/2))/a/(-a^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5367, 4512, 3868, 2739, 632, 210}

$$\int \frac{\csc^{-1}(a+bx)}{x^2} \, dx = -\frac{2b\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{b\csc^{-1}(a+bx)}{a} - \frac{\csc^{-1}(a+bx)}{x}$$

```
[In] Int[ArcCsc[a + b*x]/x^2,x]
```

```
[Out] -((b*ArcCsc[a + b*x])/a) - ArcCsc[a + b*x]/x - (2*b*ArcTan[(a - Tan[ArcCsc[
a + b*x]/2])/Sqrt[1 - a^2]])/(a*Sqrt[1 - a^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(b\text{Subst}\left(\int \frac{x\cot(x)\csc(x)}{(-a+\csc(x))^2}\,dx, x, \csc^{-1}(a+bx)\right)\right)\\
&= -\frac{\csc^{-1}(a+bx)}{x} + b\text{Subst}\left(\int \frac{1}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)\\
&= -\frac{b\csc^{-1}(a+bx)}{a} - \frac{\csc^{-1}(a+bx)}{x} + \frac{b\text{Subst}\left(\int \frac{1}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{a}\\
&= -\frac{b\csc^{-1}(a+bx)}{a} - \frac{\csc^{-1}(a+bx)}{x} + \frac{(2b)\text{Subst}\left(\int \frac{1}{1-2ax+x^2}\,dx, x, \tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)\right)}{a}
\end{aligned}$$

$$= -\frac{b\csc^{-1}(a+bx)}{a} - \frac{\csc^{-1}(a+bx)}{x} - \frac{(4b)\text{Subst}\left(\int \frac{1}{-4(1-a^2)-x^2}\,dx, x, -2a+2\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)\right)}{a}$$

$$= -\frac{b\csc^{-1}(a+bx)}{a} - \frac{\csc^{-1}(a+bx)}{x} - \frac{2b\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.67

$$\int \frac{\csc^{-1}(a+bx)}{x^2}\,dx = -\frac{\csc^{-1}(a+bx)}{x} + \frac{b\left(-\arcsin\left(\frac{1}{a+bx}\right) + \frac{i\log\left(\frac{2\left(-\frac{ia(-1+a^2+abx)}{\sqrt{1-a^2}} - a(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{bx}\right)}{\sqrt{1-a^2}}\right)}{a}$$

```
[In] Integrate[ArcCsc[a + b*x]/x^2,x]
```

```
[Out] -(ArcCsc[a + b*x]/x) + (b*(-ArcSin[(a + b*x)^(-1)] + (I*Log[(2*(((-I)*a*(-1
 + a^2 + a*b*x))/Sqrt[1 - a^2] - a*(a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2
*x^2)/(a + b*x)^2]))/(b*x)])/Sqrt[1 - a^2]))/a
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.84

method	result
derivativedivides	$b\left(-\frac{\operatorname{arccsc}(bx+a)}{bx}-\frac{\sqrt{(bx+a)^2-1}\left(\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)\sqrt{a^2-1}-\ln\left(\frac{2\sqrt{a^2-1}\sqrt{(bx+a)^2-1}+2(bx+a)a-2}{bx}\right)\right)}{\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)a\sqrt{a^2-1}}\right)$
default	$b\left(-\frac{\operatorname{arccsc}(bx+a)}{bx}-\frac{\sqrt{(bx+a)^2-1}\left(\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)\sqrt{a^2-1}-\ln\left(\frac{2\sqrt{a^2-1}\sqrt{(bx+a)^2-1}+2(bx+a)a-2}{bx}\right)\right)}{\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)a\sqrt{a^2-1}}\right)$
parts	$-\frac{\operatorname{arccsc}(bx+a)}{x}-\frac{b\sqrt{b^2x^2+2abx+a^2-1}\left(\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{a^2-1}-\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\sqrt{b^2x^2+2abx+}}{x}\right.\right.}{\sqrt{\frac{b^2x^2+2abx+a^2-1}{(bx+a)^2}}\,(bx+a)a\sqrt{a^2-1}}$

```
[In] int(arccsc(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/b/x*arccsc(b*x+a)-((b*x+a)^2-1)^(1/2)*(arctan(1/((b*x+a)^2-1)^(1/2))*
(a^2-1)^(1/2)-ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x))/((
(b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(63) = 126.

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.03

$$\int \frac{\csc^{-1}(a+bx)}{x^2}\,dx = \left[\frac{2\,(a^2-1)bx\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{a^2-1}bx\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2+\sqrt{\right.}}{x}\right.}{(a^3-a)x}\right.$$

```
[In] integrate(arccsc(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [(2*(a^2 - 1)*b*x*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sq
rt(a^2 - 1)*b*x*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2
 + sqrt(a^2 - 1)*a - 1) + (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) - (a^3 -
a)*arccsc(b*x + a))/((a^3 - a)*x), (2*(a^2 - 1)*b*x*arctan(-b*x - a + sqrt(
b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*sqrt(-a^2 + 1)*b*x*arctan(-(sqrt(-a^2 + 1
)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) - (a^3
 - a)*arccsc(b*x + a))/((a^3 - a)*x)]
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{acsc}(a+bx)}{x^2}\,dx$$

```
[In] integrate(acsc(b*x+a)/x**2,x)
```

```
[Out] Integral(acsc(a + b*x)/x**2, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{arccsc}(bx+a)}{x^2}\,dx$$

```
[In] integrate(arccsc(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -(x*integrate((b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))
/(b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x)*e
^(log(b*x + a + 1) + log(b*x + a - 1))), x) + arctan2(1, sqrt(b*x + a + 1)*
sqrt(b*x + a - 1)))/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.39

$$\int \frac{\csc^{-1}(a+bx)}{x^2}\,dx = -b\left(\frac{2\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}a} - \frac{\arcsin\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{a\left(\frac{a}{bx+a}-1\right)}\right)$$

```
[In] integrate(arccsc(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] -b*(2*arctan(((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) + a)/sqrt(-a^2 + 1))
/(sqrt(-a^2 + 1)*a) - arcsin(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/(a*(a/(b
*x + a) - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{a+b\,x}\right)}{x^2}\,dx$$

```
[In] int(asin(1/(a + b*x))/x^2,x)
[Out] int(asin(1/(a + b*x))/x^2, x)
```

3.24 $\int \frac{\csc^{-1}(a+bx)}{x^3} dx$

Optimal result . . . 157
Rubi [A] (verified) . . . 157
Mathematica [C] (verified) . . . 160
Maple [B] (verified) . . . 161
Fricas [A] (verification not implemented) . . . 161
Sympy [F] . . . 162
Maxima [F] . . . 162
Giac [B] (verification not implemented) . . . 162
Mupad [F(-1)] . . . 163

Optimal result

Integrand size = 10, antiderivative size = 123

$$\int \frac{\csc^{-1}(a+bx)}{x^3} dx = -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)x} + \frac{b^2\csc^{-1}(a+bx)}{2a^2} - \frac{\csc^{-1}(a+bx)}{2x^2} + \frac{(1-2a^2)b^2\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{a^2(1-a^2)^{3/2}}$$

```
[Out] 1/2*b^2*arccsc(b*x+a)/a^2-1/2*arccsc(b*x+a)/x^2+(-2*a^2+1)*b^2*arctan((a-ta
n(1/2*arccsc(b*x+a)))/(-a^2+1)^(1/2))/a^2/(-a^2+1)^(3/2)-1/2*b*(b*x+a)*(1-1
/(b*x+a)^2)^(1/2)/a/(-a^2+1)/x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5367, 4512, 3870, 4004, 3916, 2739, 632, 210}

$$\int \frac{\csc^{-1}(a+bx)}{x^3} dx = \frac{(1-2a^2)b^2\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{a^2(1-a^2)^{3/2}} + \frac{b^2\csc^{-1}(a+bx)}{2a^2} - \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)x} - \frac{\csc^{-1}(a+bx)}{2x^2}$$

```
[In] Int[ArcCsc[a + b*x]/x^3,x]
```

```
[Out] -1/2*(b*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(a*(1 - a^2)*x) + (b^2*ArcCsc[a
 + b*x])/(2*a^2) - ArcCsc[a + b*x]/(2*x^2) + ((1 - 2*a^2)*b^2*ArcTan[(a - T
an[ArcCsc[a + b*x]/2])/Sqrt[1 - a^2]])/(a^2*(1 - a^2)^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(b^2\text{Subst}\left(\int \frac{x\cot(x)\csc(x)}{(-a+\csc(x))^3}\,dx, x, \csc^{-1}(a+bx)\right)\right)$$

$$= -\frac{\csc^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{(-a+\csc(x))^2}\,dx, x, \csc^{-1}(a+bx)\right)$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} - \frac{\csc^{-1}(a+bx)}{2x^2} - \frac{b^2\text{Subst}\left(\int \frac{1-a^2-a\csc(x)}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{2a\left(1-a^2\right)}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} + \frac{b^2\csc^{-1}(a+bx)}{2a^2} - \frac{\csc^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{\csc(x)}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{2a^2\left(1-a^2\right)}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} + \frac{b^2\csc^{-1}(a+bx)}{2a^2} - \frac{\csc^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{2a^2\left(1-a^2\right)}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} + \frac{b^2\csc^{-1}(a+bx)}{2a^2} - \frac{\csc^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{1-2ax+x^2}\,dx, x, \tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)\right)}{a^2\left(1-a^2\right)}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} + \frac{b^2\csc^{-1}(a+bx)}{2a^2} - \frac{\csc^{-1}(a+bx)}{2x^2} + \frac{\left(2\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{-4\left(1-a^2\right)-x^2}\,dx, x, -2a+2\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)\right)}{a^2\left(1-a^2\right)}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\,(1-a^2)\,x} + \frac{b^2\csc^{-1}(a+bx)}{2a^2} - \frac{\csc^{-1}(a+bx)}{2x^2} + \frac{(1-2a^2)\,b^2\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{a^2\,(1-a^2)^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.62

$$\int \frac{\csc^{-1}(a+bx)}{x^3}\,dx$$

$$= \frac{\frac{bx(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}}{a(-1+a^2)} - \csc^{-1}(a+bx) + \frac{b^2x^2\arcsin\left(\frac{1}{a+bx}\right)}{a^2} + \frac{i(-1+2a^2)b^2x^2\log\left(\frac{4(-1+a)a^2(1+a)\left(\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}+(a+bx)\sqrt{\cdots}\right.}{(-1+2a^2)b^2x}\right.}{a^2(1-a^2)^{3/2}}}{2x^2}$$

```
[In] Integrate[ArcCsc[a + b*x]/x^3,x]
```

```
[Out] ((b*x*(a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])/(a*(-1 +
a^2)) - ArcCsc[a + b*x] + (b^2*x^2*ArcSin[(a + b*x)^(-1)])/a^2 + (I*(-1 + 2
*a^2)*b^2*x^2*Log[(4*(-1 + a)*a^2*(1 + a)*((I*(-1 + a^2 + a*b*x))/Sqrt[1 -
a^2] + (a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]))/((-1 +
2*a^2)*b^2*x)])/(a^2*(1 - a^2)^(3/2)))/(2*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(109) = 218$.

Time = 0.71 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.59

method	result
parts	$-\frac{\operatorname{arccsc}(bx+a)}{2x^2}+\frac{b\sqrt{b^2x^2+2abx+a^2-1}\left((a^2-1)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)a^2bx-2\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\sqrt{}}{x}\right.\right.}{}$
derivativedivides	$b^2\left(-\frac{\operatorname{arccsc}(bx+a)}{2b^2x^2}-\frac{\sqrt{(bx+a)^2-1}\left(\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)(a^2-1)^{\frac{3}{2}}a^3-\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)(a^2-1)^{\frac{3}{2}}a^2(bx+\right.}{}\right.$
default	$b^2\left(-\frac{\operatorname{arccsc}(bx+a)}{2b^2x^2}-\frac{\sqrt{(bx+a)^2-1}\left(\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)(a^2-1)^{\frac{3}{2}}a^3-\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)(a^2-1)^{\frac{3}{2}}a^2(bx+\right.}{}\right.$

```
[In] int(arccsc(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccsc(b*x+a)/x^2+1/2*b*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*((a^2-1)^(3/2)*a
rctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1/2))*a^2*b*x-2*ln(2*(a*b*x+(a^2-1)^(1/2)*
(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^4*b*x-b*arctan(1/(b^2*x^2+2*a*b*x
+a^2-1)^(1/2))*x*(a^2-1)^(3/2)+(a^2-1)^(3/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*
a+3*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^2*b
*x-b*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*x)/(
(b^2*x^2+2*a*b*x+a^2-1)/(b*x+a)^2)^(1/2)/(b*x+a)/a^2/(a^2-1)^(5/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.48

$$\int \frac{\csc^{-1}(a+bx)}{x^3}\,dx$$
$$=\left[\frac{(2a^2-1)\sqrt{a^2-1}b^2x^2\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2-1}\left(a^2-\sqrt{a^2-1}a-1\right)-(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)-2(a^4-2a^2+1}{}\right.$$

```
[In] integrate(arccsc(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*((2*a^2 - 1)*sqrt(a^2 - 1)*b^2*x^2*log((a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1)*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^
2 - 1) - a)/x) - 2*(a^4 - 2*a^2 + 1)*b^2*x^2*arctan(-b*x - a + sqrt(b^2*x^2
```

```
 + 2*a*b*x + a^2 - 1)) + (a^3 - a)*b^2*x^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)*(a^3 - a)*b*x - (a^6 - 2*a^4 + a^2)*arccsc(b*x + a))/((a^6 - 2*a^4 + a^
2)*x^2), 1/2*(2*(2*a^2 - 1)*sqrt(-a^2 + 1)*b^2*x^2*arctan(-(sqrt(-a^2 + 1)*
b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) - 2*(a^4
 - 2*a^2 + 1)*b^2*x^2*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))
+ (a^3 - a)*b^2*x^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^3 - a)*b*x - (a^
6 - 2*a^4 + a^2)*arccsc(b*x + a))/((a^6 - 2*a^4 + a^2)*x^2)]
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{acsc}(a+bx)}{x^3}\,dx$$

```
[In] integrate(acsc(b*x+a)/x**3,x)
```

```
[Out] Integral(acsc(a + b*x)/x**3, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{arccsc}(bx+a)}{x^3}\,dx$$

```
[In] integrate(arccsc(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x^2*integrate(1/2*(b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b
*x + a - 1))/(b^2*x^4 + 2*a*b*x^3 + (a^2 - 1)*x^2 + (b^2*x^4 + 2*a*b*x^3 +
(a^2 - 1)*x^2)*e^(log(b*x + a + 1) + log(b*x + a - 1))), x) + arctan2(1, sq
rt(b*x + a + 1)*sqrt(b*x + a - 1)))/x^2
```

Giac [B] (verification not implemented)

Leave count of result is larger than twice the leaf count of optimal. 217 vs. $2(108) = 216$.

Time = 0.35 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.76

$$\int \frac{\csc^{-1}(a+bx)}{x^3}\,dx$$
$$= \frac{1}{2}b\left(\frac{2\,(2\,a^2b-b)\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{(a^4-a^2)\sqrt{-a^2+1}} + \frac{2\left((bx+a)ab\left(\sqrt{-\frac{1}{(bx+a)^2}+1\right.\right.}{\left((bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2+2\,(bx+a)a\left(\right.\right.}\right.$$

```
[In] integrate(arccsc(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*(2*(2*a^2*b - b)*arctan(((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) + a
)/sqrt(-a^2 + 1))/((a^4 - a^2)*sqrt(-a^2 + 1)) + 2*((b*x + a)*a*b*(sqrt(-1/
(b*x + a)^2 + 1) - 1) + b)/(((b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 +
 2*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)*(a^3 - a)) - (2*a*b/(b*x
 + a) - b)*arcsin(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/(a^2*(a/(b*x + a) -
 1)^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{a+b\,x}\right)}{x^3}\,dx$$

```
[In] int(asin(1/(a + b*x))/x^3,x)
```

```
[Out] int(asin(1/(a + b*x))/x^3, x)
```

3.25 $\int \frac{\csc^{-1}(a+bx)}{x^4}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 180

$$\int \frac{\csc^{-1}(a+bx)}{x^4}\,dx = -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} + \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\csc^{-1}(a+bx)}{3a^3} - \frac{\csc^{-1}(a+bx)}{3x^3} - \frac{(2-5a^2+6a^4)b^3\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{3a^3(1-a^2)^{5/2}}$$

```
[Out] -1/3*b^3*arccsc(b*x+a)/a^3-1/3*arccsc(b*x+a)/x^3-1/3*(6*a^4-5*a^2+2)*b^3*ar
ctan((a-tan(1/2*arccsc(b*x+a)))/(-a^2+1)^(1/2))/a^3/(-a^2+1)^(5/2)-1/6*b*(b
*x+a)*(1-1/(b*x+a)^2)^(1/2)/a/(-a^2+1)/x^2+1/6*(-5*a^2+2)*b^2*(b*x+a)*(1-1/
(b*x+a)^2)^(1/2)/a^2/(-a^2+1)^2/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5367, 4512, 3870, 4145, 4004, 3916, 2739, 632, 210}

$$\int \frac{\csc^{-1}(a+bx)}{x^4}\,dx = -\frac{b^3\csc^{-1}(a+bx)}{3a^3} + \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} - \frac{(6a^4-5a^2+2)b^3\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{3a^3(1-a^2)^{5/2}} - \frac{\csc^{-1}(a+bx)}{3x^3}$$

```
[In] Int[ArcCsc[a + b*x]/x^4,x]
```

```
[Out] -1/6*(b*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(a*(1 - a^2)*x^2) + ((2 - 5*a^2
)*b^2*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(6*a^2*(1 - a^2)^2*x) - (b^3*ArcC
sc[a + b*x])/(3*a^3) - ArcCsc[a + b*x]/(3*x^3) - ((2 - 5*a^2 + 6*a^4)*b^3*A
rcTan[(a - Tan[ArcCsc[a + b*x]/2])/Sqrt[1 - a^2]])/(3*a^3*(1 - a^2)^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(b^3\text{Subst}\left(\int \frac{x\cot(x)\csc(x)}{(-a+\csc(x))^4}\,dx, x, \csc^{-1}(a+bx)\right)\right)\\
&= -\frac{\csc^{-1}(a+bx)}{3x^3}+\frac{1}{3}b^3\text{Subst}\left(\int \frac{1}{(-a+\csc(x))^3}\,dx, x, \csc^{-1}(a+bx)\right)\\
&= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\left(1-a^2\right)x^2}-\frac{\csc^{-1}(a+bx)}{3x^3}\\
&\quad -\frac{b^3\text{Subst}\left(\int \frac{2\left(1-a^2\right)-2a\csc(x)-\csc^2(x)}{(-a+\csc(x))^2}\,dx, x, \csc^{-1}(a+bx)\right)}{6a\left(1-a^2\right)}\\
&= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\left(1-a^2\right)x^2}+\frac{\left(2-5a^2\right)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\left(1-a^2\right)^2x}\\
&\quad -\frac{\csc^{-1}(a+bx)}{3x^3}+\frac{b^3\text{Subst}\left(\int \frac{2\left(1-a^2\right)^2-a\left(1-4a^2\right)\csc(x)}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{6a^2\left(1-a^2\right)^2}
\end{aligned}
$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} + \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\csc^{-1}(a+bx)}{3a^3} - \frac{\csc^{-1}(a+bx)}{3x^3} + \frac{((2-5a^2+6a^4)b^3)\,\text{Subst}\left(\int\frac{\csc(x)}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{6a^3(1-a^2)^2}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} + \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\csc^{-1}(a+bx)}{3a^3} - \frac{\csc^{-1}(a+bx)}{3x^3} + \frac{((2-5a^2+6a^4)b^3)\,\text{Subst}\left(\int\frac{1}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{6a^3(1-a^2)^2}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} + \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\csc^{-1}(a+bx)}{3a^3} - \frac{\csc^{-1}(a+bx)}{3x^3} + \frac{((2-5a^2+6a^4)b^3)\,\text{Subst}\left(\int\frac{1}{1-2ax+x^2}\,dx, x, \tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)\right)}{3a^3(1-a^2)^2}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} + \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\csc^{-1}(a+bx)}{3a^3} - \frac{\csc^{-1}(a+bx)}{3x^3} - \frac{(2(2-5a^2+6a^4)b^3)\,\text{Subst}\left(\int\frac{1}{-4(1-a^2)-x^2}\,dx, x, -2a+2\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)\right)}{3a^3(1-a^2)^2}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} + \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\csc^{-1}(a+bx)}{3a^3} - \frac{\csc^{-1}(a+bx)}{3x^3} - \frac{(2-5a^2+6a^4)b^3\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{3a^3(1-a^2)^{5/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.34

$$\int \frac{\csc^{-1}(a+bx)}{x^4}\,dx = \frac{1}{6}\left(\frac{b\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\left(a^4+abx-4a^3bx+2b^2x^2-a^2(1+5b^2x^2)\right)}{a^2\left(-1+a^2\right)^2x^2}-\frac{2\csc^{-1}(a+bx)}{x^3}-\frac{2b^3\arcsin\left(\frac{1}{a+bx}\right)}{a^3}+\frac{i(2-5a^2+6a^4)\,b^3\log\left(\frac{12a^3(-1+a^2)^2\left(-\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}-(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{(2-5a^2+6a^4)b^3x}\right)}{a^3\left(1-a^2\right)^{5/2}}\right)$$

```
[In] Integrate[ArcCsc[a + b*x]/x^4,x]
```

```
[Out] ((b*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(a^4 + a*b*x - 4*a^3*b
*x + 2*b^2*x^2 - a^2*(1 + 5*b^2*x^2)))/(a^2*(-1 + a^2)^2*x^2) - (2*ArcCsc[a
 + b*x])/x^3 - (2*b^3*ArcSin[(a + b*x)^(-1)])/a^3 + (I*(2 - 5*a^2 + 6*a^4)*
b^3*Log[(12*a^3*(-1 + a^2)^2*(((-I)*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2] - (a
+ b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]))/((2 - 5*a^2 + 6*a
^4)*b^3*x)])/(a^3*(1 - a^2)^(5/2)))/6
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(160) = 320.

Time = 0.72 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.93

method	result
parts	$-\frac{\operatorname{arccsc}(bx+a)}{3x^3}-\frac{b\sqrt{b^2x^2+2abx+a^2-1}\left(2\left(a^2-1\right)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)a^4b^2x^2-6\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\,\sqrt{}}{x}\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(arccsc(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arccsc(b*x+a)/x^3-1/6*b*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(2*(a^2-1)^(3/2)
*arctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1/2))*a^4*b^2*x^2-6*ln(2*(a*b*x+(a^2-1)^
(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^6*b^2*x^2-4*(a^2-1)^(3/2)*a
rctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1/2))*a^2*b^2*x^2+5*(b^2*x^2+2*a*b*x+a^2-1
)^(1/2)*(a^2-1)^(3/2)*a^3*b*x+11*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x
+a^2-1)^(1/2)+a^2-1)/x)*a^4*b^2*x^2+2*b^2*arctan(1/(b^2*x^2+2*a*b*x+a^2-1)^
(1/2))*x^2*(a^2-1)^(3/2)-(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a^4-2*
(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a*b*x-7*ln(2*(a*b*x+(a^2-1)^(1/
2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^2*b^2*x^2+(b^2*x^2+2*a*b*x+a^2
-1)^(1/2)*(a^2-1)^(3/2)*a^2+2*b^2*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2-1)^(1/2)+a^2-1)/x)*x^2)/((b^2*x^2+2*a*b*x+a^2-1)/(b*x+a)^2)^(1/2)/(b*
x+a)/a^3/(a^2-1)^(7/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.04

$$\int \frac{\csc^{-1}(a+bx)}{x^4}\,dx = \left[\frac{(6a^4-5a^2+2)\sqrt{a^2-1}b^3x^3\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2+\sqrt{a^2-1}a-1\right)+(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)+4\,(a^6-3 \cdots}{\cdots}\right.$$

$$-\frac{2\,(6\,a^4-5\,a^2+2)\sqrt{-a^2+1}b^3x^3\arctan\left(-\frac{\sqrt{-a^2+1}bx-\sqrt{b^2x^2+2\,abx+a^2-1}\sqrt{-a^2+1}}{a^2-1}\right)-4\,(a^6-3\,a^4+3\,a^2-1) \cdots}{\cdots}$$

```
[In] integrate(arccsc(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((6*a^4 - 5*a^2 + 2)*sqrt(a^2 - 1)*b^3*x^3*log((a^2*b*x + a^3 + sqrt(b
^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 + sqrt(a^2 - 1)*a - 1) + (a*b*x + a^2 - 1)
*sqrt(a^2 - 1) - a)/x) + 4*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*arctan(-b*x -
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (5*a^5 - 7*a^3 + 2*a)*b^3*x^3 - 2*
(a^9 - 3*a^7 + 3*a^5 - a^3)*arccsc(b*x + a) - ((5*a^5 - 7*a^3 + 2*a)*b^2*x^
2 - (a^6 - 2*a^4 + a^2)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^9 - 3*a
^7 + 3*a^5 - a^3)*x^3), -1/6*(2*(6*a^4 - 5*a^2 + 2)*sqrt(-a^2 + 1)*b^3*x^3*
arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 +
 1))/(a^2 - 1)) - 4*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*arctan(-b*x - a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (5*a^5 - 7*a^3 + 2*a)*b^3*x^3 + 2*(a^9 -
3*a^7 + 3*a^5 - a^3)*arccsc(b*x + a) + ((5*a^5 - 7*a^3 + 2*a)*b^2*x^2 - (a^
6 - 2*a^4 + a^2)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^9 - 3*a^7 + 3*
a^5 - a^3)*x^3)]
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{acsc}(a+bx)}{x^4}\,dx$$

```
[In] integrate(acsc(b*x+a)/x**4,x)
[Out] Integral(acsc(a + b*x)/x**4, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{arccsc}(bx+a)}{x^4}\,dx$$

```
[In] integrate(arccsc(b*x+a)/x^4,x, algorithm="maxima")
[Out] -1/3*(3*x^3*integrate(1/3*(b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b
*x + a - 1))/(b^2*x^5 + 2*a*b*x^4 + (a^2 - 1)*x^3 + (b^2*x^5 + 2*a*b*x^4 +
(a^2 - 1)*x^3)*e^(log(b*x + a + 1) + log(b*x + a - 1))), x) + arctan2(1, sq
rt(b*x + a + 1)*sqrt(b*x + a - 1)))/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(156) = 312$.

Time = 0.37 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.50

$$\int \frac{\csc^{-1}(a+bx)}{x^4}\,dx = -\frac{1}{3}\,b\left(\frac{(6\,a^4b^2 - 5\,a^2b^2 + 2\,b^2)\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{(a^7 - 2\,a^5 + a^3)\sqrt{-a^2+1}} + \frac{4\,(bx+a)^3a^3b^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^3}{\cdots}\right.$$

```
[In] integrate(arccsc(b*x+a)/x^4,x, algorithm="giac")
[Out] -1/3*b*((6*a^4*b^2 - 5*a^2*b^2 + 2*b^2)*arctan(((b*x + a)*(sqrt(-1/(b*x + a
)^2 + 1) - 1) + a)/sqrt(-a^2 + 1))/((a^7 - 2*a^5 + a^3)*sqrt(-a^2 + 1)) + (
4*(b*x + a)^3*a^3*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 10*(b*x + a)^2*a^4
*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 - (b*x + a)^3*a*b^2*(sqrt(-1/(b*x + a
)^2 + 1) - 1)^3 + (b*x + a)^2*a^2*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 16
*(b*x + a)*a^3*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1) - 2*(b*x + a)^2*b^2*(sqrt
```

```
(-1/(b*x + a)^2 + 1) - 1)^2 - 7*(b*x + a)*a*b^2*(sqrt(-1/(b*x + a)^2 + 1) -
 1) + 5*a^2*b^2 - 2*b^2)/((a^6 - 2*a^4 + a^2)*((b*x + a)^2*(sqrt(-1/(b*x +
a)^2 + 1) - 1)^2 + 2*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)^2) + (
3*a*b^2/(b*x + a) - 3*a^2*b^2/(b*x + a)^2 - b^2)*arcsin(-1/((b*x + a)*(a/(b
*x + a) - 1) - a))/(a^3*(a/(b*x + a) - 1)^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)}{x^4}\,dx = \int \frac{\mathrm{asin}\left(\frac{1}{a+b\,x}\right)}{x^4}\,dx$$

```
[In] int(asin(1/(a + b*x))/x^4,x)
```

```
[Out] int(asin(1/(a + b*x))/x^4, x)
```

3.26 $\int \frac{\csc^{-1}(a+bx)}{x^5} dx$

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Optimal result

Integrand size = 10, antiderivative size = 239

$$\begin{aligned}\int \frac{\csc^{-1}(a+bx)}{x^5}\,dx &= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a\,(1-a^2)\,x^3}+\frac{(3-8a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^2\,(1-a^2)^2\,x^2}\\&\quad-\frac{(6-17a^2+26a^4)\,b^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^3\,(1-a^2)^3\,x}\\&\quad+\frac{b^4\csc^{-1}(a+bx)}{4a^4}-\frac{\csc^{-1}(a+bx)}{4x^4}\\&\quad+\frac{(2-7a^2+8a^4-8a^6)\,b^4\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{4a^4\,(1-a^2)^{7/2}}\end{aligned}$$

```
[Out] 1/4*b^4*arccsc(b*x+a)/a^4-1/4*arccsc(b*x+a)/x^4+1/4*(-8*a^6+8*a^4-7*a^2+2)*
b^4*arctan((a-tan(1/2*arccsc(b*x+a)))/(-a^2+1)^(1/2))/a^4/(-a^2+1)^(7/2)-1/
12*b*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/a/(-a^2+1)/x^3+1/24*(-8*a^2+3)*b^2*(b*x+
a)*(1-1/(b*x+a)^2)^(1/2)/a^2/(-a^2+1)^2/x^2-1/24*(26*a^4-17*a^2+6)*b^3*(b*x
+a)*(1-1/(b*x+a)^2)^(1/2)/a^3/(-a^2+1)^3/x
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used

$= \{5367, 4512, 3870, 4145, 4004, 3916, 2739, 632, 210\}$

$$\int \frac{\csc^{-1}(a+bx)}{x^5}\,dx = \frac{b^4\csc^{-1}(a+bx)}{4a^4} + \frac{(3-8a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^2\,(1-a^2)^2\,x^2} - \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a\,(1-a^2)\,x^3} + \frac{(-8a^6+8a^4-7a^2+2)\,b^4\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{4a^4\,(1-a^2)^{7/2}} - \frac{(26a^4-17a^2+6)\,b^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^3\,(1-a^2)^3\,x} - \frac{\csc^{-1}(a+bx)}{4x^4}$$

```
[In] Int[ArcCsc[a + b*x]/x^5,x]
```

```
[Out] -1/12*(b*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(a*(1 - a^2)*x^3) + ((3 - 8*a^
2)*b^2*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(24*a^2*(1 - a^2)^2*x^2) - ((6 -
 17*a^2 + 26*a^4)*b^3*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(24*a^3*(1 - a^2)
^3*x) + (b^4*ArcCsc[a + b*x])/(4*a^4) - ArcCsc[a + b*x]/(4*x^4) + ((2 - 7*a
^2 + 8*a^4 - 8*a^6)*b^4*ArcTan[(a - Tan[ArcCsc[a + b*x]/2])/Sqrt[1 - a^2]])
/(4*a^4*(1 - a^2)^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
```

```
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\left(b^4 \text{Subst}\left(\int \frac{x\cot(x)\csc(x)}{(-a+\csc(x))^5}\,dx, x, \csc^{-1}(a+bx)\right)\right)$$

$$= -\frac{\csc^{-1}(a+bx)}{4x^4} + \frac{1}{4}b^4\text{Subst}\left(\int \frac{1}{(-a+\csc(x))^4}\,dx, x, \csc^{-1}(a+bx)\right)$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a(1-a^2)x^3} - \frac{\csc^{-1}(a+bx)}{4x^4} - \frac{b^4\text{Subst}\left(\int \frac{3(1-a^2)-3a\csc(x)-2\csc^2(x)}{(-a+\csc(x))^3}\,dx, x, \csc^{-1}(a+bx)\right)}{12a(1-a^2)}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a(1-a^2)x^3} + \frac{(3-8a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^2(1-a^2)^2x^2} - \frac{\csc^{-1}(a+bx)}{4x^4} + \frac{b^4\text{Subst}\left(\int \frac{6(1-a^2)^2-2a(1-6a^2)\csc(x)-(3-8a^2)\csc^2(x)}{(-a+\csc(x))^2}\,dx, x, \csc^{-1}(a+bx)\right)}{24a^2(1-a^2)^2}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a(1-a^2)x^3} + \frac{(3-8a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^2(1-a^2)^2x^2} - \frac{(6-17a^2+26a^4)b^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^3(1-a^2)^3x} - \frac{\csc^{-1}(a+bx)}{4x^4} - \frac{b^4\text{Subst}\left(\int \frac{6(1-a^2)^3-3a(1-2a^2+6a^4)\csc(x)}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{24a^3(1-a^2)^3}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a(1-a^2)x^3} + \frac{(3-8a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^2(1-a^2)^2x^2} - \frac{(6-17a^2+26a^4)b^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^3(1-a^2)^3x} + \frac{b^4\csc^{-1}(a+bx)}{4a^4} - \frac{\csc^{-1}(a+bx)}{4x^4} - \frac{((2-7a^2+8a^4-8a^6)b^4)\text{Subst}\left(\int \frac{\csc(x)}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{8a^4(1-a^2)^3}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a(1-a^2)x^3} + \frac{(3-8a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^2(1-a^2)^2x^2} - \frac{(6-17a^2+26a^4)b^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^3(1-a^2)^3x} + \frac{b^4\csc^{-1}(a+bx)}{4a^4} - \frac{\csc^{-1}(a+bx)}{4x^4} - \frac{((2-7a^2+8a^4-8a^6)b^4)\text{Subst}\left(\int \frac{1}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{8a^4(1-a^2)^3}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a(1-a^2)x^3} + \frac{(3-8a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^2(1-a^2)^2x^2} - \frac{(6-17a^2+26a^4)\,b^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^3(1-a^2)^3x} + \frac{b^4\csc^{-1}(a+bx)}{4a^4} - \frac{\csc^{-1}(a+bx)}{4x^4} - \frac{((2-7a^2+8a^4-8a^6)\,b^4)\,\text{Subst}\left(\int\frac{1}{1-2ax+x^2}\,dx, x, \tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)\right)}{4a^4(1-a^2)^3}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a(1-a^2)x^3} + \frac{(3-8a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^2(1-a^2)^2x^2} - \frac{(6-17a^2+26a^4)\,b^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^3(1-a^2)^3x} + \frac{b^4\csc^{-1}(a+bx)}{4a^4} - \frac{\csc^{-1}(a+bx)}{4x^4} + \frac{((2-7a^2+8a^4-8a^6)\,b^4)\,\text{Subst}\left(\int\frac{1}{-4(1-a^2)-x^2}\,dx, x, -2a+2\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)\right)}{2a^4(1-a^2)^3}$$

$$= -\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12a(1-a^2)x^3} + \frac{(3-8a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^2(1-a^2)^2x^2} - \frac{(6-17a^2+26a^4)\,b^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{24a^3(1-a^2)^3x} + \frac{b^4\csc^{-1}(a+bx)}{4a^4} - \frac{\csc^{-1}(a+bx)}{4x^4} + \frac{(2-7a^2+8a^4-8a^6)\,b^4\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{4a^4(1-a^2)^{7/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.28

$$\int \frac{\csc^{-1}(a+bx)}{x^5}\,dx$$

$$= \frac{1}{8}\left(\frac{b\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}(2a^7-6a^6bx+3ab^2x^2+6b^3x^3+a^3(2-6b^2x^2)+2a^5(-2+9b^2x^2)+a^4bx(7+26b\right.$$

$$-\frac{2\csc^{-1}(a+bx)}{x^4}+\frac{2b^4\arcsin\left(\frac{1}{a+bx}\right)}{a^4}$$

$$\left.+\frac{i(-2+7a^2-8a^4+8a^6)\,b^4\log\left(\frac{16a^4(-1+a^2)^3\left(\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}+(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{(-2+7a^2-8a^4+8a^6)b^4x}\right)}{a^4\,(1-a^2)^{7/2}}\right)$$

```
[In] Integrate[ArcCsc[a + b*x]/x^5,x]
```

```
[Out] ((b*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(2*a^7 - 6*a^6*b*x + 3
*a*b^2*x^2 + 6*b^3*x^3 + a^3*(2 - 6*b^2*x^2) + 2*a^5*(-2 + 9*b^2*x^2) + a^4
*b*x*(7 + 26*b^2*x^2) - a^2*(b*x + 17*b^3*x^3)))/(3*a^3*(-1 + a^2)^3*x^3) -
 (2*ArcCsc[a + b*x])/x^4 + (2*b^4*ArcSin[(a + b*x)^(-1)])/a^4 + (I*(-2 + 7*
a^2 - 8*a^4 + 8*a^6)*b^4*Log[(16*a^4*(-1 + a^2)^3*((I*(-1 + a^2 + a*b*x))/S
qrt[1 - a^2] + (a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]))
/((-2 + 7*a^2 - 8*a^4 + 8*a^6)*b^4*x)])/(a^4*(1 - a^2)^(7/2)))/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(215) = 430$.

Time = 0.72 (sec) , antiderivative size = 795, normalized size of antiderivative = 3.33

method	result
parts	$-\frac{\operatorname{arccsc}(bx+a)}{4x^4}+\frac{b\sqrt{b^2x^2+2abx+a^2-1}\left(6(a^2-1)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)a^6b^3x^3-24\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-}\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int(arccsc(b*x+a)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arccsc(b*x+a)/x^4+1/24*b*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(6*(a^2-1)^(3/2
)*arctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1/2))*a^6*b^3*x^3-24*ln(2*(a*b*x+(a^2-1
)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^8*b^3*x^3-18*(a^2-1)^(3/2
)*arctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1/2))*a^4*b^3*x^3+26*(b^2*x^2+2*a*b*x+a
^2-1)^(1/2)*(a^2-1)^(3/2)*a^5*b^2*x^2+48*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2
+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^6*b^3*x^3-8*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)
*(a^2-1)^(3/2)*a^6*b*x+18*(a^2-1)^(3/2)*arctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1
/2))*a^2*b^3*x^3+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a^7-17*(b^2*
x^2+2*a*b*x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a^3*b^2*x^2-45*ln(2*(a*b*x+(a^2-1)^(
1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^4*b^3*x^3+11*(b^2*x^2+2*a*b*
x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a^4*b*x-6*b^3*arctan(1/(b^2*x^2+2*a*b*x+a^2-1)
^(1/2))*x^3*(a^2-1)^(3/2)-4*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a^5
+6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a*b^2*x^2+27*ln(2*(a*b*x+(a^
2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^2*b^3*x^3-3*(b^2*x^2+2
*a*b*x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a^2*b*x+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(
a^2-1)^(3/2)*a^3-6*b^3*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1
/2)+a^2-1)/x)*x^3)/((b^2*x^2+2*a*b*x+a^2-1)/(b*x+a)^2)^(1/2)/(b*x+a)/a^4/(a
^2-1)^(9/2)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.82

$$\int \frac{\csc^{-1}(a+bx)}{x^5}\,dx$$

$$= \left[\frac{3\left(8a^6-8a^4+7a^2-2\right)\sqrt{a^2-1}b^4x^4\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2-1}\left(a^2-\sqrt{a^2-1}a-1\right)-(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)-12}{}\right.$$

```
[In] integrate(arccsc(b*x+a)/x^5,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(8*a^6 - 8*a^4 + 7*a^2 - 2)*sqrt(a^2 - 1)*b^4*x^4*log((a^2*b*x + a
^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x
 + a^2 - 1)*sqrt(a^2 - 1) - a)/x) - 12*(a^8 - 4*a^6 + 6*a^4 - 4*a^2 + 1)*b^
4*x^4*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (26*a^7 - 43*a
^5 + 23*a^3 - 6*a)*b^4*x^4 - 6*(a^12 - 4*a^10 + 6*a^8 - 4*a^6 + a^4)*arccsc
(b*x + a) + ((26*a^7 - 43*a^5 + 23*a^3 - 6*a)*b^3*x^3 - (8*a^8 - 19*a^6 + 1
4*a^4 - 3*a^2)*b^2*x^2 + 2*(a^9 - 3*a^7 + 3*a^5 - a^3)*b*x)*sqrt(b^2*x^2 +
2*a*b*x + a^2 - 1))/((a^12 - 4*a^10 + 6*a^8 - 4*a^6 + a^4)*x^4), 1/24*(6*(8
*a^6 - 8*a^4 + 7*a^2 - 2)*sqrt(-a^2 + 1)*b^4*x^4*arctan(-(sqrt(-a^2 + 1)*b*
x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) - 12*(a^8
```

```
- 4*a^6 + 6*a^4 - 4*a^2 + 1)*b^4*x^4*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1)) + (26*a^7 - 43*a^5 + 23*a^3 - 6*a)*b^4*x^4 - 6*(a^12 - 4*a^1
0 + 6*a^8 - 4*a^6 + a^4)*arccsc(b*x + a) + ((26*a^7 - 43*a^5 + 23*a^3 - 6*a
)*b^3*x^3 - (8*a^8 - 19*a^6 + 14*a^4 - 3*a^2)*b^2*x^2 + 2*(a^9 - 3*a^7 + 3*
a^5 - a^3)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^12 - 4*a^10 + 6*a^8
- 4*a^6 + a^4)*x^4)]
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)}{x^5}\,dx = \int \frac{\operatorname{acsc}(a+bx)}{x^5}\,dx$$

```
[In] integrate(acsc(b*x+a)/x**5,x)
```

```
[Out] Integral(acsc(a + b*x)/x**5, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)}{x^5}\,dx = \int \frac{\operatorname{arccsc}(bx+a)}{x^5}\,dx$$

```
[In] integrate(arccsc(b*x+a)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*x^4*integrate(1/4*(b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b
*x + a - 1))/(b^2*x^6 + 2*a*b*x^5 + (a^2 - 1)*x^4 + (b^2*x^6 + 2*a*b*x^5 +
(a^2 - 1)*x^4)*e^(log(b*x + a + 1) + log(b*x + a - 1))), x) + arctan2(1, sq
rt(b*x + a + 1)*sqrt(b*x + a - 1)))/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(209) = 418$.

Time = 0.37 (sec) , antiderivative size = 841, normalized size of antiderivative = 3.52

$$\int \frac{\csc^{-1}(a+bx)}{x^5}\,dx = \text{Too large to display}$$

```
[In] integrate(arccsc(b*x+a)/x^5,x, algorithm="giac")
```

```
[Out] 1/12*b*(3*(8*a^6*b^3 - 8*a^4*b^3 + 7*a^2*b^3 - 2*b^3)*arctan(((b*x + a)*(sq
rt(-1/(b*x + a)^2 + 1) - 1) + a)/sqrt(-a^2 + 1))/((a^10 - 3*a^8 + 3*a^6 - a
^4)*sqrt(-a^2 + 1)) + (18*(b*x + a)^5*a^5*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1
)^5 + 84*(b*x + a)^4*a^6*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^4 + 104*(b*x +
```

```
a)^3*a^7*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 - 6*(b*x + a)^5*a^3*b^3*(sqrt
(-1/(b*x + a)^2 + 1) - 1)^5 - 12*(b*x + a)^4*a^4*b^3*(sqrt(-1/(b*x + a)^2 +
 1) - 1)^4 + 88*(b*x + a)^3*a^5*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 3*(b
*x + a)^5*a*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^5 + 228*(b*x + a)^2*a^6*b^3*
(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 - 3*(b*x + a)^4*a^2*b^3*(sqrt(-1/(b*x + a)
^2 + 1) - 1)^4 - 78*(b*x + a)^3*a^3*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 -
114*(b*x + a)^2*a^4*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 6*(b*x + a)^4*b^
3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^4 + 138*(b*x + a)*a^5*b^3*(sqrt(-1/(b*x +
a)^2 + 1) - 1) + 36*(b*x + a)^3*a*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 24
*(b*x + a)^2*a^2*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 - 96*(b*x + a)*a^3*b^
3*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 26*a^4*b^3 + 12*(b*x + a)^2*b^3*(sqrt(-1
/(b*x + a)^2 + 1) - 1)^2 + 33*(b*x + a)*a*b^3*(sqrt(-1/(b*x + a)^2 + 1) - 1
) - 17*a^2*b^3 + 6*b^3)/((a^9 - 3*a^7 + 3*a^5 - a^3)*((b*x + a)^2*(sqrt(-1/
(b*x + a)^2 + 1) - 1)^2 + 2*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)
^3) - 3*(4*a*b^3/(b*x + a) - 6*a^2*b^3/(b*x + a)^2 + 4*a^3*b^3/(b*x + a)^3
- b^3)*arcsin(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/(a^4*(a/(b*x + a) - 1)^
4))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)}{x^5}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{a+b\,x}\right)}{x^5}\,dx$$

```
[In] int(asin(1/(a + b*x))/x^5,x)
[Out] int(asin(1/(a + b*x))/x^5, x)
```

3.27 $\int x^3 \csc^{-1}(a+bx)^2 \, dx$

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Optimal result

Integrand size = 12, antiderivative size = 366

$$\begin{aligned}\int x^3 \csc^{-1}(a+bx)^2 \, dx = & -\frac{ax}{b^3}+\frac{(a+bx)^2}{12b^4}+\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^4} \\ & +\frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} \\ & -\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} \\ & +\frac{(a+bx)^3\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{6b^4}-\frac{a^4\csc^{-1}(a+bx)^2}{4b^4} \\ & +\frac{1}{4}x^4\csc^{-1}(a+bx)^2-\frac{2a\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4} \\ & -\frac{4a^3\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4}+\frac{\log(a+bx)}{3b^4} \\ & +\frac{3a^2\log(a+bx)}{b^4}+\frac{ia\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^4} \\ & +\frac{2ia^3\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^4} \\ & -\frac{ia\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^4}-\frac{2ia^3\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^4}\end{aligned}$$

```
[Out] -a*x/b^3+1/12*(b*x+a)^2/b^4-1/4*a^4*arccsc(b*x+a)^2/b^4+1/4*x^4*arccsc(b*x+
a)^2-2*a*arccsc(b*x+a)*arctanh(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^4-4*a^3*a
rccsc(b*x+a)*arctanh(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^4+1/3*ln(b*x+a)/b^4
```

```
+3*a^2*ln(b*x+a)/b^4+I*a*polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b^4-2*
I*a^3*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^4+2*I*a^3*polylog(2,-I/(
b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b^4-I*a*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(
1/2))/b^4+1/3*(b*x+a)*arccsc(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4+3*a^2*(b*x+a)
*arccsc(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4-a*(b*x+a)^2*arccsc(b*x+a)*(1-1/(b*
x+a)^2)^(1/2)/b^4+1/6*(b*x+a)^3*arccsc(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5367, 4512, 4275, 4268, 2317, 2438, 4269, 3556, 4270}

$$\begin{aligned}\int x^3 \csc^{-1}(a+bx)^2\,dx = &-\frac{a^4\csc^{-1}(a+bx)^2}{4b^4}-\frac{4a^3\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4}\\ &+\frac{2ia^3\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^4}\\ &-\frac{2ia^3\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^4}+\frac{3a^2\log(a+bx)}{b^4}\\ &+\frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4}\\ &-\frac{2a\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4}\\ &+\frac{ia\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^4}-\frac{ia\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^4}\\ &+\frac{(a+bx)^2}{12b^4}+\frac{\log(a+bx)}{3b^4}-\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4}\\ &+\frac{(a+bx)^3\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{6b^4}\\ &+\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^4}-\frac{ax}{b^3}+\frac{1}{4}x^4\csc^{-1}(a+bx)^2\end{aligned}$$

```
[In] Int[x^3*ArcCsc[a + b*x]^2,x]
```

```
[Out] -((a*x)/b^3) + (a + b*x)^2/(12*b^4) + ((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*A
rcCsc[a + b*x])/(3*b^4) + (3*a^2*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcCsc[
a + b*x])/b^4 - (a*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]*ArcCsc[a + b*x])/b^
4 + ((a + b*x)^3*Sqrt[1 - (a + b*x)^(-2)]*ArcCsc[a + b*x])/(6*b^4) - (a^4*A
rcCsc[a + b*x]^2)/(4*b^4) + (x^4*ArcCsc[a + b*x]^2)/4 - (2*a*ArcCsc[a + b*x
```

```
]*ArcTanh[E^(I*ArcCsc[a + b*x])])/b^4 - (4*a^3*ArcCsc[a + b*x]*ArcTanh[E^(I
*ArcCsc[a + b*x])])/b^4 + Log[a + b*x]/(3*b^4) + (3*a^2*Log[a + b*x])/b^4 +
 (I*a*PolyLog[2, -E^(I*ArcCsc[a + b*x])])/b^4 + ((2*I)*a^3*PolyLog[2, -E^(I
*ArcCsc[a + b*x])])/b^4 - (I*a*PolyLog[2, E^(I*ArcCsc[a + b*x])])/b^4 - ((2
*I)*a^3*PolyLog[2, E^(I*ArcCsc[a + b*x])])/b^4
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^2 \cot(x)\csc(x)(-a+\csc(x))^3\,dx, x, \csc^{-1}(a+bx)\right)}{b^4} \\
&= \frac{1}{4}x^4\csc^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x(-a+\csc(x))^4\,dx, x, \csc^{-1}(a+bx)\right)}{2b^4} \\
&= \frac{1}{4}x^4\csc^{-1}(a+bx)^2 \\
&\quad - \frac{\text{Subst}\left(\int \left(a^4x-4a^3x\csc(x)+6a^2x\csc^2(x)-4ax\csc^3(x)+x\csc^4(x)\right)\,dx, x, \csc^{-1}(a+bx)\right)}{2b^4} \\
&= -\frac{a^4\csc^{-1}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\csc^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x\csc^4(x)\,dx, x, \csc^{-1}(a+bx)\right)}{2b^4} \\
&\quad + \frac{(2a)\text{Subst}\left(\int x\csc^3(x)\,dx, x, \csc^{-1}(a+bx)\right)}{b^4} \\
&\quad - \frac{\left(3a^2\right)\text{Subst}\left(\int x\csc^2(x)\,dx, x, \csc^{-1}(a+bx)\right)}{b^4} \\
&\quad + \frac{\left(2a^3\right)\text{Subst}\left(\int x\csc(x)\,dx, x, \csc^{-1}(a+bx)\right)}{b^4}
\end{aligned}
$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} - \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{6b^4} - \frac{a^4\csc^{-1}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\csc^{-1}(a+bx)^2 - \frac{4a^3\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4} - \frac{\operatorname{Subst}\left(\int x\csc^2(x)\,dx, x, \csc^{-1}(a+bx)\right)}{3b^4} + \frac{a\operatorname{Subst}\left(\int x\csc(x)\,dx, x, \csc^{-1}(a+bx)\right)}{b^4} - \frac{(3a^2)\operatorname{Subst}\left(\int \cot(x)\,dx, x, \csc^{-1}(a+bx)\right)}{b^4} - \frac{(2a^3)\operatorname{Subst}\left(\int \log\left(1-e^{ix}\right)\,dx, x, \csc^{-1}(a+bx)\right)}{b^4} + \frac{(2a^3)\operatorname{Subst}\left(\int \log\left(1+e^{ix}\right)\,dx, x, \csc^{-1}(a+bx)\right)}{b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^4} + \frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} - \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{6b^4} - \frac{a^4\csc^{-1}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\csc^{-1}(a+bx)^2 - \frac{2a\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4} - \frac{4a^3\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4} + \frac{3a^2\log(a+bx)}{b^4} - \frac{\operatorname{Subst}\left(\int \cot(x)\,dx, x, \csc^{-1}(a+bx)\right)}{3b^4} - \frac{a\operatorname{Subst}\left(\int \log\left(1-e^{ix}\right)\,dx, x, \csc^{-1}(a+bx)\right)}{b^4} + \frac{a\operatorname{Subst}\left(\int \log\left(1+e^{ix}\right)\,dx, x, \csc^{-1}(a+bx)\right)}{b^4} + \frac{(2ia^3)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)}{b^4} - \frac{(2ia^3)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)}{b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^4} + \frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} - \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{6b^4} - \frac{a^4\csc^{-1}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\csc^{-1}(a+bx)^2 - \frac{2a\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4} - \frac{4a^3\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4} + \frac{\log(a+bx)}{3b^4} + \frac{3a^2\log(a+bx)}{b^4} + \frac{2ia^3\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^4} - \frac{2ia^3\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^4} + \frac{(ia)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^4} - \frac{(ia)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^4} + \frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} - \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{6b^4} - \frac{a^4\csc^{-1}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\csc^{-1}(a+bx)^2 - \frac{2a\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4} - \frac{4a^3\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^4} + \frac{\log(a+bx)}{3b^4} + \frac{3a^2\log(a+bx)}{b^4} + \frac{ia\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^4} + \frac{2ia^3\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^4} - \frac{ia\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^4} - \frac{2ia^3\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^4}$$

Mathematica [A] (warning: unable to verify)

Time = 3.95 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.24

$$\int x^3 \csc^{-1}(a+bx)^2 \, dx$$

$$= \frac{-16(6a-2(1+9a^2)\csc^{-1}(a+bx)+3(a+2a^3)\csc^{-1}(a+bx)^2)\cot\left(\frac{1}{2}\csc^{-1}(a+bx)\right)+2(2-24a\csc^{-1}(}{}$$

```
[In] Integrate[x^3*ArcCsc[a + b*x]^2,x]
```

```
[Out] (-16*(6*a - 2*(1 + 9*a^2)*ArcCsc[a + b*x] + 3*(a + 2*a^3)*ArcCsc[a + b*x]^2
)*Cot[ArcCsc[a + b*x]/2] + 2*(2 - 24*a*ArcCsc[a + b*x] + (3 + 36*a^2)*ArcCs
c[a + b*x]^2)*Csc[ArcCsc[a + b*x]/2]^2 + 3*ArcCsc[a + b*x]^2*Csc[ArcCsc[a +
 b*x]/2]^4 - (2*ArcCsc[a + b*x]*(-1 + 6*a*ArcCsc[a + b*x])*Csc[ArcCsc[a + b
*x]/2]^4)/(a + b*x) - 64*(1 + 9*a^2)*(Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(
-2)])] + Log[Sqrt[1 - (a + b*x)^(-2)]]) + 192*(a + 2*a^3)*(ArcCsc[a + b*x]*
(Log[1 - E^(I*ArcCsc[a + b*x])] - Log[1 + E^(I*ArcCsc[a + b*x])]) + I*(Poly
Log[2, -E^(I*ArcCsc[a + b*x])] - PolyLog[2, E^(I*ArcCsc[a + b*x])])) + 2*(2
 + 24*a*ArcCsc[a + b*x] + (3 + 36*a^2)*ArcCsc[a + b*x]^2)*Sec[ArcCsc[a + b*
x]/2]^2 + 3*ArcCsc[a + b*x]^2*Sec[ArcCsc[a + b*x]/2]^4 - 32*(a + b*x)^3*Arc
Csc[a + b*x]*(1 + 6*a*ArcCsc[a + b*x])*Sin[ArcCsc[a + b*x]/2]^4 - 16*(6*a +
 2*(1 + 9*a^2)*ArcCsc[a + b*x] + 3*(a + 2*a^3)*ArcCsc[a + b*x]^2)*Tan[ArcCs
c[a + b*x]/2])/(192*b^4)
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{\frac{2\ln\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{3}-\frac{\ln\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}-1\right)}{3}+\frac{(bx+a)^2}{12}-(bx+a)a-\operatorname{arccsc}(bx+a)^2a^3(bx+a)+\frac{3\operatorname{arccsc}(bx+a)^2}{2}}{}$
default	$\frac{\frac{2\ln\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{3}-\frac{\ln\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}-1\right)}{3}+\frac{(bx+a)^2}{12}-(bx+a)a-\operatorname{arccsc}(bx+a)^2a^3(bx+a)+\frac{3\operatorname{arccsc}(bx+a)^2}{2}}{}$

```
[In] int(x^3*arccsc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(-1/3*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+1/12*(b*x+a)^2-(b*x+a)*a+
2/3*ln(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))-1/3*ln(I/(b*x+a)+(1-1/(b*x+a)^2)^(1
/2)-1)+2*ln(1-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))*a^3*arccsc(b*x+a)-2*ln(1+I/(
b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a^3*arccsc(b*x+a)+2*I*polylog(2,-I/(b*x+a)-(1
-1/(b*x+a)^2)^(1/2))*a^3-2*I*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a^3
```

```
+ln(1-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))*a*arccsc(b*x+a)-ln(1+I/(b*x+a)+(1-1/
(b*x+a)^2)^(1/2))*a*arccsc(b*x+a)+I*polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1
/2))*a-I*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a-arccsc(b*x+a)^2*a^3*(
b*x+a)+3/2*arccsc(b*x+a)^2*a^2*(b*x+a)^2-arccsc(b*x+a)^2*a*(b*x+a)^3+1/6*ar
ccsc(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)^3-3*I*a^2*arccsc(b*x+a)
+1/3*arccsc(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)+3*arccsc(b*x+a)*
(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*a^2*(b*x+a)-arccsc(b*x+a)*(((b*x+a)^2-1)/(b
*x+a)^2)^(1/2)*a*(b*x+a)^2+6*ln(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a^2-3*ln(1
+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a^2-3*ln(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2)-
1)*a^2+1/4*arccsc(b*x+a)^2*(b*x+a)^4-1/3*I*arccsc(b*x+a))
```

Fricas [F]

$$\int x^3 \csc^{-1}(a+bx)^2\,dx = \int x^3 \operatorname{arccsc}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^3*arccsc(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^3*arccsc(b*x + a)^2, x)
```

Sympy [F]

$$\int x^3 \csc^{-1}(a+bx)^2\,dx = \int x^3 \operatorname{acsc}^2\left(a+bx\right)dx$$

```
[In] integrate(x**3*acsc(b*x+a)**2,x)
[Out] Integral(x**3*acsc(a + b*x)**2, x)
```

Maxima [F]

$$\int x^3 \csc^{-1}(a+bx)^2\,dx = \int x^3 \operatorname{arccsc}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^3*arccsc(b*x+a)^2,x, algorithm="maxima")
[Out] 1/4*x^4*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/16*x^4*log(b^
2*x^2 + 2*a*b*x + a^2)^2 + integrate(1/4*(2*sqrt(b*x + a + 1)*sqrt(b*x + a
- 1)*b*x^4*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - 4*(b^3*x^6 + 3
*a*b^2*x^5 + (3*a^2 - 1)*b*x^4 + (a^3 - a)*x^3)*log(b*x + a)^2 + (b^3*x^6 +
 2*a*b^2*x^5 + (a^2 - 1)*b*x^4 + 4*(b^3*x^6 + 3*a*b^2*x^5 + (3*a^2 - 1)*b*x
^4 + (a^3 - a)*x^3)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 +
3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \csc^{-1}(a+bx)^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*arccsc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \csc^{-1}(a+bx)^2\,dx = \int x^3\,\mathrm{asin}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

```
[In] int(x^3*asin(1/(a + b*x))^2,x)
```

```
[Out] int(x^3*asin(1/(a + b*x))^2, x)
```

3.28 $\int x^2 \csc^{-1}(a+bx)^2 \, dx$

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Optimal result

Integrand size = 12, antiderivative size = 272

$$\begin{aligned}\int x^2 \csc^{-1}(a+bx)^2 \, dx =& \frac{x}{3b^2} - \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^3} \\ &+ \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^3} + \frac{a^3\csc^{-1}(a+bx)^2}{3b^3} \\ &+ \frac{1}{3}x^3\csc^{-1}(a+bx)^2 + \frac{2\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{3b^3} \\ &+ \frac{4a^2\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{2a\log(a+bx)}{b^3} \\ &- \frac{i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{3b^3} - \frac{2ia^2\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} \\ &+ \frac{i\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{3b^3} + \frac{2ia^2\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3}\end{aligned}$$

```
[Out] 1/3*x/b^2+1/3*a^3*arccsc(b*x+a)^2/b^3+1/3*x^3*arccsc(b*x+a)^2+2/3*arccsc(b*
x+a)*arctanh(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^3+4*a^2*arccsc(b*x+a)*arcta
nh(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^3-2*a*ln(b*x+a)/b^3-1/3*I*polylog(2,-
I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b^3-2*I*a^2*polylog(2,-I/(b*x+a)-(1-1/(b*x
+a)^2)^(1/2))/b^3+1/3*I*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^3+2*I*
a^2*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^3-2*a*(b*x+a)*arccsc(b*x+a
)*(1-1/(b*x+a)^2)^(1/2)/b^3+1/3*(b*x+a)^2*arccsc(b*x+a)*(1-1/(b*x+a)^2)^(1/
2)/b^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5367, 4512, 4275, 4268, 2317, 2438, 4269, 3556, 4270}

$$\begin{aligned}\int x^2\csc^{-1}(a+bx)^2\,dx=&\frac{a^3\csc^{-1}(a+bx)^2}{3b^3}+\frac{4a^2\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3}\\&-\frac{2ia^2\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3}\\&+\frac{2ia^2\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3}\\&+\frac{2\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{3b^3}\\&-\frac{i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{3b^3}+\frac{i\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{3b^3}\\&-\frac{2a\log(a+bx)}{b^3}-\frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^3}\\&+\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\csc^{-1}(a+bx)^2+\frac{x}{3b^2}\end{aligned}$$

```
[In] Int[x^2*ArcCsc[a + b*x]^2,x]
```

```
[Out] x/(3*b^2) - (2*a*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcCsc[a + b*x])/b^3 +
((a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]*ArcCsc[a + b*x])/(3*b^3) + (a^3*ArcCs
c[a + b*x]^2)/(3*b^3) + (x^3*ArcCsc[a + b*x]^2)/3 + (2*ArcCsc[a + b*x]*ArcT
anh[E^(I*ArcCsc[a + b*x])])/(3*b^3) + (4*a^2*ArcCsc[a + b*x]*ArcTanh[E^(I*A
rcCsc[a + b*x])])/b^3 - (2*a*Log[a + b*x])/b^3 - ((I/3)*PolyLog[2, -E^(I*Ar
cCsc[a + b*x])])/b^3 - ((2*I)*a^2*PolyLog[2, -E^(I*ArcCsc[a + b*x])])/b^3 +
 ((I/3)*PolyLog[2, E^(I*ArcCsc[a + b*x])])/b^3 + ((2*I)*a^2*PolyLog[2, E^(I
*ArcCsc[a + b*x])])/b^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^2\cot(x)\csc(x)(-a+\csc(x))^2\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{1}{3}x^3\csc^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int x(-a+\csc(x))^3\,dx,x,\csc^{-1}(a+bx)\right)}{3b^3}$$

$$= \frac{1}{3}x^3\csc^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int\left(-a^3x+3a^2x\csc(x)-3ax\csc^2(x)+x\csc^3(x)\right)\,dx,x,\csc^{-1}(a+bx)\right)}{3b^3}$$

$$= \frac{a^3\csc^{-1}(a+bx)^2}{3b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int x\csc^3(x)\,dx,x,\csc^{-1}(a+bx)\right)}{3b^3} + \frac{(2a)\text{Subst}\left(\int x\csc^2(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3} - \frac{(2a^2)\,\text{Subst}\left(\int x\csc(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^3} + \frac{a^3\csc^{-1}(a+bx)^2}{3b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)^2 + \frac{4a^2\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{\text{Subst}\left(\int x\csc(x)\,dx,x,\csc^{-1}(a+bx)\right)}{3b^3} + \frac{(2a)\text{Subst}\left(\int\cot(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3} + \frac{(2a^2)\,\text{Subst}\left(\int\log\left(1-e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3} - \frac{(2a^2)\,\text{Subst}\left(\int\log\left(1+e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}$$

$$=\frac{x}{3b^2}-\frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^3}+\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^3}$$
$$+\frac{a^3\csc^{-1}(a+bx)^2}{3b^3}+\frac{1}{3}x^3\csc^{-1}(a+bx)^2+\frac{2\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{3b^3}$$
$$+\frac{4a^2\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3}-\frac{2a\log(a+bx)}{b^3}$$
$$+\frac{\text{Subst}\left(\int\log\left(1-e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{3b^3}$$
$$-\frac{\text{Subst}\left(\int\log\left(1+e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{3b^3}$$
$$-\frac{(2ia^2)\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$
$$+\frac{(2ia^2)\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$

$$=\frac{x}{3b^2}-\frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^3}+\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^3}$$
$$+\frac{a^3\csc^{-1}(a+bx)^2}{3b^3}+\frac{1}{3}x^3\csc^{-1}(a+bx)^2+\frac{2\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{3b^3}$$
$$+\frac{4a^2\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3}-\frac{2a\log(a+bx)}{b^3}$$
$$-\frac{2ia^2\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3}+\frac{2ia^2\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$
$$-\frac{i\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{3b^3}+\frac{i\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{3b^3}$$

$$=\frac{x}{3b^2}-\frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^3}+\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{3b^3}$$
$$+\frac{a^3\csc^{-1}(a+bx)^2}{3b^3}+\frac{1}{3}x^3\csc^{-1}(a+bx)^2+\frac{2\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{3b^3}$$
$$+\frac{4a^2\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3}-\frac{2a\log(a+bx)}{b^3}$$
$$-\frac{i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{3b^3}-\frac{2ia^2\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$
$$+\frac{i\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{3b^3}+\frac{2ia^2\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$

Mathematica [A] (warning: unable to verify)

Time = 4.46 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.28

$$\int x^2 \csc^{-1}(a+bx)^2\,dx =$$

$$-\frac{-2(2-12a\csc^{-1}(a+bx)+(1+6a^2)\csc^{-1}(a+bx)^2)\cot\left(\frac{1}{2}\csc^{-1}(a+bx)\right)+2\csc^{-1}(a+bx)\left(-1+3a\right.}{}$$

```
[In] Integrate[x^2*ArcCsc[a + b*x]^2,x]
```

```
[Out] -1/24*(-2*(2 - 12*a*ArcCsc[a + b*x] + (1 + 6*a^2)*ArcCsc[a + b*x]^2)*Cot[Ar
cCsc[a + b*x]/2] + 2*ArcCsc[a + b*x]*(-1 + 3*a*ArcCsc[a + b*x])*Csc[ArcCsc[
a + b*x]/2]^2 - (ArcCsc[a + b*x]^2*Csc[ArcCsc[a + b*x]/2]^4)/(2*(a + b*x))
- 48*a*(Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])] + Log[Sqrt[1 - (a + b*x
)^(-2)]]) + 8*(1 + 6*a^2)*(ArcCsc[a + b*x]*(Log[1 - E^(I*ArcCsc[a + b*x])]
- Log[1 + E^(I*ArcCsc[a + b*x])]) + I*(PolyLog[2, -E^(I*ArcCsc[a + b*x])] -
 PolyLog[2, E^(I*ArcCsc[a + b*x])])) + 2*ArcCsc[a + b*x]*(1 + 3*a*ArcCsc[a
+ b*x])*Sec[ArcCsc[a + b*x]/2]^2 - 8*(a + b*x)^3*ArcCsc[a + b*x]^2*Sin[ArcC
sc[a + b*x]/2]^4 - 2*(2 + 12*a*ArcCsc[a + b*x] + (1 + 6*a^2)*ArcCsc[a + b*x
]^2)*Tan[ArcCsc[a + b*x]/2])/b^3
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.84

method	result
derivativedivides	$\frac{\operatorname{arccsc}(bx+a)^2 a^2(bx+a)-\operatorname{arccsc}(bx+a)^2 a(bx+a)^2+\frac{\operatorname{arccsc}(bx+a)^2(bx+a)^3}{3}-2\operatorname{arccsc}(bx+a)\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,a(bx+a)+\frac{\operatorname{arccs}}{}}{}$
default	$\frac{\operatorname{arccsc}(bx+a)^2 a^2(bx+a)-\operatorname{arccsc}(bx+a)^2 a(bx+a)^2+\frac{\operatorname{arccsc}(bx+a)^2(bx+a)^3}{3}-2\operatorname{arccsc}(bx+a)\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,a(bx+a)+\frac{\operatorname{arccs}}{}}{}$

```
[In] int(x^2*arccsc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(arccsc(b*x+a)^2*a^2*(b*x+a)-arccsc(b*x+a)^2*a*(b*x+a)^2+1/3*arccsc(b
*x+a)^2*(b*x+a)^3-2*arccsc(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*a*(b*x+a)
+1/3*arccsc(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)^2-1/3*I*polylog(
2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))+1/3*b*x+1/3*a+1/3*arccsc(b*x+a)*ln(1+I/
(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+2*I*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2
))*a^2-1/3*arccsc(b*x+a)*ln(1-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))-2*I*polylog(
2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))*a^2+2*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1
/2))*a-4*ln(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a+2*ln(I/(b*x+a)+(1-1/(b*x+a)^
```

```
2)^(1/2)-1)*a+2*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a^2*arccsc(b*x+a)-2*l
n(1-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))*a^2*arccsc(b*x+a)+2*I*a*arccsc(b*x+a)+
1/3*I*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2)))
```

Fricas [F]

$$\int x^2 \csc^{-1}(a+bx)^2 \, dx = \int x^2 \operatorname{arccsc}\left(bx+a\right)^2 \, dx$$

```
[In] integrate(x^2*arccsc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccsc(b*x + a)^2, x)
```

Sympy [F]

$$\int x^2 \csc^{-1}(a+bx)^2 \, dx = \int x^2 \operatorname{acsc}^2\left(a+bx\right) dx$$

```
[In] integrate(x**2*acsc(b*x+a)**2,x)
```

```
[Out] Integral(x**2*acsc(a + b*x)**2, x)
```

Maxima [F]

$$\int x^2 \csc^{-1}(a+bx)^2 \, dx = \int x^2 \operatorname{arccsc}\left(bx+a\right)^2 \, dx$$

```
[In] integrate(x^2*arccsc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/12*x^3*log(b^
2*x^2 + 2*a*b*x + a^2)^2 + integrate(1/3*(2*sqrt(b*x + a + 1)*sqrt(b*x + a
- 1)*b*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - 3*(b^3*x^5 + 3
*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a)^2 + (b^3*x^5 +
 2*a*b^2*x^4 + (a^2 - 1)*b*x^3 + 3*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x
^3 + (a^3 - a)*x^2)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 +
3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

Giac [F]

$$\int x^2 \csc^{-1}(a+bx)^2 \, dx = \int x^2 \operatorname{arccsc}(bx+a)^2 \, dx$$

```
[In] integrate(x^2*arccsc(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*arccsc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \csc^{-1}(a+bx)^2 \, dx = \int x^2 \operatorname{asin}\left(\frac{1}{a+b\,x}\right)^2 \, dx$$

```
[In] int(x^2*asin(1/(a + b*x))^2,x)
[Out] int(x^2*asin(1/(a + b*x))^2, x)
```

3.29 $\int x\csc^{-1}(a+bx)^2\,dx$

Optimal result 198
Rubi [A] (verified) 198
Mathematica [A] (verified) 201
Maple [A] (verified) 202
Fricas [F] 202
Sympy [F] 202
Maxima [F] 203
Giac [F] 203
Mupad [F(-1)] 203

Optimal result

Integrand size = 10, antiderivative size = 145

$$\begin{aligned}\int x\csc^{-1}(a+bx)^2\,dx=&\ \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^2}\\&-\frac{a^2\csc^{-1}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\csc^{-1}(a+bx)^2\\&-\frac{4a\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{\log(a+bx)}{b^2}\\&+\frac{2ia\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^2}-\frac{2ia\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^2}\end{aligned}$$

```
[Out] -1/2*a^2*arccsc(b*x+a)^2/b^2+1/2*x^2*arccsc(b*x+a)^2-4*a*arccsc(b*x+a)*arct
anh(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^2+ln(b*x+a)/b^2+2*I*a*polylog(2,-I/(
b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b^2-2*I*a*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)
^(1/2))/b^2+(b*x+a)*arccsc(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used

= {5367, 4512, 4275, 4268, 2317, 2438, 4269, 3556}

$$\int x \csc^{-1}(a+bx)^2\,dx = -\frac{a^2\csc^{-1}(a+bx)^2}{2b^2} - \frac{4a\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2} + \frac{2ia\,\mathrm{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^2} - \frac{2ia\,\mathrm{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^2} + \frac{\log(a+bx)}{b^2} + \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^2} + \frac{1}{2}x^2\csc^{-1}(a+bx)^2$$

```
[In] Int[x*ArcCsc[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcCsc[a + b*x])/b^2 - (a^2*ArcCsc[a +
b*x]^2)/(2*b^2) + (x^2*ArcCsc[a + b*x]^2)/2 - (4*a*ArcCsc[a + b*x]*ArcTanh[
E^(I*ArcCsc[a + b*x])])/b^2 + Log[a + b*x]/b^2 + ((2*I)*a*PolyLog[2, -E^(I*
ArcCsc[a + b*x])])/b^2 - ((2*I)*a*PolyLog[2, E^(I*ArcCsc[a + b*x])])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^2\cot(x)\csc(x)(-a+\csc(x))\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}\\
&= \frac{1}{2}x^2\csc^{-1}(a+bx)^2-\frac{\text{Subst}\left(\int x(-a+\csc(x))^2\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}\\
&= \frac{1}{2}x^2\csc^{-1}(a+bx)^2-\frac{\text{Subst}\left(\int \left(a^2x-2ax\csc(x)+x\csc^2(x)\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}\\
&= -\frac{a^2\csc^{-1}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\csc^{-1}(a+bx)^2-\frac{\text{Subst}\left(\int x\csc^2(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}\\
&\quad+\frac{(2a)\text{Subst}\left(\int x\csc(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}\\
&= \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^2}-\frac{a^2\csc^{-1}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\csc^{-1}(a+bx)^2\\
&\quad-\frac{4a\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2}-\frac{\text{Subst}\left(\int\cot(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}\\
&\quad-\frac{(2a)\text{Subst}\left(\int\log\left(1-e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}\\
&\quad+\frac{(2a)\text{Subst}\left(\int\log\left(1+e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}
\end{aligned}$$

$$= \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^2} - \frac{a^2\csc^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\csc^{-1}(a+bx)^2 - \frac{4a\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2} + \frac{\log(a+bx)}{b^2} + \frac{(2ia)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)}{b^2} - \frac{(2ia)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)}{b^2}$$

$$= \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)}{b^2} - \frac{a^2\csc^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\csc^{-1}(a+bx)^2 - \frac{4a\csc^{-1}(a+bx)\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2} + \frac{\log(a+bx)}{b^2} + \frac{2ia\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^2} - \frac{2ia\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.47

$$\int x\csc^{-1}(a+bx)^2\,dx$$

$$= \frac{2a\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\csc^{-1}(a+bx) + 2bx\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\csc^{-1}(a+bx) - a^2\csc^{-1}(a+bx)^2 + b^2x^2\csc^{-1}}{}$$

```
[In] Integrate[x*ArcCsc[a + b*x]^2,x]
```

```
[Out] (2*a*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*ArcCsc[a + b*x] + 2*b
*x*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*ArcCsc[a + b*x] - a^2*A
rcCsc[a + b*x]^2 + b^2*x^2*ArcCsc[a + b*x]^2 + 4*a*ArcCsc[a + b*x]*Log[1 -
E^(I*ArcCsc[a + b*x])] - 4*a*ArcCsc[a + b*x]*Log[1 + E^(I*ArcCsc[a + b*x])]
 - 2*Log[(a + b*x)^(-1)] + (4*I)*a*PolyLog[2, -E^(I*ArcCsc[a + b*x])] - (4*
I)*a*PolyLog[2, E^(I*ArcCsc[a + b*x])])/(2*b^2)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.46

method	result
derivativedivides	$-a\left(\operatorname{arccsc}(bx+a)^2(bx+a)-2\operatorname{arccsc}(bx+a)\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)+2\operatorname{arccsc}(bx+a)\ln\left(1+\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right.$
default	$-a\left(\operatorname{arccsc}(bx+a)^2(bx+a)-2\operatorname{arccsc}(bx+a)\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)+2\operatorname{arccsc}(bx+a)\ln\left(1+\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right.$

```
[In] int(x*arccsc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-a*(arccsc(b*x+a)^2*(b*x+a)-2*arccsc(b*x+a)*ln(1-I/(b*x+a)-(1-1/(b*x
+a)^2)^(1/2))+2*arccsc(b*x+a)*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))-2*I*dil
og(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+2*I*dilog(1-I/(b*x+a)-(1-1/(b*x+a)^2)
^(1/2)))+1/2*arccsc(b*x+a)^2*(b*x+a)^2+arccsc(b*x+a)*(((b*x+a)^2-1)/(b*x+a)
^2)^(1/2)*(b*x+a)-ln(1/(b*x+a)))
```

Fricas [F]

$$\int x\csc^{-1}(a+bx)^2\,dx=\int x\operatorname{arccsc}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x*arccsc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*arccsc(b*x + a)^2, x)
```

Sympy [F]

$$\int x\csc^{-1}(a+bx)^2\,dx=\int x\operatorname{acsc}^2\left(a+bx\right)dx$$

```
[In] integrate(x*acsc(b*x+a)**2,x)
```

```
[Out] Integral(x*acsc(a + b*x)**2, x)
```

Maxima [F]

$$\int x\csc^{-1}(a+bx)^2\,dx=\int x\operatorname{arccsc}(bx+a)^2\,dx$$

```
[In] integrate(x*arccsc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/8*x^2*log(b^2
*x^2 + 2*a*b*x + a^2)^2 + integrate(1/2*(2*sqrt(b*x + a + 1)*sqrt(b*x + a -
 1)*b*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - 2*(b^3*x^4 + 3*
a*b^2*x^3 + (3*a^2 - 1)*b*x^2 + (a^3 - a)*x)*log(b*x + a)^2 + (b^3*x^4 + 2*
a*b^2*x^3 + (a^2 - 1)*b*x^2 + 2*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2 - 1)*b*x^2
+ (a^3 - a)*x)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b
^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

Giac [F]

$$\int x\csc^{-1}(a+bx)^2\,dx=\int x\operatorname{arccsc}(bx+a)^2\,dx$$

```
[In] integrate(x*arccsc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccsc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\csc^{-1}(a+bx)^2\,dx=\int x\,\operatorname{asin}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

```
[In] int(x*asin(1/(a + b*x))^2,x)
```

```
[Out] int(x*asin(1/(a + b*x))^2, x)
```

3.30 $\int \csc^{-1}(a+bx)^2\,dx$

Optimal result 204
Rubi [A] (verified) 204
Mathematica [A] (verified) 206
Maple [A] (verified) 207
Fricas [F] 207
Sympy [F] 207
Maxima [F] 208
Giac [F] 208
Mupad [F(-1)] 208

Optimal result

Integrand size = 8, antiderivative size = 86

$$\int \csc^{-1}(a+bx)^2\,dx = \frac{(a+bx)\csc^{-1}(a+bx)^2}{b} + \frac{4\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{2i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{2i\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b}$$

```
[Out] (b*x+a)*arccsc(b*x+a)^2/b+4*arccsc(b*x+a)*arctanh(I/(b*x+a)+(1-1/(b*x+a)^2)
^(1/2))/b-2*I*polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b+2*I*polylog(2,I
/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5361, 5325, 3843, 4268, 2317, 2438}

$$\int \csc^{-1}(a+bx)^2\,dx = \frac{4\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{2i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{2i\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{(a+bx)\csc^{-1}(a+bx)^2}{b}$$

```
[In] Int[ArcCsc[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*ArcCsc[a + b*x]^2)/b + (4*ArcCsc[a + b*x]*ArcTanh[E^(I*ArcCsc[a
+ b*x])])/b - ((2*I)*PolyLog[2, -E^(I*ArcCsc[a + b*x])])/b + ((2*I)*PolyLog
[2, E^(I*ArcCsc[a + b*x])])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3843

```
Int[Cot[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)
*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^p/(b*n*p)),
 x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csc[a + b*x^n]^p, x], x] /; F
reeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5325

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Sub
st[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[n, 0]
```

Rule 5361

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCsc[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \csc^{-1}(x)^2 \, dx, x, a+bx\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int x^2 \cot(x) \csc(x)\, dx, x, \csc^{-1}(a+bx)\right)}{b}$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int x \csc(x)\, dx, x, \csc^{-1}(a+bx)\right)}{b}$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)^2}{b} + \frac{4\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{2\text{Subst}\left(\int \log\left(1-e^{ix}\right)\, dx, x, \csc^{-1}(a+bx)\right)}{b} - \frac{2\text{Subst}\left(\int \log\left(1+e^{ix}\right)\, dx, x, \csc^{-1}(a+bx)\right)}{b}$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)^2}{b} + \frac{4\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{(2i)\text{Subst}\left(\int \frac{\log(1-x)}{x}\, dx, x, e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{(2i)\text{Subst}\left(\int \frac{\log(1+x)}{x}\, dx, x, e^{i\csc^{-1}(a+bx)}\right)}{b}$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)^2}{b} + \frac{4\csc^{-1}(a+bx)\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{2i\,\text{PolyLog}\left(2, -e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{2i\,\text{PolyLog}\left(2, e^{i\csc^{-1}(a+bx)}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \csc^{-1}(a+bx)^2\, dx$$

$$= \frac{\csc^{-1}(a+bx)\left((a+bx)\csc^{-1}(a+bx) - 2\log\left(1-e^{i\csc^{-1}(a+bx)}\right) + 2\log\left(1+e^{i\csc^{-1}(a+bx)}\right)\right) - 2i\,\text{PolyLog}}{b}$$

```
[In] Integrate[ArcCsc[a + b*x]^2,x]
```

```
[Out] (ArcCsc[a + b*x]*((a + b*x)*ArcCsc[a + b*x] - 2*Log[1 - E^(I*ArcCsc[a + b*x
])] + 2*Log[1 + E^(I*ArcCsc[a + b*x])]) - (2*I)*PolyLog[2, -E^(I*ArcCsc[a +
 b*x])] + (2*I)*PolyLog[2, E^(I*ArcCsc[a + b*x])])/b
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

method	result
derivativedivides	$\frac{\operatorname{arccsc}(bx+a)^{2}(bx+a)-2\operatorname{arccsc}(bx+a)\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^{2}}}\right)+2\operatorname{arccsc}(bx+a)\ln\left(1+\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^{2}}}\right)-2i}{b}$
default	$\frac{\operatorname{arccsc}(bx+a)^{2}(bx+a)-2\operatorname{arccsc}(bx+a)\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^{2}}}\right)+2\operatorname{arccsc}(bx+a)\ln\left(1+\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^{2}}}\right)-2i}{b}$

```
[In] int(arccsc(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(arccsc(b*x+a)^2*(b*x+a)-2*arccsc(b*x+a)*ln(1-I/(b*x+a)-(1-1/(b*x+a)^2)
^(1/2))+2*arccsc(b*x+a)*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))-2*I*dilog(1+I
/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+2*I*dilog(1-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2)
))
```

Fricas [F]

$$\int \csc^{-1}(a+bx)^2\,dx = \int \operatorname{arccsc}\left(bx+a\right)^2\,dx$$

```
[In] integrate(arccsc(b*x+a)^2,x, algorithm="fricas")
[Out] integral(arccsc(b*x + a)^2, x)
```

Sympy [F]

$$\int \csc^{-1}(a+bx)^2\,dx = \int \operatorname{acsc}^2\left(a+bx\right)dx$$

```
[In] integrate(acsc(b*x+a)**2,x)
[Out] Integral(acsc(a + b*x)**2, x)
```

Maxima [F]

$$\int \csc^{-1}(a+bx)^2\,dx = \int \operatorname{arccsc}\left(bx+a\right)^2\,dx$$

```
[In] integrate(arccsc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/4*x*log(b^2*x^2 + 2
*a*b*x + a^2)^2 + integrate((2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*b*x*arct
an2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - (b^3*x^3 + 3*a*b^2*x^2 + a^3
+ (3*a^2 - 1)*b*x - a)*log(b*x + a)^2 + (b^3*x^3 + 2*a*b^2*x^2 + (a^2 - 1)*
b*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a))*log
(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x -
 a), x)
```

Giac [F]

$$\int \csc^{-1}(a+bx)^2\,dx = \int \operatorname{arccsc}\left(bx+a\right)^2\,dx$$

```
[In] integrate(arccsc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arccsc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^{-1}(a+bx)^2\,dx = \int \operatorname{asin}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

```
[In] int(asin(1/(a + b*x))^2,x)
```

```
[Out] int(asin(1/(a + b*x))^2, x)
```

3.31 $\int \frac{\csc^{-1}(a+bx)^2}{x}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 324

$$\begin{aligned}\int \frac{\csc^{-1}(a+bx)^2}{x}\,dx =&\ \csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\ &+\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\ &-\csc^{-1}(a+bx)^2\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)\\ &-2i\csc^{-1}(a+bx)\,\mathrm{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\ &-2i\csc^{-1}(a+bx)\,\mathrm{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\ &+i\csc^{-1}(a+bx)\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)\\ &+2\,\mathrm{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\ &+2\,\mathrm{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{2i\csc^{-1}(a+bx)}\right)\end{aligned}$$

```
[Out] -arccsc(b*x+a)^2*ln(1-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)+arccsc(b*x+a)^2*
ln(1+I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arccsc(b*x+a
)^2*ln(1+I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+I*arccsc
(b*x+a)*polylog(2,(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)-2*I*arccsc(b*x+a)*po
lylog(2,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))-2*I*arcc
sc(b*x+a)*polylog(2,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2
)))-1/2*polylog(3,(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)+2*polylog(3,-I*a*(I/
```

```
(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+2*polylog(3,-I*a*(I/(b*x
+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5367, 4648, 4625, 3798, 2221, 2611, 2320, 6724, 4615}

$$\begin{aligned}\int \frac{\csc^{-1}(a+bx)^2}{x}\,dx &= -2i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\&\quad-2i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)\\&\quad+2\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\&\quad+2\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)\\&\quad+\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\&\quad+\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)\\&\quad+i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)\\&\quad-\frac{1}{2}\operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(a+bx)}\right)-\csc^{-1}(a+bx)^2\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)\end{aligned}$$

```
[In] Int[ArcCsc[a + b*x]^2/x,x]
```

```
[Out] ArcCsc[a + b*x]^2*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])]
+ ArcCsc[a + b*x]^2*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])
] - ArcCsc[a + b*x]^2*Log[1 - E^((2*I)*ArcCsc[a + b*x])] - (2*I)*ArcCsc[a +
 b*x]*PolyLog[2, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])] - (2*I
)*ArcCsc[a + b*x]*PolyLog[2, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a
^2])] + I*ArcCsc[a + b*x]*PolyLog[2, E^((2*I)*ArcCsc[a + b*x])] + 2*PolyLog
[3, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])] + 2*PolyLog[3, ((-I
)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] - PolyLog[3, E^((2*I)*ArcCs
c[a + b*x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4625

```
Int[(Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cot[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cos[c + d*x]*(Cot[c + d*x]^(n -
1)/(a + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Rule 4648

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/(Csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> In
```

```
t[(e + f*x)^m*Sin[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Sin[c + d*x]))
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\cot(x)\csc(x)}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)\\
&= -\text{Subst}\left(\int \frac{x^2\cot(x)}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)\\
&= -\left(a\text{Subst}\left(\int \frac{x^2\cos(x)}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)\right)\\
&\quad - \text{Subst}\left(\int x^2\cot(x)\,dx, x, \csc^{-1}(a+bx)\right)\\
&= 2i\text{Subst}\left(\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx, x, \csc^{-1}(a+bx)\right)\\
&\quad - a\text{Subst}\left(\int \frac{e^{ix}x^2}{1-\sqrt{1-a^2}+iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)\\
&\quad - a\text{Subst}\left(\int \frac{e^{ix}x^2}{1+\sqrt{1-a^2}+iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)
\end{aligned}$$

$$= \csc^{-1}(a+bx)^2 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\csc^{-1}(a+bx)^2 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$-\csc^{-1}(a+bx)^2 \log\left(1-e^{2i\csc^{-1}(a+bx)}\right)$$
$$-2\text{Subst}\left(\int x\log\left(1+\frac{iae^{ix}}{1-\sqrt{1-a^2}}\right)dx, x, \csc^{-1}(a+bx)\right)$$
$$-2\text{Subst}\left(\int x\log\left(1+\frac{iae^{ix}}{1+\sqrt{1-a^2}}\right)dx, x, \csc^{-1}(a+bx)\right)$$
$$+2\text{Subst}\left(\int x\log\left(1-e^{2ix}\right)dx, x, \csc^{-1}(a+bx)\right)$$

$$= \csc^{-1}(a+bx)^2 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\csc^{-1}(a+bx)^2 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$-\csc^{-1}(a+bx)^2 \log\left(1-e^{2i\csc^{-1}(a+bx)}\right)$$
$$-2i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$
$$-2i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$+i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)$$
$$-i\text{Subst}\left(\int \text{PolyLog}\left(2,e^{2ix}\right)dx, x, \csc^{-1}(a+bx)\right)$$
$$+2i\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{iae^{ix}}{1-\sqrt{1-a^2}}\right)dx, x, \csc^{-1}(a+bx)\right)$$
$$+2i\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{iae^{ix}}{1+\sqrt{1-a^2}}\right)dx, x, \csc^{-1}(a+bx)\right)$$

$$= \csc^{-1}(a+bx)^2 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)^2 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$- \csc^{-1}(a+bx)^2 \log\left(1-e^{2i\csc^{-1}(a+bx)}\right)$$
$$- 2i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$
$$- 2i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$+ i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)$$
$$- \frac{1}{2}\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2i\csc^{-1}(a+bx)}\right)$$
$$+ 2\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{iax}{-1+\sqrt{1-a^2}}\right)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)$$
$$+ 2\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-\frac{iax}{1+\sqrt{1-a^2}}\right)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)$$

$$= \csc^{-1}(a+bx)^2 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)^2 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$- \csc^{-1}(a+bx)^2 \log\left(1-e^{2i\csc^{-1}(a+bx)}\right)$$
$$- 2i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$
$$- 2i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$+ i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right) + 2\,\text{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$
$$+ 2\,\text{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \frac{1}{2}\,\text{PolyLog}\left(3,e^{2i\csc^{-1}(a+bx)}\right)$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.26

$$\int \frac{\csc^{-1}(a+bx)^2}{x}\,dx = \frac{i\pi^3}{6} - \frac{1}{3}i\csc^{-1}(a+bx)^3 - \csc^{-1}(a+bx)^2\log\left(1-e^{-i\csc^{-1}(a+bx)}\right) - \csc^{-1}(a+bx)^2\log\left(1+e^{i\csc^{-1}(a+bx)}\right) + \csc^{-1}(a+bx)^2\log\left(1-\frac{iae^{i\csc^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - 2i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{-i\csc^{-1}(a+bx)}\right) + 2i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right) - 2i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{iae^{i\csc^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) - 2i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - 2\operatorname{PolyLog}\left(3,e^{-i\csc^{-1}(a+bx)}\right) - 2\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right) + 2\operatorname{PolyLog}\left(3,\frac{iae^{i\csc^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) + 2\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

```
[In] Integrate[ArcCsc[a + b*x]^2/x,x]
```

```
[Out] (I/6)*Pi^3 - (I/3)*ArcCsc[a + b*x]^3 - ArcCsc[a + b*x]^2*Log[1 - E^((-I)*Ar
cCsc[a + b*x])] - ArcCsc[a + b*x]^2*Log[1 + E^(I*ArcCsc[a + b*x])] + ArcCsc
[a + b*x]^2*Log[1 - (I*a*E^(I*ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] + Arc
Csc[a + b*x]^2*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] - (
2*I)*ArcCsc[a + b*x]*PolyLog[2, E^((-I)*ArcCsc[a + b*x])] + (2*I)*ArcCsc[a
+ b*x]*PolyLog[2, -E^(I*ArcCsc[a + b*x])] - (2*I)*ArcCsc[a + b*x]*PolyLog[2
, (I*a*E^(I*ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] - (2*I)*ArcCsc[a + b*x]
*PolyLog[2, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] - 2*PolyLog
[3, E^((-I)*ArcCsc[a + b*x])] - 2*PolyLog[3, -E^(I*ArcCsc[a + b*x])] + 2*Po
lyLog[3, (I*a*E^(I*ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] + 2*PolyLog[3, (
(-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])]
```

Maple [F]

$$\int \frac{\operatorname{arccsc}(bx+a)^2}{x}dx$$

```
[In] int(arccsc(b*x+a)^2/x,x)
[Out] int(arccsc(b*x+a)^2/x,x)
```

Fricas [F]

$$\int \frac{\csc^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arccsc}(bx+a)^2}{x}\,dx$$

```
[In] integrate(arccsc(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(arccsc(b*x + a)^2/x, x)
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{acsc}^2(a+bx)}{x}\,dx$$

```
[In] integrate(acsc(b*x+a)**2/x,x)
[Out] Integral(acsc(a + b*x)**2/x, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arccsc}(bx+a)^2}{x}\,dx$$

```
[In] integrate(arccsc(b*x+a)^2/x,x, algorithm="maxima")
[Out] integrate(arccsc(b*x + a)^2/x, x)
```

Giac [F]

$$\int \frac{\csc^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arccsc}\left(bx+a\right)^2}{x}\,dx$$

```
[In] integrate(arccsc(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(arccsc(b*x + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{a+b\,x}\right)^2}{x}\,dx$$

```
[In] int(asin(1/(a + b*x))^2/x,x)
[Out] int(asin(1/(a + b*x))^2/x, x)
```

3.32 $\int \frac{\csc^{-1}(a+bx)^2}{x^2}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 254

$$\int \frac{\csc^{-1}(a+bx)^2}{x^2}\,dx = -\frac{b\csc^{-1}(a+bx)^2}{a}-\frac{\csc^{-1}(a+bx)^2}{x} - \frac{2ib\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{2ib\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{2b\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

```
[Out] -b*arccsc(b*x+a)^2/a-arccsc(b*x+a)^2/x-2*I*b*arccsc(b*x+a)*ln(1+I*a*(I/(b*x
+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+2*I*b*arccs
c(b*x+a)*ln(1+I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(
-a^2+1)^(1/2)-2*b*polylog(2,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2
+1)^(1/2)))/a/(-a^2+1)^(1/2)+2*b*polylog(2,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^
(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5367, 4512, 4276, 3404, 2296, 2221, 2317, 2438}

$$\int \frac{\csc^{-1}(a+bx)^2}{x^2}\,dx = -\frac{2b\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2b\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}} - \frac{2ib\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{2ib\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}} - \frac{b\csc^{-1}(a+bx)^2}{a}-\frac{\csc^{-1}(a+bx)^2}{x}$$

```
[In] Int[ArcCsc[a + b*x]^2/x^2,x]
```

```
[Out] -((b*ArcCsc[a + b*x]^2)/a) - ArcCsc[a + b*x]^2/x - ((2*I)*b*ArcCsc[a + b*x]
*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]
) + ((2*I)*b*ArcCsc[a + b*x]*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[
1 - a^2])])/(a*Sqrt[1 - a^2]) - (2*b*PolyLog[2, ((-I)*a*E^(I*ArcCsc[a + b*x
]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (2*b*PolyLog[2, ((-I)*a*E^(I*
ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(b\text{Subst}\left(\int \frac{x^2\cot(x)\csc(x)}{(-a+\csc(x))^2}\,dx, x, \csc^{-1}(a+bx)\right)\right)\\
&= -\frac{\csc^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \frac{x}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)\\
&= -\frac{\csc^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \left(-\frac{x}{a}+\frac{x}{a(1-a\sin(x))}\right)\,dx, x, \csc^{-1}(a+bx)\right)
\end{aligned}$$

$$= -\frac{b\csc^{-1}(a+bx)^2}{a} - \frac{\csc^{-1}(a+bx)^2}{x} + \frac{(2b)\text{Subst}\left(\int \frac{x}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{a}$$

$$= -\frac{b\csc^{-1}(a+bx)^2}{a} - \frac{\csc^{-1}(a+bx)^2}{x} + \frac{(4b)\text{Subst}\left(\int \frac{e^{ix}x}{-ia+2e^{ix}+iae^{2ix}}\,dx, x, \csc^{-1}(a+bx)\right)}{a}$$

$$= -\frac{b\csc^{-1}(a+bx)^2}{a} - \frac{\csc^{-1}(a+bx)^2}{x} + \frac{(4ib)\text{Subst}\left(\int \frac{e^{ix}x}{2-2\sqrt{1-a^2}+2iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}} - \frac{(4ib)\text{Subst}\left(\int \frac{e^{ix}x}{2+2\sqrt{1-a^2}+2iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}$$

$$= -\frac{b\csc^{-1}(a+bx)^2}{a} - \frac{\csc^{-1}(a+bx)^2}{x} - \frac{2ib\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{2ib\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{(2ib)\text{Subst}\left(\int \log\left(1+\frac{2iae^{ix}}{2-2\sqrt{1-a^2}}\right)\,dx, x, \csc^{-1}(a+bx)\right)}{a\sqrt{1-a^2}} - \frac{(2ib)\text{Subst}\left(\int \log\left(1+\frac{2iae^{ix}}{2+2\sqrt{1-a^2}}\right)\,dx, x, \csc^{-1}(a+bx)\right)}{a\sqrt{1-a^2}}$$

$$= -\frac{b\csc^{-1}(a+bx)^2}{a} - \frac{\csc^{-1}(a+bx)^2}{x} - \frac{2ib\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{2ib\csc^{-1}(a+bx)\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{(2b)\text{Subst}\left(\int \frac{\log\left(1+\frac{2iax}{2-2\sqrt{1-a^2}}\right)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)}{a\sqrt{1-a^2}} - \frac{(2b)\text{Subst}\left(\int \frac{\log\left(1+\frac{2iax}{2+2\sqrt{1-a^2}}\right)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)}{a\sqrt{1-a^2}}$$

$$= -\frac{b \csc^{-1}(a+bx)^2}{a} - \frac{\csc^{-1}(a+bx)^2}{x} - \frac{2ib \csc^{-1}(a+bx) \log\left(1 + \frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$+ \frac{2ib \csc^{-1}(a+bx) \log\left(1 + \frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$- \frac{2b \operatorname{PolyLog}\left(2, -\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{2b \operatorname{PolyLog}\left(2, -\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 802 vs. $2(254) = 508$.

Time = 2.49 (sec) , antiderivative size = 802, normalized size of antiderivative = 3.16

$$\int \frac{\csc^{-1}(a+bx)^2}{x^2}\,dx =$$
$$-\frac{b\left(\frac{(a+bx)\csc^{-1}(a+bx)^2}{bx} + \frac{2\pi\arctan\left(\frac{a-\tan\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right)}{\sqrt{1-a^2}} + \frac{2\left(-2\arccos\left(\frac{1}{a}\right)\operatorname{arctanh}\left(\frac{(1+a)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(a+bx)\right)\right)}{\sqrt{-1+a^2}}\right)+(\pi-2\operatorname{cs}\right.}{}\right.}{}$$

```
[In] Integrate[ArcCsc[a + b*x]^2/x^2,x]
```

```
[Out] -((b*(((a + b*x)*ArcCsc[a + b*x]^2)/(b*x) + (2*Pi*ArcTan[(a - Tan[ArcCsc[a
+ b*x]/2])/Sqrt[1 - a^2]])/Sqrt[1 - a^2] + (2*(-2*ArcCos[a^(-1)]*ArcTanh[((
1 + a)*Cot[(Pi + 2*ArcCsc[a + b*x])/4])/Sqrt[-1 + a^2]] + (Pi - 2*ArcCsc[a
+ b*x])*ArcTanh[((-1 + a)*Tan[(Pi + 2*ArcCsc[a + b*x])/4])/Sqrt[-1 + a^2]]
+ (ArcCos[a^(-1)] + (2*I)*(-ArcTanh[((1 + a)*Cot[(Pi + 2*ArcCsc[a + b*x])/4
])/Sqrt[-1 + a^2]] + ArcTanh[((-1 + a)*Tan[(Pi + 2*ArcCsc[a + b*x])/4])/Sqr
t[-1 + a^2]]))*Log[((1/2 + I/2)*Sqrt[-1 + a^2])/(Sqrt[a]*E^((I/2)*ArcCsc[a
+ b*x])*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[a^(-1)] + (2*I)*ArcTanh[((1 +
a)*Cot[(Pi + 2*ArcCsc[a + b*x])/4])/Sqrt[-1 + a^2]] - (2*I)*ArcTanh[((-1 +
a)*Tan[(Pi + 2*ArcCsc[a + b*x])/4])/Sqrt[-1 + a^2]])*Log[((1/2 - I/2)*Sqrt[
-1 + a^2]*E^((I/2)*ArcCsc[a + b*x]))/(Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] -
(ArcCos[a^(-1)] - (2*I)*ArcTanh[((1 + a)*Cot[(Pi + 2*ArcCsc[a + b*x])/4])/S
qrt[-1 + a^2]])*Log[((-1 + a)*(I + I*a + Sqrt[-1 + a^2])*(-I + Cot[(Pi + 2*
ArcCsc[a + b*x])/4]))/(a*(-1 + a + Sqrt[-1 + a^2]*Cot[(Pi + 2*ArcCsc[a + b*
x])/4]))] - (ArcCos[a^(-1)] + (2*I)*ArcTanh[((1 + a)*Cot[(Pi + 2*ArcCsc[a +
 b*x])/4])/Sqrt[-1 + a^2]])*Log[((-1 + a)*(-I - I*a + Sqrt[-1 + a^2])*(I +
Cot[(Pi + 2*ArcCsc[a + b*x])/4]))/(a*(-1 + a + Sqrt[-1 + a^2]*Cot[(Pi + 2*A
rcCsc[a + b*x])/4]))] + I*(-PolyLog[2, ((1 - I*Sqrt[-1 + a^2])*(1 - a + Sqr
t[-1 + a^2]*Cot[(Pi + 2*ArcCsc[a + b*x])/4]))/(a*(-1 + a + Sqrt[-1 + a^2]*C
```

```
ot[(Pi + 2*ArcCsc[a + b*x])/4]))] + PolyLog[2, ((1 + I*Sqrt[-1 + a^2])*(1 -
 a + Sqrt[-1 + a^2]*Cot[(Pi + 2*ArcCsc[a + b*x])/4]))/(a*(-1 + a + Sqrt[-1
+ a^2]*Cot[(Pi + 2*ArcCsc[a + b*x])/4]))])))/Sqrt[-1 + a^2]))/a)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.19

method	result
derivativedivides	$b\left(-\frac{(bx+a)\operatorname{arccsc}(bx+a)^2}{abx}-\frac{2\operatorname{arccsc}(bx+a)\ln\left(\frac{-\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)a+\sqrt{a^2-1}+i}{i+\sqrt{a^2-1}}\right)}{a\sqrt{a^2-1}}+\frac{2\operatorname{arccsc}(bx+a)\ln\left(\right.}{}\right.$
default	$b\left(-\frac{(bx+a)\operatorname{arccsc}(bx+a)^2}{abx}-\frac{2\operatorname{arccsc}(bx+a)\ln\left(\frac{-\left(\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)a+\sqrt{a^2-1}+i}{i+\sqrt{a^2-1}}\right)}{a\sqrt{a^2-1}}+\frac{2\operatorname{arccsc}(bx+a)\ln\left(\right.}{}\right.$

```
[In] int(arccsc(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-(b*x+a)*arccsc(b*x+a)^2/a/b/x-2/a*arccsc(b*x+a)/(a^2-1)^(1/2)*ln((-(I/(
b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a+(a^2-1)^(1/2)+I)/(I+(a^2-1)^(1/2)))+2/a*arc
csc(b*x+a)/(a^2-1)^(1/2)*ln((-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a+I-(a^2-1)
^(1/2))/(I-(a^2-1)^(1/2)))+2*I/a/(a^2-1)^(1/2)*dilog((-(I/(b*x+a)+(1-1/(b*x
+a)^2)^(1/2))*a+(a^2-1)^(1/2)+I)/(I+(a^2-1)^(1/2)))-2*I/a/(a^2-1)^(1/2)*dil
og((-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a+I-(a^2-1)^(1/2))/(I-(a^2-1)^(1/2))
))
```

Fricas [F]

$$\int \frac{\csc^{-1}(a+bx)^2}{x^2}\,dx=\int \frac{\operatorname{arccsc}\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(arccsc(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccsc(b*x + a)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{acsc}^2{(a+bx)}}{x^2}\,dx$$

```
[In] integrate(acsc(b*x+a)**2/x**2,x)
[Out] Integral(acsc(a + b*x)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arccsc}\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(arccsc(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] -1/4*(4*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 + 4*x*integrate((
2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*b*x*arctan2(1, sqrt(b*x + a + 1)*sqrt
(b*x + a - 1)) + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*
x + a)^2 + (b^3*x^3 + 2*a*b^2*x^2 + (a^2 - 1)*b*x - (b^3*x^3 + 3*a*b^2*x^2
+ a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b
^3*x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2), x) - log(b^2*x^2
 + 2*a*b*x + a^2)^2)/x
```

Giac [F]

$$\int \frac{\csc^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arccsc}\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(arccsc(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(arccsc(b*x + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{a+b\,x}\right)^2}{x^2}\,dx$$

```
[In] int(asin(1/(a + b*x))^2/x^2,x)
[Out] int(asin(1/(a + b*x))^2/x^2, x)
```

3.33 $\int x^2 \csc^{-1}(a+bx)^3 \, dx$

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Sympy [F] 237
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Giac [F] 238
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Optimal result

Integrand size = 12, antiderivative size = 464

$$\int x^2 \csc^{-1}(a+bx)^3\,dx = \frac{(a+bx)\csc^{-1}(a+bx)}{b^3} - \frac{3ia\csc^{-1}(a+bx)^2}{b^3} - \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{b^3} + \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^3} + \frac{a^3\csc^{-1}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)^3 + \frac{\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^3} + \frac{6a\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{6ia^2\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6ia^2\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{3ia\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\text{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2\,\text{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{\text{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{6a^2\,\text{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$

```
[Out] (b*x+a)*arccsc(b*x+a)/b^3+6*I*a^2*arccsc(b*x+a)*polylog(2,I/(b*x+a)+(1-1/(b
*x+a)^2)^(1/2))/b^3+1/3*a^3*arccsc(b*x+a)^3/b^3+1/3*x^3*arccsc(b*x+a)^3+arc
csc(b*x+a)^2*arctanh(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^3+6*a^2*arccsc(b*x+
a)^2*arctanh(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^3+arctanh((1-1/(b*x+a)^2)^(
```

```
1/2))/b^3+6*a*arccsc(b*x+a)*ln(1-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)/b^3-6
*I*a^2*arccsc(b*x+a)*polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b^3-3*I*a*
arccsc(b*x+a)^2/b^3-3*I*a*polylog(2,(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)/b^
3+I*arccsc(b*x+a)*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^3-I*arccsc(b
*x+a)*polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b^3+polylog(3,-I/(b*x+a)-
(1-1/(b*x+a)^2)^(1/2))/b^3+6*a^2*polylog(3,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2)
)/b^3-polylog(3,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^3-6*a^2*polylog(3,I/(b*x
+a)+(1-1/(b*x+a)^2)^(1/2))/b^3-3*a*(b*x+a)*arccsc(b*x+a)^2*(1-1/(b*x+a)^2)^
(1/2)/b^3+1/2*(b*x+a)^2*arccsc(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules

used = {5367, 4512, 4275, 4268, 2611, 2320, 6724, 4269, 3798, 2221, 2317, 2438, 4271, 3855}

$$\int x^2 \csc^{-1}(a+bx)^3\,dx = \frac{a^3\csc^{-1}(a+bx)^3}{3b^3} + \frac{6a^2\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{6ia^2\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6ia^2\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2\,\text{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{6a^2\,\text{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^3} + \frac{\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{3ia\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\text{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{\text{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^3} - \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{b^3} - \frac{3ia\csc^{-1}(a+bx)^2}{b^3} + \frac{(a+bx)\csc^{-1}(a+bx)}{b^3} + \frac{6a\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)^3$$

```
[In] Int[x^2*ArcCsc[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcCsc[a + b*x])/b^3 - ((3*I)*a*ArcCsc[a + b*x]^2)/b^3 - (3*a*(a
 + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcCsc[a + b*x]^2)/b^3 + ((a + b*x)^2*Sqrt
[1 - (a + b*x)^(-2)]*ArcCsc[a + b*x]^2)/(2*b^3) + (a^3*ArcCsc[a + b*x]^3)/(
3*b^3) + (x^3*ArcCsc[a + b*x]^3)/3 + (ArcCsc[a + b*x]^2*ArcTanh[E^(I*ArcCsc
[a + b*x])])/b^3 + (6*a^2*ArcCsc[a + b*x]^2*ArcTanh[E^(I*ArcCsc[a + b*x])])
/b^3 + ArcTanh[Sqrt[1 - (a + b*x)^(-2)]]/b^3 + (6*a*ArcCsc[a + b*x]*Log[1 -
 E^((2*I)*ArcCsc[a + b*x])])/b^3 - (I*ArcCsc[a + b*x]*PolyLog[2, -E^(I*ArcC
```

```
sc[a + b*x])])/b^3 - ((6*I)*a^2*ArcCsc[a + b*x]*PolyLog[2, -E^(I*ArcCsc[a +
 b*x])])/b^3 + (I*ArcCsc[a + b*x]*PolyLog[2, E^(I*ArcCsc[a + b*x])])/b^3 +
((6*I)*a^2*ArcCsc[a + b*x]*PolyLog[2, E^(I*ArcCsc[a + b*x])])/b^3 - ((3*I)*
a*PolyLog[2, E^((2*I)*ArcCsc[a + b*x])])/b^3 + PolyLog[3, -E^(I*ArcCsc[a +
b*x])]/b^3 + (6*a^2*PolyLog[3, -E^(I*ArcCsc[a + b*x])])/b^3 - PolyLog[3, E^
(I*ArcCsc[a + b*x])]/b^3 - (6*a^2*PolyLog[3, E^(I*ArcCsc[a + b*x])])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^3\cot(x)\csc(x)(-a+\csc(x))^2\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}\\
&= \frac{1}{3}x^3\csc^{-1}(a+bx)^3-\frac{\text{Subst}\left(\int x^2(-a+\csc(x))^3\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}\\
&= \frac{1}{3}x^3\csc^{-1}(a+bx)^3\\
&\quad -\frac{\text{Subst}\left(\int\left(-a^3x^2+3a^2x^2\csc(x)-3ax^2\csc^2(x)+x^2\csc^3(x)\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}\\
&= \frac{a^3\csc^{-1}(a+bx)^3}{3b^3}+\frac{1}{3}x^3\csc^{-1}(a+bx)^3-\frac{\text{Subst}\left(\int x^2\csc^3(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}\\
&\quad +\frac{(3a)\text{Subst}\left(\int x^2\csc^2(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}\\
&\quad -\frac{(3a^2)\,\text{Subst}\left(\int x^2\csc(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}\\
&= \frac{(a+bx)\csc^{-1}(a+bx)}{b^3}-\frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{b^3}\\
&\quad +\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^3}+\frac{a^3\csc^{-1}(a+bx)^3}{3b^3}\\
&\quad +\frac{1}{3}x^3\csc^{-1}(a+bx)^3+\frac{6a^2\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3}\\
&\quad -\frac{\text{Subst}\left(\int x^2\csc(x)\,dx,x,\csc^{-1}(a+bx)\right)}{2b^3}-\frac{\text{Subst}\left(\int\csc(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}\\
&\quad +\frac{(6a)\text{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}\\
&\quad +\frac{(6a^2)\,\text{Subst}\left(\int x\log\left(1-e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}\\
&\quad -\frac{(6a^2)\,\text{Subst}\left(\int x\log\left(1+e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}
\end{aligned}$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)}{b^3} - \frac{3ia\csc^{-1}(a+bx)^2}{b^3}$$

$$- \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{b^3} + \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^3}$$

$$+ \frac{a^3\csc^{-1}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)^3 + \frac{\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$

$$+ \frac{6a^2\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^3}$$

$$- \frac{6ia^2\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$

$$+ \frac{6ia^2\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$

$$+ \frac{\text{Subst}\left(\int x\log\left(1-e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}$$

$$- \frac{\text{Subst}\left(\int x\log\left(1+e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}$$

$$- \frac{(12ia)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}$$

$$+ \frac{(6ia^2)\,\text{Subst}\left(\int\text{PolyLog}\left(2,-e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}$$

$$- \frac{(6ia^2)\,\text{Subst}\left(\int\text{PolyLog}\left(2,e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)}{b^3} - \frac{3ia\csc^{-1}(a+bx)^2}{b^3} - \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{b^3} + \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^3} + \frac{a^3\csc^{-1}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)^3 + \frac{\csc^{-1}(a+bx)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2\csc^{-1}(a+bx)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^3} + \frac{6a\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{6ia^2\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6ia^2\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{i\text{Subst}\left(\int\operatorname{PolyLog}\left(2,-e^{ix}\right)dx,x,\csc^{-1}(a+bx)\right)}{b^3} - \frac{i\text{Subst}\left(\int\operatorname{PolyLog}\left(2,e^{ix}\right)dx,x,\csc^{-1}(a+bx)\right)}{b^3} - \frac{(6a)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)dx,x,\csc^{-1}(a+bx)\right)}{b^3} + \frac{(6a^2)\,\text{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{(6a^2)\,\text{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$

$$
\begin{aligned}
&= \frac{(a+bx)\csc^{-1}(a+bx)}{b^3} - \frac{3ia\csc^{-1}(a+bx)^2}{b^3} \\
&\quad - \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{b^3} + \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^3} \\
&\quad + \frac{a^3\csc^{-1}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)^3 + \frac{\csc^{-1}(a+bx)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} \\
&\quad + \frac{6a^2\csc^{-1}(a+bx)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^3} \\
&\quad + \frac{6a\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{b^3} \\
&\quad - \frac{i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} \\
&\quad - \frac{6ia^2\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} \\
&\quad + \frac{i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} \\
&\quad + \frac{6ia^2\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} \\
&\quad - \frac{6a^2\operatorname{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^3} \\
&\quad - \frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^3} \\
&\quad + \frac{(3ia)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(a+bx)}\right)}{b^3}
\end{aligned}
$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)}{b^3} - \frac{3ia\csc^{-1}(a+bx)^2}{b^3} - \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{b^3} + \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^3} + \frac{a^3\csc^{-1}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\csc^{-1}(a+bx)^3 + \frac{\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^3} + \frac{6a\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{6ia^2\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6ia^2\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{3ia\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{\text{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2\,\text{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{\text{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^3} - \frac{6a^2\,\text{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^3}$$

Mathematica [A] (warning: unable to verify)

Time = 7.18 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.41

$$\int x^2\csc^{-1}(a+bx)^3\,dx = -\frac{72ia\csc^{-1}(a+bx)^2 - 12\csc^{-1}(a+bx)\cot\left(\frac{1}{2}\csc^{-1}(a+bx)\right) + 36a\csc^{-1}(a+bx)^2\cot\left(\frac{1}{2}\csc^{-1}(a+bx)\right)}{}$$

```
[In] Integrate[x^2*ArcCsc[a + b*x]^3,x]
```

```
[Out] -1/24*((72*I)*a*ArcCsc[a + b*x]^2 - 12*ArcCsc[a + b*x]*Cot[ArcCsc[a + b*x]/
2] + 36*a*ArcCsc[a + b*x]^2*Cot[ArcCsc[a + b*x]/2] - 2*ArcCsc[a + b*x]^3*Co
t[ArcCsc[a + b*x]/2] - 12*a^2*ArcCsc[a + b*x]^3*Cot[ArcCsc[a + b*x]/2] - 3*
ArcCsc[a + b*x]^2*Csc[ArcCsc[a + b*x]/2]^2 + 6*a*ArcCsc[a + b*x]^3*Csc[ArcC
sc[a + b*x]/2]^2 - (ArcCsc[a + b*x]^3*Csc[ArcCsc[a + b*x]/2]^4)/(2*(a + b*x
```

)) + 12*ArcCsc[a + b*x]^2*Log[1 - E^(I*ArcCsc[a + b*x])] + 72*a^2*ArcCsc[a
+ b*x]^2*Log[1 - E^(I*ArcCsc[a + b*x])] - 12*ArcCsc[a + b*x]^2*Log[1 + E^(I
*ArcCsc[a + b*x])] - 72*a^2*ArcCsc[a + b*x]^2*Log[1 + E^(I*ArcCsc[a + b*x])
] - 144*a*ArcCsc[a + b*x]*Log[1 - E^((2*I)*ArcCsc[a + b*x])] + 24*Log[Tan[A
rcCsc[a + b*x]/2]] + (24*I)*(1 + 6*a^2)*ArcCsc[a + b*x]*PolyLog[2, -E^(I*Ar
cCsc[a + b*x])] - (24*I)*(1 + 6*a^2)*ArcCsc[a + b*x]*PolyLog[2, E^(I*ArcCsc
[a + b*x])] + (72*I)*a*PolyLog[2, E^((2*I)*ArcCsc[a + b*x])] - 24*PolyLog[3
, -E^(I*ArcCsc[a + b*x])] - 144*a^2*PolyLog[3, -E^(I*ArcCsc[a + b*x])] + 24
*PolyLog[3, E^(I*ArcCsc[a + b*x])] + 144*a^2*PolyLog[3, E^(I*ArcCsc[a + b*x
])] + 3*ArcCsc[a + b*x]^2*Sec[ArcCsc[a + b*x]/2]^2 + 6*a*ArcCsc[a + b*x]^3*
Sec[ArcCsc[a + b*x]/2]^2 - 8*(a + b*x)^3*ArcCsc[a + b*x]^3*Sin[ArcCsc[a + b
*x]/2]^4 - 12*ArcCsc[a + b*x]*Tan[ArcCsc[a + b*x]/2] - 36*a*ArcCsc[a + b*x]
^2*Tan[ArcCsc[a + b*x]/2] - 2*ArcCsc[a + b*x]^3*Tan[ArcCsc[a + b*x]/2] - 12
*a^2*ArcCsc[a + b*x]^3*Tan[ArcCsc[a + b*x]/2])/b^3

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 749, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-6i\,\text{polylog}\left(2,-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)a-\frac{\text{arccsc}(bx+a)^2\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{2}-6i\,\text{polylog}\left(2,-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right.}{}$
default	$\frac{-6i\,\text{polylog}\left(2,-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)a-\frac{\text{arccsc}(bx+a)^2\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{2}-6i\,\text{polylog}\left(2,-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right.}{}$

[In] int(x^2*arccsc(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/b^3*(-6*I*polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))*a-1/2*arccsc(b*x+a)
^2*ln(1-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))-6*I*polylog(2,-I/(b*x+a)-(1-1/(b*x
+a)^2)^(1/2))*a^2*arccsc(b*x+a)-polylog(3,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+
1/2*arccsc(b*x+a)^2*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))-I*arccsc(b*x+a)*p
olylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))+polylog(3,-I/(b*x+a)-(1-1/(b*x+a
)^2)^(1/2))+2*arctanh(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+6*ln(1-I/(b*x+a)-(1-
1/(b*x+a)^2)^(1/2))*a*arccsc(b*x+a)+6*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))
*a*arccsc(b*x+a)+I*arccsc(b*x+a)*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))
-6*I*a*arccsc(b*x+a)^2+6*I*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a^2*a
rccsc(b*x+a)-3*ln(1-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))*a^2*arccsc(b*x+a)^2+1/
6*arccsc(b*x+a)*(6*arccsc(b*x+a)^2*a^2*(b*x+a)-6*arccsc(b*x+a)^2*a*(b*x+a)^
2+2*arccsc(b*x+a)^2*(b*x+a)^3-18*arccsc(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1
/2)*a*(b*x+a)+3*arccsc(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)^2+18*
I*a*arccsc(b*x+a)+6*b*x+6*a)+3*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a^2*ar
ccsc(b*x+a)^2-6*I*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a-6*polylog(3,
I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))*a^2+6*polylog(3,-I/(b*x+a)-(1-1/(b*x+a)^2)

```
^(1/2))*a^2)
```

Fricas [F]

$$\int x^2 \csc^{-1}(a+bx)^3 \, dx = \int x^2 \operatorname{arccsc}(bx+a)^3 \, dx$$

```
[In] integrate(x^2*arccsc(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^2*arccsc(b*x + a)^3, x)
```

Sympy [F]

$$\int x^2 \csc^{-1}(a+bx)^3 \, dx = \int x^2 \operatorname{acsc}^3(a+bx) \, dx$$

```
[In] integrate(x**2*acsc(b*x+a)**3,x)
[Out] Integral(x**2*acsc(a + b*x)**3, x)
```

Maxima [F]

$$\int x^2 \csc^{-1}(a+bx)^3 \, dx = \int x^2 \operatorname{arccsc}(bx+a)^3 \, dx$$

```
[In] integrate(x^2*arccsc(b*x+a)^3,x, algorithm="maxima")
[Out] 1/3*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 1/4*x^3*arctan2
(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - i
ntegrate(1/4*(12*(b^3*x^5*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) +
 3*a*b^2*x^4*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (3*a^2*arcta
n2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*s
qrt(b*x + a - 1)))*b*x^3 + (a^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a -
 1)) - a*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*x^2)*log(b*x + a)
^2 - (4*b*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x^3*log
(b^2*x^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - 4*(b^3*x
^5*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 2*a*b^2*x^4*arctan2(1,
 sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (a^2*arctan2(1, sqrt(b*x + a + 1)*s
qrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x^3
+ 3*(b^3*x^5*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3*a*b^2*x^4*
arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (3*a^2*arctan2(1, sqrt(b*
x + a + 1)*sqrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a -
 1)))*b*x^3 + (a^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - a*arct
an2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*x^2)*log(b*x + a))*log(b^2*x^2
 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

Giac [F]

$$\int x^2 \csc^{-1}(a+bx)^3\,dx = \int x^2 \operatorname{arccsc}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^2*arccsc(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^2*arccsc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \csc^{-1}(a+bx)^3\,dx = \int x^2 \operatorname{asin}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

```
[In] int(x^2*asin(1/(a + b*x))^3,x)
[Out] int(x^2*asin(1/(a + b*x))^3, x)
```

3.34 $\int x \csc^{-1}(a+bx)^3 \, dx$

Optimal result 239
Rubi [A] (verified) 240
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Sympy [F] 246
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Giac [F] 247
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Optimal result

Integrand size = 10, antiderivative size = 264

$$\begin{aligned}\int x \csc^{-1}(a+bx)^3 \, dx = {} & \frac{3i \csc^{-1}(a+bx)^2}{2b^2} + \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^2} \\ & - \frac{a^2 \csc^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2 \csc^{-1}(a+bx)^3 \\ & - \frac{6a \csc^{-1}(a+bx)^2 \operatorname{arctanh}\left(e^{i \csc^{-1}(a+bx)}\right)}{b^2} \\ & - \frac{3 \csc^{-1}(a+bx) \log\left(1-e^{2i \csc^{-1}(a+bx)}\right)}{b^2} \\ & + \frac{6ia \csc^{-1}(a+bx) \operatorname{PolyLog}\left(2,-e^{i \csc^{-1}(a+bx)}\right)}{b^2} \\ & - \frac{6ia \csc^{-1}(a+bx) \operatorname{PolyLog}\left(2,e^{i \csc^{-1}(a+bx)}\right)}{b^2} \\ & + \frac{3i \operatorname{PolyLog}\left(2,e^{2i \csc^{-1}(a+bx)}\right)}{2b^2} \\ & - \frac{6a \operatorname{PolyLog}\left(3,-e^{i \csc^{-1}(a+bx)}\right)}{b^2} + \frac{6a \operatorname{PolyLog}\left(3,e^{i \csc^{-1}(a+bx)}\right)}{b^2}\end{aligned}$$

```
[Out] 3/2*I*arccsc(b*x+a)^2/b^2-1/2*a^2*arccsc(b*x+a)^3/b^2+1/2*x^2*arccsc(b*x+a)
^3-6*a*arccsc(b*x+a)^2*arctanh(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^2-3*arccs
c(b*x+a)*ln(1-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)/b^2+6*I*a*arccsc(b*x+a)*
polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b^2-6*I*a*arccsc(b*x+a)*polylog
(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^2+3/2*I*polylog(2,(I/(b*x+a)+(1-1/(b*
x+a)^2)^(1/2))^2)/b^2-6*a*polylog(3,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b^2+6
```

```
*a*polylog(3,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b^2+3/2*(b*x+a)*arccsc(b*x+a)
^2*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {5367, 4512, 4275, 4268, 2611, 2320, 6724, 4269, 3798, 2221, 2317, 2438}

$$\int x\csc^{-1}(a+bx)^3\,dx=-\frac{a^2\csc^{-1}(a+bx)^3}{2b^2}-\frac{6a\csc^{-1}(a+bx)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{6ia\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^2}-\frac{6ia\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{3i\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)}{2b^2}-\frac{6a\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{6a\operatorname{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^2}+\frac{3i\csc^{-1}(a+bx)^2}{2b^2}-\frac{3\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{1}{2}x^2\csc^{-1}(a+bx)^3$$

```
[In] Int[x*ArcCsc[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*ArcCsc[a + b*x]^2)/b^2 + (3*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*A
rcCsc[a + b*x]^2)/(2*b^2) - (a^2*ArcCsc[a + b*x]^3)/(2*b^2) + (x^2*ArcCsc[a
 + b*x]^3)/2 - (6*a*ArcCsc[a + b*x]^2*ArcTanh[E^(I*ArcCsc[a + b*x])])/b^2 -
 (3*ArcCsc[a + b*x]*Log[1 - E^((2*I)*ArcCsc[a + b*x])])/b^2 + ((6*I)*a*ArcC
sc[a + b*x]*PolyLog[2, -E^(I*ArcCsc[a + b*x])])/b^2 - ((6*I)*a*ArcCsc[a + b
*x]*PolyLog[2, E^(I*ArcCsc[a + b*x])])/b^2 + (((3*I)/2)*PolyLog[2, E^((2*I)
*ArcCsc[a + b*x])])/b^2 - (6*a*PolyLog[3, -E^(I*ArcCsc[a + b*x])])/b^2 + (6
*a*PolyLog[3, E^(I*ArcCsc[a + b*x])])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^3 \cot(x)\csc(x)(-a+\csc(x))\,dx, x, \csc^{-1}(a+bx)\right)}{b^2}\\
&= \frac{1}{2}x^2\csc^{-1}(a+bx)^3 - \frac{3\text{Subst}\left(\int x^2(-a+\csc(x))^2\,dx, x, \csc^{-1}(a+bx)\right)}{2b^2}\\
&= \frac{1}{2}x^2\csc^{-1}(a+bx)^3 - \frac{3\text{Subst}\left(\int \left(a^2x^2-2ax^2\csc(x)+x^2\csc^2(x)\right)\,dx, x, \csc^{-1}(a+bx)\right)}{2b^2}\\
&= -\frac{a^2\csc^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\csc^{-1}(a+bx)^3 - \frac{3\text{Subst}\left(\int x^2\csc^2(x)\,dx, x, \csc^{-1}(a+bx)\right)}{2b^2}\\
&\quad + \frac{(3a)\text{Subst}\left(\int x^2\csc(x)\,dx, x, \csc^{-1}(a+bx)\right)}{b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^2}-\frac{a^2\csc^{-1}(a+bx)^3}{2b^2} \\
&\quad+\frac{1}{2}x^2\csc^{-1}(a+bx)^3-\frac{6a\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2} \\
&\quad-\frac{3\text{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2} \\
&\quad-\frac{(6a)\text{Subst}\left(\int x\log\left(1-e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2} \\
&\quad+\frac{(6a)\text{Subst}\left(\int x\log\left(1+e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3i\csc^{-1}(a+bx)^2}{2b^2}+\frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^2}-\frac{a^2\csc^{-1}(a+bx)^3}{2b^2} \\
&\quad+\frac{1}{2}x^2\csc^{-1}(a+bx)^3-\frac{6a\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2} \\
&\quad+\frac{6ia\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^2} \\
&\quad-\frac{6ia\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^2} \\
&\quad+\frac{(6i)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\csc^{-1}(a+bx)\right)}{b^2} \\
&\quad-\frac{(6ia)\text{Subst}\left(\int\text{PolyLog}\left(2,-e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2} \\
&\quad+\frac{(6ia)\text{Subst}\left(\int\text{PolyLog}\left(2,e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}
\end{aligned}
$$

$$=\frac{3i\csc^{-1}(a+bx)^2}{2b^2}+\frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^2}-\frac{a^2\csc^{-1}(a+bx)^3}{2b^2}+\frac{1}{2}x^2\csc^{-1}(a+bx)^3-\frac{6a\csc^{-1}(a+bx)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2}-\frac{3\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{6ia\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^2}-\frac{6ia\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{3\operatorname{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b^2}-\frac{(6a)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{(6a)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b^2}$$

$$=\frac{3i\csc^{-1}(a+bx)^2}{2b^2}+\frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^2}-\frac{a^2\csc^{-1}(a+bx)^3}{2b^2}+\frac{1}{2}x^2\csc^{-1}(a+bx)^3-\frac{6a\csc^{-1}(a+bx)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2}-\frac{3\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{6ia\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^2}-\frac{6ia\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^2}-\frac{6a\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^2}+\frac{6a\operatorname{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^2}-\frac{(3i)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(a+bx)}\right)}{2b^2}$$

$$= \frac{3i\csc^{-1}(a+bx)^2}{2b^2} + \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\csc^{-1}(a+bx)^2}{2b^2} - \frac{a^2\csc^{-1}(a+bx)^3}{2b^2}$$
$$+ \frac{1}{2}x^2\csc^{-1}(a+bx)^3 - \frac{6a\csc^{-1}(a+bx)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b^2}$$
$$- \frac{3\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{b^2}$$
$$+ \frac{6ia\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b^2}$$
$$- \frac{6ia\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b^2} + \frac{3i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)}{2b^2}$$
$$- \frac{6a\,\text{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b^2} + \frac{6a\,\text{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.19

$$\int x\csc^{-1}(a+bx)^3\,dx$$
$$= \frac{3i\csc^{-1}(a+bx)^2 + 3a\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\csc^{-1}(a+bx)^2 + 3bx\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\csc^{-1}(a+bx)^2 - a^2\csc^{-1}}{}$$

```
[In] Integrate[x*ArcCsc[a + b*x]^3,x]
```

```
[Out] ((3*I)*ArcCsc[a + b*x]^2 + 3*a*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x
)^2]*ArcCsc[a + b*x]^2 + 3*b*x*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x
)^2]*ArcCsc[a + b*x]^2 - a^2*ArcCsc[a + b*x]^3 + b^2*x^2*ArcCsc[a + b*x]^3
+ 6*a*ArcCsc[a + b*x]^2*Log[1 - E^(I*ArcCsc[a + b*x])] - 6*a*ArcCsc[a + b*x
]^2*Log[1 + E^(I*ArcCsc[a + b*x])] - 6*ArcCsc[a + b*x]*Log[1 - E^((2*I)*Arc
Csc[a + b*x])] + (12*I)*a*ArcCsc[a + b*x]*PolyLog[2, -E^(I*ArcCsc[a + b*x])
] - (12*I)*a*ArcCsc[a + b*x]*PolyLog[2, E^(I*ArcCsc[a + b*x])] + (3*I)*Poly
Log[2, E^((2*I)*ArcCsc[a + b*x])] - 12*a*PolyLog[3, -E^(I*ArcCsc[a + b*x])]
 + 12*a*PolyLog[3, E^(I*ArcCsc[a + b*x])])/(2*b^2)
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccsc}(bx+a)^2\left(2\operatorname{arccsc}(bx+a)a(bx+a)-\operatorname{arccsc}(bx+a)(bx+a)^2-3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)+3i\right)}{2}+3a\operatorname{arccsc}(bx+a)^2\ln\left(1-\frac{i}{bx+a}-\sqrt{\right.$
default	$\frac{-\frac{\operatorname{arccsc}(bx+a)^2\left(2\operatorname{arccsc}(bx+a)a(bx+a)-\operatorname{arccsc}(bx+a)(bx+a)^2-3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)+3i\right)}{2}+3a\operatorname{arccsc}(bx+a)^2\ln\left(1-\frac{i}{bx+a}-\sqrt{\right.$

```
[In] int(x*arccsc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-1/2*arccsc(b*x+a)^2*(2*arccsc(b*x+a)*a*(b*x+a)-arccsc(b*x+a)*(b*x+a
)^2-3*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)+3*I)+3*a*arccsc(b*x+a)^2*ln(1
-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))-3*a*arccsc(b*x+a)^2*ln(1+I/(b*x+a)+(1-1/(
b*x+a)^2)^(1/2))-3*arccsc(b*x+a)*ln(1-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))+6*a*
polylog(3,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))-3*arccsc(b*x+a)*ln(1+I/(b*x+a)+(
1-1/(b*x+a)^2)^(1/2))-6*a*polylog(3,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))-6*I*a
*arccsc(b*x+a)*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+6*I*a*arccsc(b*x+
a)*polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))+3*I*arccsc(b*x+a)^2+3*I*poly
log(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+3*I*polylog(2,-I/(b*x+a)-(1-1/(b*x+a
)^2)^(1/2)))
```

Fricas [F]

$$\int x\csc^{-1}(a+bx)^3\,dx=\int x\operatorname{arccsc}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x*arccsc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arccsc(b*x + a)^3, x)
```

Sympy [F]

$$\int x\csc^{-1}(a+bx)^3\,dx=\int x\operatorname{acsc}^3\left(a+bx\right)dx$$

```
[In] integrate(x*acsc(b*x+a)**3,x)
```

```
[Out] Integral(x*acsc(a + b*x)**3, x)
```

Maxima [F]

$$\int x\csc^{-1}(a+bx)^3\,dx = \int x\,\mathrm{arccsc}\,(bx+a)^3\,dx$$

[In] integrate(x*arccsc(b*x+a)^3,x, algorithm="maxima")

[Out] 1/2*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3/8*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integrate(3/8*(8*(b^3*x^4*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3*a*b^2*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (3*a^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x^2 + (a^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - a*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*x)*log(b*x + a)^2 - (4*b*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x^2*log(b^2*x^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - 4*(b^3*x^4*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 2*a*b^2*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (a^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x^2 + 2*(b^3*x^4*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3*a*b^2*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (3*a^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x^2 + (a^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - a*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*x)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)

Giac [F]

$$\int x\csc^{-1}(a+bx)^3\,dx = \int x\,\mathrm{arccsc}\,(bx+a)^3\,dx$$

[In] integrate(x*arccsc(b*x+a)^3,x, algorithm="giac")

[Out] integrate(x*arccsc(b*x + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int x \csc^{-1}(a+bx)^3\,dx = \int x\,\mathrm{asin}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

```
[In] int(x*asin(1/(a + b*x))^3,x)
[Out] int(x*asin(1/(a + b*x))^3, x)
```

3.35 $\int \csc^{-1}(a+bx)^3\,dx$

Optimal result 249
Rubi [A] (verified) 249
Mathematica [A] (verified) 253
Maple [A] (verified) 253
Fricas [F] 254
Sympy [F] 254
Maxima [F] 254
Giac [F] 255
Mupad [F(-1)] 255

Optimal result

Integrand size = 8, antiderivative size = 140

$$\int \csc^{-1}(a+bx)^3\,dx = \frac{(a+bx)\csc^{-1}(a+bx)^3}{b} + \frac{6\csc^{-1}(a+bx)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{6i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{6i\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{6\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{6\operatorname{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b}$$

```
[Out] (b*x+a)*arccsc(b*x+a)^3/b+6*arccsc(b*x+a)^2*arctanh(I/(b*x+a)+(1-1/(b*x+a)^
2)^(1/2))/b-6*I*arccsc(b*x+a)*polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b
+6*I*arccsc(b*x+a)*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/b+6*polylog(3
,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2))/b-6*polylog(3,I/(b*x+a)+(1-1/(b*x+a)^2)^
(1/2))/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used

$= \{5361, 5325, 3843, 4268, 2611, 2320, 6724\}$

$$\int \csc^{-1}(a+bx)^3\,dx = \frac{6\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{6i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{6i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{6\,\text{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{6\,\text{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{(a+bx)\csc^{-1}(a+bx)^3}{b}$$

```
[In] Int[ArcCsc[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcCsc[a + b*x]^3)/b + (6*ArcCsc[a + b*x]^2*ArcTanh[E^(I*ArcCsc[
a + b*x])])/b - ((6*I)*ArcCsc[a + b*x]*PolyLog[2, -E^(I*ArcCsc[a + b*x])])/
b + ((6*I)*ArcCsc[a + b*x]*PolyLog[2, E^(I*ArcCsc[a + b*x])])/b + (6*PolyLo
g[3, -E^(I*ArcCsc[a + b*x])])/b - (6*PolyLog[3, E^(I*ArcCsc[a + b*x])])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3843

```
Int[Cot[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)
*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^p/(b*n*p)),
 x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csc[a + b*x^n]^p, x], x] /; F
reeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5325

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Sub
st[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[n, 0]
```

Rule 5361

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCsc[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \csc^{-1}(x)^3\,dx, x, a+bx\right)}{b}\\
&= -\frac{\text{Subst}\left(\int x^3\cot(x)\csc(x)\,dx, x, \csc^{-1}(a+bx)\right)}{b}\\
&= \frac{(a+bx)\csc^{-1}(a+bx)^3}{b} - \frac{3\text{Subst}\left(\int x^2\csc(x)\,dx, x, \csc^{-1}(a+bx)\right)}{b}\\
&= \frac{(a+bx)\csc^{-1}(a+bx)^3}{b} + \frac{6\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b}\\
&\quad + \frac{6\text{Subst}\left(\int x\log\left(1-e^{ix}\right)\,dx, x, \csc^{-1}(a+bx)\right)}{b}\\
&\quad - \frac{6\text{Subst}\left(\int x\log\left(1+e^{ix}\right)\,dx, x, \csc^{-1}(a+bx)\right)}{b}
\end{aligned}$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)^3}{b} + \frac{6\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{6i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{6i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{(6i)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b} - \frac{(6i)\text{Subst}\left(\int \text{PolyLog}\left(2,e^{ix}\right)\,dx,x,\csc^{-1}(a+bx)\right)}{b}$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)^3}{b} + \frac{6\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{6i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{6i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{6\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{6\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{i\csc^{-1}(a+bx)}\right)}{b}$$

$$= \frac{(a+bx)\csc^{-1}(a+bx)^3}{b} + \frac{6\csc^{-1}(a+bx)^2\text{arctanh}\left(e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{6i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{6i\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,e^{i\csc^{-1}(a+bx)}\right)}{b} + \frac{6\,\text{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)}{b} - \frac{6\,\text{PolyLog}\left(3,e^{i\csc^{-1}(a+bx)}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16

$$\int \csc^{-1}(a+bx)^3\,dx$$
$$= \frac{a\csc^{-1}(a+bx)^3 + bx\csc^{-1}(a+bx)^3 - 3\csc^{-1}(a+bx)^2\log\left(1-e^{i\csc^{-1}(a+bx)}\right) + 3\csc^{-1}(a+bx)^2\log\left(1+\right.}{}$$

```
[In] Integrate[ArcCsc[a + b*x]^3,x]
```

```
[Out] (a*ArcCsc[a + b*x]^3 + b*x*ArcCsc[a + b*x]^3 - 3*ArcCsc[a + b*x]^2*Log[1 -
E^(I*ArcCsc[a + b*x])] + 3*ArcCsc[a + b*x]^2*Log[1 + E^(I*ArcCsc[a + b*x])]
 - (6*I)*ArcCsc[a + b*x]*PolyLog[2, -E^(I*ArcCsc[a + b*x])] + (6*I)*ArcCsc[
a + b*x]*PolyLog[2, E^(I*ArcCsc[a + b*x])] + 6*PolyLog[3, -E^(I*ArcCsc[a +
b*x])] - 6*PolyLog[3, E^(I*ArcCsc[a + b*x])])/b
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{\operatorname{arccsc}(bx+a)^3(bx+a)-3\operatorname{arccsc}(bx+a)^2\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)+6i\operatorname{arccsc}(bx+a)\operatorname{polylog}\left(2,\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right.}{}$
default	$\frac{\operatorname{arccsc}(bx+a)^3(bx+a)-3\operatorname{arccsc}(bx+a)^2\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)+6i\operatorname{arccsc}(bx+a)\operatorname{polylog}\left(2,\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right.}{}$

```
[In] int(arccsc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arccsc(b*x+a)^3*(b*x+a)-3*arccsc(b*x+a)^2*ln(1-I/(b*x+a)-(1-1/(b*x+a)^
2)^(1/2))+6*I*arccsc(b*x+a)*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))-6*po
lylog(3,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+3*arccsc(b*x+a)^2*ln(1+I/(b*x+a)+(
1-1/(b*x+a)^2)^(1/2))-6*I*arccsc(b*x+a)*polylog(2,-I/(b*x+a)-(1-1/(b*x+a)^2
)^(1/2))+6*polylog(3,-I/(b*x+a)-(1-1/(b*x+a)^2)^(1/2)))
```

Fricas [F]

$$\int \csc^{-1}(a+bx)^3\,dx=\int \operatorname{arccsc}\left(bx+a\right)^3\,dx$$

[In] integrate(arccsc(b*x+a)^3,x, algorithm="fricas")

[Out] integral(arccsc(b*x + a)^3, x)

Sympy [F]

$$\int \csc^{-1}(a+bx)^3\,dx=\int \operatorname{acsc}^3\left(a+bx\right)dx$$

[In] integrate(acsc(b*x+a)**3,x)

[Out] Integral(acsc(a + b*x)**3, x)

Maxima [F]

$$\int \csc^{-1}(a+bx)^3\,dx=\int \operatorname{arccsc}\left(bx+a\right)^3\,dx$$

[In] integrate(arccsc(b*x+a)^3,x, algorithm="maxima")

[Out] x*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3/4*x*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integrate(3/4*(4*(b^3*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3*a*b^2*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + a^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (3*a^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x - a*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*log(b*x + a)^2 - (4*b*x*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x*log(b^2*x^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - 4*(b^3*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 2*a*b^2*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (a^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x + (b^3*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3*a*b^2*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + a^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (3*a^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x - a*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)

Giac [F]

$$\int \csc^{-1}(a+bx)^3\,dx = \int \operatorname{arccsc}\left(bx+a\right)^3\,dx$$

```
[In] integrate(arccsc(b*x+a)^3,x, algorithm="giac")
[Out] integrate(arccsc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^{-1}(a+bx)^3\,dx = \int \operatorname{asin}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

```
[In] int(asin(1/(a + b*x))^3,x)
[Out] int(asin(1/(a + b*x))^3, x)
```

3.36 $\int \frac{\csc^{-1}(a+bx)^3}{x}\,dx$

Optimal result 256
Rubi [A] (verified) 257
Mathematica [A] (verified) 265
Maple [F] 266
Fricas [F] 266
Sympy [F] 266
Maxima [F] 267
Giac [F] 267
Mupad [F(-1)] 267

Optimal result

Integrand size = 12, antiderivative size = 448

$$\begin{aligned}\int \frac{\csc^{-1}(a+bx)^3}{x}\,dx &= \csc^{-1}(a+bx)^3\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\ &\quad+\csc^{-1}(a+bx)^3\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\ &\quad-\csc^{-1}(a+bx)^3\log\left(1-e^{2i\csc^{-1}(a+bx)}\right) \\ &\quad-3i\csc^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\ &\quad-3i\csc^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\ &\quad+\frac{3}{2}i\csc^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right) \\ &\quad+6\csc^{-1}(a+bx)\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\ &\quad+6\csc^{-1}(a+bx)\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\ &\quad-\frac{3}{2}\csc^{-1}(a+bx)\operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(a+bx)}\right) \\ &\quad+6i\operatorname{PolyLog}\left(4,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\ &\quad+6i\operatorname{PolyLog}\left(4,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)-\frac{3}{4}i\operatorname{PolyLog}\left(4,e^{2i\csc^{-1}(a+bx)}\right)\end{aligned}$$

```
[Out] -arccsc(b*x+a)^3*ln(1-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)+arccsc(b*x+a)^3*
ln(1+I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arccsc(b*x+a
)^3*ln(1+I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+3/2*I*ar
ccsc(b*x+a)^2*polylog(2,(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)-3*I*arccsc(b*x
+a)^2*polylog(2,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))-
3*I*arccsc(b*x+a)^2*polylog(2,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a
^2+1)^(1/2)))-3/2*arccsc(b*x+a)*polylog(3,(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))
^2)+6*arccsc(b*x+a)*polylog(3,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a
^2+1)^(1/2)))+6*arccsc(b*x+a)*polylog(3,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/
2))/(1+(-a^2+1)^(1/2)))-3/4*I*polylog(4,(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2
)+6*I*polylog(4,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+
6*I*polylog(4,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules

used = {5367, 4648, 4625, 3798, 2221, 2611, 6744, 2320, 6724, 4615}

$$\begin{aligned}\int \frac{\csc^{-1}(a+bx)^3}{x}\,dx = &-3i\csc^{-1}(a+bx)^2\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&-3i\csc^{-1}(a+bx)^2\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)\\
&+6\csc^{-1}(a+bx)\,\text{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&+6\csc^{-1}(a+bx)\,\text{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)\\
&+6i\,\text{PolyLog}\left(4,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&+6i\,\text{PolyLog}\left(4,-\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)\\
&+\csc^{-1}(a+bx)^3\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&+\csc^{-1}(a+bx)^3\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)\\
&+\frac{3}{2}i\csc^{-1}(a+bx)^2\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)\\
&-\frac{3}{2}\csc^{-1}(a+bx)\,\text{PolyLog}\left(3,e^{2i\csc^{-1}(a+bx)}\right)\\
&-\frac{3}{4}i\,\text{PolyLog}\left(4,e^{2i\csc^{-1}(a+bx)}\right)\\
&-\csc^{-1}(a+bx)^3\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)\end{aligned}$$

```
[In] Int[ArcCsc[a + b*x]^3/x,x]
```

```
[Out] ArcCsc[a + b*x]^3*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])]
+ ArcCsc[a + b*x]^3*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])
] - ArcCsc[a + b*x]^3*Log[1 - E^((2*I)*ArcCsc[a + b*x])] - (3*I)*ArcCsc[a +
 b*x]^2*PolyLog[2, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])] - (3
*I)*ArcCsc[a + b*x]^2*PolyLog[2, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1
 - a^2])] + ((3*I)/2)*ArcCsc[a + b*x]^2*PolyLog[2, E^((2*I)*ArcCsc[a + b*x]
)] + 6*ArcCsc[a + b*x]*PolyLog[3, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[
1 - a^2])] + 6*ArcCsc[a + b*x]*PolyLog[3, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1
 + Sqrt[1 - a^2])] - (3*ArcCsc[a + b*x]*PolyLog[3, E^((2*I)*ArcCsc[a + b*x]
)])/2 + (6*I)*PolyLog[4, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])
] + (6*I)*PolyLog[4, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] -
((3*I)/4)*PolyLog[4, E^((2*I)*ArcCsc[a + b*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4625

```
Int[(Cot[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cot[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cos[c + d*x]*(Cot[c + d*x]^(n -
1)/(a + b*Sin[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
```

```
0] && IGtQ[n, 0]
```

Rule 4648

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/(Csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> In
t[(e + f*x)^m*Sin[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Sin[c + d*x]))
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^3\cot(x)\csc(x)}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right)\\
&= -\text{Subst}\left(\int \frac{x^3\cot(x)}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)\\
&= -\left(a\text{Subst}\left(\int \frac{x^3\cos(x)}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)\right)\\
&\quad -\text{Subst}\left(\int x^3\cot(x)\,dx, x, \csc^{-1}(a+bx)\right)
\end{aligned}
$$

$$= 2i\text{Subst}\left(\int \frac{e^{2ix}x^3}{1-e^{2ix}}\,dx, x, \csc^{-1}(a+bx)\right)$$
$$- a\text{Subst}\left(\int \frac{e^{ix}x^3}{1-\sqrt{1-a^2}+iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)$$
$$- a\text{Subst}\left(\int \frac{e^{ix}x^3}{1+\sqrt{1-a^2}+iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)$$

$$= \csc^{-1}(a+bx)^3\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)^3\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$- \csc^{-1}(a+bx)^3\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)$$
$$- 3\text{Subst}\left(\int x^2\log\left(1+\frac{iae^{ix}}{1-\sqrt{1-a^2}}\right)\,dx, x, \csc^{-1}(a+bx)\right)$$
$$- 3\text{Subst}\left(\int x^2\log\left(1+\frac{iae^{ix}}{1+\sqrt{1-a^2}}\right)\,dx, x, \csc^{-1}(a+bx)\right)$$
$$+ 3\text{Subst}\left(\int x^2\log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(a+bx)\right)$$

$$= \csc^{-1}(a+bx)^3\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)^3\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$- \csc^{-1}(a+bx)^3\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)$$
$$- 3i\csc^{-1}(a+bx)^2\,\text{PolyLog}\left(2, -\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$
$$- 3i\csc^{-1}(a+bx)^2\,\text{PolyLog}\left(2, -\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$+ \frac{3}{2}i\csc^{-1}(a+bx)^2\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(a+bx)}\right)$$
$$- 3i\text{Subst}\left(\int x\,\text{PolyLog}\left(2, e^{2ix}\right)\,dx, x, \csc^{-1}(a+bx)\right)$$
$$+ 6i\text{Subst}\left(\int x\,\text{PolyLog}\left(2, -\frac{iae^{ix}}{1-\sqrt{1-a^2}}\right)\,dx, x, \csc^{-1}(a+bx)\right)$$
$$+ 6i\text{Subst}\left(\int x\,\text{PolyLog}\left(2, -\frac{iae^{ix}}{1+\sqrt{1-a^2}}\right)\,dx, x, \csc^{-1}(a+bx)\right)$$

$$
\begin{aligned}
&= \csc^{-1}(a+bx)^3 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)^3 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\
&\quad - \csc^{-1}(a+bx)^3 \log\left(1-e^{2i\csc^{-1}(a+bx)}\right) \\
&\quad - 3i\csc^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\
&\quad - 3i\csc^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\
&\quad + \frac{3}{2}i\csc^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right) \\
&\quad + 6\csc^{-1}(a+bx) \operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\
&\quad + 6\csc^{-1}(a+bx) \operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\
&\quad - \frac{3}{2}\csc^{-1}(a+bx) \operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(a+bx)}\right) \\
&\quad + \frac{3}{2}\text{Subst}\left(\int \operatorname{PolyLog}\left(3,e^{2ix}\right)\,dx, x, \csc^{-1}(a+bx)\right) \\
&\quad - 6\text{Subst}\left(\int \operatorname{PolyLog}\left(3,-\frac{iae^{ix}}{1-\sqrt{1-a^2}}\right)\,dx, x, \csc^{-1}(a+bx)\right) \\
&\quad - 6\text{Subst}\left(\int \operatorname{PolyLog}\left(3,-\frac{iae^{ix}}{1+\sqrt{1-a^2}}\right)\,dx, x, \csc^{-1}(a+bx)\right)
\end{aligned}
$$

$$= \csc^{-1}(a+bx)^3 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \csc^{-1}(a+bx)^3 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$- \csc^{-1}(a+bx)^3 \log\left(1-e^{2i\csc^{-1}(a+bx)}\right)$$
$$- 3i\csc^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$
$$- 3i\csc^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$+ \frac{3}{2}i\csc^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)$$
$$+ 6\csc^{-1}(a+bx) \operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$
$$+ 6\csc^{-1}(a+bx) \operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$- \frac{3}{2}\csc^{-1}(a+bx) \operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(a+bx)}\right)$$
$$- \frac{3}{4}i\text{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\,dx, x, e^{2i\csc^{-1}(a+bx)}\right)$$
$$+ 6i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,\frac{iax}{-1+\sqrt{1-a^2}}\right)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)$$
$$+ 6i\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,-\frac{iax}{1+\sqrt{1-a^2}}\right)}{x}\,dx, x, e^{i\csc^{-1}(a+bx)}\right)$$

$$
\begin{aligned}
&= \csc^{-1}(a+bx)^3 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\csc^{-1}(a+bx)^3 \log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\
&\quad - \csc^{-1}(a+bx)^3 \log\left(1-e^{2i\csc^{-1}(a+bx)}\right) \\
&\quad - 3i\csc^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\
&\quad - 3i\csc^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\
&\quad + \frac{3}{2}i\csc^{-1}(a+bx)^2 \operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right) \\
&\quad + 6\csc^{-1}(a+bx) \operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\
&\quad + 6\csc^{-1}(a+bx) \operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\
&\quad - \frac{3}{2}\csc^{-1}(a+bx) \operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(a+bx)}\right)+6i\operatorname{PolyLog}\left(4,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) \\
&\quad + 6i\operatorname{PolyLog}\left(4,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)-\frac{3}{4}i\operatorname{PolyLog}\left(4,e^{2i\csc^{-1}(a+bx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.24

$$\int \frac{\csc^{-1}(a+bx)^3}{x}\,dx = \frac{i\pi^4}{8} - \frac{1}{4}i\csc^{-1}(a+bx)^4 - \csc^{-1}(a+bx)^3\log\left(1-e^{-i\csc^{-1}(a+bx)}\right)$$
$$-\csc^{-1}(a+bx)^3\log\left(1+e^{i\csc^{-1}(a+bx)}\right)$$
$$+\csc^{-1}(a+bx)^3\log\left(1-\frac{iae^{i\csc^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)$$
$$+\csc^{-1}(a+bx)^3\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$-3i\csc^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,e^{-i\csc^{-1}(a+bx)}\right)$$
$$+3i\csc^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(a+bx)}\right)$$
$$-3i\csc^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,\frac{iae^{i\csc^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)$$
$$-3i\csc^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$-6\csc^{-1}(a+bx)\operatorname{PolyLog}\left(3,e^{-i\csc^{-1}(a+bx)}\right)$$
$$-6\csc^{-1}(a+bx)\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(a+bx)}\right)$$
$$+6\csc^{-1}(a+bx)\operatorname{PolyLog}\left(3,\frac{iae^{i\csc^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)$$
$$+6\csc^{-1}(a+bx)\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$
$$+6i\operatorname{PolyLog}\left(4,e^{-i\csc^{-1}(a+bx)}\right)-6i\operatorname{PolyLog}\left(4,-e^{i\csc^{-1}(a+bx)}\right)$$
$$+6i\operatorname{PolyLog}\left(4,\frac{iae^{i\csc^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)$$
$$+6i\operatorname{PolyLog}\left(4,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

```
[In] Integrate[ArcCsc[a + b*x]^3/x,x]
```

```
[Out] (I/8)*Pi^4 - (I/4)*ArcCsc[a + b*x]^4 - ArcCsc[a + b*x]^3*Log[1 - E^((-I)*Ar
cCsc[a + b*x])] - ArcCsc[a + b*x]^3*Log[1 + E^(I*ArcCsc[a + b*x])] + ArcCsc
[a + b*x]^3*Log[1 - (I*a*E^(I*ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] + Arc
Csc[a + b*x]^3*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] - (
```

```
3*I)*ArcCsc[a + b*x]^2*PolyLog[2, E^((-I)*ArcCsc[a + b*x])] + (3*I)*ArcCsc[
a + b*x]^2*PolyLog[2, -E^(I*ArcCsc[a + b*x])] - (3*I)*ArcCsc[a + b*x]^2*Pol
yLog[2, (I*a*E^(I*ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] - (3*I)*ArcCsc[a
+ b*x]^2*PolyLog[2, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] - 6
*ArcCsc[a + b*x]*PolyLog[3, E^((-I)*ArcCsc[a + b*x])] - 6*ArcCsc[a + b*x]*P
olyLog[3, -E^(I*ArcCsc[a + b*x])] + 6*ArcCsc[a + b*x]*PolyLog[3, (I*a*E^(I*
ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] + 6*ArcCsc[a + b*x]*PolyLog[3, ((-I
)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] + (6*I)*PolyLog[4, E^((-I)*
ArcCsc[a + b*x])] - (6*I)*PolyLog[4, -E^(I*ArcCsc[a + b*x])] + (6*I)*PolyLo
g[4, (I*a*E^(I*ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] + (6*I)*PolyLog[4, (
(-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])]
```

Maple [F]

$$\int \frac{\operatorname{arccsc}(bx+a)^3}{x}dx$$

```
[In] int(arccsc(b*x+a)^3/x,x)
[Out] int(arccsc(b*x+a)^3/x,x)
```

Fricas [F]

$$\int \frac{\csc^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arccsc}(bx+a)^3}{x}\,dx$$

```
[In] integrate(arccsc(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(arccsc(b*x + a)^3/x, x)
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{acsc}^3(a+bx)}{x}\,dx$$

```
[In] integrate(acsc(b*x+a)**3/x,x)
[Out] Integral(acsc(a + b*x)**3/x, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arccsc}\left(bx+a\right)^3}{x}\,dx$$

```
[In] integrate(arccsc(b*x+a)^3/x,x, algorithm="maxima")
[Out] integrate(arccsc(b*x + a)^3/x, x)
```

Giac [F]

$$\int \frac{\csc^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arccsc}\left(bx+a\right)^3}{x}\,dx$$

```
[In] integrate(arccsc(b*x+a)^3/x,x, algorithm="giac")
[Out] integrate(arccsc(b*x + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{a+b\,x}\right)^3}{x}\,dx$$

```
[In] int(asin(1/(a + b*x))^3/x,x)
[Out] int(asin(1/(a + b*x))^3/x, x)
```

3.37 $\int \frac{\csc^{-1}(a+bx)^3}{x^2} dx$

Optimal result . . . 268
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Maple [F] . . . 273
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Sympy [F] . . . 274
Maxima [F] . . . 274
Giac [F] . . . 275
Mupad [F(-1)] . . . 275

Optimal result

Integrand size = 12, antiderivative size = 378

$$\begin{aligned}\int \frac{\csc^{-1}(a+bx)^3}{x^2} dx = & -\frac{b\csc^{-1}(a+bx)^3}{a}-\frac{\csc^{-1}(a+bx)^3}{x} \\ & -\frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\ & +\frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\ & -\frac{6b\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\ & +\frac{6b\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\ & -\frac{6ib\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{6ib\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\end{aligned}$$

```
[Out] -b*arccsc(b*x+a)^3/a-arccsc(b*x+a)^3/x-3*I*b*arccsc(b*x+a)^2*ln(1+I*a*(I/(b
*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+3*I*b*arc
csc(b*x+a)^2*ln(1+I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
/a/(-a^2+1)^(1/2)-6*b*arccsc(b*x+a)*polylog(2,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^
2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+6*b*arccsc(b*x+a)*polylog(2,
-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
-6*I*b*polylog(3,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))
/a/(-a^2+1)^(1/2)+6*I*b*polylog(3,-I*a*(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))/(1
+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5367, 4512, 4276, 3404, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{\csc^{-1}(a+bx)^3}{x^2}\,dx = -\frac{6b\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{6b\csc^{-1}(a+bx)\,\text{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}} - \frac{6ib\,\text{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{6ib\,\text{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}} - \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}} - \frac{b\csc^{-1}(a+bx)^3}{a} - \frac{\csc^{-1}(a+bx)^3}{x}$$

```
[In] Int[ArcCsc[a + b*x]^3/x^2,x]
```

```
[Out] -((b*ArcCsc[a + b*x]^3)/a) - ArcCsc[a + b*x]^3/x - ((3*I)*b*ArcCsc[a + b*x]
^2*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^
2]) + ((3*I)*b*ArcCsc[a + b*x]^2*Log[1 + (I*a*E^(I*ArcCsc[a + b*x]))/(1 + S
qrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*ArcCsc[a + b*x]*PolyLog[2, ((-I)*a
*E^(I*ArcCsc[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (6*b*ArcC
sc[a + b*x]*PolyLog[2, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])])
/(a*Sqrt[1 - a^2]) - ((6*I)*b*PolyLog[3, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1
- Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + ((6*I)*b*PolyLog[3, ((-I)*a*E^(I*Arc
Csc[a + b*x]))/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4512

```
Int[Cot[(c_.) + (d_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)]*(Csc[(c_.) + (d_.)*(x_)
]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e +
f*x)^m)*((a + b*Csc[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(
n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Csc[c + d*x])^(n + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5367

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csc[x]*Cot[
```

```
x]*(d*e - c*f + f*Csc[x])^m, x], x, ArcCsc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(b\text{Subst}\left(\int \frac{x^3\cot(x)\csc(x)}{(-a+\csc(x))^2}\,dx, x, \csc^{-1}(a+bx)\right)\right) \\
&= -\frac{\csc^{-1}(a+bx)^3}{x} + (3b)\text{Subst}\left(\int \frac{x^2}{-a+\csc(x)}\,dx, x, \csc^{-1}(a+bx)\right) \\
&= -\frac{\csc^{-1}(a+bx)^3}{x} + (3b)\text{Subst}\left(\int \left(-\frac{x^2}{a} + \frac{x^2}{a(1-a\sin(x))}\right)dx, x, \csc^{-1}(a+bx)\right) \\
&= -\frac{b\csc^{-1}(a+bx)^3}{a} - \frac{\csc^{-1}(a+bx)^3}{x} + \frac{(3b)\text{Subst}\left(\int \frac{x^2}{1-a\sin(x)}\,dx, x, \csc^{-1}(a+bx)\right)}{a} \\
&= -\frac{b\csc^{-1}(a+bx)^3}{a} - \frac{\csc^{-1}(a+bx)^3}{x} + \frac{(6b)\text{Subst}\left(\int \frac{e^{ix}x^2}{-ia+2e^{ix}+iae^{2ix}}\,dx, x, \csc^{-1}(a+bx)\right)}{a} \\
&= -\frac{b\csc^{-1}(a+bx)^3}{a} - \frac{\csc^{-1}(a+bx)^3}{x} \\
&\quad + \frac{(6ib)\text{Subst}\left(\int \frac{e^{ix}x^2}{2-2\sqrt{1-a^2}+2iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}} \\
&\quad - \frac{(6ib)\text{Subst}\left(\int \frac{e^{ix}x^2}{2+2\sqrt{1-a^2}+2iae^{ix}}\,dx, x, \csc^{-1}(a+bx)\right)}{\sqrt{1-a^2}} \\
&= -\frac{b\csc^{-1}(a+bx)^3}{a} - \frac{\csc^{-1}(a+bx)^3}{x} - \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\
&\quad + \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\
&\quad + \frac{(6ib)\text{Subst}\left(\int x\log\left(1+\frac{2iae^{ix}}{2-2\sqrt{1-a^2}}\right)dx, x, \csc^{-1}(a+bx)\right)}{a\sqrt{1-a^2}} \\
&\quad - \frac{(6ib)\text{Subst}\left(\int x\log\left(1+\frac{2iae^{ix}}{2+2\sqrt{1-a^2}}\right)dx, x, \csc^{-1}(a+bx)\right)}{a\sqrt{1-a^2}}
\end{aligned}$$

$$= -\frac{b\csc^{-1}(a+bx)^3}{a} - \frac{\csc^{-1}(a+bx)^3}{x} - \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$+ \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$- \frac{6b\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$+ \frac{6b\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$+ \frac{(6b)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2iae^{ix}}{2-2\sqrt{1-a^2}}\right)dx,x,\csc^{-1}(a+bx)\right)}{a\sqrt{1-a^2}}$$
$$- \frac{(6b)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2iae^{ix}}{2+2\sqrt{1-a^2}}\right)dx,x,\csc^{-1}(a+bx)\right)}{a\sqrt{1-a^2}}$$

$$= -\frac{b\csc^{-1}(a+bx)^3}{a} - \frac{\csc^{-1}(a+bx)^3}{x} - \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$+ \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$- \frac{6b\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$+ \frac{6b\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$- \frac{(6ib)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{iax}{-1+\sqrt{1-a^2}}\right)}{x}dx,x,e^{i\csc^{-1}(a+bx)}\right)}{a\sqrt{1-a^2}}$$
$$+ \frac{(6ib)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{iax}{1+\sqrt{1-a^2}}\right)}{x}dx,x,e^{i\csc^{-1}(a+bx)}\right)}{a\sqrt{1-a^2}}$$

$$= -\frac{b\csc^{-1}(a+bx)^3}{a} - \frac{\csc^{-1}(a+bx)^3}{x} - \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$+ \frac{3ib\csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$- \frac{6b\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$+ \frac{6b\csc^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$
$$- \frac{6ib\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{6ib\operatorname{PolyLog}\left(3,-\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.76

$$\int \frac{\csc^{-1}(a+bx)^3}{x^2}\,dx =$$
$$-\frac{\frac{(a+bx)\csc^{-1}(a+bx)^3}{x} + \frac{3ib\left(\csc^{-1}(a+bx)^2\log\left(1-\frac{iae^{i\csc^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) - \csc^{-1}(a+bx)^2\log\left(1+\frac{iae^{i\csc^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - 2i\csc^{-1}(a+bx)\operatorname{PolyLog}(\ldots}{\ldots}}{a}$$

```
[In] Integrate[ArcCsc[a + b*x]^3/x^2,x]
```

```
[Out] -((((a + b*x)*ArcCsc[a + b*x]^3)/x + ((3*I)*b*(ArcCsc[a + b*x]^2*Log[1 - (I
*a*E^(I*ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] - ArcCsc[a + b*x]^2*Log[1 +
 (I*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] - (2*I)*ArcCsc[a + b*x]*P
olyLog[2, (I*a*E^(I*ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] + (2*I)*ArcCsc[
a + b*x]*PolyLog[2, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])] + 2
*PolyLog[3, (I*a*E^(I*ArcCsc[a + b*x]))/(-1 + Sqrt[1 - a^2])] - 2*PolyLog[3
, ((-I)*a*E^(I*ArcCsc[a + b*x]))/(1 + Sqrt[1 - a^2])]))/Sqrt[1 - a^2])/a)
```

Maple [F]

$$\int \frac{\operatorname{arccsc}(bx+a)^3}{x^2}dx$$

```
[In] int(arccsc(b*x+a)^3/x^2,x)
```

```
[Out] int(arccsc(b*x+a)^3/x^2,x)
```

Fricas [F]

$$\int \frac{\csc^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arccsc}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(arccsc(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(arccsc(b*x + a)^3/x^2, x)
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{acsc}^3\left(a+bx\right)}{x^2}\,dx$$

```
[In] integrate(acsc(b*x+a)**3/x**2,x)
[Out] Integral(acsc(a + b*x)**3/x**2, x)
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arccsc}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(arccsc(b*x+a)^3/x^2,x, algorithm="maxima")
[Out] -1/4*(4*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3*arctan2(1, sq
rt(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - 4*x*int
egrate(-3/4*(4*(b^3*x^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3
*a*b^2*x^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + a^3*arctan2(1,
 sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (3*a^2*arctan2(1, sqrt(b*x + a + 1)
*sqrt(b*x + a - 1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x
- a*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*log(b*x + a)^2 + (4*b*
x*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x*log(b^2*x^2 + 2*a
*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*(b^3*x^3*arctan2(1,
sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 2*a*b^2*x^2*arctan2(1, sqrt(b*x + a
+ 1)*sqrt(b*x + a - 1)) + (a^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a -
1)) - arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x - (b^3*x^3*arcta
n2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3*a*b^2*x^2*arctan2(1, sqrt(b*
x + a + 1)*sqrt(b*x + a - 1)) + a^3*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x +
 a - 1)) + (3*a^2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - arctan2
(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))*b*x - a*arctan2(1, sqrt(b*x + a +
 1)*sqrt(b*x + a - 1)))*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^
5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2), x))/x
```

Giac [F]

$$\int \frac{\csc^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arccsc}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(arccsc(b*x+a)^3/x^2,x, algorithm="giac")
[Out] integrate(arccsc(b*x + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{a+b\,x}\right)^3}{x^2}\,dx$$

```
[In] int(asin(1/(a + b*x))^3/x^2,x)
[Out] int(asin(1/(a + b*x))^3/x^2, x)
```

3.38 $\int x^3 \csc^{-1}(a+bx^4)\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 48

$$\int x^3 \csc^{-1}(a+bx^4)\,dx = \frac{(a+bx^4)\csc^{-1}(a+bx^4)}{4b} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx^4)^2}}\right)}{4b}$$

```
[Out] 1/4*(b*x^4+a)*arccsc(b*x^4+a)/b+1/4*arctanh((1-1/(b*x^4+a)^2)^(1/2))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6847, 5359, 379, 272, 65, 212}

$$\int x^3 \csc^{-1}(a+bx^4)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx^4)^2}}\right)}{4b} + \frac{(a+bx^4)\csc^{-1}(a+bx^4)}{4b}$$

```
[In] Int[x^3*ArcCsc[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcCsc[a + b*x^4])/(4*b) + ArcTanh[Sqrt[1 - (a + b*x^4)^(-2)]]
/(4*b)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5359

```
Int[ArcCsc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcCsc[c + d*x]
/d), x] + Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \csc^{-1}(a+bx)\,dx, x, x^4\right)\\
&= \frac{(a+bx^4)\csc^{-1}(a+bx^4)}{4b} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\,dx, x, x^4\right)\\
&= \frac{(a+bx^4)\csc^{-1}(a+bx^4)}{4b} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx, x, a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\csc^{-1}(a+bx^4)}{4b} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{(a+bx^4)^2}\right)}{8b}\\
&= \frac{(a+bx^4)\csc^{-1}(a+bx^4)}{4b} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{(a+bx^4)^2}}\right)}{4b}
\end{aligned}
$$

$$= \frac{(a+bx^4)\csc^{-1}(a+bx^4)}{4b} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx^4)^2}}\right)}{4b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. 2(48) = 96.

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.65

$$\int x^3 \csc^{-1}\left(a+bx^4\right) dx$$
$$= \frac{(a+bx^4)\csc^{-1}(a+bx^4)}{4b} + \frac{\sqrt{-1+(a+bx^4)^2}\left(-\log\left(1-\frac{a+bx^4}{\sqrt{-1+(a+bx^4)^2}}\right)+\log\left(1+\frac{a+bx^4}{\sqrt{-1+(a+bx^4)^2}}\right)\right)}{8b(a+bx^4)\sqrt{1-\frac{1}{(a+bx^4)^2}}}$$

```
[In] Integrate[x^3*ArcCsc[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcCsc[a + b*x^4])/(4*b) + (Sqrt[-1 + (a + b*x^4)^2]*(-Log[1 -
 (a + b*x^4)/Sqrt[-1 + (a + b*x^4)^2]] + Log[1 + (a + b*x^4)/Sqrt[-1 + (a +
 b*x^4)^2]]))/(8*b*(a + b*x^4)*Sqrt[1 - (a + b*x^4)^(-2)])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{\operatorname{arccsc}(b\,x^4+a)\,(b\,x^4+a)+\ln\left(b\,x^4+a+(b\,x^4+a)\,\sqrt{1-\frac{1}{(b\,x^4+a)^2}}\right)}{4b}$	54
default	$\frac{\operatorname{arccsc}(b\,x^4+a)\,(b\,x^4+a)+\ln\left(b\,x^4+a+(b\,x^4+a)\,\sqrt{1-\frac{1}{(b\,x^4+a)^2}}\right)}{4b}$	54

```
[In] int(x^3*arccsc(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*(arccsc(b*x^4+a)*(b*x^4+a)+ln(b*x^4+a+(b*x^4+a)*(1-1/(b*x^4+a)^2)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(42) = 84.

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.83

$$\int x^3 \csc^{-1}(a+bx^4)\,dx = \frac{bx^4 \operatorname{arccsc}(bx^4+a) - 2\,a \arctan\left(-bx^4 - a + \sqrt{b^2x^8 + 2\,abx^4 + a^2 - 1}\right) - \log\left(-bx^4 - a + \sqrt{b^2x^8 + 2\,ab\right.}{4\,b}$$

```
[In] integrate(x^3*arccsc(b*x^4+a),x, algorithm="fricas")
[Out] 1/4*(b*x^4*arccsc(b*x^4 + a) - 2*a*arctan(-b*x^4 - a + sqrt(b^2*x^8 + 2*a*b
*x^4 + a^2 - 1)) - log(-b*x^4 - a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)))/b
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \csc^{-1}(a+bx^4)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*acsc(b*x**4+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int x^3 \csc^{-1}(a+bx^4)\,dx = \frac{2\,(bx^4+a) \operatorname{arccsc}(bx^4+a) + \log\left(\sqrt{-\frac{1}{(bx^4+a)^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{(bx^4+a)^2}+1}+1\right)}{8\,b}$$

```
[In] integrate(x^3*arccsc(b*x^4+a),x, algorithm="maxima")
[Out] 1/8*(2*(b*x^4 + a)*arccsc(b*x^4 + a) + log(sqrt(-1/(b*x^4 + a)^2 + 1) + 1)
- log(-sqrt(-1/(b*x^4 + a)^2 + 1) + 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int x^3 \csc^{-1}\left(a+bx^4\right)\,dx = \frac{1}{8}\,b\left(\frac{2\left(bx^4+a\right)\arcsin\left(-\frac{1}{\left(bx^4+a\right)\left(\frac{a}{bx^4+a}-1\right)-a}\right)}{b^2}+\frac{\log\left(\sqrt{-\frac{1}{\left(bx^4+a\right)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{\left(bx^4+a\right)^2}+1}+1\right)}{b^2}\right.$$

```
[In] integrate(x^3*arccsc(b*x^4+a),x, algorithm="giac")
[Out] 1/8*b*(2*(b*x^4 + a)*arcsin(-1/((b*x^4 + a)*(a/(b*x^4 + a) - 1) - a))/b^2 +
 (log(sqrt(-1/(b*x^4 + a)^2 + 1) + 1) - log(-sqrt(-1/(b*x^4 + a)^2 + 1) + 1
))/b^2)
```

Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int x^3 \csc^{-1}\left(a+bx^4\right)\,dx = \frac{\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{\left(b\,x^4+a\right)^2}}}\right)}{4\,b}+\frac{\operatorname{asin}\left(\frac{1}{b\,x^4+a}\right)\,\left(b\,x^4+a\right)}{4\,b}$$

```
[In] int(x^3*asin(1/(a + b*x^4)),x)
[Out] atanh(1/(1 - 1/(a + b*x^4)^2)^(1/2))/(4*b) + (asin(1/(a + b*x^4))*(a + b*x^
4))/(4*b)
```

3.39 $\int x^{-1+n} \csc^{-1}(a+bx^n) \, dx$

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Rubi [A] (verified) 281
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Optimal result

Integrand size = 14, antiderivative size = 48

$$\int x^{-1+n} \csc^{-1}(a+bx^n) \, dx = \frac{(a+bx^n)\csc^{-1}(a+bx^n)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

```
[Out] (a+b*x^n)*arccsc(a+b*x^n)/b/n+arctanh((1-1/(a+b*x^n)^2)^(1/2))/b/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6847, 5359, 379, 272, 65, 212}

$$\int x^{-1+n} \csc^{-1}(a+bx^n) \, dx = \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn} + \frac{(a+bx^n)\csc^{-1}(a+bx^n)}{bn}$$

```
[In] Int[x^(-1 + n)*ArcCsc[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcCsc[a + b*x^n])/(b*n) + ArcTanh[Sqrt[1 - (a + b*x^n)^(-2)]]
/(b*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5359

```
Int[ArcCsc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcCsc[c + d*x]
/d), x] + Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \csc^{-1}(a+bx)\,dx, x, x^n\right)}{n}\\
&= \frac{(a+bx^n)\csc^{-1}(a+bx^n)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\,dx, x, x^n\right)}{n}\\
&= \frac{(a+bx^n)\csc^{-1}(a+bx^n)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}x}\,dx, x, a+bx^n\right)}{bn}\\
&= \frac{(a+bx^n)\csc^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, \frac{1}{(a+bx^n)^2}\right)}{2bn}\\
&= \frac{(a+bx^n)\csc^{-1}(a+bx^n)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}
\end{aligned}
$$

$$= \frac{(a+bx^n)\csc^{-1}(a+bx^n)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 130 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.71

$$\int x^{-1+n}\csc^{-1}(a+bx^n)\,dx$$
$$= \frac{(a+bx^n)\csc^{-1}(a+bx^n)}{bn} + \frac{\sqrt{-1+(a+bx^n)^2}\left(-\log\left(1-\frac{a+bx^n}{\sqrt{-1+(a+bx^n)^2}}\right)+\log\left(1+\frac{a+bx^n}{\sqrt{-1+(a+bx^n)^2}}\right)\right)}{2bn(a+bx^n)\sqrt{1-\frac{1}{(a+bx^n)^2}}}$$

```
[In] Integrate[x^(-1 + n)*ArcCsc[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcCsc[a + b*x^n])/(b*n) + (Sqrt[-1 + (a + b*x^n)^2]*(-Log[1 -
 (a + b*x^n)/Sqrt[-1 + (a + b*x^n)^2]] + Log[1 + (a + b*x^n)/Sqrt[-1 + (a +
 b*x^n)^2]]))/(2*b*n*(a + b*x^n)*Sqrt[1 - (a + b*x^n)^(-2)])
```

Maple [F]

$$\int x^{-1+n}\operatorname{arccsc}(a+b\,x^n)\,dx$$

```
[In] int(x^(-1+n)*arccsc(a+b*x^n),x)
```

```
[Out] int(x^(-1+n)*arccsc(a+b*x^n),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(46) = 92$.

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int x^{-1+n}\csc^{-1}(a+bx^n)\,dx$$
$$= \frac{bx^n\operatorname{arccsc}(bx^n+a) - 2\,a\arctan\left(-bx^n-a+\sqrt{b^2x^{2n}+2\,abx^n+a^2-1}\right) - \log\left(-bx^n-a+\sqrt{b^2x^{2n}+2\right.}{bn}$$

```
[In] integrate(x^(-1+n)*arccsc(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (b*x^n*arccsc(b*x^n + a) - 2*a*arctan(-b*x^n - a + sqrt(b^2*x^(2*n) + 2*a*b
*x^n + a^2 - 1)) - log(-b*x^n - a + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)
))/(b*n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n} \csc^{-1}(a + bx^n) \, dx = \text{Timed out}$$

```
[In] integrate(x**(-1+n)*acsc(a+b*x**n),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int x^{-1+n} \csc^{-1}(a + bx^n) \, dx = \frac{2(bx^n + a)\,\text{arccsc}(bx^n + a) + \log\left(\sqrt{-\frac{1}{(bx^n+a)^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{(bx^n+a)^2} + 1} + 1\right)}{2\,bn}$$

```
[In] integrate(x^(-1+n)*arccsc(a+b*x^n),x, algorithm="maxima")
[Out] 1/2*(2*(b*x^n + a)*arccsc(b*x^n + a) + log(sqrt(-1/(b*x^n + a)^2 + 1) + 1)
- log(-sqrt(-1/(b*x^n + a)^2 + 1) + 1))/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int x^{-1+n} \csc^{-1}(a + bx^n) \, dx = \frac{b\left(\frac{2(bx^n+a)\arcsin\left(\frac{1}{bx^n+a}\right)}{b^2} + \frac{\log\left(\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)}{b^2}\right)}{2\,n}$$

```
[In] integrate(x^(-1+n)*arccsc(a+b*x^n),x, algorithm="giac")
[Out] 1/2*b*(2*(b*x^n + a)*arcsin(1/(b*x^n + a))/b^2 + (log(sqrt(-1/(b*x^n + a)^2
 + 1) + 1) - log(-sqrt(-1/(b*x^n + a)^2 + 1) + 1))/b^2)/n
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int x^{-1+n}\csc^{-1}(a+bx^n)\,dx=\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{(a+b\,x^n)^2}}}\right)+\operatorname{asin}\left(\frac{1}{a+b\,x^n}\right)\,(a+b\,x^n)}{b\,n}$$

```
[In] int(x^(n - 1)*asin(1/(a + b*x^n)),x)
[Out] (atanh(1/(1 - 1/(a + b*x^n)^2)^(1/2)) + asin(1/(a + b*x^n))*(a + b*x^n))/(b
*n)
```

3.40 $\int \csc^{-1}\left(ce^{a+bx}\right)\,dx$

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Mathematica [B] (verified) 288
Maple [A] (verified) 289
Fricas [F(-2)] 289
Sympy [F] 290
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Giac [F] 290
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Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \csc^{-1}\left(ce^{a+bx}\right)\,dx=\frac{i\csc^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\csc^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2i\csc^{-1}\left(ce^{a+bx}\right)}\right)}{b}+\frac{i\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

```
[Out] 1/2*I*arccsc(c*exp(b*x+a))^2/b-arccsc(c*exp(b*x+a))*ln(1-(I/c/exp(b*x+a)+(1
-1/c^2/exp(b*x+a)^2)^(1/2))^2)/b+1/2*I*polylog(2,(I/c/exp(b*x+a)+(1-1/c^2/e
xp(b*x+a)^2)^(1/2))^2)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2320, 5327, 4721, 3798, 2221, 2317, 2438}

$$\int \csc^{-1}\left(ce^{a+bx}\right)\,dx=\frac{i\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}\left(ce^{a+bx}\right)}\right)}{2b}+\frac{i\csc^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\csc^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2i\csc^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

```
[In] Int[ArcCsc[c*E^(a + b*x)],x]
```

```
[Out] ((I/2)*ArcCsc[c*E^(a + b*x)]^2)/b - (ArcCsc[c*E^(a + b*x)]*Log[1 - E^((2*I)
*ArcCsc[c*E^(a + b*x)])])/b + ((I/2)*PolyLog[2, E^((2*I)*ArcCsc[c*E^(a + b*
x)])])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5327

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\csc^{-1}(cx)}{x}\,dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx, x, e^{-a-bx}\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int x\cot(x)\,dx, x, \arcsin\left(\frac{e^{-a-bx}}{c}\right)\right)}{b}$$
$$= \frac{i\arcsin\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} + \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \arcsin\left(\frac{e^{-a-bx}}{c}\right)\right)}{b}$$
$$= \frac{i\arcsin\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\arcsin\left(\frac{e^{-a-bx}}{c}\right)\log\left(1-e^{2i\arcsin\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx, x, \arcsin\left(\frac{e^{-a-bx}}{c}\right)\right)}{b}$$
$$= \frac{i\arcsin\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\arcsin\left(\frac{e^{-a-bx}}{c}\right)\log\left(1-e^{2i\arcsin\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} - \frac{i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\arcsin\left(\frac{e^{-a-bx}}{c}\right)}\right)}{2b}$$
$$= \frac{i\arcsin\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\arcsin\left(\frac{e^{-a-bx}}{c}\right)\log\left(1-e^{2i\arcsin\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{i\,\text{PolyLog}\left(2, e^{2i\arcsin\left(\frac{e^{-a-bx}}{c}\right)}\right)}{2b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(85) = 170.

Time = 0.42 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.29

$$\int \csc^{-1}\left(ce^{a+bx}\right)\,dx = x\csc^{-1}\left(ce^{a+bx}\right)$$
$$+ \frac{e^{-a-bx}\left(4\sqrt{-1+c^2e^{2(a+bx)}}\arctan\left(\sqrt{-1+c^2e^{2(a+bx)}}\right)\left(2bx-\log\left(c^2e^{2(a+bx)}\right)\right)+\sqrt{1-c^2e^{2(a+bx)}}\left(\log^2\left(c\right.\right.\right.}{}$$

[In] Integrate[ArcCsc[c*E^(a + b*x)],x]

[Out] x*ArcCsc[c*E^(a + b*x)] + (E^(-a - b*x)*(4*Sqrt[-1 + c^2*E^(2*(a + b*x))]*ArcTan[Sqrt[-1 + c^2*E^(2*(a + b*x))]]*(2*b*x - Log[c^2*E^(2*(a + b*x))]) + Sqrt[1 - c^2*E^(2*(a + b*x))]*(Log[c^2*E^(2*(a + b*x))]^2 - 4*Log[c^2*E^(2*(a + b*x))]*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2] + 2*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2]^2) - 4*Sqrt[1 - c^2*E^(2*(a + b*x))]*PolyLog[2, (1 - Sqrt[1 - c^2*E^(2*(a + b*x))])/2]))/(8*b*c*Sqrt[1 - 1/(c^2*E^(2*(a + b*x))]])

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.21

method	result
derivativedivides	$\frac{\frac{i\,\mathrm{arccsc}\left(\mathrm{e}^{bx+a}c\right)^2}{2}-\mathrm{arccsc}\left(\mathrm{e}^{bx+a}c\right)\ln\left(1-\frac{i\mathrm{e}^{-bx-a}}{c}-\sqrt{1-\frac{\mathrm{e}^{-2bx-2a}}{c^2}}\right)+i\,\mathrm{polylog}\left(2,\frac{i\mathrm{e}^{-bx-a}}{c}+\sqrt{1-\frac{\mathrm{e}^{-2bx-2a}}{c^2}}\right)-\mathrm{arccsc}}{b}$
default	$\frac{\frac{i\,\mathrm{arccsc}\left(\mathrm{e}^{bx+a}c\right)^2}{2}-\mathrm{arccsc}\left(\mathrm{e}^{bx+a}c\right)\ln\left(1-\frac{i\mathrm{e}^{-bx-a}}{c}-\sqrt{1-\frac{\mathrm{e}^{-2bx-2a}}{c^2}}\right)+i\,\mathrm{polylog}\left(2,\frac{i\mathrm{e}^{-bx-a}}{c}+\sqrt{1-\frac{\mathrm{e}^{-2bx-2a}}{c^2}}\right)-\mathrm{arccsc}}{b}$

[In] int(arccsc(exp(b*x+a)*c),x,method=_RETURNVERBOSE)

[Out] 1/b*(1/2*I*arccsc(exp(b*x+a)*c)^2-arccsc(exp(b*x+a)*c)*ln(1-I/exp(b*x+a)/c-(1-1/c^2/exp(b*x+a)^2)^(1/2))+I*polylog(2,I/c/exp(b*x+a)+(1-1/c^2/exp(b*x+a)^2)^(1/2))-arccsc(exp(b*x+a)*c)*ln(1+I/c/exp(b*x+a)+(1-1/c^2/exp(b*x+a)^2)^(1/2))+I*polylog(2,-I/exp(b*x+a)/c-(1-1/c^2/exp(b*x+a)^2)^(1/2)))

Fricas [F(-2)]

Exception generated.

$$\int \csc^{-1}\left(ce^{a+bx}\right)\,dx = \text{Exception raised: TypeError}$$

[In] integrate(arccsc(c*exp(b*x+a)),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

$$\int \csc^{-1}\left(ce^{a+bx}\right)\,dx = \int \operatorname{acsc}\left(ce^{a+bx}\right)dx$$

```
[In] integrate(acsc(c*exp(b*x+a)),x)
[Out] Integral(acsc(c*exp(a + b*x)), x)
```

Maxima [F]

$$\int \csc^{-1}\left(ce^{a+bx}\right)\,dx = \int \operatorname{arccsc}\left(ce^{(bx+a)}\right)\,dx$$

```
[In] integrate(arccsc(c*exp(b*x+a)),x, algorithm="maxima")
[Out] 1/2*(2*b^2*c^2*integrate(x*e^(2*b*x + 2*a + 1/2*log(c*e^(b*x + a) + 1) + 1/
2*log(c*e^(b*x + a) - 1))/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a) - 1)*
e^(log(c*e^(b*x + a) + 1) + log(c*e^(b*x + a) - 1)) - 1), x) - 2*I*b^2*c^2*
integrate(x*e^(2*b*x + 2*a)/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a) - 1
)*e^(log(c*e^(b*x + a) + 1) + log(c*e^(b*x + a) - 1)) - 1), x) + I*b^2*x^2
- I*b*x*log(c^2*e^(2*b*x + 2*a)) + I*b*x*log(c*e^(b*x + a) + 1) + I*b*x*log
(-c*e^(b*x + a) + 1) - 2*((-I*a - arctan2(1, sqrt(c*e^(b*x + a) + 1)*sqrt(c
*e^(b*x + a) - 1)))*b - I*b*log(c))*x + I*dilog(c*e^(b*x + a)) + I*dilog(-c
*e^(b*x + a)))/b
```

Giac [F]

$$\int \csc^{-1}\left(ce^{a+bx}\right)\,dx = \int \operatorname{arccsc}\left(ce^{(bx+a)}\right)\,dx$$

```
[In] integrate(arccsc(c*exp(b*x+a)),x, algorithm="giac")
[Out] integrate(arccsc(c*e^(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \csc^{-1}\left(ce^{a+bx}\right)\,dx=\frac{\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{\mathrm{e}^{-a-b\,x}}{c}\right)\,2\mathrm{i}}\right)\,1\mathrm{i}}{2\,b}+\frac{{\mathrm{asin}\left(\frac{\mathrm{e}^{-a-b\,x}}{c}\right)}^2\,1\mathrm{i}}{2\,b}-\frac{\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{\mathrm{e}^{-a-b\,x}}{c}\right)\,2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{\mathrm{e}^{-a-b\,x}}{c}\right)}{b}$$

```
[In] int(asin(exp(- a - b*x)/c),x)
```

```
[Out] (polylog(2, exp(asin(exp(- a - b*x)/c)*2i))*1i)/(2*b) + (asin(exp(- a - b*x
)/c)^2*1i)/(2*b) - (log(1 - exp(asin(exp(- a - b*x)/c)*2i))*asin(exp(- a -
b*x)/c))/b
```

3.41 $\int e^{\csc^{-1}(ax)}x^2\,dx$

Optimal result . . . 292
Rubi [A] (verified) . . . 292
Mathematica [A] (verified) . . . 294
Maple [F] . . . 294
Fricas [F] . . . 294
Sympy [F] . . . 295
Maxima [F] . . . 295
Giac [F] . . . 295
Mupad [F(-1)] . . . 295

Optimal result

Integrand size = 10, antiderivative size = 95

$$\int e^{\csc^{-1}(ax)}x^2\,dx = \frac{\left(\frac{4}{5}-\frac{12i}{5}\right)e^{(1+3i)\csc^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{i}{2},3,\frac{5}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^3} - \frac{\left(\frac{8}{5}-\frac{24i}{5}\right)e^{(1+3i)\csc^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{i}{2},4,\frac{5}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^3}$$

```
[Out] (4/5-12/5*I)*exp((1+3*I)*arccsc(a*x))*hypergeom([3, 3/2-1/2*I],[5/2-1/2*I],
(I/a/x+(1-1/a^2/x^2)^(1/2))^2)/a^3+(-8/5+24/5*I)*exp((1+3*I)*arccsc(a*x))*h
ypergeom([4, 3/2-1/2*I],[5/2-1/2*I],(I/a/x+(1-1/a^2/x^2)^(1/2))^2)/a^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5375, 12, 4559, 2283}

$$\int e^{\csc^{-1}(ax)}x^2\,dx = \frac{\left(\frac{4}{5}-\frac{12i}{5}\right)e^{(1+3i)\csc^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{i}{2},3,\frac{5}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^3} - \frac{\left(\frac{8}{5}-\frac{24i}{5}\right)e^{(1+3i)\csc^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{i}{2},4,\frac{5}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^3}$$

```
[In] Int[E^ArcCsc[a*x]*x^2,x]
```

```
[Out] ((4/5 - (12*I)/5)*E^((1 + 3*I)*ArcCsc[a*x])*Hypergeometric2F1[3/2 - I/2, 3,
 5/2 - I/2, E^((2*I)*ArcCsc[a*x])])/a^3 - ((8/5 - (24*I)/5)*E^((1 + 3*I)*Ar
cCsc[a*x])*Hypergeometric2F1[3/2 - I/2, 4, 5/2 - I/2, E^((2*I)*ArcCsc[a*x])
])/a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4559

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 5375

```
Int[(u_.)*(f_)^(ArcCsc[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Csc[x]/b)*f^(c*x^n)*Csc[x]*Cot[x], x],
 x, ArcCsc[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{e^x \cot(x)\csc^3(x)}{a^2}\,dx, x, \csc^{-1}(ax)\right)}{a}\\
&= -\frac{\text{Subst}\left(\int e^x \cot(x)\csc^3(x)\,dx, x, \csc^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{16e^{(1+3i)x}}{(-1+e^{2ix})^4}+\frac{8e^{(1+3i)x}}{(-1+e^{2ix})^3}\right)\,dx, x, \csc^{-1}(ax)\right)}{a^3}\\
&= -\frac{8\text{Subst}\left(\int \frac{e^{(1+3i)x}}{(-1+e^{2ix})^3}\,dx, x, \csc^{-1}(ax)\right)}{a^3}-\frac{16\text{Subst}\left(\int \frac{e^{(1+3i)x}}{(-1+e^{2ix})^4}\,dx, x, \csc^{-1}(ax)\right)}{a^3}
\end{aligned}$$

$$= \frac{\left(\frac{4}{5}-\frac{12i}{5}\right)e^{(1+3i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{i}{2},3,\frac{5}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^3} - \frac{\left(\frac{8}{5}-\frac{24i}{5}\right)e^{(1+3i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{i}{2},4,\frac{5}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^3}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int e^{\csc^{-1}(ax)}x^2\,dx = \frac{e^{\csc^{-1}(ax)}\left((4+4i)e^{i\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)+a^3x^3(5-\cos\left(2\csc^{-1}(ax)\right)\right.}{12a^3}$$

```
[In] Integrate[E^ArcCsc[a*x]*x^2,x]
[Out] (E^ArcCsc[a*x]*((4 + 4*I)*E^(I*ArcCsc[a*x])*Hypergeometric2F1[1/2 - I/2, 1,
 3/2 - I/2, E^((2*I)*ArcCsc[a*x])] + a^3*x^3*(5 - Cos[2*ArcCsc[a*x]] + Sin[
2*ArcCsc[a*x]])))/(12*a^3)
```

Maple [F]

$$\int e^{\operatorname{arccsc}(ax)}x^2dx$$

```
[In] int(exp(arccsc(a*x))*x^2,x)
[Out] int(exp(arccsc(a*x))*x^2,x)
```

Fricas [F]

$$\int e^{\csc^{-1}(ax)}x^2\,dx = \int x^2e^{(\operatorname{arccsc}(ax))}\,dx$$

```
[In] integrate(exp(arccsc(a*x))*x^2,x, algorithm="fricas")
[Out] integral(x^2*e^(arccsc(a*x)), x)
```

Sympy [F]

$$\int e^{\csc^{-1}(ax)}x^2\,dx = \int x^2 e^{\operatorname{acsc}(ax)}\,dx$$

```
[In] integrate(exp(acsc(a*x))*x**2,x)
[Out] Integral(x**2*exp(acsc(a*x)), x)
```

Maxima [F]

$$\int e^{\csc^{-1}(ax)}x^2\,dx = \int x^2 e^{(\operatorname{arccsc}(ax))}\,dx$$

```
[In] integrate(exp(arccsc(a*x))*x^2,x, algorithm="maxima")
[Out] integrate(x^2*e^(arccsc(a*x)), x)
```

Giac [F]

$$\int e^{\csc^{-1}(ax)}x^2\,dx = \int x^2 e^{(\operatorname{arccsc}(ax))}\,dx$$

```
[In] integrate(exp(arccsc(a*x))*x^2,x, algorithm="giac")
[Out] integrate(x^2*e^(arccsc(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\csc^{-1}(ax)}x^2\,dx = \int x^2\,\mathrm{e}^{\operatorname{asin}\left(\frac{1}{a\,x}\right)}\,dx$$

```
[In] int(x^2*exp(asin(1/(a*x))),x)
[Out] int(x^2*exp(asin(1/(a*x))), x)
```

3.42 $\int e^{\csc^{-1}(ax)}x\,dx$

Optimal result . . . 296
Rubi [A] (verified) . . . 296
Mathematica [A] (verified) . . . 298
Maple [F] . . . 298
Fricas [F] . . . 298
Sympy [F] . . . 298
Maxima [F] . . . 299
Giac [F] . . . 299
Mupad [F(-1)] . . . 299

Optimal result

Integrand size = 8, antiderivative size = 87

$$\int e^{\csc^{-1}(ax)}x\,dx = \frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(1-\frac{i}{2},2,2-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^2} - \frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(1-\frac{i}{2},3,2-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^2}$$

```
[Out] (8/5+4/5*I)*exp((1+2*I)*arccsc(a*x))*hypergeom([2, 1-1/2*I],[2-1/2*I],(I/a/
x+(1-1/a^2/x^2)^(1/2))^2)/a^2-(16/5+8/5*I)*exp((1+2*I)*arccsc(a*x))*hyperge
om([3, 1-1/2*I],[2-1/2*I],(I/a/x+(1-1/a^2/x^2)^(1/2))^2)/a^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5375, 12, 4559, 2283}

$$\int e^{\csc^{-1}(ax)}x\,dx = \frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(1-\frac{i}{2},2,2-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^2} - \frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(1-\frac{i}{2},3,2-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a^2}$$

```
[In] Int[E^ArcCsc[a*x]*x,x]
```

```
[Out] ((8/5 + (4*I)/5)*E^((1 + 2*I)*ArcCsc[a*x])*Hypergeometric2F1[1 - I/2, 2, 2
- I/2, E^((2*I)*ArcCsc[a*x])])/a^2 - ((16/5 + (8*I)/5)*E^((1 + 2*I)*ArcCsc[
a*x])*Hypergeometric2F1[1 - I/2, 3, 2 - I/2, E^((2*I)*ArcCsc[a*x])])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4559

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 5375

```
Int[(u_.)*(f_)^(ArcCsc[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Csc[x]/b)*f^(c*x^n)*Csc[x]*Cot[x], x],
 x, ArcCsc[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{e^x \cot(x)\csc^2(x)}{a}\,dx, x, \csc^{-1}(ax)\right)}{a}\\
&= -\frac{\text{Subst}\left(\int e^x \cot(x)\csc^2(x)\,dx, x, \csc^{-1}(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{8ie^{(1+2i)x}}{\left(-1+e^{2ix}\right)^3} - \frac{4ie^{(1+2i)x}}{\left(-1+e^{2ix}\right)^2}\right)\,dx, x, \csc^{-1}(ax)\right)}{a^2}\\
&= \frac{(4i)\text{Subst}\left(\int \frac{e^{(1+2i)x}}{\left(-1+e^{2ix}\right)^2}\,dx, x, \csc^{-1}(ax)\right)}{a^2} + \frac{(8i)\text{Subst}\left(\int \frac{e^{(1+2i)x}}{\left(-1+e^{2ix}\right)^3}\,dx, x, \csc^{-1}(ax)\right)}{a^2}\\
&= \frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(1-\frac{i}{2}, 2, 2-\frac{i}{2}, e^{2i\csc^{-1}(ax)}\right)}{a^2}\\
&\quad - \frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(1-\frac{i}{2}, 3, 2-\frac{i}{2}, e^{2i\csc^{-1}(ax)}\right)}{a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int e^{\csc^{-1}(ax)}x\,dx = \frac{\left(\frac{1}{5}+\frac{i}{10}\right)e^{\csc^{-1}(ax)}\left((2-i)ax\left(\sqrt{1-\frac{1}{a^2x^2}}+ax\right)+(1+2i)\,\text{Hypergeometric2F1}\left(-\frac{i}{2},1,1-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)+}{a^2}$$

```
[In] Integrate[E^ArcCsc[a*x]*x,x]
[Out] ((1/5 + I/10)*E^ArcCsc[a*x]*((2 - I)*a*x*(Sqrt[1 - 1/(a^2*x^2)] + a*x) + (1
 + 2*I)*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, E^((2*I)*ArcCsc[a*x])] + E^((
2*I)*ArcCsc[a*x])*Hypergeometric2F1[1, 1 - I/2, 2 - I/2, E^((2*I)*ArcCsc[a*
x])]))/a^2
```

Maple [F]

$$\int e^{\operatorname{arccsc}(ax)}x dx$$

```
[In] int(exp(arccsc(a*x))*x,x)
[Out] int(exp(arccsc(a*x))*x,x)
```

Fricas [F]

$$\int e^{\csc^{-1}(ax)}x\,dx = \int xe^{\left(\operatorname{arccsc}(ax)\right)}\,dx$$

```
[In] integrate(exp(arccsc(a*x))*x,x, algorithm="fricas")
[Out] integral(x*e^(arccsc(a*x)), x)
```

Sympy [F]

$$\int e^{\csc^{-1}(ax)}x\,dx = \int xe^{\operatorname{acsc}(ax)}\,dx$$

```
[In] integrate(exp(acsc(a*x))*x,x)
[Out] Integral(x*exp(acsc(a*x)), x)
```

Maxima [F]

$$\int e^{\csc^{-1}(ax)}x\,dx = \int xe^{(\operatorname{arccsc}(ax))}\,dx$$

```
[In] integrate(exp(arccsc(a*x))*x,x, algorithm="maxima")
[Out] integrate(x*e^(arccsc(a*x)), x)
```

Giac [F]

$$\int e^{\csc^{-1}(ax)}x\,dx = \int xe^{(\operatorname{arccsc}(ax))}\,dx$$

```
[In] integrate(exp(arccsc(a*x))*x,x, algorithm="giac")
[Out] integrate(x*e^(arccsc(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\csc^{-1}(ax)}x\,dx = \int x\,\mathrm{e}^{\operatorname{asin}\left(\frac{1}{a\,x}\right)}\,dx$$

```
[In] int(x*exp(asin(1/(a*x))),x)
[Out] int(x*exp(asin(1/(a*x))), x)
```

3.43 $\int e^{\csc^{-1}(ax)}\,dx$

Optimal result . . . 300
Rubi [A] (verified) . . . 300
Mathematica [A] (verified) . . . 302
Maple [F] . . . 302
Fricas [F] . . . 302
Sympy [F] . . . 302
Maxima [F] . . . 303
Giac [F] . . . 303
Mupad [F(-1)] . . . 303

Optimal result

Integrand size = 6, antiderivative size = 87

$$\int e^{\csc^{-1}(ax)}\,dx = -\frac{(1-i)e^{(1+i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a} + \frac{(2-2i)e^{(1+i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},2,\frac{3}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a}$$

```
[Out] (-1+I)*exp((1+I)*arccsc(a*x))*hypergeom([1, 1/2-1/2*I],[3/2-1/2*I],(I/a/x+(
1-1/a^2/x^2)^(1/2))^2)/a+(2-2*I)*exp((1+I)*arccsc(a*x))*hypergeom([2, 1/2-1
/2*I],[3/2-1/2*I],(I/a/x+(1-1/a^2/x^2)^(1/2))^2)/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5375, 4559, 2283}

$$\int e^{\csc^{-1}(ax)}\,dx = \frac{(2-2i)e^{(1+i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},2,\frac{3}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a} - \frac{(1-i)e^{(1+i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)}{a}$$

```
[In] Int[E^ArcCsc[a*x],x]
```

```
[Out] ((-1 + I)*E^((1 + I)*ArcCsc[a*x])*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2
, E^((2*I)*ArcCsc[a*x])])/a + ((2 - 2*I)*E^((1 + I)*ArcCsc[a*x])*Hypergeome
tric2F1[1/2 - I/2, 2, 3/2 - I/2, E^((2*I)*ArcCsc[a*x])])/a
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4559

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 5375

```
Int[(u_.)*(f_)^(ArcCsc[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Csc[x]/b)*f^(c*x^n)*Csc[x]*Cot[x], x],
 x, ArcCsc[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int e^x \cot(x)\csc(x)\,dx, x, \csc^{-1}(ax)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{2e^{(1+i)x}}{1-e^{2ix}} - \frac{4e^{(1+i)x}}{(-1+e^{2ix})^2}\right)dx, x, \csc^{-1}(ax)\right)}{a} \\
&= -\frac{2\text{Subst}\left(\int \frac{e^{(1+i)x}}{1-e^{2ix}}\,dx, x, \csc^{-1}(ax)\right)}{a} + \frac{4\text{Subst}\left(\int \frac{e^{(1+i)x}}{(-1+e^{2ix})^2}\,dx, x, \csc^{-1}(ax)\right)}{a} \\
&= -\frac{(1-i)e^{(1+i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2}, 1, \frac{3}{2}-\frac{i}{2}, e^{2i\csc^{-1}(ax)}\right)}{a} \\
&\quad + \frac{(2-2i)e^{(1+i)\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2}, 2, \frac{3}{2}-\frac{i}{2}, e^{2i\csc^{-1}(ax)}\right)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int e^{\csc^{-1}(ax)}\,dx = \frac{e^{\csc^{-1}(ax)}\left(ax+(1+i)e^{i\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)\right)}{a}$$

```
[In] Integrate[E^ArcCsc[a*x],x]
[Out] (E^ArcCsc[a*x]*(a*x + (1 + I)*E^(I*ArcCsc[a*x])*Hypergeometric2F1[1/2 - I/2
, 1, 3/2 - I/2, E^((2*I)*ArcCsc[a*x])]))/a
```

Maple [F]

$$\int \mathrm{e}^{\operatorname{arccsc}(ax)}dx$$

```
[In] int(exp(arccsc(a*x)),x)
[Out] int(exp(arccsc(a*x)),x)
```

Fricas [F]

$$\int e^{\csc^{-1}(ax)}\,dx = \int e^{(\operatorname{arccsc}(ax))}\,dx$$

```
[In] integrate(exp(arccsc(a*x)),x, algorithm="fricas")
[Out] integral(e^(arccsc(a*x)), x)
```

Sympy [F]

$$\int e^{\csc^{-1}(ax)}\,dx = \int e^{\operatorname{acsc}(ax)}\,dx$$

```
[In] integrate(exp(acsc(a*x)),x)
[Out] Integral(exp(acsc(a*x)), x)
```

Maxima [F]

$$\int e^{\csc^{-1}(ax)}\,dx = \int e^{(\operatorname{arccsc}(ax))}\,dx$$

```
[In] integrate(exp(arccsc(a*x)),x, algorithm="maxima")
[Out] integrate(e^(arccsc(a*x)), x)
```

Giac [F]

$$\int e^{\csc^{-1}(ax)}\,dx = \int e^{(\operatorname{arccsc}(ax))}\,dx$$

```
[In] integrate(exp(arccsc(a*x)),x, algorithm="giac")
[Out] integrate(e^(arccsc(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\csc^{-1}(ax)}\,dx = \int \mathrm{e}^{\operatorname{asin}\left(\frac{1}{a\,x}\right)}\,dx$$

```
[In] int(exp(asin(1/(a*x))),x)
[Out] int(exp(asin(1/(a*x))), x)
```

3.44 $\int \frac{e^{\csc^{-1}(ax)}}{x} dx$

Optimal result . . . 304
Rubi [A] (verified) . . . 304
Mathematica [A] (verified) . . . 306
Maple [F] . . . 306
Fricas [F] . . . 306
Sympy [F] . . . 306
Maxima [F] . . . 307
Giac [F] . . . 307
Mupad [F(-1)] . . . 307

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{e^{\csc^{-1}(ax)}}{x} dx = -ie^{\csc^{-1}(ax)} + 2ie^{\csc^{-1}(ax)} \text{Hypergeometric2F1}\left(-\frac{i}{2}, 1, 1-\frac{i}{2}, e^{2i\csc^{-1}(ax)}\right)$$

```
[Out] -I*exp(arccsc(a*x))+2*I*exp(arccsc(a*x))*hypergeom([1, -1/2*I],[1-1/2*I],(I
/a/x+(1-1/a^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5375, 12, 4528, 2225, 2283}

$$\int \frac{e^{\csc^{-1}(ax)}}{x} dx = 2ie^{\csc^{-1}(ax)} \text{Hypergeometric2F1}\left(-\frac{i}{2}, 1, 1-\frac{i}{2}, e^{2i\csc^{-1}(ax)}\right) - ie^{\csc^{-1}(ax)}$$

```
[In] Int[E^ArcCsc[a*x]/x,x]
```

```
[Out] (-I)*E^ArcCsc[a*x] + (2*I)*E^ArcCsc[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I
/2, E^((2*I)*ArcCsc[a*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 5375

```
Int[(u_.)*(f_)^(ArcCsc[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Csc[x]/b)*f^(c*x^n)*Csc[x]*Cot[x], x],
 x, ArcCsc[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int ae^x\cot(x)\,dx, x, \csc^{-1}(ax)\right)}{a}\\
&= -\text{Subst}\left(\int e^x\cot(x)\,dx, x, \csc^{-1}(ax)\right)\\
&= i\text{Subst}\left(\int\left(-e^x-\frac{2e^x}{-1+e^{2ix}}\right)\,dx, x, \csc^{-1}(ax)\right)\\
&= -\left(i\text{Subst}\left(\int e^x\,dx, x, \csc^{-1}(ax)\right)\right)-2i\text{Subst}\left(\int\frac{e^x}{-1+e^{2ix}}\,dx, x, \csc^{-1}(ax)\right)\\
&= -ie^{\csc^{-1}(ax)}+2ie^{\csc^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{i}{2},1,1-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.74

$$\int \frac{e^{\csc^{-1}(ax)}}{x} \, dx = -i\left(-e^{\csc^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{i}{2},1,1-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)-\left(\frac{1}{5}-\frac{2i}{5}\right)e^{(1+2i)\csc^{-1}(ax)}\text{Hypergeometric2F1}\left(1,1-\frac{i}{2},2-\frac{i}{2},e^{2i\csc^{-1}(ax)}\right)\right)$$

```
[In] Integrate[E^ArcCsc[a*x]/x,x]
```

```
[Out] (-I)*(-(E^ArcCsc[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, E^((2*I)*ArcCsc
[a*x])]) - (1/5 - (2*I)/5)*E^((1 + 2*I)*ArcCsc[a*x])*Hypergeometric2F1[1, 1
 - I/2, 2 - I/2, E^((2*I)*ArcCsc[a*x])])
```

Maple [F]

$$\int \frac{e^{\operatorname{arccsc}(ax)}}{x} dx$$

```
[In] int(exp(arccsc(a*x))/x,x)
```

```
[Out] int(exp(arccsc(a*x))/x,x)
```

Fricas [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x} \, dx = \int \frac{e^{(\operatorname{arccsc}(ax))}}{x} \, dx$$

```
[In] integrate(exp(arccsc(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(e^(arccsc(a*x))/x, x)
```

Sympy [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x} \, dx = \int \frac{e^{\operatorname{acsc}(ax)}}{x} \, dx$$

```
[In] integrate(exp(acsc(a*x))/x,x)
```

```
[Out] Integral(exp(acsc(a*x))/x, x)
```

Maxima [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x}\,dx = \int \frac{e^{(\operatorname{arccsc}(ax))}}{x}\,dx$$

```
[In] integrate(exp(arccsc(a*x))/x,x, algorithm="maxima")
[Out] integrate(e^(arccsc(a*x))/x, x)
```

Giac [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x}\,dx = \int \frac{e^{(\operatorname{arccsc}(ax))}}{x}\,dx$$

```
[In] integrate(exp(arccsc(a*x))/x,x, algorithm="giac")
[Out] integrate(e^(arccsc(a*x))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\csc^{-1}(ax)}}{x}\,dx = \int \frac{e^{\operatorname{asin}\left(\frac{1}{a\,x}\right)}}{x}\,dx$$

```
[In] int(exp(asin(1/(a*x)))/x,x)
[Out] int(exp(asin(1/(a*x)))/x, x)
```

3.45 $\int \frac{e^{\csc^{-1}(ax)}}{x^2}\,dx$

Optimal result 308
Rubi [A] (verified) 308
Mathematica [A] (verified) 309
Maple [F] 309
Fricas [A] (verification not implemented) 310
Sympy [F] 310
Maxima [F] 310
Giac [A] (verification not implemented) 310
Mupad [F(-1)] 311

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{e^{\csc^{-1}(ax)}}{x^2}\,dx=-\frac{1}{2}ae^{\csc^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}-\frac{e^{\csc^{-1}(ax)}}{2x}$$

```
[Out] -1/2*exp(arccsc(a*x))/x-1/2*a*exp(arccsc(a*x))*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5375, 12, 4518}

$$\int \frac{e^{\csc^{-1}(ax)}}{x^2}\,dx=-\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}e^{\csc^{-1}(ax)}-\frac{e^{\csc^{-1}(ax)}}{2x}$$

```
[In] Int[E^ArcCsc[a*x]/x^2,x]
```

```
[Out] -1/2*(a*E^ArcCsc[a*x]*Sqrt[1 - 1/(a^2*x^2)]) - E^ArcCsc[a*x]/(2*x)
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 4518</u>

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 5375

```
Int[(u_.)*(f_)^(ArcCsc[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Csc[x]/b)*f^(c*x^n)*Csc[x]*Cot[x], x],
 x, ArcCsc[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int a^2 e^x \cos(x)\,dx, x, \csc^{-1}(ax)\right)}{a}\\ &= -\left(a\text{Subst}\left(\int e^x \cos(x)\,dx, x, \csc^{-1}(ax)\right)\right)\\ &= -\frac{1}{2}ae^{\csc^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}-\frac{e^{\csc^{-1}(ax)}}{2x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{e^{\csc^{-1}(ax)}}{x^2}\,dx = -\frac{1}{2}ae^{\csc^{-1}(ax)}\left(\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{ax}\right)$$

```
[In] Integrate[E^ArcCsc[a*x]/x^2,x]
[Out] -1/2*(a*E^ArcCsc[a*x]*(Sqrt[1 - 1/(a^2*x^2)] + 1/(a*x)))
```

Maple [F]

$$\int \frac{\mathrm{e}^{\operatorname{arccsc}(ax)}}{x^2}dx$$

```
[In] int(exp(arccsc(a*x))/x^2,x)
[Out] int(exp(arccsc(a*x))/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int \frac{e^{\csc^{-1}(ax)}}{x^2}\,dx = -\frac{\left(\sqrt{a^2x^2-1}+1\right)e^{(\operatorname{arccsc}(ax))}}{2\,x}$$

```
[In] integrate(exp(arccsc(a*x))/x^2,x, algorithm="fricas")
[Out] -1/2*(sqrt(a^2*x^2 - 1) + 1)*e^(arccsc(a*x))/x
```

Sympy [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{\operatorname{acsc}(ax)}}{x^2}\,dx$$

```
[In] integrate(exp(acsc(a*x))/x**2,x)
[Out] Integral(exp(acsc(a*x))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{(\operatorname{arccsc}(ax))}}{x^2}\,dx$$

```
[In] integrate(exp(arccsc(a*x))/x^2,x, algorithm="maxima")
[Out] integrate(e^(arccsc(a*x))/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{e^{\csc^{-1}(ax)}}{x^2}\,dx = -\frac{1}{2}\left(\sqrt{-\frac{1}{a^2x^2}+1}e^{\left(\arcsin\left(\frac{1}{ax}\right)\right)}+\frac{e^{\left(\arcsin\left(\frac{1}{ax}\right)\right)}}{ax}\right)a$$

```
[In] integrate(exp(arccsc(a*x))/x^2,x, algorithm="giac")
[Out] -1/2*(sqrt(-1/(a^2*x^2) + 1)*e^(arcsin(1/(a*x))) + e^(arcsin(1/(a*x)))/(a*x
))*a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\csc^{-1}(ax)}}{x^2}\,dx = \int \frac{\mathrm{e}^{\mathrm{asin}\left(\frac{1}{a\,x}\right)}}{x^2}\,dx$$

```
[In] int(exp(asin(1/(a*x)))/x^2,x)
[Out] int(exp(asin(1/(a*x)))/x^2, x)
```

3.46 $\int \frac{e^{\csc^{-1}(ax)}}{x^3} dx$

Optimal result . . . 312
Rubi [A] (verified) . . . 312
Mathematica [A] (verified) . . . 313
Maple [F] . . . 314
Fricas [A] (verification not implemented) . . . 314
Sympy [F] . . . 314
Maxima [F] . . . 314
Giac [F] . . . 315
Mupad [F(-1)] . . . 315

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{e^{\csc^{-1}(ax)}}{x^3} dx = \frac{1}{5}a^2e^{\csc^{-1}(ax)}\cos\left(2\csc^{-1}(ax)\right) - \frac{1}{10}a^2e^{\csc^{-1}(ax)}\sin\left(2\csc^{-1}(ax)\right)$$

```
[Out] 1/5*a^2*exp(arccsc(a*x))*cos(2*arccsc(a*x))-1/10*a^2*exp(arccsc(a*x))*sin(2
*arccsc(a*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5375, 12, 4557, 4517}

$$\int \frac{e^{\csc^{-1}(ax)}}{x^3} dx = \frac{1}{5}a^2e^{\csc^{-1}(ax)}\cos\left(2\csc^{-1}(ax)\right) - \frac{1}{10}a^2e^{\csc^{-1}(ax)}\sin\left(2\csc^{-1}(ax)\right)$$

```
[In] Int[E^ArcCsc[a*x]/x^3,x]
```

```
[Out] (a^2*E^ArcCsc[a*x]*Cos[2*ArcCsc[a*x]])/5 - (a^2*E^ArcCsc[a*x]*Sin[2*ArcCsc[
a*x]])/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5375

```
Int[(u_.)*(f_)^(ArcCsc[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Csc[x]/b)*f^(c*x^n)*Csc[x]*Cot[x], x],
 x, ArcCsc[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int a^3 e^x \cos(x)\sin(x)\,dx, x, \csc^{-1}(ax)\right)}{a}\\
&= -\left(a^2\text{Subst}\left(\int e^x \cos(x)\sin(x)\,dx, x, \csc^{-1}(ax)\right)\right)\\
&= -\left(a^2\text{Subst}\left(\int \frac{1}{2}e^x \sin(2x)\,dx, x, \csc^{-1}(ax)\right)\right)\\
&= -\left(\frac{1}{2}a^2\text{Subst}\left(\int e^x \sin(2x)\,dx, x, \csc^{-1}(ax)\right)\right)\\
&= \frac{1}{5}a^2 e^{\csc^{-1}(ax)}\cos\left(2\csc^{-1}(ax)\right) - \frac{1}{10}a^2 e^{\csc^{-1}(ax)}\sin\left(2\csc^{-1}(ax)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{e^{\csc^{-1}(ax)}}{x^3}\,dx = -\frac{1}{10}a^2 e^{\csc^{-1}(ax)}\left(-2\cos\left(2\csc^{-1}(ax)\right)+\sin\left(2\csc^{-1}(ax)\right)\right)$$

```
[In] Integrate[E^ArcCsc[a*x]/x^3,x]
[Out] -1/10*(a^2*E^ArcCsc[a*x]*(-2*Cos[2*ArcCsc[a*x]] + Sin[2*ArcCsc[a*x]]))
```

Maple [F]

$$\int \frac{\mathrm{e}^{\operatorname{arccsc}(ax)}}{x^3} dx$$

```
[In] int(exp(arccsc(a*x))/x^3,x)
[Out] int(exp(arccsc(a*x))/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{e^{\csc^{-1}(ax)}}{x^3} dx = \frac{\left(a^2x^2 - \sqrt{a^2x^2-1} - 2\right)e^{(\operatorname{arccsc}(ax))}}{5\,x^2}$$

```
[In] integrate(exp(arccsc(a*x))/x^3,x, algorithm="fricas")
[Out] 1/5*(a^2*x^2 - sqrt(a^2*x^2 - 1) - 2)*e^(arccsc(a*x))/x^2
```

Sympy [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^3} dx = \int \frac{e^{\operatorname{acsc}(ax)}}{x^3}\, dx$$

```
[In] integrate(exp(acsc(a*x))/x**3,x)
[Out] Integral(exp(acsc(a*x))/x**3, x)
```

Maxima [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^3} dx = \int \frac{e^{(\operatorname{arccsc}(ax))}}{x^3}\, dx$$

```
[In] integrate(exp(arccsc(a*x))/x^3,x, algorithm="maxima")
[Out] integrate(e^(arccsc(a*x))/x^3, x)
```

Giac [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^3}\,dx = \int \frac{e^{(\mathrm{arccsc}(ax))}}{x^3}\,dx$$

```
[In] integrate(exp(arccsc(a*x))/x^3,x, algorithm="giac")
[Out] integrate(e^(arccsc(a*x))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\csc^{-1}(ax)}}{x^3}\,dx = \int \frac{\mathrm{e}^{\mathrm{asin}\left(\frac{1}{a\,x}\right)}}{x^3}\,dx$$

```
[In] int(exp(asin(1/(a*x)))/x^3,x)
[Out] int(exp(asin(1/(a*x)))/x^3, x)
```

3.47 $\int \frac{e^{\csc^{-1}(ax)}}{x^4}\,dx$

Optimal result 316
Rubi [A] (verified) 316
Mathematica [A] (verified) 318
Maple [F] 318
Fricas [A] (verification not implemented) 318
Sympy [F] 318
Maxima [F] 319
Giac [F] 319
Mupad [F(-1)] 319

Optimal result

Integrand size = 10, antiderivative size = 84

$$\int \frac{e^{\csc^{-1}(ax)}}{x^4}\,dx = -\frac{1}{8}a^3e^{\csc^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^2e^{\csc^{-1}(ax)}}{8x}+\frac{1}{40}a^3e^{\csc^{-1}(ax)}\cos\left(3\csc^{-1}(ax)\right)+\frac{3}{40}a^3e^{\csc^{-1}(ax)}\sin\left(3\csc^{-1}(ax)\right)$$

```
[Out] -1/8*a^2*exp(arccsc(a*x))/x+1/40*a^3*exp(arccsc(a*x))*cos(3*arccsc(a*x))+3/
40*a^3*exp(arccsc(a*x))*sin(3*arccsc(a*x))-1/8*a^3*exp(arccsc(a*x))*(1-1/a^
2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5375, 12, 4557, 4518}

$$\int \frac{e^{\csc^{-1}(ax)}}{x^4}\,dx = \frac{1}{40}a^3e^{\csc^{-1}(ax)}\cos\left(3\csc^{-1}(ax)\right)+\frac{3}{40}a^3e^{\csc^{-1}(ax)}\sin\left(3\csc^{-1}(ax)\right)-\frac{a^2e^{\csc^{-1}(ax)}}{8x}-\frac{1}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}e^{\csc^{-1}(ax)}$$

```
[In] Int[E^ArcCsc[a*x]/x^4,x]
```

```
[Out] -1/8*(a^3*E^ArcCsc[a*x]*Sqrt[1 - 1/(a^2*x^2)]) - (a^2*E^ArcCsc[a*x])/(8*x)
+ (a^3*E^ArcCsc[a*x]*Cos[3*ArcCsc[a*x]])/40 + (3*a^3*E^ArcCsc[a*x]*Sin[3*Ar
cCsc[a*x]])/40
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5375

```
Int[(u_.)*(f_)^(ArcCsc[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Csc[x]/b)*f^(c*x^n)*Csc[x]*Cot[x], x],
 x, ArcCsc[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int a^4 e^x \cos(x)\sin^2(x)\,dx, x, \csc^{-1}(ax)\right)}{a}\\
&= -\left(a^3\text{Subst}\left(\int e^x \cos(x)\sin^2(x)\,dx, x, \csc^{-1}(ax)\right)\right)\\
&= -\left(a^3\text{Subst}\left(\int \left(\frac{1}{4}e^x\cos(x) - \frac{1}{4}e^x\cos(3x)\right)\,dx, x, \csc^{-1}(ax)\right)\right)\\
&= -\left(\frac{1}{4}a^3\text{Subst}\left(\int e^x\cos(x)\,dx, x, \csc^{-1}(ax)\right)\right) + \frac{1}{4}a^3\text{Subst}\left(\int e^x\cos(3x)\,dx, x, \csc^{-1}(ax)\right)\\
&= -\frac{1}{8}a^3 e^{\csc^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^2 e^{\csc^{-1}(ax)}}{8x}\\
&\quad + \frac{1}{40}a^3 e^{\csc^{-1}(ax)}\cos\left(3\csc^{-1}(ax)\right) + \frac{3}{40}a^3 e^{\csc^{-1}(ax)}\sin\left(3\csc^{-1}(ax)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \frac{e^{\csc^{-1}(ax)}}{x^4} dx$$
$$= \frac{1}{40} a^3 e^{\csc^{-1}(ax)} \left(-5\sqrt{1 - \frac{1}{a^2x^2}} - \frac{5}{ax} + \cos\left(3\csc^{-1}(ax)\right) + 3\sin\left(3\csc^{-1}(ax)\right) \right)$$

```
[In] Integrate[E^ArcCsc[a*x]/x^4,x]
[Out] (a^3*E^ArcCsc[a*x]*(-5*Sqrt[1 - 1/(a^2*x^2)] - 5/(a*x) + Cos[3*ArcCsc[a*x]]
 + 3*Sin[3*ArcCsc[a*x]]))/40
```

Maple [F]

$$\int \frac{e^{\operatorname{arccsc}(ax)}}{x^4} dx$$

```
[In] int(exp(arccsc(a*x))/x^4,x)
[Out] int(exp(arccsc(a*x))/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.49

$$\int \frac{e^{\csc^{-1}(ax)}}{x^4} dx = \frac{\left(a^2x^2 - (a^2x^2 + 1)\sqrt{a^2x^2 - 1} - 3\right)e^{(\operatorname{arccsc}(ax))}}{10\,x^3}$$

```
[In] integrate(exp(arccsc(a*x))/x^4,x, algorithm="fricas")
[Out] 1/10*(a^2*x^2 - (a^2*x^2 + 1)*sqrt(a^2*x^2 - 1) - 3)*e^(arccsc(a*x))/x^3
```

Sympy [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^4} dx = \int \frac{e^{\operatorname{acsc}(ax)}}{x^4} dx$$

```
[In] integrate(exp(acsc(a*x))/x**4,x)
[Out] Integral(exp(acsc(a*x))/x**4, x)
```

Maxima [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^4}\,dx = \int \frac{e^{(\mathrm{arccsc}(ax))}}{x^4}\,dx$$

```
[In] integrate(exp(arccsc(a*x))/x^4,x, algorithm="maxima")
[Out] integrate(e^(arccsc(a*x))/x^4, x)
```

Giac [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^4}\,dx = \int \frac{e^{(\mathrm{arccsc}(ax))}}{x^4}\,dx$$

```
[In] integrate(exp(arccsc(a*x))/x^4,x, algorithm="giac")
[Out] integrate(e^(arccsc(a*x))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\csc^{-1}(ax)}}{x^4}\,dx = \int \frac{\mathrm{e}^{\mathrm{asin}\left(\frac{1}{a\,x}\right)}}{x^4}\,dx$$

```
[In] int(exp(asin(1/(a*x)))/x^4,x)
[Out] int(exp(asin(1/(a*x)))/x^4, x)
```

3.48 $\int \frac{e^{\csc^{-1}(ax)}}{x^5} dx$

Optimal result . . . 320
Rubi [A] (verified) . . . 320
Mathematica [A] (verified) . . . 322
Maple [F] . . . 322
Fricas [A] (verification not implemented) . . . 322
Sympy [F] . . . 322
Maxima [F] . . . 323
Giac [F] . . . 323
Mupad [F(-1)] . . . 323

Optimal result

Integrand size = 10, antiderivative size = 81

$$\int \frac{e^{\csc^{-1}(ax)}}{x^5} dx = \frac{1}{10}a^4e^{\csc^{-1}(ax)}\cos\left(2\csc^{-1}(ax)\right) - \frac{1}{34}a^4e^{\csc^{-1}(ax)}\cos\left(4\csc^{-1}(ax)\right) - \frac{1}{20}a^4e^{\csc^{-1}(ax)}\sin\left(2\csc^{-1}(ax)\right) + \frac{1}{136}a^4e^{\csc^{-1}(ax)}\sin\left(4\csc^{-1}(ax)\right)$$

```
[Out] 1/10*a^4*exp(arccsc(a*x))*cos(2*arccsc(a*x))-1/34*a^4*exp(arccsc(a*x))*cos(
4*arccsc(a*x))-1/20*a^4*exp(arccsc(a*x))*sin(2*arccsc(a*x))+1/136*a^4*exp(a
rccsc(a*x))*sin(4*arccsc(a*x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5375, 12, 4557, 4517}

$$\int \frac{e^{\csc^{-1}(ax)}}{x^5} dx = \frac{1}{10}a^4e^{\csc^{-1}(ax)}\cos\left(2\csc^{-1}(ax)\right) - \frac{1}{34}a^4e^{\csc^{-1}(ax)}\cos\left(4\csc^{-1}(ax)\right) - \frac{1}{20}a^4e^{\csc^{-1}(ax)}\sin\left(2\csc^{-1}(ax)\right) + \frac{1}{136}a^4e^{\csc^{-1}(ax)}\sin\left(4\csc^{-1}(ax)\right)$$

```
[In] Int[E^ArcCsc[a*x]/x^5,x]
```

```
[Out] (a^4*E^ArcCsc[a*x]*Cos[2*ArcCsc[a*x]])/10 - (a^4*E^ArcCsc[a*x]*Cos[4*ArcCsc
[a*x]])/34 - (a^4*E^ArcCsc[a*x]*Sin[2*ArcCsc[a*x]])/20 + (a^4*E^ArcCsc[a*x]
*Sin[4*ArcCsc[a*x]])/136
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5375

```
Int[(u_.)*(f_)^(ArcCsc[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Csc[x]/b)*f^(c*x^n)*Csc[x]*Cot[x], x],
 x, ArcCsc[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int a^5 e^x \cos(x)\sin^3(x)\,dx, x, \csc^{-1}(ax)\right)}{a}\\
&= -\left(a^4\text{Subst}\left(\int e^x\cos(x)\sin^3(x)\,dx, x, \csc^{-1}(ax)\right)\right)\\
&= -\left(a^4\text{Subst}\left(\int \left(\frac{1}{4}e^x\sin(2x)-\frac{1}{8}e^x\sin(4x)\right)\,dx, x, \csc^{-1}(ax)\right)\right)\\
&= \frac{1}{8}a^4\text{Subst}\left(\int e^x\sin(4x)\,dx, x, \csc^{-1}(ax)\right)-\frac{1}{4}a^4\text{Subst}\left(\int e^x\sin(2x)\,dx, x, \csc^{-1}(ax)\right)\\
&= \frac{1}{10}a^4e^{\csc^{-1}(ax)}\cos\left(2\csc^{-1}(ax)\right)-\frac{1}{34}a^4e^{\csc^{-1}(ax)}\cos\left(4\csc^{-1}(ax)\right)\\
&\quad -\frac{1}{20}a^4e^{\csc^{-1}(ax)}\sin\left(2\csc^{-1}(ax)\right)+\frac{1}{136}a^4e^{\csc^{-1}(ax)}\sin\left(4\csc^{-1}(ax)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{e^{\csc^{-1}(ax)}}{x^5}\,dx = -\frac{1}{680}a^4e^{\csc^{-1}(ax)}\left(-68\cos\left(2\csc^{-1}(ax)\right)+20\cos\left(4\csc^{-1}(ax)\right)\right.$$
$$\left.+34\sin\left(2\csc^{-1}(ax)\right)-5\sin\left(4\csc^{-1}(ax)\right)\right)$$

```
[In] Integrate[E^ArcCsc[a*x]/x^5,x]
[Out] -1/680*(a^4*E^ArcCsc[a*x]*(-68*Cos[2*ArcCsc[a*x]] + 20*Cos[4*ArcCsc[a*x]] +
 34*Sin[2*ArcCsc[a*x]] - 5*Sin[4*ArcCsc[a*x]]))
```

Maple [F]

$$\int \frac{\mathrm{e}^{\mathrm{arccsc}(ax)}}{x^5}dx$$

```
[In] int(exp(arccsc(a*x))/x^5,x)
[Out] int(exp(arccsc(a*x))/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int \frac{e^{\csc^{-1}(ax)}}{x^5}\,dx = \frac{\left(6\,a^4x^4+3\,a^2x^2-(6\,a^2x^2+5)\sqrt{a^2x^2-1}-20\right)e^{(\mathrm{arccsc}(ax))}}{85\,x^4}$$

```
[In] integrate(exp(arccsc(a*x))/x^5,x, algorithm="fricas")
[Out] 1/85*(6*a^4*x^4 + 3*a^2*x^2 - (6*a^2*x^2 + 5)*sqrt(a^2*x^2 - 1) - 20)*e^(ar
ccsc(a*x))/x^4
```

Sympy [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^5}\,dx = \int \frac{e^{\operatorname{acsc}(ax)}}{x^5}\,dx$$

```
[In] integrate(exp(acsc(a*x))/x**5,x)
[Out] Integral(exp(acsc(a*x))/x**5, x)
```

Maxima [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^5}\,dx = \int \frac{e^{(\operatorname{arccsc}(ax))}}{x^5}\,dx$$

```
[In] integrate(exp(arccsc(a*x))/x^5,x, algorithm="maxima")
[Out] integrate(e^(arccsc(a*x))/x^5, x)
```

Giac [F]

$$\int \frac{e^{\csc^{-1}(ax)}}{x^5}\,dx = \int \frac{e^{(\operatorname{arccsc}(ax))}}{x^5}\,dx$$

```
[In] integrate(exp(arccsc(a*x))/x^5,x, algorithm="giac")
[Out] integrate(e^(arccsc(a*x))/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\csc^{-1}(ax)}}{x^5}\,dx = \int \frac{\mathrm{e}^{\operatorname{asin}\left(\frac{1}{a\,x}\right)}}{x^5}\,dx$$

```
[In] int(exp(asin(1/(a*x)))/x^5,x)
[Out] int(exp(asin(1/(a*x)))/x^5, x)
```

3.49 $\int \frac{\csc^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal result 324
Rubi [A] (verified) 324
Mathematica [A] (verified) 326
Maple [A] (verified) 327
Fricas [F] 327
Sympy [F] 327
Maxima [F] 328
Giac [A] (verification not implemented) 328
Mupad [F(-1)] 328

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\csc^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{i\csc^{-1}(a+bx)^2}{2d} - \frac{\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{d} + \frac{i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)}{2d}$$

```
[Out] 1/2*I*arccsc(b*x+a)^2/d-arccsc(b*x+a)*ln(1-(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2)
)^2)/d+1/2*I*polylog(2,(I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))^2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5365, 12, 5327, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{\csc^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(a+bx)}\right)}{2d} + \frac{i\csc^{-1}(a+bx)^2}{2d} - \frac{\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)}{d}$$

```
[In] Int[ArcCsc[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*ArcCsc[a + b*x]^2)/d - (ArcCsc[a + b*x]*Log[1 - E^((2*I)*ArcCsc[a +
b*x])])/d + ((I/2)*PolyLog[2, E^((2*I)*ArcCsc[a + b*x])])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5327

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule 5365

```
Int[((a_.) + ArcCsc[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCsc[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{b\csc^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\csc^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{\arcsin(x)}{x}\,dx, x, \frac{1}{a+bx}\right)}{d}\\
&= -\frac{\text{Subst}\left(\int x\cot(x)\,dx, x, \arcsin\left(\frac{1}{a+bx}\right)\right)}{d}\\
&= \frac{i\arcsin\left(\frac{1}{a+bx}\right)^2}{2d} + \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \arcsin\left(\frac{1}{a+bx}\right)\right)}{d}\\
&= \frac{i\arcsin\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\arcsin\left(\frac{1}{a+bx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{a+bx}\right)}\right)}{d}\\
&\quad + \frac{\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx, x, \arcsin\left(\frac{1}{a+bx}\right)\right)}{d}\\
&= \frac{i\arcsin\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\arcsin\left(\frac{1}{a+bx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{a+bx}\right)}\right)}{d}\\
&\quad - \frac{i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\arcsin\left(\frac{1}{a+bx}\right)}\right)}{2d}\\
&= \frac{i\arcsin\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\arcsin\left(\frac{1}{a+bx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{i\,\text{PolyLog}\left(2, e^{2i\arcsin\left(\frac{1}{a+bx}\right)}\right)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{\csc^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$$
$$= \frac{-\csc^{-1}(a+bx)\log\left(1-e^{2i\csc^{-1}(a+bx)}\right)+\frac{1}{2}i\left(\csc^{-1}(a+bx)^2+\text{PolyLog}\left(2, e^{2i\csc^{-1}(a+bx)}\right)\right)}{d}$$

```
[In] Integrate[ArcCsc[a + b*x]/((a*d)/b + d*x),x]
[Out] (-(ArcCsc[a + b*x]*Log[1 - E^((2*I)*ArcCsc[a + b*x])]) + (I/2)*(ArcCsc[a +
b*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[a + b*x])]))/d
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.42

method	result
derivativedivides	$\frac{\frac{ib\,\mathrm{arccsc}(bx+a)^2}{2d}-\frac{b\,\mathrm{arccsc}(bx+a)\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{d}+\frac{ib\,\mathrm{polylog}\left(2,\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{d}-\frac{b\,\mathrm{arccsc}(bx+a)\ln\left(1+\frac{i}{bx+\cdots}\right.}{d}\cdots}{b}$
default	$\frac{\frac{ib\,\mathrm{arccsc}(bx+a)^2}{2d}-\frac{b\,\mathrm{arccsc}(bx+a)\ln\left(1-\frac{i}{bx+a}-\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{d}+\frac{ib\,\mathrm{polylog}\left(2,\frac{i}{bx+a}+\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{d}-\frac{b\,\mathrm{arccsc}(bx+a)\ln\left(1+\frac{i}{bx+\cdots}\right.}{d}\cdots}{b}$

```
[In] int(arccsc(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*I*b/d*arccsc(b*x+a)^2-b/d*arccsc(b*x+a)*ln(1-I/(b*x+a)-(1-1/(b*x+a
)^2)^(1/2))+I*b/d*polylog(2,I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))-b/d*arccsc(b*x
+a)*ln(1+I/(b*x+a)+(1-1/(b*x+a)^2)^(1/2))+I*b/d*polylog(2,-I/(b*x+a)-(1-1/(
b*x+a)^2)^(1/2)))
```

Fricas [F]

$$\int \frac{\csc^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arccsc}\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

```
[In] integrate(arccsc(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccsc(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]

$$\int \frac{\csc^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{b\int \frac{\operatorname{acsc}\left(a+bx\right)}{a+bx}\,dx}{d}$$

```
[In] integrate(acsc(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acsc(a + b*x)/(a + b*x), x)/d
```

Maxima [F]

$$\int \frac{\csc^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{arccsc}\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

```
[In] integrate(arccsc(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*d*integrate(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*log(b*x + a)/(b^3*
d*x^3 + 3*a*b^2*d*x^2 + (3*a^2 - 1)*b*d*x + (a^3 - a)*d), x) - 2*I*b*d*inte
grate(log(b*x + a)/(b^3*d*x^3 + 3*a*b^2*d*x^2 + (3*a^2 - 1)*b*d*x + (a^3 -
a)*d), x) + (2*arctan2(1, sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + I*log(-b*x
 - a + 1))*log(b*x + a) - I*log(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + I*l
og(b*x + a + 1)*log(b*x + a) + I*log(b*x + a)^2 + I*dilog(b*x + a) + I*dilo
g(-b*x - a))/d
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.65

$$\int \frac{\csc^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = -\frac{1}{4}b^2\left(\frac{2\,(bx+a)^2\arcsin\left(\frac{1}{\left((bx+a)\left(\frac{a}{bx+a}-1\right)-a\right)\left(\frac{a}{bx+a}-1\right)+a}\right)}{b^3d} + \frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)-\frac{1}{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)}}{b^3d}\right)$$

```
[In] integrate(arccsc(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] -1/4*b^2*(2*(b*x + a)^2*arcsin(1/(((b*x + a)*(a/(b*x + a) - 1) - a)*(a/(b*x
 + a) - 1) + a))/(b^3*d) + ((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) - 1/((
b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1)))/(b^3*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\mathrm{asin}\left(\frac{1}{a+b\,x}\right)}{d\,x+\frac{a\,d}{b}}\,dx$$

```
[In] int(asin(1/(a + b*x))/(d*x + (a*d)/b),x)
```

```
[Out] int(asin(1/(a + b*x))/(d*x + (a*d)/b), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 329

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```